Experimental Validation of the Triangle Zig-Zag Transition Model

Aleksandar Kavčić and José M. F. Moura

Data Storage Systems Center, Carnegie Mellon University, Pittsburgh, PA 15213-3890

Abstract—The triangle zig-zag transition (TZ-ZT) model is a stochastic zig-zag transition model. It is a fast alternative to micromagnetic modeling. Thus far, the TZ-ZT model has been compared only to the micromagnetic model. Here we compare the model to real data and validate its signal and noise modeling capability.

I. INTRODUCTION

Stochastic zig-zag transition models in longitudinal magnetic recording have gained attention in signal processing applications through recently proposed models [1], [2]. The most attractive feature of the stochastic zig-zag models is the computational simplicity with which they model transitions and media noise. This makes these models useful in error rate simulations where thousands of transitions need to be created. The triangle zig-zag transition (TZ-ZT) model [1], for example, offers 4 orders of magnitude in computational savings over the micromagnetic model [3].

The stochastic zig-zag transition models describe the transition boundary as a cross-track stochastic process. Pioneering work on these models has been done by Arnoldussen and Tong [4] and Middleton and Miles [5]. Analytical results linking signal and noise to zig-zag properties are given in [6]. These early models exhibit problems in relating the model parameters to media/recoding properties and in cross-track stability, see details in [7].

Improving on these models, we introduced the triangle zig-zag transition model in [1]. In the TZ-ZT model, the zig-zag line is constructed from sides of isosceles triangles, making the model stable, while the structured geometry and some results from renewal theory [7] have been exploited to relate the model to media/recording properties. An alternative approach is the microtrack model [2], where the zig-zag line is simplified to a stochastic square wave.

In [1], the TZ-ZT model has been validated against the micromagnetic model. In this paper, we explore the accuracy of the TZ-ZT model by comparing it to the data obtained through spin-stand measurements. Our major comparison criteria are waveform and media noise prediction accuracy.

II. TRIANGLE ZIG-ZAG TRANSITION MODELING

The triangle zig-zag transition (TZ-ZT) model is a stochastic model of the zig-zag line that separates two oppositely magnetized regions of the magnetic medium. The TZ-ZT model
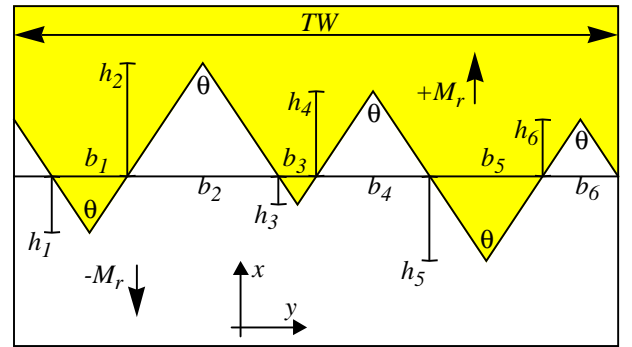
Fig. 1. The triangle zig-zag transition model.

(illustrated in Figure 1) is constructed by placing side-by-side isosceles triangles of alternating orientations on the line representing the nominal transition center. The triangle heights $(h_1, h_2, \ldots)$ are independent random variables drawn from a probability density function (pdf) $f_H(h)$. The vertex angle θ is chosen to be constant. For relationships relating the pdf $f_H(h)$ and the vertex angle θ to the transition profile shape and the cross-track correlation width, respectively, see [1], [8]. While the above formulation of the TZ-ZT model is suited only for modeling isolated transitions, equations governing intertransition interactions are also given in [1], [8], making the model suitable for modeling sequences of interacting transitions.

III. MODEL VALIDATION PROCEDURE AND RESULTS

We validate the model in two steps. First we consider only isolated transitions. After determining the model parameters for isolated transitions we evaluate the model predictions for isolated waveform shapes and media noise for isolated transitions. In the second step we determine the parameters for the interacting transitions (dibits) and show the model prediction results.

