Triangle zig-zag transition modeling*

Aleksandar Kavčić and José M. F. Moura

Data Storage Systems Center
Department of Electrical and Computer Engineering
Carnegie Mellon University, Pittsburgh, PA 15213

ABSTRACT

Designing read-back subsystems in magnetic recording requires precise knowledge about the signal picked up by the reading head. As areal densities in longitudinal magnetic recording increase, the read-back signal becomes more corrupted by intersymbol interference, media noise, and intertrack interference. Due to the spatial distribution of transitions on a disk surface and the nonlinear character of bit interactions, two dimensional media plane models are widely used to model the write process. If a two dimensional (or three dimensional) read-head model is utilized, intertrack interferences are also observed. Micromagnetic media modeling, coupled with appropriate read-head models, have been successfully used to model the "raw" magnetic recording channel. However, due to its high computational complexity, micromagnetic modeling is an impractical tool in statistical signal analyses such as error rate studies where thousands of transitions need to be created. We propose a much simpler, yet realistic, two dimensional write process model. We call it the triangle zig-zag transition (TZ-ZT) model since the transition boundary is modeled by lateral sides of isosceles triangles of alternating orientations truncated on a common basis line across the track width. Formulas are presented that relate the parameters of the model, the probability density function of triangle heights and the constant vertex angle, to the magnetization transition profile of an isolated transition and to the cross track correlation width, respectively. Although stochastic zig-zag models have been proposed in the past, our model has the advantage that it is stable across the track, that is, it is not an independent increment process and it therefore doesn't exhibit a cross track drift. Compared to micromagnetic modeling, the TZ-ZT model offers computational savings of 4 orders of magnitude, while transition shapes and media noise are modeled with comparable accuracy, as our results show. For these reasons, the TZ-ZT model, combined with an appropriate head-sensitivity function, is an attractive "raw channel" model for applications such as statistical performance analyses where large numbers of bits are needed.

Keywords: media noise, transition noise, jitter, cross-track correlation width, cross-track autocorrelation function, generalized telegraphic signal, micromagnetic modeling

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1 INTRODUCTION

As linear densities in magnetic recording increase, we need more sophisticated signal detection schemes. This is a result of increased intersymbol interference (ISI) between signals generated by neighboring bits (magnetization transitions). The traditional peak-detection,[1] which is essentially a simple bit-by-bit detector, is being replaced by maximum likelihood sequence detection (MLSD) algorithms.[2] Examples of sub-optimal but realizable MLSD detectors are partial response maximum likelihood (PRML) algorithms and fixed delay tree search with decision feedback (FDTS/DF).[3,4] PRML is based on shaping the read-back signal to one of the PR (partial response) targets,[5,6] followed by a Viterbi detector.[7,8] FDTS/DF is a hybrid of MLSD and decision feedback equalization (DFE) and has the advantage over PRML detectors because the signal need not be shaped to a prespecified PR target.

Originally, both PRML detection and FDTS/DF were derived for the case of additive white Gaussian noise (AWGN) and linear ISI. While these assumptions hold to reasonable accuracy at moderately high recording densities, they fail at the ultra high densities anticipated for future magnetic recording systems. A common approach is to modify existing detection schemes to improve performance when the noise is signal dependent and spatially correlated, and/or when ISI is non-linear, but it is not clear how close to the optimal detector this is in the presence of typical noise and interferences at high densities. Deriving new detectors fine-tuned to high-density noise, on the other hand, requires knowledge of the physics involved in magnetic recording and requires usage of signal models that reflect these physical properties. In this paper we present a statistical signal generation model (statistical media noise model followed by a read-back model) developed for fast signal generation while preserving the statistics of signal and noise in the read-back signal. We show that the statistical properties of our model (we call it the TZ-ZT model) match those of much more complex models (micromagnetic models), while the computational savings reach 4 orders of magnitude. This makes the TZ-ZT model an ideal tool to use in system evaluations, such as on-track and off-track bit error rate predictions.

The paper is organized as follows. In section 2 we give a brief introduction to media noise and media noise models. In section 3 we present the triangle zig-zag transition (TZ-ZT) model. Comparison of the statistics of transitions generated by the TZ-ZT model and the micromagnetic model is conducted in section 4. We conclude the paper with key remarks in section 5.

