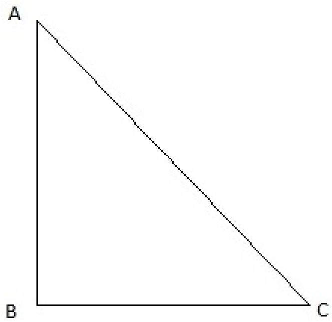
Chapter : 14. HEIGHTS AND DISTANCES

Exercise : 14

Question: 1

A tower stands ve

Solution:



Let AB be the tower and C be the point on the ground 20 m away from the foot of the tower B from where the angle of elevation of the top of the tower is 60°. Now, draw a line from C to A. Join B and C. We get a triangle ABC with right angle at B. We are to find the height of the tower, that is AB. We will be using trigonometric ratios involving AB(height) and BC(base). So, ∠ACB = 60° BC = 20m.

Now in ΔABC,

$$\tan(\angle ACB) = \tan 60° = \frac{AB}{BC} = \frac{AB}{20}$$

or,

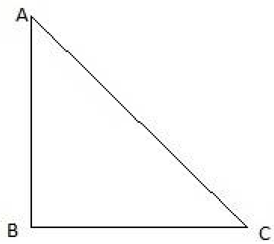
AB = 20√3 = 34.64m.

The height of the tower is 34.64 m.

Question: 2

A kite is flying

Solution:



Let A be the position of the kite in the sky. Let C be the position on the ground from where a string is attached to the kite. Since we have assumed that there is no slack in the string, we take AC to be a straight line making 60° angle with the ground. Draw a perpendicular from A on the ground which meets at point B. The kite is flying at a height of 75 m above the ground. So, AB = 75 m. Join B and C. We thus get a triangle ABC with right angle at B. We are to find the length of the string that is AC. We will use the trigonometric ratio sine which uses the perpendicular AB and the hypotenuse AC to find AC. Now, ∠ ACB = 60°

From ΔABC,

$$\sin(\angle ACB) = \sin 60° = \frac{AB}{BC} = \frac{75}{BC}$$

or,

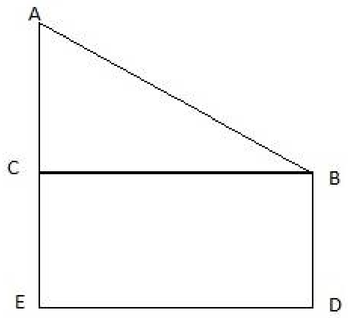
$$BC = \frac{75 \times 2}{\sqrt{3}} = 86.60\text{m}.$$

The length of the string is 86.60 m.

Question: 3

An observer 1.5 m

Solution:



In the figure, let BD be the height of the man, i.e. BD = 1.5m. Let AE be the chimney in the figure. Join B and C. We get a triangle which is right angled at C. Clearly, BD = CE. Also, given that ∠ ABC = 60°. We use the trigonometric ratio tan which uses AC as height and BC as a base to find the height of the chimney AE.

Now, clearly CE = 1.5m. The height of the chimney is AE.

From ΔABC,

$$\tan\angle ABC = \tan 60° = \frac{AC}{BC} = \frac{AC}{30}$$

or, $AC = \sqrt{3} \times 30 = 51.96$

Thus, AC = 51.96m and CE = 1.5m. The height of the chimney

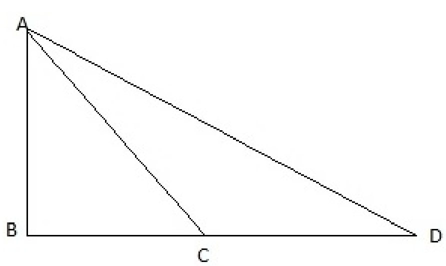
AE = AC + CE = 51.96 + 1.5 = 53.46 m.

Hence, the answer is 53.46 m.

Question: 4

The angles of ele

Solution:



Let AB be the tower in the figure and C and D be two points on the straight line BD, at distances 5m and 20m from the foot of the tower AB. Now, join B, C, and C, D. We get two right-angled triangles both right angled at B. We use the trigonometric ratio tan by using AB as height and BC as a base(for ΔABC) and AB as height and BD as a base(for ΔABD) to find the height of the tower AB. By the problem we have, ∠ACB + ∠ADB = 90°.

In ΔABC, we have

$$\tan\angle ACB = \tan(90°\text{-}\angle ABD) = \frac{AB}{BC} = \frac{AB}{5}$$

or, $AB = 5\cot\angle ABD$

or, $\tan\angle ABD = \frac{5}{AB}$

Also, from ΔABD,

$$\tan\angle ABD = \frac{AB}{BD} = \frac{AB}{20}$$

or,

$$AB = 20\tan\angle ABD$$

Putting the value of tan∠ABD in the equation we get

$$AB = \frac{20 \times 5}{AB}$$

or, $AB^2 = 100$

or, $AB = 10$

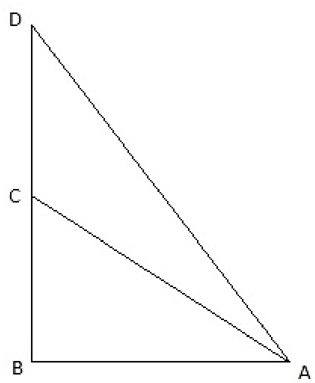
a positive value is taken since the height of the tower can't be negative.

The height of the tower AB = 10m.

Question: 5

The angle of elev

Solution:



Let BC be the tower and CD be the flagstaff. Join C, A and D, A and A, B. We get two right-angled triangles ABC and BAD which are right-angled at B. By the problem, it is clear that ∠ BAC = 45° and ∠ BAD = 60°. We use trigonometric ratio tan for both the triangles using BC as height and AB as a base(for ΔABC) and BD as height and AB as a base(for ΔABD) to find the height of the flagstaff CD.

Let BC be x.

In ΔABC we have,

$$\tan\angle BAC = \frac{BC}{AB} = \frac{x}{120}$$

or,

$$\tan 45° = \frac{x}{120}$$

or,

x = 120

So, we get BC = 120m. In ΔABD,

$$\tan\angle BAD = \frac{BD}{AB} = \frac{DC + BC}{120} = \frac{DC + 120}{120}$$

or,

$$\tan 60° = \frac{DC + 120}{120}$$

or,

$$\sqrt{3} = \frac{DC + 120}{120}$$

or,

$$DC = 120 \times \left(\sqrt{3}-1\right) = 120 \times 0.732 = 87.84$$

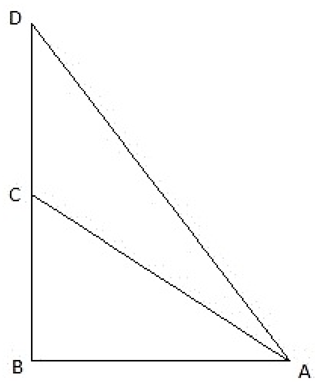
So, height of the flagstaff = DC = 87.84m.

Question: 6

From a point on t

Solution:

In the figure assume, A to be the point 40m away from the foot of the tower BC. Join A,B and A,C and A,D. Let DC be the water tank and BC be the tower. We get two right-angled triangles ABC and ABD, right angled at B. Also, ∠ BAC = 30° and ∠ BAD = 45°. We use trigonometric ratios tan using AB as base and BC as height(for ΔABC) and AB as base and CD as height(for ΔABD). Let BC be x.



In ΔABC,

$$\tan \angle BAC = \tan 30° = \frac{BC}{AB} = \frac{x}{40}$$

or,

$$x = \frac{40}{\sqrt{3}} = \frac{40}{1.732} = 23.09$$

So, BC = 23.09m.

In ΔABD,

$$\tan \angle BAD = \tan 45° = \frac{BD}{AB} = \frac{DC + \frac{40}{\sqrt{3}}}{40}$$

or,

$$DC = 40 - \frac{40}{\sqrt{3}} = \frac{40(\sqrt{3}-1)}{\sqrt{3}} = \frac{40 \times 0.732}{1.732} = 16.9$$

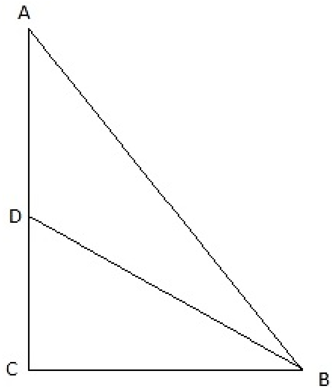
i. Height of the tower = BC = 23.1m

ii. Depth of the tank = DC = 16.9m

Question: 7

A vertical tower

Solution:



Let AD be the Flagstaff, 6m high and DC be the tower. B be the point on the plane from where the angles of elevation of the top(A) and bottom of the flagstaff(D) are 60° and 30° respectively. Join C and B. Then we get two triangles ABC and BCD with right angle at C. We are to find the height of the tower DC. Now, to summarize, we are given, AD = 6m, ∠ ABC = 60°, and ∠ DBC = 30°. To find DC we will first find BC from the triangle ABC and again from triangle BCD and equate the expression for BC in both the cases. Then we will solve for DC from the equation.

In ΔABC,

$$\tan\angle ABC = \tan 60° = \frac{AC}{BC} = \frac{AD + DC}{BC} = \frac{6 + DC}{BC}$$

or,

$$BC = \frac{6 + DC}{\sqrt{3}}$$

In ΔBCD,

$$\tan\angle DBC = \tan 30° = \frac{DC}{BC}$$

or,

$$BC = \frac{DC}{\tan 30°} = DC\sqrt{3}$$

Equating the values of BC,

$$DC\sqrt{3} = \frac{6 + DC}{\sqrt{3}}$$

or,

$$3DC = 6 + DC$$

or,

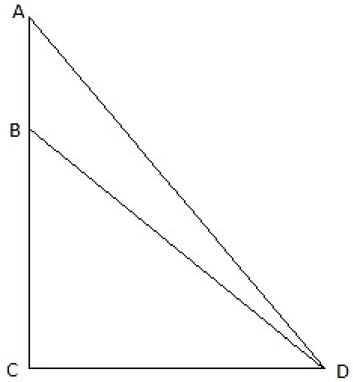
DC = 3m

Hence the height of the tower is 3m.

Question: 8

A statue 1.46 m t

Solution:



In the figure, let AB be the statue of height 1.46 m, BC the pedestal. Let D be a point on the ground from which the angles of elevation of the top of the statue and the top of the pedestal are 60° and 45° respectively. We are to find the height of the pedestal, which is BC. Join C and D. We get two triangles ACD and BCD with right angle at C. To find BC, we use trigonometric ratios to find the expression of CD from ACD and BCD and then equate them. Now, ∠BDC = 45°, ∠ADC = 60°, AB = 1.46 m.

