Chapter : 7. TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES

Exercise : 7

Question: 1 A

Without using tri

Solution:

$\frac{\sin 16^\circ}{\cos 74^\circ} = \frac{\sin 16^\circ}{\cos(90-16)^\circ} = \frac{\sin 16^\circ}{\sin 16^\circ} = 1$

($\because$ cos (90-θ) = sin θ and (90-θ) lies in the first quadrant where

all the angles are taken as positive.)

Question: 1 B

Solve

Solution:

$\frac{\sec 11^\circ}{cosec\, 79^\circ} = \frac{1/\cos 11^\circ}{1/\sin 79^\circ} = \frac{\sin 79^\circ}{\cos 11^\circ} = \frac{\sin 79^\circ}{\cos(90-79)^\circ} = \frac{\sin 79^\circ}{\sin 79^\circ} = 1$

Question: 1 C

Solve

Solution:

$\frac{\tan 27^\circ}{\cot 63^\circ} = \frac{\tan 27^\circ}{\cot(90-27)^\circ} = \frac{\tan 27^\circ}{\tan 27^\circ} = 1$ ($\because$ cot (90-θ) = tan θ)

Question: 1 D

Solve

Solution:

$\frac{\cos 35^\circ}{\sin 55^\circ} = \frac{\cos 35^\circ}{\sin(90-35)^\circ} = \frac{\cos 35^\circ}{\cos 35^\circ} = 1$

Question: 1 E

Solve

Solution:

$\frac{cosec\, 42^\circ}{\sec 48^\circ} = \frac{1/\sin 42^\circ}{1/\cos 48^\circ} = \frac{\cos 48^\circ}{\sin 42^\circ} = \frac{\cos 48^\circ}{\sin(90-48)^\circ} = \frac{\cos 48^\circ}{\sin 48^\circ} = 1$

Question: 1 F

Solve

Solution:

$\frac{\cot 38^\circ}{\tan 52^\circ} = \frac{\cot 38^\circ}{\tan(90-38)^\circ} = \frac{\cot 38^\circ}{\cot 38^\circ} = 1$ ($\because$ tan (90-θ) = cot θ)

Question: 2 A

Without using tri

Solution:

Consider cos 81° - sin 9° = cos 81° - sin (90 - 81)°

= cos 81° - cos 81°

= 0

Hence, proved.

Question: 2 B

Without using tri

Solution:

Consider $\tan 71° - \cot 19° = \tan 71° - \cot (90 - 71)°$

$= \tan 71° - \tan 71°$

$= 0$

Hence, proved.

Question: 2 C

Without using tri

Solution:

Consider $\operatorname{cosec} 80° - \sec 10° = \operatorname{cosec} 80° - \sec (90 - 10)°$

$= \operatorname{cosec} 80° - \operatorname{cosec} 80°$

$= 0$

Hence, proved.

Question: 2 D

Without using tri

Solution:

Consider $\operatorname{cosec}^2 72° - \tan^2 18° = \operatorname{cosec}^2 72° - \tan^2 (90 - 72)°$

$= \operatorname{cosec}^2 72° - \cot^2 72°$

$= 1$

$(\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta = 1)$

Hence, proved.

Question: 2 E

Without using tri

Solution:

Consider $\cos^2 75° + \cos^2 15° = \cos^2 75° + \cos^2 (90 - 75)°$

$= \cos^2 75° + \sin^2 75°$

$= 1$

$(\because \cos^2\theta + \sin^2 \theta = 1)$

Hence, proved.

Question: 2 F

Without using tri

Solution:

Consider $\tan^2 66° - \cot^2 24° = \tan^2 66° - \cot^2 (90 - 66)°$

$= \tan^2 66° - \tan^2 66°$

$= 0$

Hence, proved.

Question: 2 G

Without using tri

Solution:

Consider $\sin^2 48° + \sin^2 42° = \sin^2 48° + \sin^2 (90 - 48)°$

$= \sin^2 48° + \cos^2 48°$

$= 1$

$(\because \cos^2\theta + \sin^2 \theta = 1)$

Hence, proved.

Question: 2 H

Without using tri

Solution:

Consider $\cos^2 57° - \sin^2 33° = \cos^2 57° - \sin^2 (90 - 57)°$

$= \cos^2 57° - \cos^2 57°$

$= 0$

Hence, proved.

