

6. Algebraic Expressions and Identities

Exercise 6.1

1. Question

Identify the terms, their coefficients for each of the following expressions.

(i) $7x^2yz - 5xy$

(ii) $x^2 + x + 1$

(iii) $3x^2y^2 - 5x^2y^2z^2 + z^2$

(iv) $9 - ab + bc - ca$

(v) $\frac{a}{2} + \frac{b}{2} - ab$

(vi) $0.2x - 0.3xy + 0.5y$

Answer

(i) $7x^2yz - 5xy$

This equation consists of two terms that are:

$7x^2yz$ and $-5xy$

The coefficient of $7x^2yz$ is 7

The coefficient of $-5xy$ is -5

(ii) $x^2 + x + 1$

This equation consists of three terms that are:

x^2 , x , 1

The coefficient of x^2 is 1

The coefficient of x is 1

The coefficient of 1 is 1

(iii) $3x^2y^2 - 5x^2y^2z^2 + z^2$

This equation consists of three terms that are:

$3x^2y^2$, $-5x^2y^2z^2$ and z^2

The coefficient of $3x^2y^2$ is 3

The coefficient of $-5x^2y^2z^2$ is -5

The coefficient of z^2 is 1

(iv) $9 - ab + bc - ca$

The term	Coefficient
9	9
-ab	-1
bc	1
ca	-1

(v) $\frac{a}{2} + \frac{b}{2} - ab$

The term	Coefficient
$\frac{a}{2}$	1
$\frac{b}{2}$	1
$-ab$	1

(vi) $0.2x - 0.3xy + 0.5y$

The term	Coefficient
$0.2x$	0.2
$-0.3xy$	-0.3
$0.5y$	0.5

2. Question

Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any category?

(i) $x+y$

(ii) 1000

(iii) $x+x^2+x^3+x^4$

(iv) $7+a+5b$

(v) $2b-3b^2$

(vi) $2y-3y^2+4y^3$

(vii) $5x-4y+3x$

(viii) $4a-15a^2$

(ix) $xy+yz+zt+tx$

(x) pqr

(xi) p^2q+pq^2

(xii) $2p+2q$

Answer

(i) $x+y$

This expression contains two terms x and y

So, it is called 'Binomial'

(ii) 1000

It contains one term 1000

So, it is called monomial

(iii) $x+x^2+x^3+x^4$

It contains four terms

So, it is not a monomial, binomial and trinomial

(iv) $7+a+5b$

It contains three terms

So, it is called trinomial

(v) $2b-3b^2$

It contains two terms

So, it is called binomial

(vi) $2y-3y^2+4y^3$

It contains three terms

So, it is called trinomial

(vii) $5x-4y+3x$

$8x - 4y$

It contains two terms

So, it is called binomial

(viii) $4a-15a^2$

It contains two terms

So, it is called binomial

(ix) $xy+yz+zt+tx$

It contains four terms

So, it is not a monomial, binomial and trinomial

(x) pqr

It contains one term

So, it is called monomial

(xi) p^2q+pq^2

It contains two terms

So, it is called binomial

(xii) $2p+2q$

It contains two terms

So, it is called monomial

Exercise 6.2

1. Question

Add the following algebraic expressions:

(i) $3a^2b, -4a^2b, 9a^2b$

(ii) $\frac{2}{3}a, \frac{3}{5}a, -\frac{6}{5}a$

(iii) $4xy^2 - 7x^2y, 12x^2y - 6xy^2, -3x^2y + 5xy^2$

(iv) $\frac{3}{2}a - \frac{5}{4}b + \frac{2}{5}c, \frac{2}{3}a - \frac{7}{2}b + \frac{7}{2}c, \frac{5}{3}a + \frac{5}{2}b - \frac{5}{4}c$

(v) $\frac{11}{2}xy + \frac{12}{5}y + \frac{13}{7}x, -\frac{11}{2}y - \frac{12}{5}x - \frac{13}{7}xy$

(vi) $\frac{7}{2}x^3 - \frac{1}{2}x^2 + \frac{5}{3}, \frac{3}{2}x^3 + \frac{7}{4}x^2 - x + \frac{1}{3}, \frac{3}{2}x^2 - \frac{5}{2}x - 2$

Answer

$$(i) 3a^2b, -4a^2b, 9a^2b$$

$$= 3a^2b + (-4a^2b) + 9a^2b$$

$$= 3a^2b - 4a^2b + 9a^2b$$

$$= 3a^2b$$

$$(ii) \frac{2}{3}a, \frac{3}{5}a, -\frac{6}{5}a$$

$$= \frac{2}{3}a + \frac{3}{5}a - \frac{6}{5}a$$

Taking L.C.M 3, 5, 5 is 15

$$= 11 \frac{10a+9a-18a}{15}$$

$$= \frac{19a-18a}{15}$$

$$= \frac{a}{15}$$

$$(iii) 4xy^2 - 7x^2y, 12x^2y - 6xy^2, -3x^2y + 5xy^2$$

$$= 4xy^2 - 7x^2y + 12x^2y - 6xy^2 - 3x^2y + 5xy^2$$

$$= 4x^2 + 12x^2y - 3x^2y - 7x^2y - 6xy^2 + 5xy^2$$

$$= 3xy^2 + 2x^2y$$

$$(iv) \frac{3}{2}a - \frac{5}{4}b + \frac{2}{5}c, \frac{2}{3}a - \frac{7}{2}b + \frac{7}{2}c, \frac{5}{3}a + \frac{5}{2}b - \frac{5}{4}c$$

Adding all, we get

$$= \frac{3}{2}a - \frac{5}{4}b + \frac{2}{5}c + \frac{2}{3}a - \frac{7}{2}b + \frac{7}{2}c + \frac{5}{3}a + \frac{5}{2}b - \frac{5}{4}c$$

$$= \frac{9a+4a+10a}{6} + \frac{-5b-14b+10b}{4} + \frac{8c+70c-25c}{20}$$

$$= \frac{23a}{6} - \frac{9b}{4} + \frac{53c}{20}$$

$$(v) \frac{11}{2}xy + \frac{12}{5}y + \frac{13}{7}x, -\frac{11}{2}y - \frac{12}{5}x - \frac{13}{7}xy$$

Adding all, we get

$$= \frac{11}{2}xy + \frac{11}{5}y + \frac{13}{7}x + \frac{-11}{2}y - \frac{12}{5}x - \frac{13}{7}xy$$

$$= \frac{11xy-12xy}{14} + \frac{24y-55y}{10} + \frac{65x-84x}{35}$$

$$= \frac{51xy}{14} - \frac{19x}{35} - \frac{31y}{10}$$

$$(vi) \frac{7}{2}x^3 - \frac{1}{2}x^2 + \frac{5}{3}, \frac{3}{2}x^3 + \frac{7}{4}x^2 - x + \frac{1}{3}, \frac{3}{2}x^2 - \frac{5}{2}x - 2$$

Adding all, we get

$$= \frac{7}{2}x^3 - \frac{1}{2}x^2 + \frac{5}{3} + \frac{3}{2}x^3 + \frac{7}{4}x^2 - x + \frac{1}{3} + \frac{3}{2}x^2 - \frac{5}{2}x - 2$$

$$= \frac{10}{2}x^3 + \frac{11}{4}x^2 - \frac{7x}{2} + \frac{0}{6}$$

$$= 5x^3 + \frac{11}{4}x^2 - \frac{7x}{2}$$

2. Question

Subtract:

(i) $-5xy$ from $12xy$

(ii) $2a^2$ from $-7a^2$

(iii) $2a-b$ from $3a-5b$

(iv) $2x^3 - 4x^2 + 3x + 5$ from $4x^3 + x^2 + x + 6$

(v) $\frac{3}{2}y^3 - \frac{2}{7}y^2 - 5$ from $\frac{1}{3}y^3 + \frac{5}{7}y^2 + y - 2$

(vi) $\frac{3}{2}x - \frac{5}{4}y - \frac{7}{2}z$ from $\frac{2}{3}x + \frac{3}{2}y - \frac{4}{3}z$

(vii) $x^2 - \frac{4}{5}xy^2 + \frac{4}{3}xy$ from $\frac{2}{3}x^2y + \frac{3}{2}xy^2 - \frac{1}{3}xy$

(viii) $\frac{ab}{7} - \frac{35}{3}bc + \frac{6}{5}ac$ from $\frac{3}{5}bc - \frac{4}{5}ac$

Answer

(i) $-5xy$ from $12xy$

After subtracting, we get

$$= 12xy - (-5xy)$$

$$= 5xy + 12xy$$

$$= 17xy$$

(ii) $2a^2$ from $-7a^2$

After subtracting, we get

$$= 2a^2 + (-7a^2)$$

$$= -2a^2 + 7a^2$$

$$= -9a^2$$

(iii) $2a-b$ from $3a-5b$

After subtracting, we get

$$= -(2a - b) + (3a - 5b)$$

$$= -2a + b + 3a - 5b$$

$$= a - 4b$$

(iv) $2x^3 - 4x^2 + 3x + 5$ from $4x^3 + x^2 + x + 6$

After subtracting, we get

$$= -(2x^3 - 4x^2 + 3x + 5) + (4x^3 + x^2 + x + 6)$$

$$= -2x^3 + 4x^2 - 3x - 5 + 4x^3 + x^2 + x + 6$$

$$= 2x^3 + 5x^2 - 2x + 1$$

(v) $\frac{3}{2}y^3 - \frac{2}{7}y^2 - 5$ from $\frac{1}{3}y^3 + \frac{5}{7}y^2 + y - 2$

After subtracting, we get

$$= \frac{1}{3}y^3 + \frac{5}{7}y^2 + y - 2 - \left(\frac{3}{2}y^3 - \frac{2}{7}y^2 - 5\right)$$

$$= \frac{-1}{3}y^3 + \frac{7}{7}y^2 + y + 3$$

$$= \frac{-1}{3}y^3 + y^2 + y + 3$$

$$(vi) \frac{3}{2}x - \frac{5}{4}y - \frac{7}{2}z \text{ from } \frac{2}{3}x + \frac{3}{2}y - \frac{4}{3}z$$

After subtracting, we get

$$= \frac{2}{3}x + \frac{3}{2}y - \frac{4}{3}z - (\frac{3}{2}x - \frac{5}{4}y - \frac{7}{2}z)$$

$$= \frac{2}{3}x - \frac{3}{2}x + \frac{3}{2}y + \frac{5}{4}y - \frac{4}{3}z + \frac{7}{2}z$$

$$= \frac{4x-9x}{6} + \frac{6y+5y}{4} + \frac{-8z+21z}{6}$$

$$= \frac{-5x}{6} + \frac{11y}{4} + \frac{13z}{6}$$

$$(vii) x^2 - \frac{4}{5}xy^2 + \frac{4}{3}xy \text{ from } \frac{2}{3}x^2y + \frac{3}{2}xy^2 - \frac{1}{3}xy$$

$$= \frac{2}{3}x^2y + \frac{3}{2}xy^2 - \frac{1}{3}xy - (x^2 - \frac{4}{5}xy^2 + \frac{4}{3}xy)$$

$$= \frac{2}{3}x^2y - x^2y + \frac{3}{2}xy^2 + \frac{4}{5}xy^2 - \frac{1}{3}xy - \frac{4}{3}xy$$

$$= \frac{-1}{3}x^2y + \frac{25}{10}xy^2 - \frac{5}{3}xy$$

$$(viii) \frac{ab}{7} - \frac{35}{3}bc + \frac{6}{5}ac \text{ from } \frac{3}{5}bc - \frac{4}{5}ac$$

After subtracting, we get

$$= \frac{3}{5}bc - \frac{4}{5}ac - (\frac{ab}{7} - \frac{35}{3}bc + \frac{6}{5}ac)$$

$$= \frac{3}{5}bc + \frac{35}{3}bc - \frac{4}{5}ac - \frac{6}{5}ac - \frac{ab}{7}$$

$$= \frac{9bc+175bc}{15} + \frac{-4ac-6ac}{5} - \frac{ab}{7}$$

$$= \frac{184bc}{15} + \frac{-10ac}{5} - \frac{ab}{7}$$

$$= \frac{184bc}{15} - \frac{10ac}{5} - \frac{ab}{7}$$

$$= \frac{184bc}{15} - 2ac - \frac{ab}{7}$$

3. Question

Take away:

$$(i) \frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{5}{6} + \frac{3}{2}x \text{ from } \frac{x^3}{3} - \frac{5}{2}x^2 + \frac{3}{5}x + \frac{1}{4}$$

$$(ii) \frac{5a^2}{2} + \frac{3a^3}{2} + \frac{a}{3} - \frac{6}{5} \text{ from } \frac{1}{3}a^3 - \frac{3}{4}a^2 - \frac{5}{2}$$

$$(iii) \frac{7}{4}x^3 + \frac{3}{5}x^2 + \frac{1}{2}x + \frac{9}{2} \text{ from } \frac{7}{2} - \frac{x}{3} - \frac{x^2}{5}$$

$$(iv) \frac{y^3}{3} + \frac{7}{3}y^2 + \frac{1}{2}y + \frac{1}{2} \text{ from } \frac{1}{3} - \frac{5}{3}y^2$$

$$(v) \frac{2}{3}ac - \frac{5}{7}ab + \frac{2}{3}bc \text{ from } \frac{3}{2}ab - \frac{7}{4}ac - \frac{5}{6}bc$$

