8. Solution of Simultaneous Linear Equations

Exercise 8.1

1 A. Question

Solve the following system of equations by matrix method:

$5x + 7y + 2 = 0$

$4x + 6y + 3 = 0$

Answer

The above system of equations can be written as

$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ B = $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 30 - 28 = 2$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\ 6 = 6$

$C_{12} = (-1)^{1+2}\ 4 = -4$

$C_{21} = (-1)^{2+1}\ 7 = -7$

$C_{22} = (-1)^{2+2}\ 5 = 5$

Also, adj A = $\begin{bmatrix} 6 & -4 \\ -7 & 5 \end{bmatrix}^T$

$= \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$

Now, $X = A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -12 + 21 \\ 8 - 15 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ \frac{-7}{2} \end{bmatrix}$

Hence, $X = \frac{9}{2}$ $Y = \frac{-7}{2}$

1 B. Question

Solve the following system of equations by matrix method:

5x + 2y = 3

3x + 2y = 5

Answer

The above system of equations can be written as

$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 10 – 6 = 4

So, the above system has a unique solution, given by

X = $A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\ 2 = 2$

$C_{12} = (-1)^{1+2}\ 3 = -3$

$C_{21} = (-1)^{2+1}\ 2 = -2$

$C_{22} = (-1)^{2+2}\ 2 = 5$

Also, adj A = $\begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}^{T}$

= $\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj A}$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

Now, X = $A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6-10 \\ -9+25 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

Hence, X = -1 Y = 4

1 C. Question

Solve the following system of equations by matrix method:

3x + 4y = 5

x – y = – 3

Answer

The above system of equations can be written as

$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = – 3 – 4 = – 7

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1} - 1 = -1$

$C_{12} = (-1)^{1+2}\ 1 = -1$

$C_{21} = (-1)^{2+1}\ 4 = -4$

$C_{22} = (-1)^{2+2}\ 3 = 3$

Also, adj A = $\begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7} \begin{bmatrix} -5 + 12 \\ -5 - 9 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 7 \\ -14 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Hence, X = 1 Y = – 2

1 D. Question

Solve the following system of equations by matrix method:

3x + y = 19

3x – y = 23

Answer

The above system of equations can be written as

$\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}$ B = $\begin{bmatrix} 19 \\ 23 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = – 3 – 3 = – 6

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1} - 1 = -1$

$C_{12} = (-1)^{1+2}\, 3 = -3$

$C_{21} = (-1)^{2+1}\, 1 = -1$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} -1 & -3 \\ -1 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj A}$

$A^{-1} = \frac{1}{-6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -19 - 23 \\ -57 + 69 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-6}\begin{bmatrix} -42 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

Hence, X = 7 Y = – 2

1 E. Question

Solve the following system of equations by matrix method:

$3x + 7y = 4$

$x + 2y = -1$

Answer

The above system of equations can be written as

$\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

$|A| = 6 - 7 = -1$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 2 = 2$

$C_{12} = (-1)^{1+2}\, 1 = -1$

$C_{21} = (-1)^{2+1}\, 7 = -7$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix}^T$

$= \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{-1} \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 8 + 7 \\ -4 - 3 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 15 \\ -7 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -15 \\ 7 \end{bmatrix}$

Hence, X = – 15 Y = 7

1 F. Question

Solve the following system of equations by matrix method:

3x + y = 7

5x + 3y = 12

Answer

The above system of equations can be written as

$\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 7 \\ 12 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 9 – 5 = 4

So, the above system has a unique solution, given by

$X = A^{-1}B$

Let C_{ij} be the cofactor of a_{ij} in A, then

$C_{11} = (-1)^{1+1}\, 3 = 3$

$C_{12} = (-1)^{1+2}\, 5 = -5$

$C_{21} = (-1)^{2+1}\, 1 = -1$

$C_{22} = (-1)^{2+2}\, 3 = 3$

Also, adj A = $\begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$

Now, $X = A^{-1}B$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 21 - 12 \\ -35 + 36 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 9 \\ 1 \end{bmatrix}$

$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{4} \end{bmatrix}$

Hence, $X = \frac{9}{4}$ $Y = \frac{1}{4}$

2 A. Question

Solve the following system of equations by matrix method:

x + y –z = 3

2x + 3y + z = 10

3x – y – 7z = 1

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$ or A X = B

A = $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

Now, |A| = $1\begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$

= (– 20) – 1(– 17) – 1(11)

= – 20 + 17 + 11 = 8

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1} - 21 + 1 = -20$

$\mathbf{C_{21}} = (-1)^{2+1} - 7 - 1 = 8$

$\mathbf{C_{31}}$ = (– 1)$^{3 + 1}$ 1 + 3 = 4

$\mathbf{C_{12}}$ = (– 1)$^{1 + 2}$ – 14 – 3 = 17

$\mathbf{C_{22}}$ = (– 1)$^{2 + 1}$ – 7 + 3 = – 4

$\mathbf{C_{32}}$ = (– 1)$^{3 + 1}$ 1 + 2 = – 3

$\mathbf{C_{13}}$ = (– 1)$^{1 + 2}$ – 2 – 9 = – 11

$\mathbf{C_{23}}$ = (– 1)$^{2 + 1}$ – 1 – 3 = 4

$\mathbf{C_{33}}$ = (– 1)$^{3 + 1}$ 3 – 2 = 1

adj A = $\begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^{T}$

= $\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$

Now, X = A^{-1}B = $\frac{1}{8}\begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$

X = $\frac{1}{8}\begin{bmatrix} -60 + 80 + 4 \\ 51 - 40 - 3 \\ -33 + 40 + 1 \end{bmatrix}$

X = $\frac{1}{8}\begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$

Hence, X = 3,Y = 1 and Z = 1

2 B. Question

Solve the following system of equations by matrix method:

x + y + z = 3

2x – y + z = – 1

2x + y – 3z = – 9

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$ or A X = B

A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$

Now, |A| = 1$\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 2 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}$

= (3 – 1) – 1(– 6 – 2) + 1(2 + 2)

= 2 + 8 + 4

= 14

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}\, 3 - 1 = 2$

$\mathbf{C_{21}} = (-1)^{2+1} - 3 - 1 = 4$

$\mathbf{C_{31}} = (-1)^{3+1}\, 1 + 1 = 2$

$\mathbf{C_{12}} = (-1)^{1+2} - 6 - 2 = 8$

$\mathbf{C_{22}} = (-1)^{2+1} - 3 - 2 = -5$

$\mathbf{C_{32}} = (-1)^{3+1}\, 1 - 2 = 1$

$\mathbf{C_{13}} = (-1)^{1+2}\, 2 + 2 = 4$

$\mathbf{C_{23}} = (-1)^{2+1}\, 1 - 2 = 1$

$\mathbf{C_{33}} = (-1)^{3+1} - 1 - 2 = -3$

adj A = $\begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$

Now, $X = A^{-1}B = \frac{1}{14}\begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$

$X = \frac{1}{14}\begin{bmatrix} -16 \\ 20 \\ 38 \end{bmatrix}$

$X = \frac{1}{7}\begin{bmatrix} \frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7} \end{bmatrix}$

Hence, $X = \frac{-8}{7}$, $Y = \frac{10}{7}$ and $Z = \frac{19}{7}$

2 C. Question

Solve the following system of equations by matrix method:

$6x - 12y + 25z = 4$

$4x + 15y - 20z = 3$

$2x + 18y + 15z = 10$

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$ or A X = B

$A = \begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$

Now, |A| = $6\begin{vmatrix}15 & -20\\18 & 15\end{vmatrix} - 12\begin{vmatrix}4 & -20\\2 & 15\end{vmatrix} + 25\begin{vmatrix}4 & 15\\2 & 18\end{vmatrix}$

= 6(225 + 360) + 12(60 + 40) + 25(72 – 30)

= 3510 + 1200 + 1050

= 5760

So, the above system has a unique solution, given by

X = $A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ (225 + 360) = 585

$\mathbf{C_{21}}$ = $(-1)^{2+1}$ (– 180 – 450) = 630

$\mathbf{C_{31}}$ = $(-1)^{3+1}$ (240 – 375) = – 135

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ (60 + 40) = – 100

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ (90 – 50) = 40

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ (– 120 – 100) = 220

$\mathbf{C_{13}}$ = $(-1)^{1+2}$ (72 – 30) = 42

$\mathbf{C_{23}}$ = $(-1)^{2+1}$(108 + 24) = – 132

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ (90 + 48) = 138

adj A = $\begin{bmatrix}585 & -100 & 42\\630 & 40 & -132\\-135 & 220 & 138\end{bmatrix}^T$

= $\begin{bmatrix}585 & 630 & -135\\-100 & 40 & 220\\42 & -132 & 138\end{bmatrix}$

Now, X = $A^{-1}B = \frac{1}{5760}\begin{bmatrix}585 & 630 & -135\\-100 & 40 & 220\\42 & -132 & 138\end{bmatrix}\begin{bmatrix}4\\3\\10\end{bmatrix}$

X = $\frac{1}{5760}\begin{bmatrix}2880\\1920\\1152\end{bmatrix}$

X = $\frac{1}{7}\begin{bmatrix}\frac{1}{2}\\\frac{1}{3}\\\frac{1}{5}\end{bmatrix}$

Hence, X = $\frac{1}{2}$,Y = $\frac{1}{3}$ and Z = $\frac{1}{5}$

2 D. Question

Solve the following system of equations by matrix method:

3x + 4y + 7z = 14

2x – y + 3z = 4

X + 2y – 3z = 0

Answer

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$ or A X = B

$A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

Now, $|A| = 3\begin{vmatrix} -1 & 3 \\ 2 & -3 \end{vmatrix} - 4\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} + 7\begin{vmatrix} 2 & 3 \\ 2 & -3 \end{vmatrix}$

$= 3(3 - 6) - 4(-6 - 3) + 7(4 + 1)$

$= -9 + 36 + 35$

$= 62$

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}\ 3 - 6 = -3$

$\mathbf{C_{21}} = (-1)^{2+1} - 12 - 14 = 26$

$\mathbf{C_{31}} = (-1)^{3+1} 12 + 7 = 19$

$\mathbf{C_{12}} = (-1)^{1+2} - 6 - 3 = 9$

$\mathbf{C_{22}} = (-1)^{2+1} - 3 - 7 = -10$

$\mathbf{C_{32}} = (-1)^{3+1}\ 9 - 14 = 5$

$\mathbf{C_{13}} = (-1)^{1+2}\ 4 + 1 = 5$

$\mathbf{C_{23}} = (-1)^{2+1}\ 6 - 4 = -2$

$\mathbf{C_{33}} = (-1)^{3+1} - 3 - 8 = -11$

$$\text{adj } A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -5 & -2 \\ 19 & 5 & -11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, $X = A^{-1}B = \frac{1}{62}\begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}\begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$

$$X = \frac{1}{62}\begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix}$$

$$X = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$$

$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Hence, X = 1,Y = 1 and Z = 1

2 E. Question

Solve the following system of equations by matrix method:

