Solutions to HW3

Problem 2.6:

From the Wiener-Hopf equations (2.34) and Eqn. (2.49) of the minimum mean-square error J_{min}, we have:

$$R_u \underline{w}_0 = \underline{p} \quad \text{and} \quad J_{min} = \sigma_d^2 - \underline{p}^H \underline{w}_0 \quad \text{where} \begin{cases} R_u = E[\underline{u}(n)\underline{u}^H(n)] \\ \underline{p} = E[\underline{u}(n) d^*(n)] \\ \sigma_d^2 = E[d(n) d^*(n)] \end{cases}$$

The correlation matrix of $\begin{bmatrix} d(n) \\ \underline{u}(n) \end{bmatrix}$ is

$$A = E\left[\begin{pmatrix} d(n) \\ \underline{u}(n) \end{pmatrix} \cdot \left(d^*(n), \ \underline{u}^H(n)\right)\right] = \begin{bmatrix} E[d(n)d^*(n)] & E[d(n)\underline{u}^H(n)] \\ E[\underline{u}(n) d^*(n)] & E[\underline{u}(n)\underline{u}^H(n)] \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_d^2 & \underline{p}^H \\ \underline{p} & R_u \end{bmatrix}.$$

Hence

$$A \cdot \begin{bmatrix} 1 \\ -\underline{w}_0 \end{bmatrix} = \begin{bmatrix} \sigma_d^2 - \underline{p}^H \underline{w}_0 \\ \underline{p} - R_u \underline{w}_0 \end{bmatrix} = \begin{bmatrix} J_{min} \\ \underline{0} \end{bmatrix}. \quad \#$$

Problem 2.7:

By applying eigendecompositions to the correlation matrix R, we have

$$R = Q \Lambda Q^H, \text{ where } QQ^H = I \text{ (identity matrix) and } \Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_M)$$

$$\begin{cases} \lambda_1, \lambda_2, \ldots, \lambda_M \text{ are the eigenvalues of } R. \\ Q = (\underline{q}_1, \underline{q}_2 \cdots \underline{q}_M), \ \underline{q}_i \text{ is the unit eigenvector corresponding to } \lambda_i. \end{cases}$$

Hence, $\underline{p}^H R^{-1} \underline{p} = \underline{p}^H (Q\Lambda Q^H)^{-1} \underline{p} = \underline{p}^H (Q \Lambda^{-1} Q^H) \underline{p} = (\underline{p}^H Q) \cdot \Lambda^{-1} \cdot (\underline{p}^H Q)^H$

$$= (\underline{p}^H \underline{q}_1, \underline{p}^H \underline{q}_2, \cdots \underline{p}^H \underline{q}_M) \Lambda^{-1} \cdot \begin{pmatrix} \underline{q}_1^H \underline{p} \\ \underline{q}_2^H \underline{p} \\ \vdots \\ \underline{q}_M^H \underline{p} \end{pmatrix}$$

We evaluate $\underline{p}^H R^{-1} \underline{p}$ further,

$$\underline{p}^H R^{-1} \underline{p} = \left(\underline{p}^H \underline{q}_1, \ \underline{p}^H \underline{q}_2 \cdots \underline{p}^H \underline{q}_M\right) \begin{pmatrix} \frac{1}{\lambda_1} & 0 & \cdots & 0 \\ 0 & & & \\ \vdots & & 0 & \\ 0 & & & \end{pmatrix} \begin{pmatrix} \underline{q}_1^H \underline{p} \\ \underline{q}_2^H \underline{p} \\ \vdots \\ \underline{q}_M^H \underline{p} \end{pmatrix}$$

$$+ \cdots$$

$$+ \left(\underline{p}^H \underline{q}_1 \ \underline{p}^H \underline{q}_2 \cdots \underline{p}^H \underline{q}_M\right) \begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 1/\lambda_M \end{pmatrix} \begin{pmatrix} \underline{q}_1^H \underline{p} \\ \underline{q}_2^H \underline{p} \\ \vdots \\ \underline{q}_M^H \underline{p} \end{pmatrix}$$

$$= \sum_{k=1}^{M} \frac{1}{\lambda_k} \cdot (\underline{p}^H \underline{q}_k) \cdot (\underline{q}_k^H \underline{p})$$

$$= \sum_{k=1}^{M} \frac{1}{\lambda_k} \cdot (\underline{q}_k^H \underline{p})^H (\underline{q}_k^H \underline{p})$$

$$= \sum_{k=1}^{M} \frac{1}{\lambda_k} \left|\underline{q}_k^H \underline{p}\right|^2 .$$

