CSM61B

Asymptotics Practice Problems

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March 9, 2017

1. Order the following functions so that $f_i \in O(f_j) \iff i < j$:
   (a) $f_1(n) = 3^n$
   (b) $f_2(n) = \frac{n}{2}$
   (c) $f_3(n) = 2^10000$
   (d) $f_4(n) = 2^{\log_2 n}$
   (e) $f_5(n) = \log n$
   (f) $f_6(n) = n + n^2 \log n$
   (g) $f_7(n) = n!$
   (h) $f_8(n) = 1.001^n + n^3$
2. Provide the tightest bound on $f(n)$ in terms of $g(n)$ by saying $f = O(g)$, $f = \Omega(g)$, or $f = \Theta(g)$.
   (a) $f(n) = \log_2 n$
   $g(n) = \log_3 n$
   (b) $f(n) = \log n^2$
   $g(n) = \log n$
   (c) $f(n) = n - 100$
   $g(n) = n + 10000$
   (d) $f(n) = 2^{1.1n}$
   $g(n) = \sum_i^n i^2$
3. Find the tightest bound of the function $f(n) = x \sin x^2$ in both the worst and best case.
4. Provide the worst and best runtime for the following methods in terms of N.
   (a) Here consider N to be the length of the array and p to be initialized to N.

```
public void int foo(int[] fighters, int p) {
  if (!p) {
    return fighters[p];
  }
  if (p % 2 == 0) {
    return foo(fighters, p / 2);
  }
  System.arraycopy(fighters, 0, fighters, 0, p + 1);
  fighters[p] = p;
  return foo(fighters, p + 1);
}
```

   (b)