From ΔADC,

$$\tan\angle ADC = \tan 60° = \frac{AC}{DC} = \frac{1.46 + BC}{CD}$$

or,

$$CD = \frac{1.46 + BC}{\sqrt{3}}$$

Again, from ΔBCD,

$$\tan\angle BDC = \tan 45° = \frac{BC}{CD}$$

or,

BC = CD

or,

$$\frac{1.46 + BC}{\sqrt{3}} = BC$$

or,

$$\sqrt{3}BC = 1.46 + BC$$

or,

$$(\sqrt{3} - 1)BC = 1.46$$

or,

$$0.732 \times BC = 1.46$$

or,

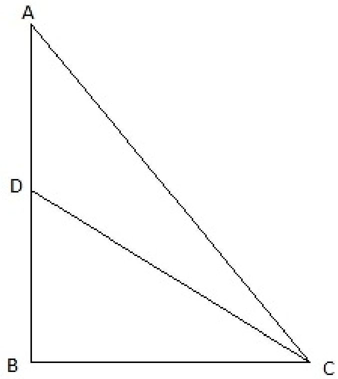
$$BC = \frac{1.46}{0.73} = 2\ \mathrm{m}$$

Hence the answer is 2m.

Question: 9

The angle of elev

Solution:



In the above figure, let DB be the unfinished tower. Let C be the point on the ground, 75 m from the base B from which the angle of elevation of the top D is 30°. Let A be the point to which the tower must be raised such that the angle of elevation of it from C becomes 60°. Join B and C. We get two triangles ABC and BCD with right angle at B. We need to find the excess height that is an AD. We have, BC = 75 m, ∠BCD = 30° and ∠ACB = 60°. To find the AD, we will find AB and BD from the triangles ABC and BCD respectively using the trigonometric ratio tan and then subtract BD from AB.

Now, from ΔBCD,

$$\tan \angle BCD = \tan 30° = \frac{DB}{BC} = \frac{DB}{75}$$

or,

$$DB = \frac{75}{\sqrt{3}}$$

Again, from ΔABC,

$$\tan\angle ACB = \tan 60° = \frac{AB}{BC} = \frac{AB}{75}$$

or,

$$AB = 75 \times \sqrt{3}$$

Now, we have got both AB and DB. Then we are asked to find the AD.

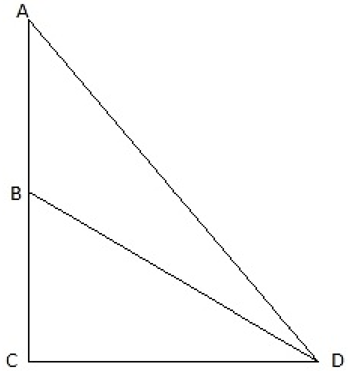
$$AD = AB - DB = 75 \times \sqrt{3} - \frac{75}{\sqrt{3}} = \frac{75(3-1)}{\sqrt{3}} = \frac{75 \times 2}{1.732} = 86.60 \text{ m.}$$

Hence the answer is 86.6 m.

Question: 10

On a horizontal p

Solution:



In the above figure, let BC be the tower, AB is the flagpole. Let D be the point 9 m away from C such that the angle of elevations of the top and bottom of the flagpole AB are 60° and 30° respectively. Join C and D. We get two triangles ADC and BCD with right angle at C. We have to find the height of the tower BC and the height of the flagpole AB. For this, we use trigonometric ratio tan for triangles ACD and BCD to find AC and BC respectively. Subtract BC from AC to find AB.

Now we have, DC = 9 m, ∠ADC = 60°, and ∠BDC = 30°. We are to find AB and BC.

From ΔACD,

$$\tan\angle ADC = \tan 60° = \frac{AC}{CD} = \frac{AC}{9}$$

or,

$$AC = 9 \times \sqrt{3}$$

From ΔBDC,

$$\tan\angle BDC = \tan 30° = \frac{BC}{CD} = \frac{BC}{9}$$

or,

$$BC = \frac{9}{\sqrt{3}}$$

Now,

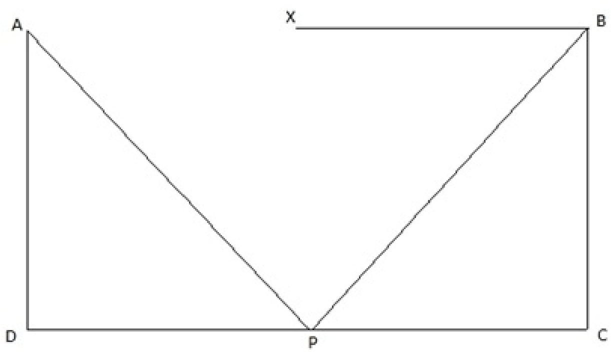
$$AB = AC - BC = 9 \times \sqrt{3} - \frac{9}{\sqrt{3}} = \frac{9(3-1)}{\sqrt{3}} = \frac{9 \times 2}{1.73} = 10.40 \text{ m.}$$

And, BC = 9/1.73 = 5.20 m

Question: 11

Two poles of equa

Solution:



Let AD and BC be the two poles of equal height standing on the two sides of the road of width 80 m. Join DC. Then DC = 80 m. P is a point on the road from which the angle of elevation of the top of tower AD is 60°. Also, the angle of depression of the point P from the point B is 30°. Draw a line BX from B parallel to the ground. Then ∠XBP = ∠BPC = 30°. We are to find the height of the poles and the distance of the point P from both the poles. Also, ∠APD = 60°.

Let DP = x. Then PC = 80-x.

In ΔAPD,

$$\tan\angle APD = \tan 60° = \frac{AD}{PD} = \frac{AD}{x}$$

or,

$$AD = \sqrt{3}x$$

Now, from ΔAPD,

$$\tan\angle BPC = \tan 30° = \frac{BC}{PC} = \frac{BC}{80 - x}$$

or,

$$BC = \frac{80 - x}{\sqrt{3}}$$

Now, BC = AD

So,

$$\frac{80 - x}{\sqrt{3}} = \sqrt{3}x$$

or,

$$3x = 80 - x$$

or,

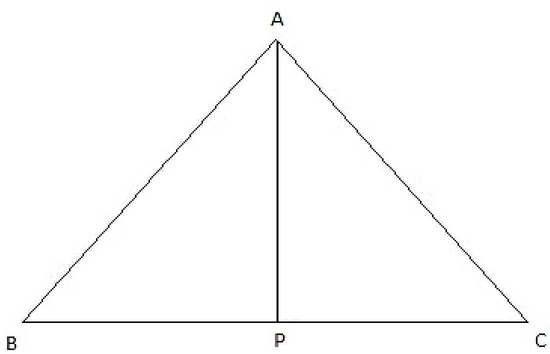
x = 20 m

So,PD = 20 m. Hence, PC = 80-20 = 60 m. Also, BC = AD = $20\sqrt{3}$ m.

Question: 12

Two men are on op

Solution:



Let the positions of the two men are B and C. Let AP represent the tower in the figure. Join B, P, and C, P.We have, $\angle ABP = 30°$ and $\angle ACP = 45°$. Given that AP = 50 m.We get two right-angled triangles ΔAPB and ΔAPC, both right angled at P. We use trigonometric ratio tan for both the triangles using BP as a base and AP as height(for ΔAPB) and PC as base and AP as height(for ΔAPC). We get BP and PC. The distance between the men is BC.

We have to find BC. Let BP = x. In ΔAPB,

$$\tan\angle ABP = \tan 30° = \frac{AP}{BP} = \frac{50}{x}$$

or,

$$x = \frac{50}{\tan 30°} = 50\sqrt{3}$$

Again, in ΔAPC,

$$\tan\angle ACP = \tan 45° = \frac{AP}{PC} = \frac{50}{PC}$$

or,

PC = 50

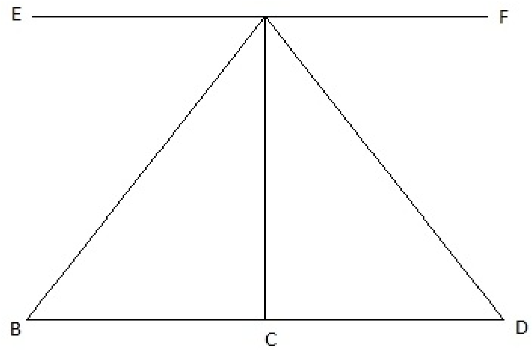
So, distance between the two men is

BC = BP + PC = $50\sqrt{3} + 50 = 50(\sqrt{3} + 1)$ = 136.6m.

Question: 13

From the top of a

Solution:



In the figure, let AC be the tower of height 100m. The angles of depression are ∠BAE and ∠DAF. Given, AC = 100m. Since the line EF is parallel to the line BD, we have ∠BAE = ∠BAC = 30°, ∠DAF = ∠DAC = 45°, where B, D are the positions of the cars. Join B, C, and C, D. We get two right-angled triangles ΔABC and ΔADC, both right angled at C. We use trigonometric ratio tan for both the triangles, where BC is the base and Ac is the height (for ΔABC) and CD is the base and AC is the height in ΔACD. We are to find the distance between the two cars, i.e. BD.

In ΔABC,

$$\tan\angle BAC = \tan 30° = \frac{AC}{BC} = \frac{100}{BC}$$

or,

$$BC = \frac{100}{\tan 30°} = 100\sqrt{3}$$

In ΔADC,

$$\tan\angle DAC = \tan 45° = \frac{AC}{DC} = \frac{100}{DC}$$

or,

DC = 100

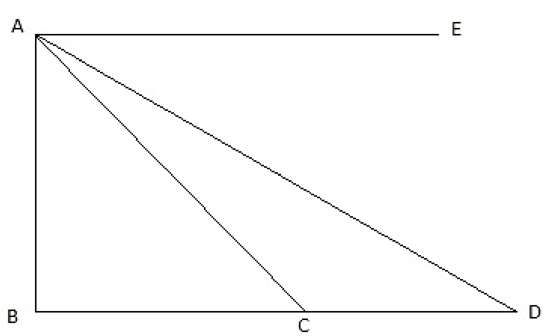
Therefore, the distance between the two cars is

BD = BC + CD = 100√3 + 100 = 100(√3 + 1) = 273 m.

