EE 342

HW3 - SOLUTIONS

1)

$\boxed{a}$ CH 3, PROBLEM 29

LET'S NAME EVENTS CORRESPONDING TO FIRST DRAW

A_0 = {AMONG THE 3 CHOSEN BALLS, 0 ARE NEW}

A_1 = {AMONG THE 3 CHOSEN BALLS, 1 IS NEW}

A_2 = {AMONG THE 3 CHOSEN BALLS, 2 ARE NEW}

A_3 = {AMONG THE 3 CHOSEN BALLS, ALL 3 ARE NEW}

$$P(A_0) = \frac{\binom{6}{3}}{\binom{15}{3}} = 0.044 \qquad P(A_1) = \frac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = 0.2967$$

$$P(A_2) = \frac{\binom{6}{1}\binom{9}{2}}{\binom{15}{3}} = 0.4747 \qquad P(A_3) = \frac{\binom{9}{3}}{\binom{15}{3}} = 0.1846$$

NOW, LET'S NAME EVENTS CORRESPONDING TO THE SECOND DRAW

B_3 = {ALL 3 CHOSEN BALLS ARE NEW}

$$P(B_3|A_0) = \frac{\binom{9}{3}}{\binom{15}{3}} = 0.1846 \qquad P(B_3|A_1) = \frac{\binom{8}{3}}{\binom{15}{3}} = 0.1231$$

$$P(B_3|A_2) = \frac{\binom{7}{3}}{\binom{15}{3}} = 0.0769 \qquad P(B_3|A_3) = \frac{\binom{6}{3}}{\binom{15}{3}} = 0.0440$$

$$P(B_3) = P(B_3|A_0)P(A_0) + P(B_3|A_1)P(A_1) + P(B_3|A_2)P(A_2) + P(B_3|A_3)\cdot P(A_3)$$

$$= 0.1846 \times 0.044 + 0.1231 \times 0.2967 + 0.0769 \times 0.4747 + 0.044 \times 0.1846 = \underline{0.0893}$$

b CH3, PROBLEM 60

a) SINCE SMITH'S SISTER HAS BLUE EYES,
AND HER PARENTS HAVE BROWN EYES,
BOTH PARENTS MUST HAVE 1 Be-EYED GENE AND
1 Br-EYED GENE

SMITH'S MOM (Be, Br)

SMITH'S DAD (Be, Br)

SMITH

POSSIBILITIES FOR SMITH'S GENE PAIRS:

~~(Be, Be)~~ ← IMPOSSIBLE BECAUSE SMITH HAS BROWN EYES

(Be, Br)
(Br, Be)
(Br, Br)

IN 2 OUT OF THESE 3 POSSIBLE OUTCOMES SMITH POSSESSES A Be-EYED GENE

So, $\boxed{P(\text{Smith Possesses Be-eyed Gene}) = \frac{2}{3}}$

b) IF THE FIRST CHILD IS TO HAVE BLUE EYES, THEN SMITH MUST PASS A Be-EYED GENE TO THE CHILD. DEFINE THE FOLLOWING EVENTS

$X = \{\text{Smith passes Be-eyed gene}\}$

$Y = \{\text{Smith possesses Be-eyed gene}\}$

we need to find $P(X)$

$$P(X) = P(X|Y) \cdot P(Y) + P(X|\bar{Y}) \cdot P(\bar{Y})$$

$$= P(X|Y) \cdot \frac{2}{3} + P(X|\bar{Y}) \cdot \frac{1}{3}$$

from part a)

