

## Chapter : 28. DIFFERENTIATION

### Exercise : 28A

#### Question: 1

Differentiation

#### Solution:

(i)  $x^{-3}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{-3} = -3x^{-3-1}$$

$$= -3x^{-4}$$

(ii)  $\sqrt[3]{x} = x^{\frac{1}{3}}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{\frac{1}{3}} = \frac{1}{3} x^{\frac{1}{3}-1}$$

$$= \frac{1}{3} x^{-\frac{2}{3}}$$

#### Question: 2

(i)  $\frac{1}{x} = x^{-1}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{-1} = -1x^{-1-1}$$

$$= -x^{-2}$$

(ii)  $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{-\frac{1}{2}} = \frac{-1}{2} x^{-\frac{1}{2}-1}$$

$$= \frac{-1}{2} x^{-\frac{3}{2}}$$

(iii)  $\frac{1}{\sqrt[3]{x}} = x^{-\frac{1}{3}}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating w.r.t x,

$$\frac{d}{dx} x^{-\frac{1}{3}} = \frac{-1}{3} x^{-\frac{1}{3}-1}$$

$$= -\frac{1}{3} x^{-\frac{4}{3}}$$

**Question: 3**

(i)  $3x^{-5}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx} 3x^{-5} = 3(-5)x^{-5-1}$$

$$= -15x^{-6}$$

(ii)  $\frac{1}{5}x = \frac{1}{5}x^{-1}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\frac{1}{5} \frac{d}{dx} x^{-1} = \frac{-1}{5} x^{-1-1}$$

$$= \frac{1}{5} x^{-2}$$

(iii)  $6\sqrt[3]{x^2} = 6x^{\frac{2}{3}}$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx} 6x^{\frac{2}{3}} = 6 \times \frac{2}{3} x^{\frac{2}{3}-1}$$

$$= 4x^{-\frac{1}{3}}$$

**Question: 4**

(i)  $6x^5 + 4x^3 - 3x^2 + 2x - 7$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\frac{d}{dx} (6x^5 + 4x^3 - 3x^2 + 2x - 7) = 30x^{5-1} + 12x^{3-1} - 6x^{2-1} + 2x^{1-1} + 0$$

$$= 30x^4 + 12x^2 - 6x^1 + 2x$$

$$(ii) 5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x}$$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\begin{aligned}\frac{d}{dx} (5x^{-3/2} + \frac{4}{\sqrt{x}} + \sqrt{x} - \frac{7}{x}) \\ = 5x^{-\frac{3}{2}-1} + 4x^{-\frac{1}{2}-1} + \frac{1}{2}x^{\frac{1}{2}-1} - 7x^{-1-1} \\ = -\frac{15}{2}x^{-\frac{5}{2}} - 2x^{-\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + 7x^{-2}\end{aligned}$$

$$(iii) ax^3 + bx^2 + cx + d, \text{ where } a, b, c, d \text{ are constants}$$

Formula:-

$$\frac{d}{dx} x^n = nx^{n-1}$$

Differentiating with respect to x,

$$\begin{aligned}\frac{d}{dx} (ax^3 + bx^2 + cx + d) &= 3ax^{3-1} + 2bx^{2-1} + cx^{1-1} + d \times 0 \\ &= 3ax^2 + 2bx + c\end{aligned}$$

### Question: 5

$$(i) 4x^3 + 3 \cdot 2^x + 6 \cdot \sqrt[8]{x^{-4}} + 5 \cot x$$

$$= 4x^3 + 3 \cdot 2^x + 6x^{-\frac{1}{2}} + 5 \cot x$$

Formulae:

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} a^x = \log_n(a) \times a^x$$

Differentiating with respect to x,

$$\begin{aligned}\frac{d}{dx} (4x^3 + 3 \cdot 2^x + 6x^{-\frac{1}{2}} + 5 \cot x) \\ = 4 \cdot 3x^{3-1} + 3 \cdot \log_2(2) \cdot 2^x + 6 \times \frac{1}{2}x^{-\frac{1}{2}-1} + 5 \times -\operatorname{cosec}^2 x \\ = 12x^2 + 3 \cdot \log_2(2) \cdot 2^x - 3x^{-\frac{3}{2}} - 5 \operatorname{cosec}^2 x\end{aligned}$$

$$\begin{aligned}(ii) \frac{x}{3} - \frac{3}{x} + \sqrt{x} - \frac{1}{\sqrt{x}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6 \\ = \frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-2/3} - \frac{2}{3}x^6\end{aligned}$$

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} a^x = \log_n(a) \times a^x$$

Differentiating with respect to x,

$$\frac{d}{dx} \left( \frac{x}{3} - 3x^{-1} + x^{\frac{1}{2}} - x^{-\frac{1}{2}} + x^2 - 2^x + 6x^{-\frac{2}{3}} - \frac{2}{3}x^6 \right)$$

$$\begin{aligned} &= \frac{1}{3} - (-1) \times 3x^{-1-1} + \frac{1}{2}x^{\frac{1}{2}-1} - \left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1} + 2x^{2-1} - \log(2) \cdot 2^x + \\ &\quad 6 \left(-\frac{2}{3}\right)x^{-\frac{2}{3}-1} - \frac{2}{3} \times 6x^{6-1} \\ &= \frac{1}{3} + 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} + 2x^1 - \log(2) \cdot 2^x - 4x^{-\frac{5}{3}} - 4x^5 \end{aligned}$$

### Question: 6

Formulae: -

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} k = 0, k \text{ is constant}$$

$$\begin{aligned} (i) \quad &4 \cot x - \frac{1}{2} \cos x + \frac{2}{\cos x} - \frac{3}{\sin x} + \frac{6 \cot x}{\operatorname{cosec} x} + 9 \\ &= 4 \cot x - \frac{1}{2} \cos x + 2 \sec x - 3 \operatorname{cosec} x + 6 \cos x + 9 \end{aligned}$$

Differentiating with respect to x,

$$\begin{aligned} \frac{d}{dx} (4 \cot x - \frac{1}{2} \cos x + 2 \sec x - 3 \operatorname{cosec} x + 6 \cos x + 9) \\ &= 4(-\operatorname{cosec}^2 x) - \frac{1}{2}(-\sin x) + 2 \sec x \tan x - 3(-\operatorname{cosec} x \cot x) + 6(-\sin x) + 0 \\ &= -4 \operatorname{cosec}^2 x + \frac{1}{2} \sin x + 2 \sec x \tan x + 3 \operatorname{cosec} x \cot x - 6 \sin x \end{aligned}$$

$$(ii) -5 \tan x + 4 \tan x \cos x - 3 \cot x \sec x + 2 \sec x - 13$$

$$= -5 \tan x + 4 \sin x - 3 \operatorname{cosec} x + 2 \sec x - 13$$

Differentiating with respect to x,

$$\begin{aligned} \frac{d}{dx} (-5 \tan x + 4 \sin x - 3 \operatorname{cosec} x + 2 \sec x - 13) \\ &= -5 \sec^2 x + 4 \cos x - 3(-\operatorname{cosec} x \cot x) + 2 \sec x \tan x - 0 \\ &= -5 \sec^2 x + 4 \cos x + 3 \operatorname{cosec} x \cot x + 2 \sec x \tan x \end{aligned}$$

### Question: 7

Formula:

$$\frac{d}{dx} f(g(x)) = \frac{d}{dg} f(g) \frac{d}{dx} g$$

Chain rule -

$$\frac{d}{dx}(uv) = u \frac{d}{dx}v + v \frac{d}{dx}u$$

Where  $u$  and  $v$  are the functions of  $x$ .

$$(i) (2x + 3)(3x - 5)$$

Applying, Chain rule

$$\text{Here, } u = 2x + 3$$

$$V = 3x - 5$$

$$\begin{aligned}\frac{d}{dx}(2x + 3)(3x - 5) &= (2x + 3)\frac{d}{dx}(3x - 5) + (3x - 5)\frac{d}{dx}(2x + 3) \\ &= (2x + 3)(3x^{1-1} + 0) + (3x - 5)(2x^{1-1} + 0) \\ &= 6x + 9 + 6x - 10\end{aligned}$$

$$= 12x - 1$$

$$(ii) x(1 + x)^3$$

Applying, Chain rule

$$\text{Here, } u = x$$

$$V = (1 + x)^3$$

$$\begin{aligned}\frac{d}{dx}x(1 + x)^3 &= x\frac{d}{dx}(1 + x)^3 + (1 + x)^3\frac{d}{dx}(x) \\ &= x \times 3 \times (1 + x)^2 + (1 + x)^3(1) \\ &= (1 + x)^2(3x + x + 1) \\ &= (1 + x)^2(4x + 1)\end{aligned}$$

$$(iii) \left(\sqrt{x} + \frac{1}{x}\right)\left(x - \frac{1}{\sqrt{x}}\right) = (x^{1/2} + x^{-1})(x - x^{-1/2})$$

Applying, Chain rule

$$\text{Here, } u = (x^{1/2} + x^{-1})$$

$$V = (x - x^{-1/2})$$

$$\begin{aligned}\frac{d}{dx}(x^{1/2} + x^{-1})(x - x^{-1/2}) &= (x^{1/2} + x^{-1})\frac{d}{dx}(x - x^{-1/2}) + (x - x^{-1/2})\frac{d}{dx}(x^{1/2} + x^{-1}) \\ &= (x^{1/2} + x^{-1})(1 + \frac{1}{2}x^{-3/2}) + (x - x^{-1/2})(\frac{1}{2}x^{-1/2} - x^{-2}) \\ &= x^{1/2} + x^{-1} + \frac{1}{2}x^{-1} + \frac{1}{2}x^{-5/2} + \frac{1}{2}x^{1/2} - x^{-1} - \frac{1}{2}x^{-1} + x^{-5/2} \\ &= \frac{3}{2}x^{1/2} + \frac{3}{2}x^{-5/2}\end{aligned}$$

$$(iv) \left(x - \frac{1}{x}\right)^2$$

Differentiation of composite function can be done by

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\frac{d}{dx}g$$

$$\text{Here, } f(g) = g^2, g(x) = x - \frac{1}{x}$$

$$\frac{d}{dx}\left(x - \frac{1}{x}\right)^2 = 2g \times (1 + \frac{1}{x^2})$$

$$= 2\left(x - \frac{1}{x}\right) \left(1 + \frac{1}{x^2}\right)$$

$$= 2\left(x + \frac{1}{x} - \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= 2\left(x + \frac{1}{x^2}\right)$$

$$(v) \left(x^2 - \frac{1}{x^2}\right)^3$$

Differentiation of composite function can be done by

$$\frac{d}{dx} f(g(x)) = \frac{d}{dg} f(g) \frac{d}{dx} g$$

$$\text{Here, } f(g) = g^3, g(x) = x^2 - \frac{1}{x^2}$$

$$\frac{d}{dx} \left(x^2 - \frac{1}{x^2}\right)^3 = 3g^2 \times (2x - \frac{2}{x^3})$$

$$= 3\left(x^2 - \frac{1}{x^2}\right)^2 (2x - \frac{2}{x^3})$$

$$= 3(2x^3 - \frac{2}{x} \cdot \frac{2}{x} + \frac{2}{x^5})$$

$$= 3(2x^3 - \frac{4}{x} + \frac{2}{x^5})$$

$$(vi) (2x^2 + 5x - 1)(x - 3)$$

Applying, Chain rule

$$\text{Here, } u = (2x^2 + 5x - 1)$$

$$V = (x - 3)$$

$$\frac{d}{dx}(2x^2 + 5x - 1)(x - 3)$$

$$= (2x^2 + 5x - 1) \frac{d}{dx}(x - 3) + (x - 3) \frac{d}{dx}(2x^2 + 5x - 1)$$

$$= (2x^2 + 5x - 1) \times 1 + (x - 3)(4x + 5)$$

$$= 2x^2 + 5x - 1 + 4x^2 - 7x - 15$$

$$= 6x^2 - 2x - 16$$

### Question: 8

Formula:

$$\frac{d}{dx} \frac{u}{v} = \frac{v \frac{du}{dx} - u \frac{dv}{dx} v}{u^2}$$

$$(i) \frac{3x^2+4x-5}{x}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{3x^2 + 4x - 5}{x} = \frac{x \frac{d}{dx}(3x^2 + 4x - 5) - (3x^2 + 4x - 5) \frac{d}{dx} x}{x^2}$$

$$= \frac{x(6x+4) - (3x^2 + 4x - 5)1}{x^2}$$

$$= \frac{6x^2 + 4x - (3x^2 + 4x - 5)}{x^2}$$

$$= \frac{3x^2 + 5}{x^2}$$

$$(ii) \frac{(x^2+1)(x-2)}{x^2}$$

Applying, quotient rule

$$\begin{aligned} \frac{d}{dx} \frac{(x^3+1)(x-2)}{x^2} &= \frac{x^2 \frac{d}{dx}(x^3+1)(x-2) - (x^3+1)(x-2) \frac{d}{dx} x^2}{x^4} \\ &= \frac{x^2 \{[(x^2+1)\frac{d}{dx}(x-2)+(x-2)\frac{d}{dx}(x^2+1)] - (x^3+1)(x-2)2x\}}{x^4} \\ &= \frac{x^2 \{[(x^2+1)+(x-2)3x^2] - (x^3+1)(x-2)2x\}}{x^4} \\ &= \frac{x^2 \{x^3+1+3x^3-6x^2\} - 2(x^4+x)(x-2)}{x^4} \\ &= \frac{4x^5-6x^4+x^2-2(x^5-2x^4+x^2-2x)}{x^4} \\ &= \frac{2x^5-2x^4-x^2+4x}{x^4} \end{aligned}$$

$$(iii) \frac{x-4}{2\sqrt{x}}$$

Applying, quotient rule

$$\begin{aligned} \frac{d}{dx} \frac{x-4}{2\sqrt{x}} &= \frac{2\sqrt{x} \frac{d}{dx}(x-4) - (x-4) \frac{d}{dx} 2\sqrt{x}}{4x} \\ &= \frac{2\sqrt{x} - (x-4)2\frac{1}{2}x^{-\frac{1}{2}}}{4x} \\ &= \frac{2\sqrt{x} - (x-4)x^{-\frac{1}{2}}}{4x} \\ &= \frac{2\sqrt{x} - x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}}{4x} \\ &= \frac{\sqrt{x} + 4x^{-\frac{1}{2}}}{4x} \end{aligned}$$

$$(iv) \frac{(1+x)\sqrt{x}}{\sqrt[3]{x}}$$

Applying, quotient rule

$$\begin{aligned} \frac{d}{dx} \frac{(1+x)\sqrt{x}}{\sqrt[3]{x}} &= \frac{\sqrt[3]{x} \frac{d}{dx}(1+x)\sqrt{x} - (1+x)\sqrt{x} \frac{d}{dx} \sqrt[3]{x}}{x^{\frac{2}{3}}} \\ &= \frac{\sqrt[3]{x} \left\{ (1+x) \frac{d}{dx} \sqrt{x} + \sqrt{x} \frac{d}{dx} (1+x) \right\} - (1+x)\sqrt{x} \cdot \frac{1}{3} x^{-\frac{2}{3}}}{x^{\frac{2}{3}}} \\ &= \frac{\sqrt[3]{x} \left\{ (1+x) \cdot \frac{1}{2} x^{-\frac{1}{2}} + \sqrt{x} \right\} - (1+x) \cdot \frac{1}{3} x^{-\frac{1}{6}}}{x^{\frac{2}{3}}} \\ &= \frac{\sqrt[3]{x} \left( \frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} x^{\frac{1}{2}} + \sqrt{x} \right) - \frac{1}{3} \left( x^{-\frac{1}{6}} + x^{\frac{5}{6}} \right)}{x^{\frac{2}{3}}} \\ &= \frac{\sqrt[3]{x} \left( \frac{1}{2} x^{-\frac{1}{2}} + \frac{1}{2} x^{\frac{1}{2}} + \sqrt{x} \right) - \frac{1}{3} \left( x^{-\frac{1}{6}} + x^{\frac{5}{6}} \right)}{x^{\frac{2}{3}}} \end{aligned}$$

$$= \frac{\frac{1}{2}x^{\frac{1}{6}} + \frac{1}{2}x^{\frac{5}{6}} + \sqrt{x} - \frac{1}{3}\left(x^{\frac{-1}{6}} + x^{\frac{5}{6}}\right)}{x^{\frac{2}{3}}}$$

$$= \frac{\frac{1}{2}x^{\frac{1}{6}} + \frac{1}{2}x^{\frac{5}{6}} + \sqrt{x}}{x^{\frac{2}{3}}}$$

$$(v) \frac{ax^2 + bx + c}{\sqrt{x}}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{ax^2 + bx + c}{\sqrt{x}} = \frac{\sqrt{x} \frac{d}{dx}(ax^2 + bx + c) - (ax^2 + bx + c) \frac{d}{dx}\sqrt{x}}{x}$$

$$= \frac{\sqrt{x}(2ax+b) - \frac{1}{2}(ax^2 + bx + c)x^{-\frac{1}{2}}}{x}$$

$$= \frac{\frac{3}{2}ax^{\frac{3}{2}} + \frac{1}{2}bx^{\frac{1}{2}} - \frac{1}{2}cx^{-\frac{1}{2}}}{x}$$

$$(vi) \frac{a+b \cos x}{\sin x}$$

Applying, quotient rule

$$\frac{d}{dx} \frac{a+b \cos x}{\sin x} = \frac{\sin x \frac{d}{dx}(a+b \cos x) - (a+b \cos x) \frac{d}{dx}\sin x}{\sin^2 x}$$

$$= \frac{\sin x(-b \sin x) - (a+b \cos x)\cos x}{\sin^2 x}$$

$$= \frac{-b \sin^2 x - a \cos x - b \cos^2 x}{\sin^2 x}$$

$$= \frac{-b(1) - a \cos x}{\sin^2 x}$$

### Question: 9

Formulae:

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \sin x = \cos x$$

$$(i) \text{ If } y = 6x^5 - 4x^4 - 2x^2 + 5x - 9, \text{ find } \frac{dy}{dx} \text{ at } x = -1.$$

Differentiating with respect to x,

$$\frac{d}{dx} (6x^5 - 4x^4 - 2x^2 + 5x - 9)$$

$$= 30x^4 - 16x^3 - 4x + 5$$

$$\text{substituting } x = -1$$

$$(\frac{dy}{dx})_{x=-1} = 30(-1)^4 - 16(-1)^3 - 4(-1) + 5$$

$$= 30 + 16 + 4 + 5$$

$$= 55$$

(ii) If  $y = (\sin x + \tan x)$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{3}$ .

Differentiating with respect to x,

$$\frac{d}{dx}(\sin x + \tan x) = \cos x + \sec^2 x$$

$$\text{Substituting } x = \frac{\pi}{3}$$

$$\left(\frac{dy}{dx}\right)_{x=\pi/3} = \cos \frac{\pi}{3} + \sec^2 \frac{\pi}{3}$$

$$= \frac{1}{2} + 4$$

$$= \frac{5}{2}$$

(iii) If  $y = \frac{(2-3\cos x)}{\sin x}$ , find  $\frac{dy}{dx}$  at  $x = \frac{\pi}{4}$ .

Differentiating with respect to x,

$$\frac{d}{dx}(2\cosec x - 3\cot x) = 2(-\cosec x \cot x) - 3(-\cosec^2 x)$$

$$\text{Substituting } x = \frac{\pi}{4}$$

$$\left(\frac{dy}{dx}\right)_{x=\pi/4} = 2(-\cosec \frac{\pi}{4} \cot \frac{\pi}{4}) - 3(-\cosec^2 \frac{\pi}{4})$$

$$= -2 \times \sqrt{2} + 3 \times 2$$

$$= 6 - 2 \times \sqrt{2}$$

### Question: 10

If To show:  $2x \cdot \frac{dy}{dx} + y = 2\sqrt{x}$

Differentiating with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right) = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$$

Now,

$$\text{LHS} = 2x \cdot \frac{dy}{dx} + y$$

$$\text{LHS} = 2x \times \left( \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}} \right) + \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\text{LHS} = \sqrt{x} - \frac{1}{\sqrt{x}} + \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\text{LHS} = 2\sqrt{x}$$

$$\therefore \text{LHS} = \text{RHS}$$

### Question: 11

If To prove:  $(2xy) \left( \frac{dy}{dx} \right) = \left( \frac{x}{a} - \frac{a}{x} \right)$ .

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \left( \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) = \frac{1}{2\sqrt{ax}} - \frac{\sqrt{a}}{2x^{3/2}}$$

Now,

$$\text{LHS} = (2xy) \left( \frac{dy}{dx} \right)$$

$$\text{LHS} = 2x \left( \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) \left( \frac{1}{2\sqrt{ax}} - \frac{\sqrt{a}}{2x^2} \right)$$

$$\text{LHS} = \left( \sqrt{\frac{x}{a}} + \sqrt{\frac{a}{x}} \right) \left( \sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}} \right)$$

$$\text{LHS} = \left( \frac{x}{a} - \frac{a}{x} \right)$$

$\therefore \text{LHS} = \text{RHS}$

### Question: 12

$$\text{If } y = \sqrt{\frac{1 + \cos 2x}{1 - \cos 2x}}$$

Formula:

Using double angle formula:

$$\cos 2x = 2\cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\therefore 1 + \cos 2x = 2\cos^2 x$$

$$1 - \cos 2x = 2\sin^2 x$$

$$\therefore y = \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$$

$$= \sqrt{\cot^2 x}$$

$$= \cot x$$

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} (\cot x)$$

$$= -\operatorname{cosec}^2 x$$

### Question: 13

Formula:

Using Half angle formula,

$$\cos x = \frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)}$$

$$\therefore y = \cos x$$

Differentiating y with respect to x

$$\frac{dy}{dx} = \frac{d}{dx} \cos x$$

$$= -\sin x$$

## Exercise : 28B

### Question: 1

Find the derivati

**Solution:**

Let  $f(x) = ax + b$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = ax + b$$

$$f(x + h) = a(x + h) + b$$

$$= ax + ah + b$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{ax + ah + b - (ax + b)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{ax + ah + b - ax - b}{h}$$

$$= \lim_{h \rightarrow 0} \frac{ah}{h}$$

$$= \lim_{h \rightarrow 0} a$$

$$f'(x) = a$$

Hence,  $f'(x) = a$

**Question: 2**

Find the derivative

**Solution:**

$$\text{Let } f(x) = ax^2 + \frac{b}{x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = ax^2 + \frac{b}{x}$$

$$f(x + h) = a(x + h)^2 + \frac{b}{(x + h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\left[a(x + h)^2 + \frac{b}{(x + h)}\right] - \left[ax^2 + \frac{b}{x}\right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a(x + h)^2 + \frac{b}{(x + h)} - ax^2 - \frac{b}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[(x + h)^2 - x^2] + b\left[\frac{1}{x + h} - \frac{1}{x}\right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[x^2 + h^2 + 2xh - x^2] + b\left[\frac{x - (x + h)}{x(x + h)}\right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[h^2 + 2xh] + b \left[ \frac{x-x-h}{x(x+h)} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{a[h^2 + 2xh] + b \left[ \frac{-h}{x(x+h)} \right]}{h}$$

$$= \lim_{h \rightarrow 0} \left[ \frac{ah(h+2x)}{h} + \frac{b(-h)}{hx(x+h)} \right]$$

Taking 'h' common from both the numerator and denominator, we get

$$= \lim_{h \rightarrow 0} \left[ a(h+2x) - \frac{b}{x(x+h)} \right]$$

Putting  $h = 0$ , we get

$$\begin{aligned} &= a[(0) + 2x] - \frac{b}{x(x+0)} \\ &= 2ax - \frac{b}{x^2} \end{aligned}$$

$$\text{Hence, } f'(x) = 2ax - \frac{b}{x^2}$$

### Question: 3

Find the derivative

#### Solution:

$$\text{Let } f(x) = 3x^2 + 2x - 5$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = 3x^2 + 2x - 5$$

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 5$$

$$= 3(x^2 + h^2 + 2xh) + 2x + 2h - 5$$

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= 3x^2 + 3h^2 + 6xh + 2x + 2h - 5$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - (3x^2 + 2x - 5)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - 3x^2 - 2x + 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3h^2 + 6xh + 2h}{h}$$

$$= \lim_{h \rightarrow 0} 3h + 6x + 2$$

Putting  $h = 0$ , we get

$$f'(x) = 3(0) + 6x + 2$$

$$= 6x + 2$$

Hence,  $f'(x) = 6x + 2$

**Question: 4**

Find the derivative

**Solution:**

$$\text{Let } f(x) = x^3 - 2x^2 + x + 3$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = x^3 - 2x^2 + x + 3$$

$$f(x+h) = (x+h)^3 - 2(x+h)^2 + (x+h) + 3$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - [x^3 - 2x^2 + x + 3]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^3 - 2(x+h)^2 + (x+h) + 3 - x^3 + 2x^2 - x - 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(x+h)^3 - x^3] - 2[(x+h)^2 - x^2] + [x+h - x]}{h}$$

Using the identities:

$$(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$= \lim_{h \rightarrow 0} \frac{[x^3 + h^3 + 3xh^2 + 3x^2h - x^3] - 2[x^2 + h^2 + 2xh - x^2] + h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[h^3 + 3xh^2 + 3x^2h] - 2[h^2 + 2xh] + h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h[h^2 + 3xh + 3x^2] - 2h[h + 2x] + h}{h}$$

$$= \lim_{h \rightarrow 0} h^2 + 3xh + 3x^2 - 2h - 4x + 1$$

Putting  $h = 0$ , we get

$$f'(x) = (0)^2 + 2x(0) + 3x^2 - 2(0) - 4x + 1$$

$$= 3x^2 - 4x + 1$$

$$\text{Hence, } f'(x) = 3x^2 - 4x + 1$$

**Question: 5**

Find the derivative

**Solution:**

$$\text{Let } f(x) = x^8$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = x^8$$

$$f(x + h) = (x + h)^8$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x + h)^8 - x^8}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x + h)^8 - x^8}{(x + h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \rightarrow x} \frac{z^8 - x^8}{z - x} \text{ where } z = x + h \text{ and } z \rightarrow x \text{ as } h \rightarrow 0$$

$$= 8x^{8-1} \left[ \because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= 8x^7$$

$$\text{Hence, } f'(x) = 8x^7$$

### Question: 6

Find the derivative

#### Solution:

$$\text{Let } f(x) = \frac{1}{x^3}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{x^3}$$

$$f(x + h) = \frac{1}{(x + h)^3}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{(x + h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x + h)^{-3} - x^{-3}}{(x + h) - x}$$

[Add and subtract x in denominator]

$$= \lim_{z \rightarrow x} \frac{z^{-3} - x^{-3}}{z - x} \text{ where } z = x + h \text{ and } z \rightarrow x \text{ as } h \rightarrow 0$$

$$= (-3)x^{-3-1} \left[ \because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= -3x^{-4}$$

$$= -\frac{3}{x^4}$$

$$\text{Hence, } f'(x) = -\frac{3}{x^4}$$

### Question: 7

Find the derivative

**Solution:**

Let  $f(x) = \frac{1}{x^5}$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{x^5}$$

$$f(x+h) = \frac{1}{(x+h)^5}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^5} - \frac{1}{x^5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^{-5} - x^{-5}}{(x+h) - x}$$

[Add and subtract  $x$  in denominator]

$$= \lim_{z \rightarrow x} \frac{z^{-5} - x^{-5}}{z - x} \text{ where } z = x + h \text{ and } z \rightarrow x \text{ as } h \rightarrow 0$$

$$= (-5)x^{-5-1} \left[ \because \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1} \right]$$

$$= -5x^{-6}$$

$$= -\frac{5}{x^6}$$

Hence,  $f'(x) = -\frac{5}{x^6}$

**Question: 8**

Find the derivative

**Solution:**

Let  $f(x) = \sqrt{ax + b}$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \sqrt{ax + b}$$

$$f(x+h) = \sqrt{a(x+h) + b}$$

$$= \sqrt{ax + ah + b}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{ax + ah + b} - \sqrt{ax + b}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of

$$\sqrt{ax + ah + b} - \sqrt{ax + b}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{ax + ah + b} - \sqrt{ax + b}}{h} \times \frac{\sqrt{ax + ah + b} + \sqrt{ax + b}}{\sqrt{ax + ah + b} + \sqrt{ax + b}}$$

Using the formula:

$$(a + b)(a - b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{ax + ah + b})^2 - (\sqrt{ax + b})^2}{h(\sqrt{ax + ah + b} + \sqrt{ax + b})}$$

$$= \lim_{h \rightarrow 0} \frac{ax + ah + b - ax - b}{h(\sqrt{ax + ah + b} + \sqrt{ax + b})}$$

$$= \lim_{h \rightarrow 0} \frac{ah}{h(\sqrt{ax + ah + b} + \sqrt{ax + b})}$$

$$= \lim_{h \rightarrow 0} \frac{a}{\sqrt{ax + ah + b} + \sqrt{ax + b}}$$

Putting  $h = 0$ , we get

$$= \frac{a}{\sqrt{ax + a(0) + b} + \sqrt{ax + b}}$$

$$= \frac{a}{\sqrt{ax + b} + \sqrt{ax + b}}$$

$$= \frac{a}{2\sqrt{ax + b}}$$

$$\text{Hence, } f'(x) = \frac{a}{2\sqrt{ax+b}}$$

### Question: 9

Find the derivative

#### Solution:

$$\text{Let } f(x) = \sqrt{5x - 4}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \sqrt{5x - 4}$$

$$f(x+h) = \sqrt{5(x+h) - 4}$$

$$= \sqrt{5x + 5h - 4}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{5x + 5h - 4} - \sqrt{5x - 4}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{5x + 5h - 4} - \sqrt{5x - 4}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{5x + 5h - 4} - \sqrt{5x - 4}}{h} \times \frac{\sqrt{5x + 5h - 4} + \sqrt{5x - 4}}{\sqrt{5x + 5h - 4} + \sqrt{5x - 4}}$$

Using the formula:

$$\begin{aligned}
 & (a+b)(a-b) = (a^2 - b^2) \\
 &= \lim_{h \rightarrow 0} \frac{(\sqrt{5x+5h-4})^2 - (\sqrt{5x-4})^2}{h(\sqrt{5x+5h-4} + \sqrt{5x-4})} \\
 &= \lim_{h \rightarrow 0} \frac{5x+5h-4 - 5x+4}{h(\sqrt{5x+5h-4} + \sqrt{5x-4})} \\
 &= \lim_{h \rightarrow 0} \frac{5h}{h(\sqrt{5x+5h-4} + \sqrt{5x-4})} \\
 &= \lim_{h \rightarrow 0} \frac{5}{\sqrt{5x+5h-4} + \sqrt{5x-4}}
 \end{aligned}$$

Putting  $h = 0$ , we get

$$\begin{aligned}
 &= \frac{5}{\sqrt{5x+5(0)-4} + \sqrt{5x-4}} \\
 &= \frac{5}{\sqrt{5x-4} + \sqrt{5x-4}} \\
 &= \frac{5}{2\sqrt{5x-4}}
 \end{aligned}$$

$$\text{Hence, } f'(x) = \frac{5}{2\sqrt{5x-4}}$$

### Question: 10

Find the derivative

#### Solution:

$$\text{Let } f(x) = \frac{1}{\sqrt{x+2}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{\sqrt{x+2}}$$

$$f(x+h) = \frac{1}{\sqrt{x+h+2}}$$

Putting values in (i), we get

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h+2}} - \frac{1}{\sqrt{x+2}}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{x+2} - \sqrt{x+h+2}}{(\sqrt{x+h+2})(\sqrt{x+2})}}{h}
 \end{aligned}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{x+2} - \sqrt{x+h+2}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{x+h+2}}{h(\sqrt{x+h+2})(\sqrt{x+2})} \times \frac{\sqrt{x+2} + \sqrt{x+h+2}}{\sqrt{x+2} + \sqrt{x+h+2}}$$

