Department of Electrical Engineering, University of Hawaii

EE 342: Probability and Statistics **Fall 2016**

<u>Homerwork Set 7</u>

Due date: October 19, 2016

(1) a) Chapter 6, problem 6
b) Chapter 6, problem 19
c) Chapter 6, problem 20
d) Chapter 6, problem 23
e) Chapter 6, theoretical exercise 18

(2) A Rayleigh random variable X with parameter σ has the following pdf:

$$f_X(x) = (x/\sigma^2)\cdot\exp[-x^2/(2\sigma^2)]\cdot u(x)$$

a) Find the moment generating function of X.
b) Find the mean of X.
c) Find the variance of X.

(3) Assume that the random variable X is uniformly distributed on the interval [0,1]:
a) Find the pdf of Y=exp(X).
b) Use matlab to make plots of the PDF and CDF of Y. Compute the mean and variance of Y.
c) Use Matlab to generate random numbers from Y. From these random numbers, generate plots of the sample PDF and CDF. Also find the sample mean and variance.

(4) Repeat Problem (3) for $Z=\sin(\pi X/2)$.

(5) Repeat Problem (3) for $W=(2X-1)^2$.

6.6. A bin of 5 transistors is known to contain 2 that are defective. The transistors are to be tested, one at a time, until the defective ones are identified. Denote by N_1 the number of tests made until the first defective is identified and by N_2 the number of additional tests until the second defective is identified. Find the joint probability mass function of N_1 and N_2.

6.19. Show that $f(x, y) = 1/x$, $0 < y < x < 1$, is a joint density function. Assuming that f is the joint density function of X, Y, find
(a) the marginal density of Y;
(b) the marginal density of X;
(c) $E[X]$;
(c) $E[Y]$.

6.20. The joint density of X and Y is given by

$$f(x, y) = \begin{cases} xe^{-(x+y)} & x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Are X and Y independent? If, instead, $f(x, y)$ were given by

$$f(x, y) = \begin{cases} 2 & 0 < x < y,\ 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

would X and Y be independent?

6.23. The random variables X and Y have joint density function

$$f(x, y) = 12xy(1 - x) \quad 0 < x < 1,\ 0 < y < 1$$

and equal to 0 otherwise.

(a) Are X and Y independent?
(b) Find $E[X]$.
(c) Find $E[Y]$.
(d) Find $\mathrm{Var}(X)$.
(e) Find $\mathrm{Var}(Y)$.

6.18. Suppose X and Y are both integer-valued random variables. Let

$$p(i|j) = P(X = i|Y = j)$$

and

$$q(j|i) = P(Y = j|X = i)$$

Show that

$$P(X = i, Y = j) = \frac{p(i|j)}{\sum_i \frac{p(i|j)}{q(j|i)}}$$