Decision Feedback Equalization in Channels with Signal-Dependent Media Noise

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***Abstract*—Since maximum likelihood sequence detection in channels with media noise requires too many trellis states, it is advantageous to seek simpler detectors. This paper formulates the signal-dependent decision feedback equalizer (SD-DFE) for these channels. For the purpose of designing the feed-forward equalizer, the channel is treated as linear and the noise as stationary. These assumptions guide the design of the minimum-mean-square-error feed-forward equalizer. The feedback structure is derived using the autoregressive signal/noise model. In 90% jitter noise dominated channels, the SD-DFE performs as well as a 16-state signal-dependent Viterbi detector, at a fraction of the complexity. A drawback of this method is error propagation due to the feedback.**

***Key words*—media noise, signal-dependent noise, decision feedback equalization, intersymbol interference**

I. Introduction

The signal-dependent maximum likelihood sequence detector (SD-MSLD) and the maximum likelihood symbol detector have recently been formulated for channels with signal-dependent noise [1], [2]. Since maximum likelihood sequence (and symbol) detection involves many trellis states, in some applications it is advantageous to seek simpler signal-dependent detectors.

Decision feedback equalization (DFE) detectors in magnetic recording have been proposed as simpler alternatives to partial response maximum likelihood (PRML) detectors [3]. Hybrid detectors in the form of fixed-delay tree-search [4] and reduced-state sequence estimators (RSE) [5] have also been proposed. Random access memory (RAM) realizations were proposed for channels with nonlinearities RAM-DFE [6], RAM-RSE [7]. Recently, the decision aided equalizer (a version of a soft-output DFE) was proposed for iterative decoding [8]. In this paper, we formulate a DFE detector for recording channels with signal-dependent (media) noise and compare its performance to signal-dependent Viterbi detectors.

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Notation: Underlined characters represent column vectors, uppercase boldface characters represent matrices, and the superscript $^{\mathrm{T}}$ denotes the matrix and vector transposition. If $\{x_k\}$ is a discrete-time sequence (k denotes time), then for $i \leq j$ the column vector of samples $x_i, x_{i+1}, \ldots, x_j$ is denoted by

$$\underline{x}_j^i = \begin{bmatrix} x_i \\ x_{i+1} \\ \vdots \\ x_j \end{bmatrix}. \tag{1}$$

II. The feed-forward equalizer

For good performance of a DFE detector, it is required that the feed-forward-equalized pulse be of minimum phase in order to maximize the energy in the first non-zero coefficient [9]. In a linear time-invariant channel with additive stationary Gaussian noise, this is achieved with a linear time-invariant feed-forward filter [9]. In magnetic recording, where the channel response is generally nonlinear and where the noise is non-stationary (more precisely, signal-dependent), it is not clear how to formulate an optimal feed-forward equalizer. Here we adopt the following ad hoc strategy.

Let $a_k \in \{-1, 1\}$, represent the input symbol at time instant k. Let the output of the recording channel be

$$\xi_k = g\left(\underline{a}_k^{k-D}\right) + \nu_k \tag{2}$$

where g is an arbitrary (possibly nonlinear) function and ν_k is a sample of an additive Gaussian correlated (possibly signal-dependent) noise sequence. We can approximate the equation in (2) with a linear fit

$$\xi_k \approx \sum_{i=0}^{D} \hat{h}_i a_{k-i} + \eta_k, \tag{3}$$

where the dipulse response coefficients $\hat{h}_i$ are obtained by the least-squares fit (see, e.g., [10], p. 365, eq. 9.21)

$$\begin{bmatrix} \hat{h}_D & \cdots & \hat{h}_1 & \hat{h}_0 \end{bmatrix}^{\mathrm{T}} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\underline{\xi}_N^D. \tag{4}$$

The entry in location (i, j) of the $(N-D+1) \times (D+1)$ matrix $\mathbf{A}$ is $A_{ij} = a_{i+j-2}$, and a_k are realizations of equally likely independent binary random training variables, $0 \leq k \leq N$. The estimated covariance of the noise

η_k in (3) is

$$\mathrm{E}\left[\eta_k \eta_{k+j}\right] \approx \hat{r}_j \;=\; \frac{1}{N-D-j+1} \sum_{k=D}^{N-j} \left(\xi_k - \sum_{i=0}^{D} \hat{h}_i a_{k-i} \right) \times \left(\xi_{k+j} - \sum_{i=0}^{D} \hat{h}_i a_{k+j-i} \right). \tag{5}$$

We proceed as if (3) were the signal model and as if the noise η_k in (3) were Gaussian with the covariance $\hat{r}_j$ given in (5). We formulate the standard minimum-mean-square-error (MMSE) feed-forward equalizer with coefficients f_0 through f_Q given by the following system of linear equations (see, e.g., [9] for details)

$$\sum_{q=0}^{Q} \Psi_{pq} f_q = \hat{h}_p, \tag{6}$$

where for $p, q \in \{0, 1, \ldots, Q\}$, we have

$$\Psi_{pq} = \sum_{m=0}^{p} \hat{h}_m \hat{h}_{m-p+q} + \hat{r}_{p-q}. \tag{7}$$

The output of the feed-forward equalizer is thus given by

$$z_k = \sum_{q=0}^{Q} f_q \xi_{k-q}. \tag{8}$$

III. The feedback architecture

The feed-forward equalizer described above does not remove the channel nonlinearities nor does it whiten the noise. To deal with the residual nonlinearities and the noise signal-dependence, we employ feedback.

