Conference Package

for the

28^{th} Canadian Undergraduate Mathematics Conference

19–22 August 2021

Hosted virtually by Western University

Contact information

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About CUMC 2021

Welcome to CUMC 2021! We're glad to have you with us.

The Canadian Undergraduate Mathematics Conference (CUMC) is a national mathematics conference designed to introduce research topics and relevant skills to undergraduate students. The CUMC was first held at McGill University in 1994, and has been held annually since then, sponsored by the Canadian Mathematical Society (CMS) and its associated student committee, Studc.

Students are encouraged to present their mathematical topic of interest in a friendly and non-competitive environment, and to also gain exposure to new areas of mathematics. Keynote lectures by established professors give attendees a glimpse into mathematics as it is practised in both academia and industry, while talks by fellow attendees help motivate undergraduates to explore mathematics beyond the curriculum. Planned social events will also encourage connections among students that will provide support for their future academic and professional ambitions. The conference also features a student poster session, and a variety of panels and workshops.

Conference Schedule

*** All times are in Eastern Daylight Time (EDT). ***

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<tr><td>11:00am – 11:30am</td><td rowspan="2">Keynote lecture: Dr. Jeremy Quastel</td><td rowspan="2">Keynote lecture: Dr. Malabika Pramanik</td><td rowspan="2">Keynote lecture: Dr. Francesca Vidotto</td><td rowspan="2">Keynote lecture: Dr. Gord Sinnamon</td></tr>
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8:30pm – 9:00pm		Student social	Student social	
9:00pm – 9:30pm				
9:30pm – 10:00pm				
10:00pm – 10:30pm				

Keynote Lectures – Abstracts

The mathematics of random growth

Dr. Jeremy Quastel
Department of Mathematics
University of Toronto
http://www.math.toronto.edu/quastel/

In the past two decades there has been considerable progress on understanding the dynamics of random interfaces. The talk will be an introduction to the results and methods used to study the KPZ (Kardar-Parisi-Zhang) equation and its universality class which include stochastic analysis, integrable partial differential equations, complex analysis and algebra.

Colouring trees

Dr. Stephanie van Willigenburg
Department of Mathematics
University of British Columbia
https://secure.math.ubc.ca/~steph/

Trees are used to model a myriad of concepts from data storage to sentence structure. One particular example is to model hydrocarbons, such as methane and propane, for which a formula to calculate the exact number is still not known. One potential way to calculate this number is to use a generalization of the chromatic polynomial, which itself was invented in 1912 by Birkhoff when he was trying to solve the famous four colour map problem. This generalization is called the chromatic symmetric function, and in this talk we will be introduced to it, and, amongst other things, discover which trees it can distinguish to help us count hydrocarbons.

So you think you can solve $x + y = 2z$?

Dr. Malabika Pramanik
Department of Mathematics
University of British Columbia
https://secure.math.ubc.ca/~malabika/index.html

A linear equation is an integral part of our introduction to high school algebra. An underdetermined system of homogeneous linear equations (in other words, where the number of equations is fewer than the number of unknowns and the constant terms are all zero) is easy to solve with infinitely many solutions ... or so we are told. This talk aims to provide a counterpoint. Depending on which equations you are solving and where you are looking for solutions, you may find some, plenty or none. This line of inquiry has a rich history in mathematics with far-reaching consequences, interfacing with several areas of analysis, number theory, combinatorics and geometry. We will look at some of the landmark results in the subject, and work out a couple of elementary proofs in real time.

Any two circles meet in 4 points

Dr. Joel Kamnitzer
Department of Mathematics
University of Toronto
http://www.math.toronto.edu/~jkamnitz/

If you have ever drawn two circles on a piece of paper, then you will probably say that the title of this talk is false. However, I will convince you of the truth of this statement and explain that it is a special case of Bezout's theorem on the intersection of plane curves. We will see that Bezout's theorem gives us an entry into modern algebraic geometry, including projective space, schemes, intersection theory, and derived algebraic geometry.

The mathematics of quantum gravity

Dr. Francesca Vidotto
Department of Physics and Astronomy
Western University
http://loopy.space/vidotto/

Loop Quantum Gravity has a beautiful and rich mathematical structure. I review some of the main mathematical tools that are at the core of the theory, in particular in its covariant formulation. I discuss their physical motivation in order to construct a background independent quantum field theory. I conclude by mentioning some applications to cosmology and black holes.

The World of Primes

Dr. Sujatha Ramdorai
Department of Mathematics
University of British Columbia
https://secure.math.ubc.ca/~sujatha/

Prime Numbers have been studied for centuries as building blocks in Number Theory. In the last decade, they found tremendous applications in Cryptography and Internet Security. In this talk, we will discuss the fascinating results that are known about prime numbers, their applications as well as a few other conjectures in Number Theory.

A Cesàro Operator Inequality via Calculus

Dr. Gord Sinnamon
Department of Mathematics
Western University
https://www.math.uwo.ca/faculty/sinnamon/

For each $p > 1$ we will find the smallest constant C_p such that the inequality

$$\sum_{n=1}^{\infty} \left| x_n - \frac{1}{n} \sum_{k=1}^{n} x_k \right|^p \leq C_p \sum_{n=1}^{\infty} |x_n|^p$$

is true for all sequences (x_n) for which the right-hand side is finite. This problem was posed in 1996 and solved earlier this year. Despite being an open problem for 25 years, the solution involves only undergraduate Calculus.

On Hadwiger's conjecture

Dr. Luke Postle
Department of Combinatorics and Optimization
University of Waterloo
https://www.math.uwaterloo.ca/~lpostle/

In 1943, Hadwiger conjectured that every graph with no K_t minor is $(t-1)$-colourable for every $t \geq 1$. This is a vast generalization of the famous Four Colour Theorem which is implied by the $t = 5$ case. In the 1980s, Kostochka and Thomason independently proved that every graph with no K_t minor has average degree $O(t\sqrt{\log t})$ and hence is $O(t\sqrt{\log t})$-colourable. Recently, Norin, Song and I showed that every graph with no K_t minor is $O(t(\log t)^{\beta})$-colourable for every $\beta > 1/4$, making the first improvement on the order of magnitude of the $O(t\sqrt{\log t})$ bound. Here we discuss this and my more recent result that every graph with no K_t minor is $O(t(\log t)^{\beta})$-colourable for every $\beta > 0$; more specifically, they are $O(t(\log \log t)^{6})$-colourable.

