Chapter : 8. TRIGONOMETRIC IDENTITIES

Exercise : 8A

Question: 1 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 - \cos^2\theta) \times \text{cosec}^2\theta$

= $(\sin^2\theta) \times \text{cosec}^2\theta$ ($\because \sin^2\theta + \cos^2\theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 1 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 + \cot^2\theta) \times \sin^2\theta$

= $(\text{cosec}^2\theta) \times \sin^2\theta$ ($\because 1 + \cot^2\theta = \text{cosec}^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(\sec^2\theta - 1) \times \cot^2\theta$

= $(\tan^2\theta) \times \cot^2\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(\sec^2\theta - 1)(\text{cosec}^2\theta - 1)$

= $(\tan^2\theta) \times \cot^2\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$)

= 1

= R.H.S.

Hence, proved.

Question: 2 C

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 - \cos^2 \theta) \sec^2 \theta$

= $(\sin^2\theta) \times (1/\cos^2\theta)$ (∵ $\sin^2 \theta + \cos^2 \theta = 1$

= $\tan^2 \theta$

= R.H.S.

Hence, proved.

Question: 3 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$

= $(\sin^2\theta) + (1/\sec^2 \theta)$ (∵ $1 + \tan^2 \theta = \sec^2 \theta$)

= $(\sin^2\theta) + (\cos^2\theta)$ (∵ $\sin^2 \theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 3 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{1}{1 + \tan^2 \theta} + \frac{1}{1 + \cot^2 \theta}$

= $(1/\sec^2\theta) + (1/\text{cosec}^2 \theta)$ (∵ $1 + \tan^2 \theta = \sec^2 \theta$ and $1 + \tan^2 \theta = \sec^2 \theta$)

= $(\cos^2 \theta) + (\sin^2\theta)$ (∵ $\sin^2 \theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 4 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $(1 + \cos \theta)(1 - \cos \theta)(1 + \cot^2 \theta)$

= $(1 - \cos^2 \theta) \times \text{cosec}^2 \theta$ (∵ $1 + \cot^2 \theta = \text{cosec}^2 \theta$)

= $(\sin^2 \theta) \times \text{cosec}^2 \theta$ (∵ $\sin^2\theta + \cos^2 \theta = 1$)

= 1

= R.H.S.

Hence, proved.

Question: 4 B

Prove each of the

Solution:

To prove: (cosec θ)(1 + cos θ)(cosec θ – cot θ) = 1**Proof:**Consider the left – hand side:

(cosec θ) (1 + cos θ)(cosec θ – cot θ)⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) = (cosec θ + cosec θ cos θ)(cosec θ – cot θ)since cosec θ = 1/sin θ⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ)= $\left(cosec\ \theta+\frac{\cos\theta}{\sin\theta}\right)(\operatorname{cosec}\theta-\cot\theta)$ Also $\cot\theta=\frac{cos\theta}{sin\theta}$ So,⇒ (cosec θ) (1 + cos θ) (cosec θ – cot θ) =(cosec θ + cot θ)(cosec θ – cot θ)

Use the formula (a + b)(a - b) = $a^2 - b^2$

⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) =($\operatorname{cosec}^2\theta - \cot^2\theta$)

Since $\operatorname{cosec}^2\theta - \cot^2\theta = 1$⇒ (cosec θ) (1 + cos θ)(cosec θ – cot θ) =1

= R.H.S.

Hence, proved.

Question: 5 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\cot^2\theta - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$

$= \frac{\cos^2\theta - 1}{\sin^2\theta}$

= $(-\sin^2\theta) \times \sin^2\theta$ ($\because \sin^2\theta + \cos^2\theta = 1$)

= – 1

= R.H.S.

Hence, proved.

Question: 5 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\tan^2\theta - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$

$= \frac{\sin^2\theta - 1}{\cos^2\theta}$

= $(-\cos^2\theta) \times \cos^2\theta$ ($\because \sin^2\theta + \cos^2\theta = 1$)

= – 1

= R.H.S.

Hence, proved.

