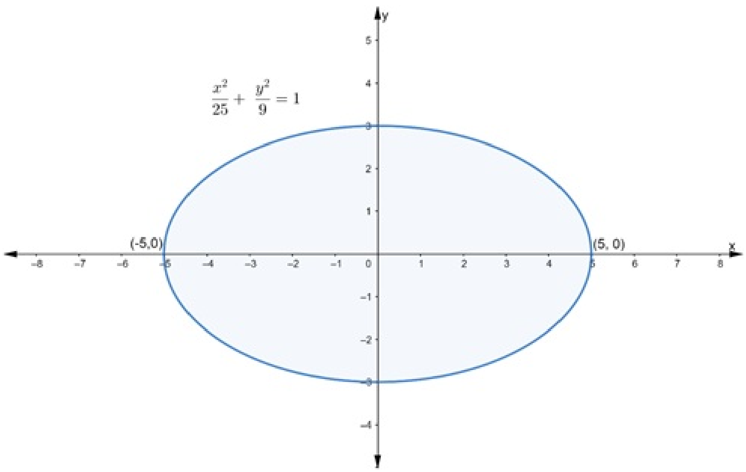
Chapter : 23. ELLIPSE

Exercise : 23

Question: 1

Find the (i) leng

Solution:



Given:

$\frac{x^2}{25} + \frac{y^2}{9} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 25 > 9

So, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 9$

⇒ $a = \sqrt{25}$ and $b = \sqrt{9}$

⇒ a = 5 and b = 3

(i) <u>To find</u>: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 5

= 10 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (5, 0) and (-5, 0)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = $(\pm c, 0)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

$= 25 - 9$

$c^2 = 16$

$c = \sqrt{16}$

$c = 4$ ...(I)

∴ Coordinates of foci = $(\pm 4, 0)$

(iv) To find: Eccentricity

We know that,

$\text{Eccentricity} = \frac{c}{a}$

$\Rightarrow e = \frac{4}{5}$ [from (I)]

(v) To find: Length of the Latus Rectum

We know that,

$\text{Length of Latus Rectum} = \frac{2b^2}{a}$

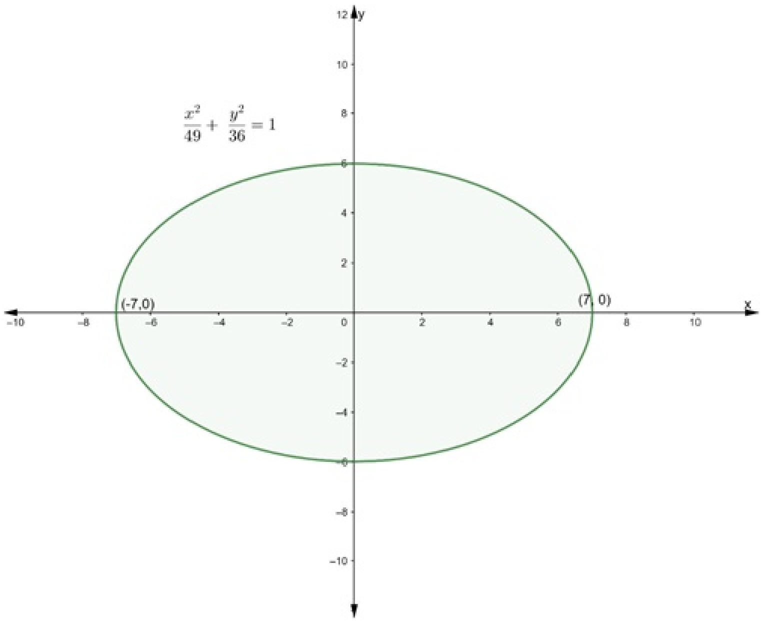
$= \frac{2 \times (3)^2}{5}$

$= \frac{18}{5}$

Question: 2

Find the (i) leng

Solution:



Given:

$\frac{x^2}{49} + \frac{y^2}{36} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 49 > 36

So, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 49$ and $b^2 = 36$

⇒ a = √49 and b = √36

⇒ a = 7 and b = 6

(i) To find: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 7

= 14 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (7, 0) and (-7, 0)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 49 – 36

$c^2 = 13$

c = √13 ...(I)

∴ Coordinates of foci = (±√13, 0)

(iv) To find: Eccentricity

We know that,

$\text{Eccentricity} = \frac{c}{a}$

$\Rightarrow e = \frac{\sqrt{13}}{7}$ [from (I)]

(v) To find: Length of the Latus Rectum

We know that,

Length of Latus Rectum $= \frac{2b^2}{a}$

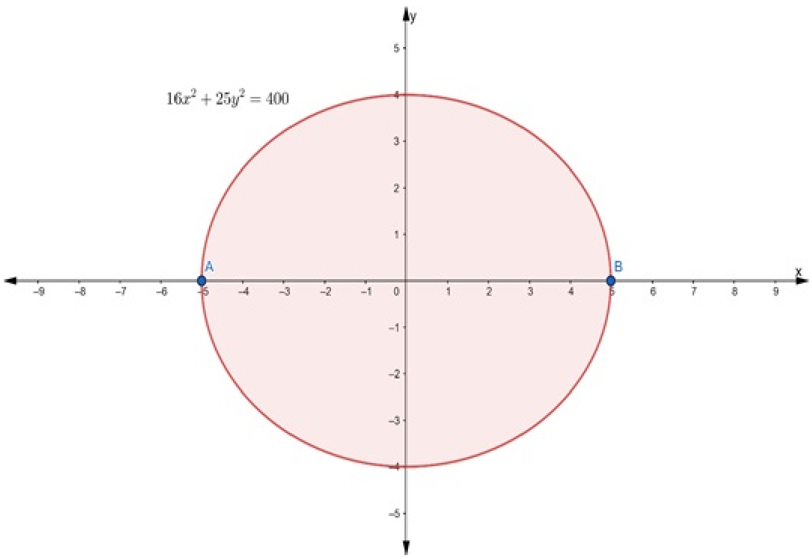
$= \frac{2 \times (6)^2}{7}$

$= \frac{72}{7}$

Question: 3

Find the (i) leng

Solution:



Given:

$16x^2 + 25y^2 = 400$

Divide by 400 to both the sides, we get

$\frac{16}{400}x^2 + \frac{25}{400}y^2 = 1$

$\frac{x^2}{25} + \frac{y^2}{4} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 25 > 4

So, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 4$

$\Rightarrow a = \sqrt{25}$ and $b = \sqrt{4}$

$\Rightarrow a = 5$ and $b = 2$

(i) To find: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 5

= 10 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (5, 0) and (-5, 0)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 25 – 4

$c^2 = 21$

c = √21 …(I)

∴ Coordinates of foci = (±√21, 0)

(iv) To find: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{21}}{5} \text{ [from (I)]}$$

(v) To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

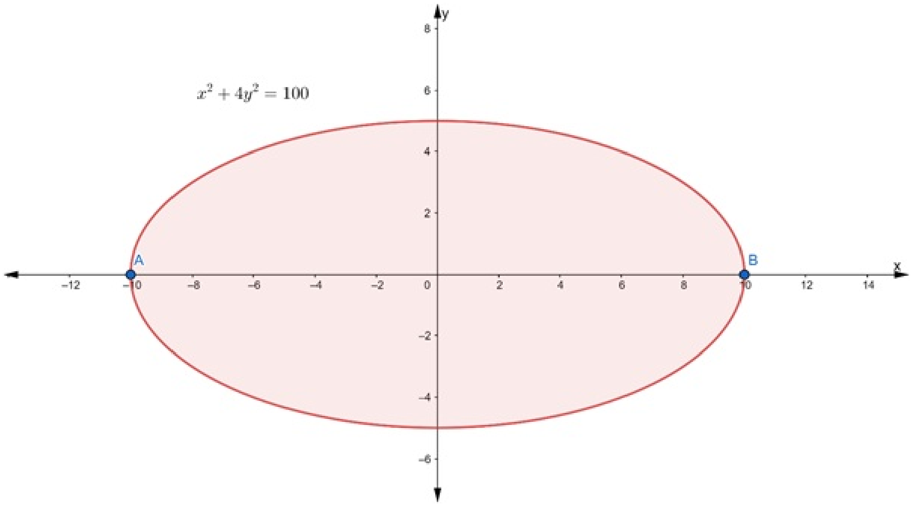
$$= \frac{2 \times (4)^2}{5}$$

$$= \frac{32}{5}$$

Question: 4

Find the (i) leng

Solution:



Given:

$x^2 + 4y^2 = 100$

Divide by 100 to both the sides, we get

$$\frac{1}{100}x^2 + \frac{4}{100}y^2 = 1$$

$$\frac{x^2}{100} + \frac{y^2}{25} = 1 \text{ ...(i)}$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 100 > 25