- NOW, IF SMITH POSSES A B_l-EYED GENE, THEN HIS GENE PAIR IS EITHER (B_l, B_r) OR (B_r, B_l). IN EITHER CASE, THE PROBABILITY OF PASSING ON A B_l-EYED GENE IS $\frac{1}{2}$. SO,

$$\boxed{P(X|Y) = \frac{1}{2}}$$

- ON THE OTHER HAND, IF SMITH DOES NOT POSSESS A B_l-EYED GENE, THEN HE CANNOT PASS IT, SO

$$\boxed{P(X|\bar{Y}) = 0}$$

HENCE:

$$P(X) = \frac{1}{2} \cdot \frac{2}{3} + 0 \cdot \frac{1}{3} \Rightarrow \boxed{P(X) = \frac{1}{3}}$$

c) LET'S FIRST NAME SOME EVENTS

Y = {SMITH POSSESSES B_l-EYED GENE}

Z_1 = {SMITH PASSES B_r-EYED GENE TO FIRST CHILD}

Z_2 = {SMITH PASSES B_r-EYED GENE TO SECOND CHILD}

SINCE SMITH'S WIFE HAS BLUE EYES, THE CHILDREN WILL HAVE BROWN EYES ONLY IF SMITH PASSES ON B_r-EYED <u>GENES. SO</u>, WE NEED TO FIND

$$P(Z_2 | Z_1)$$

$$P(Z_2|Z_1) = \frac{P(Z_1 \cap Z_2)}{P(Z_1)}$$

- LET'S FIRST FIND $P(Z_1)$

$$P(Z_1) = P(Z_1|Y)\cdot P(Y) + P(Z_1|\bar{Y})\cdot P(\bar{Y})$$

$$= P(Z_1|Y)\cdot \frac{2}{3} + P(Z_1|\bar{Y})\cdot \frac{1}{3}$$

$P(Z_1|Y) = \frac{1}{2}$

$P(Z_1|\bar{Y}) = 1$

$$= \frac{1}{2}\cdot\frac{2}{3} + 1\cdot\frac{1}{3}$$

$$\boxed{P(Z_1) = \frac{2}{3}}$$

We could also have gotten this from part b) because $P(Z_1) = P(\bar{X})$
$= 1 - P(X)$

- LET'S NOW FIND $P(Z_1 Z_2) = P(Z_1 \cap Z_2)$

THIS IS THE PROBABILITY THAT SMITH PASSES THE Br-EYED GENE TO BOTH CHILDREN

$$P(Z_1 Z_2) = P(Z_1 Z_2|Y)\cdot P(Y) + P(Z_1 Z_2|\bar{Y})\cdot P(\bar{Y})$$

$$= \left(\frac{1}{2}\right)^2 \cdot \frac{2}{3} + (1)^2 \cdot \frac{1}{3}$$

$$\boxed{P(Z_1 Z_2) = \frac{1}{2}}$$

FINALLY, $\boxed{P(Z_2|Z_1) = \frac{P(Z_1 Z_2)}{P(Z_1)} = \frac{3}{4}}$

$\boxed{C}$ CH 3, PROB. 63

LET'S FIRST NAME EVENTS

$R_n = \{\text{ROUND } n \text{ TAKES PLACE}\} \Rightarrow P(R_n) = \left[(1-p_A)(1-p_B)\right]^{n-1}$

$A_n = \{A \text{ IS HIT IN ROUND } n\} \Rightarrow P(A_n|R_n) = p_B$

$B_n = \{B \text{ IS HIT IN ROUND } n\} \Rightarrow P(B_n|R_n) = p_A$

$P(A_n B_n | R_n) = p_A p_B$

a) $P(\{A \text{ not hit}\}) = P(\{A \text{ not hit}\} \cap \{B \text{ is hit}\})$

$$= \sum_{n=1}^{\infty} P(\{A \text{ not hit}\} \cap \{B \text{ is hit}\} | R_n) \cdot P(R_n)$$

$$= \sum_{n=1}^{\infty} P(\bar{A}_n B_n | R_n) \cdot P(R_n)$$

$$= \sum_{n=1}^{\infty} (1-p_B) \cdot p_A \cdot \left[(1-p_A)(1-p_B)\right]^{n-1}$$

