A Signal-Dependent Autoregressive Channel Model

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***Abstract*—A fast and accurate channel model is presented that captures intersymbol interference, signal non-linearities and signal-dependent noise correlation. The noise is modeled as the output of a signal-dependent autoregressive filter. This channel model is well suited for detector designers who desire realistic simulators with short run-times.**

I. Introduction

Modeling high density recording effects, particularly intersymbol interference (ISI), signal nonlinearities and media noise correlation has been the subject of intensive research interest in recent years. The typical tradeoff is speed for accuracy, with detailed but computationally intensive models [1], [2] on one end of the spectrum, media stochastic models [3], [4] in the middle and fast parametric models [5], [6] on the other end. To achieve both speed and accuracy in a single model, the signal-dependent autoregressive (AR) model is proposed, which generalizes ideas presented in [6] to *correlated* signal-dependent noise.

Given a set of conditional, signal-dependent first and second order waveform statistics, i.e., means and covariances, the model generates a process with the same conditional means and covariances. Hence, the model matches the observed waveform up to the second order statistics; not the whole probability distribution. However, when the noise is Gaussian, as is well known, the first and second order statistics uniquely define the probability distribution.

The proposed model has several distinct features. (i) It is conceptually simple and computationally efficient. (ii) Estimation of the model parameters is simple, resting on the computations of the empirical signal-dependent covariance matrices which can be obtained either from real waveforms, or from more detailed media noise models [1]-[4]. (iii) Agreement with experimental data is very good, not only in the parametric comparison sense (first and second order statistics comparison), but also in the detection error-rate comparison sense. (iv) The model rests on a Markov noise assumption which is the necessary condition for formulating optimal (but also suboptimal) signal-dependent sequence detectors [7].

In Section II the channel model is presented. Section III discusses how to generate the channel model parameters from a *seed* waveform which, in turn, is obtained either experimentally or from a slower model. Section IV compares the modeled waveform to an experimental waveform. Finally, in Section V, connections between the proposed AR channel model, the detection algorithms and the recording physics are discussed.

Manuscript received March, 1999.
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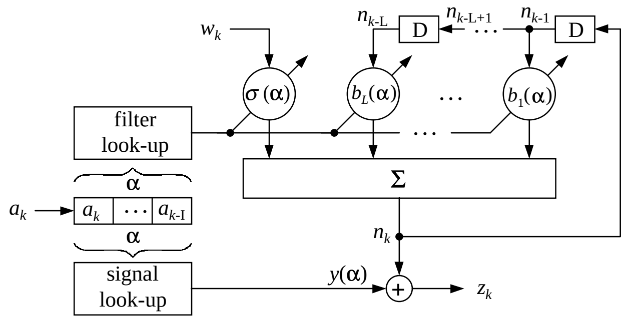

Fig. 1. Autoregressive channel with the data memory length I and signal-dependent Gaussian noise with Markov memory length L.

Notation Throughout the paper, lowercase boldface characters represent column vectors, uppercase boldface characters represent matrices, and the superscript $^{\mathrm{T}}$ denotes the matrix and vector transposition. If $\{x_k\}$ denotes a discrete-time sequence (k denotes time), then for $i \leq j$ the column vector collecting samples $x_i, x_{i+1}, \ldots, x_j$ is denoted by

$$\mathbf{x}_i^j = [x_i \ x_{i+1} \ \cdots \ x_j]^{\mathrm{T}}. \quad (1)$$

The vector $\boldsymbol{\alpha}$ denotes a binary column vector of size $I+1$, where $I \geq 0$ is defined in the text.

II. The model

The channel model is depicted in Figure 1. The sampled channel output is

$$z_k = y\left(\mathbf{a}_{k-I}^k\right) + n_k, \quad (2)$$

where for $k \geq 0$, $\{a_k\}$ is the channel input sequence of binary symbols, $y(\boldsymbol{\alpha})$ is the noiseless channel output which depends on the $I+1$ input symbols $\boldsymbol{\alpha} = \mathbf{a}_{k-I}^k = [a_{k-I}, a_{k-I+1}, \ldots, a_k]^{\mathrm{T}}$, I is the *data memory length*, and $\{n_k\}$ is the additive noise sequence. To include channel non-linearities, $y(\boldsymbol{\alpha})$ is constructed as a look-up-table rather than the more common convolution between the $I+1$ input symbols and a linear channel response. The noise term n_k is the output of a signal-dependent autoregressive filter whose input is a zero-mean unit-variance white Gaussian noise sequence $\{w_k\}$

