Appendix B

Mathematics of the WAS Model

WAS Mathematical Formulation

The WAS model is written in the Generalized Algebraic Modeling System (GAMS) language (the full text is presented in Appendix D). As the objective function is not linear, the MINOS non-linear operating system is used to solve the WAS model. The model is presented below in the standard form for optimization, namely, the objective function followed by the constraints. In mathematical terms, the model[1] is as follows:

$$Max \quad Z = \sum_i \sum_d \left(\frac{B_{id} \times (QD_{id} + QFRY_{id})^{ALPHA_{id}+1}}{ALPHA_{id}+1} \right) - \sum_i \sum_s (QS_{is} \times CS_{is})$$

$$- \sum_i \sum_j (QTR_{ij} \times CTR_{ij}) - \sum_i \sum_j (QRY_{ij} \times CR_{ij})$$

$$- \sum_i \sum_j (QTRY_{ij} \times CTRY_{ij}) - \sum_i \sum_j [CE_{id} \times (QD_{id} + QFRY_{id})]$$

Subject to[2]:

$$\sum_d QD_{id} = \sum_s QS_{is} + \sum_j QTR_{ji} - \sum_j QTR_{ij} \quad \forall i$$

$$\sum_d QFRY_{id} = \sum_d QRY_{id} + \sum_j QTRY_{ji} - \sum_j QTRY_{ij} \quad \forall i$$

$$QRY_{id} = PR_{id} \times (QD_{id} + QFRY_{id}) \quad \forall i,d$$

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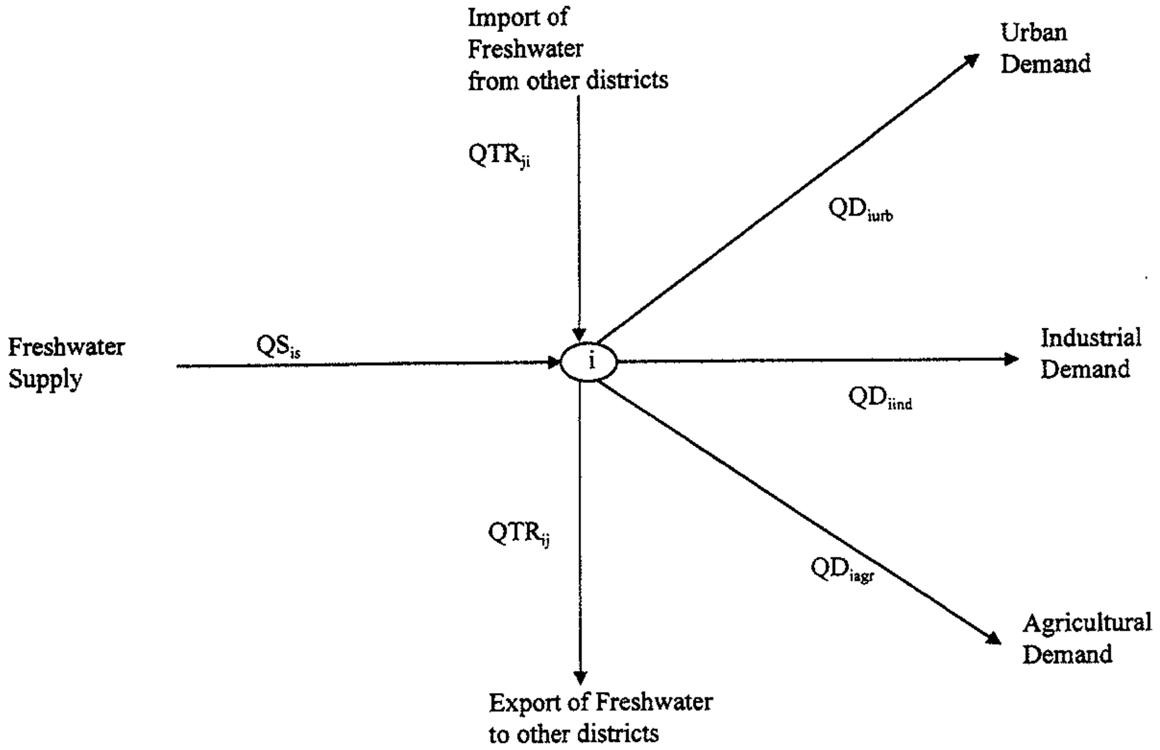
[1] Note that the first term of the objective function is the integral of the inverse demand function:
$P_{id} = B_{id} \times (QD_{id} + QFRY_{id})^{ALPHA_{id}}$