$$= (1-p_B) p_A \sum_{n=1}^{n} \left[(1-p_A)(1-p_B)\right]^{n-1}$$

$$= (1-p_B) p_A \cdot \frac{1}{1-(1-p_A)(1-p_B)}$$

$$\boxed{P(\{A \text{ not hit}\}) = \frac{p_A - p_A p_B}{p_A + p_B - p_A p_B}}$$

b) $P(\{A \text{ hit}\} \cap \{B \text{ hit}\}) = \sum_{n=1}^{\infty} P(A_n B_n | R_n) \cdot P(R_n)$

$$= \sum_{n=1}^{\infty} p_A \cdot p_B \cdot \left[(1-p_A)(1-p_B)\right]^{n-1}$$

$$\boxed{P(\{A \text{ hit}\} \cap \{B \text{ hit}\}) = \frac{p_A p_B}{p_A + p_B - p_A p_B}}$$

c) $P(A_n \cup B_n | R_n) \cdot P(R_n) = P((A_n \cup B_n) R_n)$

probability that Round n takes place

Probability that at least one is hit in round n, given that round n takes place

THE DUEL ENDS IN ROUND n ONLY IF BOTH OF THESE EVENTS TAKE PLACE

$$P(A_n \cup B_n | R_n) P(R_n) = [P(A_n \bar{B}_n) + P(\bar{A}_n B_n) + P(A_n B_n)] \cdot P(R_n)$$

$$= [p_B(1-p_A) + p_A(1-p_B) + p_A p_B] \cdot [(1-p_A)(1-p_B)]^{n-1}$$

$$= [p_A + p_B - p_A p_B] \cdot [(1-p_A)(1-p_B)]^{n-1}$$

d) We need to find

$$P((A_n \cup B_n) R_n | \{A \text{ not Hit}\}) = \frac{P((A_n \cup B_n) R_n \{A \text{ not Hit}\})}{P(\{A \text{ not Hit}\})}$$

$$P(\{A \text{ not Hit}\}) = \frac{p_A - p_A p_B}{p_A + p_B - p_A p_B} \quad \text{(see part a)}$$

$$P((A_n \cup B_n) R_n \{A \text{ not Hit}\}) = P(R_n \{B \text{ hit}, A \text{ not hit}\})$$

$$= (1-p_A)^{n-1}(1-p_B)^{n-1} \cdot p_A \cdot (1-p_B)$$

Finally

$$P((A_n \cup B_n) R_n | \{A \text{ not hit}\}) = (1-p_A)^{n-1}(1-p_B)^{n-1}(p_A + p_B - p_A p_B)$$

e) Similarly to part d), we need to find

$$P((A_n \cup B_n) R_n \mid \{\text{Both hit}\})$$

$$= \frac{P((A_n \cup B_n) R_n \{\text{Both hit}\})}{P(\{\text{Both hit}\})}$$

$$= \frac{(1-p_A)^{n-1}(1-p_B)^{n-1} p_A p_B}{\frac{p_A p_B}{p_A + p_B - p_A p_B}}$$

$$= (1-p_A)^{n-1}(1-p_B)^{n-1}(p_A + p_B - p_A p_B)$$

$\boxed{d}$ CH 3, PROBLEM 64

a) IF EITHER OF THE TWO IS CHOSEN TO ANSWER THE QUESTION, THE PROBABILITY OF GIVING THE RIGHT ANSWER IS

$$\boxed{P_a = p}$$

b) LET'S NAME THE FOLLOWING EVENT

$$C = \{\text{two answers coincide}\}$$

$$P_b = P(\{\text{correct answer}\})$$

$$= P(\{\text{correct answer}\} | C)\, P(C) + P(\{\text{correct answer}\} | \bar{C}) \cdot P(\bar{C})$$

$$P(C) = p^2 + (1-p)^2$$

$$P(\bar{C}) = 2p(1-p)$$

$$P_b = P(\{\text{correct answer}\} | C) \cdot [p^2 + (1-p)^2] + P(\{\text{correct answer}\} | \bar{C}) \cdot 2p(1-p)$$