Question: 14

A straight highwa

Solution:



In the figure, let AB be the tower and D be the position of the car. Let C be the position of the car 6seconds later. The angles of depression are ∠DAE and ∠CAE. Join B, C, and C, D and A, E. Since AE is parallel to BD, we must have, ∠ADB = ∠DAE = 30°, ∠CAE = ∠ACB = 60°. We get two right-angled triangles ΔABC and ΔABD, both right angled at B. We use trigonometric ratio tan for both the triangles using BC as base and AB as height for ΔABC and BD as base and AB as height in ΔABD. We find the CD.

Let BC = x.

In ΔABC,

$$\tan \angle ACB = \tan 60° = \frac{AB}{BC} = \frac{AB}{x}$$

or,

$$AB = x \tan 60° = x\sqrt{3}$$

Again, in ΔABD,

$$\tan \angle ADB = \tan 30° = \frac{AB}{DB} = \frac{AB}{BC + CD} = \frac{x\sqrt{3}}{x + CD}$$

or,

CD = 2x

Now, the speed of the car = CD/6 = 2x/6 = x/3.

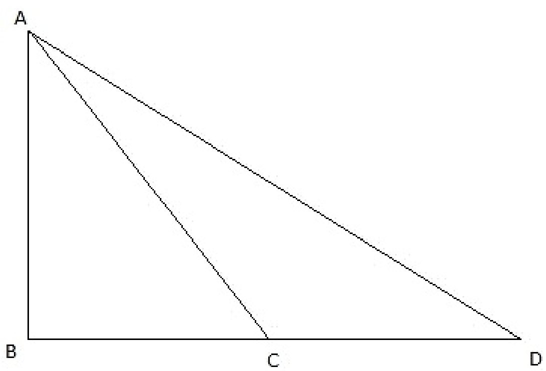
Hence, time taken by the car to reach the foot of the tower is

= BC/(Speed of the car) = 3 secs.

Question: 15

A TV tower stands

Solution:



In the figure, let AB be the tower and BC be the canal. C is the point on the other side of the canal directly opposite the tower. Join B and C. Let D be another point, 20m away from C. Join C and D. We get two right-angled triangles ΔABC and ΔABD, both right angled at B. Given that, ∠ACB = 60° and ∠ADB = 30°. DC = 20m. We use trigonometric ratio tan using AB as height and BC as a base(for ΔABC) and AB as height and BD as a base(for ΔABD) to find AB and BC.

In ΔABC,

$$\tan \angle ACB = \tan 60° = \frac{AB}{BC}$$

or,

$$AB = BC \times \tan 60° = BC\sqrt{3}$$

In ΔABD,

$$\tan \angle ADB = \tan 30° = \frac{AB}{BD} = \frac{AB}{BC + CD} = \frac{AB}{BC + 20}$$

or,

$$AB = (BC + 20)\tan 30° = \frac{BC + 20}{\sqrt{3}}$$

Equating the values of AB in ΔABC and ΔABD,

$$BC\sqrt{3} = \frac{BC + 20}{\sqrt{3}}$$

or,

$$3BC = BC + 20$$

or,

$$2BC = 20$$

or,

$$BC = 10$$

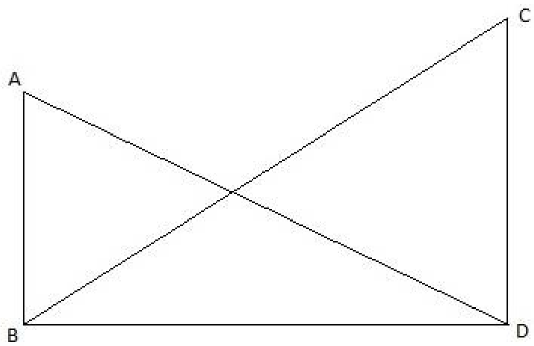
So, the width of the canal = BC = 10m. We found that AB = BC√3 = 10√3m.

So, height of the tower = AB = $10\sqrt{3}$m.

Question: 16

The angle of elev

Solution:



In the figure, let CD be the tower and AB be the building. Join the points A, D, and B, C. We get two right-angled triangles ΔABD and ΔBCD, which are right-angled at B and D respectively. We are given that the angle of elevation of the top of the building from the foot of the tower is 30° and that of the top of the tower from the foot of the building is 60°. So, ∠CBD = 60°, ∠ADB = 30°. We are also given that the height of the tower is 60 m. Hence CD = 60 m. We need to find the height of the building, that is AB. For this, we will first find BD from ΔBDC using the trigonometric ratio tan. Using this value of BD, we will find the value of AB from ΔABD using the trigonometric ratio tan.

In ΔBCD,

$$\tan\angle CBD = \tan 60° = \frac{CD}{BD} = \frac{60}{BD}$$

or,

$$BD = \frac{60}{\tan 60°} = \frac{60}{\sqrt{3}}$$

So, BD = 60/√3m.

In ΔABD,

$$\tan\angle ADB = \tan 30° = \frac{AB}{BD} = \frac{AB}{\frac{60}{\sqrt{3}}}$$

or,

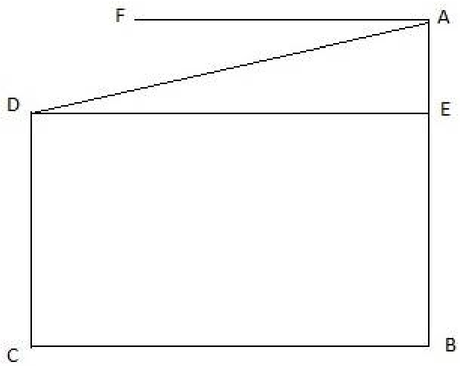
$$AB = BD \times \tan 30° = \frac{60}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{60}{3} = 20m$$

So, height of the building = AB = 20m.

Question: 17

The horizontal di

Solution:



In the figure, let CD be the first tower and AB, the second tower. We are given that the distance between the two towers is 60 m. Join C and B. We thus get CB = 60 m. Again, we are given that the angle of depression of the top of the first tower from the top of the second tower is 30°. We draw a horizontal line parallel to BC from A to F. Then we get ∠FAD = 30°. Now, we draw a line from the top of the first tower onto the second tower parallel to BC to the point E. We get a right-angled triangle ADE with right angle at E. Then ∠ ADE = 30°.We are also given that the height of the second tower is 90 m. So, AB = 90m. We are told to find the height of the first tower, that is CD.

Note that, BC = ED = 60m.

To find CD, we use the trigonometric ratio tan on ΔADE to find AE.

In ΔAED,

$$\tan \angle ADE = \tan 30^\circ = \frac{AE}{DE} = \frac{AE}{60}$$

or,

$$AE = 60 \times \times \tan 30^\circ = 60/\sqrt{3}m$$

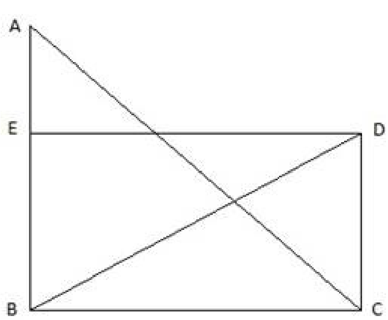
Now we just see that we can get the height of the first tower by subtracting the value of AE from AB to get BE which is equal to CD.

Height of the first tower = DC = AB-AE = 90-60/√3 = 55.35m.

Question: 18

The angle of elev

Solution:



Let AB be the chimney and DC be the tower. We are given that the angle of elevation of the top of the chimney from the foot of the tower DC is 60°. Join A and C. We get a right-angled ΔABC with right angle at B and ∠ACB = 60°. We are again told that the angle of depression of the foot of the chimney from the top of the tower is 30°. If we join B and D, then we get a right-angled ΔBCD with right angle at C. Draw a line from D to a point on AB parallel to BC. So, ∠EDB = 30° and ∠DBC = 30°. The height of the tower is given to be 40 m. So, DC = 40m. We will first find the height of the chimney AB. If AB is less then 100 m, then it does not meet the pollution norms and if otherwise, then it does not meet the pollution norms.

First, we will find the value of BC from ΔBCD using the trigonometric ratio tan. Then using the value of BC, we will find the value of AB from ΔABC using the trigonometric ratio tan.

In ΔBDC,

$$\tan\angle DBC = \tan 30° = \frac{DC}{BC} = \frac{40}{BC}$$

or,

$$BC = \frac{40}{\tan 30°} = 40 \times \sqrt{3}m$$

In ΔABC,

$$\tan\angle ACB = \tan 60° = \frac{AB}{BC} = \frac{AB}{40\sqrt{3}} = \frac{AB}{40\sqrt{3}}$$

or,

$$AB = 40\sqrt{3} \times \sqrt{3} = 120\text{ m}$$

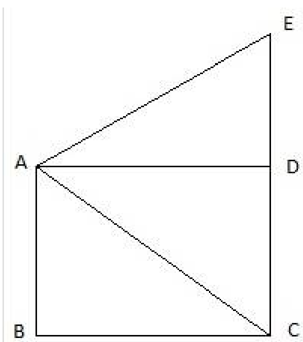
So, the height of the chimney is AB = 120m, which is greater than 100 m.

The height satisfies the pollution norms.

Question: 19

From the top of a

Solution:



Let AB be the building and CE be the cable tower. We are given that, the angle of elevation of the top of the cable tower from the top of the building is 60° and the angle of depression of the bottom of the tower from the top of the building is 45°. Join C, E to A. Also draw a line from A to EC at the point D parallel to BC. We get two right-angled triangles ABC and AED with right angles at B and D respectively. Also, ∠EAD = 60°. ∠DAC = ∠ACB = 45°. Also given that the height of the building is 7 m. That is AB = 7m. We are to find the height of the tower CE.

We first find the value of BC from ΔABC using the trigonometric ratio tan.

In ΔABC,

$$\tan\angle ACB = \tan 45° = \frac{AB}{BC} = \frac{7}{BC}$$

or,

$$BC = \frac{7}{\tan 45°} = 7m$$

Now, AD = BC.

In ΔDAE,

$$\tan\angle DAE = \tan 60° = \frac{DE}{AD} = \frac{DE}{7}$$

or,

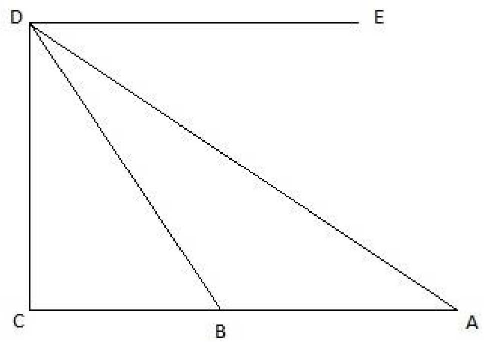
$DE = 7 \times \tan 60° = 7\sqrt{3}m$

So, the height of the cable tower = CE = CD + DE = 7m + 7√3m = 19.12m.