Question: 2 I

Without using tri

Solution:

Consider $(\sin 65° + \cos 25°)(\sin 65° - \cos 25°)$

$= \sin^2 65° - \cos^2 25°$

$= \sin^2 65° - \cos^2 (90 - 65)°$

$= \sin^2 65° - \sin^2 65°$

$= 0$

Hence, proved.

Question: 3 A

Without using tri

Solution:

Consider $(\sin 53° \cos 37°) + (\cos 53° \sin 37°)$

$= (\sin 53° \cos (90-53)°) + (\cos 53° \sin (90-53)°)$

$= \sin^2 53° + \cos^2 53°$

$= 1$

Hence, proved.

Question: 3 B

$\cos 54° \cos 36° -$

Solution:

Consider $(\cos 54° \cos 36°) - (\sin 54° \sin 36°)$

$= (\cos 54° \cos (90-54)°) - (\sin 54° \sin (90-54)°)$

$= (\cos 54° \sin 54°) - (\sin 54° \cos 54°)$

= 0

Hence, proved.

Question: 3 C

sec 70° sin 20° +

Solution:

Consider L.H.S.

= (sec 70° sin 20°) + (cos 20° cosec 70°)

= (sec (90-20)° sin 20°) + (cos 20° cosec (90-20)°)

= (cosec 70° sin70°) + (cos 20° sec 20°)

= 1 + 1 (∵ cosec θ = 1/sinθ and sec θ = 1/cos θ)

= 2 = R.H.S.

Hence, proved.

Question: 3 D

sin 35° sin 55° -

Solution:

Consider L.H.S. = (sin 35° sin 55°) - (cos 35° cos 55°)

= (sin 35° sin (90-35)°) - (cos 35° cos (90-35)°)

= (sin 35° cos 35°) - (cos 35° sin 35°)

= 0 = R.H.S.

Hence, proved.

Question: 3 E

(sin 72° + cos 18

Solution:

Consider (sin 72° + cos 18°)(sin 72° - cos 18°)

= $\sin^2 72° - \cos^2 18°$

= $\sin^2 72° - \cos^2 (90-72)°$

= $\sin^2 72° - \sin^2 72°$

[since, cos(90 - θ) = sinθ]

= 0

Hence, proved.

Question: 3 F

tan 48° tan 23° t

Solution:

Consider L.H.S.

= tan 48° tan 23° tan 42° tan 67°

= tan 48° tan 42° tan 23° tan 67°

= tan 48° tan (90-48)° tan 23° tan (90-23)°

= (tan 48° cot 48°) (tan 23° cot 23°)

= (1) × (1) = 1 = R.H.S

Hence, proved.

Question: 4 A

Prove that:

Solution:

Consider L.H.S

$= \frac{\sin 70°}{\cos 20°} + \frac{cosec\, 20°}{\sec 70°}$ - 2 cos 70° cosec 20°

$= \frac{\sin 70°}{\cos(90-70)°} + \frac{cosec\,(90-70)°}{\sec 70°}$ - 2 cos 70° cosec (90-70)°

$= \frac{\sin 70°}{\sin 70°} + \frac{sec\ 70°}{\sec 70°}$ - 2 cos 70° sec 70°

= 1 + 1 + - 2 = 0 = R.H.S.

Hence, proved.

Question: 4 B

Prove

Solution:

Consider L.H.S.

$= \frac{\cos 80°}{\sin 10°}$ + cos 59° cosec 31°

$= \frac{\cos 80°}{\sin(90-80)°}$ + cos 59° cosec (90-59)°

$= \frac{\cos 80°}{\cos 80°}$ + cos 59° sec 59°

= 1 + 1 = 2 = R.H.S.

Hence, proved.

Question: 4 C

Prove

Solution:

Consider L.H.S.

$$= \frac{2\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45°\tan 20°\tan 40°\tan 50°\tan 70°}{5}$$

$$= \frac{2\sin 68°}{\cos(90-68)°} - \frac{2\cot 15°}{5\tan(90-15)°} - \frac{3\tan 45°\tan 20°\tan 40°\tan(90-50)°\tan(90-20)°}{5}$$

$$= \frac{2\sin 68°}{\sin 68°} - \frac{2\cot 15°}{5\cot 15°} - \frac{3\tan 45°\tan 20°\tan 40°\cot 40°\cot 20°}{5}$$

= 2 –(2/5) – 3(1 × 1 × 1)/5

= 2 – (2/5) – (3/5)

= (10 – 2 - 3)/5

= 5/5

= 1 = R.H.S.