Student Talks – Schedule

*** All times are in Eastern Daylight Time (EDT). ***

THURSDAY, AUGUST 19TH

	Room 1	Room 2	Room 3
3:30pm – 4:00pm	Information Theory and its Relevance to Biological Signaling Pathways *Julie Midroni*	Probability Concentration Inequalities *Danil Platonov*	Limit Theorems for Permutations with Restricted Cycles: An Experimental Journey *Tiadora Ruza*
4:00pm – 4:30pm	Cursed Fractions and Hyperbolic Geometry *Kunal Chawla*	PageRank: The Billion Dollar Algorithm *Devam Sisodraker*	Baire's Category Theorem And The Banach-Steinhaus Theorem *William Kimoto*

	Room 1	Room 2	Room 3
6:30pm – 7:00pm	An Introduction to the Poincare Upper Half Plane and Modular Forms *Jiyuan (Maki) Lu*	A Graph-Theoretical Representation of the Natural Numbers *Christopher Frank Stephan Maligec*	Symétries exhibées par des ensembles du type Mandelbrot pour certaines familles polynomiales *Charlotte Lavoie-Bel*
7:00pm – 7:30pm	Field and Ring of Fractions *Shilong Yu*	Dissipative-Dispersive Wave Equations *Mahima Siali*	Simple Colouring Oriented Graphs *Alexander Clow*
7:30pm – 8:00pm	Paul Lévy's Construction of Brownian Motion *Mathew Cater Benavides*	On the center of the cohomology algebra of a closed orientable 3-manifold *Qiu Shi Wang*	Influence Centrality *Rakan Omar*
8:00pm – 8:30pm	Hinges and Snowballs *Yuveshen Mooroogen*	Enumerating Positions of Distance Games *Lexi Nash*	Extended chromatic symmetric functions and equaliity of ribbon Schur functions *Victor Wang*

Friday, August 20th

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<tr><td>12:00pm – 12:30pm</td><td rowspan="2">Examining the Gröbner Basis of a Symmetric Function Ideal Within the Context of Catalan Numbers

Xavier Mootoo,
Vedarth Vyas</td><td>A Tale of Two Triangles

Kieran Bhaskara</td><td>Cross-frequency coupling studies of intracranial EEG data of epilepsy patients using time-frequency distributions

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Sylvie Dornez</td><td>Building galaxies from the bottom up: how uniquely does galaxy assembly determine their present day properties?

Sina Babaei Zadeh</td></tr>
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	Room 1	Room 2	Room 3
6:30pm – 7:00pm	Quantum Walks on Graphs: Perfect State Transfer and Periodicity *Antonio Acuaviva*	Integrated data driven approaches for understanding immunological data *Geneviève Bistodeau-Gagnon*	The Mathematics of Tiling a Soccer Ball *Amie Ga Yun Su*
7:00pm – 7:30pm	Reflecting on Our Conceptions of Proofs While Solving a Sudoku *Julia McClellan*	A Mathematical Model of Brain Tumour Growth *Blanche Mongeon*	Framelets on Manifold and Graphs with Applications *Songlin Jin*
7:30pm – 8:00pm	Reciprocity Laws: Uncovering the Secrets Behind Primes *Kiera Sobolewski*	Plank theorems and their applications *William Verreault*	How to Solve a Cryptarithm *Vinicius Santos*
8:00pm – 8:30pm	Exceptional Gegenbauer Polynomials and Confluent Darboux Transformations *James Elgin Munday*	Zero Forcing Sets and Forts in Trees *Johnna Parenteau*	Spectral Graph Theory: Structure & Characteristic Polynomial *Lewis Glabush*

SATURDAY, AUGUST 21ST

	Room 1	**Room 2**	**Room 3**
6:30pm – 7:00pm	An Introduction to Vinogradov's Mean Value Theorem *Tuan Hiep Do, Josué Kurke, Jihanjie (Andrew) Luo*	Understanding Non-Equilibrium Steady States Adopting Methods From Quantum Field Theory *Gustavo Cicchini Santos*	Introduction to Bayesian Statistics *Ashley Zhang*
7:00pm – 7:30pm		An introduction to Elliptic curves and variants of the Lang-Trotter conjecture *Seraphim Jarov*	Various Methods for Optimization *Younwoo (Ethan) Choi*
7:30pm – 8:00pm		From Field Theory to Algebraic Number Theory *Amar Venga*	Introduction on the min-max construction of minimal surfaces *Xinze Li*
8:00pm – 8:30pm	Spatiotemporal transmission dynamics of COVID-19 using mathematical models *Crystal Wai*	A mathematical definition of entanglement and its measurement *Alex Kazachek*	

Sunday, August 22nd

	Room 1	**Room 2**	**Room 3**
6:30pm – 7:00pm	Bayesian Disease Mapping of COVID-19 Cases Across the Boroughs of Montreal *Leo Vanciu*	Existence of an Equilibrium in Mixed Networks of Coordinators and Anti-coordinators *Achintya Raya Polavarapu*	Determining Galois Group Containment with Invariants *Curtis Wilson*
7:00pm – 7:30pm	Rapid testing for COVID-19 and SIQR model *Jiawei Chen*	Introduction to minimal surfaces and submanifolds and some results for conformal automorphisms of the unit ball *Karan Bains*	Number Theory meets Machine Learning *Brendon James Thomson*
7:30pm – 8:00pm	Billiards, Illumination and the Irrationality Of It All *Uttara Sheila Rajagopalan*	The Birthday Paradox *Veer Prakash Bhatia*	Introducing… Determinantal Facet Ideals! (With a Connection to Kazhdan-Lusztig Ideals) *Ryan Edwards*
8:00pm – 8:30pm	Distorted Projections: Do you see what I see? *Patrick Cyr*	How to solve $a/(b+c)+b/(a+c)+c/(a+b)=4$ *Xinran Zhou*	Determinants of COVID-19 vaccine acceptance in 143 countries around the world *Mehrdad Kazemi*

