Achievable Information Rates for Channels with Insertions, Deletions and Intersymbol Interference with i.i.d. Inputs

Jun Hu, Tolga M. Duman, M. Fatih Erden, and Aleksandar Kavcic

***Abstract*—We propose to use various trellis structures to characterize different types of insertion and deletion channels. We start with binary independent and identically distributed (i.i.d.) insertion or deletion channels, propose a trellis representation and develop a simulation based algorithm to estimate the corresponding information rates with independent and uniformly distributed inputs. This approach is then generalized to other cases, including channels with additive white Gaussian noise, channels with both insertions and deletions, and channels with intersymbol interference (ISI) where the latter model is motivated by the recent developments on bit-patterned media recording. We demonstrate that the proposed algorithm is an efficient and flexible technique to closely estimate the achievable information rates for channels with insertions and/or deletions with or without intersymbol interference when i.i.d. inputs are employed while we also provide some notes on the achievable information rates when Markov inputs are used. We emphasize that our method is useful for evaluating information rates for channels with insertion/deletions with additional impairments where there does not seem to be a hope of obtaining fully analytical results.**

***Index Terms*—Information rates, deletion channel, insertion channel, intersymbol interference, synchronization errors, bit-patterned media recording.**

I. Introduction

FOR over half a century various digital communication techniques have been developed to address reliable communications over imperfect links. A large body of research is available on information theoretical characterization and practical signaling for various channel models with impairments including additive noise, fading and intersymbol interference. Primarily due to the possible mis-synchronization of the transmitter and receiver clocks, channels with random insertions and/or deletions are also of significant concern in practical systems. Despite their importance, channels with such impairments prove to be very difficult to analyze, and they are far from being fully understood. For instance, the capacity of a binary input i.i.d. deletion channel, which is one of the simplest models, is still unknown.

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Over the past several decades, insertion/deletion channels have attracted a lot of interest, especially in terms of their information theoretic characterization. Different upper and lower bounds on the channel capacity limits are developed. For instance, Gallager [1] and Zigangirov [2] consider convolutional coding and sequential decoding over binary insertion/deletion/substitution channels, and derive lower bounds for the corresponding achievable information rates. Ullman [3] derives a combinatorial upper bound for channels with synchronization errors assuming that no decoding errors are allowed, that is, the bound is on the zero error capacity, not on the channel capacity in the usual sense even though in previous literature it was often misused as an upper bound on the Shannon capacity. Other more recent examples of the related work include the paper [4] which considers both i.i.d. and first order Markov codebooks, and employs a simple subsequence matching decoder to find lower bounds on the deletion channel capacity, the work in [5]–[7] which further improves the lower bound by considering codewords with blocks of ones and zeros (where the run-length is chosen according to a given distribution) and a stronger decoder. Compared to the lower bounds, there are only a few works that develop upper bounds. Diggavi et. al. [8] consider a genie-aided decoder and develop two non-trivial upper bounds on the capacity of deletion channels. More recently, Fertonani et. al. [9], [10] provide other genie-aided approaches which convert the insertion/deletion channels to memoryless channels and provide tighter upper bounds and, in some cases, lower bounds on the channel capacity. In general, most of these bounds are derived for very specific channel models and it is not straightforward to extend them to other more complicated channels.

In this paper, our interest is on achievable information rates over channels with deletions and/or insertions, and other impairments including additive noise and intersymbol interference. We mainly focus on the cases that exist in most practical systems, namely systems in which the insertion or deletion probabilities are small, and we calculate achievable rates with independent and uniformly distributed (i.u.d.) inputs. In principle, the ideas can also be generalized to channels with Markov inputs. In terms of the ISI channel model, we focus on the bit-patterned media (BPM) recording channels with write synchronization errors. Bit-patterned media is a new technology proposed to replace the conventional film media to further increase the recording densities by discretization of the recording media [11]. The main idea is to prepattern the recording medium with small magnetically stable islands, and then employ a write head which flies over to magnetize them in one direction or another depending on the written bit. For these types of recording channels, one important and interesting problem is the write synchronization [12], which is not an issue in the conventional recording channels. That is, due to possible mis-synchronization between the write head and the islands, timing error and writing failure may result in written-in errors, which greatly degrade the read channel performance. In [13], the effect of written-in errors caused by mis-synchronization during the writing process has been studied. Another problem arising as a result of mis-synchronization is the existence of insertion and deletion errors. Let us give a simple explanation. Usually the frequency of the write head and the frequency of the islands are not exactly the same. During the writing process, the position where the head is aligned to the island will experience a continuous shift. If this shift is not compensated for, a "cycle slip" will occur causing insertions and/or deletions of bits in the write process. Usually, it is hard to adjust the head to each island, therefore, the cycle slips are unavoidable. The specific insertion/deletion ISI channel model adoped in this paper is based on this particular system, and it is different from other systems with ISI (for instance in traditional recording channels) when the insertions/deletions occur due to the mis-synchronization at the receiver.

For practical BPM recording channels, the insertion and deletion probabilities are usually very small. When this is the case, we demonstrate in this paper that we can develop a very simple and flexible approach, namely a trellis-based computation technique, to calculate lower bounds on the capacity of insertion/deletion channels, and their various extensions (e.g., such channels with additive Gaussian noise and even ISI). In the past few years, simulation based approaches have been proposed for achievable information rate calculation of channels with memory [14]–[17]. They are shown to be flexible and efficient for calculation of information rates for many channel models including channels with read synchronization errors. Specifically, [18] proposes a reduced-state technique to obtain a lower bound on achievable information rates for insertion/deletion channels, while [19] models the timing errors as a Markov process and employs the simulation-based approach to derive bounds on achievable information rates for ISI channels with timing errors and additive white Gaussian noise (AWGN).

