

8. Division of Algebraic Expressions

Exercise 8.1

1. Question

Write the degree of each of the following polynomials:

(i) $2x^3 + 5x^2 - 7$

(ii) $5x^2 - 3x + 2$

(iii) $2x + x^2 - 8$

(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$

(v) $3x^3 + 1$

(vi) 5

(vii) $20x^3 + 12x^2y^2 + 20$

Answer

(i) $2x^3 + 5x^2 - 7$

Degree is the highest power of the variable of a polynomial. In the given polynomial highest power is 3.

Therefore degree of the polynomial is 3.

(ii) $5x^2 - 3x + 2$

Degree is the highest power of the variable of a polynomial. In the given polynomial highest power is 2.

Therefore degree of the polynomial is 2.

(iii) $2x + x^2 - 8$

Degree is the highest power of the variable of a polynomial. In the given polynomial highest power is 2.

Therefore degree of the polynomial is 2.

(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$

Degree is the highest power of the variable of a polynomial. In the given polynomial highest power is 7.

Therefore degree of the polynomial is 7.

(v) $3x^3 + 1$

Degree is the highest power of the variable of a polynomial. In the given polynomial highest power is 3.

Therefore degree of the polynomial is 3.

(vi) 5

Degree is the highest power of the variable of a polynomial. In the given polynomial there is no variable term.

Therefore degree of the polynomial is 0.

(vii) $20x^3 + 12x^2y^2 + 20$

Degree is the highest power of the variable of a polynomial. In the given polynomial highest power is 4.

Therefore degree of the polynomial is 4.

2. Question

Which of the following expressions are not polynomials?

(i) $x^2 + 2x^{-2}$

(ii) $\sqrt{a}x + x^2 - x^3$

(iii) $3y^3 - \sqrt{5}y + 9$

(iv) $ax^{1/2} + ax + 9x^2 + 4$

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

Answer

(i) $x^2 + 2x^{-2}$

A polynomial never has negative or fractional power. In the given expression x has negative power.

Therefore it is not a polynomial.

(ii) $\sqrt{a}x + x^2 - x^3$

A polynomial always has positive power.

Therefore the given expression is a polynomial.

(iii) $3y^3 - \sqrt{5}y + 9$

A polynomial always has positive power.

Therefore the given expression is a polynomial.

(iv) $ax^{1/2} + ax + 9x^2 + 4$

A polynomial never has negative or fractional power. In the given expression x has fractional power.

Therefore it is not a polynomial.

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

A polynomial never has negative or fractional power. In the given expression x has negative power.

Therefore it is not a polynomial.

3. Question

Write each of the following polynomials in the standard form. Also, write their degree:

(i) $x^2 + 3 + 6x + 5x^4$

(ii) $a^2 + 4 + 5a^6$

(iii) $(x^3 - 1)(x^3 - 4)$

(iv) $(y^3 - 2)(y^3 + 11)$

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

Answer

(i) $x^2 + 3 + 6x + 5x^4$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5x^4 + x^2 + 6x + 3$ or $3 + 6x + x^2 + 5x^4$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 4

(ii) $a^2 + 4 + 5a^6$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5a^6 + a^2 + 4$ or $4 + a^2 + 5a^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(iii) $(x^3 - 1)(x^3 - 4)$

$$(x^3 - 1)(x^3 - 4) = x^6 - 4x^3 - x^3 + 4$$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $x^6 - 5x^3 + 4$ or $4 - 5x^3 + x^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(iv) $(y^3 - 2)(y^3 + 11)$

$$(y^3 - 2)(y^3 + 11) = y^6 + 11y^3 - 2y^3 - 22$$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $y^6 + 9y^3 - 22$ or $-22 + 9y^3 + y^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

$$\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right) = a^6 + \frac{16a^3}{17} - \frac{3a^3}{8} - \frac{6}{17}$$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^6 + \frac{11a^3}{17} - \frac{6}{17}$ or $-\frac{6}{17} + \frac{11a^3}{17} + a^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

$$\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right) = a^2 + \frac{4a}{3} + \frac{3a}{4} + 1$$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^2 + \frac{25a}{12} + 1$ or $1 + \frac{25a}{12} + a^2$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 2

Exercise 8.2

1. Question

Divide:

$$6x^3y^3z^2 \text{ by } 3x^2yz$$

Answer

$$\frac{6x^3y^3z^2}{3x^2yz} = \left(\frac{6}{3}x^{3-2}y^{3-1}z^{2-1}\right) = 2xy^2z \text{ [Using } a^n \div a^m = a^{n-m}]$$

2. Question

Divide:

$$15m^3n^3 \text{ by } 5m^2n^2$$

Answer

$$\frac{15m^3n^3}{5m^2n^2} = \left(\frac{15}{5}m^{3-2}n^{3-2}\right) = 3mn \text{ [Using } a^n \div a^m = a^{n-m}]$$

3. Question

Divide:

$$24a^3b^3 \text{ by } -8ab$$

Answer

$$\frac{24a^3b^3}{-8ab} = \left(\frac{24}{-8}a^{3-1}b^{3-1}\right) = -3a^2b^2 \text{ [Using } a^n \div a^m = a^{n-m}]$$