Student Talks – Abstracts

Information Theory and its Relevance to Biological Signaling Pathways

Julie Midroni
University of Toronto

Information theory provides a way by which to measure the 'quality' of a signal once it has been received – that is, how much of the original message has been lost or distorted, and if it is reasonably possible to decode this message despite loss. This concept is directly relevant to biological signalling pathways, which transmit information from the outside of the cell to the inside in a noisy and often bottlenecked environment. A brief explanation of the basics of information theory will be provided, followed by a review of this concept in the context of biological signalling, and a few original results.

Probability Concentration Inequalities

Danil Platonov
University of British Columbia

I will introduce probability concentration inequalities, deriving some of them, and explaining their uses in the context of non-asymptotic probability theory. The talk will be based on the first two chapters of Vershynin's *High-Dimensional Probability* book.

Limit Theorems for Permutations with Restricted Cycles: An Experimental Journey

Tiadora Ruza
University of Waterloo

Analytic combinatorics in several variables is a field of study focused on the derivation of limit behaviours of multivariate sequences. This talk will walk through proving a local central limit theorem for a family of permutations with restricted cycles, using techniques from the theory of analytic combinatorics in several variables. The focus of the talk will be not only on general methods for proving a local central limit theorem, but also on the benefits of experimentation with mathematical software to provide insight on how to apply the tools provided by analytic combinatorics in several variables. As a key part of the proof, we compute a symbolic determinant using a powerful (and somewhat general) method involving LU-factorization. This talk assumes basic familiarity with matrices, but no prior knowledge of analytic combinatorics or of limit theorems is required.

Cursed Fractions and Hyperbolic Geometry

Kunal Chawla
University of Toronto

When learning how to add fractions, a common error is to add the numerators and denominators: $a/b + c/d = (a + b)/(c + d)$. Together, we'll see some interesting math in which this is a natural operation, and we'll look at some cool pictures along the way. No formal prerequisites, but if you've heard the words 'continued fraction', 'geometric group theory', or 'hyperbolic geometry', then I hope you'll enjoy the talk.

PageRank: The Billion Dollar Algorithm

Devam Sisodraker
University of British Columbia

The PageRank algorithm was invented to help rank the relevance of various websites during the early days of the internet. Larry Page and Sergey Brin, the founders of Google, designed this algorithm to improve search results on the internet. It recursively defines the popularity of a website based on how often other sites reference it. In this talk, we'll discuss how it came to life, some web history, the algorithm's inner workings and its optimization for large-scale use.

Baire's Category Theorem And The Banach-Steinhaus Theorem

William Kimoto
Queen's University

In this talk we will introduce the Baire Category Theorem and study its application to the Banach-Steinhaus Theorem, sometimes known as the principal of uniform boundedness. We begin by recalling the notions of category, density relative to a set, meager and non-meager sets, and completeness. Topological notions such as these will play a pivotal role in the conceptualization and proof of these statements. We will present the Banach-Steinhaus Theorem as a result of the Baire Category Theorem, and highlight its significance in the study of mathematical analysis.

An Introduction to the Poincare Upper Half Plane and Modular Forms

Jiyuan (Maki) Lu
University of Toronto

The Poincaré Upper Half Plane, $\mathcal{H}$, together with its corresponding metric (the Poincaré metric), form one of the first models in hyperbolic geometry. The special linear group $\mathrm{SL}(2, \mathbb{R})$ acts on $\mathcal{H}$ transitively and isometrically by fractional linear transformations. A modular form is a holomorphic function with certain growth condition satisfying a symmetry condition under some group actions. The theory of modular forms, as a branch of complex analysis, plays an important role in number theory; it also has many relationships with geometry. This talk will cover the basic notions in the subject, inviting the audience to the modern theory of modular forms.

A Graph-Theoretical Representation of the Natural Numbers

Christopher Frank Stephan Maligec
Western University

The present short paper aims at providing an alternative way of looking at natural numbers in terms of the basic human abilities of perceiving identity and distinctness (sameness and difference). Numbers are defined in terms of the relations of identity and distinctness since we perceive pluralities in terms of something being distinct from something else, and that, in turn, as being distinct from something else, and so on. Although the results are not as elegant and simple as other definitions, and are in fact based on basic graph theory, they represent an interesting way of looking at the concept.

Symétries exhibées par des ensembles du type Mandelbrot pour certaines familles polynomiales

Symmetries appearing in Mandelbrot-like sets for some polynomial families

Charlotte Lavoie-Bel
Université Laval

L’objectif de cette présentation est de décrire des aspects du comportement dynamique de certaines familles polynomiales complexes. À cet égard, il est difficile d’oublier l’archiconnu ensemble de Mandelbrot. Ici, l’accent est mis sur les ensembles du type Mandelbrot pour d’autres polynômes, et en particulier sur les symétries que ces ensembles exhibent. La présentation commence par un aperçu de certains concepts fondamentaux reliés aux systèmes dynamiques et à la théorie du chaos. Elle se conclut par une étude de cas d’une famille polynomiale en particulier. Préparez-vous à contempler de magnifiques images et graphiques!

La présentation et le Beamer sont en anglais. Les questions peuvent être posées en français et en anglais.

The goal of this presentation is to describe aspects of the dynamical behavior of some complex polynomial families. When one thinks about these concepts, it is difficult to forget the ultra-famous Mandelbrot set. Here, attention will be focused on Mandelbrot-like sets for other polynomial functions, and especially on the symmetries appearing in these sets. The presentation begins with an overview of some fundamental concepts related to dynamical systems and chaos theory. It concludes with a case study of a precise family of polynomials. Get ready to contemplate many beautiful pictures and graphics!