Specifically, in this paper, we study a series of insertion/deletion channels, from the simplest i.i.d insertion/deletion channels to significantly more complicated BPM recording systems with write synchronization errors. For each channel model, we first develop suitable trellis structures, then detail the corresponding trellis-based algorithm for estimation of the joint probabilities of the input and output sequences for a given run of simulation, and finally estimate the achievable information rates. We note that our results are guaranteed to be lower bounds (with large simulation lengths) on the corresponding channel capacities. One reason is that we are using i.u.d. binary inputs as opposed to optimized inputs for the insertion/deletion channel. The other reason is that the numerical calculations of the joint entropy of the input and the output are performed by reduced state techniques (on a mismatched trellis) to simplify the calculations (which results in an upper bound), while the individual entropies are calculated precisely [20], [21]. To clarify our contribution with respect to [20], [21], we note that while [20], [21] consider channels with memory and develop provable upper and lower bounds on the achievable information rates, they only focus on finite state channels with perfect synchronization between the transmitter and receiver clocks. Here, motivated by possible synchronization errors in a communication system, we focus on the insertion/deletion channels which have infinitely many (but countable) number of states, and derive the relevant BCJR type algorithm for estimating the mutual information between the input and the output. We refer to the results in [20], [21] to show that our results are provable lower bounds on the information rates. We emphasize that our method is useful for evaluating achievable information rates for channels with insertion/deletions where there does not seem to be a hope of obtaining fully analytical results.

The organization of the paper is as follows. In the next section, we first briefly introduce the channel model and present a detailed discussion on the trellis construction and achievable information rate calculations for insertion/deletion channels. In Section III, we extend the technique to channels with both insertions and deletions. Then in Section IV, we consider another extension by studying channels with intersymbol interference. Finally, we provide our conclusions in Section V.

II. Information Rates for Insertion/Deletion Channels

A. Basic Channel Model

The basic model we consider is an i.i.d. binary insertion (deletion) channel characterized by the insertion probability P_I (deletion probability P_D) [4]. The channel input sequence is $X_1^N = (X_1, \ldots, X_N)$ where each input symbol takes the values 0 or 1. During the transmission, random symbols are inserted independently with probability P_I (for the insertion channel) or the transmitted symbols are deleted independently with probability P_D (for the deletion channel). At the receiver end, the binary output sequence is denoted by $Y_1^{N'} = (Y_1, \ldots, Y_{N'})$, where N' may be larger/smaller than N due to the presence of insertions/deletions.

Since the insertion/deletion process is memoryless, information stability holds [22], and the channel capacity equals

$$C = \lim_{N\to\infty} \frac{1}{N} \sup_{P(X_1^N)} I(X_1^N; Y_1^{N'}), \qquad (1)$$

where $I(\cdot;\cdot)$ denotes the mutual information and it is maximized over the joint distribution of the input sequence. Here, we are interested in computing the achievable information rates for i.i.d. binary insertion/deletion channels with i.u.d.

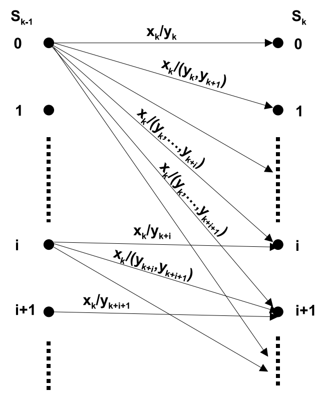

Fig. 1. Trellis diagram for insertion channels, kth time instant.

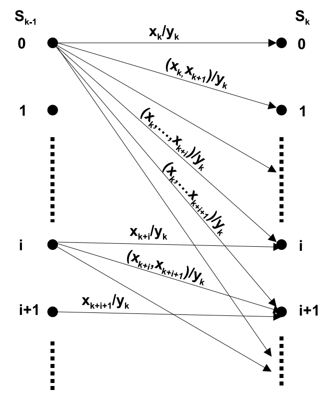

Fig. 2. Trellis diagram for deletion channels, kth time instant.

inputs X_1^N, given by

$$C_{i.u.d.} = \lim_{N\to\infty} \frac{1}{N} I(X_1^N; Y_1^{N'}). \tag{2}$$

In general, since the input is constrained, $C_{i.u.d.}$ is smaller than or equal to C.

B. Binary Symmetric Insertion Channels

The first model we consider is a binary symmetric insertion channel which is characterized by the insertion probability P_I and the substitution error probability P_e. That is, in this model, in addition to insertions, each bit may be toggled with probability P_e. The input symbols and inserted symbols are both independent and uniformly distributed. We adopt the insertion model in [23] where back to back insertions are possible as opposed to the model in [1]. In the following, we first develop a trellis representation for the channel, and present the calculation of the achievable information rates based on the proposed trellis.

1) Trellis: The proposed trellis representation is depicted in Fig. 1 where the trellis segment at the kth time instant is shown. $S_k = i$ denotes the state where there are i insertions before the transmission of X_k. The labels on the branches denote the possible input and output combinations at the kth time instant. If there are m insertions ($m = 0, 1, \ldots$) between X_{k-1} and X_k, a transition from the state $S_{k-1} = i$ to $S_k = j = i + m$ with the corresponding output segment of Y_{k+i}^{k+i+m} will occur. It is easy to see that, with i.i.d. insertions, for each transition (from $i \to j$), the transition probabilities are given by

$$P(S_k = j|S_{k-1} = i) = P_I^{j-i}(1 - P_I) \quad \text{for} \quad j \geq i. \tag{3}$$

2) Information Rates: Let us now calculate the achievable information rates for this channel. We assume that N bits are transmitted, but N' bits are received where N' is known. Arguing from law of large numbers, it is clear that assuming the total number of insertions is known (for large N) is not a problem in obtaining a lower bound on the achievable rates. More formally, as done in [9] for the deletion-only channel, it can be shown that the mutual information between the input and the output sequences without the knowledge of N' differs by at most $H(N')$ (i.e., discrete entropy of output sequence length) from the value calculated with conditioning on N'. Therefore, the inaccuracy in the achievable rate calculation is bounded by $H(N')/N$. Invoking the large N regime, N' can be approximated as Gaussian, we can write the following

$$\lim_{N\to\infty} \frac{H(N')}{N} = \lim_{N\to\infty} \frac{1}{2N} \ln(2\pi e N P_I(1-P_I)) = 0.$$

Clearly, we can also write

$$\lim_{N\to\infty} \frac{1}{N} I(X_1^N; Y_1^{N'}|N') = \lim_{N\to\infty} \frac{1}{N} I(X_1^N; Y_1^{N'}).$$

For instance, for the case of an insertion channel with probability $P_I = 0.1$, and $N = 1{,}000{,}000$, $H(N')/N$ amounts to roughly 10^{-5}. This means that taking the length of the received sequence as known will only affect the accuracy of the results in the fifth digit after the decimal point. We note that this amount is significantly smaller than the accuracy of the Monte Carlo based techniques which is in the order of $1/\sqrt{N}$, i.e., around 10^{-3} for the specific example considered. It is clear that given the insertion probability, one can easily determine the simulation length required for any desired accuracy in the results.