So, above equation is of the form,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ...(ii)}$$

Comparing eq. (i) and (ii), we get

$a^2 = 100$ and $b^2 = 25$

⇒ $a = \sqrt{100}$ and $b = \sqrt{25}$

⇒ a = 10 and b = 5

(i) To find: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 10

= 20 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (10, 0) and (-10, 0)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 100 – 25

$c^2 = 75$

$c = \sqrt{75}$

$c = 5\sqrt{3}$ ...(I)

∴ Coordinates of foci = $(\pm 5\sqrt{3}, 0)$

(iv) To find: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2} \text{ [from (I)]}$$

(v) To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

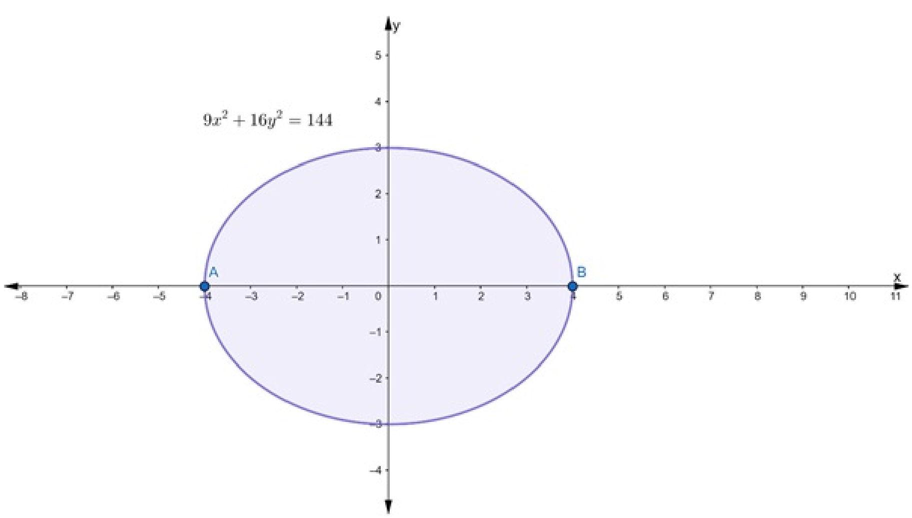
$$= \frac{2 \times (4)^2}{5}$$

$$= \frac{32}{5}$$

Question: 5

Find the (i) leng

Solution:



Given:

$9x^2 + 16y^2 = 144$

Divide by 144 to both the sides, we get

$$\frac{9}{144}x^2 + \frac{16}{144}y^2 = 1$$

$\frac{x^2}{16}+\frac{y^2}{9}=1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	(± c, 0)	(± a, 0)	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	(0,± c)	(0,± a)	2a	2b	$e=\frac{c}{a}$	$\frac{2b^2}{a}$

Since, 16 > 9

So, above equation is of the form,

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 16$ and $b^2 = 9$

⇒ a = √16 and b = √9

⇒ a = 4 and b = 3

(i) To find: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

= 2 × 4

= 8 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

= (4, 0) and (-4, 0)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 16 – 9

$c^2 = 7$

c = √7 ...(I)

∴ Coordinates of foci = (±√7, 0)

(iv) To find: Eccentricity

We know that,

$\text{Eccentricity} = \frac{c}{a}$

$\Rightarrow e = \frac{\sqrt{7}}{4}$ [from (I)]

(v) To find: Length of the Latus Rectum

We know that,

Length of Latus Rectum $= \frac{2b^2}{a}$

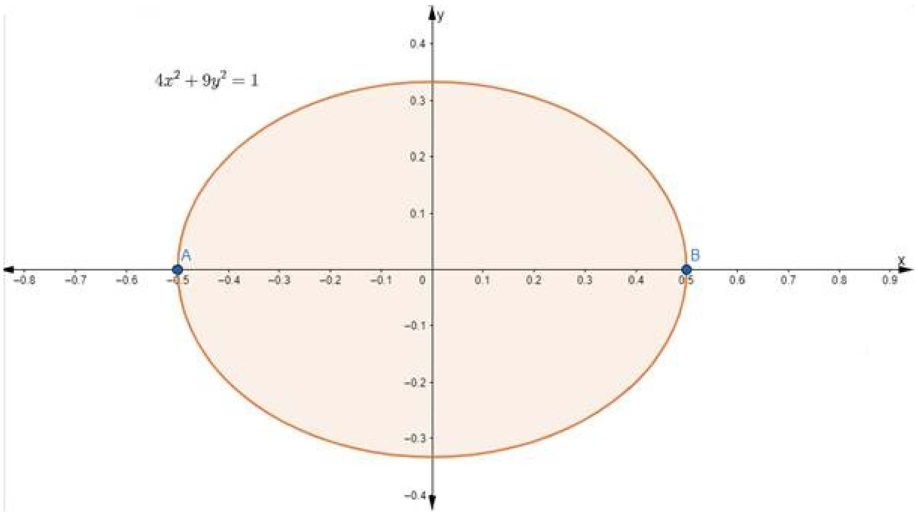
$= \frac{2 \times (3)^2}{4}$

$= \frac{9}{2}$

Question: 6

Find the (i) leng

Solution:



Given:

$4x^2 + 9y^2 = 1$

$\frac{x^2}{\frac{1}{4}} + \frac{y^2}{\frac{1}{9}} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, $\frac{1}{4} > \frac{1}{9}$

So, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{9}$

$\Rightarrow a = \sqrt{\frac{1}{4}}$ and $b = \sqrt{\frac{1}{9}}$

$\Rightarrow a = \frac{1}{2}$ and $b = \frac{1}{3}$

(i) To find: Length of major axes

Clearly, a > b, therefore the major axes of the ellipse is along x axes.

∴Length of major axes = 2a

$= 2 \times \frac{1}{2}$

= 1 unit

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (a, 0) and (-a, 0)

$= \left(\frac{1}{2}, 0\right) \text{and} \left(-\frac{1}{2}, 0\right)$

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = (±c, 0) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

$= \frac{1}{4} - \frac{1}{9}$

$c^2 = \frac{9-4}{36}$

$c^2 = \frac{5}{36}$

$c = \frac{\sqrt{5}}{6}$ ...(I)

$\therefore$ Coordinates of foci $= \left(\pm\frac{\sqrt{5}}{6}, 0\right)$

(iv) To find: Eccentricity

We know that,

Eccentricity $= \frac{c}{a}$

$\Rightarrow e = \frac{\frac{\sqrt{5}}{6}}{\frac{1}{2}} = \frac{\sqrt{5}}{6} \times 2 = \frac{\sqrt{5}}{3}$ [from (I)]

(v) To find: Length of the Latus Rectum

We know that,

Length of Latus Rectum $= \frac{2b^2}{a}$

$= \frac{2 \times \left(\frac{1}{3}\right)^2}{\frac{1}{2}}$

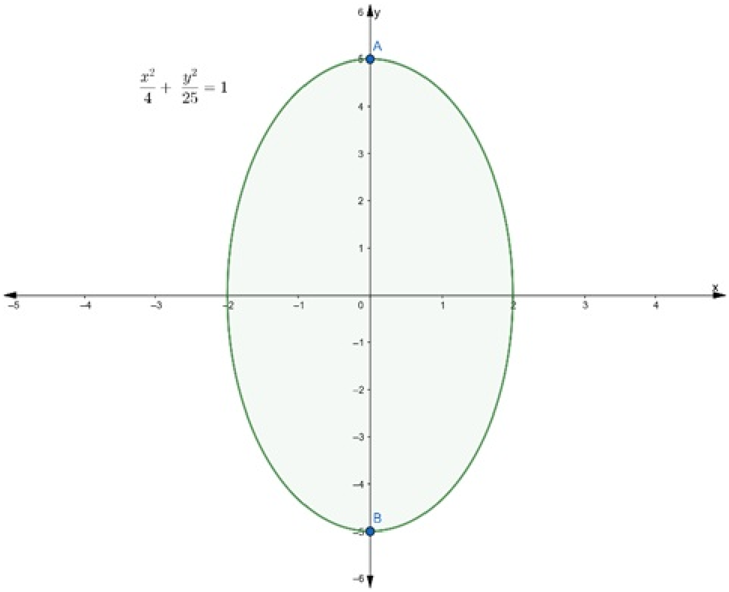
$= \frac{\frac{2}{9}}{\frac{1}{2}}$

$= \frac{2}{9} \times 2$

Question: 7

Find the (i) leng

Solution:



Given:

$\frac{x^2}{4}+\frac{y^2}{25}=1$ …(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 4 < 25