5x + 3y + z = 16

2x + y + 3z = 19

X + 2y + 4z = 25

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$ or A X = B

$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$

Now, $|A| = 5\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$

= 5(4 – 6) – 3(8 – 3) + 1(4 – 2)

= – 10 – 15 + 3

= – 22

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}(4-6) = -2$

$\mathbf{C_{21}} = (-1)^{2+1}(12-2) = -10$

$\mathbf{C_{31}} = (-1)^{3+1}(9-1) = 8$

$\mathbf{C_{12}} = (-1)^{1+2}(8-3) = -5$

$\mathbf{C_{22}} = (-1)^{2+1}\ 20 - 1 = 19$

$\mathbf{C_{32}} = (-1)^{3+1}\ 15 - 2 = -13$

$\mathbf{C_{13}} = (-1)^{1+2}(4-2) = 2$

$\mathbf{C_{23}} = (-1)^{2+1}\ 10 - 3 = -7$

$\mathbf{C_{33}} = (-1)^{3+1}\ 5 - 6 = -1$

$\text{adj } A = \begin{bmatrix} -2 & -5 & -3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T$

$= \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & 13 \\ 3 & -7 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}\begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$

$X = \frac{1}{-22}\begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix}$

$X = \frac{1}{-22}\begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

Hence, X = 1,Y = 2 and Z = 5

2 F. Question

Solve the following system of equations by matrix method:

3x + 4y + 2z = 8

2y – 3z = 3

x – 2y + 6z = – 2

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$ or A X = B

$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$

Now, $|A| = 3\begin{vmatrix} 2 & -3 \\ -2 & 6 \end{vmatrix} - 4\begin{vmatrix} 0 & -3 \\ 1 & 6 \end{vmatrix} + 2\begin{vmatrix} 0 & 2 \\ 1 & -2 \end{vmatrix}$

= 3(12 – 6) – 4(0 + 3) + 2(0 – 2)

= 18 – 12 – 4

= 2

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}(12 - 6) = 6$

$\mathbf{C_{21}} = (-1)^{2+1}(24 + 4) = -28$

$\mathbf{C_{31}} = (-1)^{3+1}(-12 - 4) = -16$

$\mathbf{C_{12}} = (-1)^{1+2}(0 + 3) = -3$

$\mathbf{C_{22}} = (-1)^{2+1}\ 18 - 2 = 16$

$\mathbf{C_{32}} = (-1)^{3+1} - 9 - 0 = 9$

C_{13} = $(-1)^{1+2}$ (0 – 2) = – 2

C_{23} = $(-1)^{2+1}$ (– 6 – 4) = 10

C_{33} = $(-1)^{3+1}$ 6 – 0 = 6

adj A = $\begin{bmatrix} 6 & -3 & 2 \\ -28 & 16 & 10 \\ -16 & -9 & 6 \end{bmatrix}^T$

= $\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, X = $A^{-1}B = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & -9 \\ 2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$

X = $\frac{1}{2}\begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix}$

X = $\frac{1}{2}\begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$

X = $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

Hence, X = – 2,Y = 3 and Z = 1

2 G. Question

Solve the following system of equations by matrix method:

2x + y + z = 2

x + 3y –z = 5

3x + y – 2z = 6

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$ or A X = B

A = $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

Now, |A| = $2\begin{vmatrix} 3 & -1 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$

= 2(– 6 + 1) – 1(– 2 + 3) + 1(1 – 9)

= – 10 – 1 – 8

= – 19

So, the above system has a unique solution, given by

X = A^{-1}B

Cofactors of A are:

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ – 6 + 1 = – 5

$\mathbf{C_{21}}$ = $(-1)^{2+1}$(24 + 4) = – 28

$\mathbf{C_{31}}$ = $(-1)^{3+1}$ – 1 – 3 = – 4

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ – 2 + 3 = – 1

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ – 4 – 3 = – 7

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ – 2 – 1 = 3

$\mathbf{C_{13}}$ = $(-1)^{1+2}$1 – 9 = – 8

$\mathbf{C_{23}}$ = $(-1)^{2+1}$2 – 3 = – 1

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ 6 – 1 = 5

adj A = $\begin{bmatrix} -5 & -1 & -8 \\ 3 & -7 & 1 \\ -4 & 3 & 5 \end{bmatrix}^{T}$

= $\begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

Now, $X = A^{-1}B = \frac{1}{-19}\begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$

$X = \frac{1}{-19}\begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix}$

$X = \frac{1}{-19}\begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

Hence, X = 1,Y = 1 and Z = – 1

2 H. Question

Solve the following system of equations by matrix method:

2x + 6y = 2

3x – z = – 8

2x – y + z = – 3

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$ or A X = B

A = $\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$

Now, |A| = $2\begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} - 6\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} + 0$

= 2(0 – 1) – 6(3 + 2)

= – 2 – 30

= – 32

So, the above system has a unique solution, given by

X = $A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ 0 – 1 = – 1

$\mathbf{C_{21}}$ = $(-1)^{2+1}$6 + 0 = – 6

$\mathbf{C_{31}}$ = $(-1)^{3+1}$ – 6 = – 6

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ 3 + 2 = 5

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ 2 – 0 = 2

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ – 2 – 0 = 2

$\mathbf{C_{13}}$ = $(-1)^{1+2}$ – 3 – 0 = – 3

$\mathbf{C_{23}}$ = $(-1)^{2+1}$ – 2 – 12 = 14

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ 0 – 18 = – 18

adj A = $\begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T$

= $\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj A}$

Now, X = $A^{-1}B = \frac{1}{-32}\begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}\begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$

X = $\frac{1}{62}\begin{bmatrix} -2 + 48 + 18 \\ -10 - 16 - 6 \\ -6 - 112 + 54 \end{bmatrix}$

X = $\frac{1}{62}\begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix}$

X = $\begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

Hence, X = – 2,Y = 1 and Z = 2

2 I. Question

Solve the following system of equations by matrix method:

2y – z = 1

x – y + z = 2

2x – y = 0

Answer

The given system can be written in matrix form as:

$$\begin{bmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

AX = B

Now, |A| = 0 $-2\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$

= 0 + 4 – 1

= 3

So, the above system has a unique solution, given by

X = $A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ 1 – 0 = 1

$\mathbf{C_{21}}$ = $(-1)^{2+1}$1 – 2 = 1

$\mathbf{C_{31}}$ = $(-1)^{3+1}$0 + 1 = 1

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ – 2 – 0 = 2

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ – 1 – 0 = – 1

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ 0 – 2 = 2

$\mathbf{C_{13}}$ = $(-1)^{1+2}$ 4 – 0 = 4

$\mathbf{C_{23}}$ = $(-1)^{2+1}$ 2 – 0 = – 2

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ – 1 + 2 = 1

adj A = $\begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -2 \\ 1 & 2 & 1 \end{bmatrix}^T$

= $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj A}$

Now, X = $A^{-1}B = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

X = $\frac{1}{3}\begin{bmatrix} 2 + 0 + 1 \\ 4 - 0 + 2 \\ 8 - 0 + 1 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix}3\\6\\9\end{bmatrix}$

$X = \begin{bmatrix}1\\2\\3\end{bmatrix}$

Hence, X = 1,Y = 2 and Z = 3

2 J. Question

Solve the following system of equations by matrix method:

8x + 4y + 3z = 18

2x + y + z = 5

X + 2y + z = 5

Answer

The given system can be written in matrix form as:

$$\begin{bmatrix}8 & 4 & 3\\2 & 1 & 1\\1 & 2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}18\\5\\5\end{bmatrix}$$

AX = B

Now, $|A| = 8\begin{vmatrix}1 & 1\\2 & 1\end{vmatrix} - 4\begin{vmatrix}2 & 1\\1 & 1\end{vmatrix} + 3\begin{vmatrix}2 & 1\\1 & 2\end{vmatrix}$

= 8(– 1) – 4(1) + 3(3)

= – 8 – 4 + 9

= – 3

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}\ 1 - 2 = -1$

$\mathbf{C_{21}} = (-1)^{2+1}\ 4 - 6 = 2$

$\mathbf{C_{31}} = (-1)^{3+1}\ 4 - 3 = 1$

$\mathbf{C_{12}} = (-1)^{1+2}\ 2 - 1 = -1$

$\mathbf{C_{22}} = (-1)^{2+1}\ 8 - 3 = 5$

$\mathbf{C_{32}} = (-1)^{3+1}\ 8 - 6 = -2$

$\mathbf{C_{13}} = (-1)^{1+2}\ 4 - 1 = 3$

$\mathbf{C_{23}} = (-1)^{2+1}\ 16 - 4 = -12$

$\mathbf{C_{33}} = (-1)^{3+1}\ 8 - 8 = 0$

adj $A = \begin{bmatrix}-1 & -1 & 3\\2 & 5 & -12\\1 & -2 & 0\end{bmatrix}^T$

$= \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{-3}\begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}\begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$

$X = \frac{1}{3}\begin{bmatrix} -18 + 10 + 5 \\ -18 + 25 - 10 \\ 54 - 60 + 0 \end{bmatrix}$

$X = \frac{1}{-3}\begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix}$

$X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Hence, X = 1,Y = 1 and Z = 2

2 K. Question

Solve the following system of equations by matrix method:

x + y + z = 6

x + 2z = 17

3x + y + z = 12

Answer

The given system can be written in matrix form as:

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 17 \\ 12 \end{bmatrix}$

A X = B

Now, $|A| = 1\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

= 1(– 2) – 1(1 – 6) + 1(1)

= – 2 + 5 + 1

= 4

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}\, 0 - 2 = -2$

$\mathbf{C_{21}} = (-1)^{2+1}\, 1 - 1 = 0$

$\mathbf{C_{31}} = (-1)^{3+1}\, 2 - 0 = 2$

$\mathbf{C_{12}} = (-1)^{1+2}\, 1 - 6 = 5$

$\mathbf{C_{22}} = (-1)^{2+1}\, 1 - 3 = -2$

C_{32} = $(-1)^{3+1}$ 2 – 1 = – 1

C_{13} = $(-1)^{1+2}$ 1 – 0 = 1

C_{23} = $(-1)^{2+1}$ 1 – 3 = 2

C_{33} = $(-1)^{3+1}$ 0 – 1 = – 1

adj A = $\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}^T$

$= \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

Now, $X = A^{-1}B = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} -12 + 0 + 24 \\ 30 - 14 - 12 \\ 6 + 14 - 12 \end{bmatrix}$

$X = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}$

$X = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

Hence, X = 3,Y = 1 and Z = 2

2 L. Question

Solve the following system of equations by matrix method:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2; x, y, z \neq 0$$

Answer

The given system can be written in matrix form as:

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

AX = B

Now,

|A| = 2(75) – 3(– 110) + 10(72)

= 150 + 330 + 720

= 1200

So, the above system has a unique solution, given by

X = A $^{-1}$B

Cofactors of A are:

C_{11} = (– 1)$^{1+1}$ 120 – 45 = 75

C_{21} = (– 1)$^{2+1}$ – 60 – 90 = 150

C_{31} = (– 1)$^{3+1}$ 15 + 60 = 75

C_{12} = (– 1)$^{1+2}$ – 80 – 30 = 110

C_{22} = (– 1)$^{2+1}$ – 40 – 60 = – 100

C_{32} = (– 1)$^{3+1}$ 10 – 40 = 30

C_{13} = (– 1)$^{1+2}$ 36 + 36 = 72

C_{23} = (– 1)$^{2+1}$ 18 – 18 = 0

C_{33} = (– 1)$^{3+1}$ – 12 – 12 = – 24

adj A = $\begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$

= $\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

A $^{-1}$ = $\frac{1}{|A|}\text{adj A}$

Now, X = A $^{-1}$B = $\frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

X = $\frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$

$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$

Hence, X = 2,Y = 3 and Z = 5

2 M. Question

Solve the following system of equations by matrix method:

x – y + 2z = 7

3x + 4y – 5z = – 5

2x – y + 3z = 12

Answer

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

A X = B

Now,

|A| = 1(12 – 5) + 1(9 + 10) + 2(– 3 – 8)

= 7 + 19 – 22

= 4

So, the above system has a unique solution, given by

$X = A^{-1}B$

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}\ 12 - 5 = 7$

$\mathbf{C_{21}} = (-1)^{2+1} - 3 + 2 = 1$

$\mathbf{C_{31}} = (-1)^{3+1}\ 5 - 8 = -3$

$\mathbf{C_{12}} = (-1)^{1+2}\ 9 + 10 = -19$

$\mathbf{C_{22}} = (-1)^{2+1}\ 3 - 4 = -1$

$\mathbf{C_{32}} = (-1)^{3+1} - 5 - 6 = 11$

$\mathbf{C_{13}} = (-1)^{1+2} - 3 - 8 = -11$

$\mathbf{C_{23}} = (-1)^{2+1} - 1 + 2 = -1$

$\mathbf{C_{33}} = (-1)^{3+1}\ 4 + 3 = 7$

$$\text{adj } A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & -7 \end{bmatrix}^T$$

$$= \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$X = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, X = 2,Y = 1 and Z = 3

3 A. Question

Show that each of the following systems of linear equations is consistent and also find their

6x + 4y = 2

9x + 6y = 3

Answer

The above system of equations can be written as

$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix}$ B = $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 36 – 36 = 0

So, A is singular, Now X will be consistence if (Adj A)xB = 0

$C_{11} = (-1)^{1+1} 6 = 6$

$C_{12} = (-1)^{1+2} 9 = -9$

$C_{21} = (-1)^{2+1} 4 = -4$

$C_{22} = (-1)^{2+2} 6 = 6$

Also, adj A = $\begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T$

= $\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

= $\begin{bmatrix} 12 - 12 \\ -18 + 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Thus, AX = B will be infinite solution,

Let y = k

Hence, 6x = 2 – 4k or 9x = 3 – 6k

X = $\frac{1-2k}{3}$

Hence, X = $\frac{1-2k}{3}$, Y = k

3 B. Question

Show that each of the following systems of linear equations is consistent and also find their

2x + 3y = 5

6x + 9y = 15

Answer

The above system of equations can be written as

$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ 15 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 18 – 18 = 0

So, A is singular,Now X will be consistence if (Adj A)xB = 0

$C_{11} = (-1)^{1+1} 9 = 9$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1}\, 3 = -3$

$C_{22} = (-1)^{2+2}\, 2 = 2$

Also, adj A = $\begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$

$= \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

(Adj A).B = $\left[\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\right]\begin{bmatrix} 5 \\ 15 \end{bmatrix}$

$= \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Thus, AX = B will be infinite solution,

Let y = k

Hence,

2x = 5 – 3k or X = $\frac{5-3k}{2}$

x = 15 – 9k or X = $\frac{5-3k}{2}$

Hence, X = $\frac{5-3k}{2}$, Y = k

3 C. Question

Show that each of the following systems of linear equations is consistent and also find their

Solutions :

5x + 3y + 7z = 4

3x + 26y + 2z = 9

7x + 2y + 10z = 5

Answer

This can be written as:

$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$

|A| = 5(260 – 4) – 3(30 – 14) + 7(6 – 182)

= 5(256) – 3(16) + 7(176)

|A| = 0

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

(Adj A)x B≠0 or (Adj A)x B = 0

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}\, 260 - 4 = 256$

$\mathbf{C_{21}} = (-1)^{2+1}\, 30 - 14 = -16$

$C_{31} = (-1)^{3+1}\ 6 - 182 = -176$

$C_{12} = (-1)^{1+2}\ 30 - 14 = -16$

$C_{22} = (-1)^{2+1}\ 50 - 49 = 1$

$C_{32} = (-1)^{3+1}\ 10 - 21 = 11$

$C_{13} = (-1)^{1+2}\ 6 - 182 = -176$

$C_{23} = (-1)^{2+1}\ 10 - 21 = 11$

$C_{33} = (-1)^{3+1}\ 130 - 9 = 121$

adj A $= \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T$

$= \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$

Adj A x B $= \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, AX = B has infinite many solution

Let z = k

Then, 5x + 3y = 4 – 7k

3x + 26y = 9 – 2k

This can be written as:

$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$

|A| = 121

Adj A $= \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$

Now, X = A^{-1}B $= \frac{1}{|A|} \text{Adj A} \times \text{B}$

$= \frac{1}{121} \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$

$= \frac{1}{121} \begin{bmatrix} 77 - 176k \\ 11k + 33 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{7 - 16k}{11} \\ \frac{k + 3}{11} \end{bmatrix}$

There values of x,y,z satisfy the third equation

3 D. Question

Show that each of the following systems of linear equations is consistent and also find their

Solution:

x + y + z = 6

x + 2y + 3z = 14

x + 4y + 7z = 30

Answer

This can be written as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

|A| = 1(2) – 1(4) + 1(2)

= 2 – 4 + 2

|A| = 0

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

(Adj A)x B≠0 or (Adj A)x B = 0

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1}\ 14 - 12 = 2$

$\mathbf{C_{21}} = (-1)^{2+1}\ 7 - 4 = -3$

$\mathbf{C_{31}} = (-1)^{3+1}\ 3 - 2 = 1$

$\mathbf{C_{12}} = (-1)^{1+2}\ 7 - 3 = -4$

$\mathbf{C_{22}} = (-1)^{2+1}\ 7 - 1 = 6$

$\mathbf{C_{32}} = (-1)^{3+1}\ 3 - 1 = 2$

$\mathbf{C_{13}} = (-1)^{1+2}\ 4 - 2 = 2$

$\mathbf{C_{23}} = (-1)^{2+1}\ 4 - 1 = -3$

$\mathbf{C_{33}} = (-1)^{3+1}\ 2 - 1 = 1$

adj A = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 6 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T$

= $\begin{bmatrix} 2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1 \end{bmatrix}$

Adj A x B = $\begin{bmatrix} 2 & -3 & 1 \\ -4 & 1 & -2 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, AX = B has infinite many solution

Let z = k

Then, x + y = 6 – k

X + 2y = 14 – 3k

This can be written as:

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix}$$

|A| = 1

Adj A = $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

Now, X = $A^{-1}B = \frac{1}{|A|}\text{Adj A} \times \text{B}$

$= \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6-k \\ 14-3k \end{bmatrix}$

$= \frac{1}{1}\begin{bmatrix} 12-2k-14+3k \\ -6+k+14-3k \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+k \\ 8-2k \end{bmatrix}$

There values of x,y,z satisfy the third equation

Hence, x = k – 2, y = 8 – 2k, z = k

3 E. Question

Show that each of the following systems of linear equations is consistent and also find their

Solution:

2x + 2y – 2z = 1

4x + 4y – z = 2

6x + 6y + 2z = 3

Answer

$\begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & -1 \\ 6 & 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

|A| = 2(14) – 2(14) – 2(0)

|A| = 0

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solution according to as:

(Adj A)x B≠0 or (Adj A)x B = 0

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1} 8 + 6 = 14$

$\mathbf{C_{21}} = (-1)^{2+1} 4 + 12 = -16$

$\mathbf{C_{31}} = (-1)^{3+1} -2 + 8 = 6$

$\mathbf{C_{12}} = (-1)^{1+2} 8 + 6 = -14$

$\mathbf{C_{22}} = (-1)^{2+1} 4 + 12 = 16$

$\mathbf{C_{32}} = (-1)^{3+1} -2 + 8 = -6$

$\mathbf{C_{13}} = (-1)^{1+2} 24 - 24 = 0$

$\mathbf{C_{23}} = (-1)^{2+1} 12 - 12 = 0$

$\mathbf{C_{33}} = (-1)^{3+1} 8 - 8 = 0$

adj A = $\begin{bmatrix} 14 & -14 & 6 \\ -16 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}^T$

= $\begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$

Adj A x B = $\begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now, AX = B has infinite many solution

Let z = k

Then, 2x + 2y = 1 + 2k

4x + 4y = 2 + k

This can be written as:

$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$

Hence, |A| = 0 z = 0

Hence, The given equation doesn't satisfy .

4 A. Question

Show that each one of the following systems of linear equations is inconsistent

2x + 5y = 7

6x + 15y = 13

Answer

The above system of equations can be written as

$\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}$ B = $\begin{bmatrix} 7 \\ 13 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 30 – 30 = 0

So, A is singular,Now X will be consistence if (Adj A)xB = 0

$C_{11} = (-1)^{1+1}\, 15 = 15$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1}\, 5 = -5$

$C_{22} = (-1)^{2+2}\, 2 = 2$

Also, adj A = $\begin{bmatrix} 15 & -6 \\ -5 & 2 \end{bmatrix}^T$

= $\begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 15 & -5 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 13 \end{bmatrix}$

= $\begin{bmatrix} 105 - 65 \\ -35 + 26 \end{bmatrix} = \begin{bmatrix} 40 \\ -16 \end{bmatrix}$

$= \neq$

Hence, The given system is inconsistent.

4 B. Question

Show that each one of the following systems of linear equations is inconsistent

2x + 3y = 5

6x + 9y = 10

Answer

The above system of equations can be written as

$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ B = $\begin{bmatrix} 5 \\ 10 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = 18 – 18 = 0

So, A is singular,Now X will be consistence if (Adj A)xB = 0

$C_{11} = (-1)^{1+1} 9 = 9$

$C_{12} = (-1)^{1+2} 6 = -6$

$C_{21} = (-1)^{2+1} 3 = -3$

$C_{22} = (-1)^{2+2} 2 = 2$

Also, adj A = $\begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T$

$= \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \end{bmatrix}$

$= \begin{bmatrix} 45 - 30 \\ -30 + 20 \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$

$= \neq 0$

Hence, The given system is inconsistent.

4 C. Question

Show that each one of the following systems of linear equations is inconsistent

4x – 2y = 3

6x – 3y = 5

Answer

The above system of equations can be written as

$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ or AX = B

Where A = $\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and X = $\begin{bmatrix} X \\ Y \end{bmatrix}$

|A| = – 12 + 12 = 0

So, A is singular,Now X will be consistence if (Adj A)xB = 0

$C_{11} = (-1)^{1+1} - 3 = -3$

$C_{12} = (-1)^{1+2}\, 6 = -6$

$C_{21} = (-1)^{2+1} - 2 = 2$

$C_{22} = (-1)^{2+2}\, 4 = 4$

Also, adj A = $\begin{bmatrix} -3 & -2 \\ -6 & 4 \end{bmatrix}^T$

= $\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$

(Adj A).B = $\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

= $\begin{bmatrix} -9 + 10 \\ -18 + 20 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Hence, The given system is inconsistent.