Question: 5 C

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\cos^2 \theta + \frac{1}{1 + \cot^2 \theta}$

= $\cos^2 \theta + \frac{1}{\mathrm{cosec}^2 \theta}$ ($\because 1 + \cot^2 \theta = \mathrm{cosec}^2 \theta$)

= $\cos^2 \theta + \sin^2 \theta$

= $(-\cos^2 \theta) \times \cos^2 \theta$ ($\because \sin^2\theta + \cos^2 \theta = 1$)

= – 1

= R.H.S.

Hence, proved.

Question: 6

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

= $\frac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$

= $\frac{2}{\cos^2\theta}$

= $2\sec^2 \theta$

= R.H.S.

Hence, proved.

Question: 7 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\sec \theta(1 - \sin \theta)(\sec \theta + \tan \theta)$

= $\left(\frac{1}{\cos\theta}\right) \times (1 - \sin \theta) \times \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$

= $\left(\frac{1}{\cos\theta}\right) \times (1 - \sin \theta) \times \left(\frac{1+\sin\theta}{\cos\theta}\right)$

= $\frac{1-\sin^2\theta}{\cos^2\theta}$

= $\frac{\cos^2\theta}{\cos^2\theta}$

= 1

= R.H.S.

Hence, proved.

Question: 7 B

Prove each of the

Solution:

To prove: $\sin \theta (1 + \tan \theta) + \cos \theta (1 + \cot \theta) = (\sec \theta + \operatorname{cosec} \theta)$**Proof:**Consider the left – hand side:

L.H.S. = $\sin \theta (1 + \tan \theta) + \cos \theta (1 + \cot \theta)$

$= \sin \theta \left(1 + \frac{\sin \theta}{\cos \theta}\right) + \cos \theta \left(1 + \frac{\cos \theta}{\sin \theta}\right)$

$= \sin \theta \left(\frac{\cos\theta + \sin\theta}{\cos\theta}\right) + (\cos \theta) \times \left(\frac{\sin\theta + \cos\theta}{\sin \theta}\right)$

$= (\cos \theta + \sin \theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$

$= (\cos \theta + \sin \theta)\left(\frac{(\cos^2\theta + \sin^2\theta)}{\cos\theta \sin\theta}\right)$We know $\cos^2\theta + \sin^2\theta = 1$

$= \left(\frac{\cos\theta + \sin\theta}{\cos\theta \sin\theta}\right)$

$= \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)$

$= \operatorname{cosec} \theta + \sec \theta$

= R.H.S.

Hence, proved.

Question: 8 A

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. $= 1 + \frac{\cot^2\theta}{1 + \operatorname{cosec}\theta}$

$= 1 + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1 + \frac{1}{\sin\theta}}$

$= 1 + \frac{\cos^2\theta}{1 + \sin\theta} \times \frac{\sin\theta}{\sin^2\theta}$

$= 1 + \frac{\cos^2\theta}{(1 + \sin\theta)\sin\theta}$

$= \frac{\sin\theta + \sin^2\theta + \cos^2\theta}{\sin\theta + \sin^2\theta}$

$= \frac{\sin\theta + 1}{\sin\theta(1 + \sin\theta)}$

$= 1/\sin \theta$

$= \operatorname{cosec} \theta$

= R.H.S.

Hence, proved.

Question: 8 B

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $1 + \frac{\tan^2\theta}{1+\sec\theta}$

$= 1 + \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{1}{\cos\theta}}$

$= 1 + \frac{\sin^2\theta}{1+\cos\theta} \times \frac{\cos\theta}{\cos^2\theta}$

$= 1 + \frac{\sin^2\theta}{(1+\cos\theta)\cos\theta}$

$= \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{\cos\theta + \cos^2\theta}$

$= \frac{\cos\theta+1}{\cos\theta(1+\cos\theta)}$

= 1/cos θ

= sec θ

= R.H.S.

Hence, proved.

Question: 9

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta}$

$= \frac{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\times\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin^2\theta}}$

$= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} \times \sin^2\theta$

$= 1 \times \frac{\sin\theta}{\cos\theta}$

= tan θ

= R.H.S.

Hence, proved.

