Chapter : 1. REAL NUMBER

Exercise : 1A

Question: 1

What are ra

Solution:

Rational number is a number which can be written in the form $\frac{p}{q}$, where p and q both are integers but q is not equals to zero.

Examples $= 0, \frac{1}{9}, \frac{4}{7}, \frac{3}{5}, \frac{6}{11}, \frac{13}{12}, \frac{23}{16}, \frac{4}{5}, \frac{56}{57}, 1$

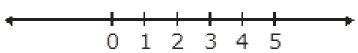
Question: 2

Represent e

Solution:

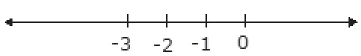
(i) Given number = 5

First draw a line and mark the origin Zero. The given number 5 is positive so we are going to locate it on the right side of the zero.



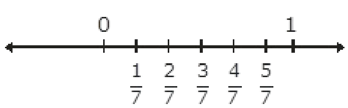
(ii) Given = -3

First draw a line and mark the origin Zero. The given number is negative (-3) so we are going to locate it on the left side of the zero.



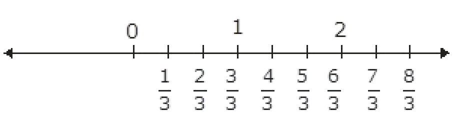
(iii) Given = 5/7

First draw a line and mark the origin Zero. The given number $\frac{5}{7}$ is positive so we are going to locate it on the right side of the zero.



(iv) Given = 8/3

First draw a line and mark the origin Zero. The given number $\frac{8}{3}$ is positive so we are going to locate it on the right side of the zero.

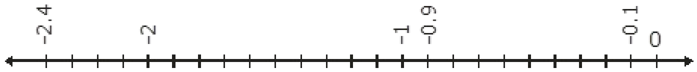


(v) Given = 1.3

First draw a line and mark the origin Zero. The given number 1.3 is positive so we are going to locate it on the right side of the zero.

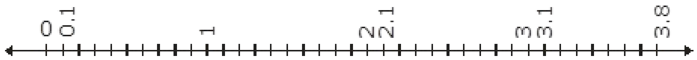
(vi) Given = -2.4

First draw a line and mark the origin Zero. The given number -2.4 is negative so we are going to locate it on the left side of the zero.



(vii) Given = 23/6

First draw a line and mark the origin Zero. The given number $\frac{23}{6} = 3.8$ is positive so we are going to locate it on the right side of the zero.



Question: 3

Find a rati

Solution:

(i) $\frac{1}{4} \, and \, \frac{1}{3}$

Let suppose,

$a = \dfrac{1}{4} \text{ and } b = \dfrac{1}{3}$

As we can see that a ˂ b

So, the rational number lying between a and b

$= \frac{1}{2}(a + b)$

$= \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}\right)$

$= \frac{1}{2} \times \left(\frac{3+4}{12}\right)$

$= \dfrac{1}{2} \times \left(\dfrac{7}{12}\right) = \dfrac{7}{24}$

So, we can say that $\frac{7}{24}$ is a rational number lying between 1/4 and 1/3.

(ii) $\frac{3}{8} \, and \, \frac{2}{5}$

Let's take $a = \frac{3}{8} \, and \, b = \frac{2}{5}$

As we can see a˂b

A rational number lying between $\frac{3}{8} \, and \, \frac{2}{5}$

$= \dfrac{1}{2}\,(a + b)$

$= \dfrac{1}{2}\left(\dfrac{3}{8} + \dfrac{2}{5}\right)$

$= \dfrac{1}{2}\left(\dfrac{15 + 16}{40}\right)$

$= \dfrac{1}{2}\left(\dfrac{31}{40}\right) = \dfrac{31}{80}$

$\frac{31}{80}$ is the number lying between $\frac{3}{8} \, and \, \frac{2}{5}$

(iii) 1.3 and 1.4

By taking a = 1.3 and b = 1.4

As we can see a ˂ b

The rational number between a and b,

$$=\frac{1}{2}(a + b)$$

$$=\frac{1}{2}(1.3 + 1.4)$$

$$=\frac{1}{2}(2.7)$$

$$=\frac{1}{2} \times 2.7 = 1.35$$

1.35 is the number lying between 1.3 and 1.4

(iv) 0.75 and 1.2

Let's take a = 0.75 and b = 1.2

As we can see a˂b,

The rational number between a and b,

$$=\frac{1}{2}(a + b)$$

$$=\frac{1}{2}(0.75 + 1.2)$$

$$=\frac{1}{2}(1.95)$$

$$=\frac{1}{2} \times 1.95 = 0.975$$

0.975 is the number between 0.75 and 1.2

(v) – 1 $and\frac{1}{2}$

Let's take $a = -1\ and\ b = \frac{1}{2}$

The rational number between a and b,

$$=\frac{1}{2}(a + b)$$

$$=\frac{1}{2}\left(-1 + \frac{1}{2}\right)$$

$$=\frac{1}{2}\left(-\frac{1}{2}\right) = -\frac{1}{4}$$

$-\frac{1}{4}$ is the number between -1 and $\frac{1}{2}$

(vi) $-\frac{3}{4}\ and\ -\frac{2}{5}$

Let's take $a = -\frac{3}{4}\ and\ b = -\frac{2}{5}$

The rational number between a and b,

$$=\frac{1}{2}(a + b)$$

$$=\frac{1}{2}\left(-\frac{3}{4} + \left(-\frac{2}{5}\right)\right)$$

$$= \frac{1}{2}\left(-\frac{3}{4} - \frac{2}{5}\right)$$

$$= \frac{1}{2}\left(\frac{-15-8}{20}\right)$$

$$= \frac{1}{2}\left(\frac{-23}{20}\right) = -\frac{23}{40}$$

So, the rational number between $-\frac{3}{4}$ and $-\frac{2}{5}$ is $-\frac{23}{40}$

Question: 4

Find three

Solution:

Let's take $a = \frac{1}{5}, b = \frac{1}{4}$ and n = 3

n= numbers required to find out

So, $d = \frac{b-a}{n+1} = \frac{\left(\frac{1}{4}-\frac{1}{5}\right)}{3+1} = \frac{\frac{5-4}{20}}{4} = \frac{1}{80}$

Thus, three rational numbers are:

(a + d), (a + 2d), and (a + 3d)

$$(a + d) = \left(\frac{1}{5} + \frac{1}{80}\right) = \left(\frac{16 + 1}{80}\right) = \frac{17}{80}$$

$$(a + 2d) = \left(\frac{1}{5} + 2 \times \frac{1}{80}\right) = \left(\frac{1}{5} + \frac{2}{80}\right) = \frac{16 + 2}{80} = \frac{18}{80}$$

$$(a + 3d) = \left(\frac{1}{5} + \left(3 \times \frac{1}{80}\right)\right) = \left(\frac{1}{5} + \frac{3}{80}\right) = \left(\frac{16 + 3}{80}\right) = \frac{19}{80}$$

Hence, three rational numbers lying between 1/5 and 1/4 are $\frac{17}{80}, \frac{18}{80}, \frac{19}{80}$

Question: 5

Find five r

Solution:

Let's take $a = \frac{2}{5}, b = \frac{3}{4}$ $and\ n = 5$

n= numbers required to be find out

So, $d = \frac{b-a}{n+1} = \frac{\left(\frac{3}{4}-\frac{2}{5}\right)}{5+1} = \frac{\frac{15-8}{20}}{6} = \frac{7}{6 \times 20} = \frac{7}{120}$

Thus, five rational numbers are:

(a + d), (a + 2d), (a + 3d), (a + 4d) and (a + 5d)

$$(a + d) = \left(\frac{2}{5} + \frac{7}{120}\right) = \left(\frac{48+7}{120}\right) = \frac{55}{120}$$

$$(a + 2d) = \left(\frac{2}{5} + 2 \times \frac{7}{120}\right) = \left(\frac{2}{5} + \frac{14}{120}\right) = \frac{48+14}{120} = \frac{62}{120}$$

$$(a + 3d) = \left(\frac{2}{5} + \left(3 \times \frac{7}{120}\right)\right) = \left(\frac{2}{5} + \frac{21}{120}\right) = \left(\frac{48+21}{80}\right) = \frac{69}{120}$$

$$(a + 4d) = \left(\frac{2}{5} + \left(4 \times \frac{7}{120}\right)\right) = \left(\frac{2}{5} + \frac{28}{120}\right) = \left(\frac{48+28}{80}\right) = \frac{76}{120}$$

$(a + 5d) = \left(\frac{2}{5} + \left(5 \times \frac{7}{120}\right)\right) = \left(\frac{2}{5} + \frac{35}{120}\right) = \left(\frac{48 + 35}{80}\right) = \frac{83}{120}$

Hence, five rational numbers lying between $\frac{2}{5}$ and $\frac{3}{4}$ are $\frac{55}{120}, \frac{62}{120}, \frac{69}{120}, \frac{76}{120}$ and $\frac{83}{120}$

Question: 6

Insert six

Solution:

Let's take a = 3, b = 4 and n = 6

n= numbers required to be find out

So, $d = \frac{b-a}{n+1} = \frac{(4-3)}{6+1} = \frac{1}{7}$

Thus, six rational numbers are:

(a + d), (a + 2d), (a + 3d), (a + 4d), (a + 5d) and (a + 6d)

$(a + d) = \left(3 + \frac{1}{7}\right) = \left(\frac{21+1}{7}\right) = \frac{22}{7}$

$(a + 2d) = \left(3 + \left(2 \times \frac{1}{7}\right)\right) = \left(3 + \frac{2}{7}\right) = \left(\frac{21+2}{7}\right) = \frac{23}{7}$

$(a + 3d) = \left(3 + \left(3 \times \frac{1}{7}\right)\right) = \left(3 + \frac{3}{7}\right) = \left(\frac{21+3}{7}\right) = \frac{24}{7}$

$(a + 4d) = \left(3 + \left(4 \times \frac{1}{7}\right)\right) = \left(3 + \frac{4}{7}\right) = \left(\frac{21+4}{7}\right) = \frac{25}{7}$

$(a + 5d) = \left(3 + \left(5 \times \frac{1}{7}\right)\right) = \left(3 + \frac{5}{7}\right) = \left(\frac{21 + 5}{7}\right) = \frac{26}{7}$

$(a + 6d) = \left(3 + \left(6 \times \frac{1}{7}\right)\right) = \left(3 + \frac{6}{7}\right) = \left(\frac{21+6}{7}\right) = \frac{27}{7}$

Hence, six rational numbers lying between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ and $\frac{27}{7}$

Question: 7

Insert 16 r

Solution:

Let's take $a = 2.1$, $b = 2.2$ and $n = 16$

n= numbers required to be find out

So, $d = \frac{b-a}{n+1} = \frac{2.2-2.1}{16+1} = \frac{0.1}{17} = \frac{1}{170} = 0.005$