So, above equation is of the form,

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ …(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 4$

⇒ $a = \sqrt{25}$ and $b = \sqrt{4}$

⇒ a = 5 and b = 2

(i) <u>To find</u>: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 5

= 10 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 5) and (0, -5)

(iii) <u>To find</u>: Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 25 – 4

$c^2 = 21$

$c = \sqrt{21}$ ...(I)

∴ Coordinates of foci = $(0, \pm\sqrt{21})$

(iv) To find: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{21}}{5} \text{ [from (I)]}$$

(v) To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

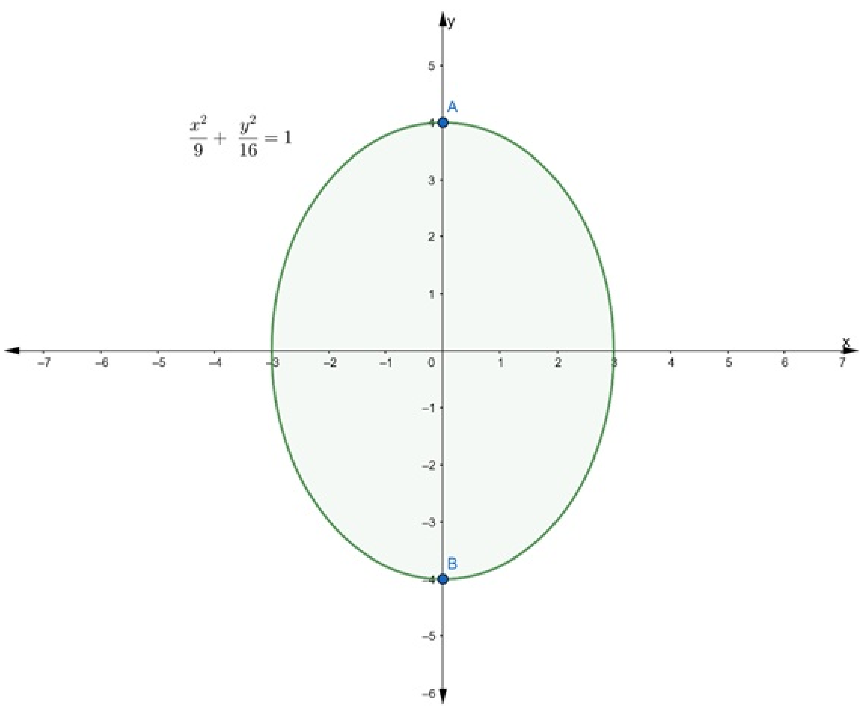
$$= \frac{2 \times (2)^2}{5}$$

$$= \frac{8}{5}$$

Question: 8

Find the (i) leng

Solution:



Given:

$\frac{x^2}{9} + \frac{y^2}{16} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 9 < 16

So, above equation is of the form,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 16$ and $b^2 = 9$

⇒ $a = \sqrt{16}$ and $b = \sqrt{9}$

⇒ a = 4 and b = 3

(i) To find: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 4

= 8 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 4) and (0, -4)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = $(0, \pm c)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 16 – 9

$c^2 = 7$

$c = \sqrt{7}$ ...(I)

∴ Coordinates of foci = $(0, \pm\sqrt{7})$

(iv) To find: Eccentricity

We know that,

$\text{Eccentricity} = \frac{c}{a}$

⇒ $e = \frac{\sqrt{7}}{4}$ [from (I)]

(v) To find: Length of the Latus Rectum

We know that,

Length of Latus Rectum $= \frac{2b^2}{a}$

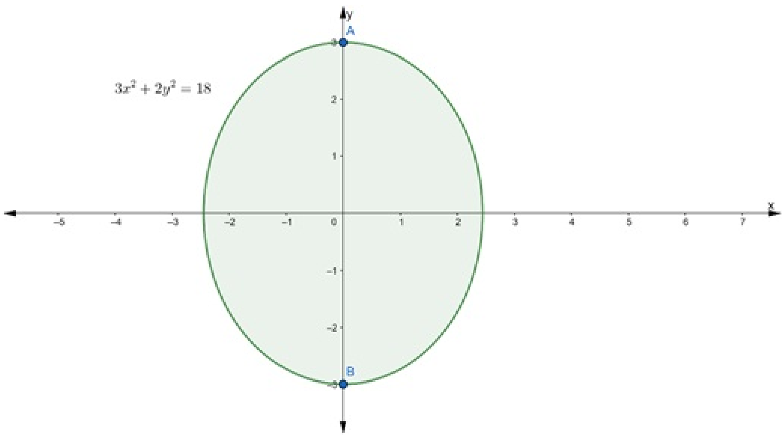
$= \frac{2 \times (3)^2}{4}$

$= \frac{9}{2}$

Question: 9

Find the (i) leng

Solution:



Given:

$3x^2 + 2y^2 = 18$

Divide by 18 to both the sides, we get

$\frac{3}{18}x^2 + \frac{2}{18}y^2 = 1$

$\frac{x^2}{6} + \frac{y^2}{9} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 6 < 9

So, above equation is of the form,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 9$ and $b^2 = 6$

$\Rightarrow a = \sqrt{9}$ and $b = \sqrt{6}$

$\Rightarrow a = 3$ and $b = \sqrt{6}$

(i) To find: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

$\therefore$Length of major axes = 2a

$= 2 \times 3$

= 6 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 6) and (0, -6)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 9 – 6

$c^2 = 3$

c = √3 …(I)

∴ Coordinates of foci = (0, ±√3)

(iv) To find: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$\Rightarrow e = \frac{\sqrt{3}}{3}$ [from (I)]

(v) To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times \left(\sqrt{6}\right)^2}{3}$$

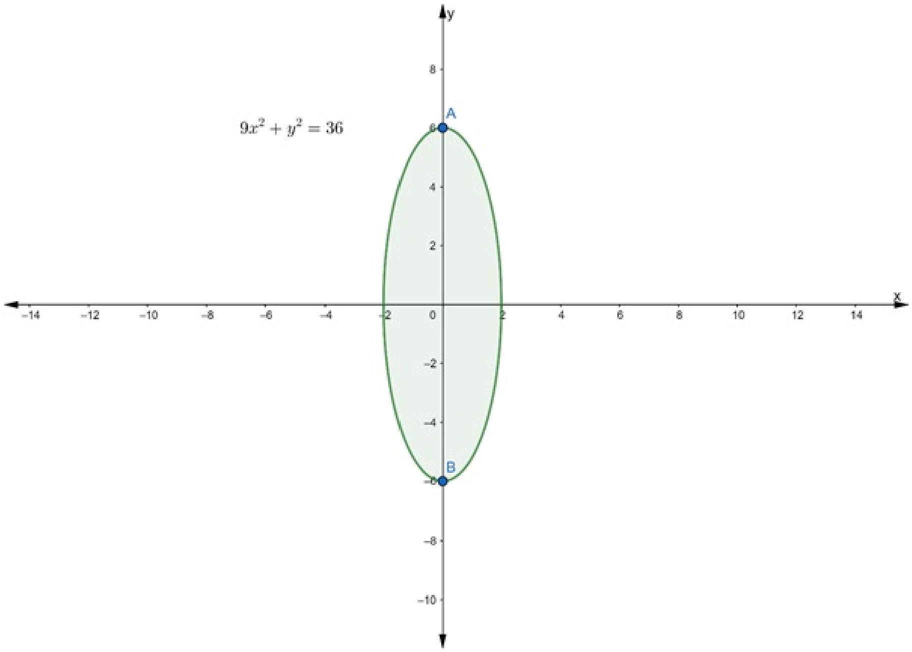
$$= \frac{2 \times 6}{3}$$

= 4

Question: 10

Find the (i) leng

Solution:



Given:

$9x^2 + y^2 = 36$

Divide by 36 to both the sides, we get

$$\frac{9}{36}x^2 + \frac{1}{36}y^2 = 1$$

$$\frac{x^2}{4} + \frac{y^2}{36} = 1 \text{ ...(i)}$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 4 < 36

So, above equation is of the form,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \text{ ...(ii)}$$

Comparing eq. (i) and (ii), we get

$a^2 = 36$ and $b^2 = 4$

⇒ $a = \sqrt{36}$ and $b = \sqrt{4}$

⇒ a = 6 and b = 2

(i) <u>To find</u>: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 6

= 12 units

(ii) <u>To find</u>: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 6) and (0, -6)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = $(0, \pm c)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

$= 36 - 4$

$c^2 = 32$

$c = \sqrt{32}$ ...(I)

∴ Coordinates of foci = $(0, \pm\sqrt{32})$

(iv) To find: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{32}}{6} \text{ [from (I)]}$$