4 D. Question

Show that each one of the following systems of linear equations is inconsistent

4x – 5y – 2z = 2

5x – 4y + 2z = – 2

2x + 2y + 8z = – 1

Answer

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

|A| = 4(– 36) + 5(36) – 2(18)

|A| = 0

Cofactors of A are:

$\mathbf{C_{11}} = (-1)^{1+1} - 32 - 4 = -36$

$\mathbf{C_{21}} = (-1)^{2+1} - 40 + 4 = -36$

$\mathbf{C_{31}} = (-1)^{3+1} - 10 - 8 = -18$

$\mathbf{C_{12}} = (-1)^{1+2}\, 40 - 4 = -36$

$\mathbf{C_{22}} = (-1)^{2+1}\, 32 + 4 = 36$

$\mathbf{C_{32}} = (-1)^{3+1}\, 8 + 10 = -18$

$\mathbf{C_{13}} = (-1)^{1+2}\, 10 + 8 = 18$

$\mathbf{C_{23}} = (-1)^{2+1}\, 8 + 10 = -18$

$\mathbf{C_{33}} = (-1)^{3+1} - 16 + 25 = 9$

adj A = $\begin{bmatrix} -36 & -34 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}^T$

$$= \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

Adj A x B = $\begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

$$= \begin{bmatrix} -72 - 72 + 18 \\ -72 - 72 + 18 \\ 36 + 36 - 9 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, The above system is inconsistent.

4 E. Question

Show that each one of the following systems of linear equations is inconsistent

3x – y – 2z = 2

2y – z = – 1

3x – 5y = 3

Answer

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

|A| = 3(– 5) + 1(3) – 2(– 6)

|A| = 0

Cofactors of A are:

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ 0 – 5 = – 5

$\mathbf{C_{21}}$ = $(-1)^{2+1}$ 0 – 10 = 10

$\mathbf{C_{31}}$ = $(-1)^{3+1}$ 1 + 4 = 5

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ 0 + 3 = – 3

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ 0 + 6 = 6

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ – 3 – 0 = 3

$\mathbf{C_{13}}$ = $(-1)^{1+2}$ 0 – 6 = – 6

$\mathbf{C_{23}}$ = $(-1)^{2+1}$ – 15 + 3 = 12

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ 6 – 0 = 6

adj A = $\begin{bmatrix} -5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}^T$

$$= \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

Adj A x B = $\begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

$$= \begin{bmatrix} -10 - 10 + 15 \\ 6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, The above system is inconsistent.

4 F. Question

Show that each one of the following systems of linear equations is inconsistent

x + y – 2z = 5

x – 2y + z = – 2

– 2x + y + z = 4

Answer

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

|A| = 1(– 3) – 1(3) – 2(– 3) = – 3 – 3 + 6

|A| = 0

Cofactors of A are:

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ – 2 – 1 = – 3

$\mathbf{C_{21}}$ = $(-1)^{2+1}$ 1 + 2 = – 3

$\mathbf{C_{31}}$ = $(-1)^{3+1}$ 1 – 4 = – 3

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ 1 + 2 = – 3

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ 1 – 4 = – 3

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ 1 + 2 = – 3

$\mathbf{C_{13}}$ = $(-1)^{1+2}$ 1 – 4 = – 3

$\mathbf{C_{23}}$ = $(-1)^{2+1}$ 1 + 2 = – 3

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ – 2 – 1 = – 3

$$\text{adj } A = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$\text{Adj } A \times B = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -15 + 6 - 12 \\ -15 + 6 - 12 \\ -15 + 6 - 12 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, The above system is inconsistent.

5. Question

If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, find A B and hence solve the system of linear equations: X – y = 3, 2x + 3y + 4z = 17, y + 2z = 7.

Answer

$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$

$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Now, we can see that it is AB = 6I. where I is the unit Matrix

Or, $A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Now the given equation can be written as:

$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

A X B

Or, $X = A^{-1}B$

$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$

$X = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

Hence, x = 2,y = – 1 and z = 4

6. Question

If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations:2 x – 3 y + 5z = 11, 3x + 2y – 4z = – 5, x + y – 2z = – 3

Answer

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

|A| = 2(0) + 3(– 2) + 5(1)

= – 1

Now, the cofactors of A

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ – 4 + 4 = 0

$\mathbf{C_{21}}$ = $(-1)^{2+1}$ 6 – 5 = – 1

$\mathbf{C_{31}}$ = $(-1)^{3+1}$ 12 – 10 = 2

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ – 6 + 4 = 2

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ – 4 – 5 = – 9

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ – 8 – 15 = 23

$\mathbf{C_{13}}$ = $(-1)^{1+2}$ 3 – 2 = 1

$\mathbf{C_{23}}$ = $(-1)^{2+1}$ 2 + 3 = – 5

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ 4 + 9 = 13

adj A = $\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

A X B

Or, X = A^{-1}B

$$= \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 + 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, x = 1,y = 2 and z = 3

7. Question

Find A $^{-1}$, if $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$. Hence, solve the following system of linear equations:

x + 2y + 5z = 10, x – y – z = – 2,

2x + 3y – z = – 11

Answer

A = $\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

|A| = 1(1 + 3) + 2(– 1 + 2) + 5(3 + 2)

= 4 + 2 + 25

= 27

Now, the cofactors of A

$\mathbf{C_{11}}$ = $(-1)^{1+1}$ 1 + 3 = 4

$\mathbf{C_{21}}$ = $(-1)^{2+1}$ – 2 – 15 = 17

$\mathbf{C_{31}}$ = $(-1)^{3+1}$ – 2 + 5 = 3

$\mathbf{C_{12}}$ = $(-1)^{1+2}$ – 1 + 2 = – 1

$\mathbf{C_{22}}$ = $(-1)^{2+1}$ – 1 – 10 = – 11

$\mathbf{C_{32}}$ = $(-1)^{3+1}$ – 1 – 5 = 6

$\mathbf{C_{13}}$ = $(-1)^{1+2}$ 3 + 2 = 5

$\mathbf{C_{23}}$ = $(-1)^{2+1}$ 3 – 4 = 1

$\mathbf{C_{33}}$ = $(-1)^{3+1}$ – 1 – 2 = – 3

adj A = $\begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T = \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

A $^{-1}$ = $\frac{1}{|A|}$ adj A

A $^{-1}$ = $\frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

A X B

Or, X = A $^{-1}$B

$$= \frac{1}{27}\begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}\begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27}\begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix}$$

$$X = \frac{1}{27}\begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix}$$

Hence, x = – 1,y = – 2 and z = 3

8 A. Question

Solve the following questions.

If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of linear equations:

x – 2y = 10, 2x + y + 3z = 8, – 2y + z = 7.

Answer

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$$

|A| = 1(1 + 6) + 2(2 – 0) + 0

= 7 + 4

= 11

Now, the cofactors of A

$\mathbf{C_{11}} = (-1)^{1+1}\ 1 + 6 = 7$

$\mathbf{C_{21}} = (-1)^{2+1} - 2 - 0 = 2$

$\mathbf{C_{31}} = (-1)^{3+1} - 6 - 0 = -6$

$\mathbf{C_{12}} = (-1)^{1+2}\ 2 - 0 = -2$

$\mathbf{C_{22}} = (-1)^{2+1}\ 1 - 0 = 1$

$\mathbf{C_{32}} = (-1)^{3+1}\ 3 - 0 = -3$

$\mathbf{C_{13}} = (-1)^{1+2} - 4 - 0 = -4$

$\mathbf{C_{23}} = (-1)^{2+1} - 2 - 0 = 2$

$\mathbf{C_{33}} = (-1)^{3+1}\ 1 + 4 = 5$

$$\text{adj } A = \begin{bmatrix} 7 & 2 & -4 \\ -2 & 1 & -3 \\ -6 & 2 & 5 \end{bmatrix}^T = \begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A$$

$$A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}$$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X B

Or, X = $A^{-1}B$

$$= \frac{1}{11}\begin{bmatrix} 7 & -2 & -6 \\ 2 & 1 & 2 \\ -4 & -3 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$

$$X = \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, x = 4,y = – 3 and z = 1

8 B. Question

Solve the following questions.

$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

3x – 4y + 2z = – 1, 2x + 3y + 5z = 7, x + z = 2.

Answer

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

|A| = 3(3 – 0) + 4(2 – 5) + 2(0 – 3)

= 9 – 12 – 6

= – 9

Now, the cofactors of A

$\mathbf{C_{11}} = (-1)^{1+1}\, 3 - 0 = 3$

$\mathbf{C_{21}} = (-1)^{2+1} - 4 - 0 = 4$

$\mathbf{C_{31}} = (-1)^{3+1} - 20 - 6 = -26$

$\mathbf{C_{12}} = (-1)^{1+2}\, 2 - 5 = 3$

$\mathbf{C_{22}} = (-1)^{2+1}\, 3 - 2 = 1$

$\mathbf{C_{32}} = (-1)^{3+1}\, 15 - 4 = -11$

$\mathbf{C_{13}} = (-1)^{1+2}\, 0 - 3 = -3$

$\mathbf{C_{23}} = (-1)^{2+1}\, 0 + 4 = -4$

C_{33} = $(-1)^{3+1}$ 9 + 8 = 17

adj A = $\begin{bmatrix} 3 & 3 & -3 \\ 4 & 1 & -4 \\ -26 & -4 & 27 \end{bmatrix}^T = \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A$

$A^{-1} = \frac{1}{-9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$

Now the given equation can be written as:

$\begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$

A X B

Or, X = $A^{-1}B$

$= \frac{1}{-9}\begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & 11 \\ -3 & -4 & 17 \end{bmatrix}\begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-9}\begin{bmatrix} -3 + 28 - 52 \\ 21 + 7 + 22 \\ 3 - 28 + 34 \end{bmatrix}$

X = $\frac{1}{-9}\begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix}$

X = $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

Hence, x = 3,y = 2 and z = – 1

8 C. Question

Solve the following questions.

$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB. Hence, solve the system of equations:

x – 2y = 10, 2x + y + 3z = 8 and – 2y + z = 7

Answer

A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

AB = $\begin{bmatrix} 7 + 4 - 0 & 2 - 2 + 0 & -6 + 6 + 0 \\ 14 - 2 - 12 & 4 + 1 + 6 & -12 - 3 + 15 \\ 0 - 4 + 4 & 0 - 2 + 2 & 0 + 6 + 5 \end{bmatrix}$

AB = $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$

Now, we can see that it is AB = 11I. where I is the unit Matrix

Or, $A^{-1} = \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X B

Or, $X = A^{-1}B$

$$= \frac{1}{11}\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, x = 4,y = – 3 and z = 1

8 D. Question

Solve the following questions.

If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of linear equations

x – 2y = 10, 2x – y – z = 8, – 2y + z = 7.