Therefore:

$$\underline{J_{min} = \sigma_d^2 - \underline{p}^H R^{-1} \underline{p} = \sigma_d^2 - \sum_{k=1}^{M} \frac{\left|\underline{q}_k^H \cdot \underline{p}\right|^2}{\lambda_k}} \ . \ \#$$

Problem 2.9:

(a) For the linear regression model, the Wiener-Hopf equation is.

$$R_M \underline{a}_M = \underline{p}_M$$

$$\begin{bmatrix} R_m & \underline{r}_{M-m} \\ \underline{r}_{M-m}^H & R_{M-m, M-m} \end{bmatrix} \cdot \begin{bmatrix} \underline{a}_m \\ \underline{0}_{M-m} \end{bmatrix} = \begin{bmatrix} R_m \underline{a}_m \\ \underline{r}_{M-m}^H \cdot \underline{a}_m \end{bmatrix} = \begin{bmatrix} \underline{p}_m \\ \underline{p}_{M-m} \end{bmatrix}$$

$\therefore$ $R_m \underline{a}_m = \underline{p}_m \Rightarrow \underline{a}_m = R_m^{-1} \underline{p}_m$

$\underline{r}_{M-m}^H \underline{a}_m = \underline{p}_{M-m} \Rightarrow \underline{p}_{M-m} = \underline{r}_{M-m}^H \cdot R_m^{-1} \underline{p}_m$ ------ (*)

Hence the condition in Eqn. (*) satisfies the Wiener-Hopf equation.

(b) From the example in Section 2.7, we have

$M=4, \quad m=3, \quad \underline{r}^H_{M-m} = [-0.05 \quad 0.1 \quad 0.5]$

$$R_m = R_3 = \begin{bmatrix} 1.1 & 0.5 & 0.1 \\ 0.5 & 1.1 & 0.5 \\ 0.1 & 0.5 & 1.1 \end{bmatrix}, \qquad \underline{P}_m = \underline{P}_3 = \begin{bmatrix} 0.5272 \\ -0.4458 \\ -0.1003 \end{bmatrix}$$

Hence, using the result in part (a), we know that ~~the~~ last entry in $\underline{P}_4$ is

$$P_{M-m} = \underline{r}^H_{M-m} \cdot (R_3)^{-1} \cdot \underline{P}_3 = -0.0127. \quad \#$$

Problem 2.13:

(a) $u(n) = A_1 e^{-j\omega_1 n} + v(n)$

$$r(l) = E[u(n) \cdot u^*(n-l)] = E\left[(A_1 e^{-j\omega_1 n} + v(n))(A_1^* e^{j\omega_1(n-l)} + v^*(n-l))\right]$$

$$= E\left[|A_1|^2 e^{-j\omega_1 l} + v(n) \cdot A_1^* e^{j\omega_1(n-l)} + A_1 v^*(n-l) \cdot e^{-j\omega_1 n} + v(n) v^*(n-l)\right]$$

$$= E[|A_1|^2] \cdot e^{-j\omega_1 l} + r_v(l) \qquad (\because A_1 \text{ and } v(n) \text{ are independent, and } E[v(n)]=0)$$

$$= \sigma_1^2 \cdot e^{-j\omega_1 l} + \sigma_v^2 \cdot \delta(l) \qquad (\because v(n) \text{ is white})$$

