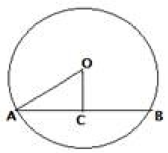
Chapter : 11. CIRCLES

Exercise : 11A

Question: 1

Let AB be a chord of a circle with center O. OC$\perp$AB, then

AB = 16 cm, and OA = 10 cm.



OC$\perp$AB

Therefore,

OC bisects AB at C

AC = (1/2) AB

⇒ AC = (1/2) 16

⇒ AC = 8 cm

In triangle OAC,

$OA^2 = OC^2 + AC^2$

⇒ $10^2 = OC^2 + 8^2$

⇒ $100 = OC^2 + 64$

⇒ $OC^2 = 36$

⇒ OC= 6

Question: 2

Let distance OC = 3 cm

Radius = OA = 5 cm

Draw OC$\perp$AB

In triangle OCA,

$OA^2 = OC^2 + AC^2$

⇒ $5^2 = 3^2 + AC^2$

⇒ $AC^2 = 16$

⇒ AC = 4 cm __________________ (i)

Now,

AB = 2 AC

⇒ AB = 8 cm [From equation (i)]

Hence, length of a chord = 8 cm.

Question: 3

Let distance OC = 8 cm

Chord AB = 30 cm

Draw $OC \perp AB$

Therefore,

OC bisects AB at C

AC = (1/2) AB

⇒ AC = (1/2) 30

⇒ AC = 15 cm

In triangle OCA,

$OA^2 = OC^2 + AC^2$

$\Rightarrow OA^2 = 8^2 + 15^2$

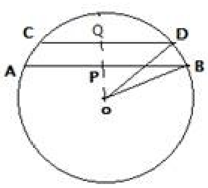
$\Rightarrow OA^2 = 64 + 225$

$\Rightarrow OA^2 = 289$

⇒ OA = 17 cm

Hence, radius of the circle = 17 cm.

Question: 4

(i) 

Let radius OB = OD = 5 cm

Chord AB = 8 cm

Chord CD = 6 cm

BP = (1/2) AB

⇒ BP = (1/2) 8 = 4 cm

DQ = (1/2) CD

⇒ DQ = (1/2) 6 = 3 cm

In triangle OPB,

$OP^2 = OB^2 - BP^2$

$\Rightarrow OP^2 = 5^2 - 4^2$

$\Rightarrow OP^2 = 25 - 16$

$\Rightarrow OP^2 = 9$

⇒ OP = 3 cm

In triangle OQD,

$OQ^2 = OD^2 - DQ^2$

$\Rightarrow OQ^2 = 5^2 - 3^2$

$\Rightarrow OQ^2 = 25 - 9$

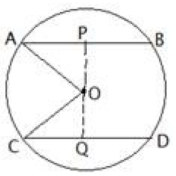
$\Rightarrow OQ^2 = 16$

⇒ OQ = 4 cm

Now,

PQ = OQ – OP = 4 – 3 = 1

Hence, distance between chords = 1 cm.

(ii) 

Let radius OA = OC = 5 cm

Chord AB = 8 cm

Chord CD = 6 cm

AP = (1/2) AB

⇒ AP = (1/2) 8 = 4 cm

CQ = (1/2) CD

⇒ CQ = (1/2) 6 = 3 cm

In triangle OAP,

$OP^2 = OA^2 - AP^2$

⇒ $OP^2 = 5^2 - 4^2$

⇒ $OP^2 = 25 - 16$

⇒ $OP^2 = 9$

⇒ OP = 3 cm

In triangle OQD,

$OQ^2 = OC^2 - CQ^2$

⇒ $OQ^2 = 5^2 - 3^2$

⇒ $OQ^2 = 25 - 9$

⇒ $OQ^2 = 16$

⇒ OQ = 4 cm

Now,

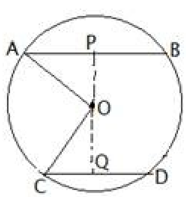
PQ = OP + OQ = 3 + 4 = 7

Hence, distance between chords = 7 cm.

Question: 5

Let radius OA = OC = 17 cm

Chord AB = 30 cm and CD = 16 cm



Draw OL and OM

Therefore,

AP = (1/2) AB

⇒ AP = (1/2) 30 = 15 cm

CQ = (1/2) CD

⇒ CQ = (1/2) 16 = 8 cm

In triangle OAP,

$OP^2 = OA^2 - AP^2$

⇒ $OP^2 = 17^2 - 15^2$

⇒ $OP^2 = 289 - 225$

⇒ $OP^2 = 64$

⇒ OP = 8 cm

In triangle OQD,

$OQ^2 = OC^2 - CQ^2$

⇒ $OQ^2 = 17^2 - 8^2$

⇒ $OQ^2 = 289 - 64$

⇒ $OQ^2 = 225$

⇒ OQ = 15 cm

Now,

PQ = OP + OQ = 8 + 15 = 23

Hence, distance between chords = 23 cm.

Question: 6

Let radius OA = OC = OD = r

Chord AB = 12 cm

OE = OC – CE

⇒ OE = r – 3

AE = (1/2) AB

⇒ AE = (1/2) 12 = 6 cm

In triangle AOE,

$OA^2 = AE^2 + OE^2$

⇒ $r^2 = 6^2 + (r - 3)^2$

⇒ $r^2 = 36 + r^2 + 9 - 6r$

⇒ 6r = 45

⇒ r = 7.5 cm

Hence, radius of circle = 7.5 cm.

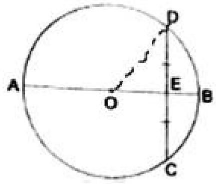
Question: 7

Let radius OA = OB = OD = r

DE = 8 cm

OE = OB – BE

⇒ OE = r – 4



In triangle ODE,

$OD^2 = DE^2 + OE^2$

$\Rightarrow r^2 = 8^2 + (r - 4)^2$

$\Rightarrow r^2 = 64 + r^2 + 16 - 8r$

$\Rightarrow 8r = 80$

$\Rightarrow r = 10$ cm

Hence, radius of circle = 10 cm.

Question: 8

Given $OD \perp AB$

In triangle ABC,

D is the mid-point of AB

$\therefore$ AD = DB

O is the mid-point of BC

$\therefore$ OC = OB

We say, AC||OD

(1/2) AC = OD [Mid-point theorem in triangle ABC]

$\Rightarrow AC = 2 \times OD$ Proved.

Question: 9

Proof

In ΔOEP and ΔOFP,

$\angle OEP = \angle OFP$ [equal to 90°]

OP = OP [common]

$\angle OPE = \angle OPF$ [OP bisects $\angle BPD$]

Therefore,

ΔOEP = ΔOFP [By angle-side-angle]

$\therefore$ OE = OF

AB = CD [Chords are equidistant from the center]

Hence, AB = CD Proved.

Question: 10

90°]

$\therefore PF \perp CD$ and $OF \perp CD$

We know that the perpendicular from the center of a circle to chord, bisect the chord.

Therefore,

CF = FD Proved.

Question: 11

Let two different circles intersect at three distinct points A, B and C.

Then, these points are already non-collinear.

A unique circle can be drawn to pass through these points. This is a contradiction.

Hence, two different circles cannot intersect each other at more than two points.

Question: 12

Let,

Radius OA = 10 cm and O'A = 8 cm

Chord AB = 12 cm

Now,

AD = (1/2) AB

⇒ AD = (1/2) 12 = 6 cm

In triangle OAD,

$OD^2 = OA^2 - AD^2$

⇒ $OD^2 = 10^2 - 6^2$

⇒ $OD^2 = 100 - 36$

⇒ $OD^2 = 64$

⇒ OD = 8 cm

In triangle O'AD,

$O'D^2 = O'A^2 - AD^2$

⇒ $O'D^2 = 8^2 - 6^2$

⇒ $O'D^2 = 64 - 36$

⇒ $O'D^2 = 28$

⇒ O'D = 2 √7 cm

Now,

OO' = OD + O'D = $\left(8+2\sqrt{7}\right)$ cm

Hence, distance between their centers = $\left(8+2\sqrt{7}\right)$ cm

Question: 13

Join PQ,

PQ is the common chord of both the circles.

Thus,

arc PCQ = arc PDQ

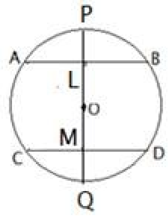
∴ ∠QAP = ∠QBP

∴ QA = QB Proved.

Question: 14

Let AB and CD are two chords of a circle with center O.

Diameter POQ bisect s them at L and M.



Then,

OL$\perp$ AB and OM$\perp$ CD

$\therefore$ $\angle$ALM = $\angle$LMD

$\therefore$ AB||CD [Alternate angles]

Question: 15

Join AP.

Let PQ intersect AB at L,

Then, AB = 5 – 3 = 2 cm

PQ is the perpendicular bisector of AB,

Then,

AL = (1/2) AB

⇒ AL = (1/2) 2 = 1 cm

In triangle APL,

$PL^2 = PA^2 - AL^2$

⇒ $PL^2 = 5^2 - 1^2$

⇒ $PL^2 = 25 - 1$

⇒ $PL^2 = 24$

⇒ PL = 2 $\sqrt{6}$ cm

Now,

PQ = 2 PL

⇒ PQ = 2 × 2 $\sqrt{6}$

⇒ PQ = 4 $\sqrt{6}$ cm

Question: 16

Given, OB = OC

Then, $\angle$BOC = $\angle$BCO = y°

External $\angle$OBA = $\angle$BOC + $\angle$BCO = (2y)°

Now,

OA = OB

Then, $\angle$OAB = $\angle$OBA = (2y)°

External $\angle$AOD = $\angle$OAC + $\angle$ACO

= $\angle$OAB + $\angle$BCO = (3y)°

$\therefore$ x° = (3y)°[Given $\angle AOD = x°$,]

Question: 17

Given AB = AC

$\therefore$ (1/2)AB = (1/2)AC

OP $\perp$ AB and OQ $\perp$ AC

$\therefore$ MB = NC

$\Rightarrow$ $\angle$PMB = $\angle$QNC [90°]

Equal chords are equidistant from the center.

$\Rightarrow$ OM = ON

OP = OQ

$\Rightarrow$ OP – OM = OQ – ON

$\Rightarrow$ PM = QN

$\therefore$ ΔABC ≅ ΔABC [By side-angle-side criterion of congruence]

$\therefore$ PB = QC Proved.

Question: 18

Draw, OP $\perp$ AB and OQ $\perp$ CD

In triangle OBP and triangle OQC,

$\angle$OPB = $\angle$OQC [Angle = 90°]

$\angle$OBP = $\angle$OCD [Alternate angle]

OB = OC [Radius]

By side-angle-side criterion of congruence

ΔOBP ≅ ΔOQC

$\therefore$ OP = OQ

The chords equidistant from the center are equal.

$\therefore$ AB = CD Proved.

Question: 19

Let ABC be an equilateral triangle of side 9 cm.

And AD be one of its medians.

Then,

AD $\perp$ BC

BD = (1/2) BC

$\Rightarrow$ BD = (1/2) 9 = 4.5 cm

In triangle ADB,

$AD^2 = AB^2 - BD^2$

$\Rightarrow AD^2 = 9^2 – (9/2)^2$

$\Rightarrow AD^2 = 81 – (81/4)$

$\Rightarrow$ AD = (9√3)/2

In an equilateral triangle the centroid and circumcenter coincide and AO: OD = 2: 1

$\therefore$ radius AO = (2/3) AD

= (2/3) (9√3)/2

= 3√3 cm

Hence, radius of circle = 3√3 cm.

Question: 20

In triangle OAB and triangle OAC,

AB = AC [Given]

OB = CO [Radius]

OA = OA [Common]

By side-side-side criterion of congruence

ΔOAB ≅ ΔOAC

∴ ∠OAB = ∠OAC Proved.

