Asymptotics Practice Problems

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March 9, 2017

- 1. Order the following functions so that $f_i \in O(f_j) \iff i < j$:
 - (a) $f_1(n) = 3^n$
 - (b) $f_2(n) = \frac{n}{2}$
 - (c) $f_3(n) = 2^{10000}$
 - (d) $f_4(n) = 2^{\log_2 n}$
 - (e) $f_5(n) = \log n$
 - (f) $f_6(n) = n + n^2 \log n$
 - (g) $f_7(n) = n!$
 - (h) $f_8(n) = 1.001^n + n^3$
- 2. Provide the tightest bound on f(n) in terms of g(n) by saying f = O(g), $f = \Omega(g)$, or $f = \Theta(g)$.
 - (a) $f(n) = \log_2 n$
 - $g(n) = \log_3 n$
 - (b) $f(n) = \log n^2$
 - $g(n) = \log n$
 - (c) f(n) = n 100
 - g(n) = n + 10000
 - (d) $f(n) = 2^{1.1n}$
 - $g(n) = \sum_{i=1}^{n} i^2$
- 3. Find the tightest bound of the function $f(n) = x \sin x^2$ in both the worst and best case.
- 4. Provide the worst and best runtime for the following methods in terms of N.
 - (a) Here consider N to be the length of the array and p to be initialized to N.

```
public void int foo(int[] fighters, int p) {
   if (p != 0) {
      return fighters[p];
   }
   if (p % 2 == 0) {
      return foo(fighters, p / 2);
   }
   System.arraycopy(fighters, 0, fighters, 0, p + 1);
   fighters[p] = p;
   return foo(fighters, p + 1);
}
```