Thus, 16 rational numbers are:

(a + d), (a + 2d), (a + 3d), (a + 4d), (a + 5d), (a + 6d), (a + 7d), (a + 8d), (a + 9d), (a + 10d), (a + 11d), (a + 12d), (a + 13d), (a + 14d), (a + 15d) and (a + 16d)

So,

(a + d) = (2.1 + 0.005) = 2.105

(a + 2d) = [2.1 + (2 × 0.005)] = 2.110

(a + 3d) = [2.1 + (3 × 0.005)] = 2.115

(a + 4d) = [2.1 + (4 × 0.005)] = 2.120

(a + 5d) = [2.1 + (5 × 0.005)] = 2.125

(a + 6d) = [2.1 + (6 × 0.005)] = 2.130

(a + 7d) = [2.1 + (7 × 0.005)] = 2. 135

(a + 8d) = [2.1 + (8 × 0.005)] = 2. 140

(a + 9d) = [2.1 + (9 × 0.005)] = 2. 145

(a + 10d) = [2.1 + (10 × 0.005)] = 2. 150

(a + 11d) = [2.1 + (11 × 0.005)] = 2. 155

(a + 12d) = [2.1 + (12 × 0.005)] = 2. 160

(a + 13d) = [2.1 + (13 × 0.005)] = 2. 165

(a + 14d) = [2.1 + (14 × 0.005)] = 2. 170

(a + 15d) = [2.1 + (15 × 0.005)] = 2. 175

(a + 16d) = [2.1 + (16 × 0.005)] = 2. 180

Thus, the rational numbers between 2.1 and 2.2 are 2.105, 2.110, 2.115, 2.120, 2.125, 2.130, 2.135, 2.140, 2.145, 2.150, 2.155, 2.160, 2.165, 2.170, 2.175, 2.180,

Exercise : 1B

Question: 1

Without act

Solution:

First we have to know what is terminating decimal. Terminating decimal is the number which has digits that do not go on forever.

(i) $\frac{13}{80}$

Denominator 80 has factors = 2 × 2 × 2 × 2 × 5

So, 80 has no prime factors other than 2 and 5, thus $\frac{13}{80}$ is terminating decimal.

(ii) $\frac{7}{24}$

Denominator 24 has factors = 2 × 2 × 2 × 3

So, 24 has factors other than 2 and 5, thus $\frac{7}{24}$ is not a terminating decimal.

(iii) $\frac{5}{12}$

Denominator 12 has factors = 2 × 2 × 3

So, 12 has factors other than 2 and 5, thus $\frac{5}{12}$ is not a terminating decimal.

(iv) $\frac{8}{35}$

Denominator 35 has factors = 5 × 7

So, 35 has factors other than 2 and 5, thus $\frac{8}{35}$ is not a terminating decimal.

(v) $\frac{16}{125}$

Denominator 24 has factors = 5 × 5 × 5

So, 125 has no prime factors other than 2 and 5, thus $\frac{16}{125}$ is a terminating decimal.

Question: 2

Convert eac

Solution:

(i) Given $\frac{5}{8}$

By actual division method, we get

```
   0.625
8)5.000
  48
   20
   16
    40
    40
     ×
```

Thus $\frac{5}{8} = 0.625$

(ii) Given $\frac{9}{16}$

By actual division method we get:

```
    0.5625
16)9.0000
   80
   100
    96
    40
    32
     80
     80
      ×
```

Thus $\frac{9}{16} = 0.5625$

(iii) $\frac{7}{25}$

By actual division method we get:

```
    0.28
25)7.00
   50
   200
   200
    ×
```

Thus $\frac{7}{25} = 0.28$

(iv) $\frac{11}{24}$

By actual division method we get:

```
     0.45833..
24) 11.00
    96
    140
    120
     200
     192
      80
      72
      80
```

Thus $\frac{11}{24} = 0.45833$

(v) $2\frac{5}{12} = \frac{29}{12}$

By actual division method we get:

$$
\begin{array}{r|l}
 & 2.4166.... \\
12 & 29.0 \\
 & 24 \\
\hline
 & \quad 50 \\
 & \quad 48 \\
\hline
 & \qquad 20 \\
 & \qquad 12 \\
\hline
 & \qquad\; 80 \\
 & \qquad\; 72 \\
\hline
 & \qquad\;\; 8
\end{array}
$$

Thus $\frac{29}{12} = 2.4166$

Question: 3

Express each of t

Solution:

(i) Given $0.\bar{3}$

Let x equals to the repeating decimal = $0.\bar{3}$

As we can see the repeating digit is 3

x = 0.3333333……… (i)

10x = 3.333333……… (ii)

Subtracting (i) from (ii), we get

10x - x = 3.3333 – 0.3333

9x = 3

$x = \frac{3}{9} = \frac{1}{3}$

So, we can say that 0.3333333333…. is equals to the $\frac{1}{3}$.

(i) Given $1.\bar{3}$

Let x equals to the repeating decimal = $1.\bar{3}$

As we can see the repeating digit is 3

x = 1.3333333……… (i)

10x = 13.333333……… (ii)

Subtracting (i) from (ii), we get

10x-x = 13.3333 – 1.3333

9x = 12

$x = \frac{12}{9} = \frac{4}{3}$

So, we can say that 1.3333333333…. is equals to the $\frac{4}{3}$.

(ii) $0.\overline{34}$

Let x equals to the repeating decimal = $0.\overline{34}$

As we can see the repeating digit is 34

X = 0.3434343434……… (i)

10x = 3.434343434……… (ii)

100x = 34.3434343434..... (iii)

Subtracting (i) from (iii), we get

100x-x = 0.34343434 – 34.34343434

99x = 34

$$x = \frac{34}{99} =$$

So, we can say that 0.34343434343434.... is equals to the $\frac{34}{99}$.

(iii) $3.\overline{14}$

Let x equals to the repeating decimal = $3.\overline{14}$

As we can see the repeating digit is 14

X = 3.1414141414......... (i)

10x = 31.41414141414......... (ii)

100x = 314.1414141414..... (iii)

Subtracting (i) from (iii), we get

100x - x = 3.1414141414 – 314.14141414

99x = 311

$$x = \frac{311}{99}$$

So, we can say that 3.1414141414.... is equals to the $\frac{311}{99}$.

(iv) $0.\overline{324}$

Let x equals to the repeating decimal = $0.\overline{324}$

As we can see the repeating digit is 324

X = 0.324324324324324......... (i)

10x = 3.24324324324324......... (ii)

100x = 32.4324324324324..... (iii)

1000x = 324.324324324324.......(iv)

Subtracting (i) from (iv), we get

1000x - x = 0.324324324324 – 324.324324324324

999x = 324

$$x = \frac{324}{999}$$

So, we can say that 0.324324324324324324324.... is equals to the $\frac{324}{999}$.

(v) $0.1\overline{7}$

Let x equals to the repeating decimal = $0.1\overline{7}$

As we can see the repeating digit is 7

x = 0.17777777777......... (i)

10x = 1.777777777......... (ii)

100x = 17.77777777..... (iii)

Subtracting (ii) from (iii), we get

100x - 10x = 17.777777-1.777777

90x = 16

$$x = \frac{16}{90} = \frac{8}{45}$$

So, we can say that 0.17777777…. is equals to the $\frac{8}{45}$

(vi) $0.5\overline{4}$

Let x equals to the repeating decimal = $0.5\overline{4}$

As we can see the repeating digit is 4

X = 0.5444444444……… (i)

10x = 5.44444444……… (ii)

100x = 54.444444….. (iii)

Subtracting (ii) from (iii), we get

100x-10x = 54.44444-5.44444

90x = 49

$$x = \frac{49}{90}$$

So, we can say that 0.5444444…. is equals to the $\frac{49}{90}$

(vii) $0.1\overline{63}$

Let x equals to the repeating decimal = $0.1\overline{63}$

As we can see the repeating digit is 63

X = 0.16363636363……… (i)

10x =1.6363636363……… (ii)

100x = 16.363636363….. (iii)

1000x = 163.63636363…. (iv)

Subtracting (ii) from (iv), we get

1000x-10x = 163.636363-1.63636363

990x = 162

$$x = \frac{162}{990} = \frac{18}{110} = \frac{9}{55}$$

So, we can say that 0.16363636363…. is equals to the $\frac{9}{55}$

Question: 4

Write, whet

Solution:

(i) True: every natural number is the whole number because natural number starts with 1 and whole number start with 0. So, every natural number will automatically fall in the category of whole number.

(ii) False: every whole number can’t be natural number as natural number starts from 1 and whole number starts with 0.

(iii) True: Integers includes all whole numbers and their negative counterparts. Rational numbers can be expressed in the form of fractions where denominator is not equals to the zero but both, numerator and denominator are integers.

(iv) False: Rational number is the number which can be expressed in the form of fraction where denominator is not equals to zero. But whole numbers are natural numbers including zero and they can't be written in fractional form.

(v) True: Rational number is the number which can be expressed in the form of fraction where denominator is not equals to zero and terminating decimal can also be written in fraction form.

(vi) True: Yes, every repeating decimal is also the rational number because it also written in the form of fraction.

(vii) True: yes, 0 is also the rational number because it can be written in the form of fraction.

Exercise : 1C

Question: 1

What are ir

Solution:

Irrational number:- A number which can't be expressed as an terminating decimal or recurring decimal and fractional form is called irrational.

Ex: π, (3.1415926535), $\sqrt{7}$

Irrational number is different from Rational number because Rational number is a number which can be expressed as fractional form or terminating decimal form is called rational number. It is exactly the opposite of Irrational number.

Ex:- 5 , $\frac{7}{8}$, .21

Question: 2

Classify th

Solution:

(i) $\sqrt{4}$

$= \sqrt{4} = \sqrt{2 \times 2} = 2$

∵ we can express 2 as $\frac{2}{1}$ which is the quotient of the integer 2 and 1

Hence, it is a rational number.

(ii) $\sqrt{196}$

$= \sqrt{196} = \sqrt{14 \times 14} = 14.$

∵ we can express 14 as $\frac{14}{1}$ which is the quotient of the integer 14 and 1

Hence, it is a rational number.

(iii) $\sqrt{21}$

$= \sqrt{21} = \sqrt{3 \times 7} = \sqrt{3} \times \sqrt{7}$

∵we can not simplify $\sqrt{3}$ *and* $\sqrt{7}$, in the form $\frac{p}{q}$, $\quad q \neq 0$

Hence, it is an irrational number.

(iv) $\sqrt{43}$

We know that 43 is a prime number so we can not get prime factors of it and neither we can write $\sqrt{43}$ in fractional form.

Hence, it is an irrational number.

(v) $3+\sqrt{3}$

∵ $\sqrt{3}$ is irrational number and addition of an irrational number to any real number always gives irrational number.

