Expedient Media Noise Modeling: Isolated and Interacting Transitions

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***Abstract*— We propose a stochastic media model as a faster alternative to micromagnetic media modeling. The basic building block of the model are zig-zag patterns formed by triangles. We present equations that relate the statistics of the stochastic model to recording process parameters. Both isolated and interacting transitions are modeled. Simulation results are presented to show performance of the model.**

I. Introduction

There is renewed interest in finding expedient media noise models. Micromagnetic modeling [1], [2] is probably the most accurate model for media (transition) noise. Although accurate, micromagnetic modeling is not an ideal tool for statistical studies of readback systems (e.g., error rate predictions), where generation of thousands of bits (transitions) is required. Generating a single micromagnetic transition on a media sample of 50×50 grains takes about 10-20 minutes on an average workstation, which is prohibitive for signal processing purposes.

Statistical zig-zag models, on the other hand, are an attractive alternative. These models try to capture the statistical essence of the random zig-zag line (wall) that separates oppositely magnetized regions of the media. Arnoldussen and Tong [3], and Middleton and Miles [4], [5] suggest zig-zag patterns where the peak-to-peak distances are the independent random variables of the model. This leads to instability (a down-track wall drift) since the model is then an independent increment random process [6]. One way to fix this is to use deconvolution of the peak-to-peak probability density function (pdf) to obtain a zero-to-peak pdf as in [7]. In this paper, we circumvent this problem by presenting a model that involves only zero-to-peak distances in the form of triangle heights, rather than peak-to-peak distances. Thus, we solve the instability issue by avoiding altogether an independent increment random process. Furthermore, the geometry of our model allows us to find unique relationships between the model defining quantities and the recording process parameters (transition profile and cross-track correlation width). We call our model the triangle zig-zag transition (TZ-ZT) model. TZ-ZT modeling is 10^4 times faster than micromagnetic modeling. We show results comparing the TZ-ZT to micromagnetic modeling. We also propose a modification to the TZ-ZT model to incorporate high density nonlinearities.

II. Triangle Zig-zag Transition Modeling

The triangle zig-zag transition (TZ-ZT) model is a stochastic model of the zig-zag line that separates two oppositely magnetized regions of the magnetic medium. The TZ-ZT model

Manuscript received March 4, 1996.
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This work was supported in part by the National Science Foundation under Grant No. ECD-8907068. The United States government has certain rights in this material.

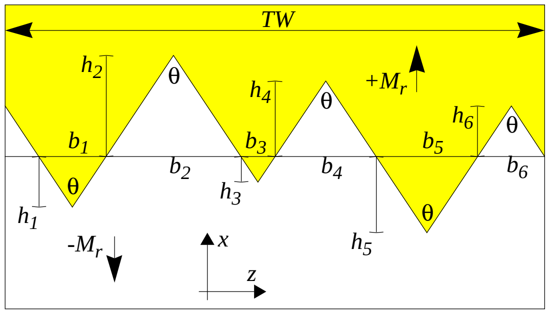

Fig. 1. The triangle zig-zag transition model.

(illustrated in Figure 1) is constructed by placing side-by-side isosceles triangles of alternating orientations on the line representing the nominal transition center. The triangle heights $(h_1, h_2, \ldots)$ are independent random variables drawn from a probability density function (pdf) $f_H(h)$. The vertex angle θ is chosen to be constant. This makes it possible to find a relationship between θ and the cross-track correlation width, which we actually use to determine θ for a given medium.

III. Isolated Transitions

A. Transition Profile and Jitter Noise Modeling

Denote by $M_x(x)$ the average down-track magnetization profile, where x denotes the down-track direction. We are assuming that for $x > 0$, $M_x(x) > 0$ and that $M_x(x)$ is odd, e.g., $M_x(x) = \frac{2M_r}{\pi} \cdot \mathrm{atan}\left(\frac{x}{a}\right)$ or $M_x(x) = M_r \cdot \mathrm{erf}\left(\frac{x}{\sqrt{2}\sigma}\right)$, where M_r is the remanent magnetization.

Theorem 1 *The average transition profile $M_x(x)$ is related to the pdf of TZ-ZT heights $f_H(h)$ for an isolated transition through*

$$f_H(h) = -\frac{M_x''(x)}{M_x'(0)} \quad \text{for} \quad h \geq 0, \tag{1}$$

where $M_x''(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left[M_x'(x)\right] = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left[M_x(x)\right]$.