The presentation and the Beamer are in English. Questions can be asked in French and English.

Field and Ring of Fractions

Shilong Yu
University of Toronto

Give a brief overview of rings, fields, definition of field and ring of fractions, as well as examples.

Dissipative-Dispersive Wave Equations

Mahima Siali
Western University

Abstract pending.

Simple Colouring Oriented Graphs

Alexander Clow
St. Francis Xavier University

Graph colouring traces its roots as far back as the 1850s with the inception of the famous Four Colour Theorem (proven by Appeal and Haken in 1976) and remains relevant to this day. Give this it can be surprising that for oriented planar graphs the the maximum number of colours needed (the maximum oriented chromatic number, χ_o, of a planar graph) cannot even be conjectured with any confidence (it is known to be $18 \leq n \leq 80$). In this talk will introduce another method of colouring oriented graphs, known as simple colouring, which is equivalent to oriented colouring in some special cases of interest.

Paul Lévy's Construction of Brownian Motion

Mathew Cater Benavides
University of Toronto

Throughout the early 1920's celebrated American mathematician Norbert Wiener produced foundational works within the now deemed field of stochastic processes. Among these works was a provision of among the first formal mathematical formulations of a physical phenomenon first observed by one Robert Browne in 1827 of pollen grains suspended in water subject to the random forces of neighbouring particles and their erratic movement as consequence; a phenomenon now kept in common parlance as Brownian motion. Wiener's first existence and uniqueness theorems for the process were soon simplified by one Paul Lévy just under three decades later (circa 1948) when he published his monograph on Brownian motion *Processus stochastiques et mouvement brownien* containing a number of fundamental results regarding the process but in particular that of his especially elegant and simple construction of Brownian motion, a construction of which is widely considered as the most beautiful the field has seen to date. This talk aims to serve as a juxtaposition between that of Wiener and Lévy's approaches, an exposition into such beauty and an unravelling of its elegance.

On the center of the cohomology algebra of a closed orientable 3-manifold

Qiu Shi Wang
McGill University

Algebraic topology uses algebraic invariants to study topological spaces up to homotopy type. I will first present the basic properties of the singular cohomology algebra of a closed orientable 3-manifold. An existence theorem of Sullivan allows us to equivalently study the space of alternating three-forms on a vector space. I will present a result of my work on the subject : a basic characterization of the center of the cohomology algebra in terms of Sullivan's three-form. Time permitting, I will frame this result in the context of Hochschild cohomology.

Influence Centrality

Rakan Omar
York University

In graph theory and network analysis, the notion of centrality refers to assigning nodes in a network an index representing the extent to which each node is central – important to the network, or well positioned in it – based on some mathematical property. There are a variety of measures of centrality, each of which measures 'centrality' differently, based on (or resulting in) a different definition of 'importance' or 'prominence'. I will define an original measure of centrality, a variation of pageRank centrality, which I call 'influence centrality', that measures the extent to which a node contributes a relation (what is represented by the arcs) to the graph in a directed weighted graph with a finite number of nodes, which are initially labelled with values. I will discuss some properties, applications, and extensions of influence centrality. I assume some familiarity with graph theory terminology and linear algebra.

Hinges and Snowballs

Yuveshen Mooroogen
University of Toronto

The isoperimetric problem asks: among all plane figures of a given perimeter, which is the one with the greatest area? In the 1830s, J. Steiner presented a number of ingenious – yet flawed – geometric solutions to this problem. Can you spot the mistake in his arguments?

Enumerating Positions of Distance Games

Lexi Nash
Concordia University of Edmonton

Distance games are two player games that are played on a graph. They consist of the players colouring vertices/placing tokens without later moving or removing them and the placement of new pieces is dependent on the distance from previously played pieces. The game ends when the current player cannot place a token, resulting in a loss for them. It was found that the independence polynomial of some graph, related to the game board, is a representation of the number of positions for that game on that board. Recent work enumerated the positions of three distances games (Snort, Col, and Cis) on paths by computing their generating functions. We extend this work to give bivariate polynomials enumerating the same games on new families of graphs. We also take a look at some new distance games on paths.

Extended chromatic symmetric functions and equaliity of ribbon Schur functions

Victor Wang
University of British Columbia

We introduce the extended chromatic symmetric function of a weighted graph, which generalizes the chromatic polynomial and satisifies a deletion-contraction recurrence. Intriguingly, we discover that two weighted paths have equal extended chromatic symmetric functions precisely when two ribbon Schur functions are equal.

Examining the Grobner Basis of a Symmetric Function Ideal Within the Context of Catalan Numbers

Xavier Mootoo, Vedarth Vyas
York University

We examine the symmetric function ideal contained within a commutative ring over a field with characteristic zero. We conjecture that the reduced Grobner basis of this ideal consists of polynomials which have leading terms equivalent to a Modified Catalan Path (MCP), which is defined as a Dyck path that stays above the diagonal (moving northeast) until it crosses the diagonal at the last step. Furthermore, the number of MCPs for a fixed path length corresponds a Catalan number as a function of that length.

A Tale of Two Triangles

Kieran Bhaskara
Dalhousie University

Pascal's triangle is a well-studied object in number theory and combinatorics. In this talk, I will describe a related array of numbers known as Stern's triangle. Stern's triangle has many remarkable properties, some of which have analogues in Pascal's triangle, and others which are completely novel. This talk will explore some of these properties and discuss current open problems concerning Stern's triangle.