4. Question

Divide:

$$-21abc^2 \text{ by } -7abc$$

Answer

$$\frac{-21abc^2}{-7abc} = \left(\frac{-21}{-7}a^{1-1}b^{1-1}c^{2-1}\right) = 3a^0b^0c = 3c \text{ [Using } a^n \div a^m = a^{n-m} \text{ and } [a^0 = 1]}$$

5. Question

Divide:

$$xyz^2 \text{ by } -9xz$$

Answer

$$\frac{xyz^2}{-9xz} = \left(\frac{1}{-9}x^{1-1}yz^{2-1}\right) = -\frac{yz}{9} = -\frac{yx}{9} \text{ [Using } a^n \div a^m = a^{n-m} \text{ and } [a^0 = 1]}$$

6. Question

Divide:

$$-72a^4b^5c^8 \text{ by } -9a^2b^2c^3$$

Answer

$$\frac{-72a^4b^5c^8}{-9a^2b^2c^3} \\ = 8a^2b^3c^5$$

7. Question

Simplify:

$$\frac{16m^3y^2}{4m^2y}$$

Answer

$$\frac{16m^3y^2}{4m^2y} = \left(\frac{16}{4}m^{3-2}y^{2-1}\right) = 4my \text{ [Using } a^n \div a^m = a^{n-m}]$$

8. Question

Simplify:

$$\frac{32m^2n^2p^2}{4mnp}$$

Answer

$$\frac{32m^2n^2p^2}{4mnp} = \left(\frac{32}{4}m^{2-1}n^{2-1}p^{2-1}\right) = 8mnp \text{ [Using } a^n \div a^m = a^{n-m}]$$

Exercise 8.3**1. Question**

Divide:

$$x + 2x^2 + 3x^4 - x^5 \text{ by } 2x$$

Answer

$$\frac{x+2x^2+3x^4-x^5}{2x} = \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^4}{2x} - \frac{x^5}{2x} = \frac{1}{2} + x + \frac{3x^3}{2} - \frac{x^4}{2} \text{ [Using } a^n \div a^m = a^{n-m}]$$

2. Question

Divide:

$$y^4 - 3y^3 + \frac{1}{2}y^2 \text{ by } 3y$$

Answer

$$\frac{y^4-3y^3+\frac{1}{2}y^2}{3y} = \frac{y^4}{3y} - \frac{3y^3}{3y} + \frac{y^2}{6y} = \frac{y^3}{3} - y^2 + \frac{y}{6} \text{ [Using } a^n \div a^m = a^{n-m}]$$

3. Question

Divide:

$$-4a^3 + 4a^2 + aby \text{ by } 2a$$

Answer

$$-\frac{4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a} = -2a^2 + 2a + \frac{1}{2} \text{ [Using } a^n \div a^m = a^{n-m}]$$

4. Question

Divide:

$$-x^6 + 2x^4 + 4x^3 + 2x^2 \text{ by } \sqrt{2}x^2$$

Answer

$$-\frac{x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} = -\frac{x^4}{\sqrt{2}} + \frac{2x^2}{\sqrt{2}} + \frac{4x}{\sqrt{2}} = -\frac{x^4}{\sqrt{2}} + \sqrt{2}x^2 + \sqrt{2}x \text{ [Using } a^n \div a^m = a^{n-m}]$$

5. Question

Divide:

$$5z^3 - 6z^2 + 7z \text{ by } 2z$$

Answer

$$\frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z} = \frac{5z^2}{2} - 3z + \frac{7}{2} \text{ [Using } a^n \div a^m = a^{n-m}]$$

6. Question

Divide:

$$\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3^2 - 6a \text{ by } 3a$$

Answer

$$\frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{9}{3a} - \frac{6a}{3a} = \frac{\sqrt{3}a^3}{3} + \frac{2\sqrt{3}a^2}{3} + \frac{3}{a} - 2 \text{ [Using } a^n \div a^m = a^{n-m}]$$

Exercise 8.4

1. Question

Divide:

$$5x^3 - 15x^2 + 25x \text{ by } 5x$$

Answer

$$\frac{5x^3}{5x} - \frac{15x^2}{5x} + \frac{25x}{5x} = 5x^2 - 3x + 5 \text{ [Using } a^n \div a^m = a^{n-m}]$$

2. Question

Divide:

$$4z^3 + 6z^2 - z \text{ by } -\frac{1}{2}z$$

Answer

$$\frac{2 \times 4z^3}{-1z} + \frac{2 \times 6z^2}{-1z} - \frac{2 \times z}{-1z} = -8z^2 - 12z + 2 \text{ [Using } a^n \div a^m = a^{n-m}]$$

3. Question

Divide:

$$9x^2y - 6xy + 12xy^2 \text{ by } -\frac{3}{2}xy$$

Answer

$$\frac{2 \times 9x^2y}{-3xy} - \frac{2 \times 6xy}{-3xy} + \frac{2 \times 12xy^2}{-3xy} = -6x^2y + 4y - 8y \text{ [Using } a^n \div a^m = a^{n-m}]$$

4. Question

Divide:

$$3x^3y^2 + 2x^2y + 15xy \text{ by } 3xy$$

Answer

$$\frac{3x^3y^2}{3xy} + \frac{2x^2y}{3xy} + \frac{15xy}{3xy} = x^2y + \frac{2x}{3} + 5 \text{ [Using } a^n \div a^m = a^{n-m}]$$