2 MEDIA NOISE

There are three sources of noise in magnetic recording read-back systems. First, there is noise that is a product of unevenness of the magnetic medium and random magnetization patterns in the media. This type of noise is referred to as media noise. Second, there is noise produced by the read head – the head noise, and finally, there is the systems electronics noise. At today's densities, media noise is the dominant source of noise in magnetic recording systems, and it is the subject of this paper.

There are two types of media noise (and a number of other interference effects). The amplitude modulation (AM) noise is a consequence of uneven magnetic properties of different regions of the disk. As the read head passes (flies) over these regions, the amplitude of the read-back signal varies, and hence the name – AM noise. This paper is not concerned with AM noise. Here, we are interested in the transition noisea[†]. In magnetic recording, bits are encoded by magnetization transitions along the track. Ideally, these transitions would occur along straight lines perpendicular to the recording direction as in Figure 1.a. In reality, these lines are actually zig-zag patterns as in Figure 1.b. These zig-zag patterns cause broadening of read-back pulses and since they are stochastic in nature, they are the source of random jitter in the read-back pulse.

[†]In a well designed disk, transition noise is the dominant media noise component and often the term 'media noise' is used when referring just to transition noise

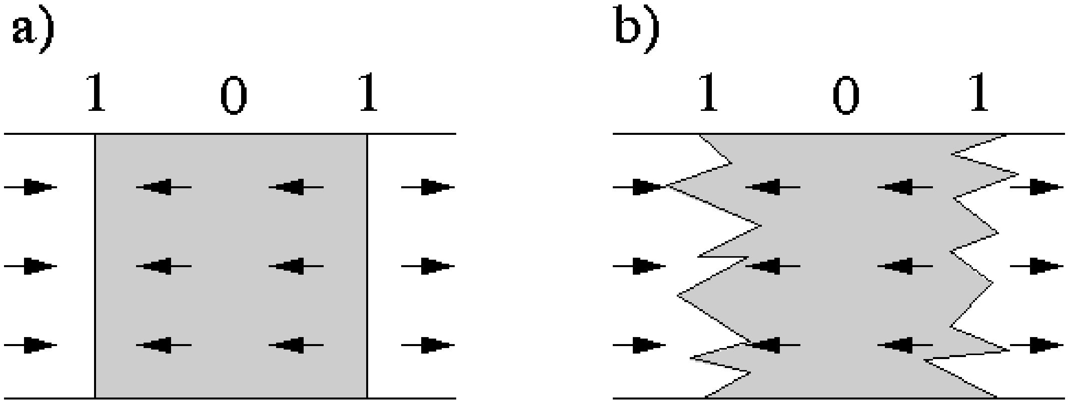

Figure 1: a) Ideal transitions across the track. b) Realistic zig-zag transitions.

Probably the best model for media (transition) noise today is micromagnetic modeling.[9,10] In micromagnetic models, the thin-film magnetic medium is divided into small, close-packed, single-domain grains. Transitions are then generated by applying a write-head field to the medium and minimizing energy terms. A typical example of such a transition is depicted in Figure 2. Although micromagnetic modeling is extremely accurate in predicting magnetization patterns, it is not an ideal tool for statistical studies of read-back systems. Statistical studies, such as error rate predictions, require generation of thousands (even millions) of bits (transitions). Having in mind that generating a single micromagnetic transition on a media sample of 50×50 grains takes about 10-20 minutes on an average workstation, it is clear why micromagnetic modeling is not exactly suited for this purpose.

Statistical zig-zag models, on the other hand, are an attractive alternative. Instead of modeling the whole media plane, these models try to capture the statistical essence of the random zig-zag line (wall) that separates oppositely magnetized regions of the media. Arnoldussen and Tong[11] used Lorentz microscopy pictures to experimentally obtain the probability density function (pdf) of peak-to-peak distances of individual zig-zags and the pdf of zig-zag vertex angles. Middleton and Miles[12] fixed the vertex angle to be a constant and derived the pdf of peak-to-peak distances of zig-zags from the hysteresis loop. If the peak-to-peak pdf is used to generate independent peak-to-peak distances, the resulting zig-zag line will exhibit a drift because it is an independent increment random process. One way to fix this is to use deconvolution of the peak-to-peak pdf to obtain the zero-to-peak pdf, by Tong and Osse.[13] This method, however, does not work with the pdf obtained by Middleton and Miles[‡]. In our TZ-ZT model, we shall use zero-to-peak distances (zig-zag triangle heights) as independent random variables. We shall also fix the vertex angle to be a constant in such a way that the cross-track correlation width (a measure that is proportional to the amount of jitter noise in the read-back signal) of the considered thin-film magnetic medium is preserved. Thus, we not only obtain a stable (non-independent increment) random process as our zig-zag line, but we can also exploit the geometry of the model to find analytic expressions relating the pdf of zig-zag triangle heights to the average transition profile and linking the cross-track correlation width to the probability distributions of the triangle heights. Also, the results of Middleton and Miles[12] can be modified to deliver triangle heights pdf for the TZ-ZT model, which solves the instability issue[14] (experimental verification can be found in the work by Sacks[15]).

