HW4 – SOLUTIONS

1 CH 3, PROBLEM 7

IF THE KING IS ONE OF 2 SIBLINGS,
THEN THE FOLLOWING 3 OUTCOMES ARE POSSIBLE
FOR THE CHILDREN IN THE FAMILY

a) BOTH ARE MALE → (M, M)

b) OLDER CHILD MALE, YOUNGER CHILD FEMALE → (M, F)

c) OLDER CHILD FEMALE, YOUNGER CHILD MALE → (F, M).

[(F, F) IS NOT POSSIBLE BECAUSE AT LEAST ONE CHILD IS MALE (KING)]

OF THE 3 POSSIBILITIES (M, M), (M, F), (F, M)
IN ONLY 2 THE MALE CHILD (KING) HAS A SISTER

$$P(\text{KING HAS SISTER}) = \frac{2}{3}$$

2 CH 3, PROBLEM 8

4 POSIBLE OUTOMES

(F, F) ←
(F, M) ←
(M, F)
(M, M)

older younger

} TWO OUTCOMES IN WHICH THE OLDER CHILD IS A FEMALE. OF THESE 2 OUTCOMES, ONLY ONE OUTCOME (F, F) HAS THE SECOND CHILD BEING A GIRL

So $\text{Prob}(\text{2nd child F} \mid \text{1st child F}) = \frac{1}{2}$

$\boxed{3}$ CH3, PROBLEM 15

S_M = {woman is a smoker}

E_P = {woman has ectopic pregnancy}

GIVEN:

a) $P(E_P | S_M) = 2\,P(E_P | \bar{S}_M)$

b) $P(S_M) = 0.32$

TASK: FIND $P(S_M | E_P)$!

$$P(S_M | E_P) = \frac{P(S_M \cap E_P)}{P(E_P)}$$

$$= \frac{P(E_P | S_M) \cdot P(S_M)}{P(E_P)}$$

$$= \frac{P(E_P | S_M) \cdot P(S_M)}{P(E_P | S_M) \cdot P(S_M) + P(E_P | \bar{S}_M) \cdot P(\bar{S}_M)}$$

$$= \frac{2\,P(E_P | \bar{S}_M) \cdot P(S_M)}{2\,P(E_P | \bar{S}_M) \cdot P(S_M) + P(E_P | \bar{S}_M) \cdot P(\bar{S}_M)}$$

$$= \frac{2\,P(S_M)}{2\,P(S_M) + P(\bar{S}_M)} = \frac{2 \times 0.32}{2 \times 0.32 + 0.68}$$

$$= \underline{\underline{0.4848}}$$

4 CH 4, PROBLEM 2

POSSIBLE VALUES OF THE PRODUCT ARE:

Product	events	PMF of X
1	(1, 1)	$p_X(1) = 1/36$
2	(1,2), (2,1)	$p_X(2) = 2/36$
3	(1,3), (3,1)	$p_X(3) = 2/36$
4	(1,4), (4,1), (2,2)	$p_X(4) = 3/36$
5	(1,5), (5,1)	$p_X(5) = 2/36$
6	(1,6), (6,1), (2,3), (3,2)	$p_X(6) = 4/36$
8	(2,4), (4,2)	$p_X(8) = 2/36$
9	(3,3)	$p_X(9) = 1/36$
10	(2,5), (5,2)	$p_X(10) = 2/36$
12	(2,6), (6,2), (3,4), (4,3)	$p_X(12) = 4/36$
15	(3,5), (5,3)	$p_X(15) = 2/36$
16	(4,4)	$p_X(16) = 1/36$
18	(3,6), (6,3)	$p_X(18) = 2/36$
20	(4,5), (5,4)	$p_X(20) = 2/36$
24	(4,6), (6,4)	$p_X(24) = 2/36$
25	(5,5)	$p_X(25) = 1/36$
30	(5,6), (6,5)	$p_X(30) = 2/36$
36	(6,6)	$p_X(36) = 1/36$

FOR ALL OTHER VALUES OF i

$$P(X = i) = p_X(i) = 0$$

$\boxed{5}$ CHAPTER 4, PROBLEM 4

1°) $P(X=1) = P(\text{FIRST POSITION OCCUPIED BY A WOMAN}) = \dfrac{\binom{5}{1}}{\binom{10}{1}} = \dfrac{5}{10} = \dfrac{1}{2}$ //

2°) $P(X=2) = P\left(\begin{array}{l}\text{FIRST POSITION OCCUPIED BY A MAN AND} \\ \text{SESOND POSITION OCCUPIED BY A WOMAN}\end{array}\right)$

$$= \frac{5 \cdot 5}{10 \cdot 9}$$

$$= \frac{5}{18} \quad //$$

3°) $P(X=3) = P\left(\begin{array}{l}\text{FIRST 2 POSITIONS OCCUPIED BY MEN} \\ \text{AND 3RD POSITION OCCUPIED BY A WOMAN}\end{array}\right)$

$$= \frac{(5 \cdot 4) \cdot 5}{10 \cdot 9 \cdot 8} = \frac{5}{2 \cdot 9 \cdot 2}$$

$$= \frac{5}{36} \quad //$$

4°) $P(X=4) = \dfrac{(5 \cdot 4 \cdot 3) \cdot 5}{10 \cdot 9 \cdot 8 \cdot 7} = \dfrac{5}{2 \cdot 3 \cdot 2 \cdot 7} =$

$$= \frac{5}{84} \quad //$$

5°) $P(X=5) = \dfrac{(5 \cdot 4 \cdot 3 \cdot 2) \cdot 5}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6} = \dfrac{5}{3 \cdot 2 \cdot 7 \cdot 6} = \dfrac{5}{252}$ //

6°) $P(X=6) = \dfrac{5! \cdot 5}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5} = \dfrac{1}{252}$ //

7°) $X=7$ is impossible so $P(X=7)=0$ //

8°) $P(X=8)=0$ //

9°) $P(X=9)=0$ //

10°) $P(X=10)=0$ //

$\boxed{6}$ CH 4, PROBLEM 18

$$p_X(0) = P(X=0) = \left(\tfrac{1}{2}\right)^4 = \frac{1}{16}$$

$$p_X(1) = P(X=1) = 4\cdot\left(\tfrac{1}{2}\right)^4 = \frac{4}{16}$$

$$p_X(2) = P(X=2) = \binom{4}{2}\left(\tfrac{1}{2}\right)^4 = \frac{6}{16}$$

$$p_X(3) = P(X=3) = \binom{4}{3}\left(\tfrac{1}{2}\right)^4 = \frac{4}{16}$$

$$p_X(4) = P(X=4) = \left(\tfrac{1}{2}\right)^4 = \frac{1}{16}$$

NOW TAKE

$$p_{X-2}(-2) = P(X-2=-2) = P(X=0) = p_X(0) = \frac{1}{16}$$

$$p_{X-2}(-1) = P(X-2=-1) = P(X=1) = p_X(1) = \frac{4}{16}$$

$$p_{X-2}(0) = P(X-2=0) = P(X=2) = p_X(2) = \frac{6}{16}$$

$$p_{X-2}(1) = P(X-2=1) = P(X=3) = p_X(3) = \frac{4}{16}$$

$$p_{X-2}(2) = P(X-2=2) = P(X=4) = p_X(4) = \frac{1}{16}$$