Department of Electrical Engineering, University of Hawaii

EE 342: Probability and Statistics **Fall 2016**

Homework Set 9

Due date: Nov. 9, 2016

(1) a) Chapter 7, problem 4
b) Chapter 7, problem 16
c) Chapter 7, problem 25
d) Chapter 7, problem 33
e) Chapter 7, problem 48
f) Chapter 7, problem 50
g) Chapter 7, problem 58

(2) Let $X_1, X_2, X_3, \ldots, X_n$ be mutually independent uniform [0,1] random variables

a) Find the PDF of $Y=\min(X_1, X_2, X_3, \ldots, X_n)$. Confirm by observing random samples using Matlab.

b) Find the PDF of $Z=\max(X_1, X_2, X_3, \ldots, X_n)$. Confirm by observing random samples using Matlab.

c) Find the joint PDF of Y and Z.

d) Are Y and Z independent?

e) Find E[Y|Z].

f) Find E[Z|Y].

g) Compute the following moments: E[Y], E[Z], Var(Y), Var(Z), Cov(Y,Z). Confirm by computing sample statistics in Matlab

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7.4. If X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} 1/y, & \text{if } 0 < y < 1,\ 0 < x < y \\ 0, & \text{otherwise} \end{cases}$$

find
(a) $E[XY]$
(b) $E[X]$
(c) $E[Y]$

7.16. Let Z be a standard normal random variable, and, for a fixed x, set

$$X = \begin{cases} Z & \text{if } Z > x \\ 0 & \text{otherwise} \end{cases}$$

Show that $E[X] = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

7.25. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed continuous random variables. Let $N \geq 2$ be such that

$$X_1 \geq X_2 \geq \cdots \geq X_{N-1} < X_N$$

That is, N is the point at which the sequence stops decreasing. Show that $E[N] = e$.
Hint: First find $P\{N \geq n\}$.

7.33. If $E[X] = 1$ and $\mathrm{Var}(X) = 5$, find
(a) $E[(2 + X)^2]$;
(b) $\mathrm{Var}(4 + 3X)$.

7.48. A fair die is successively rolled. Let X and Y denote, respectively, the number of rolls necessary to obtain a 6 and a 5. Find
(a) $E[X]$;
(b) $E[X|Y = 1]$;
(c) $E[X|Y = 5]$.

7.50. The joint density of X and Y is given by

$$f(x, y) = \frac{e^{-x/y} e^{-y}}{y}, \quad 0 < x < \infty, \quad 0 < y < \infty$$

Compute $E[X^2|Y = y]$.

7.58. A coin having probability p of coming up heads is continually flipped until both heads and tails have appeared. Find
(a) the expected number of flips;
(b) the probability that the last flip lands on heads.