[‡]Middleton and Miles[12] use centered peak-to-peak distances, such that their center lies on the nominal transition center location.

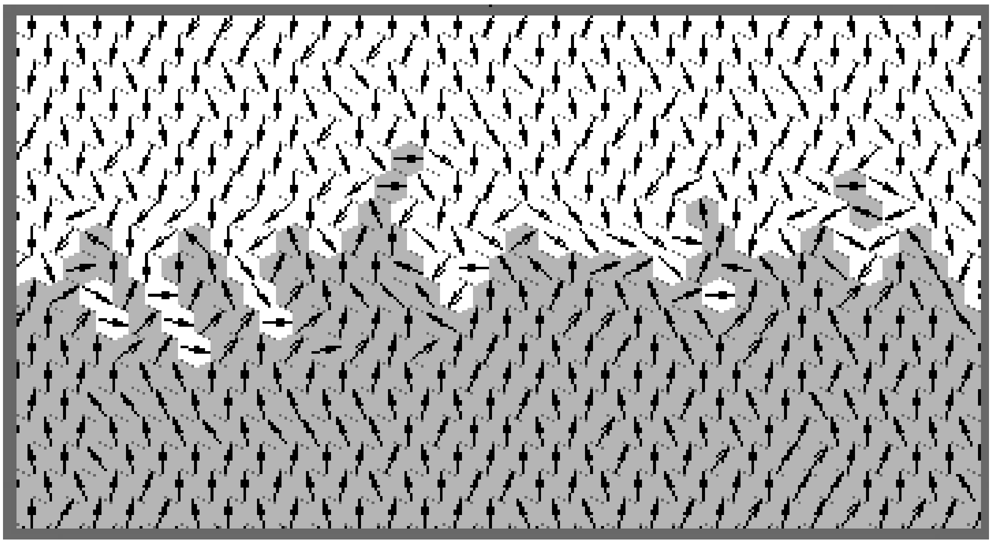

Figure 2: Sample transition generated by micromagnetic simulation.

3 THE TRIANGLE ZIG-ZAG TRANSITION MODEL

The triangle zig-zag transition (TZ-ZT) model is a stochastic model of the zig-zag line that separates two oppositely magnetized regions of the magnetic medium. Figure 3 illustrates the TZ-ZT model. The model is constructed by placing side-by-side isosceles triangles of alternating orientations(one up, the next one down, then up again and so on) on the line representing the nominal transition center (in our case the y-axis in Figure 3). The triangle heights $(h1, h2, \ldots)$ are independent random variables drawn from a pdf $f_H(h)$. The vertex angle θ is chosen to be a constant. In reality, θ is different for different triangles and should be treated as a random variable too. However, the pdf of θ is rather narrow,[16] so considering θ to be a constant is not a bad approximation. Besides, if θ is treated as a constant, it is possible to find an analytic relationship between θ and the media cross-track correlation width, which we actually use to determine θ for a given medium.

As mentioned earlier, the pdf $f_H(h)$ can be obtained by modifying the results of Middleton and Miles[12] as described by Kavčić and Moura[14] and experimentally verified by Sacks.[15] Here, however, we take a different approach in deriving $f_H(h)$. Suppose that we obtained (measured) N $(N \geq 1)$ different magnetization profiles in the along-track x-direction $M_{ix}(x)$ $(1 \leq i \leq N)$. Then we can find the average magnetization profile as

$$M_x(x) = \frac{1}{N}\sum_{i=1}^{N} M_{ix}(x) = \frac{1}{N}\sum_{i=1}^{N}\int_{-TW/2}^{TW/2} M_{ix}(x,y)\,\mathrm{d}y. \tag{1}$$

N is the number of independent transitions created on the same media; TW is the track width; $M_{ix}(x, y)$ is the x-component of the media plane magnetization corresponding to the i-th $(1 \leq i \leq N)$ transition, where y is the cross-track direction. Using the following result, we can obtain $f_H(h)$ from the average magnetization profile $M_x(x)$.

