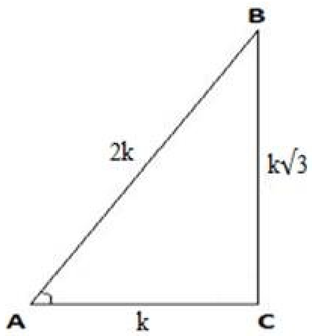
Chapter : 5. TRIGONOMETRIC RATIOS

Exercise : 5

Question: 1

If We have, $\sin\theta = \frac{\sqrt{3}}{2} = \frac{Perependicular}{Hypotenuse}$ Let Perpendicular = √3 kand hypotenuse = 2kwhere 'k' is some integer.



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$(2k)^2 = (k\sqrt{3})^2 + AC^2$

$4k^2 = 3k^2 + AC^2$

$AC^2 = (4 - 3)k^2$

$AC^2 = k^2$

→ AC = k, for some number k

Hence, the trigonometric ratios for the given θ are:

$\sin\theta = \frac{k\sqrt{3}}{2k} = \frac{\sqrt{3}}{2}$

cosθ = AC/AB = k/(2k) = 1/2

tanθ = BC/AC = sinθ /cosθ = (k√3)/k = √3

cotθ = 1/tanθ = AC/BC = k/(k√3) = 1/√3

cosecθ = 1/sinθ = AB/BC = (2k)/(k√3) = 2/√3

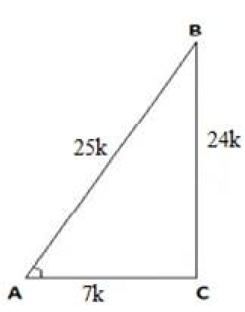
secθ = 1/cosθ = AB/AC = (2k)/k = 2

Question: 2

If cosθ = 7/25, f

Solution:

We have, cosθ = (7k)/(25k) = base/hypotenuse (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= (25k)^2 = BC^2 + (7k)^2$ (for some value of k)

$= 625k^2 = BC^2 + 49k^2$

$= BC^2 = 576k^2$

$= BC^2 = (24k)^2$

$\rightarrow BC = 24k$

Hence, the trigonometric ratios of the given θ are:

$\sin\theta = BC/AB = (24k)/(25k) = 24/25$

$\cos\theta = 7/25$

$\tan\theta = BC/AC = \sin\theta / \cos\theta = 24/7$

$\cot\theta = AC/BC = 1/\tan\theta = 7/24$

$\operatorname{cosec}\theta = AB/BC = 1/\sin\theta = 25/24$

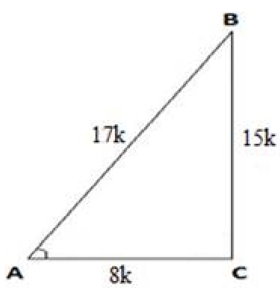
$\sec\theta = AB/AC = 1/\cos\theta = 25/7$

Question: 3

If $\tan\theta = 15/8$, f

Solution:

We have, $\tan\theta = 15k/8k$ = perpendicular/base (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$AB^2 = (15k)^2 + (8k)^2$

$AB^2 = 225k^2 + 64k^2$

$AB^2 = 289k^2 = (17k)^2$

$\rightarrow AB = 17k$

Hence, the trignometeric ratios for the given θ are:

$\sin\theta = BC/AB = (15k)/(17k) = 15/17$

$\cos\theta = AC/AB = (8k)/(17k) = 8/17$

$\tan\theta = 15/8$

$\cot\theta = AC/BC = 1/\tan\theta = 8/15$

$\operatorname{cosec}\theta = AB/BC = 1/\sin\theta = 17/15$

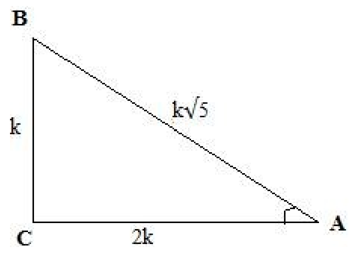
$\sec\theta = AB/AC = 1/\cos\theta = 17/8$

Question: 4

If cotθ = 2, find

Solution:

We have, cotθ = 2k/k = base/perpendicular (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$AB^2 = (k)^2 + (2k)^2$

$AB^2 = k^2 + 4k^2$

$AB^2 = 5k^2 = (k\sqrt{5})^2$

$\rightarrow AB = k\sqrt{5}$

Hence, the trignometeric ratios for the given θ are:

$\sin\theta = BC/AB = k/(k\sqrt{5}) = 1/\sqrt{5}$

$\cos\theta = AC/AB = (2k)/(k\sqrt{5}) = 2/\sqrt{5}$

$\tan\theta = BC/AC = \sin\theta /\cos\theta = k/(2k) = 1/2$

$\cot\theta = 2$

$\text{cosec}\theta = AB/BC = 1/\sin\theta = \sqrt{5}$

$\sec\theta = AB/AC = 1/\cos\theta = \sqrt{5}/2$

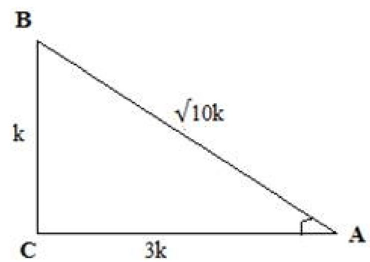
Question: 5

If cosecθ = √10,

Solution:

We have, $\text{cosec}\theta = (k\sqrt{10})/k = 1/\sin\theta$ (For some value of k)

$\rightarrow \sin\theta = k/(k\sqrt{10})$ = perpendicular/hypotenuse



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$(\sqrt{10}k)^2 = (k)^2 + AC^2$

$AC^2 = 10k^2 - k^2$

$AC^2 = 9k^2 = (3k)^2$

$\rightarrow AC = 3k$

Hence, the trignometeric ratios for the given θ are:

$\sin\theta = 1/\sqrt{10}$

$\cos\theta = AC/AB = (3k)/(k\sqrt{10}) = 3/\sqrt{10}$

$\tan\theta = BC/AC = \sin\theta / \cos\theta = 1/3$

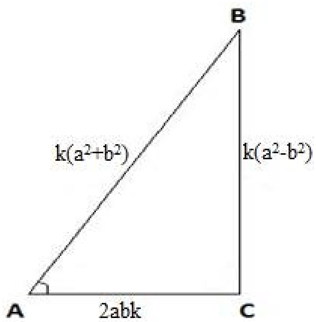
$\cot\theta = AC/BC = 1/\tan\theta = 3$

$\operatorname{cosec}\theta = \sqrt{10}$

$\sec\theta = AB/AC = 1/\cos\theta = \sqrt{10}/3$

Question: 6

If We have, $\sin\theta = \frac{(a^2 - b^2)k}{(a^2 + b^2)k}$ = perpendicular/hypotenuse (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$\{(a^2 + b^2)k\}^2 = \{(a^2 - b^2)k\}^2 + AC^2$

$a^4k^2 + b^4k^2 + 2a^2b^2k^2 = a^4k^2 + b^4k^2 - 2a^2b^2k^2 + AC^2$

$AC^2 = 4a^2b^2k^2 = (2abk)^2$

$\rightarrow AC = 2abk$

Hence, the trigonometric ratios for the given θ are:

$\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$

$\cos\theta = AC/AB = \frac{2abk}{(a^2 + b^2)k} = \frac{2ab}{a^2 + b^2}$

$\tan\theta = BC/AC = \sin\theta / \cos\theta = \frac{a^2 - b^2}{2ab}$

$\cot\theta = AC/BC = 1/\tan\theta = \frac{2ab}{a^2 - b^2}$

$\operatorname{cosec}\theta = AB/BC = 1/\sin\theta = \frac{a^2 + b^2}{a^2 - b^2}$

$\sec\theta = AB/AC = 1/\cos\theta = \frac{a^2 + b^2}{2ab}$

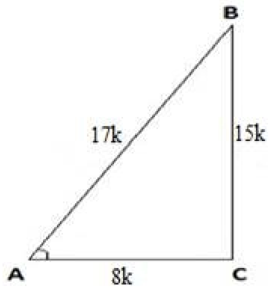
Question: 7

If 15 cotA = 8, f

Solution:

We have, 15 cotA = 8

$\rightarrow \cot A = (8k)/(15k) = 1/\tan A = AC/BC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$AB^2 = (15k)^2 + (8k)^2$

$AB^2 = 225k^2 + 64k^2$

$AB^2 = 289k^2$

$= (17k)^2$

$\rightarrow AB = 17k$

$\therefore$ sinA = BC/AB = (15k)/(17k) = 15/17

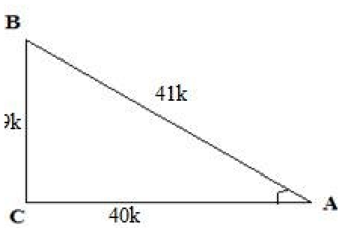
secA = 1/cosA = AB/AC = (17k)/(8k) = 17/8

