# EE 342 PROBLEM SET 1
## (SOLUTIONS)

3!·4!

**1** CH 1, PROBLEM 7

a) TOTAL 6 PEOPLE (KIDS) $\Rightarrow$ 6! SEATING ARRANGEMENTS

$$6! = \boxed{720}$$

b) 3! WAYS 3 GIRLS CAN SIT TOGETHER

3! WAYS 3 BOYS CAN SIT TOGETHER

2 WAYS TO ARRANGE (EITHER BOYS SIT FIRST OR GIRLS FIRST)

$$2 \cdot 3! \cdot 3! = 2 \cdot 6 \cdot 6 = \boxed{72}$$

c) 3! WAYS IN WHICH BOYS CAN SIT TOGETHER

4! WAYS IN WHICH GIRLS CAN SIT AROUND THEM

$$3! \cdot 4! = 6 \cdot 24 = \boxed{144}$$

d) 3! WAYS GIRLS CAN TAKE ALTERNATING SEATS

3! WAYS BOYS CAN TAKE ALTERNATING SEATS

2 WAYS TO ARRANGE (EITHER GIRLS OR BOYS TAKE ODD-NUMBERED SEATS)

$$2 \cdot 3! \cdot 3! = 2 \cdot 6 \cdot 6 = \boxed{72}$$

**2** CH 1, PROBLEM 13

NUMBER OF HAND SHAKES $\binom{20}{2} = \frac{20 \cdot 19}{2} = \boxed{190}$

3 CH 2, PROBLEM 11

$P(A) = 0.28$

$P(B) = 0.07$

$P(AB) = 0.05$

a) $P(\overline{A \cup B})$ IS THE PERCENTAGE (PROBABILITY) OF NONSMOKERS

$$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - P(A) - P(B) + P(AB) =$$

$$= 1 - 0.28 - 0.07 + 0.05$$

$$= 0.7 \quad \text{or} \quad \boxed{70\%}$$

b) $P(\bar{A}B) = P(B(A \cup \bar{A})) - P(AB)$

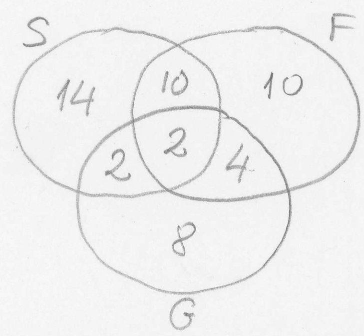
$$= P(B) - P(AB)$$

$$= 0.07 - 0.05$$

$$= 0.02 \quad \text{or} \quad \boxed{2\%}$$

4 CH 2, PROBLEM 12

VENN DIAGRAM




a) $\dfrac{\#\text{ STUDENTS NOT TAKING A LANGUAGE CLASS}}{\text{TOTAL }\#\text{ OF STUDENTS}} = \dfrac{100 - 50}{100} = \dfrac{1}{2}$

b) $\dfrac{14 + 10 + 8}{100} = \dfrac{32}{100} = 0.32$

c) $P(N) = P(\text{NEITHER STUDENT IS TAKING A LANGUAGE CLASS}) = \dfrac{50}{100} \cdot \dfrac{49}{99}$

$$P(\text{AT LEAST ONE STUDENT IS TAKING A LANG. CLASS}) = 1 - P(N)$$

$$= 1 - \frac{50}{100} \cdot \frac{49}{99}$$

$$= 1 - \frac{49}{2 \cdot 99} = \frac{2 \cdot 99 - 49}{198}$$

$$= \frac{198 - 49}{198} = \boxed{\frac{149}{198}}$$

## 5 CH 2, PROBLEM 35

TOTAL NUMBER OF POSSIBLE WAYS OF CHOOSING 7 BALLS

$$n = \binom{46}{7}$$

a) $$\frac{\binom{12}{3} \cdot \binom{16}{2} \cdot \binom{18}{2}}{\binom{46}{7}}$$

b) $$P(\text{NO RED BALL IS DRAWN}) = \frac{\binom{34}{7}}{\binom{46}{7}}$$

$$P(\text{EXACTLY 1 RED BALL IS DRAWN}) = \frac{\binom{34}{6} \cdot \binom{12}{1}}{\binom{46}{7}}$$

$$P(\text{2 OR MORE RED BALLS ARE DRAWN}) = 1 - \frac{\binom{34}{7}}{\binom{46}{7}} - \frac{\binom{34}{6}\binom{12}{1}}{\binom{46}{7}}$$

c) $$P(\text{ALL RED}) = \frac{\binom{12}{7}}{\binom{46}{7}}$$

$$P(\text{All blue}) = \frac{\binom{16}{7}}{\binom{46}{7}}$$

$$P(\text{all green}) = \frac{\binom{18}{7}}{\binom{46}{7}}$$

$$P(\text{all same color}) = \frac{\binom{12}{7} + \binom{16}{7} + \binom{18}{7}}{\binom{46}{7}}$$

d) $$P(\text{EXACTLY 3 RED}) = P(R) = \frac{\binom{12}{3} \cdot \binom{34}{4}}{\binom{46}{7}}$$

$$P(\text{EXACTLY 3 BLUE}) = P(B) = \frac{\binom{16}{3} \cdot \binom{30}{4}}{\binom{46}{7}}$$

$$P(\text{EXACTLY 3 BLUE AND EXACTLY 3 RED}) = P(RB) = \frac{\binom{12}{3} \cdot \binom{16}{3} \cdot \binom{18}{1}}{\binom{46}{7}}$$