Question: 10

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{\tan^2\theta}{1+\tan^2\theta} + \frac{\cot^2\theta}{1+\cot^2\theta}$

$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} + \frac{\frac{\cos^2\theta}{\sin^2\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$

$= \frac{\sin^2\theta}{1+\sin^2\theta} + \frac{\cos^2\theta}{1+\cos^2\theta}$

$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{(1+\sin^2\theta)(1+\cos^2\theta)}$

$= \frac{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}{\sin^2\theta+\sin^2\theta\cos^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta}$

= 1

= R.H.S.

Hence, proved.

Question: 11

Prove each of the

Solution:

Consider the left – hand side:

L.H.S. = $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$

Adding both the fractions, we get

$= \frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{\sin\theta(1+\cos\theta)}$

As $\sin^2\theta + \cos^2\theta = 1$, we have

$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)}$

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

= 2/sin θ

= 2cosec θ

= R.H.S.

Hence, proved.

Question: 12

Prove :

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\sin^2\theta}{(\sin\theta-\cos\theta)\cos\theta}+\frac{\cos^2\theta}{(\cos\theta-\sin\theta)\sin\theta}$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left(\frac{sin^2\theta}{cos\ \theta}-\frac{\cos^2\theta}{\sin\ \theta}\right)$$

$$= \frac{1}{(sin\theta-cos\theta)}\left(\frac{sin^3\theta-\cos^3\theta}{cos\theta\sin\theta}\right)$$

$= \frac{\sin^3\theta-\cos^3\theta}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$

$= \frac{\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)\sin\theta\cos\theta}$

$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$$= \frac{1}{\sin\theta\,.\cos\theta}+\frac{\sin\theta\,.\cos\theta}{\sin\theta\,.\cos\theta}$$

$$= \frac{1}{\sin\theta\,.\cos\theta}+1$$

~~= sec θ cosec θ + 1~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 13**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider the left – hand side:~~

~~L.H.S. =~~ $\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$

~~=~~ $\frac{\cos^2\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}$

~~=~~ $\frac{\cos^3\theta}{\cos\theta-\sin\theta}-\frac{\sin^3\theta}{\cos\theta-\sin\theta}$

~~=~~ $\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

~~=~~ $\frac{(\cos\theta-\sin\theta)\left(\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta\right)}{\cos\theta-\sin\theta}$

~~=~~ $\cos^2\theta+\sin^2\theta+\sin\theta\cos\theta$

~~=~~ $1+\cos\theta\sin\theta$

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 14**~~

~~Prove each of the~~

~~**Solution:**~~

$\frac{\cos\theta}{1-\tan\theta}-\frac{\sin^2\theta}{\cos\theta-\sin\theta}$ ~~=~~ $\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}-\frac{\sin^2\theta}{\cos\theta-\sin\theta}$

~~=~~ $\frac{\cos^2\theta}{\cos\theta-\sin\theta}-\frac{\sin^2\theta}{\cos\theta-\sin\theta}$

~~=~~ $\frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$

~~=~~ $\frac{(\cos\theta-\sin\theta)(\cos\theta+\sin\theta)}{(\cos\theta-\sin\theta)}$

~~=~~ $\cos\theta+\sin\theta$

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 15**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $(1+\tan^2\theta)(1+\cot^2\theta)$

~~=~~ $(\sec^2\theta)(\operatorname{cosec}^2\theta)$

~~=~~ $\frac{1}{\cos^2\theta}\times\frac{1}{\sin^2\theta}$

~~=~~ $\frac{1}{1-\sin^2\theta}\times\frac{1}{\sin^2\theta}$

~~=~~ $\frac{1}{\sin^2\theta - \sin^4\theta}$

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 16~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

~~=~~ $\frac{\frac{\sin\theta}{\cos\theta}}{\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)^2}$

~~=~~ $\frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(\frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}\right)^2}$

~~=~~ $\frac{\frac{\sin\theta}{\cos\theta}}{\left(\frac{1}{\cos^2\theta}\right)^2} + \frac{\frac{\cos\theta}{\sin\theta}}{\left(\frac{1}{\sin^2\theta}\right)^2}$

