

Chapter : 29. MATHEMATICAL REASONING

Exercise : 29A

Question: 1

Which of the foll

Solution:

(i) The sun is a star is a statement. It is a scientifically proven fact that the sun is a star and, therefore this sentence is always true. Hence it is a statement, and it is true.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(ii) $\sqrt{7}$ is an irrational number. An irrational number is any number which cannot be expressed as a fraction of two integers. $\sqrt{7}$ cannot be expressed as a fraction of two integers, so $\sqrt{7}$ is an irrational number; therefore the sentence is always true. Hence it is a statement, and it is true.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(iii) The sentence is true because the sum of 5 and 6 is not less than 10. Sum of 5 and 6 is 11, which is not less than 10. Hence it is a statement. The statement is true.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(iv) This sentence 'Go to your class' is an order. Hence it is not a statement.

Note: A sentence which is in the form of an order, exclamation and question is not a statement.

(v) Ice is always cold is a statement. It is scientifically proven the fact that ice is always cold and, therefore the sentence is always true.

Hence it is a statement, and it is true.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(vi) The sentence 'Have you ever seen the Red Fort? Is a question, hence it is not a statement.

Note: A sentence which is in the form of an order, exclamation and question is not a statement.

(vii) The sentence 'Every relation is a function' is a statement. There are relations which are not functions. Therefore the sentence is false. Hence it is a statement, and it is false.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(viii) The sentence 'The sum of any two sides of a triangle is always greater than the third side' is a statement. Because the sum of any two sides of the triangle is always greater than the third side. Hence the statement is true.

(ix) The sentence 'May God bless you!' is an exclamation. Hence it is not a statement.

Note: A sentence which is in the form of an order, exclamation and question is not a statement.

Question: 2

Which of the foll

Solution:

(i) The sentence 'Paris is in France' is a statement. Paris is located in France, so the sentence given is true, so it is a statement. The statement is true.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(ii) The sentence 'Each prime number has exactly two factors' is a statement. It is a mathematically proven fact that each prime number has exactly two factors, so the given

sentence is true. Hence it is a statement. The statement is true.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(iii) The sentence 'The equation $x^2 + 5|x| + 6 = 0$ has no real roots.' Is a statement. $x^2 + 5|x| + 6 = 0$ do not have real roots.

Case 1: ($x \geq 0$)

$$|x| = x: (x \geq 0)$$

$$x^2 + 5|x| + 6 = 0$$

$$x^2 + 5x + 6 = 0$$

$$(x + 2)(x + 3) = 0$$

$$x = -2 \text{ and } x = -3$$

But we assumed $x \geq 0$. So it is a contradiction.

Case 2: ($x < 0$)

$$|x| = -x: (x < 0)$$

$$x^2 + 5|x| + 6 = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

$$x = 2 \text{ and } x = 3$$

But we assumed $x < 0$. So it is a contradiction.

So, there are no real roots for the equation $x^2 + 5|x| + 6 = 0$

So, the given sentence is true, and it is a statement.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(iv) The sentence '(2 + $\sqrt{3}$) is a complex number' is a statement.

A number which can be expressed in the form ' $a+ib$ ' is a complex number, (2 + $\sqrt{3}$) cannot be expressed in ' $a+ib$ ' form, so 2 + $\sqrt{3}$ is not a complex number. So the given sentence is a statement, and it is false.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(v) The sentence 'Is 6 a positive integer?' is a question, so it is not a statement.

Note: A sentence which is in the form of an order, exclamation and question is not a statement.

(vi) The sentence 'The product of -3 and -2 is -6' is a statement.

Because, the product of -3 and -2 is 6 not -6, the given sentence is false. Hence the given sentence is a statement. This statement is false.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(vii) The sentence given is a statement. It is mathematically proven that the angles opposite to the equal sides of an isosceles triangle are equal. So the given sentence is true, and it is a statement.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(viii) The sentence 'Oh! It is too hot' is not a statement. It is an exclamation, and hot is subjective, it is not a fact, and it is an opinion. So, the given sentence is not a statement.