Question: 20

The angle of depr

Solution:



Let DC be the tower. Given that the angle of depression of the point A on the ground from the top of the tower DC is 30°. Join C and A. Now draw a line DE parallel to CA. Also join A and D. Then, ∠ EDA = 30°. We get a right-angled triangle ACD with right angle at C and ∠DAC = 30°. If we move 20 m from A to B towards the foot of the tower C, then the angle of depression changes to 60°. Then, AB = 20 m. Join D and B. Then we get a right-angled triangle ΔDCB with right angle at C.

We are to find the height of the tower that is DC and its distance from A, that is, AC.

Let DC = x. In ΔDCB,

$$\tan \angle DBC = \tan 60° = \frac{DC}{CB} = \frac{x}{BC}$$

or,

$$BC = \frac{x}{\tan 60°} = \frac{x}{\sqrt{3}}$$

In ΔADC,

$$\tan \angle DAC = \tan 30° = \frac{DC}{AC} = \frac{x}{AB + BC} = \frac{x}{20 + \frac{x}{\sqrt{3}}}$$

or,

$$20 + \frac{x}{\sqrt{3}} = \frac{x}{\tan 30°} = x\sqrt{3}$$

or,

$$x\sqrt{3} - \frac{x}{\sqrt{3}} = 20$$

or,

$$x = 10\sqrt{3} = 17.32m$$

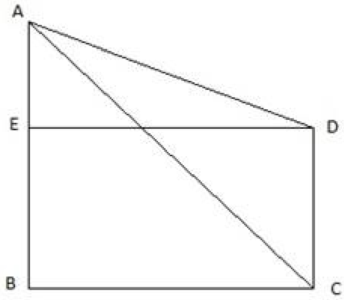
The height of the tower = DC = x = 17.32m.

We have, BC = x/√3 = 10m. So, a distance of A from the tower = AC = 20 + 10 = 30m.

Question: 21

The angle of elev

Solution:



In the figure, let AB be the tower and C be the point on the ground from which the angle of elevation of the top of the tower AB is 60°. Join A and C and B and C. Then we get a right-angled triangle ABC with right angle at B and ∠ACB = 60°. Let D be the point 10 m vertically above C. Then, CD = 10 m. Given that the angle of elevation of the top of the tower from the point D is 30°. Join D and A. Also draw a line DE from D to AB, parallel to BC. Then we get a right-angled triangle AED with right angle at E and ∠ADE = 30°. We are to find the height of the tower, that is AB.

Clearly, ED = BC. Let AE = x

In ΔAED,

$$\tan\angle ADE = \frac{AE}{DE} = \frac{x}{DE}$$

or,

$$\tan 30° = \frac{x}{DE}$$

or,

$$DE = \frac{x}{\tan 30°} = \sqrt{3}x$$

Again, in ΔABC,

$$\tan\angle ACB = \frac{AB}{BC} = \frac{AE+EB}{DE}$$

or,

$$\tan 60° = \frac{x+10}{\sqrt{3}x}$$

or,

$$\sqrt{3} = \frac{x+10}{\sqrt{3}x}$$

or,

3x = x + 10"

or,

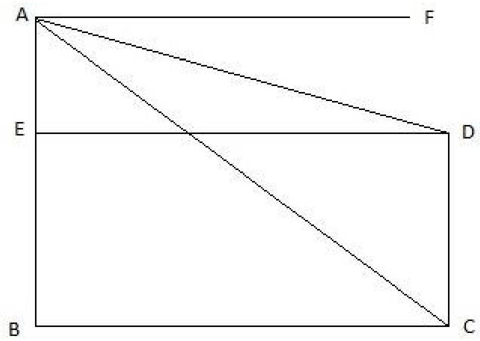
x = 5m

Height of the tower = AB = AE + EB = 5m + 10m = 15m.

Question: 22

The angles of dep

Solution:



Let AB be the cliff and CD be the tower. Join B and C. Draw a line AF parallel to BC. Now, given that the angles of depression of the top and bottom of the tower from the top of the cliff are 45° and 60° respectively. Hence, ∠FAD = 45° and ∠FAC = 60°. Draw a line DE from CD to AB parallel to BC. Also, join A, D and A, C. We get two right-angled triangles ADE and ABC with right angles at E and B respectively. Also, ∠ADE = ∠FAD = 45°, and ∠ACB = ∠FAC = 60°. We are also given that the height of the cliff AB is 60√3 m. We are to find the height of the tower, that is, CD.

We first find the value of BC from the ΔABC, using the trigonometric ratio tan.

In ΔABC,

$$\tan\angle ACB = \tan 60° = \frac{AB}{BC} = \frac{60\sqrt{3}}{BC}$$

or,

$$BC = \frac{60\sqrt{3}}{\tan 60°} = 60$$

Clearly, ED = BC. Then, we will find the value of AE from ΔADE using the trigonometric ratio tan.
In ΔADE,

$$\tan\angle ADE = \tan 45° = \frac{AE}{ED} = \frac{AE}{60}$$

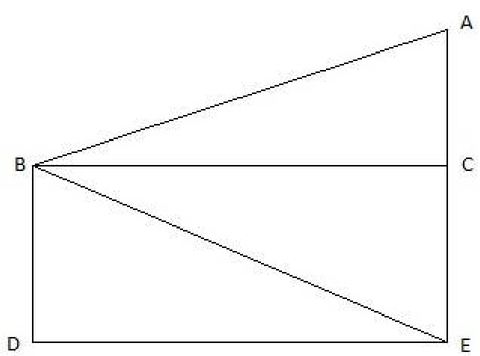
or, $AE = 60m.$

The height of the tower = DC = 60√3m-60m = 43.92 m.

Question: 23

A man on the deck

Solution:



In the above figure, let B be the position of the man and AE be the cliff. We are given that the position of the man is 16 m above water level. Draw a line BD to a point D on the water level. So, BD = 16 m. Now, we are given that the angle of elevation of the top of the tower from the position of the man is 60°. Join A and B. Also, draw a line BC on to the line AE parallel to the water level. Then we get a right angled triangle ABC with right angle at B and ∠ABC = 60°. Also, the angle of depression of the bottom of the tower from the position of the man is 30°. So, ∠CBE = 30°. Joining B and E we get a right angled triangle BDE. And, ∠BED = ∠CBE = 30°. We need to find the distance of the ship from the cliff, that is DE and the height of the cliff AE.

We first find DE from the ΔBDE by using the trigonometric ratio tan.

In ΔBDE,

$$\tan \angle BED = \frac{BD}{DE}$$

or,

$$\tan 30° = \frac{16}{DE}$$

or,

$$DE = 16\sqrt{3}m = 27.71 \text{ m.}$$

Now, BC = DE = $16\sqrt{3}$ m.

From ΔABC,

$$\tan \angle ABC = \frac{AC}{BC}$$

or,

$$\tan 60° = \frac{AC}{16\sqrt{3}}$$

or,

$$AC = 16\sqrt{3} \times \sqrt{3} = 16 \times 3 = 48 \text{ m.}$$

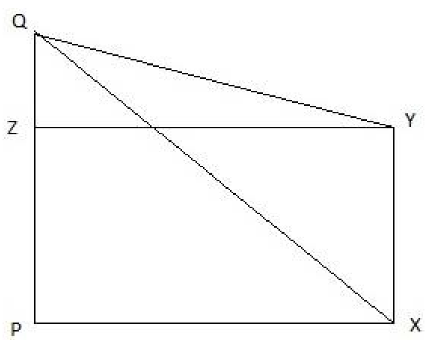
Also, CE = BD = 16 m.

Hence, the height of the cliff is AE = AC + CE = 48 + 16 = 64 m.

Question: 24

The angle of elev

Solution:



PQ is the tower and from a point X on the ground, the angle of elevation of the top of the tower is 60°. Join A and P, and Q and X. Then we get a right-angled triangle QPX with right angle at P and ∠QXP = 60°. Y is a point vertically above X. Then YX = 40 m. Draw a line YZ from Y onto the line QP parallel to PX. We are also given that the angle of elevation of the top of the tower from the point Y is 45°. Join Y and Q. We get a right-angled triangle QYZ with right angle at Z and ∠QYZ = 45°. We are to find the height of the tower PQ.

Clearly, ZY = XP, ZP = XY. Let QZ = x

In ΔQZY,

$$\tan \angle QYZ = \frac{QZ}{ZY} = \frac{x}{ZY}$$

or,

$$\tan 45° = \frac{x}{ZY}$$

or,

$$ZY = \frac{x}{\tan 45°} = x$$

In ΔPQX,

$$\tan\angle PXQ = \frac{PQ}{PX} = \frac{PZ + ZQ}{ZY} = \frac{x + 40}{x}$$

or,

$$\tan 60° = \frac{x + 40}{x}$$

or,

$$\sqrt{3}x = x + 40$$

or,

$$x = \frac{40}{\sqrt{3}-1} = \frac{40}{0.73} = 54.79m$$

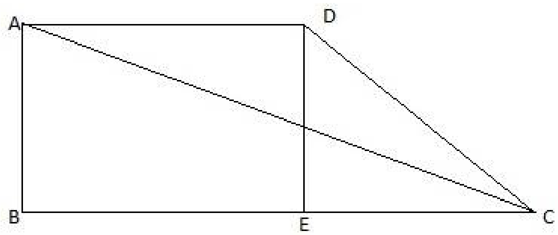
QZ = ZY = x = 54.79 m.

The height of the tower PQ = PZ + ZQ = 54.79m + 40m = 94.79m.

Question: 25

The angle of elev

Solution:



Let D be the initial position of the aeroplane, 2500 m above the ground. After 15 s, let A be the position of the aeroplane flying in the same height. Draw two lines AB and DE from A and D perpendicular to the ground. Then, AB = DE = 2500m. Let C be the point on the ground from which the angles of elevation of the two positions of the plane, viz., D and A are 45° and 30° respectively. Join A and D with C. Also join the points B,E and C on the ground. We then get two right-angled triangles ABC and DEC with right angles at B and E respectively. So, we get ∠ACB = 30° and ∠DCE = 45°. We are to find speed of the plane. The plane travels a distance AD in 15 s. Since speed = (distance/time), we need to find AD only.

We first find CE from ΔDEC using the trigonometric ratio tan θ.

In ΔDEC,

$$\tan\angle DCE = \frac{DE}{EC}$$

or,

$$\tan 45° = \frac{DE}{EC}$$

or,

$$DE = EC\tan 45° = EC$$

or,

$$DE = EC = 2500\text{ m}.$$

Now, we will again use tan to find the value of BE from ΔABC.