5. Question

Divide:

$$x^3 + 7x + 12 \text{ by } x + 4$$

Answer

$$\begin{array}{r}
 x+4 \overline{) \begin{array}{c} x^2+7x+12 \\ x^2+4x \\ \hline 3x+12 \\ 3x+12 \\ \hline 0 \end{array}} x+3
 \end{array}$$

Ans: $x+3$

6. Question

Divide:

$$4y^2 + 3y + \frac{1}{2} \text{ by } 2y + 1$$

Answer

$$\begin{array}{r}
 2y+1 \overline{) \begin{array}{c} 4y^2+3y+\frac{1}{2} \\ 4y^2+2y \\ \hline y+\frac{1}{2} \\ y+\frac{1}{2} \\ \hline 0 \end{array}} 2y+\frac{1}{2}
 \end{array}$$

7. Question

Divide:

$$3x^3 + 4x^2 + 5x + 18 \text{ by } x + 2$$

Answer

$$\begin{array}{r}
 x+2 \overline{) \begin{array}{c} 3x^3+4x^2+5x+18 \\ 3x^3+6x^2 \\ \hline -2x^2+5x \\ -2x^2-4x \\ \hline 9x+18 \\ 9x+18 \\ \hline 0 \end{array}} 3x^2-2x+9
 \end{array}$$

8. Question

Divide:

$$14x^2 - 53x + 45 \text{ by } 7x - 9$$

Answer

$$\begin{array}{r}
 7x-9 \overline{) \begin{array}{c} 14x^2-53x+45 \\ 14x^2-18x \\ \hline -35x+45 \\ -35x+45 \\ \hline 0 \end{array}} 2x-5
 \end{array}$$

9. Question

Divide:

$$-21 + 71x - 31x^2 - 24x^3 \text{ by } 3 - 8x$$

Answer

$$\begin{array}{r}
 -8x + 3 \overline{) -24x^3 - 31x^2 + 71x - 21} \quad 3x^2 + 5x - 7 \\
 \underline{+ \quad -} \\
 -40x^2 + 71x \\
 \underline{-40x^2 + 15x} \\
 + \quad - \\
 56x - 21 \\
 56x - 21 \\
 \underline{- \quad +} \\
 0
 \end{array}$$

10. Question

Divide:

$$3y^4 - 3y^3 - 4y^2 - 4y \text{ by } y^2 - 2y$$

Answer

$$\begin{array}{r}
 y^2 - 2y \overline{) 3y^4 - 3y^3 - 4y^2 - 4y} \quad 3y^2 + 3y + 2 \\
 \underline{3y^4 - 6y^3} \\
 - \quad + \\
 3y^3 - 4y^2 \\
 \underline{3y^3 - 6y^2} \\
 - \quad + \\
 2y^2 - 4y \\
 \underline{2y^2 - 4y} \\
 - \quad + \\
 0
 \end{array}$$

11. Question

Divide:

$$2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3 \text{ by } 2y^3 + 1$$

Answer

$$\begin{array}{r}
 2y^3 + 1 \overline{) 2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3} \quad y^2 + 5y + 3 \\
 \underline{2y^5} \underline{+ y^2} \\
 10y^4 + 6y^3 + 5y + 3 \\
 \underline{10y^4} \underline{+ 5y} \\
 6y^3 + 3 \\
 \underline{6y^3} \underline{+ 3} \\
 - \quad - \\
 0
 \end{array}$$

12. Question

Divide:

$$x^4 - 2x^3 + 2x^2 + x + 4 \text{ by } x^2 + x + 1$$

Answer

$$\begin{array}{r}
 x^2 + x + 1 \overline{) x^4 - 2x^3 + 2x^2 + x + 4} \quad x^2 - 3x + 4 \\
 \underline{x^4 + x^3 + x^2} \\
 -3x^3 + x^2 + x + 4 \\
 \underline{-3x^3 - 3x^2 - 3x} \\
 + \quad + \\
 4x^2 + 4x + 4 \\
 \underline{4x^2 + 4x + 4} \\
 - \quad - \quad - \\
 0
 \end{array}$$

13. Question

Divide:

$$m^3 - 14m^2 + 37m - 26 \text{ by } m^2 - 12m + 13$$

Answer

$$\begin{array}{r}
 m^2 - 12m + 13 \overline{) m^3 - 14m^2 + 37m - 26} \quad m - 2 \\
 \underline{m^3 - 12m^2 + 13m} \\
 -2m^2 + 24m - 26 \\
 \underline{-2m^2 + 24m - 26} \\
 0
 \end{array}$$

14. Question

Divide:

$$x^4 + x^2 + 1 \text{ by } x^2 + x + 1$$

Answer

$$\begin{array}{r}
 x^2 + x + 1 \overline{) x^4 + 0x^3 + x^2 + 1} \quad x^2 - x + 1 \\
 \underline{x^4 + x^3 + x^2} \\
 -x^3 + 1 \\
 \underline{-x^3} \\
 \\
 \\
 \\
 \\
 0
 \end{array}$$