Hence, it is an irrational number.

(vi) $\sqrt{7}-2$

∵ $\sqrt{7}$ is irrational number and addition of an irrational number to any real number always gives irrational number.Hence, it is an irrational number.

(vii) $\frac{2}{3}\sqrt{6}$

$= \frac{2}{3} \times \sqrt{3 \times 2} = \frac{2}{3} \times \sqrt{3} \times \sqrt{2}$

∵ As, $\sqrt{3}$ and √2 are irrational numbers and multiplication of an irrational number to a non zero rational number gives irrational number.

Hence, it is an irrational number.

(viii) $0.\bar{6}$

∵ we know that all repeating decimals are rational,

Hence, it is a rational number.

(ix) 1.232332333...

∵ The decimal expansion here is non terminating and non repeating,

Hence, it is an irrational number.

(x) 3.040040004...

∵ The decimal expansion here is non terminating and non repeating,

Hence, it is an irrational number.

(xi) 3.2576

∵ It is a terminating decimal fraction and can be expressed in form $\frac{32576}{10000}$.

Hence it is a rational number.

(xii) 2.3565656...

∵ it is a non terminating but repeating decimal form that can be written as $2.35\overline{65}$.

Hence, it is a rational number.

(xiii) π

∵ We know that π is a non terminating Decimal fraction,

Hence it is an irrational number.

(xiv) $\frac{22}{7}$

∵ it is an fractional form,

Hence it is rational.

Question: 3

Represent <

Solution:

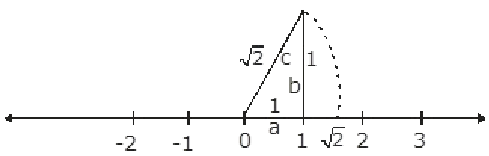
By using Pythagoras theorem,

$a^2 + b^2 = c^2$

$1^2 + 1^2 = c^2$

$c^2 = 2$

$c = \sqrt{2}$



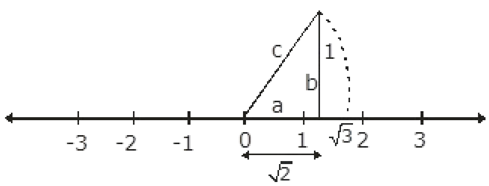
By using Pythagoras theorem,

$a^2 + b^2 = c^2$

$\left(\sqrt{2}\right)^2 + 1^2 = c^2$

$c^2 = 2 + 1 = 3$

$c = \sqrt{3}$

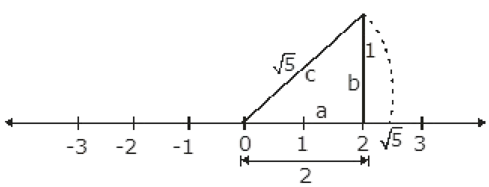


By using Pythagoras theorem,

$a^2 + b^2 = c^2$

$2^2 + 1^2 = c^2$

$c^2 = 5 \; c = \sqrt{5}$



Question: 4

Represent <

Solution:

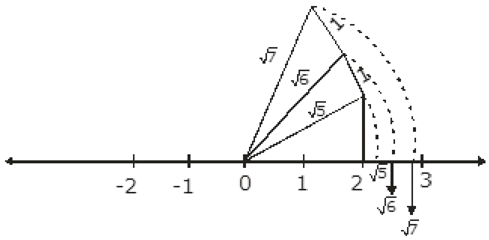
By using Pythagoras theorem,

(i) $(AB)^2 + (BC)^2 = (AC)^2$

$(AC)^2 = (2)^2 + (1)^2$

$(AC)^2 = 5$

$AC = \sqrt{5}$



(ii) $(AC)^2 + (DC)^2 = (AD)^2$

$(AD)^2 = (\sqrt{5})^2 + (1)^2$

$(AD)^2 = 5 + 1 = 6$

$AD = \sqrt{6}$

(iii) $(AD)^2 + (ED)^2 = (AE)^2$

$(AE)^2 = (\sqrt{6})^2 + (1)^2$

$(AE)^2 = 6 + 1$

$AE = \sqrt{7}$

Question: 5

Giving reas

Solution:

(i) $4+\sqrt{5}$

We cannot simplify $\sqrt{5}$,

Hence 4 + $\sqrt{5}$ is an irrational number.

(ii) $(-3+\sqrt{6})$

$= -3 + \sqrt{2 \times 3} = -3 + \sqrt{2} \times \sqrt{3}$

We can't simplify $\sqrt{2} and \sqrt{3}$.

Hence it is an irrational number.

(iii) $5\sqrt{7}$

We can't simplify $\sqrt{7}$,

Hence it is an irrational number.

(iv) $-3\sqrt{8}$

$= -3 \times \sqrt{4 \times 2} = -3 \times 2 \times \sqrt{2} = -6\sqrt{2}$

We can't simplify $\sqrt{2}$,

Hence it is an irrational number.

(v) $\frac{2}{\sqrt{5}}$

We cannot simplify $\sqrt{5}$,

Hence it is an irrational number.

(vi) $\frac{4}{\sqrt{3}}$

We cannot simplify $\sqrt{3}$,

Hence it is an irrational number.

Question: 6

State in ea

Solution:

(i) True: $= \frac{2}{3} + \frac{1}{3} = 3$, always a rational number.

(ii) False: $= \sqrt{11} + (-\sqrt{11}) = 0$, which is a rational number.

(iii) True: $= \frac{5}{8} \times \frac{2}{3} = \frac{10}{24} = \frac{5}{12}$, always a rational number.

(iv) False: $= \sqrt{3} \times \sqrt{3} = 3$, which is a rational number.

(v) True : $= 2 + \sqrt{3}$, is always irrational.

(vi) False: $= 5 \times \sqrt{3} = 5\sqrt{3}$, is always an irrational number.

(vii) False : As rational numbers are on number line and all numbers on number line is real. Hence, every rational number is also Real.

(viii) True: As both rational and irrational numbers can be presented at number line are real. Hence they may be rational or irrational.

(ix) True: π = 3.141592653...... non terminating decimal form and $\frac{22}{7}$ is a fractional form.

Exercise : 1D

Question: 1

Add:

Solution:

(i) $(2\sqrt{3} - 5\sqrt{2})\,and\,(\sqrt{3} + 2\sqrt{2})$

Adding by making pairs,

$= 2\sqrt{3} + \sqrt{3} - 5\sqrt{2} + 2\sqrt{2} = (3\sqrt{3} - 3\sqrt{2})$

(ii) $(2\sqrt{2} + 5\sqrt{3} - 7\sqrt{5})\,and\,(3\sqrt{3} - \sqrt{2} + \sqrt{5})$

Adding by making pairs,

$= 2\sqrt{2} - \sqrt{2} + 5\sqrt{3} + 3\sqrt{3} - 7\sqrt{5} + \sqrt{5} = (\sqrt{2} + 8\sqrt{3} - 6\sqrt{5})$

(iii) $\left(\frac{2}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + 6\sqrt{11}\right)\,and\,\left(\frac{1}{3}\sqrt{7} + \frac{3}{2}\sqrt{2} - \sqrt{11}\right)$

Adding by making pairs,

$= \frac{2}{3}\sqrt{7} + \frac{1}{3}\sqrt{7} - \frac{1}{2}\sqrt{2} + \frac{3}{2}\sqrt{2} + 6\sqrt{11} - \sqrt{11} = \left(\frac{3}{3}\sqrt{7} + \frac{2}{2}\sqrt{2} + 5\sqrt{11}\right)$

$= \sqrt{7} + \sqrt{2} + 5\sqrt{11}$.

Question: 2

Multiply:

Solution:

(i) $3\sqrt{5}\,by\,2\sqrt{5}$

$= 3\sqrt{5} \times 2\sqrt{5} = (3 \times 2)(\sqrt{5} \times \sqrt{5}) = 6 \times 5 = 30.$

(ii) $6\sqrt{15}\ by\ 4\sqrt{3}$

$= 6\sqrt{15} \times 4\sqrt{3} = (6 \times 4)(\sqrt{15} \times \sqrt{3})$

$= 24 \times \sqrt{45} = 24 \times \sqrt{9 \times 5}$

$= 24 \times 3\sqrt{5} = 72\sqrt{5}.$

(iii) $2\sqrt{6}\ by\ 3\sqrt{3}$

$= 2\sqrt{6} \times 3\sqrt{3} = (2 \times 3)(\sqrt{18}) = 6 \times \sqrt{9 \times 2}$

$= 6 \times 3\sqrt{2} = 18\sqrt{2}.$

(iv) $3\sqrt{8}\ by\ 3\sqrt{2}$

$= 3\sqrt{8} \times 3\sqrt{2} = (3 \times 3)(\sqrt{16}) = 9 \times 4 = 36.$

(v) $\sqrt{10}\ by\ \sqrt{40}$

$= \sqrt{10} \times \sqrt{40} = \sqrt{10 \times 40} = \sqrt{400} = 20.$

(vi) $3\sqrt{28}\ by\ 2\sqrt{7}$

$= 3\sqrt{28} \times 2\sqrt{7} = (3 \times 2)(\sqrt{4 \times 7 \times 7}) = 6 \times 2 \times 7 = 84.$

Question: 3

Divide:

Solution:

(i) $16\sqrt{6}\ by\ 4\sqrt{2}$

$= \frac{(16\sqrt{6})}{4\sqrt{2}} = \left(\frac{16}{4}\right)\left(\frac{\sqrt{6}}{\sqrt{2}}\right) = 4\sqrt{3}.$

(ii) $12\sqrt{15}\ by\ 4\sqrt{3}$

$= \frac{12\sqrt{15}}{4\sqrt{3}} = \left(\frac{12}{4}\right)\left(\frac{\sqrt{15}}{\sqrt{3}}\right) = 3\sqrt{5}.$

(iii) $18\sqrt{21}\ by\ 6\sqrt{7}$

$= \frac{18\sqrt{21}}{6\sqrt{7}} = \left(\frac{18}{6}\right)\left(\frac{\sqrt{21}}{\sqrt{7}}\right) = 3\sqrt{3}$

Question: 4

Simplify:

Solution:

(i) $(4 + \sqrt{2})(4 - \sqrt{2})$

$\because$ (a + b) (a - b) = $a^2 - b^2$

$= (4 + \sqrt{2})(4 - \sqrt{2}) = 4^2 - \sqrt{2}^2 = 16 - 2 = 14.$

(ii) $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})$

$\because$ (a + b) (a - b) = $a^2 - b^2$

$= (\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = \sqrt{5}^2 - \sqrt{3}^2 = 5 - 3 = 2.$

(iii) $\left(6-\sqrt{6}\right)\left(6+\sqrt{6}\right)$

$\because$ (a + b) (a - b) = a^2 - b^2

$= \left(6-\sqrt{6}\right)\left(6+\sqrt{6}\right) = 6^2 - \sqrt{6}^2 = 36 - 6 = 30.$

(iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{3}\right)$

$= \left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{2}-\sqrt{3}\right) = \left(\sqrt{10}-\sqrt{15}-\sqrt{4}+\sqrt{6}\right)$

$= \sqrt{10}-\sqrt{15}-2+\sqrt{6}$

(v) $\left(\sqrt{5}-\sqrt{3}\right)^2$

$\because$ (a - b)^2 = a^2 + b^2 - 2ab

$=\left(\sqrt{5}-\sqrt{3}\right)^2 = (\sqrt{5}^2 + \sqrt{3}^2 - 2 \times \sqrt{5} \times \sqrt{3} = 8 - 2\sqrt{15}.$

(vi) $\left(3-\sqrt{3}\right)^2$

$\because$ (a - b)^2 = a^2 + b^2 - 2ab

$= \left(3-\sqrt{3}\right)^2 = \left(3^2 + \sqrt{3}^2 - 2 \times 3 \times \sqrt{3}\right) = 12 - 6\sqrt{3}$

Question: 5

Represent <

Solution:

Let’s draw a line AB = 3.2 units

Extend this line from B to C by 1 unit.

