Chapter : 6. T-RATIOS OF SOME PARTICULAR ANGLES

Exercise : 6

Question: 1

Evaluate each of

Solution:

Since sin 60° = √3/2 = cos 30°

and sin 30° = 1/2 = cos 60°

∴ sin 60° cos 30° + cos 60° sin 30° = (√3/2)(√3/2) + (1/2)(1/2)

= (3/4) + (1/4)

= 4/4

= 1

Question: 2

Evaluate each of

Solution:

Since cos 60° = 1/2 = sin 30°

and cos 30° = √3/2 = sin 60°

∴ cos 60° cos 30° — sin 60° sin 30° = (1/2) × (√3/2) - (√3/2) × (1/2)

= (√3/4) - (√3/4)

= 0

Question: 3

Evaluate each of

Solution:

Since cos 45° = 1/√2 = sin 45°

cos 30° = √3/2

sin 30° = 1/2

∴ cos 45° cos 30° + sin 45° sin 30° = (1/√2)×(√3/2) + (1/√2)(1/2)

= (√3 / 2√2) + (1 / 2√2)

= (√3 + 1) / (2√2)

Question: 4

Evaluate ea

Solution:

Since sin 30° = 1/2 , sin 60° = √3/2 , sin 90° = 1

cos 30° = √3/2 , cos 45° = 1/√2 , cos 60° = 1/2

sec 60° = (1/cos 60°)= 2

tan 45° = (sin 45° /cos 45°) = (1/√2) / (1/√2) = 1

cot 45° = (1/tan 45°) = 1/1 = 1

$\therefore \frac{\sin 30^\circ}{\cos 45^\circ} + \frac{\cot 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\tan 45^\circ} + \frac{\cos 30^\circ}{\sin 90^\circ}$

$= ((1/2) / ((1/\sqrt{2})) + (1/2) – ((\sqrt{3}/2) / 1) + ((\sqrt{3}/2)/1)$

$= (1/\sqrt{2}) + (1/2) – (\sqrt{3}/2) + (\sqrt{3}/2)$

$= (1/\sqrt{2}) + (1/2)$

$= (\sqrt{2}/2) + (1/2)$

$= (\sqrt{2} + 1)/2$

Question: 5

Evaluate ea

Solution:

$\cos 30° = \sqrt{3}/2 \Rightarrow \cos^2 30° = 3/4$

$\cos 60° = 1/2 \Rightarrow \cos^2 60° = 1/4$

$\sec 30° = (1/\cos 30°) = (2/\sqrt{3}) \Rightarrow \sec^2 30° = 4/3$

$\tan 45° = 1 \Rightarrow \tan^2 45° = 1$

$\sin 30° = 1/2 \Rightarrow \sin^2 30° = 1/4$

We also know that $\sin^2 \theta + \cos^2 \theta = 1$

$$\therefore \frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\cos^2 30° + \sin^2 30°}$$

$= [(5\times(1/4)) + (4\times(4/3)) – (1)]/1$

$= (5/4) + (16/3) – 1$

$= (15 + 64 - 12)/12 = 67/12$

Question: 6

Evaluate each of

Solution:

$\cos 60° = 1/2 \Rightarrow \cos^2 60° = 1/4$

$\sin 45° = 1/\sqrt{2} \Rightarrow \sin^2 45° = 1/2$

$\sin 30° = 1/2 \Rightarrow \sin^2 30° = 1/4$

$\cos 90° = 0 \Rightarrow \cos^2 90° = 0$

$\therefore 2\cos^2 60° + 3\sin^2 45° — 3\sin^2 30° + 2\cos^2 90°$

$= 2(1/4) + 3(1/2) – 3(1/4) + 2(0)$

$= (1/2) + (3/2) – (3/4) = 2 – (3/4)$

$= 5/4$

Question: 7

Evaluate each of

Solution:

$\cos 30° = \sqrt{3}/2$, $\Rightarrow \cos^2 30° = 3/4$

$\sin 30° = 1/2$

$\therefore \operatorname{cosec} 30° = 1/\sin 30° = 2 \Rightarrow \operatorname{cosec}^2 30° = 4$

