

3. Squares and Square Roots

Exercise 3.1

1. Question

Which of the following numbers are perfect squares?

(i) 484 (ii) 625

(iii) 576 (iii) 576

(iv) 941 (v) 961

(vi) 2500

Answer

(i) 484

Resolving 484 into prime factors we get,

$$484 = 2 \times 2 \times 11 \times 11$$

Now,

Grouping the factors into pairs of equal factors, we get:

$$484 = (2 \times 2) \times (11 \times 11)$$

We observe that all are paired so,

484 is a perfect square

(ii) 625

Resolving 625 into prime factors we get,

$$625 = 5 \times 5 \times 5 \times 5$$

Now,

Grouping the factors into pairs of equal factors, we get:

$$625 = (5 \times 5) \times (5 \times 5)$$

We observe that all are paired so,

625 is a perfect square

(iii) 576

Resolving 576 into prime factors we get,

$$576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

Now,

Grouping the factors into pairs of equal factors, we get:

$$576 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (3 \times 3)$$

We observe that all are paired so,

576 is a perfect square

(iv) 941

Resolving 941 into prime factors we get,

$$941 = 941 \times 1$$

Now,

As 941 itself is a prime number

Hence,

It do not have a perfect square

(v) 961

Resolving 961 into prime factors we get,

$$961 = 31 \times 31$$

Now,

Grouping the factors into pairs of equal factors, we get:

$$961 = (31 \times 31)$$

We observe that all are paired so,

961 is a perfect square

(vi) 2500

Resolving 2500 into prime factors we get,

$$2500 = 2 \times 2 \times 5 \times 5 \times 5 \times 5$$

Now,

Grouping the factors into pairs of equal factors, we get:

$$2500 = (2 \times 2) \times (5 \times 5) \times (5 \times 5)$$

We observe that all are paired so,

2500 is a perfect square

2. Question

Show that each of the following numbers is a perfect square. Also find the number whose square is the given number in each case:

(i) 1156

(ii) 2025

(iii) 14641

(iv) 4761

Answer

(i) 1156

Resolving 1156 into prime factors we get,

$$1156 = 2 \times 2 \times 17 \times 17$$

Now, grouping the factors into pairs of equal factors

We get,

$$1156 = (2 \times 2) \times (17 \times 17)$$

As all factors are paired

Hence, 1156 is a perfect square

Again,

$$1156 = (2 \times 17) \times (2 \times 17)$$

$$= 34 \times 34$$

$$= (34)^2$$

Thus, 1156 is a square of 34

(ii) 2025

Resolving 2025 into prime factors we get,

$$2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$$

Now, grouping the factors into pairs of equal factors

We get,

$$2025 = (3 \times 3) \times (3 \times 3) \times (5 \times 5)$$

As all factors are paired

Hence, 2025 is a perfect square

Again,

$$2025 = (3 \times 3 \times 5) \times (3 \times 3 \times 5)$$

$$= 45 \times 45$$

$$= (45)^2$$

Thus, 2025 is a square of 45

(iii) 14641

Resolving 14641 into prime factors we get,

$$14641 = 11 \times 11 \times 11 \times 11$$

Now, grouping the factors into pairs of equal factors

We get,

$$14641 = (11 \times 11) \times (11 \times 11)$$

As all factors are paired

Hence, 14641 is a perfect square

Again,

$$14641 = (11 \times 11) \times (11 \times 11)$$

$$= 121 \times 121$$

$$= (121)^2$$

Thus, 14641 is a square of 121

(iv) 4761

Resolving 4761 into prime factors we get,

$$4761 = 3 \times 3 \times 23 \times 23$$

Now, grouping the factors into pairs of equal factors

We get,

$$4761 = (3 \times 3) \times (23 \times 23)$$

As all factors are paired

Hence, 4761 is a perfect square

Again,

$$4761 = (3 \times 23) \times (3 \times 23)$$

$$= 69 \times 69$$

$$= (69)^2$$

Thus, 4761 is a square of 69

3. Question

Find the smallest number by which the given number must be multiplied so that the product is a perfect square:

(i) 23805

(ii) 12150

(iii) 7688

Answer

(i) 23805

Resolving 23805 into prime factors, we get

$$23805 = 3 \times 3 \times 23 \times 23 \times 5$$

Obtained factors can be paired into equal factors except for 5

To pair it equally multiply with 5

$$23805 \times 5 = 3 \times 3 \times 5 \times 5 \times 23 \times 23$$

Again,

$$23805 \times 5 = (3 \times 5 \times 23) \times (3 \times 5 \times 23)$$

$$= 345 \times 345$$

$$= (345)^2$$

Therefore, product is the square of 345

(ii) 12150

Resolving 12150 into prime factors, we get

$$12150 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 2$$

Obtained factors can be paired into equal factors except for 2

To pair it equally multiply with 2

$$12150 \times 2 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 3$$

Again,

$$12150 \times 2 = (5 \times 3 \times 2 \times 2 \times 2) \times (5 \times 3 \times 2 \times 2 \times 2)$$

$$= 120 \times 120$$

$$= (120)^2$$

Therefore, product is the square of 120

(iii) 7688

Resolving 7688 into prime factors, we get

$$7688 = 2 \times 2 \times 31 \times 31 \times 2$$

Obtained factors can be paired into equal factors except for 2

To pair it equally multiply with 2

$$7688 \times 2 = 2 \times 2 \times 2 \times 2 \times 31 \times 31$$

Again,

$$7688 \times 2 = (2 \times 2 \times 31) \times (2 \times 2 \times 31)$$

$$= 124 \times 124$$

$$= (124)^2$$

Therefore, product is the square of 124

4. Question

Find the smallest number by which the given number must be divided so that the resulting number is a perfect square:

(i) 12283

(ii) 1800

(iii) 2904

Answer

(i) 12283

Resolving 14283 into prime factors, we get

$$14283 = 3 \times 3 \times 3 \times 23 \times 23$$

Obtained factors can be paired into equal factors except for 3

So, eliminate 3 by dividing the number with 3

$$\frac{14283}{3} = (3 \times 3) \times (23 \times 23)$$

Again,

$$\frac{14283}{3} = (3 \times 23) \times (3 \times 23)$$

$$= 69 \times 69$$

$$= (69)^2$$

Therefore,

The resultant is the square of 69

(ii) 1800

Resolving 1800 into prime factors, we get

$$1800 = 2 \times 2 \times 5 \times 5 \times 3 \times 3 \times 2$$

Obtained factors can be paired into equal factors except for 2

So, eliminate 2 by dividing the number with 2

$$\frac{1800}{2} = (2 \times 2) \times (3 \times 3) \times (5 \times 5)$$

Again,

$$\frac{1800}{2} = (2 \times 3 \times 5) \times (2 \times 3 \times 5)$$

$$= 30 \times 30$$

$$= (30)^2$$

Therefore,

The resultant is the square of 30

(iii) 2904

Resolving 2904 into prime factors, we get

$$2904 = 2 \times 2 \times 11 \times 11 \times 2 \times 3$$

Obtained factors can be paired into equal factors except for 2 and 3

So, eliminate 6 by dividing the number with 6

$$\frac{2904}{6} = (2 \times 2) \times (11 \times 11)$$

Again,

$$\frac{2904}{6} = (2 \times 11) \times (2 \times 11)$$

$$= 22 \times 22$$

$$= (22)^2$$

Therefore,

The resultant is the square of 22

5. Question

Which of the following numbers are perfect squares?

11, 12, 16, 32, 36, 50, 64, 79, 81, 111, 121

Answer

11: Since 11 is a prime number,

Hence, it is not a perfect square

12: Since, 12 is ending with 2,

Hence, not a perfect square

16: Since, $16 = 4 \times 4$

$$= (4)^2$$

Therefore, it is a perfect square

32: Since, 32 is ending with 2,

Hence, not a perfect square

36: Since, $36 = 6^2$

Hence, it is a perfect square

50: Since, $50 = 5^2 \times 2$

Hence, it is not a perfect square

64: Since, $64 = 8^2$

Hence, it is a perfect square

79: Since it is a prime number so it cannot be a perfect square

81: Since, $81 = 9^2$

Hence, it is a perfect square

111: Since, 111 is a prime number so it cannot be a perfect square

121: Since, $121 = 11^2$

Hence, it is perfect square

6. Question

Using prime factorization method, find which of the following numbers are perfect squares?

189, 225, 2048, 343, 441, 2961, 11025, 3549

Answer

Since,

$$189 = 3^2 \times 3 \times 7$$

It cannot be written as pair of two equal factors, so 189 is not a perfect square

Since,

$$225 = (5 \times 5) \times (3 \times 3)$$

It can be written as pair of two equal factors, so 22 is a perfect square

Since,

$$2048 = (2 \times 2) \times (2 \times 2) \times (2 \times 2) (2 \times 2) \times (2 \times 2) \times 2$$

All the factors cannot be written as pair of two equal factors, so 189 is not a perfect square

Since,

$$343 = (7 \times 7) \times 7$$

It cannot be written as pair of two equal factors, so 343 is not a perfect square

Since,

$$441 = (7 \times 7) \times (3 \times 3)$$

It can be written as pair of two equal factors, so 441 is a perfect square

Since,

$$2916 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (2 \times 2)$$

It can be written as pair of two equal factors, so 2916 is a perfect square

Since,

$$11025 = (5 \times 5) \times (3 \times 3) \times (7 \times 7)$$

It can be written as pair of two equal factors, so 11025 is a perfect square

Since,

$$3549 = (13 \times 13) \times 3 \times 7$$

It cannot be written as pair of two equal factors, so

3549 is not a perfect square

7. Question

By what number should each of the following numbers be multiplied to get a perfect square in each case? Also find the number whose square is the new number.

(i) 8820 (ii) 3675

(iii) 605 (iv) 2880

(v) 4056 (vi) 3468

(vii) 7776

Answer

(i) 8820

$$8820 = (2 \times 2) \times (3 \times 3) \times (7 \times 7) \times 5$$

In the above factors only 5 is unpaired

So, multiply the number with 5 to make it paired

Again,

$$8820 \times 5 = 2 \times 2 \times 3 \times 3 \times 7 \times 7 \times 5 \times 5$$

$$= (2 \times 2) \times (3 \times 3) \times (7 \times 7) (5 \times 5)$$

$$= (2 \times 3 \times 7 \times 5) \times (2 \times 3 \times 7 \times 5)$$

$$= 210 \times 210$$

$$= (210)^2$$

So, the product is the square of 210

(ii) 3675

$$3675 = (5 \times 5) \times (7 \times 7) \times 3$$

In the above factors only 3 is unpaired

So, multiply the number with 3 to make it paired

Again,

$$3675 \times 3 = 5 \times 5 \times 7 \times 7 \times 3 \times 3$$

$$= (5 \times 5) \times (7 \times 7) \times (3 \times 3)$$

$$= (3 \times 5 \times 7) \times (3 \times 5 \times 7)$$

$$= 105 \times 105$$

$$= (105)^2$$

So, the product is the square of 105

(iii) 605

$$605 = 5 \times (11 \times 11)$$

In the above factors only 5 is unpaired

So, multiply the number with 5 to make it paired

Again,

$$605 \times 5 = 5 \times 5 \times 11 \times 11$$

$$= (5 \times 5) \times (11 \times 11)$$

$$= (5 \times 11) \times (5 \times 11)$$

$$= 55 \times 55$$

$$= (55)^2$$

So, the product is the square of 55

(iv) 2880

$$2880 = 5 \times (3 \times 3) \times (2 \times 2) \times (2 \times 2) \times (2 \times 2)$$