The proof of Theorem 1 escapes the length constraints of this paper, but can be found in [6]. The tricky part in the proof is to relate the zig-zag patterns to renewal theory. To avoid errors made in previous attempts to derive similar relationships, we need to recognize the paradox of residual life in renewal theory [8]. Due to this paradox, the pdf $p(w)$ in equation (18) of [4] should be replaced by $w \cdot p(w)/\mathrm{E}[w]$. This will change the relationship between the magnetization profile and the sawtooth pdf $p(w)$ in Equations (23) in [4] and (5) in [5]. See [6] for details.

The cross-track correlation width s (defined in [9]) is correlated with the jitter present in the readback signal. We therefore find an expression for the cross-track correlation width s_{TZZT} of the TZ-ZT model.

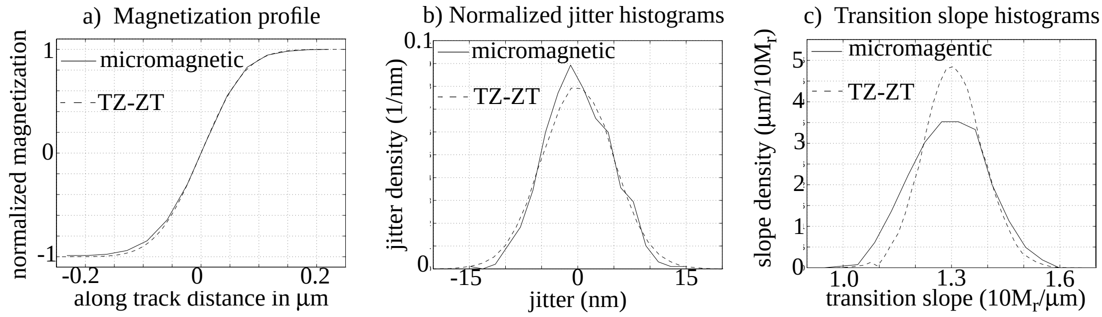

Fig. 2. Comparison of TZ-ZT and micromagnetic statistics.

Theorem 2 *Denote by B the random variable that represents the bases of the zig-zag triangles used in the TZ-ZT model. The TZ-ZT cross-track correlation width is then*

$$s_{\text{TZZT}} = \frac{\mathrm{Var}\,(B)}{\mathrm{E}\,[B]}, \tag{2}$$

where $\mathrm{E}\,[B]$ is the mean and $\mathrm{Var}\,(B)$ is the variance of B.

Due to space constraints, we relegate the proof to [10]. We should note that Equation (2) in Theorem 2 holds for other types of models too. In particular, in [10], we apply Theorem 2 to the microtrack model of [11] to show that the cross-track correlation width, as computed by (2), equals the microtrack width. We interpret Equation (2). Let B be the random variable representing the cross-track size of a cluster. Let the magnetization of adjacent clusters point in opposite down-track directions, both with intensity M_r. Then Equation (2) gives the cross-track correlation width. Since the cross-track correlation width is correlated with the jitter noise level, to have low jitter noise we either reduce the cluster size variance $\mathrm{Var}\,(B)$ or increase the average cluster size $\mathrm{E}\,[B]$. The latter, however, is not an option in high density recording because it requires increasing the track width.

Corollary 2.1 *The TZ-ZT modeled medium and the thin film medium being modeled have the same cross-track correlation width s if the TZ-ZT vertex angle θ is*

$$\theta = 2\mathrm{atan}\left[\frac{s \cdot \mathrm{E}\,[H]}{2 \cdot \mathrm{Var}\,(H)}\right], \tag{3}$$

where H represents the random TZ-ZT heights.

Proof: From Figure 1, the bases b_i are related to the heights h_i as $b_i = h_i \cdot 2\tan\frac{\theta}{2}$. If we substitute $B = H \cdot 2\tan\frac{\theta}{2}$ into (2) and solve the resulting equation for θ, we get (3).

B. Modeling Results

We modeled a magnetic thin-film with the following characteristics: remanent magnetization $M_r = 625\text{emu/cm}^3$, coercivity $H_c = 1670\text{Oe}$, media thickness $\delta = 400\text{Å}$, orientation ratio O.R.= 1.3. The chosen track width was $TW = 4.8\mu\text{m}$. We used a Karlqvist writing head with gap length $g = 0.28\mu\text{m}$ and flying height (magnetic spacing) $d = 0.1\mu\text{m}$.