$$\therefore R = E[\underline{u}(n)\underline{u}^H(n)] = \begin{bmatrix} r(0) & r(1) & \cdots & r(M-1) \\ r(-1) & r(0) & \cdots & r(M-2) \\ \vdots & \vdots & & \vdots \\ r(1-M) & r(2-M) & \cdots & r(0) \end{bmatrix}$$

$$= \sigma_1^2 \cdot \underline{S}(\omega_1)\underline{S}^H(\omega_1) + \sigma_v^2 \cdot I$$

where I is the identity matrix and $\underline{S}(\omega_1) = [1, e^{-j\omega_1}, e^{-j\omega_1 2}, \cdots e^{-j\omega_1(M-1)}]^T$.

(b) The tap-weight vector of the Wiener filter is:

$\underline{w}_0 = R^{-1} \cdot \underline{P}$, where $R = \sigma_v^2 I + \underline{S}(\omega_1) \cdot \sigma_1^2 \cdot \underline{S}^H(\omega_1)$

According to the matrix inversion lemma (see Page 440, Section 9.2), i.e.

if $A = B^{-1} + CD^{-1}C^H$, then $A^{-1} = B - BC \cdot (D + C^H B C)^{-1} C^H B$,

we have:

$$R^{-1} = \frac{1}{\sigma_v^2} I - \frac{1}{\sigma_v^2} I \cdot \underline{S}(\omega_1) \cdot \left(\frac{1}{\sigma_1^2} + \underline{S}^H(\omega_1) \cdot \frac{1}{\sigma_v^2} I \cdot \underline{S}(\omega)\right)^{-1} \cdot \underline{S}^H(\omega) \cdot \frac{1}{\sigma_v^2} I,$$

$$R^{-1} = \frac{1}{\sigma_v^2} I - \frac{1}{\sigma_v^2} \cdot \frac{1}{\sigma_v^2} \cdot \underline{s}(w_1) \cdot \left[\frac{1}{\sigma_1^2} + \frac{1}{\sigma_v^2} \underline{s}^H(w_1) \cdot \underline{s}(w_1) \right]^{-1} \cdot \underline{s}^H(w)$$

$$= \frac{1}{\sigma_v^2} I - \frac{1}{\sigma_v^4} \cdot \underline{s}(w_1) \left[\frac{1}{\sigma_1^2} + \frac{M}{\sigma_v^2} \right]^{-1} \underline{s}^H(w_1) \quad (\because \underline{s}^H(w_1) \cdot \underline{s}(w_1) = M)$$

$$= \frac{1}{\sigma_v^2} I - \frac{1}{\sigma_v^2 + M\sigma_1^2} \cdot \frac{\sigma_1^2}{\sigma_v^2} \cdot \underline{s}(w_1)\, \underline{s}^H(w_1)$$

Hence

$$w_0 = R^{-1} \cdot \underline{p} \quad \text{where } \underline{p} = \sigma_0^2\, \underline{s}(w_0)$$

$$= \frac{\sigma_0^2}{\sigma_v^2} \underline{s}(w_0) - \frac{1}{\sigma_v^2 + M\sigma_1^2} \cdot \frac{\sigma_1^2 \cdot \sigma_0^2}{\sigma_v^2} \cdot \underline{s}(w_1)\, \underline{s}^H(w_1) \cdot \underline{s}(w_0). \qquad \#$$

Problem 2.15:

This is an optimization problem. We want to find $\underline{w}$ such that

$$\begin{cases} \min & E[|e(n)|^2] \\ \text{s.t.} & S^H \underline{w} = D^{1/2} \cdot 1 . \end{cases}$$

Define the following function:

$$J(\underline{w}) = E[|e(n)|^2] + \Lambda^H (S^H \underline{w} - D^{1/2} \cdot 1) \quad \text{where } \Lambda = [\lambda_1, \lambda_2, \cdots \lambda_k]^T.$$

$$= E[e(n) \cdot e^*(n)] + \Lambda^H S^H \underline{w} - \Lambda^H D^{1/2} \cdot 1$$

$$= \underline{w}^H R \cdot \underline{w} + \Lambda^H S^H \underline{w} - \Lambda^H D^{1/2} 1 \qquad (\because e(n) = \underline{w}^H \underline{u}(n))$$