$$\begin{aligned} n_k &= \sum_{i=1}^{L} b_i\left(\mathbf{a}_{k-I}^k\right) n_{k-i} + \sigma\left(\mathbf{a}_{k-I}^k\right) w_k \\ &= \mathbf{b}(\mathbf{a}_{k-I}^k)^{\mathrm{T}} \mathbf{n}_{k-L}^{k-1} + \sigma\left(\mathbf{a}_{k-I}^k\right) w_k, \quad (3) \end{aligned}$$

where the coefficients of the filter at time k, i.e., the standard deviation $\sigma(\boldsymbol{\alpha})$ and the vector of tap-weights $\mathbf{b}(\boldsymbol{\alpha}) = [b_L(\boldsymbol{\alpha}), \cdots, b_1(\boldsymbol{\alpha})]^{\mathrm{T}}$, depend on $\boldsymbol{\alpha} = \mathbf{a}_{k-I}^k$. This makes the noise sequence $\{n_k\}$ both signal-dependent and correlated (where the correlation is also signal-dependent). Note that, due to the autoregressive filter, the noise sample n_k at time k is, in fact, dependent on $\mathbf{a}_0^k$. However, if the previous L noise samples n_{k-L} through n_{k-1} are known, the dependence may be written as in (3). This is the signal-dependent Markov noise assumption. The length L is the *Markov memory length*.

The pair (I, L) specifies the *size* of the model. Given the channel parameters $y(\boldsymbol{\alpha}), \sigma(\boldsymbol{\alpha})$ and $\mathbf{b}(\boldsymbol{\alpha})$, using equations (2) and (3), we can generate (in a reasonably short time) a sufficiently large number of samples needed to observe low-probability channel error events.

III. Parameter estimation from waveforms

In this section, the channel model parameters are estimated given the model size pair (I, L) and a known seed $\{\mathbf{a}_0^{N-1}, \mathbf{z}_0^{N-1}\}$, with $N >> 2^{I+1}$, i.e., given a known data sequence $\mathbf{a}_0^{N-1}$ and its associated waveform $\mathbf{z}_0^{N-1}$. The waveform may be obtained either experimentally or from a slower model [1]-[4].

From $\{\mathbf{a}_0^{N-1}, \mathbf{z}_0^{N-1}\}$, first the empirical conditional means and covariance matrices are computed. Then, from these conditional first and second order statistics the channel model parameters $y(\boldsymbol{\alpha}), \sigma(\boldsymbol{\alpha})$ and $\mathbf{b}(\boldsymbol{\alpha})$ are derived.

The signal values $y(\boldsymbol{\alpha})$ are conditional mean values, defined as $y(\boldsymbol{\alpha}) = \mathrm{E}\left[z_k | \boldsymbol{\alpha} = \mathbf{a}_{k-I}^k\right]$, for $k \geq I + L$. These conditional mean values are computed empirically as

$$\hat{y}(\boldsymbol{\alpha}) = \frac{1}{N_{\boldsymbol{\alpha}}} \sum_{k:\mathbf{a}_{k-I}^k = \boldsymbol{\alpha}} z_k \tag{4}$$

where $N_{\boldsymbol{\alpha}}$ is the number of occurrences of the vector $\boldsymbol{\alpha}$ in the sequence $\mathbf{a}_0^{N-1}$. Assuming $\mathbf{z}_0^{N-1}$ is ergodic, as $N \to \infty$, we can set $y(\boldsymbol{\alpha}) = \hat{y}(\boldsymbol{\alpha})$; precisely, $\hat{y}(\boldsymbol{\alpha}) \to y(\boldsymbol{\alpha})$ in probability.

The conditional $(L+1) \times (L+1)$ noise covariance matrix is defined as $\mathbf{C}_{L+1}(\boldsymbol{\alpha}) = \{c_{p,q}(\boldsymbol{\alpha})\}$ where $c_{L-i,L-j}(\boldsymbol{\alpha}) = \mathrm{E}\left[n_{k-i} n_{k-j} | \boldsymbol{\alpha} = \mathbf{a}_{k-I}^k\right]$, $0 \leq i, j \leq L$, and $k \geq I + L$. Let $\hat{n}_k = z_k - \hat{y}\left(\mathbf{a}_{k-I}^k\right)$, then the empirical conditional covariance matrices are $\hat{\mathbf{C}}_{L+1}(\boldsymbol{\alpha}) = \{\hat{c}_{p,q}(\boldsymbol{\alpha})\}$ where

$$\hat{c}_{L-i,L-j}(\boldsymbol{\alpha}) = \frac{1}{N_{\boldsymbol{\alpha}}} \sum_{k:\mathbf{a}_{k-I}^k = \boldsymbol{\alpha}} \hat{n}_{k-i} \hat{n}_{k-j}. \tag{5}$$

Similar to the mean approximation, take $\mathbf{C}_{L+1}(\boldsymbol{\alpha}) = \hat{\mathbf{C}}_{L+1}(\boldsymbol{\alpha})$ as $N \to \infty$; precisely, $\hat{\mathbf{C}}_{L+1}(\boldsymbol{\alpha}) \to \mathbf{C}_{L+1}(\boldsymbol{\alpha})$ in probability.