Question: 21

In triangle OPX and triangle ORY,

OX = OY [Radius]

∠OPX = ∠ORY [Common]

OP = OR [Sides of square]

By side-angle-side criterion of congruence,

ΔOPX ≅ ΔORY

∴ PX = RY

⇒ PQ – PX = QR – RY [PQ = QR]

⇒ QX = QY Proved.

Exercise : 11B

Question: 1

(i) Join OB.

∠OAB = ∠OBA = 40°[Because OB = OA]

∠OCB = ∠OBC = 30°[Because OB = OC]

∠ABC = ∠OBA + ∠OBC

⇒ ∠ABC = 40°+ 30°

⇒ ∠ABC = 70°

∠AOC = 2 × ∠ABC

⇒ ∠AOC = 2 × ∠ABC

⇒ ∠AOC = 2 × 70°

⇒ ∠AOC = 140°

(ii) $\angle BAC = 80°$

∠BOC = 360° - (∠AOB + ∠AOC) [Sum of all angles at a point = 360°]

⇒ ∠BOC = 360° - (90° + 110°)

⇒ ∠BOC = 360° - 200°

⇒ ∠BOC = 160°

We know that ∠BOC = 2 × ∠BAC

⇒ ∠BAC = (1/2) × ∠BOC

⇒ ∠BAC = (1/2) × 160°

⇒ ∠BAC = 80°

Question: 2

AOC + $\angle AOB = 180°$[Because BC is a straight line]

$\Rightarrow \angle AOC + 70° = 180°$

$\Rightarrow \angle AOC + 70° = 180°$

$\Rightarrow \angle AOC = 110°$

OA = OC [Radius]

$\therefore \angle OAC = \angle OCA$ ________________ (i)

In triangle AOC,

$\angle OAC + \angle OCA + \angle AOC = 180°$[Sum of angles of triangle]

$\Rightarrow 2\ \angle OCA + 110° = 180°$[From equation (i)]

$\Rightarrow 2\ \angle OCA = 70°$

$\Rightarrow 2\ \angle OCA = 70°$

$\Rightarrow \angle OCA = 35°$

(ii) $\angle OAC = 35°$

$\angle AOC + \angle AOB = 180°$[Because BC is a straight line]

$\Rightarrow \angle AOC + 70° = 180°$

$\Rightarrow \angle AOC + 70° = 180°$

$\Rightarrow \angle AOC = 110°$

OA = OC [Radius]

$\therefore \angle OAC = \angle OCA$ ________________ (i)

In triangle AOC,

$\angle OAC + \angle OCA + \angle AOC = 180°$[Sum of angles of triangle]

$\Rightarrow 2\ \angle OAC + 110° = 180°$[From equation (i)]

$\Rightarrow 2\ \angle OAC = 70°$

$\Rightarrow 2\ \angle OAC = 70°$

$\Rightarrow \angle OAC = 35°$

Question: 3

BPC + $\angle APB = 180°$[Because APC is a straight line]

$\Rightarrow \angle BPC + 110° = 180°$

$\Rightarrow \angle BPC = 70°$

In triangle BPC,

$\angle BPC + \angle PBC + \angle PCB = 180°$[Sum of angles of triangle]

$\Rightarrow 70° + 25° + \angle PCB = 180°$

$\Rightarrow \angle PCB = 85°$

$\therefore \angle ADB = \angle PCB = 85°$[Angles in the same segment of a circle]

Question: 4

In triangle ABD,

$\angle ABD + \angle BAD + \angle ADB = 180°$[Sum of angles of triangle]

$\Rightarrow 35° + 90° + \angle ADB = 180°$

$\Rightarrow \angle ADB = 55°$

$\therefore \angle ACB = \angle ADB = 55°$[Angles in the same segment of a circle]

Question: 5

$AOB = 2 \times \angle ACB$

$\Rightarrow \angle AOB = 2 \times 50°$

$\Rightarrow \angle AOB = 100°$

OA =OB [Radius of the circle]

$\therefore \angle OAB = \angle OBA$ _________________ (i)

In triangle AOB,

$\angle OAB + \angle OBA + \angle AOB = 180°$[Sum of angles of triangle]

$\Rightarrow 2\ \angle OAB + 100° = 180°$[From equation (i)]

$\Rightarrow 2\ \angle OAB = 80°$

$\Rightarrow \angle OAB = 40°$

Question: 6

(i) $\angle ACD = 54°$

$\angle ABD$ and $\angle ACD$ are in the segment AD.

$\therefore \angle ACD = \angle ABD$ [Angles in the same segment of a circle]

$\angle ACD = 54°$

(ii) $\angle BAD = 43°$

$\angle BAD$ and $\angle BCD$ are in the segment BD.

$\therefore \angle BAD = \angle BCD$ [Angles in the same segment of a circle]

$\angle BAD = 43°$

(iii) $\angle BDA = 83°$

In triangle ABD,

$\angle ABD + \angle BAD + \angle BAD = 180°$[Sum of angles of triangle]

$\Rightarrow 54° + 43° + \angle BAD = 180°$

$\Rightarrow 97° + \angle BAD = 180°$

$\Rightarrow \angle BAD = 83°$

Question: 7

CAD and $\angle CBD$ are in the segment BD.

$\therefore \angle CAD = \angle CBD$ [Angles in the same segment of a circle]

$\angle CAD = 60°$

In triangle ACD,

$\angle CAD + \angle ADC + \angle ACD = 180°$[Sum of angles of triangle]

$\Rightarrow 60° + 90° + \angle ACD = 180°$

$\Rightarrow 150° + \angle ACD = 180°$

$\Rightarrow \angle ACD = 30°$

$\therefore \angle CDE = \angle ACD = 30°$[Alternate angles]

Question: 8

Join OC and OD.

$\angle ABC = \angle BCD = 25°$[Alternate angles]

The angle subtended by an arc of a circle at the center is double the angle subtended by the arc at any point on the circumference.

$\therefore \angle BOD = 2 \times \angle BCD$

$\Rightarrow \angle BOD = 2 \times 25°$

$\Rightarrow \angle BOD = 50°$

Similarly,

$\angle AOC = 2 \times \angle ABC$

$\Rightarrow \angle AOC = 2 \times 25°$

$\Rightarrow \angle AOC = 50°$

Now,

$\angle AOB = 180°$ [AOB is a straight line]

$\Rightarrow \angle AOC + \angle COD + \angle BOD = 180°$

$\Rightarrow 50° + \angle COD + 50° = 180°$

$\Rightarrow 100° + \angle COD = 180°$

$\Rightarrow \angle COD = 80°$

$\therefore \angle CED = (1/2)\ \angle COD$

$\Rightarrow \angle CED = (1/2)\ 80°$

$\Rightarrow \angle CED = 40°$

Question: 9

(i) $\angle DCE = 50°$

In triangle CDE,

$\angle CDE + \angle CED + \angle DCE = 180°$[Sum of angles of triangle]

$\Rightarrow 40° + 90° + \angle DCE = 180°$

$\Rightarrow 130° + \angle DCE = 180°$

$\Rightarrow \angle DCE = 50°$

(ii) $\angle ABC = 30°$

$\angle AOC + \angle BOC = 180°$[Because AOB is a straight line]

$\Rightarrow 80° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 100°$

In triangle BOC,

$\angle OCB + \angle BOC + \angle OBC = 180°$[Sum of angles of triangle]

$\Rightarrow 50° + 100° + \angle OBC = 180°$[$\angle DCE = 50°$]

$\Rightarrow 150° + \angle OBC = 180°$

$\Rightarrow \angle OBC = 30°$

$\therefore \angle ABC = \angle OBC = 30°$

Question: 10

DCB = (1/2) $\angle AOB$ [$\angle DCB = \angle ACB$]

⇒ ∠DCB = (1/2) 40°

⇒ ∠DCB = 20°

In triangle BCD,

∠BDC + ∠DCB + ∠DBC = 180°[Sum of angles of triangle]

⇒ 100° + 20° + ∠OBC = 180°

⇒ 120° + ∠DBC = 180°

⇒ ∠DBC = 60°

∴ ∠OBC = ∠DBC = 60°

Question: 11

Join OB,

∴ OA = OB [Radius]

∴ ∠OAB = ∠OBA = 25°

In triangle AOB,

∠AOB + ∠OAB + ∠OBA = 180°[Sum of angles of triangle]

⇒ ∠AOB + 25° + 25° = 180°

⇒ ∠AOB + 50° = 180°

⇒ ∠AOB = 130°

Now,

∠ACB = (1/2) ∠AOB

⇒ ∠ACB = (1/2) 130°

⇒ ∠ACB = 65°

In triangle BEC,

∠EBC + ∠ECB + ∠BEC = 180°[Sum of angles of triangle]

⇒ ∠EBC + 65° + 90° = 180°

⇒ ∠EBC + 155° = 180°

⇒ ∠EBC = 25°

Question: 12

(i) $\angle BOC = 70^\circ$

OB = OC [Radius]

∴ ∠OBC = ∠OCB = 55°

In triangle OCB,

∠OBC + ∠OCB + ∠BOC = 180°[Sum of angles of triangle]

⇒ 55° + 55° + ∠BOC = 180°

⇒ 110° + ∠BOC = 180°

⇒ ∠BOC = 70°

(ii) $\angle AOC = 70^\circ$

OA = OB [Radius]

∴ ∠OBA = ∠OAB = 20°

In triangle AOB,

$\angle OBA + \angle OAB + \angle AOB = 180°$[Sum of angles of triangle]

$\Rightarrow 20° + 20° + \angle AOB = 180°$

$\Rightarrow 40° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 140°$

$\therefore \angle AOC = \angle AOB - \angle BOC$

$\Rightarrow \angle AOC = 140° - 70°$

$\Rightarrow \angle AOC = 70°$

Question: 13

BOC = $2 \times \angle BAC$

$\Rightarrow \angle BOC = 2 \times 30°$

$\Rightarrow \angle BOC = 60°$______________ (i)

OB = OC

$\therefore \angle OBC = \angle OCB$ ________________ (ii)

In triangle AOB,

$\angle OBC + \angle OCB + \angle BOC = 180°$[Sum of angles of triangle]

$\Rightarrow 2\ \angle OCB + 60° = 180°$

$\Rightarrow 2\ \angle OCB = 120°$

$\Rightarrow \angle OCB = 60°$

$\therefore \angle OBC = 60°$[From equation (ii)]

From equation (i) and (ii),

$\angle OBC = \angle OCB = \angle BOC = 60°$

$\therefore$ BOC is an equilateral triangle.

$\therefore$ OB = OC = BC

Hence, BC is the radius of the circumcircle.

