Cycle-Slip-Detector-Aided Iterative Timing Recovery

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***Abstract*— The latest developments in low-density parity-check (LDPC) codes have shown performances very close to the channel capacity. However, in order to use these capacity approaching codes in the magnetic recording channel, the cycle-slip problem in the synchronizer has to be solved. In this paper, we develop a cycle-slip detector (CSD) for the binary intersymbol interference (ISI) channel that uses the channel soft-output information as its input. This CSD is then used in an iterative timing recovery scheme to eliminate cycle-slips. Our simulation results show that utilizing the CSD improves the convergence speed of iterative timing recovery.**

***Index Terms*— cycle-slip detection, iterative timing recovery, LDPC, intersymbol interference.**

I. Introduction

RECENT research on LDPC codes and turbo codes has shown that properly constructed codes can approach the capacities of both the additive white Gaussian noise (AWGN) channel [1], [2] and the partial response (PR) channel [3]. Since these capacity approaching codes can operate at very low signal-to-noise ratios (SNRs), symbol synchronization at low SNRs becomes critical in order to utilize their full coding gain potentials. In the magnetic recording channel, conventional timing recovery devices (synchronizers) tend to experience cycle-slips at low SNRs, which always results in a burst of symbol detection errors and thus a burst of decoding errors. In [4], we developed a Bayesian detection rule to detect cycle-slips in the memoryless AWGN channel by using soft-output information. Here, we generalize the method and develop the CSD for the ISI channel. We investigate the binary input PR4 ($1-D^2$) channel where a rate 4/5 LDPC code with block length 5120 is used.

When the channel experiences a slowly time-varying phase drift, a typical phase tracking solution is the Mueller and Müller (M&M) synchronizer [5]. This synchronizer may be used in an iterative decoding and timing recovery scheme, as in [6]. In this paper, we show how to accelerate the decoding convergence by utilizing the cycle slip detector (CSD).

The paper is organized as follows. In section II we give the channel model for an ISI channel with timing errors. Section III derives an approximate analytic form for the symbol-by-symbol *a posteriori* probability (APP) and formulates the CSD for the ISI channel. Section IV introduces a CSD-aided iterative timing recovery algorithm and provides simulation results. Section V concludes the paper.

This work was supported by the National Science Foundation under Grant No. CCR-9984297.

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Notation: Throughout the paper, the acronym "i.i.d." stands for "independent identically distributed", $x \sim \mathcal{N}\left(m, \sigma^2\right)$ denotes that the random variable x is normally distributed with mean m and variance σ^2, and "pdf" stands for probability density function. $E\left[x\right]$ and $\mathrm{Var}\left[x\right]$ stand for the mean and variance of the random variable x, respectively. A vector is denoted by $\underline{x}_1^N$, i.e., $\underline{x}_1^N = [x_1, x_2, \cdots, x_N]$.

II. Channel Timing Model

Let the binary input sequence $\{a_m\}$ be a sequence of equally probable i.i.d. symbols that take values from $\{-1, 1\}$. Assuming the channel is equalized to an ideal PR4 response and corrupted by additive white Gaussian noise, the received noisy pulse amplitude modulated (PAM) waveform is given by

$$z\left(t\right) = \sum_{m=-\infty}^{\infty} \left(a_m - a_{m-2}\right) h\left(t - mT - \varepsilon_m T\right) + \eta\left(t\right). \quad (1)$$

Here $\eta\left(t\right)$ is zero mean white Gaussian noise, and $\varepsilon_m = \varepsilon\left(mT\right)$, where $\varepsilon\left(t\right)$ is a *slowly* varying phase drift. For the derivations in Section II, it is irrelevant what functional or stochastic form $\varepsilon\left(t\right)$ takes, as long as it is slowly time varying. We assume the pulse to be $h\left(t\right) = \mathrm{sinc}\left(\frac{t}{T}\right) = \frac{\sin(\pi t/T)}{\pi t/T}$. Since ε_m is unknown to the receiver, a timing recovery device [5] is used in the receiver to estimate the unknown phase drift in order to determine the sampling instants.