15. Question

Divide:

$$x^5 + x^4 + x^3 + x^2 + x + 1 \text{ by } x^3 + 1$$

Answer

$$\begin{array}{r}
 x^3 + 1 \overline{) x^5 + x^4 + x^3 + x^2 + x + 1} \quad x^2 + x + 1 \\
 \underline{x^5} \\
 x^4 + x^3 + x^2 + x + 1 \\
 \underline{x^4} \\
 \\
 \\
 \\
 0
 \end{array}$$

16. Question

Divide each of the following and find the quotient and remainder:

$$14x^3 - 5x^2 + 9x - 1 \text{ by } 2x - 1$$

Answer

$$\begin{array}{r}
 2x - 1 \overline{) 14x^3 - 5x^2 + 9x - 1} \quad 7x^2 + x + 5 \\
 \underline{14x^3 - 7x^2} \\
 2x^2 + 9x - 1 \\
 \underline{2x^2 - x} \\
 \\
 \\
 \\
 \\
 4
 \end{array}$$

Quotient: $7x^2 + x + 5$

Remainder: 4

17. Question

Divide each of the following and find the quotient and remainder:

$$3x^3 - x^2 - 10x - 3 \text{ by } x - 3$$

Answer

$$\begin{array}{r} x-3 \overline{) 3x^3 - x^2 - 10x - 3} \quad 3x^2 + 8x + 14 \\ \underline{3x^3 - 9x^2} \\ 8x^2 - 10x - 3 \\ \underline{8x^2 - 24x} \\ 14x - 3 \\ \underline{14x - 42} \\ 39 \end{array}$$

Quotient: $3x^2 + 8x + 14$

Remainder: 39

18. Question

Divide each of the following and find the quotient and remainder:

$$6x^3 + 11x^2 - 39x - 65 \text{ by } 3x^2 + 13x + 13$$

Answer

$$\begin{array}{r} 3x^2 + 13x + 13 \overline{) 6x^3 + 11x^2 - 39x - 65} \quad 2x - 5 \\ \underline{6x^3 + 26x^2 + 26x} \\ -15x^2 - 65x - 65 \\ \underline{-15x^2 - 65x - 65} \\ 0 \end{array}$$

Quotient: $2x - 5$

Remainder: 0

19. Question

Divide each of the following and find the quotient and remainder:

$$30x^4 + 11x^3 - 82x^2 - 12x + 48 \text{ by } 3x^2 + 2x - 4$$

Answer

$$\begin{array}{r} 3x^2 + 2x - 4 \overline{) 30x^4 + 11x^3 - 82x^2 - 12x + 48} \quad 10x^2 - 3x - 12 \\ \underline{30x^4 + 20x^3 - 40x^2} \\ -9x^3 - 42x^2 - 12x + 48 \\ \underline{-9x^3 - 6x^2 + 12x} \\ -36x^2 - 24x + 48 \\ \underline{-36x^2 - 24x + 48} \\ 0 \end{array}$$

Quotient: $10x^2 - 3x - 12$

Remainder: 0

20. Question

Divide each of the following and find the quotient and remainder:

$$9x - 4x^2 + 4 \text{ by } 3x^2 - 4x + 2$$

Answer

$$\begin{array}{r}
 3x^2 - 4x + 2 \overline{) 9x^4 + 0x^3 - 4x^2 + 4} \quad \left(3x^2 + 4x + 2 \right. \\
 \underline{9x^4 - 12x^3 + 6x^2} \\
 12x^3 - 10x^2 + 4 \\
 \underline{12x^3 - 16x^2 + 8x} \\
 6x^2 - 8x + 4 \\
 \underline{6x^2 - 8x + 4} \\
 0
 \end{array}$$

Quotient: $3x^2 + 4x + 2$

Remainder: 0

21. Question

Verify division algorithm i.e. Dividend = Divisor \times Quotient + Remainder, in each of the following. Also, write the quotient and remainder;

Dividend	Divisor
(i) $14x^2 + 13x - 15$	$7x - 4$
(ii) $15z^3 - 20z^2 + 13z - 12$	$3z - 6$
(iii) $6y^5 - 28y^3 + 3y^2 + 30y - 9$	$2x^2 - 6$
(iv) $34x - 22x^3 - 12x^4 - 10x^2 - 75$	$3x + 7$
(v) $15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6$	$3y - 2$
(vi) $4y^3 + 8y + 8y^2 + 7$	$2y^2 - y + 1$
(vii) $6y^4 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$	$2y^3 + 1$

Answer

(i)

$$\begin{array}{r}
 7x - 4 \overline{) 14x^2 + 13x - 15} \quad \left(2x + 3 \right. \\
 \underline{14x^2 - 8x} \\
 21x - 15 \\
 \underline{21x - 12} \\
 -3
 \end{array}$$

Dividend = Divisor \times Quotient + Remainder

$$14x^2 + 13x - 15 = (7x - 4) \times (2x + 3) + (-3)$$

$$14x^2 + 13x - 15 = 14x^2 + 21x - 8x - 12 - 3$$

$$14x^2 + 13x - 15 = 14x^2 + 13x - 15$$

(ii)

$$\begin{array}{r}
 3z - 6 \overline{) 15z^3 - 20z^2 + 13z - 12} \left(5z^2 + \frac{10z}{3} + 11 \right. \\
 \underline{15z^3 - 30z^2} \\
 10z^2 + 13z - 12 \\
 \underline{10z^2 - 20z} \\
 33z - 12 \\
 \underline{33z - 66} \\
 54
 \end{array}$$