In the above factors only 5 is unpaired

So, multiply the number with 5 to make it paired

Again,

$$2880 \times 5 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$$

$$= (2 \times 2) \times (2 \times 2) \times (2 \times 2) (3 \times 3) \times (5 \times 5)$$

$$= (2 \times 2 \times 2 \times 3 \times 5) \times (2 \times 2 \times 2 \times 3 \times 5)$$

$$= 120 \times 120$$

$$= (120)^2$$

So, the product is the square of 120

(v) 4056

$$4056 = (2 \times 2) \times (13 \times 13) \times 2 \times 3$$

In the above factors only 2 and 3 are unpaired

So, multiply the number with 6 to make it paired

Again,

$$4056 \times 6 = 2 \times 2 \times 13 \times 13 \times 2 \times 2 \times 3 \times 3$$

$$= (2 \times 2) \times (13 \times 13) \times (2 \times 2) (3 \times 3)$$

$$= (2 \times 2 \times 3 \times 13) \times (2 \times 2 \times 3 \times 13)$$

$$= 156 \times 156$$

$$= (156)^2$$

So, the product is the square of 156

(vi) 3468

$$3468 = (2 \times 2) \times 3 \times (17 \times 17)$$

In the above factors only 3 are unpaired

So, multiply the number with 3 to make it paired

$$3468 \times 3 = (2 \times 2) \times (3 \times 3) \times (17 \times 17)$$

$$= (2 \times 3 \times 17) \times (2 \times 3 \times 17)$$

$$= 102 \times 102$$

$$= (102)^2$$

So, the product is the square of 102

(vii) 7776

$$7776 = (2 \times 2) \times (2 \times 2) \times (3 \times 3) \times (3 \times 3) \times 2 \times 3$$

In the above factors only 2 and 3 are unpaired

So, multiply the number with 6 to make it paired

Again,

$$7776 \times 6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= (2 \times 2) \times (2 \times 2) \times (2 \times 2) (3 \times 3) \times (3 \times 3) \times (3 \times 3)$$

$$= (2 \times 2 \times 2 \times 3 \times 3 \times 3) \times (2 \times 2 \times 2 \times 3 \times 3 \times 3)$$

$$= 216 \times 216$$

$$= (216)^2$$

So, the product is the square of 216

8. Question

By What numbers should each of the following be divided to get a perfect square in each case? Also, find the number whose square is the new number.

(i) 16562

(ii) 3698

(iii) 5103

(iv) 3174

(v) 1575

Answer

(i) 16562

$$16562 = (7 \times 7) \times (13 \times 13) \times 2$$

$$\frac{16562}{2} = (7 \times 7) \times (13 \times 13)$$

$$\frac{16562}{2} = (7 \times 13) \times (7 \times 13)$$

$$= 91 \times 91$$

$$= 91^2$$

Therefore, the resultant is the square of 91

(ii) 3698

$$3698 = 2 \times (43 \times 43)$$

$$\frac{3698}{2} = 43 \times 43$$

$$= 43^2$$

Therefore, the numbers must be divided by 2 and resultant is square of 43

(iii) 5103

$$5103 = (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times 7$$

$$\frac{5103}{7} = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$$

$$= 27 \times 27$$

$$= 27^2$$

Therefore, the number must be divided by 7 and resultant is square of 27

(iv) 3174

$$3174 = 2 \times 3 \times (23 \times 23)$$

$$\frac{3174}{6} = 23 \times 23$$

$$= 23^2$$

Therefore, the number must be divided by 6 and the resultant is square of 23

(v) 1575

$$1575 = 3 \times 3 \times 5 \times 5 \times 7$$

$$\frac{1575}{7} = 3 \times 3 \times 5 \times 5$$

$$= (3 \times 5) \times (3 \times 5)$$

$$= 15 \times 15$$

$$= 15^2$$

Therefore, the number must be divided by 7 and the resultant is square of 15

9. Question

Find the greatest number of two digits which is a perfect square.

Answer

Greatest 2 digit number = 99

$$\begin{array}{r} 9 \\ 9 \overline{) 99} \\ \underline{81} \\ 18 \end{array}$$

Hence, greatest 2 digit perfect square number is:

$$99 - 18 = 81$$

10. Question

Find the least number of three digits which is perfect square.

Answer

Smallest 3 digit number = 100

At first we will find the square root of 100

$$\begin{array}{r} 10 \\ 1 \overline{) 100} \\ \underline{1} \\ 00 \end{array}$$

Hence, the least number that is a perfect square is 100 itself

11. Question

Find the smallest number by which 4851 must be multiplied so that the product becomes a perfect square.

Answer

Factors of 4851 are:

$$4851 = 3 \times 3 \times 7 \times 7 \times 11$$

$$\text{Pairs} = 3^2 \times 7^2$$

Hence, 4851 should be multiplied by 11 in order to get a perfect square when smallest number multiplied to 4851

12. Question

Find the smallest number by which 28812 must be divided so that it becomes a perfect square. Also find the number whose square is the resulting number.

Answer

Factors of 28812 are:

$$28812 = 2 \times 2 \times 3 \times 3 \times 3 \times 17 \times 17$$

$$\text{Pairs} = 2^2 \times 3^2 \times 17^2$$

Hence, 28812 should be divided by 3 in order to get a perfect square when divided by the least number

The square root will be:

$$2 \times 3 \times 17 = 102$$

13. Question

Find the smallest number by which 1152 must be divided so that it becomes a perfect square. Also find the number whose square is the resulting number.

Answer

Factors of 1152 are:

$$1152 = 2^7 \times 3^2$$

$$\text{Pairs} = 2^6 \times 3^2$$

Hence, 1152 should be divided by 2 in order to get the perfect square.

$$\text{Hence the number after division by 2} = 1152/2 = 576$$

$$\text{Factors of 576 are} = 2^6 \times 3^2 = 24^2$$

Hence, resulting number is the square of 24.

Exercise 3.2

1. Question

The following numbers are not perfect squares. Give reason.

(i) 1547 (ii) 45743

(iii) 8948 (iv) 333333

Answer

Numbers ending with 2, 3, 7 or 8 are not perfect squares. So,

(i) 1547

(ii) 45743

(iii) 8948

(iv) 333333 are not perfect squares

2. Question

Show that the following numbers are not, perfect squares:

(i) 9327 (ii) 4058

(iii) 22453 (iv) 743522

Answer

Hence, 7, 8, 3, 2 as ending numbers respectively. As mentioned above ending with 2, 3, 7, 8 are not perfect square. So, these given numbers are not perfect squares

3. Question

The square of which of the following numbers would be an old number?

(i) 731 (ii) 3456

(iii) 5559 (iv) 42008

Answer

Square of an odd number is an odd number

Square of an even number is an even number

(i) 731: It is an odd number so its square is also odd number

(ii) 3456: It is an even number so its square is also even number

(iii) 5559: It is an odd number so its square is also odd number

(iv) 42008: It is an even number so its square is also even number

4. Question

What will be the units digit of the squares of the following numbers?

(i) 52 (ii) 977

(iii) 4583 (iv) 78367

(v) 52698 (vi) 99880

(vii) 12796 (viii) 55555

(ix) 53924

Answer

(i) 52

Unit digit of $(52)^2 = \text{unit digit of } (2)^2 = 4$

(ii) 977

Unit digit of $(977)^2 = \text{unit digit of } (7)^2 = 9$

(iii) 4583

Unit digit of $(4583)^2 = \text{unit digit of } (3)^2 = 9$

(iv) 78367

Unit digit of $(78367)^2 = \text{unit digit of } (7)^2 = 9$

(v) 52698

Unit digit of $(52698)^2 = \text{unit digit of } (8)^2 = 4$

(vi) 99880

Unit digit of $(99880)^2 = \text{unit digit of } (0)^2 = 0$

(vii) 12796

Unit digit of $(12796)^2 = \text{unit digit of } (6)^2 = 6$

(viii) 55555

Unit digit of $(55555)^2 = \text{unit digit of } (5)^2 = 5$

(ix) 53924

Unit digit of $(53924)^2 = \text{unit digit of } (4)^2 = 6$

5. Question

Observe the following pattern

$$1 + 3 = 2^2$$

$$1 + 3 + 5 = 3^2$$

$$1 + 3 + 5 + 7 = 4^2$$

And write the value of $1+3+5+7+9+\dots$ upto n terms.

Answer

The pattern here is the square of the number on the Right-hand side is equal to the sum of all the numbers on the left-hand side.

Thus, for n terms,

$$1 + 3 + 5 + \dots n \text{ terms} = n^2 \text{ [As there are } n \text{ terms]}$$

6. Question

Observe the following pattern

$$2^2 - 1^2 = 2 + 1$$

$$3^2 - 2^2 = 3 + 2$$

$$4^2 - 3^2 = 4 + 3$$

$$5^2 - 4^2 = 5 + 4$$

And find the value of

(i) $100^2 - 99^2$ (ii) $111^2 - 109^2$

(iii) $99^2 - 96^2$

Answer

(i) $100^2 - 99^2$

$$= 100 + 99$$

$$= 199$$

(ii) $111^2 - 109^2$

$$= 111^2 - 110^2 + 110^2 - 109^2$$

$$= (111 + 110) + (110 + 109)$$

$$= 440$$

(iii) $99^2 - 96^2$

$$= 99^2 - 98^2 + 98^2 - 97^2 + 97^2 - 96^2$$

$$= (99 + 98) + (98 + 97) + (97 + 96)$$

$$= 585$$

7. Question

Which of the following triplets are Pythagorean?

(i) (8, 15, 17)

(ii) (18, 80, 82)

(iii) (14, 48, 51)

(iv) (10, 24, 26)

(v) (16, 63, 65)

(vi) (12, 35, 38)

Answer

(i) (8, 15, 17)

$$\text{L.H.S} = 8^2 + 15^2 = 289$$

$$\text{R.H.S} = 17^2 = 289$$

$$\text{L.H.S} = \text{R.H.S}$$

So, it is Pythagoras

(ii) (18, 80, 82)

$$\text{L.H.S} = 18^2 + 80^2 = 6724$$

$$\text{R.H.S} = 82^2 = 6724$$

$$\text{L.H.S} = \text{R.H.S}$$

So, it is Pythagoras

(iii) (14, 48, 51)

$$\text{L.H.S} = 14^2 + 48^2 = 2500$$

$$\text{R.H.S} = 51^2 = 2601$$

$$\text{L.H.S} \neq \text{R.H.S}$$

So, it is not Pythagoras

(iv) (10, 24, 26)

$$\text{L.H.S} = 10^2 + 24^2 = 676$$

$$\text{R.H.S} = 26^2 = 676$$

$$\text{L.H.S} = \text{R.H.S}$$

So, it is Pythagoras

(v) (16, 63, 65)

$$\text{L.H.S} = 16^2 + 63^2 = 4225$$

$$\text{R.H.S} = 65^2 = 4225$$

$$\text{L.H.S} = \text{R.H.S}$$

So, it is Pythagoras

(vi) (12, 35, 38)

$$\text{L.H.S} = 12^2 + 35^2 = 1369$$

$$\text{R.H.S} = 38^2 = 1444$$

$$\text{L.H.S} \neq \text{R.H.S}$$

So, it is Pythagoras

8. Question

Observe the following pattern

$$(1 \times 2) + (2 \times 3) = \frac{2 \times 3 \times 4}{3}$$

$$(1 \times 2) + (2 \times 3) + (3 \times 4) = \frac{3 \times 4 \times 5}{3}$$

$$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) = \frac{4 \times 5 \times 6}{3}$$

And find the value of

$$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6)$$

Answer

From observation:

$$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6) = \frac{5 \times 6 \times 7}{3} \\ = 70$$

9. Question

Observe the following pattern

$$1 = \frac{1}{2} \{1 \times (1 + 1)\} \\ 1 + 2 = \frac{1}{2} \{2 \times (2 + 1)\} \\ 1 + 2 + 3 = \frac{1}{2} \{3 \times (3 + 1)\} \\ 1 + 2 + 3 + 4 = \frac{1}{2} \{4 \times (4 + 1)\}$$

And find the values of each of the following:

(i) $1 + 2 + 3 + 4 + 5 + \dots + 50$

(ii) $31 + 32 + \dots + 50$

Answer

$$\text{R.H.S} = \frac{1}{2} [\text{No. of terms in L.H.S} \times (\text{No. of terms} + 1)] \text{ (Therefore, only when L.H.S starts with 1)}$$

Therefore,

(i) $1 + 2 + 3 + \dots + 50 = \frac{1}{2} [50 \times (50 + 1)]$

$$= 25 \times 51$$

$$= 1275$$

(ii) $31 + 32 + \dots + 50 = (1 + 2 + 3 + \dots + 50) - (1 + 2 + \dots + 30)$

$$= 1275 - \left[\frac{1}{2} (30 \times 30 + 1) \right]$$

$$= 1275 - 465$$

$$= 810$$

10. Question

Observe the following pattern

$$1^2 = \frac{1}{6} [1 \times (1 + 1) \times (2 \times 1 + 1)] \\ 1^2 + 2^2 = \frac{1}{2} [2 \times (2 + 1) \times (2 \times 2 + 1)] \\ 1^2 + 2^2 + 3^2 = \frac{1}{6} [3 \times (3 + 1) \times (2 \times 3 + 1)] \\ 1^2 + 2^2 + 3^2 + 4^2 = \frac{1}{6} [4 \times (4 + 1) \times (2 \times 4 + 1)]$$

And find the values of each of the following.