With this argument, in what follows we treat N', i.e., received sequence length as known (but random). To obtain $C_{i.u.d.}$, we simply approximate $I(X_1^N; Y_1^{N'})$ as

$$\begin{aligned} & I(X_1^N; Y_1^{N'}|N') \\ &\quad = H(X_1^N|N') + H(Y_1^{N'}|N') - H(X_1^N, Y_1^{N'}|N'), && (4) \\ &\quad = N + E[N'] + E[\log p(x_1^N, y_1^{N'})], && (5) \end{aligned}$$

where $E[\cdot]$ denotes the expectation operator and $p(x_1^N, y_1^{N'}) := P(X_1^N = x_1^N, Y_1^{N'} = y_1^{N'})$ (we follow the same terminology in the rest of the paper as well). $H(X_1^N)$ equals to N simply due to the use of an i.u.d. input sequence. Furthermore, since the inputs are i.u.d., the insertions are randomly selected to be 0 or 1 with equal probability and the channel is symmetric, it is obvious that the channel output is also an i.u.d. sequence. Therefore we obtain $H(Y_1^{N'}) = N'$. The only issue that remains is the calculation of $\log(p(x_1^N, y_1^{N'}))$. To accomplish this, we take a Monte Carlo based approach [14], [15] where we first generate a long sequence of inputs and then based on the channel model, we conduct a Monte Carlo simulation to generate the corresponding output sequences. Using these channel observations, we rely on the methods presented in the following section to estimate the joint probabilities. Let us demonstrate how these computations can be made in an efficient manner, for a given input and output sequence [24].

3) Calculation of $p(x_1^N, y_1^{N'})$: To calculate $p(x_1^N, y_1^{N'})$, we first define the following forward recursion

$$\begin{aligned} \varphi_k(j) &= p(x_1^k, y_1^{k+j}, S_k = j), \\ &= \sum_{i=0}^{j} p(x_1^{k-1}, y_1^{k-1+i}, S_{k-1} = i) \\ &\qquad \cdot p(x_1^k, y_1^{k+j}, S_k = j | x_1^{k-1}, y_1^{k-1+i}, S_{k-1} = i), \\ &= \sum_{i=0}^{j} p(x_1^{k-1}, y_1^{k-1+i}, S_{k-1} = i) \\ &\qquad \cdot p(x_k, y_{k+i}^{k+j}, S_k = j | S_{k-1} = i), \end{aligned} \tag{6}$$

$$= \sum_{i=0}^{j} \varphi_{k-1}(i)\theta_k(i,j), \quad (7)$$

where the third equation follows from the fact that x_1^{k-1} and y_1^{k-1+i} provide no additional information with the conditioning, and

$$\begin{aligned}\theta_k(i,j) &= p(y_{k+i}^{k+j}, x_k, S_k = j | S_{k-1} = i),\\ &= p(y_{k+i}^{k+j}, x_k | S_{k-1} = i, S_k = j) p(S_k = j | S_{k-1} = i),\\ &= p(y_{k+j}, x_k | S_{k-1} = i, S_k = j)\\ &\quad \cdot p(y_{k+i}^{k+j-1} | S_{k-1} = i, S_k = j) P(S_k = j | S_{k-1} = i).\end{aligned}$$

The third equation follows the fact that, given the state transition ($S_{k-1} = i \rightarrow S_k = j$), y_{k+j} is the observation of the original transmitted symbol x_k, and y_{k+i}^{k+j-1} are the observations of the inserted symbols and they are independent.

Let us consider the first term of $\theta_k(i,j)$, which is calculated as

$$\begin{aligned}&p(y_{k+j}, x_k | S_{k-1} = i, S_k = j)\\ &\quad = p(y_{k+j} | S_{k-1} = i, S_k = j, x_k) P(x_k),\\ &\quad = \begin{cases} \frac{1}{2}(1 - P_e) & \text{if} \quad y_{k+j} = x_k \\ \frac{1}{2} P_e & \text{if} \quad y_{k+j} \neq x_k \end{cases}. \end{aligned} \quad (8)$$

For the second term, y_{k+i}^{k+j-1} corresponds to the $j-i$ insertions between $(k-1)$th and kth transmitted symbols. We assume that the inserted symbols are independent, and take values 0 or 1 with probability $1/2$ each. Thus, we have

$$p(y_{k+i}^{k+j-1} | S_{k-1} = i, S_k = j) = \frac{1}{2^{j-i}}. \quad (9)$$

The third term $P(S_k = j | S_{k-1} = i)$ is given by (3).

The final joint probability is given by

$$p(x_1^N, y_1^{N'}) = \varphi_N(N' - N). \quad (10)$$

One practical constraint of this approach is that we can have a huge number of states so that actual simulation becomes infeasible. To remedy this problem we proceed as follows. After a very long simulation of the channel is generated using the exact model, we run the forward recursion of the BCJR algorithm detailed above using a reduced state approach [20], [21]. Specifically, at each time step we consider only a reasonable number of channel states with the largest metrics, and ignore the rest. It is proven in [20], [21] that this results in an upper bound on the entropy being estimated (i.e., the joint input-output entropy $H(X_1^N, Y_1^{N'})$), hence the final result is guaranteed to be a lower bound on the i.u.d. achievable information rate given in (5).

C. Binary Symmetric Deletion Channels

For the binary symmetric deletion channels, the channel model is similar to the insertion case, except that the parameter P_I is replaced with P_D (deletion probability) and N' cannot be greater than N. In this case, we consider the dual approach to the one used for the insertion channels. Note that, similar to the previous case, the output of this type of channels is also an i.u.d. sequence. Therefore we also only need to calculate the joint probability (or equivalently, the joint entropy).