The signal in (1) is matched-filtered and sampled at time instants $kT + \hat{\varepsilon}_k T$ to produce an output sample (where "$\approx$" can be substituted by "$=$" only if $\hat{\varepsilon}_k - \varepsilon_m = 0$)

$$z_k \approx \sum_{m=-\infty}^{\infty} \left(a_m - a_{m-2} + n_m\right) \mathrm{sinc}\left[k - m - \left(\hat{\varepsilon}_k - \varepsilon_m\right)\right]. \quad (2)$$

Here, $\hat{\varepsilon}_k$ is the estimate of ε_k, and n_m is i.i.d. $\mathcal{N}\left(0, \sigma^2\right)$. Denote the timing error by $\epsilon_k = \hat{\varepsilon}_k - \varepsilon_k$. Since ε_m is slowly time-varying, for values of m near k, we have $\epsilon_k \approx \hat{\varepsilon}_k - \varepsilon_m$. For values $m >> k$, the value of $\mathrm{sinc}(k - m - \epsilon_k)$ is negligible, so (2) becomes

$$z_k \approx \sum_{m=-\infty}^{\infty} \left(a_m - a_{m-2} + n_m\right) \mathrm{sinc}\left[k - m - \epsilon_k\right]. \quad (3)$$

Since $z(t)$ is cyclostationary with period T, see [7], the synchronizer has stable operating points at $\epsilon_k \in \mathbb{Z}$. At the start of the waveform, at $k = 0$, we assume a perfectly synchronized system $\epsilon_0 = 0$. A cycle-slip happens when the value of ϵ_k changes from near 0 to near ± 1. Usually, ϵ_k takes continuous values. For the purpose of classification, we discretize this continuum into two hypotheses H_0 and H_1

$$\begin{aligned} H_0 &: |\epsilon_k| = \epsilon^{(0)} && \text{no cycle} - \text{slip} \\ H_1 &: |\epsilon_k| = \epsilon^{(1)} = \frac{1}{2} && \text{cycle} - \text{slip}, \end{aligned}$$

where $\epsilon^{(0)}$ and $\epsilon^{(1)}$ are predetermined constants. The task of the CSD at any time k is to decide whether H_0 or H_1 is a better hypothesis after observing the received waveform.

III. Cycle-slip Detection for ISI Channels

In [4], we developed a CSD for memoryless AWGN channels that uses the channel soft-output information as its input. A similar approach can also be applied to an ISI channel. We define the soft information μ_k for the symbol a_k as the log-likelihood ratio of the *a posteriori* probabilities of a_k. Assuming perfect timing, i.e., $\epsilon_k = 0$,

$$\mu_k = \ln\left[\frac{p\left(a_k = +1 | \underline{z}_1^N\right)}{p\left(a_k = -1 | \underline{z}_1^N\right)}\right], \tag{4}$$

where N is the block length. The value μ_k can be calculated by the BCJR algorithm [8]. It is very hard to find an analytic form for the probability density function (pdf) of μ_k, which prevents us from applying directly the approach developed in [4] to the ISI channel. We therefore use an approximate expression for μ_k. We assume that the ISI channel has $l+1$ tap-coefficients $h_0, \cdots, h_l$. An approximate expression for μ_k was derived in [9]

$$\mu_k \approx \frac{2}{\sigma^2} \sum_{m=k}^{k+l} h_{m-k}\left(z_m - \sum_{j=0, j\neq m-k}^{l} h_j \hat{a}_{m-j}\right). \tag{5}$$

Here, $\hat{a}_i$ is the hard decision (i.e. an estimate) of a_i. For the PR4 channel, $l = 2$ and $h_0 = 1, h_1 = 0, h_2 = -1$. Then

$$\mu_k \approx \frac{2}{\sigma^2}\left(z_k - z_{k+2} + \hat{a}_{k+2} + \hat{a}_{k-2}\right). \tag{6}$$

