$$
\begin{array}{ll}
M ::= x \mid c \mid \texttt{(lambda [\$}x\ \cdots\texttt{]}\ M\texttt{)} \mid \texttt{(}M\ M\ \cdots\texttt{)} & p ::= \_ \mid \texttt{\$}x \mid \texttt{,}M \mid \texttt{<}C\ p\ \cdots\texttt{>} \\
\quad \mid [M \cdots] \mid \{M \cdots\} \mid \texttt{<}C\ M\ \cdots\texttt{>} & \phi ::= \texttt{[}pp\ M\ \texttt{\{[}dp\ M\texttt{]}\ \cdots\texttt{\}]} \\
\quad \mid \texttt{(match-all}\ M\ M\ \texttt{[}p\ M\texttt{])} & pp ::= \texttt{\$} \mid \texttt{,\$}x \mid \texttt{<}C\ pp\ \cdots\texttt{>} \\
\quad \mid \texttt{something} \mid \texttt{(matcher}\ \{\phi\ \cdots\}\texttt{)} & dp ::= \texttt{\$}x \mid \texttt{<}C\ dp\ \cdots\texttt{>}
\end{array}
$$

Figure 1: Syntax of Egison

Evaluation of `matcher` and `match-all`:

$$
\frac{}{\Gamma, \texttt{(matcher [}pp_i\ \ M_i\ \ \texttt{[}dp_{ij}\ \ N_{ij}\texttt{]}_j\texttt{]}_i\texttt{)} \Downarrow ([pp_i, M_i, [dp_{ij}, N_{ij}]_j]_i, \Gamma)}
$$

$$
\frac{\Gamma, M \Downarrow v \qquad \Gamma, N \Downarrow m \qquad [[[p \sim_m v], \Gamma, \emptyset]] \Rrightarrow [\Delta_i]_i \qquad \Gamma \cup \Delta_i, L \Downarrow v_i \quad (\forall i)}{\Gamma, \texttt{(match-all}\ M\ N\ \texttt{[}p\ L\texttt{])} \Downarrow [v_i]_i}
$$

Matching states:

$$
\frac{}{\epsilon \to \texttt{none}, \texttt{none}, \texttt{none}} \qquad \frac{}{(\epsilon, \Gamma, \Delta) : \vec{s} \to (\texttt{some}\ \Delta), \texttt{none}, (\texttt{some}\ \vec{s})}
$$

$$
\frac{p \sim_m^{\Gamma \cup \Delta} v \Downarrow [\vec{a}_i]_i, \Delta'}{((p \sim_m v) : \vec{a}, \Gamma, \Delta) : \vec{s} \to \texttt{none}, (\texttt{some}[\vec{a}_i + \vec{a}, \Gamma, \Delta \cup \Delta']_i), (\texttt{some}\ \vec{s})}
$$

$$
\frac{\vec{s}_i \to \texttt{opt}\,\Gamma_i, \texttt{opt}\,\vec{s'}_i, \texttt{opt}\,\vec{s''}_i \quad (\forall i)}{[\vec{s}_i]_i \Rrightarrow \sum_i (\texttt{opt}\,\Gamma_i), \sum_i (\texttt{opt}\,\vec{s'}_i) + \sum_i (\texttt{opt}\,\vec{s''}_i)} \qquad \frac{}{\epsilon \Rrightarrow \epsilon} \qquad \frac{\vec{\vec{s}} \Rrightarrow \vec{\Gamma}, \vec{\vec{s'}} \qquad \vec{\vec{s'}} \Rrightarrow \vec{\Delta}}{\vec{\vec{s}} \Rrightarrow \vec{\Gamma} + \vec{\Delta}}
$$

Matching atoms:

$$
\frac{}{\texttt{\$x} \sim^{\Gamma}_{\texttt{something}} v \Downarrow [\epsilon], \{x \mapsto v\}} \qquad \frac{pp \approx^{\Gamma} p \Downarrow \textbf{fail} \qquad p \sim^{\Gamma}_{(\vec{\phi}, \Delta)} v \Downarrow \vec{\vec{a}}, \Gamma'}{p \sim^{\Gamma}_{((pp, M, \vec{\sigma}) : \vec{\phi}, \Delta)} v \Downarrow \vec{\vec{a}}, \Gamma'}
$$

$$
\frac{pp \approx^{\Gamma} p \Downarrow [p'_i]_i, \Delta' \qquad dp \approx v \Downarrow \textbf{fail} \qquad p \sim^{\Gamma}_{((pp, M, \vec{\sigma}) : \vec{\phi}, \Delta)} v \Downarrow \vec{\vec{a}}, \Gamma'}{p \sim^{\Gamma}_{((pp, M, (dp, N) : \vec{\sigma}) : \vec{\phi}, \Delta)} v \Downarrow \vec{\vec{a}}, \Gamma'}
$$

