# EE 615: Problem set 6 (*due on Wednesday, April 20*)

Problems 3.22, 3.23, 10.5 in 4th edition (14.5 in 5th edition), 10.X1

## *Problem 10.X1(due on Wednesday, April 20)*

Prove that (10.54) and (10.55) in the 4th edition textbook are equivalent. In the 5th edition, the corresponding equations are (14.54) and (14.55).

## *Computer Assignment (due Monday, April 25)*

**The answer to the computer assignment is program listings and relevant plots.**

Implement in Matlab the Kalman Filter as outlined in table 10.2, with the modification that the output should be the filtered estimate $\hat{\mathbf{x}}\left(n \middle| Y_n\right)$ and not the predicted estimate $\hat{\mathbf{x}}\left(n+1 \middle| Y_n\right)$. In the following let $n$ be the dimension of the state vector, $m$ the dimension of the observation vector and $N$ the total number of time steps. The Kalman Filter has the following input and output

Input

1. State transition matrices $\mathbf{F}(k+1,k)$, $k = 1..N$. Use multidimensional arrays.
2. Measurement matrices $\mathbf{C}(k)$, $k = 1..N$. Use multidimensional arrays.
3. Correlation matrices $\mathbf{Q}_1(k)$, $k = 1..N$. Use multidimensional arrays.
4. Correlation matrices $\mathbf{Q}_1(k)$, $k = 1..N$. Use multidimensional arrays.
5. Observation vectors $\mathbf{y}(k)$, $k = 1..N$. Combined into a matrix of dimension $(m \times N)$.
6. Initial values for $\mathbf{x}$ and $\mathbf{K}$.

Output

1. $\hat{\mathbf{x}}\left(k \middle| Y_k\right), k = 1..N$ combined into one matrix of dimension $(n \times N)$.
2. Filtered error covariance matrix $\mathbf{K}(k)$ , $k = 1..N$. Use multidimensional arrays.

### Application example

Consider a vehicle moving in two dimensions. The available measurement data are noisy measurements of the (x,y) position.

The state equations of the vehicle are

$$\begin{bmatrix} p_x(n+1) \\ p_y(n+1) \\ v_x(n+1) \\ v_y(n+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x(n) \\ p_y(n) \\ v_x(n) \\ v_y(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ u_x(n) \\ u_y(n) \end{bmatrix}$$

and the measurement equations

$$\begin{bmatrix} m_x(n) \\ m_y(n) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x(n) \\ p_y(n) \\ v_x(n) \\ v_y(n) \end{bmatrix} + \begin{bmatrix} n_x(n) \\ n_y(n) \end{bmatrix}$$

where

$\Delta T$ : Sampling time.

$p_x(n)$ : x - position at time $n\Delta T$

$p_y(n)$ : y - position at time $n\Delta T$

$v_x(n)$ : x - velocity at time $n\Delta T$

$v_y(n)$ : y - velocity at time $n\Delta T$

$m_x(n)$ : measured x - position at time $n\Delta T$

$m_y(n)$ : measured y - position at time $n\Delta T$

$u_x(n)$ : external disturbance (accelleration), random variable with variance $\sigma_u{}^2$

$u_y(n)$ : external disturbance (accelleration), random variable with variance $\sigma_u{}^2$

$n_x(n)$ : measurement noise, random variable with variance $\sigma_n{}^2$

$n_y(n)$ : measurement noise, random variable with variance $\sigma_n{}^2$

In the following we put $\Delta T = 1$, and $\sigma_u{}^2 = 0.0001, \sigma_n{}^2 = 0.1$, and the noise variables are mutually uncorrelated. Furthermore we consider 100 points in time, $n = 1..100$.

1. Make a simulation model of the given system. For the noise variables, use Gaussian (normal) distrubted random numbers. The initial (actual) state is

$$\mathbf{x}[0] = [10 \quad -5 \quad -0.2 \quad 0.2]^T$$

2. Apply the Kalman filter to the output of the simulation model. The initial condition used for the Kalman filter is

$$\hat{\mathbf{x}}[1\,|\,0] = [5 \quad 5 \quad 0 \quad 0]^T$$
$$\mathbf{K}[1\,|\,0] = 100\mathbf{I}$$

3. For a few realizations, plot the actual (x,y) postions, the measured (x,y) positions and the (x,y) positions estimated by the Kalman filter in the same plot, $n = 1..100$.
4. Calculate the variance of the Kalman estimate of the x-position as a function of $n$. This can be done be running at least 100 simulations (with different noise realizations), and using the Matlab function *cov* for each value of $n$.

5 Plot the variance calculated in 4 together with the Kalman filter covariance of x, i.e. $\mathbf{K}[n]_{1,1}$. Are they equal?