Now find the mid-point M of AC.

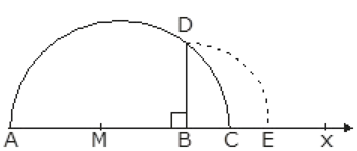
Take M as the center and MA as radius draw a semicircle.

Draw BD ⊥ AC intersecting the semi circle at D.

Then BD = $\sqrt{3.2}\,units$

With B as center and BD as radius, draw an arc, meeting AC produced at E.

Then BE = BD = $\sqrt{3.2}\,units.$



Question: 6

Represent <

Solution:

Lets draw a line AB = 7.28 units

Extend this line from B to C by 1 unit.

Now find the mid-point M of AC.

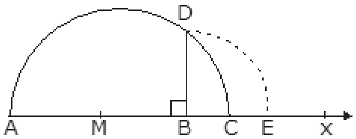
Take M as the center and MA as radius draw a semicircle.

Draw BD ⊥ AC intersecting the semi circle at D.

Then BD = $\sqrt{7.28}\,units$

With B as center and BD as radius, draw an arc, meeting AC produced at E.

Then BE = BD = $\sqrt{7.28}\ units$.



Question: 7

Mention the

Solution:

Closure property of addition of rational numbers:

The sum of two rational numbers is always a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\left(\frac{a}{b} + \frac{c}{d}\right)$ is also a rational number.

Example:

Consider the rational numbers $\frac{1}{3}$ and $\frac{3}{4}$ Then,

$= \frac{1}{3} + \frac{3}{4} = \frac{4+9}{12} = \frac{13}{12}$, is a rational number

Commutative property of addition of rational numbers:

Two rational numbers can be added in any order.

Thus for any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have

$= \left(\frac{a}{b} + \frac{c}{d}\right) = \left(\frac{c}{d} + \frac{a}{b}\right)$

Example:

$= \frac{1}{2} + \frac{3}{4} = \frac{3}{4} + \frac{1}{2}$

$= \frac{2+3}{4} = \frac{3+2}{4}$

$= \frac{5}{4} = \frac{5}{4}$

Existence of additive identity property of addition of rational numbers:

0 is a rational number such that the sum of any rational number and 0 is the rational number itself.

Thus,

$= \frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}$, for every rational number $\frac{a}{b}$

0 is called the additive identity for rationals.

Example:

$= \left(\frac{2}{5} + 0\right) = \left(\frac{2}{5} + \frac{0}{5}\right) = \frac{2+0}{5} = \frac{2}{5}$,

Existence of additive inverse property of addition of rational numbers:

For every rational number $\frac{a}{b}$, there exists a rational number $-\frac{a}{b}$

such that $\left(\frac{a}{b} + \left(-\frac{a}{b}\right)\right) = \frac{\{a + (-a)\}}{b} = \frac{0}{b} = 0$ and similarly, $\left(-\frac{a}{b} + \frac{a}{b}\right) = 0$.

Thus, $\left(\frac{a}{b}+\left(-\frac{a}{b}\right)\right)=\left(-\frac{a}{b}+\frac{a}{b}\right)=0$

$=-\frac{a}{b}$ is called the additive inverse of $\frac{a}{b}$

Example:

$=\left(\frac{3}{5}+\left(-\frac{3}{5}\right)\right)=\frac{\{3+(-3)\}}{5}=\frac{0}{5}=0$ and similarly, $\left(-\frac{3}{5}+\frac{3}{5}\right)=0$

Thus, $\frac{3}{5}$ and $-\frac{3}{5}$ are additive inverses of each other.

Associative property of addition of rational numbers:

While adding three rational numbers, they can be grouped in any order.

Thus, for any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$, we have

$=\left(\frac{a}{b}+\frac{c}{d}\right)+\left(\frac{e}{f}\right)=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$

Example:

Consider three rational numbers, $-\frac{2}{3}, \frac{5}{7}$ and $\frac{1}{6}$ Then,

$=\left(-\frac{2}{3}+\frac{5}{7}\right)+\frac{1}{6}=-\frac{2}{3}+\left(\frac{5}{7}+\frac{1}{6}\right)$

$=\frac{-14+15}{21}+\frac{1}{6}=-\frac{2}{3}+\frac{30+7}{42}$

$=\frac{2+7}{42}=\frac{-28+37}{42}$

$=\frac{9}{42}=\frac{9}{42}$.

Closure property of multiplication of rational numbers:

The product of two rational numbers is always a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational number.

Example:

Consider the rational numbers $\frac{1}{3}$ and $\frac{2}{7}$. Then,

Commutative property of multiplication of rational numbers:

Two rational numbers can be multiplied in any order.

Thus, for any rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we have:

$=\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$

Example:

Let us consider the rational numbers $\frac{4}{5}$ and $\frac{2}{7}$ Then,

$=\left(\frac{4}{5}\times\frac{2}{7}\right)=\left(\frac{2}{7}\times\frac{4}{5}\right)=\frac{8}{35}$ and $\left(\frac{4}{5}\times\frac{2}{7}\right)=\frac{4\times2}{5\times7}=\frac{8}{35}$.

Therefore, $\left(\frac{4}{5}\times\frac{2}{7}\right)=\left(\frac{2}{7}\times\frac{4}{5}\right)$

Associative property of multiplication of rational numbers:

While multiplying three or more rational numbers, they can be grouped in any order.

Thus, for any rationals $\frac{a}{b}, \frac{c}{d}$, and $\frac{e}{f}$ we have:

$= \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$

Example:

Consider the rationals $-\frac{5}{2}$, $-\frac{7}{4}$ and $\frac{1}{3}$, we have

$=\left(-\frac{5}{2} \times -\frac{7}{4}\right) \times \frac{1}{3} = \left\{\left(-\frac{5}{7}\right) \times \left(-\frac{7}{4} \times \frac{1}{3}\right)\right\}$

$= \left(\frac{35}{8}\right) \times \frac{1}{3} = \left(-\frac{5}{7}\right)\left(-\frac{7}{12}\right)$

$= \frac{35}{24} = \frac{35}{24}.$

Existence of multiplicative identity property:

For any rational number $\frac{a}{b}$, we have $\left(\frac{a}{b} \times 1\right) = \left(1 \times \frac{a}{b}\right)$

1 is called the multiplicative identity for rationals.

Example:

Consider the rational number $\frac{3}{4}$. Then, we have

$= \left(\frac{3}{4} \times 1\right) = \left(\frac{3}{4} \times \frac{1}{1}\right) = \frac{3 \times 1}{4 \times 1} = \frac{3}{4}$

Existence of multiplicative inverse property:

Every nonzero rational number $\frac{a}{b}$ has its multiplicative inverse $\frac{b}{a}$.

Thus, $\left(\frac{a}{b} \times \frac{b}{a}\right) = \left(\frac{b}{a} \times \frac{a}{b}\right) = 1$

$= \frac{b}{a}$ is called the reciprocal of $\frac{a}{b}$.

Clearly, zero has no reciprocal.

Reciprocal of 1 is 1 and the reciprocal of (-1) is (-1)

Exercise : 1E

Question: 1

Rationalise

Solution:

$\frac{1}{\sqrt{7}}$

By rationalization the denominator, we get,

$= \frac{1}{\sqrt{7}} = \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}}{7}$

Question: 2

Rationalise

Solution:

$\frac{\sqrt{5}}{2\sqrt{3}}$

By rationalization the denominator, we get,

$= \frac{\sqrt{5}}{2\sqrt{3}} = \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}} = \frac{2 \times \sqrt{5} \times \sqrt{3}}{\left(2\sqrt{3}\right)^2} = \frac{2\sqrt{15}}{12} = \frac{\sqrt{15}}{6}.$

Question: 3

Rationalise

Solution:

$\frac{1}{(2+\sqrt{3})}$

By rationalization the denominator, we get,

$= \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}.$

Question: 4

Rationalise

Solution:

$\frac{1}{(\sqrt{5}-2)}$

By rationalization the denominator, we get,

$= \frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{\sqrt{5}^2-2^2} = \frac{\sqrt{5}+2}{5-4} = \sqrt{5} + 2\,.$

Question: 5

Rationalise

Solution:

$\frac{1}{(5+3\sqrt{2})}$

By rationalization the denominator, we get,

$= \frac{1}{5+3\sqrt{2}} = \frac{1}{5+3\sqrt{2}} \times \frac{(5-3\sqrt{2})}{5-3\sqrt{2}} = \frac{5-3\sqrt{2}}{5^2-(3\sqrt{2})^2} = \frac{(5-3\sqrt{2})}{25-18} = \frac{5-3\sqrt{2}}{7}.$

Question: 6

Rationalise

Solution:

$\frac{1}{(\sqrt{6}-\sqrt{5})}$

By rationalization the denominator, we get,

$= \frac{1}{\sqrt{6}-\sqrt{5}} = \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}^2-\sqrt{5}^2} = \frac{\sqrt{6}+\sqrt{5}}{6-5} = \sqrt{6} + \sqrt{5}.$

Question: 7

Rationalise

Solution:

$\frac{4}{(\sqrt{7}+\sqrt{3})}$

By rationalization the denominator, we get,

$= \frac{4}{\sqrt{7}+\sqrt{3}} = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \left(\frac{4(\sqrt{7}-\sqrt{3})}{\sqrt{7}^2-\sqrt{3}^2}\right) = \frac{4(\sqrt{7}-\sqrt{3})}{4} = \sqrt{7}-\sqrt{3}.$

Question: 8

Rationalise

Solution:

$\frac{\sqrt{3}-1}{\sqrt{3}+1}$

By rationalization the denominator, we get,$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3}-1)^2}{\sqrt{3}^2-1^2} = \frac{(3+1-2\sqrt{3})}{3-1} = \frac{2(2-\sqrt{3})}{2} = 2-\sqrt{3}.$

Question: 9

Rationalise

Solution:

$\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$

By rationalization the denominator, we get,

$= \frac{(3-2\sqrt{2})}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} = \frac{(3-2\sqrt{2})^2}{3^2-(2\sqrt{2})^2} = \frac{9+8-12\sqrt{2}}{9-8} = \frac{17-12\sqrt{2}}{1} = 17-12\sqrt{2}.$

Question: 10

Find the va

Solution:

By rationalizing the L.H.S we get,

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{\sqrt{3}^2-1^2} = \frac{3+1+2\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{2} = \left(\frac{2(2+\sqrt{3})}{2}\right) = 2+\sqrt{3}$

Putting LHS = RHS, we get,

= 2 + √3 = a + b√3

Clearly, a = 2 and b = 1.