In ΔABC,

$$\tan\angle ACB = \frac{AB}{BC} = \frac{AB}{BE + EC} = \frac{2500}{BE + 2500}$$

or,

$$\tan 30° = \frac{2500}{BE + 2500}$$

or,

$$2500\sqrt{3} = BE + 2500$$

or,

$$BE = 2500\left(\sqrt{3}\text{-}1\right) = 1830m$$

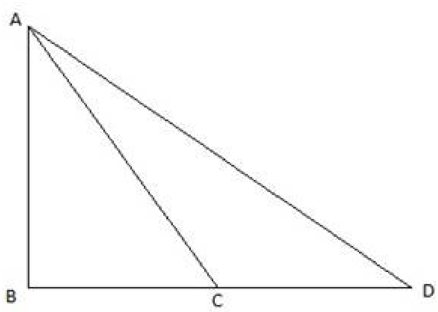
Now, BE = AD = 1830 m.

Speed of the aeroplane = $\frac{BE}{15} = \frac{1830}{15} = 122\ m/s = 439.2\ km/hr$.

Question: 26

The angle of elev

Solution:



In the given figure, let AB be the tower. Let D be the point on the same level as the foot of the tower from which the angle of elevation of the top of the tower is 30°. Join B,C and A,D. Then we get a right-angled triangle ABD with right angle at B and ∠ADB = 30°. Let C be the point on the same level of the ground as Band D, 150 m from D towards B, from which the angle of elevation of the top of the tower is 60°. Joining A and C, we get a right angled triangle ABC, with right angle at B and ∠ACB = 60°. We are to show that the height of the tower AB is 129.9 m.

In ΔABC,

$$\tan\angle ACB = \tan 60° = \frac{AB}{BC}$$

or,

$$AB = \tan 60° \times BC = BC\sqrt{3}$$

In ΔABD,

$$\tan\angle ADB = \tan 30° = \frac{AB}{BD} = \frac{BC\sqrt{3}}{BC + CD} = \frac{BC\sqrt{3}}{BC + 150}$$

or,

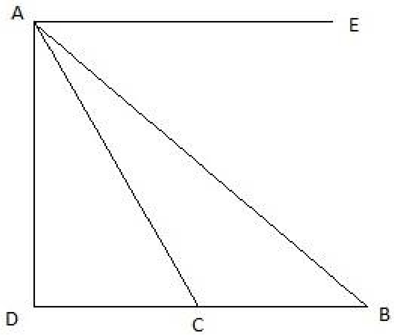
$$3BC = BC + 150$$

So, BC = 75m. Now, AB = BC√3 = 75 × 1.732m = 129.9m. Hence proved.

Question: 27

As observed from

Solution:



In the above figure, let AD be the lighthouse and B and C be the two consecutive positions of the ship in view from it. The top of the lighthouse is 100 m above sea level. So, AD = 100 m. Also given that the angles of depression of the ship from the point A at the positions B and C are 30° and 60° respectively. Join D, C, B. Also join B, C to A. Then we get two right angled triangles ABD and ACD with right angles at D. Now, draw a line AE parallel to the sea level. Then, ∠EAB = ∠ABD = 30° and ∠EAC = ∠ACD = 60°. We are to find the distance travelled by the ship during the period of observation, that is, BC. We just use the trigonometric ratio tan from triangles ABD and ADC to find DB and DC respectively.

From ΔACD,

$$\tan\angle ACD = \tan 60° = \frac{AD}{CD} = \frac{100}{CD}$$

or,

$$CD = \frac{100}{\sqrt{3}}$$

Again, from ΔABD,

$$\tan\angle ABD = \frac{AD}{BD}$$

or,

$$\tan 30° = \frac{100}{BD}$$

or,

$$BD = 100\sqrt{3}$$

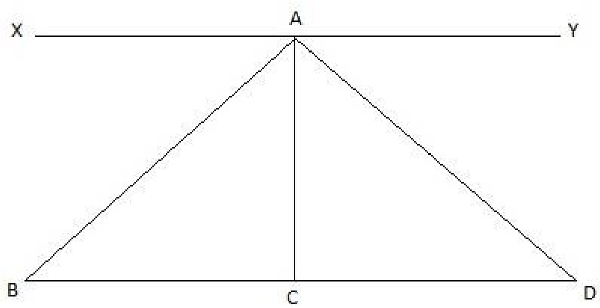
Hence, distance travelled by ship during the observation is

$$BC = BD\text{-}CB = 100\sqrt{3}\text{-}\frac{100}{\sqrt{3}} = \frac{100 \times 2}{1.732} = 115.47 \text{ m.}$$

Question: 28

From a point on a

Solution:



In the above figure, let XY be the bridge and A be the point on the bridge from which two points, say B and D, on opposite sides of the river are observed. Join B and C. Given that the angles of depression of B and C from the point A are 30° and 45° respectively. Join B, D to A. So, ∠XAB = ∠ABD = 30° and also ∠YAD = ∠ADB = 45°. Again, draw a line AC from A perpendicular to the ground. Then, we get two right-angled triangles ABC and ACD with right angles at C. Now, given that the height of the bridge is AC = 2.5 m. We have to find the width of the river, that is BD.

From ΔACD,

$$\tan\angle ADC = \tan 45° = \frac{AC}{CD} = \frac{2.5}{CD}$$

CD = 2.5m.

Again from ΔABC,

$$\tan\angle ABC = \tan 30° = \frac{AC}{BC} = \frac{2.5}{BC}$$

or,

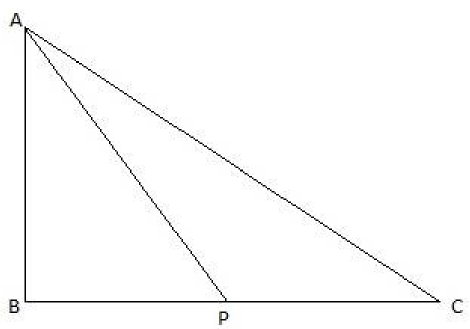
$$BC = 2.5\sqrt{3} = 2.5 \times 1.732 = 4.33\ \text{m}.$$

Hence, the width of the river is BD = BC + CD = 4.33 + 2.5 = 6.83m, which is the required solution.

Question: 29

The angles of ele

Solution:



In the given figure, let AB be the tower. Let P and C be the points on the ground 4 m and 9 m from the foot of the tower. Join B, P and C. So, BC = 9m, BP = 4m. Again join P and C with A. Then we get two right-angled triangles ABP and ABC with right angle at B. Also, ∠APB and ∠ACB are given to be complementary. So, ∠APB + ∠ACB = 90°. We have to show that the height of the tower is 6 m.We find the value of AB from ΔABC using tan and also using the fact that tan(90-θ) = cotθ.

In ΔABC,

$$\tan\angle ACB = \frac{AB}{BC} = \frac{AB}{9}$$

or,

$$AB = 9 \times \tan(90° - \angle APB) = 9\cot\angle APB$$

We will now use the above-found value of AB in ΔAPB.

From ΔAPB,

$$\tan\angle APB = \frac{AB}{BP} = \frac{9\cot\angle APB}{4}$$

or,

$$\frac{\tan\angle APB}{\cot\angle APB} = (\tan\angle APB)^2 = \frac{9}{4}$$

or,

$$\tan \angle APB = \frac{3}{2}$$

or,

$$\cot \angle APB = \frac{2}{3}$$

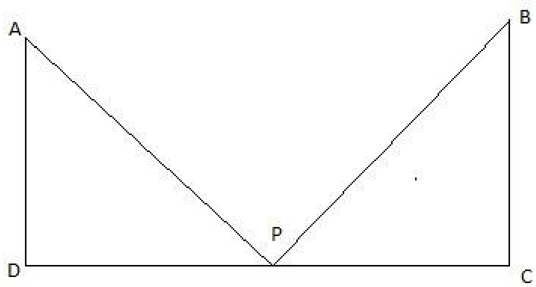
Now, $AB = 9 \cot \angle APB = 9 \times \frac{2}{3} = 3 \times 2 = 6$ m.

Hence, proved.

Question: 30

A ladder of lengt

Solution:



In the above figure, AD and BC are the two opposite walls of the same room. The ladder is fixed at the position say P on the floor. Join D,P and C. At first let it leans against the wall BC making an angle of 45° with the floor. Join B and P. We get a right-angled triangle BPC with right angle at C and ∠BPC = 45°. Again, when the ladder leans against the second wall AD, it makes an angle of 60° with the floor. Joining A and P, we get a right-angled triangle APD with right angle at D and ∠APD = 60°. We are to find the distance between the two walls, that is DC. Also given that, AP = PB = length of the ladder = 6 m.

To find DC, we separately find DP and PC from triangles APD and BPC respectively by using the trigonometric ratio cosine.

From ΔAPD,

$$\cos \angle APD = \cos 60° = \frac{DP}{AP} = \frac{DP}{6}$$

or,

$$DP = \frac{1}{2} \times 6 = 3 \text{ m.}$$

Again, from ΔBPC,

$$\cos \angle BPC = \cos 45° = \frac{PC}{PB} = \frac{PC}{6}$$

or,

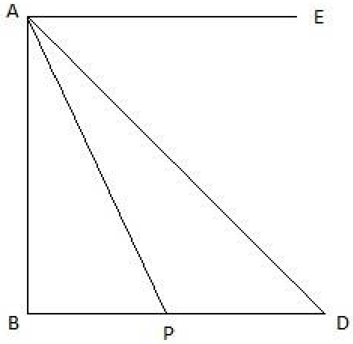
$$PC = 6 \times \frac{1}{\sqrt{2}} = 3\sqrt{2} = 3 \times 1.41 = 4.23 \text{ m.}$$

Hence, the distance between the two walls is, DC = DP + PC = 3 + 4.23 = 7.23m.

Question: 31

From the top of a

Solution:



In the above figure, let AB be the tower and P and D be the positions of the two cars at an instant, observed from A. join A and E. The angles of depression are ∠DAE and ∠PAE. Given that, PD = 100 m. Since AE is parallel to BD, so, ∠ADB = ∠DAE = 45° and ∠APB = ∠PAE = 60°. Join P,D and A,B. We get two right-angled triangles ΔABD and ΔABP. We are to find AB. We use trigonometric ratio tan for both the triangles, using AB as height and BP as base for ΔABP and AB as height and BD as base for ΔABD.

