

MODULE <i>HermesRMWs</i>	
EXTENDS	<i>Hermes</i>
VARIABLES	<i>Rmsgs</i> , <i>nodeFlagRMW</i> , <i>committedRMWs</i> , <i>committedWrites</i>
<i>HRMessage</i>	$\triangleq$ Invalidation <i>msgs</i> exchanged by the <i>Hermes</i> Protocol w/ <i>RMWs</i> $[type : \{ "RINV" \}, \quad flagRMW : \{0, 1\}, \quad RMW \text{ change}$ $\quad epochID : 0 \dots (Cardinality(H\_NODES) - 1),$ $\quad sender : H\_NODES,$ $\quad version : 0 \dots H\_MAX\_VERSION,$ $\quad tieBreaker : H\_NODES]$
<i>HRTs</i>	$\triangleq [version : 0 \dots H\_MAX\_VERSION,$ $\quad tieBreaker : H\_NODES]$
<i>HRTypеOK</i>	$\triangleq$ The type correctness invariant $\wedge HTypeOK$ $\wedge Rmsgs \subseteq HRMessage$ $\wedge nodeFlagRMW \in [H\_NODES \rightarrow \{0, 1\}]$ $\wedge committedRMWs \subseteq HRTs$ $\wedge committedWrites \subseteq HRTs$
<i>HRSemanticsRMW</i>	$\triangleq$ The invariant that an we cannot have two operations committed with same versions ( <i>i.e.</i> , that read the same value unless they are both writes) $\wedge \forall x \in committedRMWs :$ $\quad \forall y \in committedWrites : \wedge x.version \neq y.version$ $\quad \wedge x.version \neq y.version - 1$ $\wedge \forall x, y \in committedRMWs : \forall x.version \neq y.version$ $\quad \wedge x.tieBreaker = y.tieBreaker$
<i>HRInit</i>	$\triangleq$ The initial predicate $\wedge HInit$ $\wedge Rmsgs = \{ \}$ $\wedge committedRMWs = \{ \}$ $\wedge committedWrites = \{ \}$ $\wedge nodeFlagRMW = [n \in H\_NODES \mapsto 0] \quad RMW \text{ change}$
<p>A buffer maintaining all Invalidation messages. Messages are only appended to this variable (not removed once delivered) intentionally to check protocols tolerance in duplicates and reorderings</p> <p><math>HRsend(m) \triangleq Rmsgs' = Rmsgs \cup \{m\}</math></p> <p><math>smallerTS(v1, tb1, v2, tb2) \triangleq</math>  <math>\quad \wedge \neg equalTS(v1, tb1, v2, tb2)</math></p>	

$$\begin{aligned}
& \wedge \neg \text{greaterTS}(v1, tb1, v2, tb2) \\
hr\_upd\_nothing & \triangleq \\
& \wedge \text{UNCHANGED} \langle nodeFlagRMW, Rmsgs, committedRMWs, committedWrites \rangle \\
hr\_completeWrite(ver, tieB) & \triangleq \\
& \wedge committedWrites' = committedWrites \cup \{[version \mapsto ver, tieBreaker \mapsto tieB]\} \\
& \wedge \text{UNCHANGED} \langle Rmsgs, nodeFlagRMW, committedRMWs \rangle \\
hr\_completeRMW(ver, tieB) & \triangleq \\
& \wedge committedRMWs' = committedRMWs \cup \{[version \mapsto ver, tieBreaker \mapsto tieB]\} \\
& \wedge \text{UNCHANGED} \langle Rmsgs, nodeFlagRMW, committedWrites \rangle
\end{aligned}$$


