Simulation-Based Computation of Information Rates: Upper and Lower Bounds

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Abstract — **It has recently become feasible to compute information rates of finite-state source/channel models with memory. Such methods can also be used to compute upper and lower bounds on the information rate of very general (non-finite-state) channels with memory by means of finite-state approximations. Further numerical bounds are obtained from reduced-state computations.**

In [1], it was shown how the information rate of a finite-state source/channel model can be computed numerically by sampling a long sequence of channel input and the corresponding channel output, followed by a forward sum-product recursion on the joint source/channel trellis. Essentially the same idea was independently proposed also by Sharma and Singh [2] and by Pfister et al. [3].

A significant extension of this method to compute upper and lower bounds on the information rate of very general (not necessarily finite-state) channels was proposed in [4]. The general character of these algorithms is as follows.

1. Choose a finite-state auxiliary channel that somehow approximates the actual "difficult" channel. (The validity of the bounds does not depend on the accuracy of this approximation.)
2. Sample a "very long" channel input sequence and the corresponding channel output sequence of the actual channel.
3. Use these sequences for a computation using the auxiliary channel. (The computation is a forward-only sum-product recursion in the style of [1].)

The lower bound can be computed from simulated (or measured) channel input/output data alone; the upper bound, requires also an analytical lower bound on the conditional entropy rate of the channel output sequence conditioned on the channel input sequence.

Another (new) type of upper bound on the information rate of a finite-input channel (fed from a finite-state source) is obtained as follows. At the heart of the method of [1] lies the recursive computation of state metrics

$$\begin{aligned}\mu_k(s_k) &= \sum_{x_k}\sum_{s_{k-1}} \mu_{k-1}(s_{k-1})\, p(x_k, y_k, s_k|s_{k-1}) && (1)\\ &= \sum_{x^k}\sum_{s_0^{k-1}} p(x^k, y^k, s^k) && (2)\end{aligned}$$

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for $k = 1, 2, 3, \ldots$, from which the desired quantity

$$p(y^n) = \sum_{s_n} \mu_n(s_n), \tag{3}$$

is obtained as the sum of all final state metrics. By restricting the sum (1) to a subset of the states s_{k-1}, a lower bound on $p(y^n)$ and thus an upper bound on $h(Y) = -\frac{1}{n}\log p(Y^n)$ is obtained.

This upper bound can also be applied to non-finite state channels as follows. Consider, e.g., the autoregressive channel of Fig. 1 and assume that, at time zero, the channel is in some fixed initial state. At time one, there will be two states; at time two, there will be four states, etc. We track all these states according to (1) until there are too many of them, and then we switch to the reduced-state recursion.

For the channel of Fig. 1 (and for many much more complex channels), accurate information rates can now be computed for all signal-to-noise ratios. For the auxiliary-channel bounds, we may either truncate the impulse response or insert a quantizer in the feedback loop, cf. [4]. The reduced-state upper bound can be applied directly to the channel of Fig. 1 and yields excellent numerical results with very few states.

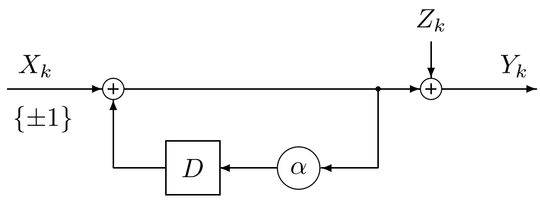

Figure 1: Autoregressive-filter channel.

References

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