Chapter : 2. POLYNOMIALS

Exercise : 2A

Question: 1

Which of th

Solution:

(i) $x^5 - 2x^3 + x + 7$

Yes,

The given expression is a polynomial

This is because all the variables have integer exponents that are positive.

Since, the highest power of the variable is 5.

Hence, the degree of the polynomial is 5.

(ii) $y^3 - \sqrt{3}y$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 3

Hence, the degree of the polynomial is 3

(iii) $t^2 - \frac{2}{5}t + \sqrt{2}$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 2

Hence, the degree of the polynomial is 2

(iv) $5\sqrt{z} - 6$

No,

The given expression is not a polynomial

Since, the term has a fractional exponent.

(v) $x - \frac{1}{x}$

No,

The given expression is not a polynomial

Since, the term has a negative exponent.

(vi) $x^{108} - 1$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 108

Hence, the degree of the polynomial is 108

(vii) $\sqrt[3]{x} - 27$

No,

The given expression is not a polynomial

Since, the term has a fractional exponent.

(viii) $\frac{1}{\sqrt{2}} x^2 - \sqrt{2x} + 2$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 2

Hence, the degree of the polynomial is 2

(ix) $x^{-2} + 2x^{-1} + 3$

No,

The given expression is not a polynomial

Since, the term has a negative exponent.

(x) 1

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 0

Hence, the degree of the polynomial is 0

(xi) $-\frac{3}{5}$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 0

Hence, the degree of the polynomial is 0

(xii) $\sqrt[3]{2} y^2 - 8$

Yes,

The given expression is a polynomial

This is because all of the variables have integer exponents that are positive.

Since, the highest power of the variable is 2

Hence, the degree of the polynomial is 2

Question: 2

Write the d

Solution:

(i) $2x - \sqrt{5}$

Since,

In the given polynomial, the highest power of the variable is 1

Hence,

The degree of the polynomial is 1

(ii) $3 - x + x^2 - 6x^3$

Since,

In the given polynomial, the highest power of the variable is 3

Hence,

The degree of the polynomial is 3

(iii) 9

Since,

In the given polynomial, the highest power of the variable is 0

Hence,

The degree of the polynomial is 0

(iv) $8x^4 - 36x + 5x^7$

Since,

In the given polynomial, the highest power of the variable is 7

Hence,

The degree of the polynomial is 7

(v) $x^9 - x^5 + 3x^{10} + 8$

Since,

In the given polynomial, the highest power of the variable is 10

Hence,

The degree of the polynomial is 10

(vi) $2 - 3x^2$

Since,

In the given polynomial, the highest power of the variable is 2

Hence,

The degree of the polynomial is 2

Question: 3

Write:

Solution:

(i) $2x + x^2 - 5x^3 + x^4$

Hence,

The Coefficient of x^3 in the given polynomial is -5

(ii) $\sqrt{3} - 2\sqrt{2x} + 4x^2$

Hence,

The Coefficient of x in the given polynomial is -22

(iii) $\frac{\pi}{3}x^2 + 7x - 3$

Hence,

The Coefficient of x^2 in the given polynomial is $\frac{\pi}{3}$

(iv) 3x – 5

Since,

There isn't any variable with exponent as 2

Hence,

The Coefficient of x^2 in the given polynomial, is 0

Question: 4 A

Give an exa

Solution:

An example of a binomial of degree 27 is a two-term polynomial with highest degree 27.

Hence,

The suitable example for the question can be $y^{27} - 29$.

Question: 4 B

Give an exa

Solution:

An example of a monomial of degree 16 is a single term polynomial with highest degree 16.

Hence,

The suitable example for the question can be y^{16}

Question: 4 C

Give an exa

Solution:

An example of a trinomial of degree is a three-term polynomial with highest degree 3.

Hence,

The suitable example for the question can be $y^3 - y^2 + 29$

Question: 5

Classify th

Solution:

(i) $2x^2 + 4x$

Since,

The degree of the given polynomial is 2

Hence,

The polynomial is a quadratic polynomial.

(ii) $x - x^3$

Since,

The degree of the given polynomial is 3

Hence,

The polynomial is a cubic polynomial.

(iii) $2-y-y^2$

Since,

The degree of the given polynomial is 2

Hence,

The polynomial is a quadratic polynomial.

(iv) -7+z

Since,

The degree of the given polynomial is 1

Hence,

The polynomial is a linear polynomial.

(v) $5t$

Since,

The degree of the given polynomial is 1

Hence,

The polynomial is a linear polynomial.

(vi) p^3

Since,

The degree of the given polynomial is 3

Hence,

The polynomial is a cubic polynomial.

Exercise : 2B

Question: 1

If

Solution:

(i) We have,

$p(x)=5-4x+2x^2,$

Now,

Put x = 0

p(0) = 5 - 4(0) + 2(0)^2

= 5 - 4(0) + 2(0)

= 5 – 0 + 0

= 5

Hence,

P(0) = 5

(ii) We have,

$p(x)=5-4x+2x^2,$

Now,

Put x = 3

$p(3) = 5 - 4(3) + 2(3)^2$

= 5 - 4(3) + 2(9)

= 5 – 12 + 18

= 11

Hence,

P (3) = 11

(iii) We have,

$p(x) = 5 - 4x + 2x^2,$

Now,

Put x = -2

$p(-2) = 5 - 4(-2) + 2(-2)^2$

= 5 - 4(-2) + 2(4)

= 5 + 8 + 8

= 21

Hence,

P (-2) = 21

Question: 2

If

Solution:

(i) We have,

$p(y) = 4 + 3y - y^2 + 5y^3.$

Now,

Put y = 0

$p(0) = 4 + 3(0) - (0)^2 + 5(0)^3$

= 4 - 3(0) - (0) + 5(0)

= 4 – 0 – 0 + 0

= 4

Hence,

P (0) = 4

(ii) We have,

$p(y) = 4 + 3y - y^2 + 5y^3.$

Now,

Put y = 2

$p(0) = 4 + 3(2) - (2)^2 + 5(2)^3$

$= 4 + 3(2) - (4) + 5(8)$

$= 4 + 6 - 4 + 40$

$= 46$

Hence,

P (2) = 46

(iii) We have,

$p(y) = 4 + 3y - y^2 + 5y^3$

Now,

Put y = -1

$p(-1) = 4 + 3(-1) - (-1)^2 + 5(-1)^3$

$= 4 + 3(-1) - (1) + 5(-1)$

$= 4 - 3 - 1 - 5$

$= -5$

Hence,

p(-1) = -5

Question: 3

If

Solution:

(i) We have,

$f(t) = 4t^2 - 3t + 6,$

Now,

Put t = 0

$f(0) = 4(0)^2 - 3(0) + 6$

$= 4(0) - 3(0) + 6$

$= 0 - 0 + 6$

$= 6$

Hence,

f(0) = 6

(ii) We have,

$f(t) = 4t^2 - 3t + 6,$

Now,

Put t = 4

$f(4) = 4(4)^2 - 3(4) + 6$

$= 4(16) - 3(4) + 6$

$= 64 - 12 + 6$

$= 58$

Hence,

f(4) = 58

(iii) We have,

$f(t) = 4t^2 - 3t + 6,$

Now,

Put t = -5

$f(-5)=4(-5)^2 - 3(-5) + 6$

$= 4(25) - 3(-5) + 6$

$= 100 + 15 + 6$

$= 121$

Hence,

$f(0) = 6$

Question: 4

Find the ze

Solution:

(i) At first,

In order to find the zero of the polynomial we will,

Put $p(x) = 0$

Now,

We have,

$P(x)=x-5$

$0 = x – 5$

$x = 5$

Hence, 5 is the zero of the given polynomial.

(ii) At first,

In order to find the zero of the polynomial we will,

Put $q(x) = 0$

Now,

We have,

$q(x)=x+4$

$0 = x + 4$

$x = - 4$

Hence, -4 is the zero of the given polynomial.

(iii) At first,

In order to find the zero of the polynomial we will,

Put $p(t) = 0$

Now,

We have,

$P(t)= 2t - 3$

$0 = 2t – 3$

$2t = 3$

$t = 3/2$

Hence, 3/2 is the zero of the given polynomial.

(iv) At first,

In order to find the zero of the polynomial we will,

Put f(x) = 0

Now,

We have,

f(x)= 3x + 1

0 = 3x + 1

3x = -1

x = $\frac{-1}{3}$

Hence, -1/3 is the zero of the given polynomial.

(v) At first,

In order to find the zero of the polynomial we will,

Put g(x) = 0

Now,

We have,

g(x)= 5 – 4x

0 = 5 – 4x

4x = 5

x = 5/4

Hence, 5/4 is the zero of the given polynomial.

(vi) At first,

In order to find the zero of the polynomial we will,

Put h(x) = 6x - 1

Now,

We have,

h(x)= 6x- 1

0 = 6x – 1

6x = 1

x = 1/6

Hence, 1/6 is the zero of the given polynomial.

(vii) At first,

In order to find the zero of the polynomial we will,

Put p(x) = 0

Now,

We have,

p(x)= ax + b

0 = ax + b

ax = -b

x = (-b)/a

Hence, (-b)/a is the zero of the given polynomial.

(viii) At first,

In order to find the zero of the polynomial we will,

Put q(x) = 0

Now,

We have,

q(x)= 4x

0 = 4x

x = 0

Hence, 0 is the zero of the given polynomial.

(ix) At first,

In order to find the zero of the polynomial we will,

Put p(x) = 0

Now,

We have,

p(x)= ax

0 = ax

x = 0

Hence, 0 is the zero of the given polynomial.

Question: 5

Verify that:

Solution:

(i) We have, p(x) = x – 4

In order to verify the zero of the polynomial,Put p(x) = 4put x = 4 in the expression , we get, p(4) = 4 – 4p(4) = 0Since p(4) = 0

Hence, 4 is a zero of the polynomial p(x).

(ii) We have, p(x) = x + 3

In order to verify the zero of the polynomial,Put p(x) = -3p(-3) = - 3 + 3p(-3) = 0Since p(-3) = 0

Hence, -3 is a zero of the polynomial p(x).

(iii) We have, p(y) = 2y + 1

In order to verify the zero of the polynomial,Put p(y) = 1/2

p(1/2) = 2(1/2) + 1

p(1/2) = 1 + 1p(1/2) = 2

Since p(1/2) ≠ 0

Hence, 1/2 is not a zero of the polynomial p(y)

(iv) We have, p(x) = 2 - 5x

In order to verify the zero of the polynomial,Put p(x) = 2/5p(2/5) = 2 - 5 (2/5)p(2/5)= 2 - 2p(2/5)= 0

Since p(2/5) = 0

Hence, 2/5 is a zero of the polynomial p(x)

(v) We have, p(x) = (x-1)(x-2)

In order to verify the zero of the polynomial,

Case 1: Put x = 1, we get,

p(1) = (1-1)(1-2)p(1) = 0(-1)p(1) = 0Hence, 1 is a zero of the polynomial p(x)

Case 2: Put x = 2, we get,

p(3) = (2-1)(2-2)p(3) = (1)0p(3) = 0since p(3) = 0Hence, 2 is a zero of the polynomial p(x)

(vi) We have, p(x) = x^2 -3xIn order to verify the zero of the polynomial,

Case 1: Put x = 0

p(0) = $(0)^2$ – 3(0)p(0) = 0 - 0p(0) = 0

Since p(0) = 0Hence, 0 is a zero of the polynomial p(x)

Case 2: Put x = 3p(3) = $(3)^2$ – 3(3)p(3) = 9 - 9p(3) = 0

Since p(3) = 0Hence, 3 is a zero of the polynomial p(x)

(vii) We have, p(x) =x^2 + x - 6

In order to verify the zero of the polynomial,

Case 1: Put x = 2p(2) = $(2)^2$ + 2 - 6p(2) = 4 + 2 - 6p(2) = 0

Since p(2) = 0

Hence, 2 is a zero of the polynomial p(x)

Case 2: Put x = -3

p(3) = $(-3)^2$ + (–3) – 6p(3) = 9 - 3 - 6p(3) = 0

Since p(-3) = 0

Hence, -3 is a zero of the polynomial p(x)

Exercise : 2C

Question: 1

Solution:

Let, f(x) = x^3 – 6x^2 + 9x + 3

Now,

As per the question,

x – 1 = 0

x = 1

Using Remainder theorem,

We know that when f(x)is divided by (x – 1), the remainder so obtained will be f(1).

Hence,

f(1) = $(1)^3$ – 6$(1)^2$ + 9(1) + 3

= 1 – 6 + 9 + 3

= 13 – 6

= 7

Therefore,

The required remainder is 7

Question: 2

Solution:

Let, $f(x) = 2x^3 – 5x^2 + 9x - 8$

Now,

As per the question,

$x – 3 = 0$

$x = 3$

Using Remainder theorem,

We know that when f(x)is divided by (x – 3), the remainder so obtained will be f(3).

Hence,

$f(3) = 2(3)^3 – 5(3)^2 + 9(3) - 8$

$= 2(27) – 5(9) + 27 - 8$

$= 54 – 45 + 19$

$= 28$

Therefore,

The required remainder is 28

Question: 3

Solution:

Let, $f(x) = 3x^4 – 6x^2 - 8x + 2$

Now,

As per the question,

$x – 2 = 0$

$x = 2$

Using Remainder theorem,

We know that when f(x)is divided by (x – 2), the remainder so obtained will be f(2).

Hence,

$f(2) = 3(2)^4 – 6(2)^2 - 8(2) + 2$

$= 3(16) – 6(4) -16 + 2$

$= 48 -24 – 14$

$= 10$

Therefore,

The required remainder is 10.

Question: 4

Solution:

Let, $f(x) = x^3 – 7x^2 + 6x + 4$

Now,

As per the question,

x – 6 = 0

x = 6

Using Remainder theorem,

We know that when f(x) is divided by (x – 6), the remainder so obtained will be f(6).

Hence,

$f(6) = (6)^3 – 7(6)^2 + 6(6) + 4$

= (216) – 7(36) + 36 + 4

= 256 -252

= 4

Therefore,

The required remainder is 4.

Question: 5

Solution:

Let, $f(x) = x^3 – 6x^2 + 13x + 60$

Now,

As per the question,

x + 2 = 0

x = -2

Using Remainder theorem,

We know that when f(x)is divided by (x + 2), the remainder so obtained will be f(-2).

Hence,

$f(-2) = (-2)^3 – 6(-2)^2 + 13(-2) + 60$

= - 8 – 6(4) - 26 + 60

= 60 - 58

= 2

Therefore,

The required remainder is 2.

Question: 6

Solution:

Let, $f(x) = 2x^4 + 6x^3 + 2x^2 + x - 8$

Now,

As per the question,

x - 3 = 0

x = 3

Using Remainder theorem,

We know that when f(x)is divided by (x – 3), the remainder so obtained will be f(3).

Hence,

$f(3) = 2(3)^4 + 6(3)^3 + 2(3)^2 + 3 - 8$

$= 2(81) + 6(27) + 18 – 5$

$= 18 - 11$

$= 7$

Therefore,

The required remainder is 7.

Question: 7

Solution:

Let, $f(x) = 4x^3 – 12x^2 + 11x - 5$

Now,

As per the question,

$2x – 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

Using Remainder theorem,

We know that when f(x)is divided by (2x – 1), the remainder so obtained will be $f\left(\frac{1}{2}\right)$.