Note: A sentence which is in the form of an order, exclamation and question is not a statement.

(ix) The sentence 'Monica is a beautiful girl' is not a statement. The given sentence is an opinion;

this can be true for some cases, false for some other case. So, the given sentence is not a statement.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

(x) The given sentence is a statement.

Because not every quadratic equation will have a real root. So the given sentence is false. It is a statement. This statement is false.

Note: A sentence is called a mathematically acceptable statement if it is either true or false but not both.

Question: 3

Which of the foll

Solution:

(i) p: $\sqrt{11}$ is an irrational number is a TRUE statement.

An irrational number is any number which cannot be expressed as a fraction of two integers. $\sqrt{11}$ cannot be expressed as a fraction of two integers, so $\sqrt{11}$ is an irrational number.

(ii) q: Circle is a particular case of an ellipse is a TRUE statement.

A circle is a particular case of an ellipse with the same radius in all points.

The equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

When $a = b$, we will get the equation of the circle, $x^2 + y^2 = 1$

(iii) r: Each radius of a circle is a chord of the circle is a FALSE statement.

A chord intersects the circle at two points, but radius intersects the circle only at one point. So the radius is not a chord of the circle.

(iv) S: The center of a circle bisects each chord of the circle is a FALSE statement.

The only diameter of a circle is bisected by the center of the circle. Except for diameter, no other chords are bisected the center of the circle. The only center lies on the diameter of the circle.

(v) t: If a and b are integers such that $a < b$, then $-a > -b$ is a TRUE statement.

$a < b$, then $-a > -b$, is TRUE by the rule of inequality.

(vi) y: The quadratic equation $x^2 + x + 1 = 0$ have no real roots is a TRUE statement.

General form of a quadratic equation is $ax^2 + bx + c = 0$.

If $b^2 - 4ac < 0$, there is no real solution.

In the given equation; $x^2 + x + 1 = 0$

$a = 1$; $b = 1$; $c = 1$

$b^2 - 4ac = 1 - 4 \times 1 \times 1 = -3 < 0$

So, there is no real root.

Question: 4

Write the negatio

Solution:

(i) The negation of the given statement is:

It is false that every natural number is greater than 0.

(Or)

Every natural number is not greater than 0.

(Or)

There exists a natural number which is not greater than 0.

(ii) The negation of the given statement is:

It is false that both the diagonals of a rectangle are equal.

(Or)

There exists at least one rectangle whose both the diagonals are not equal.

(iii) The negation of the given statement is:

It is false that the sum of 4 and 5 is 8.

(Or)

The sum of 4 and 5 is not 8.

(iv) The negation of the given statement is:

It is false that the number 6 is greater than 4.

(Or)

The number 6 is not greater than 4.

(v) The negation of the given statement is:

It is false that every natural number is an integer.

(Or)

Every natural number is not an integer.

(Or)

There exists at least one natural number which is not an integer.

(vi) The negation of the given statement is:

It is false that the number -5 is a rational number.

(Or)

The number -5 is not a rational number.

(vii) The negation of the given statement is:

It is false that all cats scratch.

(Or)

There exists a cat which does not scratch.

(viii) The negation of the given statement is:

It is false that there exists a rational number x such that $x^2 = 3$.

(Or)

There does not exist a rational number x such that $x^2 = 3$

(ix) The negation of the given statement is:

It is false that all students study mathematics at the elementary level.

(Or)

It is not the case that all students study mathematics at the elementary level.

(x) The negation of the given statement is:

It is false that every student has paid the fees.

(Or)

It is not the case that every student has paid the fees.

(Or)

There exists at least a student who does not pay the fees.

(xi) The negation of the given statement is:

It is false that there is some integer k for which $2k = 6$.