Question: 11

Find the va

Solution:

By rationalizing the L.H.S we get,

$= \frac{(3+\sqrt{2})}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{(3+\sqrt{2})^2}{3^2-\sqrt{2}^2} = \frac{9+2+6\sqrt{2}}{9-2} = \frac{11+6\sqrt{2}}{7}$

Putting LHS = RHS, we get,

$= \frac{11+6\sqrt{2}}{7} = a + b\sqrt{2}$

$= \frac{11}{7} + \frac{6}{7}\sqrt{2} = a + b\sqrt{2}$

Clearly, a = $\frac{11}{7}$ $and\ b = \frac{6}{7}$.

Question: 12

Find the va

Solution:

By rationalizing the L.H.S we get,

$= \frac{5-\sqrt{6}}{5+\sqrt{6}} \times \frac{5-\sqrt{6}}{5-\sqrt{6}} = \frac{(5-\sqrt{6})^2}{5^2-\sqrt{6}^2} = \frac{25+6-10\sqrt{6}}{25-6} = \frac{31-10\sqrt{6}}{19}$

Putting LHS = RHS, we get,

$= \frac{31-10\sqrt{6}}{19} = a - b\sqrt{6}$

$= \frac{31}{19} - \frac{10}{19}\sqrt{6} = a - b\sqrt{6}$

Clearly, $a = \frac{31}{19}$ and $b = \frac{10}{19}$.

Question: 13

Find the va

Solution:

By rationalizing the L.H.S we get,

$= \frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}} = \frac{35-20\sqrt{3}+14\sqrt{3}-24}{7^2-(4\sqrt{3})^2} = \frac{11-6\sqrt{3}}{49-48} = 11 - 6\sqrt{3}.$

Putting LHS = RHS, we get,

= 11 - 6√3 = a - b√3

Clearly a = 11 and b = 6.

Question: 14

Simplify: <

Solution:

By taking LCM,

$\frac{\{(\sqrt{5}-1)(\sqrt{5}-1) + (\sqrt{5}+1)(\sqrt{5}+1)\}}{\sqrt{5}^2-1^2}$

$= \frac{\{(\sqrt{5}-1)^2 + (\sqrt{5}+1)^2\}}{4}$

$= \frac{\{(5+1-2\sqrt{5}) + (5+1+2\sqrt{5})\}}{4}$

$= \frac{12}{4} = 3.$

Question: 15

Simplify: <

Solution:

By taking LCM,

$= \left(\frac{4+\sqrt{5}}{4-\sqrt{5}} + \frac{4-\sqrt{5}}{4+\sqrt{5}}\right) = \frac{\{(4+\sqrt{5})^2 + (4-\sqrt{5})^2\}}{4^2-\sqrt{5}^2} = \frac{\{21+8\sqrt{5}+21-8\sqrt{5}\}}{11} = \frac{42}{11}.$

Question: 16

If

Solution:

Given that, $x = \left(4 - \sqrt{15}\right)$ $so, \frac{1}{x} = \frac{1}{4-\sqrt{15}}$

By rationalizing $\frac{1}{x}$, we get,

$= \frac{1}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}} = \frac{4+\sqrt{15}}{4^2-\sqrt{15}^2} = \frac{4+\sqrt{15}}{16-15} = 4 + \sqrt{15}$

Hence, $x + \frac{1}{x} = 4 - \sqrt{15} + 4 + \sqrt{15} = 4 + 4 = 8.$

Question: 17

If

Solution:

Given that, $x = \left(2 + \sqrt{3}\right) so, \frac{1}{x} = \frac{1}{2+\sqrt{3}}$

By Rationalizing $\frac{1}{x}$ $we\ get,$

$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{2^2-\sqrt{3}^2} = \frac{2-\sqrt{3}}{1} = 2 - \sqrt{3}$

Now, $x^2 + \frac{1}{x^2} = \left(2 + \sqrt{3}\right)^2 + \left(2 - \sqrt{3}\right)^2 = 7 + 4\sqrt{3} + 7 - 4\sqrt{3}$

= 7 + 7 = 14

Question: 18

Show that <

Solution:

By rationalizing LHS we get,

$= \left\{\left(\frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}}\right) - \left(\frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}}\right) + \left(\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}\right) - \left(\frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right) + \left(\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}\right)\right\}$

$= \frac{3+\sqrt{8}}{(9-8)} - \frac{\sqrt{8}+\sqrt{7}}{(8-7)} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$

$= 3 + \sqrt{8} - \sqrt{8} - \sqrt{7} + \sqrt{7} + \sqrt{6} - \sqrt{6} - \sqrt{5} + \sqrt{5} + 2 = 3 + 2 = 5$

Clearly, LHS = RHS,

Hence, proved.

Exercise : 1F

Question: 1

Simplify:

Solution:

(i) $(6^{2/5} \times 6^{3/5})$

We know that powers get added in multiplication, so,

$= \left(6^{\frac{2}{5}} \times 6^{\frac{3}{5}}\right)$

$= 6^{\left(\frac{2}{5}\right)+\left(\frac{3}{5}\right)}$

$= 6^{\frac{2+3}{5}}$

$= 6^{\frac{5}{5}} = 6^1 = 6.$

(ii) $(3^{1/2} \times 3^{1/3})$

We know that powers get added in multiplication, so,

$= 3^{\frac{1}{2}} \times 3^{\frac{1}{3}}$

$= 3^{\frac{3+2}{6}}$

$= 3^{\frac{5}{6}}$.

(iii) $(7^{5/6} \times 7^{2/3})$

We know that powers get added in multiplication, so,

$= 7^{\frac{5}{6}} \times 7^{\frac{2}{3}}$

$= 7^{\left(\frac{5}{6}\right)+\left(\frac{2}{3}\right)}$

$= 7^{\frac{5+4}{6}}$

$= 7^{\frac{9}{6}}$

$= 7^{\frac{3}{2}}$

Question: 2

Simplify:

Solution:

(i) $\frac{6^{1/4}}{6^{1/5}}$

We know that powers get subtracted in dividing, so,

$\frac{6^{\frac{1}{4}}}{6^{\frac{1}{5}}}$

$= 6^{\frac{1}{4}-\frac{1}{5}}$

$= 6^{\left(\frac{5-4}{20}\right)}$

$= 6^{\frac{1}{20}}$

(ii) $\frac{8^{1/2}}{8^{2/3}}$

We know that powers get subtracted in dividing, so,

$= \frac{8^{\frac{1}{2}}}{8^{\frac{2}{3}}}$

$= 8^{\frac{1}{2}-\frac{2}{3}}$

$= 8^{\frac{3-4}{6}}$

$= 8^{-\frac{1}{6}}$

(iii) $\frac{5^{6/7}}{5^{2/3}}$

We know that powers get subtracted in dividing, so,

$= \frac{5^{\frac{6}{7}}}{5^{\frac{2}{3}}}$

$= 5^{\frac{6}{7}-\frac{2}{3}}$

$= 5^{\frac{(18-14)}{21}}$

$= 5^{\frac{4}{21}}$

Question: 3

Simplify:

Solution:

(i) $3^{1/4} \times 5^{1/4}$

We know that when the powers are same then only numbers get multiplied, so,

$= 3^{\frac{1}{4}} \times 5^{\frac{1}{4}} = (3 \times 5)^{\frac{1}{4}} = 15^{\frac{1}{4}}$

(ii) $2^{5/8} \times 3^{5/8}$

We know that when the powers are same then only numbers get multiplied, so,

$= 2^{\frac{5}{8}} \times 3^{\frac{5}{8}} = (2 \times 3)^{\frac{5}{8}} = 6^{\frac{5}{8}}$

(iii) $6^{1/2} \times 7^{1/2}$

We know that when the powers are same then only numbers get multiplied, so,

$= 6^{\frac{1}{2}} \times 7^{\frac{1}{2}} = (6 \times 7)^{\frac{1}{2}} = 42^{\frac{1}{2}}$

Question: 4

Simplify:

Solution:

(i) $(3^4)^{1/4}$

$= (3^4)^{\frac{1}{4}} = 3^{4 \times \frac{1}{4}} = 3^1 = 3.$

(ii) $(3^{1/3})^{1/4}$

$= \left(3^{\frac{1}{3}}\right)^{\frac{1}{4}} = (3)^{\frac{1}{3} \times \frac{1}{4}} = 3^{\frac{1}{12}}.$

(iii) $\left(\frac{1}{3^4}\right)^{1/2}$

$= \left(\frac{1}{3^4}\right)^{\frac{1}{2}} = (3^{-4})^{\frac{1}{2}} = (3)^{-4 \times \frac{1}{2}} = 3^{-2}.$

Question: 5

Evaluate:

Solution:

(i) $(49)^{\frac{1}{2}} = (7^2)^{\frac{1}{2}} = (7)^{2 \times \frac{1}{2}} = 7^1 = 7.$

(ii) $(125)^{\frac{1}{3}} = (5^3)^{\frac{1}{3}} = (5)^{3 \times \frac{1}{3}} = 5^1 = 5.$

(iii) $(64)^{\frac{1}{6}} = (2^6)^{\frac{1}{6}} = (2)^{6 \times \frac{1}{6}} = 2^1 = 2.$

Question: 6

Evaluate:

Solution:

(i) $(25)^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = (5)^{2 \times \frac{3}{2}} = 5^3 = 125.$

(ii) $(32)^{\frac{2}{5}} = (2^5)^{\frac{2}{5}} = (2)^{5 \times \frac{2}{5}} = 2^2 = 4.$

(iii) $(81)^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = (3)^{\left(4 \times \frac{3}{4}\right)} = 3^3 = 27.$

Question: 7

Evaluate:

Solution:

(i) $(64)^{-\frac{1}{2}} = (8^2)^{-\frac{1}{2}} = (8)^{2 \times -\frac{1}{2}} = 8^{-1} = \frac{1}{8}$.

