Department of Electrical Engineering, University of Hawaii

EE 342: Probability and Statistics **Fall 2016**

Homework Set 10

Due date: November 16, 2016

(1) a) Chapter 7, problem 38
b) Chapter 7, problem 53
c) Chapter 7, problem 67
d) Chapter 7, problem 73
e) Chapter 7, theoretical exercise 38
f) Chapter 7, theoretical exercise 39

(2) Let X and Y be random variables. The joint PDF of X and Y is

$$f_{X,Y}(x,y) = e^{-x} \quad \text{if } 0 \le y < x < \infty$$

a) Find the marginal PDF of X
b) Find the marginal PDF of Y
c) Are X and Y independent?
d) Find the MMSE (minimum mean square estimate) of X without observing Y.
e) Find the LMMSE (linear minimum mean square estimate) of X after observing Y
f) Find the MMSE (minimum mean square estimate) of X after observing Y.

(3) Repeat problem 2 if X and Y are jointly Gaussian random variables, where E[X]=1 and Var(X)=1 and E[Y] = -1 and Var(Y)=4, and Cov(X,Y) = -1.

(4) Let X and Y be independent Gaussian random variables, both with mean zero and unit variance. Let Z = X+1 and W = X+Y.
a) Find E[Z] and E[W]
b) Find the Covariance matrix of the vector $[Z \quad W]^T$
c) Find the joint PDF of Z and W.
d) Are Z and W independent?
e) Determine a linear transform to get X and Y back from Z and W.

7.38. The random variables X and Y have a joint density function given by

$$f(x, y) = \begin{cases} 2e^{-2x}/x & 0 \le x < \infty, 0 \le y \le x \\ 0 & \text{otherwise} \end{cases}$$

Compute $\text{Cov}(X, Y)$.

7.53. A prisoner is trapped in a cell containing 3 doors. The first door leads to a tunnel that returns him to his cell after 2 days' travel. The second leads to a tunnel that returns him to his cell after 4 days' travel. The third door leads to freedom after 1 day of travel. If it is assumed that the prisoner will always select doors 1, 2, and 3 with respective probabilities .5, .3, and .2, what is the expected number of days until the prisoner reaches freedom?

7.67. Consider a gambler who, at each gamble, either wins or loses her bet with respective probabilities p and $1 - p$. A popular gambling system known as the Kelley strategy is to always bet the fraction $2p - 1$ of your current fortune when $p > \frac{1}{2}$. Compute the expected fortune after n gambles of a gambler who starts with x units and employs the Kelley strategy.

7.73. In Example 6b, let S denote the signal sent and R the signal received.

(a) Compute $E[R]$.
(b) Compute $\text{Var}(R)$.
(c) Is R normally distributed?
(d) Compute $\text{Cov}(R, S)$.

7.38. The best linear predictor of Y with respect to X_1 and X_2 is equal to $a + bX_1 + cX_2$, where a, b, and c are chosen to minimize

$$E[(Y - (a + bX_1 + cX_2))^2]$$

Determine a, b, and c.

7.39. The best quadratic predictor of Y with respect to X is $a + bX + cX^2$, where a, b, and c are chosen to minimize $E[(Y - (a + bX + cX^2))^2]$. Determine a, b, and c.