A similar channel trellis is illustrated in Fig. 2. In this case, each trellis segment corresponds to a received symbol and $S_k = i$ denotes the state where there are i deletions before the reception of y_k. If there are m deletions (where $m = 0, 1, \ldots, N - (i + k - 1)$) between Y_{k-1} and Y_k [1], a transition from the state $S_{k-1} = i$ to $S_k = j = i + m$ with the corresponding input segment of X_{k+i}^{k+i+m} will occur.

Based on this trellis representation, we can use a similar forward recursion technique as in the previous section. In the following, we only summarize the results and omit the details:

$$p(x_1^N, y_1^{N'}) = \xi_{N'}(N - N'), \quad (11)$$

$$\begin{aligned}\xi_k(j) &= p(x_1^{k+j}, y_1^k, S_k = j),\\ &= \sum_{i=0}^{j} \xi_{k-1}(i)\pi_k(i,j), \end{aligned} \quad (12)$$

$$\begin{aligned}\pi_k(i,j) &= p(y_k, x_{k+i}^{k+j} | S_{k-1} = i, S_k = j)\\ &\quad \cdot p(S_k = j | S_{k-1} = i),\\ &= p(y_k, x_{k+j} | S_{k-1} = i, S_k = j)\\ &\quad \cdot p(x_{k+i}^{k+j-1} | S_{k-1} = i, S_k = j)\\ &\quad \cdot P(S_k = j | S_{k-1} = i),\\ &= p(y_k, x_{k+j} | S_{k-1} = i, S_k = j)\\ &\quad \cdot p(S_k = j | S_{k-1} = i)/2^{j-i}, \end{aligned} \quad (13)$$

where

$$\begin{aligned}&p(y_k, x_{k+j} | S_{k-1} = i, S_k = j)\\ &\quad = \begin{cases} \frac{1}{2}(1 - P_e) & \text{if} \quad y_k = x_{k+j} \\ \frac{1}{2} P_e & \text{if} \quad y_k \neq x_{k+j} \end{cases}, \end{aligned} \quad (14)$$

and $P(S_k = j | S_{k-1} = i)$ is given by

$$P(S_k = j | S_{k-1} = i) = P_D^{j-i}(1 - P_D) \quad \text{for} \quad j \geq i. \quad (15)$$

D. Extension to the Additive White Gaussian Noise Case

Previously, we have considered the simplest communication model where the channel impairment is characterized by a single cross-over probability P_e. Another common example of communication channel model is the AWGN channel. Assuming BPSK modulation, the discrete time representation of the received signal is given by

$$Y_i = (2\tilde{X}_i - 1) + Z_i, \quad (16)$$

where $\tilde{X}_i \in \{0, 1\}$ (sequence of bits corresponding to the non-deleted or inserted bits). Z_i is a Gaussian random variable with zero mean and a variance of $N_0/2$. We define the signal-to-noise ratio (SNR) as $1/N_0$.

The mutual information between the channel input sequence and output sequence is given by

$$\begin{aligned} I(X_1^N; Y_1^{N'}) &= H(X_1^N) + h(Y_1^{N'}) - h(X_1^N, Y_1^{N'}), \quad (17)\\ &= N + h(Y_1^{N'}) + E[\log p(X_1^N, Y_1^{N'})], \quad (18)\end{aligned}$$

where $h(Y_1^{N'})$ can be easily computed by using Monte-Carlo simulation given the fact that Y_k is a hidden Markov process. In the following, we show the necessary modification of the previous algorithms to calculate the joint probability for AWGN insertion/deletion channels.

[1] Note that we can only have up to $N - (i + k - 1)$ deletions at this stage because we already have i symbols deleted, $(k - 1)$ symbols transmitted and only have $N - (i + k - 1)$ symbols left.

1) Insertion Channels: For insertion channels, the definitions of $\gamma_k(i,j)$, $\theta_k(i,j)$ are the same. The only revisions are in the calculations of each term. Hence, Eqs. (8) and (9) are replaced by

$$\begin{aligned} p(y_{k+j}, x_k|S_{k-1}=i, S_k=j) \\ &= p(y_{k+j}|S_{k-1}=i, S_k=j, x_k)p(x_k), \quad (19) \\ &= \frac{1}{2}\frac{1}{\sqrt{\pi N_0}}e^{-\frac{\left(y_{k+j}-(2x_k-1)\right)^2}{N_0}}, \quad (20) \end{aligned}$$

$$\begin{aligned} p(y_{k+i}^{k+j-1}|S_{k-1}=i, S_k=j) \\ &= \frac{1}{2^{j-i}}\sum_{D_{k+i}^{k+j-1}\in\Psi}\frac{1}{\sqrt{\pi N_0}}e^{-\frac{\left\|y_{k+i}^{k+j-1}-D_{k+i}^{k+j-1}\right\|^2}{N_0}}, \quad (21) \end{aligned}$$

where D_{k+i}^{k+j-1} denotes a dummy vector of length $j-i$, and Ψ denotes the set of all the possible noiseless output segments that correspond to all possible inserted bit patterns. Note that this equation is only applicable to the computation of $\theta_k(i,j)$ when $j > i$. If $j = i$, i.e., when the transition does not correspond to an insertion, the term $p(y_{k+i}^{k+j-1}|S_{k-1}=i, S_k=j)$ does not appear.

2) Deletion Channels: For deletion channels, there is only one change in (14), i.e., it is revised as

$$p(y_k, x_{k+j}|S_{k-1}=i, S_k=j) = \frac{1}{2}\frac{1}{\sqrt{\pi N_0}}e^{-\frac{\left(y_k-(2x_{k+j}-1)\right)^2}{N_0}}.$$

E. Examples

Let us illustrate the proposed techniques via three examples. We show the achievable information rates for an i.i.d. binary deletion channel with no substitution errors ($P_e = 0$) in Fig. 3. For this binary deletion channel, the exact characterization of the channel capacity is generally not possible, and only lower and upper bounds were available (several of the well-known bounds: random lower bound [2], Ullman's upper/lower bounds [3], Markov lower bound [4], geometric lower bound [5] and side information upper bound of [8] are also included in the figure). We note that the lower bound we have computed using the trellis based approach is in agreement with the existing analytical results which only exist for significantly simple scenarios (e.g., deletion only channel). Furthermore, our approach is very flexible and simple to use for very complicated models for which there are no analytical methods as will be evident later in the paper.