Hence, proved.

Question: 4 D

Prove

Solution:

To prove: $\frac{\sin 18°}{\cos 72°}+\sqrt{3}\left(\tan 10°\ \tan 30°\ \tan 40°\ \tan 50°\ \tan 80°\right)=2$ **Proof:**Consider L.H.S.

$= \frac{\sin 18°}{\cos 72°}+\sqrt{3}(\tan 10° \tan 30° \tan 40° \tan 50° \tan 80°)$Now look for the pairs whose angles give sum of 90°.Here the sum of angles of tan 10° and tan 80° gives 90°.Also the sum of angles of tan 40° and tan 50° gives 90°.So, now change tan 80° into tan(90-10°) and tan 50° into tan(90-40°)

$= \frac{\sin 18°}{\cos(90-18)°}+\sqrt{3}(\tan 10° \tan 30° \tan 40° \tan(90-40)° \tan(90-10°)$We know tan(90-θ) = cot θtan 30° = 1/√3

$= \frac{\sin 18°}{\sin 18°}+\sqrt{3}\left(\tan 10° \times \frac{1}{\sqrt{3}} \times \tan 40°\ \cot 40°\ \cot 10°\right)$

$= 1 + \sqrt{3}\left(\tan 10° \cot 10° \times \frac{1}{\sqrt{3}} \times \tan 40°\ \cot 40°\ \right)$Since tan θ = 1/cot θ

$= 1 + \sqrt{3}\left(1\times \frac{1}{\sqrt{3}} \times 1\right)$

= 1 + 1 = 2 = R.H.S.

Hence, proved.**Note:** In such questions take the pairs whose angles give sum of 90° ,change one of them in the form of 90-θ and substitute remaining known values.Like in this case tan 50° is changed into tan(90-40)° so that it will can be written as cot 50° andtan 50° × cot 50° gives us value 1.

Question: 4 E

Prove

Solution:

Consider L.H.S.

$$= \frac{7\cos 55°}{3\sin 35°}-\frac{4\left(\cos 70° cosec\, 20°\right)}{3\left(\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°\right)}$$

$$= \frac{7\cos 55°}{3\sin(90-55)°}-\frac{4\left(\cos 70° cosec(90-70)°\right)}{3\left(\tan 5° \tan 25° \tan 45° \tan(90-25)° \tan(90-5°)\right)}$$

$$= \frac{7\cos 55°}{3\cos 55°}-\frac{4\left(\cos 70° sec\, 70°\right)}{3\left(\tan 5° \tan 25° \times 1 \times \cot 25° \cot 5°\right)}$$

$$= \frac{7\cos 55°}{3\cos 55°}-\frac{4\left(\cos 70° sec\, 70°\right)}{3\left(\tan 5° \cot 5° \times 1 \times \tan 25° \cot 25°\right)}$$

= (7/3) – (4/3) = 43/3 = 1 = R.H.S.

Hence, proved.

Question: 5 A

Prove that:

Solution:

Consider L.H.S.

= sin θ cos (90° - θ) + sin (90° - θ) cos θ

= sin θ sin θ + cos θ cos θ

= $\sin^2\theta + \cos^2\theta$

= 1 = R.H.S.

Hence, proved.

Question: 5 B

Prove

Solution:

Consider L.H.S.

$$= \frac{\sin\theta}{\cos(90-\theta)} + \frac{\cos\theta}{\sin(90-\theta)}$$

$$= \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\cos\theta}$$

= 1 +1 = 2 = R.H.S.

Hence, proved.

Question: 5 C

Prove

Solution:

Consider L.H.S.

$$= \frac{\sin\theta\cos(90°-\theta)\cos\theta}{\sin(90°-\theta)} + \frac{\cos\theta\sin(90°-\theta)\sin\theta}{\cos(90°-\theta)}$$

$$= \frac{\sin\theta\sin\theta\cos\theta}{\cos\theta} + \frac{\cos\theta\cos\theta\sin\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta\cos\theta}{\cos\theta} + \frac{\cos^2\theta\sin\theta}{\sin\theta}$$

$$= \frac{\sin^3\theta\cos\theta + \cos^3\theta\sin\theta}{\cos\theta\sin\theta}$$

$$= \frac{\cos\theta\sin\theta\ (\cos^2\theta + \sin^2\theta)}{\cos\theta\sin\theta}$$

$$= \frac{\cos\theta\sin\theta\ (1)}{\cos\theta\sin\theta} = 1 = R.H.S.$$

Hence, proved.