Cross-frequency coupling studies of intracranial EEG data of epilepsy patients using time-frequency distributions

Daniel Girvitz
University of Calgary

In this presentation, I address the relevance of time-frequency distributions (TFD) for cross-frequency coupling studies, an area of interest in computational neuroscience. Cross-frequency coupling (CFC) is a phenomenon whereby a lower frequency band of neuronal signals from one region of the brain modulates higher frequency bands of the same and/or

other regions. This is suggested to be one of the key mechanisms to understand information processing in the brain. Here, I study human epilepsy by analyzing and interpreting intracranial electrical encephalogram (iEEG) data recorded on or inside the brain. I focus on the hippocampus, specifically left-anterior hippocampus, which is known for its role in short- and long-term memory. The iEEG data are intrinsically non-stationary, which traditional Fourier transform methods fail to analyze. To resolve this, I adapt time-frequency distributions to study cross-frequency coupling and in particular, phase-amplitude coupling (PAC). I show, with examples of time-frequency distributions, reduced-interference Rihaczek TFD and how the phase and amplitude are extracted, and the phase-amplitude coupling metrics that are studied with human epilepsy data.

De la géométrie euclidienne aux géométries non euclidiennes

Sylvie Dornez
Université de Saint Boniface

Les théorèmes de Pythagore, Ceva, Menelaus, Pappus et Desargues sont cinq théorèmes bien connus de la géométrie euclidienne classique. Cependant, restent-ils valides dans le contexte de la géométrie sphérique et hyperbolique? Avec un point de vue synthétique, nous explorerons les analogues de ces théorèmes dans le contexte de ces géométries, ou des contre-exemples lorsque tels résultats n'existent pas.

Building galaxies from the bottom up: how uniquely does galaxy assembly determine their present day properties?

Sina Babaei Zadeh
University of Toronto

An important question in astrophysics is how to predict the future state of galaxies, and to determine if two evolving galaxies will share the same properties given their assembly history. In an attempt to answer this, we look at the EAGLE simulation (Evolution and Assembly of GaLaxies and their Environments) which contains data on about one million galaxies, and their defining properties. Our research utilizes the assembly history in order to answer this question. By analyzing and grouping similar assembly histories, we look to analyze the properties of the galaxies in a particular group to see if their likeness in assembly history can predict their likeness in other properties. An important aspect of the research involves the criteria that we use to find similar assembly histories, and defining the mathematical properties that define our criteria. Using this bottom-up approach, we hope to be able to predict the properties of galaxies like our own Milky Way in a few billion years in the future.

This analysis complements the work done in the reverse direction, which attempts to predict the assembly history given the present-day properties of a particular galaxy.

Quantum Walks on Graphs: Perfect State Transfer and Periodicity

Antonio Acuaviva
Complutense University of Madrid

In recent years, the rise of quantum computing has brought a newfound interest to the study of quantum walk, a natural generalization of random walks on graphs. In this talk, we will introduce the concept of continuous-time quantum walk and some of its properties. Particularly, we will focus on perfect state transfer and periodicity in quantum walks and the relation between these two phenomena.

Integrated data driven approaches for understanding immunological data

Geneviève Bistodeau-Gagnon
Université de Montréal

Methodologies currently applied to the synthesis of large biomedical datasets remain, generally speaking, poorly equipped to provide a mechanistic/causal understanding of time-varying biological processes across the multiple scales present within data. The research project presented in this talk is focused on developing a data science pipeline in which complex biomedical data of diseases can be visualized, understood and predicted, using data from a rare blood disorder called cyclic thrombocytopenia and patients hospitalized with COVID-19. We will discuss approaches based on empirical dynamics and data exploration methods, which allow us to get a better understanding of immunological data.

The Mathematics of Tiling a Soccer Ball

Amie Ga Yun Su
University of British Columbia

With the rise in excitement around sports in 2021, whether it's from the UEFA games or the Olympics, one may begin thinking about the mathematics behind the pentagonal and hexagonal tiling on the surface of a ball in order to achieve the same pattern as seen in a standard soccer ball. In this presentation, we discuss how we can be certain why the pattern always looks the way it does and the consequences that occur when changes are made in the polygons we choose to tile a sphere with. This talk demonstrates a fun application of geometry, graph theory, and topology that is seen in the real world.

Reflecting on Our Conceptions of Proofs While Solving a Sudoku

Julia McClellan
University of Toronto

We all know the age-old question in math classes of "When am I EVER going to use this outside of class?". As we progress through our journey in mathematics, we see there are many applications of concrete mathematical concepts, such as statistical analysis, computer programming, and engineering, to name a few. On the other hand, in a proofs-based mathematics class, we may perceive the techniques we use to construct rigorous proofs as only applicable in the mathematics universe. However, many of the valuable skills gained through our development in proof construction can be used in various aspects of everyday life. In this talk I will ask attendees to question their own conceptions of proofs and proofs-based mathematics by presenting a connection between solving a Sudoku puzzle and a few select proof techniques.

A Mathematical Model of Brain Tumour Growth

Blanche Mongeon
Université de Montréal

The incidence of glioblastoma (GBM), the most complex and deadliest brain tumour of all, is 4 per 100 000 people in Canada, according to the Brain Tumour Registry of Canada. Patients can expect a mean survival of 12 to 18 months, with the most recent treatment plans having only been able to increase the prognostic by a few months. Mathematical modeling can be an effective tool in oncology: by studying the interactions between the immune system and the cancer cells, and the effects of various medical treatments, we can simulate the tumour evolution and establish treatment plans that consider patient heterogeneity. In this presentation, we'll become familiar with mathematical oncology with a brief description of our immune system and its components. We will then look into the different steps necessary to the elaboration of a mathematical model, from the schematization of the interactions to the simulations, including the description of important well-known biomathematical models. All of those steps will be conceptualized into a model describing glioblastoma growth.

Framelets on Manifold and Graphs with Applications

Songlin Jin
University of British Columbia

Graph signal decomposition and reconstruction.