~~=~~ $\left(\frac{\sin\theta}{\cos\theta} \times \cos^4\theta\right) + \left(\frac{\cos\theta}{\sin\theta} \times \sin^4\theta\right)$

~~$= \sin\theta\ (\cos^3\theta) + \cos\theta\ (\sin^3\theta)$~~

~~$= \sin\theta\cos\theta\ (\cos^2\theta + \sin^2\theta)$~~

~~$= \sin\theta\cos\theta$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 17 A~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. = $\sin^6\theta + \cos^6\theta$~~

~~$= (\sin^2\theta)^3 + (\cos^2\theta)^3$~~

~~$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$~~

~~[Using $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]~~

~~$= (\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$~~

~~$(\because \sin^2\theta + \cos^2\theta = 1)$~~

~~$= [\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta) - \sin^2\theta\cos^2\theta]$~~

~~$(\because (a^2 + b^2) = (a + b)^2 - 2ab))$~~

~~$= [1 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta]$~~

~~$= 1 - 3\sin^2\theta\cos^2\theta$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 17 B~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. = $\sin^2 \theta + \cos^4 \theta$~~

~~$= (\sin^2 \theta) + (\cos^2 \theta)^2$~~

~~$= (\sin^2 \theta) + (1 - \sin^2 \theta)^2$~~

~~$= (\sin^2 \theta) + 1 + \sin^4 \theta - 2\sin^2 \theta$~~

~~$= 1 - \sin^2 \theta + \sin^4 \theta$~~

~~$= \cos^2 \theta + \sin^4 \theta$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 17 C~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. = $\text{cosec}^4 \theta - \text{cosec}^2 \theta$~~

~~$= (\text{cosec}^2 \theta)^2 - (\text{cosec}^2 \theta)$~~

~~$= (1 + \cot^2 \theta)^2 - (\text{cosec}^2 \theta)$~~

~~$= 1 + \cot^4 \theta + 2\cot^2 \theta - (\text{cosec}^2 \theta)$~~

~~$= 1 + \cot^4 \theta + \cot^2 \theta - (\text{cosec}^2 \theta - \cot^2 \theta)$~~

~~$= 1 + \cot^4 \theta + \cot^2 \theta - 1$~~

~~$= \cot^4 \theta + \cot^2 \theta$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 18 A~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. = $\frac{1-\tan^2 \theta}{1+\tan^2 \theta}$~~

~~$= \frac{1-\frac{\sin^2 \theta}{\cos^2 \theta}}{1+\frac{\sin^2 \theta}{\cos^2 \theta}}$~~

~~$= \frac{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}}$~~

~~$= \frac{\cos^2 \theta - \sin^2 \theta}{\sin^2 \theta + \cos^2 \theta}$~~

~~$= \cos^2 \theta - \sin^2 \theta$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 18 B~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{1-\tan^2\theta}{\cot^2\theta-1}$

$= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta}{\sin^2\theta}-1}$

$= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}\times\frac{\sin^2\theta}{\cos^2\theta}$

~~= sin^2 θ/ cos^2 θ~~

~~= tan^2 θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 19 A~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\tan\theta}{\sec\theta-1}+\frac{\tan\theta}{\sec\theta+1}$

$= \frac{\tan\theta(\sec\theta+1)+\tan\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$

$= \frac{\tan\theta\sec\theta+\tan\theta+\tan\theta\sec\theta-\tan\theta}{\sec^2\theta-1}$

$= \frac{2\tan\theta\sec\theta}{\tan^2\theta}$

$= \frac{2\sec\theta}{\tan\theta}$

~~= [2 (1/cos θ)]/[sin θ /cos θ]~~

~~= [2/sin θ]~~

~~= 2 cosec θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 19 B~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\cot\theta}{\operatorname{cosec}\theta+1}+\frac{\operatorname{cosec}\theta+1}{\cot\theta}$

$= \frac{\cot^2\theta+(\operatorname{cosec}\theta+1)^2}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{\cot^2\theta+\operatorname{cosec}^2\theta+1+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{\operatorname{cosec}^2\theta+\operatorname{cosec}^2\theta+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{2\operatorname{cosec}^2\theta+2\operatorname{cosec}\theta}{(\operatorname{cosec}\theta+1)(\cot\theta)}$