Question: 5 D

Prove

Solution:

Consider L.H.S.

$$= \frac{\cos(90°-\theta)\sec(90°-\theta)\tan\theta}{\operatorname{cosec}(90°-\theta)\sin(90°-\theta)\cot(90°-\theta)} + \frac{\tan(90°-\theta)}{\cot\theta}$$

$$= \frac{(\sin\theta\operatorname{cosec}\theta)\tan\theta}{(\sec\theta\cos\theta)\tan\theta} + \frac{\cot\theta}{\cot\theta}$$

$$= \frac{1\times\tan\theta}{1\times\tan\theta} + 1 = 1+1 = 2 = \text{R.H.S.}$$

Hence, proved.

Question: 5 E

Prove

Solution:

Consider L.H.S.

$$= \frac{cos(90° - \theta)}{1 + \sin(90° - \theta)} + \frac{1 + \sin(90° - \theta)}{cos(90° - \theta)}$$

$$= \frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1 + \cos\theta)^2}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{\sin\theta(1 + \cos\theta)} = \frac{(\sin^2\theta + \cos^2\theta) + 1 + 2\cos\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{1 + 1 + 2\cos\theta}{\sin\theta(1 + \cos\theta)} = \frac{2(1 + \cos\theta)}{\sin\theta(1 + \cos\theta)}$$

$= \frac{2}{\sin\theta}$ = 2 cosec θ = R.H.S.

Hence, proved.

Question: 5 F

Prove

Solution:

Consider L.H.S.

$$= \frac{\sec(90 - \theta) cosec\,\theta - \tan(90 - \theta)\cot\theta + \cos^2 25° + \cos^2 65°}{3\tan 27° \tan 63°}$$

$$= \frac{\text{cosec}\,\theta\, cosec\,\theta - \cot\theta\cot\theta + \cos^2 25° + \cos^2(90 - 25)°}{3\tan 27° \tan(90 - 27)°}$$

$$= \frac{(\text{cosec}^2\theta - \cot^2\theta) + (\cos^2 25° + \sin^2 25°)}{3\tan 27° \cot 27°}$$

$= \frac{1 + 1}{3 \times 1}$ = 2/3 = R.H.S.

Hence, proved.

Question: 5 G

cot θ tan (90° -θ

Solution:

Consider L.H.S.

= cot θ tan (90° -θ) - sec (90° - θ)cosec θ +√3tan 12°tan 60°tan 78°

= cot θ cot θ - cosec θ cosec θ +√3 tan 60° tan 12° tan 78°

= $\cot^2$ θ – cosec^2 θ +√3 tan 60° tan 12° tan(90-12)°

$= -(\text{cosec}^2 \theta - \cot^2 \theta) + \sqrt{3} \tan 60° \tan 12° \cot 12°$

$= -1 + \sqrt{3}(\sqrt{3} \times \tan 12° \cot 12°)$

$= -1 + \sqrt{3}(\sqrt{3} \times 1)$

$= -1 + 3$

= 2 = R.H.S.

Hence, proved.

Question: 6 A

Prove that:

Solution:

Consider L.H.S.

$= \tan 5° \tan 25° \tan 30° \tan 65° \tan 85°$

$= \tan 5° \tan 85° \tan 25° \tan 65° \tan 30°$

$= \tan 5° \tan (90-5)° \tan 25° \tan (90-25)° \tan 30°$

$= (\tan 5° \cot 5°) (\tan 25° \cot 25°) \tan 30°$

$= 1 \times 1 \times (1/\sqrt{3})$

$= 1/\sqrt{3}$ = R.H.S.

Hence, proved.

Question: 6 B

cot 12° cot 38° c

Solution:

Consider L.H.S.