Dividend = Divisor \times Quotient + Remainder

$$15z^3 - 20z^2 + 13z - 12 = (3z - 6) \times \left(5z^2 + \frac{10z}{3} + 11 \right) + 54$$

$$15z^3 - 20z^2 + 13z - 12 = 15z^3 + 10z^2 + 33z - 30z^2 - 20z + 54$$

$$15z^3 - 20z^2 + 13z - 12 = 15z^3 - 20z^2 + 13z - 12$$

(iii)

$$\begin{array}{r}
 2y^2 - 6 \overline{) 6y^5 - 28y^3 + 3y^2 + 30y - 9} \left(3y^3 - 5y + \frac{3}{2} \right. \\
 \underline{6y^5 - 18y^3} \\
 -10y^3 + 3y^2 + 30y - 9 \\
 \underline{-10y^3 + 30y} \\
 3y^2 - 9 \\
 \underline{3y^2 - 9} \\
 0
 \end{array}$$

Dividend = Divisor \times Quotient + Remainder

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = (2y^2 - 6) \times \left(3y^3 - 5y + \frac{3}{2} \right) + 0$$

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = 6y^5 - 10y^3 + 3y^2 - 18y^3 + 30y - 9$$

$$6y^5 - 28y^3 + 3y^2 + 30y - 9 = 6y^5 - 28y^3 + 3y^2 + 30y - 9$$

(iv)

$$\begin{array}{r}
 3x + 7 \overline{) -12x^4 - 22x^3 - 10x^2 + 34x - 75} \left(-4x^3 + 2x^2 - 8x + 30 \right. \\
 \underline{-12x^4 - 28x^3} \\
 6x^3 - 10x^2 + 34x - 75 \\
 \underline{6x^3 + 14x^2} \\
 -24x^2 + 34x - 75 \\
 \underline{-24x^2 - 56x} \\
 90x - 75 \\
 \underline{90x + 210} \\
 -285
 \end{array}$$

Dividend = Divisor \times Quotient + Remainder

$$-12x^4 - 22x^3 - 10x^2 + 34x - 75 = (3x + 7) \times (-4x^3 + 2x^2 - 8x + 30) - 285$$

$$-12x^4 - 22x^3 - 10x^2 + 34x - 75 = -12x^4 + 6x^3 - 24x^2 - 28x^3 + 14x^2 + 90x - 56x + 210 - 285$$

$$-12x^4 - 22x^3 - 10x^2 + 34x - 75 = -12x^4 - 22x^3 - 10x^2 + 34x - 75$$

(v)

$$\begin{array}{r}
 3y - 2 \overline{) 15y^4 - 16y^3 + 9y^2 - \frac{10y}{3} + 6} \overline{) 5y^3 - 2y^2 + \frac{5y}{3}} \\
 \underline{15y^4 - 10y^3} \phantom{+ 9y^2 - \frac{10y}{3} + 6} \\
 -6y^3 + 9y^2 - \frac{10y}{3} + 6 \\
 \underline{-6y^3 + 4y^2} \phantom{- \frac{10y}{3} + 6} \\
 5y^2 - \frac{10y}{3} + 6 \\
 \underline{5y^2 - \frac{10y}{3}} \\
 6
 \end{array}$$

Dividend = Divisor \times Quotient + Remainder

$$15y^4 - 16y^3 + 9y^2 - \frac{10y}{3} + 6 = (3y - 2) \times \left(5y^3 - 2y^2 + \frac{5y}{3}\right) + 6$$

$$15y^4 - 16y^3 + 9y^2 - \frac{10y}{3} + 6 = 15y^4 - 6y^3 + 5y^2 - 10y^3 + 4y^2 - \frac{10y}{3} + 6$$

$$15y^4 - 16y^3 + 9y^2 - \frac{10y}{3} + 6 = 15y^4 - 16y^3 + 9y^2 - \frac{10y}{3} + 6$$

(vi)

$$\begin{array}{r}
 2y^2 - y + 1 \overline{) 4y^3 + 8y^2 + 8y + 7} \overline{) 2y + 5} \\
 \underline{4y^3 - 2y^2 + 2y} \\
 10y^2 + 6y + 7 \\
 \underline{10y^2 - 5y + 5} \\
 11y + 2
 \end{array}$$

Dividend = Divisor \times Quotient + Remainder

$$4y^3 + 8y^2 + 8y + 7 = (2y^2 - y + 1) \times (2y + 5) + 11y + 2$$

$$4y^3 + 8y^2 + 8y + 7 = 4y^3 + 10y^2 - 2y^2 - 5y + 2y + 5 + 11y + 2$$

$$4y^3 + 8y^2 + 8y + 7 = 4y^3 + 8y^2 + 8y + 7$$

(vii)

$$\begin{array}{r}
 2y^3 + 1 \overline{) 6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6} \overline{) 3y^2 + 2y + 2} \\
 \underline{6y^5} \\
 4y^4 + 4y^3 + 4y^2 + 27y + 6 \\
 \underline{4y^4} \\
 4y^3 + 4y^2 + 25y + 6 \\
 \underline{4y^3} \\
 4y^2 + 25y + 4
 \end{array}$$

Dividend = Divisor \times Quotient + Remainder

$$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6 = (2y^3 + 1) \times (3y^2 + 2y + 2) + 4y^2 + 25y + 4$$

$$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6 = 6y^5 + 4y^4 + 4y^3 + 3y^2 + 2y + 2 + 4y^2 + 25y + 4$$

$$6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6 = 6y^5 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$$

22. Question

Divide $15y^4 + 16y^3 + \frac{10}{3}y - 9y^2 - 6$ by $3y - z$ Write down the coefficients of the terms in the quotient.