- LET'S COMPUTE $P(\{\text{correct answer}\} | C)$

$$P(\{\text{correct answer}\} | C) = \frac{P(\{\text{correct answer}\} \cap C)}{P(C)}$$

$$\boxed{P(\{\text{correct answer}\} | C) = \frac{p^2}{p^2 + (1-p)^2}}$$

– NEXT COMPUTE $P(\{\text{correct answer}\} \mid \bar{C})$

$$P(\{\text{correct answer}\} \mid \bar{C}) = \frac{1}{2}$$

because it is chosen by a coin toss

FINALLY

$$P_b = \frac{p^2}{[p^2 + (1-p)^2]} \cdot [p^2 + (1-p)^2] + \frac{1}{2}[2 \cdot p(1-p)]$$

$$P_b = p^2 + p(1-p)$$

$$\boxed{P_b = p}$$

SINCE $P_a = P_b = p$, THE TWO STRATEGIES ARE EQUALLY GOOD, SO WE CAN CHOOSE ANY ONE OF THE TWO!

$\boxed{e}$ CH 3., TH. EXCERCISE 3

LET'S FIRST PROVE THE INEQUALITY IN THE HINT:

$$\sum_{i=1}^{k} i n_i \sum_{j=1}^{k} \frac{n_j}{j} \geq \sum_{i=1}^{k} n_i \sum_{j=1}^{k} n_j$$

NOW NOTICE $\displaystyle\sum_{i=1}^{k} i n_i \sum_{j=1}^{k} \frac{n_j}{j} = \sum_{i=1}^{k} n_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{i n_i n_j}{j} + \frac{j n_j n_i}{i} \right)$

ALSO NOTICE $\displaystyle\sum_{i=1}^{k} n_i \sum_{j=1}^{k} n_j = \sum_{i=1}^{k} n_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (n_i n_j + n_j n_i)$

SO, EQUIVALENTLY WE NEED TO PROVE

$$\sum_{i=1}^{k} n_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{i n_i n_j}{j} + \frac{j n_j n_i}{i} \right) \geq \sum_{i=1}^{k} n_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (n_i n_j + n_j n_i)$$

AND THIS IS EASY BECAUSE

$$\underline{\frac{i n_i n_j}{j} + \frac{j n_j n_i}{i} \geq n_i n_j + n_j n_i} \qquad \text{AND} \qquad \underline{n_i^2 \geq n_i^2}$$

$$\frac{i}{j} + \frac{j}{i} \geq 2$$

$$\frac{i^2 + j^2}{ij} \geq 2$$

$$i^2 + j^2 \geq 2ij$$

$$i^2 - 2ij + j^2 \geq 0$$

$$(i-j)^2 \geq 0$$

NOW, LET'S REWRITE THE PROVED INEQUALITY AS

$$\left(\sum_{i=1}^{k} i n_i\right)\left(\sum_{j=1}^{k} \frac{n_j}{j}\right) \geq \underbrace{\left(\sum_{i=1}^{k} n_i\right)}_{m}\underbrace{\left(\sum_{j=1}^{k} n_j\right)}_{m} = m^2$$

OR EQUIVALENTLY

$$\boxed{\sum_{i=1}^{k} \frac{1}{i} \cdot \frac{n_i}{m} \geq \frac{m}{\sum_{i=1}^{k} i \cdot n_i}} \quad (\text{!})$$