Answer

$$(i) \frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{5}{6} + \frac{3}{2}x \text{ from } \frac{x^3}{3} - \frac{5}{2}x^2 + \frac{3}{5}x + \frac{1}{4}$$

$$= \frac{1}{3}x^3 - \frac{5}{2}x^2 + \frac{3}{5}x + \frac{1}{4} - \left(\frac{6}{5}x^2 - \frac{4}{5}x^3 + \frac{5}{6} + \frac{3}{2}x\right)$$

$$= \frac{1}{3}x^3 + \frac{4}{5}x^3 - \frac{5}{2}x^2 - \frac{6}{5}x^2 + \frac{3}{5}x - \frac{3}{2}x + \frac{1}{4} - \frac{5}{6}$$

$$= \frac{17}{15}x^3 - \frac{37}{10}x^2 - \frac{9x}{10} - \frac{14}{24}$$

$$= \frac{17}{15}x^3 - \frac{37}{10}x^2 - \frac{9}{10}x - \frac{7}{12}$$

$$(ii) \frac{5a^2}{2} + \frac{3a^3}{2} + \frac{a}{3} - \frac{6}{5} \text{ from } \frac{1}{3}a^3 - \frac{3}{4}a^2 - \frac{5}{2}$$

$$= \frac{1}{3}a^3 - \frac{3}{4}a^2 - \frac{5}{2} - \left(\frac{5}{2}a^2 + \frac{3}{2}a^2 + \frac{a}{3} - \frac{6}{5}\right)$$

$$= \frac{1}{3}a^3 - \frac{3}{2}a^3 - \frac{3}{4}a^2 - \frac{5}{2}a^2 - \frac{a}{3} - \frac{5}{2} + \frac{6}{5}$$

$$= \frac{1}{6}(2a^3 - 9a^3) - \frac{1}{4}(3a^2 - 10a^2) - \frac{a}{3} + \frac{-25+12}{10}$$

$$= -\frac{7}{6}a^3 - \frac{13}{4}a^2 - \frac{a}{3} - \frac{13}{10}$$

$$(iii) \frac{7}{4}x^3 + \frac{3}{5}x^2 + \frac{1}{2}x + \frac{9}{2} \text{ from } \frac{7}{2} - \frac{x}{3} - \frac{x^2}{5}$$

$$= \frac{7}{2} - \frac{x}{3} - \frac{1}{5}x^2 - \left(\frac{7}{4}x^3 + \frac{3}{5}x^2 + \frac{1}{2}x + \frac{9}{2}\right)$$

$$= -\frac{7}{4}x^3 - \frac{1}{5}x^2 - \frac{3}{5}x^2 - \frac{x}{3} - \frac{1}{2}x + \frac{7}{2} - \frac{9}{2}$$

$$= -\frac{7}{4}x^3 - \frac{4}{5}x^2 - \frac{2x-3x}{6} + \frac{7-9}{2}$$

$$= -\frac{7}{4}x^3 - \frac{4}{5}x^2 - \frac{5x}{6} - 1$$

$$(iv) \frac{y^3}{3} + \frac{7}{3}y^2 + \frac{1}{2}y + \frac{1}{2} \text{ from } \frac{1}{3} - \frac{5}{3}y^2$$

$$= \frac{1}{3} - \frac{5}{3}y^2 - \left(\frac{1}{3}y^3 + \frac{7}{3}y^2 + \frac{1}{2}y + \frac{1}{2}\right)$$

$$= -\frac{1}{3}y^3 - \frac{5}{3}y^2 - \frac{7}{3}y^2 - \frac{1}{2}y + \frac{1}{3} - \frac{1}{2}$$

$$= -\frac{1}{3}y^3 + \frac{1}{3}(-5y^2 - 7y^2) - \frac{1}{2}y + \frac{2-3}{2}$$

$$= -\frac{1}{3}y^3 - \frac{12}{3}y^2 - \frac{1}{2}y - \frac{1}{2}$$

$$(v) \frac{2}{3}ac - \frac{5}{7}ab + \frac{2}{3}bc \text{ from } \frac{3}{2}ab - \frac{7}{4}ac - \frac{5}{6}bc$$

$$= \frac{3}{2}ab - \frac{7}{4}ac - \frac{5}{6}bc - \left(\frac{2}{3}ac - \frac{5}{7}ab + \frac{2}{3}bc\right)$$

$$= \frac{3}{2}ab - \frac{5}{7}ab - \frac{7}{4}ac - \frac{2}{3}ac - \frac{5}{6}bc - \frac{2}{3}bc$$

$$= \frac{21ab-10ab}{14} - \frac{21ac-8ac}{12} - \frac{5bc+4bc}{6}$$

$$= \frac{11}{14}ab - \frac{29}{12}ac - \frac{3}{2}bc$$

4. Question

Subtract $3x-4y-7z$ from the sum of $x-3y+2z$ and $-4x+9y-11z$.

Answer

The sum of $x - 3y + 2z$ and $-4x + 9y - 11z$ is calculated as below:

$$= (x - 3y + 2z) + (-4x + 9y - 11z)$$

$$= x - 4x - 3y + 9y + 2z - 11z$$

$$= -3x + 6y - 9z$$

Now, The expression $3x-4y-7z$ has to be subtracted from the resultant expression i.e. $-3x + 6y - 9z$

$$= (-3x + 6y - 9z) - (3x - 4y - 7z)$$

$$= -3x - 3x + 6y + 4y - 9z + 7z$$

$$= -6x + 10y - 2z$$

5. Question

Subtract the sum of $3l-4m-7n^2$ and $2l+3m-4n^2$ from the sum of $9l+2m-3n^2$ and $-3l+m+4n^2$

Answer

Subtract the sum of $3l-4m-7n^2$ and $2l+3m-4n^2$ from the sum of $9l+2m-3n^2$ and $-3l+m+4n^2$

Sum of $9l + 2m - 3n^2$ and $-3l + m + 4n^2$

$$= 9l + 2m - 3n^2 + (-3l + m + 4n^2)$$

$$= 9l - 3l + 2m + m - 3n^2 + 4n^2$$

$$= 6l + 3m + n^2 \text{ (i)}$$

Sum of $3l - 4m - 7n^2$ and $2l + 5m - 4n^2$

$$= 3l - 4m - 7n^2 + 2l + 5m - 4n^2$$

$$= 5l - m - 11n^2 \text{ (ii)}$$

Subtract (i) and (ii), we get

$$= 6l + 3m + n^2 - (5l - m - 11n^2)$$

$$= 6l - 5l + 3m + m + n^2 + 11n^2$$

$$= l + 4m + 12n^2$$

6. Question

Subtract the sum of $2x-x^2+5$ and $-4x-3+7x^2$ from 5.

Answer

As given in the question, the Sum of $2x - x^2 + 5$ and $-4x - 3 + 7x^2$ is given as:

$$= 2x - x^2 + 5 - 4x - 3 + 7x^2$$

$$= 2x - 4x - x^2 + 7x^2 + 5 - 3$$

$$= -2x + 6x^2 + 2 \text{ (i)}$$

Now subtracting equation (i) from 5 we get,

Subtracting (ii) from (i), we get

$$= 5 - (-2x + 6x^2 + 2)$$

$$= 5 + 2x - 6x^2 - 2$$

$$= 3 + 2x - 6x^2$$

Therefore, the resultant expression is $3 + 2x - 6x^2$

7. Question

Simplify each of the following:

$$(i) \ x^2 - 3x + 5 - \frac{1}{2}(3x^2 - 5x + 7)$$

$$(ii) \ [5 - 3x + 2y - (2x - y)] - (3x - 7y + 9)$$

$$(iii) \ \frac{11}{2}x^2y - \frac{9}{4}xy^2 + \frac{1}{4}xy - \frac{1}{14}y^2x + \frac{1}{15}yx^2 + \frac{1}{2}xy$$

$$(iv) \ \left(\frac{1}{3}y^2 - \frac{4}{7}y + 11\right) - \left(\frac{1}{7}y - 3 + 2y^2\right) - \left(\frac{2}{7}y - \frac{2}{3}y^2 + 2\right)$$

$$(v) \ -\frac{1}{2}a^2b^2c + \frac{1}{3}ab^2c - \frac{1}{4}abc^2 - \frac{1}{5}cb^2a^2 + \frac{1}{6}cb^2a - \frac{1}{7}c^2ab + \frac{1}{8}ca^2b.$$

Answer

$$(i) \ x^2 - 3x + 5 - \frac{1}{2}(3x^2 - 5x + 7)$$

$$= x^2 - \frac{1}{2}3x^2 - 3x + \frac{1}{2}5x + 5 - \frac{1}{2}7$$

$$= \frac{1}{2}(2x^2 - 3x^2) - \frac{1}{2}(6x + 5x) + \frac{10-7}{2}$$

$$= \frac{-1}{2}x^2 - \frac{x}{2} + \frac{3}{2}$$

$$= \frac{-1}{2}x^2 - \frac{1}{2}x + \frac{3}{2}$$

$$(ii) \ [5 - 3x + 2y - (2x - y)] - (3x - 7y + 9)$$

$$= 5 - 3x + 2y - 2x + y - 3x + 7y - 9$$

$$= -8x + 10y - 4$$

$$(iii) \ \frac{11}{2}x^2y - \frac{9}{4}xy^2 + \frac{1}{4}xy - \frac{1}{14}y^2x + \frac{1}{15}yx^2 + \frac{1}{2}xy$$

$$= \frac{11}{2}x^2y + \frac{1}{15}x^2y - \frac{9}{4}xy^2 - \frac{1}{14}xy^2 + \frac{1}{4}xy + \frac{1}{2}xy$$

$$= \frac{1}{30}(165x^2y + 2x^2y) + \frac{1}{56}(-126xy^2 - 4xy^2) + \frac{xy+2xy}{4}$$

$$= \frac{167}{30}x^2y - \frac{130}{56}xy^2 + \frac{3}{4}xy$$

$$= \frac{167}{30}x^2y - \frac{65}{28}xy^2 + \frac{3}{4}xy$$

$$(iv) \ \left(\frac{1}{3}y^2 - \frac{4}{7}y + 11\right) - \left(\frac{1}{7}y - 3 + 2y^2\right) - \left(\frac{2}{7}y - \frac{2}{3}y^2 + 2\right)$$

$$= \frac{1}{3}y^2 - 2y^2 - \frac{2}{3}y^2 - \frac{4}{7}y - \frac{1}{7}y - \frac{2}{7}y + 11 + 3 - 2$$

$$= \frac{1}{3}(y^2 - 6y^2 + 2y^2) - \frac{1}{7}(4y - y - 2y) + 14 - 2$$

$$= \frac{-3}{3}y^2 - \frac{7}{7}y + 12$$

$$= -y^2 - y + 12$$

$$(v) -\frac{1}{2}a^2b^2c + \frac{1}{3}ab^2c - \frac{1}{4}abc^2 - \frac{1}{5}cb^2a + \frac{1}{6}cb^2a - \frac{1}{7}c^2ab + \frac{1}{8}ca^2b.$$

$$= \frac{-1}{2}a^2b^2c - \frac{1}{5}a^2b^2c + \frac{1}{3}ab^2c + \frac{1}{6}ab^2c - \frac{1}{4}abc^2 - \frac{1}{7}abc^2 + \frac{1}{8}a^2bc$$

$$= \frac{-7}{10}a^2b^2c + \frac{1}{2}ab^2c - \frac{11}{28}abc^2 + \frac{1}{8}a^2bc$$