Next, given $y(\boldsymbol{\alpha})$ and $\mathbf{C}_{L+1}(\boldsymbol{\alpha})$, the channel model parameters $\sigma^2(\boldsymbol{\alpha})$ and $\mathbf{b}(\boldsymbol{\alpha})$ are obtained. For a given $\boldsymbol{\alpha}$, the conditional covariance matrix is partitioned as

$$\mathbf{C}_{L+1}(\boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{C}_L(\boldsymbol{\alpha}) & \mathbf{c}(\boldsymbol{\alpha}) \\ \mathbf{c}(\boldsymbol{\alpha})^{\mathrm{T}} & c_{L,L}(\boldsymbol{\alpha}) \end{bmatrix}, \tag{6}$$

where $\mathbf{C}_L(\boldsymbol{\alpha})$ is the $L \times L$ principal minor of $\mathbf{C}_{L+1}(\boldsymbol{\alpha})$ and $\mathbf{c}(\boldsymbol{\alpha}) = [c_{0,L}(\boldsymbol{\alpha}), c_{1,L}(\boldsymbol{\alpha}), \ldots, c_{L-1,L}(\boldsymbol{\alpha})]^{\mathrm{T}}$.

Multiply both sides of (3) by $\mathbf{n}_{k-L}^k$ and then take the conditional expectation, conditioned on $\boldsymbol{\alpha}$. We get the signal-dependent Yule-Walker equations (also known as the normal equations or the augmented Wiener-Hopf equations) [8]

$$\begin{bmatrix} \mathbf{C}_L(\boldsymbol{\alpha}) & \mathbf{c}(\boldsymbol{\alpha}) \\ \mathbf{c}(\boldsymbol{\alpha})^{\mathrm{T}} & c_{L,L}(\boldsymbol{\alpha}) \end{bmatrix} \begin{bmatrix} -\mathbf{b}(\boldsymbol{\alpha}) \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \sigma^2(\boldsymbol{\alpha}) \end{bmatrix}, \tag{7}$$

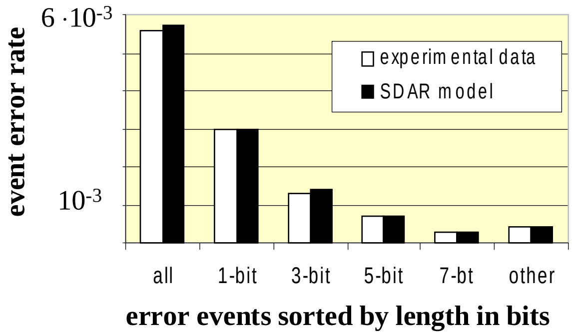

Fig. 2. Error rate comparison between signal-dependent AR (SDAR) model and experiment for several error event lengths.

whose solutions are well-known to be

$$\mathbf{b}(\boldsymbol{\alpha}) = \mathbf{C}_L(\boldsymbol{\alpha})^{-1} \mathbf{c}(\boldsymbol{\alpha}) \tag{8}$$

and

$$\sigma^2(\boldsymbol{\alpha}) = c_{L,L}(\boldsymbol{\alpha}) - \mathbf{c}(\boldsymbol{\alpha})^{\mathrm{T}} \mathbf{C}_L(\boldsymbol{\alpha})^{-1} \mathbf{c}(\boldsymbol{\alpha}) = \frac{\det \mathbf{C}_{L+1}(\boldsymbol{\alpha})}{\det \mathbf{C}_L(\boldsymbol{\alpha})}. \tag{9}$$

The signal-dependent AR model parameter estimates are given by (4), (8) and (9), where (5) approximates $\mathbf{C}_{L+1}(\boldsymbol{\alpha})$ in (6).

IV. Model performance

Obviously, the first and second order statistics of the model and the seed waveform match perfectly since the model parameters were extracted from the very same first and second order statistics of the seed waveform. The real test is the error rate comparison shown in Figure 2. The channel was first equalized to the PR4 target and then the signal-dependent AR model parameters were estimated from real data for the model size of $(I, L) = (9, 15)$, where the size was deliberately overestimated. The error rates in Figure 2 reflect the raw (no error correction code) performance of the PR4 detector applied on a very noisy *real* channel. The first bar compares the overall event error rate, while the subsequent bars compare event error rates for specific event error lengths.