Question: 14

In triangle PQR,

$\angle QPR + \angle PQR + \angle PRQ = 180°$[Sum of angles of triangle]

$\Rightarrow \angle QPR + 65° + 90° = 180°$

$\Rightarrow \angle QPR + 155° = 180°$

$\Rightarrow \angle QPR = 25°$______________ (i)

In triangle PMQ,

$\angle QPM + \angle PMQ + \angle PQM = 180°$[Sum of angles of triangle]

$\Rightarrow \angle QPM + 90° + 50° = 180°$

$\Rightarrow \angle QPM + 140° = 180°$

$\Rightarrow \angle QPM = 40°$

Now,

$\angle PRS = \angle QPR = 25°$[Alternate angles]

Exercise : 11C

Question: 1

(i) $\angle BCD = 80°$

$\angle BAC = \angle BDC = 40°$[Angles in the same segment]

In triangle BCD,

$\angle BCD + \angle DBC + \angle BDC = 180°$[Sum of angles of triangle]

$\Rightarrow \angle BCD + 60° + 40° = 180°$

$\Rightarrow \angle BCD + 100° = 180°$

$\Rightarrow \angle BCD = 80°$

(ii) $\angle BCD = 80°$

$\angle CAD = \angle CBD$ [Angles in the same segment]

$\Rightarrow \angle CAD = 40°$

Question: 2

In cyclic quadrilateral PQRS,

$\angle PSR + \angle PQR = 180°$[Opposite angles]

$\Rightarrow 150° + \angle PQR = 180°$

$\Rightarrow \angle PQR = 30°$

In triangle PQR,

$\angle RPQ + \angle PQR + \angle PRQ = 180°$[Sum of angles of triangle]

$\Rightarrow \angle RPQ + 30° + 90° = 180°$

$\Rightarrow \angle RPQ + 120° = 180°$

$\Rightarrow \angle RPQ = 60°$

Question: 3

(i) $\angle BCD = 80°$

$\angle BAD + \angle BCD = 180°$[Opposite angles of a cyclic quadrilateral arc supplementary]

$\Rightarrow 100° + \angle BCD = 180°$

$\Rightarrow \angle BCD = 80°$

(ii) $\angle ADC = 80°$

$\angle BAD + \angle ADC = 180°$[Interior angles of same side]

$\Rightarrow 100° + \angle ADC = 180°$

$\Rightarrow \angle ADC = 80°$

(iii) $\angle ABC = 100°$

$\angle BCD + \angle ABC = 180°$[Interior angles of same side]

$\Rightarrow 80° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 100°$

Question: 4

Reflex $\angle AOC = 360° - \angle AOC$

$= 360° - 130°$

$= 230°$

$\therefore \angle ABC = (1/2)\ \angle AOC$

$\Rightarrow \angle ABC = (1/2)\ 230°$

$\Rightarrow \angle ABC = 115°$

Now,

$\angle ABC + \angle PBC = 180°$[Because ABP is a straight line]

$\Rightarrow 115° + \angle PBC = 180°$

$\Rightarrow \angle PBC = 65°$

Question: 5

ABCD is cyclic quadrilateral.

$\therefore \angle ABC + \angle ADC = 180°$

$\Rightarrow 92° + \angle ADC = 180°$

$\Rightarrow \angle ADC = 88°$

AE || CD

$\therefore \angle EAD = \angle ADC = 88°$

Now,

$\angle BCD = 180° - \angle DAB \Rightarrow \angle BCD = \angle DAF = \angle EAD + \angle EAF$

$\Rightarrow \angle BCD = 88° + 20°$

$\Rightarrow \angle BCD = 108°$

Question: 6

BD = CD

$\therefore \angle CBD = \angle BCD = 30°$

In triangle BCD,

$\angle BDC + \angle BCD + \angle CBD = 180°$[Sum of angles of triangle]

$\Rightarrow \angle BDC + 30° + 30° = 180°$

$\Rightarrow \angle BDC + 60° = 180°$

$\Rightarrow \angle BDC = 120°$

Now,

$\angle BDC + \angle BAC = 180°$[ABCD is a cyclic quadrilateral]

$\Rightarrow 120° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 60°$

Question: 7

ADC = (1/2) $\angle AOC$

$\Rightarrow \angle ADC = (1/2)\ 100°$

$\Rightarrow \angle ADC = 50°$

Now,

$\angle ADC + \angle ABC = 180°$[ABCD is a cyclic quadrilateral]

$\Rightarrow 50° + \angle ABC = 180°$

$\Rightarrow \angle ABC = 130°$

Question: 8

(i) $\angle BDC = 60°$

ABC is equilateral triangle.

$\therefore \angle ABC = \angle ACB = \angle BAC = 60°$ ____________________ (i)

$\angle BDC = \angle BAC = 60°$[Angles in the same segment of a circle are equal]

(ii) $\angle BEC = 120°$

ABCD is a cyclic quadrilateral

$\therefore \angle BAC + \angle BEC = 180°$

$\Rightarrow 60° + \angle BEC = 180°$

$\Rightarrow \angle BEC = 120°$

Question: 9

ABCD is a cyclic quadrilateral

$\therefore \angle BCD + \angle BAD = 180°$[Opposite angle of a cyclic quadrilateral are supplementary]

$\Rightarrow 100° + \angle BAD = 180°$

$\Rightarrow \angle BAD = 80°$

In triangle ABD,

$\angle ADB + \angle ABD + \angle BAD = 180°$[Sum of angles of triangle]

$\Rightarrow \angle ADB + 50° + 80° = 180°$

$\Rightarrow \angle ADB + 130° = 180°$

$\Rightarrow \angle ADB = 50°$

Question: 10

Reflex $\angle BOD = (360° – \angle BOD)$

$\Rightarrow$ Reflex $\angle BOD = (360° – 150°)$

$\Rightarrow$ Reflex $\angle BOD = 210°$

Now,

$X = (1/2)$ (Reflex $\angle BOD$)

$\Rightarrow X = (1/2)\ 210°$

$\Rightarrow X = 105°$

$X + Y = 180°$

$\Rightarrow 105° + Y = 180°$

$\Rightarrow Y = 75°$

Question: 11

OA = OB [Radius]

$\therefore \angle OAB = \angle OBC = 50°$

In triangle AOB,

$\angle AOB + \angle OAB + \angle OBC = 180°$[Sum of angles of triangle]

$\Rightarrow \angle AOB + 50° + 50° = 180°$

$\Rightarrow \angle AOB + 100° = 180°$

$\Rightarrow \angle AOB = 80°$

$\therefore x = 180° – \angle AOB$ [AOD is a straight line]

$\Rightarrow x = 180° - 80°$

$\Rightarrow x = 100°$

∴ X + Y = 180°[Opposite angle of a cyclic quadrilateral are supplementary]

⇒ 100° + Y = 180°

⇒ Y = 80°

Question: 12

ABC + ∠CBF = 180°[Because ABF is a straight line]

⇒ ∠ABC + 130° = 180°

⇒ ∠ABC = 50°

∴ x = ∠ABC = 50°[Exterior angle = interior opposite angle]

Question: 13

(i) $\angle BAD = 60^\circ$

ABCD is a cyclic quadrilateral.

∴ ∠BAD + ∠BCD = 180°

⇒ ∠BAD + 120° = 180°

⇒ ∠BAD = 60°

(ii) $\angle ABD = 30^\circ$

∠BDA = 90°[Angle in a semi-circle]

In triangle ABD,

∠ABD + ∠BDA + ∠BAD = 180°[Sum of angles of triangle]

⇒ ∠ABD + 90° + 60° = 180°

⇒ ∠ABD + 150° = 180°

⇒ ∠ABD = 30°

(iii) $\angle CBD = 30^\circ$

OD = OA [Radius]

∴ ∠OAD = ∠ODA = ∠BAD = 180°

∴∠ODB = 90° - ∠ODA

⇒ ∠ODB = 90° - 60°

⇒ ∠ODB = 30°

(iv) $\angle ADC = 120^\circ$

∠ADC = ∠ADB + ∠CDB

⇒ ∠ADC = 90° + 30°

⇒ ∠ADC = 120°

In triangle AOD,

∠AOD + ∠OAD + ∠ODA = 180°[Sum of angles of triangle]

⇒ ∠AOD + 60° + 60° = 180°

⇒ ∠AOD + 120° = 180°

⇒ ∠AOD = 60°

∴ Triangle AOD is an equilateral triangle.

Question: 14

Two chords AB and CD of a circle intersect each other at P outside the circle.

$\therefore$ AP × BP = CP × PD

$\Rightarrow$ (AB + BP) × BP = (CD + PD) × PD

$\Rightarrow$ (6 + 2) × 2 = (CD + 2.5) × 2.5

$\Rightarrow$ 16 = 2.5 CD + 6.25

$\Rightarrow$ 2.5 CD = 9.75

$\Rightarrow$ CD = 3.9 cm

Question: 15

(i) $\angle EDB = 50°$

$\angle$BOD + $\angle$AOD = 180°[AOB is a straight line]

$\Rightarrow$ $\angle$BOD + 140° = 180°

$\Rightarrow$ $\angle$BOD = 40°

OB = OD

$\therefore$ $\angle$OBD = $\angle$ODB

In triangle AOD,

$\angle$BOD + $\angle$OBD + $\angle$ODB = 180°[Sum of angles of triangle]

$\Rightarrow$ 40°+ 2 $\angle$OBD = 180°

$\Rightarrow$ 2 $\angle$OBD = 140°

$\Rightarrow$ $\angle$OBD = 70°

$\therefore$ $\angle$OBD = $\angle$ODB = 70°

ABDC is a cyclic quadrilateral.

$\therefore$ $\angle$CAB + $\angle$BDC = 180°

$\Rightarrow$ $\angle$CAB + $\angle$ODB + $\angle$ODC = 180°

$\Rightarrow$ 50°+ 70°+ $\angle$ODC = 180°

$\Rightarrow$ $\angle$ODC = 60°

Now,

$\angle$EDB = 180° – $\angle$BDC [Because CDE is a straight line]

$\Rightarrow$ $\angle$EDB = 180° – ($\angle$ODB + $\angle$ODC)

$\Rightarrow$ $\angle$EDB = 180° – (70°+ 60°)

$\Rightarrow$ $\angle$EDB = 180° – 130°

$\Rightarrow$ $\angle$EDB = 50°

(ii) $\angle EBD = 110°$

$\angle$BOD + $\angle$AOD = 180°[AOB is a straight line]

$\Rightarrow$ $\angle$BOD + 140° = 180°

$\Rightarrow$ $\angle$BOD = 40°

OB = OD

$\therefore$ $\angle$OBD = $\angle$ODB

In triangle AOD,

$\angle$BOD + $\angle$OBD + $\angle$ODB = 180°[Sum of angles of triangle]

$\Rightarrow$ 40°+ 2 $\angle$OBD = 180°

$\Rightarrow 2\ \angle OBD = 140°$

$\Rightarrow \angle OBD = 70°$

$\therefore\ \angle OBD = \angle ODB = 70°$

Now,

$\angle EBD + \angle OBD = 180°$[Because OBE is a straight line]

$\Rightarrow \angle EBD + 70° = 180°$

$\Rightarrow \angle EBD = 110°$

Question: 16

In ΔEBC and ΔEDA,

$\angle EBC = \angle CDA$

$\Rightarrow \angle EBC = \angle CDA$ ________________ (i)

$\angle ECB = \angle BAD$

$\Rightarrow \angle ECB = \angle EAD$ ________________ (ii)

$\angle BEC = \angle DEA$ ________________ (iii)

From equation (i), (ii) and (iii),

$\Delta EBC \cong \Delta EDA$ Proved.

Question: 17

Given AB = AC

$\therefore\ \angle ACB = \angle ABC$

Ext. $\angle ADE = \angle ACB = \angle ABC$

$\therefore\ \angle ADE = \angle ABC$

$\therefore$ DE || BC Proved.

Question: 18

Given, ABC is an isosceles triangle in which AB = AC. D and E are midpoints of AB and AC respectively.

$\therefore$ DE || BC

$\Rightarrow \angle ADE = \angle ABC$ ______________ (i)

AB = AC

$\Rightarrow \angle ABC = \angle ACB$ ______________ (ii)

Now,

$\angle ADE + \angle EDB = 180°$[Because ADB is a straight line]

$\Rightarrow \angle ACB + \angle EDB = 180°$

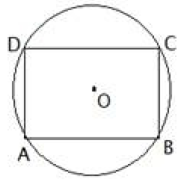
The opposite angles are supplementary.

$\therefore$ D, B, C, E are concyclic.

Question: 19

Let, ABCD be a cyclic quadrilateral and O be the center of

the circle passing through A, B, C, and D.



Then,

Each of AB, BC, CD and DA being a chord of the

circle, its right bisector must pass through O.

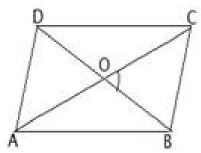
Therefore,

The right bisectors of AB, BC, CD and DA pass through and are concurrent.

Question: 20

Let diagonals BD and AC of the rhombus ABCD intersect at O.

We know that the diagonals of a rhombus bisect each other at right angles.



∴ ∠BOC = 90°

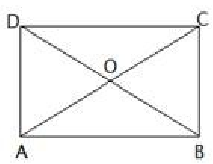
∴∠BOC lies in a circle.