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**Helper functions**

$$\begin{aligned}
hr\_upd\_state(n, newVersion, newTieBreaker, newState, newAcks, flagRMW) & \triangleq \\
& \wedge nodeFlagRMW' = [nodeFlagRMW \text{ EXCEPT } ![n] = flagRMW] \text{ RMW change} \\
& \wedge h\_upd\_state(n, newVersion, newTieBreaker, newState, newAcks) \\
hr\_send\_inv(n, newVersion, newTieBreaker, flagRMW) & \triangleq \\
& \wedge HRsend([type \mapsto \text{"RINV"}, \\
& \quad epochID \mapsto epochID, \text{ we always use the latest epochID} \\
& \quad flagRMW \mapsto flagRMW, \text{ RMW change} \\
& \quad sender \mapsto n, \\
& \quad version \mapsto newVersion, \\
& \quad tieBreaker \mapsto newTieBreaker]) \\
hr\_upd\_actions(n, newVersion, newTieBreaker, newState, newAcks, flagRMW) & \triangleq \text{Execute a write} \\
& \wedge hr\_upd\_state(n, newVersion, newTieBreaker, newState, newAcks, flagRMW) \\
& \wedge hr\_send\_inv(n, newVersion, newTieBreaker, flagRMW) \\
& \wedge \text{UNCHANGED} \langle aliveNodes, epochID, msgs, committedRMWs, committedWrites \rangle \\
hr\_upd\_replay\_actions(n, acks) & \triangleq \text{Apply a write-replay using same TS (version, Tie Breaker)} \\
& \text{and either reset acks or keep already gathered acks} \\
& \wedge hr\_upd\_actions(n, nodeTS[n].version, nodeTS[n].tieBreaker, \text{"replay"}, acks, nodeFlagRMW[n])
\end{aligned}$$


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**Coordinator functions**

$$\begin{aligned}
HRWrite(n) & \triangleq \text{Execute a write} \\
& \wedge nodeState[n] \in \{\text{"valid"}, \text{"invalid"}\} \\
& \text{writes in invalid state are also supported as an optimization} \\
& \wedge nodeState[n] = \text{"valid"} \\
& \wedge nodeTS[n].version + 2 \leq H\_MAX\_VERSION \text{ condition to bound execution} \\
& \wedge hr\_upd\_actions(n, nodeTS[n].version + 2, n, \text{"write"}, \{\}, 0)
\end{aligned}$$

$$\begin{aligned}
HRRMW(n) &\triangleq \text{Execute an RMW} \\
&\wedge \text{nodeState}[n] = \text{"valid"} \\
&\wedge \text{nodeTS}[n].\text{version} + 1 \leq H\_MAX\_VERSION \quad \text{condition to bound execution} \\
&\wedge \text{hr\_upd\_actions}(n, \text{nodeTS}[n].\text{version} + 1, n, \text{"write"}, \{\}, 1) \\
\\
HRWriteReplay(n) &\triangleq \text{Execute a write-replay} \\
&\wedge \text{nodeState}[n] \in \{\text{"write"}, \text{"replay"}\} \\
&\wedge \text{nodeWriteEpochID}[n] < \text{epochID} \\
&\wedge \neg \text{receivedAllAcks}(n) \quad \text{optimization to not replay when we have gathered acks from all alive} \\
&\wedge \text{nodeFlagRMW}[n] = 0 \\
&\wedge \text{hr\_upd\_replay\_actions}(n, \text{nodeRcvdAcks}[n]) \\
\\
HRRMWReplay(n) &\triangleq \text{Execute an RMW-replay} \\
&\wedge \text{nodeState}[n] \in \{\text{"write"}, \text{"replay"}\} \\
&\wedge \text{nodeWriteEpochID}[n] < \text{epochID} \\
&\wedge \neg \text{receivedAllAcks}(n) \quad \text{optimization to not replay when we have gathered acks from all alive} \\
&\wedge \text{nodeFlagRMW}[n] = 1 \\
&\wedge \text{hr\_upd\_replay\_actions}(n, \{\}) \\
\\
\text{Keep the } HRead, HRcvAck \text{ and } HSendVals \text{ the same as } Hermes \text{ w/o RMWs} \\
HRead(n) &\triangleq \\
&\wedge HRead(n) \\
&\wedge \text{hr\_upd\_nothing} \\
\\
HRcvAck(n) &\triangleq \\
&\wedge HRcvAck(n) \\
&\wedge \text{hr\_upd\_nothing} \\
\\
HSendValsRMW(n) &\triangleq \\
&\wedge \text{nodeFlagRMW}[n] = 1 \\
&\wedge HSendVals(n) \\
&\wedge \text{hr\_completeRMW}(\text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker}) \\
HSendValsWrite(n) &\triangleq \\
&\wedge \text{nodeFlagRMW}[n] = 0 \\
&\wedge HSendVals(n) \\
&\wedge \text{hr\_completeWrite}(\text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker}) \\
\\
HSendVals(n) &\triangleq \\
&\vee HSendValsRMW(n) \\
&\vee HSendValsWrite(n) \\
\\
HRCoordinatorActions(n) &\triangleq \text{Actions of a read/write/RMW coordinator} \\
&\vee HRead(n) \\
&\vee HRRMWReplay(n) \\
&\vee HRWriteReplay(n) \\
&\vee HRWrite(n) \\
&\vee HRRMW(n)
\end{aligned}$$