Exercise 6.3

1. Question

Find each of the following products:

$$5x^2 \times 4x^3$$

Answer

$$5 \times x \times x \times 4 \times x \times x \times x$$

$$= 5 \times 4 \times x^5$$

$$= 20 \times x^5$$

$$= 20x^5$$

2. Question

Find each of the following products:

$$-3a^2 \times 4b^4$$

Answer

$$- 3 \times 4 - a^2 \times b^2$$

$$= -12 \times a^2 \times b^2$$

$$= -12a^2b^2$$

3. Question

Find each of the following products:

$$(-5xy) \times (-3x^2yz)$$

Answer

$$(-5) \times (-5) \times x \times x^2 \times y \times y \times z$$

$$= 15 \times x^3 \times y^2 \times z$$

$$= 15x^3y^2z$$

4. Question

Find each of the following products:

$$\frac{1}{2}xy \times \frac{2}{3}x^2yz^2$$

Answer

$$\frac{1}{2} \times \frac{2}{3} \times x \times x^2 \times y \times y \times z^2$$

$$= \frac{1}{3} \times x^3 \times y^2 \times z^2$$

$$= \frac{1}{3} x^3 y^2 z^2$$

5. Question

Find each of the following products:

$$\left(-\frac{7}{5}xy^2z\right) \times \left(\frac{13}{3}x^2yz^2\right)$$

Answer

$$\frac{-7}{5} \times \frac{13}{3} \times x \times x \times x^2 \times y^2 \times y \times z \times z^2$$

$$= \frac{-91}{15} \times x^3 \times y^3 \times z^3$$

$$= \frac{-91}{15} x^3 y^3 z^3$$

6. Question

Find each of the following products:

$$\left(\frac{-24}{25}x^3z\right) \times \left(-\frac{15}{16}xz^2y\right)$$

Answer

$$\frac{-24}{25} \times \frac{-15}{16} \times x^3 \times x \times x \times z \times z^2 \times y$$

$$= \frac{18}{20} \times x^4 \times z^3 \times y$$

$$= \frac{9}{10} x^4 z^3 y$$

7. Question

Find each of the following products:

$$\left(-\frac{1}{27}a^2b^2\right) \times \left(\frac{9}{2}a^3b^2c^2\right)$$

Answer

$$\frac{-1}{27} \times \frac{9}{2} \times a^2 \times a^3 \times b^2 \times b^2 \times c^2$$

$$= \frac{-1}{6} \times a^5 \times b^4 \times c^2$$

$$= \frac{-1}{6} a^5 b^4 c^2$$

8. Question

Find each of the following products:

$$(-7xy) \times \left(\frac{1}{4}x^2yz\right)$$

Answer

$$-7 \times \frac{1}{4} \times x \times x \times y \times x^2 \times y \times z$$

$$= \frac{-7}{4} \times x^3 \times y^2 \times z$$

$$= \frac{-7}{4}x^3y^2z$$

9. Question

Find each of the following products:

$$(7ab) \times (-5ab^2c) \times (6abc^2)$$

Answer

$$7 \times -5 \times 6 \times a \times a \times a \times b \times b^2 \times b \times c \times c^2$$

$$= 210 \times a^3 \times b^4 \times c^3$$

$$= 210a^3b^4c^3$$

10. Question

Find each of the following products:

$$(-5a) \times (-10a^2) \times (-2a^3)$$

Answer

$$(-5) \times (-10) \times (-2) \times a \times a^2 \times a^3$$

$$= -100 \times a^6$$

$$= -100a^6$$

11. Question

Find each of the following products:

$$(-4x^2) \times (-6xy^2) \times (-3yz^2)$$

Answer

$$(-4) \times (-6) \times (-3) \times x^2 \times x \times y^2 \times y \times z^2$$

$$= -72 \times x^3 \times y^3 \times z^2$$

$$= -72x^3y^3z^2$$

12. Question

Find each of the following products:

$$\left(-\frac{2}{7}a^4\right) \times \left(-\frac{3}{4}a^2b\right) \times \left(-\frac{14}{5}b^2\right)$$

Answer

$$\frac{-2}{7} \times \frac{-3}{4} \times \frac{-14}{5} \times a \times a^2 \times b \times b^2$$

$$= \frac{-6}{10} \times a^6 \times b^3$$

$$= \frac{-3}{5}a^6b^3$$

13. Question

Find each of the following products:

$$\left(\frac{7}{9}ab^2\right) \times \left(\frac{15}{7}ac^2b\right) \times \left(-\frac{3}{5}a^2c\right)$$

Answer

$$\frac{7}{9} \times \frac{15}{7} \times \frac{-3}{5} \times a \times a \times a^2 \times b^2 \times b \times c^2 \times c$$

$$= -a^4 \times b^3 \times c^3$$

$$= -a^4b^3c^3$$

14. Question

Find each of the following products:

$$\left(\frac{4}{3}u^2vw\right) \times (-5uvw^2) \times \left(\frac{1}{3}v^2wu\right)$$

Answer

$$\frac{4}{3} \times -5 \times \frac{1}{3} \times u^2 \times u \times u \times v \times v \times v^2 \times w \times w^2 \times w$$

$$= \frac{-20}{9} \times u^4 \times v^4 \times w^4$$

$$= \frac{-20}{9}u^4v^4w^4$$

15. Question

Find each of the following products:

$$(0.5x) \times \left(\frac{1}{3}xy^2z^4\right) \times (24x^2yz)$$

Answer

$$0.5 \times \frac{1}{3} \times 24 \times x \times x \times x \times y^2 \times y \times x^2 \times z^4 \times z$$

$$= \frac{12}{3} \times x^4 \times y^3 \times z^5$$

$$= 4x^4 \times y^3 \times z^5$$

$$= 4x^4y^3z^5$$

16. Question

Find each of the following products:

$$\left(\frac{4}{3}pq^2\right) \times \left(-\frac{1}{4}p^2r\right) \times (16p^2q^2r^2)$$

Answer

$$\frac{4}{3} \times \frac{1}{4} \times 16 \times p \times p^2 \times p^2 \times q^2 \times q^2 \times r \times r^2$$

$$= \frac{-16}{3} \times p^5 \times q^4 \times r^3$$

$$= \frac{-16}{3}p^5q^4r^3$$

17. Question

Find each of the following products:

$$(2.3xy) \times (0.1x) \times (0.16)$$

Answer

$$2.3 \times 0.1 \times 0.16 \times x \times x \times y$$

$$= 0.0368 \times x^2 \times y$$

$$= 0.0368x^2y$$

18. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

$$(3x) \times (4x) \times (-5x)$$

Answer

$$3 \times 4 \times -5 \times x \times x \times x$$

$$= -60 \times x^3$$

$$= -60x^3$$

19. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

$$(4x^2) \times (-3x) \times \left(\frac{4}{5}x^3\right)$$

Answer

$$4 \times -3 \times \frac{4}{5} \times x^2 \times x \times x^3$$

$$= \frac{-48}{5} \times x^6$$

$$= \frac{-48}{5}x^6$$

20. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

$$(5x^4) \times (x^2)^3 \times (2x)^2$$

Answer

$$5x^4 \times x^6 \times 4 \times x^2$$

$$= 5 \times 4 \times x^4 \times x^6 \times x^2$$

$$= 20 \times x^{12}$$

$$= 20x^{12}$$

21. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

$$(x^2)^3 \times (2x) \times (-4x) \times (5)$$

Answer

$$x^6 \times 2x \times (-4x) \times 5$$

$$= 2 \times -4 \times 5 \times x^6 \times x \times x$$

$$= -40 \times x^8$$

$$= -40x^8$$

22. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

Write down the product of $8x^2y^6$ and $-20xy$ verify the product for $x=2.5$, $y=1$

Answer

$$\begin{aligned}
 & -8 \times -2 \times x^2 \times x \times y^6 \times y \\
 & = 16 \times x^3 \times y^7 \\
 & = 16x^3y^7
 \end{aligned}$$

Verification is when, $x = 2.5$ and $y = 1$

$$\begin{aligned}
 \text{R.H.S} &= 16 (2.5)^3 \times (1)^7 \\
 &= 16 \times 15.625 \\
 &= 250
 \end{aligned}$$

$$\begin{aligned}
 \text{L.H.S} &= -8 \times 2.5^2 \times 1^6 \times -20 \times 1 \times 2.5 \\
 &= 250
 \end{aligned}$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

23. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

Evaluate $(3.2x^6y^3) \times (2.1x^2y^2)$ when $x=1$ and $y=0.5$

Answer

$$\begin{aligned}
 & 3.2 \times 2.1 \times x^6 \times x^2 \times y^3 \times y^2 \\
 & = 6.72 \times x^8 \times y^5 \\
 & = 6.72x^8y^5
 \end{aligned}$$

Verify:

When $x = 1$ and $y = 0.5$

$$\begin{aligned}
 \text{R.H.S} &= 6.72x^3y^5 \\
 &= 6.72 \times 1^8 \times 0.5^5 \\
 &= 0.21
 \end{aligned}$$

$$\text{L.H.S} = 3.2 \times 1^6 \times (-.5)^3 \times 2.1 \times 1^2 \times 0.5^2 = 0.21$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

24. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

Find the value of $(5x^6) \times (-1.5x^2y^3) \times (-12xy^2)$ when $x = 1, y=0.5$

Answer

$$\begin{aligned}
 & 5 \times -1.5 \times -12 \times x^6 \times x^2 \times x \times y^3 \times y^2 \\
 & = 90 \times x^9 \times y^5 \\
 & = 90x^9y^5
 \end{aligned}$$

Verification:

$x = 1$ and $y = 0.5$

$$\text{R.H.S} = 90x^9y^5$$

$$= 90 (1)^9 (05)^5$$

$$= 2.8125$$

$$\text{L.H.S} = 2.8125$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

25. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

Evaluate $(2.3a^5b^2) \times (1.2a^2b^2)$ when $a=1$ and $b = 0.5$

Answer

$$2.3a^5b^2 \times 1.2a^2b^2$$

$$= 2.3 \times 1.2 \times a^5 \times a^2 \times b^2 \times b^2$$

$$= 2.76 \times a^7 \times b^4$$

$$= 2.76a^7b^4$$

Verification:

$$a = 1 \text{ and } b = 0.5$$

$$2.76 a^7 b^4 = 2.76 (1)^7 (0.5)^4$$

$$= 2.76 \times 1 \times 0.0025$$

$$= 0.1725$$

26. Question

Express each of the following products as a monomials and verify the result in each case for $x=1$:

Evaluate $(-8x^2y^6) \times (-20xy)$ for $x = 2.5$ and $y=1$.

Answer

$$-8 \times -20 \times x^2 \times x \times y^6 \times y$$

$$= 160x^3y^7$$

Verify:

$$\text{When, } x = 2.5 \text{ and } y = 1$$

$$\text{R.H.S} = 160x^3y^7$$

$$= 160 \times (2.5)^3 \times (1)^7$$

$$= 2500$$

$$\text{L.H.S} = -8 \times 2.5^2 \times 1 \times -20 \times 1 \times 2.5$$

$$= 2500$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

27. Question

Express each of the following products as a monomials and verify the result for $x=1$, $y= 2$:

$$(-xy^3) \times (yx^3) \times (xy)$$

Answer

$$-x \times x^3 \times x \times y^3 \times y \times y$$

$$= -x^5y^5$$

Verify:

When $x = 1$ and $y = 2$

$$\text{R.H.S} = -x^5y^5$$

$$= (-1)^5 \times 2^5$$

$$= -1 \times 32$$

$$= -32$$

$$\text{L.H.S} = (-1) \times 2^3 \times 2 \times 1^3 \times 1 \times 2$$

$$= -32$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

28. Question

Express each of the following products as a monomials and verify the result for $x=1$, $y= 2$:

$$\left(\frac{1}{8}x^2y^4\right) \times \left(\frac{1}{4}x^4y^2\right) \times (xy) \times 5$$

Answer

$$\frac{1}{8} \times \frac{1}{4} \times 5 \times x^2 \times x^4 \times x \times y^4 \times y^2 \times y$$

$$= \frac{5}{32} \times x^6 \times y^6$$

$$= \frac{5}{32}x^6y^6$$

Verification:

When $x = 1$ and $y = 2$

$$\text{R.H.S} = \frac{5}{32} \times 1^6 \times 2^6$$

$$= \frac{5}{32} \times 64$$

$$= 5 \times 2$$

$$= 10$$

$$\text{L.H.S} = \frac{1}{8} \times 1^2 \times 2^4 \times \frac{1}{4} \times 1^4 \times 2^2 \times 1 \times 2 \times 5$$

$$= 10$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

29. Question

Express each of the following products as a monomials and verify the result for $x=1$, $y= 2$:

$$\left(\frac{2}{5}a^2b\right) \times (-15b^2ac) \times \left(-\frac{1}{2}c^2\right)$$

Answer

$$\frac{2}{5} \times 15 \times \frac{1}{2} \times a^2 \times a \times b \times b^2 \times c \times c^3$$

$$= 3a^3 \times b^3 \times c^3$$

$$= 3a^3b^3c^3$$

30. Question

Express each of the following products as a monomials and verify the result for x=1, y= 2:

$$\left(-\frac{4}{7}a^2b\right) \times \left(-\frac{2}{3}b^2c\right) \times \left(-\frac{7}{6}c^2a\right)$$

Answer

$$\frac{-4}{7} \times \frac{-2}{3} \times \frac{-7}{6} \times a^2 \times a \times b \times b^2 \times c \times c^2$$

$$= \frac{-4}{9} \times a^3 \times b^3 \times c^3$$

$$= \frac{-4}{9}a^3b^3c^3$$

31. Question

Express each of the following products as a monomials and verify the result for x=1, y= 2:

$$\left(\frac{4}{9}abc^3\right) \times \left(-\frac{27}{5}a^3b^3\right) \times (-8b^3c)$$

Answer

$$\frac{4}{9} \times \frac{-27}{5} \times -8 \times a \times a^3 \times b \times b^2 \times b^3 \times c^3 \times c$$

$$= \frac{-96}{5} \times a^4 \times b^6 \times c^4$$

$$= \frac{-96}{5}a^4b^6c^4$$

32. Question

Evaluate each of the following when x=2, y =-1

$$(2xy) \times \left(\frac{x^2y}{4}\right) \times (x^2) \times (y^2)$$

Answer

$$2 \times \frac{1}{4} \times x \times x \times x^2 \times x^2 \times y \times y^2 \times y$$

$$= \frac{1}{2}x^5y^5$$

$$= \frac{1}{2}x^5y^5$$

Verification:

When x = 2 and y = 1

$$\text{R.H.S} = \frac{1}{2}x^5y^5$$

$$= \frac{1}{2}(2)^5 \times (-1)^5$$

$$= \frac{1}{2} \times 32 \times -1$$

$$= -16$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

33. Question

Evaluate each of the following when $x=2$, $y = -1$

$$\left(\frac{3}{5}x^2y\right) \times \left(-\frac{15}{4}xy^2\right) \times \left(\frac{7}{9}x^2y^2\right)$$