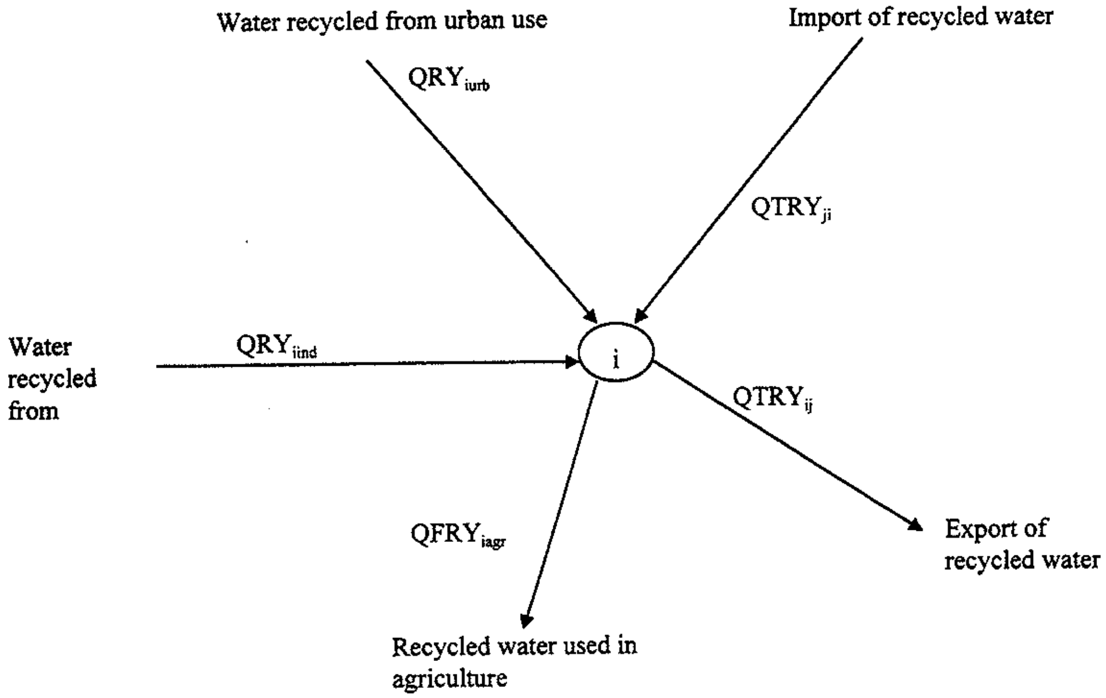
[2] See Figures 1a and 1b for an illustration of continuity for freshwater and recycled water, respectively, as given in the first two equations listed below.

Figure 1: Continuity of Flows at Node i

a) Freshwater Continuity



b) Recycled Water Continuity



$$\left(QD_{id} + QFRY_{id}\right) \geq \left(\frac{PMAX}{B_{id}}\right)^{1/ALPHA_{id}} \quad \forall\, i,d$$

With the following bounds:

$$QS_{is} \leq QSMAX_{is} \quad \forall\, i,s$$

$$PR_{id} \leq PRMAX_{id} \quad \forall\, i,d$$

All Variables Positive

Where:

Indices

i = district (Israel: I1,I3…I15; Jordan: J1…J8; Palestinian Authority: P1…P10; Golan: GOL; Jerusalem: JER)

d = demand type (urban, industrial, or agricultural)

s = supply source or steps (S1…S5)

Parameters

$ALPHA_{id}$	Exponent of inverse demand function for demand d in district i
B_{id}	Coefficient of inverse demand curve for demand d in district i
CE_{id} ($/m^3)	Unit environmental cost of water discharged by demand sector d in district i

CR_{id}	Unit recycling cost of water supplied from demand sector d in district i ($\$/m^3$)
CS_{is}	Unit cost of water supplied from supply step s in district i ($\$/m^3$)
CTR_{id}	Unit cost of water transported by demand sector d in district i ($\$/m^3$)
$CTRY_{id}$	Unit cost of recycled water transported by demand sector d in district i ($\$/m^3$)
$PMAX_{id}$	Maximum price of water from demand sector d in district i
$PRMAX_{id}$	Maximum percent of water from demand sector d that can be recycled in district i
$QSMAX_{is}$	Maximum amount of water from supply step s in district i (MCM)
P_{id}	Shadow price of water for demand sector d in district i (computed) in $