A. Isolated Transitions

The quantities that govern an isolated transition of the TZ-ZT model are the pdf of the triangle heights $f_H(h)$ and the vertex angle θ. We first go over the procedure for choosing these quantities. The recording system we considered had the following characteristics. The media was an isotropic cobalt-alloy thin film medium with coercivity $H_c = 2050 \text{Oe}$ and remanence-thickness product $M_r \delta = 2.14 \text{memu/cm}^2$. An inductive head was used for both reading and writing to avoid additional nonlinearities introduced by a magnetoresistive (MR) head. The head flew at a flying height $d = 0.807 \mu \text{in}$, writing a track of width $TW = 5.7 \mu \text{m}$. All measurements were taken at the inner radius of the disk so that, even at moderately low writing frequencies, the nonlinear writing effects could be observed.
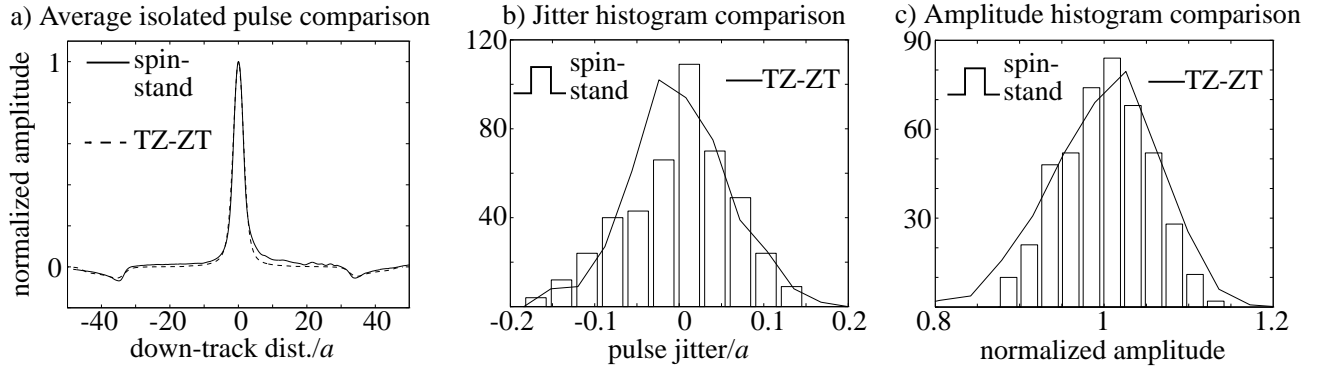
Fig. 2. Comparison of the experimental and TZ-ZT statistics for isolated pulses.

We wrote a waveform consisting of $N = 450$ isolated transitions. The electronics noise in the waveform was suppressed by recording an average of 1000 independent acquisitions of the waveform. After aligning the 450 isolated pulses, we found their average. From the average isolated pulse we deconvolved the head-sensitivity function to obtain the curve dM/dx, where x is the down-track direction and M is the down-track magnetization component normalized to the remanent magnetization M_r. The head-sensitivity function that we used here was obtained by running a detailed finite-element model of the utilized thin-film head. We next fitted a curve of the form

$$\frac{dM(x)}{dx} = \frac{A}{1 + |x/a|^k} \tag{1}$$

to the data, where, in our case, $k = 2.7$ and the transition parameter $a = 0.11\mu$m. The amplitude A was determined to match the amplitude of the data. By taking the first derivative of (1), we applied Theorem 1 from [1] to obtain the triangle heights pdf $f_H(h)$. Using the least-squares parabola-fitting method described in [1], [8], we next determined the cross-track correlation width $s = 312$Å. Substituting this value in Corollary 2.1 in [1], we found the vertex angle to be $\theta = 14.1°$.

Having determined the TZ-ZT defining quantities $f_H(h)$ and θ, we created $N = 450$ isolated transitions using the TZ-ZT model. Figures 2a through 2c compare the statistics of the spin-stand transitions to the modeled transitions. Figure 2a compares the average pulse shapes, Figure 2b compares the histograms of jitter noise, while Figure 2c compares the histograms of amplitude variations. In all three cases we can conclude that the match is very good.

