Define risk (disease) odds difference : $\omega_{1i} - \omega_{0i}$ on ind. i

Define incidence/risk/disease odds ratio : $\omega_{1i} / \omega_{0i}$ on ind. i.

The expected number of individuals over the risk period :

	Exposed	Not exposed	
Disease occurs	$A = \sum_1 r_{1i}$	$B = \sum_0 r_{0i}$	
Disease not occur	$C = \sum_1 s_{1i}$	$D = \sum_0 s_{0i}$	
	N_1	N_0	

s = survivors
r = prob. disease will occur

Incidence proportion ^{1}exposure : $\frac{A}{N_1} = \frac{\sum_1 r_{1i}}{N_1}$

Incidence proportion ^{No}exposure : $\frac{B}{N_0} = \frac{\sum_0 r_{0i}}{N_0}$

} measures of average risk

Disease odds ^{1} exposure : $\frac{A}{C} = \frac{\sum_1 r_{1i} / N_1}{\sum_1 s_{1i} / N_1}$

Disease odds ^{No} exposure : $\_ = \frac{\sum_0 r_{0i} / N_0}{\sum_0 s_{0i} / N_0}$

} Ratio of average risk to average survival probability.

Individual level risk odds.

$\omega_{1i} = r_{1i} / s_{1i}$
$\omega_{0i} = r_{0i} / s_{0i}$
} on ith individual

$\Rightarrow \frac{\sum r_{1i}}{\sum s_{1i}} \neq \frac{\sum r_{1i}}{\sum s_{1i}} \Leftarrow$ Average disease odds

$\sum \omega_{1i} / N_1$

$\sum \omega_{0i} / N_0$

Ratio of average risk to average survival probability. $\neq$ Average disease odds

It's NOT equal

odds are complicated and hard to explain!!

Assume NO CONFOUNDING (i.e. group comparability)

then the average risk in the exposed and unexposed subcohorts would be the same if the exposed group was unexposed.

This is a critical counterfactual assumption.

$$\frac{\sum_0 r_{0i}}{N_0} = \frac{\sum_1 r_{0i}}{N_1}$$

The incidence proportion (risk) difference

$$\frac{A}{N_1} - \frac{B}{N_0} = \frac{\sum_1 r_{1i}}{N_1} - \frac{\sum_0 r_{0i}}{N_0} = \frac{\sum_1 r_{1i}}{N_1} - \frac{\sum_1 r_{0i}}{N_1} \Rightarrow$$

This only holds if NO CONFOUNDING.

Hypothetical unexposed r_{0i} and exposed r_{1i} in the same cohort N_1

Absolute change in the average risk of the exposed subcohort produced by exposure

$$= \frac{\sum_1 (r_{1i} - r_{0i})}{N_1} \Rightarrow$$

Average absolute change in risk produced by exposure in exposed individuals.

This is a statement with CAUSAL MEANING!!

That's why we need to avoid bias, in order to have answers that have causal significance.

Incidence proportion ratio:

$$\frac{\frac{A}{N_1}}{\frac{B}{N_0}} = \frac{\sum_1 r_{1i}/N_1}{\sum_0 r_{0i}/N_0} = \frac{\sum_1 r_{1i}/N_1}{\sum_1 r_{0i}/N_1} =$$

proportionate change in the average risk ($\sum_1 r_{1i}/N_1$, $\sum_0 r_{0i}/N_1$) of the exposed subcohort (N_1) produced by exposure (r_0 vs r_1)

$\neq$ average proportionate change in risk produced by exposure

$$\frac{\sum_1 (r_{1i}/r_{0i})}{N_1}$$

The causal interpretation looks at what happen to the exposed cohort if they had not been exposed. ⇒ this is the point of the unexposed cohort. It's a proxy for the exposed cohort if no exposure had occured.

The disease odds ratio

$$\frac{A/C}{B/D} = \frac{\sum_1 r_{1i} / \sum_1 s_{1i}}{\sum_0 r_{0i} / \sum_0 s_{0i}}$$

NO CONFOUNDING ASSUMPTION ⇓ =

$$\frac{\sum_1 r_{1i} / \sum_1 s_{1i}}{\sum_1 r_{0i} / \sum_1 s_{0i}} = \frac{\sum w_{1i}}{\sum w_{0i}}$$

The proportionate change in the incidence odds (i.e the ratio of the average risk to the average survival probability) in the exposed population produced by exposure (r_0 vs r_1, s_0 vs s_1)

This is the correct interpretation of the OR, but it is hard to comprehend.

$$\frac{\sum_1 w_{1i} / N_1}{\sum_1 w_{0i} / N_1}$$ = proportionate change in the average odds in the exposed population produced by exposure

$$\frac{\sum_1 \left(\frac{w_{1i}}{w_{0i}}\right)}{N_1}$$ = average of the individual odds ratios among the exposed

These are more intuitive but incorrect interpretations of the OR

Thus, the incidence odds ratio lacks any simple interpretation in terms of

average risk or average odds or

an average exposure effect on individual risk or odds.