Answer

$$\begin{array}{r}
 3y - 2 \overline{) 15y^4 + 16y^3 - 9y^2 + \frac{10y}{3} - 6} \left(5y^3 + \frac{26y^2}{3} + \frac{25y}{9} + \frac{80}{27} \right. \\
 \underline{15y^4 - 10y^3} \phantom{- 9y^2 + \frac{10y}{3} - 6} \\
 26y^3 - 9y^2 + \frac{10y}{3} - 6 \\
 \underline{26y^3 - \frac{52y^2}{3}} \phantom{+ \frac{10y}{3} - 6} \\
 \frac{25y^2}{3} + \frac{10y}{3} - 6 \\
 \underline{\frac{25y^2}{3} - \frac{50y}{9}} \\
 -\frac{80y}{9} - 6 \\
 \underline{-\frac{80y}{9} - \frac{160}{27}} \\
 -\frac{2}{27}
 \end{array}$$

Quotient: $5y^3 + \frac{26y^2}{3} + \frac{25y}{9} + \frac{80}{27}$

Coefficient of $y^3 = 5$; Coefficient of $y^2 = \frac{26}{3}$; Coefficient of $y = \frac{25}{9}$; Constant term = Coefficient of $y^2 = \frac{80}{27}$

23. Question

Using division of polynomials state whether

- (i) $x + 6$ is a factor of $x^2 - x - 42$
- (ii) $4x-1$ is a factor of $4x^2 - 13x - 12$
- (iii) $2y-5$ is a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$
- (iv) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$
- (v) $z^2 + 3$ is a factor of $z^5 - 9z$
- (vi) $2x^2 - x + 3$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

Answer

- (i) $x + 6$ is a factor of $x^2 - x - 42$

$$\begin{array}{r}
 x + 6 \overline{) x^2 - x - 42} \left(x - 7 \right. \\
 \underline{x^2 + 6x} \\
 -7x - 42 \\
 \underline{-7x - 42} \\
 0
 \end{array}$$

Quotient: $x - 7$

Remainder: 0

Since remainder is 0 therefore $(x + 6)$ is a factor of $x^2 - x - 42$

- (ii) $4x-1$ is a factor of $4x^2 - 13x - 12$

$$\begin{array}{r}
 4x - 1 \overline{) 4x^2 - 13x - 12} \left(x - 3 \right. \\
 \underline{4x^2 - x} \\
 -12x - 12 \\
 \underline{-12x + 3} \\
 15
 \end{array}$$

Quotient: $x - 3$

Remainder: 15

Since remainder is 15 therefore $(4x - 1)$ is **NOT** a factor of $4x^2 - 13x - 12$

(iii) $2y-5$ is a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$

$$\begin{array}{r}
 2y - 5 \overline{) 4y^4 - 10y^3 - 10y^2 + 30y - 15} \left(2y^3 - 5y + \frac{5}{2} \right. \\
 \underline{4y^4 - 10y^3} \\
 -10y^2 + 30y - 15 \\
 \underline{-10y^2 + 25y} \\
 + -5 \\
 \underline{-5y - 15} \\
 5y - \frac{25}{2} \\
 \underline{-5y - \frac{25}{2}} \\
 \phantom{-5y - \frac{25}{2}} -\frac{5}{2}
 \end{array}$$

Quotient: $2y^3 - 5y + \frac{5}{2}$

Remainder: $-\frac{5}{2}$

Since remainder is $-\frac{5}{2}$ therefore $(2y - 5)$ is **NOT** a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$

(iv) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

$$\begin{array}{r}
 3y^2 + 5 \overline{) 6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35} \left(2y^3 + 2y + \frac{4}{3} \right. \\
 \underline{6y^5 + 15y^4} \\
 -16y^3 + 4y^2 + 10y - 35 \\
 \underline{-16y^3 + 10y^2} \\
 -10y^2 + 10y - 35 \\
 \underline{-10y^2 + 20y} \\
 -15y - 35 \\
 \underline{-15y - 25} \\
 -10 \\
 \underline{-10} \\
 -\frac{125}{3}
 \end{array}$$

Quotient: $2y^3 + 2y + \frac{4}{3}$

Remainder: $-\frac{125}{3}$

Since remainder is $-\frac{125}{3}$ therefore $(3y^2 + 5)$ is **NOT** a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

(v) $z^2 + 3$ is a factor of $z^5 - 9z$

$$\begin{array}{r}
 z^2 + 3 \overline{) z^5 - 9z} \left(z^3 - 3z \right. \\
 \underline{z^5 + 3z^3} \\
 -3z^3 - 9z \\
 \underline{-3z^3 - 9z} \\
 0
 \end{array}$$