The circle drawn with BC as diameter will pass through O.

Question: 21

Let O be the point of intersection of the diagonals BD and AC of rectangle ABCD.

Since, the diagonals of a rectangle are equal and bisect each other.



∴ OA = OB = OC = OD

Hence, O is the center of the circle through A, B, C, D.

Question: 22

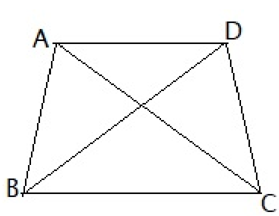
Let A, B, C, D be the given points.

With B as center and radius equal to AC draw an arc.

With C as center and AB as radius draw another arc.

Which cuts the previous arc at D,

Then, D is the required point BD and CD.



In ΔABC and ΔDCB,

AB = DC

AC = DB

BC = CB

∴ $\Delta EBC \cong \Delta EDA$

⇒ ∠BAC = ∠CDB

Thus, BC subtends equal angles, ∠BAC and ∠CDB on the same side of it.

Therefore, points A, B, C, D are concyclic.

Question: 23

Given, ∠B - ∠D = 60°____________________ (i)

ABCD is a cyclic quadrilateral,

∴ ∠B + ∠D = 180°____________________ (ii)

From equation (i) and (ii),

2 ∠B = 240°

⇒ ∠B = 120°________________ (iii)

From equation (ii),

∠B + ∠D = 180°

⇒ 120° + ∠D = 180°[From equation (iii)]

⇒ ∠D = 60°

Hence, the smaller of the two angle ∠D = 60°.

Question: 24

In ΔADE and ΔBCF,

AD = BC

∠AED = ∠BFC

∠ADE = ∠BCF [∠ADC - 90° = ∠BCD - 90°]

∴ $\Delta ADE \cong \Delta BCF$

The Cross ponding parts of the congruent triangles are equal.

∴ ∠A = ∠B

Now,

∠A + ∠B + ∠C + ∠D = 360°

⇒ 2 ∠B + 2 ∠D = 360°

⇒ ∠B + ∠D = 180°

∴ ABCD is a cyclic quadrilateral.

Question: 25

DCF = ∠DAB

⇒ x = 75° [Exterior angle is equal to the interior opposite angle.]

Now,

∠DCF + ∠DEF = 180° [Opposite angles of a cyclic quadrilateral]

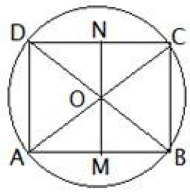
⇒ x + y = 180°

⇒ 75° + y = 180°

⇒ y = 105°

Question: 26

Given: Let ABCD be a cyclic quadrilateral, diagonals AC and BD intersect at O at right angles.



∠OCN = ∠OBM [Angles in the same segment] ___________ (i)

∠OBM + ∠BOM = 90° [Because ∠OLB = 90°] ______________ (ii)

∠BOM + ∠CON = 90°[LOM is a straight line and ∠BOC = 90°] ______________ (iii)

From equation (ii) and (iii),

∠OBM + ∠BOM = ∠BOM + ∠CON

⇒ ∠OBM = ∠CON

Thus, ∠OCN = ∠OBM and ∠OBM = ∠CON

⇒ ∠OCN = ∠CON

∴ON = CN ____________________ (iv)

Similarly, ON = ND ___________________ (v)

From equation (iv) and (v),

CN = ND Proved.

Question: 27

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

Chord AB of a circle is produced to E.

∴ ext. ∠BDE = ∠BAC = ∠EAC ___________________ (i)

Chord CD of a circle is produced to E.

∴ ext. ∠DBE = ∠ACD = ∠ACE ____________________ (ii)

In ΔEDB and ΔEAC,

∠BDE = ∠CAE [From equation (i)]

∠DBE = ∠ACE [From equation (ii)]

∠E = ∠E [Common angle]

∴ ΔEDB ~ ΔEAC Proved.

Question: 28

Given: AB and CD are two parallel chords of a circle.

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

∴ ext. ∠DCE = ∠B and ext. ∠EDC = ∠A

A || B

∴ ∠EDC = ∠B and ∠DCE = ∠A

∴∠A = ∠B

Hence, ΔAEB is isosceles.

Question: 29

(i) $\angle DBC = 115°$

∠BDA = 90°= ∠EDB [Semi circle angle]

In triangle EBD,

∠DBE + ∠EDB + ∠BED = 180°

⇒ ∠DBE + 90° + 25° = 180°

⇒ ∠DBE + 115° = 180°

⇒ ∠DBE = 65°

Now,

∠DBC + ∠DBE = 180°[CBE is a straight line]

⇒ ∠DBC + 65° = 180°

⇒ ∠DBC = 115°

(ii) $\angle DCB = 35°$

∠DCB = ∠BAD [Angle in the same segment]

∴∠DCB = 35°

(iii) ∠BDC = 30°

In triangle BCD,

∠BDC + ∠DCB + ∠DBC = 180°

⇒ ∠BDC + 35° + 115° = 180°

⇒ ∠BDC + 150° = 180°

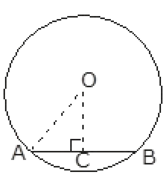
⇒ ∠BDC = 30°

Exercise : CCE QUESTIONS

Question: 1

The radius of a c

Solution:



Given radius(AO) = 13cm

Length of the chord (AB) = 10cm

Draw a perpendicular bisector from center to the chord and name it OC.

∴ AC = BC = 5cm

Now in Δ AOC,

Using Pythagoras theorem

$AO^2 = AC^2 + OC^2$

$13^2 = 5^2 + OC^2$

$OC^2 = 13^2 - 5^2$

$OC^2 = 169 - 25$

$OC^2 = 144$

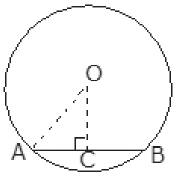
$OC = 12cm$

∴ The distance of the chord from the centre is 12cm.

Question: 2

A chord is at a d

Solution:



Given radius(AO) = 17cm

Length of the chord (AB) = x

distance of the chord from the centre is 8cm.

Draw a perpendicular bisector from center to the chord and name it OC.

∴ AC = BC

Now in Δ AOC

Using Pythagoras theorem

$AO^2 = AC^2 + OC^2$

$17^2 = AC^2 + 8^2$

$AC^2 = 17^2 – 8^2$

$AC^2 = 289 – 64$

$AC^2 = 225$

$AC = 15cm$

∴ BC = 15cm

∴ The length of the chord is AC + BC = 15 + 15 = 30 cm.

Question: 3

In the given figu

Solution:

Given: BOC is the diameter of the circle

AB = AC

Here, BAC forms a semicircle.

We know that angle in a semicircle is always 90°

∴ ∠BAC = 90°

Here ∠ABC = ∠ACB (since angles opposite equal sides are equal in a triangle)

We know that sum of all the angles in the triangle is 180°

That is

∠ABC + ∠ACB + ∠BAC = 180°

⇒ 2 × ∠ABC + ∠BAC = 180°

$\Rightarrow 2 \times \angle ABC + 90 = 180^\circ$

$\Rightarrow 2 \times \angle ABC = 180^\circ - 90^\circ$

$\Rightarrow 2 \times \angle ABC = 90^\circ$

$\Rightarrow \angle ABC = 45^\circ$

Question: 4

In the given figu

Solution:

Given: $\angle ACB = 30^0$.

We know that

$2 \times \angle ACB = \angle AOB$ ($\because$The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

$\therefore 2 \times 30^\circ = \angle AOB$

$\angle AOB = 60^\circ$.

$\therefore \angle AOB = 60^\circ$

Question: 5

In the given figu

Solution:

In Δ AOB OA = OB(radius)

$\angle OAB = \angle OBA$ (Angles opposite to equal sides are equal)

$\therefore \angle OBA = 40^\circ$

By angle sum property

$\angle OAB + \angle OBA + \angle AOB = 180^\circ$

$\angle AOB = 180^\circ - \angle OAB - \angle OBA$

$\angle AOB = 180^\circ - 40^\circ - 40^\circ = 100^\circ$

We know that

$2 \times \angle ACB = \angle AOB$ ($\because$The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

$\therefore 2 \times \angle ACB = 100^\circ$

$\angle ACB = \frac{100}{2}$

$\therefore \angle ACB = 50^\circ$

Question: 6

In the given figu

Solution:

Given: AB 34 cm and CD = 30 cm

Here OL is the perpendicular bisector to CD

$\therefore$ CL = LD = 15 cm

Construction: Join OD(radius)

OD = 17cm

Now in Δ ODL

By Pythagoras theorem

$OD^2 = OL^2 + LD^2$

$17^2 = OL^2 + 15^2$

$OL^2 = 17^2 + 15^2$

$OL^2 = 289—225$

$OL^2 = 64$

OL = 8

$\therefore$The distance of CD from AB is = OL = 8cm

Question: 7

AB and CD are two

Solution:

Given: $\angle AOB = 80^\circ$,

AB = CD

We know that angles subtended from equal chords at center are equal.

$\therefore \angle AOB = \angle COD$

$\therefore \angle COD = 80^\circ$

Question: 8

In the given figu

Solution:

Given: AB = 12cm, CE = 3cm

AB = AE + EB

AE = EB (OC is perpendicular bisector to AB)

$\therefore$ AE = 6 cm

Let CD = 2x (diameter)

AO = OC = x (radius)

In Δ AOE

$AO^2 = AE^2 + OE^2$

$x^2 = 6^2 + (OC – EC)^2$

$x^2 = 6^2 + (x – 3)^2$

$x^2 = 6^2 + x^2 + 3^2 – 2(x)(3)$

$x^2 = 36+ x^2 + 9 – 6x$

$6x = 36+ 9 + x^2 – x^2$

6x = 45

$x = \frac{45}{6} = 7.5$

$\therefore$ Radius = x = 7.5 cm

Question: 9

In the given figu

Solution:

Given: CE = ED = 8 cm and EB = 4 cm

Construction: Join OC (OC is radius)

Let AB = 2x (diameter)

OB = OC = x (radius)

In Δ COE

$CO^2 = CE^2 + OE^2$

$x^2 = 8^2 + (OB – EB)^2$

$x^2 = 8^2 + (x – 4)^2$

$x^2 = 8^2 + x^2 + 4^2 – 2(x)(4)$

$x^2 = 64 + x^2 + 16 – 8x$

$8x = 64 + 16 + x^2 – x^2$

8x = 80

$x = \frac{80}{8} = 10$

∴ Radius = x = 10 cm

Question: 10

In the given figu

Solution:

Given: AB||CD and AB = 10cm

Construction: Drop perpendiculars OE and OF on to AB and CD respectively.