Answer

$$\frac{3}{5} \times \frac{-15}{4} \times \frac{7}{9} \times x^2 \times x \times x^2 \times y \times y^2 \times y^2$$

$$= \frac{-7}{4} \times x^5 \times y^5$$

$$= \frac{-7}{4}x^5y^5$$

Verification:

When $x = 2$ and $y = -1$

$$\text{R.H.S} = \frac{-7}{4}x^5y^5$$

$$= \frac{-7}{4}(2)^5(-1)^5$$

$$= \frac{-7}{4} \times 32 \times -1$$

$$= 56$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

Exercise 6.4

1. Question

Find the following products:

$$2a^3(3a + 5b)$$

Answer

$$2a^3(3a + 5b)$$

$$= 2a^3 \times 3a + 2a^3 \times 5b$$

$$= 6 \times a^4 + 10a^3b$$

2. Question

Find the following products:

$$-11a(3a + 2b)$$

Answer

$$-11a(3a + 2b)$$

$$= (-11a \times 3a) + (-11a \times 2b)$$

$$= -33a^2 - 2 \times 11 \times a \times b$$

$$= -33a^2 - 22ab$$

3. Question

Find the following products:

$$-5a(7a - 2b)$$

Answer

$$-5a(7a - 2b)$$

$$= -5a \times 7a - (-5a) \times 2b$$

$$= -5 \times 7 \times a \times a + 5 \times 2 \times a \times b$$

$$= -35a^2 + 10ab$$

4. Question

Find the following products:

$$-11y^2(3y + 7)$$

Answer

$$-11y^2(3y + 7)$$

$$= -11y^2 \times 3y - 11y^2 \times 7$$

$$= -11 \times 3 \times y^2 \times y - 11y^2 \times 7$$

$$= -33y^3 - 77y^2$$

5. Question

Find the following products:

$$\frac{6x}{5}(x^3 + y^3)$$

Answer

$$\frac{6}{5}x(x^3 + y^3)$$

$$= \frac{6}{5}x \times x^3 + \frac{6}{5}x \times y^3$$

$$= \frac{6}{5}x^4 + \frac{6}{5}xy^3$$

6. Question

Find the following products:

$$xy(x^3 - y^3)$$

Answer

$$xy(x^3 - y^3)$$

$$= xy \times x^3 - xy \times y^3$$

$$= x^4y - xy^4$$

7. Question

Find the following products:

$$0.1y(0.1x^5 + 0.1y)$$

Answer

$$\begin{aligned} &0.1y(0.1x^5 + 0.1y) \\ &= 0.1y \times 0.1x^5 + 0.1y \times 0.1y \\ &= 0.01 \times x^5 \times y + 0.01 \times y^2 \\ &= 0.01x^5y + 0.01y^2 \end{aligned}$$

8. Question

Find the following products:

$$\left(-\frac{7}{4}ab^2c - \frac{6}{25}a^2c^2\right)(-50a^2b^2c^2)$$

Answer

$$\begin{aligned} &\left(-\frac{7}{4}ab^2c - \frac{6}{25}a^2c^2\right)(-50a^2b^2c^2) \\ &= \frac{-7}{4}ab^2c \times -50a^2b^2c^2 - \frac{6}{25}a^2c^2 \times -50a^2b^2 \times c^2 \\ &= \frac{7}{4} \times 50 \times a^3 \times b^4 \times c^3 - \frac{6}{25} \times -50 \times a^4 \times b^2 \times c^4 \\ &= \frac{350}{4}a^3b^4c^3 + 12a^4b^2c^4 \\ &= \frac{175}{2}a^3b^4c^3 + 12a^4b^2c^4 \end{aligned}$$

9. Question

Find the following products:

$$-\frac{8}{27}xyz\left(\frac{3}{2}xyz^2 - \frac{9}{4}xy^2z^3\right)$$

Answer

$$\begin{aligned} &-\frac{8}{27}xyz\left(\frac{3}{2}xyz^2 - \frac{9}{4}xy^2z^3\right) \\ &= \frac{-8}{27}xyz \times \frac{3}{2}xyz^2 - \frac{8}{27}xyz \times \frac{-9}{4}xy^2z^3 \\ &= \frac{-4}{9} \times x^2 \times y^2 \times z^3 + \frac{2}{3} \times x^2 \times y^3 \times z^4 \\ &= \frac{-4}{9}x^2y^2z^3 + \frac{2}{3}x^2y^3z^4 \end{aligned}$$

10. Question

Find the following products:

$$-\frac{4}{27}xyz\left(\frac{9}{2}x^2yz - \frac{3}{4}xyz^2\right)$$

Answer

$$\begin{aligned} &-\frac{4}{27}xyz\left(\frac{9}{2}x^2yz - \frac{3}{4}xyz^2\right) \\ &= \frac{-4}{27}xyz \times \frac{9}{2}x^2yz - \frac{4}{27}xyz \times \frac{-3}{4}xyz^2 \\ &= \frac{-2}{3} \times x^3 \times y^2 \times z^2 + 9 \times x^2 \times y^2 \times z^3 \end{aligned}$$

$$= \frac{-2}{3}x^3y^2z^2 + 9x^2y^2z^3$$

11. Question

Find the following products:

$$1.5x(10x^2y - 100xy^2)$$

Answer

$$1.5x(10x^2y - 100xy^2)$$

$$= 1.5x \times 10x^2y - 1.5x \times 100xy^2$$

$$= 15 \times x^3 \times y - 150 \times x^2 \times y^2$$

$$= 15x^3y - 150x^2y^2$$

12. Question

Find the following products:

$$4.1xy(1.1x - y)$$

Answer

$$4.1xy(1.1x - y)$$

$$= 4.1xy \times 1.1x - 4.1xy \times y$$

$$= 4.51x^2y - 4.1xy^2$$

13. Question

Find the following products:

$$250.5xy \left(xz + \frac{y}{10} \right)$$

Answer

$$250 \times 5 (x^2yz + \frac{xy*y}{10})$$

$$= 250 (5x^2yz + \frac{x*y*y}{2})$$

$$= 250 \times 5x^2yz + 125xy^2$$

14. Question

Find the following products:

$$\frac{7}{5}x^2y \left(\frac{3}{5}xy^2 + \frac{2}{5}x \right)$$

Answer

$$\frac{7}{5} \left(\frac{3}{5}x^3y^3 + \frac{2}{5}x^3y \right)$$

$$= \frac{21}{25}x^3y^3 + \frac{14}{25}x^3y$$

15. Question

Find the following products:

$$\frac{4}{5}a(a^2 + b^2 - 3c^2)$$

Answer

$$\frac{4}{5}(a^3 + ab^2 - 3ac^2)$$

$$= \frac{4}{5}a^3 + \frac{4}{5}ab^2 - \frac{12}{5}ac^2$$

16. Question

Find the product $24x^2(1-2x)$ and evaluate its value for $x=3$

Answer

$$24x^2(1 - 2x)$$

$$= 24x^2 - 48x^3$$

According to question,

When $x = 3$

$$= 24x^2 - 48x^3$$

$$= 24(3)^2 - 48(3)^3$$

$$= 24(9) - 48(27)$$

$$= 216 - 1296$$

$$= -1080$$

17. Question

Find the product $-3y(xy+y^2)$ and find its value for $x = 4$ and $y = 5$

Answer

$$-3y(xy + y^2)$$

$$= -3xy^2 - 3y^3$$

According to question:

When $x = 4$ and $y = 5$

$$= -3xy^2 - 3y^3$$

$$= -3(4)(5)^2 - 3(5)^3$$

$$= -300 - 375$$

$$= -675$$

18. Question

Multiply $-\frac{3}{2}x^2y^3bx(2x-y)$ and verify the answer for $x = 1$ and $y = 2$

Answer

$$-\frac{3}{2}x^2y^3bx(2x-y)$$

$$= -3x^3y^3bx + \frac{3}{2}x^2y^4bx$$

$$= -3x^4y^3b + \frac{3}{2}x^3y^4b$$

According to question:

When $x = 1$ and $y = 2$

$$= -3 (1)^4 (2)^3 b + \frac{3}{2} (1)^3 (2)^4 b$$

$$= -3 (8) b + 3 (8) b$$

$$= 0$$

19. Question

Multiply the monomial by the binomial and find the value of each for $x=-1$, $y=0.25$ and $z=0.005$:

(i) $15y^2 (2-3x)$

(ii) $-3x (y^2+z^2)$

(iii) $z^2 (x-y)$

(iv) $xz(x+y^2)$

Answer

(i) $15y^2 (2 - 3x)$

$$= 30y^2 - 45xy^2$$

Putting $x = -1$, $y = \frac{25}{100}$ and $z = \frac{5}{1000}$

$$= 30 \left(\frac{25}{100}\right)^2 - 45 (-1) \left(\frac{25}{100}\right)^2$$

$$= 30 \left(\frac{1}{16}\right) + 45 \left(\frac{1}{16}\right)$$

$$= \frac{15}{8} + \frac{45}{16}$$

$$= \frac{30+45}{16}$$

$$= \frac{75}{16}$$

(ii) $-3x (y^2+z^2)$

Putting $x = -1$, $y = \frac{25}{100}$ and $z = \frac{5}{1000}$

$$= -3 (-1) \left(\frac{25}{100}\right)^2 - 3 (-1) \left(\frac{5}{1000}\right)^2$$

$$= \frac{3 \times 25 \times 25}{100 \times 100} + \frac{3 \times 5 \times 5}{1000 \times 1000}$$

$$= \frac{3}{16} + \frac{3}{40000}$$

$$= \frac{39}{200}$$

(iii) $z^2 (x-y)$

Putting $x = -1$, $y = \frac{25}{100}$ and $z = \frac{5}{1000}$

$$z^2 (x - y)$$

$$= \left(\frac{5}{1000}\right)^2 \left(-1 - \frac{25}{100}\right)$$

$$= \left(\frac{1}{40000}\right) \left(\frac{-5}{4}\right)$$

$$= \frac{-1}{32000}$$

$$(iv) \ xz(x+y^2)$$

$$\text{Putting } x = -1, y = \frac{25}{100} \text{ and } z = \frac{5}{1000}$$

$$= (-1)^2 \left(\frac{5}{1000}\right) + (-1) \left(\frac{25}{100}\right)^2 \left(\frac{5}{1000}\right)$$

$$= \frac{1}{200} - \frac{1}{16} \left(\frac{1}{200}\right)$$

$$= \frac{16-1}{3200}$$

$$= \frac{3}{640}$$

20. Question

Simplify:

$$(i) \ 2x^2(x^3 - x) - 3x(x^4 + 2x) - 2(x^4 - 3x^2)$$

$$(ii) \ x^3y(x^2 - 2x) + 2xy(x^3 - x^4)$$

$$(iii) \ 3a^2 + (a + 2) - 3a(2a + 1)$$

$$(iv) \ x(x + 4) + 3x(2x^2 - 1) + 4x^2 + 4$$

$$(v) \ a(b - c) - b(c - a) - c(a - b)$$

$$(vi) \ a(b - c) + b(c - a) + c(a - b)$$

$$(vii) \ 4ab(a - b) - 6a^2(b - b^2) - 3b^2(2a^2 - a) + 2ab(b - a)$$

$$(viii) \ x^2(x^2 + 1) - x^3(x + 1) - x(x^3 - x)$$

$$(ix) \ 2a^2 + 3a(1 - 2a^3) + a(a + 1)$$

$$(x) \ a^2(2a - 1) + 3a + a^3 - 8$$

$$(xi) \ \frac{3}{2}x^2(x^2 - 1) + \frac{1}{4}x^2(x^2 + x) - \frac{3}{4}x(x^3 - 1)$$

$$(xii) \ a^2b(a - b^2) + ab^2(4ab - 2a^2) - a^3b(1 - 2b)$$

$$(xiii) \ a^2b(a^3 - a + 1) - ab(a^4 - 2a^2 + 2a) - b(a^3 - a^2 - 1)$$

Answer

$$(i) \ 2x^2(x^3 - x) - 3x(x^4 + 2x) - 2(x^4 - 3x^2)$$

$$= 2x^5 - 2x^3 - 3x^5 - 6x^2 - 2x^4 + 6x^2$$

$$= -x^5 - 2x^4 - 2x^3$$

$$(ii) \ x^3y(x^2 - 2x) + 2xy(x^3 - x^4)$$

$$= x^5y - 2x^4y + 2x^4y - 2x^5y$$

$$= -x^5y$$

$$(iii) \ 3a^2 + (a + 2) - 3a(2a + 1) = 3a^2 + a + 2 - 6a^2 - 3a$$

$$= -3a^2 - 2a + 2$$

$$(iv) \ x(x + 4) + 3x(2x^2 - 1) + 4x^2 + 4$$

$$= x^2 + 4x + 6x^3 - 3x + 4x^2 + 4$$

$$= 6x^3 + 5x^2 + x + 4$$

$$(v) \ a(b - c) - b(c - a) - c(a - b)$$

$$= ab - ac - bc + ab - ca + bc$$

$$= 2ab - 2ac$$

$$(vi) a(b - c) + b(c - a) + c(a - b)$$

$$= ab - ac + bc - ab + ac - bc$$

$$= 0$$

$$(vii) 4ab(a - b) - 6a^2(b - b^2) - 3b^2(2a^2 - a) + 2ab(b - a)$$

$$= 4a^2b - 4ab^2 - 6a^2b + 6a^2b^2 - 6a^2b^2 + 3ab^2 + 2ab^2 - 2a^2b$$

$$= 3ab^2$$

$$(viii) x^2(x^2 + 1) - x^3(x + 1) - x(x^3 - x)$$

$$= x^4 + x^2 - x^4 - x^3 - x^4 + x^2$$

$$= 2x^2 - 2x^3$$

$$(ix) 2a^2 + 3a(1 - 2a^3) + a(a + 1)$$

$$= 2a^2 + 3a - 6a^4 + a^2 + a$$

$$= -6a^4 + 3a^2 + 4a$$

$$(x) a^2(2a - 1) + 3a + a^3 - 8$$

$$= 2a^3 - a^2 + 3a + a^3 - 8$$

$$(xi) \frac{3}{2}x^2(x^2 - 1) + \frac{1}{4}x^2(x^2 + x) - \frac{3}{4}x(x^3 - 1)$$