$\vee \text{HRRcvAck}(n)$   
 $\vee \text{HRSendVals}(n)$

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**Follower functions**

$\text{HRRcvWriteInv}(n) \triangleq$  Process a received invalidation for a write  
 $\exists m \in \text{Rmsgs} :$   
 $\wedge m.\text{type} = \text{"RINV"}$   
 $\wedge m.\text{epochID} = \text{epochID}$   
 $\wedge m.\text{sender} \neq n$   
 $\wedge m.\text{flagRMW} = 0$  RMW change  
always acknowledge a received invalidation (irrelevant to the timestamp)  
 $\wedge h.\text{send\_inv\_or\_ack}(n, m.\text{version}, m.\text{tieBreaker}, \text{"ACK"})$   
 $\wedge \vee \wedge \text{greaterTS}(m.\text{version},$   
 $\quad m.\text{tieBreaker},$   
 $\quad \text{nodeTS}[n].\text{version},$   
 $\quad \text{nodeTS}[n].\text{tieBreaker})$   
 $\wedge \text{nodeFlagRMW}' = [\text{nodeFlagRMW} \text{ EXCEPT } ![n] = m.\text{flagRMW}]$  RMW change  
 $\wedge \text{nodeLastWriter}' = [\text{nodeLastWriter} \text{ EXCEPT } ![n] = m.\text{sender}]$   
 $\wedge \text{nodeTS}' = [\text{nodeTS} \text{ EXCEPT } ![n].\text{version} = m.\text{version},$   
 $\quad ![n].\text{tieBreaker} = m.\text{tieBreaker}]$   
 $\wedge \vee \wedge \text{nodeState}[n] \in \{\text{"valid"}, \text{"invalid"}, \text{"replay"}\}$   
 $\wedge \text{nodeState}' = [\text{nodeState} \text{ EXCEPT } ![n] = \text{"invalid"}]$   
 $\vee \wedge \text{nodeState}[n] \in \{\text{"write"}, \text{"invalid\_write"}\}$   
 $\wedge \text{nodeFlagRMW}[n] = 0$  RMW change  
 $\wedge \text{nodeState}' = [\text{nodeState} \text{ EXCEPT } ![n] = \text{"invalid\_write"}]$   
 $\vee \wedge \text{nodeState}[n] = \text{"write"}$  RMW change  
 $\wedge \text{nodeFlagRMW}[n] = 1$  RMW change  
 $\wedge \text{nodeState}' = [\text{nodeState} \text{ EXCEPT } ![n] = \text{"invalid"}]$  RMW change  
 $\vee \wedge \neg \text{greaterTS}(m.\text{version}, m.\text{tieBreaker},$   
 $\quad \text{nodeTS}[n].\text{version}, \text{nodeTS}[n].\text{tieBreaker})$   
 $\wedge \text{UNCHANGED } \langle \text{nodeState}, \text{nodeTS}, \text{nodeLastWriter}, \text{nodeFlagRMW} \rangle$   
 $\wedge \text{UNCHANGED } \langle \text{nodeLastWriteTS}, \text{aliveNodes}, \text{nodeRcvdAcks}, \text{Rmsgs},$   
 $\quad \text{epochID}, \text{nodeWriteEpochID}, \text{committedRMWs}, \text{committedWrites} \rangle$   
  
 $\text{HRRcvRMWInv}(n) \triangleq$  Process a received invalidation for a write  
 $\exists m \in \text{Rmsgs} :$   
 $\wedge m.\text{type} = \text{"RINV"}$   
 $\wedge m.\text{epochID} = \text{epochID}$   
 $\wedge m.\text{sender} \neq n$   
 $\wedge m.\text{flagRMW} = 1$   
 $\wedge \vee \wedge \text{greaterTS}(m.\text{version},$   
 $\quad m.\text{tieBreaker},$   
 $\quad \text{nodeTS}[n].\text{version},$