Reciprocity Laws: Uncovering the Secrets Behind Primes

Kiera Sobolewski
University of Calgary

A reciprocity law is a theorem in Number Theory that tells us about relationships between primes in different number fields. The Quadratic Reciprocity Law is typically the first reciprocity law students will encounter and it was first proved by Gauss. This theorem tells us about whether a number is a square modulo p where p is a prime. There are more general reciprocity laws such as Hilbert Reciprocity and Artin Reciprocity in which Quadratic Reciprocity can be recovered from. These laws give us intuition on the behaviour of primes in more general number fields. The goal of this talk is to share why Artin Reciprocity is important and how to get Quadratic Reciprocity from the idelic Artin map.

Plank theorems and their applications

William Verreault
Université Laval

Plank problems concern the covering of convex bodies by planks in Euclidean space and contain some famous open problems in convex geometry. In this talk, I will introduce plank problems and present the applications of plank theorems to problems in various areas of mathematics such as number theory and harmonic analysis.

How to Solve a Cryptarithm

Vinicius Santos
University of British Columbia

A cryptarithm is a mathematical puzzle where an equation is written with letters instead of numbers. The objective is to find an assignment of numbers on letters such that the equation is true. In this talk, I will first define cryptarithms. Then, I will give examples of this puzzle and present methods for solving them. Finally, I will explain how the backtracking algorithm can be used to solve cryptarithms.

Exceptional Gegenbauer Polynomials and Confluent Darboux Transformations

James Elgin Munday
Dalhousie University

Classical orthogonal polynomials are families of orthogonal polynomials that arise as solutions to Sturm-Liouville eigenvalue problems, and are entirely classified by the classical families of Hermite, Laguerre, and Jacobi polynomials. Exceptional orthogonal polynomials also arise as solutions to Sturm-Liouville eigenvalue problems; however, we allow the polynomial sequences to miss a finite number of "exceptional" degrees, resulting in new orthogonal polynomials that are generalizations of the classical families. This talk will introduce a construction for multi-parameter exceptional Gegenbauer polynomials via the isospectral deformation of the classical Gegenbauer operator. We are able to obtain a fully explicit description of the operators and polynomials in question through the use of confluent Darboux transformations.

Zero Forcing Sets and Forts in Trees

Johnna Parenteau
University of Regina

Zero forcing is a combinatorial game played on a graph G where the goal is to fill the unfilled vertices of a graph at minimal cost, denoted as $Z(G)$, using two specified actions: (1) at any point, a vertex can be filled for a cost of one token, and (2) at no cost, the player can apply the filling rule. So how do we optimally choose these vertices? In this presentation, I will introduce forts and hidden triangles as a way to predict the vertices of interest as well as some relevant results developed in recent years.

Spectral Graph Theory: Structure & Characteristic Polynomial

Lewis Glabush
Western University

I will summarize research taking place at Muller Lab, involving studying relationships between graph structures and their associated characteristic polynomial.

An Introduction to Vinogradov's Mean Value Theorem

Tuan Hiep Do, Josué Kurke, Jihanjie (Andrew) Luo
University of Waterloo

In this talk, we introduce the main conjecture of Vinogradov's Mean Value Theorem and discuss, at a high level, number theoretic approaches to tackle this problem. We then give a general outline of the infrastructure for nested efficient congruencing as well as the conditioning process, which will be utilized in order to prove the main conjecture in Vinogradov's Mean Value Theorem.

Understanding Non-Equilibrium Steady States Adopting Methods From Quantum Field Theory

Gustavo Cicchini Santos
Ryerson University

Physical systems are characterized by their response to perturbations. The Fluctuation-Dissipation Theorem predicts the behaviour of systems in equilibrium. Can an expression be derived using methods from quantum field theory to describe the vertex response to a perturbation, and is the Fluctuation-Dissipation Theorem modified as a result of these perturbations? Using Berezin integration and properties of determinants we derive said expression. The derivation yields the same result as the less rigorous methods. We learn the Fluctuation-Dissipation Theorem has an equilibrium-like response to a vertex perturbation making the Fluctuation Dissipation theorem a bad indicator of whether a system is in equilibrium or out of equilibrium.

Introduction to Bayesian Statistics

Ashley Zhang
University of Toronto

Bayesian inference is an important method in statistical inference in which Bayes' theorem is used to estimate probability of an event occurrence based on prior knowledge. In contrast to frequentist statistics, Bayesian statistics use observed data to make an inference. This presentation will be a gentle introduction to Bayesian statistics and the role of prior distributions.

An introduction to Elliptic curves and variants of the Lang-Trotter conjecture

Seraphim Jarov
University of British Columbia

Elliptic curves are of major interest in areas of number theory and for real-world applications such as cryptography and integer factorization. We provide an introduction into the theory behind elliptic curves and describe a conjecture relating the number of solutions of a given curve over finite fields by Lang and Trotter. We discuss our work in progress in proving a variant of the Lang-Trotter conjecture on average.

Various Methods for Optimization

Younwoo (Ethan) Choi
University of Toronto

Optimization is a branch of applied mathematics which is used widely in many disciplines such as computer science and economics. There are many types of optimization problems and thus methods to find the solution has been developed for centuries. The talk will introduce both basic finite and infinite dimensional optimization problems and various optimization algorithms such as steepest gradient descent, stochastic optimization, and adaptive gradient descent.

From Field Theory to Algebraic Number Theory

Amar Venga
Western University

Prerequisites: A first course in abstract algebra

This talk will introduce the notion of an algebraic extension, and the degree and basis of such an extension. We will then discuss some early but important results regarding minimal polynomials, splitting fields, and algebraic closures. Finally, we will use all of these concepts to derive some early results about number fields and number rings.

Introduction on the min-max construction of minimal surfaces

Xinze Li
University of Toronto

Minimal Surface is the mathematical models of soap bubbles, which is also widely used in architecture, computer graphics and medical image. The idea of using min-max argument to construct minimal surface goes back to Brikhoff who use sweep-outs to find simple closed geodesic on spheres. In this talk, we introduce a generalization of such method that obtain embedded minimal surfaces in 3-manifold based on the survey of Tobias H. Closing and Camilo De Lellis in 2003.