THEOREM 3.1. *Suppose that we have an infinitely wide track ($TW \to \infty$), and that the zig-zag line separating two regions of opposite magnetization is formed by truncating isosceles triangles on a common transition center line as in Figure 3. Then the magnetization profile $M_x(x)$ of an isolated transition on this track is related to the*

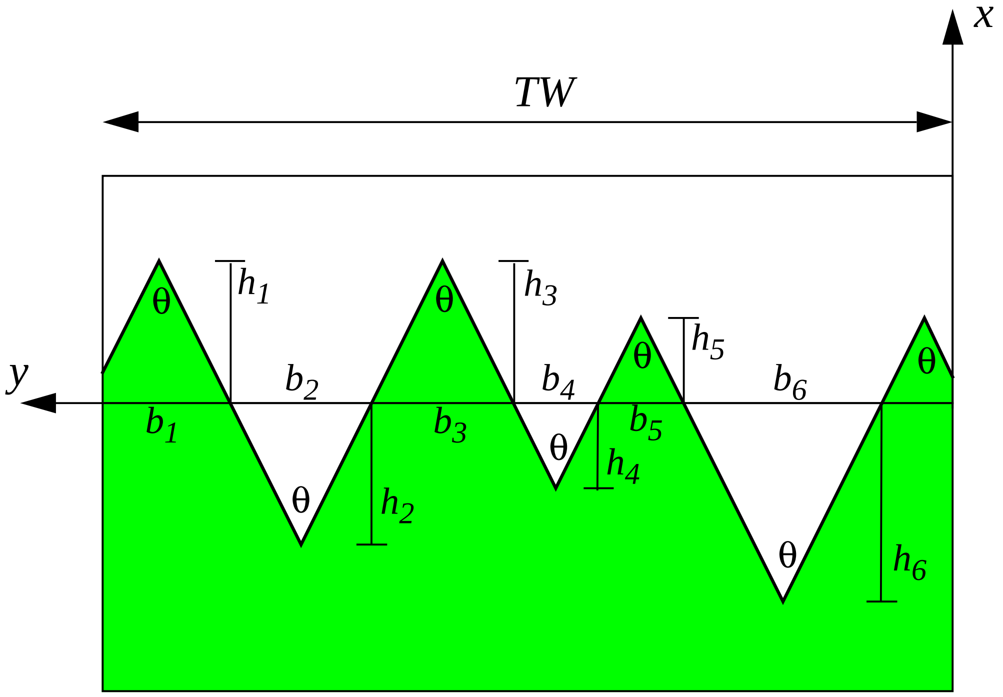

Figure 3: The triangle zig-zag transition model. A sample of a random zig-zag line is shown. The line consists of triangles placed next to each other. the heights of the triangles are independent random variables and the vertex angle θ is constant and the same for all triangles. It is assumed that regions on either side of the zig-zag line are oppositely magnetized.

pdf of the triangle heights $f_H(h)$ through

$$f_H(h) = \begin{cases} -\frac{M_x''(h)}{M_x'(0)} & \text{for } h \geq 0 \\ 0 & \text{for } h < 0 \end{cases} , \tag{2}$$

where $M_x''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left[M_x'(x)\right] = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[M_x(x)\right]$. Notice that here it is assumed that $M_x(x)$ is an odd function and that $M_x(x) > 0$ for $x > 0$ (for example $M_x(x) = M_r \cdot \frac{2}{\pi}\arctan\left(\frac{x}{a}\right)$ or $M_x(x) = M_r \cdot \mathrm{erf}\left(\frac{x}{\sqrt{2}\sigma}\right)$).

The proof of Theorem 3.1 is rather lengthy for this paper, but can be found in full in a separate report.[14] Thus, from (2), we see that, if we have $M_x(x)$, we can easily find the pdf $f_H(h)$.