1) LET'S FIND THE PROBABILITY OF CHOOSING A FIRSTBORN UNDER THE FIRST METHOD

$F_1 = \{$CHOOSE FIRSTBORN UNDER METHOD 1$\}$

$N_i = \{$CHOOSE FAMILY OF i CHILDREN$\}$

$$P(F_1) = \sum_{i=1}^{k} P(F_1 | N_i) \cdot P(N_i)$$

WITH $P(F_1|N_i) = \frac{1}{i}$

AND $P(N_i) = \frac{n_i}{m}$

$$\boxed{P(F_1) = \sum_{i=1}^{k} \frac{1}{i} \cdot \frac{n_i}{m}}$$

2) LET'S FIND THE PROBABILITY OF CHOOSING A FIRSTBORN UNDER THE SECOND METHOD

\# OF CHILDREN $\Rightarrow \sum_{i=1}^{k} i \cdot n_i$

\# OF FIRSTBORNS $\Rightarrow \sum_{i=1}^{k} n_i = m$

$$\Rightarrow \boxed{P(F_2) = \frac{m}{\sum_{i=1}^{k} i \cdot n_i}}$$

NOW APPLY INEQUALITY (!) AND WE GET $\boxed{P(F_1) \geq P(F_2)}$

$\boxed{f}$ CH3, TH. EXCERCISE 9

WE NEED TO SHOW

i) $P(AB) = P(A)P(B)$

ii) $P(AC) = P(A) \cdot P(C)$

iii) $P(BC) = P(B) \cdot P(C)$

BUT

iv) $P(ABC) \neq P(A)P(B)P(C)$

So, FIRST VERIFY $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{1}{2}$

Now, AB IS THE EVENT $\{(H,H)\}$ WHOSE PROBABILITY IS $P(AB) = \frac{1}{4}$

AC IS THE EVENT $\{(H,H)\}$ WHOSE PROBABILITY IS $P(AC) = \frac{1}{4}$

BC IS THE EVENT $\{(H,H)\}$ WHOSE PROBABILITY IS $P(BC) = \frac{1}{4}$

So, CLEARLY

$$\boxed{\frac{1}{4} = P(AB) = P(BC) = P(AC) = P(A)P(B) = P(B)P(C) = P(A)P(C)}$$

NEXT NOTICE $ABC = \{(H,H)\}$ SO $P(ABC) = \frac{1}{4}$

BUT $P(A)P(B)P(C) = \frac{1}{8}$

So $\boxed{\frac{1}{4} = P(ABC) \neq P(A)P(B)P(C) = \frac{1}{8}}$

9 CH3, TH. EXCERCISE 10

a) LET ALL 4 PEOPLE BE DISTINCT, $i \neq j$ $j \neq s$ $r \neq s$
$i \neq s$ $j \neq r$
$i \neq r$

THEN $P(A_{ij} A_{rs}) = P\left\{\begin{array}{l}\text{persons } i \,\&\, j \text{ have same b-day and} \\ \text{persons } r \,\&\, s \text{ have same b-day}\end{array}\right\}$

\# of ways i&j have same b-day and r&s have same b-day $= (365)^2$

\# of b-day 4-tuples $= (365)^4$

$$\Rightarrow P(A_{ij} A_{rs}) = \frac{(365)^2}{(365)^4} = \left(\frac{1}{365}\right)^2 = P(A_{ij}) \cdot P(A_{rs})$$

b) LET i, j, r, s CORRESPOND TO 3 DISTINCT PEOPLE

FOR EXAMPLE $i \neq j = r \neq s \neq i$

THEN $P(A_{ij} A_{rs}) = P(A_{ij} A_{js}) = P\left\{\begin{array}{l}\text{Persons } i, j, s \text{ have} \\ \text{the same b-day}\end{array}\right\}$

\# OF WAYS i&j&s have same b-day $= 365$

\# OF b-day 3-tuples $= (365)^3$

$$\Rightarrow P(A_{ij} A_{js}) = \frac{365}{(365)^3} = \left(\frac{1}{365}\right)^2 = P(A_{ij}) \cdot P(A_{js})$$

$$\boxed{\text{So } P(A_{ij} A_{rs}) = P(A_{ij}) P(A_{rs}) \quad \text{FOR ALL } A_{ij} \neq A_{rs}}$$

Pairwise independence holds

NOW VERIFY $P(A_{12} A_{23} A_{13}) = P\{\text{Persons } 1,2,3 \text{ have same b-day}\} = \dfrac{365}{(365)^3}$