$= \cot 12° \cot 38° \cot 52° \cot 60° \cot 78°$

$= (\cot 12° \cot 78°) (\cot 38° \cot 52°) \cot 60°$

$= (\cot 12° \cot (90-12)°) (\cot 38° \cot (90-38)°) \cot 60°$

$= (\cot 12° \tan 12°) (\cot 38° \tan 38°) \cot 60°$

$= 1 \times 1 \times (1/\sqrt{3})$

$= 1/\sqrt{3}$ = R.H.S.

Hence, proved.

Question: 6 C

cos 15° cos 35° c

Solution:

Consider L.H.S. $= \cos 15° \cos 35° \text{cosec } 55° \cos 60° \text{cosec } 75°$

$= \cos 15° \text{cosec } 75° \cos 35° \text{cosec } 55° \cos 60°$

$= \cos 15° \text{cosec } (90-15)° \cos 35° \text{cosec } (90-35)° \cos 60°$

$= (\cos 15° \sec 15°) \times (\cos 35° \sec 35°) \times \cos 60°$

$= (1) \times (1) \times (1/2)$

$= 1/2$ = R.H.S.

Hence, proved.

Question: 6 D

cos 1° cos 2° cos

Solution:

Consider L.H.S. = cos 1° cos 2° cos 3° ... cos 180°

= cos 1° cos 2° cos 3° ...× cos 90° × ... cos 180°

= cos 1° cos 2° cos 3° ... × 0 × cos 180°

= 0 (∵ cos 90° = 0)

Hence, proved.

Question: 6 E

Prove

Solution:

Consider L.H.S.

$$= \left(\frac{\sin 49°}{\cos 41°}\right)^2 + \left(\frac{\cos 41°}{\sin 49°}\right)^2$$

$$= \left(\frac{\sin(90-41)°}{\cos 41°}\right)^2 + \left(\frac{\cos 41°}{\sin(90-41)°}\right)^2$$

$$= \left(\frac{\cos 41°}{\cos 41°}\right)^2 + \left(\frac{\cos 41°}{\cos 41°}\right)^2$$

= $1^2 + 1^2$

1 + 1 = 2 = R.H.S.

Hence, proved.

Question: 7 A

Prove that:

Solution:

Consider L.H.S.

= sin (70°+θ) — cos (20° — θ)

= sin (70°+θ) — cos [90°-(70° + θ)]

= sin (70°+θ) — sin (70° + θ)

= 0 = R.H.S.

Hence, proved.

Question: 7 B

tan (55° — θ) — c

Solution:

Consider L.H.S.

= tan (55° — θ) — cot (35° + θ)

= tan (90°-(35° +θ)) — cot (35° + θ)

= cot (35° + θ) - cot (35° + θ)

= 0

Hence, proved.

Question: 7 C

cosec (67° + θ) —

Solution:

Consider L.H.S.

= cosec (67° + θ) — sec (23° — θ)

= cosec (67° + θ) — sec (90° —(23° + θ))

= cosec (67° + θ) - cosec (67° + θ)

= 0

Hence, proved.

Question: 7 D

cosec (65° + θ) —

Solution:

Consider L.H.S.

= cosec (65° + θ) — sec (25° —θ) — tan (55° — θ) + cot (35° + θ)

= cosec (65° + θ) — sec (90° - (65° + θ)) — tan (90° - (35° + θ)) + cot (35° + θ)

= cosec (65° + θ) — cosec (65° + θ)) — cot (35° + θ) + cot (35° + θ)

= 0 = R.H.S.

Hence, proved.

Question: 7 E

sin (50° +θ) — co

Solution:

Consider L.H.S.

= sin (50° +θ) — cos (40° — θ) + tan 1° tan 10° tan 80° tan 89°

= sin ((90°-(40° - θ)) — cos (40° — θ) + (tan 1° tan 89°)(tan10° tan 80°)

= cos (40° - θ) - cos (40° - θ) + [tan 1° tan (90°-1°)][tan10° tan(90°-10°)]

= 0 + [(tan 1° cot 1°] [tan10° cot 10°]

= 0 + [1] × [1]

= 0 + 1 = 1 = R.H.S.

Hence, proved.