Question: 8

If sinA = 9/41, f

Solution:

We have, sinA = (9k)/(41k) = BC/AB (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= (41k)^2 = (9k)^2 + AC^2$

$= 1681k^2 = 81k^2 + AC^2$

$AC^2 = 1600k^2$

$= (40k)^2$

$\rightarrow AC = 40k$

$\therefore$ cosA = AC/AB = (40k)/(41k) = 40/41

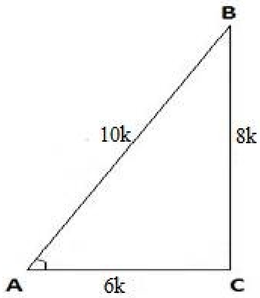
tanA = BC/AC = sinA/cosA = 9/40

Question: 9

If cosθ = 0.6, sh

Solution:

We have cosθ = 0.6 = (6k)/(10k) = AC/AB (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= (10k)^2 = BC^2 + (6k)^2$

$= 100k^2 = BC^2 + 36k^2$

$= BC^2 = 64k^2$

$= (8k)^2$

$\rightarrow BC = 8k$

$\therefore \sin\theta = BC/AB = (8k)/(10k) = 0.8$

$\tan\theta = \sin\theta / \cos\theta = 0.8/0.6$

consider, the LHS,

$5\sin\theta - 3\tan\theta = 5(0.8) - 3(0.8/0.6)$

$= 4 - 3(0.4/0.3)$

$= 4(0.3) - 3(0.4)$

$= 1.2 - 1.2$

$= 0$

$=$ RHS

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Question: 10

If cosecθ =

Solution:

We have, $\operatorname{cosec}\theta = 2 = 1/\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$ $\Rightarrow \theta = 30°$ $\Rightarrow \cos\theta = \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow \cot\theta = \cot 30 = \sqrt{3}$

consider LHS$= \cot\theta + \frac{\sin\theta}{1 + \cos\theta} = \sqrt{3} + \frac{1/2}{1 + \frac{\sqrt{3}}{2}}$

$= \sqrt{3} + \frac{1}{2 + \sqrt{3}}$

$= \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}}$

$= \frac{4 + 2\sqrt{3}}{2 + \sqrt{3}}$

$= \frac{2(2+\sqrt{3})}{2+\sqrt{3}}$

= 2

= RHS

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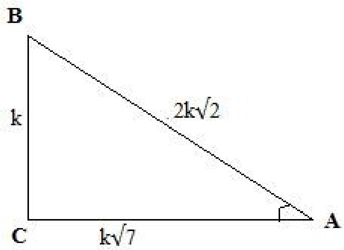
Question: 11

If $\tan\theta = 1/\sqrt{7}$,

Solution:

We have, $\tan\theta = k/(k\sqrt{7}) = BC/AC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= AB^2 = (k)^2 + (k\sqrt{7})^2$

$= AB^2 = k^2 + 7k^2$

$= 8k^2 = (2k\sqrt{2})^2$

$\rightarrow AB = 2k\sqrt{2}$

$\therefore \operatorname{cosec}\theta = AB/BC = 2k\sqrt{2}$

$\sec\theta = AB/AC = \frac{2k\sqrt{2}}{k\sqrt{7}} = \frac{2\sqrt{2}}{\sqrt{7}}$

consider the LHS,

LHS $= \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$

$= \frac{8-\frac{8}{7}}{8+\frac{8}{7}}$

$= \frac{56-8}{56+8}$

= 48/64

= 3/4

= RHS

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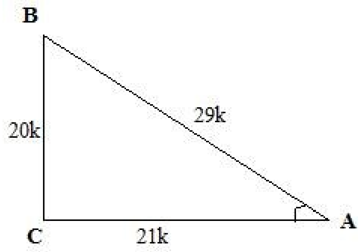
Question: 12

If $\tan\theta = 20/21$,

Solution:

We have, $\tan\theta = (20k)/(21k) = BC/AC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= AB^2 = (20k)^2 + (21k)^2$