$= \frac{2\operatorname{cosec}\theta(\operatorname{cosec}\theta+1)}{(\operatorname{cosec}\theta+1)}$

~~= 2 cosec θ/cot θ~~

~~= 2 (1/sin θ)/(cos θ/sin θ)~~

~~= 2/ cos θ~~

~~= 2 sec θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 20 A**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\sec\theta-1}{\sec\theta+1}$

~~Multiply and divide by (sec θ + 1):~~

$=\frac{\sec\theta-1}{\sec\theta+1}\times\frac{\sec\theta+1}{\sec\theta+1}$

$=\frac{\sec^2\theta-1}{(\sec\theta+1)^2}$

$=\frac{\tan^2\theta}{(1+\sec\theta)^2}$

$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\left(\frac{1+\cos\theta}{\cos\theta}\right)^2}$

$=\frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{(1+\cos\theta)^2}{\cos^2\theta}}$

$=\frac{\sin^2\theta}{(1+\cos\theta)^2}$

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 20 B**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}$

~~Multiply and divide by (sec θ + tan θ):~~

$=\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}\times\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

$=\frac{\sec^2\theta-\tan^2\theta}{(\sec\theta+\tan\theta)^2}$

$=\frac{1}{\left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)^2}$

$=\frac{1}{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$=\frac{\cos^2\theta}{(1+\sin\theta)^2}$

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 21 A~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

~~Multiply and divide by (1 + sin θ):~~

~~=~~ $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}\times\frac{1+\sin\theta}{1+\sin\theta}}$

~~=~~ $\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

~~=~~ $\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

~~= (1 + sin θ)/cos θ~~

~~= (1/cos θ) + (sin θ/cos θ)~~

~~= sec θ + tan θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 21 B~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

~~Multiply and divide by (1 – cos θ):~~

~~=~~ $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}$

~~=~~ $\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

~~=~~ $\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

~~= (1 – cos θ)/sin θ~~

~~= (1/sin θ) – (cos θ/sin θ)~~

~~= cosec θ – cot θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 21 C~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

~~Multiply and divide by (1 + cos θ) in first part and (1 – cos θ) in the second part:~~

~~=~~ $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}\times\frac{1-\cos\theta}{1-\cos\theta}}$

~~=~~ $\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$

~~=~~ $\sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}+\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}}$

~~= [(1 + cos θ)/sin θ] + [(1 – cos θ)/sin θ]~~

~~= [(1/sin θ) + (cos θ/sin θ)] + [(1/sin θ) – (cos θ/sin θ)]~~

~~= [cosec θ + cot θ] + [cosec θ – cot θ]~~

~~= 2 cosec θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 22~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}+\frac{\cos^3\theta-\sin^3\theta}{\cos\theta-\sin\theta}$

~~Using identities $(a^3 + b^3) = (a + b)(a^2 + b^2 – ab)$ and $(a^3 – b^3) = (a – b)(a^2 + b^2 + ab)$~~

~~∴ L.H.S. =~~ $\frac{(\cos\theta+\sin\theta)(\cos^2\theta+\sin^2\theta-\cos\theta\sin\theta)}{(\cos\theta+\sin\theta)}+\frac{(\cos\theta-\sin\theta)(\cos^2\theta+\sin^2\theta+\cos\theta\sin\theta)}{(\cos\theta-\sin\theta)}$

~~= $(\cos^2\theta + \sin^2\theta – \cos\theta\sin\theta) + (\cos^2\theta + \sin^2\theta + \cos\theta\sin\theta)$~~

~~= (1 – cos θ sin θ) + (1 + cos θ sin θ)~~

~~= 2~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 23~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\sin\theta}{(\cot\theta+\operatorname{cosec}\theta)}-\frac{\sin\theta}{(\cot\theta+\operatorname{cosec}\theta)}$