(i) $1^2 + 2^2 + 3^2 + 4^2 + \dots + 10^2$

(ii) $5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2 + 12^2$

Answer

$$\text{R.H.S} = \frac{1}{6} [(\text{No. of terms in L.H.S}) \times (\text{No.} + 1) \times (2 \times \text{No.} + 1)]$$

(i) $1^2 + 2^2 + 3^2 + 4^2 + \dots + 10^2 = \frac{1}{6} [10 (10 + 1) \times (2 \times 10 + 1)]$

$$= \frac{1}{6} [2310]$$

$$= 385$$

$$(ii) 5^2 + 6^2 + \dots + 12^2 = 1^2 + 2^2 + \dots + 12^2 - (1^2 + 2^2 + 3^2 + 4^2)$$

$$= \frac{1}{6} [12 \times (12 + 1) \times (2 \times 12 + 1)] - \frac{1}{6} [4 \times (4 + 1) \times (2 \times 4 + 1)]$$

$$= 650 - 30$$

$$= 620$$

11. Question

Which of the following numbers are squares of even numbers?

121, 225, 256, 324, 1296, 6561, 5476, 4489, 373758

Answer

Only even numbers be the square of even numbers

So, 256, 324, 1296, 5476, 373758 can be square of even numbers but 373758 is not a perfect square

So, 256, 324, 1296, 5476 are numbers

12. Question

By just examining the units digits, can you tell which of the following cannot be whole squares?

(i) 1026 (ii) 1028

(iii) 1024 (iv) 1022

(v) 1023 (vi) 1027

Answer

Numbers ending with 2, 3, 7, 8 cannot be perfect square. So,

1028 (iv) 1022 (v) 1023 (vi) 1027 cannot be whole squares.

13. Question

Which of the numbers for which you cannot decide whether they are squares.

Answer

All the natural numbers whose unit digit is 0, 1, 4, 5, 6 or 9 can not be said surely if they are square numbers or not

14. Question

Write five numbers which you cannot decide whether they are square just by looking at the unit's digit.

Answer

Any natural number ending with 0, 1, 4, 5, 6 or 9 can be or cannot be a square number.

Hence,

The five examples are:

(i) 2061

The ending digit is 1. Hence, it may or may not be a square number

(ii) 1069

The ending digit is 9. Hence, it may or may not be a square number

(iii) 1234

The ending digit is 4. Hence, it may or may not be a square number

(iv) 56790

The ending digit is 0. Hence, it may or may not be a square number

(v) 76555

The ending digit is 5. Hence, it may or may not be a square number

15. Question

Write true (T) or false (F) for the following statements.

(i) The number of digits in a square number is even.

(ii) The square of a prime number is prime

(iii) The sum of two square numbers is a square number.

(iv) The difference of two square numbers is a square number.

(v) The product of two square numbers is a square number.

(vi) No square number is negative.

(vii) There is no square number between 50 and 56.

(viii) There are fourteen square number upto 200.

Answer

(i) False: Because 169 is square number with odd digit

(ii) False: Square of 3 (Prime) is 9 (not prime)

(iii) False: Sum of 2^2 and 3^2 is 13 which is not square number

(iv) False: Difference of 3^2 and 2^2 is 5, which is not square number

(v) True: Because the square of 2^2 and 3^2 is 36 which is square of 6

(vi) True: As $(-2)^2$ is 4, i.e. not negative

(vii) True: As there is no square number between them

(viii) True: The fourteen numbers upto 200 are: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196

Exercise 3.3

1. Question

Find the squares of the following numbers using column method. Verify the result by finding the square using the usual multiplication:

(i) 25

(ii) 37

(iii) 54

(iv) 71

(v) 96

Answer

(i) 25

Here, $a = 2$, $b = 5$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
4	20	<u>25</u>
+2	+2	
<u>6</u>	<u>22</u>	
<hr/>		
6	2	5

$$25^2 = 625$$

And,

$$25^2 = 25 \times 25 = 625$$

(ii) 37

Here, $a = 3$, $b = 7$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
9	42	<u>49</u>
+4	+4	
<u>13</u>	<u>46</u>	
<hr/>		
13	6	9

$$37^2 = 1369$$

And,

$$37^2 = 37 \times 37 = 1369$$

(iii) 54

Here, $a = 5$, $b = 4$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
25	40	<u>16</u>
+4	+1	
<u>29</u>	<u>41</u>	
<hr/>		
29	1	6

$$54^2 = 2916$$

And,

$$54^2 = 54 \times 54 = 2916$$

(iv) 71

Here, $a = 7$, $b = 1$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
49	14	<u>01</u>
1	0	
<u>49</u>	<u>14</u>	
<hr/>		
49	4	1

$$71^2 = 4941$$

And,

$$71^2 = 71 \times 71 = 4941$$

(v) 96

Here, $a = 9$, $b = 6$

Column 1	Column 2	Column 3
a^2	$2ab$	b^2
81	108	36
11	3	
<u>92</u>	<u>111</u>	
92	1	6

$$96^2 = 9216$$

And,

$$96^2 = 96 \times 96 = 9216$$

2. Question

Find the squares of the following numbers using diagonal method:

(i) 98

(ii) 273

(iii) 348

(iv) 295

(v) 171

Answer

(i) 98

Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.

Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n - 1)$ horizontal and $(n - 1)$ vertical lines.

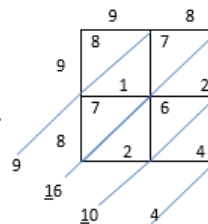
Step III: Draw the diagonals of each sub-square.

Step IV: Write the digits of the number to be squared along left vertical side and top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding sub-square.

Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.

Step VII: Obtain the required square by writing the digits from the left-most side.



$$(98)^2 = 9604$$

(ii) 273

Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.

Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n - 1)$ horizontal and $(n - 1)$ vertical lines.

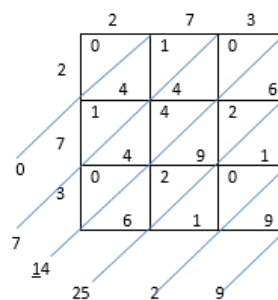
Step III: Draw the diagonals of each sub-square.

Step IV: Write the digits of the number to be squared along left vertical side and top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding sub-square.

Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.

Step VII: Obtain the required square by writing the digits from the left-most side.



$$(273)^2 = 74529$$

(iii) 348

Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.

Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n - 1)$ horizontal and $(n - 1)$ vertical lines.

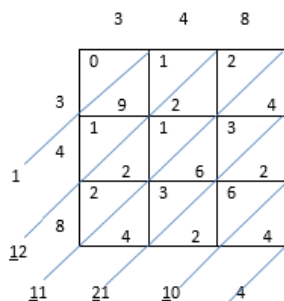
Step III: Draw the diagonals of each sub-square.

Step IV: Write the digits of the number to be squared along left vertical side and top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding sub-square.

Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.

Step VII: Obtain the required square by writing the digits from the left-most side.



$$348^2 = 121104$$

(iv) 295

Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.

Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n - 1)$ horizontal and $(n - 1)$ vertical lines.

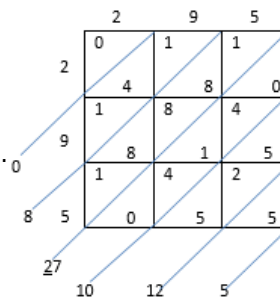
Step III: Draw the diagonals of each sub-square.

Step IV: Write the digits of the number to be squared along left vertical side and top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding sub-square.

Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.

Step VII: Obtain the required square by writing the digits from the left-most side.



$$(295)^2 = 87025$$

(v) 171

Step I: Obtain the number and count the number of digits in it. Let there be n digits in the number to be squared.

Step II: Draw square and divide it into n^2 sub-squares of the same size by drawing $(n - 1)$ horizontal and $(n - 1)$ vertical lines.

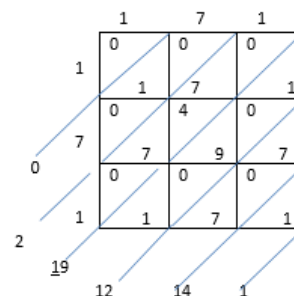
Step III: Draw the diagonals of each sub-square.

Step IV: Write the digits of the number to be squared along left vertical side and top horizontal side of the squares as shown below.

Step V: Multiply each digit on the left of the square with each digit on top of the column one-by-one. Write the units digit of the product below the diagonal and tens digit above the diagonal of the corresponding sub-square.

Step VI: Starting below the lowest diagonal sum the digits along the diagonals so obtained. Write the units digit of the sum and take carry, the tens digit (if any) to the diagonal above.

Step VII: Obtain the required square by writing the digits from the left-most side.



$$(171)^2 = 29241$$

3. Question

Find the squares of the following numbers:

(i) 127 (ii) 503

(iii) 450 (iv) 862

(v) 265

Answer

$$(i) (127)^2 = 127 \times 127$$

$$= 16129$$

$$(ii) (503)^2 = 503 \times 503$$

$$= 253009$$

$$(iii) (451)^2 = 451 \times 451$$

$$= 203401$$

$$(iv) (862)^2 = 862 \times 862$$

$$= 743044$$

$$(v) (265)^2 = 265 \times 265$$

$$= 70225$$

4. Question

Find the squares of the following numbers:

(i) 425 (ii) 575

(iii) 405 (iv) 205

(v) 95 (vi) 745

(vii) 512 (viii) 995

Answer

(i) 425

We know that,

The square of 425 is:

$$(425)^2 = 425 \times 425$$

$$= 180625$$

Hence, the square of 425 is 180625

(ii) 575

We know that,

The square of 575 is:

$$(575)^2 = 575 \times 575$$

$$= 330625$$

Hence, the square of 575 is 330625

(iii) 405

We know that,

The square of 405 is:

$$(405)^2 = 405 \times 405$$

$$= 164025$$

Hence, the square of 405 is 164025

(iv) 205

We know that,

The square of 205 is:

$$(205)^2 = 205 \times 205$$

$$= 42025$$

Hence, the square of 205 is 42025

(v) 95

We know that,

The square of 95 is:

$$(95)^2 = 95 \times 95$$

$$= 9025$$

Hence, the square of 95 is 9025

(vi) 745

We know that,

The square of 745 is:

$$(745)^2 = 745 \times 745$$

$$= 555025$$

Hence, the square of 745 is 555025

(vii) 512

We know that,

The square of 512 is:

$$(512)^2 = 512 \times 512$$

$$= 262144$$

Hence, the square of 512 is 262144

(viii) 995

We know that,

The square of 995 is:

$$(995)^2 = 995 \times 995$$

$$= 990025$$

Hence, the square of 995 is 990025

5. Question

Find the squares of the following numbers using the identity $(a + b)^2 = a^2 + 2ab + b^2$:

(i) 405

(ii) 510

(iii) 1001

(iv) 209

(v) 605

Answer

(i) 405

We have,

$$\begin{aligned}(405)^2 &= (400 + 5)^2 \\&= (400)^2 + 5^2 + 2 (400) (5) \\&= 160000 + 25 + 4000 \\&= 164025\end{aligned}$$

(ii) 510

We have,

$$\begin{aligned}(510)^2 &= (500 + 10)^2 \\&= 250000 + 100 + 10000 \\&= 260100\end{aligned}$$

(iii) 1001

We have,

$$\begin{aligned}(1001)^2 &= (1000 + 1)^2 \\&= (1000)^2 + 1 + 2 (1000) \\&= 1000000 + 1 + 2000 \\&= 1002001\end{aligned}$$

(iv) 209

We have,

$$\begin{aligned}(209)^2 &= (200 + 9)^2 \\&= (200)^2 + 9^2 + 2 (200) (9) \\&= 40000 + 81 + 3600 \\&= 43681\end{aligned}$$

(v) 605

We have,

$$\begin{aligned}(605)^2 &= (600 + 5)^2 \\&= (600)^2 + 5^2 + 2 (600) (5) \\&= 360000 + 25 + 6000 \\&= 366025\end{aligned}$$

6. Question

Find the squares of the following numbers using the identity $(a - b)^2 = a^2 - 2ab + b^2$:

(i) 395 (ii) 995

(iii) 495 (iv) 498

(v) 99 (vi) 999

(vii) 599

Answer

(i) 395

$$\begin{aligned} 395 &= (400 - 5)^2 \\ &= (400)^2 + 5^2 - 2 (400) (5) \\ &= 160000 + 25 - 4000 \\ &= 156025 \end{aligned}$$

(ii) 995

$$\begin{aligned} 995 &= (1000 - 5)^2 \\ &= (1000)^2 + 5^2 - 2 (1000) (5) \\ &= 1000000 + 25 - 10000 \\ &= 990025 \end{aligned}$$

(iii) 495

$$\begin{aligned} 495 &= (500 - 5)^2 \\ &= (500)^2 + 5^2 - 2 (500) (5) \\ &= 250000 + 25 - 5000 \\ &= 245025 \end{aligned}$$

(iv) 498

$$\begin{aligned} 498 &= (500 - 2)^2 \\ &= (500)^2 + 2^2 - 2 (500) (2) \\ &= 250000 + 4 - 2000 \\ &= 248004 \end{aligned}$$

(v) 99

$$\begin{aligned} 99 &= (100 - 1)^2 \\ &= (100)^2 + 1^2 - 2 (100) (1) \\ &= 10000 + 1 - 200 \\ &= 9799 \end{aligned}$$

(vi) 999

$$\begin{aligned} 999 &= (1000 - 1)^2 \\ &= (1000)^2 + 1^2 - 2 (1000) (1) \\ &= 1000000 + 1 - 2000 \\ &= 998001 \end{aligned}$$

(vii) 599

$$\begin{aligned} &(600 - 1)^2 \\ &= (600)^2 + 1^2 - 2 (600) (1) \\ &= 360000 + 1 - 1200 \\ &= 358801 \end{aligned}$$

7. Question

Find the squares of the following numbers by visual method:

(i) 52 (ii) 95

(iii) 505 (iv) 702

(v) 99

Answer

$$\begin{aligned}\text{(i) } 52, (52)^2 &= (50 + 2)^2 \\ &= 50^2 + 2^2 + (2 \times 50 \times 2) \\ &= 2500 + 4 + 200 \\ &= 2704\end{aligned}$$

$$\begin{aligned}\text{(ii) } 95, (95)^2 &= (100 - 5)^2 \\ &= 100^2 + 5^2 - (2 \times 5 \times 100) \\ &= 10000 + 25 - 1000 \\ &= 9025\end{aligned}$$

$$\begin{aligned}\text{(iii) } 505, (505)^2 &= (500 + 5)^2 \\ &= 500^2 + 5^2 + (2 \times 500 \times 5) \\ &= 250000 + 25 + 5000 \\ &= 255025\end{aligned}$$

$$\begin{aligned}\text{(iv) } 702, (702)^2 &= (700 + 2)^2 \\ &= 700^2 + 2^2 + (2 \times 700 \times 2) \\ &= 140000 + 4 + 2800 \\ &= 142804\end{aligned}$$

$$\begin{aligned}\text{(v) } 99, (99)^2 &= (100 - 1)^2 \\ &= 100^2 + 1^2 - (2 \times 100 \times 1) \\ &= 10000 + 1 - 200 \\ &= 9301\end{aligned}$$

Exercise 3.4

1. Question

Write the possible unit's digits of the square root of the following numbers. Which of these numbers are odd square roots?

(i) 9801

(ii) 99856

(iii) 998001

(iv) 657666025

Answer

(i) 9801

Unit digit = 1

Unit digit of square root = 1 or 9

As number is odd, square root is also odd

(ii) 99856

Unit digit = 6

Unit digit of square root = 4 or 6

As number is even, square root is also even

(iii) 998001

Unit digit = 1

Unit digit of square root = 1 or 9

As number is odd, square root is also odd

(iv) 657666025

Unit digit = 5

Unit digit of square root = 5

As number is odd, square root is also odd

2. Question

Find the square root of each of the following by prime factorization.

(i) 441 (ii) 196

(iii) 529 (iv) 1764

(v) 1156 (vi) 4096

(vii) 7056 (viii) 8281

(ix) 11664 (x) 47089

(xi) 24336 (xii) 190969

(xiii) 586756 (xiv) 27225

(xv) 3013696

Answer

(i) 441

$$441 = 3^2 \times 7^2$$

$$\sqrt{441} = 3 \times 7$$

$$= 21$$

$$\begin{array}{r|l} 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

(ii) 196

$$196 = 2^2 \times 7^2$$

$$\sqrt{196} = 2 \times 7$$

$$= 14$$

$$\begin{array}{r|l}
 2 & 196 \\
 2 & 98 \\
 7 & 49 \\
 & 7
 \end{array}$$

(iii) 529

$$529 = 23^2$$

$$\sqrt{529} = 23$$

$$\begin{array}{r|l}
 23 & 529 \\
 & 23
 \end{array}$$

(iv) 1764

$$1764 = 2^2 \times 3^2 \times 7^2$$

$$\sqrt{1764} = 2 \times 3 \times 7$$

$$= 42$$

$$\begin{array}{r|l}
 2 & 1764 \\
 2 & 882 \\
 3 & 441 \\
 3 & 147 \\
 3 & 49 \\
 7 & 7
 \end{array}$$

(v) 1156

$$1156 = 2^2 \times 17^2$$

$$\sqrt{1156} = 2 \times 17$$

$$= 34$$

$$\begin{array}{r|l}
 2 & 1156 \\
 2 & 578 \\
 17 & 289 \\
 & 17
 \end{array}$$

(vi) 4096

$$4096 = 2^{12}$$

$$\sqrt{4096} = 2^6$$

$$= 64$$

$$\begin{array}{r|l}
 2 & 4096 \\
 2 & 2048 \\
 2 & 1024 \\
 2 & 512 \\
 2 & 256 \\
 2 & 128 \\
 2 & 64 \\
 2 & 32 \\
 2 & 16 \\
 2 & 8 \\
 2 & 4 \\
 2 & 2
 \end{array}$$

(vii) 7056

$$7056 = 2^2 \times 3^2 \times 7^2$$

$$\sqrt{7056} = 2 \times 3 \times 7$$

$$= 84$$

$$\begin{array}{r|l} 2 & 7056 \\ \hline 2 & 3528 \\ \hline 2 & 1764 \\ \hline 2 & 882 \\ \hline 2 & 441 \\ \hline 21 & 21 \end{array}$$

(viii) 8281

$$8281 = 91^2$$

$$\sqrt{8281} = 91$$

$$\begin{array}{r|l} 91 & 8281 \\ \hline & 91 \end{array}$$

(ix) 11664

$$11664 = 2^2 \times 2^2 \times 3^2 \times 3^2 \times 3^2$$

$$\sqrt{11664} = 2 \times 2 \times 3 \times 3 \times 3$$

$$= 108$$

$$\begin{array}{r|l} 2 & 11664 \\ \hline 2 & 5832 \\ \hline 2 & 2916 \\ \hline 2 & 1458 \\ \hline 2 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \end{array}$$

(x) 47089

$$47089 = 217^2$$

$$\sqrt{47089} = 217$$

(xi) 24336

$$24336 = 2^2 \times 2^2 \times 3^2 \times 13^2$$

$$\sqrt{24336} = 2 \times 2 \times 3 \times 13$$

$$= 156$$

$$\begin{array}{r|l} 2 & 24336 \\ \hline 2 & 12168 \\ \hline 2 & 6084 \\ \hline 2 & 3042 \\ \hline 3 & 1521 \\ \hline 3 & 507 \\ \hline 13 & 169 \\ \hline & 13 \end{array}$$

(xii) 190969

$$190969 = 23^2 \times 19^2$$

$$\sqrt{190969} = 23 \times 19$$

$$= 437$$

23	190969
23	8303
19	361
	19

(xiii) 586756

$$586756 = 2^2 \times 383^2$$

$$\sqrt{586756} = 2 \times 383$$

$$= 766$$

(xiv) 27225

$$27225 = 5^2 \times 3^2 \times 11^2$$

$$\sqrt{27225} = 5 \times 3 \times 11$$

$$= 165$$

(xv) 3013696

$$3013696 = 2^6 \times 217^2$$

$$\sqrt{3013696} = 2^3 \times 217$$

$$= 1736$$

2	3013696
2	1506848
2	753424
2	376712
2	188356
2	94178
2	47089
217	217

3. Question

Find the smallest number by which 180 must be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

Answer

$$180 = 2^2 \times 3^2 \times 5$$

$$= (2 \times 2) \times (3 \times 3) \times 5$$

To make the unpaired 5 into paired, multiply the number with 5

Therefore,

$$180 \times 5 = 2^2 \times 3^2 \times 5^2$$

$$\text{Hence, square root of number} = \sqrt{180} \times \sqrt{5} = 2 \times 3 \times 5$$

$$= 30$$

4. Question

Find the smallest number by which 147 must be multiplied so that it becomes a perfect square. Also, find the square root of the number so obtained.

Answer

$$147 = 7^2 \times 3$$

To make the unpaired 3 into paired, multiply the number with 3

Therefore,

$$147 \times 3 = 7^2 \times 3^2$$

Hence, square root of number = $\sqrt{147} \times \sqrt{3} = 7 \times 3$

$$= 21$$

5. Question

Find the smallest number by which 3645 must be divided so that it becomes a perfect square. Also, find the square root of the resulting number.

Answer

$$3645 = 5 \times (3 \times 3) \times (3 \times 3) \times 3$$

Here 5 and 3 are unpaired so we have to divide 3645 with $5 \times 3 = 15$

Therefore,

$$\frac{3645}{15} = 3^2 \times 3^2$$

Hence,

$$\text{Square root of numbers} = \sqrt{\frac{3645}{15}} = 3 \times 3$$

$$= 9$$

6. Question

Find the smallest number by which 1152 must be divided so that it becomes a square. Also, find the square root of the number so obtained.

Answer

$$1152 = (2 \times 2) \times (2 \times 2) \times 2 \times (3 \times 3)$$

Here 2 is unpaired so we have to divide 1152 with 2

Therefore,

$$\frac{1152}{2} = 2^2 \times 2^2 \times 2^2 \times 3^2$$

Hence,

$$\text{Square root of numbers} = \sqrt{\frac{1152}{2}} = 2 \times 2 \times 2 \times 3$$

$$= 24$$

7. Question

The product of two numbers is 1296. If one number is 16 times the other, find the numbers.

Answer

Let a and b be two numbers

$$a \times b = 1296$$

$$a = 16b$$

$$= 16b \times b$$

$$= 1296$$

$$b^2 = 81$$

$$b = 9$$

Therefore,

$$a = 144 \text{ and } b = 9$$

8. Question

A welfare association collected Rs 202500 as donation from the residents. If each paid as many rupees as there were residents, find the number of residents.