V. Physical versus detector parameters

A fundamental open challenge to researchers is finding clear relationships between physical parameters of storage systems on one extreme and detector architectures on the other.

As presented thus far, the proposed AR channel model accomplishes speed, accuracy and flexibility. Hence, it is useful either to proliferate waveforms or to expedite a slower model. In this section we discuss whether the role of the channel model can be further extended to act as a connection between the worlds of the recording physics and the detector design. It is argued that whereas the AR channel model gives a wealth of information to the detector designers, finding similar connections between the parameters of recording physics and the AR channel model parameters remains a challenge.

In Subsection A, the relationship between the AR channel model and the detector is discussed. A detailed analysis reaches beyond the scope of this paper; see [7]. In Subsection B, the relationship between the AR channel model parameters and the physics of magnetic recording is discussed.

A. Model-based detector design

In Figure 1 the noise correlation is induced by the AR filters $\mathbf{b}(\boldsymbol{\alpha})$. Formulating the sequence detector (Viterbi detector) optimally tuned to the model in Figure 1 is conceptually very simple. For each branch denoted by $\boldsymbol{\alpha}$, the branch metric is the squared output of a finite impulse response (FIR) filter that is the exact inverse of the AR filter in Figure 1; for details, see [7]. The FIR filter can thus be considered to be a branch-dependent noise "whitening" filter. It is important to realize (in theory at least) that this branch-dependent noise "whitening" must be done with an FIR filter (for if it were not FIR, we would get an infinitely long ISI tail, requiring an infinitely complex Viterbi detector for optimal detection). Thus, when the channel model is the AR model depicted in Figure 1, the optimal detector design is canonically obtained by simply inverting the AR filters to obtain a bank of "whitening" FIR filters.

The AR channel model is also very useful for designing not just optimal, but also suboptimal detectors. For example, the NPML detector is based on the same FIR filters as the optimal detector, but the state reduction in the NPML detector is achieved by feeding back processed past decisions into the FIR filter [9]. Other suboptimal detectors (e.g., the K-step detector [10] and the Q-AR detector [7]) may be obtained by fitting low-order AR filters to the waveforms and utilizing their corresponding inverse filters (i.e., low-order FIR filters) in the detector; for details, see [7].

Finally, when the channel noise is memoryless, i.e., in the absence of AR filters, the parameters $y(\boldsymbol{\alpha})$ and $\sigma(\boldsymbol{\alpha})$ determine the Viterbi detector branch metrics.

B. AR model parameters and recording physics

Intuitively pleasing models of media noise that relate well to physical properties of the recording system are achieved through Karhunen-Loeve decompositions of covariance matrices [11] which identify pulse jitter and amplitude variations as the dominant media noise modes [12]. However, Karhunen-Loeve decompositions are *static* matrix decompositions that lead to good signal-processing algorithms in *static* detection problems where infinite time (in practice, a relatively long time) is available to reach a decision (a typical example is the radar detection problem [13]). For *dynamic* sequence detectors/estimators where decisions are made on the fly, the more suitable matrix decomposition is the Cholesky decomposition of the covariance matrix *inverse*, which actually is the AR noise model [14].

Hence, the challenge of linking physical and model parameters is equivalent to toggling between the Karhunen-Loeve decomposition and the Cholesky decomposition. However, the Karhunen-Loeve decomposition is based on the eigenvalue decomposition, which is reached by solving polynomial equations, and cannot be expressed in closed-form for polynomial orders greater than 5. A further difficulty is that the Cholesky decomposition is performed on the covariance matrix *inverse* which brings into play the uncomfortable task of taking the parameters of recording physics through a matrix inversion.

VI. Conclusion

The signal-dependent autoregressive model for magnetic recording channels was presented with intersymbol interference signal nonlinearities and signal-dependent and correlated media noise. Several attractive features of the model were demonstrated: simplicity, direct relationship to optimal detector design, straight-forward parameter estimation procedure, and good error rate comparisons to experimental data. The model parameters can be estimated from any experimentally obtained waveform and the optimal detector can canonically be designed based on those estimates. An interesting, but a challenging problem for future research is to relate the signal-dependent AR model parameters to physical properties of recording systems: media coercivity, remanence, and thickness, head geometry, MR sensitivity, non-linear transition shift, flying height (magnetic separation), etc.

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