We stress again that our result is the achievable information rate under the i.u.d. input constraints. As pointed out in [4], [5], by introducing memory into the input sequence, the lower bounds of Shannon capacity can be improved. Similarly, by incorporating different input constraints (e.g., Markov inputs) into our trellis-based approach, we may achieve higher achievable information rates.

Several other comments are in order. First, to generate the results, we have used an input sequence length $N = 10^5$ and a total number of states $B_D = 100$. The choice of B_D provides a tradeoff between the complexity of calculation and the tightness of the lower bound. For small values of B_D, the computational complexity is significantly reduced, however, the resulting lower bound may not be very tight. In any case,

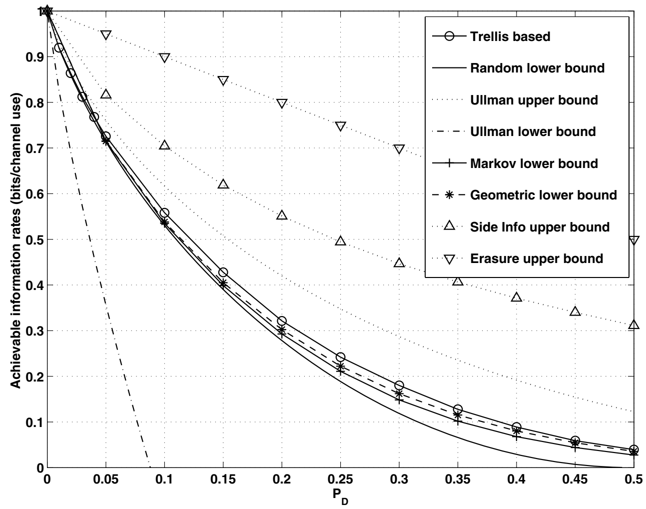

Fig. 3. Information rates for i.i.d. deletion channels.

the result obtained is a provable lower bound (within the Monte Carlo simulation error). We further note that for low values of insertion/deletion probabilities, a small value of B_D would be sufficient to obtain relatively tight results, for larger values of the deletion/insertion probabilities, many more states may need to be included in the calculation of the joint entropy to obtain useful bounds.

As another example, we provide the achievable information rates for the insertion channels with different levels of substitution errors in Fig. 4. We observe that, combination of insertion and substitution errors greatly degrades the achievable information rates for reliable communication. We also would like to emphasize that, although earlier results are for channels with deletions or insertions only (without substitutions) except for very limited cases (e.g. [1]), our technique can easily incorporate substitution errors. We do not provide a direct comparison with the results of [1] in terms the resulting bounds since the insertion models are not identical (e.g., in [1] one transmitted bit may be replaced with only two bits while in our case this random number is geometrically distributed).

One last example we want to present is for deletion channels with AWGN and BPSK modulation. We show the achievable information rates for AWGN channels with different deletion error probabilities in Fig. 5. We observe that, unless $P_D = 0$, existence of deletions greatly limits the transmission capabilities, and no matter how high the SNR is, reliable communication is not possible without the help of channel coding.

III. INFORMATION RATES FOR CHANNELS WITH BOTH INSERTIONS AND DELETIONS

In the previous section, our discussion was mainly focused on channels with insertions only or deletions only. A more general problem is the computation of channel capacity when there are both insertions and deletions during the transmission. The resulting channel is then characterized by three parameters: P_I, P_D and P_e. We study the case where insertions and deletions occur independently. At each time instant, each information bit is either transmitted or deleted (with probability

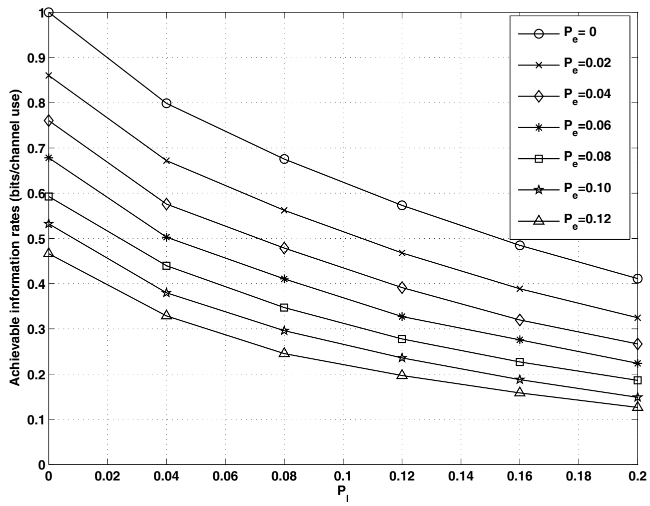

Fig. 4. Information rates for i.i.d. insertion channels.

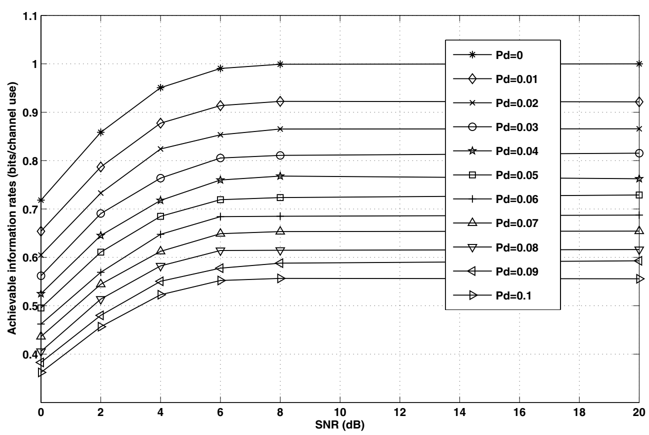

Fig. 5. Information rates for i.i.d. deletion channels with AWGN noise.

P_D). Between a pair of transmitted bits, there may also exist random insertions (with probability P_I for each insertion). Furthermore, all the transmitted bits go through an imperfect channel where substitutions may occur with error probability P_e. We will consider the extension of the approach in the previous section to this case and develop suitable algorithms to compute the achievable information rates. We will use the BSC as an example. For other cases, such as AWGN channels, the proposed technique can similarly be used.