Spatiotemporal transmission dynamics of COVID-19 using mathematical models

Crystal Wai
Mount Royal University

Introduction

We present COVIDEpisim, an R Shiny app that integrates mathematical modelling and open-source tools for generating disease prevalence and incidence maps for the spatial spread of COVID-19. It is essential to understand what future epidemic trends will be, as well as the effectiveness and potential impact of public health intervention measures. The goal of this research is to provide insight that would support public health officials towards informed, data-driven decision making.

Methods

We present spatial compartmental models of epidemiology (ex: SEIR, SEIRD, SVEIRD) to capture the transmission dynamics of the spread of COVID-19. Our interactive R Shiny app can be used to output and visualize how an infectious disease spreads across a large geographical area. The rate of spread of the disease is influenced by changing the model parameters and human mobility patterns. We use the freely available population count data downloaded as a gridded raster map from WorldPop.org. Each grid cell has a population count, which is divided into disease compartments. Each grid cell can transmit disease to its neighbours, with probabilities that decline exponentially with the Euclidean distance.

Results

Predicting the transmission dynamics of COVID-19 using mathematical models is challenging and comes with a lot of uncertainty. First, we run the spatial simulations under the worst-case scenario, in which there are no major public health interventions. Next, we account for mitigation efforts including strict mask wearing and social distancing mandates, targeted lockdowns, and widespread vaccine rollout to vaccinate priority groups. Projections for disease prevalence with and without mitigation efforts are presented via time-series graphs for the epidemic compartments. As a test case the Czech Republic is selected and the projected number of newly infected and death cases up to October 1, 2021 are estimated and presented. All simulations are carried out using the R programming language.

Conclusions

In this research we seek primarily to clarify mathematical ideas, rather than to offer definitive medical answers. Our analyses may shed light more broadly on how COVID-19 spreads in a large geographical area with places where no empirical data is recorded or observed.

A mathematical definition of entanglement and its measurement

Alex Kazachek
Western University

Entanglement has an intuitive moral definition: that some system cannot be understood as a combination of individual components, only holistically as an inseparable whole. We will build up the linear algebra needed to mathematically formalize this notion, discussing Hilbert spaces and tensor products, and introduce the technique of convex roof constructions to measure how entangled a system is.

Bayesian Disease Mapping of COVID-19 Cases Across the Boroughs of Montreal

Leo Vanciu
Marianopolis College

Disease mapping attempts to estimate spatial patterns of disease risk across different areas while accounting for the effects of covariates. The number of cases of a disease is often modelled as following a Poisson distribution. The mean of the Poisson may be set as the product between an offset and the relative risk of the disease. Commonly, the log relative risk is modelled as a linear function of covariates. However, this Poisson distribution assumes that the mean and variance are the same, which is rarely true in this kind of data. Following the Bayesian paradigm, we can include latent effects in the log relative risk that allow for overdispersion and spatial correlation using a conditional autoregressive prior. These latent effects account for possible unobserved structures that might be present in the data and are not captured by the available covariates. We fit different models to estimate the relative risk of infection and mortality of COVID-19 across the boroughs of Montreal, which has been the epicentre of the COVID-19 pandemic in Canada with the highest number of deaths in the country. The models explore the association between the boroughs' average income, age, and educational level with the relative risk of COVID-19. Results show that the importance of the covariates is related to the inclusion or not of local random effects and their prior specification. We discuss these results and point to future avenues of research. This work was conducted as part of a summer internship under the supervision of Victoire Michal and Alexandra M. Schmidt, from the Department of Epidemiology, Biostatistics, and Occupational Health at McGill University.

Existence of an Equilibrium in Mixed Networks of Coordinators and Anti-coordinators

Achintya Raya Polavarapu
University of Alberta

A network game is a game over a graph where each vertex is associated with a player and the edges in the graph represent the neighbours of each player. In a network game with binary decisions, players might prefer one action over the other based on the number of neighbours that are choosing it. For instance, a player could take an action if enough neighbours are using that action, as in the spread of innovations and fashion. This player is considered a coordinating player. Alternatively, an anti-coordinating player could take an action if enough neighbours are choosing the opposite action, as in dispersion of crowds and traffic. A network game is said to be at equilibrium when every player in the network reaches a decision with which they are all satisfied. A fundamental question is if a particular network game will ever reach an equilibrium. While it has been proved in a recent paper that in every network game consisting purely of either coordinating or anti-coordinating players there exists an equilibrium, this is not the case when both types of players coexist in a network. In this article, we establish the conditions for the existence of an equilibrium in certain types of elementary networks which contain both coordinating and anti-coordinating players, using a linear-threshold model.

Determining Galois Group Containment with Invariants

Curtis Wilson
Queen's University

We develop a working understanding of invariant theory from the perspective of invariant polynomials. We provide a way to recover the invariant polynomials given a ring of invariants. We then introduce a question from Galois theory and answer it using invariants.

Rapid testing for COVID-19 and SIQR model

Jiawei Chen
University of Toronto

Rapid testing identifies cases that are contagious with no symptoms yet. SIQR model can help us analyze how effective the test needs to be in order to contain the pandemic.

Introduction to minimal surfaces and submanifolds and some results for conformal automorphisms of the unit ball

Karan Bains
University of British Columbia

Minimal surfaces and submanifolds have important applications to general relativity, topology, and eigenvalue problems. For example, free boundary minimal surfaces have connections with the Steklov eigenvalue problem on surfaces with boundary. In this talk, I introduce minimal surfaces and mention their important connections with eigenvalue problems. I also present an analytic result about the images of totally geodesic free boundary submanifolds under conformal automorphisms of the unit ball.