Answer

Let total residents be a

Therefore, each paid Rs. a

$$\text{Total collection} = a(a) = a^2$$

$$\text{given, Total Collection} = 202500$$

Hence,

$$a = \sqrt{202500} \quad a = \sqrt{(2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5)} \quad a = 2 \times 3 \times 3 \times 5 \times 5 = 450$$

Therefore,

$$\text{Total residents} = 450$$

9. Question

A society collected Rs 92.16. Each member collected as many paise as there were members. How many members were there and how much did each contribute?

Answer

Let there were a members

Therefore, each attributed a paise

Therefore,

$$a(a), \text{ i.e. total cost collected} = 9216 \text{ paise}$$

$$a^2 = 9216$$

$$a = \sqrt{9216}$$

$$= 2 \times 2 \times 2 \times 12$$

$$= 96$$

Therefore, there were 96 members and each contributed 96 paise

10. Question

A society collected Rs 2304 as fees from its students. If each student paid as many paise as there were students in the school, how many students were there in the school?

Answer

Let, a be number of school students

Therefore, each student contributed a paise

$$\text{Total money obtained} = a^2 \text{ paise}$$

$$= 230400 \text{ paise}$$

$$a = \sqrt{230400}$$

$$= \sqrt{2304} \times \sqrt{100}$$

$$= 10 \sqrt{2304}$$

$$a = 10 \times 2 \times 2 \times 12$$

$$a = 480$$

Therefore, there were 480 students

11. Question

The area of a square field is 5184 m^2 . A rectangular field, whose length is twice its breadth has its perimeter equal to the perimeter of the square field. Find the area of the rectangular field.

Answer

Let 'a' be the side of square field

Therefore,

$$a^2 = 5184 \text{ m}^2$$

$$a = \sqrt{5184} \text{ m}$$

$$a = 2 \times 2 \times 2 \times 9$$

$$= 72 \text{ m}$$

$$\text{Perimeter of square} = 4a$$

$$= 288 \text{ m}$$

$$\text{Perimeter of rectangle} = 2(l + b)$$

$$= 288 \text{ m}$$

$$2(2b + b) = 288$$

$$b = 48 \text{ and } l = 96$$

$$\text{Area of rectangle} = 96 \times 48 \text{ m}^2$$

$$= 4608 \text{ m}^2$$

12. Question

Find the least square number, exactly divisible by each one of the numbers: (i) 6, 9, 15 and 20 (ii) 8, 12, 15 and 20

Answer

(i) 6, 9, 15 and 20

L.C.M of given 4 numbers is 180

$$180 = 2^2 \times 3^2 \times 5$$

To make it a perfect square, we have to multiply the number with 5

Therefore,

$$180 \times 5 = 2^2 \times 3^2 \times 5^2$$

900 is the least square number divisible by 6, 9, 15 and 20

3600 is the least square number divisible by 8, 12, 15 and 20

(ii) 8, 2, 15 and 20

L.C.M of given 4 numbers is 360

$$360 = 2^2 \times 3^2 \times 2 \times 5$$

To make it a perfect square, we have to multiply the number with $2 \times 5 = 10$

Therefore,

$$360 \times 10 = 2^2 \times 3^2 \times 5^2 \times 2^2$$

13. Question

Find the square roots of 121 and 169 by the method of repeated subtraction.

Answer

$$121 - 1 = 120$$

$$120 - 3 = 117$$

$$117 - 5 = 112$$

$$112 - 7 = 105$$

$$105 - 9 = 96$$

$$96 - 11 = 85$$

$$85 - 13 = 72$$

$$72 - 15 = 57$$

$$57 - 17 = 40$$

$$40 - 19 = 21$$

$$21 - 21 = 0$$

Clearly, we have performed operation 11 times

Therefore,

$$\sqrt{121} = 11$$

$$169 - 1 = 168$$

$$168 - 3 = 165$$

$$165 - 5 = 160$$

$$160 - 7 = 153$$

$$153 - 9 = 144$$

$$144 - 11 = 133$$

$$133 - 13 = 120$$

$$120 - 15 = 105$$

$$105 - 17 = 88$$

$$88 - 19 = 69$$

$$69 - 21 = 48$$

$$48 - 23 = 25$$

$$25 - 25 = 0$$

Clearly, we have performed subtraction 13 times

Therefore,

$$\sqrt{169} = 13$$

14. Question

Write the prime factorization of the following numbers and hence find their square roots.

(i) 7744

(ii) 9604

(iii) 5929

(iv) 7056

Answer

(i) 7744

$$7744 = 2^2 \times 2^2 \times 2^2 \times 11^2$$

$$\sqrt{7744} = 2 \times 2 \times 2 \times 11$$

$$= 88$$

(ii) 9604

$$9604 = 2^2 \times 7^2 \times 7^2$$

$$\sqrt{9604} = 2 \times 7 \times 7$$

$$= 98$$

(iii) 5929

$$5929 = 11^2 \times 7^2$$

$$\sqrt{5929} = 11 \times 7$$

$$= 77$$

(iv) 7056

$$7056 = 2^2 \times 2^2 \times 7^2 \times 3^2$$

$$\sqrt{7056} = 2 \times 2 \times 7 \times 3$$

$$= 84$$

15. Question

The students of class VIII of a school donated Rs 2401 for PM's National Relief Fund. Each student donated as many rupees as the number of students in the class, Find the number of students in the class.

Answer

Let a be the number of students

Therefore,

Each student donated a rupee

So,

Total amount collected = a × a rupees

$$= 2401$$

$$a^2 = 2401$$

$$a = 49$$

Therefore,

There are 49 students in the class

16. Question

A PT teacher wants to arrange maximum possible number of 6000 students in a field such that the number of rows is equal to the number of columns. Find the number of rows if 71 were left out after arrangement.

Answer

Let a be number of rows

Therefore,

No. of columns = a

Total number of students who sat in field = a^2

Total students = $a^2 + 71$

= 6000

$a^2 = 5929$

$a = \sqrt{5929}$

$a = 11 \times 7$

= 77

Therefore, total number of rows = 77

Exercise 3.5

1. Question

Find the square root of each of the following by long division method:

(i) 12544 (ii) 97344

(iii) 286225 (iv) 390625

(v) 363609 (vi) 974169

(vii) 120409 (viii) 1471369

(ix) 291600 (x) 9653449

(xi) 1745041 (xii) 4008004

(xiii) 20657025 (xiv) 152547201

(xv) 20421361 (xvi) 62504836

(xvii) 82264900 (xviii) 3226694416

(xix) 6407522209 (xx) 3915380329

Answer

(i) 12544

$$\begin{array}{r} 112 \\ 1 \overline{) 12544} \\ \underline{1} \\ 21 \\ \underline{25} \\ 24 \\ \underline{222} \\ 444 \\ \underline{444} \\ 0 \end{array}$$

Therefore,

$$\sqrt{12544} = 112$$

(ii) 97344

	312	
3	97344	
	9	
61	73	
	61	
622	1244	
	1244	
	0	

Therefore,

$$\sqrt{97344} = 312$$

(iii) 286225

	535	
5	286225	
	25	
103	362	
	309	
1065	5325	
	5325	
	0	

Therefore,

$$\sqrt{286225} = 535$$

(iv) 390625

	625	
6	390625	
	36	
122	306	
	244	
1245	6225	
	6225	
	0	

$$\sqrt{390625} = 625$$

(v) 363609

	603	
6	363609	
	36	
1203	3609	
	3609	
	0	

Therefore,

$$\sqrt{363609} = 603$$

(vi) 974169

	987	
9	974169	
	81	
188	1641	
	1504	
1967	13769	
	13769	
	0	

Therefore,

$$\sqrt{974169} = 987$$

(vii) 120409

	347
3	<u>120409</u>
	9
64	<u>304</u>
	256
687	<u>4809</u>
	4809
	0

Therefore,

$$\sqrt{120409} = 347$$

(viii) 1471369

	1213
1	<u>1471369</u>
	1
22	<u>47</u>
	44
241	<u>313</u>
	241
2423	<u>7269</u>
	7269
	0

Therefore,

$$\sqrt{1471369} = 1213$$

(ix) 291600

	540
5	<u>291600</u>
	25
104	<u>416</u>
	416
1080	<u>00</u>
	00
	0

Therefore,

$$\sqrt{291600} = 540$$

(x) 9653449

	3107
3	<u>9653449</u>
	9
61	<u>65</u>
	61
6107	<u>43449</u>
	43449
	0

Therefore,

$$\sqrt{9653449} = 3107$$

(xi) 1745041

	1321
1	<u>1745041</u>
	1
23	<u>74</u>
	69
262	<u>550</u>
	524
2641	<u>2641</u>
	2641
	0

Therefore,

$$\sqrt{1745041} = 1321$$

(xii) 4008004

	2002
2	4008004
	4
40	000
	0
400	080
	0
4002	8004
	8004
	0

Therefore,

$$\sqrt{4008004} = 2002$$

(xiii) 20657025

	4545
4	20657025
	16
85	465
	425
904	4070
	3616
9085	45425
	45425
	0

$$\sqrt{20657025} = 4545$$

(xiv) 152547201

	12351
1	152547201
	1
22	52
	44
243	854
	729
2465	12572
	12325
27701	24701
	24701
	0

Therefore,

$$\sqrt{152547201} = 12351$$

(xv) 20421361

	4519
4	20421361
	16
85	442
	425
901	1713
	901
9029	81261
	81261
	0

Therefore,

$$\sqrt{20421361} = 4519$$

(xvi) 62504836

	7906
7	62504836
	49
149	1350
	1341
1580	948
15806	94836
	94836
	0

Therefore,

$$\sqrt{62504836} = 7906$$

(xvii) 82264900

	9070
9	82264900
	81
180	126
1807	12649
	12649
14140	00
	0
	x

Therefore,

$$\sqrt{82264900} = 9070$$

(xviii) 3226694416

	56804
5	3226694416
	25
106	726
	636
1128	9069
	9024
11360	4544
	0
113604	454416
	454416
	x

$$\sqrt{3226694416} = 56804$$

(xix) 6407522209

	80047
8	6407522209
	64
160	07
	0
1600	752
	00
16004	75222
	64016
160087	1120609
	1120609
	x

Therefore,

$$\sqrt{6407522209} = 80047$$

(xx) 3915380329

	62573
6	3915380329
	36
122	315
	244
1245	07138
	6225
12507	91303
	87549
125143	0375429
	375429
	x

$$\sqrt{3915380329} = 62573$$

2. Question

Find the least number which must be subtracted from the following numbers to make them a perfect square:

(i) 2361

(ii) 194491

(iii) 26535

(iv) 161605

(v) 4401624

Answer

(i) 2361

Hence,

57 must be subtracted from 2361 in order to get a perfect square

	48
4	2361
	16
88	761
	704
	57

(ii) 194491

Hence,

10 must be subtracted from 194491 in order to get a perfect square

	441
4	194491
	16
84	344
	336
881	891
	881
	10

(iii) 26535

Hence,

291 must be subtracted from 26535 in order to get a perfect square

	162
1	26535
	1
26	165
	156
322	935
	644
	291

(iv) 161605

Hence,

1 must be subtracted from 161605 in order to get a perfect square

$$\begin{array}{r} 402 \\ 9 \overline{) 161605} \\ \underline{16} \\ 802 \\ \underline{1605} \\ 1604 \\ \underline{1604} \\ 1 \end{array}$$

(v) 4401624

Hence,

20 must be subtracted from 4401624 in order to get a perfect square number

$$\begin{array}{r} 2098 \\ 2 \overline{) 4401624} \\ \underline{4} \\ 40 \\ \underline{40} \\ 409 \\ \underline{3681} \\ 4188 \\ \underline{33524} \\ 33504 \\ \underline{33504} \\ 20 \end{array}$$

3. Question

Find the least number which must be added to the following numbers to make them a perfect square:

(i) 5607

(ii) 4931

(iii) 4515600

(iv) 37460

(v) 506900

Answer

(i) 5607

$$\begin{array}{r} 74 \\ 7 \overline{) 5607} \\ \underline{49} \\ 144 \\ \underline{707} \\ 576 \\ \underline{576} \\ 131 \end{array}$$

The remainder is 131

Hence, $(74)^2 < 5607$

The next perfect square number is:

$(75)^2 = 5625 > 5607$

Hence, the number to be added = $5625 - 5607$

= 18

(ii) 4931

$$\begin{array}{r} 70 \\ 7 \overline{) 4931} \\ \underline{49} \\ 140 \\ \underline{31} \\ 0 \\ \underline{0} \\ 31 \end{array}$$

The remainder is 31

Hence, $(70)^2 < 4931$

The next perfect square number is:

$$(71)^2 = 5041 > 4931$$

Hence, the number to be added = $5041 - 4931$

$$= 110$$

(iii) 4515600

	2124	
2	4515600	
	4	
41	51	
	41	
422	1056	
	844	
4244	21200	
	16976	
	4224	

The remainder is 4224

$$\text{Hence, } (2124)^2 < 4515600$$

The next perfect square number is:

$$(2125)^2 = 4515625 > 4515600$$

Hence, the number to be added = $4515625 - 4515600$

$$= 25$$

(iv) 37460

	193	
1	37460	
	1	
29	274	
	261	
383	1360	
	1149	
	211	

The remainder is 211

$$\text{Hence, } (193)^2 < 37460$$

The next perfect square number is:

$$(194)^2 = 37636 > 37460$$

Hence, the number to be added = $37636 - 37460$

$$= 176$$

(v) 506900

	711	
7	506900	
	49	
141	169	
	141	
1421	2800	
	1421	
	1379	

The remainder is 1379

$$\text{Hence, } (711)^2 < 506900$$

The next perfect square number is:

$$(712)^2 = 506944 > 506900$$

Hence, the number to be added = $506944 - 506900$

$$= 44$$

4. Question

Find the greatest number of 5 digits which is a perfect square.

Answer

We know that,

Greatest 5 digit number = 99999

$$\begin{array}{r} 316 \\ 3 \overline{) 99999} \\ \underline{9} \\ 61 \quad 99 \\ \underline{61} \\ 626 \quad 3899 \\ \underline{3766} \\ 143 \end{array}$$

The remainder is 143

Therefore,

The greatest 5 digit perfect square number is:

$$99999 - 143$$

$$= 99856$$

Hence, 99856 is the required greatest 5 digit perfect square number

5. Question

Find the least number of 4 digits which is a perfect square.

Answer

We know that,

Least 4 digit number = 1000

$$\begin{array}{r} 31 \\ 3 \overline{) 1000} \\ \underline{9} \\ 61 \quad 100 \\ \underline{61} \\ 39 \end{array}$$

The remainder is 39

Therefore,

$$(31)^2 < 1000$$

Hence,

The next perfect square number is:

$$(32)^2 = 1024 > 1000$$

Hence, 1024 is the required number

6. Question

Find the least number of six digits which is a perfect square.

Answer

We know that,

Least 6 digit number = 100000

$$\begin{array}{r} 316 \\ 3 \overline{) 100000} \\ \underline{9} \\ 61 \\ \underline{61} \\ 626 \\ \underline{3900} \\ 3756 \\ \underline{3756} \\ 144 \end{array}$$

The remainder is 144

Therefore,

$$(316)^2 < 100000$$

Hence, the next perfect square

$$(317)^2 = 100489 > 100000$$

Hence, 100489 is the required number

7. Question

Find the greatest number of 4 digits which is a perfect square.

Answer

We know that,

Greatest 4 digit number = 9999

$$\begin{array}{r} 91 \\ 9 \overline{) 9999} \\ \underline{81} \\ 89 \\ \underline{1899} \\ 1701 \\ \underline{1701} \\ 198 \end{array}$$

The remainder is 198

Hence,

The greatest 4 digit perfect square number = 9999 - 198

$$= 9801$$

8. Question

A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers be 8160, find the number of soldiers in each row.

Answer

Total number of soldiers = 8160

Number of soldiers left out = 60

Number of soldiers arranged in rows to form a perfect square = 8160 - 60

$$= 8100$$

Hence, number of soldiers in each row = $\sqrt{8100}$

$$= \sqrt{9 * 9 * 10 * 10}$$

$$= 90$$

9. Question

The area of a square field is 60025m². A man cycles along its boundary at 18 Km/hr. In how much time will he return at the starting point?

Answer

Area of square field = 60025 m^2

Speed of cyclist = 18 km/h

$$= 18 \times \frac{1000}{60 \times 60}$$

$$= 5 \text{ m/s}^2$$

Area = 60025 m^2

$$\text{Side}^2 = 60025$$

$$\text{Side} = \sqrt{60025}$$

$$= 245$$

Therefore,

Total length of boundary = $4 \times \text{Side}$

$$= 4 \times 245$$

$$= 980 \text{ m}$$

Hence,

$$\text{Time taken} = \frac{980}{5}$$

$$= 196 \text{ seconds}$$

$$= 3 \text{ minutes and } 16 \text{ seconds}$$

10. Question

The cost of leveling and turning a square lawn at Rs 2.50 per m^2 is Rs13322.50 Find the cost of fencing it at Rs 5 per metre.

Answer

Rate of leveling and turning a square lawn = 2.50 per m^2

Total cost of leveling and turning = Rs. 13322.50

$$\text{Total area of square lawn} = \frac{13322.50}{2.50}$$

$$= 5329 \text{ m}^2$$

$$\text{Side of square lawn} = \sqrt{5329}$$

$$= 73 \text{ m}$$

Total length of lawn = 4×73

$$= 292 \text{ m}$$

Cost of fencing the lawn at Rs 5 per metre = 292×5

$$= \text{Rs. } 1460$$

11. Question

Find the greatest number of three digits which is a perfect square.

Answer

We know that,

Largest 3 digit number = 999

$$\begin{array}{r} 31 \\ 3 \overline{) 999} \\ \underline{9} \\ 61 \\ \underline{61} \\ 38 \end{array}$$

The remainder is 38

Hence,

The greatest 3-digit perfect square number = $999 - 38$

= 961

12. Question

Find the smallest number which must be added to 2300 so that it becomes a perfect square.

Answer

At first we have to find,

The square root of 2300

So, the square root of 2300 is:

$$\begin{array}{r} 47 \\ 4 \overline{) 2300} \\ \underline{16} \\ 87 \\ \underline{700} \\ 700 \\ \underline{91} \end{array}$$

The remainder is 91

Hence,

$$(47)^2 < 2300$$

Now, the next perfect square number is $(48)^2 = 2304 > 2300$

Hence,

The smallest number that must be added to 2300 to get a perfect square is:

$$2304 - 2300$$

$$= 4$$

Exercise 3.6

1. Question

Find the square root of:

(i) $\frac{441}{961}$ (ii) $\frac{324}{841}$

(iii) $4\frac{29}{29}$ (iv) $2\frac{14}{25}$

(v) $2\frac{137}{196}$ (vi) $23\frac{26}{121}$

(vii) $25\frac{544}{729}$ (viii) $75\frac{46}{49}$

(ix) $3\frac{942}{2209}$ (x) $3\frac{334}{3025}$

$$(xi) \ 21\frac{2797}{3364} \quad (xii) \ 38\frac{11}{25}$$

$$(xiii) \ 23\frac{394}{729} \quad (xiv) \ 21\frac{51}{169}$$

$$(xv) \ 10\frac{151}{225}$$

Answer

$$(i) \ \frac{441}{961}$$

$$\frac{\sqrt{441}}{\sqrt{961}} = \frac{21}{31}$$

$$(ii) \ \frac{324}{841}$$

$$\frac{\sqrt{324}}{\sqrt{841}} = \frac{18}{29}$$

$$(iii) \ \frac{225}{49}$$

$$\frac{\sqrt{225}}{\sqrt{49}} = \frac{15}{7}$$

$$(iv) \ \frac{64}{25}$$

$$\frac{\sqrt{64}}{\sqrt{25}} = \frac{8}{5}$$

$$(v) \ \frac{529}{196}$$

$$\frac{\sqrt{529}}{\sqrt{196}} = \frac{23}{14}$$

$$(vi) \ \frac{2809}{121}$$

$$\frac{\sqrt{2809}}{\sqrt{121}} = \frac{53}{11}$$

$$(vii) \ \frac{18769}{729}$$

$$\frac{\sqrt{18769}}{\sqrt{729}} = \frac{137}{27}$$

$$(viii) \ \frac{3721}{49}$$

$$\frac{\sqrt{3721}}{\sqrt{49}} = \frac{61}{7}$$

$$(ix) \ \frac{7569}{2209}$$

$$\frac{\sqrt{7569}}{\sqrt{2209}} = \frac{87}{47}$$

$$(x) \ \frac{9409}{3025}$$

$$\frac{\sqrt{9409}}{\sqrt{3025}} = \frac{97}{55}$$

$$(xi) \ \frac{73441}{3364}$$

$$\frac{\sqrt{73441}}{\sqrt{3364}} = \frac{271}{58}$$

$$(xii) \frac{961}{25}$$

$$\frac{\sqrt{961}}{\sqrt{25}} = \frac{31}{5}$$

$$(xiii) \frac{17161}{729}$$

$$\frac{\sqrt{17161}}{\sqrt{729}} = \frac{131}{27}$$

$$(xiv) \frac{3600}{169}$$

$$\frac{\sqrt{3600}}{\sqrt{169}} = \frac{60}{13}$$

$$(xv) \frac{2401}{225}$$

$$\frac{\sqrt{2401}}{\sqrt{225}} = \frac{49}{15}$$

2. Question

Find the value of:

$$(i) \frac{\sqrt{80}}{\sqrt{405}}$$

$$(ii) \frac{\sqrt{441}}{\sqrt{625}}$$

$$(iii) \frac{\sqrt{1587}}{\sqrt{1728}}$$

$$(iv) \sqrt{72} \times \sqrt{338}$$

$$(v) \sqrt{45} \times \sqrt{20}$$

Answer

$$(i) \frac{\sqrt{80}}{\sqrt{405}} = \frac{\sqrt{16}}{\sqrt{81}} \text{ (Cancelling numerator and denominator with 5)}$$

$$= \frac{4}{9} \text{ (Therefore, } \sqrt{16} = 4, \sqrt{81} = 9 \text{)}$$

$$(ii) \frac{\sqrt{441}}{\sqrt{625}}$$

$$= \frac{\sqrt{16}}{\sqrt{81}} = \frac{21}{25} \text{ (Therefore, } \sqrt{441} = 21, \sqrt{625} = 25 \text{)}$$

$$(iii) \frac{\sqrt{1587}}{\sqrt{1728}} = \frac{\sqrt{529}}{\sqrt{576}} \text{ (Cancelling numerator and denominator with 3)}$$

$$= \frac{23}{24} \text{ (Therefore, } \sqrt{529} = 23, \sqrt{576} = 24 \text{)}$$

$$(iv) \sqrt{72} * \sqrt{338}$$

$$= \sqrt{2 * 2 * 2 * 3 * 3} \times \sqrt{2 * 13 * 13}$$

We know that,

$$\sqrt{a} \times \sqrt{b} = \sqrt{a * b}$$

$$\sqrt{2 * 2 * 2 * 2 * 3 * 3 * 13 * 13} = 2^2 \times 3 \times 13$$

$$= 156$$

$$(v) \sqrt{45} * \sqrt{20}$$

$$= \sqrt{5 * 9 * 9} \times \sqrt{5 * 2 * 2}$$

We know that,

$$\sqrt{a} \times \sqrt{b} = \sqrt{a * b}$$

$$\sqrt{5 * 5 * 9 * 9 * 2 * 2} = 5 \times 9 \times 2$$

$$= 90$$

3. Question

The area of a square field is $80\frac{244}{729}$ square metres. Find the length of each side of the field.