First, we note that the channel output is still an i.u.d. sequence as in the case of insertion/deletion only channels. Therefore, we only need to calculate the joint probability. Next, we develop a different trellis whose state is defined by a two-element vector $\mathbf{i} = (i_1, i_2)$, where i_1 denotes the total number of insertions and i_2 denotes the total number of deletions until the current time instant. The trellis is illustrated in Fig. 6. For each trellis segment, only one bit from the original input sequence is actually transmitted, i.e., there may be insertions and deletions before this transmission. Let us consider the transition from state $S_{k-1} = \mathbf{i} = (i_1, i_2)$ to state $S_k = \mathbf{j} = (j_1, j_2)$ where $j_1 = i_1 + m_1$ and $j_2 = i_2 + m_2$. At the kth time instant, X_{k+j_2} is transmitted and m_2 deletions (of

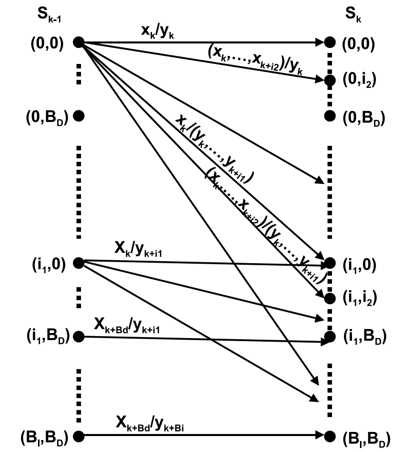

Fig. 6. Trellis diagram for channels with both insertions and deletions.

the input segment $X_{k+i_2}^{k+j_2-1}$) occur before this transmission. At the receiver side, $Y_{k+i_1}^{k+j_1}$ is the corresponding output sequence segment (including m_1 insertions).

The state transition probabilities are now given by

$$P(S_k = \mathbf{j} | S_{k-1} = \mathbf{i}) = P_I^{j_1 - i_1} P_D^{j_2 - i_2} (1 - P_I)(1 - P_D)$$
$$\text{for} \quad j_1 \geq i_1 \quad \text{and} \quad j_2 \geq i_2.$$

As we can see, this new trellis employs a combination of the ideas used in the trellises for the insertion only and the deletion only channels. Therefore, the computation of the achievable information rates will also be the integration of the earlier proposed algorithms.

The forward recursion is now defined as

$$\begin{aligned} \varphi_k(\mathbf{j}) &= p(x_1^{k+j_2}, y_1^{k+j_1}, S_k = \mathbf{j}), \\ &= \sum_{i_1=0}^{j_1} \sum_{i_2=0}^{j_2} \varphi_{k-1}(\mathbf{i}) \theta_k(\mathbf{i}, \mathbf{j}), \end{aligned} \tag{22}$$

where

$$\begin{aligned} \theta_k(\mathbf{i}, \mathbf{j}) &= p(y_{k+i_1}^{k+j_1}, x_{k+i_2}^{k+j_2} | S_{k-1} = \mathbf{i}, S_k = \mathbf{j}) \\ &\quad \cdot P(S_k = \mathbf{j} | S_{k-1} = \mathbf{i}), \\ &= p(y_{k+j_1}, x_{k+j_2} | S_{k-1} = \mathbf{i}, S_k = \mathbf{j}) \\ &\quad \cdot P(S_k = \mathbf{j} | S_{k-1} = \mathbf{i}) / 2^{j_1 - i_1} / 2^{j_2 - i_2}, \end{aligned} \tag{23}$$

and

$$\begin{aligned} &p(y_{k+j_1}, x_{k+j_2} | S_{k-1} = \mathbf{i}, S_k = \mathbf{j}) \\ &= \begin{cases} \frac{1}{2}(1 - P_e) & \text{if} \;\; y_{k+j_1} = x_{k+j_2} \\ \frac{1}{2} P_e & \text{if} \;\; y_{k+j_1} \neq x_{k+j_2} \end{cases}. \end{aligned} \tag{24}$$

The joint probabilities is then given by

$$p(x_1^N, y_1^{N'}) \;\; = \;\; \sum_{\mathbf{j} \in \Omega} \varphi_{N''}(\mathbf{j}), \tag{25}$$

where $N'' = \min(N, N')$, and Ω is defined as the set that includes all the possible combinations of insertion and deletion patterns which could result in a received sequence with length N'.

Let us now give an example. Fig. 7 shows the achievable information rates for binary channels with both insertions and

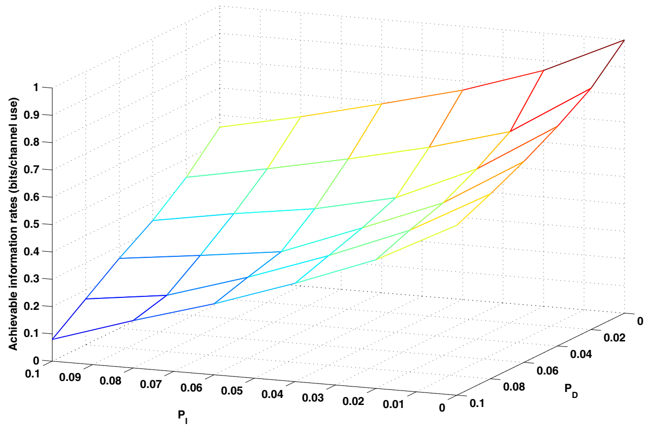

Fig. 7. Information rates for channel with both insertions and deletions.

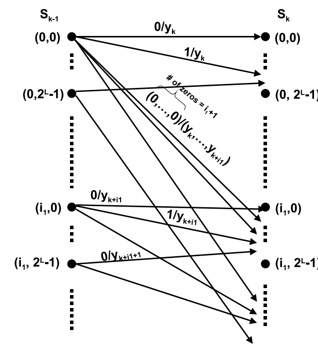

Fig. 8. Trellis diagram for insertion channels with ISI, kth time instant.