From ΔABD,

$$\tan\angle ADB = \tan 45° = \frac{AB}{BD}$$

or, AB = BD

From ΔAPB,

$$\tan\angle APB = \tan 60° = \frac{AB}{BP}$$

or, BD = BP√3

or, BP + 100 = BP√3

or, BP(√3-1) = 100

or,

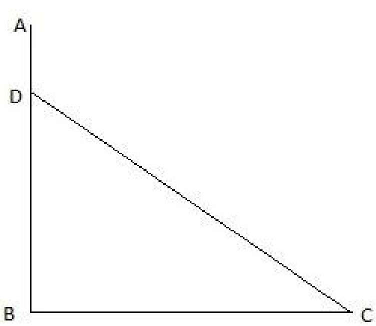
$$BP = \frac{100}{(\sqrt{3}-1)} = 136.61\text{ m.}$$

Hence, the height of the tower is, AB = BD = BP + 100 = 136.61 + 100 = 236.61m

Question: 32

An electrician ha

Solution:



Let AB be the pole and D be the 1m below A. The person needs to reach the point D. So, BD = 3m. Let DC be the ladder. Join D and C. We get a right-angled triangle ΔDBC, right angled at B. The angle of elevation of the ladder, ∠BCD = 60°. We use trigonometric ratio sin, using DC as hypotenuse and DB as height to find the length of the ladder DC.

In ΔBDC,

$$\sin\angle BCD = \sin 60° = \frac{BD}{DC} = \frac{3}{DC}$$

or,

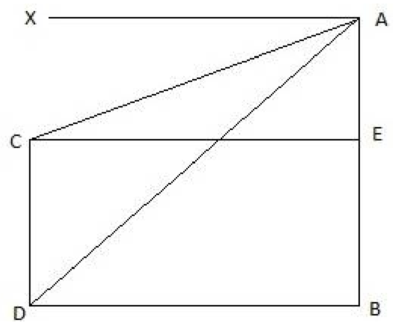
$$DC = \frac{2 \times 3}{\sqrt{3}} = 2\sqrt{3} = 3.46m$$

Hence, the length of the ladder will be 3.46m.

Question: 33

From the top of a

Solution:



In the above figure, let AB be the building such that AB = 60 m. Join A and X. The angles of depression are ∠CAX = 30° and ∠DAX = 60°. Let CD be the lamp post. Join C,D and B,D and C,E. Since AX is parallel to DB, we must have ∠XAC = ∠ACE = 30° and ∠XAD = ∠ADB = 60°. We get two right-angled triangles ΔABD and ΔCAE. We use trigonometric angle tan in both the triangles with AB as height and DB as base (for ΔABD) and AE as height and CE as base(for ΔCAE).

We have to find, (i)BD, (ii)CD, and (iii) AB-CD.

From ΔABD,

$$\tan \angle ADB = \tan 60° = \frac{AB}{DB} = \frac{60}{DB}$$

or,

$$DB = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m.}$$

DB = CE.

From ΔACE,

$$\tan \angle ACE = \frac{AE}{CE}$$

or,

$$\tan 30° = \frac{AE}{20\sqrt{3}}$$

or,

$$AE = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m.}$$

Hence,

$$CD = AB\text{-}AE = 60\text{-}20 = 40 \text{ m.}$$

And, the difference between the heights of the building and the lamp post is,

$$AE = 20 \text{ m.}$$

Thus our solutions are,

(i) The horizontal distance between AB and CD = 34.64 m.

(ii) The height of the lamp post = 40 m.

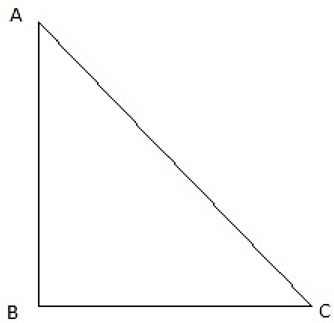
(iii) Difference between the heights of the building and the lamp post = 20 m.

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

If the height of

Solution:



In the figure let AB be the pole and BC is its shadow. Join A and C. We get a right-angled triangle ΔABC. Angle of elevation of the sun is ∠ACB. Given that, AB = BC. We use trigonometric ratio tan taking AB as height and BC as base to find the angle ∠BCA.

In ΔABC,

$$\tan\angle ACB = \frac{AB}{BC} = 1$$

or,

$$\angle ACB = 45°.$$

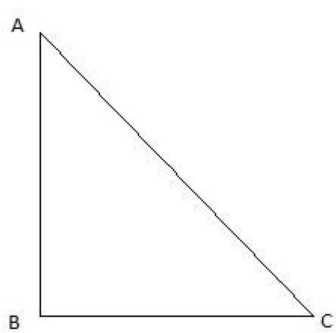
since we know that tanθ is 1 whenever θ = 45°.

So, the correct option is (C).

Question: 2

If the height of

Solution:



Let AB be the pole and BC be the shadow. Join A and C. We get a right-angled triangle right angled at B. The angle of elevation of the sun is ∠ ACB (= θ, say). We use the trigonometric ratio tan for the triangle taking AB as height and BC as the base. By the problem, AB = √3BC.

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}BC}{BC} = \sqrt{3}$$

or,

θ = 60°

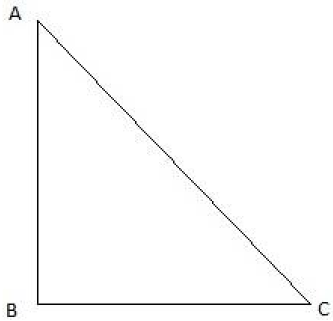
since we know that tanθ is √3 whenever θ = 60°.

So, the correct option is (C).

Question: 3

If the length of

Solution:



Let AB be the tower and BC be the shadow. Join A and C. We get a right-angled triangle right-angled at B. The angle of elevation of the sun is ∠ACB(θ, say). Given that, BC = √3AB. We will use trigonometric ratio tan for the triangle taking AB as height and BC as the base.

In ΔABC,

$$\tan\theta = \frac{AB}{BC} = \frac{AB}{\sqrt{3}AB}$$

or,

θ = 30°

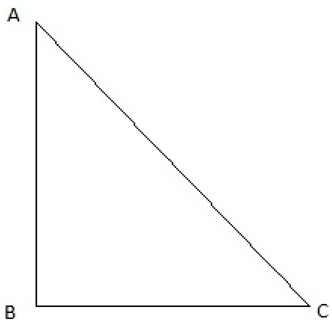
since we know that tanθ is 1/√3 whenever θ = 30°.

So, the correct option is (B).

Question: 4

If a pole 12 m hi

Solution:



Let AB be the pole and BC be its shadow. Join A and C. Given that, AB = 12m, BC = 4√3m. We get a right-angled triangle ΔABC, right angled at B. ∠ACB is the angle of elevation of the sun. We will use the trigonometric ratio tan taking AB as the height and BC as the base.

In ΔABC,

$$\tan\angle ACB = \frac{AB}{BC} = \frac{12}{4\sqrt{3}} = \sqrt{3}$$

or,

$$\angle ACB = 60°.$$

since we know that tanθ is √3 whenever θ = 60°.

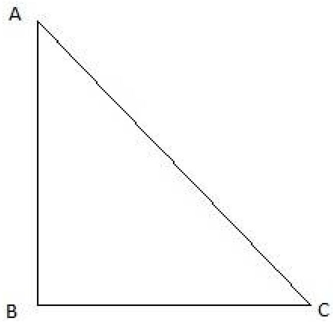
So, the correct option is (A).

Question: 5

The shadow of a 5

Solution:

All we need to find is the angle of elevation of the sun. In the figure, AB is 5m long stick and BC is 2m long shadow of the stick. We join A and C to get a right-angled triangle ΔABC. Angle of elevation of the sun is ∠ACB.

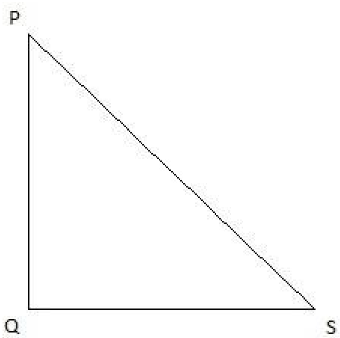


In ΔABC,

$$\tan\angle ACB = \frac{AB}{BC} = \frac{5}{2} = 2.5$$

or,

$$\tan\angle ACB = 2.5$$



In this figure, PQ is the tree and QS is its shadow. ∠PSQ is the angle of elevation of the sun. So, ∠PSQ = ∠ACB. Join P and S. We get a right-angled triangle ΔPQS. Given, PQ = 12.5m. We use trigonometric ratio tan taking PQ as the height and QS as the base.

In ΔPQS,

$$\tan\angle PSQ = \tan\angle ACB = \frac{PQ}{SQ} = \frac{12.5}{SQ}$$

or,

$$2.5 = \frac{12.5}{SQ}$$

or,

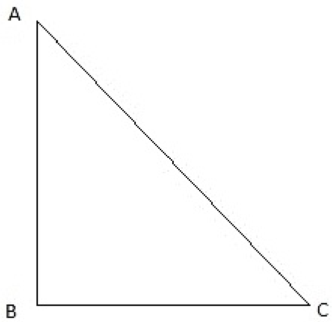
$$SQ = \frac{12.5}{2.5} = 5m$$

So, the correct option is (D).

Question: 6

A ladder makes an

Solution:



In the figure, let AC be the ladder and AB is the wall. BC is the distance of the foot of the ladder from wall AB. Join B and C. We get right-angled triangle ΔABC. Given that, BC = 2m, ∠ACB = 60°. We use trigonometric ratio tan taking AB as the height and BC as the base.

In ΔABC,

$$\cos \angle ACB = \cos 60° = \frac{BC}{AC} = \frac{2}{AC}$$

or,

$$AC = \frac{2}{\cos 60°} = 4m.$$

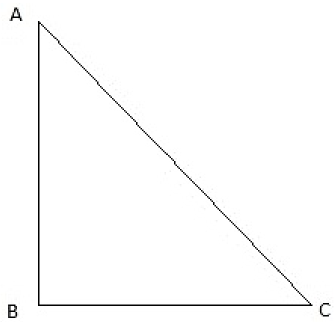
where cos60° = 1/2.

So, the correct option is (D).

Question: 7

A ladder 15 m lon

Solution:



In the figure, let AC be the ladder and AB is the wall. Join B and C. We get a right-angled triangle ΔABC, right angled at B. Given, AC = 15m, ∠BAC = 60°. We use trigonometric ratio cos taking AC as the hypotenuse and AB as the base. In ΔABC,

$$\cos \angle BAC = \cos 60° = \frac{AB}{AC} = \frac{AB}{15}$$

or,

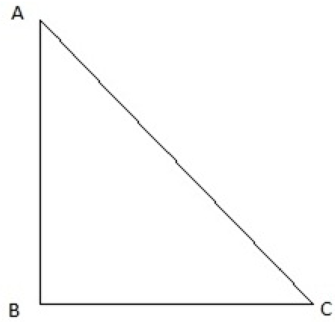
$$AB = 15 \times \cos 60° = \frac{15}{2} m$$

So, the correct option is (C).