Variables

Z	Net Benefit in from water in Million $
QS_{is}	Quantity supplied by source s in district i in MCM
QD_{id}	Quantity demanded by sector d in district i in MCM
QTRij	Quantity of freshwater transported from district i to j in MCM
QTRYij	Quantity of recycled water transported from district i to j in MCM
QRYij	Quantity of water recycled from use d in district i in MCM
QFRYij	Quantity of recycled water supplied to use d in district i in MCM
PR_{id}	Percent of water recycled from sector d in district i in MCM

Appendix C

National Policy Implementation in WAS

FIXED PRICE POLICIES

1.INTRODUCTION

This appendix describes the functioning of fixed price policies in the Harvard Middle East Water Project Water Allocation System, versions 3.2 (WAS 3.2) and 3.2a (forthcoming). The reader is assumed to be familiar with the working of WAS 3.2. (See the user manual). Version 3.2a differs from version 3.2 in that it has some new features regarding fixed price policies that will be described below.

A system of fixed price policies is a system in which the user is allowed to specify a set of fixed prices for all quantities of water. Consumers will then pay the specified price, even if the cost of extracting and transporting water turns out to be lower. Such a fixed price policy can serve as a means for either subsidizing or taxing consumers.

WAS 3.2 allows users to specify a set of fixed prices that must increase as water consumption increases. Moreover, a fixed price policy specifies prices at which consumers can purchase water of any type, without distinction between fresh and recycled water. The optimal solution will have consumers purchase the type of water that is cheaper to produce. The forthcoming version of WAS, version 3.2a, will allow users to specify prices that may decrease as water consumption increases, and will also allow separate price policies for fresh and recycled water.

The next section will describe the functioning of fixed price policies in WAS 3.2, including basic examples and special cases. Section 3 will describe the innovations in WAS 3.2a.

2. FIXED PRICE POLICIES IN WAS 3.2

A system of fixed prices is a system in which the user specifies a set of fixed prices at which consumers must purchase each quantity of water. It is possible to specify different systems of fixed prices for different districts and different demand sectors. For example, the user may wish to let urban consumers in Amman purchase any quantity of water at a fixed price of $0.20/m^3. Alternatively, one may wish to let agricultural users in the Negev district buy 50 million cubic meters (mcm) at $0.15/m^3, 50 mcm of fresh water at $0.25/m^3, and 200 mcm at

\$0.30/m³. The user is allowed to specify up to 5 different such "steps", but the prices must increase as water consumption increases.

2.1 The standard case: a "complete" system

The simplest example of fixed price policies is one in which consumers specify a "complete" system of fixed prices for any type of water, i.e., a system in which a fixed price is set for each quantity of water (either fresh or recycled) that consumers wish to purchase. As a very simple illustration, consider Figure 1. Water is available in this district at a cost of P_0. The extraction cost curve is denoted by S, and the demand curve by D.

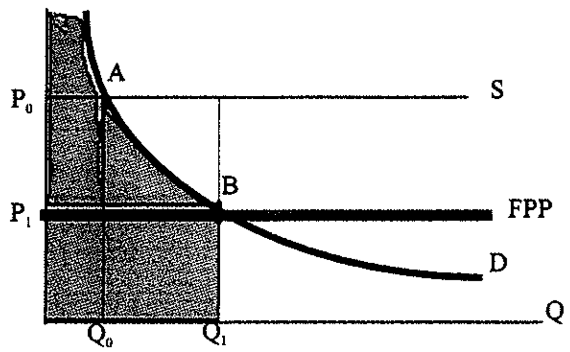


Figure 1

In this example the user has specified a single fixed price of \$$P_1$ for every quantity that consumers wish to purchase. With no such fixed price, the optimal quantity of water would have been Q_0, and the price would have been P_0. However, given the lower price that consumers face, the actual quantity demanded under the fixed price policy is Q_1. We can also use Figure 1 to determine consumer surplus and government costs. Consumer surplus is, as usual, the whole area under the demand curve up to Q_1 (the shaded area), minus the expenditure on water P_1* Q_1; government costs are given by the rectangle Q_1*(P_0- P_1).

This basic framework can be extended to more complex cases, where the user sets different prices for different quantities of water. The user is allowed to set a policy made up of up to 5 different steps, which must be increasing in price. As an example, see Figure 2, where four different steps are used.