The second parameter of the TZ-ZT model that we need is the vertex angle θ (the first one was $f_H(h)$). We shall derive the angle θ of the TZ-ZT model such that the cross-track correlation width of the TZ-ZT model equals the cross-track correlation width of the media we are modeling. The cross-track correlation width s is defined as the area under the cross-track correlation function $\rho_Y(y)$,[17]

$$s = \int_{-\infty}^{\infty} \rho_Y(\tau)\,\mathrm{d}\tau, \tag{3}$$

where the cross-track correlation function is[17]

$$\rho_Y(\tau) = \frac{1}{M_r^2}\mathrm{E}\left[M_x(0,y)M_x(0,y-\tau)\right]. \tag{4}$$

Experimentally, we can obtain a value for the cross-track correlation width s_e from the same set of data as in (1). The index 'e' shows that we are dealing with an experimental value. We first find a sample variance $\hat{\sigma}_M^2(x)$ as

$$\hat{\sigma}_M^2(x) = \frac{1}{N-1}\frac{1}{M_r^2}\sum_{i=1}^{N}\left[M_{ix}(x) - M_x(x)\right]^2. \tag{5}$$

Bertram and Che[18] have shown that the variance is

$$\sigma_M^2(x) = \frac{s}{TW}\left[1 - \left(\frac{M_x(x)}{M_r}\right)^2\right]. \tag{6}$$

Thus, we can obtain the value $\frac{s_e}{TW}$ by fitting a curve of the form (6) to the curve of $\hat{\sigma}_M^2(x)$. Knowing the track width TW, we easily get s_e.

The experimental cross-track correlation width s_e needs to be equated to the TZ-ZT cross-track correlation width in order to find an equation that determines θ. Let us first find an expression for the cross-track auto-correlation function of the TZ-ZT model. If we observe the x-component of the magnetization along the line representing the nominal transition location (see Figure 3), we will notice that it is a generalized telegraphic wave[§]. The lengths between switching intervals are the bases of the isosceles triangles and equal (See Figure 3 to verify this relationship.)

$$b_i = 2h_i \tan\frac{\theta}{2}. \tag{7}$$

The pdf of these bases (i.e., the pdf of switching interval lengths) is

$$f_B(b) = \frac{1}{c} f_H\left(\frac{b}{c}\right), \tag{8}$$

where $c = 2\tan\frac{\theta}{2}$ and $f_H(h)$ is given in (2). From the theory of generalized telegraphic waves,[14] we have

THEOREM 3.2. *If $f_B(b)$ is the pdf of the switching intervals of a generalized telegraphic wave, then the autocorrelation function of that telegraphic wave is*

$$\begin{aligned}\rho_B(y) \;=\; & \left[1 - \mathrm{P}\{\chi < y\}\right] - \left[\mathrm{P}\{\chi < y\} - \mathrm{P}\{\chi + b_1 < y\}\right] \\ & + \sum_{k=1}^{\infty}(-1)^{k+1}\left[\mathrm{P}\left\{\chi + \sum_{i=1}^{k} b_i < y\right\} - \mathrm{P}\left\{\chi + \sum_{i=1}^{k+1} b_i < y\right\}\right].\end{aligned} \tag{9}$$

Here, $\mathrm{P}\{A\}$ *denotes the probability of an event A. The random variables b_i ($1 \leq i < \infty$) are independent identically distributed with a pdf $f_B(b)$. χ is a random variable independent of all b_i and has a pdf*

$$f_\chi(\chi) = \frac{1 - F_B(\chi)}{\mathrm{E}(B)}, \tag{10}$$

where $\mathrm{F}_B(\chi) = \int_0^\chi f_B(b)\,\mathrm{d}b$ *and* $\mathrm{E}(B) = F_B(\infty)$.

Again, the proof of Theorem 3.2 is lengthy. It can be found in full in.[14]

We now observe that the TZ-ZT cross-track correlation function $\rho_Y(y)$ is actually $\rho_B(y)$, i.e.

$$\rho_Y(y) = \rho_B(y) \tag{11}$$

[§]A generalized telegraphic signal is a square wave whose distances between switching intervals are distributed according to a positive-sided pdf. This is a generalization of a classic telegraph wave whose switching interval pdf is a positive sided exponential.