Differentitate $J(\underline{w})$ w.r.t. $\underline{w}$ and set the result equal to zero:

$$\frac{\partial J}{\partial \underline{w}} = \begin{bmatrix} \partial J / \partial w_0 \\ \partial J / \partial w_1 \\ \vdots \\ \partial J / \partial w_{M-1} \end{bmatrix} = 2R\underline{w} + S \cdot \Lambda = 0 \Rightarrow \underline{w} = -\frac{1}{2} R^{-1} S \cdot \Lambda .$$

From the constraint $S^H \underline{w} = D^{1/2} 1$, we have $-\frac{1}{2} S^H R^{-1} S \Lambda = D^{1/2} \cdot 1$.

Therefore the Lagrange multipliers are $\Lambda = -2 (S^H R^{-1} S)^{-1} D^{1/2} \cdot 1$.

Hence, the optimal tap-weight vector is $\underline{w}_0 = R^{-1} S \cdot (S^H R^{-1} S)^{-1} D^{1/2} \cdot 1 .$ #

Problem 2.19:

(a) For the noncausal case, the Wiener-Hopf equations are

$$\sum_{i=-\infty}^{+\infty} W_i\, r(i-k) = p(-k) \iff \sum_{i=-\infty}^{+\infty} W_i\, r^*(k-i) = p(-k) \quad \text{--- } (*)$$

Applying z-transform to Eqn. (*), we have

$$\left(\sum_{i=-\infty}^{+\infty} W_i z^{-i}\right)\cdot\left(\sum_{j=-\infty}^{+\infty} r^*(j) z^{-j}\right) = \sum_{k=-\infty}^{+\infty} p(-k)\cdot z^{-k} \quad \text{---- } (**)$$

Define $S(z) = \sum_{k=-\infty}^{+\infty} r(k) z^{-k}$. $\Rightarrow S(\frac{1}{z}) = \sum_{k=-\infty}^{+\infty} r(k) z^{k} = \sum_{j=-\infty}^{+\infty} r(-j) z^{-j} = \sum_{j=-\infty}^{+\infty} r^*(j) z^{-j}$ $(\because r(-j) = r^*(j))$

Define $P(z) = \sum_{k=-\infty}^{+\infty} p(k) z^{-k}$. $\Rightarrow P(\frac{1}{z}) = \sum_{k=-\infty}^{+\infty} p(k) z^{k} = \sum_{k=-\infty}^{+\infty} p(-k) z^{-k}$

Define $H_u(z) = \sum_{k=-\infty}^{+\infty} W_k z^{-k}$.

Hence,

Eqn. (**) $\iff H_u(z) = \dfrac{P(1/z)}{S(1/z)}$. (It is different from that in the textbook, which has a typo or mistake.)

(b) Suppose

$$\begin{cases} P(z) = \dfrac{0.36}{(1-0.2z^{-1})(1-0.2z)} \\[2ex] S(z) = \dfrac{1.37(1-0.146z^{-1})(1-0.146z)}{(1-0.2z^{-1})(1-0.2z)} \end{cases}$$

We have $\begin{cases} P(\frac{1}{z}) = P(z) \\ S(1/z) = S(z) \end{cases}$

Hence

$$H_u(z) = \frac{P(1/z)}{S(1/z)} = \frac{0.36}{1.37(1-0.146z^{-1})(1-0.146z)} = \frac{0.36z^{-1}}{1.37(1-0.146z^{-1})(z^{-1}-0.146)}$$

$$= \frac{0.2685}{1-0.146z^{-1}} + \frac{0.0392}{z^{-1}-0.146} = \frac{0.2685}{1-0.146z^{-1}} - \frac{0.2685}{1-\underbrace{\frac{1}{0.146}}_{6.8493}z^{-1}} \quad \text{---- } (***)$$