Using the formula:

$$(a + b)(a - b) = (a^2 - b^2)$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{(\sqrt{x+2})^2 - (\sqrt{x+h+2})^2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})} \\ &= \lim_{h \rightarrow 0} \frac{x+2-x-h-2}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})} \\ &= \lim_{h \rightarrow 0} \frac{-h}{h(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})} \\ &= \lim_{h \rightarrow 0} \frac{-1}{(\sqrt{x+h+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+h+2})} \end{aligned}$$

Putting  $h = 0$ , we get

$$\begin{aligned} &= \frac{-1}{(\sqrt{x+0+2})(\sqrt{x+2})(\sqrt{x+2} + \sqrt{x+0+2})} \\ &= \frac{-1}{(\sqrt{x+2})^2(2\sqrt{x+2})} \\ &= \frac{-1}{2(\sqrt{x+2})^3} \end{aligned}$$

$$\text{Hence, } f'(x) = \frac{-1}{2(\sqrt{x+2})^3}$$

### Question: 11

Find the derivative

#### Solution:

$$\text{Let } f(x) = \frac{1}{\sqrt{2x+3}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{\sqrt{2x+3}}$$

$$f(x+h) = \frac{1}{\sqrt{2x+2h+3}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{2x+2h+3}} - \frac{1}{\sqrt{2x+3}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{(\sqrt{2x+2h+3})(\sqrt{2x+3})}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{2x+3} - \sqrt{2x+2h+3}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2x+3} - \sqrt{2x+2h+3}}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})} \times \frac{\sqrt{2x+3} + \sqrt{2x+2h+3}}{\sqrt{2x+3} + \sqrt{2x+2h+3}}$$

Using the formula:

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2x+3})^2 - (\sqrt{2x+2h+3})^2}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h \rightarrow 0} \frac{2x+3 - 2x - 2h - 3}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{h(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(\sqrt{2x+2h+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+2h+3})}$$

Putting  $h = 0$ , we get

$$= \frac{-2}{(\sqrt{2x+0+3})(\sqrt{2x+3})(\sqrt{2x+3} + \sqrt{2x+0+3})}$$

$$= \frac{-2}{(\sqrt{2x+3})^2 (2\sqrt{2x+3})}$$

$$= \frac{-2}{2(\sqrt{2x+3})^3}$$

$$= \frac{-1}{(\sqrt{2x+3})^3}$$

$$\text{Hence, } f'(x) = \frac{-1}{(\sqrt{2x+3})^3}$$

### Question: 12

Find the derivative

#### Solution:

$$\text{Let } f(x) = \frac{1}{\sqrt{6x-5}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{\sqrt{6x-5}}$$

$$f(x+h) = \frac{1}{\sqrt{6x+6h-5}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{6x+6h-5}} - \frac{1}{\sqrt{6x-5}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{6x-5} - \sqrt{6x+6h-5}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{6x-5} - \sqrt{6x+6h-5}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{6x-5} - \sqrt{6x+6h-5}}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})} \times \frac{\sqrt{6x-5} + \sqrt{6x+6h-5}}{\sqrt{6x-5} + \sqrt{6x+6h-5}}$$

Using the formula:

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{6x-5})^2 - (\sqrt{6x+6h-5})^2}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

$$= \lim_{h \rightarrow 0} \frac{6x-5 - 6x-6h+5}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

$$= \lim_{h \rightarrow 0} \frac{-6h}{h(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

$$= \lim_{h \rightarrow 0} \frac{-6}{(\sqrt{6x+6h-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6h-5})}$$

Putting  $h = 0$ , we get

$$= \frac{-6}{(\sqrt{6x+6(0)-5})(\sqrt{6x-5})(\sqrt{6x-5} + \sqrt{6x+6(0)-5})}$$

$$= \frac{-6}{(\sqrt{6x-5})^2 (2\sqrt{6x-5})}$$

$$= \frac{-6}{2(\sqrt{6x-5})^3}$$

$$= \frac{-3}{(\sqrt{6x-5})^3}$$

$$\text{Hence, } f'(x) = \frac{-3}{(\sqrt{6x-5})^3}$$

### Question: 13

Find the derivative

#### Solution:

$$\text{Let } f(x) = \frac{1}{\sqrt{2-3x}}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{1}{\sqrt{2-3x}}$$

$$f(x+h) = \frac{1}{\sqrt{2-3(x+h)}} = \frac{1}{\sqrt{2-3x-3h}}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{2-3x-3h}} - \frac{1}{\sqrt{2-3x}}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{h(\sqrt{2-3x})}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{2-3x} - \sqrt{2-3x-3h}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{2-3x} - \sqrt{2-3x-3h}}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})} \times \frac{\sqrt{2-3x} + \sqrt{2-3x-3h}}{\sqrt{2-3x} + \sqrt{2-3x-3h}}$$

Using the formula:

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2-3x})^2 - (\sqrt{2-3x-3h})^2}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \rightarrow 0} \frac{2-3x-2+3x+3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \rightarrow 0} \frac{3h}{h(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

$$= \lim_{h \rightarrow 0} \frac{3}{(\sqrt{2-3x-3h})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3h})}$$

Putting  $h = 0$ , we get

$$= \frac{3}{(\sqrt{2-3x-3(0)})(\sqrt{2-3x})(\sqrt{2-3x} + \sqrt{2-3x-3(0)})}$$

$$= \frac{3}{(\sqrt{2-3x})^2 (2\sqrt{2-3x})}$$

$$= \frac{3}{2(\sqrt{2-3x})^3}$$

$$\text{Hence, } f'(x) = \frac{3}{2(\sqrt{2-3x})^3}$$

#### Question: 14

Find the derivative

**Solution:**

$$\text{Let } f(x) = \frac{2x+3}{3x+2}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \dots(i)$$

$$f(x) = \frac{2x+3}{3x+2}$$

$$f(x+h) = \frac{2(x+h)+3}{3(x+h)+2} = \frac{2x+2h+3}{3x+3h+2}$$

Putting values in (i), we get

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{2x+2h+3}{3x+3h+2} - \frac{2x+3}{3x+2}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2x+2h+3)(3x+2) - (2x+3)(3x+3h+2)}{h((3x+3h+2)(3x+2))} \\ &= \lim_{h \rightarrow 0} \frac{6x^2 + 4x + 6xh + 4h + 9x + 6 - [6x^2 + 6xh + 4x + 9x + 9h + 6]}{h((3x+3h+2)(3x+2))} \\ &= \lim_{h \rightarrow 0} \frac{6x^2 + 4x + 6xh + 4h + 9x + 6 - 6x^2 - 6xh - 4x - 9x - 9h - 6}{h((3x+3h+2)(3x+2))} \\ &= \lim_{h \rightarrow 0} \frac{-5h}{h((3x+3h+2)(3x+2))} \\ &= \lim_{h \rightarrow 0} \frac{-5}{((3x+3h+2)(3x+2))} \end{aligned}$$

Putting  $h = 0$ , we get

$$\begin{aligned} &= \frac{-5}{((3x+3(0)+2)(3x+2))} \\ &= \frac{-5}{(3x+2)(3x+2)} \\ &= \frac{-5}{(3x+2)^2} \end{aligned}$$

$$\text{Hence, } f'(x) = \frac{-5}{(3x+2)^2}$$

### Question: 15

Find the derivative

#### Solution:

$$\text{Let } f(x) = \frac{5-x}{5+x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \frac{5-x}{5+x}$$

$$f(x+h) = \frac{5-(x+h)}{5+(x+h)} = \frac{5-x-h}{5+x+h}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5-x-h}{5+x+h} - \frac{5-x}{5+x}}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{(5-x-h)(5+x) - (5-x)(5+x+h)}{(5+x+h)(5+x)} \\
&= \lim_{h \rightarrow 0} \frac{25 + 5x - 5x - h - [25 + 5x + 5h - 5x - x^2 - xh]}{h(5+x+h)(5+x)} \\
&= \lim_{h \rightarrow 0} \frac{25 - x^2 - 5h - xh - 25 - 5h + x^2 + xh}{h(5+x+h)(5+x)} \\
&= \lim_{h \rightarrow 0} \frac{-10h}{h(5+x+h)(5+x)} \\
&= \lim_{h \rightarrow 0} \frac{-10}{(5+x+h)(5+x)}
\end{aligned}$$

Putting  $h = 0$ , we get

$$\begin{aligned}
&= \frac{-10}{(5+x+0)(5+x)} \\
&= \frac{-10}{(5+x)(5+x)} \\
&= \frac{-10}{(5+x)^2}
\end{aligned}$$

Hence,  $f'(x) = \frac{-10}{(5+x)^2}$

### Question: 16

Find the derivative

#### Solution:

Let  $f(x) = \frac{x^2+1}{x}$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \dots (i)$$

$$f(x) = \frac{x^2+1}{x}$$

$$f(x+h) = \frac{(x+h)^2+1}{x+h} = \frac{x^2+h^2+2xh+1}{x+h}$$

Putting values in (i), we get

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{x^2+h^2+2xh+1}{x+h} - \frac{x^2+1}{x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{\frac{(x^2+h^2+2xh+1)x - (x^2+1)(x+h)}{(x+h)x}}{h} \\
&= \lim_{h \rightarrow 0} \frac{x^3+xh^2+2x^2h+x - [x^3+x^2h+x+h]}{h(x+h)x} \\
&= \lim_{h \rightarrow 0} \frac{x^3+xh^2+2x^2h+x - x^3 - x^2h - x - h}{h(x+h)x}
\end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{xh^2 + x^2h - h}{h(x+h)(x)}$$

$$= \lim_{h \rightarrow 0} \frac{xh + x^2 - 1}{(x+h)(x)}$$

Putting  $h = 0$ , we get

$$= \frac{x(0) + x^2 - 1}{(x+0)(x)}$$

$$= \frac{x^2 - 1}{(x)^2}$$

$$\text{Hence, } f'(x) = \frac{x^2 - 1}{x^2}$$

### Question: 17

Find the derivative

#### Solution:

$$\text{Let } f(x) = \sqrt{\cos 3x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots(i)$$

$$f(x) = \sqrt{\cos 3x}$$

$$f(x+h) = \sqrt{\cos 3(x+h)}$$

$$= \sqrt{\cos(3x+3h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{\cos(3x+3h)} - \sqrt{\cos 3x}}{h} \times \frac{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}$$

Using the formula:

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{\cos(3x+3h)})^2 - (\sqrt{\cos 3x})^2}{h(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x})}$$

$$= \lim_{h \rightarrow 0} \frac{\cos(3x+3h) - \cos 3x}{h(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x})}$$

Using the formula:

$$\cos A - \cos B = -2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin \frac{3x+3h+3x}{2} \sin \frac{3x+3h-3x}{2}}{h(\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x})}$$

$$= \lim_{h \rightarrow 0} \frac{-2 \sin \frac{6x+3h}{2} \sin \frac{3h}{2}}{h\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}$$

$$= -2 \lim_{h \rightarrow 0} \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2} \lim_{h \rightarrow 0} \sin \left( \frac{6x+3h}{2} \right) \times \lim_{h \rightarrow 0} \frac{1}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}}$$

[Here, we multiply and divide by  $\frac{3}{2}$ ]

$$\begin{aligned} &= -2 \times \frac{3}{2} \lim_{h \rightarrow 0} \frac{\sin \frac{3h}{2}}{\frac{3h}{2}} \times \lim_{h \rightarrow 0} \sin \left( \frac{6x+3h}{2} \right) \times \lim_{h \rightarrow 0} \frac{1}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}} \\ &= -3 \times (1) \times \lim_{h \rightarrow 0} \sin \left( \frac{6x+3h}{2} \right) \times \lim_{h \rightarrow 0} \frac{1}{\sqrt{\cos(3x+3h)} + \sqrt{\cos 3x}} \end{aligned}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Putting  $h = 0$ , we get

$$\begin{aligned} &= -3 \times \sin \left[ \frac{6x+3(0)}{2} \right] \times \frac{1}{\sqrt{\cos(3x+3(0))} + \sqrt{\cos 3x}} \\ &= -3 \sin 3x \times \frac{1}{2\sqrt{\cos 3x}} \\ &= -\frac{3 \sin 3x}{2(\cos 3x)^{\frac{1}{2}}} \end{aligned}$$

$$\text{Hence, } f'(x) = -\frac{3 \sin 3x}{2(\cos 3x)^{\frac{1}{2}}}$$

### Question: 18

Find the derivative

#### Solution:

$$\text{Let } f(x) = \sqrt{\sec x}$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \sqrt{\sec x}$$

$$f(x+h) = \sqrt{\sec(x+h)}$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{\sec(x+h)} - \sqrt{\sec x}}{h}$$

Now rationalizing the numerator by multiplying and divide by the conjugate of  $\sqrt{\sec(x+h)} - \sqrt{\sec x}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{\sec(x+h)} - \sqrt{\sec x}}{h} \times \frac{\sqrt{\sec(x+h)} + \sqrt{\sec x}}{\sqrt{\sec(x+h)} + \sqrt{\sec x}}$$

Using the formula:

$$(a+b)(a-b) = (a^2 - b^2)$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{\sec(x+h)})^2 - (\sqrt{\sec x})^2}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \rightarrow 0} \frac{\sec(x+h) - \sec x}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{\cos x - \cos(x+h)}{\cos(x+h)\cos x}}{h(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \rightarrow 0} \frac{\cos x - \cos(x+h)}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

Using the formula:

$$\cos A - \cos B = 2 \sin\left(\frac{A+B}{2}\right) \sin\left(\frac{B-A}{2}\right)$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \frac{x+(x+h)}{2} \sin \frac{(x+h)-x}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2x+h}{2} \sin \frac{h}{2}}{h(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= 2 \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \\ \times \frac{1}{2} \lim_{h \rightarrow 0} \sin\left(\frac{2x+h}{2}\right) \\ \times \lim_{h \rightarrow 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

[Here, we multiply and divide by  $\frac{1}{2}$ ]

$$= 2 \times \frac{1}{2} \lim_{h \rightarrow 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} \\ \times \lim_{h \rightarrow 0} \sin\left(\frac{h}{2}\right) \times \lim_{h \rightarrow 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$= (1) \times \lim_{h \rightarrow 0} \sin\left(\frac{h}{2}\right) \times \lim_{h \rightarrow 0} \frac{1}{(\cos(x+h)\cos x)(\sqrt{\sec(x+h)} + \sqrt{\sec x})}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Putting  $h = 0$ , we get

$$\begin{aligned}
&= \sin[x + \frac{0}{2}] \times \frac{1}{\cos(x+0) \cos x (\sqrt{\sec(x+0)} + \sqrt{\sec x})} \\
&= \sin x \times \frac{1}{\cos x \cos x (\sqrt{\sec x} + \sqrt{\sec x})} \\
&= \frac{\sin x}{\cos^2 x (2\sqrt{\sec x})} \\
&= \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \times \frac{1}{2\sqrt{\sec x}} \\
&= \tan x \times \sec x \times \frac{1}{2\sqrt{\sec x}} \left[ \because \frac{\sin x}{\cos x} = \tan x \right] \& \left[ \frac{1}{\cos x} = \sec x \right] \\
&= \frac{1}{2} \tan x \sqrt{\sec x}
\end{aligned}$$

Hence,  $f'(x) = \frac{1}{2} \tan x \sqrt{\sec x}$

### Question: 19

Find the derivative

#### Solution:

Let  $f(x) = \tan^2 x$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \tan^2 x$$

$$f(x+h) = \tan^2(x+h)$$

Putting values in (i), we get

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{\tan^2(x+h) - \tan^2 x}{h} \\
&= \lim_{h \rightarrow 0} \frac{[\tan(x+h) - \tan x][\tan(x+h) + \tan x]}{h}
\end{aligned}$$

