EE615: Problem Set 1

Problems 1.6, 1.7, 1.11, 1.13, 1.15, computer assignment

Computer Assignment

The purpose of the computer assignment is to generate a complete set of tools for AR process generation. The answer should contain
- A description of the implementation with possible theoretical derivations, e.g., how can you calculate the autocorrelation function of question 2 for an AR process of arbitrary order?
- Program listings
- Plots or listings of results

In grading, it will also be evaluated if the programs are efficiently and compactly implemented and easy to read and understand.

1. Implement a function to generate *N* samples of an AR process of arbitrary order:
$$u = argen(a,s2,N),$$
   for a *real process*, and
$$u = argen(a,s2,N,1),$$
   for a *complex process*. Here a are the AR parameters $(a_1,...,a_M)$, s2 is the power of the noise process $v(n)$, and *N* is the number of output points. Use Gaussian noise. You can use the Matlab built in function `filter`. Notice that the first samples produced this way contains a transient and should therefore be disregarded. You function should also check that the AR parameters give an asymptotically stationary process, and give an error message if not.
2. Implement a function to calculate the first N values of the autocorrelation function of an AR process of arbitrary order
$$r = arcorr(a,s2,N)$$
   where r=[r(0),r(1),...,r(N)]. You only have to solve the problem for real a.
3. Implement a function to calculate the input power σ_v^2 of the noise process $v(n)$ given the output power σ_u^2,
$$si2 = aripower(a,so2)$$
   You only have to solve the problem for real a.
4. Prove that the PSD of an AR process is given by

$$S(\omega) = \frac{\sigma_v^2}{\left|1 + \sum_{k=1}^{M} a_k^* \exp(-j\omega k)\right|^2}$$

   Implement a function to calculate the PSD of an AR-process at *N* points in the interval $(-\pi, \pi]$ (make sure that $\omega = 0$ is one of the points),
$$S = arspec(a,s2,N)$$

5a. Test your functions by calculating the 100 first values of the autocorrelation function for the AR parameters in table 1.1 in the textbook (real processes). Calculate the autocorrelation using the function in 2, and by generating the AR processes using the function in 3, and then calculating the statistical autocorrelation function (e.g., using `XCORR` in Matlab). Compare the results and justify that they are identical.

5b. Test your functions by calculating the PSD for the AR processes in table 2.1 by using the function in 4 and by using statistical estimation on the generated AR process (e.g., by using `SPECTRUM` in Matlab).