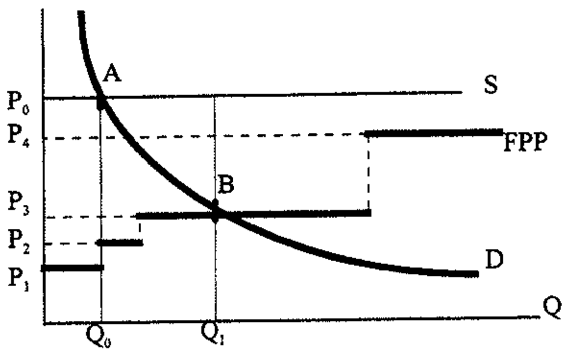


Figure 2

2.2 Incomplete Systems

Sometimes a user may wish to provide only a limited quantity of water at a fixed price, and then let the system determine the optimal price and quantity by itself. Several possibilities can then occur , and they are described below. The user is warned, however, to use incomplete systems with care, as they may lead to possibly misleading results.

2.2.1 Case 1: The market sets the price

This case is depicted in Figure 3.

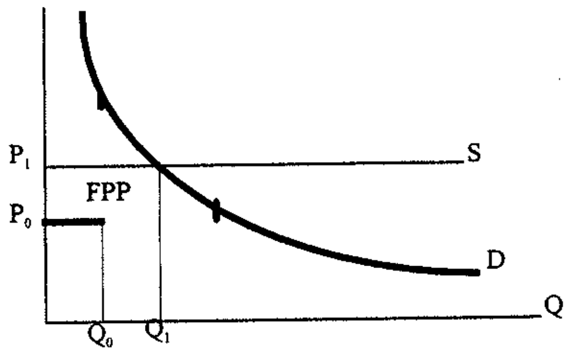


Figure 3

In this case the government allows consumers to purchase quantity Q_0 at price P_0. However, if consumers are willing to purchase a larger quantity than Q_0, they must pay the going market price. In the case depicted in Figure 3, we see that it is optimal for consumers to purchase a total quantity of Q_1. They will pay \$ P_0 for the first Q_0 units, and pay the going market price P_1 for the remaining quantity. The calculation of consumer surplus and government costs is straightforward.

2.2.2 Consumers purchase exactly the quantity set by the government

This case is depicted in Figure 4.

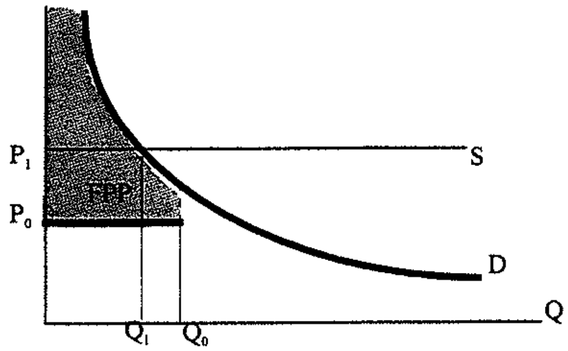


Figure 4

Again the government lets consumers purchase any quantity up to Q_0 at price P_0. In this case, however, it is optimal for consumers to purchase exactly quantity Q_0. It is easily seen that consumer surplus(the shaded area) is maximized at Q_0. Any quantity in excess of Q_0 will entail higher marginal extraction costs ($\$P_1$ per mcm) than the marginal benefit to consumers.

2.2.3 A "perverse" case

In the examples described above, it is always the case that the government effectively subsidizes water consumption, i.e., allows consumers to purchase water at a price lower than the going market price. However, the user does not know in advance whether the fixed price set will end up being higher or lower than the market price. With an incomplete price system, it is possible that WAS will report some misleading results. This can happen when a user thinks he/she is subsidizing, but ends up actually taxing consumers.

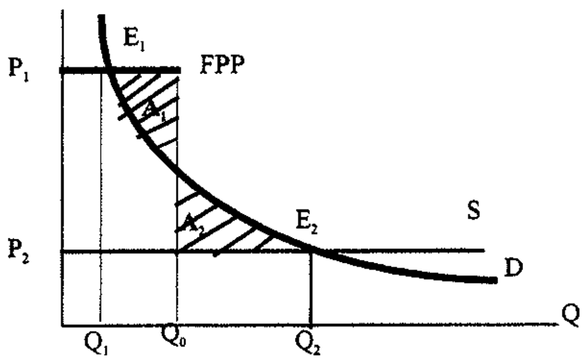


Figure 6

Take the example depicted in Figure 6. The user specifies a fixed price policy in which consumers can purchase quantity Q_0 at price P_1. In the example, there are two candidate equilibrium points, E_1 and E_2. The point that actually maximizes consumer surplus depends on the relative size of the two shaded areas A_1 and A_2. If A_1 is larger than A_2, then the cost of moving from E_1 to E_2 outweighs the benefit, and it will be optimal to purchase only quantity Q_1. Otherwise, it will be optimal to move to the higher equilibrium point E_2, and purchase a total quantity Q_2: the first Q_0 units at price P_1, and the remaining units at the going market price P_2. *However, WAS 3.2 will automatically report E_1 as the equilibrium point, but will issue a warning that the result is possibly misleading.* In later versions this may be fixed up.