$\cot 30° = (\cos 30°/\sin 30°) = \sqrt{3} \Rightarrow \cot^2 30° = 3$

$\cos 45° = 1/\sqrt{2}$

$\therefore$ sec 45° = 1/cos 45° = √2 ⇒ sec^2 45° = 2

$\therefore$ $cot^2 30°$ — $2cos^2 30°$ — $(3/4)sec^2 45°$ + $(1/4)cosec^2 30°$

= 3 – 2(3/4) – (3/4) × 2 + (1/4) × 4

= 3 – 1.5 – 1.5 + 1

= 1

Question: 8

Evaluate each of

Solution:

sin 30° = 1/2 ⇒ sin^2 30° = 1/4

cos 45° = 1/√2 = sin 45°

cot 45° = 1 ⇒ cot^2 45° = 1

cos 60° = 1/2 ⇒ sec 60° = 2 ⇒ sec^2 60° = 4

cos 30° = √3/2 ⇒ sec 30° = 2/√3 ⇒ sec^2 30° = 4/3

cosec 45° = 1/sin 45° = √2 ⇒ $cosec^2$ 45° = 2

$\therefore$ ($sin^2 30°$ + 4 $cot^2 45°$ — $sec^2 60°$)($cosec^2 45°$ $sec^2 30°$)

= ((1/4) + 4(1) – 4) × (2)(4/3)

= (1/4) × (8/3)

= 8/12

= 2/3

Question: 9

Evaluate ea

Solution:

sin 30° = 1/2 , ⇒ sin^2 30° = 1/4 ⇒ (1/sin^2 30°) = 4

cos 30° = √3/2 ,

cot 30° = (cos 30°/ sin 30°) = √3 ⇒ cot^2 30° = 3

cos 45° = 1/√2 ⇒ cos^2 45° = 1/2

sin 0° = 0

$\frac{4}{\cot^2 30°} + \frac{1}{\sin^2 30°}$ - 2 cos^2 45° - sin^2 0° = (4/3) + (4) – 2(1/2) – 0

= (4/3) + 4 – 1

= 13/3

Question: 10

Show that:

Solution:

(i) Consider L.H.S. = $\frac{1-\sin 60°}{\cos 60°} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ = (2-√3)

Consider R.H.S. = $\frac{\tan 60°-1}{\tan 60°+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

= $\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$ (Rationalizing the denominator)

$= \frac{4-2\sqrt{3}}{2}$

= 2 - √3

L.H.S. = R.H.S.

Hence, verified.

(ii) L.H.S. = $\frac{\cos 30° + \sin 60°}{1 + \sin 30° + \cos 60°} = \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\sqrt{3}}{2}$

R.H.S. = cos 30° = √3/2

L.H.S. = R.H.S.

Hence, verified.

Question: 11

Verify each of th

Solution:

(i) Consider L.H.S. = sin 60° cos 30° — cos 60° sin 30°

= (√3/2) × (√3/2) – (1/2)(1/2)

= (3/4) – (1/4)

= 2/4

=1/2

Consider R.H.S. = sin 30° = 1/2

L.H.S. = R.H.S.

Hence, verified.

(ii) Consider L.H.S. = cos 60° cos 30° + sin 60° sin 30°

= (1/2) × (√3/2) + (√3/2)(1/2)

= (√3/4) + (√3/4)

= √3/2 = cos 30° = R.H.S.

∴ L.H.S. = R.H.S.

Hence, verified.

(iii) Consider L.H.S. = 2 sin 30° cos 30°

= 2 × (1/2) × (√3/2)

= √3/2 = sin 60° = R.H.S.

∴ L.H.S. = R.H.S.

Hence, verified.

(iv) Consider L.H.S. = 2 sin 45° cos 45°

= 2 × (1/√2) × (1/√2)

= (2 × 1/2)

= 1 = sin 90° = R.H.S.

∴ L.H.S. = R.H.S.

Hence, verified.

Question: 12

If A = 45°, verif

Solution:

(i) To show: sin 2A = 2 sin A cos A

A = 45°

∴ To show: sin 90° = 2 sin 45° cos 45°

Consider R.H.S. = 2 sin 45° cos 45°

= 2 × (1/√2) × (1/√2)

= (2 × 1/2)

= 1 = sin 90° = L.H.S.

∴ L.H.S. = R.H.S.

Hence, verified.

(ii) To show: cos 2A = 2 $\cos^2$A —1 = 1— 2 $\sin^2$A

A = 45°

∴ To show: cos 90° = 2 $\cos^2$ 45° —1 = 1— 2 $\sin^2$ 45°

Consider 2 $\cos^2$ 45° —1 = 1— 2 $\sin^2$ 45°

= 2 × (1/√2) – 1

= 1 – 1

= 0

= cos 90° = R.H.S.

