Approaching the Capacity of the MIMO Rayleigh Flat-Fading Channel w/ QAM Constellations, Independent across Antennas and Dimensions

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Abstract — **This study explores the use of the Quadrature Amplitude Modulation (QAM) signal set over the Rayleigh, flat-fading, multiple-input/multiple-output (MIMO) channel. We compute the maximum information rate attainable with this signal set and the distribution that achieves it. This optimal distribution is shown to factor into the product of identical distributions over each transmit dimension. Trellis shaping is used independently on each dimension to approach the channel capacity.**

I. Introduction

In [1], it was shown that, under a transmission power constraint, using independent, identically distributed (i.i.d.), circularly symmetric, Gaussian random variables on each transmit antenna achieves the capacity of the Rayleigh, flat-fading, MIMO channel. When constraining the channel input to a finite-size constellation, however, a closed-form expression for the channel input distribution that maximizes the mutual information is not known. As such, we use the modified version of the Arimoto-Blahut Algorithm developed in [2] to calculate the maximum information rate achievable using an M-QAM constellation ($C_{\text{M-QAM}}$) and the input distribution that achieves it. These results are compared to the maximum information rate achievable using independent, uniformly distributed, channel inputs ($C_{\text{i.u.d.}}$). We give the increase in mutual information that using the optimal distribution affords, the *shaping gap*, and design trellis codes to *shape* the input symbol distribution to as close to optimal as possible.

II. Problem Formulation & Solution

The Rayleigh, flat-fading, MIMO channel with T transmit and R receive antennas is modeled, as given in [1], by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}. \tag{1}$$

In (1), the received vector $\mathbf{y} \in \mathbb{C}^R$ is an affine transform of the transmitted vector $\mathbf{x} \in \mathbb{C}^T$. The channel matrix $\mathbf{H} \in \mathbb{C}^{R \times T}$ and the noise vector $\mathbf{n} \in \mathbb{C}^R$ have i.i.d., zero-mean, complex Gaussian elements and the channel is assumed quasi-static (constant over a number of consecutive transmissions) and known to the receiver.

We assume the transmitted signal is of the form $\mathbf{x} = \alpha\mathbf{s}$, where $\alpha \in \mathbb{R}^+$ and $\mathbf{s} \in \mathcal{S}^T_{\text{M-QAM}}$, the T-fold Cartesian product of the M-QAM signal set. Imposing the transmit power constraint $E\left[|\mathbf{x}|^2\right] \le P$ and denoting the probability mass function (pmf) of $\mathbf{s}$ over $\mathcal{S}^T_{\text{M-QAM}}$ as $p_{\mathbf{s}}(\cdot)$, the capacity computation problem is given as

$$\begin{aligned}\left[\alpha^*, p^*_{\mathbf{s}}(\cdot)\right] &= \arg\max_{[\alpha, p_{\mathbf{s}}(\cdot)]} \mathcal{I}(\mathbf{x};\mathbf{y}|\mathbf{H}) \\ C_{\text{M-QAM}} &= \mathcal{I}(\mathbf{x};\mathbf{y}|\mathbf{H})\,|_{[\alpha=\alpha^*, p_{\mathbf{s}}(\cdot)=p^*_{\mathbf{s}}(\cdot)]}.\end{aligned} \tag{2}$$

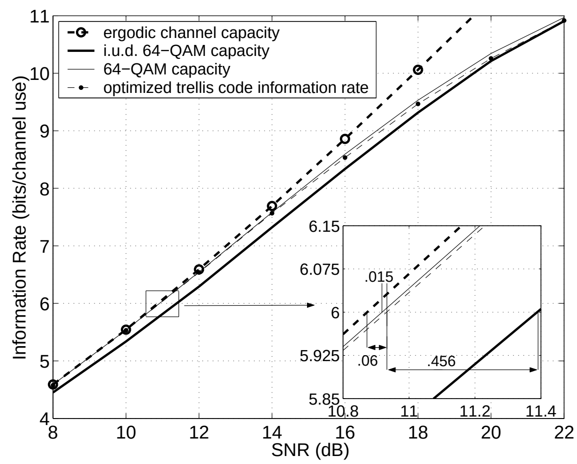

Fig. 1: Optimized trellis code information rate plotted with $C_{\text{64-QAM}}$ and $C_{\text{i.u.d.}}$. At 6 [b/cu], the trellis code is within .015 dB of $C_{\text{64-QAM}}$ (.06 dB from the channel capacity).

The modified Arimoto-Blahut algorithm presented in [2] is used to perform this optimization. This procedure, however, requires over optimization $M^T - 1$ free variables.

The k^{th} element of $\mathbf{s}$ may be written as $s_{kr} + js_{ki}$, where s_{kr}, s_{ki} are from the $\sqrt{M}$-PAM signal set $\mathcal{S}_{\sqrt{M}\text{-PAM}}$. Denoting $q^*_s(\cdot)$ as the pmf over $\mathcal{S}_{\sqrt{M}\text{-PAM}}$ that maximizes $\mathcal{I}(\mathbf{x};\mathbf{y}|\mathbf{H})$, we make the following conjecture that was numerically verified for the (2x2) MIMO channel.

<u>*Conjecture 1:*</u> The optimal pmf $p^*_{\mathbf{s}}(\cdot)$ is of the form:

$$p^*_{\mathbf{s}}(\mathbf{s}) = \prod_{m \in \{r,i\}} \prod_{k=1}^{T} q^*_s(s_{km})$$

We, therefore, perform the optimization of (2) over a single dimension of the transmit vector with the average power split equally between the input symbol dimensions. Taking the constellation symmetry into account, the number of degrees of freedom for this optimization, is reduced to $\frac{\sqrt{M}}{2} - 1$.

Using *Conjecture 1* we performed the optimization of (2) for a 64-QAM constellation in a (2x2) MIMO system. Using the heuristic rules of designing trellis codes to *shape* the channel input distribution given in [2], an 8-state code was designed and used independently on each transmit dimension. The results (Fig. 1) show the *shaping gap* is .471 dB at 6 [b/cu]. The designed trellis code has an information rate that is within .015 dB of $C_{\text{64-QAM}}$ and .456 dB inside of $C_{\text{i.u.d.}}$

References

[1] E. Telatar, “Capacity of multi-antenna Gaussian channels,” *European Trans. on Telecom.*, vol. 10, pp. 585-595, 1999.

[2] A. Kavčić, X. Ma, N. Varnica, “Matched Information Rate Codes for Partial Response Channels,” *Submitted to IEEE Trans. on Info. Theory*, 2002.