Insertion/deletion channels: Reduced-state lower bounds on channel capacities

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Abstract — **In this paper, we use Monte-Carlo methods to compute lower bounds on information rates of insertion/deletion channels. The information rate computation methods we propose are based on reduced-state trellis techniques. We compare the newly obtained bounds to some previously derived bounds (specifically for the deletion channel) and show that the new bounds are (much) sharper. In some regions, the new bound almost meets a known upper bound.**

I. Introduction

A combined insertion/deletion channel occurs if transmitted symbols are randomly deleted and random symbols are inserted in the resulting subsequence. In [1], lower bounds on information rates of a K-ary deletion channel were computed. In [2], simulation based methods to obtain upper and lower bounds on the information rate of general channels by finite-state and reduced-state approximations were presented. In this article, we use the reduced-state technique to obtain lower bounds on information rates of insertion/deletion channels. Due to space limitations, we only provide the results for the deletion channel.

II. Bounds on Information Rates

The bounds that we develop in this section hold for any stationary channel in which the deletion statistics follow a Markov rule. Let the transmitted symbol at time $t \in Z$ be denoted by the random variable X_t and the received symbol be denoted by Y_t both of which take binary values. Let the transmitted symbol sequence of length m be denoted by the vector $X_1^m = [X_1, X_2, \ldots, X_m]$. The deletion channel randomly deletes any symbol X_t with probability δ, independent of any other transmitted symbols.

The channel information rate *per received symbol* can be expressed as

$$\mathcal{I}^{(r)}(X;Y) = \lim_{k\to\infty} \frac{1}{k} \lim_{m\to\infty} I\left(X_1^m; Y_1^k\right) \quad \left[\frac{\text{bits}}{\text{received symbol}}\right].$$

$$\mathcal{I}^{(r)}(X;Y) \quad = \quad H^{(r)}(Y) - H^{(r)}(Y|X) \quad \left[\frac{\text{bits}}{\text{received symbol}}\right].$$

The information rate in bits per transmitted symbol is then

$$\mathcal{I}^{(t)}(X;Y) = (1-\delta)\cdot \mathcal{I}^{(r)}(X;Y) \quad [\text{bits/transmitted symbol}].$$

For Markov inputs X_t, the output process Y_t is a hidden Markov process ([1], see Eqn. (22)). Generally, if X_t is a Markov process, the output Y_t is *hidden* Markov, and the entropy rate $H^{(r)}(Y)$ may be computed using Monte-Carlo simulation methods.

The conditional entropy rate $H^{(r)}(Y|X)$ is difficult to compute exactly. However, we may utilize the reduced-state trellis technique [2] to upper bound $H^{(r)}(Y|X)$. Substituting this upper bound instead of $H^{(r)}(Y|X)$, we automatically get a lower bound on the information rate $\mathcal{I}^{(r)}(X;Y)$.

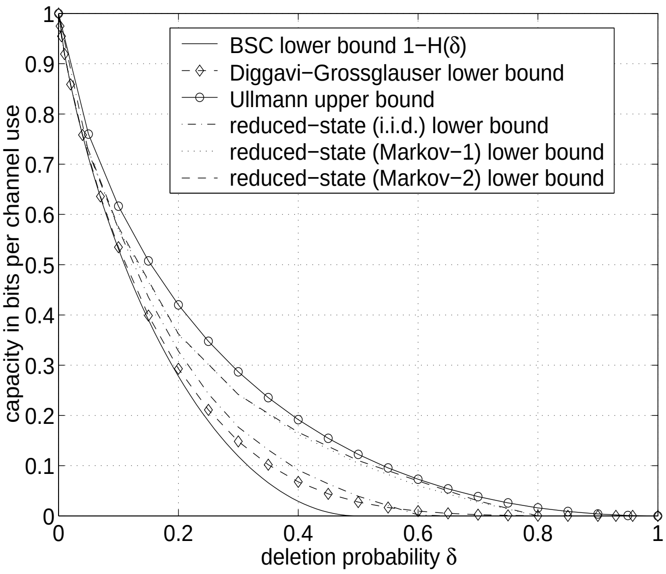

Figure 1: Bounds on information rates for the deletion channel.

The lower bound can be further tightened by varying the parameters of the input Markov process, and taking the maximum. Thereby, we can vary the order of the Markov process X_t as well its transition probabilities.

If $\mathbf{P}$ is the $2^M \times 2^M$ transition probability matrix of the M-th order Markov X_t, then the output process Y_t is a hidden Markov process driven by a Markov state sequence S_t with transition probability matrix $\mathbf{Q}$ given by $\mathbf{Q}(\mathbf{1}-\delta)\left[\mathbf{I}-\delta\mathbf{P}\right]^{-\mathbf{1}}\mathbf{P}$.

Since the process Y_t is a hidden Markov process, the quantity $-\frac{1}{k}\log p(y_1^k)$ converges almost surely to $H^{(r)}(Y)$. For a given sample of the output process, the quantity $\log p(y_1^k)$ can be computed using the BCJR algorithm. To compute $H^{(r)}(Y|X)$, we need to evaluate $-\log p(y_1^k|x_1^\infty)$, where x_1^∞ and y_1^k are samples of the input and output process, respectively. This can be done by constructing a trellis, and using a simulation-based method to compute the information rate. For the deletion channel, simulations were performed for different values of δ. The results are plotted in Fig. 1.

References

[1] S. Diggavi and M. Grossglauser, *"On transmission over deletion channels,"* Allerton Conference, Monticello, Illinois, October 2001.

[2] D. Arnold, A. Kavcic, H.-A. Loeliger, P. Vontobel, W. Zeng *"Simulation Based Computation of Information Rates: Upper and Lower Bounds by Reduced-State Techniques,"* Proc. ISIT, pp. 119, June 2003.