~~=~~ $\frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}+\frac{1}{\sin\theta}\right)}-\frac{\sin\theta}{\left(\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)}$

$$\begin{aligned}
&=\frac{\sin^2\theta}{1+\cos\theta}-\frac{\sin^2\theta}{\cos\theta-1}\\
&=\frac{\sin^2\theta}{1+\cos\theta}+\frac{\sin^2\theta}{1-\cos\theta}\\
&=\sin^2\theta\left(\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}\right)\\
&=\sin^2\theta\left(\frac{1-\cos\theta+1+\cos\theta}{1-\cos^2\theta}\right)\\
&=\sin^2\theta\times\frac{2}{\sin^2\theta}\quad[As, \sin^2\theta+\cos^2\theta=1]\\
&=2
\end{aligned}$$

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 24 A~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$

~~=~~ $\frac{(\sin\theta-\cos\theta)^2+(\sin\theta+\cos\theta)^2}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$

~~=~~ $\frac{\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

~~=~~ $\frac{1-2\sin\theta\cos\theta+1+2\sin\theta\cos\theta}{\sin^2\theta-(1-\sin^2\theta)}$

~~=~~ $\frac{2}{2\sin^2\theta-1}$

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 24 B~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

~~=~~ $\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

~~=~~ $\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

~~=~~ $\frac{1+2\sin\theta\cos\theta+1-2\sin\theta\cos\theta}{1-\cos^2\theta-\mathrm{coss}^2\theta}$

~~=~~ $\frac{2}{1-2\cos^2\theta}$

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 25~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}$

~~=~~ $\frac{\left(1-\sin^2\theta\right)+\cos\theta}{\sin\theta(1+\cos\theta)}$

~~=~~ $\frac{\left(\cos^2\theta\right)+\cos\theta}{\sin\theta(1+\cos\theta)}$

~~=~~ $\frac{\cos\theta(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

~~= cos θ/sin θ~~

~~= cot θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 26 A~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}$

~~Multiply and divide by (cosec θ + cot θ):~~

~~=~~ $\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta-\cot\theta}\times\frac{\operatorname{cosec}\theta+\cot\theta}{\operatorname{cosec}\theta+\cot\theta}$

~~=~~ $\frac{(\operatorname{cosec}\theta+\cot\theta)^2}{\operatorname{cosec}^2\theta-\cot^2\theta}$

~~= $(\operatorname{cosec}\theta + \cot\theta)^2$~~

~~Thus, proved.~~

~~Also, consider $(\operatorname{cosec}\theta + \cot\theta)^2 = \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$~~

~~$= 1 + \cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$ ($\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$)~~

~~$= (1 + 2\cot^2\theta + 2\operatorname{cosec}\theta\cot\theta)$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 26 B~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}$

~~Multiply and divide by (sec θ + tan θ):~~

~~=~~ $\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}\times\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}$

~~=~~ $\frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}$

~~= $(\sec\theta + \tan\theta)^2$~~

~~Thus, proved.~~

~~Also, consider $(\sec\theta + \tan\theta)^2 = \sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$~~

~~$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$ ($\because 1 + \tan^2\theta = \sec^2\theta$)~~

~~$= (1 + 2\tan^2\theta + 2\sec\theta\tan\theta)$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 27 A~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}$

~~Multiply and divide by ((1 + cos θ) + sin θ):~~

~~=~~ $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}\times\frac{1+\cos\theta+\sin\theta}{1+\cos\theta+\sin\theta}$

~~=~~ $\frac{(1+\cos\theta+\sin\theta)^2}{(1+\cos\theta)^2-\sin^2\theta}$

$=\frac{1+\cos^2\theta+\sin^2\theta+2\cos\theta+2\sin\theta+2\cos\theta\sin\theta}{1+\cos^2\theta+2\cos\theta-(1-\cos^2\theta)}$

$=\frac{1+1+2\cos\theta+2\sin\theta(1+\cos\theta)}{2\cos^2\theta+2\cos\theta}$

$=\frac{2(1+\cos\theta)+2\sin\theta(1+\cos\theta)}{2\cos\theta(1+\cos\theta)}$

$=\frac{2(1+\cos\theta)(1+\sin\theta)}{2\cos\theta(1+\cos\theta)}$

$=\frac{1+\sin\theta}{\cos\theta}$

~~= R.H.S.~~

~~Thus, proved.~~

~~Question: 27 B~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}$

~~Multiply and divide by (cos θ + 1) + sin θ):~~

$=\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}\times\frac{\cos\theta+1+\sin\theta}{\cos\theta+1+\sin\theta}$