(Or)

It is not the case there is some integer k for which $2k = 6$

(xii) The negation of the given statement is:

It is false that none of the students in this class has passed.

(Or)

It is not the case that none of the students of this class has passed.

Exercise : 29B

Question: 1

Split each of the

Solution:

p: A line is straight.

q: A line extends indefinitely in both the directions.

Both the simple sentences are TRUE; therefore, the given sentence is TRUE.

(ii) p: A point occupies a position.

q: Its location can be determined.

Both the simple sentences are TRUE; therefore, the given sentence is TRUE.

(iii) p: The sand heats up quickly in the sun.

q: The sand does not cool down fast at night.

The first sentence is TRUE and the second sentence is FALSE.

Therefore the given sentence is FALSE.

(iv) p: 32 is divisible by 8.

q: 32 is divisible by 12.

First sentence is TRUE. Second sentence is FALSE.

Therefore the given sentence is FALSE.

(v) p: $x = 1$ is a root of the equation $x^2 - x - 2 = 0$

q: $x = 2$ is a root of the equation $x^2 - x - 2 = 0$

So, the first sentence is FALSE. Second sentence is TRUE.

Therefore, the given sentence is FALSE.

(vi) p: 3 is rational.

q: $\sqrt{3}$ is irrational.

First sentence is TRUE. Second sentence is TRUE.

Both the sentences are TRUE; therefore the given sentence is TRUE.

(vii) p: All integers are rational numbers.

q: All rational numbers are not real numbers.

First sentence is TRUE. Second sentence is FALSE.

Therefore, the given sentence is FALSE.

(viii) p: Lucknow is in Uttar Pradesh.

q: Kanpur is in Uttarakhand.

First sentence is TRUE. Second sentence is FALSE.

Therefore, the given sentence is FALSE.

Question: 2

Split each of the

Solution:

(i) p: The sum of 3 and 7 is 10 or 11.

q: The sum of 3 and 7 is 10.

r: The sum of 3 and 7 is 11.

First sentence is TRUE. Second sentence is FALSE.

Or used is 'Exclusive or'.

(ii) p: $(1 + i)$ is a real or a complex number.

q: $(1 + i)$ is a real number.

r: $(1 + i)$ is a complex number.

First sentence is TRUE. Second sentence is FALSE.

Or used is 'Exclusive or'.

(iii) p: Every quadratic equation has one or two real roots.

q: Every quadratic equation has one real root.

r: Every quadratic equation has two real roots.

Both the sentences are FALSE. So, the compound sentence itself is FALSE.

(iv) p: You are wet when it rains, or you are in a river.

q: You are wet when it rains.

r: You are wet when you are in a river.

Both the component sentences are TRUE.

Or used is 'Inclusive or' because you can get wet either it rains or when you are in the river.

(v) p: 24 is a multiple of 5 or 8.

q: 24 is a multiple of 5.

r: 24 is a multiple of 8.

First sentence is FALSE. Second sentence is TRUE.

Or used is 'Exclusive or'.

(vi) p: Every integer is rational or irrational.

q: Every integer is rational.

r: Every integer is irrational.

The first sentence is TRUE. The second sentence is TRUE. But both cannot be TRUE at the same time.

'Or' used is 'Exclusive or'.

(vii) p: For getting a driving license you should have a ration card or a passport.

q: For getting a driving license you should have a ration card.

r: For getting a driving license you should have a passport.

Both sentences are TRUE.

Or used is 'Inclusive or', because one can get a driving license with ration card or with passport or when they have both.

(viii) p: 100 is a multiple of 6 or 8.

q: 100 is a multiple of 6.

r: 100 is a multiple of 8.

Both the sentences are FALSE. So the compound sentence itself is FALSE.

(ix) p: Square of an integer is positive or negative.

q: Square of an integer is positive.

r: Square of an integer is negative.

First sentence is TRUE. Second sentence is FALSE.

Or used is 'Exclusive or'.