because the magnetization pattern across the track is a generalized telegraphic wave. On the other hand, from (7), we have

$$\rho_B(y) = \rho_H\left(\frac{y}{2\tan\frac{\theta}{2}}\right), \tag{12}$$

where $\rho_H(y)$ can be obtained by substituting H for B, and h for b respectively in the text of Theorem 3.2 and in equations (9) and (10). Thus, from (11) and (12), we have for our TZ-ZT model

$$\rho_Y(y) = \rho_H\left(\frac{y}{2\tan\frac{\theta}{2}}\right) \tag{13}$$

If we integrate $\rho_Y(y)$, we obtain the cross-track correlation width $s_{\mathrm{TZ-ZT}}$ of the TZ-ZT model

$$\begin{aligned} s_{\mathrm{TZ-ZT}} &= \int_{-\infty}^{\infty} \rho_Y(y)\,\mathrm{d}y = \int_{-\infty}^{\infty} \rho_H\left(\frac{y}{2\tan\frac{\theta}{2}}\right)\,\mathrm{d}y \\ &= 2\tan\frac{\theta}{2}\int_{-\infty}^{\infty} \rho_H(y)\,\mathrm{d}y. \end{aligned} \tag{14}$$

Equating (14) with the experimentally obtained s_e for the media we are modeling, we find an equation for θ

$$\theta = 2\arctan\left[\frac{s_e}{2\int_{-\infty}^{\infty}\rho_H(y)\,\mathrm{d}y}\right]. \tag{15}$$

Notice that all quantities on the right hand side of (15) are known. The parameter s_e is the experimentally obtained cross-track correlation width. The function $\rho_H(y)$ is the autocorrelation function of a generalized telegraphic wave (Theorem 3.2) whose switching intervals have a pdf $f_H(h)$, and $f_H(h)$ is known from (2).

4 NOISE MODELING RESULTS

In this section we compare the media noise results obtained by TZ-ZT modeling to those obtained by micromagnetic modeling. We modeled a magnetic thin-film with the following characteristics: remanent magnetization M_r = 625emu/cc, coercivity H_c = 1670Oe, media thickness δ = 400Å, orientation ratio O.R.= 1.3. The track width chosen was TW = 4.8μm. We used a Karlqvist inductive head[19] for both reading and writing, with the following parameters: gap length g = 0.28μm, flying height (magnetic spacing) d = 0.1μm. The track width is a bit high for systems used today but we still chose to show results for this media because experimental results for a medium similar (virtually identical) to this one are documented by Zhu and Wang.[20] In this way, the TZ-ZT results are not confined to comparison to the micromagnetic modeling, but can be compared to experimental findings as well.

First, via micromagnetic modeling, we obtained N = 500 independent magnetization transition profile samples (normalized to M_r) $M_{ix}(x)$ $(0 \le i \le N)$. The 500 runs of the micromagnetic model took a week. Their average is

$$M_x(x) = \frac{1}{N}\sum_{i=1}^{N} M_{ix}(x). \tag{16}$$

It is plotted in Figure 4 with a solid line. It can easily be verified that

$$M_x(x) \approx \mathrm{erf}\left(\frac{x}{\sqrt{2}\sigma}\right), \tag{17}$$

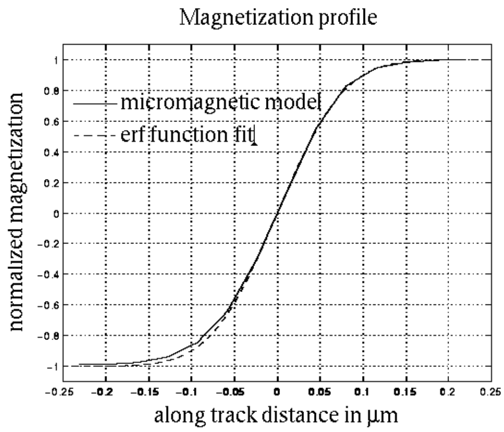

Figure 4: Average micromagnetic magnetization profile (solid line), and fitted error function (erf) profile (dashed line).