Question: 8

From a point on t

Solution:



Let AB be the tower and C be the point on the ground, 30m away from B. Given that, BC = 30m, the angle of elevation, ∠ACB = 30°. Join A, C and B,C. We get a right-angled triangle. We use trigonometric ratio tan taking AB as the height and BC as the base. In ΔABC,

$$\tan\angle ACB = \tan 30° = \frac{AB}{BC} = \frac{AB}{30}$$

or,

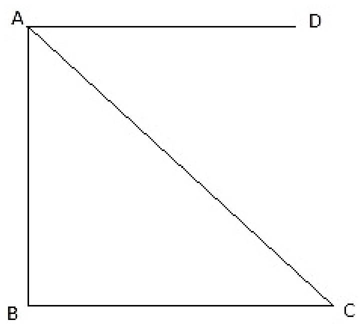
$$AB = 30 \times \tan 30° = \frac{30}{\sqrt{3}} = 10\sqrt{3}\text{m}$$

So, the correct option is (B).

Question: 9

The angle of depr

Solution:



In the figure, AB is the tower and car is at the point C. Join A and B to C. So we get a right-angled triangle ABC with right angle at B. Also height of the tower is 150 m. So, AB = 150 m. Again, draw a line AD parallel to BC. The angle of depression of the car from the top of AB is 30°. So, ∠DAC = ∠ACB = 30°. We have to find the distance of the car from the tower, that is BC. For this, we will use the trigonometric ratio tan in ΔABC.

So from ΔABC,

$$\tan\angle BAC = \tan 30° = \frac{BC}{AB} = \frac{BC}{150}$$

or,

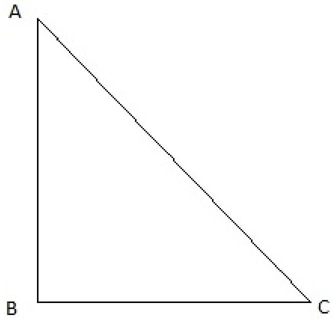
$$BC = 150 \times \tan 30° = \frac{150}{\sqrt{3}} = 50\sqrt{3}\text{m}$$

So, the correct option is (B).

Question: 10

A kite is flying

Solution:



Let the kite be at point A. Draw a line perpendicular to the ground at say point B. Since we are assuming no slack in the string, we can assume AC to be the string as in the figure. Join B and C to get a right-angled triangle ABC. Given that the kite is flying 30 m above the ground. So AC = 60 m. So, AB = 30 m. Also the length of the string is 60 m. We are to find the angle made by the string AC with the ground that is, ∠ACB.

In ΔABC,

$$\sin\angle ACB = \frac{AB}{AC} = \frac{30}{60} = \frac{1}{2}$$

Since sin30° =1/2

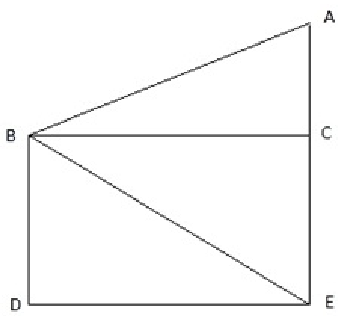
∠ACB = 30°.

So, the correct option is (B).

Question: 11

From the top of a

Solution:



Let BD be the cliff and AE be the tower. Join D and E. Now, given that the angle of elevation of the top of the tower is same as the angle of depression of the foot of the tower as seen from the top of the cliff. Join A with B and E with B. Also draw a line BC from B onto AE parallel to DE. We get two right-angled triangles ABC and BDE with right angles at C and D respectively. The angle of elevation of the top of the tower AE from the point B is ∠ABC. And the angle of depression of the foot of the tower AE from the point B is ∠CBE which is same as ∠BED. Given, BD = 20m. We are to find the height of the tower, that is BD. Clearly, BD = CE = 20m, BC = DE. In ΔBCE,

$$\tan\angle CBE = \frac{CE}{BC} = \frac{20}{BC} = \tan\angle ABC$$

or,

$$BC = \frac{20}{\tan\angle ABC}$$

In ΔABC,

$$\tan\angle ABC = \frac{AC}{BC} = \frac{AC \times \tan\angle ABC}{20}$$

or,

AC = 20m

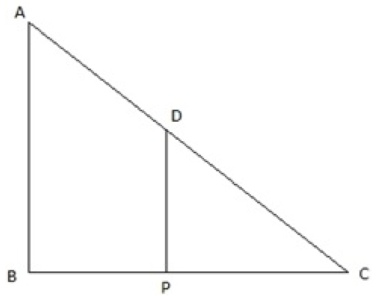
So, height of the tower is AE = AC + CE = 20m + 20m = 40m

The correct option is (B).

Question: 12

If a 1.5-m-tall g

Solution:



In the above figure, let PD be the girl, AB be the lamp post. Given that, the girl stands at a distance of 3 m from the lamp post. So BP = 3 m. Also the height of the girl is 1.5 m. So PD = 1.5 m. The shadow of the girl cast is of the length 4.5 m. So PC = 4.5 m. Then we have to find the height of the lamp post, that is, AB.

Now, BC = BP + PC = 3 + 3.5 = 7.5 m.

From ΔDPC,

$$\tan\angle DCP = \frac{DP}{PC} = \frac{1.5}{4.5} = \frac{1}{3}$$

$\angle ACB = \angle DCP$. So, $\tan\angle ACB = \frac{1}{3}$.

From, ΔABC,

$$\tan\angle ACB = \frac{AB}{BC}$$

or,

$$\frac{1}{3} = \frac{AB}{7.5}$$

or,

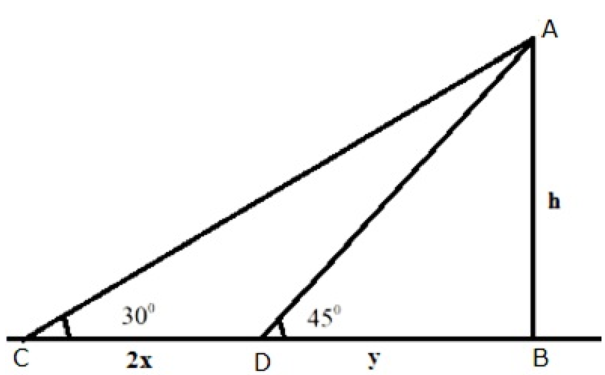
$$AB = \frac{7.5}{3} = 2.5 \text{ m.}$$

Hence the correct choice is C.

Question: 13

The length of the

Solution:



In the above figure, let AB be the tower and BC and BD are the consecutive shadows of AB for two different positions of the sun.

From ΔDBA,

$$\tan 45° = \frac{h}{y}$$

⇒ 1 = h/y

⇒ h = y

or,

DB = BA

Now, from ΔACB,

$$\tan \angle ACB = \frac{AB}{CB}$$

or, $\tan 30° = \frac{h+y}{2x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h+y}{2x}$

On cross multiplying we get,

⇒ 2x = √3(h+y)

⇒ 2x + y = √3

⇒ 2x + h = (√3h)

⇒ 2x = (√3 – 1)h

⇒ x = (√3 – 1)h

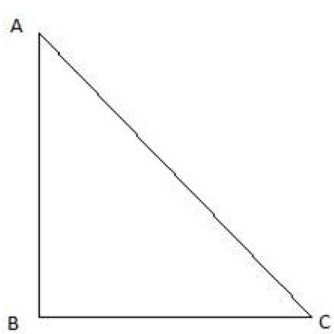
$\Rightarrow h = \frac{2x}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

⇒ h = (√3 + 1)x

Question: 14

The lengths of a

Solution:



Let AB be the vertical rod and BC be its shadow. Given that,

$$\frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

We have to find ∠ACB.

We have,

$$\tan \angle ACB = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$$

So,

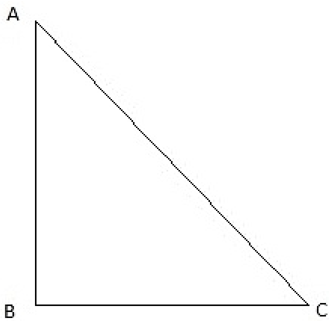
∠ACB = 30°

So the correct choice is A.

Question: 15

A pole casts a sh

Solution:



In the above figure, let AB be the pole and BC be its shadow. Given that BC = $2\sqrt{3}$ m and $\angle ACB = 60°$. We have to find AB.

$$\tan\angle ACB = \frac{AB}{BC}$$

or,

$$\tan 60° = \frac{AB}{2\sqrt{3}}$$

or,

$AB = 2\sqrt{3} \times \sqrt{3} = 2 \times 3 = 6$ m

Hence the correct choice is B.

Question: 16

In the given figu

Solution:

Given, AB = 20 m, its shadow BC = $20\sqrt{3}$ m. We have to find $\angle ACB$.

$$\tan\angle ACB = \frac{AB}{BC} = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

So,

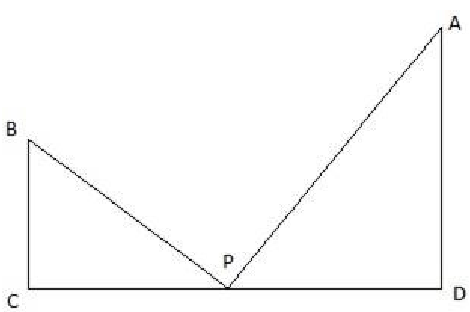
$\angle ACB = 30°$

The correct choice is A.

Question: 17

The tops of two t

Solution:



Let BC = x and AD = y be the two towers. They subtend angles 30° and 60° at the centre of the line joining their feet say P. Join C, P, D. Also join B, P and A, P. So, $\angle BPC = 30°$ and $\angle APD = 60°$.

Also, we get two right-angled triangles BPC and APD with right angles at C and D respectively.
We have to find x:y.

From ΔBPC,

$$\tan \angle BPC = \frac{BC}{PC}$$

or,

$$\tan 30° = \frac{x}{PC}$$

or,

$$x = \frac{PC}{\sqrt{3}}$$

From ΔAPD,

$$\tan \angle APD = \frac{AD}{PD}$$

or,

$$\tan 60° = \frac{y}{PD}$$

or,

y = PD√3

So, taking the ratio we get,

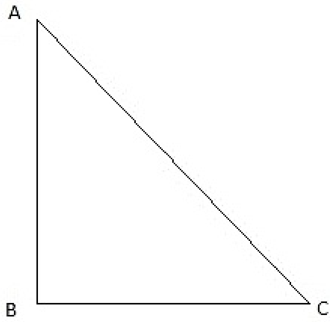
$$\frac{x}{y} = \frac{\frac{PC}{\sqrt{3}}}{PD\sqrt{3}} = \frac{PC}{PC \times 3} = \frac{1}{3}$$

Hence, the correct choice is C.