$$= 3a^3 - a^2 + 3a - 8 = \frac{3}{2}x^4 - \frac{3}{2}x^2 + \frac{1}{4}x^4 + \frac{1}{4}x^3 + \frac{3}{4}x^4 + \frac{3}{4}x \quad (xii) a^2b(a - b^2) + ab^2(4ab - 2a^2) - a^3b(1 - 2b)$$

$$= x^4 - \frac{3}{2}x^2 + \frac{1}{4}x^3 + \frac{3}{4}x$$

$$= a^3b - a^2b^3 + 4a^2b^3 - 2a^3b^2 - a^3b + 2a^3b^2$$

$$= -a^2b^3 + 4a^2b^3$$

$$= 3a^2b^3$$

$$(xiii) a^2b(a^3 - a + 1) - ab(a^4 - 2a^2 + 2a) - b(a^3 - a^2 - 1)$$

$$= a^5b - a^3b + a^2b - a^5b + 2a^3b - 2a^2b - ba^3 + a^2b + b$$

$$= b$$

Exercise 6.5

1. Question

Multiply:

$$(5x + 3) \text{ by } (7x + 2)$$

Answer

$$(5x + 3) \times (7x + 2)$$

$$= 5x(7x + 2) + 3(7x + 2)$$

$$= 35x^2 + 10x + 21x + 6$$

$$= 35x^2 + 31x + 6$$

2. Question

Multiply:

$$(2x + 8) \text{ by } (x - 3)$$

Answer

$$(2x + 8) \times (x - 3)$$

$$= 2x(x - 3) + 8(x - 3)$$

$$= 2x^2 - 6x + 8x - 24$$

$$= 2x^2 - 2x - 24$$

3. Question

Multiply:

$$(7x + y) \text{ by } (x + 5y)$$

Answer

$$(7x + y) \times (x + 5y)$$

$$= 7x(x + 5y) + y(x + 5y)$$

$$= 7x^2 + 35xy + xy + 5y^2$$

$$= 7x^2 + 36xy + 5y^2$$

4. Question

Multiply:

$$(a - 1) \text{ by } (0.1a^2 + 3)$$

Answer

$$(a - 1) \times (0.1a^2 + 3)$$

$$= a(0.1a^2 + 3) - 1(0.1a^2 + 3)$$

$$= 0.1a^3 + 3a - 0.1a^2 - 3$$

5. Question

Multiply:

$$(3x^2 + y^2) \text{ by } (2x^2 + 3y^2)$$

Answer

$$(3x^2 + y^2) \times (2x^2 + 3y^2)$$

$$= 3x^2(2x^2 + 3y^2) + y^2(2x^2 + 3y^2)$$

$$= 6x^4 + 9x^2y^2 + 2x^2y^2 + 3y^4$$

$$= 6x^4 + 11x^2y^2 + 3y^4$$

6. Question

Multiply:

$$\left(\frac{3}{5}x + \frac{1}{2}y\right) \text{ by } \left(\frac{5}{6}x + 4y\right)$$

Answer

$$\left(\frac{3}{5}x + \frac{1}{2}y\right) \times \left(\frac{5}{6}x + 4y\right)$$

$$= \frac{3}{5}x \left(\frac{5}{6}x + 4y\right) + \frac{1}{2}y \left(\frac{5}{6}x + 4y\right)$$

$$= \frac{1}{2}x^2 + \frac{12}{5}xy + \frac{5}{12}xy + 2y^2$$

$$= \frac{1}{2}x^2 + \frac{169}{60}xy + 2y^2$$

7. Question

Multiply:

$$(x^6 - y^6) \text{ by } (x^2 + y^2)$$

Answer

$$(x^6 - y^6) \times (x^2 + y^2)$$

$$= x^6 (x^2 + y^2) - y^6 (x^2 + y^2)$$

$$= x^8 + x^6y^2 - x^2y^6 - y^8$$

8. Question

Multiply:

$$(x^2 - y^2) \text{ by } (3a + 2b)$$

Answer

$$(x^2 - y^2) \times (3a + 2b)$$

$$= x^2 (3a + 2b) - y^2 (3a + 2b)$$

$$= 3ax^2 + 2bx^2 - 3ay^2 - 2by^2$$

9. Question

Multiply:

$$[-3d + (7f)] \text{ by } (5f + f)$$

Answer

$$(-3d + 7f) \times (5d + f)$$

$$= -3d (5d + f) + 7f (5d + f)$$

$$= -15d^2 - 3df + 35df + 7f^2$$

$$= -15d^2 + 32df + 7f^2$$

10. Question

Multiply:

$$(0.8a - 0.5b) \text{ by } (1.5a - 3b)$$

Answer

$$(0.8a - 0.5b) \times (1.5a - 3b)$$

$$= 0.8a (1.5a - 3b) - 0.5b (1.5a - 3b)$$

$$= 1.2a^2 - 2.4ab - 7.5ab + 1.5b^2$$

$$= 1.2a^2 - 9.9ab + 1.5b^2$$

11. Question

Multiply:

$$(2x^2y^2 - 5xy^2) \text{ by } (x^2 - y^2)$$

Answer

$$\begin{aligned}(2x^2y^2 - 5xy^2) \times (x^2 - y^2) \\&= 2x^2y^2 (x^2 - y^2) - 5xy^2 (x^2 - y^2) \\&= 2x^4y^2 - 2x^2y^4 - 5x^3y^2 + 5xy^4\end{aligned}$$

12. Question

Multiply:

$$\left(\frac{x}{7} + \frac{x^2}{2}\right) \text{ by } \left(\frac{2}{5} + \frac{9x}{4}\right)$$

Answer

$$\begin{aligned}\left(\frac{x}{7} + \frac{x \cdot x}{2}\right) \times \left(\frac{2}{5} + \frac{9x}{4}\right) \\&= \frac{x}{7} \left(\frac{2}{5} + \frac{9x}{4}\right) + \frac{x \cdot x}{2} \left(\frac{2}{5} + \frac{9x}{4}\right) \\&= \frac{2x}{35} + \frac{9x \cdot x}{28} + \frac{x \cdot x}{5} + \frac{9x \cdot x \cdot x}{8} \\&= \frac{2x}{35} + \frac{73}{140}x^2 + \frac{9}{8}x^3\end{aligned}$$

13. Question

Multiply:

$$\left(-\frac{a}{7} + \frac{a^2}{9}\right) \text{ by } \left(\frac{b}{2} - \frac{b^2}{3}\right)$$

Answer

$$\begin{aligned}\left(-\frac{a}{7} + \frac{a \cdot a}{9}\right) \times \left(\frac{b}{2} - \frac{b \cdot b}{3}\right) \\&= \frac{-a}{7} \left(\frac{b}{2} - \frac{b \cdot b}{3}\right) + \frac{a \cdot a}{9} \left(\frac{b}{2} - \frac{b \cdot b}{3}\right) \\&= \frac{-ab}{14} + \frac{a \cdot b \cdot b}{21} + \frac{a \cdot a \cdot b}{18} - \frac{a \cdot a \cdot b \cdot b}{27}\end{aligned}$$

14. Question

Multiply:

$$(3x^2y - 5xy^2) \text{ by } \left(\frac{1}{5}x^2 + \frac{1}{3}y^2\right)$$

Answer

$$\begin{aligned}(3x^2y - 5xy^2) \times \left(\frac{1}{5}x^2 + \frac{1}{3}y^2\right) \\&= 3x^2y \left(\frac{1}{5}x^2 + \frac{1}{3}y^2\right) - 5xy^2 \left(\frac{1}{5}x^2 + \frac{1}{3}y^2\right) \\&= \frac{3}{5}x^4y + 3x^2y^3 - x^3y^2 + \frac{5}{3}xy^4\end{aligned}$$

15. Question

Multiply:

$$(2x^2 - 1) \text{ by } (4x^3 + 5x^2)$$

Answer

$$\begin{aligned} & (2x^2 - 1) \times (4x^3 + 5x^2) \\ &= 2x^2 (4x^3 + 5x^2) - 1 (4x^3 + 5x^2) \\ &= 8x^5 + 10x^3 - 4x^3 - 5x^2 \\ &= 8x^5 + 6x^3 - 5x^2 \end{aligned}$$

16. Question

Multiply:

$$(2xy + 3y^2)(3y^2 - 2)$$

Answer

$$\begin{aligned} & (2xy + 3y^2) \times (3y^2 - 2) \\ &= 2xy (3y^2 - 2) + 3y^2 (3y^2 - 2) \\ &= 6xy^3 - 4xy + 3y^4 - 6y^2 \end{aligned}$$

17. Question

Find the following products and verify the result for $x=-1$, $y=-2$:

$$(3x - 5y)(x + y)$$

Answer

$$\begin{aligned} & (3x - 5y) \times (x + y) \\ &= x (3x - 5y) + y (3x - 5y) \\ &= 3x^2 - 5xy + 3xy - 5y^2 \\ &= 3x^2 - 2xy - 5y^2 \end{aligned}$$

Putting $x = -1$ and $y = -2$, we have

$$\begin{aligned} & [3(-1) - 5(-2)][(-1) + (-2)] = 3(-1)^2 - 2(-1)(-2) - 5(-2)^2 \\ & (-3 + 10)(-1 - 2) = 3 - 4 - 20 \\ & -21 = -21 \end{aligned}$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

Hence, verified

18. Question

Find the following products and verify the result for $x=-1$, $y=-2$:

$$(x^2y - 1)(3 - 2x^2y)$$

Answer

$$\begin{aligned} & x^2y (3 - 2x^2y) - 1 (3 - 2x^2y) \\ &= 3x^2y - 2x^4y^2 - 3 + 2x^2y \\ &= 2x^4y^2 + 5x^2y - 3 \end{aligned}$$

Putting $x = -1$ and $y = -2$, we have

$$= [(-1)^2 (-2) - 1] [3 - 2 (-1)^2 (-2)] = [-2 (-1)^4 (-2)^2 + 5 (-1)^2 (2) - 3]$$

$$= (-2 - 1) (3 + 4) = -8 - 10 - 3$$

$$-21 = -21$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

Hence, verified

19. Question

Find the following products and verify the result for $x=-1$, $y=-2$:

$$\left(\frac{1}{3}x - \frac{y^2}{5}\right)\left(\frac{1}{3}x + \frac{y^2}{5}\right)$$

Answer

$$\left(\frac{1}{3}x - \frac{y^2}{5}\right)^2$$

$$= \left(\frac{1}{3}x - \frac{y^2}{5}\right) \left(\frac{1}{3}x + \frac{y^2}{5}\right)$$

$$= \frac{1}{9}x^2 - \frac{1}{25}y^4$$

Putting $x = -1$ and $y = -2$, we have

$$\left(\frac{1}{3}(-1) - \frac{(-2)(-25)}{5}\right) = \left(\frac{1}{9}(-1)^2 - \frac{-2*-2*-2*-2}{25}\right)$$

$$= \left(\frac{-1}{3} - \frac{4}{5}\right) \left(\frac{-1}{3} + \frac{4}{5}\right) = \left(\frac{1}{9} - \frac{16}{25}\right)$$

$$= \left(\frac{-17}{15}\right) \left(\frac{7}{15}\right) = \frac{-119}{225}$$

$$= \frac{-119}{225} = \frac{-119}{225}$$

Therefore,

$$\text{L.H.S} = \text{R.H.S}$$

Hence, verified

20. Question

Simplify:

$$x^2(x+2y)(x-3y)$$

Answer

$$x^2(x^2 - 3xy + 2xy - 3y^2)$$

$$= x^2(x^2 - xy - 6y^2)$$

$$= x^4 - x^3y - 6x^2y^2$$

21. Question

Simplify:

$$(x^2 - 2y^2)(x + 4y)x^2y^2$$

Answer

$$(x^3 + 4x^2y - 2xy^2 - 8y^3) \times x^2y^2$$

$$= x^5y^2 + 4x^4y^3 - 2x^3y^4 - 8x^2y^5$$

22. Question

Simplify:

$$a^2b^2(a + 2b)(3a + b)$$

Answer

$$a^2b^2(3a^2 + ab + 6ab + 2b^2)$$

$$= a^2b^2(3a^2 + 7ab + 2b^2)$$

$$= 3a^4b^2 + 7a^3b^3 + 2a^2b^4$$

23. Question

Simplify:

$$x^2(x - y)y^2(x + 2y)$$

Answer

$$x^2y^2(x - y)(x + 2y)$$

$$= x^2y^2(x^2 + 2xy - xy - 2y^2)$$

$$= x^2y^2(x^2 + xy - 2y^2)$$

$$= x^4y^2 + x^3y^3 - 2x^2y^4$$

24. Question

Simplify:

$$(x^3 - 2x^2 + 5x - 7)(2x - 3)$$

Answer

$$2x^4 - 4x^3 + 4x^2 - 14x - 3x^3 + 6x^2 - 6x + 21$$

$$= 2x^4 - 7x^3 + 10x^2 - 20x + 21$$

25. Question

Simplify:

$$(5x + 3)(x - 1)(3x - 2)$$

Answer

$$(5x^2 - 2x - 3)(3x - 2)$$

$$= 15x^3 - 6x^2 - 9x - 10x^2 + 4x + 6$$

$$= 15x^3 - 16x^2 - 5x + 6$$

26. Question

Simplify:

$$(5 - x)(6 - 5x)(2 - x)$$

Answer

$$(x^2 - 7x + 10)(6 - 5x)$$

$$= -5x^3 + 35x^2 - 50x + 6x^2 - 42x + 60$$

$$= -5x^2 + 41x^2 - 92x + 60$$

27. Question

Simplify:

$$(2x^2 + 3x - 5)(3x^2 - 5x + 4)$$

Answer

$$6x^4 + 9x^3 - 15x^2 - 10x^3 - 15x^2 + 25x + 8x^2 + 12x - 20$$

$$= 6x^4 - x^3 - 22x^2 + 37x - 20$$

28. Question

Simplify:

$$(3x - 2)(2x - 3) + (5x - 3)(x + 1)$$

Answer

$$6x^2 - 9x - 4x + 6 + 5x^2 + 5x - 3x - 3$$

$$= 11x^2 - 11x + 3$$

29. Question

Simplify:

$$(5x - 3)(x + 2) - (2x + 5)(4x - 3)$$

Answer

$$5x^2 + 10x - 3x - 6 - 8x^2 + 6x - 20x + 15$$

$$= -3x^2 - 7x + 9$$

30. Question

Simplify:

$$(3x + 2y)(4x + 3y) - (2x - y)(7x - 3y)$$

Answer

$$12x^2 + 9xy + 8xy$$

$$= 12x^2 + 9xy + 8xy + 6y^2 - 14x^2 + 6xy + 7xy - 3y^2$$

$$= -2x^2 + 30xy + 3y^2$$

31. Question

Simplify:

$$(x^2 - 3x + 2)(5x - 2) - (3x^2 + 4x - 5)(2x - 1)$$

Answer

$$5x^4 - 15x^2 + 10x - 2x^3 + 6x - 4 - (6x^3 + 8x^2 - 10x - 3x^2 - 4x + 5)$$

$$= 5x^4 - 15x^2 - 2x^3 + 16x - 4 - 6x^3 - 5x^2 + 14x - 5$$

$$= 5x^4 - 8x^3 - 20x^2 + 30x - 9$$

32. Question

Simplify:

$$(x^3 - 2x^2 + 3x - 4)(x - 1) - (2x - 3)(x^2 - x + 1)$$

Answer

$$x^4 - 2x^3 + 3x^2 - 4x - x^3 + 2x^2 - 3x + 4 - (2x^3 - 2x^2 + 2x - 3x^2 + 3x - 3)$$

$$= x^4 - 3x^3 + 5x^2 - 7x + 4 - 2x^3 + 5x^2 - 5x + 3$$

$$= x^4 - 5x^3 + 10x^2 - 12x + 7$$

Exercise 6.6

1. Question

Write the following squares of binomials as trinomias:

(i) $(x + 2)^2$

(ii) $(8x + 3b)^2$

(iii) $(2m + 1)^2$

(iv) $\left(9a + \frac{1}{6}\right)^2$

(v) $\left(x + \frac{x^2}{2}\right)^2$

(vi) $\left(\frac{x}{4} - \frac{y}{3}\right)^2$

(vii) $\left(3x - \frac{1}{3x}\right)^2$

(viii) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$

(ix) $\left(\frac{3a}{2} - \frac{5b}{4}\right)^2$

(x) $(a^2b - bc^2)^2$

(xi) $\left(\frac{2a}{3b} + \frac{2b}{3a}\right)^2$

(xii) $(x^2 - ay)^2$

Answer

(i) $(x + 2)^2$

$$x^2 + 2(x)(2) + 2^2$$

$$= x^2 + 4x + 4$$

(ii) $(8x + 3b)^2$

$$(8x)^2 + 2(8x)(3b) + (3b)^2$$

$$= 16x^2 + 48xb + 9b^2$$

(iii) $(2m + 1)^2$

$$(2m)^2 + 2 (2m) (1) + 1^2$$

$$= 4m^2 + 4m + 1$$

$$(iv) \left(9a + \frac{1}{6} \right)^2$$

$$(9a)^2 + 2 (9a) \left(\frac{1}{6} \right) + \left(\frac{1}{6} \right)^2$$

$$= 81a^2 + 3a + \frac{1}{36}$$

$$(v) \left(x + \frac{x^2}{2} \right)^2$$

$$(x)^2 + 2 (x) \left(\frac{x^2}{2} \right) + \left(\frac{x^2}{2} \right)^2$$

$$= x^2 + x^3 + \frac{1}{4}x^4$$

$$(vi) \left(\frac{x}{4} - \frac{y}{3} \right)^2$$

$$\left(\frac{x}{4} \right)^2 - 2 \left(\frac{x}{4} \right) \left(\frac{y}{3} \right) + \left(\frac{y}{3} \right)^2$$

$$= \frac{1}{16}x^2 - \frac{xy}{6} + \frac{1}{9}y^2$$

$$(vii) \left(3x - \frac{1}{3x} \right)^2$$

$$(3x)^2 - 2 (3x) \left(\frac{1}{3x} \right) + \left(\frac{1}{3x} \right)^2$$

$$= 9x^2 - 2 + \frac{1}{9x^2}$$

$$(viii) \left(\frac{x}{y} - \frac{y}{x} \right)^2$$

$$\left(\frac{x}{y} \right)^2 - 2 \left(\frac{x}{y} \right) \left(\frac{y}{x} \right) + \left(\frac{y}{x} \right)^2$$

$$= \frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$$

$$(ix) \left(\frac{3a}{2} - \frac{5b}{4} \right)^2$$

$$\left(\frac{3a}{2} \right)^2 - 2 \left(\frac{3a}{2} \right) \left(\frac{5b}{4} \right) + \left(\frac{5b}{4} \right)^2$$

$$= \frac{9}{4}a^2 - \frac{15}{4}ab + \frac{25}{16}b^2$$

$$(x) (a^2b - bc^2)^2$$

$$(a^2b)^2 - 2 (a^2b) (bc^2) + (bc^2)^2$$

$$= a^4b^2 - 2a^2b^2c^2 + b^2c^4$$

$$(xi) \left(\frac{2a}{3b} + \frac{2b}{3a} \right)^2$$

$$\left(\frac{2a}{3b} \right)^2 + 2 \left(\frac{2a}{3b} \right) \left(\frac{2b}{3a} \right) + \left(\frac{2b}{3a} \right)^2$$

$$= \frac{4a^2}{9b^2} + \frac{8}{9} + \frac{4b^2}{9a^2}$$

$$(xii) (x^2 - ay)^2$$

$$(x^2)^2 - 2(x^2)(ay) + (ay)^2$$

$$= x^4 - 2x^2ay + a^2y^2$$

2. Question

Find the product of the following binomials:

$$(i) (2x + y)(2x + y)$$

$$(ii) (a + 2b)(a - 2b)$$

$$(iii) (a^2 + bc)(a^2 - bc)$$

$$(iv) \left(\frac{4x}{5} - \frac{3y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$$

$$(v) \left(2x + \frac{3}{y}\right)\left(2x - \frac{3}{4}\right)$$

$$(vi) (2a^3 + b^3)(2a^3 - b^3)$$

$$(vii) \left(x^4 + \frac{2}{x^2}\right)\left(x^4 - \frac{2}{x^2}\right)$$

$$(viii) \left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$$

Answer

$$(i) (2x + y)(2x + y)$$

$$2x(2x + y) + y(2x + y)$$

$$= 4x^2 + 2xy + 2xy + 3y$$

$$= 4x^2 + 4xy + 3y$$

$$(ii) (a + 2b)(a - 2b)$$

$$a(a - 2b) + 2b(a - 2b)$$

$$= a^2 - 2ab + 2ab - 4b^2$$

$$= a^2 - 4b^2$$

$$(iii) (a^2 + bc)(a^2 - bc)$$

$$a^2(a^2 - bc) + bc(a^2 - bc)$$

$$= a^4 - a^2bc + bca^2 - b^2c^2$$

$$= a^4 - b^2c^2$$

$$(iv) \left(\frac{4x}{5} - \frac{3y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$$

$$\frac{4x}{5}\left(\frac{4x}{5} + \frac{3y}{4}\right) - \frac{3y}{4}\left(\frac{4x}{5} + \frac{3y}{4}\right)$$

$$= \frac{16}{25}x^2 + \frac{12}{20}yx - \frac{12xy}{20} - \frac{9y*y}{16}$$

$$= \frac{16}{25}x^2 - \frac{9}{16}y^2$$

$$(v) \left(2x + \frac{3}{y}\right)\left(2x - \frac{3}{4}\right)$$

$$2x \left(2x - \frac{3}{y}\right) + \frac{3}{y} \left(2x - \frac{3}{y}\right)$$

$$= 4x^2 - \frac{6x}{y} + \frac{6x}{y} - \frac{9}{y^2}$$

$$= 4x^2 - \frac{9}{y^2}$$

$$(vi) (2a^3 + b^3)(2a^3 - b^3)$$

$$2a^3 (2a^3 - b^3) + b^3 (2a^3 - b^3)$$

$$= 4a^6 - 2a^3b^3 + 2a^3b^3 - b^6$$

$$= 4a^6 - b^6$$

$$(vii) \left(x^4 + \frac{2}{x^2}\right)\left(x^4 - \frac{2}{x^2}\right)$$

$$x^4 \left(x^4 - \frac{2}{x^2}\right) + \frac{2}{x^2} \left(x^4 - \frac{2}{x^2}\right)$$

$$= x^8 - 2x^2 + 2x^2 - \frac{4}{x^2}$$

$$= (x^8 - \frac{4}{x^2})$$

$$(viii) \left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$$

$$x^3 \left(x^3 - \frac{1}{x^3}\right) + \frac{1}{x^3} \left(x^3 - \frac{1}{x^3}\right)$$

$$= x^6 - 1 + 1 - \frac{1}{x^6}$$

$$= x^6 - \frac{1}{x^6}$$

3. Question

Using the formula for squaring a binomial, evaluate the following:

$$(i) (102)^2$$

$$(ii) (99)^2$$

$$(iii) (1001)^2$$

$$(iv) (999)^2$$

$$(v) (703)^2$$

Answer

$$(i) (102)^2$$

This can be written as:

$$(100 + 2)^2$$

$$= (100)^2 + 2 (100) (2) + 2^2$$

$$= 10000 + 400 + 4$$

$$= 10404$$

$$(ii) (99)^2$$

This can be written as:

$$\begin{aligned}(100 - 1)^2 \\&= (100)^2 - 2 (100) (1) + 1^2 \\&= 10000 - 200 + 1 \\&= 9801\end{aligned}$$

(iii) $(1001)^2$

This can be written as:

$$\begin{aligned}(1000 + 1)^2 \\&= (1000)^2 + 2 (1000) (1) + 1^2 \\&= 1000000 + 2000 + 1 \\&= 1002001\end{aligned}$$

(iv) $(999)^2$

This can be written as:

$$\begin{aligned}(1000 - 1)^2 \\&= (1000)^2 - 2 (1000) (1) + 1^2 \\&= 1000000 - 2000 + 1 \\&= 998001\end{aligned}$$

(v) $(703)^2$

This can be written as:

$$\begin{aligned}(700 + 3)^2 \\&= (700)^2 + 2 (700) (3) + 3^2 \\&= 490000 + 4200 + 9 \\&= 494209\end{aligned}$$

4. Question

Simplify the following using the formula: $(a - b)(a + b) = a^2 - b^2$:

(i) $(82)^2 - (18)^2$

(ii) $(467)^2 - (33)^2$

(iii) $(79)^2 - (69)^2$

(iv) 197×203

(v) 113×87

(vi) 95×105

(vii) 1.8×2.2

(viii) 9.8×10.2

Answer

(i) $(82)^2 - (18)^2$

Using formula:

$$\begin{aligned}(a - b) (a + b) &= a^2 - b^2, \text{ we get} \\ &= (82 - 18) (82 + 18) \\ &= 64 \times 100 \\ &= 6400\end{aligned}$$

(ii) $(467)^2 - (33)^2$

Using formula:

$$\begin{aligned}(a - b) (a + b) &= a^2 - b^2, \text{ we get} \\ &= (467 - 33) (467 + 33) \\ &= (434) (500) \\ &= 217000\end{aligned}$$

(iii) $(79)^2 - (69)^2$

Using formula:

$$\begin{aligned}(a - b) (a + b) &= a^2 - b^2, \text{ we get} \\ &= (79 + 69) (79 - 69) \\ &= (148) (10) \\ &= 1480\end{aligned}$$