Quotient: $z^3 - 3z$

Remainder: 0

Since remainder is 0 therefore $(z^2 - 3z)$ is a factor of $z^5 - 9z$

(vi) $2x^2 - x + 3$ is a factor of $6x^5 - x^4 + 4x^3 - 5x^2 - x - 15$

$$\begin{array}{r}
 2x^2 - x + 3 \overline{) 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15} \left(3x^3 + x^2 - 2x - 3 \right. \\
 \underline{6x^5 - 3x^4 + 9x^3} \\
 2x^4 - 5x^3 - 5x^2 - x - 15 \\
 \underline{2x^4 - x^3 + 3x^2} \\
 -4x^3 - 8x^2 - x - 15 \\
 \underline{-4x^3 - 2x^2 - 6x} \\
 -6x^2 + 5x - 15 \\
 \underline{-6x^2 + 3x - 9} \\
 2x - 6
 \end{array}$$

Quotient: $3x^3 + x^2 - 2x - 3$

Remainder: $2x^2 - x + 3$

Since remainder is $2x - 6$ therefore $(3y^2 + 5)$ is **NOT** a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

24. Question

Find the value of a, if $x+2$ is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$.

Answer

$x + 2$ is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

$$x + 2 = 0$$

$x = -2$ Therefore substitute $x = -2$ in the given equation we get,

$$4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$$

$$64 - 16 - 12 - 16 + 5a = 0$$

$$20 + 5a = 0$$

$$5a = -20$$

$$a = -\frac{20}{5} = -4$$

$$a = -4$$

25. Question

What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.

Answer

$$\begin{array}{r}
 x^2 + 2x - 3 \overline{) x^4 + 2x^3 - 2x^2 + x - 1} \left(x^2 + 1 \right. \\
 \underline{x^4 + 2x^3 - 3x^2} \\
 x^2 + x - 1 \\
 \underline{x^2 + 2x - 3} \\
 -x + 2
 \end{array}$$

Quotient: $x^2 + 1$

Remainder: $-x + 2$

Therefore $x - 2$ to be added.

Exercise 8.5

1. Question

Divide the first polynomial by the second polynomial in each of the following Also write the quotient and remainder:

(i) $3x^2 + 4x + 5, x - 2$

(ii) $10x^2 - 7x + 8, 5x - 3$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

(iv) $x^4 - x^3 + 5x, x - 1$

(v) $y^4 + y^2, y^2 - 2$

Answer

(i) $3x^2 + 4x + 5, x - 2$

$$\begin{array}{r} x-2 \overline{) 3x^2 + 4x + 5} \\ \underline{3x^2 - 6x} \\ 10x + 5 \\ \underline{10x - 20} \\ 25 \end{array}$$

Quotient: $3x + 10$

Remainder: 25

(ii) $10x^2 - 7x + 8, 5x - 3$

$$\begin{array}{r} 5x-3 \overline{) 10x^2 - 7x + 8} \phantom{+ \frac{1}{5}} \\ \underline{10x^2 - 6x} \phantom{+ \frac{1}{5}} \\ -x + 8 \\ \underline{-x + \frac{3}{5}} \\ 8 - \frac{3}{5} = \frac{37}{5} \end{array}$$

Quotient: $2x - \frac{1}{5}$

Remainder: $\frac{37}{5}$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

$$\begin{array}{r} 5y-1 \overline{) 5y^3 - 6y^2 + 6y - 1} \\ \underline{5y^3 - y^2} \\ -5y^2 + 6y - 1 \\ \underline{-5y^2 + y} \\ 5y - 1 \\ \underline{5y - 1} \\ 0 \end{array}$$

Quotient: $y^2 - y + 1$

Remainder: 0

(iv) $x^4 - x^3 + 5x, x - 1$

$$\begin{array}{r}
 x-1 \overline{) x^4 - x^3 + 5x} \\
 \underline{x^4 - x^3} \\
 5x \\
 \underline{5x - 5} \\
 5
 \end{array}$$

Quotient: $x^3 + 5$

Remainder: 5

(v) $y^4 + y^2, y^2 - 2$

$$\begin{array}{r}
 y^2 - 2 \overline{) y^4 - y^2} \\
 \underline{y^4 - 2y^2} \\
 y^2 \\
 \underline{y^2 - 2} \\
 2
 \end{array}$$

Quotient: $y^2 + 1$

Remainder: 2

2. Question

Find Whether or not the first polynomial is a factor of the second:

(i) $x + 1, 2x^2 + 5x + 4$

(ii) $y - 2, 3y^3 + 5y^2 + 5y + 2$

(iii) $4x^2 - 5, 4x^4 + 7x^2 + 15$

(iv) $4 - z, 3z^2 - 13z + 4$

(v) $2a - 3, 10a^2 - 9a - 5$

(vi) $4y + 1, 8y^2 - 2y + 1$

Answer

(i) $x + 1, 2x^2 + 5x + 4$

$$\begin{array}{r}
 x+1 \overline{) 2x^2 + 5x + 4} \\
 \underline{2x^2 + 2x} \\
 3x + 4 \\
 \underline{3x + 3} \\
 1
 \end{array}$$

Quotient: $2x + 3$

Remainder: 1

Since remainder is 1 therefore the first polynomial is **NOT** a factor of the second polynomial.