We use the signal-dependent autoregressive model [11] to describe the signal

$$z_k = y\left(a_k, \underline{a}_{k-1}^{k-I}\right) + n_k. \tag{9}$$

The noise-free part of the signal $y\left(a_k, \underline{a}_{k-1}^{k-I}\right)$ is dependent on the binary input symbols a_{k-I} through a_k. The noise n_k is Gaussian (since ν_k is Gaussian) and signal-dependent, given by the recursion

$$n_k = \underline{b}\left(a_k, \underline{a}_{k-1}^{k-I}\right)^{\mathrm{T}} \underline{n}_{k-1}^{k-L} + \sigma\left(a_k, \underline{a}_{k-1}^{k-I}\right) w_k \tag{10}$$

where $\underline{b}\left(a_k, \underline{a}_{k-1}^{k-I}\right)$ is a vector of signal-dependent autoregressive coefficients, $\sigma\left(a_k, \underline{a}_{k-1}^{k-I}\right)$ is a signal-dependent standard deviation term and w_k is a unit-variance white Gaussian noise process. The details of fitting autoregressive models to waveforms are covered in [11].

Let $\hat{a}_{k-I-L}, \ldots, \hat{a}_{k-1}$ denote the $I + L$ symbols detected (estimated) prior to the symbol a_k at time instant k. Denote by σ_{-1} the standard deviation $\sigma\left(-1, \hat{\underline{a}}_{k-1}^{k-I}\right)$. Similarly, $\sigma_1 = \sigma\left(1, \hat{\underline{a}}_{k-1}^{k-I}\right)$. Denote

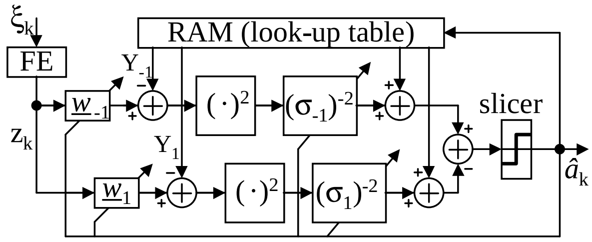

Fig. 1. The unequalized samples ξ_k pass through the feed-forward equalizer (FE) to produce samples z_k. The FIR filters $\underline{W}_{-1}$ and $\underline{W}_1$, the scaling constants σ_{-1}^2 and σ_1^2, and the noise-free sample values Y_{-1} and Y_1 are dependent on the past decisions $\hat{a}_{k-1}, \hat{a}_{k-2}, \ldots, \hat{a}_{k-I-L}$. All these constants are available from a random access memory (RAM) look-up table.

by $\underline{W}_{-1}$ the vector $\left[-\underline{b}\left(-1, \hat{\underline{a}}_{k-1}^{k-I}\right)^{\mathrm{T}}, 1 \right]^{\mathrm{T}}$. Similarly, $\underline{W}_1 = \left[-\underline{b}\left(1, \hat{\underline{a}}_{k-1}^{k-I}\right)^{\mathrm{T}}, 1 \right]^{\mathrm{T}}$. Finally, denote by Y_i (where $i \in \{-1, 1\}$), the value

$$Y_i = \underline{W}_i^{\mathrm{T}} \cdot \begin{bmatrix} y\left(\hat{a}_{k-L}, \hat{\underline{a}}_{k-1-L}^{k-I-L}\right) \\ y\left(\hat{a}_{k-L+1}, \hat{\underline{a}}_{k-L}^{k-I-L+1}\right) \\ \vdots \\ y\left(\hat{a}_{k-1}, \hat{\underline{a}}_{k-2}^{k-I-1}\right) \\ y\left(i, \hat{\underline{a}}_{k-1}^{k-I}\right) \end{bmatrix}. \tag{11}$$

Motivated by the expression for branch metrics in signal-dependent systems [1], we formulate the decision feedback detector. It is given by the following decision strategy.

$$\ln \sigma_{-1}^2 + \frac{\left(\underline{W}_{-1}^{\mathrm{T}} \underline{z}_k^{k-L} - Y_{-1}\right)^2}{\sigma_{-1}^2} \quad \underset{\hat{a}_k = 1}{\overset{\hat{a}_k = -1}{\lessgtr}} \quad \ln \sigma_1^2 + \frac{\left(\underline{W}_1^{\mathrm{T}} \underline{z}_k^{k-L} - Y_1\right)^2}{\sigma_1^2}. \tag{12}$$

Equation (12) reads: decide $\hat{a}_k = -1$ if the left-hand side is smaller than the right-hand side, otherwise decide $\hat{a}_k = 1$. The decision strategy is depicted in Figure 1. We refer to it as the signal-dependent DFE (SD-DFE). It can be verified that when $L = 0$ and when $\sigma_{-1} = \sigma_1$, the rule in (12) is a RAM-DFE [6]. In addition, if Y_i can be represented as a linear combination of past decisions, then the decision in (12) is a regular MMSE-DFE [9].