(iv) 197×203

Using formula:

$$\begin{aligned}(a - b) (a + b) &= a^2 - b^2, \text{ we get} \\ &= (200 - 3) (200 + 3) \\ &= (200)^2 - (3)^2 \\ &= 40000 - 9 \\ &= 39991\end{aligned}$$

(v) 113×87

Using formula:

$$\begin{aligned}(a - b) (a + b) &= a^2 - b^2, \text{ we get} \\ &= (100 + 3) (100 - 3) \\ &= (100)^2 - (3)^2 \\ &= 10000 - 9 \\ &= 9991\end{aligned}$$

(vi) 95×105

Using formula:

$$\begin{aligned}(a - b) (a + b) &= a^2 - b^2, \text{ we get} \\ &= (100 - 5) (100 + 5) \\ &= (100)^2 - (5)^2\end{aligned}$$

$$= 10000 - 25$$

$$= 9975$$

$$(vii) 1.8 \times 2.2$$

Using formula:

$$(a - b)(a + b) = a^2 - b^2, \text{ we get}$$

$$= (2 - 0.2)(2 + 0.2)$$

$$= (2)^2 - (0.2)^2$$

$$= 4 - 0.04$$

$$= 3.96$$

$$(viii) 9.8 \times 10.2$$

Using formula:

$$(a - b)(a + b) = a^2 - b^2, \text{ we get}$$

$$= (10 - 0.2)(10 + 0.2)$$

$$= (10)^2 - (0.2)^2$$

$$= 100 - 0.04$$

$$= 99.96$$

5. Question

Simplify the following using the identities:

$$(i) \frac{58^2 - 42^2}{16}$$

$$(ii) 178 \times 178 - 22 \times 22$$

$$(iii) \frac{198 \times 198 - 102 \times 102}{96}$$

$$(iv) 1.73 \times 1.73 - 0.27 \times 0.27$$

$$(v) \frac{8.63 \times 8.63 - 1.37 \times 1.37}{0.726}$$

Answer

$$(i) \frac{58^2 - 42^2}{16}$$

$$\frac{(58 - 42)(58 + 42)}{4 \times 4}$$

$$= \frac{16(100)}{16}$$

$$= 100$$

$$(ii) 178 \times 178 - 22 \times 22$$

$$(178)^2 - (22)^2$$

$$= (178 + 22)(178 - 22)$$

$$= 200 \times 156$$

$$= 31200$$

$$(iii) \frac{198 \times 198 - 102 \times 102}{96}$$

$$\frac{(198 - 102)(198 + 102)}{96}$$

$$= \frac{96 \times 300}{96}$$

$$= 300$$

$$(iv) 1.73 \times 1.73 - 0.27 \times 0.27$$

$$(1.73)^2 - (0.27)^2$$

$$= (1.73 + 0.27)(1.73 - 0.27)$$

$$= 2 (1.46)$$

$$= 2.92$$

$$(v) \frac{8.63 \times 8.63 - 1.37 \times 1.37}{0.726}$$

$$\frac{(8.63 + 1.37)(8.63 - 1.37)}{0.726}$$

$$= \frac{10 \times 7.26}{0.726}$$

$$= 100$$

6. Question

Find the value of x, if:

$$(i) 4x = (52)^2 - (48)^2$$

$$(ii) 14x = (47)^2 - (33)^2$$

$$(iii) 5x = (50)^2 - (40)^2$$

Answer

$$(i) 4x = 52^2 - 48^2$$

$$4x = (52 - 48)(52 + 48)$$

$$4x = 4 \times 100$$

$$4x = 400$$

$$x = 100$$

$$(ii) 14x = (47)^2 - (33)^2$$

$$14x = (47 - 33)(47 + 33)$$

$$14x = 14 \times 80$$

$$x = 80$$

$$(iii) 5x = (50)^2 - (40)^2$$

Using formula:

$$a^2 - b^2 = (a - b)(a + b), \text{ we get}$$

$$5x = (50 - 40)(50 + 40)$$

$$5x = 10 \times 90$$

$$5x = 900$$

$$x = 180$$

7. Question

If $x + \frac{1}{x} = 20$, find the value of $x^2 + \frac{1}{x^4}$.

Answer

Given that,

$$x + \frac{1}{x} = 20$$

Squaring both sides, we get

$$(x + \frac{1}{x})^2 = (20)^2$$

$$x^2 + 2 \times x \times \frac{1}{x} + (\frac{1}{x})^2 = 400$$

$$x^2 + 2 + \frac{1}{x^2} = 400$$

$$x^2 + \frac{1}{x^2} = 398$$

8. Question

If $x - \frac{1}{x} = 3$, find the values of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$.

Answer

(i) Given that,

$$x - \frac{1}{x} = 3$$

Squaring both sides, we get

$$(x - \frac{1}{x})^2 = (3)^2$$

$$x^2 - 2 \times x \times \frac{1}{x} + (\frac{1}{x})^2 = 9$$

$$x^2 - 2 + \frac{1}{x^2} = 9$$

$$x^2 + \frac{1}{x^2} = 11$$

(ii) Squaring both sides, we get

$$(x^2 + \frac{1}{x^2})^2 = (11)^2$$

$$(x^2)^2 + 2 \times x^2 \times \frac{1}{x^2} + (\frac{1}{x^2})^2 = 121$$

$$x^4 + 2 + \frac{1}{x^4} = 121$$

$$x^4 + \frac{1}{x^4} = 119$$

9. Question

If $x^2 + \frac{1}{x^2} = 18$, find the values of $x + \frac{1}{x}$ and $x - \frac{1}{x}$.

Answer

$$x^2 + \frac{1}{x^2} = 18$$

Adding 2 on both sides, we get

$$x^2 + \frac{1}{x^2} + 2 = 18 + 2$$

$$x^2 + \frac{1}{x^2} + 2 \times x \times \frac{1}{x} = 20$$

$$(x + \frac{1}{x})^2 = 20$$

$$x + \frac{1}{x} = 2\sqrt{5}$$

Given that,

$$x^2 + \frac{1}{x^2} = 18$$

Subtracting 2 from both sides, we get

$$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 18 - 2$$

$$(x - \frac{1}{x})^2 = 16$$

$$x - \frac{1}{x} = 4$$

10. Question

If $x+y = 4$ and $xy=2$, find the value of x^2+y^2

Answer

Given that,

$$x + y = 4 \text{ and } xy=2$$

We take the equation: $x + y = 4$ and on squaring both sides, we get

$$(x + y)^2 = 4^2$$

$$x^2 + y^2 + 2xy = 16$$

$$x^2 + y^2 + 2(2) = 16 \text{ (Because } xy=2 \text{ is given)}$$

$$x^2 + y^2 + 4 = 16$$

$$x^2 + y^2 = 16 - 4$$

$$x^2 + y^2 = 12$$

Therefore, the value of $x^2 + y^2$ is 12

11. Question

If $x-y = 7$ and $xy = 9$, find the value of x^2+y^2

Answer

Given that, $x - y = 7$

Squaring both sides, we get

$$(x - y)^2 = (7)^2$$

$$x^2 + y^2 - 2xy = 49$$

Its given that $xy = 9$,

$$x^2 + y^2 - 2(9) = 49$$

$$x^2 + y^2 = 49 + 18$$

$$x^2 + y^2 = 67$$

12. Question

If $3x+5y = 11$ and $xy = 2$, find the value of $9x^2+25y^2$

Answer

Given that,

$$3x + 5y = 11$$

Squaring both sides, we get

$$(3x + 5y)^2 = (11)^2$$

$$(3x)^2 + (5y)^2 + 2(3x)(5y) = 121$$

$$9x^2 + 25y^2 + 30xy = 121$$

$$9x^2 + 25y^2 + 30(2) = 121$$

$$9x^2 + 25y^2 = 121 - 60$$

$$9x^2 + 25y^2 = 61$$

13. Question

Find the values of the following expressions:

(i) $16x^2 + 24x + 9$, when $x = \frac{7}{4}$

(ii) $64x^2 + 81y^2 + 144xy$, when $x = 11$ and $y = \frac{4}{3}$

(iii) $81x^2 + 16y^2 - 72xy$, when $x = \frac{2}{3}$ and $y = \frac{3}{4}$

Answer

(i) $16x^2 + 24x + 9$, when $x = \frac{7}{4}$

$$(4x)^2 + 2(4x)(3) + 3^2$$

$$= (4x + 3)^2$$

Putting $x = \frac{7}{4}$

$$= [4(\frac{7}{4}) + 3]^2$$

$$= (7 + 3)^2$$

$$= 100$$

(ii) $64x^2 + 81y^2 + 144xy$, when $x = 11$ and $y = \frac{4}{3}$

$$(8x)^2 + 2(8x)(9y) + (9y)^2$$

$$= (8x + 9y)^2$$

Putting $x = 11$ and $y = \frac{4}{3}$

$$= [8 (11) + 9 \left(\frac{4}{3}\right)]^2$$

$$= (88 + 12)^2$$

$$= (100)^2$$

$$= 10000$$

$$(iii) \ 81x^2 + 16y^2 - 72xy, \text{ when } x = \frac{2}{3} \text{ and } y = \frac{3}{4}$$

$$(9x)^2 + (4y)^2 - 2 (9x) (4y)$$

$$= (9x - 4y)^2$$

$$\text{Putting } x = \frac{2}{3} \text{ and } y = \frac{3}{4}$$

$$= [9 \left(\frac{2}{3}\right) - 4 \left(\frac{3}{4}\right)]^2$$

$$= (6 - 3)^2$$

$$= 3^2$$

$$= 9$$

14. Question

$$\text{If } x + \frac{1}{x} = 9, \text{ find the value of } x^4 + \frac{1}{x^4}$$

Answer

Given that,

$$x + \frac{1}{x} = 9$$

Squaring both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = 9^2$$

$$x^2 + \frac{1}{x^2} + 2 = 81$$

$$x^2 + \frac{1}{x^2} = 79$$

Again,

Squaring both sides, we get

$$\left(x^2 + \frac{1}{x^2}\right)^2 = 79^2$$

$$x^4 + \frac{1}{x^4} + 2 = 6241$$

$$x^4 + \frac{1}{x^4} = 6239$$

15. Question

$$\text{If } x + \frac{1}{x} = 12, \text{ find the value of } x - \frac{1}{x}.$$

Answer

Given that,

$$x + \frac{1}{x} = 12$$

Squaring both sides, we get

$$\left(x + \frac{1}{x}\right)^2 = 12^2$$

$$x^2 + \left(\frac{1}{x}\right)^2 + 2 \times x \times \frac{1}{x} = 144$$

$$x^2 + \frac{1}{x^2} = 142$$

Subtract 2 from both sides, we get

$$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 142 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 140$$

$$x - \frac{1}{x} = \pm \sqrt{140}$$

16. Question

If $2x+3y=14$ and $2x-3y=2$, find value of xy . [Hint: Use $(2x+3y)^2 - (2x-3y)^2 = 24xy$]

Answer

Given that,

$$2x + 3y = 14 \dots\dots\dots(1)$$

$$2x - 3y = 2 \dots\dots\dots(2)$$

Now, on squaring both the equation and subtracting (2) from (1), we get,

$$(2x + 3y)^2 - (2x - 3y)^2 = (14)^2 - (2)^2$$

$$4x^2 + 9y^2 + 12xy - 4x^2 - 9y^2 + 12xy = 196 - 4$$

(The positive and negative terms gets cancelled)

$$24xy = 192$$

$$xy = 8$$

Therefore, the value of "xy" is 8.

17. Question

if $x^2+y^2 = 29$ and $xy = 2$, find the value of

(i) $x+y$

(ii) $x-y$

(iii) x^4+y^4

Answer

(i) $x+y$

Given that,

$$x^2 + y^2 = 29$$

$$x^2 + y^2 + 2xy - 2xy = 29$$

$$(x + y)^2 - 2(2) = 29$$

$$(x + y)^2 = 29 + 4$$

$$x + y = \sqrt{33}$$

(ii) $x - y$

$$x^2 + y^2 = 29$$

$$x^2 + y^2 + 2xy - 2xy = 29$$

$$(x - y)^2 + 2(2) = 29$$

$$(x - y)^2 + 4 = 29$$

$$(x - y)^2 = 25$$

$$(x - y) = 5$$

(iii) $x^4 + y^4$

$$x^2 + y^2 = 29$$

Squaring both sides, we get

$$(x^2 + y^2)^2 = (29)^2$$

$$x^4 + y^4 + 2x^2y^2 = 841$$

$$x^4 + y^4 + 2(2)^2 = 841$$

$$x^4 + y^4 = 841 - 8$$

$$x^4 + y^4 = 833$$

18. Question

What must be added each of the following expression to make it a whole square?

