Maximum Likelihood Sequence Estimation in Channels with Intersymbol Interference and Noise Memory

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Abstract — **We extend the Viterbi algorithm to observable sequences with memory and demonstrate that the complexity of the solution remains only linearly proportional to the sequence length, as long as the memory of the observable process is Markov.**

I. Introduction

The Viterbi algorithm, originally introduced as a method for decoding convolutional codes, has become one of the mostly used detectors in digital communications. Forney [1] has shown in 1972 that the algorithm solves maximum likelihood sequence detection (MLSD) of a pulse amplitude modulated (PAM) sequence of symbols with finite intersymbol interference (ISI) and memoryless noise. The algorithm has earned its place in almost every modern digital communications textbook where it is recognized as the optimal sequence detector in *memoryless* noise.

While many modifications, applications and implementations of the algorithm have been discussed in the literature, the belief remains that the algorithm provides the optimal solution only in memoryless channels. It was noticed in [2] that the Viterbi algorithm solves MLSD of a state sequence when the noise is additive Markov. In this paper, we take this idea a step further and show that not only can the noise be additive Markov, but the observable process itself can be correlated, Markov and exhibit signal-dependent memory.

Let the state sequence of a finite-state machine be represented by a sequence x_k, where x_k takes one of $M > 0$ possible values. We assume that the sequence is Markov, i.e., the state transition probabilities exhibit no memory

$$\mathrm{P}\left(x_k|x_{k-1}, x_{k-2}, \ldots, x_{-\infty}\right) = \mathrm{P}\left(x_k|x_{k-1}\right). \qquad (1)$$

Let the observable sequence be represented by z_k. For a memoryless channel, the probability density function of the sample z_k, conditioned on the infinite state-transition history, depends only on the latest transition (x_k, x_{k+1}), i.e.,

$$f\left(z_k|x_{-\infty}, \ldots, x_{\infty}\right) = f\left(z_k|x_k, x_{k+1}\right). \qquad (2)$$

It is well known that the Viterbi algorithm is the optimal detector when conditions (1) and (2) hold.

Consider now a channel *with* memory. We define a channel with Markov memory of length $L \geq 0$ by

$$f\left(z_k|z_{k-1}, \ldots, z_{-\infty}, x_{-\infty}, \ldots, x_{\infty}\right) = f\left(z_k|z_{k-1}, \ldots, z_{k-L}, x_k, x_{k+1}\right). \qquad (3)$$

An important channel that exhibits these characteristics is a channel with ISI and additive correlated noise which exhibits a signal-dependent (state-path dependent) memory.

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II. Optimal Correlation-Sensitive Sequence Estimation

In [3], we derive the MLSD for channels in which conditions (1) and (3) hold. We refer to it as the correlation-sensitive signal-dependent sequence detector. It extends the Viterbi algorithm to channels with ISI and signal-dependent noise with memory. The two most distinct features of the detector are the following. First, the number of states in the detector (assuming binary signaling) is expanded to 2^{I+L}, where I is the ISI length and L is the Markov memory length. Second, the branch metrics are computed from a sliding window of $L+1$ observations and the branch metric computation formula is different for every branch since the noise has signal-dependent memory.

In [3], we show that if the channel noise is correlated Gaussian (not necessarily stationary), the branch metric computation is performed by a finite impulse response (FIR) filter. For optimal sequence detection in this signal-dependent noise environment, the detector requires a distinct FIR filter for every branch of the trellis. The FIR filter weights are related to the covariance statistics of the signal-dependent noise through nonstationary Yule-Walker equations. These statistics can be adaptively estimated form the data (sampled waveforms) on the fly [4].

III. Conclusion

We have extended the Viterbi algorithm to solve a much wider class of sequence detection problems than just detection in memoryless noise. In fact, the Viterbi algorithm is the optimal sequence estimator in a channel with memory, as long as the memory is Markov. In practice, this corresponds to a channel with a finite intersymbol interference (ISI) length and correlated additive Gauss-Markov noise with a finite state-path dependent memory. In such a channel, the optimal solution is to expand the states to account for the finite Markov memory and then employ a distinct uncorrelating finite impulse response (FIR) filter in each branch of the Viterbi trellis.

References

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