BUT $P(A_{12}) P(A_{23}) \cdot P(A_{13}) = \left(\frac{1}{365}\right)^3$

SO $\boxed{\text{FOR } n=3 \text{ WE HAVE } P(A_{12} A_{23} A_{13}) \neq P(A_{12}) P(A_{23}) P(A_{13})}$

THIS ONE COUNTEREXAMPLE IS ENOUGH TO PROVE THAT $\binom{n}{2}$ EVENTS ARE NOT INDEPENDENT h

LET H_1 DENOTE {HIT FIRST TARGET}
LET H_2 DENOTE {HIT SECOND TARGET}

LET'S LIST ALL OUTCOMES OF AT MOST 3 SHOTS

$(\bar{H}_1, \bar{H}_1, \bar{H}_1)$

$(\bar{H}_1, \bar{H}_1, H_1)$

$(\bar{H}_1, H_1, \bar{H}_2)$

$(\bar{H}_1, H_1, H_2) \quad P(\bar{H}_1, H_1, H_2) = \left(\frac{1}{3}\right)\cdot\left(\frac{2}{3}\right)^2$

$(H_1, \bar{H}_2, \bar{H}_2)$

$(H_1, \bar{H}_2, H_2) \quad P(H_1, \bar{H}_2, H_2) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$

$(H_1, H_2, -) \quad P(H_1, H_2, -) = \left(\frac{2}{3}\right)^2$

OUTCOMES THAT RESULT IN THE 2ND TARGET BEING HIT

$P(\text{target 2 hit in at most 3 shots}) =$

$$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = \frac{4}{27} + \frac{4}{27} + \frac{12}{27} = \frac{20}{27}$$

$$P(\text{target 2 missed in at most 3 shots}) = 1 - \frac{20}{27} = \boxed{\frac{7}{27}}$$

i

a) Let D_i denote event $D_i = \{i\text{-th guess correct}\}$

$$P\left(\begin{smallmatrix}\text{guess correctly in}\\ \text{at most 3 trials}\end{smallmatrix}\right) = P(D_1) + P(D_2 | \bar{D}_1)P(\bar{D}_1) + P(D_3 | \bar{D}_1\bar{D}_2)P(\bar{D}_1\bar{D}_2)$$

$$= \frac{1}{10} + \frac{1}{9}\cdot\frac{9}{10} + \frac{1}{8}\cdot\frac{8}{9}\cdot\frac{9}{10}$$

$$= \frac{3}{10} \;//$$

b) $$P\left(\begin{smallmatrix}\text{guess correctly}\\ \text{in 3 trials}\end{smallmatrix}\right) = P(D_1) + P(D_2|\bar{D}_1)P(\bar{D}_1) + P(D_3 | \bar{D}_1\bar{D}_2)P(\bar{D}_1\bar{D}_2)$$

$$= \frac{1}{5} + \frac{1}{4}\cdot\frac{4}{5} + \frac{1}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}$$