B. Interacting Transitions

Using the same recording system as above, we wrote $N = 450$ isolated dibits for each one of different dibit separations, ranging between a and $3a$, where a is the transition width parameter. During the writing process, we used write-precompensation to cancel nonlinear bit-shift. That left the nonlinear amplitude loss as the dominant nonlinear writing effect. The dashed line in Figure 3 shows the drop of the dibits amplitude as a function of the inverse dibit separation. As a reference, in Figure 3, the solid line shows the amplitude drop when the dibit is formed as a linear superposition of two isolated pulses of Figure 2a. We see that when the dibit sepa-
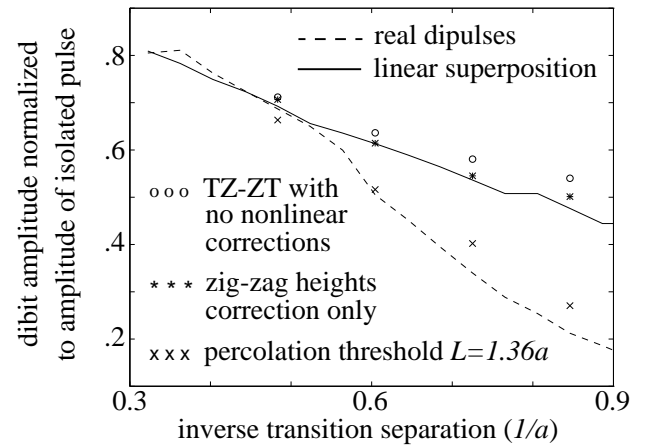

Fig. 3. Dibit amplitude drop plot.

ration falls to approximately $2a$, the amplitude drops nonlinearly.

To model this effect with the TZ-ZT model, we tried three approaches. First, we ran the TZ-ZT model ignoring all nonlinear effects, that is, we ran the model for isolated pulses, placing transitions next to each other without correcting the triangle heights. Percolation effects were modeled only on the portions where zig-zags from neighboring transitions touch each other. The result of this model is depicted with 'o's in Figure 3. Next, we introduced a correction of the zig-zag triangle heights according to equation (7) of [1]. This gave us the plot of points marked with '*'s in Figure 3. Since this correction was still not adequate for modeling the nonlinear amplitude drop, we introduced a percolation threshold L proposed in [9], thereby still retaining the correction of equation (7) in [1]. For details on how to apply the percolation threshold on the TZ-ZT model, see [8]. Contrary to the findings in [9], we found that the best-fitting percolation threshold L is dependent on the dibit separation. A reason for this dependence might be that we used real write precompensation to cancel nonlinear bit-shift, whereas the authors of [9] canceled the nonlinear bit-shift online only after the spin-stand acquisitions. This probably led to a better accuracy, since any real write precompensator leaves some residual bit-shift in the written data, which may be removed using additional fitting parameters. Nevertheless, we found a value for the percolation threshold L that best fits the
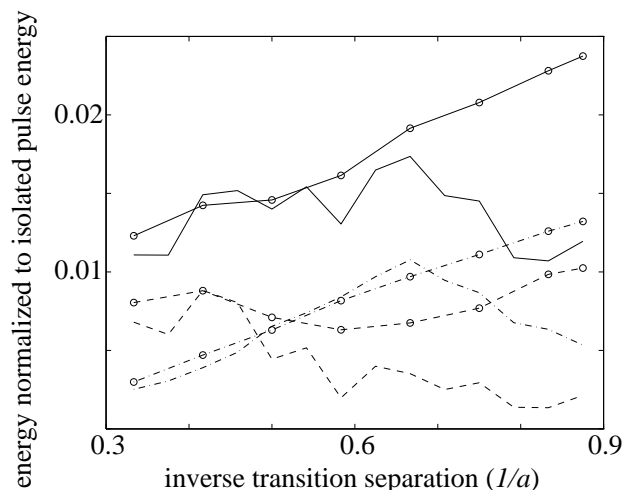
Fig. 4. Media noise energy plots. Lines marked with 'o's correspond to the TZ-ZT model. Unmarked lines correspond to spin-stand data. Solid lines: total media noise energy. Dashed lines: energy in the shift-in-unison (jitter) noise mode only. Dash-dotted lines: energy in the amplitude-variations noise mode only.

data. This value was determined to be $L = 1.36a$, where a is the transition width parameter. In Figure 3, the points marked with 'x's represent the amplitude drop obtained with the TZ-ZT model and the percolation threshold correction. We see a fairly good match there, with slight deviations as the dibit separation drops below $(0.7/a)^{-1}$, i.e., when the dibit separation becomes comparable to the percolation threshold L.