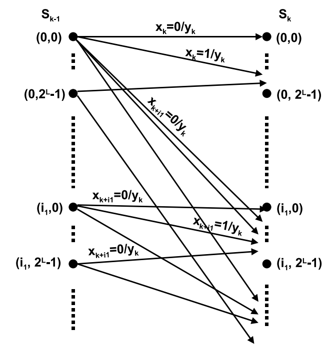

Fig. 9. Trellis diagram for deletion channels with ISI, kth time instant.

deletions. In this 3-D plot, we can see that, the achievable information rates decrease rapidly when the insertion and deletion error probabilities are increased. We note that the results here are obtained using the reduced state simulation based approach, and they are lower bounds on the achievable information rates with i.u.d. inputs, hence lower bounds on the capacity.

IV. Information Rates for Insertion/Deletion Channels with Intersymbol Interference

We now consider extensions to channels with intersymbol interference motivated by bit patterned media recording technology [11], [12]. Let us first introduce the communication model formally. As discussed before, the insertion/deletion errors are characterized by the insertion/deletion error probability P_I/P_D. During the transmission (referred to as the "writing process" in the magnetic recording nomenclature), bits are randomly inserted into/deleted from the original input bit sequence. Later, this new input sequence $\tilde{X}_1^{N'}$ (of $+1$s and -1s) incorporating the effect of insertions/deletions are received through an ISI channel (called the "readback process" in magnetic recording). More formally, the model is described by

$$Y_k = \sum_{l=0}^{L} h_l \tilde{X}_{k-l} + Z_k, \tag{26}$$

where L is the length of the channel memory, i.e., $L+1$ is the number of channel taps. $\{h_l\}_{l=0}^{L}$ are the channel coefficients which are normalized so that $\sum_{l=0}^{L} |h_l|^2 = 1$. z_k is the additive white Gaussian noise with a mean of zero and a variance of $N_0/2$. The signal-to-noise ratio is defined as $1/N_0$.

To determine the mutual information as in (17), we need to compute both $h(Y_1^{N'})$ and $h(X_1^N, Y_1^{N'})$. Since the input is assumed to be i.u.d. binary, the sequence after insertion/deletions (input to the ISI channel) is also an i.u.d. binary sequence. Thus, the output sequence (with the ISI and AWGN) is hidden Markov [4] where the states are determined by the states of the underlying ISI channel only. Therefore, the entropy $h(Y_1^{N'})$ can be calculated with an arbitrary precision using the ISI channel state trellis using the standard approaches now widely used in the literature (e.g. [14]–[17], [21]). On the other hand, the joint entropy $h(X_1^N, Y_1^{N'})$ is calculated using reduced state techniques as described before for the earlier channel models, i.e., it is overestimated. Therefore, the achievable information rate result is a provable lower bound on the actual channel capacity for this case as well.

In the following, we will present the details of modifications in calculating $h(X_1^N, Y_1^{N'})$ with respect to Section II-B and Section II-C, respectively.

A. Insertion ISI Channels

In this case, we basically follow the same idea developed in the previous sections, but employ a slightly different trellis. The state at the kth time instant is represented by the 2-element vector $S_k = \mathfrak{i} = (i_1, i_2)$, where i_1 denotes the number of insertions up to time k and i_2 is the ISI channel state value (there are 2^L different channel state values) (see Figs. 8 and 9).

Similarly, let us consider the transition from $S_{k-1} = \mathfrak{i} = (i_1, i_2)$ to $S_k = \mathfrak{j} = (j_1, j_2)$, where $j_1 = i_1 + m$ and $m = 0, 1, \ldots$. Based on the i.u.d. input distribution, we have the following transition probability (for valid transitions)

$$P(S_k = \mathfrak{j} | S_{k-1} = \mathfrak{i}) = \frac{P_I^{j_1 - i_1}(1 - P_I)}{2^{j_1 - i_1 + 1}} \text{ for } j_1 \geq i_1. \tag{27}$$

Compared to (3), each transition probability is divided by an additional factor $2^{j_1 - i_1 + 1}$. This is because, the channel state is determined jointly by the original bit and the inserted bits and the transition probability is determined by both the insertion probability and all the inputs at this time instant. For each possible insertion condition (e.g., m insertions), there are altogether $m+1$ input bits that excite the channel state transition and thus 2^{m+1} branches with equal probability (under the i.u.d. input distribution).

The forward recursion is now given by

$$\begin{aligned}
\varphi_k(\mathfrak{j}) &= p(x_1^k, y_1^{k+j_1}, S_k = \mathfrak{j}), \\
&= \sum_{i_1=0}^{j_1} \sum_{i_2} \varphi_{k-1}(\mathfrak{i}) \theta_k(\mathfrak{i}, \mathfrak{j}), && (28) \\
\theta_k(\mathfrak{i}, \mathfrak{j}) &= p(y_{k+i_1}^{k+j_1}, x_k | S_{k-1} = \mathfrak{i}, S_k = \mathfrak{j}) \\
&\quad \cdot P(S_k = \mathfrak{j} | S_{k-1} = \mathfrak{i}), && (29)
\end{aligned}$$

where

$$p(y_{k+i_1}^{k+j_1}, x_k | S_{k-1} = \mathfrak{i}, S_k = \mathfrak{j}) = \begin{cases} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left\| y_{k+i_1}^{k+j_1} - \hat{y}_{k+i_1}^{k+j_1} \right\|^2}{N_0}}, & \text{if } x_k = \hat{x}_k \\ 0, & \text{if } x_k \neq \hat{x}_k \end{cases}. \quad (30)$$

Here, $\hat{y}_k$ is the noiseless channel output and $\hat{x}_k$ is the input corresponding to each branch.

The joint probability is then given by

$$p(x_1^N, y_1^{N'}) = \sum_{j_2} \varphi_N(\mathfrak{j}), \quad (31)$$

where $j_1 = N' - N$.

B. Deletion ISI Channels

Compared to the insertion ISI channels, some simplifications can be incorporated into the algorithms for the deletion ISI channels. The reason is as follows: The deleted symbols do not participate in the channel state transitions, only the undeleted bits will contribute. Therefore, for each branch, we only consider the last bit which is actually transmitted. Based on the i.u.d. input distribution, we have the following transition probability

$$P(S_k = \mathfrak{j} | S_{k-1} = \mathfrak{i}) = \frac{P_D^{j_1 - i_1}(1 - P_D)}{2} \text{ for } j_1 \geq i_1. \quad (32)$$

Note that, the additional division by 2 is needed because only the transmitted bit matters and it has two possible values with equal probabilities.