Now,

Consider ΔBOE and ΔCOF

Here,

OB = OC (radius)

∠OEB = ∠OFC (right angle)

∠COF = ∠BOE (vertically opposite angles)

∴ By AAS congruency ΔBOE ≅ ΔCOF

∴ OE = OF (by congruent parts of congruent triangles)

Chords equidistant from center are equal in length

That is CD = AB = 10cm

∴ CD = 10cm

Question: 11

In the given figu

Solution:

Given: BC = OB and ∠ACD = 25°

Here in Δ OBC

∠BOC = ∠BCO (angles opposite to equal sides are equal)

$\therefore \angle BOC = 25°$

By angle sum property

$\angle BOC + \angle BCO + \angle OBC = 180°$

$25° + 25° + \angle OBC = 180°$

$50° + \angle OBC = 180°$

$\angle OBC = 180° – 50°$

$\therefore \angle OBC = 130°$

Here

$\angle ABC = \angle ABO + \angle OBC = 180°$

$\angle ABO + 130° = 180°$

$\angle ABO = 180° – 130°$

$\therefore \angle ABO = 50°$

Now, in ΔAOB

OB = OA (radius)

$\angle ABO = \angle BAO = 50°$ (angles opposite to equal sides are equal)

By angle sum property

$\angle ABO + \angle BAO + \angle AOB = 180°$

$50° + 50° + \angle AOB = 180°$

$\angle AOB = 180° – (50° + 50°) = 180° – 100° = 80°$

$\therefore \angle AOB = 80°$

Here

$\angle DOC = \angle AOD + \angle AOB + \angle BOC = 180°$

$\angle AOD + 80° + 25° = 180°$

$\angle AOD + 105° = 180°$

$\angle AOD = 180° – 105°$

$\angle AOD = 75°$

$\therefore \angle AOD = 75°$

Question: 12

In the given figu

Solution:

Given: $OD \perp AB$ and OD = 6cm

Here OB is radius

Let OB = x cm

In ΔBOD, By Pythagoras theorem

$OB^2 = BD^2 + OD^2$

$x^2 = BD^2 + 6^2$

$x^2 = BD^2 + 36$

$BD^2 = x^2 – 36$

Now consider Δ ABC

Here BC = 2x

By Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$(2x)^2 = 4(x^2 - 36) + AC^2$

$4x^2 = 4x^2 - 144 + AC^2$

$AC^2 = 144$

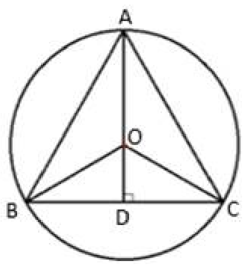
AC = 12 cm

∴ AC = 12 cm

Question: 13

An equilateral tr

Solution:



Given: Equilateral triangle of side 9 cm is inscribed in a circle.

Construction: Join OA, OB, OC and drop a perpendicular bisector from center O to BC.

Here,

Area (ΔABC) = 3× area (ΔOBC)

Area (ΔABC) = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 9^2 = \frac{81\sqrt{3}}{4}$

Now,

Area (ΔOBC) = $\frac{1}{2} \times AC \times OD = \frac{1}{2} \times 9 \times OD$

We know that,

Area (ΔABC) = 3× area (ΔOBC)

$\frac{81\sqrt{3}}{4} = \frac{1}{2} \times 9 \times OD$

$OD = \frac{3\sqrt{3}}{2}$

Now, in ΔODC

By Pythagoras theorem

$OC^2 = OD^2 + DC^2$

$OC^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{9}{2}\right)^2$

$OC^2 = \frac{27}{4} + \frac{81}{4} = \frac{108}{4} = 27$

$OC = 3\sqrt{3}$

∴ Radius = OC = $3\sqrt{3}$

Question: 14

The angle in a se

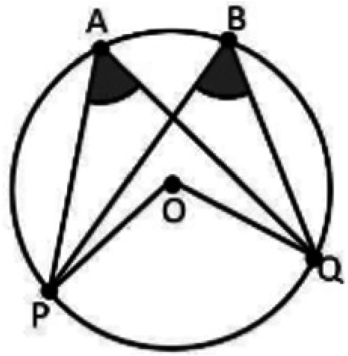
Solution:

Angle in a semicircle measures 90°

Question: 15

Angles in the sam

Solution:

Angles in the same segment of a circle are always equal. 

Proof: As we know angle subtended by an arc is double the angle subtended at any other point.So, ∠POQ = 2∠PAQ .... (1)∠POQ= 2∠PBQ .... (2)From (1) and (2), ∠PAQ = ∠PBQHence proved

Question: 16

In the given figu

Solution:

Given: Two triangles ΔABC and ΔBCD, ∠BAC = 60° and ∠DBC = 50°

We know that ∠BAC = ∠BDC = 60° (∵ angles in the same segment drawn from same chord are equal).

Now consider ΔBCD

By angle sum property

∠DBC + ∠BDC + ∠BCD = 180°

50° + 60° + ∠BCD = 180°

∠BCD = 180° – 50° – 60°

∠BCD = 70°

∴ ∠BCD = 70°

Question: 17

In the given figu

Solution:

Given: $\angle BCA = 30^\circ$,

Here,

∠BAC = 90° (angle in the semicircle)

Now, in ΔABC

By angle sum property

∠BCA + ∠BAC + ∠ABC = 180°

30° + 90° + ∠ABC = 180°

∠ABC = 180° – 30° – 90°

∠ABC = 60°

Here,

$\angle ABC = \angle ADC$ (angles in the same segment)

$\therefore \angle CDA = 60°$

Question: 18

In the given figu

Solution:

Given: $\angle OAC = 50°$

Consider ΔAOC

$\angle OAC = \angle OCA = 50°$ ($\because$ OA = OC = radius, angles opposite to equal sides are equal)

Now, by angle sum property

$\angle OAC + \angle OCA + \angle AOC = 180°$

$50° + 50° + \angle AOC = 180°$

$\angle AOC = 180° – 50° – 50°$

$\angle AOC = 80°$

Now angle $\angle BOD = \angle AOC = 80°$ (vertically opposite angles)

Now, consider ΔBOD

Here,

OB = OD (radius)

$\angle OBD = \angle ODB$ (angles opposite to equal angles are equal)

Let $\angle ODB = x$

By angle sum property

$\angle ODB + \angle OBD + \angle BOD = 180°$

$x + x + 80° = 180°$

$2x = 180° – 80°$

$2x = 100°$

$x = 50°$

$\therefore \angle ODB = 50°$

Question: 19

In the given figu

Solution:

Given: $\angle OBA = 20^{0}$ and $\angle OCA = 30^{0}$.

Consider ΔOAB

Here,

OA = OB (radius)

$\angle OBA = \angle OAB = 20°$ (angles opposite to equal sides are equal)

By angle sum property

$\angle AOB + \angle OBA + \angle OAB = 180°$

$\angle AOB + 20° + 20° = 180°$

$\angle AOB = 180° – 20° – 20°$

$\angle AOB = 140°$

Similarly, in ΔAOC

$OA = OC$ (radius)

$\angle OCA = \angle OAC = 30°$ (angles opposite to equal sides are equal)

By angle sum property

$\angle AOC + \angle OCA + \angle OAC = 180°$

$\angle AOC + 30° + 30° = 180°$

$\angle AOC = 180° – 30° – 30°$

$\angle AOC = 120°$

Here,

$\angle CAB = \angle OAB + \angle OAC = 50°$

Here,

$2\angle CAB = \angle BOC$ ($\because$The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

$\therefore 2\angle CAB = \angle BOC$

$\therefore 2 \times 50° = \angle BOC$

$\angle BOC = 100°$.

$\therefore \angle BOC = 100°$

Question: 20

In the given figu

Solution:

Given: $\angle AOB = 100°$ and $\angle AOC = 90°$,

Consider ΔOAB

Here,

$OA = OB$ (radius)

Let $\angle OBA = \angle OAB = x$ (angles opposite to equal sides are equal)

By angle sum property

$\angle AOB + \angle OBA + \angle OAB = 180°$

$100° + x + x = 180°$

$2x = 180° – 100°$

$2x = 80°$

$x = 40°$

Similarly, in ΔAOC

$OA = OC$ (radius)

Let $\angle OCA = \angle OAC = y$ (angles opposite to equal sides are equal)

By angle sum property

$\angle AOC + \angle OCA + \angle OAC = 180°$

$90° + y + y = 180°$

$2y = 180° – 90°$

$2y = 90°$

$y = 45°$

Here,

$\angle BAC = \angle OAB + \angle OAC = x + y = 40° + 45° = 85°$

$\therefore \angle BAC = 85°$

Question: 21

In the given figu

Solution:

Given: $\angle AOB = 100^0$ and $\angle AOC = 90^0$,

In ΔOAB

Here,

OA = OB (radius)

Let $\angle OBA = \angle OAB = x$ (angles opposite to equal sides are equal)

By angle sum property

$\angle AOB + \angle OBA + \angle OAB = 180°$

$50° + x + x = 180°$

$2x = 180° – 50°$

$2x = 130°$

$x = 60°$

$\therefore \angle OAB = 60°$

Question: 22

In the given figu

Solution:

Given: $\angle AOC = 120°$

Construction: Join OD

We know that,

$\angle AOC = 2 \times \angle ADC$

$120° = 2\ \angle ADC$

$\angle ADC = 60°$

Here,

$\angle ADB = 90°$ (angle in a semicircle)

$\angle ADB = \angle ADC + \angle CDB = 90°$

$\angle ADC + \angle CDB = 90°$

$60° + \angle CDB = 90°$

$\angle CDB = 90° – 60°$

$\angle CDB = 30°$

$\therefore \angle BDC = 30°$

Question: 23

In the given figu

Solution:

Given: $\angle OAB = 50°$

Construction: Join AC

Here,

In ΔAOB

OA = OB (radius)

$\angle OAB = \angle OBA$ (angles opposite to equal sides are equal)

$\therefore \angle OBA = 50°$

$\angle OBA = \angle CDA$ (angles in the same segment)

$\therefore \angle CDA = 50°$

Question: 24

In the give figur

Solution:

Given: $\angle CAB = 40°$ and $\angle BCD = 80°$

Here,

$\angle CAB = \angle CDB = 40°$ ($\because$ angles in the same segment drawn from same chord are equal).

Now, in ΔBCD

By angle sum property

$\angle BCD + \angle CDB + \angle CBD = 180°$

$80° + 40° + \angle CBD = 180°$

$\angle CBD = 180° – 40° – 80°$

$\angle CBD = 60°$

$\therefore \angle CBD = 60°$

Question: 25

In the given figu

Solution:

Given: $\angle AEB = 110°$ and $\angle CBE = 30°$

$\angle AEC = \angle AEB + \angle BEC = 180°$

$\angle AEB + \angle BEC = 180°$

$110° + \angle BEC = 180°$

$\angle BEC = 180° – 110°$

$\angle BEC = 70°$

In ΔBEC

By angle sum property

$\angle CBE + \angle BEC + \angle ECB = 180°$

$30° + 70° + \angle ECB = 180°$

$\angle ECB = 180° – 30° – 70°$

$\angle ECB = 80°$

Here,

$\angle ECB = \angle ADB$ (angles in the same segment)

$\therefore \angle ECB = \angle ADB = 80°$

$\therefore \angle ADB = 80°$

Question: 26

In the given figu

Solution:

Given: $\angle OAB = 20°$ and $\angle OCB = 50°$

Here,

In ΔAOB

OA = OB (radius)

$\angle OAB = \angle OBA$ (angles opposite to equal sides are equal)

$\therefore \angle OBA = 20°$

Now, by angle sum property

$\angle AOB + \angle OBA + \angle OAB = 180°$

$\angle AOB + 20° + 20° = 180°$

$\angle AOB = 180° – 20° – 20°$

$\angle AOB = 140°$

Now, Consider Δ BOC

OC = OB (radius)

$\angle OCB = \angle OBC$ (angles opposite to equal sides are equal)

$\therefore \angle OBA = 50°$

Now, by angle sum property

$\angle COB + \angle OBC + \angle OCB = 180°$

$\angle COB + 50° + 50° = 180°$

$\angle COB = 180° – 50° – 50°$

$\angle COB = 80°$

Here,

$\angle AOB = \angle AOC + \angle COB$

$140° = \angle AOC + 80°$

$\angle AOC = 140° – 80°$

$\angle AOC = 60°$

$\therefore \angle AOC = 60°$

Question: 27

In the given figu

Solution:

Given: ABCD is cyclic quadrilateral and $\angle ADC = 120°$

Here,

$\angle ADC + \angle ABC = 180°$ (opposite angles in cyclic quadrilateral are supplementary)

$120° + \angle ABC = 180°$

$\angle ABC = 180° – 120°$

$\angle ABC = 60°$

Here,

$\angle ACB = 90°$ (angle in semicircle)

Now, consider ΔABC

By angle sum property

$\angle BAC + \angle ABC + \angle ACB = 180°$

$\angle BAC + 60° + 90° = 180°$

$\angle BAC = 180° – 60° – 90°$

$\angle BAC = 30°$

Question: 28

In the given figu

Solution:

Given: ABCD is a cyclic quadrilateral, AB||DC and $\angle BAD = 100°$

Here,

$\angle BAD + \angle BCD = 180°$ (opposite angles in cyclic quadrilateral are supplementary)

$100° + \angle BCD = 180°$

$\angle BCD = 180° – 100°$

$\angle BCD = 80°$

Here, AB||DC and BC is the transversal

$\angle ABC + \angle BCD = 180°$ (interior angles along the transversal are supplementary)

$\angle ABC + 80° = 180°$

$\angle ABC = 180° – 80° = 100°$

$\therefore \angle ABC = 100°$

Question: 29

In the given figu

Solution:

Given: $\angle AOC = 130°$

Here,

(Exterior $\angle AOC$) = 360° – (interior $\angle AOC$)

(Exterior $\angle AOC$) = 360° – 130°

(Exterior $\angle AOC$) = 230°

We know that,

(Exterior $\angle AOC$) = $2 \times \angle ABC$

$230° = 2 \times \angle ABC$

$\angle ABC = \frac{230}{2} = 115°$

$\therefore \angle ABC = 115°$

Question: 30

In the given figu

Solution:

Given: CD||AB and $\angle BAD = 30°$

Consider ΔABD

$\angle ADB = 90°$ (angle in semicircle)

Now, by angle sum property

$\angle ABD + \angle BAD + \angle ADB = 180°$

$\angle ABD + 30° + 90° = 180°$

$\angle ABD = 180° – 30° – 90°$

$\angle ABD = 60°$

Here,

$\angle ABD + \angle ACD = 180°$ (opposite angles in cyclic quadrilateral are supplementary)

$60° + \angle ACD = 180°$

$\angle BCD = 180° – 60°$

$\angle BCD = 120°$

Here, CD||AB and AC is the transversal

$\angle CAB + \angle ACD = 180°$ (interior angles along the transversal are supplementary)

$\angle CAB + 120° = 180°$

$\angle ABC = 180° – 120° = 60°$

$\angle ABC = 60°$

$\angle ABC = \angle CAD + \angle DAB$

$60° = \angle CAD + 30°$

$\angle CAD = 60° – 30° = 30°$

$\therefore \angle CAD = 30°$

Question: 31

In the given figu

Solution:

Given: $\angle AOC = 100°$

Here,

(Exterior $\angle AOC$) = 360° – (interior $\angle AOC$)

(Exterior $\angle AOC$) = 360° – 100°

(Exterior $\angle AOC$) = 260°

We know that,

(Exterior $\angle AOC$) = $2 \times \angle ADC$

$260° = 2 \times \angle ABC$

$\angle ABC = \frac{260}{2} = 130°$

$\therefore \angle ABC = 130°$

Here,

$\angle ABD = \angle ABC + \angle CBD$

$180° = 130° + \angle CBD$

∠CBD = 180° – 130° = 50°

∴ ∠CBD = 50°

Question: 32

In the given figu

Solution:

Given: ∠OAB = 50°

Consider ΔAOB

Here,

OA = OB (radius)

∠OAB = ∠OBA = 50° (In a triangle, angles opposite to equal sides are equal)

By angle sum property

∠AOB + ∠OAB + ∠OBA = 180°

∠AOB + 50° + 50° = 180°

∠AOB = 180° – 50° – 50°

∠AOB = 80°

Here,

∠AOD = ∠AOB + ∠BOD

180° = 80° + ∠BOD

∠BOD = 180° – 80° = 100°

∴ ∠BOD = 100°

Question: 33

In the given figu

Solution:

Given: CB = CD and ∠CBD = 35°

Consider ΔBCD

Here,

CB = CD (given)

∠CBD = ∠CDB = 35° (In a triangle, angles opposite to equal sides are equal)

By angle sum property

∠BCD + ∠CBD + ∠CDB = 180°

∠BCD + 35° + 35° = 180°

∠BCD = 180° – 35° – 35° = 110°

We know that,

In a cyclic quadrilateral opposite angles are supplementary

∴ ∠BCD + ∠BAD = 180°

110° + ∠BAD = 180°

∠BAD = 180° – 110° = 70°

∴ ∠BAD = 70°

Question: 34

In the given figu

Solution:

Given: ΔABS is equilateral

In ΔABC

$\angle BAC = 60°$ (All angles in equilateral triangle are equal to 60°)

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$\therefore \angle BAC + \angle BDC = 180°$

$60° + \angle BDC = 180°$

$\angle BDC = 180° - 60° = 120°$

$\therefore \angle BDC = 120°$

Question: 35

In the given figu

Solution:

Given: $\angle CBE = 100°$

Here,

$\angle ABE = \angle ABC + \angle CBE$

$180° = \angle ABC + 100°$

$\angle ABC = 180° - 100° = 80°$

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$\therefore \angle ABC + \angle ADC = 180°$

$80° + \angle ADC = 180°$

$\angle ADC = 180° - 80° = 100°$

Here,

$\angle ADF = \angle ADC + \angle CDF$

$180° = 100° + \angle CDF$

$\angle CDF = 180° - 100° = 80°$

$\therefore \angle CDF = 80°$

Question: 36

In the given figu

Solution:

Given: $\angle AOB = 140°$

Here,

(Exterior $\angle AOB$) = 360° – (interior $\angle AOB$)

(Exterior $\angle AOB$) = 360° – 140°

(Exterior $\angle AOB$) = 220°

We know that,

(Exterior $\angle AOB$) = $2 \times \angle ACB$

$220° = 2 \times \angle ACB$

$\angle ACB = \frac{220}{2} = 110°$

$\therefore \angle ACB = 110°$

Question: 37

In the given figu

Solution:

Given: $\angle AOB = 130°$

Here,

(Exterior $\angle AOB$) = 360° – (interior $\angle AOB$)

(Exterior $\angle AOB$) = 360° – 130°

(Exterior $\angle AOB$) = 230°

We know that,

(Exterior $\angle AOB$) = $2 \times \angle ACB$

$230° = 2 \times \angle ACB$

$\angle ACB = \frac{230}{2} = 115°$

$\therefore \angle ACB = 115°$

Question: 38

In the given figu

Solution:

Given: ABCD, ABEF are two cyclic quadrilaterals and $\angle BCD = 110°$

In Quadrilateral ABCD

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$\therefore \angle BCD + \angle BAD = 180°$

$110° + \angle BAD = 180°$

$\angle BAD = 180° - 110° = 70°$

Similarly in Quadrilateral ABEF

$\therefore \angle BAD + \angle BEF = 180°$

$70° + \angle BEF = 180°$

$\angle BEF = 180° - 70° = 110°$

$\therefore \angle BEF = 110°$

Question: 39

In the given figu

Solution:

Given: $ABCD$ is a cyclic quadrilateral, CF||AB, $\angle ADC = 95^0$ and $\angle ECF = 20^0$.

Here, CF|| AB

Hence BC is transversal

$\therefore \angle ABC = \angle BCF = 85°$ (Alternate interior angles)

Here,

$\angle DCB + \angle BCF + \angle ECF = \angle DCE$

$\angle DCB + 85° + 20° = 180°$

$\angle DCB = 180° - 85° - 20° = 75°$

We know that,

In a cyclic quadrilateral opposite angles are supplementary

$\therefore \angle DCB + \angle BAD = 180°$

$75° + \angle BAD = 180°$

$\angle BAD = 180° - 75° = 105°$

$\therefore \angle BAD = 105°$

Question: 40

Two chords AB

Solution:

Given: AB = 11cm, BE = 3cm and DE = 3.5cm

Construction: Join AC

Here,

AE: CE = DE: BE

$AE \times BE = DE \times CE$

$(AB + BE) \times BE = DE \times (CD + DE)$

$(11 + 3) \times 3 = 3.5 \times (CD + 3.5)$

$14 \times 3 = 3.5 \times (CD + 3.5)$

$3.5 \times (CD + 3.5) = 42$

$(CD + 3.5) = \frac{42}{3.5} = 12$

CD = 12—3.5 = 8.5

$\therefore$ CD = 8.5

Question: 41

In the given figu

Solution:

Given: $AB = 4$ cm, two circles having radii 6 cm and 3 cm

Construction: join AP

Consider ΔABP

Here,

$AP^2 = AB^2 + BP^2$

$5^2 = 4^2 + 3^2$

25 = 16 + 9

25 = 25

$\therefore \Delta ABP$ is right angled triangle

$PQ = 2 \times BP$

PQ = 2×3 = 6cm

∴PQ = 6cm

Question: 42

In the given figu

Solution:

Given: $\angle AOB = 90^0$ and $\angle ABC = 30^0$.

Construction: join CD

We know that,

∠AOB = 2 × ∠ACB

90° = 2 × ∠ACB

∠ACB = $\frac{90}{2}$ = 45°

Similarly,

∠COA = 2 × ∠CBA

∠COA = 2 × 30

∠COA = 60°

Here,

∠COD + ∠COA = ∠AOD

∠COD + 60° = 180°

∠COD = 180° – 60° = 120°

Again

∠COD = 2 × ∠CAO

∠CAO = $\frac{120}{2}$ = 60°

∴∠CAO = 60°

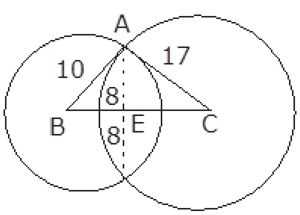
Question: 43

Three statements

Solution:

Here, Clearly I and III are correct.

Let us check for II statement



Construction: Let B and C be the centers of two circles having radii 10cm and 17 cm respectively and let AD be the common chord cutting BC at E.

Here,

AE = ED = 8cm

Now, in ΔABE

$BE^2 = AB^2 – AE^2$

$BE^2 = (10)^2 - (8)^2$

$BE^2 = 100 - 64 = 36$

BE = 6cm

Now, in ΔAEC

$EC^2 = AC^2 - AE^2$

$EC^2 = (17)^2 - (8)^2$

$EC^2 = 289 - 64 = 225$

EC = 25cm

Here,

BC = BE + EC = 6 + 15 = 21cm

But, it is given BC = 23cm

∴ Statement II is false

Question: 44

Two statements I

Solution:

Here,

ABCD is said to be cyclic quadrilateral

If either of any point is satisfied

i)Points A, B, C and D lie on a circle.

ii)$\angle B + \angle C = 180°$

Question: 45

Two statements I

Solution:

Here,

ΔABC right – angled at B

If both the conditions satisfy

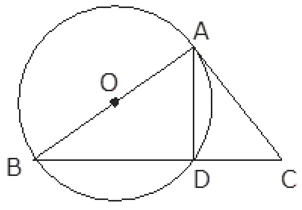
i)$ABCD$ is a cyclic quadrilateral

ii) $\angle D = 90°$.

Question: 46

The question cons

Solution:



Assertion (A):

Construction: Draw a Δ ABC in which AB = AC, Let O be the midpoint of AB and with O as centre and OA as radius draw a circle, meeting BC at D

Now, In Δ ABD

$\angle$ ADB = 90° (angle in semicircle)

Also, $\angle$ ADB + $\angle$ ADC = 180°

90° + $\angle$ ADC = 180°

$\angle$ ADC = 180° – 90°

$\angle$ ADC = 90°

Consider Δ ADB and Δ ADC

Here,

AB = AC (given)

AD = AD (common)

$\angle$ ADB = $\angle$ ADC (90°)

$\therefore$ By SAS congruency, Δ ADB$\cong$ Δ ADC

So, BD = DC(C.P.C.T)

Thus, the given circle bisects the base. So, Assertion (A) is true

Reason (R) :

Let $\angle$ BAC be an angle in a semicircle with centre O and diameter BOC

Now, the angle subtended by arc BOC at the centre is $\angle$ BOC = 2× 90°

$\angle$ BOC = 2× $\angle$ BAC = 2× 90°

So, $\angle$ BAC = 90° (right angle)

So, reason (R) is true

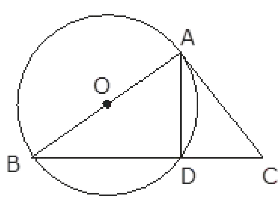
Clearly, reason (R) gives assertion (A)

Hence, correct choice is A

Question: 47

The question cons

Solution:



Assertion (A) :

Let O be the centre of the circle and AB be the chord

Construction: Draw, L is the midpoint of AB

Here,

OA = 10cm

AL = $\frac{1}{2}$ AB = 8cm

In ΔOAL,

$OL^2 = OA^2 – AL^2$

$OL^2 = (10)^2 – (8)^2$

$OL^2 = 100 – 64$

$OL = \sqrt{36} = 6cm$

Thus, Assertion (A) is true.

