Department of Electrical Engineering, University of Hawaii

EE 342: Probability and Statistics **Fall 2016**

Homework Set 11

Due date: December 7, 2016

(1) a) Chapter 8, problem 2
b) Chapter 8, problem 5
c) Chapter 8, problem 7
d) Chapter 8, problem 23

(2) Let U_1, U_2, … U_n, … be independent random variables, each uniformly distributed on the interval [-0.5 , 0.5].

Let $S_n = [U_1 + U_2 + \cdots + U_n]$.

Use Matlab to find and plot the PDFs of S_2, S_3, S_4, S_5, S_{10}, S_{20} and S_{40}. Compare each of these pdfs to the Gaussian PDF with the same mean and same variance. What can you conclude?

(3) Let $X_1, X_2, \ldots, X_{1000}$ be independent Bernoulli random variables with

$P(X_i=0) = P(X_i=1) = 0.5$.

Let $A=[X_1+X_2+\ldots X_{1000}]/1000$.

Find the probability that A is greater than 0.55.
[Hint: Use the central limit theorem.]

(4) Let $X_1, X_2, \ldots, X_n$, … be independent and identically distributed random variables and let $m_X(t)$ be the MGF of any X_i.

Let $A_n=[X_1+X_2+\ldots+X_n]/n$.

a) Derive the formula for the Chernoff bound for A_n.
b) Determine the equation for determining the constant t_0 that tightens the Chernoff bound.

8.2. From past experience, a professor knows that the test score of a student taking her final examination is a random variable with mean 75.

(a) Give an upper bound for the probability that a student's test score will exceed 85. Suppose, in addition, that the professor knows that the variance of a student's test score is equal to 25.

(b) What can be said about the probability that a student will score between 65 and 85?

(c) How many students would have to take the examination to ensure, with probability at least .9, that the class average would be within 5 of 75? Do not use the central limit theorem.

8.5. Fifty numbers are rounded off to the nearest integer and then summed. If the individual round-off errors are uniformly distributed over $(-.5, .5)$, approximate the probability that the resultant sum differs from the exact sum by more than 3.

8.7. A person has 100 light bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working bulb after 525 hours.

8.23. Let X be a Poisson random variable with mean 20.

(a) Use the Markov inequality to obtain an upper bound on

$$p = P\{X \geq 26\}$$

(b) Use the one-sided Chebyshev inequality to obtain an upper bound on p.

(c) Use the Chernoff bound to obtain an upper bound on p.

(d) Approximate p by making use of the central limit theorem.

(e) Determine p by running an appropriate program.