Using the micromagnetic model, we obtained 30 independent isolated magnetization profiles (normalized to M_r). We found their average to be $M_x(x) \approx \mathrm{erf}\left(\frac{x}{\sqrt{2}\sigma}\right)$, with $\sigma = 610\text{Å}$. Applying Theorem 1 to an erf magnetization profile, we find the triangle heights pdf $f_H(h)$ to be a Rayleigh pdf

$$f_H(h) = -\frac{M_x''(x)}{M_x'(0)} \approx \frac{h}{\sigma^2}\exp\left(-\frac{h^2}{2\sigma^2}\right) \quad \text{for} \quad h \geq 0. \tag{4}$$

The shape of a Rayleigh pdf matches well the experimental findings [3]. To find the vertex angle θ of the TZ-ZT model, we first calculate the sample magnetization variance $\hat{\sigma}_M^2(x)$ from the same 30 isolated micromagnetic profiles. The magnetization variance σ_M^2 is related to the normalized magnetization M through the equality $\sigma_M^2 = \frac{s}{TW}\left(1 - M^2\right)$, where s is the cross-track correlation width [12]. By least-squares fitting a parabola to the curve of $\hat{\sigma}_M^2(x)$ versus $M_x(x)$, we obtain $s = 197\text{Å}$. Corollary 2.1 yields then $\theta = 50.7^\circ$, where we used $\mathrm{E}\,[H] = \sigma\sqrt{\pi}/\sqrt{2}$ and $\mathrm{Var}(H) = (2 - \pi/2)\sigma^2$ for a Rayleigh distributed random variable H as in (4).

Figure 2 compares the statistical properties for isolated pulses of the TZ-ZT model with those of the micromagnetic model. The plots are based on 500 independent runs of the micromagnetic model and 50,000 runs of the TZ-ZT model. The transition shape and jitter histograms match almost perfectly, see Figures 2-a and 2-b. The transition slope variance shows a 30% mismatch, see Figure 2-c. This is a second order effect with little impact on the total noise power since the amplitude (slope) variations are weaker than the jitter for isolated pulses [13].

IV. Interacting Transitions

A. Nonlinear Bit Shift and Amplitude Loss Modeling

We model nonlinear bit shift and partial signal erasure (percolation) by modifying the existing TZ-ZT model for isolated transitions. Our model for interacting transitions relies on results derived by Bertram [9]. Equation (9.7) in [9] shows that the bit shift is

$$\Delta x = \frac{4M_r\delta\,(d + \delta/2)^3}{\pi Q H_c B^3}, \tag{5}$$

where B is the nominal separation between the current and the previous transition and Q is the head-field gradient factor. We apply this formula by writing the current transition closer to the previous one by Δx.

The standard deviation (square root of variance) of the transition location changes with the distance between transitions. A simple model for this change is Equation (12.33) in [9], where

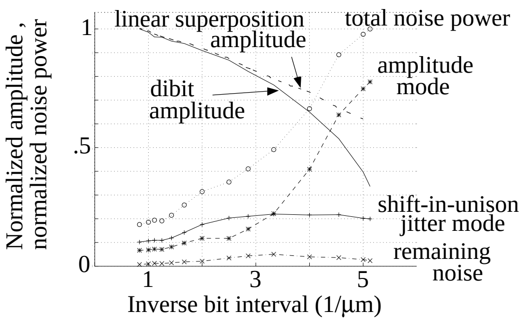

Fig. 3. Normalized dibit amplitude and normalized media noise mode powers as a function of inverse transition spacing.

it is shown that the standard deviation of interacting transitions is larger than that of isolated transitions by a factor

$$\left(1 - \frac{8M_r\delta d^2}{\pi B^3 Q H_c}\right)^{-1/2} . \tag{6}$$

This is easily incorporated into the TZ-ZT model by correcting the TZ-ZT heights for isolated transitions by the same factor (6). After making adjustments in (5) and (6), we model partial erasure by letting the magnetization percolate through the region where zig-zags of neighboring transitions overlap. Another way to model partial erasure is to let the magnetization percolate whenever the distance between adjacent zig-zags is less than some distance L [11], [14], but it yet needs to be determined how L depends on media parameters.