(v) To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (2)^2}{6}$$

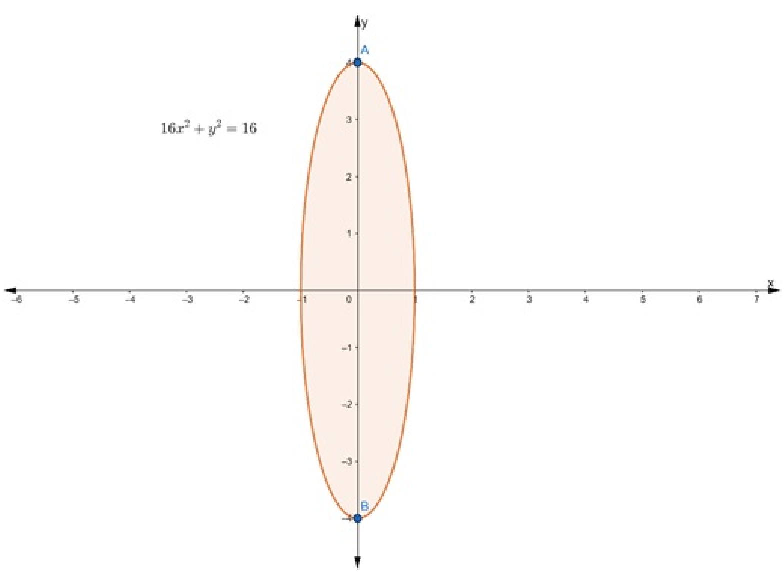
$$= \frac{8}{6}$$

$$= \frac{4}{3}$$

Question: 11

Find the (i) leng

Solution:



Given:

$16x^2 + y^2 = 16$

Divide by 16 to both the sides, we get

$\frac{16}{16}x^2 + \frac{1}{16}y^2 = 1$

$\frac{x^2}{1} + \frac{y^2}{16} = 1$ ...(i)

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 1 < 16

So, above equation is of the form,

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 16$ and $b^2 = 1$

⇒ a = √16 and b = √1

⇒ a = 4 and b = 1

(i) To find: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

= 2 × 4

= 8 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 4) and (0, -4)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = (0, ±c) where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

= 16 – 1

$c^2 = 15$

c = √15 ...(I)

∴ Coordinates of foci = (0, ±√15)

(iv) To find: Eccentricity

We know that,

$\text{Eccentricity} = \frac{c}{a}$

$\Rightarrow e = \frac{\sqrt{15}}{4}$ [from (I)]

(v) To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times (1)^2}{4}$$

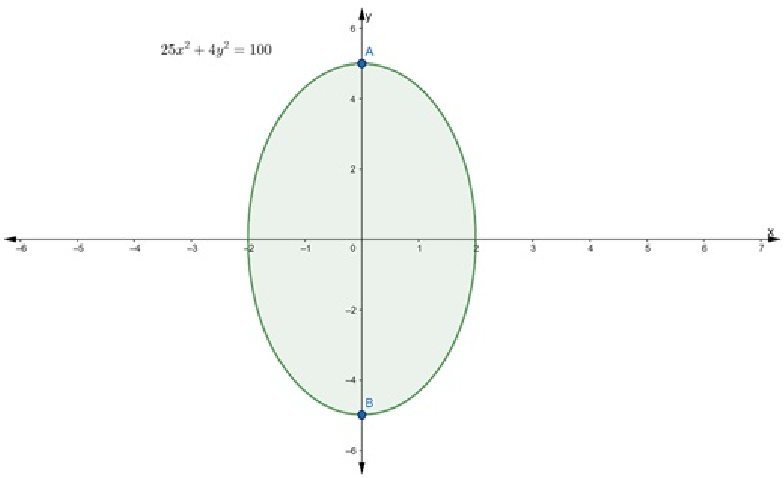
$$= \frac{2 \times 1}{4}$$

$$= \frac{1}{2}$$

Question: 12

Find the (i) leng

Solution:



Given:

$25x^2 + 4y^2 = 100$

Divide by 100 to both the sides, we get

$$\frac{25}{100}x^2 + \frac{4}{100}y^2 = 1$$

$$\frac{x^2}{4} + \frac{y^2}{25} = 1 \text{ ...(i)}$$

Equation	Major Axis		Coordinates of foci	Vertices	Major Axis	Minor Axis	Eccentricity	Latus Rectum
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	x-axis	$c^2 = a^2 - b^2$	$(\pm c, 0)$	$(\pm a, 0)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$
$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$	y-axis	$c^2 = a^2 - b^2$	$(0, \pm c)$	$(0, \pm a)$	2a	2b	$e = \frac{c}{a}$	$\frac{2b^2}{a}$

Since, 4 < 25

So, above equation is of the form,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \text{ ...(ii)}$$

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 4$

⇒ $a = \sqrt{25}$ and $b = \sqrt{4}$

⇒ a = 5 and b = 2

(i) To find: Length of major axes

Clearly, a < b, therefore the major axes of the ellipse is along y axes.

∴Length of major axes = 2a

$= 2 \times 5$

= 10 units

(ii) To find: Coordinates of the Vertices

Clearly, a > b

∴ Coordinate of vertices = (0, a) and (0, -a)

= (0, 5) and (0, -5)

(iii) To find: Coordinates of the foci

We know that,

Coordinates of foci = $(0, \pm c)$ where $c^2 = a^2 - b^2$

So, firstly we find the value of c

$c^2 = a^2 - b^2$

$= 25 - 4$

$c^2 = 21$

$c = \sqrt{21}$ ...(I)

∴ Coordinates of foci = $(0, \pm\sqrt{21})$

(iv) To find: Eccentricity

We know that,

$$\text{Eccentricity} = \frac{c}{a}$$

$$\Rightarrow e = \frac{\sqrt{21}}{5} \text{ [from (I)]}$$

(v) To find: Length of the Latus Rectum

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

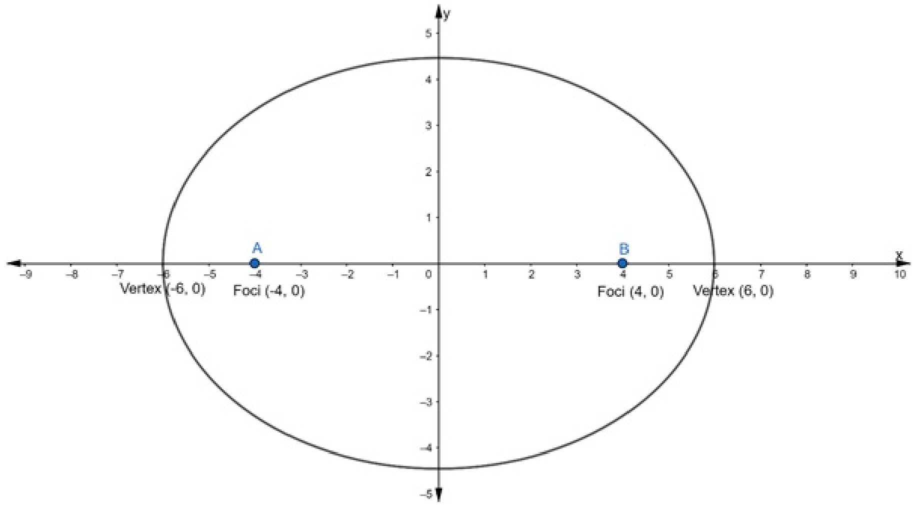
$$= \frac{2 \times (2)^2}{5}$$

$$= \frac{8}{5}$$

Question: 13

Find the equation

Solution:



Given: Vertices = (±6, 0) …(i)

The vertices are of the form = (±a, 0) …(ii)

Hence, the major axis is along x – axis

∴ From eq. (i) and (ii), we get

a = 6

⇒ a^2 = 36

and We know that, if the major axis is along x – axis then the equation of Ellipse is of the form of

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Also, given coordinate of foci = (±4, 0) …(iii)

We know that,

Coordinates of foci = (±c, 0) …(iv)