Hence,

$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) - 5$

$= \frac{1}{2} - 3 + \frac{11}{2} - 5$

$= -4/2$

$= -2$

Therefore,

The required remainder is -2.

Question: 8

Solution:

Let, $f(x) = 81x^4 + 54x^3 – 9x^2 - 3x + 2$

Now,

As per the question,

$3x + 2 = 0$

$3x = -2$

$x = \frac{-2}{3}$

Using Remainder theorem,

We know that when f(x)is divided by (2x – 1), the remainder so obtained will be $f\left(\frac{-2}{3}\right)$.

Hence,

$f\left(\frac{-2}{3}\right) = 81\left(\frac{-2}{3}\right)^4 + 54\left(\frac{-2}{3}\right)^3 - 9\left(\frac{-2}{3}\right)^2 - 3\left(\frac{-2}{3}\right) + 2$

$= 81\left(\frac{16}{81}\right) + 54\left(\frac{-8}{27}\right) - 9\left(\frac{4}{9}\right) + 2 + 2$

= 16 – 16 + 4 - 4

= 0

Therefore,

The required remainder is 0.

Question: 9

Solution:

Let, f(x) = x^3 - ax^2 + 2x - a

Now,

As per the question,

x - a = 0

x = a

Using Remainder theorem,

We know that when f(x)is divided by (x – a), the remainder so obtained will be f(a).

Hence,

f(a) = a^3 – $a(a)^2$ + 2(a) – a

= a^3 – a^3 + 2a – a

= 2a - a

= a

Therefore,

The required remainder is a.

Question: 10

The polynom

Solution:

Let f(x) = ax^3 + $3x^2$ – 3

And,

g(x) = $2x^3$ - 5x + a

Now,

f(4) = $a(4)^3$ + $3(4)^2$ – 3

= 64a + 48 - 3

= 64a + 45

And,

g(4) = $2(4)^3$ – 5(4) + a

= 128 – 20 + a

= 108 + a

According to the question,

f(4) = g(4)

64a + 45 = 108 + a

$64a - a = 108 - 45$

$63a = 63$

$a = 1$

Hence, the value of 'a' is 1.

Question: 11

The Polynomial f(

Solution:

Let $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$

Now,

$f(1) = 1^4 - 2(1)^3 + 3(1)^2 - a(1) + b$

$5 = 1 - 2 + 3 - a + b$

$3 = - a + b$ (i)

And,

$f(-1) = (-1)^4 - 2(-1)^3 + 3(-1)^2 - a(-1) + b$

$19 = 1 + 2 + 3 + a + b$

$13 = a + b$ (ii)

Now,

Adding (i) and (ii),

$8 + 2b = 24$

$2b = 16$

$b = 8$

Now,

Using the value of b in (i)

$3 = - a + 8$

$a = 5$

Hence,

$a = 5$ and $b = 8$

Hence,

$f(x) = x^4 - 2(x)^3 + 3(x)^2 - a(x) + b$

$= x^4 - 2x^3 + 3x^2 - 5x + 8$

$f(2) = (2)^4 - 2(2)^3 + 3(2)^2 - 5(2) + 8$

$= 16 - 16 + 12 - 10 + 8$

$= 20 - 10$

$= 10$

Therefore, remainder is 10

Exercise : 2D

Question: 1

(x-2) is a

Solution:

From the factor theorem, we have

(x – 2) is the factor of f(x) if f(2) = 0

Here, we have

$f(2) = (2)^3 - 8$

$= 8 - 8$

$= 0$

Therefore,

(x – 2) is a factor of $(x^3 - 8)$

Question: 2

(x-3) is a

Solution:

From the factor theorem, we have

(x – 3) is the factor of f(x) if f(3) = 0

Here, we have

$f(3) = 2 \times 3^3 + 7 \times 3^2 - 24 \times 3 - 45$

$= 54 + 63 - 72 - 45$

$= 117 - 117$

$= 0$

Therefore,

(x – 3) is a factor of $(2x^3 + 7x^2 - 24x - 45)$

Question: 3

(x-1) is a

Solution:

From the factor theorem, we have

(x – 1) is the factor of f(x) if f(1) = 0

Here, we have

$f(1) = 2 \times 1^4 + 9 \times 1^3 + 6 \times 1^2 - 11 \times 1 - 6$

$= 2 + 9 + 6 - 11 - 6$

$= 17 - 17$

$= 0$

Therefore,

(x – 1) is the factor of $(2x^4 + 9x^3 + 6x^2 - 11x - 6)$

Question: 4

(x+2) is a

Solution:

From the factor theorem, we have

(x + 2) is the factor of f(x) if (f-2) = 0

Here, we have

$f(-2) = (-2)^4 – (-2)^2 – 12$

$= 16 – 4 – 12$

$= 16 – 16$

$= 0$

Therefore,

$(x + 2)$ is a factor of $(x^4 – x^2 – 12)$

Question: 5

(x+5) is a

Solution:

From the factor theorem, we have

$(x + 5)$ is the factor of $f(x)$ if $f(-5) = 0$

Here, we have

$f(-5) = 2(-5)^3 + 9(-5)^2 – 11(-5) – 30$

$= -250 + 225 + 55 – 30$

$= -280 + 280$

$= 0$

Therefore,

$(x + 5)$ is the factor of $(2x^3 + 9x^2 – 11x – 30)$

Question: 6

(2x-3) is a

Solution:

From the factor theorem, we have

$(x – a)$ is the factor of $f(x)$ if $f(a) = 0$

Here, we have

$2x – 3 = 0$

$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^4 + \left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right) + 6$

$= 2 \times \frac{81}{16} + \frac{27}{8} – 8 \times \frac{9}{4} - \frac{3}{2} + 6$

$= \frac{81}{8} + \frac{27}{8} – 18 - \frac{3}{2} + 6$

$= \frac{81+27-144-12+48}{8} = \frac{156-156}{8}$

$= 0$

Therefore,

$(2x – 3)$ is a factor of $(2x^4 + x^3 – 8x^2 – x + 6)$

Question: 7

Solution:

From the factor theorem, we have

(x – a) is the factor of f(x) if f(a) = 0

Here, we have

$f(\sqrt{2}) = 7(\sqrt{2})^2 - 4\sqrt{2} \times \sqrt{2} - 6$

= 14 – 8 – 6

= 14 – 14

= 0

Therefore,

(x - $\sqrt{2}$) is a factor of ($7x^2$ - 4√2x – 6)

Question: 8

Solution:

From the factor theorem, we have

(x – a) is the factor of f(x) if f(a) = 0

Here, we have

$f(-\sqrt{2}) = 2\sqrt{2}\,(-\sqrt{2})^2 + 5(-\sqrt{2}) + \sqrt{2}$

$= 2\sqrt{2} \times 2 - 5\sqrt{2} + \sqrt{2}$

$= 5\sqrt{2} - 5\sqrt{2}$

= 0

Therefore,

(x + √2) is the factor of ($2\sqrt{2}x^2$ + 5x + √2)

Question: 9

Find the va

Solution:

Let, f(x) = $2x^3 + 9x^2 + x + k$

Now, we have

x – 1 = 0

x = 1

Hence,

f(1) = $2 \times 1^3 + 9 \times 1^2 + 1 + k$

= 2 + 9 + 1 + k

= 12 + k

As per the question,

(x – 1) is the factor of f(x)

Now, by using factor theorem we get

(x – a) will be a factor of f(x) only if f(a) = 0 and hence f(1) = 0

So,

f(1) = 0

0 = 12 + k

$k = -12$

Question: 10

Find the va

Solution:

Let, $f(x) = 2x^3 – 3x^2 – 18x + a$

Now, we have

$x - 4 = 0$

$x = 4$

Hence,

$f(4) = 2(4)^3 – 3(4)^2 – 18 \times 4 + a$

$= 128 – 48 – 72 + a$

$= 128 – 120 + 8$

$= 8 + a$

As per question,

$(x - 4)$ is the factor of $f(x)$

Now, by using factor theorem we get

$(x – a)$ will be the factor of $f(x)$ if $f(a) = 0$ and hence $f(4) = 0$

So,

$f(4) = 8 + a = 0$

$a = -8$

Question: 11

Find the va

Solution:

Let, $f(x) = x^4 – x^3 – 11x^2 - x + a$

Now, we have

$x + 3 = 0$

$x = -3$

Hence,

$f(-3) = (-3)^4 – (-3)^3 – 11(-3)^2 – (-3) + a$

$= 81 + 27 – 11 \times 9 + 3 + a$

$= 81 + 27 – 99 + 3 + a$

$= 111 – 99 + a$

$= 12 + a$

As per question,

$(x + 3)$ is the factor of $f(x)$

Now, by using factor theorem we get

$(x – a)$ will be the factor of $f(x)$ if $f(a) = 0$ and hence $f(-3) = 0$

So,

$f(-3) = 12 + a = 0$

a = -12

Question: 12

For what va

Solution:

Let, f(x) = $2x^3 + ax^2 + 11x + a + 3$

Now, we have

2x – 1 = 0

x = 1/2

As per the question,

f(x) is exactly divisible by 2x – 1 which means that 2x – 1 is a factor of f(x)

Hence,

Using factor theorem,

(x – a) will be a factor of f(x) if f(a) = 0 and hence,

$f(\frac{1}{2}) \neq 0$

Hence,

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + 11 \times \frac{1}{2} + a + 3 = 0$$

$$= 2 \times \frac{1}{8} + a \times \frac{1}{4} + \frac{11}{2} + a + 3 = 0$$

$$= \frac{1}{4} + \frac{1}{4}a + \frac{11}{2} + a + 3 = 0$$

$$= \frac{1+a+22+4a+12}{4} = 0$$

$$= \frac{5a+35}{4} = 0$$

= 5a = -35

a = -7

Thus, the value of a is -7

Question: 13

Find the va

Solution:

Let f(x) = $x^3 - 10x^2 + ax + b$

Now,

By using factor theorem,

(x – 1) and (x - 2) will be the factors of f(x) if f(1) = 0 and f(2) = 0

Hence,

f(1) = $1^3 - 10(1)^2 + a \times 1 + b0 = 1 - 10 + a + b$

a + b = 9 (i)

And,

f(2) = $2^3 - 10 \times 2^2 + a \times 1 + b$

0 = 8 – 40 + 2a + b

$2a + b = 32$ (ii)

Now, subtracting (i) from (ii)

$a = 23$

Using the value of a in (i), we get

$23 + b = 9$

$b = 9 – 23$

$b = -14$

Hence,

$a = 23$ and $b = -14$

Question: 14

Find the va

Solution:

Let $f(x) = x^4 + ax^3 – 7x^2 - 8x + b$

Now,

$(x + 2) = 0$

$x = -2$

And,

$(x + 3) = 0$

$x = -3$

Now,

By using factor theorem,

$(x + 2)$ and $(x + 3)$ will be the factors of $f(x)$ if $f(-2) = 0$ and $f(-3) = 0$

Hence,

$f(-2) = (-2)^4 + a(-2)^3- 7 (-2)^2 -8 (-2) + b0 = 16 – 8a - 28 + 16 + b$

$8a - b = 4$ (i)

And,

$f(-3) = (-3)^4 + a (-3)^3 – 10 (-3)^2 – 8 (-3) + b$

$0 = 81 – 27a – 63 + 24 + b$

$27a – b = 42$ (ii)

Now, subtracting (i) from (ii)

$19a = 38$

$a = 2$

Using the value of a in (i), we get

$8 (2) – b = 4$

$16 – b = 4$

$b = 12$

Therefore,

$a = 2$ and $b = 12$

Question: 15

Without act

Solution:

Let $f(x) = x^3 – 3x^2 – 13x + 15$

Now, we have

$x^2 + 2x – 3 = 0$

$x^2 + 3x – x – 3 = 0$

$(x + 3)(x – 1)$

Hence, f(x) will be exactly divisible by $x^2 + 2x – 3 = (x + 3)(x – 1)$

Now,

Using factor theorem,

If (x + 3) and (x – 1) are both the factors of f(x), then f(-3) = 0 and f(1) = 0

Now,

$f(-3) = (-3)^3 – 3(-3)^2 – 13(-3) + 15$

$= - 27 – 27 + 39 + 15$

$= - 54 + 54$

$= 0$

And,

$f(1) = (1)^3 – 3(1)^2 – 13(1) + 15$

$= 1 – 3 - 13 + 15$

$= 16 - 16$

$= 0$

Now,

Since, f(-3) = 0 and f(1) = 0

Therefore, $x^2 + 2x -3$ divides f(x) completely.

Question: 16

If

Solution:

Let $f(x) = (x^3 + ax^2 + bx + 6)$

By using remainder theorem,

If we divide f(x) by (x – 3) then it will leave a remainder as f(3)

So,

$f(3) = 3^2 + a \times 3^2 + b \times 3 + 6 = 3$

$27 + 9a + 3b + 6 = 3$

$9a + 3b + 33 = 3$

$9a + 3b = 3 – 33$

$9a + 3b = -30$

$3a + b = -10$ (i)

It is also given that,

(x – 2) is a factor of f(x)

Therefore,

By using factor theorem, we get

(x – a) is the factor of f(x) if f(a) = 0and also f(2) = 0

Now,

$f(2) = 2^3 + a \times 2^2 + b \times 2 + 6 = 0$

8 + 4a + 2b + 6 = 0

4a + 2b = -14

2a + b = -7 (ii)

Now by subtracting (ii) from (i), we get

a = -3

Putting the value of a in (i), we get

3(-3) + b = -10

-9 + b = -10

b = -10 + 9

b = -1

Therefore,

b = -1 and a = -3

Exercise : 2E

Question: 1

Factorize:

Solution:

We have,

$9x^2 + 12xy$

At first, we'll take common from the expression

$3x(3x + 4y)$

Hence,

The given expression can be factorized as:

3x(3x + 4y)

Question: 2

Factorize:

Solution:

We have,

$18x^2y - 24xyz$

At first we'll take common from the expression

$6xy(3x - 4z)$

Hence,

The given expression can be factorized as:

$6xy(3x-4z)$

Question: 3

Factorize:

Solution:

We have,

$27a^3b^3 - 45a^4b^2$

At first we'll take common from the expression

$9a^3b^2(3b-5a)$

Hence,

The given expression can be factorized as:

$9a^3b^2(3b-5a)$

Question: 4

Factorize:

Solution:

We have,

$2a(x+y) - 3b(x+y)$

At first, we'll take common from the expression

$(x+y)(2a-3b)$

Hence,

The given expression can be factorized as:

$(x+y)(2a-3b)$

Question: 5

Factorize:

Solution:

We have,

$2x(p^2+q^2)+4y(p^2+q^2)$

At first, we'll take common from the expression

$=2[x(p^2+q^2)+2y(p^2+q^2)]$

$=2(p^2+q^2)(x+2y)$

Hence,

The given expression can be factorized as:

$2(p^2+q^2)(x+2y)$

Question: 6

Factorize:

Solution:

We have,

$x(a-5)+y(5-a)$

At first we’ll take common from the expression

= (a – 5) (x – y)

Hence,

The given expression can be factorized as:

$(a-5)(x-y)$

Question: 7

Factorize:

Solution:

We have,

$4(a+b)-6(a+b)^2$

At first, we’ll take (a+b) common from the expression.