Using:

$$\begin{aligned}
\tan x &= \frac{\sin x}{\cos x} \\
&= \lim_{h \rightarrow 0} \frac{\left[ \frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x} \right] \left[ \frac{\sin(x+h)}{\cos(x+h)} + \frac{\sin x}{\cos x} \right]}{h} \\
&= \lim_{h \rightarrow 0} \frac{\left[ \frac{\sin(x+h)\cos x - \sin x \cos(x+h)}{\cos(x+h)\cos x} \right] \left[ \frac{\sin(x+h)\cos x + \sin x \cos(x+h)}{\cos(x+h)\cos x} \right]}{h} \\
&= \lim_{h \rightarrow 0} \frac{\{\sin[(x+h)-x]\}\{\sin[(x+h)+x]\}}{h[\cos^2(x+h)\cos^2 x]}
\end{aligned}$$

$[\because \sin A \cos B - \sin B \cos A = \sin(A-B)]$

$\& \sin A \cos B + \sin B \cos A = \sin(A+B)]$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{[\sin h][\sin(2x+h)]}{h[\cos^2(x+h)\cos^2 x]} \\
&= \frac{1}{\cos^2 x} \lim_{h \rightarrow 0} \frac{\sin h}{h} \times \lim_{h \rightarrow 0} \sin(2x+h) \times \lim_{h \rightarrow 0} \frac{1}{\cos^2(x+h)} \\
&= \frac{1}{\cos^2 x} \times (1) \times \lim_{h \rightarrow 0} \sin(2x+h) \times \lim_{h \rightarrow 0} \frac{1}{\cos^2(x+h)} \\
&\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]
\end{aligned}$$

Putting  $h = 0$ , we get

$$\begin{aligned}
&= \frac{1}{\cos^2 x} \times \sin(2x+0) \times \frac{1}{\cos^2(x+0)} \\
&= \frac{1}{\cos^2 x} \times \sin 2x \times \frac{1}{\cos^2 x} \\
&= \frac{1}{\cos^2 x} \times 2 \sin x \cos x \times \sec^2 x \\
&[\because \sin 2x = 2 \sin x \cos x] \\
&= 2 \frac{\sin x}{\cos x} \times \sec^2 x \left[ \because \frac{1}{\cos x} = \sec x \right] \\
&= 2 \tan x \sec^2 x \\
&\left[ \because \frac{\sin x}{\cos x} = \tan x \right]
\end{aligned}$$

Hence,  $f'(x) = 2 \tan x \sec^2 x$

### Question: 20

Find the derivative

#### Solution:

Let  $f(x) = \sin(2x + 3)$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \sin(2x + 3)$$

$$f(x+h) = \sin[2(x+h) + 3]$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sin[2(x+h) + 3] - \sin(2x+3)}{h}$$

Using the formula:

$$\begin{aligned}
\sin A - \sin B &= 2 \sin \frac{A-B}{2} \cos \frac{A+B}{2} \\
&= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2(x+h)+3-(2x+3)}{2} \cos \frac{2(x+h)+3+2x+3}{2}}{h} \\
&= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2x+2h+3-2x-3}{2} \cos \frac{2x+2h+6+2x}{2}}{h}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{2 \sin \frac{2h}{2} \cos \frac{4x + 2h + 6}{2}}{h} \\
&= \lim_{h \rightarrow 0} \frac{2 \sin(h) \cos(2x + h + 3)}{h} \\
&= 2 \lim_{h \rightarrow 0} \frac{\sin h}{h} \times \lim_{h \rightarrow 0} \cos(2x + h + 3) \\
&= 2(1) \times \lim_{h \rightarrow 0} \cos(2x + h + 3)
\end{aligned}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

Putting  $h = 0$ , we get

$$= 2 \cos(2x + 0 + 3)$$

$$= 2 \cos(2x + 3)$$

$$\text{Hence, } f'(x) = 2 \cos(2x + 3)$$

### Question: 21

Find the derivative

#### Solution:

$$\text{Let } f(x) = \tan(3x + 1)$$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \dots (i)$$

$$f(x) = \tan(3x + 1)$$

$$f(x+h) = \tan[3(x+h) + 1]$$

Putting values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{\tan[3(x+h) + 1] - \tan[3x + 1]}{h}$$

Using the formula:

$$\tan A - \tan B = \frac{\sin(A - B)}{\cos A \cos B}$$

$$= \lim_{h \rightarrow 0} \frac{\sin[3(x+h) + 1 - (3x+1)]}{\cos[3(x+h) + 1] \cos[3x + 1]}$$

$$= \lim_{h \rightarrow 0} \frac{\sin[3x + 3h + 1 - 3x - 1]}{\cos[3(x+h) + 1] \cos[3x + 1]}$$

$$= \lim_{h \rightarrow 0} \frac{\sin 3h}{h [\cos[3(x+h) + 1] \cos[3x + 1]]}$$

$$= \lim_{h \rightarrow 0} \frac{\sin 3h}{h} \times \lim_{h \rightarrow 0} \frac{1}{\cos[3(x+h) + 1] \cos[3x + 1]}$$

$$= \lim_{h \rightarrow 0} \frac{\sin 3h}{3h} \times 3 \times \lim_{h \rightarrow 0} \frac{1}{\cos[3(x+h) + 1] \cos[3x + 1]}$$

$$= 3(1) \times \lim_{h \rightarrow 0} \frac{1}{\cos[3(x+h) + 1] \cos[3x + 1]}$$

$$\left[ \because \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = 1 \right]$$

Putting h = 0, we get

$$= 3 \times \frac{1}{\cos[3(x+0)+1]\cos[3x+1]}$$

$$= \frac{3}{\cos[3x+1]\cos[3x+1]}$$

$$= \frac{3}{\cos^2(3x+1)}$$

$$= 3\sec^2(3x+1) \left[ \because \frac{1}{\cos x} = \sec x \right]$$

Hence,  $f'(x) = 3\sec^2(3x+1)$

## Exercise : 28C

### Question: 1

Differentiate:

#### Solution:

To find: Differentiation of  $x^2 \sin x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

$$(iii) \frac{d\sin x}{dx} = \cos x$$

Let us take  $u = x^2$  and  $v = \sin x$

$$u = \frac{du}{dx} = \frac{d(x^2)}{dx} = 2x$$

$$v = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$(x^2 \sin x)' = 2x \times \sin x + x^2 \times \cos x$$

$$= 2x \sin x + x^2 \cos x$$

$$\text{Ans) } 2x \sin x + x^2 \cos x$$

### Question: 2

Differentiation of  $e^x \cos x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

$$(ii) \frac{de^x}{dx} = e^x$$

$$(iii) \frac{d\cos x}{dx} = -\sin x$$

Let us take  $u = e^x$  and  $v = \cos x$

$$u = \frac{du}{dx} = \frac{de^x}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d\cos x}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$(e^x \cos x)' = e^x \times \cos x + e^x \times -\sin x$$

$$= e^x \cos x - e^x \sin x$$

$$= e^x (\cos x - \sin x)$$

$$\text{Ans) } e^x (\cos x - \sin x)$$

### Question: 3

Differentiation of  $e^x \cot x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

$$(ii) \frac{de^x}{dx} = e^x$$

$$(iii) \frac{dcotx}{dx} = -\operatorname{cosec}^2 x$$

Let us take  $u = e^x$  and  $v = \cot x$

$$u' = \frac{du}{dx} = \frac{de^x}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{dcotx}{dx} = -\operatorname{cosec}^2 x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$(e^x \cot x)' = e^x \times \cot x + e^x \times -\operatorname{cosec}^2 x$$

$$= e^x \cot x - e^x \operatorname{cosec}^2 x$$

$$= e^x (\cot x - \operatorname{cosec}^2 x)$$

$$\text{Ans) } e^x (\cot x - \operatorname{cosec}^2 x)$$

### Question: 4

Differentiation of  $x^n \cot x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

$$(iii) \frac{dcotx}{dx} = -\operatorname{cosec}^2 x$$

Let us take  $u = x^n$  and  $v = \cot x$

$$u' = \frac{du}{dx} = \frac{dx^n}{dx} = nx^{n-1}$$

$$v' = \frac{dv}{dx} = \frac{dcotx}{dx} = -\operatorname{cosec}^2 x$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$(x^n \cot x)' = nx^{n-1} \times \cot x + x^n \times -\operatorname{cosec}^2 x$$

$$= nx^{n-1} \cot x - x^n \operatorname{cosec}^2 x$$

$$= x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$$

Ans)  $x^n (nx^{-1}\cot x - \operatorname{cosec}^2 x)$

**Question: 5**

Differentiation of  $x^3 \sec x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

(iii)  $\frac{d\sec x}{dx} = \sec x \tan x$

Let us take  $u = x^3$  and  $v = \sec x$

$$u' = \frac{du}{dx} = \frac{dx^3}{dx} = 3x^2$$

$$v' = \frac{dv}{dx} = \frac{d\sec x}{dx} = \sec x \tan x$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$(x^3 \sec x)' = 3x^2 \times \sec x + x^3 \times \sec x \tan x$$

$$= 3x^2 \sec x + x^3 \sec x \tan x$$

$$= x^2 \sec x (3 + x \tan x)$$

Ans)  $x^2 \sec x (3 + x \tan x)$

**Question: 6**

Differentiation of  $(x^2 + 3x + 1) \sin x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

(iii)  $\frac{d\sin x}{dx} = \cos x$

Let us take  $u = x^2 + 3x + 1$  and  $v = \sin x$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 3x + 1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d\sin x}{dx} = \cos x$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$[(x^2 + 3x + 1) \sin x]' = (2x + 3) \times \sin x + (x^2 + 3x + 1) \times \cos x$$

$$= \sin x (2x + 3) + \cos x (x^2 + 3x + 1)$$

Ans)  $(2x + 3) \sin x + (x^2 + 3x + 1) \cos x$

**Question: 7**

Differentiation of  $x^4 \tan x$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

$$(iii) \frac{d \tan x}{dx} = \sec^2 x$$

Let us take  $u = x^4$  and  $v = \tan x$

$$u = \frac{du}{dx} = \frac{d(x^4)}{dx} = 4x^3$$

$$v = \frac{dv}{dx} = \frac{d \tan x}{dx} = \sec^2 x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$(x^4 \tan x)' = 4x^3 \times \tan x + x^4 \times \sec^2 x$$

$$= 4x^3 \tan x + x^4 \sec^2 x$$

$$= x^3 (4 \tan x + x \sec^2 x)$$

$$\text{Ans) } x^3 (4 \tan x + x \sec^2 x)$$

### Question: 8

Differentiation of  $(3x - 5)(4x^2 - 3 + e^x)$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

$$(iii) \frac{de^x}{dx} = e^x$$

Let us take  $u = (3x - 5)$  and  $v = (4x^2 - 3 + e^x)$

$$u = \frac{du}{dx} = \frac{d(3x - 5)}{dx} = 3$$

$$v = \frac{dv}{dx} = \frac{d(4x^2 - 3 + e^x)}{dx} = (8x + e^x)$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$[(3x - 5)(4x^2 - 3 + e^x)]' = 3 \times (4x^2 - 3 + e^x) + (3x - 5) \times (8x + e^x)$$

$$= 12x^2 - 9 + 3e^x + 24x^2 + 3xe^x - 40x - 5e^x$$

$$= 36x^2 + x(3e^x - 40) - 9 - 2e^x$$

$$\text{Ans) } 36x^2 + x(3e^x - 40) - 9 - 2e^x$$

### Question: 9

Differentiation of  $(x^2 - 4x + 5)(x^3 - 2)$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (x^2 - 4x + 5)$  and  $v = (x^3 - 2)$

$$u = \frac{du}{dx} = \frac{d(x^2 - 4x + 5)}{dx} = 2x - 4$$

$$v = \frac{dv}{dx} = \frac{d(x^3 - 2)}{dx} = 3x^2$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$\begin{aligned} [(x^2 - 4x + 5)(x^3 - 2)]' &= (2x - 4) \times (x^3 - 2) + (x^2 - 4x + 5) \times (3x^2) \\ &= 2x^4 - 4x^3 + 8 + 3x^4 - 12x^3 + 15x^2 \\ &= 5x^4 - 16x^3 + 15x^2 - 4x + 8 \end{aligned}$$

Ans)  $5x^4 - 16x^3 + 15x^2 - 4x + 8$

### Question: 10

Differentiation of  $(x^2 + 2x - 3)(x^2 + 7x + 5)$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

Let us take  $u = (x^2 + 2x - 3)$  and  $v = (x^2 + 7x + 5)$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 2x - 3)}{dx} = 2x + 2$$

$$v' = \frac{dv}{dx} = \frac{d(x^2 + 7x + 5)}{dx} = 2x + 7$$

Putting the above obtained values in the formula :-

$$(uv)' = u'v + uv'$$

$$\begin{aligned} [(x^2 + 2x - 3)(x^2 + 7x + 5)]' &= (2x + 2) \times (x^2 + 7x + 5) + (x^2 + 2x - 3) \times (2x + 7) \\ &= 2x^3 + 14x^2 + 10x + 2x^2 + 14x + 10 + 2x^3 + 7x^2 + 4x^2 + 14x - 6x - 21 \\ &= 4x^3 + 27x^2 + 32x - 11 \end{aligned}$$

Ans)  $4x^3 + 27x^2 + 32x - 11$

### Question: 11

Differentiation of  $(\tan x + \sec x)(\cot x + \cosec x)$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

(ii)  $\frac{d\tan x}{dx} = \sec^2 x$

(iii)  $\frac{d\sec x}{dx} = \sec x \tan x$

(iv)  $\frac{d\cot x}{dx} = -\cosec^2 x$

(v)  $\frac{d\cosec x}{dx} = -\cosec x \cot x$

Let us take  $u = (\tan x + \sec x)$  and  $v = (\cot x + \cosec x)$

$$u' = \frac{du}{dx} = \frac{d(\tan x + \sec x)}{dx} = \sec^2 x + \sec x \tan x = \sec x (\sec x + \tan x)$$

$$v' = \frac{dv}{dx} = \frac{d(\cot x + \cosec x)}{dx}$$

$$= -\cosec^2 x + (-\cosec x \cot x) = -\cosec x (\cosec x + \cot x)$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$[(\tan x + \sec x)(\cot x + \cosec x)]'$

$$= [\sec x (\sec x + \tan x)] \times [(\cot x + \csc x)] + [(\tan x + \sec x)] \times [-\csc x (\csc x + \cot x)]$$

$$= (\sec x + \tan x) [\sec x (\cot x + \csc x) - \csc x (\csc x + \cot x)]$$

$$= (\sec x + \tan x) (\sec x - \csc x) (\cot x + \csc x)$$

$$\text{Ans) } (\sec x + \tan x) (\sec x - \csc x) (\cot x + \csc x)$$

### Question: 12

Differentiation of  $(x^3 \cos x - 2^x \tan x)$

Formula used: (i)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

$$(iii) \frac{d\cos x}{dx} = -\sin x$$

$$(iv) \frac{da^x}{dx} = a^x \log a$$

$$(v) \frac{dtan x}{dx} = \sec^2 x$$

Here we have two function  $(x^3 \cos x)$  and  $(2^x \tan x)$

We have two differentiate them separately

Let us assume  $g(x) = (x^3 \cos x)$

And  $h(x) = (2^x \tan x)$

Therefore,  $f(x) = g(x) - h(x)$

$$\Rightarrow f'(x) = g'(x) - h'(x) \dots (i)$$

Applying product rule on  $g(x)$

Let us take  $u = x^3$  and  $v = \cos x$

$$u = \frac{du}{dx} = \frac{d(x^3)}{dx} = 3x^2$$

$$v = \frac{dv}{dx} = \frac{d(\cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$[x^3 \cos x]' = 3x^2 \times \cos x + x^3 \times -\sin x$$

$$= 3x^2 \cos x - x^3 \sin x$$

$$= x^2 (3\cos x - x \sin x)$$

$$g'(x) = x^2 (3\cos x - x \sin x)$$

Applying product rule on  $h(x)$

Let us take  $u = 2^x$  and  $v = \tan x$

$$u = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x \log 2$$

$$v = \frac{dv}{dx} = \frac{d(\tan x)}{dx} = \sec^2 x$$

Putting the above obtained values in the formula:-

$$(uv)' = u'v + uv'$$

$$[2^x \tan x]' = 2^x \log 2 \times \tan x + 2^x \times \sec^2 x$$

$$= 2^x (\log 2 \tan x + \sec^2 x)$$

$$h'(x) = 2^x (\log 2 \tan x + \sec^2 x)$$

Putting the above obtained values in eqn. (i)

$$f'(x) = x^2 (3 \cos x - x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$$

$$\text{Ans) } x^2 (3 \cos x - x \sin x) - 2^x (\log 2 \tan x + \sec^2 x)$$

## Exercise : 28D

### Question: 1

Differentiate

#### Solution:

To find: Differentiation of  $\frac{2^x}{x}$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule) (ii)  $\frac{da^x}{dx} = a^x \log a$