$= AB^2 = 400k^2 + 441k^2$

$= AB^2 = 841k^2$

$= (29k)^2$

→ AB = 29k

∴ sinθ = BC/AB = (20k)/(29k) = 20/29

cosθ = AC/AB = (21k)/(29k) = 21/29

consider, the LHS

LHS = $\frac{1-\sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta} = \frac{1-\frac{20}{29}+\frac{21}{29}}{1+\frac{20}{29}+\frac{21}{29}}$

$= \frac{29-20+21}{29+20+21}$

= 30/70

= 3/7

= RHS

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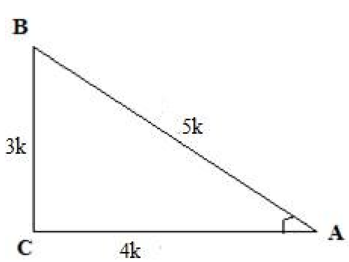
Question: 13

If secθ = 5/4, sh

Solution:

We have, secθ = 5/4 = 1/cosθ

→ cosθ = (4k)/(5k) = AC/AB (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= (5k)^2 = BC^2 + (4k)^2$

$= 25k^2 = BC^2 + 16k^2$

$= BC^2 = 9k^2$

→ BC = 3k

∴ $\sin\theta = BC/AB = (3k)/(5k) = 3/5$

consider the LHS

$$\text{LHS} = \frac{\sin\theta - 2\cos\theta}{\tan\theta + \cot\theta} = \frac{\sin\theta - 2\cos\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{(\sin\theta - 2\cos\theta)\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\left\{\frac{3}{5} - 2\left(\frac{4}{5}\right)\right\}\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)}{\frac{9}{25} - \frac{16}{25}}$$

$$= \frac{\frac{-5}{5} \times \frac{3}{5} \times \frac{4}{5}}{\frac{-7}{25}}$$

$$= \frac{-12}{-7}$$

= 12/7

= RHS

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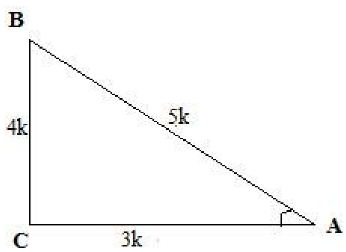
Question: 14

If $\cot\theta = 3/4$, sh

Solution:

We have, $\cot\theta = (3k)/(4k) = AC/BC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= AB^2 = (4k)^2 + (3k)^2$

$= AB^2 = 16k^2 + 9k^2$

$= AB^2 = 25k^2$

$= (5k)^2$

→ AB = 5k

∴ $\sin\theta = BC/AB = (4k)/(5k) = 4/5$

$\cos\theta = AC/AB = (3k)/(5k) = 3/5$

consider the LHS

$$\text{LHS} = \sqrt{\frac{\sec\theta - \operatorname{cosec}\theta}{\sec\theta + \operatorname{cosec}\theta}} = \sqrt{\frac{\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}}$$

$$= \sqrt{\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}}$$

$= \sqrt{\frac{\frac{4}{5}-\frac{3}{5}}{\frac{4}{5}+\frac{3}{5}}}$

$= \sqrt{\frac{1}{7}}$

= 1/√7

= RHS

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Question: 15

If sinθ = 3/4, sh

Solution:

We have, sinθ = (3k)/(4k) = BC/AB (For some value of k)

Consider the LHS

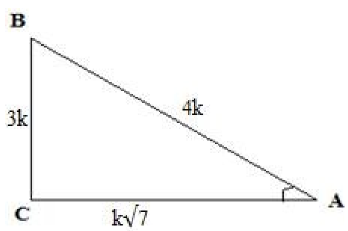
LHS = $\sqrt{\frac{\operatorname{cosec}^2\theta-\cot^2\theta}{\sec^2\theta-1}} = \sqrt{\frac{\frac{1}{\sin^2\theta}-\frac{\cos^2\theta}{\sin^2\theta}}{\frac{1}{\cos^2\theta}-1}}$

$= \sqrt{\frac{(1-\cos^2\theta)\cos^2\theta}{(1-\cos^2\theta)\sin^2\theta}}$

$= \sqrt{\frac{\cos^2\theta}{\sin^2\theta}}$

= cosθ /sinθ

= cotθ



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= (4k)^2 = (3k)^2 + AC^2$

$= 16k^2 = 9k^2 + AC^2$

$= AC^2 = 7k^2$

→ AC = k√7

∴ cotθ = AC/BC = k/(k√7) = 1/√7

∴ LHS = RHS

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Question: 16

If sinθ = a/b, sh

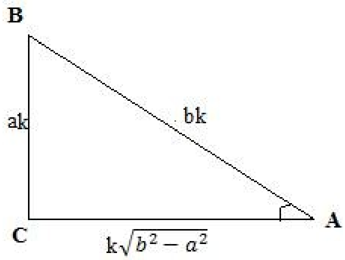
Solution:

Consider LHS,

LHS = secθ + tanθ = $\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

= $\frac{1 + \sin\theta}{\cos\theta}$ (1)

We have, sinθ = (ak)/(bk) = BC/AB (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

= $(bk)^2 = (ak)^2 + AC^2$

= $AC^2 = b^2k^2 - a^2k^2$

→ AC = $k\sqrt{b^2 - a^2}$

∴ cosθ = AC/AB = $\frac{k\sqrt{b^2-a^2}}{bk} = \frac{\sqrt{b^2-a^2}}{b}$

∴ from(1)

LHS = $\frac{1 + \sin\theta}{\cos\theta} = \frac{1 + \frac{a}{b}}{\frac{\sqrt{b^2-a^2}}{b}}$

= $\frac{b + a}{\sqrt{(b-a)(b + a)}}$

= $\sqrt{\frac{b + a}{b-a}}$

= RHS

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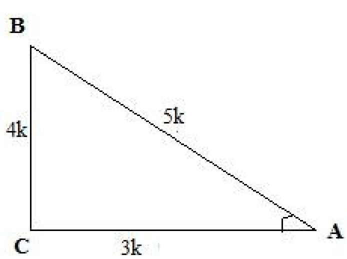
Question: 17

If cosθ = 3/5, sh

Solution:

We have, cosθ = (3k)/(5k) = AC/AB (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

= $(5k)^2 = BC^2 + (3k)^2$

= $25k^2 = BC^2 + 9k^2$

= $BC^2 = 16k^2$

$= (4k)^2$

$\rightarrow BC = 4k$

$\therefore \sin\theta = BC/AB = (4k)/(5k) = 4/5$

$\tan\theta = BC/AC = \sin\theta / \cos\theta = (4k)/(3k) = 4/3$

$\cot\theta = 1/\tan\theta = 3/4$

consider the LHS

LHS $= \frac{\sin\theta - \cot\theta}{2\tan\theta} = \frac{\frac{4}{5} - \frac{3}{4}}{2\left(\frac{4}{3}\right)}$

$= \frac{(16-15)3}{20(8)}$

$= 3/160$

= RHS

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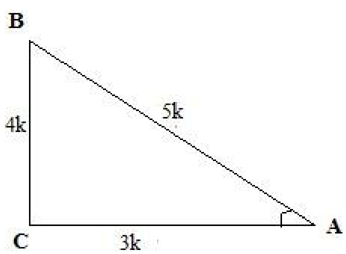
Question: 18

If $\tan\theta = 4/3$, sh

Solution:

We have, $\tan\theta = (4k)/(3k) = BC/AC$ (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= AB^2 = (4k)^2 + (3k)^2$

$= AB^2 = 16k^2 + 9k^2$

$= AB^2 = 25k^2$

$= (5k)^2$

$\rightarrow AB = 5k$

$\sin\theta = BC/AB = (4k)/(5k) = 4/5$

$\cos\theta = AC/AB = (3k)/(5k) = 3/5$

consider LHS $= \sin\theta + \cos\theta = \frac{4}{5} + \frac{3}{5}$

$= 7/5$

= RHS

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Question: 19

If $\tan\theta = a/b$, sh

Solution:

Consider the LHS = $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

(Dividing the numerator and denominator by cos θ)

$$= \frac{\frac{a\sin\theta}{\cos\theta} - \frac{b\cos\theta}{\cos\theta}}{\frac{a\sin\theta}{\cos\theta} + \frac{b\cos\theta}{\cos\theta}}$$

$= \frac{a\tan\theta - b}{a\tan\theta + b}$

$= \frac{a\left(\frac{a}{b}\right) - b}{a\left(\frac{a}{b}\right) + b}$

$= \frac{a^2 - b^2}{a^2 + b^2}$

= RHS

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Question: 20

If 3 tan &th

Solution:

We have, 3 tan θ = 4

→ tan θ = 4/3 (1)

Consider the LHS = $\frac{4\cos\theta - \sin\theta}{2\cos\theta + \sin\theta}$

(Dividing the numerator and denominator by cosθ)

$$= \frac{4 - \tan\theta}{2 + \tan\theta}$$

$= \frac{4 - \frac{4}{3}}{2 + \frac{4}{3}}$ (from(1))