= (a + b) [4 – 6 (a + b)]

Take 2 common out of [4 - 6(a+b)]

= 2(a + b) (2 – 3a – 3b)

Hence,

The given expression can be factorized as:

$2(a+b)(2-3a-3b)$

Question: 8

Factorize:

Solution:

We have,

$8(3a-2b)^2-10(3a-2b)$

At first we’ll take common from the expression

= (3a – 2b) [8 (3a – 2b) – 10]

= (3a – 2b) 2[4 (3a – 2b) – 5]

= 2 (3a - 2b) (12a – 8b – 5)

Hence,

The given expression can be factorized as:

$2(3a-2b)(12a-8b-5)$

Question: 9

Factorize:

Solution:

We have,

$x(x+y)^3-3x^2y(x+y)$

At first we’ll take common from the expression

= x (x + y) [(x + y)^2 – 3xy]

$= x (x + y) (x^2 + y^2 + 2xy - 3xy)$

$= x (x + y) (x^2 + y^2 - xy)$

Hence,

The given expression can be factorized as:

$x(x+y)(x^2+y^2-xy)$

Question: 10

Factorize:

Solution:

We have,

$x^3+2x^2+5x+10$

At first we'll take common from the expression

$= x^2 (x + 2) + 5 (x + 2)$

$= (x^2 + 5) (x + 2)$

Hence,

The given expression can be factorized as:

$(x+2)(x^2+5)$

Question: 11

Factorize:

Solution:

We have,

$x^2+xy-2xz-2yz$

At first we'll take common from the expression

$= x (x + y) - 2z (x + y)$

$= (x + y) (x - 2z)$

Hence,

The given expression can be factorized as:

$(x+y)(x-2z)$

Question: 12

Factorize:

Solution:

We have,

$a^3b-a^2b+5ab-5b$

At first we'll take common from the expression

$= a^2b (a - 1) + 5b (a - 1)$

$= (a - 1) (a^2b + 5b)$

$= (a - 1) b (a^2 + 5)$

$= b (a - 1) (a^2 + 5)$

Hence,

The given expression can be factorized as:

$b(a-1)(x^2+5)$

Question: 13

Factorize:

Solution:

We have,

$8-4a-2a^3+a^4$

At first we'll take common from the expression

$= 4(2 - a) - a^3(2 - a)$

$= (2 - a)(4 - a^3)$

Hence,

The given expression can be factorized as:

$(2-a)(4-a^3)$

Question: 14

Factorize:

Solution:

We Have,

$x^3-2x^2y+3xy^2-6y^3$

At first we'll take common from the expression

$= x^2(x - 2y) + 3y^2(x - 2y)$

$= (x - 2y)(x^2 + 3y^2)$

Hence,

The given expression can be factorized as:

$(x-2y)(x^2+3y^2)$

Question: 15

Factorize:

Solution:

We have,

$px-5q+pq-5x$

At first we'll take common from the expression

$= p(x + q) - 5(q + x)$

$= (x + q)(p - 5)$

Hence,

The given expression can be factorized as:

$(x+q)(p-5)$

Question: 16

Factorize:

Solution:

We have,

$x^2 + y - xy - x$

At first we'll take common from the expression

= x (x – y) – 1 (x – y)

= (x – y) (x – 1)

Hence,

The given expression can be factorized as:

$(x-y)(x-1)$

Question: 17

Factorize:

Solution:

We have,

$(3a-1)^2 - 6a + 2$

At first, we'll take 2 common from - 6a + 2 in the expression.

= $(3a – 1)^2$ – 2 (3a – 1)

Now take (3a-1) common from above to get,

= (3a – 1) [(3a – 1) – 2]

= (3a – 1) (3a – 3)

= 3(3a – 1) (a – 1)

Hence,

The given expression can be factorized as:

$3(3a-1)(a-1)$

Question: 18

Factorize:

Solution:

We have,

$(2x-3)^2 - 8x + 12$

At first we'll take common from the expression

= $(2x – 3)^2$ – 4 (2x – 3)

= (2x – 3) (2x – 3 – 4)

= (2x – 3) (2x – 7)

Hence,

The given expression can be factorized as:

$(2x-3)(2x-7)$

Question: 19

Factorize:

Solution:

We have,

$a^2 + a - 3a^2 - 3$

At first we'll take common from the expression

= a (a^2 + 1) – 3 (a^2 + 1)

= (a – 3) (a^2 + 1)

Hence,

The given expression can be factorized as:

$(a-3)(a^2+1)$

Question: 20

Factorize:

Solution:

We have,

$3ax - 6ay - 8by + 4bx$

At first we'll take common from the expression

= 3a (x – 2y) + 4b (x – 2y)

= (x – 2y) (3a + 4b)

Hence,

The given expression can be factorized as:

$(3a+4b)(x-2y)$

Question: 21

Factorize:

Solution:

We have,

$abx^2 + a^2x + b^2x + ab$

At first we'll take common from the expression

= ax (bx + a) + b (bx + a)

= (bx + a) (ax + b)

Hence,

The given expression can be factorized as:

$(bx+a)(ax+b)$

Question: 22

Factorize:

Solution:

We have,

$x^3 - x^2 + ax + x - a - 1$

At first we'll take common from the expression

$= x^3 – x^2 + ax – a + x – 1$

$= x^2 (x – 1) + a (x – 1) + 1 (x – 1)$

$= (x – 1) (x^2 + a + 1)$

Hence,

The given expression can be factorized as:

$(x-1)(x^2+a+1)$

Question: 23

Factorize:

Solution:

We have,

$2x+4y-8xy-1$

At first we’ll take common from the expression

$= 2x – 1 – 8xy + 4y$

$= (2x – 1) – 4y (2x – 1)$

$= (2x – 1) (1 – 4y)$

Hence,

The given expression can be factorized as:

$(1-4y)(2x-1)$

Question: 24

Factorize:

Solution:

We have,

$ab(x^2+y^2)-xy(a^2+b^2)$

At first we’ll take common from the expression

$= abx^2 + aby^2 – a^2xy – b^2xy$

$= abx^2 – a^2xy + aby^2 – b^2xy$

$= ax (bx – ay) + by (ay – bx)$

$= (bx – ay) (ax – by)$

Hence,

The given expression can be factorized as:

$(bx-ay)(ax-by)$

Question: 25

Factorize:

Solution:

We have,

$a^2+ab(b+1)+b^3$

At first we’ll take common from the expression

$= a^2 + ab^2 + ab + b^3$

$= a^2 + ab + ab^2 + b^3$

$= a (a + b) + b^2 (a + b)$

$= (a + b) (a + b^2)$

Hence,

The given expression can be factorized as:

$(a+b)(a+b^2)$

Question: 26

Factorize:

Solution:

We have,

$a^3 + ab(1-2a) - 2b^2$

At first we'll take common from the expression

$= a^3 + ab - 2a^2b - 2b^2$

$= a (a^2 + b) - 2b (a^2 + b)$

$= (a^2 + b) (a - 2b)$

Hence,

The given expression can be factorized as:

$(a-2b)(a^2+b)$

Question: 27

Factorize:

Solution:

We have,

$2a^2 + bc - 2ab - ac$

At first we'll take common from the expression

$= 2a^2 - 2ab - ac + bc$

$= 2a (a - b) - c (a - b)$

$= (a - b) (2a - c)$

Hence,

The given expression can be factorized as:

$(2a-c)(a-b)$

Question: 28

Factorize:

Solution:

We have,

$(ax+by)^2 + (bx-ay)^2$

At first we'll take common from the expression

$= a^2x^2 + b^2y^2 + 2abxy + b^2x^2 + a^2y^2 - 2abxy$

$= a^2x^2 + b^2y^2 + b^2x^2 + a^2y^2$

$= a^2x^2 + b^2x^2 + b^2y^2 + a^2y^2$

$= x^2 (a^2 + b^2) + y^2 (a^2 + b^2)$

$= (a^2 + b^2) (x^2 + y^2)$

Hence,

The given expression can be factorized as:

$\left(a^2+b^2\right)\left(x^2+y^2\right)$

Question: 29

Factorize:

Solution:

We have,

$a(a+b-c)-bc$

At first we'll take common from the expression

$= a^2 + ab - ac - bc$

$= a (a + b) - c (a + b)$

$= (a - c) (a + b)$

Hence,

The given expression can be factorized as:

$(a-c)(a+b)$

Question: 30

Factorize:

Solution:

We have,

$a(a-2b-c)+2bc$

At first we'll take common from the expression

$= a^2 - 2ab - ac + 2bc$

$= a (a - 2b) - c (a - 2b)$

$= (a - 2b) (a - c)$

Hence,

The given expression can be factorized as:

$(a-c)(a-2b)$

Question: 31

Factorize:

Solution:

We have,

$a^2x^2+\left(ax^2+1\right)x+a$

At first, we'll take common from the expression

= $a^2x^2 + ax^3 + x + a$

= $ax^2 (a + x) + 1 (x + a)$

= $(ax^2 + 1) (a + x)$

Hence,

The given expression can be factorized as:

$(a+x)(ax^2+1)$

Question: 32

Factorize:

Solution:

We have,

$ab(x^2+1)+x(a^2+b^2)$

At first, we'll take common from the expression

= $abx^2 + ab + a^2x + b^2x$

= $abx^2 + a^2x + ab + b^2x$

= $ax (bx + a) + b (bx + a)$

= $(bx + a) (ax + b)$

Hence,

The given expression can be factorized as:

$(ax+b)(bx+a)$

Question: 33

Factorize:

Solution:

We have,

$x^2-(a+b)x+ab$

At first we'll take common from the expression

= $x^2 - ax – bx + ab$

= $x (x – a) – b (x – a)$

= $(x – a) (x – b)$

Hence,

The given expression can be factorized as:

$(x-a)(x-b)$

Question: 34

Factorize:

Solution:

We have,

$x^2+\frac{1}{x^2}-2-3x+\frac{3}{x}$

At first, we'll take common from the expression

= $(x - 1/x)^2 - 3(x - 1/x)$

= $\left(x-\frac{1}{x}\right)\left(x-\frac{1}{x}-3\right)$

Hence,

The given expression can be factorized as:

$\left(x-\frac{1}{x}\right)\left(x-\frac{1}{x}-3\right)$

Exercise : 2F

Question: 1

Factorize:

Solution:

We have,

$25x^2-64y^2$

We can also write the expression as:

$(5x)^2 - (8y)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(5x-8y)(5x+8y)$

Hence,

The given expression can be factorized as: $(5x-8y)(5x+8y)$

Question: 2

Factorize:

Solution:

We have,

$100-9x^2$

We can also write the expression as:

$(10)^2 - (3x)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(10-3x)(10+3x)$

Hence,

The given expression can be factorized as: $(10-3x)(10+3x)$

Question: 3

Factorize:

Solution:

We have,

$100-9x^2$

We can also write the expression as:

$(\sqrt{5}x)^2 - (\sqrt{7}y)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$\left(\sqrt{5}x-\sqrt{7}y\right)\left(\sqrt{5}x+\sqrt{7}y\right)$

Hence,

The given expression can be factorized as: $\left(\sqrt{5}x-\sqrt{7}y\right)\left(\sqrt{5}x+\sqrt{7}y\right)$

Question: 4

Factorize:

Solution:

We have,

$(3x+5y)^2-4z^2$

We can also write the expression as:

$(3x + 5y)^2 - (2z)^2$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(3x+5y-2z)(3x+5y+2z)$

Hence,

The given expression can be factorized as: $(3x+5y-2z)(3x+5y+2z)$

Question: 5

Factorize:

Solution:

We have,

$150-6x^2$

$= 6(25 - x^2)$

We can also write the expression as:

$6[(5\ )^2 - (x)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$6(5 - x)\ (5 + x)$

Hence,

The given expression can be factorized as: $6(5 - x)(5 + x)$

Question: 6

Factorize:

Solution:

We have,

$20x^2 - 45$

= 5(x^2 – 9)

We can also write the expression as:

= 5(x – 9)(x + 9)

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$5(2x-3)(2x+3)$

Hence,

The given expression can be factorized as: $5(2x-3)(2x+3)$

Question: 7

Factorize:

Solution:

We have,

$3x^3 - 48$

= 3x(x^2 - 16)

We can also write the expression as:

$3x[(x)^2 - (4)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$3x(x-4)(x+4)$

Hence,

The given expression can be factorized as: $3x(x-4)(x+4)$

Question: 8

Factorize:

Solution:

We have,

$2 - 50x^2$

= 2(1 – 25x^2)

We can also write the expression as:

= $2[(1)^2 - (25x)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$2(1-5x)(1+5x)$

Hence,

The given expression can be factorized as: $2(1-5x)(1+5x)$

Question: 9

Factorize:

Solution:

We have,

$27a^2 - 48b^2$

= $3(9a^2 – 16b^2)$

We can also write the expression as:

= $3[(3a)^2 – (4b)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$3(3a - 4b)(3a + 4b)$

Hence,

The given expression can be factorized as: $3(3a - 4b)(3a + 4b)$

Question: 10

Factorize:

Solution:

We have,

$x - 64x^3$

= $x(1 – 64x^2)$

We can also write the expression as:

= $x[(1)^2 – (8x)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$x(1 - 8x)(1 + 8x)$

Hence,

The given expression can be factorized as: $x(1 - 8x)(1 + 8x)$

Question: 11

Factorize:

Solution:

We have,

$8ab^2 - 18a^3$

= $2a(4b^2 – 9a^2)$

We can also write the expression as:

= $2a[(2b)^2 – (3a)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$2a(2b - 3a)(2b + 3a)$

Hence,

The given expression can be factorized as: $2a(2b - 3a)(2b + 3a)$

Question: 12

Factorize:

Solution:

We have,

$3a^3b - 243ab^3$

= $3ab(a^2 - 81b^2)$

We can also write the expression as:

= $3ab[(a)^2 - (9b)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$3ab(a-9b)(a+9b)$

Hence,

The given expression can be factorized as: $3ab(a-9b)(a+9b)$

Question: 13

Factorize:

Solution:

We have,

$(a+b)^3 - a - b$

= $(a + b)^3 - (a + b)$

= $(a + b)\,[(a + b)^2 - 1]$

We can also write the expression as:

= $(a + b)[(a + b)^2 - (1)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(a+b)(a+b-1)(a+b+1)$

Hence,

The given expression can be factorized as: $(a+b)(a+b-1)(a+b+1)$

Question: 14

Factorize:

Solution:

We have,

$108a^2 - 3(b-c)^2$

= $3[36a^2 - (b - c)^2]$

We can also write the expression as:

= $3[(6a)^2 - (b - c)^2]$

Now,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$3(6a-b+c)(6a+b-c)$

Hence,

The given expression can be factorized as: $3(6a-b+c)(6a+b-c)$

Question: 15

Factorize:

Solution:

We have,

x^3-5x^2-x+5

= $x^2(x – 5) – 1(x – 5)$

= $(x - 5)[x^2 – (1)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$(x-5)(x-1)(x+1)$

Hence,

The given expression can be factorized as: $(x-5)(x-1)(x+1)$

Question: 16

Factorize:

Solution:

We have,

$a^2+2ab+b^2-9c^2$

= $(a + b)^2 – 9c^2$

We can also write the expression as:

= $[(a + b)^2 – (3c)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$(a+b-3c)(a+b+3c)$

Hence,

The given expression can be factorized as: $(a+b-3c)(a+b+3c)$

Question: 17

Factorize:

Solution:

We have,

$9-a^2+2ab-b^2$

= $[9 – (a^2 – 2ab + b)^2]$

We can also write the expression as:

= $[(3)^2 – (a – b)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$(3+a-b)(3-a+b)$

Hence,

The given expression can be factorized as: $(3+a-b)(3-a+b)$

Question: 18

Factorize:

Solution:

We have,

$a^2-b^2-4ac+4c^2$

$= a^2 – 2(a)(2c) + c^2 – b^2$

$= (a – c)^2 – (b)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$(a-2c+b)(a-2c-b)$

Hence,

The given expression can be factorized as: $(a-2c+b)(a-2c-b)$

Question: 19

Factorize:

Solution:

We have,

$9a^2+3a-8b-64b^4$

$= 9\ a^2 – 64b^2 + 3a – 8b$

We can also write this as:

$= (3a)^2 –(8b)^2 + (3a – 8b)$

Now,

Using identity: $a^2 - b^2 = (a – b)(a + b)$

$(3a-8b)(3a+8b+1)$

Question: 20

Factorize:

Solution:

We have,

x^2-y^2+6y-9

$= [x^2 – (y^2 – 2(y)(3) + 3^2]$

We can also write the expression as:

$= [(x)^2 – (y – 3)^2]$

Now,

Using identity: $a^2 – b^2 = (a – b)(a + b)$

$(x+y-3)(x-y+3)$

Hence,

The given expression can be factorized as: $(x+y-3)(x-y+3)$

Question: 21

Factorize:

Solution:

We have,

$4x^2-9y^2-2x-3y$

We can also write this as:

= $(4x)^2$ –$(9y)^2$ – (2x + 3y)

Now,

Using identity: a^2 - b^2 = (a – b)(a + b)

$(2x+3y)(2x-3y-1)$

Question: 22

Factorize:

Solution:

We have:

x^4-1

We can also write this as:

=$(x^2)^2$ - 1^2

Using identity: a^2 - b^2 = (a – b)(a + b)

=$(x^2 + 1)(x^2 – 1)$

Again,

Using identity: a^2 - b^2 = (a – b)(a + b)

=$(x^2 + 1)(x + 1)(x – 1)$

Question: 23

Factorize:

Solution:

We have:

$a-b-a^2+b^2$

= (a – b) – $(a^2 - b^2)$

= (a – b) – (a – b)(a + b)

Hence,

The factorization of the given expression is,

$(a-b)(1-a-b)$

Question: 24

Factorize:

Solution:

We have:

$x^4 - 625$

We can also write this as:

$=(x^2)^2 - (25)^2$

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$=(x^2 + 25)(x^2 - 25)$

Again,

Using identity: $a^2 - b^2 = (a - b)(a + b)$

$(x-5)(x+5)(x^2+25)$

Exercise : 2G

Question: 1

Factorize:

Solution:

We have,

$x^2 + 11x + 30$

Now by using middle-term splitting, we get

$= x^2 + 6x + 5x + 30$

$= x (x + 6) + 5 (x + 6)$

$= (x + 6) (x + 5)$

Hence,

The given expression can be factorized as:

$(x+6)(x+5)$

Question: 2

Factorize:

Solution:

We have,

$x^2 + 18x + 32$

Now by using middle-term splitting, we get

$= x^2 + 16x + 2x + 32$

$= x (x + 16) + 2 (x + 16)$

$= (x + 16) (x + 2)$

Hence,

The given expression can be factorized as:

$(x+16)(x+2)$

Question: 3

Factorize:

Solution:

We have,

$x^2+7x-18$

Now by using middle-term splitting, we get

$= x^2 + 9x - 2x - 18$

$= x (x + 9) - 2 (x + 9)$

$= (x + 9) (x - 2)$

Hence,

The given expression can be factorized as:

$(x+9)(x-2)$

Question: 4

Factorize:

Solution:

We have,

x^2+5x-6

Now by using middle-term splitting, we get

$= x^2 + 6x - x - 6$

$= x (x + 6) - 1 (x + 6)$

$= (x + 6) (x - 1)$

Hence,

The given expression can be factorized as:

$(x+6)(x-1)$

Question: 5

Factorize:

Solution:

We have,

y^2-4y+3

Now by using middle-term splitting, we get

$= y^2 – 3y - y + 3$

$= y (y - 3) - 1 (y - 3)$

$= (y - 3) (y - 1)$

Hence,

The given expression can be factorized as:

$(y-3)(y-1)$

Question: 6

Factorize:

Solution:

We have,

$x^2 - 21x + 108$

Now by using middle-term splitting, we get

= x^2 - 12x - 9x + 108

= x (x -12) - 9 (x -12)

= (x -12) (x - 9)

Hence,

The given expression can be factorized as:

$(x-12)(x-9)$

Question: 7

Factorize:

Solution:

We have,

$x^2 - 11x - 80$

Now by using middle-term splitting, we get

= x^2 - 16x + 5x - 80

= x (x - 16) + 5 (x - 16)

= (x - 16) (x + 5)

Hence,

The given expression can be factorized as:

$(x-16)(x+5)$

Question: 8

Factorize:

Solution:

We have,

$x^2 - x - 156$

Now by using middle-term splitting, we get

= x^2 - 13x + 12x - 156

= x (x - 13) + 12 (x - 13)

= (x - 13) (x + 12)

Hence,

The given expression can be factorized as:

$(x-13)(x+12)$

Question: 9

Factorize:

Solution:

We have,

$z^2 - 32z - 105$

Now by using middle-term splitting, we get

$= z^2 – 35z + 3z - 105$

$= z (z - 35) + 3 (z - 35)$

$= (z - 35) (z + 3)$

Hence,

The given expression can be factorized as:

$(z-35)(z+3)$

Question: 10

Factorize:

Solution:

We have,

$40+3x-x^2$

Now by using middle-term splitting, we get

$= 40 + 8x – 5x – x^2$

$= 8 (5 + x) – x (5 + x)$

$= (5 + x) (8 – x)$

Hence,

The given expression can be factorized as:

$(5+x)(8-x)$

Question: 11

Factorize:

Solution:

We have,

$6-x-x^2$

Now by using middle-term splitting, we get

$= 6 + 2x – 3x – x^2$

$= 2 (3 + x) – x (3 + x)$

$= (3 + x) (2 – x)$

Hence,

The given expression can be factorized as:

$(2-x)(3+x)$

Question: 12

Factorize:

Solution:

We have,

$7x^2+49x+84$

Now by using middle-term splitting, we get

$= 7 (x^2 +7x + 12)$

$= 7 [x^2 + 4x + 3x + 12]$

= 7 [x (x + 4) + 3 (x + 4)]

= 7 (x + 4) (x + 3)

Hence,

The given expression can be factorized as:

$7(x+4)(x+3)$

Question: 13

Factorize:

Solution:

We have,

$m^2+17mn-84$

Now by using middle-term splitting, we get

= m^2 + 21mn– 4mn – $84n^2$

= m (m + 21n) – 4n (m + 21n)

= (m + 21n) (m – 4n)

Hence,

The given expression can be factorized as:

$(m+21n)(m-4n)$

Question: 14

Factorize:

Solution:

We have,

$5x^2+16x+3$

Now by using middle-term splitting, we get

= $5x^2$ + 15x + x + 3

= 5x (x + 3) + 1 (x + 3)

= (5x + 1) (x + 3)

Hence,

The given expression can be factorized as:

$(x+3)(5x+1)$

Question: 15

Factorize:

Solution:

We have,

$6x^2+17x+12$

Now by using middle-term splitting, we get

= $6x^2$ + 9x + 8x + 12

= 3x (2x + 3) + 4 (2x + 3)

= (2x + 3) (3x + 4)

Hence,

The given expression can be factorized as:

$(3x+4)(2x+3)$

Question: 16

Factorize:

Solution:

We have,

$9x^2+18x+8$

Now by using middle-term splitting, we get

$= 9x^2 + 12x + 6x + 8$

$= 3x (3x + 4) + 2 (3x + 4)$

$= (3x + 4) (3x + 2)$

Hence,

The given expression can be factorized as:

$(3x+4)(3x+2)$

Question: 17

Factorize:

Solution:

We have,

$14x^2+9x+1$

Now by using middle-term splitting, we get

$= 14x^2 + 7x + 2x + 1$

$= 7x (2x + 1) + 1 (2x + 1)$

$= (7x + 1) (2x + 1)$

Hence,

The given expression can be factorized as:

$(2x+1)(7x+1)$

Question: 18

Factorize:

Solution:

We have,

$2x^2+3x-90$

Now by using middle-term splitting, we get

$= 2x^2 - 12x + 15x - 90$

$= 2x (x - 6) + 15 (x - 6)$

$= (x - 6) (2x + 15)$

Hence,

The given expression can be factorized as:

$(2x+15)(x-6)$

Question: 19

Factorize:

Solution:

We have,

$2x^2+11x-21$

Now by using middle-term splitting, we get

$= 2x^2 + 14x - 3x - 21$

$= 2x (x + 7) - 3 (x + 7)$

$= (x + 7) (2x - 3)$

Hence,

The given expression can be factorized as:

$(x+7)(2x-3)$

Question: 20

Factorize:

Solution:

We have,

$3x^2-14x+8$

Now by using middle-term splitting, we get

$= 3x^2 - 12x - 2x + 8$

$= 3x (x - 4) - 2 (x - 4)$

$= (x - 4) (3x - 2)$

Hence,

The given expression can be factorized as:

$(x-4)(3x-2)$

Question: 21

Factorize:

Solution:

We have,

$18x^2+3x-10$

Now by using middle-term splitting, we get

$= 18x^2 - 12x + 15x - 10$

$= 6x (3x - 2) + 5 (3x - 2)$

$= (6x + 5) (3x - 2)$

Hence,

The given expression can be factorized as:

$(6x+5)(3x-2)$

Question: 22

Factorize:

Solution:

We have,

$15x^2 + 2x - 8$

Now by using middle-term splitting, we get

$= 15x^2 - 10x + 12x - 8$

$= 5x (3x - 2) + 4 (3x - 2)$

$= (5x + 4) (3x - 2)$

Hence,

The given expression can be factorized as:

$(5x+4)(3x-2)$

Question: 23

Factorize:

Solution:

We have,

$6x^2 + 11x - 10$

Now by using middle-term splitting, we get

$= 6x^2 + 15x - 4x - 10$

$= 3x (2x + 5) - 2 (2x + 5)$

$= (2x + 5) (3x - 2)$

Hence,

The given expression can be factorized as:

$(2x+5)(3x-2)$

Question: 24

Factorize:

Solution:

We have,

$30x^2 + 7x - 15$

Now by using middle-term splitting, we get

$= 30x^2 - 18x + 25x - 15$

$= 6x (5x - 3) + 5 (5x - 3)$

$= (5x - 3) (6x + 5)$

Hence,

The given expression can be factorized as:

$(6x+5)(5x-3)$

Question: 25

Factorize:

Solution:

We have,

$24x^2 - 41x + 12$

Now by using middle-term splitting, we get

= $24x^2$ - 32x - 9x + 12

= 8x (3x - 4) - 3 (3x - 4)

= (3x - 4) (8x - 3)

Hence,

The given expression can be factorized as:

$(3x-4)(8x-3)$

Question: 26

Factorize:

Solution:

We have,

$2x^2 - 7x - 15$

Now by using middle-term splitting, we get

= $2x^2$ - 10x + 3x - 15

= 2x (x - 5) + 3 (x - 5)

= (x - 5) (2x + 3)

Hence,

The given expression can be factorized as:

$(x-5)(2x+3)$

Question: 27

Factorize:

Solution:

We have,

$6x^2 - 5x - 21$

Now by using middle-term splitting, we get

= $6x^2$ + 9x - 14x - 21

= 3x (2x + 3) - 7 (2x + 5)

= (3x - 7) (2x + 3)

Hence,

The given expression can be factorized as:

$(3x-7)(2x+3)$

Question: 28

Factorize:

Solution:

We have,

$10x^2 - 9x - 7$

Now by using middle-term splitting, we get

= $10x^2 + 5x - 14x - 7$

= $5x (2x + 1) - 7 (2x + 1)$

= $(2x + 1) (5x - 7)$

Hence,

The given expression can be factorized as:

$(5x-7)(2x+1)$

Question: 29

Factorize:

Solution:

We have,

$5x^2-16x-21$

Now by using middle-term splitting, we get

= $5x^2 + 5x - 21x - 21$

= $5x (x + 1) - 21 (x + 1)$

= $(x + 1) (5x - 21)$

Hence,

The given expression can be factorized as:

$(5x-21)(x+1)$

Question: 30

Factorize:

Solution:

We have,

$2x^2-x-21$

Now by using middle-term splitting, we get

= $2x^2 + 6x - 7x - 21$

= $2x (x + 3) - 7 (x + 3)$

= $(x + 3) (2x - 7)$

Hence,

The given expression can be factorized as:

$(2x-7)(x+3)$

Question: 31

Factorize:

Solution:

We have,

$15x^2-x-28$

Now by using middle-term splitting, we get

= $15x^2 + 20x - 21x - 28$

= 5x (3x + 4) - 7 (3x + 4)

= (3x + 4) (5x - 7)

Hence,

The given expression can be factorized as:

$(5x-7)(3x+4)$

Question: 32

Factorize:

Solution:

We have,

$8a^2-27ab+9b^2$

Now by using middle-term splitting, we get

= $8a^2$ – 24ab – 3ab + $9b^2$

= 8a (a – 3b) – 3b (a – 3b)

= (a – 3b) (8a – 3b)

Hence,

The given expression can be factorized as:

$(a-3b)(8a-3b)$

Question: 33

Factorize:

Solution:

We have,

$5x^2+33xy-14y^2$

Now by using middle-term splitting, we get

= $5x^2$ + 35xy – 2xy – $14y^2$

= 5x (x + 7y) – 2y (x + 7y)

= (x + 7y) (5x – 2y)

Hence,

The given expression can be factorized as:

$(x+7y)(5x-2y)$

Question: 34

Factorize:

Solution:

We have,

$3x^3-x^2-10x$

Now by using middle-term splitting, we get

= x ($3x^2$ – x – 10)

= x [$3x^2$ – 6x + 5x – 10]

= x [3x (x – 2) + 5 (x – 2)]

= x (x – 2) (3x + 5)

Hence,

The given expression can be factorized as:

$x(x-2)(3x+5)$

Question: 35

Factorize:

Solution:

We have,

$\frac{1}{3}x^2-2x-9$

Now by using middle-term splitting, we get

= 1/3 x^2 – 3x + x – 9

= x (x/3 – 3) + (x – 9)

= x/3 (x – 9) + 1 (x – 9)

= (x – 9) (x/3 + 1)

= (x – 9) × (x – 3)/3

= 1/3 (x – 9) (x + 3)

Hence,

The given expression can be factorized as:

$\frac{1}{3}(x-9)(x+3)$

Question: 36

Factorize:

Solution:

We have,

$x^2-2x+\frac{7}{16}$

Now by using middle-term splitting, we get

= 1/16 ($16x^2$ – 32x + 7)

= 1/16 ($16x^2$ – 4x – 28x + 7)

= 1/16 [4x (4x – 1) – 7 (4x – 1)]

= 1/16 (4x – 1) (4x – 7)

Hence,

The given expression can be factorized as:

$\frac{1}{16}(4x-7)(4x-1)$

Question: 37

Factorize:

Solution:

We have,

$\sqrt{2}x^2+3x+\sqrt{2}$

Now by using middle-term splitting, we get

$=\sqrt{2}x^2 + x + 2x + \sqrt{2}$

= x (√2 x+1) + √2 (√2 x+1)

= $\left(x+\sqrt{2}\right)\left(\sqrt{2}x+1\right)$

Hence,

The given expression can be factorized as:

$\left(x+\sqrt{2}\right)\left(\sqrt{2}x+1\right)$

Question: 38

Factorize:

Solution:

We have,

$\sqrt{5}x^2+2x-3\sqrt{5}$

Now by using middle-term splitting, we get

= √5 × x × x + 5x – 3x - 3√5

= √5 x (x + √5) – 3 (x + √5)

= (√5 x – 3) (x + √5)

Hence,

The given expression can be factorized as:

$\left(x+\sqrt{5}\right)\left(\sqrt{5}x-3\right)$

Question: 39

Factorize:

Solution:

We have,

$2x^2+3\sqrt{3}x+3$

Now by using middle-term splitting, we get

= 2 × x × x + 2√3x + √3x + 3

= 2x (x+ √3) + √3 (x + √3)

= (x + √3) (2x + √3)

Hence,

The given expression can be factorized as:

$\left(x+\ \sqrt{3}\right)(2x+\sqrt{3})$

Question: 40

Factorize:

Solution:

We have,

$2\sqrt{3}x^2+x-5\sqrt{3}$

Now by using middle-term splitting, we get

= 2√3 * x * x + 6x - 5x - 5√3

= 2 √3 x (x + √3) - 5 (x+ √3)

= (x + √3)(2√3x - 5)

Hence,

The given expression can be factorized as:

$(x+\sqrt{3})(2\sqrt{3}x-5)$

Question: 41

Factorize:

Solution:

We have,

$5\sqrt{5}x^2+20x+3\sqrt{5}$

Now by using middle-term splitting, we get

= 5√5 * x * x + 15x + 5x + 3√5

= 5x (√5x + 3) + √5 (√5x + 3)

= (√5x + 3)(5x + √5)

Hence,

The given expression can be factorized as:

$(\sqrt{5}x+3)(5x+\sqrt{5})$

Question: 42

Factorize:

Solution:

We have,

$7\sqrt{2}x^2-10x-4\sqrt{2}$

Now by using middle-term splitting, we get

= 7√2 * x * x - 14x + 4x - 4√2

= 7√2x (x - √2) + 4 (x- √2)

= (x - √2) (7√2 x+4)

Hence,

The given expression can be factorized as:

$(x-\sqrt{2})(7\sqrt{2}x+4)$

Question: 43

Factorize:

Solution:

We have,

$6\sqrt{3}x^2-47x+5\sqrt{3}$

Now by using middle-term splitting, we get

= 6√3 * x * x - 45x - 2x + 5√3

$= 3\sqrt{3}x(2x - 5\sqrt{3}) - 1(2x - 5\sqrt{3})$

$= (2x - 5\sqrt{3})(3\sqrt{3}x - 1)$

Hence,

The given expression can be factorized as:

$(2x - 5\sqrt{3})(3\sqrt{3}x - 1)$

Question: 44

Factorize:

Solution:

We have,

$7x^2 + 2\sqrt{14}x + 2$

Now by using middle-term splitting, we get

$= 7*x*x + \sqrt{2} \times \sqrt{7}x + \sqrt{2} \times \sqrt{7}x + 2$

$= \sqrt{7}x(\sqrt{7}x + \sqrt{2}) + \sqrt{2}(\sqrt{7}x + \sqrt{2})$

$= (\sqrt{7}x + \sqrt{2})(\sqrt{7}x + \sqrt{2})$

$= (\sqrt{7}x + \sqrt{2})^2$

Hence,

The given expression can be factorized as:

$(\sqrt{7}x + \sqrt{2})^2$

Question: 45

Factorize:

Solution:

We have,

$2(x+y)^2 - 9(x+y) - 5$

Let $x + y = z$

Then,

$2(x + y)^2 - 9(x + y) - 5$

Now by using middle-term splitting, we get

$= 2z^2 - 9z - 5$

$= 2z^2 - 10z + z - 5$

$= 2z(z - 5) + 1(z - 5)$

$= (z - 5)(2z + 1)$

Now,

Replacing z by (x + y), we get

$2(x + y)^2 - 9(x + y) - 5$

$= [(x + y) - 5][2(x + y) + 1]$

$= (x + y - 5)(2x + 2y + 1)$

Hence,

The given expression can be factorized as:

$(x+y-5)(2x+2y+1)$

Question: 46

Factorize:

Solution:

We have,

$9(2a-b)^2-4(2a-b)-13$

Let 2a – b = c

Then,

$9\ (2a - b)^2 – 4\ (2a - b) - 13$

Now by using middle-term splitting, we get

$= 9c^2 – 4c - 13$

$= 9c^2 – 13c + 9c – 13$

= c (9c – 13) + 1 (9c – 13)

= (c + 1) (9c - 13)

Now,

Replacing c by (2a - b) - 13, we get

$9\ (2a – b)^2 – 4\ (2a – b) - 13$

= (2a – b + 1) [9 (2a – b) – 13]

= (2a – b + 1) (18a – 9b – 13)

Hence,

The given expression can be factorized as:

$(18a-9b-13)(2a-b+1)$

Question: 47

Factorize:

Solution:

We have,

$7(x-2y)^2-25(x-2y)+12$

Let x - 2y = z

Then,

$7\ (x - 2y)^2 – 25\ (x - 2y) + 12$

Now by using middle-term splitting, we get

$= 7z^2 - 25z + 12$

$= 7z^2 – 21z - 4z + 12$

= 7z (z – 3) - 4 (z – 3)

= (z – 3) (7z - 4)

Now,

Replacing z by (x - 2y), we get

$7 (x - 2y)^2 - 25 (x - 2y) + 12$

$= (x - 2y – 3) [7 (x - 2y) - 4]$

$= (x - 2y – 3) (7x - 14y - 4)$

Hence,

The given expression can be factorized as:

$(x-2y-3)(7x-14y-4)$

Question: 48

Factorize:

Solution:

We have,

$4x^4+7x^2-2$

Now by using middle-term splitting, we get

$= 4y^2 + 7y – 2$

$= 4y^2 + 8y – y – 2$

$= 4y (y + 2) – (y + 2)$

$= (y + 2) (4y – 1)$

Now,

Replacing y by x^2, we get

$4x^4 + 7x^2 – 2$

$= (x^2 + 2) (4x^2 – 1)$ [Therefore, $a^2 – b^2 = (a – b) (a + b)$]

$= (x^2 + 2) (2x + 1) (2x – 1)$

Hence,

The given expression can be factorized as:

$(x^2+2)(2x-1)(2x+1)$

Exercise : 2H

Question: 1

Expand:

Solution:

(i) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (a)^2 + (2b)^2 + (5c)^2 + 2(a) (2b) + 2 (2b) (5c) + 2 (5c) (a)$

$= a^2 + 4b^2 + 25c^2 + 4ab + 20bc + 10ac$

(ii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (2a)^2 + (-b)^2 + (c)^2 + 2(2a) (-b) + 2 (-b) (c) + 2 (c) (2a)$

$= 4a^2 + b^2 + c^2 - 4ab - 2bc + 4ac$

(iii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (a)^2 + (-2b)^2 + (-3c)^2 + 2(a)(-2b) + 2(-2b)(-3c) + 2(-3c)(a)$

$= a^2 + 4b^2 + 9c^2 - 4ab + 12bc - 6ac$

Question: 2

Expand

Solution:

(i) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (2a)^2 + (-5b)^2 + (-7c)^2 + 2(2a)(-5b) + 2(-5b)(-7c) + 2(-7c)(2a)$

$= 4a^2 + 25b^2 + 49c^2 - 20ab + 70bc - 28ac$

(ii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (-3a)^2 + (4b)^2 + (-5c)^2 + 2(-3a)(4b) + 2(4b)(-5c) + 2(-5c)(-3a)$

$= 9a^2 + 16b^2 + 25c^2 - 24ab - 40bc + 30ac$

(iii) We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Using this formula, we get

$= (1/2a)^2 + (-1/4b)^2 + (2)^2 + 2(1/2a)(-1/4b) + 2(-1/4b)(2) + 2(2)(1/2a)$

$= \frac{a^2}{4} + \frac{b^2}{16} + 4 - \frac{ab}{4} - b + 2a$

Question: 3

Factorize:

Solution:

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2za$

Using this formula, we get

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y – 4z)^2$

Question: 4

Factorize:

Solution:

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2za$

Using this formula, we get

$= (-3x)^2 + (4y)^2 + (2z)^2 + 2(-3x)(4y) + 2(4y)(2z) + 2(2z)(-3x)$

$= (-3x + 4y + 2z)^2$

Question: 5

Factorize:

Solution:

We know that,

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2za$

Using this formula, we get

$= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3z)(5x)$

$= (5x - 2y + 3z)^2$

Question: 6

Evaluate:

Solution:

(i) We know that,

$(a - b)^2 = a^2 - 2ab + b^2$

Using this formula, we get

$= (100 - 1)^2$

$= (100)^2 - 2(100)(1) + (1)^2$

$= 10000 - 200 + 1$

$= 9801$

(ii) We know that,

$(a - b)^2 = a^2 - 2ab + b^2$

Using this formula, we get

$= (1000 - 2)^2$

$= (1000)^2 - 2(1000)(2) + (2)^2$

$= 1000000 - 4000 + 4$

$= 996004$

Exercise : 2I

Question: 1

Expand:

Solution:

(i) We know that,

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Using this formula, we get

$= (3x)^3 + (2)^3 + 3 \times 3x \times 2(3x + 2)$

$= 27x^3 + 8 + 18x(3x + 2)$

$= 27x^3 + 8 + 54x^2 + 36x$

(ii) We know that,

$(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$

Using this formula, we get

$= (3a)^3 - (2b)^3 - 3 \times 3a \times 2b\ (3a - 2)$

$= 27a^3 – 8b^3 – 18ab\ (3a – 2b)$

$= 27a^3 – 8b^3 – 54a^2b + 36ab^2$

(iii) We know that,

$(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$

Using this formula, we get

$= (2/3x)^3 + (1)^3 + 3 \times 2/3x \times 1\ (2/3x + 1)$

$= 8/27x^3 + 1 + 2x\ (2/3x + 1)$

$= \frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$

Question: 2

Expand:

Solution:

(i) We know that,

$(a – b)^3 = a^3 – b^3 – 3ab\ (a – b)$

Using this formula, we get

$= (2x)^3 – (2/x)^3 – 3 \times 2x \times 2/x\ (2x – 2/x)$

$= 8x^3 – 8/x^3 – 12\ (2x – 2/x)$

$= 8x^3 - \frac{8}{x^3} - 24x + \frac{24}{x}$

(ii) We know that,

$(a – b)^3 = a^3 – b^3 – 3ab\ (a – b)$

Using this formula, we get

$= (3a)^3 – (1/4b)^3 + 3 \times 3a \times 1/4b\ (3a + 1/4b)$

$= 27a^3 + 1/64b^3 + 9a/4b\ (3a + 1/4b)$

$= 27a^3 + \frac{1}{64b^3} + \frac{27a^2}{4b} + \frac{9a}{16b^2}$

(iii) We know that,

$(a – b)^3 = a^3 – b^3 – 3ab\ (a – b)$

Using this formula, we get

$= (4/5x)^3 – (2)^3 - 3 \times 4x/5 \times 2\ (3a + 1/4b)$

$= 64/125a^3 – 8 - 24/5x\ (4/5x - 2)$

$= \frac{64}{125}x^3 - 8 - \frac{96}{25}x^2 + \frac{48}{5}x$

Question: 3

Evaluate:

Solution:

(i) We know that,

$(a - b)^3 = a^3 - b^3 - 3ab\ (a - b)$

Using this formula, we get

$= (100 - 5)^3$

$= (100)^3 - (5)^3 - 3 \times 100 \times 5\ (100 - 5)$

$= 1000000 - 125 - (1500 \times 95)$

$= 857375$

(ii) We know that,

$(a - b)^3 = a^3 - b^3 - 3ab\ (a - b)$

Using this formula, we get

$= (1000 - 1)^3$

$= (1000)^3 - (1)^3 - 3 \times 1000 \times 1\ (1000 - 1)$

$= 1000000000 - 1 - 3000\ (1000 - 1)$

$= 1000000000 - 1 - 3000 \times 1000 + 3000 \times 1$

$= 997002999$

Exercise : 2J

Question: 1

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)\ (a^2 - a \times b + b^2)$

Using this formula, we get

$= x^3 + 3^3$

$= (x + 3)\ (x^2 - 3x + 9)$

Question: 2

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)\ (a^2 - a \times b + b^2)$

Using this formula, we get

$= (2x)^3 + (3y)^3$

$= (2x + 3y)\ [(2x)^2 - (2x)\ (3y) + (3y)^2]$

$= (2x + 3y)\ (4x^2 - 6xy + 9y^2)$

Question: 3

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (7)^3 + (5b)^3$

$= (7 + 5b)[(7)^2 - (7)(5b) + (5b)^2]$

$= (7 + 5b)(49 - 35b + 25b^2)$

Question: 4

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (1)^3 + (4x)^3$

$= (1 + 4x)[(1)^2 - 1(4x) + (4x)^2]$

$= (1 + 4x)(1 - 4x + 16x^2)$

Question: 5

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (5a)^3 + (\frac{1}{2})^3$

$= (5a + \frac{1}{2})[(5a)^2 - 5a \times \frac{1}{2} + (\frac{1}{2})^2]$

$= (5a + \frac{1}{2})(25a^2 - \frac{5a}{2} + \frac{1}{4})$

Question: 6

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (6x)^3 + (\frac{1}{5})^3$

$= (6x + \frac{1}{5})[(6x)^2 - 6x \times \frac{1}{5} + (\frac{1}{5})^2]$

$= (6x + \frac{1}{5})(36x^2 - \frac{6x}{5} + \frac{1}{25})$

Question: 7

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= 2x(8x^3 + 27)$

$= 2x[(2x)^3 + (3)^3]$

$= 2x(2x + 3)[(2x)^2 - 2x(3) + 3^2]$

$= 2x(2x + 3)(4x^2 - 6x + 9)$

Question: 8

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= 7(a^3 + 8b^3)$

$= 7(a + 2b)(a^2 - a \times 2b + (2b)^2]$

$= 7(a + 2b)(a^2 - 2ab + 4b^2)$

Question: 9

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= x^2(x^3 + 1)$

$= x^2(x + 1)[(x^2) - x(1) + (1)^2]$

$= x^2(x + 1)(x^2 - x + 1)$

Question: 10

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (a)^3 + (0.2)^3$

$= (a + 0.2)[(a)^2 - a(0.2) + (0.2)^2]$

$= (a + 0.2)(a^2 - 0.2a + 0.04)$

Question: 11

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (x^2)^3 + (y^2)^3$

$= (x^2 + y^2)[(x^2)^2 - x^2(y^2) + (y^2)^2]$

$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)$

Question: 12

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= 2(a^3 + 8b^3) - 5(a + 2b)$

$= 2[(a)^3 + (2b)^3] - 5(a + 2b)$

$= 2(a + 2b)[(a)^2 - a(2b) + (2b)^2] - 5(a + 2b)$

$= (a + 2b)[2(a^2 - 2ab + 4b^2) - 5]$

Question: 13

Factorize:

Solution:

We know that,

$a^3 + b^3 = (a + b)(a^2 - a \times b + b^2)$

Using this formula, we get

$= (x)^3 - (8)^3$

$= (x - 8)[(x)^2 + x(8) + (8)^2]$

$= (x - 8)(x^2 + 8x + 64)$

Question: 14

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (4x)^3 - (7)^3$

$= (4x - 7)[(4x)^2 + 4x(7) + (7)^2)$

$= (4x - 7)(16x^2 + 28x + 49)$

Question: 15

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (1)^3 – (3x)^3$

$= (1 – 3x) [(1)^2 + 1 (3x) + (3x)^2)$

$= (1 – 3x)(1 + 3x + 9x^2)$

Question: 16

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (x)^3 – (5y)^3$

$= (x – 5y) [(x)^2 + x (5y) + (5y)^2$

$= (x – 5y)(x^2 + 5xy + 25y^2)$

Question: 17

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2+ a \times b + b^2)$

Using this formula, we get

$= (2x)^3 – (\frac{1}{3y})^3$

$= (2x - \frac{1}{3y}) [(2x)^2 + 2x \times \frac{1}{3y} + (\frac{1}{3y})^2]$

$= \left(2x-\frac{1}{3y}\right)\left(4x^2+\frac{2x}{3y}+\frac{1}{9y^2}\right)$

Question: 18

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (a)^3 – (0.4)^3$

$= (a – 0.4) [(a)^2 + a (0.4) + (0.4)^2]$

$= (a – 0.4)(a^2 + 0.4a + 0.16)$

Question: 19

Factorize:

Solution:

$(a + b)^3 – (2)^3$

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (a + b – 2) [(a + b)^2 + (a + b) 2 + (2)^2]$

$= (a + b – 2) [a^2 + b^2 + 2ab + 2 (a + b) + 4]$

Question: 20

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= (x^2)^3 – (9)^3$

$= (x^2 – 9) [(x^2)^2 + x^2 9 + (9)^2]$

$= (x^2 – 9) (x^4 + 9x^2 + 81)$

$= (x + 3) (x – 3) [(x^2 + 9)^2 – (3x)^2]$

$= (x + 3) (x – 3) (x^2 + 3x + 9) (x^2 – 3x + 9)$

Question: 21

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= [a + b – (a – b)] [(a + b)^2 + (a + b) (a – b) + (a – b)^2]$

$= (a + b – a + b) [a^2 + b^2 + 2ab + a^2 – b^2 + a^2 + b^2 – 2ab]$

$= 2b (3a^2 + b^2)$

Question: 22

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b)(a^2 + a \times b + b^2)$

Using this formula, we get

$= x (1 – 8y^3)$

$= x [(1)^3 – (2y)^3]$

$= x (1 – 2y) [(1)^2 + 1 (2y) + (2y)^2]$

$= x (1 – 2y) (1 + 2y + 4y^2)$

Question: 23

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= 4x (8x^3 - 125)$

$= 4x [(2x)^3 - (5)^3]$

$= 4x [(2x - 5) [(2x)^2 + 2x (5) + (5)^2]$

$= 4x (2x - 5) (4x^2 + 10x + 25)$

Question: 24

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= 3a^4b (a^3 - 27b^3)$

$= 3a^4b [(a)^3 - (3b)^3]$

$= 3a^4b (a - 3b) [(a)^2 + a (3b) + (3b)^2]$

$= 3a^4b (a - 3b) (a^2 + 3ab + 9b^2)$

Question: 25

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= a^3 - 1/a^3 - 2 (a - 1/a)$

$= (a - 1/a) (a^2 + a \times 1/a + 1/a^2) - 2 (a - 1/a)$

$= (a - 1/a) (a^2 + 1 + 1/a^2 - 2)$

$= \left(a - \frac{1}{a}\right)\left(a^2 - 1 + \frac{1}{a^2}\right)$

Question: 26

Factorize:

Solution:

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= 8a^3 - b^3 - 2x (2a - b)$

$= (2a)^3 - (b)^3 - 2x (2a - b)$

$= (2a - b) [(2a)^2 + 2a (b) + (b)^2] - 2x (2a - b)$

$= (2a – b) (4a^2 + 2ab + b^2) – 2x (2a – b)$

$= (2a – b) (4a^2 + 2ab + b^2 – 2x)$

Question: 27

Factorize:

Solution:

We know that,

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

Using this formula, we get

$= (a + b)^3 – 8$

$= (a + b)^3 – (2)^3$

We know that,

$a^3 - b^3 = (a - b) (a^2 + a \times b + b^2)$

Using this formula, we get

$= (a + b – 2) [(a+ b)^2 + 2 (a + b) + 4)]$

Exercise : 2K

Question: 1

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (5a)^3 + (b)^3 + (4c)^3 – 3 (5a) (b) (4c)$

$= (5a + b + 4c) [(5a)^2 + b^2 + (4c)^2 – (5a) (b) – (b) (4c) – (5a) (4c)]$

$= (5a + b + 4c) (25a^2 + b^2 + 16c^2 – 5ab – 4bc – 20ac)$

Question: 2

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (a)^3 + (2b)^3 + (4c)^3 – 3 (a) (2b) (4c)$

$= (a + 2b + 4c) (a^2 + 4b^2 + 16c^2 – 2ab – 8bc – 4ac)$

Question: 3

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (1)^3 + (b)^3 + (2c)^3 – 3 (1) (b) (2c)$

$= (1 + b + 2c) [(1)^2 + (b)^2 + (4c)^2 – (1) (b) – (2b) (c) – (2c) (1)]$

$= (1 + b + 2c) (1 + b^2 + 4c^2 – b – 2bc – 2c)$

Question: 4

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (6)^3 + (3b)^3 + (2c)^3 – 3 (6) (3b) (2c)$

$= (6 + 3b + 2c) [(6)^2 + (3b)^2 + (2c)^2 – (6) (3b) – (3b) (2c) – (2c) (6)]$

$= (6 + 3b + 2c) (36 + 9b^2 + 4c^2 – 18ab – 6bc – 12ac)$

Question: 5

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (3a)^3 + (-b)^3 + (2c)^3 – 3 (3a) (-b) (2c)$

$= [3a + (-b) + 2c] [(3a)^2 + (-b)^2 + (2c)^2 – (3a) (-b) – (-b) (2c) – (2c) (3a)]$

$= (3a - b + 2c) (9a^2 + b^2 + 4c^2+ 3ab + 2bc – 6ca)$

Question: 6

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (2a)^3 + (5b)^3 + (-4c)^3 – 3 (2a) (5b) (-4c)$

$= (2a + 5b - 4c) [(2a)^2 + (5b)^2 + (-4c)^2 – (2a) (5b) – (5b) (-4c) – (-4c) (2a)]$

$= (2a + 5b - 4c) (4a^2 + 25b^2 + 16c^2 – 10ab + 20bc + 8ca)$

Question: 7

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (2)^3 + (-3b)^3 + (-7c)^3 – 3 (2) (-3b) (-7c)$

$= (2 - 3b - 7c) [(2)^2 + (-3b)^2 + (-7c)^2 – (2) (-3b) – (-3b) (-7c) – (-7c) (2)]$

$= (2 - 3b - 7c) (4 + 9b^2 + 49c^2 + 6b – 21bc + 14c)$

Question: 8

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (5)^3 + (-2x)^3 + (-3y)^3 – 3 (5) (-2x) (-3y)$

$= (5 – 2x – 3y) [(5)^2 + (-2x)^2 + (-3y)^2 – (5) (-2x) – (-2x) (-3y) – (-3y) (5)]$

$= (5 – 2x – 3y) (25 + 4x^2 + 9y^2 + 10x – 6xy + 15y)$

Question: 9

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + (c)^3 – 3 (\sqrt{2}a) (2\sqrt{2}b) (c)$

$= (\sqrt{2}a + 2\sqrt{2}b + c) [(\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + (c)^2 – (\sqrt{2}a) (2\sqrt{2}b) – (2\sqrt{2}b) (c) – (c) (\sqrt{2}a)]$

$= (\sqrt{2}a + 2\sqrt{2}b + c) (2a^2 + 8b^2 + c^2 – 4ab – 2\sqrt{2}bc – \sqrt{2}ac)$

Question: 10

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

$= (x)^3 + (y)^3 + (4)^3 – 3 (x) (y) (4)$

$= (x + y + 4) [(x)^2 + (y)^2 + (4)^2 – (x) (y) – (y) (4) – (4) (x)]$

$= (x + y + 4) (x^2 + y^2 + 16 –xy – 4y – 4x)$

Question: 11

Factorize:

Solution:

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Using this formula, we get

Putting $(a – b) = x$, $(b – c) = y$ and $(c – a) = z$, we get

$(a - b)^3 + (b - c)^3 + (c - a)^3= x^3 + y^3 + z^3$

Where $(x + y + z) = (a - b) + (b - c) + (c - a) = 0$

$= 3xyz$ [Since, $(x + y + z) = 0$ so $(x^3 + y^3 + z^3) = 3xyz$]

$= 3 (a - b) (b - c) (c - a)$

Question: 12

Factorize:

Solution:

We know that,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

We have,

$(3a - 2b) (2b - 5c) + (5c - 3a) = 0$

So,

$(3a - 2b)^3 + (2b - 5c)^3 + (5c - 3a)^3 = 3 (3a - 2b) (2b - 5c) (5c - 3a)$

Question: 13

Factorize:

Solution:

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

$= [a (b - c)]^3 + [b (c - a)]^3 + [c (a - b)]^3$

Since,

$a (b - c) + b (c - a) + c (a - b) = ab - ac + bc - ba + ca - bc = 0$

So,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

$= 3a (b - c) b (c - a) c (a - b)$

$= 3abc (a - b) (b - c) (c - a)$

Question: 14

Factorize:

Solution:

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Since,

$(5a – 7b) + (9c – 5a) + (7b – 9c) = 5a – 7b + 9c – 5a + 7b – 9c = 0$

Therefore,

$(5a – 7b)^3 + (9c – 5a)^3 + (7b – 9c)^3 = 3 (5a – 7b) (9c – 5a) (7b – 9c)$

Question: 15

Find the pr

Solution:

We have,

$a^3 (b – c)^3 + b^3 (c – a)^3 + c^3 (a – b)^3$

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Using this, we get

$= [x + y + (-z)] [(x)^2 + (y)^2 + (-z)^2 – (x) (y) – (y) (-z) – (-z) (x)]$

$= x^3 + y^3 – z^3 + 3xyz$

Question: 16

Find the pr

Solution:

We have,

$a^3 (b – c)^3 + b^3 (c – a)^3 + c^3 (a – b)^3$

We know that,

$a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Using this, we get

$= [x + (-2y) + 3] [(x)^2 + (-2y)^2 + (3) – (x) (-2y) – (-2y) (3) – (3) (x)]$

$= (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)$

$= a^{\#} + b^3 + c^3 – 3abc$

Where,

$x = a, b = -2y$ and $c = 3$

$(x – 2y + 3) (x^2 + 4y^2 + 2xy – 3x + 6y + 9)$

$= (x)^3 + (-2y)^3 + (3)^3 – 3 (x) (-2y) (3)$

$= x^3 – 8y^3 + 27 + 18xy$

Question: 17

Find the pr

Solution:

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Using this, we get

$= [x + (-2y) + (-z)] [(x)^2 + (-2y)^2 + (-z)^2 - (x) (-2y) - (-2y) (-z) - (-z) (x)$

$= (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc$

Where,

$x = a$, $b = -2y$ and $c = -z$

$(x - 2y - z) (x^2 + 4y^2 + z^2 + 2xy + zx - 2yz)$

$= (x)^3 + (-2y)^3 + (-z)^3 - 3 (x) (-2y) (-z)$

$= x^3 - 8y^3 - z^3 - 6xyz$

Question: 18

If

Solution:

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Given,

$x + y + 4 = 0$

We have,

$(x^3 + y^3 - 12xy + 64)$

$= (x)^3 + (y)^3 + (4)^3 - 3 (x) (y) (4) = 0$

Question: 19

If

Solution:

We have,

$a^3 (b - c)^3 + b^3 (c - a)^3 + c^3 (a - b)^3$

We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)]$

Also,

If, $(x + y + z) = 0$

Then $(x^3 + y^3 + z^3) = 3xyz$

Given, $x = 2y + 6$

Or, $x - 2y - 6 = 0$

We have,

$(x^3 - 8y^3 - 36xy - 216)$

$= (x^3 - 8y^3 - 216 - 36xy)$

$= (x)^3 + (-2y)^3 + (-6)^3 - 3 (x) (-2y) (-6)$

$= (x - 2y - 6) [(x)^2 + (-2y)^2 + (-6)^2 - (x) (-2y) - (-2y) (-6) - (-6) (x)]$

$= (x - 2y - 6) (x^2 + 4y^2 + 36 + 2xy - 12y + 8x)$

$= 0 (x^2 + 4y^2 + 36 + 2xy - 12y + 6x)$

$= 0$

Exercise : CCE QUESTIONS

Question: 1

Which of the foll

Solution:

Polynomials in one variable are algebraic expressions that consist of terms having same variable all through

$\therefore$ since in the expression $\sqrt{2}x^2 - \sqrt{3}x + 6$ the only polynomial used is x.

Hence, option C is correct

Question: 2

Which of the foll

Solution:

*Note: *A polynomial is an expression having variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables.*

Now, $\sqrt{x} - 1$ and $\frac{x-1}{x+1}$ is not a polynomial because it does not contain integer power of the variable "x".

And, $x^2 - \frac{2}{x^2} + 5$ is not a polynomial because it contains a negative power of the variable, which is not the criteria for a polynomial.

$\therefore$ $x^2 + \frac{2x^{3/2}}{\sqrt{x}} + 6$ is a polynomial it contains only integral powers of the variable (x) i.e. x^2

Hence, option D is correct

Question: 3

Which of the foll

Solution:

*Note: *A polynomial is an expression having variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables*

$\sqrt[3]{y+4}$ and $\sqrt{y-3}$ is not a polynomial because it does not contain integer power of the variable "y"

And, $\frac{1}{\sqrt{y}}+7$ is not a polynomial because it contains a negative power of the variable, which is not the criteria for a polynomial.

∴ y is a polynomial as it follows the criteria of polynomial and all the other do not follow these criteria.

Hence, option C is correct

Question: 4

Which of the foll

Solution:

As we know that,

A polynomial is an expression having variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents of variables

Out of the 4 given options, we have:

$x-\frac{1}{x}+2$ and $\frac{1}{x}+5$ are not polynomials as they have negative exponent.

$\sqrt{x}+3$ is not a polynomial as it has non integral exponent.

And -4 can be written as $-4x^0$, thus it is a polynomial with integer power.

Thus, -4 is the correct option.

Question: 5

Which of the foll

Solution:

Out of the 4 given options, D is the correct option because all the other has negative exponential value which shows that they aren't a polynomial

Question: 6

Which of the foll

Solution:

A quadratic polynomial is a polynomial of degree 2 which means that it must have a variable with degree 2.