Let us take  $u = 2^x$  and  $v = x$

$$u' = \frac{du}{dx} = \frac{d(2^x)}{dx} = 2^x \log 2$$

$$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{2^x}{x}\right)' = \frac{2^x \log 2 \times x - 2^x \times 1}{(x)^2}$$

$$= \frac{2^x(x \log 2 - 1)}{x^2}$$

$$\text{Ans) } \frac{2^x(x \log 2 - 1)}{x^2}$$

### Question: 2

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule) (ii)  $\frac{d \log x}{dx} = \frac{1}{x}$

Let us take  $u = \log x$  and  $v = x$

$$u' = \frac{du}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

$$v' = \frac{dv}{dx} = \frac{d(x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{\log x}{x}\right)' = \frac{\frac{1}{x} \times x - \log x \times 1}{(x)^2}$$

$$= \frac{1 - \log x}{x^2}$$

$$\text{Ans}) = \frac{1 - \log x}{x^2}$$

**Question: 3**

$$\text{Formula used: (i)} \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)} \quad \text{(ii)} \frac{de^x}{dx} = e^x$$

Let us take  $u = e^x$  and  $v = (1+x)$

$$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d(1+x)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{e^x}{(1+x)}\right)' = \frac{e^x \times (1+x) - e^x \times 1}{(1+x)^2}$$

$$= \frac{x e^x}{(1+x)^2}$$

$$\text{Ans}) = \frac{x e^x}{(1+x)^2}$$

**Question: 4**

$$\text{Formula used: (i)} \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)} \quad \text{(ii)} \frac{de^x}{dx} = e^x$$

$$\text{(iii)} \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = e^x$  and  $v = (1+x^2)$

$$u' = \frac{du}{dx} = \frac{d(e^x)}{dx} = e^x$$

$$v' = \frac{dv}{dx} = \frac{d(1+x^2)}{dx} = 2x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{e^x}{(1+x^2)}\right)' = \frac{e^x \times (1+x^2) - e^x \times 2x}{(1+x^2)^2}$$

$$= \frac{e^x(x^2 - 2x + 1)}{(1+x^2)^2}$$

$$= \frac{e^x(x-1)^2}{(1+x^2)^2}$$

$$\text{Ans}) = \frac{e^x(x-1)^2}{(1+x^2)^2}$$

**Question: 5**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

Let us take  $u = (2x^2 - 4)$  and  $v = (3x^2 + 7)$

$$u' = \frac{du}{dx} = \frac{d(2x^2 - 4)}{dx} = 4x$$

$$v' = \frac{dv}{dx} = \frac{d(3x^2 + 7)}{dx} = 6x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(2x^2-4)}{(3x^2+7)}\right]' = \frac{4x \times (3x^2+7) - (2x^2-4) \times 6x}{(3x^2+7)^2}$$

$$= \frac{12x^3 + 28x - 12x^3 + 24x}{(3x^2+7)^2}$$

$$= \frac{52x}{(3x^2+7)^2}$$

$$\text{Ans}) = \frac{52x}{(3x^2+7)^2}$$

**Question: 6**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

Let us take  $u = (x^2 + 3x - 1)$  and  $v = (x + 2)$

$$u' = \frac{du}{dx} = \frac{d(x^2 + 3x - 1)}{dx} = 2x + 3$$

$$v' = \frac{dv}{dx} = \frac{d(x + 2)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{x^2 + 3x - 1}{x + 2}\right)' = \frac{(2x + 3) \times (x + 2) - (x^2 + 3x - 1) \times 1}{(x + 2)^2}$$

$$= \frac{2x^2 + 7x + 6 - x^2 - 3x + 1}{(x + 2)^2}$$

$$= \frac{x^2 + 4x + 7}{(x + 2)^2}$$

$$\text{Ans}) = \frac{x^2 + 4x + 7}{(x + 2)^2}$$

**Question: 7**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

Let us take  $u = (x^2 - 1)$  and  $v = (x^2 + 7x + 1)$

$$u' = \frac{du}{dx} = \frac{d(x^2-1)}{dx} = 2x$$

$$v' = \frac{dv}{dx} = \frac{d(x^2+7x+1)}{dx} = 2x + 7$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(x^2-1)}{(x^2+7x+1)}\right]' = \frac{2x \times (x^2+7x+1) - (x^2-1) \times (2x+7)}{(x^2+7x+1)^2}$$

$$= \frac{2x^3 + 14x^2 + 2x - 2x^3 - 7x^2 + 2x + 7}{(x^2+7x+1)^2}$$

$$= \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$$

$$\text{Ans}) = \frac{7x^2 + 4x + 7}{(x^2+7x+1)^2}$$

### Question: 8

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

Let us take  $u = (5x^2 + 6x + 7)$  and  $v = (2x^2 + 3x + 4)$

$$u' = \frac{du}{dx} = \frac{d(5x^2 + 6x + 7)}{dx} = 10x + 6$$

$$v' = \frac{dv}{dx} = \frac{d(2x^2 + 3x + 4)}{dx} = 4x + 3$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{5x^2 + 6x + 7}{2x^2 + 3x + 4}\right)' = \frac{(10x+6) \times (2x^2+3x+4) - (5x^2+6x+7) \times (4x+3)}{(2x^2+3x+4)^2}$$

$$= \frac{20x^3 + 30x^2 + 40x + 12x^2 + 18x + 24 - 20x^3 - 15x^2 - 24x^2 - 18x - 28x - 21}{(2x^2 + 3x + 4)^2}$$

$$= \frac{3x^2 + 12x + 3}{(2x^2 + 3x + 4)^2}$$

$$= \frac{3(x^2 + 4x + 1)}{(2x^2 + 3x + 4)^2}$$

$$\text{Ans}) = \frac{3(x^2 + 4x + 1)}{(2x^2 + 3x + 4)^2}$$

### Question: 9

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

Let us take  $u = (x)$  and  $v = (a^2 + x^2)$

$$u' = \frac{du}{dx} = \frac{d(x)}{dx} = 1$$

$$v' = \frac{dv}{dx} = \frac{d(a^2+x^2)}{dx} = 2x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{x}{(a^2+x^2)}\right]' = \frac{1 \times (a^2+x^2) - (x) \times (2x)}{(a^2+x^2)^2}$$

$$= \frac{a^2+x^2 - 2x^2}{(a^2+x^2)^2}$$

$$= \frac{a^2 - x^2}{(a^2+x^2)^2}$$

$$\text{Ans}) = \frac{a^2 - x^2}{(a^2+x^2)^2}$$

### Question: 10

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

$$(iii) \frac{d\sin x}{dx} = \cos x$$

Let us take  $u = (x^4)$  and  $v = (\sin x)$

$$u' = \frac{du}{dx} = \frac{d(x^4)}{dx} = 4x^3$$

$$v' = \frac{dv}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{x^4}{\sin x}\right]' = \frac{4x^3 \times (\sin x) - (x^4) \times (\cos x)}{(\sin x)^2}$$

$$= \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$$

$$\text{Ans}) = \frac{x^3[4(\sin x) - x(\cos x)]}{(\sin x)^2}$$

### Question: 11

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dx^n}{dx} = nx^{n-1}$

Let us take  $u = (\sqrt{a} + \sqrt{x})$  and  $v = (\sqrt{a} - \sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(\sqrt{a} + \sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{a} - \sqrt{x})}{dx} = -\frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}\right]' = \frac{\frac{1}{2\sqrt{x}} \times (\sqrt{a} - \sqrt{x}) - (\sqrt{a} + \sqrt{x}) \times -\frac{1}{2\sqrt{x}}}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{a}}{2\sqrt{x}} - \frac{1}{2} + \frac{\sqrt{a}}{2\sqrt{x}} + \frac{1}{2}}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a} - \sqrt{x})^2}$$

$$\text{Ans) } = \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a} - \sqrt{x})^2}$$

### Question: 12

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule) (ii)  $\frac{d\cos x}{dx} = -\sin x$

$$(iii) \frac{d\log x}{dx} = \frac{1}{x}$$

Let us take  $u = (\cos x)$  and  $v = (\log x)$

$$u' = \frac{du}{dx} = \frac{d(\cos x)}{dx} = -\sin x$$

$$v' = \frac{dv}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\cos x}{\log x}\right]' = \frac{-\sin x \times (\log x) - (\cos x) \times \left(\frac{1}{x}\right)}{(\log x)^2}$$

$$= \frac{-x\sin x(\log x) - (\cos x)}{x(\log x)^2}$$

$$\text{Ans) } = \frac{-x\sin x(\log x) - (\cos x)}{x(\log x)^2}$$

### Question: 13

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule) (ii)  $\frac{dcot x}{dx} = -\operatorname{cosec}^2 x$

$$(iii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (2\cot x)$  and  $v = (\sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(2\cot x)}{dx} = -2\operatorname{cosec}^2 x$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\begin{aligned} \left[\frac{2\cot x}{\sqrt{x}}\right]' &= \frac{-2\operatorname{cosec}^2 x \times (\sqrt{x}) - (2\cot x) \times \left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2} \\ &= \frac{-2x\operatorname{cosec}^2 x - (\cot x)}{\sqrt{x}(\sqrt{x})^2} \end{aligned}$$

$$\text{Ans) } \frac{-2x\operatorname{cosec}^2 x - \cot x}{x^{3/2}}$$

#### Question: 14

$$\text{Formula used: (i) } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)} \quad (\text{ii) } \frac{d\cos x}{dx} = -\sin x$$

$$(iii) \frac{d\sin x}{dx} = \cos x$$

Let us take  $u = (\sin x)$  and  $v = (1 + \cos x)$

$$u' = \frac{du}{dx} = \frac{d(\sin x)}{dx} = \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sin x}{(1 + \cos x)}\right]' = \frac{\cos x \times (1 + \cos x) - (\sin x) \times (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2}$$

$$= \frac{1}{(1 + \cos x)}$$

$$\text{Ans) } \frac{1}{1 + \cos x}$$

#### Question: 15

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dsinx}{dx} = \cos x$

Let us take  $u = (1 + \sin x)$  and  $v = (1 - \sin x)$

$$u' = \frac{du}{dx} = \frac{d(1 + \sin x)}{dx} = \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(1 - \sin x)}{dx} = -\cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{1 + \sin x}{1 - \sin x}\right]' = \frac{\cos x \times (1 - \sin x) - (1 + \sin x) \times (-\cos x)}{(1 - \sin x)^2}$$

$$= \frac{\cos x - \cos x \sin x + \cos x + \cos x \sin x}{(1 - \sin x)^2}$$

$$= \frac{2\cos x}{(1 - \sin x)^2}$$

$$\text{Ans}) = \frac{2\cos x}{(1 - \sin x)^2}$$

### Question: 16

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{d\cos x}{dx} = -\sin x$

Let us take  $u = (1 - \cos x)$  and  $v = (1 + \cos x)$

$$u' = \frac{du}{dx} = \frac{d(1 - \cos x)}{dx} = \sin x$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \cos x)}{dx} = -\sin x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{1 - \cos x}{1 + \cos x}\right]' = \frac{\sin x \times (1 + \cos x) - (1 - \cos x) \times (-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\sin x + \sin x \cos x + \sin x - \sin x \cos x}{(1 + \cos x)^2}$$

$$= \frac{2\sin x}{(1 + \cos x)^2}$$

$$\text{Ans}) = \frac{2\sin x}{(1 + \cos x)^2}$$

### Question: 17

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{dsinx}{dx} = \cos x$

$$(iii) \frac{d\cos x}{dx} = -\sin x$$

Let us take  $u = (\sin x - \cos x)$  and  $v = (\sin x + \cos x)$

$$u' = \frac{du}{dx} = \frac{d(\sin x - \cos x)}{dx} = (\cos x + \sin x)$$

$$v' = \frac{dv}{dx} = \frac{d(\sin x + \cos x)}{dx} = (\cos x - \sin x)$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sin x - \cos x}{\sin x + \cos x}\right]' = \frac{(\cos x + \sin x) \times (\sin x + \cos x) - (\sin x - \cos x) \times (\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x - (\sin x - \cos x) \times -(\sin x - \cos x)}{(\sin x + \cos x)^2}$$

$$= \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x + \sin^2 x + \cos^2 x - 2\sin x \cos x}{(\sin x + \cos x)^2}$$

$$= \frac{2(\sin^2 x + \cos^2 x)}{\sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$= \frac{2}{1 + \sin 2x}$$

$$\text{Ans}) = \frac{2}{1 + \sin 2x}$$

### Question: 18

$$\text{Formula used: (i) } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)} \text{ (ii) } \frac{d \sec x}{dx} = \sec x \tan x$$

$$\text{(iii) } \frac{d \tan x}{dx} = \sec^2 x$$

Let us take  $u = (\sec x - \tan x)$  and  $v = (\sec x + \tan x)$

$$u' = \frac{du}{dx} = \frac{d(\sec x - \tan x)}{dx} = (\sec x \tan x - \sec^2 x)$$

$$v' = \frac{dv}{dx} = \frac{d(\sec x + \tan x)}{dx} = (\sec x \tan x + \sec^2 x)$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{\sec x - \tan x}{\sec x + \tan x}\right]' = \frac{(\sec x \tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x \tan x - \sec^2 x)(\sec x + \tan x) - (\sec x - \tan x)(\sec x)(\tan x + \sec x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\sec x \tan x - \sec^2 x) - (\sec x - \tan x)(\sec x)]}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\sec x \tan x - \sec^2 x) - (\sec^2 x - \sec x \tan x)]}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[2\sec x \tan x - 2\sec^2 x]}{(\sec x + \tan x)^2}$$

$$= \frac{2\sec x[\tan x - \sec x]}{(\sec x + \tan x)}$$

$$\text{Ans) } = \frac{2\sec x[\tan x - \sec x]}{(\sec x + \tan x)}$$

**Question: 19**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{d\sin x}{dx} = \cos x$

$$(iii) \frac{d\log x}{dx} = \frac{1}{x}$$

$$(iv) \frac{de^x}{dx} = e^x$$

Let us take  $u = (e^x + \sin x)$  and  $v = (1 + \log x)$

$$u' = \frac{du}{dx} = \frac{d(e^x + \sin x)}{dx} = (e^x + \cos x)$$

$$v' = \frac{dv}{dx} = \frac{d(1 + \log x)}{dx} = \frac{1}{x}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\begin{aligned} \left[\frac{e^x + \sin x}{1 + \log x}\right]' &= \frac{(e^x + \cos x) \times (1 + \log x) - (e^x + \sin x) \times \left(\frac{1}{x}\right)}{(1 + \log x)^2} \\ &= \frac{x(e^x + \cos x)(1 + \log x) - (e^x + \sin x)}{x(1 + \log x)^2} \end{aligned}$$

$$\text{Ans) } = \frac{x(e^x + \cos x)(1 + \log x) - (e^x + \sin x)}{x(1 + \log x)^2}$$