Answer

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

|A| = 1(– 1 – 1) – 2(– 2 – 0) + 0

= – 2 + 4

= 2

Now, the cofactors of A

$\mathbf{C_{11}} = (-1)^{1+1} - 1 - 1 = -2$

$\mathbf{C_{21}} = (-1)^{2+1}\, 2 - 0 = 2$

$\mathbf{C_{31}} = (-1)^{3+1} - 2 - 0 = -2$

$\mathbf{C_{12}} = (-1)^{1+2}\, 2 - 0 = -2$

$\mathbf{C_{22}} = (-1)^{2+1}\, 1 - 0 = 1$

$\mathbf{C_{32}} = (-1)^{3+1} - 1 - 0 = 1$

C_{13} = $(-1)^{1+2}$ – 2 – 0 = – 2

C_{23} = $(-1)^{2+1}$ – 1 – 0 = 1

C_{33} = $(-1)^{3+1}$ – 1 + 4 = 3

adj A = $\begin{bmatrix} -2 & -2 & -2 \\ 2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A$

$A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

A X B

Or, X = $A^{-1}B$

$$= \frac{1}{2}\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 10 - 16 + 0 \\ 20 - 8 - 7 \\ 0 - 16 + 7 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -6 \\ 5 \\ -9 \end{bmatrix}$$

Hence, x = – 3,y = 2.5 and z = – 4.5

8 E. Question

Solve the following questions.

Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations y + 2z = 7, x – y = 3, 2x + 3y + 4z = 17

Answer

B = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ A = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$

BA = $\begin{bmatrix} 2 + 4 - 0 & 2 - 2 + 0 & -4 + 4 + 0 \\ -4 - 12 + 16 & 4 + 6 - 4 & -8 - 12 + 20 \\ 0 - 4 + 8 & 0 - 2 + 2 & 0 - 4 + 10 \end{bmatrix}$

BA = $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

Now, we can see that it is BA = 6I. where I is the unit Matrix

Or, $B^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}$

Now the given equation can be written as:

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$$

A X B

Or, $X = B^{-1}A$

$$= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 4 & -1 & 5 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 14 + 6 - 68 \\ -28 + 6 - 68 \\ 28 - 3 + 85 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -48 \\ -90 \\ 110 \end{bmatrix}$$

$$X = \begin{bmatrix} -8 \\ -15 \\ \frac{110}{6} \end{bmatrix}$$

Hence, x = – 8,y = – 15 and z = $\frac{110}{6}$

9. Question

The sum of three numbers is 2. If twice the second number is added to the sum of first and third, the sum is 1. By adding second and third number to five times the first number, we get 6. Find the three numbers by using matrices.

Answer

Let the numbers are x, y, z

X + y + z = 2 ......(i)

Also, 2y + (x + z) + 1

X + 2y + z = 1 ......(ii)

Again,

x + z + 5(x) = 6

5x + y + z = 6 ...... (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

A X = B

|A| = 1(1) – 1(– 4) + 1(– 9)

= 1 + 4 – 9

= – 4

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(2 - 1) = 1$

$C_{12} = (-1)^{1+2}(1 - 5) = 4$

$C_{13} = (-1)^{1+3}(1 - 10) = -9$

$C_{21} = (-1)^{2+1}(1 - 1) = 0$

$C_{22} = (-1)^{2+2}(1 - 5) = -4$

$C_{23} = (-1)^{2+3}(1 - 5) = 4$

$C_{31} = (-1)^{3+1}(1 - 2) = -1$

$C_{32} = (-1)^{3+2}(1 - 1) = 0$

$C_{33} = (-1)^{3+3}(2 - 1) = 1$

$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$

$$\text{Adj } A = \begin{bmatrix} 1 & 4 & -9 \\ 0 & -4 & 4 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}$$

$$X = \frac{1}{-4}\begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

$$X = \frac{1}{-4}\begin{bmatrix} 2-6 \\ 8-4 \\ -18+4+6 \end{bmatrix}$$

$$= \frac{1}{-4}\begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix}$$

Hence, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

10. Question

An amount of ₹10,000 is put into three investments at the rate of 10, 12 and 15% per annum. The combined incomes are ₹1310 and the combined income of first and second investment is ₹ 190 short of the income from the third. Find the investment in each using matrix method.

Answer

Let the numbers are x, y,z

$x + y + z = 10,000$ ……(i)

Also,

$0.1x + 0.12y + 0.15z = 1310$ …… (ii)

Again,

$0.1x + 0.12y - 0.15z = -190$ …… (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$A X = B$

$|A| = 1(-0.036) - 1(-0.03) + 1(0)$

$= -0.006$

Hence, the unique solution given by x = $A^{-1}B$

$C_{11} = -0.036$

$C_{12} = 0.27$

$C_{13} = 0$

$C_{21} = 0.27$

$C_{22} = -0.25$

$C_{23} = -0.02$

$C_{31} = 0.03$

$C_{32} = -0.05$

$C_{33} = 0.02$

$X = A^{-1}B = \frac{1}{|A|}(\text{adj }A)\,B$

$$\text{Adj } A = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.27 & -0.25 & -0.05 \\ 0.03 & -0.02 & 0.02 \end{bmatrix}^T = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

$$X = \frac{1}{-0.006}\begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}\begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

$$X = \frac{1}{-0.006}\begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix}$$

Hence, x = Rs 2000, y = Rs 3000 and z = Rs 5000

11. Question

A company produces three products every day. Their production on a certain day is 45 tons. It is found that the production of the third product exceeds the production of a first product by 8 tons while the total production of a first and third product is twice the production of the second product. Determine the production level of each product using the matrix method.

Answer

Let the numbers are x, y, z

x + y + z = 45 ......(i)

Also,

Z – x = 8 ......(ii)

Again,

x + z = 2y ...... (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

A X = B

|A| = 1(2) – 1(– 2) + 1(2)

= 6

Hence, the unique solution given by x = $A^{-1}B$

$C_{11} = (-1)^{1+1}(0+2) = 2$

$C_{12} = (-1)^{1+2}(-1-1) = 2$

$C_{13} = (-1)^{1+3}(2-0) = 2$

$C_{21} = (-1)^{2+1}(1+2) = -3$

$C_{22} = (-1)^{2+2}(1-1) = 0$

$C_{23} = (-1)^{2+3}(-2-1) = 3$

$C_{31} = (-1)^{3+1}(1-0) = 1$

$C_{32} = (-1)^{3+2}(1+1) = -2$

$C_{33} = (-1)^{3+3}(0+1) = 1$

$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$

$\text{Adj } A = \begin{bmatrix} 2 & 2 & 2 \\ -3 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}^T = \left[\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\right]$

$X = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$

$X = \frac{1}{6}\begin{bmatrix} 66 \\ 90 \\ 114 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$

Hence, x = 11, y = 15 and z = 19

12. Question

The prices of three commodities P,Q and R and ₹ x,y and z per unit respectively. A purchases 4 units of R and sells 3 units of P and 5 units of Q. B purchases 3 units of Q and sells 2 units of P and 1 unit of R. C purchases 1 unit of Q. B purchases of Q and 6 units of R. In the process A, B and C earn ₹6000, ₹5000 and ₹13000 respectively. If selling the units is positive earning and buying the units is negative earnings, find the price per unit of three commodities by using the matrix method.

Answer

Let the numbers are x, y, z

3x + 5y – 4 z = 6000 …… (i)

Also,

2x – 3y + z = 5000 …… (ii)

Again,

– x + 4y + 6z = 13000 ……(iii)

$\begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$

A X = B

|A| = 3(– 18 – 4) – 2(30 + 16) – 1(5 – 12)

= 3(– 22) – 2(46) + 7

= – 66 – 92 + 7

= – 151

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(-18-4) = -22$

$C_{12} = (-1)^{1+2}(12+1) = -13$

$C_{13} = (-1)^{1+3}(8-3) = 5$

$C_{21} = (-1)^{2+1}(30+16) = -46$

$C_{22} = (-1)^{2+2}(18-4) = 14$

$C_{23} = (-1)^{2+3}(12+5) = -17$

$C_{31} = (-1)^{3+1}(5-12) = -7$

$C_{32} = (-1)^{3+2}(3+8) = -11$

$C_{33} = (-1)^{3+3}(-9-10) = -19$

$$\text{Adj A} = \begin{bmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj A})\,B$$

$$X = \frac{1}{6}\begin{bmatrix} -22 & 46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{bmatrix}\begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

$$X = \frac{1}{-151}\begin{bmatrix} -132000 - 23000 - 91000 \\ -78000 + 70000 - 143000 \\ -3000 - 85000 - 247000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3000 \\ 1000 \\ 2000 \end{bmatrix}$$

Hence, x = 3000, y = 1000 and z = 2000

13. Question

The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. A part from these values, namely, honesty cooperation and supervision, suggest one more value which the management must include for awards.

Answer

Let the numbers are x, y, z

3x + 5y – 4 z = 6000 …… (i)

Also,

2x – 3y + z = 5000 ……(ii)

Again,

– x + 4y + 6z = 13000 …… (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

A X = B

|A| = 1(3 + 6) – 1(2 – 3) + 1(– 4 – 3)

= 1(9) – 1(– 1) – 7

= 9 + 1 – 7

= 3

Hence, the unique solution given by x = $A^{-1}B$

$C_{11} = (-1)^{1+1}(3 + 6) = 9$

$C_{12} = (-1)^{1+2}(2 - 3) = 1$

$C_{13} = (-1)^{1+3}(-4 - 3) = -7$

$C_{21} = (-1)^{2+1}(1 + 2) = -3$

$C_{22} = (-1)^{2+2}(1 - 1) = 0$

$C_{23} = (-1)^{2+3}(-2 - 1) = 3$

$C_{31} = (-1)^{3+1}(3 - 3) = 0$

$C_{32} = (-1)^{3+2}(3 - 2) = -1$

$C_{33} = (-1)^{3+3}(3 - 2) = 1$

$$\text{Adj A} = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj A})\ B$$

$$X = \frac{1}{3}\begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 11 \\ 0 \end{bmatrix}$$

$$X = \frac{1}{-151}\begin{bmatrix} 36 - 33 + 0 \\ 1 + 0 - 1 \\ -7 + 3 + 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Hence, x = 3, y = 4 and z = 5

14. Question

A school wants to award its student for the values of Honesty, Regularity and Hard work with a total cash award of ₹6000. Three times the award money for Hard work added to that given for honesty amounts to ₹11000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award for each value, using the matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, and suggest one more value which the school must include for awards.