B. Modeling Results

We wrote TZ-ZT dibits and read it with a Lindholm head for different transition spacings. For each transition separation, we obtained 500 independent dibit waveforms. After subtracting their mean, we obtained the dibit media noise waveforms, and calculated their empirical correlation function. We decomposed the correlation function into its principal components (modes) using the Karhunen-Loeve decomposition (KLD) [13]. The KLD revealed that two basic noise modes (amplitude variation mode and shift in unison jitter mode) dominate the dibit media noise. Their relative contribution to the total noise power changes with transition separation, as shown in Figure 3. At large transition separations, jitter is the dominant noise mode, while at short spacings the amplitude variations dominate. This plot, obtained with TZ-ZT modeling, is similar to experimental results obtained in [13]. Figure 3 also shows the dibit amplitude as a function of inverse transition separation and the amplitude obtained by linear pulse superposition.

V. Conclusion

We presented a computationally efficient media and media noise model. Magnetization transitions are represented as portions of a zig-zag line across the track. The zig-zag line is a random process obtained by placing isosceles triangles on a common basis line. Formulas are presented that link the defining quantities of the model to recording parameters. For isolated transitions, the transition shape and jitter noise are accurately modeled. For interacting transitions, nonlinear amplitude loss and high density media noise are also incorporated into the model. The model can be applied to generate signals with high density nonlinearities and media noise, and is therefore useful for statistical studies (error rate studies) of readback subsystems.

Acknowledgment

We thank Pu-Ling Lu for providing us software and sharing his experience in micromagnetic modeling.

References

[1] J.-G. Zhu and N. H. Bertram, "Recording and transition noise simulations in thin film media," *IEEE Trans. Magn.*, vol. 24, pp. 2706–2708, Nov. 1988.

[2] P.-L. Lu and S. Charap, "Angular variations of coercivity in thin-film media by micromagnetic model," *IEEE Trans. Magn.*, vol. 28, pp. 986–989, March 1992.

[3] T. C. Arnoldussen and H. C. Tong, "Zigzag transition profiles, noise, and correlation statistics in highly oriented longitudinal media," *IEEE Trans. Magn.*, vol. 22, pp. 889–891, Sept. 1986.

[4] B. K. Middleton and J. J. Miles, "Sawtooth magnetization transitions and the digital recording properties of thin film recording media," in *IEE Conf. Proc.*, pp. 20–25, April 1991.

[5] B. K. Middleton and J. J. Miles, "Recorded magnetization distributions in thin film media," *IEEE Trans. Magn.*, vol. 27, pp. 4954–4959, Nov. 1991.

[6] A. Kavčić and J. M. F. Moura, "Triangle zig-zag transition modeling," tech. rep., Data Storage Systems Center, Dept. of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA, 1995. 54 pages.

[7] Y.-S. Tang and L. Osse, "Zig-zag domains and metal film disk noise," *IEEE Trans. Magn.*, vol. MAG-23, pp. 2371–2373, Sept. 1987.

[8] L. Kleinrock, *Queueing Systems.* New York: John Wiley & Sons, 1975.

[9] H. N. Bertram, *Theory of Magnetic Recording.* Cambridge: Cambridge University Press, 1994.

[10] A. Kavčić and J. M. F. Moura, "Statistical study of zig-zag transition boundaries in longitudinal digital magnetic recording," *IEEE Trans. Magn.*, vol. 33, pp. 4482–4491, Nov. 1997.

[11] J. Caroselli and J. K. Wolf, "A new model for media noise in thin film magnetic recording media," in *Proc. SPIE: Coding and Signal Processing for Information Storage*, vol. 2605, (Philadelphia), pp. 29–38, Oct. 1995.

[12] H. N. Bertram and X. Che, "General analysis of noise in recorded transitions in thin film recording media," *IEEE Trans. Magn.*, vol. 29, pp. 201–208, 1993.

[13] J.-G. Zhu and X.-G. Ye, "Impact of medium noise on various partial response channels," *IEEE Trans. Magn.*, vol. 31, pp. 3087–3089, Nov. 1995.

[14] R. D. Brandt, A. J. Armstrong, H. N. Bertram, and J. K. Wolf, "A simple statistical model of partial erasure in thin film disk recording systems," *IEEE Trans. Magn.*, vol. MAG-27, pp. 4978–4980, Nov. 1991.