**Question: 20**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)(ii)  $\frac{d\sin x}{dx} = \cos x$

$$(iii) \frac{d\sec x}{dx} = \sec x \tan x$$

$$(iv) \frac{de^x}{dx} = e^x$$

(v)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = (e^x \sin x)$  and  $v = (\sec x)$

$$u' = \frac{du}{dx} = \frac{d(e^x \sin x)}{dx}$$

Applying Product rule

$$(gh)' = g'h + gh'$$

Taking  $g = e^x$  and  $h = \sin x$

$$= e^x \sin x + e^x \cos x$$

$$u' = e^x \sin x + e^x \cos x$$

$$v' = \frac{dv}{dx} = \frac{d(\sec x)}{dx} = \sec x \tan x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\begin{aligned} \left[\frac{e^x \sin x}{\sec x}\right]' &= \frac{(e^x \sin x + e^x \cos x) \times (\sec x) - (e^x \sin x) \times (\sec x \tan x)}{(\sec x)^2} \\ &= \frac{(e^x \sin x + e^x \cos x) - (e^x \sin x) \times (\tan x)}{(\sec x)} \\ &= \cos x [(e^x \sin x + e^x \cos x) - (e^x \sin x) \times (\tan x)] \\ &= [(e^x \sin x \cos x + e^x \cos^2 x) - (e^x \sin x \cos x) \times (\tan x)] \\ &= [(e^x \sin x \cos x + e^x \cos^2 x) - (e^x \sin^2 x)] \\ &= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x) \\ &= (e^x \sin x \cos x + e^x \cos^2 x - e^x \sin^2 x) \\ &= e^x (\sin x \cos x + \cos 2x) \\ \text{Ans) } &= e^x (\sin x \cos x + \cos 2x) \end{aligned}$$

**Question: 21**

$$\text{Formula used: (i) } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)} \quad (\text{ii) } \frac{d \cot x}{dx} = -\operatorname{cosec}^2 x$$

$$(\text{iii) } \frac{dx^n}{dx} = nx^{n-1}$$

$$(\text{iv) } \frac{da^x}{dx} = a^x \log a$$

$$(\text{v) } (uv)' = u'v + uv' \text{ (Leibnitz or product rule)}$$

Let us take  $u = (2^x \cot x)$  and  $v = (\sqrt{x})$

$$u' = \frac{du}{dx} = \frac{d(2^x \cot x)}{dx}$$

Applying Product rule

$$(gh)' = g'h + gh'$$

Taking  $g = 2^x$  and  $h = \cot x$

$$= (2^x \log 2) \cot x + 2^x (-\operatorname{cosec}^2 x)$$

$$u' = (2^x \log 2) \cot x - 2^x (\operatorname{cosec}^2 x)$$

$$u' = 2^x [\log 2 \cot x - \operatorname{cosec}^2 x]$$

$$v' = \frac{dv}{dx} = \frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[ \frac{2^x \cot x}{\sqrt{x}} \right]' = \frac{\{2^x [\log 2 \cot x - \cosec^2 x] \times \sqrt{x}\} - \{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\}}{(\sqrt{x})^2}$$

$$= \frac{\{2^x [\log 2 \cot x - \cosec^2 x] \times \sqrt{x}\} - \{(2^x \cot x) \times \left(\frac{1}{2\sqrt{x}}\right)\}}{x}$$

$$= \frac{\{2^x [\log 2 \cot x - \cosec^2 x] \times \sqrt{x}\} - \{(2^{x-1} \cot x) \times \left(\frac{1}{\sqrt{x}}\right)\}}{x}$$

$$= \frac{\{x 2^x [\log 2 \cot x - \cosec^2 x]\} - \{(2^{x-1} \cot x)\}}{x \sqrt{x}}$$

$$= \frac{\{2^x [x \log 2 \cot x - x \cosec^2 x]\} - \{(2^{x-1} \cot x)\}}{x^{\frac{3}{2}}}$$

$$\text{Ans) } = \frac{\{2^x [x \log 2 \cot x - x \cosec^2 x]\} - \{(2^{x-1} \cot x)\}}{x^{\frac{3}{2}}}$$

**Question: 22**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule) (ii)  $\frac{de^x}{dx} = e^x$

$$(iii) \frac{dx^n}{dx} = nx^{n-1}$$

(iv)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = e^x(x-1)$  and  $v = (x+1)$

$$u' = \frac{du}{dx} = \frac{d[e^x(x-1)]}{dx}$$

Applying Product rule

$$(gh)' = g'h + gh'$$

Taking  $g = e^x$  and  $h = x - 1$

$$[e^x(x-1)]' = e^x(x-1) + e^x(1)$$

$$= e^x(x-1) + e^x$$

$$u' = e^x x$$

$$v' = \frac{dv}{dx} = \frac{d(x+1)}{dx} = 1$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[ \frac{e^x(x-1)}{(x+1)} \right]' = \frac{(e^x x)(x+1) - [e^x(x-1)](1)}{(x+1)^2}$$

$$= \frac{e^x x^2 + e^x x - e^x x + e^x}{(x+1)^2}$$

$$= \frac{e^x x^2 + e^x}{(x+1)^2}$$

$$= \frac{e^x(x^2 + 1)}{(x+1)^2}$$

$$\text{Ans) } = \frac{e^x(x^2 + 1)}{(x+1)^2}$$

**Question: 23**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d\sec x}{dx} = \sec x \tan x$$

$$(iii) \frac{dtan x}{dx} = \sec^2 x \quad (iii) \frac{dx^n}{dx} = nx^{n-1}$$

(iv)  $(uv)' = u'v + uv'$  (Leibnitz or product rule)

Let us take  $u = (x \tan x)$  and  $v = (\sec x + \tan x)$

$$u' = \frac{du}{dx} = \frac{d[x \tan x]}{dx}$$

Applying Product rule for finding  $u'$

$$(gh)' = g'h + gh'$$

Taking  $g = x$  and  $h = \tan x$

$$[x \tan x]' = (1)(\tan x) + x(\sec^2 x)$$

$$= \tan x + x \sec^2 x$$

$$u' = \tan x + x \sec^2 x$$

$$v' = \frac{dv}{dx} = \frac{d(\sec x + \tan x)}{dx} = \sec x \tan x + \sec^2 x$$

$$v' = \sec x (\tan x + \sec x)$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{x \tan x}{(\sec x + \tan x)}\right]' = \frac{(\tan x + x \sec^2 x)(\sec x + \tan x) - [x \tan x][\sec x(\tan x + \sec x)]}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)[(\tan x + x \sec^2 x) - (x \tan x)(\sec x)]}{(\sec x + \tan x)^2}$$

$$= \frac{[\tan x + x \sec^2 x - x \tan x \sec x]}{(\sec x + \tan x)}$$

$$= \frac{\tan x + x \sec x (\sec x - \tan x)}{(\sec x + \tan x)}$$

$$\text{Ans) } = \frac{\tan x + x \sec x (\sec x - \tan x)}{(\sec x + \tan x)}$$

**Question: 24**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{dx^n}{dx} = nx^{n-1}$$

Let us take  $u = (ax^2 + bx + c)$  and  $v = (px^2 + qx + r)$

$$u' = \frac{du}{dx} = \frac{d[ax^2 + bx + c]}{dx} = 2ax + b$$

$$v' = \frac{dv}{dx} = \frac{d(px^2 + qx + r)}{dx} = 2px + q$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\begin{aligned} \left[\frac{ax^2 + bx + c}{px^2 + qx + r}\right]' &= \frac{(2ax + b)(px^2 + qx + r) - [ax^2 + bx + c][2px + q]}{(px^2 + qx + r)^2} \\ &= \frac{2apx^3 + 2aqx^2 + 2axr + bpx^2 + bqx + br - [2apx^3 + qax^2 + 2bpx^2 + bqx + 2pcx + cq]}{(px^2 + qx + r)^2} \end{aligned}$$

$$= \frac{(aq - bp)x^2 + 2(ra - pc)x + br - cp}{(px^2 + qx + r)^2}$$

$$\text{Ans) } = \frac{(aq - bp)x^2 + 2(ra - pc)x + br - cp}{(px^2 + qx + r)^2}$$

### Question: 25

$$\text{Formula used: (i) } \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$(ii) \frac{dsinx}{dx} = \cos x$$

$$(iii) \frac{d\cos x}{dx} = -\sin x \text{ (iv) } (uv)' = u'v + uv' \text{ (Leibnitz or product rule)}$$

Let us take  $u = (\sin x - x \cos x)$  and  $v = (x \sin x + \cos x)$

$$u' = \frac{du}{dx} = \frac{d[\sin x - x \cos x]}{dx}$$

Applying Product rule for finding the term  $x \cos x$  in  $u'$

$$(gh)' = g'h + gh'$$

Taking  $g = x$  and  $h = \cos x$

$$[x \cos x]' = (1)(\cos x) + x(-\sin x)$$

$$[x \cos x]' = \cos x - x \sin x$$

Applying the above obtained value for finding  $u'$

$$u' = \cos x - (\cos x - x \sin x)$$

$$u' = x \sin x$$

$$v' = \frac{dv}{dx} = \frac{d(x \sin x + \cos x)}{dx}$$

Applying Product rule for finding the term  $x \sin x$  in  $v'$

$$(gh)' = g'h + gh'$$

Taking g = x and h = sinx

$$[x \sin x]' = (1) (\sin x) + x (\cos x)$$

$$[x \sin x]' = \sin x + x \cos x$$

Applying the above obtained value for finding v'

$$v' = \sin x + x \cos x - \sin x$$

$$v' = x \cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left[\frac{(\sin x - x \cos x)}{(x \sin x + \cos x)}\right]' = \frac{(x \sin x)(x \sin x + \cos x) - (\sin x - x \cos x)(x \cos x)}{(x \sin x + \cos x)^2}$$

$$= \frac{(x^2 \sin^2 x + x \sin x \cos x) - (x \sin x \cos x - x^2 \cos^2 x)}{(x \sin x + \cos x)^2}$$

$$= \frac{x^2 \sin^2 x + x \sin x \cos x - x \sin x \cos x + x^2 \cos^2 x}{(x \sin x + \cos x)^2}$$

$$= \frac{x^2(\sin^2 x + \cos^2 x)}{(x \sin x + \cos x)^2}$$

$$= \frac{x^2}{(x \sin x + \cos x)^2}$$

$$\text{Ans}) = \frac{x^2}{(x \sin x + \cos x)^2}$$

**Question: 26**

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

(ii)  $\frac{d \sin x}{dx} = \cos x$

(iii)  $\frac{d \cos x}{dx} = -\sin x$

We can write cot x as  $\frac{\cos x}{\sin x}$

Let us take u = cosx and v = sinx

$$u' = (\cos x)' = -\sin x$$

$$v' = (\sin x)' = \cos x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{\cos x}{\sin x}\right)' = \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \frac{-1}{\sin^2 x}$$

$$= -\operatorname{cosec}^2 x$$

Ans).  $-\operatorname{cosec}^2 x$

(ii) To find: Differentiation of  $\sec x$

Formula used: (i)  $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$  where  $v \neq 0$  (Quotient rule)

$$(ii) \frac{d\cos x}{dx} = -\sin x$$

We can write  $\sec x$  as  $\frac{1}{\cos x}$

Let us take  $u = 1$  and  $v = \cos x$

$$u' = (1)' = 0$$

$$v' = (\cos x)' = -\sin x$$

Putting the above obtained values in the formula:-

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2} \text{ where } v \neq 0 \text{ (Quotient rule)}$$

$$\left(\frac{1}{\cos x}\right)' = \frac{(0)(\cos x) - (1)(-\sin x)}{(\cos x)^2}$$

$$= \frac{\sin x}{\cos^2 x} = \sec x \tan x$$

Ans).  $-\operatorname{cosec}^2 x$

## Exercise : 28E

**Question: 1**

Differentiate the

**Solution:**

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\sin nu) = \cos(nu) \frac{d}{dx}(nu)$

Let us take  $y = \sin 4x$ .

So, by using the above formula, we have

$$\frac{d}{dx}(\sin 4x) = \cos(4x) \times \frac{d}{dx}(4x) = 4\cos 4x.$$

Differentiation of  $y = \sin 4x$  is  $4\cos 4x$

**Question: 2**

Differentia

**Solution:**

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\cos nu) = -\sin(nu) \frac{d}{dx}(nu)$ .

Let us take  $y = \cos 5x$ .

So, by using the above formula, we have

$$\frac{d}{dx}(\cos 5x) = -\sin(5x) \times \frac{d}{dx}(5x) = -5\sin 5x.$$

Differentiation of  $y = \cos 5x$  is  $-5\sin 5x$

### Question: 3

Differentia

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\tan nu) = \sec^2(nu) \cdot \frac{d}{dx}(nu)$ .

Let us take  $y = \tan 3x$

So, by using the above formula, we have

$$\frac{d}{dx}\tan 3x = \sec^2(3x) \times \frac{d}{dx}(3x) = 3\sec^2(3x)$$

Differentiation of  $y = \tan 3x$  is  $3\sec^2(3x)$

### Question: 4

Differentia

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\cos nu) = -\sin(nu) \frac{d}{dx}(nu)$  and  $\frac{d}{dx}(nx^n) = nx^{n-1}$

Let us take  $y = \cos x^3$

So, by using the above formula, we have

$$\frac{d}{dx}\cos x^3 = -\sin(x^3) \times \frac{d}{dx}(x^3) = -3x^2 \sin(x^3)$$

Differentiation of  $y = \cos x^3$  is  $-3x^2 \sin(x^3)$

### Question: 5

Differentia

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\cot^a nu) = a\cot^{a-1}(nu) \times \frac{d}{dx}(\cot nu) \times \frac{d}{dx}(nu)$  and  $\frac{d}{dx}(nx^n) = nx^{n-1}$

Let us take  $y = \cot^2 x$

So, by using the above formula, we have

$$\frac{d}{dx}\cot^2 x = 2\cot(x) \times \frac{d\cot x}{dx} \times \frac{dx}{dx} = -2\cot x (\operatorname{cosec}^2 x).$$

Differentiation of  $y = \cot^2 x$  is  $-2\cot x (\operatorname{cosec}^2 x)$

### Question: 6

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\tan^a nu) = \tan^{a-1} nu \times \frac{d(\tan nu)}{dx} \times \frac{d(nu)}{dx}$  and  $\frac{d}{dx}(x^n) = nx^{n-1}$

Let us take  $y = \tan^3 x$

So, by using the above formula, we have

$$\frac{d}{dx} \tan^3 x = 3\tan^2(x) \times \frac{d(\tan x)}{dx} \times \frac{dx}{dx} = 3\tan^2 x \times (\sec^2 x).$$

Differentiation of  $y = \tan^3 x$  is  $3\tan^2 x \times (\sec^2 x)$

### Question: 7

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\tan \sqrt{nu}) = \sec^2 \sqrt{nu} \times \frac{d}{dx}(\sqrt{nu}) \frac{d}{dx}(nu).$

Let us take  $y = \tan \sqrt{x}$

So, by using the above formula, we have

$$\frac{d}{dx} \tan \sqrt{x} = \sec^2(\sqrt{x}) \times \frac{d}{dx}(\sqrt{x}) \frac{d}{dx}(x) = \frac{1}{2} \cdot \frac{\sec^2(\sqrt{x})}{\sqrt{x}}$$

Differentiation of  $y = \tan \sqrt{x}$  is  $\frac{1}{2} \cdot \frac{\sec^2(\sqrt{x})}{\sqrt{x}}$

### Question: 8

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(e^{at}) = e^{at} \times \frac{d}{dx}(at)$  and  $\frac{d}{dx}(x^n) = nx^{n-1}$

Let us take  $y = e^{x^2}$

So, by using the above formula, we have

$$\frac{d}{dx} e^{x^2} = e^{x^2} \times \frac{d}{dx}(x^2) = 2xe^{x^2}$$

Differentiation of  $y = e^{x^2}$  is  $2xe^{x^2}$

### Question: 9

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(e^a) = e^a \times \frac{da}{dx}$  and  $\frac{d}{dx}(x^n) = nx^{n-1}$

Let us take  $y = e^{\cot x}$

So, by using the above formula, we have

$$\frac{d}{dx} e^{\cot x} = e^{\cot x} \times \frac{d \cot x}{dx} = -e^{\cot x} \operatorname{cosec}^2 x.$$

