Pierce's pattern generation procedure*

Suppose that a mill has recieved customers' orders for $r_1, r_2, \ldots, r_n$ rolls of a certain grade of paper, these rolls to be of widths $w_1, w_2, \ldots, w_n$ and (the same) diameter D. The mill has one paper machine which can manufacture the desired grade, this machine producing rolls of width L (where $L \geq w_i$ for all i). Since customer widths demanded are smaller than, or equal to, the production width of the paper machine, the production scheduler tries to find combinations of customers' widths[1] with which to fill out the width L of the paper machine rolls. Usually there will be a "side roll" of trim loss left over from such combinations which is repocessed or disposed of in some other manner. The paper trim problem, then, is to find trimming combinations of customer widths and to determine the number of machine rolls to be produced and cut according to each combination — so as to satisfy the customers' demand most efficiently [1, p. 13].

[...] A *dominating process* is defined to be one for which

$$d_k = \left(L - \sum_i a_{ik} w_i \right) < \min_i(w_i) \tag{1}$$

where [...] d_k is the trim loss for process k. Any process not satisfying (1) is then dominated in the sense that there exists at least one other process for the same paper machine which will supply at least one more customer roll of some width (and the same number of all other widths), for each machine roll manufactured. [...] The matrix of all possible processes for a problem will be termed the *exhaustive process* matrix, $\boldsymbol{A_E}$; the matrix containing only the dominating processes will be called the *dominating* matrix $\boldsymbol{A_D}$ [1, p. 18].

*Excerpt from Pierce J. F. Some large-scale production scheduling problems in the paper industry. — Englewood Cliffs : Prentice-Hall, 1964. — 255 p.

[1] To identify such combinations Pierce extensively used the term "process". Today it is more common to use term "pattern" instead.

[...] generation procedure is as follows, where customer widths w_1, w_2, $\ldots$, w_n are arranged in descending order:

Step 1. Set $a_1 = \min\left(\left\lfloor \frac{L}{w_1} \right\rfloor, r_1\right), a_2 = \min\left(\left\lfloor \frac{L-a_1 w_1}{w_2} \right\rfloor, r_2\right), \ldots,$
$a_n = \min\left(\left\lfloor \frac{L-\sum_{i=1}^{n-1} a_i w_i}{w_n} \right\rfloor, r_n\right)$
where $\lfloor g \rfloor$ is the largest integer contained in g.

Step 2. Record process in matrix $\boldsymbol{A_D}$.

Step 3. If there is an i, $1 \le i \le n-1$, such that $a_i > 0$ then let j be the largest such i, and go to **Step 4**.
If there is no such i, then terminate procedure: all dominating processes have been enumerated and recorded.

Step 4. Set $a'_1 = a_1, a'_2 = a_2, \ldots, a'_j = a_j - 1;$
$a'_{j+1} = \min\left(\left\lfloor \frac{L-\sum_{i=1}^{j} a'_i w_i}{w_{j+1}} \right\rfloor, r_{j+1}\right), \ldots,$
$a'_n = \min\left(\left\lfloor \frac{L-\sum_{i=1}^{n-1} a'_i w_i}{w_n} \right\rfloor, r_n\right)$
Go to **Step 2**.

[...] a procedure for generating the exhaustive matrix $\boldsymbol{A_E}$ is obtained from this procedure merely by changing the range in step 3 from $1 \le i \le n-1$ to $1 \le i \le n$. The resulting procedure will enumerate all processes, regardless of the ordering of customer width [1, p. 44].

References

[1] Pierce J. F. Some large-scale production scheduling problems in the paper industry. — Englewood Cliffs : Prentice-Hall, 1964. — 255 p.