Department of Electrical Engineering, University of Hawaii

EE 342: Probability and Statistics **Fall 2016**

Homerwork Set 8

Due date: Oct 26, 2016

(1) a) Chapter 6, problem 28
b) Chapter 6, problem 39
c) Chapter 6, problem 42
d) Chapter 6, problem 52
e) Chapter 6, problem 53
f) Chapter 6, problem 56
g) Chapter 6, problem 58

(2) An urn contains 6 blue and 3 red balls. Balls are drawn at random without replacement until the first blue ball is drawn. Let the random variable X be the total number of balls that are drawn until the first blue ball is drawn. After the first blue ball is drawn, balls continue to be drawn at random without replacement from the same urn until the second blue ball is drawn. Let the random variable Y be the number of balls drawn from the urn after the first blue ball is drawn until the second blue ball is drawn.

a) Find the joint PMF of X and Y.
b) Find the marginal PMFs.
c) Find E[X], E[Y], Var(X), Var(Y), Cov(X,Y).
d) Are X and Y independent?
e) Are X and Y uncorrelated?
f) Find the PMF of Z=X+Y.

(3) Let X and Y be independent random variables. Let X be an exponential random variable with parameter λ. It is known that the marginal PDF of X equal the marginal PDF of Y.

a) Find the PDF of Z=X+Y. Plot the PDF for λ=2.

b) Find the PDF of W=X-Y. Plot the PDF for λ=1.

c) Find the PDF of V=min(X,Y). Plot the PDF for λ=3.

6.28. The time that it takes to service a car is an exponential random variable with rate 1.

(a) If A. J. brings his car in at time 0 and M. J. brings her car in at time t, what is the probability that M. J.'s car is ready before A. J.'s car? (Assume that service times are independent and service begins upon arrival of the car.)

(b) If both cars are brought in at time 0, with work starting on M. J.'s car only when A. J.'s car has been completely serviced, what is the probability that M. J.'s car is ready before time 2?

6.42. The joint density of X and Y is

$$f(x, y) = c(x^2 - y^2)e^{-x} \quad 0 \le x < \infty, -x \le y \le x$$

Find the conditional distribution of Y, given $X = x$.

6.53. If X and Y are independent random variables both uniformly distributed over (0, 1), find the joint density function of $R = \sqrt{X^2 + Y^2}, \Theta = \tan^{-1} Y/X$.

6.58. If X_1 and X_2 are independent exponential random variables, each having parameter λ, find the joint density function of $Y_1 = X_1 + X_2$ and $Y_2 = e^{X_1}$.

6.39. Two dice are rolled. Let X and Y denote, respectively, the largest and smallest values obtained. Compute the conditional mass function of Y given $X = i$, for $i = 1, 2, \ldots, 6$. Are X and Y independent? Why?

6.52. Let X and Y denote the coordinates of a point uniformly chosen in the circle of radius 1 centered at the origin. That is, their joint density is

$$f(x, y) = \frac{1}{\pi} \quad x^2 + y^2 \le 1$$

Find the joint density function of the polar coordinates $R = (X^2 + Y^2)^{1/2}$ and $\Theta = \tan^{-1} Y/X$.

6.56. If X and Y are independent and identically distributed uniform random variables on (0, 1), compute the joint density of

(a) $U = X + Y, V = X/Y$;

(b) $U = X, V = X/Y$;

(c) $U = X + Y, V = X/(X + Y)$.