The forward recursions are given by

$$\begin{aligned} \pi_k(\mathfrak{i}, \mathfrak{j}) &= p(y_k, x_{k+i_1}^{k+j_1} | S_{k-1} = \mathfrak{i}, S_k = \mathfrak{j}) P(S_k = \mathfrak{j} | S_{k-1} = \mathfrak{i}), \\ &= p(y_k, x_{k+j_1} | S_{k-1} = \mathfrak{i}, S_k = \mathfrak{j}) \\ &\quad \cdot p(x_{k+i_1}^{k+j_1-1} | S_{k-1} = \mathfrak{i}, S_k = \mathfrak{j}) P(S_k = \mathfrak{j} | S_{k-1} = \mathfrak{i}), \\ &= p(y_k, x_{k+j_1} | S_{k-1} = \mathfrak{i}, S_k = \mathfrak{j}) \\ &\quad \cdot P(S_k = \mathfrak{j} | S_{k-1} = \mathfrak{i}) / 2^{j_1 - i_1}, \end{aligned} \quad (33)$$

where

$$p(y_k, x_{k+j_1} | S_{k-1} = \mathfrak{i}, S_k = \mathfrak{j}) = \begin{cases} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\|y_k - \hat{y}_k\|^2}{N_0}}, & \text{if } x_{k+j_1} = \hat{x}_{k+j_1} \\ 0, & \text{if } x_{k+j_1} \neq \hat{x}_{k+j_1} \end{cases}. \quad (34)$$

The joint probability is then given by

$$p(x_1^N, y_1^{N'}) = \sum_{j_2} \xi_{N'}(\mathfrak{j}), \quad (35)$$

where $j_1 = N - N'$.

C. Examples and Further Extensions

To give an example, we illustrate the achievable information rates for partial response deletion channels (PR4 target [25] with channel response as $h_0 = \frac{1}{\sqrt{2}}$, $h_1 = 0$, and $h_2 = \frac{-1}{\sqrt{2}}$) in Fig. 10. The general observation is similar to the result in Fig. 5, except that, at the same SNR, the achievable information rates of the ISI channel is smaller than those of the AWGN channel due to the existence of intersymbol interference.

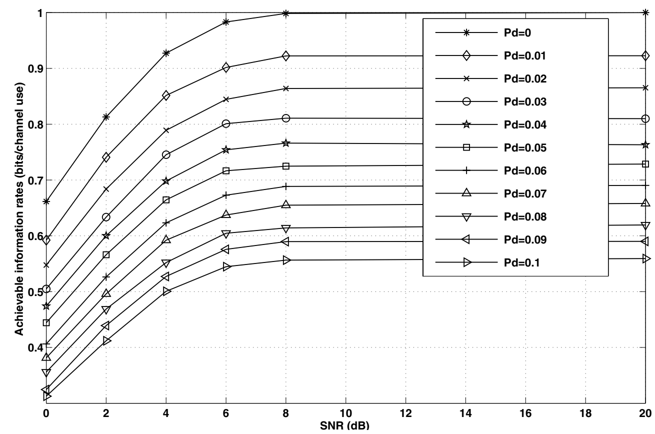

Fig. 10. Information rates for the deletion PR4 channel.

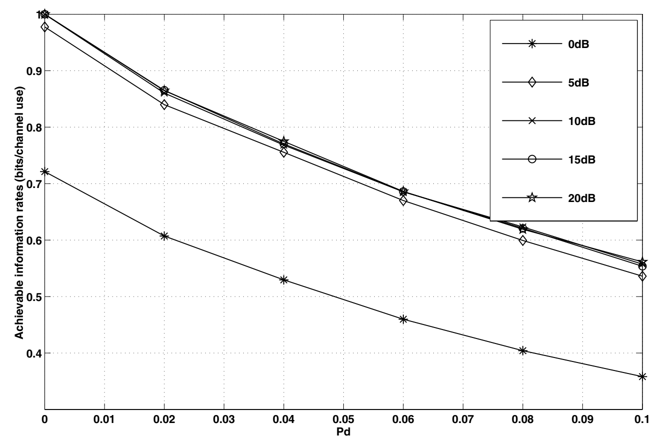

Fig. 11. Information rates for the deletion bit patterned media channel.

In Fig. 11, we show another example with a more realistic bit-patterned media recording channel model with deletions. At a certain deletion error probability, the achievable information rates increase for a certain SNR range and saturate when SNR is higher than some value. In another words, at high SNRs, the achievable information rates are limited by the existence of deletion errors.

In this section, we have considered ISI channels with only deletions or only insertions. Clearly, by using a similar approach as in Section III, we can extend the techniques to the case where there are both insertions and deletions. For example, we can use a three-element vector where the first, second and third entries represent the number of insertions, the number of deletions and the channel state value, respectively. Compared to the channel model in Section III, the number of states in the trellis is expanded by a factor of 2^L, where L is the length of the channel ISI. Once we have the trellis representation, the derivation of the achievable information rates is straightforward (though may be tedious).

V. Conclusions

We have studied the achievable information rate computation for various insertion/deletion channels. We have developed different trellis structures to characterize the insertions or deletions, and developed a forward recursion approach to estimate the achievable information rates of such channels. Specifically, we have considered insertions/deletion channels with additional substitution errors, AWGN, and with ISI. We have shown that, by modifying the trellis structures, our approach is efficient, flexible, and can easily be employed for different types of insertion and deletion channels. The results are provable lower bounds on the actual channel capacities as i.u.d. binary inputs are used, and reduced state techniques are employed in calculating the joint entropy of the input and the output sequences, while the input and output entropies are calculated either analytically or with an arbitrary precision. Based on the approach developed, we report several examples of achievable information rates for a variety of specific channel models and parameters. Although we only consider i.u.d. inputs in this work, other types of inputs, e.g. Markov inputs, can also be incorporated using a similar approach. In that case, we would need a joint trellis incorporating both the Markov characteristics of the inputs and the insertions/deletions. Meanwhile, the state transitions will be determined not only by the insertion/deletion probabilities but also by the Markov input distribution parameters.

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