EE 342
HW 2 — SOLUTIONS

1° CH 1, PROBLEM 15

OF WAYS TO CHOOSE MEN $\binom{12}{5}$

OF WAYS TO CHOOSE WOMEN $\binom{10}{5}$

OF WAYS TO FORM PAIRS $5!$

TOTAL $\boxed{\binom{12}{5}\cdot\binom{10}{5}\cdot 5!} \cong 2.395 \times 10^7$

2° CH 1. PROBLEM 16

a) $\binom{6}{2} + \binom{7}{2} + \binom{4}{2} = 15 + 21 + 6 = \boxed{42}$

b) n = # OF WAYS TO CHOOSE 2 BOOKS $\Rightarrow \binom{7+6+4}{2} = \binom{17}{2} = n$

k = # OF WAYS TO CHOOSE 2 BOOKS BOTH BEING ON SAME SUBJECT $\Rightarrow \binom{6}{2}+\binom{7}{2}+\binom{4}{2} = k$

OF WAYS TO CHOOSE 2 BOOKS ON DIFFERENT SUBJECTS $\Rightarrow n-k$

$$n-k = \binom{17}{2} - \binom{6}{2} - \binom{7}{2} - \binom{4}{2}$$

$$\boxed{n-k = 94}$$

[3] CH 1, PROBLEM 21

A PATH CONSISTS OF 7 STEPS.

EXAMPLES OF VALID PATHS ARE

U R R U U R R or
U R R R R U U or
R R U R U U R

where R denotes "RIGHT HORIZONTAL" STEP AND
U DENOTES "UPWARD VERTICAL" STEP

SO, CLEARLY WE NEED TO PLACE 3 U's ON 7 SLOTS. THERE ARE

$\binom{7}{3}$ WAYS TO DO THIS

$$\boxed{\binom{7}{3} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35}$$

Note $\binom{7}{3} = \binom{7}{4}$

[4] CH 1, PROBLEM 22

OF WAYS TO GET FROM POINT A TO CIRCLE $\Rightarrow \binom{4}{2}$

OF WAYS TO GET FROM CIRCLE TO POINT B $\Rightarrow \binom{3}{1}$

TOTAL $\binom{4}{2} \cdot \binom{3}{1} = \frac{4 \cdot 3}{2} \cdot 3 = \boxed{18}$

5 CH1, PROBLEM 28

* Let's first answer the easier of the two questions

4 GROUPS OF 2 TEACHERS

$$\frac{8!}{2!\,2!\,2!\,2!} = 2520$$

* TO ANSWER THE SECOND QUESTION, LET'S LIST ALL POSSIBLE WAYS TO DIVIDE 8 PEOPLE INTO 4 GROUPS.

GROUP SIZES	NUMBER OF PERMUTATIONS		NUMBER OF GROUP FILLINGS PER PERMUTATION
2, 2, 2, 2	1	×	$\frac{8!}{2!\,2!\,2!\,2!} = 2520$
0, 0, 0, 8	4	×	$\frac{8!}{0!\,0!\,0!\,8!} = 1$
1, 1, 1, 5	4	×	$\frac{8!}{1!\,1!\,1!\,5!} = 336$
0, 0, 4, 4	$\binom{4}{2} = 6$	×	$\frac{8!}{0!\,0!\,4!\,4!} = 70$
1, 1, 3, 3	6	×	$\frac{8!}{1!\,1!\,3!\,3!} = 1120$
0, 0, 1, 7	$\binom{4}{2} \cdot 2 = 12$	×	$\frac{8!}{0!\,0!\,1!\,7!} = 8$
0, 0, 2, 6	12	×	28
0, 0, 3, 5	12	×	56
1, 1, 0, 6	12	×	56
1, 1, 2, 4	12	×	840
2, 2, 0, 4	12	×	420
2, 2, 1, 3	12	×	1680
3, 3, 0, 2	12	×	560
0, 1, 2, 5	$4! = 24$	×	168
0, 1, 3, 4	24	×	280

TOTAL 65,536

6 CH2, PROBLEM 35

See solution to problem set 1

7 CH 2, PROBLEM 47

n = number of possible birthday distributions among 12 people

$$\boxed{n = 12^{12}}$$

k = number of possible birthday distributions among 12 people such that no two persons are born in the same month

$$\boxed{k = 12!}$$

$$P = \frac{k}{n} = \boxed{\frac{12!}{12^{12}} \cong 5.37 \times 10^{-5}}$$

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NUMBER OF WAYS 6 STAMPS CAN BE DRAWN

$$n = \binom{10+5+2}{6} = \binom{17}{6}$$

NOW, LET'S FIND ALL COMBINATIONS OF 6 STAMPS THAT ADD UP TO ¢100 = $1

$4 \times 20 + 2 \times 10$ ← $\binom{10}{4} \cdot \binom{2}{2}$ such draws

$3 \times 20 + 2 \times 15 + 1 \times 10$ ← $\binom{10}{3} \cdot \binom{5}{2} \cdot \binom{2}{1}$ such draws

$2 \times 20 + 4 \times 15$ ← $\binom{10}{2} \cdot \binom{5}{4}$ such draws

ANSWER

$$\frac{\binom{10}{4}\cdot\binom{2}{2}+\binom{10}{3}\binom{5}{2}\binom{2}{1}+\binom{10}{2}\binom{5}{4}}{\binom{17}{6}}$$

$$= \frac{210 + 2400 + 225}{12376} \cong 0.229$$

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$$B = SB$$
$$= (A \cup \bar{A})B$$
$$= (AB) \cup (\bar{A}B)$$

$(AB) \cap (\bar{A}B) = \emptyset$
so apply axiom iii

$$P(B) = P(AB) + P(\bar{A}B)$$

$$\Downarrow$$

$$\boxed{P(\bar{A}B) = P(B) - P(AB)}$$

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$$P = \frac{\binom{5}{5}}{\binom{20}{5}} = \boxed{\frac{5! \times 15!}{20!} \cong 6.45 \times 10^{-5}}$$