$$= \frac{3}{5} \;//$$

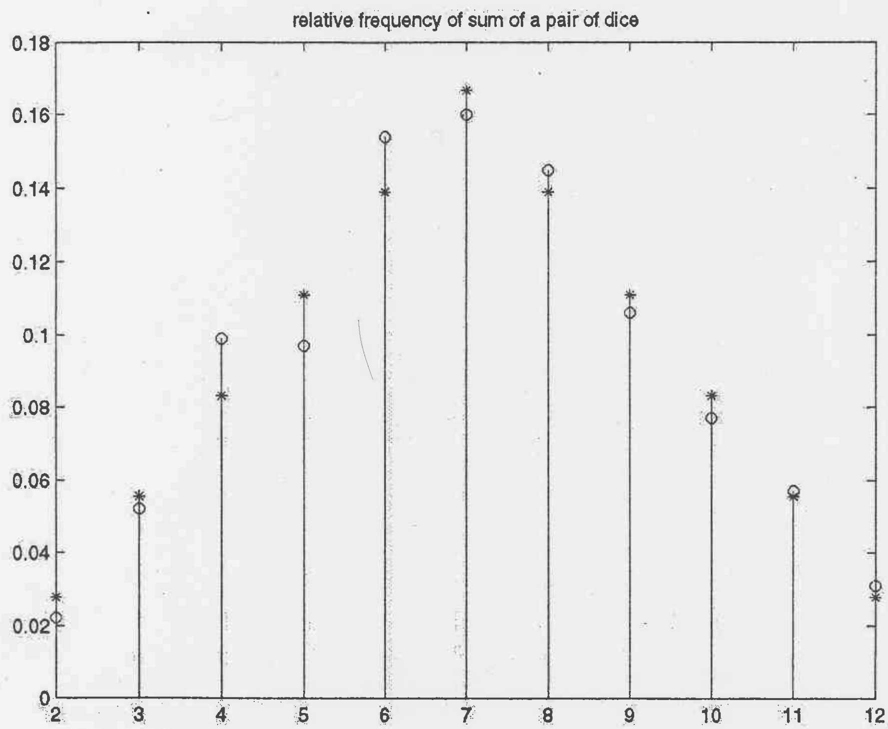
18

2a) 1000 random numbers uniform distributed between 0 and 1 are generated by
>>rand(1,1000);
Two sets of 1000 random numbers were generated with the following statistics.

Data	Mean	Deviation	Min	Max
X1	.5005	.2885	.00003989	.9995
X2	.5019	.2928	.0019	.9997

The two sets of random numbers give two different histograms, but the mean and standard deviation of each set of numbers is close to the true mean of .5 and the true

b) >> n= 1000; sum = ceil(6*rand(1,n)) + ceil(6*rand(1,n)); h= hist(sum,11);
stem(2:12,h/n); hold; stem(2:12, [1 2 3 4 5 6 5 4 3 2 1]/12,'*');
mean = 7.008



Symbol ° represents relative frequencies and symbol * represents true probability which is listed below. As the number of trials increases the relative frequencies approach the true probabilities.

S={2,3,4,5,6,7,8,9,10,11,12} , $\mathcal{F}$ = all possible subsets of S
P(2)=P(12)=1/36, P(3)=P(11)=1/18, P(4)=P(10)=1/12, P(5)=P(9)=2/9, P(6)=P(8)=5/36;
P(7)=1/6

3a) Matlab code for generating samples of the sum and maximum of two dice.

```
x1=ceil(rand(1,10000)*6);
x2=ceil(rand(1,10000)*6);
sum=x1+x2;
maximum=max(x1,x2);
sumhist=hist(sum,[2:12]);
maxhist=hist(maximum,[1:6]);
```

```
sumpmf=[1 2 3 4 5 6 5 4 3 2 1]/36;
maxpmf=[1 3 5 7 9 11]/36;
subplot(221);
stem([2:12],sumpmf);
hold;
stem([2:12],sumhist/10000,'*');
axis([2 12 0 .2]);
title('relative freq. of sum of two dice')
hold;
mean(sum)
var(sum)
subplot(222);
stem([1:6],maxpmf);
hold;
stem([1:6],maxhist/10000,'*');
axis([1 6 0 .35]);
title('relative freq. of max of two dice')
hold;
mean(maximum)
var(maximum)
```

sample mean of sum is 7.0010
sample variance of sum is 5.7426
sample mean of maximum is 4.4776
sample variance of maximum is 1.9427

Drawing 10000 samples give more accurate results of averages and histogram plots than drawing 1000 samples.

Sample space for maximum: S={1,2,3,4,5,6}
P(1)=1/36, P(2)=1/12, P(3)=5/36, P(4)=7/36, P(5)=1/4, P(6)=11/36

b) Matlab code:

```
t=1:10000;
max5=t(maximum==5);
sumcond=sum(max5); (This picks out sample sums where the max of two die is 5)
histsumcond=hist(sumcond,[6:10]);
sumcondpmf= [2 2 2 2 1]/9;
subplot(223)
stem([6:10],sumcondpmf);
hold;
stem([6:10],histsumcond/length(sumcond),'*');
axis([6 10 0 .25]);
title('relative freq. of cond. prob. of sum given max=5')
hold
mean(sumcond)
```

```
var(sumcond)
```

sample mean of conditional sum is 7.7445
sample variance of conditional sum is 1.7343