Question: 8 A

Express each of t

Solution:

Consider sin 67° + cos 75° = sin (90-23)° + cos (90-15)°

= cos 23° + sin 15°

Question: 8 B

Express each of t

Solution:

Consider cot 65° + tan 49° = cot (90-25)° + tan (90-41)°

= tan 25° + cot 41°

Question: 8 C

Express each of t

Solution:

Consider sec 78° + cosec 56° = sec (90-12)° + cosec (90-34)°

= cosec 12° + sec 34°

Question: 8 D

Express each of t

Solution:

Consider cosec 54° + sin 72° = cosec (90-36)° + sin (90-18)°

= sec 36° + cos 18°

Question: 9

If A, B and C are

Solution:

Since A, B and C are the angles of a triangle, therefore sum of the angles equals 180°.

∴ A + B + C = 180° ⇒ C + A = 180° - B

Now, consider L.H.S. = $\tan\left(\frac{C+A}{2}\right)$

= $\tan\left(\frac{180°-B}{2}\right)$

= $\tan\left(90° - \frac{B}{2}\right)$

= $\cot\left(\frac{B}{2}\right)$ = R.H.S.

Hence, proved.

Question: 10

If cos 2 θ

Solution:

given: cos 2θ = sin 4θ**To find:** the value of θ**Solution:**Consider cos 2θ = sin 4θ,Since, sin(90° - θ) = cos θ ∴We can rewrite it as: sin (90° - 2θ) = sin 4θ

On comparing both sides, we get,

90° - 2θ = 4θ

⇒ 90° = 4θ + 2θ

⇒ 6θ = 90°

⇒ θ = 15°

Question: 11

If sec 2A = cosec

Solution:

We are given that: sec 2A = cosec (A – 42°)

∴We can rewrite it as: cosec (90° - 2A) = cosec (A – 42°)

On comparing both sides, we get,

90° - 2A = A – 42 °

$\Rightarrow$ A + 2A = 90° + 42°

$\Rightarrow$ 3A = 132°

$\Rightarrow$ A = 44°

Question: 12

If sin 3A = cos (

Solution:

We are given that: sin 3A = cos (A - 26°)

∴We can rewrite it as: cos (90° - 3A) = cos (A - 26°)

On comparing both sides, we get,

90° - 3A = A - 26°

$\Rightarrow$ A + 3A = 90° + 26°

$\Rightarrow$ 4A = 116°

$\Rightarrow$ A = 29°

Question: 13

If tan 2A = cot (

Solution:

We are given that: tan 2A = cot (A – 12°)

∴We can rewrite it as: cot (90° - 2A) = cot (A – 12°)

On comparing both sides, we get,

90° - 2A = A – 12 °

$\Rightarrow$ A + 2A = 90° + 12°

$\Rightarrow$ 3A = 102°

$\Rightarrow$ A = 34°

Question: 14

If sec 4A = cosec

Solution:

We are given that: sec 4A = cosec (A – 15°)

∴We can rewrite it as: cosec (90° - 4A) = cosec (A – 15°)

On comparing both sides, we get,

90° - 4A = A – 15 °

$\Rightarrow$ A + 4A = 90° + 15°

$\Rightarrow$ 5A = 105°

$\Rightarrow$ A = 21°

Question: 15

Prove that:

Solution:

Consider L.H.S.

$= \frac{2}{3}\ \text{cosec}^2\ 58° - \frac{2}{3}\ \cot 58° \tan 32° - \frac{5}{3} \tan 13° \tan 37° \tan 45° \tan 53° \tan 77°$

$= \frac{2}{3}$ cosec^2 58°- $\frac{2}{3}$ cot 58° tan (90-58)° - $\frac{5}{3}$ × [tan 13° tan 77°] × [tan 37° tan 53°] × tan 45°

$= \frac{2}{3}$ cosec^2 58°- $\frac{2}{3}$ cot 58° cot 58° - $\frac{5}{3}$ × [tan 13° tan (90-13)°] × [tan 37° tan (90-37)°] × 1

$= \frac{2}{3}$ cosec^2 58°- $\frac{2}{3}$ cot^2 58° - $\frac{5}{3}$ × [tan 13° cot 13°] × [tan 37° cot 37°]×1

$= \frac{2}{3}$ [cosec^2 58°- cot^2 58°] - $\frac{5}{3}$ × [1] × [1] × 1

$= \frac{2}{3}$ [1] - $\frac{5}{3}$

= (2/3) – (5/3)

= (2 - 5)/3

= -3/3

= -1 = R.H.S.

Hence, proved.