Answer

Let the numbers are x, y, z be the cash awards for Honesty, Regularity and Hard Work

x + y + z = 6000 ...... (i)

Also,

x + 3z = 11000 ...... (ii)

Again,

x – 2y + z = 0 ...... (iii)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

A X = B

|A| = 1(0 + 6) – 1(1 – 3) + 1(– 2 – 0)

= 1(6) – 1(– 2) – 2

= 6 + 2 – 2

= 6

Hence, the unique solution given by x = $A^{-1}B$

$C_{11} = (-1)^{1+1}(0+6) = 6$

$C_{12} = (-1)^{1+2}(1-3) = 2$

$C_{13} = (-1)^{1+3}(-2-0) = -2$

$C_{21} = (-1)^{2+1}(1+2) = -3$

$C_{22} = (-1)^{2+2}(1-1) = 0$

$C_{23} = (-1)^{2+3}(-2-1) = 3$

$C_{31} = (-1)^{3+1}(3-0) = 3$

$C_{32} = (-1)^{3+2}(3-1) = -2$

$C_{33} = (-1)^{3+3}(0-1) = -1$

$$\text{Adj A} = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}^T = \begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}$$

$X = A^{-1}B = \frac{1}{|A|}(\text{adj A})\,B$

$$X = \frac{1}{6}\begin{bmatrix} 6 & -3 & 3 \\ 2 & 0 & -2 \\ -2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$X = \frac{1}{6}\begin{bmatrix} 36000 - 33000 + 0 \\ 12000 + 0 - 0 \\ -12000 + 33000 - 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3000 \\ 12000 \\ 21000 \end{bmatrix}$$

Hence, x = 500, y = 2000 and z = 3500

15. Question

Two institutions decided to award their employees for the three values resourcefulness, competence and determination in the from of prizes at the rate of₹x, ₹y and ₹z respectively per person. The first institutions decided to award respectively 4,3 and 2 employees with total prize money of ₹37000 and the second institution decided to award respectively 5,3 and 4 employees with total prize money of ₹47000. If all the three prizes per person together amount to ₹12000, then using matrix method find the value of x,y and z. What values are described in this equations?

Answer

Let the numbers are x, y, z be the cash awards for Resourcefulness, Competence, and Determination respectively

$4x + 3y + 2z = 37000$ …… (i)

Also,

$5x + 3y + 4z = 47000$ …… (ii)

Again,

$x + y + z = 12000$ …… (iii)

$$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

A X = B

$|A| = 4(3 - 4) - 3(5 - 4) + 2(5 - 3)$

$= 4(-1) - 3(1) + 2(2)$

$= -4 - 3 + 4$

$= -3$

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(3 - 4) = -1$

$C_{12} = (-1)^{1+2}(5 - 4) = -1$

$C_{13} = (-1)^{1+3}(5 - 3) = 2$

$C_{21} = (-1)^{2+1}(3 - 2) = -1$

$C_{22} = (-1)^{2+2}(4 - 2) = 2$

$C_{23} = (-1)^{2+3}(4 - 3) = -1$

$C_{31} = (-1)^{3+1}(12 - 6) = 6$

$C_{32} = (-1)^{3+2}(16 - 10) = -6$

$C_{33} = (-1)^{3+3}(12 - 15) = -3$

$$\text{Adj } A = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 6 & -6 & -3 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$

$$X = \frac{1}{-3}\begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}\begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$X = \frac{1}{-3}\begin{bmatrix} -37000 - 47000 + 72000 \\ -37000 + 94000 - 72000 \\ 74000 - 47000 - 36000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

Hence, x = 4000, y = 5000 and z = 3000

Thus, The value X, Y, Z describes the amount of prizes per person for Resourcefulness, Competence and Determination.

16. Question

Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of ₹x, ₹y and ₹z per persons respectively. The first factory decided to honour respectively 2, 4 and 3 employees with total prize money of ₹29000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of ₹30500. If the three prizes per person together cost ₹9500, then

i. represent the above situation by a matrix equation and form linear equations using matrix multiplication.

ii. Solve these equations using matrices.

iii. Which values are reflected in the questions?

Answer

Let the numbers are x, y,z be the prize amount per person for adaptability, carefulness and calmness respectively

As per the given data we get,

$2x + 4y + 3z = 29000$

$5x + 2y + 3z = 30500$

$X + y + z = 9500$

These three equations can be written as

$$\begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$$

A X = B

$|A| = 2(2 - 3) - 4(5 - 3) + 3(5 - 2)$

$= 2(-1) - 4(2) + 3(3)$

$= -2 - 8 + 9$

$= -1$

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(2 - 3) = -1$

$C_{12} = (-1)^{1+2}(5 - 3) = -2$

$C_{13} = (-1)^{1+3}(5 - 2) = 3$

$C_{21} = (-1)^{2+1}(4 - 3) = -1$

$C_{22} = (-1)^{2+2}(2 - 3) = -1$

$C_{23} = (-1)^{2+3}(2 - 4) = -2$

$C_{31} = (-1)^{3+1}(12 - 6) = 6$

$C_{32} = (-1)^{3+2}(6 - 15) = -9$

$C_{33} = (-1)^{3+3}(4 - 20) = -16$

Adj A = $\begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 6 & 9 & -16 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}$

X = $A^{-1}B = \frac{1}{|A|}(\text{adj A})\,B$

X = $\frac{1}{-1}\begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}\begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$

X = $\begin{bmatrix} 1 & 1 & -6 \\ - & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix}\begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$

X = $\begin{bmatrix} 29000 + 30500 - 57000 \\ 58000 + 30500 - 85500 \\ -87000 - 61000 + 152000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2500 \\ 3000 \\ 4000 \end{bmatrix}$

Hence, x = 2500, y = 3000 and z = 4000

17. Question

Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award ₹x each ₹y each and ₹ z each for the three respective values to 3, 2 and 1 students respectively with total award money of ₹16,00. School B wants to Spend ₹2,300 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is ₹900, using matrices, find the award money for each value, Apart from these three values, suggest one more value which should be considered for the award.

Answer

Let the numbers are x, y,z be the prize amount per person for sincerity, truthfulness and helpfulness respectively

As per the given data we get,

$3x + 2y + z = 1600$

$4x + y + 3z = 2300$

$x + y + z = 900$

These three equations can be written as

$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$

A X = B

$|A| = 3(1 - 3) - 2(4 - 3) + 1(4 - 1)$

$= 3(-2) - 2(1) + 1(3)$

$= -6 - 2 + 3$

$= -5$

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(1 - 3) = -2$

$C_{12} = (-1)^{1+2}(4 - 3) = -1$

$C_{13} = (-1)^{1+3}(4 - 1) = 3$

$C_{21} = (-1)^{2+1}(2 - 1) = -1$

$C_{22} = (-1)^{2+2}(3 - 1) = 2$

$C_{23} = (-1)^{2+3}(3 - 2) = -1$

$C_{31} = (-1)^{3+1}(6 - 1) = 5$

$C_{32} = (-1)^{3+2}(9 - 4) = -5$

$C_{33} = (-1)^{3+3}(3 - 8) = -5$

$$\text{Adj } A = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$$

$$X = \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$X = \begin{bmatrix} 640 + 460 - 900 \\ -320 - 920 + 900 \\ -960 + 460 + 900 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

Hence, x = 200, y = 300 and z = 400

Excellence in extra curricular activities should be another value considered for an award.

18. Question

Two schools P and Q want to award their selected students on the values of Discipline, Politeness and Punctuality. The school P wants to award ₹x each, ₹y each and ₹z each for the three respectively values to its 3, 2 and 1 students with a total award money of ₹1,000. School Q wants to spend ₹1,500 to award its 4,1 and 3 students on the respective values (by giving the same award money for three values as before.) If the total amount of awards for one prize on each value is ₹600, using matrices, find the award money for each value, Apart on each value is ₹600, using matrices, find the award money for each value, Apart from the above three values, suggest one more value for awards.

Answer

x,y and z be the prize amount per student for Discipline, Politeness and Punctuality respectively.

$3x + 2y + z = 1000$

4x + y + 3z = 1500

x + y + z = 600

These three equations can be written as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

A X = B

|A| = 3(1 – 3) – 2(4 – 3) + 1(4 – 1)

= 3(– 2) – 2(1) + 1(3)

= – 6 – 2 + 3

= – 5

Hence, the unique solution given by x = $A^{-1}B$

$C_{11} = (-1)^{1+1}(1-3) = -2$

$C_{12} = (-1)^{1+2}(4-3) = -1$

$C_{13} = (-1)^{1+3}(4-1) = 3$

$C_{21} = (-1)^{2+1}(2-1) = -1$

$C_{22} = (-1)^{2+2}(3-1) = 2$

$C_{23} = (-1)^{2+3}(3-2) = -1$

$C_{31} = (-1)^{3+1}(6-1) = 5$

$C_{32} = (-1)^{3+2}(9-4) = -5$

$C_{33} = (-1)^{3+3}(3-8) = -5$

Adj A = $\begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$

X = $A^{-1}B = \frac{1}{|A|}(\text{adj A})\,B$

X = $\frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$

X = $\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$

X = $\begin{bmatrix} 640 + 460 - 900 \\ -320 - 920 + 900 \\ -960 + 460 + 900 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$

Hence, x = 100, y = 200 and z = 300

19. Question

Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and

Leadership. The school P want to award ₹x each, ₹y each and ₹z each for the three respective values to 3,2 and 1 students respectively with total award money of ₹2,200. School Q wants to spend ₹3,100 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as school P). If the total amount of award for one prize on each values is ₹1,200, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for the award

Answer

x, y and z be the prize amount per student for Discipline, Politeness and Punctuality respectively.

$3x + 2y + z = 2200$

$4x + y + 3z = 3100$

$x + y + z = 1200$

These three equations can be written as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

A X = B

$|A| = 3(1 - 3) - 2(4 - 3) + 1(4 - 1)$

$= 3(-2) - 2(1) + 1(3)$

$= -6 - 2 + 3$

$= -5$

Hence, the unique solution given by $x = A^{-1}B$

$C_{11} = (-1)^{1+1}(1 - 3) = -2$

$C_{12} = (-1)^{1+2}(4 - 3) = -1$

$C_{13} = (-1)^{1+3}(4 - 1) = 3$

$C_{21} = (-1)^{2+1}(2 - 1) = -1$

$C_{22} = (-1)^{2+2}(3 - 1) = 2$

$C_{23} = (-1)^{2+3}(3 - 2) = -1$

$C_{31} = (-1)^{3+1}(6 - 1) = 5$

$C_{32} = (-1)^{3+2}(9 - 4) = -5$

$C_{33} = (-1)^{3+3}(3 - 8) = -5$

$$\text{Adj } A = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$$

$$X = \frac{1}{-5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -440 \\ -620 \\ -240 \end{bmatrix}$$

$$X = \begin{bmatrix} 880 + 620 - 1200 \\ 440 - 1240 + 1200 \\ -1320 + 620 + 1200 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

Hence, x = 300, y = 400 and z = 500

20. Question

A total amount of ₹7000 is deposited in three different saving bank accounts with annual interest rates of 5%, 8% and $8\frac{1}{2}\%$respectively. The total annual interest from these three accounts is ₹550. Equal amounts have been deposited in the 5% and 8% savings accounts. Find the amount deposited in each of the three accounts, with the help of matrices.

Answer

Let the deposited be x, y and z respectively.