(x) p: Sun rises or Moon sets.

q: Sun rises.

r: Moon sets.

Here both the sentences are TRUE, but only one occurs at a time.

So, Or used is 'Exclusive or'.

Question: 3

Find the truth set

Solution:

The open sentence $x + 2 < 10$ is defined on \mathbb{N} ; the set of natural numbers.

$\mathbb{N}: \{1, 2, 3, 4, \dots\}$

$$x = 1 \rightarrow x + 2 = 3 < 10$$

$$x = 2 \rightarrow x + 2 = 4 < 10$$

$$x = 3 \rightarrow x + 2 = 5 < 10$$

$$x = 4 \rightarrow x + 2 = 6 < 10$$

$$x = 5 \rightarrow x + 2 = 7 < 10$$

$$x = 6 \rightarrow x + 2 = 8 < 10$$

$$x = 7 \rightarrow x + 2 = 9 < 10$$

$$x = 8 \rightarrow x + 2 = 10$$

So, $\exists x \in \mathbb{N}$, such that $x + 2 < 10$

$x = \{1, 2, 3, 4, 5, 6, 7\}$ satisfies $x + 2 < 10$.

So, the truth set of open sentence $x + 2 < 10$ defined on \mathbb{N} is,

$\{1, 2, 3, 4, 5, 6, 7\}$

(ii) The open sentence $x + 5 < 4$ is defined on \mathbb{N} ; the set of natural numbers.

$\mathbb{N}: \{1, 2, 3, 4, \dots\}$

$$x = 1 \rightarrow 1 + 5 = 6 > 4$$

So, the truth set of open sentence $x + 5 < 4$ defined on N is an empty set, $\{\}$.

(iii) The open sentence $x + 3 > 2$ is defined on N ; the set of natural numbers.

$N: \{1, 2, 3, 4, \dots\}$

$$x = 1 \rightarrow x + 3 = 4 > 2$$

$$x = 2 \rightarrow x + 3 = 5 > 2$$

$$x = 3 \rightarrow x + 3 = 6 > 2$$

$$x = 4 \rightarrow x + 3 = 7 > 2$$

$$x = 5 \rightarrow x + 3 = 8 > 2$$

$$x = 6 \rightarrow x + 3 = 9 > 2$$

And so on...

So, $\exists x \in N$, such that $x + 3 > 2$

$x = \{1, 2, 3, 4, 5, 6, 7, \dots\}$ satisfies $x + 3 > 2$.

So, the truth set of open sentence $x + 3 > 2$ defined on N is an infinite set as there is infinite natural numbers satisfying the equation $x + 3 > 2$.

$\{1, 2, 3, 4, 5, 6, 7, \dots\}$

Question: 4

Let $A = [2, 3, 5,$

Solution:

$A = [2, 3, 5, 7]$ (given in the question).

The given statement is: $\exists x \in A$ such that $x + 3 > 9$.

So, we need to see whether there exists 'x' which belongs to 'A', such that $x + 3 > 9$.

When $x = 7 \in A$,

$$x + 3 = 7 + 3 = 10 > 9$$

So, $\exists x \in A$ and $x + 3 > 9$.

So, the given statement is TRUE.

(ii) $A = [2, 3, 5, 7]$ (given in the question).

The given statement is $\exists x \in A$ such that x is even.

So, we need to see whether there exists 'x' which belongs to 'A', such that x is even.

In the set $A = [2, 3, 5, 7]$

$x = 2$, is an even number and $2 \in A$.

$\therefore \exists x \in A$ such that x is even.

So, the given statement is TRUE.

(iii) $A = [2, 3, 5, 7]$ (given in the question).

The given statement is: $\exists x \in A$ such that $x + 2 = 6$.