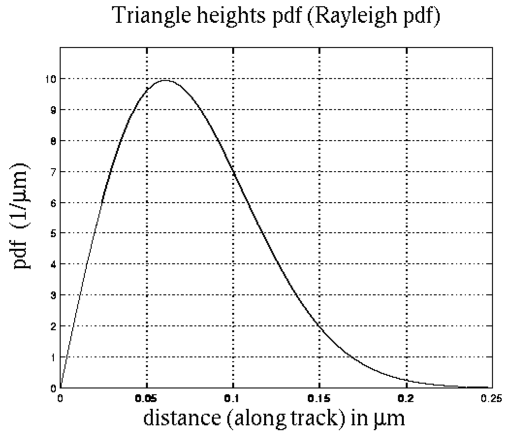

Figure 5: Probability density function (pdf) of the zig-zag triangle heights - Rayleigh pdf obtained as the second derivative of an error function (erf).

where $\sigma = 610$Å (see dashed line in Figure 4). Applying Theorem 3.1 to (17), we find that the triangle heights pdf $f_H(h)$ is a Rayleigh pdf (Figure 5)

$$f_H(h) = \begin{cases} -\frac{M_x''(h)}{M_x'(0)} \approx \frac{h}{\sigma^2}\exp\left(-\frac{h^2}{2\sigma^2}\right) & \text{for } h \geq 0 \\ 0 & \text{for } h < 0 \end{cases}. \tag{18}$$

Comparing the Rayleigh pdf to those experimentally obtained,[11,12] we find that they are similar in form.

We next find the vertex angle θ. First we obtain a sample variance

$$\hat{\sigma}_M^2(x) = \frac{1}{N-1}\sum_{i=1}^{N}\left[M_{ix}(x) - M_x(x)\right]^2. \tag{19}$$

We next plot the curve of $\hat{\sigma}_M^2(x)$ versus $M_x(x)$ in Figure 6, solid line. By least-squares-fitting a parabola of the form $\sigma_M^2 = \frac{s_e}{TW}\left(1 - M^2\right)$, (dashed line in Figure 6), to the curve of $\hat{\sigma}_M^2(x)$ versus $M_x(x)$, we obtain a value for the cross-track correlation width s_e. For the media we are considering here, the least-square fit evaluates $s_e = 197$Å. Utilizing Theorem 3.2, by Monte Carlo simulation¶, we found $\rho_H(y)$ to be as in Figure 7. Notice that $\rho_H(y)$ has a form similar to Arnoldussen's and Tong's cross-track correlation function,[11] except that their autocorrelation function is much noisier since it was obtained experimentally from only 5 sample zig-zag photographs. Now that we have $\rho_H(y)$, we find by numerical integration that

$$\int_{-\infty}^{\infty} \rho_H(y)\,\mathrm{d}y = 207.7\text{Å}. \tag{20}$$

Substituting this into (15) and using $s_e = 197$Å, we find $\theta = 50.7°$.

Having obtained both $f_H(h)$ and θ, we are ready to run the TZ-ZT model. Using the above determined parameters, we created 50000 transitions on a track of track width $TW = 4.8\mu$m using the TZ-ZT model‖. The

¶The probability expressions in (9) are hard to derive analytically, but are easily obtained via Monte Carlo simulation.

‖Notice that we are now able to create many more transitions than with the micromagnetic model due to lower computational complexity.

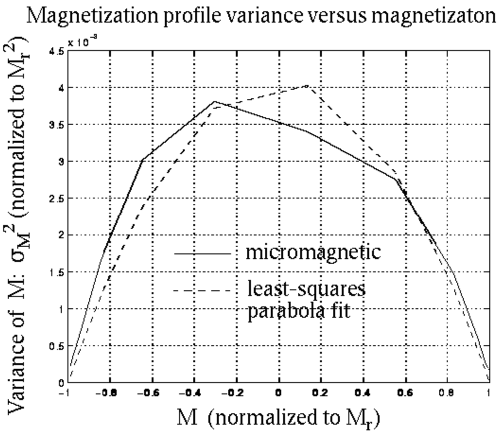

Figure 6: Magnetization variance σ_M^2 versus normalized magnetization M. Solid line - micromagnetic results. Dashed line - least-squares fitted parabola.