Now we can derive the pdf of μ_k following an approach similar to [4]. Substituting (3) into (6), we have

$$\begin{aligned} \mu_k =& \frac{2}{\sigma^2}(2a_k - a_{k-2} - a_{k+2})\mathrm{sinc}(\epsilon_k) + \frac{2}{\sigma^2}\left(\hat{a}_{k+2} + \hat{a}_{k-2}\right) \\ &+ \sum_{m\neq 0}(2a_{k-m} - a_{k-m-2} - a_{k+2-m})\mathrm{sinc}(m+\epsilon_k) + \nu_k. \end{aligned} \tag{7}$$

Here $\nu_k = \frac{2}{\sigma^2}\sum_m (n_{k-m} - n_{k+2-m})\mathrm{sinc}(m+\epsilon_k)$ is $\mathcal{N}\left(0, \frac{8}{\sigma^2}\right)$ by the sampling theorem for band-limited Gaussian random processes [10]. We assume that $\hat{a}_{k+2}$ and $\hat{a}_{k-2}$ have the same probability distribution as a_{k+2} and a_{k-2}, respectively. Then, under hypothesis H_i ($i \in \{0, 1\}$), we substitute $\epsilon_k = \epsilon^{(i)}$ into (7) and approximate the pdf $f(\mu_k | H_i, a_k)$ by a Gaussian mixture $\frac{1}{2}\mathcal{N}\left(m_i, \sigma_i^2\right) + \frac{1}{2}\mathcal{N}\left(-m_i, \sigma_i^2\right)$. It follows that

$$\begin{aligned} m_i &= E\left[\mu_k | H_i\right] = \frac{4a_k}{\sigma^2}\mathrm{sinc}\left(\epsilon^{(i)}\right) \\ \sigma_i^2 &= \mathrm{Var}\left(\mu_k | H_i, a_k\right) = \frac{8}{\sigma^4}\left(4 + \sigma^2 - 2\mathrm{sinc}\epsilon^{(i)} - 2\mathrm{sinc}^2\epsilon^{(i)}\right). \end{aligned}$$

We use the same decision rule as the one developed in [4],

$$\xi_k = \sum_{j=k-W+1}^{k}\left(|\mu_j| + C\right)^2 \underset{H_0}{\overset{H_1}{\gtrless}} \tau, \tag{8}$$

where $C = \frac{m_0\sigma_1^2 - m_1\sigma_0^2}{\sigma_0^2 - \sigma_1^2}$, τ is the threshold, and W is the number of soft-information values μ_k needed to detect cycle-slips. We assume that within the window of size W, ϵ_k stays constant. Then from (8), by the central limit theorem [10], for W large, ξ_k is approximately a Gaussian random variable distributed as

$$\left(\xi_k | H_i\right) \sim \mathcal{N}\left(W \cdot m_{H_i}, W \cdot \sigma_{H_i}^2\right), \tag{9}$$

where

$$\begin{aligned} m_{H_i} &= E\left[\left(|\mu_k| + C\right)^2 | H_i\right] && (10) \\ \sigma_{H_i}^2 &= Var\left[\left(|\mu_k| + C\right)^2 | H_i\right]. && (11) \end{aligned}$$

The values of m_{H_i} and $\sigma_{H_i}^2$ can be analytically computed, see [4]. We define the probability of detection P_D as the probability that H_1 is chosen when H_1 is true, and define the probability of false alarm P_{FA} as the probability that H_1 is chosen when H_0 is true. We can express P_D and P_{FA} as

$$\begin{aligned} P_D(\tau, W) &= \frac{1}{2}\mathrm{erfc}\left[\frac{\tau - W \cdot m_{H_1}}{\sqrt{2 \cdot W \cdot \sigma_{H_1}^2}}\right] && (12) \\ P_{FA}(\tau, W) &= \frac{1}{2}\mathrm{erfc}\left[\frac{\tau - W \cdot m_{H_0}}{\sqrt{2 \cdot W \cdot \sigma_{H_0}^2}}\right], && (13) \end{aligned}$$

where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \mathrm{e}^{-t^2}\,\mathrm{d}t$. In order to design a good CSD that reliably detects cycle-slips, we need to choose τ and W so that P_D is high and P_{FA} is low. In general, a longer window size W gives higher P_D and lower P_{FA}. However, the assumption that μ_k stays constant within the window W may become invalid if W is too long. Given (12) and (13), similar to [4], we can find the threshold τ and the lowest window size W that satisfy the desired specification of P_D and P_{FA}.