Consider 1— 2 $\sin^2$ 45° = 1 – 2(1/2)

= 1 – 1

= 0

= cos 90° = R.H.S.

∴ L.H.S. = R.H.S.

Hence, verified.

Question: 13

If A = 30°, verif

Solution:

(i) To prove:- sin 2A = $\frac{2tanA}{1+\tan^2 A}$

A = 30°

∴ To show:- sin 60° = $\frac{2tan30°}{1+\tan^2 30°}$

Consider L.H.S. = $\frac{2tan30°}{1+\tan^2 30°} = \frac{2\times\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

$= \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4}$

= √3/2

= sin 60° = R.H.S.

∴ R.H.S. = L.H.S.

Hence, verified.

(ii) To show:- cos 2A = $\frac{1-\tan^2 A}{1+\tan^2 A}$

A = 30°

∴ To show:- cos 60° = $\frac{1-\tan^2 30°}{1+\tan^2 30°}$

Consider R.H.S. = $\frac{1-\tan^2 30°}{1+\tan^2 30°} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$

= 2/4 = 1/2 = cos 60° = L.H.S.

∴ R.H.S. = L.H.S.

Hence, verified.

(iii) To show:- tan2A =$\frac{2tanA}{1-\tan^2 A}$

A = 30°

∴ To show:- tan 60° = $\frac{2tan30°}{1-\tan^2 30°}$

Consider R.H.S. = $\frac{2tan\,30°}{1-\tan^2 30°} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2/\sqrt{3}}{2/3}$

= 3/√3 = √3

= tan 60° = L.H.S.

∴ R.H.S. = L.H.S.

Hence, verified.

Question: 14

If A = 60° and B

Solution:

(i) To verify: sin (A + B) = sin A cos B + cos A sin B

If A = 60° and B = 30°, then

To verify: sin 90° = sin 60° cos 30° + cos 60° sin 30°

Consider R.H.S. = sin 60° cos 30° + cos 60° sin 30°

= (√3/2) × (√3/2) + (1/2)(1/2)

= (3/4) + (1/4)

= 4/4

=1

Consider L.H.S. = sin 90° = 1

∴ L.H.S. = R.H.S.

Hence, verified.

(ii) To verify: cos (A + B) = cos A cos B — sin A sin B

If A = 60° and B = 30°, then

To verify: cos (90°) = cos 60° cos 30° — sin60° sin 30°

Consider R.H.S. = cos 60° cos 30° - sin 60° sin 30°

= (1/2) × (√3/2) - (√3/2)(1/2)

= (√3/4) - (√3/4)

= 0 = cos 90° = L.H.S.

∴ L.H.S. = R.H.S.

Hence, verified.

Question: 15

If A = 60° and B

Solution:

(i) To verify: sin (A - B) = sin A cos B - cos A sin B

If A = 60° and B = 30°, then

Consider LHS sin(60°-30°) = sin 30° = 1/2Consider R.H.S. = sin 60° cos 30° - cos 60° sin 30°

= (√3/2) × (√3/2) - (1/2) (1/2)

= (3/4) - (1/4)

= 2/4

=1/2

∴ L.H.S. = R.H.S.

Hence, verified.

(ii) To verify: cos (A - B) = cos A cos B + sin A sin B

If A = 60° and B = 30°, then

To verify: cos (30°) = cos 60° cos 30° + sin 60° sin 30°

Consider R.H.S. = cos 60° cos 30° + sin 60° sin 30°

= (1/2) × (√3/2) + (√3/2)(1/2)

= (√3/4) + (√3/4)

= √3/2 = cos 30° = L.H.S.

∴ L.H.S. = R.H.S.

Hence, verified.

(iii) To verify:- tan(A +B) =$\frac{tanA+tanB}{1-tanA\ tanB}$

If A = 60° and B = 30°, then

To verify: tan 90° = $\frac{tan60°+tan30°}{1-tan60°tan30°}$

Consider R.H.S. = $\frac{tan60°+tan30°}{1-tan60°\ tan30°} = \frac{\sqrt{3}+\frac{1}{\sqrt{3}}}{1-\sqrt{3}\times\frac{1}{\sqrt{3}}}$

= $\frac{\frac{3+1}{\sqrt{3}}}{1-1}$ = 4/0 = ∞

= tan 90° = L.H.S.

