Department of Electrical Engineering, University of Hawaii

EE 342: Probability and Statistics   Fall 2016

Homerwork Set 3

Due date: Sep. 14, 2016

(1)   a)   Chapter 3, problem 29
       b)   Chapter 3, problem 60
       c)   Chapter 3, problem 63
       d)   Chapter 3, problem 64
       e)   Chapter 3, theoretical exercise 3
       f)   Chapter 3, theoretical exercise 9
       g)   Chapter 3, theoretical exercise 10
       h)   A shooter hits the target with probability 2/3. There are 2 targets. If she scores a hit on the 1st target in the first shot, she is allowed to fire at the 2nd target. Otherwise, she must keep firing at the 1st target until she hits it. Find the probability of failing to hit the 2nd target in at most three shots.
       i)   A man has forgotten the last digit of a telephone number and therefore has to dial it at random.
             - Find the probability that he must dial at most 3 times.
             - Find the same probability if he knows that the last digit is an odd number?

(2)   Matlab exercises: In this exercise random numbers are drawn using the "rand" command. Several Matlab commands are used. To get help for any Matlab command, type "help command"
       a)   Generate a vector x of 1000 random numbers using the "rand" command. Try the command "hist(x,10)" to observe where the generated random numbers lie. Try also "hist(x,100)". Now try commands "max(x)", "min(x)", "mean(x)", and "std(x)". What do these commands do? Repeat the experiment by drawing a second vector x of random numbers and using the above commands. Discuss your observations.
       b)   Generate a random experiment where you roll a pair of six-sided dice (use hints from lectures). Conduct this experiment 1000 times and observe the sum of the pips of the two dice. Make plots of the relative frequency of the sums of the pips, S={2,3,4,…,10,11,12}. (Use "stem" and "hist" commands to plot the relative frequencies.) Develop a probability model for this experiment.

(3)   Matlab exercises: This is a continuation of Problem 2a)
       a)   Generate the same experiment as in 2a), but conduct the experiment 10000 times. Observe the sums. Also observe the maximum of the two dice. Develop a probability model for this experiment.
       b)   From part a), let us assume that the maximum of the two dice is 5. Make relative frequency plots of the sum of two dice when the maximum of the two dice is 5. Develop a conditional probability model for this experiment.
       c)   From part a). let us assume that the sum of two dice is 7. Make relative frequency plots of the maximum of the two dice when the sum of the two dice is 7. Develop a conditional probability model for this experiment.

3.29. There are 15 tennis balls in a box, of which 9 have not previously been used. Three of the balls are randomly chosen, played with, and then returned to the box. Later, another 3 balls are randomly chosen from the box. Find the probability that none of these balls has ever been used.

3.60. The color of a person's eyes is determined by a single pair of genes. If they are both blue-eyed genes, then the person will have blue eyes; if they are both brown-eyed genes, then the person will have brown eyes; and if one of them is a blue-eyed gene and the other a brown-eyed gene, then the person will have brown eyes. (Because of the latter fact, we say that the brown-eyed gene is *dominant* over the blue-eyed one.) A newborn child independently receives one eye gene from each of its parents, and the gene it receives from a parent is equally likely to be either of the two eye genes of that parent. Suppose that Smith and both of his parents have brown eyes, but Smith's sister has blue eyes.

(a) What is the probability that Smith possesses a blue-eyed gene?

(b) Suppose that Smith's wife has blue eyes. What is the probability that their first child will have blue eyes?

(c) If their first child has brown eyes, what is the probability that their next child will also have brown eyes?

3.63. A and B are involved in a duel. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously. If one or both are hit, then the duel is over. If both shots miss, then they repeat the process. Suppose that the results of the shots are independent and that each shot of A will hit B with probability p_A, and each shot of B will hit A with probability p_B. What is

(a) the probability that A is not hit?

(b) the probability that both duelists are hit?

(c) the probability that the duel ends after the nth round of shots?

(d) the conditional probability that the duel ends after the nth round of shots given that A is not hit?

(e) the conditional probability that the duel ends after the nth round of shots given that both duelists are hit?

3.64. A true–false question is to be posed to a husband-and-wife team on a quiz show. Both the husband and the wife will independently give the correct answer with probability p. Which of the following is a better strategy for the couple?

(a) Choose one of them and let that person answer the question.

(b) Have them both consider the question, and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give.

3.3. Consider a school community of m families, with n_i of them having i children, $i = 1, \ldots, k$, $\sum_{i=1}^{k} n_i = m$.

Consider the following two methods for choosing a child:

1. Choose one of the m families at random and then randomly choose a child from that family.

2. Choose one of the $\sum_{i=1}^{k} in_i$ children at random.

Show that method 1 is more likely than method 2 to result in the choice of a firstborn child.

Hint: In solving this problem, you will need to show that

$$\sum_{i=1}^{k} in_i \sum_{j=1}^{k} \frac{n_j}{j} \geq \sum_{i=1}^{k} n_i \sum_{j=1}^{k} n_j$$

To do so, multiply the sums and show that, for all pairs i, j, the coefficient of the term $n_i n_j$ is greater in the expression on the left than in the one on the right.

3.9. Consider two independent tosses of a fair coin. Let A be the event that the first toss results in heads, let B be the event that the second toss results in heads, and let C be the event that in both tosses the coin lands on the same side. Show that the events A, B, and C are pairwise independent—that is, A and B are independent, A and C are independent, and B and C are independent—but not independent.

3.10. Consider a collection of n individuals. Assume that each person's birthday is equally likely to be any of the 365 days of the year and also that the birthdays are independent. Let $A_{i,j}$, $i \neq j$, denote the event that persons i and j have the same birthday. Show that these events are pairwise independent, but not independent. That is, show that $A_{i,j}$ and $A_{r,s}$ are independent, but the $\binom{n}{2}$ events $A_{i,j}, i \neq j$ are not independent.