∴ $x^2 + 5x + 4$ is a quadratic polynomial

Hence, option D is correct

Question: 7

Which of the foll

Solution:

x + 1 is a linear polynomial because it has degree one which means that highest power of the variable must be 1

Hence, option B is correct

Question: 8

Which of the foll

Solution:

In algebra, a binomial is a polynomial which is the sum of two terms, each of which is a monomial.
It is the simplest kind of polynomial after the monomials

$\therefore x^2 + 4$ is a binomial

Hence, option B is correct

Question: 9

Solution:

$\sqrt{3}$ is a polynomial of degree 0 this is because it do not have any variable

Hence, option D is correct

Question: 10

Degree of the zer

Solution:

The degree of the zero polynomial is left undefined

Hence, option C is correct

Question: 11

Zero of the polyn

Solution:

We have,

p (x) = 2x + 3

So, zero of the given polynomial can be calculated as follows:

0 = 2x + 3

2x = - 3

$x = -\frac{3}{2}$

Hence, option B is correct

Question: 12

Zero of the polyn

Solution:

We have,

p (x) = 2 – 5x

So, zero of the given polynomial can be calculated as follows:

0 = 2 – 5x

5x = 2

$x = \frac{2}{5}$

Hence, option A is correct

Question: 13

Zero of the zero

Solution:

Zero of a Zero polynomial can be any real number.

Hence, option C is correct

Question: 14

If p(x) = x + 4,

Solution:

We have,

p (x) = x + 4

p (-x) = - x + 4

Then, the value of p (x) + (-x) and it can be calculated as follows:

p (x) + p (-x) = x + 4 – x + 4

= 4 + 4

= 8

Hence, option D is correct

Question: 15

If $\sqrt{2}$x + 1

$$\therefore p\left(2\sqrt{2}\right) = \left(2\sqrt{2}\right)^2 - \left(2\sqrt{2}\right) \times \left(2\sqrt{2}\right) + 1$$

= 8 – 8 + 1

= 1

Hence, option B is correct

Question: 16

The zeroes of the

Solution:

We have,

p (x) = x^2 + x - 6

So, zero of the given polynomial can be calculated as follows:

x^2 + x – 6 = 0

x^2 + 3x – 2x – 6 = 0

x (x + 3) – 2 (x + 3) = 0

(x + 3) (x – 2) = 0

Now, x + 3 = 0

x = - 3

And, x – 2 = 0

x = 2

Hence, the zeros of the given polynomial are -3 and 2

∴ Option C is correct

Question: 17

The zeroes of the

Solution:

We have,

$p(x) = 2x^2 + 5x - 3$

So, zero of the given polynomial can be calculated as follows:

$2x^2 + 5x - 3 = 0$

$2x^2 + 6x - x - 3 = 0$

$2x(x + 3) - 1(x + 3) = 0$

$(2x - 1)(x + 3) = 0$

Now, $(2x - 1) = 0$

$2x = 1$

$x = \frac{1}{2}$

Also, $(x + 3) = 0$

$x = -3$

Hence, zeros of the given polynomial are $\frac{1}{2}$ and -3

∴ Option B is correct

Question: 18

If $(x^2$

Solution:

We have, $(x^2 + kx - 3) = (x - 3)(x + 1)$

So, the value of k can be calculated as follows:

$(x^2 + kx - 3) = x^2 - 3x + x - 3$

$x^2 + kx - 3 = x^2 - 2x - 3$

On comparing the coefficients, we get,

$kx = -2x$

$k = -2$

Thus, the value of $k = -2$

Hence, option B is correct

Question: 19

If $(x + 1)$ is a f

Solution:

Let $p(x) = 2x^2 + kx$

It is given that, $(x + 1)$ is a factor of $(2x^2 + kx)$

Thus, $x = -1$ is a factor of $(2x^2 + kx)$

∴ $p(-1) = 0$

$2(-1)^2 + k(-1) = 0$

$2 - k = 0$

$k = 2$

Thus, the value of $k = 2$

Hence, option C is correct

Question: 20

The coefficient o

Solution:

We have,

$2x^2 – 4x^4 + 5x^2 – x^5 + 3$

From the given polynomial,

The highest power of x = 5

Coefficient of $x^5 = - 1$

Hence, option D is correct

Question: 21

When (x^{31}

Solution:

Let, p (x) = $(x^{31} + 31)$

And, x + 1 = 0

x = - 1

It is given that, (x + 1) is a factor of p (x) so the remainder is equal to p (-1)

∴ p (-1) = $(-1)^{31} + 31$

= - 1 + 31

= 30

Hence, option C is correct

Question: 22

When p (x) = x

Solution:

We have,

$x^3 – ax^2 + x$

Let, p (x) = $x^3 – ax^2 + x$

And, x - a = 0

x = a

It is given that, (x - a) is a factor of p (x) so the remainder is equal to p (a)

∴ p (a) = $(a)^3 – a (a)^2 + a$

= $a^3 – a^3 + a$

= a

Hence, option B is correct

Question: 23

When p (x) = (x

~~**Solution:**~~

~~We have,~~

$\cancel{(x^3 + ax^2 + 2x + a)}$

Let, $p(x) = (x^3 + ax^2 + 2x + a)$

And, $x + a = 0$

$x = -a$

It is given that, $(x + a)$ is a factor of $p(x)$ so the remainder is equal to $p(-a)$

$\therefore p(-a) = (-a)^3 + a(-a)^2 + 2(-a) + a$

$= -a^3 + a^3 - 2a + a$

$= -a$

Hence, option C is correct

Question: 24

When $p(x) = x$

Solution:

We have,

$x^4 + 2x^3 - 3x^2 + x - 1$

Let, $p(x) = x^4 + 2x^3 - 3x^2 + x - 1$

And, $x - 2 = 0$

$x = 2$

It is given that, $(x - 2)$ is a factor of $p(x)$ so the remainder is equal to $p(2)$

$\therefore p(2) = (2)^4 + 2(2)^3 - 3(2)^2 + 2 - 1$

$= 16 + 16 - 12 + 2 - 1$

$= 34 - 13$

$= 21$

Hence, option D is correct

Question: 25

When $p(x) = x$

Solution:

We have,

$x^3 - 3x^2 + 4x + 32$

Let, $p(x) = x^3 - 3x^2 + 4x + 32$

And, $x + 2 = 0$

$x = -2$

It is given that, $(x + 2)$ is a factor of $p(x)$ so the remainder is equal to $p(-2)$

$\therefore p(-2) = (-2)^3 - 3(-2)^2 + 4(-2) + 32$

$= -8 - 12 - 8 + 32$

$= -28 + 32$

$= 4$

Hence, option D is correct

Question: 26

When $p(x) = 4x$

~~**Solution:**~~

~~We have,~~

~~$4x^3 - 12x^2 + 11x - 5$~~

~~Let, p (x) = $4x^3 - 12x^2 + 11x - 5$~~

~~And, $(2x - 1) = 0$~~

~~$2x = 1$~~

~~$x = \frac{1}{2}$~~

~~It is given that, (2x - 1) is a factor of p (x) so the remainder is equal to p($\frac{1}{2}$)~~

~~$\therefore$ p (2) = $4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 11\left(\frac{1}{2}\right) - 5$~~

~~$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + 11 \times \frac{1}{2} - 5$~~

~~$= \frac{1}{2} - 3 + \frac{11}{2} - 5$~~

~~$= \frac{12}{2} - 8$~~

~~$= 6 - 8$~~

~~$= -2$~~

~~Hence, option C is correct~~

~~**Question: 27**~~

~~(x +1) is a facto~~

~~**Solution:**~~

~~We have, $(x + 1) = 0$~~

~~$x = -1$~~

~~Firstly, putting $(x = -1)$ in $x^3 - 2x^2 + x + 2$ we get:~~

~~$= (-1)^3 - 2(-1)^2 + (-1) + 2$~~

~~$= -1 - 2 - 1 + 2$~~

~~$= -2$~~

~~$\therefore$ (x + 1) is not a factor of $x^3 - 2x^2 + x + 2$~~

~~Secondly, putting $(x = -1)$ in $x^3 + 2x^2 + x - 2$ we get:~~

~~$= (-1)^3 + 2(-1)^2 + (-1) - 2$~~

~~$= -1 + 2 - 1 - 2$~~

~~$= -2$~~

~~$\therefore$ (x + 1) is not a factor of $x^3 + 2x^2 + x - 2$~~

~~Thirdly, putting $(x = -1)$ in $x^3 + 2x^2 - x - 2$ we get:~~

~~$= (-1)^3 + 2(-1)^2 - (-1) - 2$~~

~~$= -1 + 2 + 1 - 2$~~

~~$= 0$~~

~~Hence, (x + 1) is a factor of $x^3 + 2x^2 + x - 2$~~

~~Thus, option C is correct~~

Question: 28

$4x^2 +$

Solution:

We have,

$4x^2 + 4x - 3$

$= 4x^2 - 2x + 6x - 3$

$= 2x\,(2x - 1) + 3\,(2x - 1)$

$= (2x - 1)\,(2x + 3)$

Hence, option D is correct

Question: 29

$6x^2 +$

Solution:

We have,

$6x^2 + 17x + 5$

$= 6x^2 + 2x + 15x + 5$

$= 2x\,(3x + 1) + 5\,(3x + 1)$

$= (2x + 5)\,(3x + 1)$

Hence, option D is correct

Question: 30

$x^2 - 4$

Solution:

We have,

$x^2 - 4x - 21$

$= x^2 + 3x - 7x - 21$

$= x\,(x + 3) - 7\,(x + 3)$

$= (x + 3)\,(x - 7)$

Hence, option C is correct

Question: 31

If $(x + 5)$ is a f

Solution:

We have,

$p\,(x) = x^3 - 20x + 5k$

It is given in the question that, $(x + 5)$ is a factor of $p\,(x)$ so:

$p\,(-5) = 0$

$(-5)^3 - 20\,(-5) + 5\,k = 0$

$-125 + 100 + 5k = 0$

$-25 + 5k = 0$

$5k = 25$

$k = \frac{25}{5}$

$k = 5$

Hence, option B is correct

Question: 32

$3x^3 +$

Solution:

We have,

$3x^3 + 2x^2 + 3x + 2$

$= x^2 (3x + 2) + 1 (3x + 2)$

$= (x^2 + 1) (3x + 2)$

Hence, option D is correct

Question: 33

If $\frac{x}{y} + \frac{y}{x} = -1$

$\frac{x^2 + y^2}{xy} = -1$

$x^2 + y^2 = -xy$

$x^2 + y^2 + xy = 0$

$\therefore$ Value of $(x^3 - y^3) = (x - y) (x^2 + y^2 + xy)$

$= (x - y) \times 0$

$= 0$

Hence, option C is correct

Question: 34

If $a + b + c = 0$,

Solution:

The correct answer is D

It is given that: $a + b + c = 0$

We know,

$(a + b + c)^3 = a^3 + b^3 + c^3 + 2(ab + bc + ac)(a + b + c)$

Put $a + b + c = 0$ in the above equation we get,

Then, $a^3 + b^3 + c^3 = 3\ abc$

Hence, option D is correct

Question: 35

$(x + 2)$ and $(x -$

Solution:

We have,

$(x^3 + 10x^2mx + n)$

Let, $p (x) = x^3 + 10x^2 + mx + n$

It is given in the question that $(x + 2)$ and $(x - 1)$ are the factors of $p (x)$

$\therefore$ p (-2) = 0

$(-2)^3 + 10(-2)^2 + m(-2) + n = 0$

$-8 + 40 - 2m + n = 0$

$32 - 2m + n = 0$

$2m - n - 32 = 0$ ....(i)

And,

p (1) = 0

$(1)^3 + 10(1)^2 + m(1)\ n$

$1 + 10 + m + n = 0$

$11 + m + n = 0$

$m + n + 11 = 0$ .... (ii)

Now, adding equation (i) and (ii) we get

$2m - n - 32 + m + n + 11 = 0$

$3m - 21 = 0$

$3m = 21$

$m = \frac{21}{3}$

$m = 7$

Now, putting the value of m in (ii) we get:

$7 + n + 11 = 0$

$18 + n = 0$

$n = -18$

$\therefore$ the value of m is 7 and that of n is -18

Hence, option B is correct

Question: 36

The value of (369

Solution:

We have,

$(369)^2 - (368)^2$

We know that,

$(a^2 - b^2) = (a + b)(a - b)$

Using this identity, we get:

$(369)^2 - (368)^2 = (369 + 368)(369 - 368)$

$= 737 \times 1$

$= 737$

Hence, option D is correct

Question: 37

104 x 96 =?

Solution:

We have,

$104 \times 96 = (100 + 4)(100 - 4)$

$= (100)^2 - (4)^2$

$= 10000 - 16$

$= 9984$

Hence, option B is correct

Question: 38

$4a^2 +$

Solution:

We have,

$4a^2 + b^2 + 4ab + 8a + 4b + 4$

We know that,

$x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x + y + z)^2$

$= (2a)^2 + (b)^2 + (2)^2 + 2(2a)(b) + 2(b)(2) + 2 \times 2(2a)$

$= (2a + b + 2)^2$

Hence, option A is correct

Question: 39

The coefficient o

Solution:

The coefficient of x in the expansion of $(x+3)^2$ can be calculated as follows:

$(x + 3)^3 = x^3 + (3)^3 + 3(x)(3)(x + 3)$

$= x^3 + 27 + 9x(x + 3)$

$= x^3 + 27 + 9x^2 + 27x$

$\therefore$ Coefficient of x is 27

Hence, option D is correct

Question: 40

If $a + b + c = 0$,

Solution:

It is given in the question that,

$a + b + c = 0$

So, $\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right) = \left(\frac{a^3 + b^3 + c^3}{abc}\right)$

$= \frac{3abc}{abc}$

$= 3$

Hence, option D is correct

Question: 41

If $x + y + z = 9$

Solution:

It is given that,

$x + y + z = 9$

And, $xy + yz + zx = 23$

As we know that,

$(x + y + z)^2 = (x^2 + y^2 + z^2 + 2xy + 2yz + 2zx)$

$\therefore (9)^2 = [x^2 + y^2 + z^2 + 2 (xy + yz + zx)]$

$x^2 + y^2 + z^2 = 81 - 2 \times 23$

$x^2 + y^2 + z^2 = 81 - 46$

$x^2 + y^2 + z^2 = 35$

We also know that:

$(x^3 + y^3 + z^3 - 3xyz) = (x + y + z) [x^2 + y^2 + z^2 - (xy + yz + zx)]$

$= 9 (35 - 23)$

$= 9 \times 12$

$= 108$

Hence, option A is correct

Question: 42

If $(x^{100}$

Solution:

Let $p (x) = x^{100} + 2x^{99} + k$

It is given in the question that, $(x + 1)$ is divisible by $(x + 1)$

So, $p (-1) = 0$

$(-1)^{100} + 2 (-1)^{99} + k = 0$

$1 + 2 (-1) + k = 0$

$1 - 2 + k = 0$

$-1 + k = 0$

$k = 1$

Thus, the value of $k = 1$

Hence, option A is correct

Question: 43

In a polynomial i

Solution:

We know that,

In any polynomial in x, the indices of x must be a non-negative integer

Hence, option C is correct

Question: 44

For what value of

Solution:

We have,

$p(x) = 2x^3 - kx^2 + 3x + 10$

It is given in the question that $(x + 2)$ is exactly divisible by $p(x)$

$\therefore p(-2) = 0$

$2(-2)^3 - k(-2)^2 + 3(-2) + 10 = 0$

$2 \times (-8) - k \times (4) - 6 + 10 = 0$

$-16 - 4k - 6 + 10 = 0$

$-22 - 4k + 10 = 0$

$-12 - 4k = 0$

$-12 = 4k$

$k = -\frac{12}{4}$

$k = -3$

Hence, option D is correct

Question: 45

207 x 193 =?

Solution:

We have,

$207 \times 193 = (200 + 7)(200 - 7)$

$= (200)^2 - (7)^2$

$= 40000 - 49$

$= 39951$

Hence, option B is correct

Question: 46

305 x 308 =?