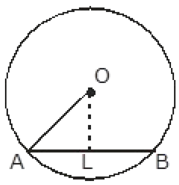
Clearly, reason (R) given Assertion (A).

Hence, the correct choice is A.

Question: 48

The question cons

Solution:



Clearly, reason (R) is true.

Assertion (A) :

OA = 13cm

OL = 12cm

In ΔOAL,

$AL^2 = OA^2 – OL^2$

$AL^2 = (13)^2 – (12)^2$

$AL^2 = 169 – 144$

$OL = \sqrt{25} = 5cm$

Now, AB = 2 × AL = 2 × 5 = 10cm

Thus, Assertion (A) is true

∴ Reason (R) and Assertion (A) are both true but reason (R) does not gives Assertion (A).

Hence, correct choice is B

Question: 49

The question cons

Solution:

Assertion (A) :

Here, in ΔABC

By angle sum property

∠ABC + ∠BCA + ∠CAB = 180°

70° + 30° + ∠CAB = 180°

∠CAB = 180° – 70° – 30° = 80°

∠CAB = ∠BDC = 80° (angles in same segment)

But given that ∠BDC = 70°

∴ Assertion(A) is wrong.

Reason (R) :

$\angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 130° = 65°$

$\angle ABC + \angle ADC = 180°$

$\angle ABC + 65° = 180°$

$\angle ABC = 180° - 65° = 115°$

Reason (R) is true

Assertion (A) :

$\angle ABC + \angle BCA + \angle BAC = 180°$

$70° + 30° + \angle BAC = 180°$

$\angle BAC = 180° - 70° - 30°$

$\angle BAC = 80°$

$\therefore \angle BDC = \angle BAC = 80°$ (angles in the same segment)

This is false.

Thus, Assertion (A) is false and Reason (R) is true.

Hence, correct choice is D

Question: 50

The question cons

Solution:

Clearly, Assertion (A) is false and Reason (R) is true.

Hence, correct choice is D

Question: 51

The question cons

Solution:

Clearly, Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

Question: 52

Write T for true

Solution:

(i) T

(ii) T

(iii) T (The region inside the circle, region outside the circle and region on the circle).

(iv) T (because point P lies inside the circle)

(v) F (A circle can have infinite number of chords)

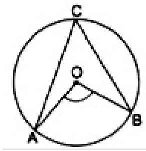
Question: 53

Match the followi

Solution:

(a) Angle in a semicircle measures – 90° (r)

(b) In the given figure, O is the centre of a circle. If $\angle AOB = 120°$, then $\angle ACB = ?$

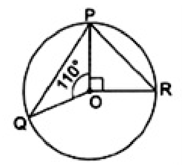


$\frac{1}{2}\angle AOB = \angle ACB$

$\angle ACB = \frac{1}{2} \times 120° = 60°$

$\angle ACB = 60°$ (s)

(c) In the given figure, O is the centre of a circle. If $\angle POR = 90^{0}$ and $\angle POQ = 110^{0}$, then $\angle QPR = ?$



Here, OP = OR = OQ (radius)

In ΔPOR

$\angle OPR = \angle ORP$ (angles opposite to equal sides are equal)

By angle sum property

$\angle POR + \angle OPR + \angle ORP = 180°$

$90° + 2\times\angle OPR = 180°$

$2\times\angle OPR = 180° - 90°$

$2\times\angle OPR = 90°$

$\angle OPR = 45°$

Similarly in Δ POQ

$\angle OPQ = \angle OQP$ (angles opposite to equal sides are equal)

By angle sum property

$\angle POQ + \angle OPQ + \angle OQP = 180°$

$110° + 2\times\angle OQP = 180°$

$2\times\angle OQP = 180° - 110°$

$2\times\angle OQP = 70°$

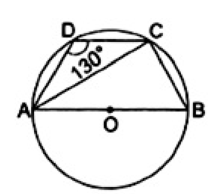
$\angle OQP = 35°$

$\angle QPR = \angle QPO + \angle OPR = 45° + 35° = 80°$

$\therefore \angle QPR = 80°$ (q)

(d) In cyclic quadrilateral $ABCD$, it is given that $\angle ADC = 130^{0}$ and AOB is a diameter of the circle

through A, B, C and D. Then, $\angle BAC = ?$



Here,

$\angle ADC + \angle ABC = 180°$ (opposite angles in cyclic quadrilateral are supplymentary)

$130° + \angle ABC = 180°$

$\angle ABC = 180° – 130° = 50°$

In ΔABC

By angle sum property

$\angle BAC + \angle ABC + \angle ACB = 180°$

$\angle BAC + 50° + 90° = 180°$

$\angle BAC = 180° – 50° – 90° = 40°$

$\therefore \angle BAC = 40°$ (p)

$\therefore$ Answers are: (a) – (r), (b) – (s), (c) – (q), (d) – (p)

Question: 54

Fill in the blank

Solution:

(i) Two circles having the same centre and different radii are called *concentric* cricles.

(ii) Diameter is the *longest* chord of a circle.

(iii) A continuous piece of a circle is called the *arc* of the circle.

(iv) An arc of a circle is called a *semicircle* if the ends of the arc are the ends of a diameter.

(v) A segment of a circle is the region between an arc and a *chord* of the circle.

(vi) A line segment joining the centre to any point on the circle is called its *radius.*

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

In the given figu

Solution:

Given: $\angle ECB = 40^{0}$ and $\angle CEB = 105^{0}$.

Here,

$\angle ACB = \angle ADB = 40°$ (angles in same segment)

$\angle BEC = \angle AED = 105°$ (vertically opposite angles)

In ΔAED

By angle sum property

$\angle ADE + \angle AED + \angle EAD = 180°$

$40° + 105° + \angle EAD = 180°$

$\angle EAD = 180° – 40° – 105° = 35°$

$\therefore \angle EAD = 35°$

Question: 2

In the given figu

Solution:

Given: $\angle AOB = 90^{0}$ and $\angle ABC = 30^{0}$.

We know that,

$\angle AOB = 2\times \angle ACB$

$\frac{1}{2}\angle AOB = \angle ACB$

$\frac{1}{2}\times 90° = \angle ACB$

$\angle ACB = 45°$

Now, consider ΔABC

By angle sum property

$\angle ACB + \angle ABC + \angle CAB = 180°$

$45° + 30° + \angle CAB = 180°$

$\angle CAB = 180° — 45° — 30° = 105°$

Consider ΔAOB

Here,

OA = OB (radius)

Let OA = OB = x

By angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180°$

$90° + x + x = 180°$

$2x = 180° – 90° = 90°$

$x = 45°$

Now,

$\angle CAB = \angle BAO + \angle CAO = 105°$

$\angle CAO = 105° – 45° = 60°$

$\therefore \angle CAO = 60°$

Question: 3

In the given figu

Solution:

Given: $\angle OAB = 40°$

Consider ΔAOB

Here,

OA = OB (radius)

$\angle OBA = \angle OAB = 40°$ (angles opposite to equal sides are equal)

By angle sum property

$\angle OBA + \angle OAB + \angle AOB = 180°$

$40° + 40° + \angle AOB = 180°$

$\angle AOB = 180° — 40° — 40° = 100°$

We know that,

$\angle AOB = 2\times \angle ACB$

$\frac{1}{2}\angle AOB = \angle ACB$

$\frac{1}{2}\times 100° = \angle ACB$

$\angle ACB = 50°$

$\therefore \angle ACB = 50°$

Question: 4

In the given figu

Solution:

Given: $\angle DAB = 60°$ and $\angle ABD = 50°$

In ΔABD

By angle sum property

$\angle DAB + \angle ABD + \angle ADB = 180°$

$60° + 50° + \angle ADB = 180°$

$110° + \angle ADB = 180°$

$\angle ADB = 180° – 110° = 70°$

Here,

$\angle ADB = \angle ACB = 70°$ (angles in same segment)

$\therefore \angle ACB = 70°$

Question: 5

In the given figu

Solution:

Given: $\angle BAO = 60^0$.

Consider ΔAOB

Here,

OA = OB (radius)

$\angle OBA = \angle OAB = 60°$ (angles opposite to equal sides are equal)

By angle sum property

$\angle OBA + \angle OAB + \angle AOB = 180°$

$60° + 60° + \angle AOB = 180°$

$\angle AOB = 180° — 60° — 60° = 60°$

Here,

$\angle BOC = \angle BOA + \angle AOC = 180°$

$60° + \angle AOC = 180°$

$\angle AOC = 180° – 60° = 120°$

We know that,

$\angle AOC = 2\times \angle ADC$

$\frac{1}{2}\angle AOC = \angle ADC$

$\frac{1}{2}\times 120° = \angle ADC$

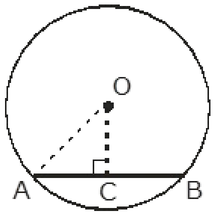
$\angle ADC = 60°$

$\therefore \angle ADC = 60°$

Question: 6

Find the length o

Solution:



Given radius(AO) = 15cm

Length of the chord (AB) = x

distance of the chord from the centre is 9cm.

Draw a perpendicular bisector from center to the chord and name it OC.

∴ AC = BC

Now in Δ AOC

Using Pythagoras theorem

$AO^2 = AC^2 + OC^2$

$15^2 = AC^2 + 9^2$

$AC^2 = 15^2 - 9^2$

$AC^2 = 225 - 81$

$AC^2 = 144$

AC = 12cm

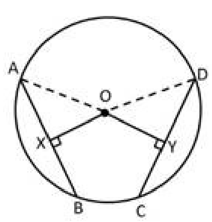
∴ BC = 12cm

∴ The length of the chord is AC + BC = 12 + 12 = 24 cm.

Question: 7

Prove that equal

Solution:



Given: AB = CD

Construction: Drop perpendiculars OX and OY on to AB and CD respectively and join OA and OD.

Here, OX ⊥AB (perpendicular from center to chord divides it into two equal halves)

$AX = BX = \frac{AB}{2}$ – – (1)

OY ⊥CD (perpendicular from center to chords divides it into equal halves

$CY = DY = \frac{CD}{2}$ – – (2)

Now, given that

AB = CD

$\therefore \frac{AB}{2} = \frac{CD}{2}$

AX = DY (from –1 and –2) – – (3)

In ΔAOX and ΔDOY

∠OXA = ∠OYD (right angle)

OA = OD (radius)

AX = DY (from –3)

∴ BY RHS congruency

ΔAOX≅ΔDOY

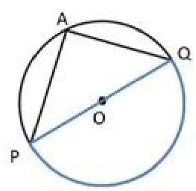
OX = OY (by C.P.C.T)

Hence proved.

Question: 8

Prove that an ang

Solution:



We know that,

∠POQ = 2∠PAQ

$\frac{\angle POQ}{2} = \angle PAQ$

$\frac{180^\circ}{2} = \angle PAQ$

90° = ∠PAQ

∠PAQ = 90°

Hence proved

Question: 9

Prove that a diam

Solution:

We know that,

A chord nearer to the center is longer than the chord which is far from the center

∴ Diameter is the longest chord in the circle (because it passes through the center and other chords are far from the center)

Question: 10

A circle with cen

Solution:

Given: $\angle OBA = 30^0$ and $\angle OCA = 40^0$.