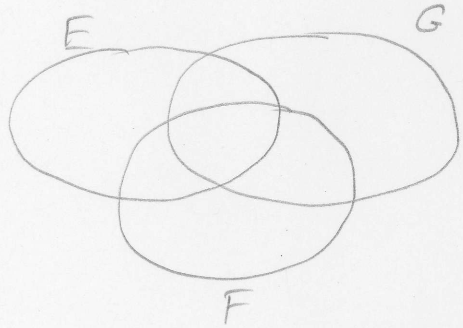
$$P\left(\begin{matrix}\text{EITHER EXACTLY 3 BLUE} \\ \text{OR EXACTLY 3 RED}\end{matrix}\right) = P(R \cup B)$$

$$= P(R) + P(B) - P(RB)$$

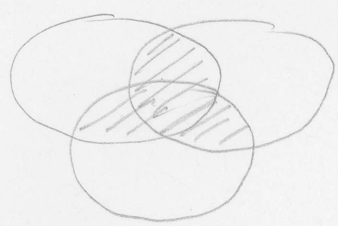
$$= \frac{\binom{12}{3}\binom{34}{4} + \binom{16}{3}\binom{30}{4} - \binom{12}{3}\binom{16}{3}\binom{18}{1}}{\binom{46}{7}}$$

6 CH 2, THEO. EXCERCISE 6

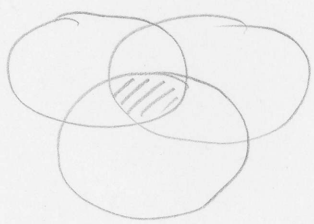
VENN DIAGRAM




d) $EG \cup GF \cup EF$



e) $EGF$



f) $\overline{E \cup G \cup F}$

7 CH 2, THEO. EXCERCISE 11

$P(S) \geq P(E \cup F)$ because $(E \cup F) \subset S$

$1 \geq P(E \cup F)$ because $P(S) = 1$

$1 \geq P(E) + P(F) - P(EF)$ because $P(E \cup F) = P(E) + P(F) - P(EF)$

$P(EF) \geq P(E) + P(F) - 1$ rearranging

$P(EF) \geq 0.9 + 0.8 - 1 = 0.7$ substitute probability values

$\boxed{8}$

a) FROM PROBLEM $\boxed{7}$, WE HAVE

$$P(AB) \geq P(A) + P(B) - 1$$

$$P(AB) \geq \frac{3}{4} + \frac{1}{3} - 1 = \frac{9+4-12}{12} = \frac{1}{12}$$

NOW $\quad P(AB) \leq P(B) = \frac{1}{3}$

AND $\quad P(AB) \leq P(A) = \frac{3}{4}$

COMBINING ALL THREE, WE GET

$$\boxed{\frac{1}{12} \leq P(AB) \leq \frac{1}{3}}$$

* $P(AB) = \frac{1}{3}$ IS POSSIBLE WHEN $B \subset A$

  BECAUSE THEN $P(B) = P(AB) = \frac{1}{3}$

* $P(AB) = \frac{1}{12}$ IS POSSIBLE WHEN $A \cup B = S$

  BECAUSE THEN

$$1 = P(S) = P(A \cup B) = P(A) + P(B) - P(AB)$$

$$= \frac{3}{4} + \frac{1}{3} - \frac{1}{12}$$

b) $$\boxed{\frac{3}{4} \leq P(A \cup B) \leq 1}$$

$\boxed{9}$ a) SIX TURNS UP ONLY ONCE IN EXACTLY 10 OUTCOMES

$$\Rightarrow \boxed{P(A) = \frac{10}{36}}$$

b) Both numbers are odd IN EXACTLY 9 OUTCOMES

$$\Rightarrow \boxed{P(B) = \frac{9}{36} = \frac{1}{4}}$$

c) THE OUTCOMES WITH SUM 4 ARE

(1,3) , (2,2) AND (3,1) $\Rightarrow$ only 3 outcomes

$$\Rightarrow \boxed{P(C) = \frac{3}{36} = \frac{1}{12}}$$

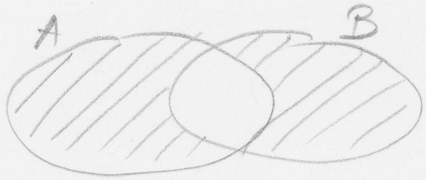
d) THE OUTCOMES WHOSE SUM IS DIVISIBLE BY 3 IS

(1,2) (1,5)
(2,1) (2,4)
(3,3) (3,6)
(4,2) (4,5)
(5,1) (5,2)
(6,3) (6,6)

$\Rightarrow$ TOTAL 12 OUTCOMES

$$\Rightarrow \boxed{P(D) = \frac{12}{36} = \frac{1}{3}}$$

10 VENN DIAGRAM. FIND THE PROBABILITY OF SHADED REGION




THE REGION OF INTEREST IS $A\bar{B} \cup \bar{B}A$

$P(A\bar{B} \cup \bar{B}A) = P(A\bar{B}) + P(\bar{B}A)$ because $(A\bar{B}) \cap (\bar{B}A) = \emptyset$

$= P(A) - P(AB) + P(\bar{B}A)$ because $(AB) \cap (A\bar{B}) = \emptyset$ and $(AB) \cup (A\bar{B}) = A$

$= P(A) - P(AB) + P(B) - P(AB)$ because $(AB) \cap (\bar{B}A) = \emptyset$ and $(AB) \cup (\bar{B}A) = B$

$= P(A) + P(B) - P(AB)$