As per the Data we get,

x + y + z = 7000

5%x + 8%y + 8.5%z = 550

i.e 5x + 8y + 8.5z = 55000

x – y = 0

These three equations can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$$

A X = B

|A| = 1(0 + 8.5) – 1(0 – 8.5) + 1(– 5 – 8)

= 1(8.5) – 1(– 8.5) + 1(– 13)

= 8.5 + 8.5 – 13

= 4

Hence, the unique solution given by x = $A^{-1}B$

$C_{11} = (-1)^{1+1}(0 + 8.5) = 8.5$

$C_{12} = (-1)^{1+2}(0 - 8.5) = 8.5$

$C_{13} = (-1)^{1+3}(-5 - 8) = -13$

$C_{21} = (-1)^{2+1}(0 + 1) = -1$

$C_{22} = (-1)^{2+2}(0 - 1) = -1$

$C_{23} = (-1)^{2+3}(-1 - 1) = 2$

$C_{31} = (-1)^{3+1}(8.5 - 8) = 0.5$

$C_{32} = (-1)^{3+2}(8.5 - 5) = -3.5$

$C_{33} = (-1)^{3+3}(8 - 5) = 3$

Adj A = $\begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}^T = \begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}$

X = A^{-1} B = $\frac{1}{|A|}(\text{adj A})\text{ B}$

X = $\frac{1}{4}\begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}\begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$

X = $\frac{1}{4}\begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}\begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$

X = $\frac{1}{4}\begin{bmatrix} 4500 \\ 4500 \\ 19000 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix}$

Hence, x = 1125, y = 1125 and z = 4750

21. Question

A shopkeeper has 3 varities of pens ‘A’, ‘B’ and ‘C’. Meenu purchased 1 pen of each variety for a total of ₹21. Jeen purchased 4 pens of ‘A’ variety, 3 pens of ‘B’ variety and 2 pens of ‘C’ variety for ₹60. While Shikha purchased 6 pens of ‘A’ variety, 2 pens of ‘B’ variety and 3 pens of ‘C’ variety for ₹70. Using matrix method find the cost of each pen.

Answer

Let the varieties of pen A, B and C be x, y and z respectively.

As per the Data we get,

x + y + z = 21

4x + 3y + 2z = 60

6x + 2y + 3z = 70

These three equations can be written as

$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

A X = B

|A| = 1(9 – 4) – 1(12 – 12) + 1(8 – 18)

= 1(5) – 1(0) + 1(– 10)

= 5 – 0 – 10

= – 5

Hence, the unique solution given by x = A^{-1}B

$C_{11} = (-1)^{1+1}(9 - 4) = 5$

$C_{12} = (-1)^{1+2}(12 - 12) = 0$

$C_{13} = (-1)^{1+3}(8 - 18) = -10$

$C_{21} = (-1)^{2+1}(3 - 2) = -1$

$C_{22} = (-1)^{2+2}(3-6) = -3$

$C_{23} = (-1)^{2+3}(2-6) = 4$

$C_{31} = (-1)^{3+1}(2-3) = -1$

$C_{32} = (-1)^{3+2}(2-4) = 2$

$C_{33} = (-1)^{3+3}(3-4) = -1$

Adj A = $\begin{bmatrix} 5 & 0 & -10 \\ -1 & -3 & 4 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$

$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)\, B$

$X = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$

$X = \frac{1}{-5}\begin{bmatrix} 105 - 60 - 70 \\ 0 - 180 + 140 \\ -210 + 240 - 70 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$

Hence, A = Rs 5, B = Rs 8 and C = Rs 8

Exercise 8.2

1. Question

Solve the following systems of homogeneous linear equations by matrix method:

Answer

$2x - y + z = 0$

$3x + 2y - z = 0$

$X + 4y + 3z = 0$

The system can be written as

$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

A X = 0

Now, $|A| = 2(6 + 4) + 1(9 + 1) + 1(12 - 2)$

$|A| = 2(10) + 10 + 10$

$|A| = 40 \neq 0$

Since, $|A| \neq 0$, hence $x = y = z = 0$ is the only solution of this homogeneous equation.

2. Question

Solve the following systems of homogeneous linear equations by matrix method:

$2x - y + 2z = 0$

5x + 3y – z = 0

X + 5y – 5z = 0

Answer

The system can be written as

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 2(– 15 + 5) + 1(– 25 + 1) + 2(25 – 3)

|A| = – 20 – 24 + 44

|A| = 0

Hence, the system has infinite solutions

Let z = k

2x – y = – 2k

5x + 3y = k

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

A X = B

|A| = 6 + 5 = 11≠0 So, A^{-1} exist

Now adj A = $\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$

X = A^{-1} B = $\frac{1}{|A|}\ (\text{adj A})B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} -2k \\ k \end{bmatrix}$

X = $\begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$

Hence, x = $\frac{-5k}{11}$, y = $\frac{12k}{11}$ and z = k

3. Question

Solve the following systems of homogeneous linear equations by matrix method:

3x – y + 2z = 0

4x + 3y + 3z = 0

5x + 7y + 4z = 0

Answer

The system can be written as

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 3(12 – 21) + 1(16 – 15) + 2(28 – 15)

|A| = – 27 + 1 + 26

|A| = 0

Hence, the system has infinite solutions

Let z = k

3x – y = – 2k

4x + 3y = – 3k

$$\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$$

A X = B

|A| = 9 + 4 = 13 ≠0 So, A^{-1} exist

Now adj A = $\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$

X = A^{-1} B = $\frac{1}{|A|}$ $(\text{adj A})B = \frac{1}{13}\begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2k \\ -3k \end{bmatrix}$

X = $\begin{bmatrix} \frac{-9k}{13} \\ \frac{-k}{13} \end{bmatrix}$

Hence, x = $\frac{-9k}{13}$, y = $\frac{-k}{13}$ and z = k

4. Question

Solve the following systems of homogeneous linear equations by matrix method:

x + y – 6z = 0

x – y + 2z = 0

– 3x + y + 2z = 0

Answer

The system can be written as

$$\begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 1(– 2 – 2) – 1(2 + 6) – 6(1 – 3)

|A| = – 4 – 8 + 12

|A| = 0

Hence, the system has infinite solutions

Let z = k

X + y = 6k

x – y = – 2k

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

A X = B

|A| = – 1 – 1 = – 2 ≠0 So, A^{-1} exist

Now adj A = $\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^T$ = $\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

X = A $^{-1}$ B = $\frac{1}{|A|}$ $(\text{adj A})B$ = $\frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 6k \\ -2k \end{bmatrix}$

X = $\frac{1}{-2}\begin{bmatrix} -6k + 2k \\ -6k - 2k \end{bmatrix}$

X = $\begin{bmatrix} -4k \\ -8k \end{bmatrix}$

Hence, x = 2k, y = 4k and z = k

5. Question

Solve the following systems of homogeneous linear equations by matrix method:

x + y + z = 0

x – y – 5z = 0

x + 2y + 4z = 0

Answer

The system can be written as

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -5 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

A X = 0

Now, |A| = 1(6) – 1(9) + 1(3) = 0

|A| = 6 – 9 + 3

|A| = 0

Hence, the system has infinite solutions

Let z = k

X + y = – k

x – y = 5k

$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -k \\ 5k \end{bmatrix}$

A X = B

|A| = – 1 – 1 = – 2 ≠0 So, A $^{-1}$ exist

Now adj A = $\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^T$ = $\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$

X = A $^{-1}$ B = $\frac{1}{|A|}$ $(\text{adj A})B$ = $\frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -k \\ 5k \end{bmatrix}$

X = $\frac{1}{-2}\begin{bmatrix} k - 5k \\ k + 5k \end{bmatrix}$

X = $\begin{bmatrix} 2k \\ -3k \end{bmatrix}$

Hence, x = 2k, y = – 3k and z = k

6. Question

Solve the following systems of homogeneous linear equations by matrix method:

x + y – z = 0

x – 2y + z = 0

3x + 6y – 5z = 0

Answer

The system can be written as

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 1(4) – 1(– 8) – 1(12) = 0

|A| = 4 + 8 – 12

|A| = 0

Hence, the system has infinite solutions

Let z = k

X + y = k

x – 2y = – k

$$\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ -k \end{bmatrix}$$

A X = B

|A| = – 2 – 1 = – 3 ≠0 So, A^{-1} exist

Now adj A = $\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$

X = A^{-1} B = $\frac{1}{|A|}$ $(\text{adj A})B = \frac{1}{-3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} k \\ -k \end{bmatrix}$

X = $\frac{1}{-3}\begin{bmatrix} -2k + k \\ -k - k \end{bmatrix}$

X = $\frac{1}{-3}\begin{bmatrix} -k \\ -2k \end{bmatrix}$

Hence, x = $\frac{k}{3}$, y = $\frac{2k}{3}$ and z = k

7. Question

Solve the following systems of homogeneous linear equations by matrix method:

3x + y – 2z = 0

x + y + z = 0

x – 2y + z = 0

Answer

The system can be written as

$$\begin{bmatrix} 3 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 3(1 + 2) – 1(1 – 1) – 2(– 2 – 1) = 0

|A| = 9 – 0 + 6

|A| = 15

Hence, the given system has only trivial solution given by x = y = z = 0

8. Question

Solve the following systems of homogeneous linear equations by matrix method:

2x + 3y – z = 0

x – y – 2z = 0

3x + y + 3z = 0

Answer

The system can be written as

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 2(– 3 + 2) – 3(3 + 6) – 1(1 + 3) = 0

|A| = – 2 – 27 – 4

|A| = – 33

Hence, the given system has only trivial solution given by x = y = z = 0

MCQ

1. Question

Mark the correct alternative in the following:

The system of equation x + y + z = 2, 3x – y + 2z = 6 and 3x + y + z = –18 has

A. a unique solution

B. no solution

C. an infinite number of solutions

D. zero solution as the only solution

Answer

The given system of equations is

x + y + z = 2

3x – y + 2z = 6

3x + y + z = -18

The matrix equation corresponding to this system is :-

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

Let A = $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix}$ = 1(-1-2) -1(3-6) + 1(3+3)

= -3+3+6 = 6

i.e., $|A| \neq 0$

A unique solution of the system exists.

2. Question

Mark the correct alternative in the following:

The number of solutions of the system of equations: x – 3y + 2z = 1, is

x + 4y – 3z = 5

A. 3

B. 2

C. 1

D. 0

Answer

The given system of equations is :-

x– 3y +2z = 1

x + 4y – 3z = 5

As there are three variables x,y,z and we have only two equations so it is impossible to find the solution.
Thus, no solution exists for this system of equations.