In our simulation, we set $\epsilon^{(0)}$=0.2, P_D=99%, P_{FA}=8%. For an SNR of 8.6dB[1], the threshold is $\tau = 6500$ and the lowest window length is $W = 90$. A forward-only APP algorithm [11] with survival memory length 6 is used as the timing loop detector. It makes preliminary decisions $\hat{a}_k$ of input symbols. The same detector with survival memory 30 is used in the iterative decoder. At each time k, ($k \geq 2$), the CSD computes μ_k and updates the value of ξ_k according to (8). If ξ_k is greater than τ, then the CSD fires. Here, "fire" means that the CSD reports a cycle-slip. In general, the CSD keeps firing for a period of time, which we call the *cycle-slip region*.

IV. CSD-aided Iterative Timing Recovery

In [6], an iterative timing recovery scheme is proposed, where the soft information is exchanged iteratively between the

[1]SNR is defined as $10\log_{10}\frac{P}{\sigma^2}$, where P is the power of the signal. For PR4 channels, we assume $P = 2$.

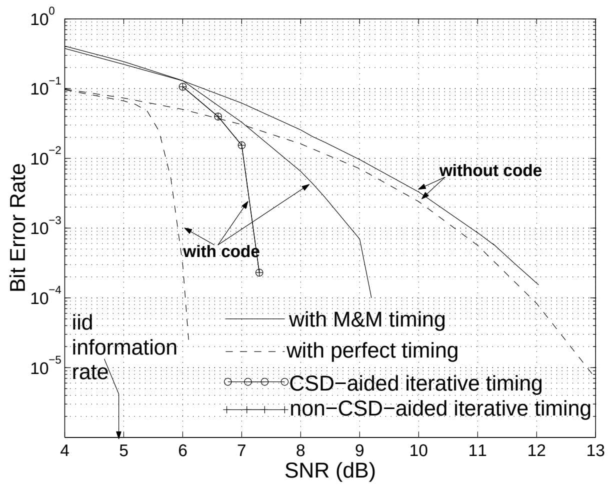

Fig. 1. Performance comparison between non-CSD-aided [6] and CSD-aided iterative timing recovery. A rate 4/5 LDPC code is used.

LDPC decoder and the synchronizer. We use this method, but improve on it using the cycle-slip detector (CSD). The CSD can detect the position of the cycle slip. We use the knowledge of the cycle-slip position to achieve faster decoding. The algorithm proceeds as follows.

CSD-Aided Iterative Timing Recovery Algorithm

1. Sample the waveform $z(t)$ to get a sequence of samples $\{z_k\}$.
2. Run the CSD.
3. **IF** the CSD fires,
   a) Record the position ν of the firing instance.
   b) Create: $z'_k = \begin{cases} z_k & \text{if } k \le \nu \\ z_{k+1} & \text{if } k > \nu \end{cases}$.
   c) Create: $z''_k = \begin{cases} z_k & \text{if } k \le \nu \\ z_{k-1} & \text{if } k > \nu \end{cases}$.
   d) Perform iterative decoding (turbo equalization and LDPC decoding) on $\{z_k\}$, $\{z'_k\}$,$\{z''_k\}$ to get $\{\hat{a}_k\}$, $\{\hat{a}'_k\}$,$\{\hat{a}''_k\}$, respectively.
   e) **if** one of the sequences $\{\hat{a}_k\}$, $\{\hat{a}'_k\}$, $\{\hat{a}''_k\}$ satisfies all parity checks, <u>stop</u>.
   else
   -resample $z(t)$ using $\{\hat{a}_k\}$ as prior information to get a new sequence $\{z_k\}$.
   -resample $z(t)$ using $\{\hat{a}'_k\}$ as prior information to get a new sequence $\{z'_k\}$.
   -resample $z(t)$ using $\{\hat{a}''_k\}$ as prior information to get a new sequence $\{z''_k\}$.
   -go back to step d).