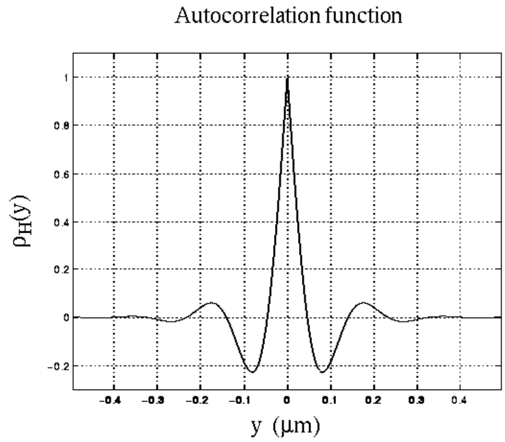

Figure 7: Autocorrelation function of a generalized telegraphic wave whose switching intervals are Rayleigh distributed. Due to Thorem 3.2, this is also the form of the cross-track autocorrelation function of a transition generated by the TZ-ZT model.

average of all 50000 TZ-ZT transitions is compared to the average of 500 micromagnetic transitions in Figure 8, and close agreement can be observed. We next computed the zero-crossing point for each transition created by the micromagnetic model and by the TZ-ZT model, and plotted the distributions (normalized histograms) of zero-crossings for both models in Figure 9. Again, close agreement can be observed. Notice that the histogram for the micromagnetic model is not as smooth as the TZ-ZT histogram. This is a consequence of using only 500 samples to obtain the micromagnetic model histogram. These histograms are actually experimental jitter pdfs which show the jitter distribution. Thus, we see that the jitter (which is how media noise exhibits itself in the read-back pulse) is accurately predicted by the TZ-ZT model.

5 CONCLUSION

In this paper, we have presented a new, computationally efficient write process model for longitudinal magnetic recording. In the model, written magnetization transitions are represented as portions of zig-zag lines across the track width. The zig-zag line is a random process obtained by truncating isosceles triangles of alternating orientations on a common basis line. Hence we named it the Triangle Zig-Zag Transition (TZ-ZT) model. The random zig-zag line is determined by two parameters: the probability density function of triangle heights, and the constant vertex angle. We provided two simple relationships relating the parameters of the TZ-ZT model to measurable properties of magnetic media. Thus, the triangle probability density function is proportional to the second derivative of the average magnetization profile of an isolated transition. The vertex angle is related to the cross-track correlation width of the thin film medium and to the autocorrelation function of an associated generalized random telegraphic signal through a simple trigonometric relationship. In simulation studies, the magnetization profile and the cross-track correlation width can be estimated with fair accuracy by writing a few (20-30) micromagnetic transitions on a desired medium. We have tested the TZ-ZT model in comparison to micromagnetic modeling. Our results reveal that the TZ-ZT model predicts magnetization profile shapes and jitter media noise as accurately as the micromagnetic model. The computational complexity of the TZ-ZT model is 4 orders of magnitude lower than that of the micromagnetic model, making the TZ-ZT model attractive for

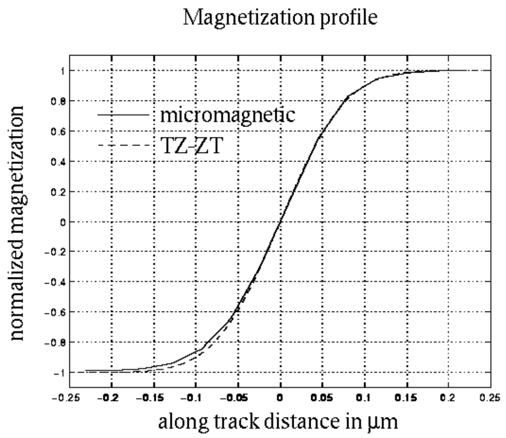

Figure 8: Average magnetization profiles. Solid line - micromagnetic model. Dashed line - TZ-ZT.

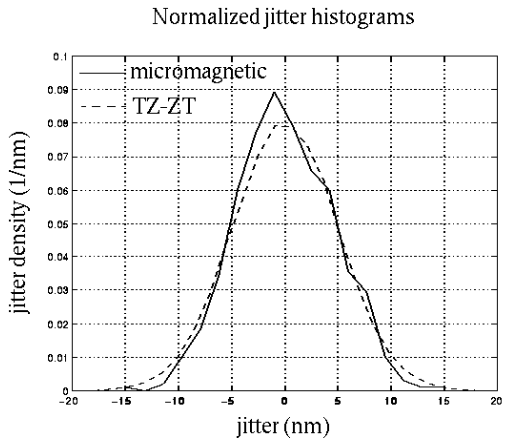

Figure 9: Jitter histograms (normalized to unit area) - experimental jitter pdfs. Solid line - micromagnetic model. Dashed line - TZ-ZT

statistical studies such as error rate performance analyses.

6 ACKNOWLEDGMENTS

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