Differentiation of  $y = e^{\cot x}$  is  $-e^{\cot x} \operatorname{cosec}^2 x$

### Question: 10

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\sqrt{\sin u}) = \frac{1}{2\sqrt{\sin u}} \times \frac{d}{dx}(\sin u) \times \frac{d}{dx}(u)$  and  $\frac{d}{dx}(x^n) = nx^{n-1}$

Let us take  $y = \sqrt{\sin x}$

So, by using the above formula, we have

$$\frac{d}{dx} \sqrt{\sin x} = \frac{1}{2\sqrt{\sin x}} \times \frac{d}{dx}(\sin x) \frac{d}{dx}(x) = \frac{1}{2\sqrt{\sin x}} \cos x$$

Differentiation of  $y = \sqrt{\sin x}$  is  $\frac{1}{2\sqrt{\sin x}} \cos x$

### Question: 11

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take  $y = (5 + 7x)^6$

So, by using the above formula, we have

$$\frac{d}{dx}(5 + 7x)^6 = 6(5 + 7x)^5 \times \frac{d}{dx}(5 + 7x) = 6(5 + 7x)^5 \times 7 = 42(5 + 7x)^5$$

Differentiation of  $y = (5 + 7x)^6$  is  $42(5 + 7x)^5$

### Question: 12

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take  $y = (3 - 4x)^5$

So, by using the above formula, we have

$$\frac{d}{dx}(3 - 4x)^5 = 4(3 - 4x)^5 \times \frac{d}{dx}(3 - 4x) = 4(3 - 4x)^5 \times (-4) = -16(3 - 4x)^5$$

Differentiation of  $y = (3 - 4x)^5$  is  $-16(3 - 4x)^5$

### Question: 13

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take  $y = (3x^2 - x + 1)^4$

So, by using the above formula, we have

$$\frac{d}{dx}(3x^2 - x + 1)^4 = 4(3x^2 - x + 1)^3 \times \frac{d}{dx}(3x^2 - x + 1) = 4(3x^2 - x + 1)^3 \times (3 \times 6x - 1) = 4(3x^2 - x + 1)^3(6x - 1)$$

Differentiation of  $y = (3x^2 - x + 1)^4$  is  $4(3x^2 - x + 1)^3(6x - 1)$

### Question: 14

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take  $y = (ax^2 + bx + c)$

So, by using the above formula, we have

$$\frac{d}{dx}(ax^2 + bx + c) = 2ax + b$$

Differentiation of  $y = (ax^2 + bx + c)$  is  $2ax + b$

### Question: 15

Differentiation

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let us take  $y = \frac{1}{(x^2 - x + 3)^3} = (x^2 - x + 3)^{-3}$

So, by using the above formula, we have

$$\frac{d}{dx}(x^2 - x + 3)^{-3} = -3(x^2 - x + 3)^{-4} \times (2x - 1) = -3\frac{1}{(x^2 - x + 3)^4} (2x - 1)$$

Differentiation of  $y = (x^2 - x + 3)^{-3}$  is  $\frac{-3(2x-1)}{(x^2-x+3)^{-4}}$

### Question: 16

Differentia

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx} \sin^2(ax + b) = 2 \sin(ax + b) \frac{d}{dx} \sin(ax + b) \frac{d}{dx}(ax + b)$

Let us take  $y = \sin^2(2x + 3)$

So, by using above formula, we have

$\frac{d}{dx} \sin^2(2x + 3) = 2 \sin(2x + 3) \frac{d}{dx} \sin(2x + 3) \frac{d}{dx}(2x + 3) = 4\sin(2x + 3)\cos(2x + 3).$

Differentiation of  $y = \sin^2(2x + 3)$  is  $4\sin(2x + 3)\cos(2x + 3)$

### Question: 17

Differentiate the

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\cos^a nu) = a\cos^{a-1}nu \frac{d}{dx}(\cos nu) \frac{d}{dx}(nu)$

Let us take  $y = \cos^2(x^3)$

So, by using the above formula, we have

$\frac{d}{dx} \cos^2(x^3) = 2 \cos x^3 (-\sin(x^3))3x^2 = -6x^2 \cos(x^3)\sin x^3$

Differentiation of  $y = \cos^2(x^3)$  is  $-6x^2 \cos(x^3)\sin x^3$

### Question: 18

Differentiate the

#### Solution:

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\sqrt{\sin u^a}) = \frac{1}{2\sqrt{\sin u^a}} \times \frac{d}{dx}(\sin u^a) \times \frac{d}{dx}(u^a)$

Let us take  $y = \sqrt{\sin x^3}$

So, by using the above formula, we have

$\frac{d}{dx} \sqrt{\sin x^3} = \frac{1}{2\sqrt{\sin x^3}} \times \frac{d}{dx}(\sin x^3) \times \frac{d}{dx}(x^3) = \frac{1}{2\sqrt{\sin x^3}} \times (\cos x^3) \times 3x^2 = \frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$

Differentiation of  $y = \sqrt{\sin x^3}$  is  $\frac{3x^2(\cos x^3)}{2\sqrt{\sin x^3}}$

### Question: 19

Differentiate the

**Solution:**

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\sqrt{usinx}) = \frac{1}{2\sqrt{usinx}} \times \frac{d}{dx}(usinx)$

Let us take  $y = \sqrt{x \sin x}$

So, by using the above formula, we have

$$\frac{d}{dx} \sqrt{x \sin x} = \frac{1}{2\sqrt{x \sin x}} \times \frac{d}{dx}(x \sin x) = \frac{1}{2\sqrt{x \sin x}} \times (\sin x + x \cos x) = \frac{(\sin x + x \cos x)}{2\sqrt{x \sin x}}$$

Differentiation of  $y = \sqrt{x \sin x}$  is  $\frac{(\sin x + x \cos x)}{2\sqrt{x \sin x}}$

**Question: 20**

Differentia

**Solution:**

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Formula used:  $\frac{d}{dx}(\sqrt{\cot \sqrt{x}}) = \frac{1}{2\sqrt{\cot \sqrt{x}}} \times \frac{d}{dx}(\cot \sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x})$

Let us take  $y = \sqrt{\cot \sqrt{x}}$

So, by using the above formula, we have

$$\frac{d}{dx} \sqrt{\cot \sqrt{x}} = \frac{1}{2\sqrt{\cot \sqrt{x}}} \times \frac{d}{dx} \cot \sqrt{x} \times \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{\cot \sqrt{x}}} \times (-\sec^2 \sqrt{x}) \times \frac{1}{2\sqrt{x}} = \frac{-\sec^2 \sqrt{x}}{4\sqrt{x} \sqrt{\cot \sqrt{x}}}$$

Differentiation of  $y = \sqrt{\cot \sqrt{x}}$  is  $\frac{-\sec^2 \sqrt{x}}{4\sqrt{x} \sqrt{\cot \sqrt{x}}}$

**Question: 21**

Differentia

**Solution:**

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Let us take  $y = \cos 3x \sin 5x$

So, by using the above formula, we have

$$\frac{d}{dx}(\cos 3x \sin 5x) = \sin 5x \frac{d(\cos 3x)}{dx} + \cos 3x \frac{d(\sin 5x)}{dx} = \sin 5x (-3\sin 3x) + \cos 3x (5\cos 5x) = 5\cos(3x)\cos(5x) - 3\sin(5x)\sin(3x)$$

Differentiation of  $y = \cos 3x \sin 5x$  is  $5\cos(3x)\cos(5x) - 3\sin(5x)\sin(3x)$

**Question: 22**

Differentia

**Solution:**

To Find: Differentiation

NOTE : When 2 functions are in the product then we used product rule i.e  $\frac{d(uv)}{dx} = v \frac{du}{dx} + u \frac{dv}{dx}$

Let us take  $y = \sin x \sin 2x$

So, by using the above formula, we have

$$\frac{d}{dx}(\sin x \sin 2x) = \sin x \frac{d(\sin 2x)}{dx} + \sin 2x \frac{d(\sin x)}{dx} = \sin x (2\cos 2x) + \sin 2x(\sin x) = 2\sin(x)\cos(2x) + \sin 2x(\sin x)$$

Differentiation of  $y = \sin x \sin 2x$  is  $2\sin(x)\cos(2x) + \sin 2x(\sin x)$

### Question: 23

Differentiate w.r

#### Solution:

Let  $y = \cos(\sin \sqrt{ax+b})$ ,  $z = \sin \sqrt{ax+b}$  and  $w = \sqrt{ax+b}$

$$\text{Formula : } \frac{d(\cos x)}{dx} = -\sin x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\sqrt{ax+b})}{dx} = \frac{1}{2} \times (ax+b)^{\frac{1}{2}-1} \times a$$

According to the chain rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dw} \times \frac{dw}{dx} \\ &= -\sin(\sin \sqrt{ax+b}) \times \cos \sqrt{ax+b} \times \frac{1}{2} \times (ax+b)^{-\frac{1}{2}} \times a \\ &= -\frac{a}{2} \sin(\sin \sqrt{ax+b}) \times \cos \sqrt{ax+b} \times (ax+b)^{-\frac{1}{2}} \end{aligned}$$

### Question: 24

Differentiate w.r

#### Solution:

Let  $y = e^{2x} \sin 3x$ ,  $z = e^{2x}$  and  $w = \sin 3x$

$$\text{Formula : } \frac{d(e^x)}{dx} = e^x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

According to product rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\ &= [\sin 3x \times (2 \times e^{2x})] + [e^{2x} \times 3 \cos 3x] \\ &= e^{2x} \times [2 \sin 3x + 3 \cos 3x] \end{aligned}$$

### Question: 25

Differentiate w.r

#### Solution:

Let  $y = e^{3x} \cos 2x$ ,  $z = e^{3x}$  and  $w = \cos 2x$

$$\text{Formula : } \frac{d(e^x)}{dx} = e^x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the product rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\ &= [\cos 2x \times (3 \times e^{3x})] + [e^{3x} \times (-2 \sin 2x)] \end{aligned}$$

$$= e^{3x} \times [3 \cos 2x - 2 \sin 2x]$$

### Question: 26

Differentiate w.r

#### Solution:

Let  $y = e^{-5x} \cot 4x$ ,  $z = e^{-5x}$  and  $w = \cot 4x$

Formula :  $\frac{d(e^x)}{dx} = e^x$  and  $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to the product rule of differentiation

$$\begin{aligned}\frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\ &= [\cot 4x \times (-5e^{-5x})] + [e^{-5x} \times (-4 \operatorname{cosec}^2 4x)] \\ &= -e^{-5x} \times [5 \cot 4x + 4 \operatorname{cosec}^2 4x]\end{aligned}$$

### Question: 27

Differentiate w.r

#### Solution:

Let  $y = \cos(x^3 \cdot e^x)$ ,  $z = x^3 \cdot e^x$ ,  $m = e^x$  and  $w = x^3$

Formula :  $\frac{d(e^x)}{dx} = e^x$ ,  $\frac{d(x^n)}{dx} = n \times x^{n-1}$  and  $\frac{d(\cos x)}{dx} = -\sin x$

According to the product rule of differentiation

$$\begin{aligned}\frac{dz}{dx} &= w \times \frac{dm}{dx} + m \times \frac{dw}{dx} \\ &= [x^3 \times (e^x)] + [e^x \times (3x^2)] \\ &= e^x \times [x^3 + 3x^2]\end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\ &= -\sin(x^3 \times e^x) \times \{ e^x \times [x^3 + 3x^2] \}\end{aligned}$$

### Question: 28

Differentiate w.r

#### Solution:

Let  $y = e^{(x \sin x + \cos x)}$ ,  $z = x \sin x + \cos x$ ,  $m = x$  and  $w = \sin x$

Formula :  $\frac{d(e^x)}{dx} = e^x$ ,  $\frac{d(\sin x)}{dx} = \cos x$  and  $\frac{d(\cos x)}{dx} = -\sin x$

According to the product rule of differentiation

$$\begin{aligned}\frac{dz}{dx} &= w \times \frac{dm}{dx} + m \times \frac{dw}{dx} + \frac{d(\cos x)}{dx} \\ &= [\sin x \times (1)] + [x \times (\cos x)] - \sin x \\ &= x \cos x\end{aligned}$$

According to the chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= e^{(x \sin x + \cos x)} \times (x \cos x)$$

### Question: 29

Differentiate w.r

#### Solution:

$$\text{Let } y = \frac{e^x + e^{-x}}{e^x - e^{-x}}, u = e^x + e^{-x}, v = e^x - e^{-x}$$

$$\text{Formula : } \frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(e^x - e^{-x}) \times (e^x - e^{-x}) - (e^x + e^{-x}) \times (e^x + e^{-x})}{(e^x - e^{-x})^2} \\ &= \frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x - e^{-x})^2} \\ &= \frac{(e^x - e^{-x} + e^x + e^{-x})(e^x - e^{-x} - e^x - e^{-x})}{(e^x - e^{-x})^2} \end{aligned}$$

$$(a^2 - b^2 = (a - b)(a + b))$$

$$\begin{aligned} &= \frac{(2e^x)(-2e^{-x})}{(e^x - e^{-x})^2} \\ &= \frac{-4}{(e^x - e^{-x})^2} \end{aligned}$$

### Question: 30

Differentiate w.r

#### Solution:

$$\text{Let } y = \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}, u = e^{2x} + e^{-2x}, v = e^{2x} - e^{-2x}$$

$$\text{Formula : } \frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(e^{2x} - e^{-2x}) \times (2e^{2x} - 2e^{-2x}) - (e^{2x} + e^{-2x}) \times (2e^{2x} + 2e^{-2x})}{(e^{2x} - e^{-2x})^2} \\ &= \frac{2(e^{2x} - e^{-2x})^2 - 2(e^{2x} + e^{-2x})^2}{(e^{2x} - e^{-2x})^2} \\ &= \frac{2(e^{2x} - e^{-2x} + e^{2x} + e^{-2x})(e^{2x} - e^{-2x} - e^{2x} - e^{-2x})}{(e^{2x} - e^{-2x})^2} \end{aligned}$$

$$(a^2 - b^2) = (a - b)(a + b)$$

$$= \frac{2(2e^{2x})(-2e^{-2x})}{(e^{2x} - e^{-2x})^2}$$

$$= \frac{-8}{(e^{2x} - e^{-2x})^2}$$

**Question: 31**

Differentiate w.r

**Solution:**

$$\text{Let } y = \sqrt{\frac{1-x^2}{1+x^2}}, u = 1-x^2, v = 1+x^2, z = \frac{1-x^2}{1+x^2}$$

$$\text{Formula : } \frac{d(x^2)}{dx} = 2x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1+x^2) \times (-2x) - (1-x^2) \times (2x)}{(1+x^2)^2}$$

$$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$$

$$= \frac{-4x}{(1+x^2)^2}$$

According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1-x^2}{1+x^2} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{-4x}{(1+x^2)^2} \right]$$

$$= \left[ \frac{-2x}{1} \times \left( \frac{1-x^2}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1+x^2)^2 \cdot \frac{1}{2}} \right]$$

$$= \left[ -2x \times (1-x^2)^{-\frac{1}{2}} \right] \times (1+x^2)^{-\frac{3}{2}}$$

**Question: 32**

Differentiate w.r

**Solution:**

$$\text{Let } y = \sqrt{\frac{a^2-x^2}{a^2+x^2}}, u = a^2 - x^2, v = a^2 + x^2, z = \frac{a^2-x^2}{a^2+x^2}$$

$$\text{Formula : } \frac{d(x^2)}{dx} = 2x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\begin{aligned}
\frac{dz}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\
&= \frac{(a^2 + x^2) \times (-2x) - (a^2 - x^2) \times (2x)}{(a^2 + x^2)^2} \\
&= \frac{-2xa^2 - 2x^3 - 2xa^2 + 2x^3}{(1+x^2)^2} \\
&= \frac{-4xa^2}{(1+x^2)^2}
\end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\
&= \left[ \frac{1}{2} \times \left( \frac{a^2 - x^2}{a^2 + x^2} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{-4xa^2}{(a^2 + x^2)^2} \right] \\
&= \left[ \frac{-2xa^2}{1} \times \left( \frac{a^2 - x^2}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(a^2 + x^2)^{2-\frac{1}{2}}} \right] \\
&= \left[ -2xa^2 \times (a^2 - x^2)^{-\frac{1}{2}} \right] \times (a^2 + x^2)^{-\frac{3}{2}}
\end{aligned}$$

### Question: 33

Differentiate w.r

#### Solution:

Let  $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$ ,  $u = 1 + \sin x$ ,  $v = 1 - \sin x$ ,  $z = \frac{1+\sin x}{1-\sin x}$

$$\text{Formula : } \frac{d(\sin x)}{dx} = \cos x$$

According to the quotient rule of differentiation

$$\begin{aligned}
\text{If } z &= \frac{u}{v} \\
\frac{dz}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\
&= \frac{(1 - \sin x) \times (\cos x) - (1 + \sin x) \times (-\cos x)}{(1 - \sin x)^2} \\
&= \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2} \\
&= \frac{2 \cos x}{(1 - \sin x)^2}
\end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\
&= \left[ \frac{1}{2} \times \left( \frac{1+\sin x}{1-\sin x} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{2 \cos x}{(1-\sin x)^2} \right]
\end{aligned}$$

$$= \left[ \frac{\cos x}{1} \times \left( \frac{1+\sin x}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1-\sin x)^{2-\frac{1}{2}}} \right]$$

$$= [\cos x \times (1 + \sin x)^{-\frac{1}{2}}] \times (1 - \sin x)^{-\frac{3}{2}}$$

**Question: 34**

Differentiate w.r.t.