(ii) $(8)^{-\frac{1}{3}} = (2^3)^{-\frac{1}{3}} = (2)^{3 \times -\frac{1}{3}} = 2^{-1} = \frac{1}{2}$.

(iii) $(81)^{-\frac{1}{4}} = (3^4)^{-\frac{1}{4}} = (3)^{4 \times -\frac{1}{4}} = 3^{-1} = \frac{1}{3}$.

Exercise : CCE QUESTIONS

Question: 1

Which of the foll

Solution:

A number which cannot be written as simple fraction is called Irrational Number.

3.141141114… is an irrational number because in this decimal is going forever without repeating.

Question: 2

Which of the foll

Solution:

$\sqrt{49} = 7 = \frac{7}{1}$

$\sqrt{\frac{9}{16}} = \frac{3}{4}$

$\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2 = \frac{2}{1}$

$\sqrt{5} = 2.236067977.....$

$\sqrt{5}$ is an irrational number because in this decimal is going forever without repeating.

Question: 3

Which of the foll

Solution:

0.3232232223… is an irrational number because decimal is going forever without repeating.

Question: 4

Which of the foll

Solution:

$\sqrt{225} = 15 = \frac{15}{1}$

So, $\sqrt{225}$ is a rational number.

Question: 5

Every rational nu

Solution:

Every rational number is a real number.

Question: 6

Between any two r

Solution:

Between any two rational numbers there are infinitely many rational numbers.

Question: 7

The decimal repre

Solution:

The decimal representation of a rational number is neither terminating nor repeating

Question: 8

The decimal repre

Solution:

The decimal representation of an irrational number is non-terminating and non-repeating. Ex. Value of pie.

Question: 9

Decimal expansion

Solution:

$\sqrt{2}$ = 1.41421356.......

So, Decimal expansion of $\sqrt{2}$ is a non-terminating and non-repeating decimal

Question: 10

The product of tw

Solution:

The product of two irrational number is sometimes rational and sometimes irrational

Case 1: $\sqrt{2}$ X $\sqrt{2}$ = 2 (this is rational)

Case 2: $\sqrt{5}\times\sqrt{2}=\sqrt{10}$ (this is irrational)

Question: 11

Which of the foll

Solution:

Every real number is either rational or irrational. If any real number can be written as fraction then it would be rational number otherwise it will be irrational number.

Question: 12

Which of the foll

Solution:

π is irrational and $\frac{22}{7}$ is rational

π = 3.14159265358979........ (this is non-terminating and non-repeating)

$\frac{22}{7}$ = 3.142857142857142857 (this is repeating)

Question: 13

A rational number

Solution:

$\sqrt{2}$ = 1.4142...

$\sqrt{3}$ = 1.7321.....

1.5 is a rational number between $\sqrt{2}\ and\ \sqrt{3}$.

Others would be 1.45, 1.55, 1.6 etc.

Question: 14

Solve the e

Solution:

$$(125)^{-1/3} = (5)^{3*\frac{-1}{3}} = 5^{-1}$$

$$= \frac{4}{2} = \frac{1}{5}$$

Question: 15

Solve the e

Solution:

$$\frac{(\sqrt{32}+\sqrt{48})}{(\sqrt{8}+\sqrt{12})} = \frac{\sqrt{16*2}+\sqrt{16*3}}{\sqrt{4*2}+\sqrt{4*3}}$$

$$= \frac{4\sqrt{2}+4\sqrt{3}}{2\sqrt{2}+2\sqrt{3}} = \frac{4(\sqrt{2}+\sqrt{3})}{2(\sqrt{2}+\sqrt{3})}$$

$$= \frac{4}{2} = 2$$

Question: 16

Solve the e

Solution:

$$\sqrt[3]{2}\times\sqrt[4]{2}\times\sqrt[12]{32} = (2)^{\frac{1}{3}}\times(2)^{\frac{1}{4}}\times(2^5)^{\frac{1}{12}}$$

$$= (2)^{\frac{1}{3}}\times(2)^{\frac{1}{4}}\times(2)^{\frac{5}{12}}$$

$$= (2)^{\frac{1}{3}+\frac{1}{4}+\frac{5}{12}}$$

$$= (2)^{\frac{12}{12}} = 2^1 = 2$$

Question: 17

Solve the e

Solution:

$$\left(\frac{81}{16}\right)^{3/4} = \frac{(81)^{\frac{3}{4}}}{(16)^{\frac{3}{4}}} = \frac{(3^4)^{\frac{3}{4}}}{(2^4)^{\frac{3}{4}}}$$

$$= \frac{3^3}{2^3} = \frac{27}{8}$$

Question: 18

Solve the e

Solution:

$$\sqrt[4]{(64)^2}$$

$$= \sqrt[4]{\left((8)^2\right)^2}$$

$$= \sqrt[4]{(8)^4} = 8$$

Question: 19

Solve the e

Solution:

$$\frac{1}{(\sqrt{4}-\sqrt{3})} = \frac{1}{(\sqrt{4}-\sqrt{3})} \times \frac{\sqrt{4}+\sqrt{3}}{\sqrt{4}+\sqrt{3}}$$

$$= \frac{1\left(\sqrt{4}+\sqrt{3}\right)}{\left(\sqrt{4}\right)^2 - \left(\sqrt{3}\right)^2} = \frac{1\left(\sqrt{4}+\sqrt{3}\right)}{4-3}$$

$$= \frac{1\left(\sqrt{4}+\sqrt{3}\right)}{1}$$

$$= 2+\sqrt{3}$$

Question: 20

Solve the e

Solution:

$$\frac{1}{(3+2\sqrt{2})}$$

$$= \frac{1}{(3+2\sqrt{2})} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3-2\sqrt{2}}{(3)^2 - \left(2\sqrt{2}\right)^2} = \frac{3-2\sqrt{2}}{9-8}$$

$$= 3-2\sqrt{2}$$

$$= \frac{1}{(3+2\sqrt{2})}$$

Question: 21

Solve the e

Solution:

$$\left((7+4\sqrt{3})+\frac{1}{(7+4\sqrt{3})}\right)=\left(\frac{(7+4\sqrt{3})^2+1}{(7+4\sqrt{3})}\right)$$

$$=\left(\frac{(7)^2+\left(4\sqrt{3}\right)^2+2\times 7\times 4\sqrt{3}+1}{(7+4\sqrt{3})}\right)$$

$$=\left(\frac{49+48+2\times 7\times 4\sqrt{3}+1}{(7+4\sqrt{3})}\right)$$

$$=\left(\frac{98+56\sqrt{3}}{(7+4\sqrt{3})}\right)$$

$$=\left(\frac{14(7+4\sqrt{3})}{(7+4\sqrt{3})}\right)$$

= 14

Question: 22

Solve the e

Solution:

$\frac{1}{\sqrt{2}}=\frac{1}{1.41}$ (When $\sqrt{2}=1.41$ only)

= 0.709

Question: 23

Solve the e

Solution:

$$\frac{1}{\sqrt{7}}=0.378$$

(When $\sqrt{7}=2.646$ only)

Question: 24

Solve the e

Solution:

$$\sqrt{10}\times\sqrt{15}=\sqrt{150}$$

$$=\sqrt{25\times 6}$$

$$=5\sqrt{6}$$

Question: 25

$(625)^{0.16}$

Solution:

$$=(625)^{0.16+0.09}$$

$$=(625)^{0.25}$$

$= \left(5^4\right)^{0.25}$

$= 5^{4\times 0.25}$

= 5

Question: 26

Solve the e

Solution:

$$\sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)}} = \sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{(2-1)}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{1}}$$

$$= \sqrt{(\sqrt{2}-1)^2}$$

$$= \sqrt{(1.414-1)^2}$$

$$= \sqrt{(0.414)^2}$$

= 0.414

Question: 27

The simplest form

Solution:

Let x = $1.\overline{6}$ then

X = 1.666 …………………(i)

10x = 16.666 …………….(ii)

On subtracting (i) from (ii) we get,

9x = 15

$x = \frac{15}{9}$

$x = \frac{5}{3}$

Question: 28

The simplest form

Solution:

Let x = $0.\overline{54}$ then

X = 0.545454………………(i)

10x = 5.45454 …………….(ii)

100x = 54.545454 ………..(iii)

On subtracting (i) from (iii) we get,

99x = 54

$x=\frac{54}{99}$

$x=\frac{6}{11}$

Question: 19

The simplest form

Solution:

Let x = $0.3\overline{2}$ then

X = 0.32222 ………………(i)

10x = 3.2222 …………….(ii)

100x = 32.222 ………..(iii)

On subtracting (ii) from (iii) we get,

90x = 29

$x=\frac{29}{90}$

Question: 30

28. The simplest

Solution:

Let x = $0.12\overline{3}$ then

X = 0.123333 ………………(i)

10x = 1.23333 …………….(ii)

100x = 12.3333 ………..(iii)

1000x = 123.3333………(iv)

On subtracting (iii) from (iv) we get,

900x = 111

$x=\frac{111}{900}$

$x=\frac{37}{300}$

Question: 31

An irrational num

Solution:

$\sqrt{5\times 6}$ = 5.477225575051…..

$\sqrt{5+6}$ = 3.316624790……..

The decimal representation of $\sqrt{5\times 6}$ is non-terminating and non-repeating.

So, $\sqrt{5\times 6}$ is an irrational number between 5 and 6.

Question: 32

An irrational num

Solution:

$\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

Now, $(\sqrt{2} + \sqrt{3})$ > $\sqrt{3}$

And, $\left(\sqrt{2} \times \sqrt{3}\right) > \sqrt{3}$

And, $5^{1/4}$ = 1.49534878122

And, $6^{1/4}$ = 1.56508458007

Both the options C & D are non-terminating and non-ending. So, both could be the answer.

Question: 33

An irrational num

Solution:

$\frac{1}{7}$ = 0.142857142857

$\frac{2}{7}$ = 0.2857142857142857

$\sqrt{\frac{1}{7} \times \frac{2}{7}}$ = 0.2020305089104421……. It is non-terminating and non-ending.

Question: 34

The question cons

Solution:

A rational number between $\frac{2}{5}$ and $\frac{3}{5}$ is : $\frac{1}{2}\left(\frac{2}{5} + \frac{3}{5}\right) = \frac{5}{10}$

Thus, the rational number between $\frac{2}{5}$ and $\frac{5}{10}$ is : $\frac{1}{2}\left(\frac{2}{5} + \frac{5}{10}\right) = \frac{9}{20}$

Thus, the rational number between $\frac{3}{5}$ and $\frac{5}{10}$ is : $\frac{1}{2}\left(\frac{3}{5} + \frac{5}{10}\right) = \frac{11}{20}$

Thus, three rational numbers between $\frac{2}{5}$ *and* $\frac{3}{5}$ *are* : $\frac{9}{10}, \frac{11}{20}$ *and* $\frac{10}{20}$

Question: 35

The question cons

Solution:

The square roots of numbers that are not a perfect square are members of the irrational numbers.

Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

Question: 36

The question cons

Solution:

e may or may not be π .

Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

Question: 37

The question cons

Solution:

The square roots of 3 is not a perfect square. So, it is an irrational number.

Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

Question: 38

Match the followi

Solution:

(a)-(r)

$6.\overline{54}\ is$ rational number.

(b)-(s)

$\frac{1}{7}$ an irrational number.

(c)-(q)

The value of $\frac{1}{7}$ is 0.142857142857142857........ So length of period is 6.

(d)-(p)

If $x=(2-\sqrt{3})$, then $\left(x^2+\frac{1}{x^2}\right)=14$

Question: 39

Match the followi

Solution:

(a)-(r)

$\sqrt[4]{(81)^{-2}}=(81)^{\frac{-2}{4}}$

$=(3^4)^{\frac{-2}{4}}$

$=3^{-2}$

$=\frac{1}{3^2}=\frac{1}{9}$

(b)-(s)

$\left(\frac{a}{b}\right)^{x-2}=\left(\frac{b}{a}\right)^{x-4}$

$\left(\frac{a}{b}\right)^{x-2}=\left(\frac{a}{b}\right)^{-(x-4)}$

x -2 = - x + 4

x + x = 4 + 2

2x = 6

X = 3

(c)-(p)

$$if\ x = (9 + 4\sqrt{5}),\ \text{then}\ \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = 4.$$

(d)-(q)

$$\left(\frac{81}{16}\right)^{-3/4} \times \left(\frac{64}{27}\right)^{-1/3} = \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left(\frac{4^3}{3^3}\right)^{-1/3}$$

$$= \left(\frac{3}{2}\right)^{4\times-3/4} \times \left(\frac{4}{3}\right)^{3\times-1/3}$$

$$= \left(\frac{3}{2}\right)^{-3} \times \left(\frac{4}{3}\right)^{-1}$$

$$= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{2}{9}$$

Question: 40

Give an example o

Solution:

Let two irrational numbers be **$10 + 2\sqrt{5}$ and $5 - 2\sqrt{5}$** (i) sum is rational**= $10 + 2\sqrt{5} + 5 - 2\sqrt{5}$**

= 15 (a rational number)(ii) product is rational are

= $(10 + 2\sqrt{5})(10 - 2\sqrt{5}) = (10)^2 - (2\sqrt{5})^2 = 100 - 20 = 80$ (a rational number)

Question: 41

If x is rational

Solution:

Let x = 10 and y = $2\sqrt{5}$

X + y = $10 + 2\sqrt{5}$ is irrational number.

Question: 42

Is the product of

Solution:

No

If you multiply any irrational number by the rational number zero, the result will be zero, which is rational.

$0 \times 2\sqrt{5} = 0$ is rational number.

Question: 43

Given an example

Solution:

$Take\, x = \sqrt[4]{3}$

Let x = $\sqrt[4]{2}$

Then $x^2 = (\sqrt[4]{2})^2 = \sqrt{2}$ is an irrational number

And $x^4 = (\sqrt[4]{2})^4 = 2$ is a rational number.

Question: 44

The number

Solution:

Let x = $4.\overline{17}$ then

X = 4.17171717………………(i)

10x = 41.7171717 …………….(ii)

100x = 417.171717 ………..(iii)

On subtracting (i) from (iii) we get,

99x = 413

X=$\frac{413}{99}$

Question: 45

if

Solution:

$x = (2+\sqrt{3})$

$$\left(x^2 + \frac{1}{x^2}\right)^2 = \left((2+\sqrt{3})^2 + \frac{1}{(2+\sqrt{3})^2})\right)^2$$

$$= \left((4+3+4\sqrt{3}) + \frac{1}{(4+3+4\sqrt{3})}\right)^2$$

$$= \left(7+4\sqrt{3} + \frac{1}{7+4\sqrt{3}}\right)^2$$

$$= \left(\frac{(7+4\sqrt{3})^2+1}{7+4\sqrt{3}}\right)^2$$

$$= \left(\frac{49+48+56\sqrt{3}+1}{7+4\sqrt{3}}\right)^2$$

$$= \left(\frac{49+48+56\sqrt{3}+1}{7+4\sqrt{3}}\right)^2$$

$$= \left(\frac{14(7+4\sqrt{3})}{7+4\sqrt{3}}\right)^2$$

$$= 14^2 = 196$$

Question: 46

If

Solution:

$$\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}$$

$$=\frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-1^2}=\frac{3+1-2\sqrt{3}}{2}$$

$$=\frac{4-2\sqrt{3}}{2}=\frac{2(2-\sqrt{3})}{2}$$

$$=2-\sqrt{3}$$

$$(a-b\sqrt{3}) = 2-\sqrt{3}$$

So, a =2 and b =1

Question: 47

If

Solution:

$$\frac{(4+\sqrt{5})}{(4-\sqrt{5})}\times\frac{(4+\sqrt{5})}{(4+\sqrt{5})}$$

$$=\frac{(4+\sqrt{5})^2}{4^2-(\sqrt{5})^2}=\frac{16+5+8\sqrt{5}}{16-5}$$

$$=\frac{21+8\sqrt{5}}{11}$$

$$(a+b\sqrt{5}) = \frac{21}{11}+\frac{8\sqrt{5}}{11}$$

So, a = $\frac{21}{11}$ and b =$\frac{8}{11}$

Question: 48

If

Solution:

$$\frac{(\sqrt{5}-1)}{(\sqrt{5}+1)}\times\frac{(\sqrt{5}-1)}{(\sqrt{5}-1)}+\frac{(\sqrt{5}+1)}{(\sqrt{5}-1)}\times\frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$$

$$=\frac{(\sqrt{5}-1)^2}{(\sqrt{5})^2-1^2}+\frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-1^2}$$

$$=\frac{5+1-2\sqrt{5}}{4}+\frac{5+1+2\sqrt{5}}{4}$$

$$=\frac{12}{4}=3$$

$(a+b\sqrt{5})$ =3

So, a = 3 and b=0.

Question: 49

If

Solution:

$$\frac{(\sqrt{2}+\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})}=\frac{(\sqrt{2}+\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})}\times\frac{(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})}$$

$$=\frac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$=\frac{6+3\sqrt{6}+2\sqrt{6}+6}{18-12}$$

$$=\frac{12+5\sqrt{6}}{6}$$

So, a = $\frac{12}{6}=2$ and b = $\frac{5}{6}$

Question: 50

If

Solution:

$$x=\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}=\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}\times\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}$$

$$=\frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3+2+2\sqrt{6}}{3-2}$$

$$=5+2\sqrt{6}$$

$$y=\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})}=\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})}\times\frac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$$

$$=\frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$=\frac{3+2-2\sqrt{6}}{3-2}$$

$$=5-2\sqrt{6}$$

Now, $(x^2+y^2)=(5+2\sqrt{6})^2+(5-2\sqrt{6})^2$

$$=25+24+20\sqrt{6}+25+24-20\sqrt{6}$$

= 98

Question: 51

If

Solution:

$x = \frac{1}{(2-\sqrt{3})} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$

$= \frac{(2+\sqrt{3})}{(2)^2 - (\sqrt{3})^2}$

$= \frac{(2+\sqrt{3})}{4-3}$

Then, x $= 2+\sqrt{3}$

Then, $x^2 = (2+\sqrt{3})^2 = 4+3+4\sqrt{3} = 7+4\sqrt{3}$

Ans, $x^3 = (2+\sqrt{3})(7+4\sqrt{3}) = 14+7\sqrt{3}+8\sqrt{3}+12 = 26+15\sqrt{3}$

Now,

$x^3 - 2x^2 - 7x + 5 =$

$= 26+15\sqrt{3} - 2(7+4\sqrt{3}) - 7(2+\sqrt{3}) + 5$

$= 26+15\sqrt{3} - 14 - 8\sqrt{3} - 14 - 7\sqrt{3} + 5$

$= 26 - 28 + 5$

= 3

Question: 52

if

Solution:

We have, $x = (3+\sqrt{8})$

$x^2 = (3+\sqrt{8})^2 = 9+8+6\sqrt{8}$

$= 17+6\sqrt{8}$

Then, $x^2 + \frac{1}{x^2} = \left((17+6\sqrt{8}) + \frac{1}{17+6\sqrt{8}}\right)$

$= \left(\frac{\left(17+6\sqrt{8}\right)^2 + 1}{17+6\sqrt{8}}\right)$

$= \frac{289+288+204\sqrt{8}+1}{17+6\sqrt{8}}$

$= \frac{578+204\sqrt{8}}{17+6\sqrt{8}} = \frac{34\left(17+6\sqrt{8}\right)}{17+6\sqrt{8}}$

= 34

Question: 53

if

Solution:

$x=(2+\sqrt{3})$

$x^2=(2+\sqrt{3})^2=4+3+4\sqrt{3}=7+4\sqrt{3}$

$x^3=(2+\sqrt{3})(7+4\sqrt{3})=14+7\sqrt{3}+8\sqrt{3}+12=26+15\sqrt{3}$

Now,

$$x^3+\frac{1}{x^3}=(26+15\sqrt{3})+\frac{1}{26+15\sqrt{3}}$$

$$=\frac{(26+15\sqrt{3})^2+1}{26+15\sqrt{3}}$$

$$=\frac{676+675+780\sqrt{3}+1}{26+15\sqrt{3}}$$

$$=\frac{1352+780\sqrt{3}}{26+15\sqrt{3}}$$

$$=\frac{52(26+15\sqrt{3})}{26+15\sqrt{3}}$$

$=52$

Question: 54

if

Solution:

$$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=\frac{x-1}{\sqrt{x}}$$

$$=\frac{3-2\sqrt{2}-1}{\sqrt{3-2\sqrt{2}}}=\frac{2-2\sqrt{2}}{\sqrt{3-2\sqrt{2}}}$$

$$=\frac{2(1-\sqrt{2})}{\sqrt{3-2\sqrt{2}}}=\frac{2(1-\sqrt{2})}{\sqrt{\sqrt{2^2}+1^2-2\sqrt{2}}}=\frac{2(1-\sqrt{2})}{\sqrt{(1-\sqrt{2})^2}}$$

$$=\frac{2(1-\sqrt{2})}{1-\sqrt{2}}=2$$

Question: 55

if

Solution:

$$\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)=\frac{x+1}{\sqrt{x}}$$

$$=\frac{5+2\sqrt{6}+1}{\sqrt{5+2\sqrt{6}}}=\frac{6+2\sqrt{6}}{\sqrt{5+2\sqrt{6}}}=\frac{6+2\sqrt{6}}{\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{6}}}$$

$$= \frac{6+2\sqrt{6}}{\sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{6}}} = \frac{6+2\sqrt{6}}{\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}}$$

$$= \frac{6+2\sqrt{6}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}(\sqrt{3}+\sqrt{2})}{\sqrt{3}+\sqrt{2}}$$

$$= 2\sqrt{3}$$

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

Find two rational

Solution:

If x and y are two rational numbers such that x<y then $\frac{1}{2}(x+y)$ is a rational number between x and y.