(i) $4x^2 - 12x + 7$

(ii) $4x^2 - 20x + 20$

Answer

(i) $4x^2 - 12x + 7$

$$(2x)^2 - 2(2x)(3) + 3^2 - 3^2 + 7$$

$$= (2x - 3)^2 - 9 + 7$$

$$= (2x - 3)^2 - 2$$

Hence, 2 must be added to the expression in order to make a whole square

(ii) $4x^2 - 20x + 20$

$$(2x)^2 - 2(2x)(5) + 5^2 - 5^2 + 20$$

$$= (2x - 5)^2 - 25 + 20$$

$$= (2x - 5)^2 - 5$$

Hence, 5 must be added to the expression in order to make it a whole square

19. Question

Simplify:

(i) $(x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

(ii) $(2x - 1)(2x + 1)(4x^2 + 1)(16x^4 + 1)$

$$(iii) (7m - 8n)^2 + (7m + 8n)^2$$

$$(iv) (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$$

$$(v) (m^2 - n^2m)^2 + 2m^3n^2$$

Answer

$$(i) (x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$$

$$(x^2 - y^2)(x^2 + y^2)(x^4 + y^4)$$

$$= [(x^2)^2 - (y^2)^2](x^4 + y^4)$$

$$= (x^4 - y^4)(x^4 + y^4)$$

$$= [(x^4)^2 - (y^4)^2]$$

$$= x^8 - y^8$$

$$(ii) (2x - 1)(2x + 1)1(4x^2 + 1)(16x^4 + 1)$$

$$[(2x)^2 - (1)^2](4x^2 + 1)(16x^4 + 1)$$

$$= (4x^2 - 1)(4x^2 + 1)(16x^4 + 1)1$$

$$= [(4x^2)^2 - (1)^2](16x^4 + 1)1$$

$$= (16x^4 - 1)(16x^4 + 1)1$$

$$= [(16x^4)^2 - (1)^2]1$$

$$= 256x^8 - 1$$

$$(iii) (7m - 8n)^2 + (7m + 8n)^2$$

$$(7m)^2 + (8n)^2 - 112mn + (7m)^2 + (8n)^2 + 112mn$$

$$= 49m^2 + 64n^2 + 49m^2 + 64n^2$$

$$= 98m^2 + 64n^2 + 64n^2$$

$$= 98m^2 + 128n^2$$

$$(iv) (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$$

$$(2.5p)^2 + (1.5q)^2 - 2(2.5p)(1.5q) - (1.5p)^2 - (2.5q)^2 + 2(1.5p)(2.5q)$$

$$= 6.25p^2 + 2.25q^2 - 2.25p^2 - 6.25q^2$$

$$= 4p^2 - 6.25q^2 + 2.25q^2$$

$$= 4p^2 - 4q^2$$

$$= 4(p^2 - q^2)$$

$$(v) (m^2 - n^2m)^2 + 2m^3n^2$$

$$(m^2)^2 - 2(m^2)(n^2)(m) + (n^2m)^2 + 2m^3n^2$$

$$= m^4 - 2m^3n^2 + (n^2m)^2 + 2m^3n^2$$

$$= m^4 + n^4m^2 - 2m^3n^2 + 2m^3n^2$$

$$= m^4 + m^2n^4$$

$$= m^2 (m^2 + n^4)$$

20. Question

Show that:

$$(i) (3x + 7)^2 - 84x = (3x - 7)^2$$

$$(ii) (9a - 5b)^2 + 180ab = (9a + 5b)^2$$

$$(iii) \left(\frac{4m}{3} - \frac{3n}{4} \right)^2 + 2mn = \frac{16m^2}{9} + \frac{9n^2}{16}$$

$$(iv) (4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

$$(v) (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

Answer

$$(i) (3x + 7)^2 - 84x = (3x - 7)^2$$

$$\text{L.H.S} = (3x + 7)^2 - 84x$$

$$= (3x)^2 + (7)^2 + 2 (3x) (7) - 84x$$

$$= (3x)^2 + (7)^2 + 42x - 84x$$

$$= (3x)^2 + (7)^2 - 42x$$

$$= (3x)^2 + (7)^2 - 2 (3x) (7)$$

$$= (3x - 7)^2$$

$$= \text{R.H.S}$$

Hence, proved

$$(ii) (9a - 5b)^2 + 180ab = (9a + 5b)^2$$

$$\text{L.H.S} = (9a - 5b)^2 + 180ab$$

$$= (9a)^2 + (5b)^2 - 2 (9a) (5b) + 180ab$$

$$= (9a)^2 + (5b)^2 - 90ab + 180ab$$

$$= (9a)^2 + (5b)^2 + 90ab$$

$$= (9a)^2 + (5b)^2 + 2 (9a) (5b)$$

$$= (9a + 5b)^2$$

$$= \text{R.H.S}$$

Hence, proved

$$(iii) \left(\frac{4m}{3} - \frac{3n}{4} \right)^2 + 2mn = \frac{16m^2}{9} + \frac{9n^2}{16}$$

$$\text{L.H.S} = \left(\frac{4m}{3} - \frac{3n}{4} \right)^2 + 2mn$$

$$= \left(\frac{4m}{3} \right)^2 + \left(\frac{3n}{4} \right)^2 - 2mn + 2mn$$

$$= \left(\frac{4m}{3} \right)^2 + \left(\frac{3n}{4} \right)^2$$

$$= \frac{16}{9}m^2 + \frac{9}{16}n^2$$

= R.H.S

Hence, verified

$$(iv) (4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

$$L.H.S = (4pq + 3q)^2 - (4pq - 3q)^2$$

$$= (4pq)^2 + (3q)^2 + 2(4pq)(3q) - (4pq)^2 - (3q)^2 + 24pq^2$$

$$= 24pq^2 + 24pq^2$$

$$= 48pq^2$$

Hence, proved

$$(v) (a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$$

$$L.H.S = (a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a)$$

Using identity:

$$(a-b)(a+b) = a^2 - b^2$$

We get,

$$= (a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2)$$

$$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$$

$$= 0$$

= R.H.S

Hence, verified

Exercise 6.7

1. Question

Find the following products:

$$(i) (x+4)(x+7) \quad (ii) (x-11)(x+4)$$

$$(iii) (x+7)(x-5) \quad (iv) (x-3)(x-2)$$

$$(v) (y^2-4)(y^2-3) \quad (vi) \left(x + \frac{4}{3}\right)\left(x + \frac{3}{4}\right)$$

$$(vii) (3x+5)(3x+11) \quad (viii) (2x^2-3)(2x^2+5)$$

$$(ix) (z^2+2)(z^2-3) \quad (x) (3x-4y)(2x-4y)$$

$$(xi) (3x^2-4xy)(3x^2-3xy) \quad (xii) \left(x + \frac{1}{5}\right)(x+5)$$

$$(xiii) \left(z + \frac{3}{4}\right)\left(z + \frac{4}{3}\right) \quad (xiv) (x^2+4)(x^2+9)$$

$$(xv) (y^2+12)(y^2+6) \quad (xvi) \left(y^2 + \frac{5}{7}\right)\left(y^2 - \frac{14}{5}\right)$$

$$(xvii) (p^2+16)\left(p^2 - \frac{1}{4}\right)$$

Answer

$$(i) (x+4)(x+7)$$

$$x(x+7)+4(x+7)$$

$$= x^2 + 7x + 4x + 28$$

$$= x^2 + 11x + 28$$

$$(ii) (x-11)(x+4)$$

$$x(x+4)-11(x+4)$$

$$= x^2 + 4x - 11x - 44$$

$$= x^2 - 7x - 44$$

$$(iii) (x+7)(x-5)$$

$$x(x-5)+7(x-5)$$

$$= x^2 - 5x + 7x - 35$$

$$= x^2 + 2x - 35$$

$$(iv) (x-3)(x-2)$$

$$x(x-2)-3(x-2)$$

$$= x^2 - 2x - 3x + 6$$

$$= x^2 - 5x + 6$$

$$(v) (y^2-4)(y^2-3)$$

$$y^2(y^2-3)-4(y^2-3)$$

$$= y^4 - 3y^2 - 4y^2 + 12$$

$$= y^4 - 7y^2 + 12$$

$$(vi) \left(x + \frac{4}{3}\right)\left(x + \frac{3}{4}\right)$$

$$x\left(x + \frac{3}{4}\right) + \frac{4}{3}\left(x + \frac{3}{4}\right)$$

$$= x^2 + \frac{3x}{4} + \frac{4x}{3} + \frac{12}{12}$$

$$= x^2 + \frac{3x}{4} + \frac{4x}{3} + 1$$

$$= x^2 + \frac{25x}{12} + 1$$

$$(vii) (3x+5)(3x+11)$$

$$3x(3x+11)+5(3x+11)$$

$$= 9x^2 + 33x + 15x + 55$$

$$= 9x^2 + 48x + 55$$

$$(viii) (2x^2-3)(2x^2+5)$$

$$2x^2(2x^2-5)-3(2x^2-5)$$

$$= 4x^4 - 10x^2 - 6x^2 + 15$$

$$= 4x^4 - 16x^2 + 15$$

$$(ix) (z^2+2)(z^2-3)$$

$$z^2 (z^2 - 3) + 2 (z^2 - 3)$$

$$= z^4 - 3z^2 + 2z^2 - 6$$

$$= z^4 - z^2 - 6$$

$$(x) (3x - 4y)(2x - 4y)$$

$$3x (2x - 4y) - 4y (2x - 4y)$$

$$= 6x^2 - 12xy - 8xy + 16y^2$$

$$= 6x^2 - 20xy + 16y^2$$

$$(xi) (3x^2 - 4xy)(3x^2 - 3xy)$$

$$3x^2 (3x^2 - 3xy) - 4xy (3x^2 - 3xy)$$

$$= 9x^4 - 9x^3y - 12x^3y + 12x^2y^2$$

$$= 9x^4 - 21x^3y + 12x^2y^2$$

$$(xii) \left(x + \frac{1}{5}\right)(x + 5)$$

$$x \left(x + \frac{1}{5}\right) + 5 \left(x + \frac{1}{5}\right)$$

$$= x^2 + \frac{x}{5} + 5x + 1$$

$$= x^2 + \frac{26}{5}x + 1$$

$$(xiii) \left(z + \frac{3}{4}\right)\left(z + \frac{4}{3}\right)$$

$$z \left(z + \frac{4}{3}\right) + \frac{3}{4} \left(z + \frac{4}{3}\right)$$

$$= z^2 + \frac{4}{3}z + \frac{3}{4}z + \frac{12}{12}$$

$$= z^2 + \frac{4}{3}z + \frac{3}{4}z + 1$$

$$= z^2 + \frac{25}{12}z + 1$$

$$(xiv) (x^2 + 4)(x^2 + 9)$$

$$x^2 (x^2 + 9) + 4 (x^2 + 9)$$

$$= x^4 + 9x^2 + 4x^2 + 36$$

$$= x^4 + 13x^2 + 36$$

$$(xv) (y^2 + 12)(y^2 + 6)$$

$$y^2 (y^2 + 6) + 12 (y^2 + 6)$$

$$= y^4 + 6y^2 + 12y^2 + 72$$

$$= y^4 + 18y^2 + 72$$

$$(xvi) \left(y^2 + \frac{5}{7}\right)\left(y^2 - \frac{14}{5}\right)$$

$$y^2 \left(y^2 - \frac{14}{5}\right) + \frac{5}{7} \left(y^2 - \frac{14}{5}\right)$$

$$= y^4 - \frac{14}{5}y^2 + \frac{5}{7}y^2 - 2$$

$$= y^4 - \frac{73}{35}y^2 - 2$$

$$(xvii) (p^2 + 16)\left(p^2 - \frac{1}{4}\right)$$

$$p^2 \left(p^2 - \frac{1}{4}\right) + 16 \left(p^2 - \frac{1}{4}\right)$$

$$= p^4 - \frac{1}{4}p^2 + 16p^2 - 4$$

$$= p^4 - \frac{63}{4}p^2 - 4$$

2. Question

Evaluate the following:

(i) 102×106

(ii) 109×107

(iii) 35×37

(iv) 53×55

(v) 103×96

(vi) 34×36

(vii) 994×1006

Answer

(i) 102×106

$$(100 + 2)(100 + 6)$$

$$= 100(100 + 6) + 2(100 + 6)$$

$$= 10000 + 600 + 200 + 12$$

$$= 10812$$

(ii) 109×107

This can be written as:

$$(100 + 9)(100 + 7)$$

$$= 100(100 + 7) + 9(100 + 7)$$

$$= 10000 + 700 + 900 + 63$$

$$= 11663$$

(iii) 35×37

This can be written as:

$$(30 + 5)(30 + 7)$$

$$= 30(30 + 7) + 5(30 + 7)$$

$$= 900 + 210 + 150 + 35$$

$$= 1295$$

(iv) 53×55

This can be written as:

$$\begin{aligned} & (50 + 3) (50 + 5) \\ &= 50 (50 + 5) + 3 (50 + 5) \\ &= 2500 + 250 + 150 + 15 \\ &= 2915 \end{aligned}$$

(v) 103×96

This can be written as:

$$\begin{aligned} & (100 + 3) (100 - 4) \\ &= 100 (100 - 4) + 3 (100 - 4) \\ &= 10000 - 400 + 300 - 12 \\ &= 10000 - 112 \\ &= 9888 \end{aligned}$$

(vi) 34×36

This can be written as:

$$\begin{aligned} & (30 + 4) (30 + 6) \\ &= 30 (30 + 6) + 4 (30 + 6) \\ &= 900 + 180 + 120 + 24 \\ &= 1224 \end{aligned}$$

(vii) 994×1006

This can be written as:

$$\begin{aligned} & (1000 - 6) (1000 + 6) \\ &= 1000 (1000 + 6) - 6 (1000 + 6) \\ &= 1000000 + 6000 - 6000 - 36 \\ &= 999964 \end{aligned}$$