∴ R.H.S. = L.H.S.

Hence, verified.

Question: 16

If A and B are ac

Solution:

Given: tan (A +B) = $\frac{tanA+tanB}{1-tanA\ tanB}$ and tan A = 1/3, tan B = 1/2

$\therefore$ tan(A +B) = $\frac{\tan A+\tan B}{1-\tan A\tan B} = \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}} = \frac{5/6}{5/6} = 1$

$\therefore$ tan(A + B) = 1

Also, A and B are acute angles, therefore both A and B are less than 90°. So A + B must be less than 180°.

Therefore, the only possible case for which tan (A+B) = 1 will be when (A + B) equals 45°.

Thus, A + B = 45°

Question: 17

Using the formula

Solution:

To find:- tan 60°

Given: tan2A =$\frac{2\tan A}{1-\tan^2 A}$ ……………………(1)

tan 30° = 1/√3

$\therefore$ Putting A = 30° in equation (1), we have the following:

tan 60° = $\frac{2\tan 30°}{1-\tan^2 30°}$

$\therefore$ tan 60° = $\frac{2\tan 30°}{1-\tan^2 30°} = \frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2/\sqrt{3}}{2/3}$

= 3/√3 = √3

$\therefore$ tan 60° = √3

Question: 18

Using the formula

Solution:

Given: cos A = $\sqrt{\frac{1+\cos 2A}{2}}$, ……………………..(1)

cos 60° = 1/2

To find: cos 30°

By putting A = 30° in equation (1), we get the following:

cos 30° = $\sqrt{\frac{1+\cos 60°}{2}}$

= $\sqrt{\frac{1+(1/2)}{2}}$

= $\sqrt{\frac{3}{4}}$ = √3/2

$\therefore$ cos 30° = √3/2

Question: 19

Using the formula

Solution:

Given: sin A = $\sqrt{\frac{1-\cos 2A}{2}}$, ……………………..(1)

cos 60° = 1/2

To find: sin 30°

By putting A = 30° in equation (1), we get the following:

$\sin 30° = \sqrt{\frac{1-\cos 60°}{2}}$

$= \sqrt{\frac{1-(1/2)}{2}}$

$= \sqrt{\frac{1}{4}} = 1/2$

∴ sin 30° = 1/2

Question: 20

In the adjoining

Solution:

Since, in a right angled triangle,

sin θ = Perpendicular / Hypotenuse ,

and cos θ = Base / Hypotenuse ,

where θ is the angle made between the hypotenuse and the base.

(i) ∴ In the given figure, sin 30° = BC/AC

⇒ 1/2 = BC/20

⇒ (1/2) × 20 = BC

⇒ BC = 10 cm

(ii) Now, In the given figure, cos 30° = AB/AC

⇒ √3/2 = AB/20

⇒ (√3/2) × 20 = AB

⇒ AB = 10√3 cm

Question: 21

In the adjoining

Solution:

Since, in a right-angled triangle,

sin θ = Perpendicular / Hypotenuse,

and cos θ = Base / Hypotenuse,

where θ is the angle made between the hypotenuse and the base.

(i) ∴ In the given figure, sin 30° = BC/AC

⇒ 1/2 = 6/AC

⇒ AC = 6 × 2

⇒ AC = 12 cm

(ii) Now, In the given figure, cos 30° = AB/AC

⇒ √3/2 = AB/12

⇒ (√3/2) × 12 = AB

⇒ AB = 6√3 cm

Aliter: Since ABC is a right-angled triangle,

∴ $(AB)^2 + (BC)^2 = (AC)^2$

$\therefore (AB)^2 = (AC)^2 - (BC)^2$

$\Rightarrow (AB)^2 = 144 – 36 = 108$

$\Rightarrow (AB) = \sqrt{108} = 6\sqrt{3}$

$\therefore AB = 6\sqrt{3}$ cm

Question: 22

In the adjoining

Solution:

Since, in a right-angled triangle,

sin θ = Perpendicular / Hypotenuse,

and cos θ = Base / Hypotenuse,

where θ is the angle made between the hypotenuse and the base.