(ii) $y - 2, 3y^3 + 5y^2 + 5y + 2$

$$\begin{array}{r}
 y - 2 \overline{) 3y^3 + 5y^2 + 5y + 2} \quad \overline{) 3y^2 + 11y + 27} \\
 \underline{3y^3 - 6y^2} \\
 11y^2 + 5y + 2 \\
 \underline{11y^2 - 22y} \\
 27y + 2 \\
 \underline{27y - 54} \\
 56
 \end{array}$$

Quotient: $3y^2 + 11y + 27$

Remainder: 56

Since remainder is 56 therefore the first polynomial is **NOT** a factor of the second polynomial.

(iii) $4x^2 - 5, 4x^4 + 7x^2 + 15$

$$\begin{array}{r}
 4x^2 - 5 \overline{) 4x^4 + 7x^2 + 15} \quad \overline{) x^2 + 3} \\
 \underline{4x^4 - 5x^2} \\
 12x^2 + 15 \\
 \underline{12x^2 - 15} \\
 30
 \end{array}$$

Quotient: $x^2 + 3$

Remainder: 30

Since remainder is 30 therefore the first polynomial is **NOT** a factor of the second polynomial.

(iv) $4 - z, 3z^2 - 13z + 4$

$$\begin{array}{r}
 -z + 4 \overline{) 3z^2 - 13z + 4} \quad \overline{) -3z + 1} \\
 \underline{3z^2 - 12z} \\
 -z + 4 \\
 \underline{-z + 1} \\
 0
 \end{array}$$

Quotient: $-3z + 1$

Remainder: 0

Since remainder is 0 therefore the first polynomial is a factor of the second polynomial.

(v) $2a - 3, 10a^2 - 9a - 5$

$$\begin{array}{r}
 2a - 3 \overline{) 10a^2 - 9a - 5} \quad \overline{) 5a + 3} \\
 \underline{10a^2 - 15a} \\
 6a - 5 \\
 \underline{6a - 9} \\
 4
 \end{array}$$

Quotient: $5a + 3$

Remainder: 4

Since remainder is 4 therefore the first polynomial is **NOT** a factor of the second polynomial.

(vi) $4y + 1, 8y^2 - 2y + 1$

$$\begin{array}{r}
 4y + 1 \overline{) 8y^2 - 2y + 1} \quad 2y - 1 \\
 \underline{8y^2 + 2y} \\
 -4y + 1 \\
 \underline{-4y - 1} \\
 + + \\
 2
 \end{array}$$

Quotient: $2y - 1$

Remainder: 2

Since remainder is 2 therefore the first polynomial is **NOT** a factor of the second polynomial.

Exercise 8.6

1. Question

Divide:

$$x^2 - 5x + 6 \text{ by } x - 3$$

Answer

$$\begin{array}{r}
 x - 3 \overline{) x^2 - 5x + 6} \quad x - 2 \\
 \underline{x^2 - 3x} \\
 -2x + 6 \\
 \underline{-2x + 6} \\
 + - \\
 0
 \end{array}$$

Quotient: $x - 2$

Remainder: 0

2. Question

Divide:

$$ax^2 - ay^2 \text{ by } ax + ay$$

Answer

$$\begin{array}{r}
 ax - ay \overline{) ax^2 - ay^2} \quad x - y \\
 \underline{ax^2 - axy} \\
 -ay^2 + axy \\
 \underline{-ay^2 + axy} \\
 + - \\
 0
 \end{array}$$

Quotient: $x - y$

Remainder: 0

3. Question

Divide:

$$x^4 - y^4 \text{ by } x^2 - y^2$$

Answer

$$\begin{array}{r}
 x^2 - y^2 \overline{) x^4 - y^4} \quad x^2 + y^2 \\
 \underline{x^4 - x^2y^2} \\
 + + \\
 -y^4 + x^2y^2 \\
 \underline{-y^4 + x^2y^2} \\
 + - \\
 0
 \end{array}$$

Quotient: $x^2 + y^2$

Remainder: 0

4. Question

Divide:

$$acx^2 + (bc + ad)x + bdy \text{ by } (ax + b)$$

Answer

$$\begin{array}{r} ax + b \overline{) acx^2 + bcx + adx + bd} \\ \underline{acx^2 + bcx} \\ adx + bd \\ \underline{adx + bd} \\ 0 \end{array}$$

Quotient: $cx + d$

Remainder: 0

5. Question

Divide:

$$(a^2 + 2ab + b^2) - (a^2 + 2ac + c^2) \text{ by } 2a + b + c$$

Answer

$$\begin{array}{r} 2a + b + c \overline{) 2ab - 2ac + b^2 - c^2} \\ \underline{2ab} \\ -2ac - c^2 - bc \\ \underline{-2ac - c^2 - bc} \\ 0 \end{array}$$

Quotient: $b - c$

Remainder: 0

6. Question

Divide:

$$\frac{1}{4}x^2 - \frac{1}{2}x - 12 \text{ by } \frac{1}{2}x - 4$$

Answer

$$\begin{array}{r} \frac{x}{2} - 4 \overline{) \frac{x^2}{4} - \frac{x}{2} - 12} \\ \underline{\frac{x^2}{4} - 2x} \\ \frac{3x}{2} - 12 \\ \underline{\frac{3x}{2} - 12} \\ 0 \end{array}$$

Quotient: $\frac{x}{2} + 3$

Remainder: 0