Consider ΔOAB

Here,

OA = OB (radius)

$\angle OBA = \angle OAB = 30°$ (angles opposite to equal sides are equal)

Similarly, in ΔAOC

$OA = OC$ (radius)

$\angle OCA = \angle OAC = 40°$ (angles opposite to equal sides are equal)

Here,

$\angle CAB = \angle OAB + \angle OAC = 30° + 40° = 70°$

Here,

$2\times\angle CAB = \angle BOC$ ($\because$The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

$\therefore 2\times\angle CAB = \angle BOC$

$\therefore 2\times 70° = \angle BOC$

$\angle BOC = 140°$.

$\therefore \angle BOC = 140°$

Question: 11

In the given figu

Solution:

Given: $AXB = \frac{1}{2}$ arc BYC.

Here,

$2\times AXB = BYC$

$\therefore 2\times\angle AOB = \angle BOC$

$\angle AOB = \frac{1}{2}\angle BOC$ –1

Here,

$\angle AOC = \angle AOB + \angle BOC = 180°$

$\frac{1}{2}\angle BOC + \angle BOC = 180°$ (from –1)

$\frac{3}{2}\angle BOC = 180°$

$\angle BOC = \frac{2}{3}\times 180° = 120°$

$\therefore \angle BOC = 120^0$

Question: 12

In the given figu

Solution:

Given: $\angle ABC = 45°$

We know that ,

$\angle AOC = 2\times\angle ABC$

$\angle AOC = 2\times 45 = 90°$

$\therefore \angle AOC = 90°$

Therefore $OA \perp OC$.

Hence proved.

Question: 13

In the given figu

Solution:

Given: $\angle ADC = 130°$, $BC = BE$

We know that,

(exterior $\angle AFC$) = $(2\times\angle ADC)$

(exterior $\angle AFC$) = (2×130)

(exterior $\angle AFC$) = 260

$\angle AFC = 360° – (\text{exterior } \angle AFC) = 360°–260° = 100°$

$\angle AFB = \angle AFC + \angle CFB = 180°$

$\angle AFC + \angle CFB = 180°$

$100° + \angle CFB = 180°$

$\angle CFB = 180° – 100° = 80°$

In quadrilateral ABCD

$\angle ADC + \angle ABC = 180°$ (opposite angles in cyclic quadrilateral are supplementary)

$130° + \angle ABC = 180°$

$\angle ABC = 180° – 130° = 50°$

In ΔBCF

By angle sum property

$\angle CBF + \angle CFB + \angle BCF = 180°$

$50° + 80° + \angle BCF = 180°$

$\angle BCF = 180° – 50° – 80° = 50°$

Now,

$\angle CFE = \angle CFB + \angle BFE = 180°$

$\angle CFB + \angle BFE = 180°$

$80° + \angle BFE = 180°$

$\angle BFE = 180° – 80° = 100°$

Here,

In ΔBCE

$BC = BE$ (given)

$\angle BCE = \angle BEC = 50°$ (angles opposite to equal sides are equal)

By angle sum property

$\angle BCE + \angle BEC + \angle CBE = 180°$

$50° + 50° + \angle CBE = 180°$

$\angle CBE = 180° – 50° – 50° = 100°$

$\therefore \angle CBE = 100^{\circ}$

Question: 14

In the given figu

Solution:

Given: $\angle ACB = 40°$

We know that ,

$\angle AOB = 2\times\angle ACB$

$\angle AOB = 2\times 40 = 80°$

$\therefore \angle AOB = 80°$

In ΔAOB

OA = OB (radius)

$\angle OAB = \angle OBA$ (angles opposite to equal sides are equal)

Let $\angle OAB = \angle OBA = x$

By angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180°$

$80 + x + x = 180°$

$80 + 2x = 180°$

$2x = 180° - 80° = 100°$

$x = \frac{100}{2} = 50°$

$\therefore \angle OAB = 50^0$

Question: 15

In the given figu

Solution:

Given: $\angle OAB = 30^0$ and $\angle OCB = 55^0$.

Here,

In ΔAOB

OA = OB (radius)

$\angle OAB = \angle OBA$ (angles opposite to equal sides are equal)

$\therefore \angle OBA = 30°$

Now, by angle sum property

$\angle AOB + \angle OBA + \angle OAB = 180°$

$\angle AOB + 30° + 30° = 180°$

$\angle AOB = 180° - 30° - 30°$

$\angle AOB = 120°$

Now, Consider ΔBOC

OC = OB (radius)

$\angle OCB = \angle OBC$ (angles opposite to equal sides are equal)

$\therefore \angle OBA = 55°$

Now, by angle sum property

$\angle BOC + \angle OBC + \angle OCB = 180°$

$\angle BOC + 55° + 55° = 180°$

$\angle BOC = 180° - 55° - 55° = 70°$

$\therefore\ \angle BOC = 70°$

Here,

$\angle AOB = \angle AOC + \angle BOC$

$120° = \angle AOC + 70°$

$\angle AOC = 120° - 70°$

$\angle AOC = 50°$

$\therefore\ \angle AOC = 50°$

$\therefore\ \angle BOC = 70^{\circ}, \angle AOC = 50^{\circ}$

Question: 16

In the given figu

Solution:

Given: $BD = OD$ and $CD \perp AB$.

In ΔOBD

$OB = OD = DB$

$\therefore\ \Delta$ OBD is equilateral

$\therefore\ \angle ODB = \angle DBO = \angle BOD = 60°$

Consider ΔDEB and $\angle BEC$

Here,

BE = BE (common)

$\angle CEB = \angle DEB$ (right angle)

CE = DE (OE is perpendicular bisector)

$\therefore$ By SAS congruency

$\angle CAB = 30^{\circ}$

$\Delta DEB \cong \angle BEC$

$\therefore \angle DEB = \angle EBC$ (C.P.C.T)

$\therefore \angle EBC = 60°$

Now, in ΔABC

$\angle EBC = 60°$

$\angle ACB = 90°$ (angle in semicircle)

By angle sum property

$\angle EBC + \angle ACB + \angle CAB = 180°$

$60° + 90° + \angle CAB = 180°$

$\angle CAB = 180° - 60° - 90° = 30°$

$\therefore \angle CAB = 30°$

Question: 17

In the given figu

Solution:

Here,

In cyclic Quadrilateral ABFE

∠ABF + ∠AEF = 180° (opposite angles in cyclic quadrilateral are supplementary) –1

In cyclic Quadrilateral ABCD

∠ABC + ∠ADC = 180° (opposite angles in cyclic quadrilateral are supplementary) –2

From –1 and –2

∠ABF + ∠AEF = ∠ABC + ∠ADC

∠AEF = ∠ADC (∠ABF = ∠ABC)

Since these are corresponding angles

We can say that EF || DC

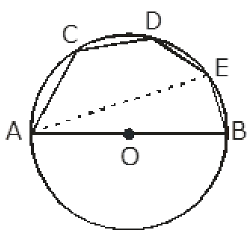
∴ EF||DC

Hence proved.

Question: 18

In the given figu

Solution:



Construction: Join AE

Consider cyclic quadrilateral ACDEA

Here,

∠ACD + ∠DEA = 180° (opposite angles in cyclic quadrilateral are supplementary)

Also,

∠AEB = 90° (angle in semicircle)

∴∠ACD + ∠DEA + ∠AEB = 180° + 90°

∠ACD + ∠BED = 270° (∠DEA + ∠AEB = ∠BED)

∴∠ACD + ∠BED = 270°

Hence proved.

Question: 19

In the given figu

Solution:

Given: $\angle BCO = 30^\circ$.

In ΔEOC

By angle sum property

∠EOC + ∠OEC + ∠OCE = 180°

∠EOC + 90° + 30° = 180°

∠EOC = 180° – 90° – 30° = 60°

∠EOC = 60°

Here,

$\angle EOD = \angle EOC + \angle COD = 90°$

$\angle EOC + \angle COD = 90°$

$60° + \angle COD = 90°$

$\angle COD = 90° – 60° = 30°$

Now,

$\angle AOC = \angle AOD + \angle COD = 90° + 30° = 120°$

We know that ,

$\angle COD = 2\times\angle CBD$

$\frac{1}{2}\angle COD = \angle CBD$

$\angle CBD = \frac{1}{2}\times 120° = 60°$

Consider ΔABE

By angle sum property

$\angle AEB + \angle ABE + \angle BAE = 180°$

$90° + 60° + \angle BAE = 180°$

$\angle BAE = 180° – 90° – 60° = 30°$

$\therefore x = 30°$

We know that ,

$\angle AOC = 2\times\angle ABC$

$\frac{1}{2}\angle AOC = \angle ABC$

$\angle ABC = \frac{1}{2}\times 30° = 15°$

$\therefore y = 15°$

$\therefore\ x = 30,\ y = 15$

Question: 20

PQ and RQ are the

Solution:

Given: chords PQ and RQ are equidistant from center.

Here consider ΔPQS and ΔRQS

Here,

QS = QS (common)

$\angle QPS = \angle QRS$ (right angle)

PQ = QS (chords equidistant from center are equal in length)

$\therefore$ By RHS congruency $\Delta PQS \cong \Delta RQS$

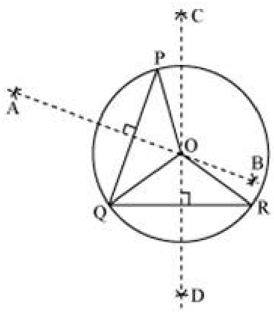
$\therefore \angle RQS = \angle SQP$ and $\angle RSQ = \angle QSP$ (by C.P.C.T)

Therefore we can say that diameter passing through Q bisects $\angle PQR$ and $\angle PSR$.

Question: 21

Prove that there

Solution:



Given: Three non collinear points P, Q and R

Construction: Join PQ and QR.

Draw perpendicular bisectors AB of PQ and CD of QR. Let the perpendicular bisectors intersect at the point O.

Now join OP, OQ and OR.

A circle is obtained passing through the points P, Q and R.

Proof:

We know that,

Every point on the perpendicular bisector of a line segment is equidistant from its ends

points.

Thus, OP = OQ (Since, O lies on the perpendicular bisector of PQ)

and OQ = OR. (Since, O lies on the perpendicular bisector of QR)

So, OP = OQ = OR.

Let OP = OQ = OR = r.

Now, draw a circle C(O, r) with O as centre and r as radius.

Then, circle C(O, r) passes through the points P, Q and R.

Next, we prove this circle is the only circle passing through the points P, Q and R.

If possible, suppose there is a another circle C(O′, t) which passes through the points P, Q, R.

Then, O′ will lie on the perpendicular bisectors AB and CD.

But O was the intersection point of the perpendicular bisectors AB and CD.

So, O ′ must coincide with the point O. **(Since, two lines cannot intersect at more than one point**)

As, O′P = t and OP = r; and O ′ coincides with O, we get t = r .

Therefore, C(O, r) and C(O, t) are congruent.

Thus, there is one and only one circle passing through three the given non – collinear points.

Question: 22

In the give figur

Solution:

Construction: Join OX and OY

In Δ OPX and ΔORY,

OX = OY (radii of the same circle)

OP = OR (sides of the square)

∴ΔOPX≅ Δ ORY (RHS rule)

∴ PX = RY (CPCT)—1

OPQR is a square

∴ PQ = RQ

∴ PX + QX = RY + QY

QX = QY (from –1)

Hence proved

Question: 23

In the given figu

Solution:

Given: AB = AC

Construction: join OA, OB and OC

Proof:

Consider ΔAOB and ΔAOC

Here,

OC = OB (radius)

OA = OA (common)

AB = AC (given)

∴ By SSS congruency

ΔAOB ≅ ΔAOC

∴ ∠OAC = ∠OAB (by C.P.C.T)

Hence, we can say that OA is the bisector of ∠BAC, that is O lies on the bisector of ∠BAC.