Answer

$$\text{Given area} = 80 \times \frac{244}{729} \text{ m}^2$$

$$= \frac{58564}{729} \text{ m}^2$$

If L is length of each side

Therefore,

$$L^2 = \frac{58564}{729}$$

$$L = \frac{\sqrt{58564}}{\sqrt{729}} \text{ (Therefore, } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{)}$$

$$= \frac{242}{27}$$

4. Question

The area of a square field is $30\frac{1}{4} \text{ m}^2$. Calculate the length of the side of the square.

Answer

$$\text{Given, area} = 30 \times \frac{1}{4} \text{ m}^2$$

$$= \frac{121}{4} \text{ m}^2$$

If L is length of each side then,

$$L^2 = \frac{121}{4}$$

$$L = \sqrt{\frac{121}{4}} = \frac{\sqrt{121}}{\sqrt{4}}$$

$$= \frac{11}{2} \text{ (Therefore, } \sqrt{121} = 11, \sqrt{4} = 2 \text{)}$$

Therefore, length is $\frac{11}{2}$

5. Question

Find the length of a side of a square playground whose area is equal to the area of a rectangular field of

dimensions 72m and 338 m.

Answer

Area of rectangular field = $l \times b$

$$= 72 \times 338 \text{ m}^2$$

$$= 24336 \text{ m}^2$$

$$\text{Area of square} = L^2 = 24336 \text{ m}^2$$

$$L = \sqrt{24336}$$

$$= 156 \text{ m}$$

Therefore, 156 m is the length of side of square playground.

Exercise 3.7

1. Question

Find the square root of the following numbers in decimal form:

84.8241

Answer

84.8241

	9.21
9	<u>848241</u>
	81
182	<u>382</u>
	364
1841	<u>1841</u>
	1841
	0

Therefore,

$$\sqrt{84.8241} = 9.21$$

2. Question

Find the square root of the following numbers in decimal form:

0.7225

Answer

0.7225

	0.85
0	<u>0.7225</u>
	0
8	<u>72</u>
	64
165	<u>825</u>
	825
	0

$$\sqrt{0.7225} = 0.85$$

3. Question

Find the square root of the following numbers in decimal form:

0.813604

Answer

0.81304

	0.902
0	0.813604
	0
9	81
	81
180	36
	0
1802	3604
	3604
	0

$$\sqrt{0.813604} = 0.902$$

4. Question

Find the square root of the following numbers in decimal form:

0.00002025

Answer

0.00002025

	0.0045
0	0.00002025
	0 0 0
4	20
	16
85	425
	425
	0

$$\sqrt{0.00002025} = 0.0045$$

5. Question

Find the square root of the following numbers in decimal form:

150.0625

Answer

150.0625

	12.25
1	150.0625
	1
22	050
	44
242	606
	484
2445	12225
	12225
	0

$$\sqrt{150.0625} = 12.25$$

6. Question

Find the square root of the following numbers in decimal form:

225.6004

Answer

225.6004

	15.02
1	<u>225.6004</u>
	1
25	<u>125</u>
	125
300	<u>060</u>
	0
3002	<u>6004</u>
	6004
	0

$$\sqrt{225.6004} = 15.02$$

7. Question

Find the square root of the following numbers in decimal form:

3600.720036

Answer

60.006

	60.006
6	<u>3600.720036</u>
	36
120	<u>000</u>
	0
1200	<u>7200</u>
	0000
12006	<u>720036</u>
	720036
	0

$$\sqrt{3600.720036} = 60.006$$

8. Question

Find the square root of the following numbers in decimal form:

236.144489

Answer

15.367

	15.367
1	<u>236.144689</u>
	1
25	<u>136</u>
	125
303	<u>1114</u>
	909
3066	<u>20546</u>
	18396
30727	<u>215089</u>
	215089
	0

$$\sqrt{236.144689} = 15.367$$

9. Question

Find the square root of the following numbers in decimal form:

0.00059049

Answer

0.00059049

	0.0243
0	0.00059049
	0
0	0
	0
2	05
	4
44	190
	176
483	1449
	1449
	0

$$\sqrt{0.00059049} = 0.0243$$

10. Question

Find the square root of the following numbers in decimal form:

176.252176

Answer

176.252176

	13.276
1	176.252176
	1
13	076
	69
262	725
	524
2647	20121
	18529
26546	159276
	159276
	0

$$\sqrt{176.252176} = 13.276$$

11. Question

Find the square root of the following numbers in decimal form:

9998.0001

Answer

9998.0001

	99.99
9	9998.0001
	81
189	1898
	1701
1989	19700
	17901
19989	179901
	179901
	0

$$\sqrt{9998.0001} = 99.99$$

12. Question

Find the square root of the following numbers in decimal form:

0.00038809

Answer

0.00038809

	0.0197
0	0.00038809
	0
0	000
	0
1	03
	1
29	288
	261
387	2709
	2709
	0

$$\sqrt{0.00038809} = 0.0197$$

13. Question

What is that fraction which when multiplied by itself gives 227.798649?

Answer

$$a = \sqrt{227.798649} = 15.093$$

	15.093
1	227.798649
	1
25	127
	125
300	279
	0
3009	27986
	27081
30183	90549
	90549
	0

14. Question

The area of a square playground is 256.6404 square meter. Find the length of one side of the playground.

Answer

$$\text{Given: area} = L^2 = 256.6 \text{ m}^2$$

$$L = \sqrt{25.6} = 16.02m$$

	16.02
1	256.6404
	1
26	156
	156
320	064
	0
3202	6404
	6404
	0

15. Question

What is the fraction which when multiplied by it self gives 0.00053361?

Answer

$$a^2 = 0.00053361$$

$$\begin{array}{r}
 0.0231 \\
 0 \overline{) 0.00053361} \\
 \underline{0} \\
 0 \\
 \underline{0} \\
 2 \\
 \underline{005} \\
 4 \\
 \underline{43} \\
 133 \\
 \underline{129} \\
 461 \\
 \underline{461} \\
 0
 \end{array}$$

Therefore,

$$a = 0.0231$$

16. Question

Simplify:

$$(i) \frac{\sqrt{59.29} - \sqrt{5.29}}{\sqrt{59.29} + \sqrt{5.29}}$$

$$(ii) \frac{\sqrt{0.2304} + \sqrt{0.1764}}{\sqrt{0.2304} - \sqrt{0.1764}}$$

Answer

$$(i) \frac{\sqrt{59.29} - \sqrt{5.29}}{\sqrt{59.29} + \sqrt{5.29}}$$

At first, we find $\sqrt{59.29}$ and $\sqrt{5.29}$

Therefore,

$$\sqrt{59.29}$$

$$= \frac{\sqrt{5929}}{\sqrt{100}}$$

$$= \frac{77}{10} = 7.7$$

And,

$$\sqrt{5.29}$$

$$= \frac{\sqrt{529}}{\sqrt{100}}$$

$$= \frac{23}{10} = 2.3$$

Now,

$$\frac{7.7 - 2.3}{7.7 + 2.3} = 0.54$$

$$(ii) \frac{\sqrt{0.2304} + \sqrt{0.1764}}{\sqrt{0.2304} - \sqrt{0.1764}}$$

At first, we find $\sqrt{0.2304}$ and $\sqrt{0.1764}$

Therefore,

$$\sqrt{0.2304}$$

$$= \frac{\sqrt{2304}}{\sqrt{10000}}$$

$$= \frac{48}{100} = 0.48$$

And,

$$\sqrt{0.1764}$$

$$= \frac{\sqrt{0.1764}}{\sqrt{10000}}$$

$$= \frac{42}{100} = 0.42$$

Now,

$$\frac{0.48 + 0.42}{0.48 - 0.42} = 15$$

17. Question

Evaluate $\sqrt{50625}$ and hence find the value of $\sqrt{506.25} + \sqrt{5.0625}$

Answer

$$\sqrt{50625} =$$

$$\begin{array}{r} 225 \\ 2 \overline{) 50625} \\ \underline{4} \\ 42 \\ \underline{106} \\ 84 \\ \underline{2225} \\ 2225 \\ \underline{0} \end{array}$$

Now,

$$\sqrt{506.25}$$

$$= \frac{\sqrt{50625}}{\sqrt{100}}$$

$$= \frac{225}{10} = 22.5$$

$$\sqrt{5.0625}$$

$$= \frac{\sqrt{50625}}{\sqrt{10000}}$$

$$= \frac{225}{100} = 2.25$$

$$\sqrt{506.25} + \sqrt{5.0625}$$

$$= 22.5 + 2.25$$

$$= 24.75$$

18. Question

Find the value of $\sqrt{103.0225}$ and hence find the value of

(i) $\sqrt{10302.25}$

(ii) $\sqrt{1.030225}$

Answer

$$\sqrt{103.0225} =$$

	10.15
1	<u>1030225</u>
	1
20	<u>003</u>
	0
201	<u>302</u>
	201
2025	<u>10125</u>
	10125
	0

Now,

$$(i) \sqrt{10302.25}$$

v

$$= \sqrt{103.0225 \times 100}$$

$$= 10 \times 10.15$$

$$(ii) \sqrt{1.030225} = \frac{10.15}{10}$$

$$= 1.015$$

Exercise 3.8

1. Question

Find the square root of each of the following correct to three places of decimal.

$$(i) 5 \quad (ii) 7$$

$$(iii) 17 \quad (iv) 20$$

$$(v) 66 \quad (vi) 427$$

$$(vii) 1.7 \quad (viii) 23.1$$

$$(ix) 2.5 \quad (x) 237.615$$

$$(xi) 15.3215 \quad (xii) 0.9$$

$$(xiii) 0.1 \quad (xiv) 0.016$$

$$(xv) 0.00064 \quad (xvi) 0.019$$

$$(xvii) \frac{7}{8} \quad (xviii) \frac{5}{12}$$

$$(xix) 2\frac{1}{2} \quad (xx) 287\frac{5}{8}$$

Answer

$$(i) 5 = 2.236$$

	2.2360
2	<u>5.000000</u>
	4
42	<u>100</u>
	84
443	<u>1600</u>
	1329
4466	<u>27100</u>
	26796
44720	<u>30400</u>

$$= 2.236$$

$$(ii) 7 = 2.647$$

	2.6457
2	<u>7.000000</u>
	4
46	<u>300</u>
	276
524	<u>2400</u>
	2096
5285	<u>30400</u>
	26425
52927	<u>397500</u>
	370489
	27011

= 2.646

(iii) 17 = 4.123

	4.123
4	<u>17.000000</u>
	16
81	<u>1.00</u>
	81
822	<u>1900</u>
	1644
8243	<u>25600</u>
	24729
82431	<u>87100</u>
	82431
	4669

= 4.123

(iv) 20 = 4.472

	4.4721
4	<u>20.000000</u>
	16
84	<u>400</u>
	336
887	<u>6400</u>
	6209
8942	<u>19100</u>
	17884
89441	<u>121600</u>
	89441
	32159

= 4.472

(v) 66 = 8.124

	8.1240
8	<u>66.000000</u>
	64
161	<u>200</u>
	161
1622	<u>3900</u>
	3244
16244	<u>65600</u>
	64976
162480	62400

= 8.124

(vi) 427 = 20.664

	20.6639
2	427000000
	4
40	027
	0
406	2700
	2436
4126	26400
	24756
41323	164400
	123969
413269	4043100
	3719421
	323679

$$= 20.664$$

$$(vii) 1.7 = 1.304$$

	1.3038
1	1700000
	1
23	0.70
	69
260	100
	0
2603	10000
	7809
26068	219100
	208544
	10556

$$= 1.304$$

$$(viii) 23.1 = 4.806$$

	4.8062
4	23100000
	16
88	710
	704
960	600
	0
9606	60000
	57636
96122	236400
	192244
	44156

$$= 4.806$$

$$(ix) 2.5 = 1.581$$

	1.5811
1	2500000
	1
25	150
	125
308	2500
	2464
3161	3600
	3161
31621	43900
	31621
	2279

$$= 1.581$$

$$(x) 237.615 = 15.415$$

	15.4147
1	237.615000
	1
25	137
	125
304	1261
	1216
3081	4550
	3081
30824	146900
	123296
308287	2360400
	2158009
	202391