**Solution:**

$$\text{Let } y = \sqrt{\frac{1+e^x}{1-e^x}}, u = 1 + e^x, v = 1 - e^x, z = \frac{1+e^x}{1-e^x}$$

$$\text{Formula : } \frac{d(e^x)}{dx} = e^x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\frac{dz}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(1 - e^x) \times (e^x) - (1 + e^x) \times (-e^x)}{(1 - e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^x + e^{2x}}{(1 - e^x)^2}$$

$$= \frac{2e^x}{(1 - e^x)^2}$$

According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1+e^x}{1-e^x} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{2e^x}{(1-e^x)^2} \right]$$

$$= \left[ \frac{e^x}{1} \times \left( \frac{1+e^x}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1-e^x)^{2-\frac{1}{2}}} \right]$$

$$= [e^x \times (1 + e^x)^{-\frac{1}{2}}] \times (1 - e^x)^{-\frac{3}{2}}$$

**Question: 35**

Differentiate w.r.t.

**Solution:**

$$\text{Let } y = \frac{e^{2x} + x^3}{\cosec 2x}, u = e^{2x} + x^3, v = \cosec 2x$$

$$\text{Formula: } \frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1} \text{ and } \frac{d(\cosec x)}{dx} = -\cosec x \cot x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$\begin{aligned}
&= \frac{(\csc 2x) \times (2e^{2x} + 3x^2) - (e^{2x} + x^3) \times (-2 \csc 2x \cot 2x)}{(\csc 2x)^2} \\
&= \frac{2e^{2x} \csc 2x + 3x^2 \csc 2x + 2e^{2x} \csc 2x \cot 2x + 2x^3 \csc 2x \cot 2x}{(\csc 2x)^2} \\
&= \frac{2e^{2x} \csc 2x (1 + \cot 2x) + 3x^2 \csc 2x (1 + \cot 2x)}{(\csc 2x)^2} \\
&= \frac{(1 + \cot 2x)(2e^x \csc 2x + 3x^2 \csc 2x)}{(\csc 2x)^2} \\
&= \frac{(1 + \cot 2x)(2e^x + 3x^2)(\csc 2x)}{(\csc 2x)^2} \\
&= \frac{(1 + \cot 2x)(2e^x + 3x^2)}{(\csc 2x)^1} \\
&= (1 + \cot 2x)(2e^x + 3x^2)(\sin 2x)
\end{aligned}$$

**Question: 36**

Find

**Solution:**

Let  $y = \sin(\sqrt{\sin x + \cos x})$ ,  $z = \sqrt{\sin x + \cos x}$

Formula :  $\frac{d(\cos x)}{dx} = -\sin x$  and  $\frac{d(\sin x)}{dx} = \cos x$

$$\diamond \frac{d(\sqrt{\sin x + \cos x})}{dx} = \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2}-1} \times (\cos x - \sin x)$$

According to the chain rule of differentiation

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\
&= \cos(\sin \sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{\frac{1}{2}-1} \times (\cos x - \sin x) \\
&= \cos(\sin \sqrt{\sin x + \cos x}) \times \frac{1}{2} \times (\sin x + \cos x)^{-\frac{1}{2}} \times (\cos x - \sin x)
\end{aligned}$$

**Question: 37**

Find

**Solution:**

Let  $y = e^x \log(\sin 2x)$ ,  $z = e^x$  and  $w = \log(\sin 2x)$

Formula :  $\frac{d(e^x)}{dx} = e^x$ ,  $\frac{d(\log x)}{dx} = \frac{1}{x}$  and  $\frac{d(\sin x)}{dx} = \cos x$

According to the product rule of differentiation

$$\begin{aligned}
\frac{dy}{dx} &= w \times \frac{dz}{dx} + z \times \frac{dw}{dx} \\
&= [\log(\sin 2x) \times (e^x)] + [e^x \times \frac{1}{\sin 2x} \times 2 \cos 2x] \\
&= e^x \times [\log(\sin 2x) + \frac{2 \cos 2x}{\sin 2x}] \\
&= e^x \times [\log(\sin 2x) + 2 \cot 2x]
\end{aligned}$$

**Question: 38**

Find

**Solution:**

$$\text{Let } y = \cos\left(\frac{1-x^2}{1+x^2}\right), u = 1-x^2, v = 1+x^2, z = \frac{1-x^2}{1+x^2}$$

$$\text{Formula : } \frac{d(x^2)}{dx} = 2x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\begin{aligned} \frac{dz}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(1+x^2) \times (-2x) - (1-x^2) \times (2x)}{(1+x^2)^2} \\ &= \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2} \\ &= \frac{-4x}{(1+x^2)^2} \end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\ &= \left[-\sin\frac{1-x^2}{1+x^2}\right] \times \left[\frac{-4x}{(1+x^2)^2}\right] \\ &= \left[\sin\frac{1-x^2}{1+x^2}\right] \times \left[\frac{4x}{(1+x^2)^2}\right] \end{aligned}$$

**Question: 39**

Find

**Solution:**

$$\text{Let } y = \sin\left(\frac{1+x^2}{1-x^2}\right), u = 1+x^2, v = 1-x^2, z = \frac{1+x^2}{1-x^2}$$

$$\text{Formula : } \frac{d(x^2)}{dx} = 2x \text{ and } \frac{d(\sin x)}{dx} = \cos x$$

According to the quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\begin{aligned} \frac{dz}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(1-x^2) \times (2x) - (1+x^2) \times (-2x)}{(1-x^2)^2} \\ &= \frac{2x - 2x^3 + 2x + 2x^3}{(1+x^2)^2} \\ &= \frac{4x}{(1+x^2)^2} \end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\ &= \left[ \cos \frac{1+x^2}{1-x^2} \right] \times \left[ \frac{4x}{(1+x^2)^2} \right] \end{aligned}$$

**Question: 40**

Find

**Solution:**

Let  $y = \frac{\sin x + x^2}{\cot 2x}$ ,  $u = \sin x + x^2$ ,  $v = \cot 2x$

Formula:  $\frac{d(\sin x)}{dx} = \cos x$ ,  $\frac{d(x^n)}{dx} = n \times x^{n-1}$  and  $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to the quotient rule of differentiation

If  $y = \frac{u}{v}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(\cot 2x) \times (\cos x + 2x) - (\sin x + x^2) \times (-2 \operatorname{cosec}^2 2x)}{(\cot 2x)^2} \\ &= \frac{\cot 2x \cos x + 2x \cot 2x + 2 \operatorname{cosec}^2 2x \sin x + 2x^2 \operatorname{cosec}^2 2x}{(\operatorname{cosec} 2x)^2} \\ &= \frac{\cot 2x (\cos x + 2x) + 2 \operatorname{cosec}^2 2x (\sin x + x^2)}{(\operatorname{cosec} 2x)^2} \\ &= \frac{2 \operatorname{cosec}^2 2x (\sin x + x^2)}{(\operatorname{cosec} 2x)^2} + \frac{\cot 2x (\cos x + 2x)}{(\operatorname{cosec} 2x)^2} \\ &= \frac{2 (\sin x + x^2)}{1} + \frac{\cos 2x (\cos x + 2x)}{\sin 2x \frac{1}{\sin^2 2x}} \\ &= 2(\sin x + x^2) + \cos 2x \sin 2x (\cos x + 2x) \end{aligned}$$

**Question: 41**

If <

**Solution:**

Let  $y = \frac{\cos x - \sin x}{\cos x + \sin x}$ ,  $u = \cos x - \sin x$ ,  $v = \cos x + \sin x$

Formula:  $\frac{d(\sin x)}{dx} = \cos x$  and  $\frac{d(\cos x)}{dx} = -\sin x$

According to the quotient rule of differentiation

If  $y = \frac{u}{v}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(\cos x + \sin x) \times (-\sin x - \cos x) - (\cos x - \sin x) \times (-\sin x + \cos x)}{(\cos x + \sin x)^2} \end{aligned}$$

$$= \frac{-(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x + \sin x)^2}$$

$$= -\frac{(\cos x + \sin x)^2}{(\cos x + \sin x)^2} - \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2}$$

$$= -\frac{1}{1} - y^2 \quad (y = \frac{\cos x - \sin x}{\cos x + \sin x})$$

$$\frac{dy}{dx} + y^2 + 1 = 0$$

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### Question: 42

If

#### Solution:

$$\text{Let } y = \frac{\cos x + \sin x}{\cos x - \sin x}, \quad u = \cos x + \sin x, v = \cos x - \sin x$$

$$\text{Formula: } \frac{d(\sin x)}{dx} = \cos x \text{ and } \frac{d(\cos x)}{dx} = -\sin x$$

According to the quotient rule of differentiation

$$\text{If } y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$$

$$= \frac{(\cos x - \sin x) \times (-\sin x + \cos x) - (\cos x + \sin x) \times (-\sin x - \cos x)}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos x - \sin x)^2 + (\cos x + \sin x)^2}{(\cos x - \sin x)^2}$$

$$= \frac{(\cos^2 x + \sin^2 x - 2 \cos x \sin x) + (\cos^2 x + \sin^2 x + 2 \cos x \sin x)}{(\cos x - \sin x)^2}$$

$$= \frac{2(\cos^2 x + \sin^2 x)}{(\cos x - \sin x)^2}$$

$$= \frac{(1)}{(\cos x - \sin x)^2 / 2} (\cos^2 x + \sin^2 x) = 1$$

$$= \frac{1}{\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{\left(\frac{\cos x \cos 45^\circ}{1} - \frac{\sin x \sin 45^\circ}{1}\right)^2}$$

$$= \frac{1}{\cos^2(x + \frac{\pi}{4})} [\cos a \cos b - \sin a \sin b = \cos(a + b)]$$

$$= \sec^2(x + \frac{\pi}{4})$$

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**Question: 43****Solution:**

$$\text{Let } y = \sqrt{\frac{1-x^4}{1+x^4}}, u = 1-x^4, v = 1+x^4, z = \frac{1-x^4}{1+x^4}$$

$$\text{Formula : } \frac{d(x^4)}{dx} = 1$$

According to quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\begin{aligned} dz/dx &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(1+x^4) \times (-1) - (1-x^4) \times (1)}{(1+x^4)^2} \end{aligned}$$

$$= \frac{-1-x^4 - 1 + x}{(1+x^4)^2}$$

$$= \frac{-2}{(1+x^4)^2}$$

According to the chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left[ \frac{1}{2} \times \left( \frac{1-x^4}{1+x^4} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{-2}{(1+x^4)^2} \right]$$

$$= \left[ \frac{-1}{1} \times \left( \frac{1-x^4}{1+x^4} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1+x^4)^2} \right]$$

$$= \left[ -1 \times \frac{(1-x^4)^{-\frac{1}{2}}}{(1+x^4)^{1-\frac{1}{2}}} \right] \times \left[ \frac{1}{(1+x^4)^1} \right] \times \frac{1-x}{1-x}$$

(Multiplying and dividing by 1-x )

$$= \left[ -1 \times \frac{(1-x^4)^{1-\frac{1}{2}}}{(1+x^4)^{\frac{1}{2}}} \right] \times \frac{1}{(1-x)(1+x)}$$

$$= \left[ -1 \times \frac{(1-x^4)^{\frac{1}{2}}}{(1+x^4)^{\frac{1}{2}}} \right] \times \frac{1}{(1-x)(1+x)} = -\frac{y}{1-x^2}$$

$$\text{Therefore } (1-x^2) \frac{dy}{dx} = -y$$

$$(1-x^2) \frac{dy}{dx} + y = 0$$

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**Question: 44****Solution:**

$$y = \sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$$

$$y = \sqrt{\frac{1-\sin x}{1+\sin x}} = \sqrt{\frac{1-\sin x}{1+\sin x}}$$

$$u = 1 - \sin x, v = 1 + \sin x, z = \frac{1-\sin x}{1+\sin x}$$

$$\text{Formula : } \frac{d(\sin x)}{dx} = \cos x$$

According to quotient rule of differentiation

$$\text{If } z = \frac{u}{v}$$

$$\begin{aligned} \frac{dz}{dx} &= \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2} \\ &= \frac{(1 + \sin x) \times (-\cos x) - (1 - \sin x) \times (\cos x)}{(1 + \sin x)^2} \\ &= \frac{-\cos x - \sin x \cos x - \cos x + \sin x \cos x}{(1 + \sin x)^2} \\ &= \frac{-2 \cos x}{(1 + \sin x)^2} \end{aligned}$$

According to the chain rule of differentiation

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{dz} \times \frac{dz}{dx} \\ &= \left[ \frac{1}{2} \times \left( \frac{1-\sin x}{1+\sin x} \right)^{\frac{1}{2}-1} \right] \times \left[ \frac{-2\cos x}{(1+\sin x)^2} \right] \\ &= \left[ -\frac{\cos x}{1} \times \left( \frac{1-\sin x}{1} \right)^{-\frac{1}{2}} \right] \times \left[ \frac{1}{(1+\sin x)^{2-\frac{1}{2}}} \right] \\ &= \left[ \cos x \times (1 + \sin x)^{-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left( \frac{1+\sin x}{1+\sin x} \right)^{\frac{3}{2}} \\ &\quad (\text{Multiplying and dividing by } (1 + \sin x)^{\frac{3}{2}}) \\ &= \left[ \cos x \times (1 + \sin x)^{\frac{3}{2}-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times \left( \frac{1}{1+\sin x} \right)^{\frac{3}{2}} \\ &= \left[ \cos x \times (1 + \sin x)^{\frac{3}{2}-\frac{1}{2}} \right] \times (1 - \sin x)^{-\frac{3}{2}} \times (1 + \sin x)^{-\frac{3}{2}} \\ &= [\cos x \times (1 + \sin x)^1] \times (1 - \sin^2 x)^{-\frac{3}{2}} \\ &= [\cos x \times (1 + \sin x)^1] \times (\cos^2 x)^{-\frac{3}{2}} \\ &= [\cos x \times (1 + \sin x)^1] \times (\cos x)^{-3} \\ &= [(1 + \sin x)^1] \times (\cos x)^{-3+1} \\ &= \frac{1+\sin x}{\cos^2 x} \\ &= \frac{1}{\cos^1 x} \times \frac{1+\sin x}{\cos^1 x} \\ &= \sec x \left( \frac{1}{\cos x} + \frac{\sin x}{\cos x} \right) \\ &= \sec x (\sec x + \tan x) \end{aligned}$$

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