Department of Electrical Engineering, University of Hawaii

EE 342: Probability and Statistics **Fall 2016**

Homerwork Set 5

Due date: Sep 28, 2016

(1)
a) Chapter 4, problem 14
b) Chapter 4, problem 19
c) Chapter 4, problem 22
d) Chapter 4, problem 25
e) Chapter 4, problem 38
f) Chapter 4, problem 58
g) Chapter 4, theoretical exercise 4
h) Chapter 4, theoretical exercise 8

(2) Matlab exercise:

a) Use Matlab to make plots of the PMF and CDF of a binomial random variable when n=20 and p=0.5. For plots of discrete random variables use "stem" for the PMF and "stairs" for the CDF. Use Matlab to compute the mean and variance of these 2 random variables. Repeat for n=20 and p=0.2.

b) Use Matlab to generate 5000 random numbers drawn from the binomial random variable with parameters n=20 and p=0.5. From these random numbers, generate plots of the sample PMF and CDF. Also, find the sample mean and variance. Compare to problem 2a). Repeat for n=20 and p=0.2.

[Hint: Binomial random variables can be generated from Bernoulli random variables as discussed in class.]

(3) Matlab exercise:

a) Repeat 2) for a geometric random variable with p=0.5.
b) Repeat 2) for a geometric random variable with p=0.2.

[Hint: Geometric random variables can be generated from Bernoulli random variables as discussed in class.]

4.19. If the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{1}{2} & 0 \le b < 1 \\ \frac{3}{5} & 1 \le b < 2 \\ \frac{4}{5} & 2 \le b < 3 \\ \frac{9}{10} & 3 \le b < 3.5 \\ 1 & b \ge 3.5 \end{cases}$$

calculate the probability mass function of X.

4.14. Five distinct numbers are randomly distributed to players numbered 1 through 5. Whenever two players compare their numbers, the one with the higher one is declared the winner. Initially, players 1 and 2 compare their numbers; the winner then compares her number with that of player 3, and so on. Let X denote the number of times player 1 is a winner. Find $P\{X = i\}, i = 0, 1, 2, 3, 4$.

4.22. Suppose that two teams play a series of games that ends when one of them has won i games. Suppose that each game played is, independently, won by team A with probability p. Find the expected number of games that are played when (a) $i = 2$ and (b) $i = 3$. Also, show in both cases that this number is maximized when $p = \frac{1}{2}$.

4.25. Two coins are to be flipped. The first coin will land on heads with probability .6, the second with probability .7. Assume that the results of the flips are independent, and let X equal the total number of heads that result.
(a) Find $P\{X = 1\}$.
(b) Determine $E[X]$.

4.58. Compare the Poisson approximation with the correct binomial probability for the following cases:
(a) $P\{X = 2\}$ when $n = 8, p = .1$;
(b) $P\{X = 9\}$ when $n = 10, p = .95$;
(c) $P\{X = 0\}$ when $n = 10, p = .1$;
(d) $P\{X = 4\}$ when $n = 9, p = .2$.

4.38. If $E[X] = 1$ and $\text{Var}(X) = 5$, find
(a) $E[(2 + X)^2]$;
(b) $\text{Var}(4 + 3X)$.

Theoretical Exercises:

4.4. For a nonnegative integer-valued random variable N, show that

$$E[N] = \sum_{i=1}^{\infty} P\{N \ge i\}$$

Hint: $\sum_{i=1}^{\infty} P\{N \ge i\} = \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} P\{N = k\}$. Now interchange the order of summation.

4.8. Find $\text{Var}(X)$ if

$$P(X = a) = p = 1 - P(X = b)$$