$$
\frac{pp \approx^{\Gamma} p \Downarrow [p'_j]_j, \Delta' \qquad dp \approx v \Downarrow \Delta'' \qquad \Delta \cup \Delta' \cup \Delta'', N \Downarrow [[v'_{ij}]_j]_i \qquad \Delta, M \Downarrow [m'_j]_j}{p \sim^{\Gamma}_{((pp, M, (dp, N) : \vec{\sigma}) : \vec{\phi}, \Delta)} v \Downarrow [[p'_j \sim_{m'_j} v'_{ij}]_j]_i, \emptyset}
$$

Pattern matching on patterns:

$$
\frac{}{\texttt{\$} \approx^{\Gamma} p \Downarrow [p], \emptyset} \qquad \frac{\Gamma, M \Downarrow v}{\texttt{,\$y} \approx^{\Gamma} \texttt{,}M \Downarrow \epsilon, \{y \mapsto v\}} \qquad \frac{pp_i \approx^{\Gamma} p_i \Downarrow \vec{p}_i, \Gamma_i \quad (\forall i)}{\texttt{<C}\ pp_1 \ldots pp_n\texttt{>} \approx^{\Gamma} \texttt{<C}\ p_1 \ldots p_n\texttt{>} \Downarrow \sum_i \vec{p}_i, \bigcup_i \Gamma_i}
$$

Pattern matching on data:

$$
\frac{}{\texttt{\$z} \approx v \Downarrow \{z \mapsto v\}} \qquad \frac{dp_i \approx v_i \Downarrow \Gamma_i \quad (\forall i)}{\texttt{<C}\ dp_1 \ldots dp_n\texttt{>} \approx \texttt{<C}\ v_1 \ldots v_n\texttt{>} \Downarrow \bigcup_i \Gamma_i}
$$

Figure 2: Formal semantics of Egison

Typing rules for terms:

$$\frac{}{\Gamma, x : T, \Delta \vdash x : T \Rightarrow \Gamma, x : T, \Delta}\text{T-Var} \qquad \frac{}{\Gamma \vdash n : \texttt{Integer} \Rightarrow \Gamma}\text{T-Num}$$

$$\frac{\Gamma \vdash M_1 : T_1 \Rightarrow \Gamma \qquad \Gamma \vdash M_2 : T_2 \Rightarrow \Gamma \ \cdots \ \Gamma \vdash M_n : T_n \Rightarrow \Gamma}{\Gamma \vdash \texttt{[}M_1\ M_2\ ...\ M_n\texttt{]} : \texttt{[}T_1\ T_2\ ...\ T_n\texttt{]} \Rightarrow \Gamma}\text{T-Tuple}$$

$$\frac{\Gamma \vdash M_1 : T \Rightarrow \Gamma \qquad \Gamma \vdash M_2 : T \Rightarrow \Gamma \ ... \ \Gamma \vdash M_n : T \Rightarrow \Gamma}{\Gamma \vdash \{M_1\ M_2\ ...\ M_n\} : \{T\} \Rightarrow \Gamma}\text{T-Collection}$$

$$\frac{\Gamma \vdash C : \texttt{[}S_1 \cdots S_n\texttt{]} \Rightarrow \Gamma \to T \qquad \Gamma \vdash N_1 : S_1 \cdots \Gamma \vdash N_n : S_n \Rightarrow \Gamma}{\Gamma \vdash \langle C\ N_1\ N_2\ ...\ N_n\rangle : T \Rightarrow \Gamma}\text{T-InductiveData}$$

$$\frac{\begin{array}{c}\Gamma \vdash M_1 : T_1 \Rightarrow \Gamma \qquad \Gamma \vdash M_2 : \texttt{(Matcher}\ T_1\texttt{)} \Rightarrow \Gamma \\ \Gamma \vdash p : \texttt{(Pattern}\ T_1\texttt{)} \Rightarrow \Gamma' \qquad \Gamma' \vdash M_3 : T_2 \Rightarrow \Gamma'\end{array}}{\Gamma \vdash \texttt{matchAll}\ M_1\ \texttt{as}\ M_2\ \texttt{with | }\ p\ M_3 \texttt{|} \cdots : \texttt{[}T_2\texttt{]} \Rightarrow \Gamma}\text{hoge}$$

$$\frac{}{\Gamma \vdash \texttt{something} : \texttt{(Matcher}\ T\texttt{)} \Rightarrow \Gamma}\text{T-Something}$$