3. Question

Mark the correct alternative in the following:

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If AX = B, then X is equal to

A. $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

B. $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$

C. $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

Answer

Given that

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

Also AX = B and we have to find the value of X ,

Pre-multiplying A^{-1} both sides we get,

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$ ($\because A^{-1}A = I$)

$X = A^{-1}B$ ($\because IX = X$) ......(i)

Now,

$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 1(0-2) +1(2-3) +2(4-0) = -2-1+8 = 5$

And $\text{adj}A = \begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}$

From (i),

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -5 \\ 10 \\ 15 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

On comparing both sides we get,

$$\boxed{\begin{array}{c} x_1 = -1 \\ x_2 = 2 \\ x_3 = 3 \end{array}}$$

4. Question

Mark the correct alternative in the following:

The number of solutions of the system of equations:

2x + y – z = 7

x – 3y + 2z = 1, is

x + 4y – 3z = 5

A. 3

B. 2

C. 1

D. 0

Answer

The given system of equations is :-

2x + y – z = 7

x – 3y + 2z = 1

x + 4y – 3z = 5

The matrix equation corresponding to the above system is

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 5 \end{bmatrix}$$

Let A = $\begin{bmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & -3 \end{bmatrix}$ and B = $\begin{bmatrix} 7 \\ 1 \\ 5 \end{bmatrix}$

∴ |A| = $\begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & -3 \end{vmatrix}$ = 2(9-8) -1(-3-2) -1(4+3) = 2+5-7 = 0

and adjA = $\begin{bmatrix} 1 & 5 & -7 \\ -1 & -5 & -7 \\ 1 & -5 & 7 \end{bmatrix}$

∴ (adjA)B = $\begin{bmatrix} 1 & 5 & -7 \\ -1 & -5 & -7 \\ 1 & -5 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} -23 \\ -47 \\ 37 \end{bmatrix} \neq 0$

So, No solution exists.

5. Question

Mark the correct alternative in the following:

The system of linear equations:

x + y + z = 2

2x + y – z = 3

3x + 2y + kz = 4 has a unique solution if

A. k ≠ 0

B. –1 < k < 1

C. –2 < k < 2

D. k = 0

Answer

The system of linear equations:

x + y + z = 2

2x + y – z = 3

3x + 2y + kz = 4

The matrix equation corresponding to the above system is

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} -23 \\ -47 \\ 37 \end{bmatrix}$$

Let A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}$ = 1(k+2) -1(2k+3) +1(4-3) = k+2-2k-3+1 = -k

For the system to have a unique solution , it is necessary that $|A| \neq 0$

$\therefore -k \neq 0$

Thus,

$\boxed{k \neq 0}$

6. Question

Mark the correct alternative in the following:

Consider the system of equations:

$a_1x + b_1y + c_1z = 0$

$a_2x + b_2y + c_2z = 0$

$a_3x + b_3y + c_3z = 0$

If $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = 0,$ then the system has

A. more than two solutions

B. one trivial and one non-trivial solutions

C. no solution

D. only trivial solution (0, 0, 0)

Answer

The given system of linear equations is :-

$a_1x + b_1y + c_1z = 0$

$a_2x + b_2y + c_2z = 0$

$a_3x + b_3y + c_3z = 0$

The matrix equation corresponding to the above system is :-

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

According to the question,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = 0$$

This matrix is a singular matrix. So, the system has infinitely many solutions including the trivial solution.

7. Question

Mark the correct alternative in the following:

Let a, b, c be positive real numbers. The following system of equations in x, y and z

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ has

A. no solution

B. unique solution

C. infinitely many solution

D. finitely many solutions

Answer

The given system of linear equations is :-

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

$- \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Let x^2 = p , y^2 = q , z^2 = r

The equation becomes,

$\frac{p}{a^2} + \frac{q}{b^2} - \frac{r}{c^2} = 1$

$\frac{p}{a^2} - \frac{q}{b^2} + \frac{r}{c^2} = 1$

$- \frac{p}{a^2} + \frac{q}{b^2} + \frac{r}{c^2} = 1$

The matrix equation corresponding to the above system of equation is

$$\begin{bmatrix} \frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ \frac{1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & \frac{1}{c^2} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ \mu \end{bmatrix}$$

Let A = $\begin{bmatrix} \frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ \frac{1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & \frac{1}{c^2} \end{bmatrix}$

$\therefore$ |A| = $\begin{vmatrix} \frac{1}{a^2} & \frac{1}{b^2} & -\frac{1}{c^2} \\ \frac{1}{a^2} & \frac{-1}{b^2} & \frac{1}{c^2} \\ -\frac{1}{a^2} & \frac{1}{b^2} & \frac{1}{c^2} \end{vmatrix}$

Taking common $\frac{1}{a^2}$ from C_1 , $\frac{1}{b^2}$ from C_2 and $\frac{1}{c^2}$ from C_3 we get,

|A| = $\frac{1}{a^2}\frac{1}{b^2}\frac{1}{c^2}\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{-4}{a^2b^2c^2} \neq 0$

$\therefore$ A unique solution exists.

8. Question

Mark the correct alternative in the following:

For the system of equations:

x + 2y + 3z = 1

2x + y + 3z = 2

5x + 5y + 9z = 4

A. there is only one solution

B. there exists infinitely many solution

C. there is no solution

D. none of these

Answer

For the system of equations:

x + 2y + 3z = 1

2x + y + 3z = 2

5x + 5y + 9z = 4

The matrix equation corresponding to the above system is

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{bmatrix}$

∴ |A| = $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 5 & 5 & 9 \end{vmatrix}$ = 1(9-8) -2(18-15) +3(10-5) = 1-6+15 = 10

i.e., |A| $\neq$ 0

Hence, a unique solution exists of the above system.

9. Question

Mark the correct alternative in the following:

The existence of the unique solution of the system of equations:

x + y + z = λ

5x – y + μz = 10

2x + 3y – z = 6 depends on

A. μ only

B. λ only

C. λ and μ both

D. neither λ nor μ

Answer

The given system of linear equation :-

x + y + z = λ

5x – y + μz = 10

2x + 3y – z = 6

The matrix equation corresponding to the above system is :-

$$\begin{bmatrix}1 & 1 & 1\\5 & -1 & \mu\\2 & 3 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}\lambda\\10\\6\end{bmatrix}$$

Let A = $\begin{bmatrix}1 & 1 & 1\\5 & -1 & \mu\\2 & 3 & -1\end{bmatrix}$

∴ |A| = $\begin{vmatrix}1 & 1 & 1\\5 & -1 & \mu\\2 & 3 & -1\end{vmatrix}$ = 1(1-3 μ) -1(-5-2 μ) +1(15+2)

= 1-3 μ +5 +2 μ +17 = 23- μ

For the existence of the unique solution, the value of |A| must be equal to 0.

Hence the existence of the unique solution merely depends on the value of μ.

10. Question

Mark the correct alternative in the following:

The system of equations:

x + y + z = 5

x + 2y + 3z = 9

x + 3y + λz = μ

has a unique solution, if

A. λ = 5, μ = 13

B. λ ≠ 5

C. λ = 5, μ ≠ 13

D. μ ≠ 13

Answer

The given system of linear equations is :-

x + y + z = 5

x + 2y + 3z = 9

x + 3y + λz = μ

The matrix equation corresponding to the above system is

$$\begin{bmatrix}1 & 1 & 1\\1 & 2 & 3\\1 & 3 & \lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}5\\9\\\mu\end{bmatrix}$$

Let A = $\begin{bmatrix}1 & 1 & 1\\1 & 2 & 3\\1 & 3 & \lambda\end{bmatrix}$

∴ |A| = $\begin{vmatrix}1 & 1 & 1\\1 & 2 & 3\\1 & 3 & \lambda\end{vmatrix}$ = 1(2 $\lambda - 9) - 1(\lambda - 3) + 1(3 - 2)$

= 2 λ-9 - λ+3+1 = $\lambda - 5$

For the existence of a unique solution |A| $\neq$ 0

∴ $\lambda - 5 \neq$ 0

∴

$\boxed{\lambda \neq 5}$

Very short answer

1. Question

If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, find x, y and z.

Answer

The given matrix equation is :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

The first matrix is an identity matrix which is denoted by I .

Let P = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

Thus the equation becomes

IP = B

As we know that any matrix when multiplied with the identity matrix will be the matrix itself. But here the order of I is 3×3 while the order of P is 3×1, so the order of the result matrix is 3×1.

IP = P = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

So, we have

P = B

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

On comparing both sides we get,

$$\boxed{\begin{matrix} x = 1 \\ y = -1 \\ z = 0 \end{matrix}}$$

2. Question

If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, find x, y and z.

Answer

The given matrix equation is :-

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Let A = $\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}$; P = $\begin{bmatrix}x\\y\\z\end{bmatrix}$ and B = $\begin{bmatrix}1\\0\\1\end{bmatrix}$

So, we can write the equation as

AP = B

Pre-multiplying A^{-1} both sides we get,

$A^{-1}AP = A^{-1}B$

$IP = A^{-1}B$ ($\because A^{-1}A = I$)

$P = A^{-1}B$ (IP = P) ....(i)

Now,

$|A| = \begin{vmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{vmatrix} = 1$

and

adj A = $\begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A$

$\therefore A^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}$

From (i) ,

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\-1\end{bmatrix}$$

Comparing both sides we get,

$$\boxed{\begin{array}{l}x = 1\\ y = 0\\ z = -1\end{array}}$$

3. Question

If $\begin{bmatrix}1 & 0 & 0\\0 & y & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\-1\\z\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix}$, find x, y and z.

Answer

The given matrix equation is :-

$$\begin{bmatrix}1 & 0 & 0\\0 & y & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\-1\\z\end{bmatrix} = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

or, $\begin{bmatrix} x \\ -y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

Comparing both sides we get,

$$\boxed{\begin{array}{l} x = 1 \\ y = 0 \\ z = 1 \end{array}}$$

4. Question

Solve $\begin{bmatrix} 3 & -4 \\ 9 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$ for x and y.

Answer

The given matrix equation is :-

$\begin{bmatrix} 3 & -4 \\ 9 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

Multiplying the two matrices in the LHS we get,

$\begin{bmatrix} 3x - 4y \\ 9x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

Comparing both sides we get,

3x-4y = 10 ....(i)

9x+y = 2 ....(ii)

Multiplying eq.(ii) by 4 and then adding both the equations we get,

$$\begin{array}{l} 3x-4y = 10 \\ 36x+4y = 8 \\ \hline 39x = 18 \end{array}$$

$x = \frac{6}{13}$

Putting this value of x in eq.(ii) we get,

$\frac{54}{13} + y = 2$

or, $y = 2 - \frac{54}{13}$

$\therefore\ y = -\frac{28}{13}$

5. Question

If $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, find x, y, z.

Answer

The given matrix equation is :-

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Multiplying the two matrices in the LHS we get,

$$\begin{bmatrix} x \\ z \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Comparing both sides we get,

$$\boxed{\begin{array}{l} x = 2 \\ y = 3 \\ z = -1 \end{array}}$$

6. Question

If $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}, X = \begin{bmatrix} n \\ 1 \end{bmatrix}, B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$ and AX = B, then find n.

Answer

It is given that

AX = B

Where

A = $\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}$; $X = \begin{bmatrix} n \\ 1 \end{bmatrix}$ and B = $\begin{bmatrix} 8 \\ 11 \end{bmatrix}$

And we have to find the value of n.

∴ AX = $\begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} n \\ 1 \end{bmatrix}$

or, AX = $\begin{bmatrix} 2n + 4 \\ 4n + 3 \end{bmatrix}$

Thus,

$$\begin{bmatrix} 2n + 4 \\ 4n + 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$$

On comparing both sides we get,

2n+4 = 8 ....(i)

4n+3 = 11 ....(ii)

From (i),

2n = 4

∴ n = 2

and from (ii) ,

4n = 8

∴ n = 2

$\boxed{n = 2}$

Hence