ELSE
   f) Perform iterative decoding operations on $\{z_k\}$ to get a decoded sequence $\{\hat{a}_k\}$.
   g) **if** the sequence $\{\hat{a}_k\}$ satisfies all parity checks, <u>stop</u>.
   else
   -resample $z(t)$ using $\{a_k\}$ as prior information to get a new sequence $\{z_k\}$.
   -go to f).

The algorithm described above assumes that at most one cycle-slip happens in each coded block. For the LDPC code with rate 4/5 and block length 5120, our simulation results showed that having two cycle-slips in one block is very unlikely. This observation ensures the simplicity of the algorithm. The iterative timing recovery scheme in [6] is simply the **ELSE** part of step 3 of our algorithm. Thus, the two algorithms have the same performance. However, by using the CSD, our proposed scheme reduces the number of resampling iterations significantly with a slight increase in the receiver complexity in order to implement the CSD. Fig. 1 and Table I present our simulation results. In Table I, the number of synchronization iterations stands for the number of times the waveform is resampled. The timing offset in these simulations was chosen to be $\varepsilon(t) = \sin(\frac{t}{1000})$.

TABLE I
CONVERGENCE RATE COMPARISON

SNR: 7.4dB to 8.6dB	average number of synchronization iterations		BER after decoding
	CS present	CS absent	
CSD-aided	3	1	$< 10^{-6}$
non-CSD-aided [6]	32	1	$< 10^{-6}$

V. CONCLUSION

We have described a cycle-slip detection algorithm for the ISI channel that uses soft-output information as its input. In general, a long delay ($W \approx 100$) is needed in order to detect cycle-slips with high reliability. Since the iterative LDPC decoder introduces a much longer delay, the cycle-slip detection algorithm works well in systems with LDPC codes. The CSD-aided iterative timing recovery algorithm is then introduced. This method uses the idea as in [6] and applies the CSD to get good initial values for iterative timing recovery. Our simulation results showed that the method accelerates the decoding convergence without compromising the performance.

REFERENCES

[1] T. Richardson, A. Shokrollahi, and R. Urbanke, "Design of capacity-approaching irregular low-density parity-check codes," *IEEE Trans. Info. Theory*, pp. 619–637, Feb. 2001.

[2] S-Y. Chung, "On the construction of some capacity-approaching coding schemes," PhD dissertation,MIT, Sept. 2000.

[3] A. Kavčić, X. Ma, and M. Mitzenmacher, "Binary intersymbol interference channels: Gallager codes, density evolution and code performance bounds," accepted for publication in *IEEE Trans.Inform. Theory*, can be downloaded at http://hrl.harvard.edu/commlab/papers/it02.ps.

[4] X. Jin and A. Kavčić, "Cycle-slip detection using soft-output information," in *Proc. IEEE ICC'01*, Finland, June. 2001, pp. 2706–2710.

[5] K. H. Mueller and M. Müller, "Timing recovery in digital synchronous data receivers," *IEEE Trans. Commun.*, vol. 24, pp. 516–531, May 1976.

[6] A. R. Nayak, J. R. Barry, and S. W. McLaughlin, "Joint timing recovery, equalization, and decoding," to appear in this issue of *IEEE Trans. Mag.* Amsterdam, the Netherlands, April 2002.

[7] H. Meyr, M. Moeneclaey, and S. A. Fechtel, *Digital Communication Receivers*, John Wiley & Sons, Inc., New York, 1997.

[8] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284–287, Sept. 1974.

[9] Z. N. Wu and J. M. Cioffi, "Turbo decision aided equalization for magnetic recording channels," in *GLOBECOM*, 1999, pp. 733–738.

[10] John G. Proakis, *Digital Communications*, McGraw-Hill, New York, 1997.

[11] X. Ma and A. Kavčić, "Forward-only path partitions and treillis algorithms," accepted for publication in *IEEE Trans. Inform. Theory*, can be downloaded at http://hrl.harvard.edu/commlab/papers/ma02.ps.