∴ From eq. (iii) and (iv), we get

c = 4

We know that,

$c^2 = a^2 – b^2$

⇒ $(4)^2 = (6)^2 – b^2$

⇒ $16 = 36 – b^2$

⇒ $b^2 = 36 – 16$

⇒ $b^2 = 20$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

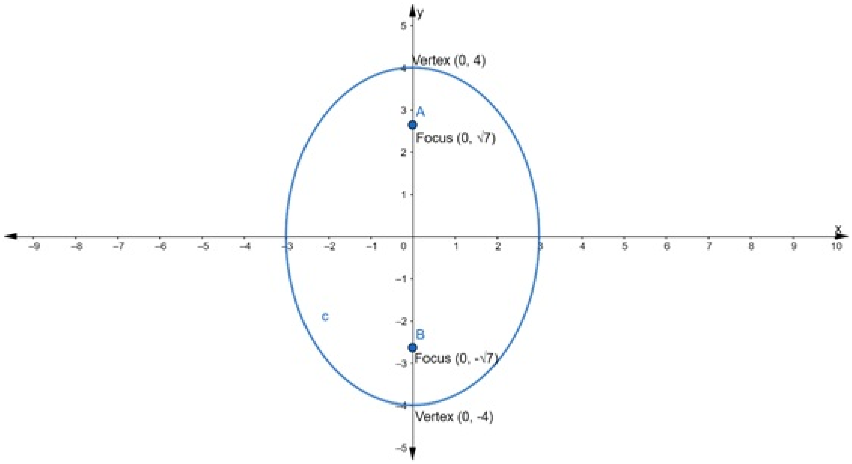
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Rightarrow \frac{x^2}{36}+\frac{y^2}{20}=1$$

Question: 14

Find the equation

Solution:



Given: Vertices = (0, ±4) …(i)

The vertices are of the form = (0, ±a) …(ii)

Hence, the major axis is along y – axis

∴ From eq. (i) and (ii), we get

a = 4

⇒ a^2 = 16

and We know that, if the major axis is along y – axis then the equation of Ellipse is of the form of

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Also, given coordinate of foci = (0, ±√7) …(iii)

We know that,

Coordinates of foci = (0, ±c) …(iv)

∴ From eq. (iii) and (iv), we get

c = √7

We know that,

$c^2 = a^2 - b^2$

⇒ $(\sqrt{7})^2 = (4)^2 - b^2$

⇒ $7 = 16 - b^2$

⇒ $b^2 = 16 - 7$

⇒ $b^2 = 9$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$$

Question: 15

Find the equation

Solution:

Given:

Ends of Major Axis = (±4, 0)

and Ends of Minor Axis = (0, ±3)

Here, we can see that the major axis is along the x – axis.

∴ The Equation of Ellipse is of the form,

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

where, a is the semi – major axis and b is the semi – minor axis.

Accordingly, a = 4 and b = 3

Substituting the value of a and b in eq. (i), we get

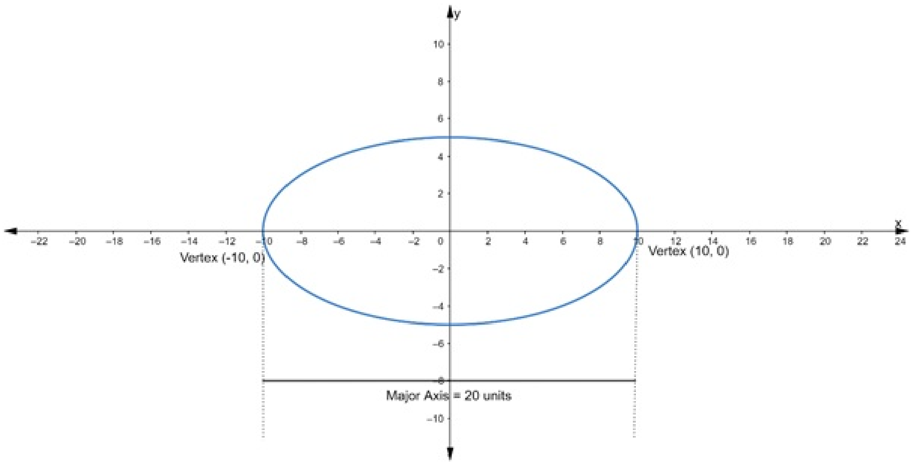
$$\frac{x^2}{(4)^2}+\frac{y^2}{(3)^2}=1$$

$$\Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$$

Question: 16

The length of the

Solution:



Let the equation of the required ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Given: Length of Major Axis = 20units ...(i)

We know that,

Length of Major Axis = 2a ...(ii)

∴ From eq. (i) and (ii), we get

2a = 20

⇒ a = 10

It is also given that,

Coordinates of foci = (±5√3, 0) ...(iii)

We know that,

Coordinates of foci = (±c, 0) ...(iv)

∴ From eq. (iii) and (iv), we get

c = 5√3

We know that,

$c^2 = a^2 - b^2$

$\Rightarrow (5\sqrt{3})^2 = (10)^2 - b^2$

$\Rightarrow 75 = 100 - b^2$

$\Rightarrow b^2 = 100 - 75$

$\Rightarrow b^2 = 25$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

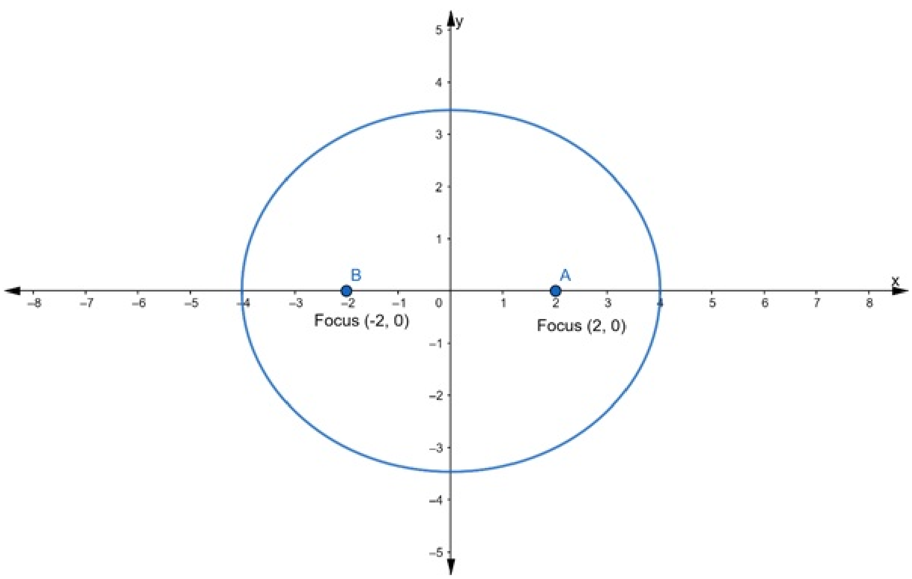
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{100} + \frac{y^2}{25} = 1$$

Question: 17

Find the equation

Solution:



Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given:

Coordinates of foci = (±2, 0) …(iii)

We know that,

Coordinates of foci = (±c, 0) …(iv)

∴ From eq. (iii) and (iv), we get

$c = 2$

It is also given that

$$\text{Eccentricity} = \frac{1}{2}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$\Rightarrow \frac{1}{2} = \frac{2}{a}$ [$\because$ c = 2]

$\Rightarrow$ a = 4

Now, we know that,

$c^2 = a^2 - b^2$

$\Rightarrow (2)^2 = (4)^2 - b^2$

$\Rightarrow 4 = 16 - b^2$

$\Rightarrow b^2 = 16 - 4$

$\Rightarrow b^2 = 12$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

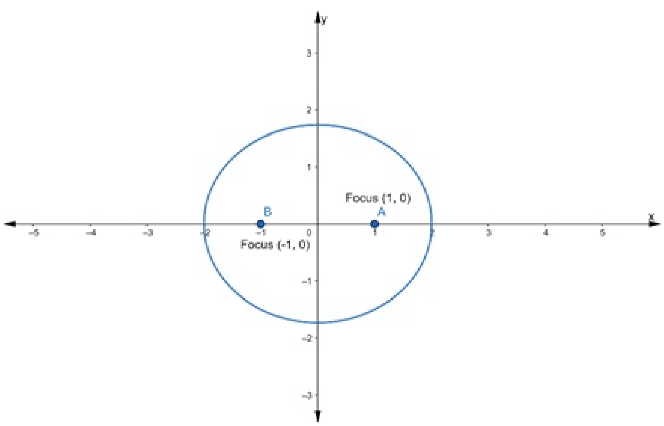
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{12} = 1$$

Question: 18

Find the equation

Solution:



Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Given:

Coordinates of foci = (±1, 0) ...(i)

We know that,

Coordinates of foci = (±c, 0) ...(ii)

$\therefore$ From eq. (i) and (ii), we get

c = 1

It is also given that

$$\text{Eccentricity} = \frac{1}{2}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$\Rightarrow \frac{1}{2} = \frac{1}{a}$ [$\because$ c = 1]