So, rational numbers will be:

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{2+3}{6}\right) = \frac{1}{2}\times\frac{5}{6} = \frac{5}{12}$$

$$\frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right) = \frac{1}{2}\left(\frac{4+5}{12}\right) = \frac{1}{2}\times\frac{9}{12} = \frac{9}{24} = \frac{3}{8}$$

$\frac{5}{12}$ *and* $\frac{3}{8}$ are two rational numbers lying between $\frac{1}{3}$ *and* $\frac{1}{2}$.

Question: 2

Find four rationa

Solution:

If x and y are two rational numbers such that x<y then $\frac{1}{2}(x+y)$ is a rational number between x and y.

So, rational numbers will be:

$$\frac{1}{2}\left(\frac{3}{5}+\frac{4}{5}\right) = \frac{1}{2}\left(\frac{3+4}{5}\right) = \frac{1}{2}\times\frac{7}{5} = \frac{7}{10}$$

$$\frac{1}{2}\left(\frac{3}{5}+\frac{7}{10}\right) = \frac{1}{2}\left(\frac{6+7}{10}\right) = \frac{1}{2}\times\frac{13}{10} = \frac{13}{20}$$

$$\frac{1}{2}\left(\frac{13}{20}+\frac{4}{5}\right) = \frac{1}{2}\left(\frac{13+16}{20}\right) = \frac{1}{2}\times\frac{29}{10} = \frac{29}{40}$$

$$\frac{1}{2}\left(\frac{4}{5}+\frac{29}{40}\right) = \frac{1}{2}\left(\frac{32+29}{40}\right) = \frac{1}{2}\times\frac{61}{40} = \frac{61}{80}$$

$\frac{7}{10}, \frac{13}{20}, \frac{29}{40}$ *and* $\frac{61}{80}$ are two rational numbers lying between $\frac{3}{5}$ *and* $\frac{4}{5}$.

Question: 3

Write four irrati

Solution:

Four irrational numbers are

0.1010010001…,

0.1212212221…,

0.13113313331…, and

0.1414414441…As they all have non terminating and non repeating decimal

Question: 4

Express

Solution:

$\sqrt[4]{1250} = \sqrt[4]{625 \times 2}$

$= \sqrt[4]{5^4 \times 2}$

$= 5\sqrt[4]{2}$

Question: 5

Express

Solution:

$\frac{2}{3}.\sqrt{18} = \sqrt{\frac{2}{3} \times \frac{2}{3} \times 18}$

$= \sqrt{\frac{2 \times 2 \times 18}{3 \times 3}}$

$= \sqrt{2 \times 2 \times 2}$

$= \sqrt{8}$

Question: 6

Divide

Solution:

$16\sqrt{75}\ \ by\ \ 5\sqrt{12}$ is given as:

$= \sqrt{\frac{16 \times 16 \times 75}{5 \times 5 \times 12}}$

$= \sqrt{\frac{16 \times 16 \times 3}{12}}$

$= \sqrt{\frac{4 \times 16 \times 3}{3}}$

$= \sqrt{4 \times 16}$

$= \sqrt{64} = 8$

Question: 7

Express

Solution:

Let x = $0.1\overline{23}$ then

X = 0.123232323………………(i)

10x = 1.23232323 …………….(ii)

100x = 123.23232323 ………..(iii)

On subtracting (ii) from (iii) we get,

90x = 122

$$X=\frac{122}{90}$$

$$X=\frac{61}{45}$$

Question: 8

If

Solution:

$$\frac{6}{(3\sqrt{2}-2\sqrt{3})}$$

$$=\frac{6}{(3\sqrt{2}-2\sqrt{3})}\times\frac{(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}+2\sqrt{3})}$$

$$=\frac{6(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$$

$$=\frac{6(3\sqrt{2}+2\sqrt{3})}{18-12}$$

$$=\frac{6(3\sqrt{2}+2\sqrt{3})}{6}$$

$$=3\sqrt{2}+2\sqrt{3}$$

So, a = 3 and b =2

Question: 9

The simplest form

Solution:

$$\left(\frac{64}{729}\right)^{-1/6}$$

$$=\left(\frac{2^6}{3^6}\right)^{-1/6}$$

$$=\left(\frac{2}{3}\right)^{6\times\frac{-1}{6}}$$

$$=\left(\frac{2}{3}\right)^{-1}$$

$= \frac{3}{2}$

Question: 10

Which of the foll

Solution:

0.1401401400014….. is an irrational number because it is non-ending and non-terminating.

Question: 11

Between two ratio

Solution:

There are infinitely many irrational numbers between two rational numbers.

Question: 12

Decimal represent

Solution:

Decimal representation of an irrational number is always non-terminating and non-repeating decimal.

Question: 13

If

Solution:

We have, $x = (7+5\sqrt{2})$

Then,

$x^2 = (7+5\sqrt{2})^2 = 49+50+70\sqrt{2}$

$= 99+70\sqrt{2}$

$= x^2 + \frac{1}{x^2} = \left((99+70\sqrt{2}) + \frac{1}{99+70\sqrt{2}} \right)$

$= \left(\frac{\left(99+70\sqrt{2}\right)^2 + 1}{99+70\sqrt{2}} \right)$

$= \frac{9801+9800+13860\sqrt{2}+1}{99+70\sqrt{2}}$

$= \frac{19602+13860\sqrt{2}}{99+70\sqrt{2}}$

$= \frac{198\left(99+70\sqrt{2}\right)}{99+70\sqrt{2}}$

$= 198$

Question: 14

Rationalize the d

Solution:

We have, $\left(\frac{5\sqrt{3}-4\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\right)$

$$=\left(\frac{5\sqrt{3}-4\sqrt{2}}{4\sqrt{3}+3\sqrt{2}}\right)\times\left(\frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}}\right)$$

$$=\left(\frac{\left(5\sqrt{3}-4\sqrt{2}\right)\left(4\sqrt{3}-3\sqrt{2}\right)}{\left(4\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2}\right)$$

$$=\left(\frac{60-16\sqrt{6}-15\sqrt{6}+24}{48-18}\right)$$

$$=\left(\frac{84-31\sqrt{6}}{30}\right)$$

Question: 15

Simplify: <

Solution:

Now, we have,

$$\frac{1}{(27)^{-1/3}}+\frac{1}{(625)^{-1/4}}=\frac{1}{(3^3)^{-1/3}}+\frac{1}{(5^4)^{-1/4}}$$

$$=\frac{1}{(3)^{-1}}+\frac{1}{(5)^{-1}}$$

= 3 + 5

= 8

Question: 16

Find the smallest

Solution:

$\sqrt[3]{6}, \sqrt[6]{24}, and\ \sqrt[4]{8}$ can be written as:

$$(6)^{\frac{1}{3}},(24)^{\frac{1}{6}} and\,(8)^{\frac{1}{4}}$$

Equalizing powers by multiplying and multiplying and dividing by 12, we get,

$$=(6)^{(12)/(3\times12)},(24)^{12/(2\times12)},(8)^{(12)/(4\times12)}$$

$$=\left(6^4\right)^{1/12},\left(24^6\right)^{1/12},\left(8^3\right)^{1/12}$$

$$=(1296)^{1/12},(191102976)^{1/12},(512)^{1/12}$$

Now, in ascending order,

$$=(512)^{1/12},(1296)^{1/12},(191102976)^{1/12}$$

So, $\sqrt[4]{8}$ is the smallest number.

Question: 17

Match the followi

Solution:

(a)-(q)

π is an irrational number.

(b)-(p)

$3.\overline{1416}$ is a rational number.

(c)-(s) $\frac{23}{99}$ = 0.23232323...... = $0.\overline{23}$

(d)-(r) $\frac{7}{30}$ = 0.23333... = $0.2\overline{3}$

Question: 18

If

Solution:

We have,

$$x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

On rationalizing we get,

$$=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$=\frac{5+3+2\sqrt{15}}{5-3}$$

$$=\frac{8+2\sqrt{15}}{2}$$

$$=4+\sqrt{15}$$

Now,

$$x^2=\left(4+\sqrt{15}\right)^2$$

$$=16+15+8\sqrt{15}$$

$$x^2=31+8\sqrt{15}$$

Now,

$$y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$= \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{5+3-2\sqrt{15}}{5-3}$

$= \frac{8-2\sqrt{15}}{2}$

$= 4-\sqrt{15}$

Now, $y^2 = \left(4-\sqrt{15}\right)^2$

$y^2 = 16+15-8\sqrt{15}$

$y^2 = 31-8\sqrt{15}$

$(x^2+y^2) = 31+8\sqrt{15}+31-8\sqrt{15}$

$= 62$

Question: 19

If

Solution:

We have,

$\frac{3}{(8\sqrt{2}+5\sqrt{5})}+\frac{2}{(8\sqrt{2}-5\sqrt{5})}$

$= \frac{24\sqrt{2}-15\sqrt{5}+16\sqrt{2}+10\sqrt{5}}{(8\sqrt{2})^2-(5\sqrt{5})^2}$

$= \frac{40\sqrt{2}-5\sqrt{5}}{128-125}$

$= \frac{40\sqrt{2}-5\sqrt{5}}{3}$

$= \frac{(40\times 1.41)-(5\times 2.24)}{3}$

$= \frac{56.4-11.2}{3}$

$= \frac{45.2}{3}$

= 15.7

Question: 20

Prove that

Solution:

We have, $\left(\frac{81}{16}\right)^{-3/4}\times\left\{\left(\frac{25}{9}\right)^{-3/2}\div\left(\frac{5}{2}\right)^{-3}\right\}$

$$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{5^2}{3^2}\right)^{-3/2} \div \left(\frac{5}{2}\right)^{-3}\right\}$$

$$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{5}{3}\right)^{2\times-\frac{3}{2}} \div \left(\frac{5}{2}\right)^{-3}\right\}$$

$$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{5}{3}\right)^{-3} \div \left(\frac{5}{2}\right)^{-3}\right\}$$

$$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\left(\frac{3}{5}\right)^{3} \div \left(\frac{2}{5}\right)^{3}\right\}$$

$$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \left\{\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{5}{2}\times\frac{5}{2}\times\frac{5}{2}\right\}$$

$$= \left(\frac{3^4}{2^4}\right)^{-3/4} \times \frac{27}{8}$$

$$= \left(\frac{3}{2}\right)^{-3} \times \frac{27}{8}$$

$$= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{27}{8}$$

= 1

Hence, Proved.