$$\frac{\begin{array}{c}\Gamma \vdash pp_i : \texttt{(PPPattern}\ T\ \texttt{[}S_k\texttt{]}_k\texttt{)} \Rightarrow \Gamma \\ \Gamma \vdash M_i : \texttt{(Matcher}\ \texttt{[}S_k\texttt{]}_k\texttt{)} \Rightarrow \Gamma \\ \Gamma \vdash dp_{ij} : \texttt{(PDPattern}\ T\texttt{)} \Rightarrow \Gamma' \\ \Gamma' \vdash N_{ij} : \{\texttt{[}S_k\texttt{]}_k\} \Rightarrow \Gamma' \qquad (\forall i, j)\end{array}}{\begin{array}{c}\Gamma \vdash \texttt{(matcher}\ \{\texttt{[}pp_i\ M_i\ \{\texttt{[}dp_{ij}\ N_{ij}\texttt{]}\}_j\texttt{]}_i\}\texttt{)} : \\ \texttt{(Matcher}\ T\texttt{)} \Rightarrow \Gamma\end{array}}\text{T-Matcher}$$

Typing rules of patterns:

$$\frac{}{\Gamma \vdash \_ : \texttt{(Pattern}\ T\texttt{)} \Rightarrow \Gamma}\text{T-Wildcard} \quad \frac{\Gamma \vdash M : T \Rightarrow \Gamma}{\Gamma \vdash \texttt{\#}M : \texttt{(Pattern}\ T\texttt{)} \Rightarrow \Gamma}\text{T-ValuePattern}$$

$$\frac{}{\Gamma \vdash \texttt{\$}x : \texttt{(Pattern}\ T\texttt{)} \Rightarrow \Gamma, \{x : T\}}\text{T-PatternVariable}$$

$$\frac{\begin{array}{c}\Gamma \vdash C_p : \texttt{[(Pattern}\ S_1\texttt{)} \cdots \texttt{(Pattern}\ S_n\texttt{)]} \to \texttt{(Pattern}\ T\texttt{)} \Rightarrow \Gamma \\ \Gamma \vdash M_1 : \texttt{(Pattern}\ S_1\texttt{)} \Rightarrow \Gamma^1 \qquad \Gamma^1 \vdash M_2 : \texttt{(Pattern}\ S_2\texttt{)} \Rightarrow \Gamma^2 \\ \Gamma^2 \vdash M_3 : \texttt{(Pattern}\ S_3\texttt{)} \Rightarrow \Gamma^3 \\ \cdots \quad \Gamma^{n-1} \vdash M_n : \texttt{(Pattern}\ S_n\texttt{)} \Rightarrow \Gamma^n\end{array}}{\Gamma \vdash \langle C_p\ M_1\ M_2\ ...\ M_n\rangle : \texttt{(Pattern}\ T\texttt{)} \Rightarrow \Gamma^n}\text{T-InductivePattern}$$

Typing rules of atomic pattern pattern:

$$\frac{}{\Gamma \vdash \texttt{\$} : \texttt{(PPPattern}\ T\ \texttt{[}T\texttt{])} \Rightarrow \Gamma}\text{T-PatternHole}$$

$$\frac{}{\Gamma \vdash \texttt{,\$}x : \texttt{(PPPattern}\ T\ \texttt{[])} \Rightarrow \Gamma}\text{T-ValuePatternPattern}$$

$$\frac{\Gamma \vdash C_{pp} : \texttt{[}T_i\texttt{]}_i \to T \Rightarrow \Gamma \qquad \Gamma \vdash pp_i : \texttt{(PPPattern}\ T_i\ S_i\texttt{)} \Rightarrow \Gamma}{\Gamma \vdash \langle C_{pp}\ pp_i\rangle_i : \texttt{(PPPattern}\ T\ \Sigma_i S_i\texttt{)} \Rightarrow \Gamma}\text{T-InductivePatternPattern}$$

Typing rules of atomic data pattern:

$$\frac{}{\Gamma \vdash \texttt{\$}x : \texttt{(PDPattern}\ T\texttt{)} \Rightarrow \Gamma +\!\!+ \{x : T\}}\text{T-DataPatternVariable}$$

$$\frac{\Gamma \vdash C_{dp} : \texttt{[}S_i\texttt{]}_i \to T \Rightarrow \Gamma \qquad \Gamma \vdash dp_i : S_i \Rightarrow \Gamma^i}{\Gamma \vdash \langle C_{dp}\ dp_i\rangle_i : T \Rightarrow \sum_i \Gamma^i}\text{T-InductiveDataPattern}$$

Figure 3: Type system of Egison