$\Rightarrow$ a = 2

Now, we know that,

$c^2 = a^2 - b^2$

$\Rightarrow (1)^2 = (2)^2 - b^2$

$\Rightarrow 1 = 4 - b^2$

$\Rightarrow b^2 = 4 - 1$

$\Rightarrow b^2 = 3$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

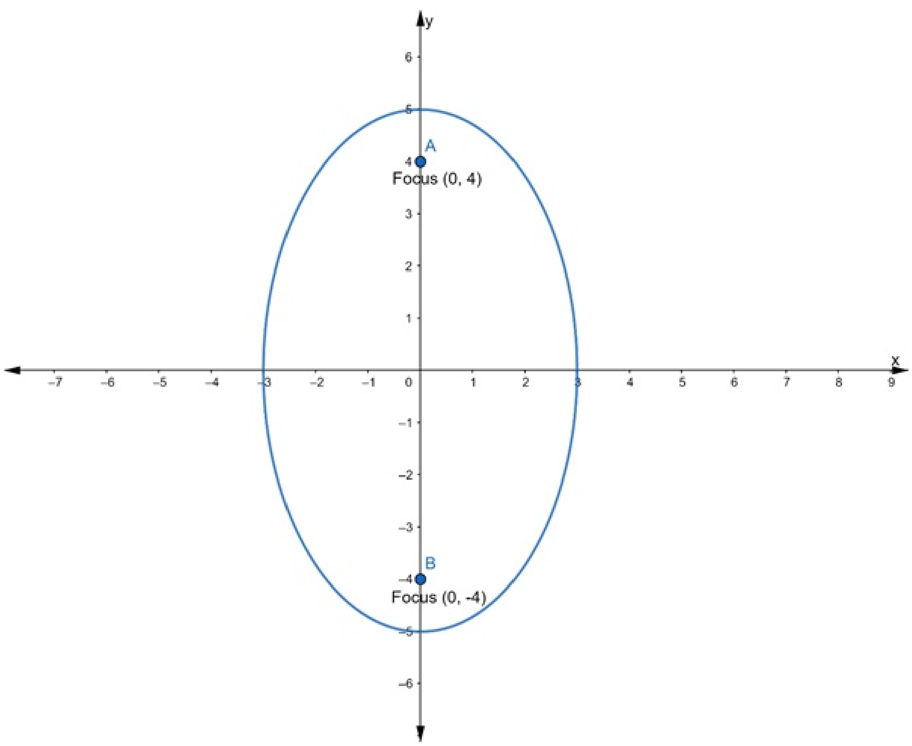
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$$

Question: 19

Find the equation

Solution:



Given:

Coordinates of foci = (0, ±4) ...(i)

We know that,

Coordinates of foci = (0, ±c) ...(ii)

The coordinates of the foci are (0, ±4). This means that the major and minor axes are along y and x axes respectively.

$\therefore$ From eq. (i) and (ii), we get

c = 4

It is also given that

$\text{Eccentricity} = \frac{4}{5}$

we know that,

$\text{Eccentricity}, e = \frac{c}{a}$

$\Rightarrow \frac{4}{5} = \frac{4}{a}$ [$\because$ c = 4]

$\Rightarrow a = 5$

Now, we know that,

$c^2 = a^2 - b^2$

$\Rightarrow (4)^2 = (5)^2 - b^2$

$\Rightarrow 16 = 25 - b^2$

$\Rightarrow b^2 = 25 - 16$

$\Rightarrow b^2 = 9$

Since, the foci of the ellipse are on y – axis. So, the Equation of Ellipse is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

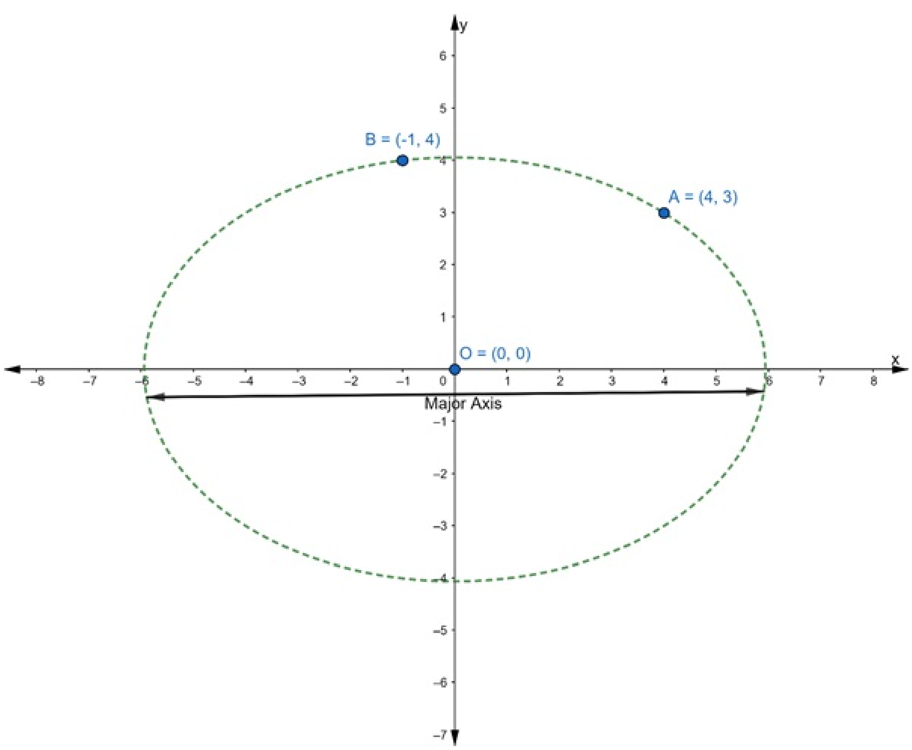
Substituting the value of a^2 and b^2 , we get

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{25} = 1$$

Question: 20

Find the equation

Solution:



Given: Center is at the origin

and Major axis is along x – axis

So, Equation of ellipse is of the form

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

Given that ellipse passing through the points (4, 3) and (-1, 4)

So, point (4, 3) and (-1, 4) will satisfy the eq. (i)

Taking point (4, 3) where x = 4 and y = 3

Putting the values in eq. (i), we get

$$\frac{(4)^2}{a^2}+\frac{(3)^2}{b^2}=1$$

$\Rightarrow \frac{16}{a^2}+\frac{9}{b^2}=1$ ...(ii)

Taking point (-1, 4) where x = -1 and y = 4

Putting the values in eq. (i), we get

$$\frac{(-1)^2}{a^2}+\frac{(4)^2}{b^2}=1$$

$\Rightarrow \frac{1}{a^2}+\frac{16}{b^2}=1$ ...(iii)

Now, we have to solve the above two equations to find the value of a and b

Multiply the eq. (iii) by 16, we get

$$\frac{16}{a^2}+\frac{16\times16}{b^2}=1\times16$$

$\Rightarrow \frac{16}{a^2}+\frac{256}{b^2}=16$ ...(iv)

Subtracting eq. (iv) from (ii), we get

$$\begin{array}{r} \frac{16}{a^2}+\frac{9}{b^2}=1 \\ \frac{16}{a^2}+\frac{256}{b^2}=16 \\ - \quad - \quad - \\ \hline \frac{9}{b^2}-\frac{256}{b^2}=1-16 \end{array}$$