Number Theory meets Machine Learning

Brendon James Thomson
Concordia University

The Collatz conjecture was formulated in 1937 and it still remains open. Like many other fascinating number theory conjectures, it admits an easy formulation. Consider a positive integer n, and define the function $f(n)$ to be $n/2$ if n is even or $(3n+1)$ if n is odd. Then, define the recursive sequence $n, f(n), f(f(n)), f(f(f(n))), \dots$. The Collatz conjecture states that this sequence reaches the value 1 in a finite number of steps, for any positive integer n. Since its formulation, there has a been a tremendous computational effort in the mathematical community aimed at verifying the conjecture for larger and larger values of n (as of July 2020, for n up to 2^{68}). This led to the production of a vast amount of simulated data. The conjecture has a notorious history of showing signs of well-behaving and then quickly leading researchers to a brick-wall-halt. It is no secret that this is a difficult problem. Regardless, there has been a monumental breakthrough recently by Terrence Tao. This presentation is a timeline of the research done over the summer for the CUSRA 2021. The humble goal of this summer's research project became to use state-of-the-art machine learning algorithms to try to extract insights from this data, to most likely throw the attempt in the weighty "attempted and failed" bin with the rest of the approaches that tried and failed, at least something new was being added to the bin! It turns out we weren't the only ones to attack this problem from a computer science angle, a recent paper, four years in the making, was uploaded to the arXiv by Yolcu, Aaronson, and Heule in this direction! In attempting the goal, another acceleration of the iterates is introduced, the Montreal iterates, a twist on the Syracuse iterates, from which a stopping time is defined, and the helper function is defined which maps each positive integer to its stopping time. The inspiration for this summer's research came from what looked like a pattern beginning to appear in the upper bounds of the helper function on intervals of length a power of 2. This idea is followed up by using data from Eric Roosendaal's website $3x+1$ and the results from the machine learning experiments will be presented. Goodstein sequences were also studied; hence they are included in the presentation as well for their remarkable properties. To conclude, the ideas that looked promising but we didn't try are mentioned, and Goodstein sequences are compared to Collatz sequences, specifically using the Montreal iterates.

Billiards, Illumination and the Irrationality Of It All

Uttara Sheila Rajagopalan
University of Toronto

An idealised billiards table or a room with mirror walls is the perfect setting for all sorts of problems in dynamics based on specular reflections. We might ask questions about the periodicity of the trajectory of a photon in such a room, or about which areas on this table are in danger of getting struck by a cue ball. I will present a few problems and try to answer why the rationality of angles even figures in any of this.

The Birthday Paradox

Veer Prakash Bhatia
University of British Columbia

August is my favourite month of the year – since that's when I was born. This talk will explore the fascinating nature of the Birthday Paradox: In a group of n people, what is the likelihood that at least two of them share the same birthday? The answer is surprisingly big! We will be deriving an approximate solution from first principles, performing graphical comparisons, and delving deeper into how the probabilities shift when we center the problem around ourselves that is: what is the probability in a group of n people that someone else has the same birthday as you?

Introducing... Determinantal Facet Ideals! (With a Connection to Kazhdan-Lusztig Ideals)

Ryan Edwards
McMaster University

An ideal in a polynomial ring generated by matrix minors is called a determinantal ideal. I will discuss two special types of determinantal ideals: determinantal facet ideals and Kazhdan-Lusztig ideals. Both are generated using combinatorial data, putting them at the intersection of combinatorics and algebra. Although these two ideals are in many ways unrelated, there is a surprising connection between them. (Some familiarity with ideals and polynomial rings is assumed.)

Distorted Projections: Do you see what I see?

From the Celestial Sphere to Tessellating Triangles via the Schwarzian Derivative

Patrick Cyr
University of Ottawa

Imagine yourself at rest in the center of a unit sphere, where each point on the surface represents a light-ray travelling towards your eye. To you, the constellations in the sky appear to have a certain shape, but I, moving relative to you, do not agree. How can you know I am accelerating? I.e., what is the relative distortion between the projected shapes on our celestial spheres, and why must one of us be moving non-uniformly? The answer to this question (and many more) is given simply by the Schwarzian derivative of the transformation associated with this motion. We will define this object and explore its remarkably versatile properties which helped Hermann Schwarz find triangular tessellations of the unit disk as solutions to the hypergeometric differential equation. This talk assumes basic knowledge of ordinary differential equations, the complex plane and Euclidean geometry in 3-D.

How to solve $a/(b+c) + b/(a+c) + c/(a+b) = 4$

Xinran Zhou
University of British Columbia

I will talk about the basic strategy of how to solve the equation in two ways as well as a small section of history of the discovery of the equation.

Determinants of COVID-19 vaccine acceptance in 143 countries around the world

Mehrdad Kazemi
Supervisor: Dr. Jude Kong
York University

Background

After COVID-19 pandemic spread across the world, countries adopted containment measures to stop the spread and overwhelming of hospitals, and limit fatalities. Many countries implemented social distancing and lockdown strategies that negatively impacted their economies and the lives of their citizens. A COVID-19 vaccine provides viable possibility for a permanent solution to controlling the pandemic. Several vaccines are currently in human trials. However, the uptake rate has become stagnant in several countries despite several appeal by the media policy makers, and community leaders.

Objective

To determine a vaccination uptake index and identify national socio-economic factors influencing vaccination uptake rate.

Methods

We first estimate the vaccination uptake rate across countries by fitting a logistic model to reported daily case numbers. Using the uptake rate, we estimate the vaccination uptake index. Next, we use Random Forest, an “off-the-shelf” machine learning algorithm, to study the association between vaccination uptake rate and socio-economic factors.

Results

We found that the mean vaccine update index is 0.015538 (S.D. 0.015727), with a range between 5.58E-05 (Democratic Republic of the Congo) and 0.0829 (Mongolia). The top three factors associated with COVID-19 uptake rate are the number of physicians per 1000 population, human development index and life expectancy.

Conclusion

Countries have different sociodemographic factors that influence their attitude towards COVID-19 vaccines. Addressing these sociodemographic determinants may help to increase the acceptance of the global vaccination program to tackle future pandemics. Targeted interventions are needed to increase the uptake of the future COVID-19 vaccine.