Applying inverse z-transform to Eqn. (***), we have

h(n) = 0.2685 * (0.146)^n * u_{step}(n) + 0.2685 * (1/0.146)^n * u_{step}(-n-1),   0.146 <= z <= 1/0.146

$$\cancel{h(n) = 0.2685\cdot(0.146)^n u_{step}(n) - 0.2685\,(6.8493)^n u_{step}(-n)} \quad (\because \tfrac{1}{0.146} = 6.8493)$$

where $u_{step}(n) = \begin{cases} 1, & n = 0, 1, 2, \cdots \\ 0 & n = -1, -2, \cdots \end{cases}$ is the unit step function.

Therefore, the impulse response of the filter is:

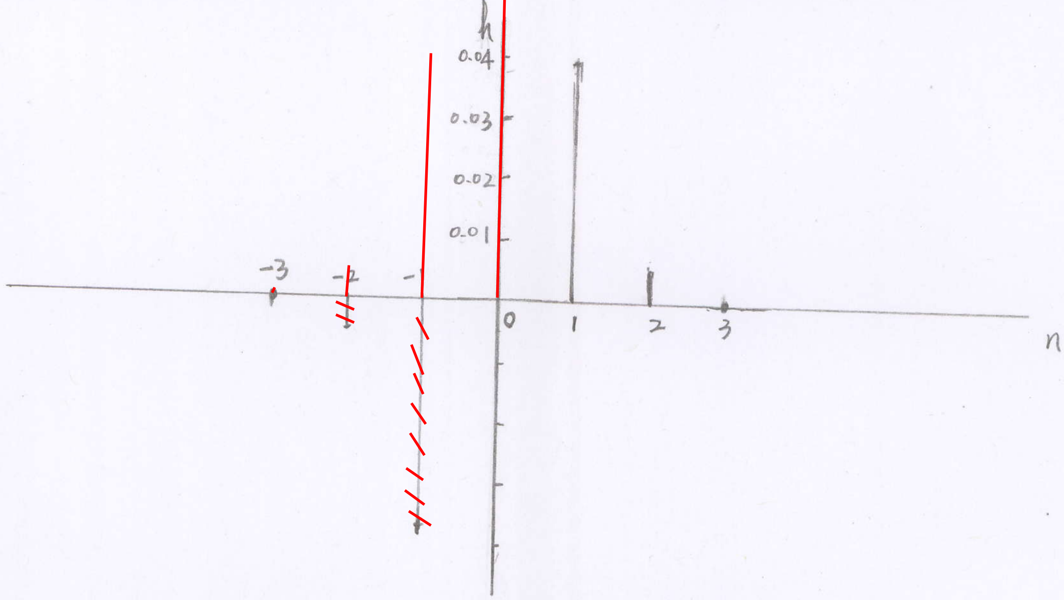
$\cancel{h(0) = 0}$ $\boxed{h(0) = 1}$

$h(1) = 0.2685 \cdot 0.146 = 0.0392$ $\qquad h(-1) = +0.2685 \cdot 6.8493^{-1} = +0.0392$

$h(2) = 0.2685 \cdot 0.146^2 = 0.0057$ $\qquad h(-2) = +0.2685 \cdot 6.8493^{-2} = +0.0057$

$h(3) = 0.2685 \cdot 0.146^3 = 0.0008$ $\qquad h(-3) = +0.2685 \cdot 6.8493^{-3} = +0.0008$

$\boxed{1}$



c) A delay of 3 time units in the impulse response will make the filter realizable.

Problem 4.1:

(a) To ensure convergence of the steepest-descent algorithm, we choose μ such that

$$0 < \mu < \frac{2}{\lambda_{max}}, \quad \text{where } \lambda_{max} \text{ is the largest eigenvalue of the correlation matrix } R.$$

$$R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \Rightarrow |\lambda I - R| = (\lambda-1)^2 - 0.25 = 0 \Rightarrow \lambda_1 = 0.5,\ \lambda_2 = 1.5 = \lambda_{max}.$$

$$\therefore \quad 0 < \mu < \frac{2}{1.5} = \frac{4}{3}.$$

A suitable value of μ may be 1.