So, we need to see whether there exists 'x' which belongs to 'A', such that $x + 2 = 6$.

$$x = 2 \rightarrow x + 2 = 4 \neq 6$$

$$x = 3 \rightarrow x + 2 = 5 \neq 6$$

$$x = 5 \rightarrow x + 2 = 7 \neq 6$$

$$x = 7 \rightarrow x + 2 = 9 \neq 6$$

So, the given statement is FALSE.

(iv) $A = [2, 3, 5, 7]$ (given in the question).

The given statement is: $\forall x \in A, x$ is prime.

So, we need to see whether for all 'x' which belongs to 'A', such that x is a prime number.

All 'x' which belongs to $A = [2, 3, 5, 7]$ is a prime number.

$\therefore \forall x \in A, x$ is prime.

So, the given statement is TRUE.

(v) $A = [2, 3, 5, 7]$ (given in the question).

The given statement is: $\forall x \in A, x + 2 < 10$.

So, we need to see whether for all 'x' which belongs to 'A', such that $x + 2 < 10$.

$A = [2, 3, 5, 7]$

$x = 2 \rightarrow x + 2 = 4 < 10$

$x = 3 \rightarrow x + 2 = 5 < 10$

$x = 5 \rightarrow x + 2 = 7 < 10$

$x = 7 \rightarrow x + 2 = 9 < 10$

$\forall x \in A, x + 2 < 10$, is a TRUE statement.

(vi) $A = [2, 3, 5, 7]$ (given in the question).

The given statement is: $\forall x \in A, x + 4 \geq 11$.

So, we need to see whether for all 'x' which belongs to 'A', such that $x + 4 \geq 11$.

$A = [2, 3, 5, 7]$

$x = 2 \rightarrow x + 4 = 6 \geq 11$

$x = 3 \rightarrow x + 4 = 7 \geq 11$

$x = 5 \rightarrow x + 4 = 9 \geq 11$

$x = 7 \rightarrow x + 4 = 11 \geq 11$

Only for $x = 7, x + 4 = 11 \geq 11$.

$\forall x \in A, x + 4 \geq 11$, is a FALSE statement.

Exercise : 29C

Question: 1

Rewrite the follo

Solution:

I. A given number is a multiple of 6; it implies that it is a multiple of 3.

II. A given number is a multiple of 6 only if it is a multiple of 3.

III. For a given number to be a multiple of 6, it is necessary that it is a multiple of 3.

IV. For a given number to be a multiple of 3, it is sufficient that the number is multiple of 6.

V. If the given number is not a multiple of 3, then it is not a multiple of 6.

Question: 2

Write each of the

Solution:

- (i) If each of the angles of a rhombus measures 90° , then the rhombus is a square.
- (ii) If a number is a multiple of 9, then the number is multiple of 3.
- (iii) If you get a job, then your credentials are good.
- (iv) If it rains, then the atmospheric humidity increases.
- (v) If a number is a multiple of 6, then it is a multiple of 3.

Question: 3

Write the convers

Solution:

- (i) If x is not an odd number, then it is not a prime.
- (ii) If the sum of the digits of a positive integer n is not divisible by 9, then n is not divisible by 9.
- (iii) If two lines intersect in the same plane, then they are not parallel.
- (iv) If the quadrilateral is not a parallelogram, then its diagonals do not bisect each other.
- (v) If A and B are subsets of X such that $(X - B)$ is not a subset of $(X - A)$, then A is not a subset of B .
- (vi) If $f(x)$ is not divisible by $(x-2)$, then $f(2) \neq 0$.
- (vii) If you are not a citizen of India, then you were not born in India.
- (viii) If I do not stay at home, then it does not rain.

Question: 4

Given below are s

Solution:

- (i) A quadrilateral is a rectangle if and only if it is equiangular.
- (ii) A number is divisible by 3 if and only if the sum of the digits of the number is divisible by 3.
- (iii) A quadrilateral is a parallelogram if and only if its diagonals bisect each other.
- (iv) $(x-a)$ is a factor of polynomial $f(x)$ if and only if $f(a) = 0$.
- (v) Square matrix A is invertible if and only if $|A|$ is nonzero.