$$\Rightarrow \frac{9-256}{b^2}=-15$$

$$\Rightarrow -\frac{247}{b^2}=-15$$

$$\Rightarrow b^2=\frac{247}{15}$$

Substituting the value of b^2 in eq. (iii), we get

$$\frac{1}{a^2}+\frac{16}{\frac{247}{15}}=1$$

$$\Rightarrow \frac{1}{a^2}+\frac{15\times16}{247}=1$$

$$\Rightarrow \frac{1}{a^2}+\frac{240}{247}=1$$

$$\Rightarrow \frac{1}{a^2}=1-\frac{240}{247}$$

$$\Rightarrow \frac{1}{a^2} = \frac{247 - 240}{247}$$

$$\Rightarrow \frac{1}{a^2} = \frac{7}{247}$$

$$\Rightarrow a^2 = \frac{247}{7}$$

Thus, $a^2 = \frac{247}{7}$ & $b^2 = \frac{247}{15}$

Substituting the value of a^2 and b^2 in eq. (i), we get

$$\frac{x^2}{\frac{247}{7}} + \frac{y^2}{\frac{247}{15}} = 1$$

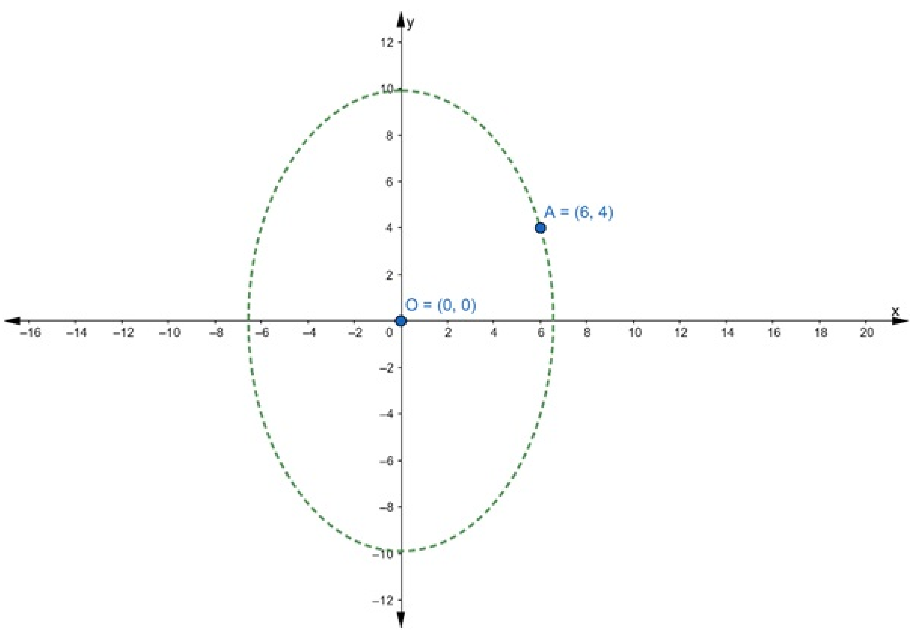
$$\Rightarrow \frac{7x^2}{247} + \frac{15y^2}{247} = 1$$

$\Rightarrow 7x^2 + 15y^2 = 247$

Question: 21

Find the equation

Solution:



Given that

$$\text{Eccentricity} = \frac{3}{4}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$$\Rightarrow \frac{3}{4} = \frac{c}{a}$$

$$\Rightarrow c = \frac{3}{4}a$$

We know that,

$c^2 = a^2 - b^2$

$$\Rightarrow \left(\frac{3a}{4}\right)^2 = a^2 - b^2$$

$$\Rightarrow \frac{9a^2}{16} = a^2 - b^2$$

$$\Rightarrow b^2 = a^2 - \frac{9a^2}{16}$$

$$\Rightarrow b^2 = \frac{16a^2 - 9a^2}{16}$$

$$\Rightarrow b^2 = \frac{7a^2}{16} \ldots\text{(i)}$$

It is also given that Coordinates of foci is on the y – axis

So, Equation of ellipse is of the form

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Substituting the value of b^2 in above eq., we get

$$\frac{x^2}{\frac{7a^2}{16}} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow \frac{16x^2}{7a^2} + \frac{y^2}{a^2} = 1 \ldots\text{(ii)}$$

Given that ellipse passing through the points (6, 4)

So, point (6, 4) will satisfy the eq. (ii)

Taking point (6, 4) where x = 6 and y = 4

Putting the values in eq. (ii), we get

$$\frac{16(6)^2}{7a^2} + \frac{(4)^2}{a^2} = 1$$

$$\Rightarrow \frac{16 \times 36}{7a^2} + \frac{16}{a^2} = 1$$

$$\Rightarrow \frac{576 + 7 \times 16}{7a^2} = 1$$

$$\Rightarrow \frac{576 + 112}{7a^2} = 1$$

$$\Rightarrow \frac{688}{7a^2} = 1$$

$$\Rightarrow a^2 = \frac{688}{7}$$

Substituting the value of a^2 in eq. (i), we get

$$b^2 = \frac{7 \times \frac{688}{7}}{16}$$

$$\Rightarrow b^2 = \frac{688}{16}$$

Substituting the value of a^2 and b^2 in the equation of an ellipse, we get

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$$\Rightarrow \frac{x^2}{\frac{688}{16}} + \frac{y^2}{\frac{688}{7}} = 1$$

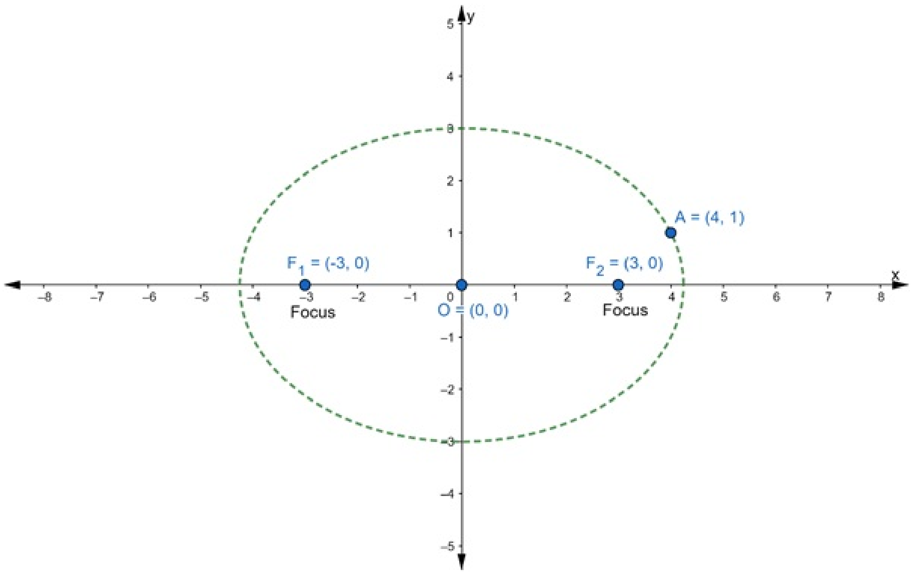
$$\Rightarrow \frac{16x^2}{688} + \frac{7y^2}{688} = 1$$

or $16x^2 + 7y^2 = 688$

Question: 22

Find the equation

Solution:



Let the equation of the required ellipse be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

Given:

Coordinates of foci = (±3, 0) ...(ii)

We know that,

Coordinates of foci = (±c, 0) ...(iii)

∴ From eq. (ii) and (iii), we get

c = 3

We know that,

$c^2 = a^2 - b^2$

$\Rightarrow (3)^2 = a^2 - b^2$

$\Rightarrow 9 = a^2 - b^2$

$\Rightarrow b^2 = a^2 - 9$ ...(iv)

Given that ellipse passing through the points (4, 1)

So, point (4, 1) will satisfy the eq. (i)

Taking point (4, 1) where x = 4 and y = 1

Putting the values in eq. (i), we get

$$\frac{(4)^2}{a^2} + \frac{(1)^2}{b^2} = 1$$

$\Rightarrow \frac{16}{a^2} + \frac{1}{b^2} = 1$

$\Rightarrow \frac{16}{a^2} + \frac{1}{a^2-9} = 1$ [from (iv)]

$\Rightarrow \frac{16(a^2-9)+a^2}{(a^2)(a^2-9)} = 1$

$\Rightarrow 16a^2 - 144 + a^2 = a^2(a^2 - 9)$

$\Rightarrow 17a^2 - 144 = a^4 - 9a^2$

$\Rightarrow a^4 - 9a^2 - 17a^2 + 144 = 0$

$\Rightarrow a^4 - 26a^2 + 144 = 0$

$\Rightarrow a^4 - 8a^2 - 18a^2 + 144 = 0$

$\Rightarrow a^2(a^2 - 8) - 18(a^2 - 8) = 0$

$\Rightarrow (a^2 - 8)(a^2 - 18) = 0$

$\Rightarrow a^2 - 8 = 0$ or $a^2 - 18 = 0$

$\Rightarrow a^2 = 8$ or $a^2 = 18$

If $a^2 = 8$ then

$b^2 = 8 - 9$

$= -1$

Since the square of a real number cannot be negative. So, this is not possible

If $a^2 = 18$ then

$b^2 = 18 - 9$

$= 9$

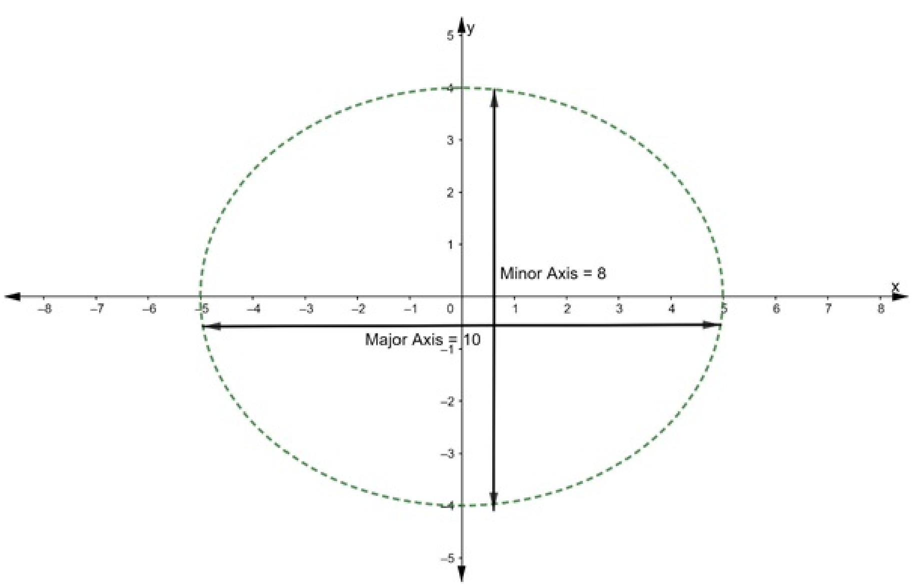
So, equation of ellipse if $a^2 = 18$ and $b^2 = 9$