(b) From the recursive relation in Eqn. (4.10) of the textbook

$$\underline{w}(n+1) = \underline{w}(n) + \mu[\underline{p} - R\underline{w}(n)] \quad \text{where } \underline{p} = \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix},\ \underline{w}(n) = \begin{bmatrix} w_1(n) \\ w_2(n) \end{bmatrix}.$$

we have (if we choose $\mu = 1$),

$$\underline{w}(n+1) = (I - R)\underline{w}(n) + \underline{p} = \begin{bmatrix} 0 & -0.5 \\ -0.5 & 0 \end{bmatrix}\begin{bmatrix} w_1(n) \\ w_2(n) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix}.$$

Then

$$\begin{cases} w_1(n+1) = -0.5\,w_2(n) + 0.5 \\ w_2(n+1) = -0.5\,w_1(n) + 0.25 \end{cases} \quad \text{and} \quad \begin{cases} w_1(0) = 0 \\ w_2(0) = 0 \end{cases}.$$

(c) In this part, we work on another vector $\underline{v}(n)$ where

$$\begin{cases} \underline{v}(0) = Q^H\underline{w}_0 \\ \underline{v}(n) = Q^H[\underline{w}_0 - \underline{w}(n)] \end{cases} \quad \text{and } Q \text{ is the unitary matrix such that } R = Q\Lambda Q^H,\ \Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

By eigendecomposition of $R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$, we have $Q = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$.

And the optimal tap-weight vector is

$$\underline{w}_0 = R^{-1}p = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}.$$

Hence $\underline{v}(0) = Q^H\underline{w}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 0.5 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2\sqrt{2}} \\ \frac{1}{2\sqrt{2}} \end{bmatrix}$, That is $v_1(0) = -\frac{1}{2\sqrt{2}},\ v_2(0) = \frac{1}{2\sqrt{2}}$

The kth natural mode of the steepest-descent algorithm is

$$v_k(n+1) = (1 - \mu\lambda_k)v_k(n) \qquad k = 1, 2$$

Therefore

$$\begin{cases} v_1(n+1) = (1 - 0.5\mu)v_1(n) = \cdots = (1-0.5\mu)^{n+1}v_1(0) \\ v_2(n+1) = (1 - 1.5\mu)v_2(n) = \cdots = (1-1.5\mu)^{n+1}v_2(0) \end{cases}, (n > 0)$$

From part (a), we choose $\mu \in (0, \frac{4}{3})$ to ensure convergence of the algorithm.

Therefore: $1-0.5\mu \in (\frac{1}{3}, 1)$ $\quad 1-1.5\mu \in (-1, 1)$

Let $1-1.5\mu_0 = 0 \Rightarrow \mu_0 = \frac{2}{3}$.

Hence ① if $0 < \mu < \mu_0 = \frac{2}{3}$, then $\begin{cases} 1-0.5\mu > 0 \\ 1-1.5\mu > 0 \end{cases}$.

Set $\mu = 0.2$, then $\begin{cases} v_1(n+1) = 0.9^{n+1} v_1(0) \\ v_2(n+1) = 0.7^{n+1} v_2(0) \end{cases} \Rightarrow$ a damped trajectory

② if $\mu_0 = \frac{2}{3} < \mu < \frac{4}{3}$ then $\begin{cases} 1-0.5\mu > 0 \\ 1-1.5\mu < 0 \end{cases}$

Set $\mu = 1.0$, then $\begin{cases} v_1(n+1) = 0.5^{n+1} v_1(0) \\ v_2(n+1) = (-0.5)^{n+1} v_1(0) \end{cases} \Rightarrow$ an oscillatory trajectory. #

Problem 4.10:

From the second-order AR process $u(n) = -0.5u(n-1) + u(n-2) + v(n)$, we have

Yule-Walker equations:

$$\begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} \begin{bmatrix} -0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} r(1) \\ r(2) \end{bmatrix} \Leftrightarrow \begin{array}{l} -0.5r(0) + r(1) = r(1) \Rightarrow r(0) = 0 \\ -0.5r(1) + r(0) = r(2) \Rightarrow r(2) = -0.5r(1) \end{array}$$

We also have: $\sigma_v^2 = r(0) + 0.5r(1) - r(2)$ & $\sigma_v^2 = 1 \Rightarrow r(1) = 1 \quad r(2) = -0.5$.