Question: 5

write each of the

Solution:

- (i) You get an A grade if and only if you do all your homework regularly.
- (ii) You watch television if and only if your mind is free.

Exercise : 29D

Question: 1

Let p : If x is a

Solution:

Let p : x is an integer and x^2 is even.

q : x is even

For contrapositive,

$\sim p$ = x is an integer and x^2 is not even.

$\sim q$ = x is not even.

The contrapositive statement is: If x is an integer and x^2 is not even, then x is not even.

Proof;

Let x be an odd/ not even integer

$$x = 2n + 1$$

{ $2n$ must be an even integer as when an integer is multiplied with an even integer, the answer is always even. Adding one ensures that the integer is odd after the multiplication.}

$$\neg x^2 = (2n+1)^2$$

$$\neg x^2 = 4n^2 + 4n + 1$$

{ $4n^2$ and $4n$ are even irrespective of integer n 's value. Adding 1 makes the not even/odd}

Thus, if x is an integer and x^2 is not even, then x is not even.

Question: 2

Consider the stat

Solution:

Let us take the numbers $a = +5$ and $b = -5$.

$$a^2 = (+5)^2 = 25$$

$$b^2 = (-5)^2 = 25$$

$$\therefore a^2 = b^2$$

But, $+5 \neq -5$, thus $a \neq b$.

\therefore q is false.

Question: 3

By giving a count

Solution:

Let us take a odd positive integer $n = +9$

Even though $(+9)$ is an odd positive integer, It is divisible by 3.

To be a prime number, a number must only have itself and 1 as its factors. Since 9 has 3 as its factor too, it is not a prime number in spite of being an odd positive integer.

\therefore The given statement p is false.

Question: 4

Use contradiction

Solution:

Let us assume that $\sqrt{3}$ is a rational number.

For a number to be rational, it must be able to express it in the form $\frac{p}{q}$ where p and q do not have any common factor, i.e. they are co-prime in nature.

Since $\sqrt{3}$ is rational, we can write it as

$$\sqrt{3} = \frac{p}{q}$$

$$\neg \frac{p}{\sqrt{3}} = q$$

[squaring both sides]

$$\rightarrow \frac{p^2}{3} = q^2$$

Thus, p^2 must be divisible by 3. Hence p will also be divisible by 3.

We can write $p = 3c$ (c is a constant), $p^2 = 9c^2$

Putting this back in the equation,

$$\frac{9c^2}{3} = q^2$$

$$\rightarrow 3c^2 = q^2$$

$$\rightarrow c^2 = \frac{q^2}{3}$$

Thus, q^2 must also be divisible by 3, which implies that q will also be divisible by 3.

This means that both p and q are divisible by 3 which proves that they are not co-prime and hence the condition for rationality has not been met. Thus, $\sqrt{3}$ is not rational.

$\therefore \sqrt{3}$ is irrational.

Hence, the statement $p: \sqrt{3}$ is irrational , is true.

Question: 5

By giving a count

Solution:

By the properties of triangles, if all the sides of a triangle are equal, then the each of the angle of the triangle will also be equal.

By the question,

All sides of the triangle are equal.

\therefore All angles of the triangle are also equal.

Let each angle of the equilateral triangle be x° . We know that the sum of all angles of a triangle is 360° .

$$x^\circ + x^\circ + x^\circ = 360^\circ$$

$$\rightarrow 3x^\circ = 360^\circ$$

$$\rightarrow x^\circ = (360 \div 3)^\circ$$

$$\therefore x^\circ = 60^\circ$$

Thus, all angles of the triangle measure 60° which is an acute angle (lying between 0° and 90° .)

Obtuse angles are those which lie between 90° and 180° .

Thus, when all sides are equal in a triangle, its angles measure 60° each. This implies that all angles are acute angles and not obtuse angles.

Thus, the statement p is false.