Question: 18

The angle of elev

Solution:



In the above figure, let AB be the tower and C be the point on the ground 30 m away from the foot of AB from where the point A is observed. Join B, C and A, C. We get a right-angled triangle ABC with right angle at B. BC = 30 m. Also, the angle of elevation of the top of the tower AB from C is 30°. So, ∠ACB = 30°. We need to find the height of the tower, that is, AB.

In ΔABC,

$$\tan \angle ACB = \frac{AB}{BC}$$

or,

$$\tan 30° = \frac{AB}{30}$$

or,

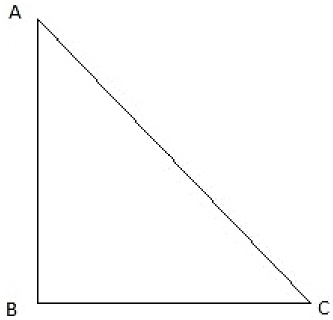
$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

Hence the correct choice is B.

Question: 19

The string of a k

Solution:



In the above figure, let A be the position of the kite. Draw a perpendicular AB from A onto the ground. Since there is no slack in the string, so we can take AC = 100 m to be the string, where C is the position where the string touches the ground. Also, the string makes an angle of 60° with the horizontal. Join B and C. So we get a right-angled triangle ABC with right angle at B and $\angle ACB = 60°$. We have to find the height of the kite from the ground, that is, AB.

$$\sin\angle ACB = \frac{AB}{AC}$$

or,

$$\sin 60° = \frac{AB}{100}$$

or,

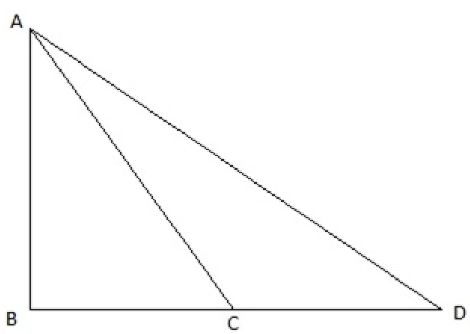
$$AB = \frac{\sqrt{3}}{2}100 = 50\sqrt{3}m.$$

Hence the correct choice is A.

Question: 20

If the angles of

Solution:



In the above figure, let AB be the tower and C and D be the two points on the ground from where A is observed. Join B, C, D. Let BC = a and BD = b. Join C, D with A. We get two right-angled triangles ABC and ABD with right angle at B. Also, the angles of elevation of the top of the tower AB from C and D are $\angle ACB$ and $\angle ADB$ respectively. These angles are complementary. So, $\angle ACB + \angle ADB = 90°$. We are to find the height of the tower, that is, AB.

From ΔABD,

$$\tan\angle ADB = \frac{AB}{DB} = \frac{AB}{b}$$

Again, from ΔABC,

$$\tan\angle ACB = \frac{AB}{BC}$$

or,

$$\tan(90-\angle ADB) = \frac{AB}{a}$$

or,

$$\cot\angle ADB = \frac{AB}{a}$$

Now, multiplying we get,

$$\tan\angle ADB \cot\angle ADB = \frac{AB}{b}\frac{AB}{a}$$

or,

$$AB^2 = ab$$

or,

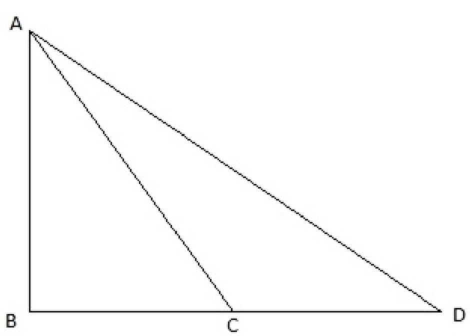
AB = √ab

So the correct choice is (B).

Question: 21

On the level grou

Solution:



Let AB be the tower and C, D are two points on the ground. Join B, C, D. Also, join A with C and D. We get two right-angled triangles ABC and ABD with right angle at B. The angles of elevation of the top of the tower from the points D and C are 30° and 60° respectively. So, ∠ADB = 30° and ∠ACB = 30°. We are to find the height of the tower. Also given that the distance from D to C is 20 m. So CD = 20 m.

In ΔABC,

$$\tan\angle ACB = \tan 60° = \frac{AB}{BC}$$

or,

$$BC = \frac{AB}{\tan 60°} = \frac{AB}{\sqrt{3}}$$

In ΔABD,

$$\tan\angle ADB = \tan 30° = \frac{AB}{BD} = \frac{AB}{BC + CD} = \frac{AB}{\frac{AB}{\sqrt{3}} + 20}$$

or,

$$\left(\frac{AB}{\sqrt{3}} + 20\right) = \sqrt{3}AB$$

or,

$$\frac{3AB\text{-}AB}{\sqrt{3}} = 20$$

or,

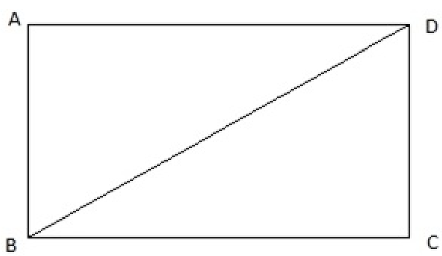
$$AB = 10\sqrt{3}m$$

So, the correct option is (B).

Question: 22

In a rectangle, t

Solution:



Let ABCD be the rectangle. Given that, BD = 8cm, $\angle DBC = 30°$.

In ΔBCD,

$$\sin\angle DBC = \sin 30° = \frac{DC}{BD} = \frac{DC}{8}$$

or,

$$DC = 8 \times \sin 30° = 4cm.$$

Again,

$$\cos\angle DBC = \cos 30° = \frac{BC}{BD} = \frac{BC}{8}$$

or,

$$BC = 8 \times \cos 30° = 4\sqrt{3}m$$

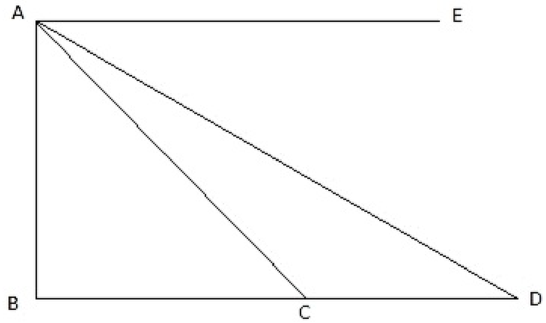
Area of the rectangle = $\left(4 \times 4\sqrt{3}\right) cm^2 = 16\sqrt{3}cm^2$.

So, the correct option is (C).

Question: 23

From the top of a

Solution:



Let AB be the hill and C, D are the km stones. Join B, C, D. Also join C, D with A. So we get two right-angled triangles ABC and ABD with right angle at B. Draw a line AE parallel to BD. Given that the angles of depression of C and D are 45° and 30° respectively. So, ∠EAD = ∠ADB = 30° and ∠EAC = ∠ACB = 45°. Now, CD = 1km. We are to find the height of the hill, that is, AB.

In ΔABC,

$$\tan \angle ACB = \tan 45° = \frac{AB}{BC}$$

or,

$$AB = BC$$

In ΔABD,

$$\tan \angle ADB = \tan 30° = \frac{AB}{BD} = \frac{AB}{BC + CD} = \frac{AB}{AB + 1}$$

or,

$$\sqrt{3}AB = AB + 1$$

or,

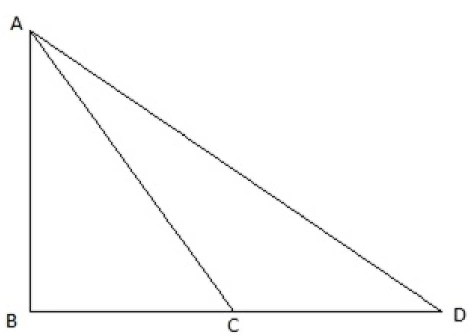
$$AB = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{3}+1}{3-1} = \frac{1}{2}(\sqrt{3}+1)$$

So, the correct option is (B).

Question: 24

If the elevation

Solution:



Let AB be the pole and BD, BC is its shadows when the angle of elevation of the sun is 30°, 60° respectively. Here, the height of the pole is 15 m. So, AB = 15m. Join C and D with A. So we get two right-angled triangles ABC and ABD with right angle at B. And ∠ADB = 30° and ∠ACB = 60°. We are to find the difference between the length of the shadows, that is, CD. For this, we find the lengths BD and BC from triangles ABD and ABC respectively using the trigonometric ratio tan. In ΔABC,

$$\tan \angle ACB = \tan 60° = \frac{AB}{BC} = \frac{15}{BC}$$

or,

$$BC = \frac{15}{\tan 60°} = \frac{15}{\sqrt{3}} = 5\sqrt{3}m$$

In ΔABD,

$$\tan \angle ADB = \tan 30° = \frac{AB}{BD} = \frac{15}{BC + CD} = \frac{15}{5\sqrt{3} + CD}$$

or,

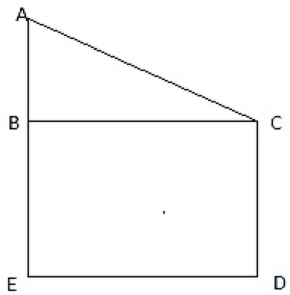
CD = 15√3 -5√3 = 10 √3 m.

So, the correct option is (C).

Question: 25

An observer 1.5 m

Solution:



Let AE be the tower and CD be the observer. Join E and D. The observer is 28.5 m away from the tower. So, ED = 28.5 m. The observer is 1.5 m tall. So, CD = 1.5 m. Given that the angle of elevation of the top of the tower from the eye of the observer is 45°. Join A and C. Also draw a line BC from C onto AE parallel to DE. We get a right-angled triangle ABC with right angle at B and ∠ ACB = 45°. We are to find the height of the tower, that is, AE.

We see that, BC = ED = 28.5 m.

In ΔABC,

$$\tan \angle ACB = \tan 45° = \frac{AB}{BC}$$

or,

$$AB = BC = 28.5m$$

So, the height of the tower is AE = EB + BA = 1.5m + 28.5m = 30m.

The correct option is (B).