$= \frac{12 - 4}{6 + 4}$

= 8/10

= 4/5

= RHS

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Question: 21

If 3cotθ = 2, sho

Solution:

We have 3cotθ = 2

→ cotθ = 2/3 (1)

Consider the LHS = $\frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta}$

(Dividing the numerator and denominator by sinθ)

$= \frac{\frac{4\sin\theta - 3\cos\theta}{\sin\theta}}{\frac{2\sin\theta + 6\cos\theta}{\sin\theta}}$

$= \frac{4-3\cot\theta}{2+6\cot\theta}$

$= \frac{4-3(\frac{2}{3})}{2+6(\frac{2}{3})}$ (from(1))

$= \frac{4-2}{2+4}$

= 2/6

= 1/3

= RHS

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Question: 22

If 3cotθ = 4, sho

Solution:

Consider the LHS = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \frac{1-\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}{1+\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$

$= \cos^2\theta - \sin^2\theta$ ($\because \cos^2\theta + \sin^2\theta = 1$)

= RHS

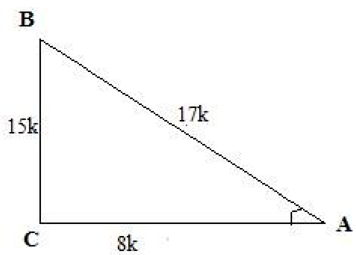
HENCE PROVED

Question: 23

If secθ = 17/8, v

Solution:

We have, secθ = (17k)/(8k) = AB/AC (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= (17k)^2 = BC^2 + (8k)^2$

$= 289k^2 = BC^2 + 64k^2$

$= BC^2 = 225k^2$

→ BC = 15k

∴ sinθ = BC/AB = (15k)/(17k) = 15/17

cosθ = AC/AB = (8k)/(17k) = 8/17

tanθ = BC/AC = sinθ /cosθ = 15/8

consider the LHS = $\frac{3-4\sin^2\theta}{4\cos^2\theta-3}$

$= \frac{3-4\left(\frac{15}{17}\right)^2}{4\left(\frac{8}{17}\right)^2-3}$

$= \frac{3(289)-4(225)}{4(64)-3(289)}$

$= \frac{867-900}{256-867}$

= (- 33)/(- 611)

= 33/611

Now consider RHS = $\frac{3-\tan^2\theta}{1-3\tan^2\theta}$

$= \frac{3-\left(\frac{15}{8}\right)^2}{1-3(15/8)^2}$

$= \frac{3(64)-225}{64-675}$

= (- 33)/(- 611)

= 33/611

∴ RHS = LHS

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Question: 24

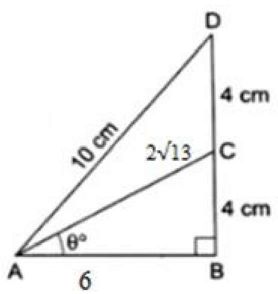
In the adjoining

Solution:

Clearly, Δ ABC and Δ ABD are right angled triangles

where AD = 10cm BC = CD = 4cm

BD = BC + CD = 8cm



Applying Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AD^2 = BD^2 + AB^2$

$= (10)^2 = (8)^2 + AB^2$

$= 100 = 64 + AB^2$

$= AB^2 = 36$

$= (6)^2$

→ AB = 6cm

Now applying Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AC^2 = BC^2 + AB^2$

$= AC^2 = (4)^2 + (6)^2$

$= AC^2 = 16 + 36$

$= 52$

$\rightarrow AC = \sqrt{52}$

$= 2\sqrt{13}cm$

i. $\sin\theta = BC/AC = \frac{4}{2\sqrt{13}}$

$= 2/\sqrt{13}$

$= (2\sqrt{13})/13$

ii. $\cos\theta = AB/AC = \frac{6}{2\sqrt{13}}$

$= 3/\sqrt{13}$

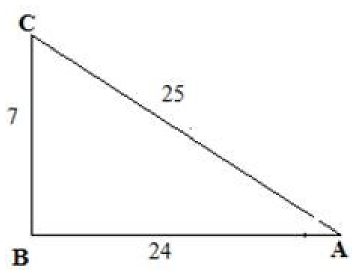
$= (3\sqrt{13})/13$

Question: 25

In a Δ ABC,

Solution:

Clearly Δ ABC is a right angled triangle,



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AC^2 = BC^2 + AB^2$

$= AC^2 = (7)^2 + (24)^2$

$= AC^2 = 49 + 576$

$= AC^2 = 625$

$\rightarrow AC = 25$

a) $\sin A = BC/AC = 7/25$

b) $\cos A = AB/AC = 24/25$

c) $\sin C = AB/AC = 24/25$

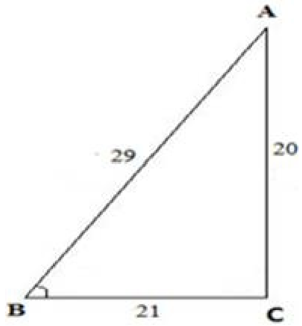
d) $\cos C = BC/AC = 7/25$

Question: 26

In a Δ ABC,

Solution:

Δ ABC is a right angled triangle



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = AC^2 + BC^2$

$= (29)^2 = AC^2 + (21)^2$

$= 841 = AC^2 + 441$

$= AC^2 = 400$

$\rightarrow AC = 20$

$\therefore \sin\theta = AC/AB = 20/29$

$\cos\theta = BC/AB = 21/29$

$\cos^2\theta - \sin^2\theta = (21/29)^2 - (20/29)^2$

$= \frac{441-400}{841}$

$= 41/841$

= RHS

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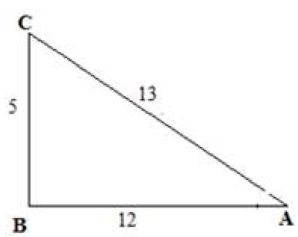
Question: 27

In a Δ ABC,

Solution:

Δ ABC is a right angled triangle



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AC^2 = BC^2 + AB^2$

$= AC^2 = (5)2 + (12)^2$

$= AC^2 = 25 + 144$

$= AC^2 = 169$

$= (13)^2$

$\rightarrow AC = 13$

i. $\cos A = AB/AC = 12/13$

ii. $\text{cosec} A = AC/BC = 13/5$

iii. cosC = BC/AC = 5/13

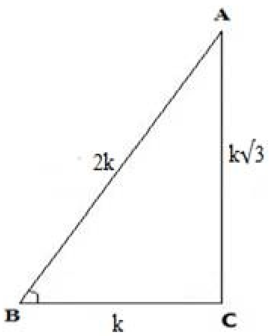
iv. cosecC = AC/AB = 13/12

Question: 28

If sinα = 1/2, pr

Solution:

We have, sinα = k/(2k) = BC/AB (For some value of k)



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AB^2 = BC^2 + AC^2$

$= (2k)^2 = (k)^2 + AC^2$

$= 4k^2 = k^2 + AC^2$

$= AC^2 = 3k^2$

→ AC = k√3

∴ cosα = AC/AB = (k√3)/(2k) = √3/2

Then, $3\cos\alpha - 4\cos^3\alpha = 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 3\left(\frac{\sqrt{3}}{2}\right)$

= 0

LHS = RHS

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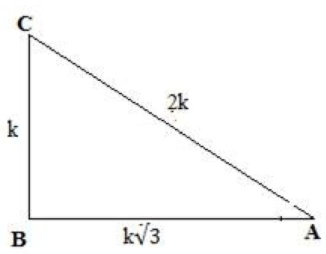
Question: 29

In a Δ ABC,

Solution:

We have, tanA = k/(k√3) = BC/AB

Δ ABC is a right angled triangle



By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AC^2 = BC^2 + AB^2$

$= AC^2 = (k)^2 + (k\sqrt{3})^2$

$= AC^2 = k^2 + 3k^2$

$= AC^2 = 4k^2$

→ AC = 2k

∴ sinA = BC/AC = k/(2k) = 1/2

cosA = AB/AC = (k√3)/(2k) = √3/2

sinC = AB/AC = (k√3)/(2k) = √3/2

cosC = = BC/AC = k/(2k) = 1/2

i. sinA cosC + cosA sinC = (1/2)(1/2) + (√3/2)(√3/2)

$= \frac{1}{4} + \frac{3}{4}$

= 4/4

= 1

∴ RHS = LHS

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ii. cosA cosC - sinA sinC = (1/2)(√3/2) - (1/2)(√3/2)

= (√3/4) - (√3/4)

= 0

∴ RHS = LHS

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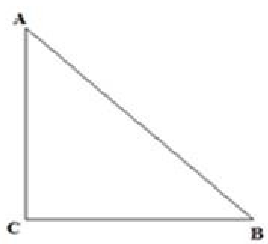
Question: 30

If Consider ΔABC to be a right - angled triangle.