Solution:

We have,

$305 \times 308 = 305 \times (300 + 8)$

$= 305 \times 300 + 305 \times 8$

$= 91500 + 2440$

$= 93940$

Hence, option C is correct

Question: 47

The zeroes of the

Solution:

We have,

$p(x) = x^2 - 3x$

$\therefore$ Zeros of $p(x)$ are:

$p(x) = 0$

$x^2 - 3x = 0$

$x (x – 3) = 0$

So, $x = 0$

And, $x – 3 = 0$

$x = 3$

Thus, zeros of the given polynomial are 0 and 3

Hence, option B is correct

Question: 48

The zeroes of the

Solution:

We have,

$p (x) = 3x^2 – 1$

$\therefore$ Zeros of p (x) are:

$p (x) = 0$

$3x^2 – 1 = 0$

$3x^2 = 1$

So, $x^2 = \frac{1}{3}$

And, $x = \pm\sqrt{\frac{1}{3}}$

Thus, zeros of the given polynomial are $\sqrt{\frac{1}{3}}$ and $\sqrt{\frac{1}{3}}$

Hence, option D is correct

Question: 49

The question cons

Solution:

The correct answer is: (a)

We have,

$p (x) = x^2 + kx + 1$

$(x – 1)$ is a factor of p (x)

$\therefore p (1) = 0$

$(1)^2 + k (1) + 1 = 0$

$1 + k + 1 = 0$

$2 + k = 0$

$k = – 2$

Hence, both Assertion (A) and Reason (R) are true also Reason (R) is the correct explanation of Assertion (A)

$\therefore$ Option A is correct

Question: 50

The question cons

Solution:

We have,

$p(x) = x^3 - ax^2 + 6x - a$

$(x - a)$ is divided by $p(x)$

$\therefore p(a) = (a)^3 - a(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a$

$= 5a$

Hence, both Assertion (A) and Reason (R) are true also Reason (R) is the correct explanation of Assertion (A)

∴ Option A is correct

Question: 51

The question cons

Solution:

The correct answer is: (B)

We have,

$p(x) = x^3 - 2x + 3k$

$(x - 2)$ is a factor of $p(x)$

$\therefore p(2) = 0$

$(2)^3 - 2(2) + 3k = 0$

$8 - 4 + 3k = 0$

$4 + 3k = 0$

$3k = -4$

$k = -\frac{4}{3}$

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

∴ Option B is correct

Question: 52

The question cons

Solution:

The correct answer is: (a)

We have,

$(25)^3 + (-16)^3 + (-9)^3$

Since, $25 - 16 - 9 = 25 - 25$

$= 0$

$\therefore 3(-25)(-16)(-9)$

$= 75 \times 144$

$= 10800$

Hence, both Assertion (A) and Reason (R) are true also Reason (R) is the correct explanation of Assertion (A)

∴ Option A is correct

Question: 53

~~Match the followi~~

~~**Solution:**~~

~~The correct match for the above is as follows:~~

Column I	**Column II**
(a) If p (x) = $x^3 - 2x^2 + 3x + 1$ is divided by (x + 1), then remainder = ……	(r) – 1
(b) If (2x – 1) is a factor of q (x) = $x^3 + 2x^2 + 3x + k$ then k = ……	(p) $\frac{3}{8}$
(c) The degree of the constant polynomial (-5) is = ……	(s) 0
(d) When $x^{51} + 51$ is divided by (x + 1), the remainder is = ……	(q) 50

~~Hence, the correct answer is:~~

~~(a) – (r)~~

~~(b) – (p)~~

~~(c) – (s)~~

~~(d) – (q)~~

~~**Question: 54**~~

~~Match the followi~~

~~**Solution:**~~

~~The correct match for the above is as follows:~~

Column I	Column II
(a) If p (x) = $81x^4 + 54x^3 - 9x^2 - 3x + 2$ is divided by (3x + 2), then remainder = ...	(r) 0
(b) p (x) = $ax^3 + 3x^2 - 3$ and q (x) = $2ax^3 - 5x + a$ divided by (x – 4) leave the same remainder. Then, a = ...	(p) 1
(c) If p (x) = $7x^2 - 4\sqrt{2}x$ + c is completely divisible by (x - $\sqrt{2}$), then c = ...	(s) – 6
(d) If q (x) = $2x^3 + bx^2 + 11x + b + 3$ is divisible by (2x – 1), then b = ...	(q) – 7

~~Hence, the correct answer is:~~

~~(a) – (r)~~

~~(b) – (p)~~

~~(c) – (s)~~

~~(d) – (q)~~

Exercise : ~~FORMATIVE ASSESSMENT (UNIT TEST)~~

~~Question: 1~~

~~Let~~

$\cancel{\therefore p\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 + 8\left(\frac{1}{2}\right)^2 - 17\left(\frac{1}{2}\right) + 10}$

$\cancel{= 4 \times \frac{1}{8} + 8 \times \frac{1}{4} - 17 \times \frac{1}{2} + 10}$

$\cancel{= \frac{1}{2} + 2 - \frac{17}{2} + 10}$

$\cancel{= \frac{1-17}{2} + 12}$

$\cancel{= -\frac{16}{2} + 12}$

$\cancel{= -8 + 12}$

$\cancel{= 4}$

~~Hence, remainder = 4~~

~~**Question: 3**~~

~~If (x – 2) is a f~~

~~**Solution:**~~

~~It is given in the question that $(x - 2)$ is a factor of $2x^3 - 7x^2 + 11x + 5a$~~

~~Let $f(x) = 2x^3 - 7x^2 + 11x + 5a$~~

~~Now, $x - 2 = 0$~~

~~$x = 2$~~

~~$\therefore f(2) = 0$~~

~~$2(2)^3 - 7(2)^2 + 11(2) + 5a = 0$~~

~~$16 - 28 + 22 + 5a = 0$~~

~~$38 - 28 + 5a = 0$~~

~~$10 + 5a = 0$~~

~~$5a = -10$~~

~~$a = -\frac{10}{5}$~~

~~$a = -2$~~

~~Hence, the value of $a = -2$~~

~~**Question: 4**~~

~~For what value of~~

~~**Solution:**~~

~~It is given in the question that $(x + 2)$ is a factor of $x^3 - 2mx^2 + 16$~~

~~$p(x) = x^3 - 2mx^2 + 16$~~

~~Now, $x + 2 = 0$~~

~~$x = -2$~~

~~$\therefore f(-2) = 0$~~

~~$(-2)^3 - 2m(-2)^2 + 16 = 0$~~

~~$-8 - 8m + 16 = 0$~~

~~$-8 + 8m = 0$~~

~~$-8 = -8m$~~

~~$m = \frac{8}{8}$~~

~~$m = 1$~~

~~Hence, the value of $m = 1$~~

~~**Question: 5**~~

~~If $(a + b + c) =$~~

~~**Solution:**~~

~~It is given in the question that,~~

~~$(a + b + c) = 8$~~

~~And, $(ab + bc + ca) = 19$~~

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$

$\therefore (8)^2 = a^2 + b^2 + c^2 + 2 \times 19$

$64 = a^2 + b^2 + c^2 + 38$

$64 - 38 = a^2 + b^2 + c^2$

$a^2 + b^2 + c^2 = 26$

Question: 6

Expand: (3a + 4b

Solution:

We have,

$(3a + 4b + 5c)^2$

We know that,

$(a + b + c)^2 = a^2 + y^2 + c^2 + 2 (xy + yz + zx)$

$\therefore (3a + 4b + 5c)^2 = (9a^2 + 16b^2 + 25c^2 + 2 \times 3a \times 4b + 2 \times 4b \times 5c + 2 \times 5c \times 3a)$

$= 9a^2 + 16b^2 + 25c^2 + 24ab + 40bc + 30ac$

Question: 7

Expand: (3x + 2)<

Solution:

We have,

$(3x + 2)^2$

We know that,

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$

$\therefore (3x + 2)^2 = (3x)^3 + (2)^3 + 3 (3x) (2) (3x + 2)$

$= 27x^3 + 8 + 18x (3x + 2)$

$= 27x^3 + 8 + 54x^2 + 36x$

$= 27x^3 + 54x^2 + 36x + 8$

Question: 8

Evaluate: {(28)

Solution:

We have,

$[(28)^3 + (-15)^3 + (-13)^3]$

Now putting a = 28, b = - 15 and c = - 13

Now, $a + b + c = 28 - 15 - 13$

$= 28 - 28$

$= 0$

So, $x^3 + y^3 + z^3 = 3xyz$

$(28)^3 + (-15)^3 + (-13)^3 = 3 \times (28) \times (-15) \times (-13)$

$= 84 \times 195$

= 16380

Question: 9

If (x^{60}

Solution:

It is given in the question that (x + 1) is divided by (x^{60} + 60)

Let f (x) = x^{60} + 60

And, (x + 1) = 0

x = - 1

∴ f (x) = x^{60} + 60

f (-1) = $(-1)^{60}$ + 60

= 1 + 60

= 61

Hence, remainder = 61

Thus, option C is correct

Question: 10

One of the factor

Solution:

We have,

$(36x^2 – 1) + (1 + 6x)^2$

$= [(6x)^2 – (1)^2] + (1 + 6x)^2$

= (6x – 1) (6x + 1) + (6x + 1) (6x + 1)

= (6x + 1) (6x – 1 + 1 + 6x)

= (6x + 1) (12x)

∴ (6x + 1) is a factor of $(36x^2 – 1) + (6x + 1)^2$

Hence, option B is correct

Question: 11

If

Solution:

It is given in the question that,

$\frac{a}{b} + \frac{b}{a} = -1$

$\frac{a \times a + b \times b}{ab} = -1$

$a^2 + b^2 = - ab$

$a^2 + b^2 + ab = 0$ (i)

We know that,

$a^3 – b^3 = (a – b) (a^2 + b^2 + ab)$

From (i), we have $a^2 + b^2 + ab = 0$

∴ $a^3 – b^3 = (a – b) \times 0$

$a^3 - b^3 = 0$

Hence, option D is correct

Question: 12

The coefficient o

Solution:

We have,

$(x + 5)^3 = x^3 + (5)^3 + 3 \times x \times 5\ (x + 5)$

$= x^3 + 125 + 15x\ (x + 5)$

$= x^3 + 125 + 15x^2 + 75x$

$\therefore$ Coefficient of x = 75

Hence, option D is correct

Question: 13

Solution:

We know that,

$\sqrt{5}$ is a polynomial of degree 0

Hence, option C is correct

Question: 14

One of the zeroes

Solution:

Let $f(x) = 2x^2 + 7x - 4$

$= 2x^2 + 8x - x - 4$

$= 2x\ (x + 4) - 1\ (x + 4)$

$= (2x - 1)\ (x + 4)$

So, $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

And, $x + 4 = 0$

$x = -4$

Thus, one of zero of the given polynomial is $\frac{1}{2}$

Hence, option B is correct

Question: 15

Zero of the zero

Solution:

We know that,

The zero of the zero polynomial is not defined

Hence, option D is correct

Question: 16

If (x + 1) and (x

Solution:

We have,

p (x) = $ax^3 + x^2 - 2x + b$

It is given in the question that,

(x + 1) and (x – 1) are the factors of p (x)

∴ p (-1) = p (1)

So, p (-1) = $a(-1)^3 + (-1)^2 - 2(-1) + b$

0 = - a + 1 + 2 + b

0 = – a + 3 + b (i)

Also, p (1) = $a(1)^3 + (1)^2 - 2(1) + b$

0 = a + 1 – 2 + b

0 = a + b – 1 (ii)

As, p (-1) = p (1)

-a + 3 + b = a + b -1

-2a = - 4

a = 2

Now, putting the value of a in (ii), we get:

2 + b – 1 = 0

1 + b = 0

b = - 1

Hence, the value of a is 2 and that of b is -1

Question: 17

If (x + 2) is a f

Solution:

We have,

p (x) = $ax^3 + bx^2 + x - 6$

It is given in the question that, (x + 1) is a factor of p (x)

x + 2 = 0

x = – 2

∴ f (-2) = 0

$a(-2)^3 + b(-2)^2 + (-2) - 6 = 0$

-8a + 4b - 2 - 6 = 0

-8a + 4b – 8 = 0

-4 (2a – b + 2) = 0

2a – b + 2 = 0 (i)

Also, it is given in the question that when p (x) is divided by (x – 2) then it leaves a remainder 4

∴ p (2) = 4

$a(2)^3 + b(2)^2 + (2) - 6 = 4$

~~$8a + 4b + 2 - 6 = 4$~~

~~$8a + 4b - 4 - 4 = 0$~~

~~$4 (2a + b - 2) = 0$~~

~~$2a + b - 2 = 0$ (ii)~~

~~Now, adding (i) and (ii) we get:~~

~~$2a - b + 2 + 2a + b - 2 = 0$~~

~~$4a = 0$~~

~~$a = 0$~~

~~Putting the value of a in (ii), we get:~~

~~$2 (0) + b - 2 = 0$~~

~~$b - 2 = 0$~~

~~$b = 2$~~

~~Hence, it is proved that the value of a is 0 and that of b is 2~~

~~Question: 18~~

~~The expanded form~~

~~Solution:~~

~~We have,~~

~~$(3x - 5)^3 = (3x)^3 - (5)^3 - 3 (3x) (5) (3x - 5)$~~

~~$= 27x^3 - 125 - 45x (3x - 5)$~~

~~$= 27x^3 - 125 - 135x^2 + 225x$~~

~~$= 27x^3 - 135x^2 + 225x - 125$~~

~~Hence, option D is correct~~

~~Question: 19~~

~~If $a + b + c = 5$~~

~~Solution:~~

~~It is given in the question that,~~

~~$a + b + c = 5$~~

~~And, $ab + bc + ca = 0$~~

~~We know that,~~

~~$(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$~~

~~Putting the given values, we get:~~

~~$(5)^2 = a^2 + b^2 + c^2 + 20$~~

~~$25 - 20 = a^2 + b^2 + c^2$~~

~~$a^2 + b^2 + c^2 = 5$ (i)~~

~~Also, we know that~~

~~$(a^3 + b^3 + c^3 - 3abc) = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$~~

~~$= (a + b + c) [(a^2 + b^2 + c^2) - (ab + bc + ca)]$~~

~~Now, putting the values we get:~~

~~$= (5) \times (5 - 10)$~~

~~$= 5 \times (-5)$~~

~~$= -25$~~

~~Hence, it is proved that~~

~~$(a^3 + b^3 + c^3 - 3abc) = -25$~~

~~**Question: 20**~~

~~If p (x) = 2x~~

~~**Solution:**~~

~~We have,~~

~~$p(x) = 2x^3 + ax^2 + 3x - 5$~~

~~$q(x) = x^3 + x^2 - 4x + a$~~

~~It is given in the question that, when p (x) and q (x) is divided by (x – 2) it leaves same remainder~~

~~$\therefore p(2) = q(2)$~~

~~$2(2)^3 + a(2)^2 + 3(2) - 5 = (2)^3 + (2)^2 - 4(2) + a$~~

~~$2 \times 8 + a \times 4 + 3 \times 2 - 5 = 8 + 4 - 4 \times 2 + a$~~

~~$16 + 4a + 6 - 5 = 12 - 8 + a$~~

~~$16 + 4a + 1 = 4 + a$~~

~~$4a - a = 4 - 17$~~

~~$3a = -13$~~

~~$a = -\frac{13}{3}$~~

~~Hence proved~~