= 15.415

(xi) 15.3215 = 3.914

	3.9142
3	15321500
	9
69	632
	621
781	1115
	781
7824	33400
	31296
78282	210400
	156564
	53836

= 3.914

(xii) 0.9 = 0.949

	0.9486
0	0.900000
	0
9	090
	81
184	900
	736
1888	16400
	15104
18966	129600
	113796
	15804

= 0.949

(xiii) 0.1 = 0.316

	0.3162
0	0.100000
	0
3	10
	9
61	100
	61
626	3900
	3756
6322	14400
	12644
	1756

= 0.316

(xiv) 0.016 = 0.126

	0.1264
0	0.016000
	0
1	001
	1
22	060
	44
246	1600
	1476
2524	12400
	10096
	2304

(xv) $0.00064 = 0.025$

	0.0252
0	0.000640
	0
0	0.00
	0
2	006
	4
45	240
	225
502	1500
	1004
	496

(xvi) $0.019 = 0.138$

	0.1378
0	0.019000
	0
1	01
	1
23	090
	69
267	2100
	1869
2748	23100
	21984
	1116

$= 0.138$

(xvii) $\frac{7}{8} = 0.875$

	0.9354
0	0.875000
	0
9	087
	81
183	650
	549
1865	10100
	9325
18704	77500
	74816
	2684

$= 0.875$

(xviii) $\frac{5}{12} = 0.416$

	0.6454
0	0.416666
	0
6	41
	36
124	566
	496
1285	7066
	6245
12904	64100
	51616
	12484

$$= 0.645$$

$$(xix) 2\frac{1}{2} = 2.500000$$

	1.5811
1	<u>2.500000</u>
	1
25	<u>150</u>
	125
308	<u>2500</u>
	2464
3161	<u>3600</u>
	3161
31621	<u>43900</u>
	31621
	12279

$$(xx) 287\frac{5}{8} = 287.62$$

	16.9593
1	<u>287.62</u>
	1
26	<u>187</u>
	156
329	<u>3162</u>
	2961
3385	<u>20100</u>
	16925
33909	<u>317500</u>
	305181
339183	<u>1231900</u>
	1017549
	214351

$$\text{Hence, } \sqrt{287.62} = 16.980$$

2. Question

Find the square root of 12.0068 correct to four decimal places.

Answer

The square root of 12.0068 is:

	3.46508
3	<u>12.0068</u>
	9
64	<u>300</u>
	256
686	<u>4468</u>
	4116
6925	<u>35200</u>
	34625
693008	<u>5750000</u>
	5544064
	205936

$$\text{Hence, } \sqrt{12.0064} = 3.4651 \text{ approx.}$$

Hence,

$$\sqrt{12.0068} = 3.4651 \text{ approx}$$

3. Question

Find the square root of 11 correct to five decimal places.

Answer

The square root of 11 is:

	3.316624
3	<u>11.000000</u>
	9
63	<u>200</u>
	189
661	<u>1100</u>
	661
6626	<u>43900</u>
	39756
66326	<u>414400</u>
	398196
663322	<u>1620400</u>
	1327444
6633244	<u>29295600</u>
	26532976
	2762624

Hence,

$$\sqrt{11} = 3.31662$$

4. Question

Give that: $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{7} = 2.646$, evaluate each of the following:

(i) $\sqrt{\frac{144}{7}}$

(ii) $\sqrt{\frac{2500}{3}}$

Answer

(i) $\sqrt{\frac{144}{7}} = \frac{\sqrt{12 \times 12}}{\sqrt{7}}$

$$= \frac{12}{2.646}$$

$$= 4.535$$

(ii) $\sqrt{\frac{2500}{3}} = \frac{\sqrt{5 \times 5 \times 10 \times 10}}{\sqrt{3}}$

$$= \frac{5 \times 10}{\sqrt{3}}$$

$$= \frac{50}{1.732}$$

$$= 28.867$$

5. Question

Given that $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$ and $\sqrt{7} = 2.646$, find the square roots of the following:

(i) $\frac{169}{75}$

(ii) $\frac{400}{63}$

(iii) $\frac{150}{7}$

(iv) $\frac{256}{5}$

(v) $\frac{276}{50}$

Answer

$$(i) \frac{169}{75}$$

$$\frac{169}{75} = \sqrt{\frac{169}{75}}$$

$$= \frac{\sqrt{13} * \sqrt{13}}{\sqrt{5} * \sqrt{5} * \sqrt{3}}$$

$$= \frac{13}{5\sqrt{3}}$$

$$= \frac{13}{5 (1.732)}$$

$$= \frac{13}{8.66}$$

$$= 1.50$$

$$(ii) \frac{400}{63}$$

$$\frac{400}{63} = \sqrt{\frac{400}{63}}$$

$$= \frac{\sqrt{2} * \sqrt{2 * 10 * 10}}{\sqrt{3} * \sqrt{3} * \sqrt{7}}$$

$$= \frac{2 * 10}{3\sqrt{7}}$$

$$= \frac{20}{3 (2.646)}$$

$$= \frac{20}{7.938}$$

$$= 2.519$$

$$(iii) \frac{150}{7}$$

$$\frac{150}{7} = \sqrt{\frac{150}{7}}$$

$$= \frac{\sqrt{3 * 5 * 5 * 2}}{\sqrt{7}}$$

$$= \frac{5 \sqrt{3} * \sqrt{2}}{\sqrt{7}}$$

$$= \frac{5 * 1.731 * 1.414}{2.646}$$

$$= \frac{12.24524}{2.646}$$

$$= 4.627$$

$$(iv) \frac{256}{5}$$

$$\frac{256}{5} = \sqrt{\frac{256}{5}}$$

$$= \frac{\sqrt{16} * \sqrt{16}}{\sqrt{5}}$$

$$= \frac{16}{\sqrt{5}}$$

$$= \frac{16}{2.236}$$

$$= 7.155$$

$$(v) \frac{276}{50}$$

$$\frac{276}{50} = \sqrt{\frac{276}{50}}$$

$$= \frac{\sqrt{2 \times 2 \times 3 \times 23}}{\sqrt{5 \times 5 \times 2}}$$

$$= \frac{2 \times \sqrt{3 \times 23}}{5\sqrt{2}}$$

$$= \frac{2 \times 1.732 \times 4.796}{5 (1.414)}$$

$$= 0.735$$

Exercise 3.9

1. Question

Using square root table, find the square roots of the following:

7

Answer

From square root table,

Square root of 7 is:

$$\sqrt{7} = 2.645$$

Therefore,

The square root of 7 is 2.645

2. Question

Using square root table, find the square roots of the following:

15

Answer

From square root table,

Square root of 15 is:

$$\sqrt{15} = 3.872$$

Therefore,

The square root of 15 is 3.872

3. Question

Using square root table, find the square roots of the following:

74

Answer

From square root table,

Square root of 74 is:

$$\sqrt{74} = 8.602$$

Therefore,

The square root of 74 is 8.602

4. Question

Using square root table, find the square roots of the following:

82

Answer

From square root table,

Square root of 82 is:

$$\sqrt{82} = 9.055$$

Therefore,

The square root of 82 is 9.055

5. Question

Using square root table, find the square roots of the following:

198

Answer

From square root table,

Square root of 198 is:

$$\sqrt{198} = 14.071$$

Therefore,

The square root of 198 is 14.071

6. Question

Using square root table, find the square roots of the following:

540

Answer

From square root table,

Square root of 540 is:

$$\sqrt{540} = 23.237$$

Therefore,

The square root of 540 is 23.237

7. Question

Using square root table, find the square roots of the following:

8700

Answer

From square root table,

Square root of 8700 is:

$$\sqrt{8700} = 93.237$$

Therefore,

The square root of 8700 is 93.237

8. Question

Using square root table, find the square roots of the following:

3509

Answer

From square root table,

Square root of 3509 is:

$$\sqrt{3509} = 59.236$$

Therefore,

The square root of 3509 is 59.236

9. Question

Using square root table, find the square roots of the following:

6929

Answer

From square root table,

Square root of 6929 is:

$$\sqrt{6929} = 83.240$$

Therefore,

The square root of 6929 is 83.240

10. Question

Using square root table, find the square roots of the following:

25720

Answer

From square root table,

Square root of 25720 is:

$$\sqrt{25720} = 160.374$$

Therefore,

The square root of 25720 is 160.374

11. Question

Using square root table, find the square roots of the following:

1312

Answer

From square root table,

Square root of 1312 is:

$$\sqrt{1312} = 36.221$$

Therefore,

The square root of 1312 is 36.221

12. Question

Using square root table, find the square roots of the following:

4192

Answer

From square root table,

Square root of 4192 is:

$$\sqrt{4192} = 64.745$$

Therefore,

The square root of 4192 is 64.745

13. Question

Using square root table, find the square roots of the following:

49555

Answer

From square root table,

Square root of 49555 is:

$$\sqrt{49555} = 222.609$$

Therefore,

The square root of 49555 is 222.609

14. Question

Using square root table, find the square roots of the following:

$$\frac{99}{144}$$

Answer

From square root table,

Square root of $\frac{99}{144}$ is:

$$\sqrt{\frac{99}{144}} = 0.829$$

Therefore,

The square root of $\frac{99}{144}$ is 0.829

15. Question

Using square root table, find the square roots of the following:

$$\frac{57}{169}$$

Answer

From square root table,

Square root of $\frac{57}{169}$ is:

$$\sqrt{\frac{57}{169}} = 0.580$$

Therefore,

The square root of $\frac{57}{169}$ is 0.580

16. Question

Using square root table, find the square roots of the following:

$$\frac{101}{169}$$

Answer

From square root table,

Square root of $\frac{101}{169}$ is:

$$\sqrt{\frac{101}{169}} = 0.773$$

Therefore,

The square root of $\frac{101}{169}$ is 0.773

17. Question

Using square root table, find the square roots of the following:

$$13.21$$

Answer

From square root table,

Square root of 13.21 is:

$$\sqrt{13.21} = 3.634$$

Therefore,

The square root of 13.21 is 3.634

18. Question

Using square root table, find the square roots of the following:

$$21.97$$

Answer

From square root table,

Square root of 21.97 is:

$$\sqrt{21.97} = 4.687$$

Therefore,

The square root of 21.97 is 4.687

19. Question

Using square root table, find the square roots of the following:

110

Answer

From square root table,

Square root of 110 is:

$$\sqrt{110} = 10.488$$

Therefore,

The square root of 110 is 10.488

20. Question

Using square root table, find the square roots of the following:

1110

Answer

From square root table,

Square root of 1110 is:

$$\sqrt{1110} = 33.316$$

Therefore,

The square root of 1110 is 33.316

21. Question

Using square root table, find the square roots of the following:

11.11

Answer

From square root table,

Square root of 11.11 is:

$$\sqrt{11.11} = 3.333$$

Therefore,

The square root of 11.11 is 3.333

22. Question

The area of a square field is 325m^2 . Find the approximate length of one side of the field.

Answer

Area of the field = 325 m^2

In order to find approximate length of the side of the field we will have to calculate the square root of 325

$$\sqrt{325} = 18.027\text{ m}$$

Hence,

The approximate length of one side of the field is 18.027 m

23. Question

Find the length of a side of a square, whose area is equal to the area of a rectangle with sides 240 m and 70 m.

Answer

According to the question,

Area of square = Area of rectangle

$$\text{Side}^2 = 240 \times 70$$

$$\text{Side} = \sqrt{240 \times 70}$$

$$= \sqrt{10 \times 10 \times 2 \times 2 \times 2 \times 3 \times 7}$$

$$= 20\sqrt{42}$$

$$= 20 \times 6.48$$

$$= 129.60 \text{ m}$$