P(sum=6|max=5)=P(sum=7|max=5)=P(sum=8|max=5)=P(sum=9|max=5)=2/9,
P(sum=10|max=5)=1/9

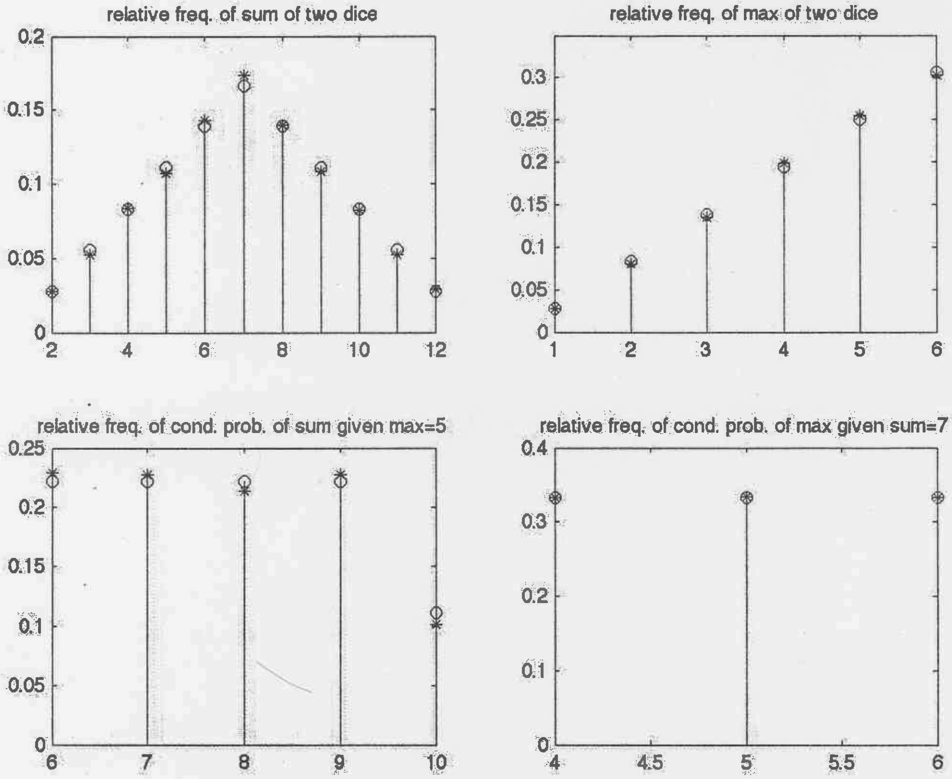
c) Matlab code:

```
sum7=t(sum==7);
maxcond=maximum(sum7); (This picks out sample maxs where the sum of two die is 7)
histmaxcond=hist(maxcond,[4:6]);
maxcondpmf=[2 2 2]/6;
subplot(224)
stem([4:6],maxcondpmf);
hold
stem([4:6],histmaxcond/length(maxcond),'*');
axis([4 6 0 .4]);
title('relative freq. of cond. prob. of max given sum=7')
hold
mean(maxcond)
var(maxcond)
```

P(max=4|sum=7)=P(max=5|sum=7)=P(max=6|sum=7)=1/3

sample mean on conditional max is 5.0006
sample variance of conditional max is .6661

For plots circles represent true probability and *s represent sampled probability.


relative freq. of sum of two dice
0.2
0.15
0.1
0.05
0
2
4
6
8
10
12
relative freq. of max of two dice
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
relative freq. of cond. prob. of sum given max=5
0.25
0.2
0.15
0.1
0.05
0
6
7
8
9
10
relative freq. of cond. prob. of max given sum=7
0.4
0.3
0.2
0.1
0
4
4.5
5
5.5
6