∴ sinA = BC/AB

sinB = AC/AB

Given that sinA = sinB

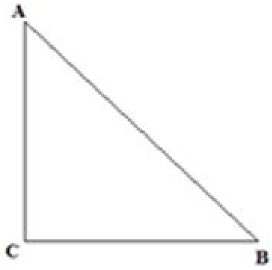
BC/AB = AC/AB

→ BC = AC

→ ∠ A = ∠ B (In a triangle, angles opposite to equal angles are equal)

Question: 31

If Consider ΔABC to be a right - angled triangle.



∴ tanA = BC/AC

tanB = AC/BC

Given that tanA = tanB

BC/AC = AC/BC

$BC^2 = AC^2$

→ BC = AC

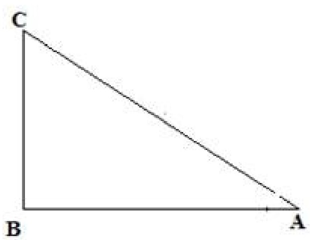
→ ∠ A = ∠ B (In a triangle, angles opposite to equal angles are equal)

Question: 32

In a right ΔABC,

Solution:

Consider ΔABC to be a right - angled triangle.



tanA = 1 = BC/AB

→ BC = AB

Also, tanA = 1 = sinA/cosA

→ sinA = cosA

By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore AC^2 = BC^2 + AB^2$

$= AC^2 = 2BC^2$

$= (AC/BC)^2 = 2$

= AC/BC = √2

→ cosecA = √2

→ sinA = 1/√2 = cosA

2 sinA cosA = 2(1/√2)(1/√2)

= 2(1/2)

= 1

∴ LHS = RHS

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Question: 33

In the given figu

Solution:

Δ PQR is a right - angled triangle

By Pythagoras theorem, $(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\therefore PR^2 = RQ^2 + PQ^2$

$= (x + 2)^2 = x^2 + PQ^2$

$= x^2 + 4 + 4x = x^2 + PQ^2$

$PQ^2 = 4 + 4x$

$\rightarrow PQ = 2\sqrt{x + 1}$

a. $\cot\phi = RQ/PQ = \frac{x}{2\sqrt{x+1}}$

$\therefore \sqrt{x + 1}\cot\phi\sqrt{x + 1}\times \frac{x}{2\sqrt{x+1}} =$

$= x/2$

b. $\tan\theta = RQ/PQ = \frac{x}{2\sqrt{x+1}}$

$\therefore \sqrt{x^3 + x^2}\tan\theta = \sqrt{x^3 + x^2} \times \frac{x}{2\sqrt{x+1}} =$

$= x\sqrt{x + 1}\times \frac{x}{2\sqrt{x+1}}$

$= x^2/2$

c. $\cos\theta = PQ/RP = \frac{2\sqrt{x+1}}{x+2}$

Question: 34

If x = cosecA + c

Solution:

x + y = cosecA + cosA + cosecA - cosA

= 2cosecA

x - y = cosecA + cosA - (cosecA - cosA)

= cosecA + cosA - cosecA + cosA

= 2cosA

Consider the LHS, $\left(\frac{2}{x+y}\right)^2 + \left(\frac{x-y}{2}\right)^2 - 1 = \left(\frac{2}{2\ \mathrm{cosecA}}\right)^2 + \left(\frac{2\cos A}{2}\right)^2 - 1$

$= \sin^2A + \cos^2A - 1$

$= 1 - 1$ ($\because \sin^2A + \cos^2A = 1$)

= 0

= RHS

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Question: 35

If x = cotA + cos

Solution:

x - y = cotA + cosA - (cotA - cosA)

x - y = cotA + cosA - cotA + cosA

x - y = 2cosA

x + y = cotA + cosA + cotA - cosA

x + y = 2cotA

Consider the LHS = $\left(\frac{x-y}{2}\right)^2 + \left(\frac{x-y}{x+y}\right)^2 = \left(\frac{2\cos A}{2}\right)^2 + \left(\frac{2\cos A}{2\cot A}\right)^2$

= $\cos^2 A + \sin^2 A$

= 1 ($\because \sin^2 A + \cos^2 A = 1$)

= RHS

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