$\frac{x^2}{18} + \frac{y^2}{9} = 1$

Question: 23

Find the equation

Solution:



Let the equation of required ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(A)

Given:

Length of Major Axis = 10units ...(i)

We know that,

Length of major axis = 2a ...(ii)

∴From eq. (i) and (ii), we get

2a = 10

⇒ a = 5

It is also given that

Length of Minor Axis = 8 units ...(iii)

We know that,

Length of minor axis = 2b ...(iv)

∴From eq. (iii) and (iv), we get

2b = 8

⇒ b = 4

Substituting the value of a and b in eq. (A), we get

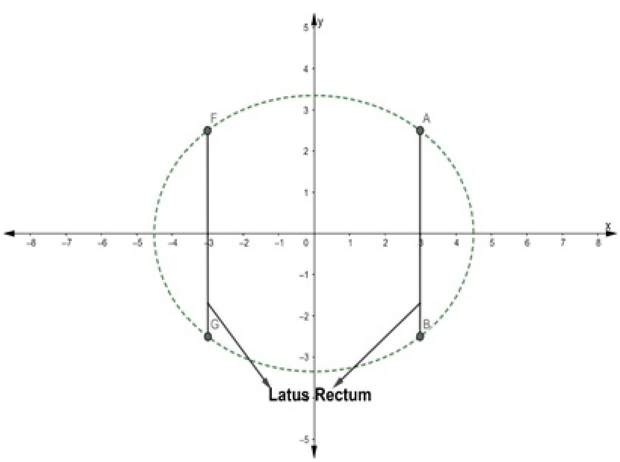
$$\frac{x^2}{(5)^2} + \frac{y^2}{(4)^2} = 1$$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1$$

Question: 24

Find the equation

Solution:



Let the equation of the required ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

Given that

$$\text{Eccentricity} = \frac{2}{3}$$

we know that,

$$\text{Eccentricity}, e = \frac{c}{a}$$

$$\Rightarrow \frac{2}{3} = \frac{c}{a}$$

$$\Rightarrow c = \frac{2}{3}a$$

We know that,

$c^2 = a^2 - b^2$

$$\Rightarrow \left(\frac{2a}{3}\right)^2 = a^2 - b^2$$

$$\Rightarrow \frac{4a^2}{9} = a^2 - b^2$$

$$\Rightarrow b^2 = a^2 - \frac{4a^2}{9}$$

$$\Rightarrow b^2 = \frac{9a^2 - 4a^2}{9}$$

$\Rightarrow b^2 = \frac{5a^2}{9}$ ...(ii)

It is also given that, Latus Rectum = 5 ...(iii)

We know that,

$$\text{Latus Rectum } = \frac{2b^2}{a}$$

$$\Rightarrow 5 = \frac{2 \times \left(\frac{5a^2}{9}\right)}{a}$$

$$\Rightarrow 5 = \frac{10a^2}{9a}$$

$$\Rightarrow 5 = \frac{10a}{9}$$

$$\Rightarrow a = \frac{5 \times 9}{10}$$

$$\Rightarrow a = \frac{9}{2}$$

$$\Rightarrow a^2 = \frac{81}{4}$$

Substituting the value of a in eq. (ii), we get

$$b^2 = \frac{5\left(\frac{9}{2}\right)^2}{9}$$

$$\Rightarrow b^2 = \frac{5 \times 9}{4}$$

$$\Rightarrow b^2 = \frac{45}{4}$$

Substituting the value of a^2 and b^2 in eq. (i), we get

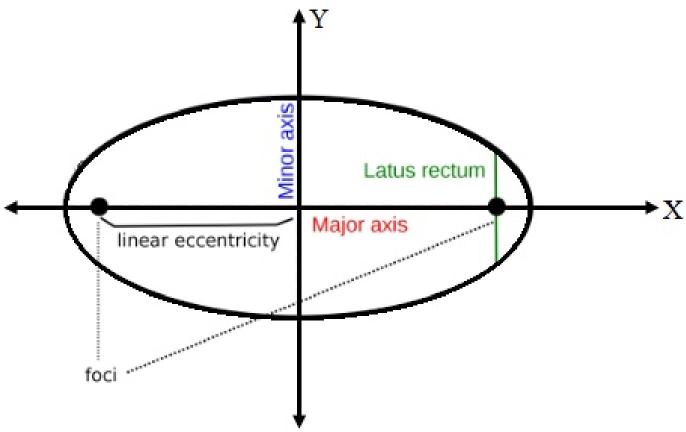
$$\frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

$$\Rightarrow \frac{4x^2}{81} + \frac{4y^2}{45} = 1$$

Question: 25

Find the eccentri

Solution:



Let the equation of the required ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(i)

It is given that,

$$\text{Length of Latus Rectum} = \frac{1}{2}\text{minor Axis}$$

We know that,

$$\text{Length of Latus Rectum } = \frac{2b^2}{a}$$

and Length of Minor Axis = 2b

So, according to the given condition,

$$\frac{2b^2}{a} = \frac{1}{2} \times 2b$$

$$\Rightarrow \frac{2b^2}{a} = b$$

$$\Rightarrow \frac{2b^2}{b} = a$$

⇒ 2b = a ...(ii)

Now, we have to find the eccentricity

We know that,

$\text{Eccentricity}, e = \frac{c}{a}$ ...(iii)

where, $c^2 = a^2 - b^2$

So, $c^2 = (2b)^2 - b^2$ [from (ii)]

$\Rightarrow c^2 = 4b^2 - b^2$

$\Rightarrow c^2 = 3b^2$

$\Rightarrow c = \sqrt{3b^2}$

$\Rightarrow c = b\sqrt{3}$

Substituting the value of c and a in eq. (iii), we get

$$\text{Eccentricity}, e = \frac{c}{a}$$

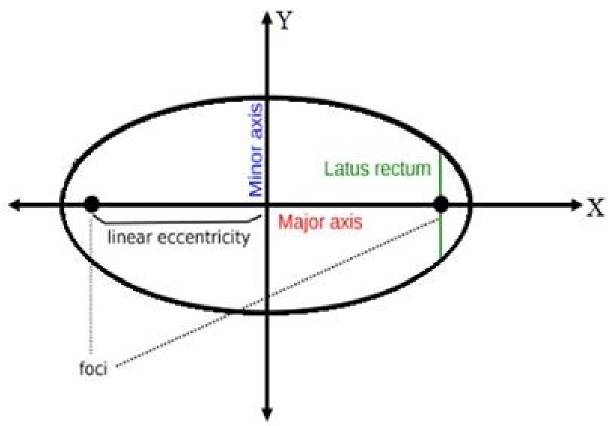
$$= \frac{b\sqrt{3}}{2b}$$

$$\therefore e = \frac{\sqrt{3}}{2}$$

Question: 26

Find the eccentri

Solution:



Let the equation of the required ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ldots(i)$$

It is given that,

$$\text{Length of Latus Rectum} = \frac{1}{2}\text{major Axis}$$

We know that,

$$\text{Length of Latus Rectum} = \frac{2b^2}{a}$$

and Length of Minor Axis = 2a

So, according to the given condition,

$$\frac{2b^2}{a} = \frac{1}{2} \times 2a$$

$$\Rightarrow \frac{2b^2}{a} = a$$

$\Rightarrow 2b^2 = a^2 \ldots(ii)$

$\Rightarrow a = \sqrt{2b^2}$

$\Rightarrow a = b\sqrt{2}$

Now, we have to find the eccentricity

We know that,

$\text{Eccentricity}, e = \frac{c}{a}$ ...(iii)

where, $c^2 = a^2 - b^2$

So, $c^2 = 2b^2 - b^2$ [from (ii)]

$\Rightarrow c^2 = b^2$

$\Rightarrow c = \sqrt{b^2}$

$\Rightarrow c = b$

Substituting the value of c and a in eq. (iii), we get

$$\text{Eccentricity}, e = \frac{c}{a}$$

$$= \frac{b}{b\sqrt{2}}$$

$$\therefore e = \frac{1}{\sqrt{2}}$$