IV. Simulation results

Figure 2 compares the simulated error rates for a recording channel with a mixture Gaussian-Lorentzian pulse and symbol density of 2.5 bits/PW50. The noise is comprised of 10% additive white Gaussian noise and 90% Gaussian jitter noise (media noise). The curve denoted by "sd-dfe" is for an SD-DFE with $I = 5$ and

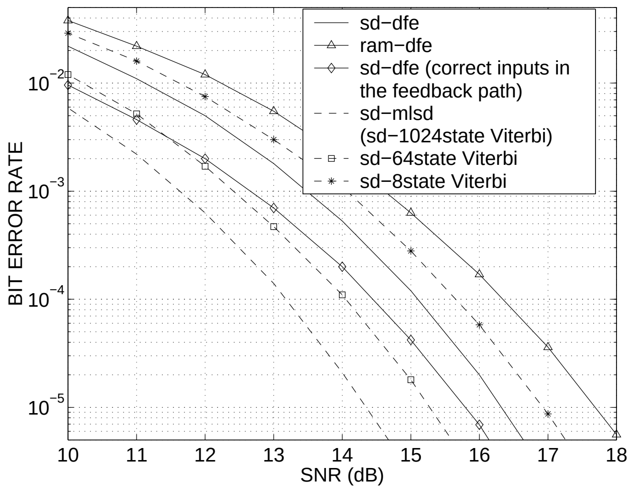

Fig. 2. Error rates for the SD-DFE, compared to RAM-DFE, the signal-dependent MLSD (SD-MLSD), and two other suboptimal signal-dependent Viterbi detectors with 8 and 64 states, plotted versus the signal-to-noise ratio (SNR). To illustrate the effect of error propagation, the performance of the SD-DFE is shown when the correct inputs are used in the feedback path.

$L = 10$. For comparison, the curve for a RAM-DFE ($I = 5$, $L = 0$) is also depicted. Also, for comparison, the performance of several suboptimal signal-dependent detectors [1] are depicted. These range in complexity from an 8-state to a 1024-state signal-dependent Viterbi detector, where the performance of the 1024-state Viterbi detector is very close to the signal-dependent maximum-likelihood sequence detector [1], hence the label "sd-mlsd" in Figure 2. In Figure 2 the performance of a 16-state signal dependent detector is not shown, but it actually coincides with the curve for the SD-DFE. Hence we conclude that the signal-dependent DFE can achieve the same error rate as the best 16-state Viterbi detector, with roughly a 16-fold complexity reduction (a precise quantitative complexity evaluation of signal-dependent Viterbi detectors can be found in [1]). Evidently, the price paid is a performance that is almost 2dB worse than the best 1024-state signal-dependent Viterbi detector. On the other hand, the SD-DFE is roughly 1.5dB better than the RAM-DFE. An obvious drawback is that the SD-DFE, like all other decision feedback detectors, suffers from error propagation. For error-propagation analysis methods, see [12], [13]. The effect of error propagation on the SD-DFE detector is illustrated in Figure 2, where an error rate curve is shown for a hypothetical SD-DFE detector that has no errors in the feedback path. Obviously, due to error propagation, SD-DFE is not a strategy that will replace the PRML detector in systems with very low SNRs (signal-to-noise ratios). However, in systems where the SNR budget is higher, and where error bursts exist anyhow due to reasons other than feedback error propagation (e.g., dropout in tape-systems), SD-DFE may be used as a replacement for a more complex Viterbi detector.

V. Conclusion

In this paper, a signal-dependent decision-feedback equalizer (SD-DFE) was developed for channels with signal-dependent media noise. The receiver consists of a MMSE feed-forward equalizer that is derived under the assumption that linearity and noise-stationarity hold. The feedback structure (derived from the signal-dependent autoregressive signal/noise model) takes care of the residual nonlinearities, intersymbol interference and the signal-dependence of the noise. In a 90% media-noise-dominated channel at a symbol density of 2.5 bits/PW50, the SD-DFE performs as well as a 16-state signal-dependent Viterbi detector, which is 2dB worse than the optimal 1024-state signal-dependent maximum likelihood sequence detector (SD-MLSD), but 1.5dB better than RAM-DFE. The advantage of the SD-DFE is a lower complexity than Viterbi detection, which may play a role if several detectors need to run in parallel (e.g., in tape-recording systems) at relatively high SNRs. The disadvantage is error propagation inherent to all decision feedback detectors.

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