(i) ∴ In the given figure, sin 45° = BC/AC

$\Rightarrow 1/\sqrt{2} = BC / (3\sqrt{2})$

$\Rightarrow BC = (1/\sqrt{2}) \times (3\sqrt{2}) = 3$

⇒ BC = 3 cm

(ii) Now, In the given figure, cos 45° = AB/AC

$\Rightarrow 1/\sqrt{2} = AB / (3\sqrt{2})$

$\Rightarrow AB = 1/\sqrt{2} \times (3\sqrt{2})$

⇒ AB = 3 cm

Question: 23

If sin (A + B) =1

Solution:

Given: (i) sin (A + B) = 1

(ii) cos (A - B) = 1

Since, sin (A + B) = 1

⇒ sin (A + B) = sin 90° (∵ 0° ≤ (A + B) ≤ 90°, sin 90° = 1)

⇒ A + B = 90° .... (1)

Also, cos (A - B) = 1

⇒ cos (A - B) = cos 0° (∵ 0° ≤ (A + B) ≤ 90°, cos 0° = 1)

⇒ A - B = 0° .... (2)

From equation (2), we get A = B

Putting this value in equation (1), we get, 2A = 90° ⇒ A = 45°

∴ B = A = 45°

∴ ∠A = ∠B = 45°

Question: 24

If sin (A - B) =

Solution:

Given: (i) sin (A - B) = 1/2

(ii) cos (A + B) = 1/2

Since, sin (A - B) = 1

⇒ sin (A - B) = sin 30° (∵ 0° < (A + B) < 90°, sin 30° = 1/2)

⇒ A - B = 30° .... (1)

Also, cos (A + B) = 1/2

⇒ cos (A + B) = cos 60° (∵ 0° < (A + B) < 90°, cos 60° = 1/2)

⇒ A + B = 60° .... (2)

On adding equation (1) and (2), we get,

2A = 90° ⇒ A = 45°

Putting this value in equation (2), we get,

B = 60° - A = 60° - 45° ⇒ B = 15°

∴ ∠A = 45°, ∠B = 15°

Question: 25

If tan (A - B) =

Solution:

Given: (i) tan (A - B) = 1/√3

(ii) tan (A + B) = √3

Since, tan (A - B) = 1/√3

⇒ tan (A - B) = tan 30°(∵ 0° < (A + B) < 90°, tan 30° = 1/√3)

⇒ A - B = 30° .... (1)

Also, tan (A + B) = √3

⇒ tan (A + B) = tan 60°(∵ 0° < (A + B) < 90°, tan 60° = √3)

⇒ A + B = 60° .... (2)

On adding equation (1) and (2), we get,

A - B + A + B = 30° + 60°

2A = 90° ⇒ A = 45°

Putting this value in equation (2), we get,

B = 60° - A = 60° - 45° ⇒ B = 15°

∴ ∠A = 45°, ∠B = 15°

Question: 26

If 3x = cosec θ a

Solution:

Given,

3x = cot θ

⇒ $9x^2 = \cot^2\theta$ [1]and $\frac{3}{x} = cosec\theta$ $\Rightarrow \frac{9}{x^2} = cosec^2\theta \quad [2]$ Subtracting [2] from [1], we get

$$9x^2 - \frac{9}{x^2} = \cot^2\theta - cosec^2\theta$$

$$\Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1 \qquad \Rightarrow 3\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{3}$$

$$\Rightarrow \left(x^2 - \frac{1}{x^2}\right) = \frac{1}{9}$$

Question: 27

If sin (A + B) =

Solution:

Given: sin (A + B) = sin A cos B + cos A sin B

cos (A - B) = cos A cos B + sin A sin B

(i) To find: sin 75°

If we put A = 30° and B = 45°, then we have:

sin 75° = sin 30° cos 45° + cos 30° sin 45°

∴ sin 75° = (1/2) × (1/√2) + (√3/2) × (1/√2)

= (1/2√2) + (√3/2√2)

= $\frac{1+\sqrt{3}}{2\sqrt{2}}$

(ii) To find: cos 715°

If we put A = 45° and B = 30°, then we have:

cos 15° = cos 45° cos 30° + sin 45° sin 30°

∴ cos 15° = (1/√2) × (√3/2) + (1/√2) × (1/2)

= (√3 / 2√2) + (1/2√2)

= $\frac{1+\sqrt{3}}{2\sqrt{2}}$

∴ (i) sin 75° = $\frac{1+\sqrt{3}}{2\sqrt{2}}$

(ii) cos 15° = $\frac{1+\sqrt{3}}{2\sqrt{2}}$