3. FIXED PRICE POLICIES IN WAS 3.2a

Two major innovations in the working of fixed price policy have been introduced in the forthcoming version of WAS, WAS 3.2a. First, users will be able to specify a system of fixed prices that may possibly decrease as water consumption increases; second, separate fixed price policies may be specified for fresh and recycled water. Clearly, a combination of both features is also possible.

3.1 Fixed Price Policies With Decreasing Prices

Fixed price policies with decreasing prices as water consumption increases are similar in most respects to fixed price policies with increasing prices. One notable exception is described in Figure 7.

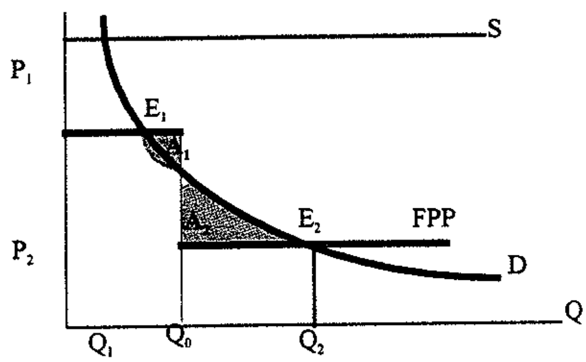


Figure 7

In this example, a set of decreasing prices generates a possible ambiguity as to which should be the optimal solution, as the fixed price policy steps cross the demand curve twice (this was not possible when prices were forced to be strictly increasing). There are now two candidate equilibrium points, E_1 and E_2. The optimal solution will depend on the relative size of the two areas, $\mathbf{A_1}$ and $\mathbf{A_2}$. If $\mathbf{A_1}$ is larger than $\mathbf{A_2}$, then the cost to consumers of purchasing the extra water will outweigh the benefits, and the optimal solution will be E_1. Otherwise, consumer surplus is maximized at E_2. WAS 3.2a will calculate in a preliminary step the optimal solution for districts and sectors with fixed price policies (i.e., will compare the two areas $\mathbf{A_1}$ and $\mathbf{A_2}$), before proceeding to the full optimization.

3.2 Fixed Price Policies For Both Fresh And Recycled Water

The discussion up to now has referred to the case in which there is a fixed price policy for only one type of water, be it fresh or recycled. A new feature of WAS 3.2a is the possibility of setting separate fixed price policies for fresh and recycled water, even in the same district and demand sector. The mechanism of the system when there is a fixed price policy for both fresh and recycled water is similar to that when there are multiple steps. Consumers, however, will always purchase water first at the cheaper step. Take the following example: the system of fixed prices specifies that consumers may purchase 50 mcm of fresh water at \$0.20, 20 mcm of recycled water at \$0.15 on the first step, and any remaining quantity of recycled water at \$0.25 on the second step. The optimal solution will have consumers purchase first the 20 mcm of recycled water at \$0.15, then eventually the 50 mcm of fresh water at \$0.20, and finally (if necessary) the remaining recycled water at \$0.25.

A few further notes on fixed price policies for recycled water:

1) Instituting a fixed price policy for recycled water is possible for the agricultural sector only, as this is the only sector that consumes recycled water in WAS 3.2a.
2) If you attempt to institute a fixed price policy for recycled water in a district that has no recycling plant, or that has no recycled water connection to a district with a recycling plant, WAS will issue a warning reminding you that there is no plant in that district. You must either allow for a recycling plant in that district, or for a recycled water connection from a district with a recycling plant, otherwise the system will explode.

3) Some care must be used when instituting a fixed price policy for recycled water. Recycled water is a product of urban and industrial fresh water consumption. If you specify too low a price, the result will be a large demand for recycled water, which can be supplied only by an even larger consumption of fresh water by urban and industrial consumers. In turn, this very large consumption can be achieved only if the price to urban and industrial consumers is very low, and if consumption is drastically reduced in other locations. All this may entail a very large distortion from the optimal solution, or even from the solution obtained by instituting the same policy for fresh water.