Hence the correlation matrix of $u(n)$ is

$$R = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

Let $|\lambda I - R| = \lambda^2 - 1 = 0. \Rightarrow \lambda_1 = -1, \quad \lambda_2 = +1 = \lambda_{max}$.

Therefore, to ensure the stability of the steepest-descent algorithm, we choose μ such that:

$$0 < \mu < \frac{2}{\lambda_{max}} = 2. \quad \#$$

Problem 4.14:

(a) From the ARMA process of order (1,1) $u(n) = -0.5\,u(n-1) + v(n) - 0.2\,v(n-1)$,

We have its Z-transforms as: $U(Z) = -0.5Z^{-1}U(Z) + V(Z) - 0.2Z^{-1}V(Z)$.

$$\Rightarrow \frac{U(Z)}{V(Z)} = \frac{1-0.2Z^{-1}}{1+0.5Z^{-1}} = \frac{1}{(1+0.5Z^{-1})(1-0.2Z^{-1})^{-1}}$$

$$\approx \frac{1}{(1+0.5Z^{-1})(1+0.2Z^{-1}+0.04Z^{-2}+0.008Z^{-3})} \quad \text{(Using Taylor's series and its first four terms)}$$

$$\approx \frac{1}{1+0.7Z^{-1}+0.14Z^{-2}+0.028Z^{-3}} \quad \text{(Approximating to third-order terms)}$$

$\Rightarrow$ The third-order AR process is:

$$\boxed{u(n) + 0.7\,u(n-1) + 0.14\,u(n-2) + 0.028\,u(n-3) = v(n)}.$$

The AR coefficients are $a_1 = 0.7$ $a_2 = 0.14$ $a_3 = 0.028$.

The Yule-Walker equations are

$$\begin{pmatrix} r(0) & r(1) & r(2) \\ r(1) & r(0) & r(1) \\ r(2) & r(1) & r(0) \end{pmatrix} \begin{pmatrix} -0.7 \\ -0.14 \\ -0.028 \end{pmatrix} = \begin{pmatrix} r(1) \\ r(2) \\ r(3) \end{pmatrix} \Longleftrightarrow \begin{cases} -0.7\,r(0) - 0.14\,r(1) - 0.028\,r(2) = r(1) \\ -0.7\,r(1) - 0.14\,r(0) - 0.028\,r(1) = r(2) \\ -0.7\,r(2) - 0.14\,r(1) - 0.028\,r(0) = r(3) \end{cases}$$

and: $1 = \sigma_v^2 = r(0) + 0.7\,r(1) + 0.14\,r(2) + 0.028\,r(3)$

$$\Rightarrow \begin{cases} r(0) = 1.6554 \\ r(1) = -1.0292 \\ r(2) = 0.5175 \\ r(3) = -0.2645 \end{cases} \Rightarrow \text{the correlation matrix is } \mathbf{R} = \begin{bmatrix} 1.6554 & -1.0292 & 0.5175 \\ -1.0292 & 1.6554 & -1.0292 \\ 0.5175 & -1.0292 & 1.6554 \end{bmatrix}$$

(b). The eigenvalues of R are $\lambda_1 = 0.4358$, $\lambda_2 = 1.1379$, $\lambda_3 = 3.3924$.

(c) To ensure the convergence of the steepest-descent algorithm, we choose μ such that

$$0 < \mu < \frac{2}{\lambda_{max}} = \frac{2}{\lambda_3} = \frac{2}{3.3924} \approx 0.5896. \quad \#$$