$$\begin{aligned}
& nodeTS[n].tieBreaker) \\
& \wedge nodeFlagRMW' = [nodeFlagRMW \text{ EXCEPT } ![n] = m.flagRMW] \quad \text{RMW change} \\
& \wedge nodeLastWriter' = [nodeLastWriter \text{ EXCEPT } ![n] = m.sender] \\
& \wedge nodeTS' = [nodeTS \text{ EXCEPT } ![n].version = m.version, \\
& \quad \quad \quad ![n].tieBreaker = m.tieBreaker] \\
& \text{acknowledge a received invalidation (w/ greater timestamp)} \\
& \wedge h\_send\_inv\_or\_ack(n, m.version, m.tieBreaker, "ACK") \\
& \wedge \vee \wedge nodeState[n] \in \{\text{"valid"}, \text{"invalid"}, \text{"replay"}\} \\
& \quad \wedge nodeState' = [nodeState \text{ EXCEPT } ![n] = \text{"invalid"}] \\
& \quad \vee \wedge nodeState[n] \in \{\text{"write"}, \text{"invalid\_write"}\} \\
& \quad \quad \wedge nodeFlagRMW[n] = 0 \quad \text{RMW change} \\
& \quad \quad \wedge nodeState' = [nodeState \text{ EXCEPT } ![n] = \text{"invalid\_write"}] \\
& \quad \vee \wedge nodeState[n] = \text{"write"} \quad \text{RMW change} \\
& \quad \quad \wedge nodeFlagRMW[n] = 1 \quad \text{RMW change} \\
& \quad \quad \wedge nodeState' = [nodeState \text{ EXCEPT } ![n] = \text{"invalid"}] \quad \text{RMW change} \\
& \wedge \text{UNCHANGED } \langle Rmsgs \rangle \\
& \vee \wedge equalTS(m.version, m.tieBreaker, \\
& \quad \quad \quad nodeTS[n].version, nodeTS[n].tieBreaker) \\
& \quad \text{acknowledge a received invalidation (w/ equal timestamp)} \\
& \quad \wedge h\_send\_inv\_or\_ack(n, m.version, m.tieBreaker, "ACK") \\
& \quad \wedge \text{UNCHANGED } \langle nodeState, nodeTS, nodeLastWriter, nodeFlagRMW, Rmsgs \rangle \\
& \vee \wedge smallerTS(m.version, m.tieBreaker, \\
& \quad \quad \quad nodeTS[n].version, nodeTS[n].tieBreaker) \\
& \quad \wedge hr\_send\_inv(n, nodeTS[n].version, nodeTS[n].tieBreaker, nodeFlagRMW[n]) \\
& \quad \wedge \text{UNCHANGED } \langle nodeState, nodeTS, nodeLastWriter, nodeFlagRMW, msgs \rangle \\
& \wedge \text{UNCHANGED } \langle nodeLastWriteTS, aliveNodes, nodeRcvdAcks, epochID, \\
& \quad \quad \quad nodeWriteEpochID, committedRMWs, committedWrites \rangle
\end{aligned}$$

Keep the  $HRcvVals$  the same as  $Hermes$  w/o  $RMWs$

$$\begin{aligned}
HRRcvVal(n) & \triangleq \\
& \wedge HRcvVal(n) \\
& \wedge hr\_upd\_nothing
\end{aligned}$$

$$\begin{aligned}
HRFollowerWriteReplay(n) & \triangleq \quad \text{Execute a write-replay when coordinator failed} \\
& \wedge nodeState[n] = \text{"invalid"} \\
& \wedge \neg isAlive(nodeLastWriter[n]) \\
& \wedge hr\_upd\_replay\_actions(n, \{\})
\end{aligned}$$

$$\begin{aligned}
HRFollowerActions(n) & \triangleq \quad \text{Actions of a write follower} \\
& \vee HRFollowerWriteReplay(n) \\
& \vee HRRcvWriteInv(n) \\
& \vee HRRcvRMWInv(n) \\
& \vee HRRcvVal(n)
\end{aligned}$$

$$\begin{aligned}
HRNodeFailure(n) &\triangleq \\
&\wedge nodeFailure(n) \\
&\wedge hr\_upd\_nothing
\end{aligned}$$

$$\begin{aligned}
HRNext &\triangleq \text{Modeling } Hermes \text{ protocol (Coordinator and Follower actions while emulating failures)} \\
&\exists n \in aliveNodes : \\
&\quad \vee HRFollowerActions(n) \\
&\quad \vee HRCoordinatorActions(n) \\
&\quad \vee HRNodeFailure(n) \text{ emulate node failures}
\end{aligned}$$


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