With the above determined percolation threshold, we finally performed an experiment to evaluate the media noise predictions of the TZ-ZT model. For eight different dibit separation ranging between a and $3a$, we created $N = 450$ isolated dibits using the TZ-ZT model. After aligning the dibits and subtracting their mean, we obtained $N = 450$ media noise waveforms in vector form, $\underline{n}_i$, where $1 \leq i \leq 450$. From these waveform samples we computed the empirical media noise autocorrelation matrix as

$$\mathbf{C} = \frac{1}{450} \sum_{i=1}^{450} \underline{n}_i \, \underline{n}_i^{\mathrm{T}} . \tag{2}$$

We next performed the Karhunen-Loeve decomposition [10] to break the noise into its principal components (modes). We followed the same procedure with the spin-stand data and compared the results in Figure 4. In Figure 4 the lines whose points are marked with 'o's represent the TZ-ZT model data, whereas the corresponding lines with no point markings represent the spin-stand data. The solid lines show the total dibit media noise energy (normalized by the energy of an isolated pulse) as a function of inverse dibit separation. The dashed lines compare the shift-in-unison (jitter) noise mode energies of the TZ-ZT model and the experimental data. The amplitude variations media noise mode energies are compared with the dash-dotted lines. In Figure 4, observe that the media noise energy starts decreasing for inverse dibit separations greater than $0.7/a$. This is because at these densities the signal itself is so small that variations in the signal itself produce small noise too. From Figure 4 we observe that the TZ-ZT model predictions are fairly accurate for dibit separations greater than $(0.65/a)^{-1} = 1.54a$, i.e., for dibit separations that are approx-

imately grater than the percolation threshold L. This covers the linear densities of interest because recording systems operate at transition separations that do not go below 3 to 2.5 times a, which is far greater than the percolation threshold $L = 1.36a$. Since the accuracy of the TZ-ZT model goes down to transition separations of $1.5a$, we conclude that the TZ-ZT model is well suited for simulating future systems that will challenge the bounds of achievable linear density.

IV. CONCLUSION

We compared the results of the triangle zig-zag transition (TZ-ZT) signal and media noise model to experimental data. We also presented a method of matching the model parameters to a real recording system. Our findings are that, after appropriate matching of the model parameters, the model is a fairly accurate representation of real data. Specifically, the TZ-ZT model is extremely accurate at predicting signal waveforms and media noise for isolated transitions. For interacting transitions, the TZ-ZT model is fairly accurate at densities of interest, that is, at linear densities that do not exceed the inverse percolation threshold length.

ACKNOWLEDGMENT

REFERENCES

[1] A. Kavčić and J. M. F. Moura, "Expedient media noise modeling: Isolated and interacting transitions," *IEEE Trans. Magn.*, vol. 32, pp. 3875–3877, Sept. 1996.

[2] J. Caroselli and J. K. Wolf, "Applications of a new simulation model for media noise limited magnetic recording channels," *IEEE Trans. Magn.*, vol. 32, pp. 3917–3919, Sept. 1996.

[3] J.-G. Zhu and N. H. Bertram, "Micromagnetic studies of thin metallic films," *J. Appl. Phys.*, vol. 63, pp. 3248–3253, April 1988.

[4] T. C. Arnoldussen and H. C. Tong, "Zigzag transition profiles, noise, and correlation statistics in highly oriented longitudinal media," *IEEE Trans. Magn.*, vol. 22, pp. 889–891, Sept. 1986.

[5] B. K. Middleton and J. J. Miles, "Recorded magnetization distributions in thin film media," *IEEE Trans. Magn.*, vol. 27, pp. 4954–4959, Nov. 1991.

[6] Y.-S. Tang, "Explicit formula for thin film disk noise based on zig-zag domain transitions," *IEEE Trans. Magn.*, vol. 24, pp. 3102–3104, Nov. 1988.

[7] A. Kavčić and J. M. F. Moura, "Statistical study of zig-zag transition boundaries in longitudinal digital magnetic recording." submitted to *IEEE Trans. Magn.*, 30 pages, Aug. 1996.

[8] A. Kavčić and J. M. F. Moura, "Signal generation model for high density magnetic recording," in *Proc. IEEE GLOBECOM 97*, (London), Nov. 1996.

[9] R. D. Brandt, A. J. Armstrong, H. N. Bertram, and J. K. Wolf, "A simple statistical model of partial erasure in thin film disk recording systems," *IEEE Trans. Magn.*, vol. MAG-27, pp. 4978–4980, Nov. 1991.

[10] J.-G. Zhu and X.-G. Ye, "Impact of medium noise on various partial response channels," *IEEE Trans. Magn.*, vol. 31, pp. 3087–3089, Nov. 1995.