$=\frac{(1+\sin\theta)^2-\cos^2\theta}{(\sin\theta+\cos\theta)^2-1}$

$=\frac{1+\sin^2\theta+2\sin\theta-(1-\sin^2\theta)}{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}$

$=\frac{2\sin\theta(1+\sin\theta)}{2\sin\theta\cos\theta}$

$=\frac{1+\sin\theta}{\cos\theta}$

~~= R.H.S.~~

~~Thus, proved.~~

~~Question: 28~~

~~Prove each of the~~

~~Solution:~~

~~Consider L.H.S. =~~ $\frac{\sin\theta}{\sec\theta+\tan\theta-1}+\frac{\cos\theta}{\operatorname{cosec}\theta+\cot\theta-1}$

$=\frac{\sin\theta}{\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}-1}+\frac{\cos\theta}{\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}-1}$

$=\frac{\sin\theta\cos\theta}{1+\sin\theta-\cos\theta}+\frac{\cos\theta\sin\theta}{1+\cos\theta-\sin\theta}$

~~= sin θ cos θ ×~~ $\left(\frac{1}{1+\sin\theta-\cos\theta}+\frac{1}{1+\cos\theta-\sin\theta}\right)$

~~= sin θ cos θ ×~~ $\left(\frac{1+\sin\theta-\cos\theta+1+\cos\theta-\sin\theta}{(1+\sin\theta-\cos\theta)(1+\cos\theta-\sin\theta)}\right)$

~~= sin θ cos θ ×~~ $\frac{2}{1-(\sin\theta-\cos\theta)^2}$

~~= sin θ cos θ ×~~ $\frac{2}{1-(\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta)}$

~~= sin θ cos θ ×~~ $\frac{2}{1-1+2\sin\theta\cos\theta}$

~~= sin θ cos θ/ sin θ cos θ~~

~~= 1~~

~~= R.H.S.~~

~~**Question: 29**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}+\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$

~~=~~ $\frac{(\sin\theta+\cos\theta)^2+(\sin\theta-\cos\theta)^2}{(\sin\theta-\cos\theta)(\sin\theta+\cos\theta)}$

~~=~~ $\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta+\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta}{\sin^2\theta-\cos^2\theta}$

~~=~~ $\frac{2}{\sin^2\theta-\cos^2\theta}$

~~Thus, prove.~~

~~Also, consider~~ $\frac{2}{\sin^2\theta-\cos^2\theta}$ ~~=~~ $\frac{2}{\sin^2\theta-(1-\sin^2\theta)}$

~~=~~ $\frac{2}{(2\sin^2\theta-1)}$

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 30**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\cos\theta\operatorname{cosec}\theta-\sin\theta\sec\theta}{\cos\theta+\sin\theta}$

~~=~~ $\frac{\cos\theta\left(\frac{1}{\sin\theta}\right)-\sin\theta\left(\frac{1}{\cos\theta}\right)}{\cos\theta+\sin\theta}$

~~=~~ $\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta(\cos\theta+\sin\theta)}$

~~=~~ $\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{\cos\theta\sin\theta(\cos\theta+\sin\theta)}$

~~=~~ $\frac{\cos\theta-\sin\theta}{\cos\theta\sin\theta}$

~~= (1/sin θ) – (1/cos θ)~~

~~= cosec θ – sec θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 31**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. = (1 + tan θ + cot θ)(sin θ – cos θ)~~

~~= sin θ – cos θ + tan θ sin θ – tan θ cos θ + cot θ sin θ – cot θ cos θ~~

~~= sin θ – cos θ + tan θ sin θ – sin θ + cos θ – cot θ cos θ~~

~~= tan θ sin θ – cot θ cos θ~~

$$= \frac{\sin\theta}{\cos\theta} \times \sin\theta - \frac{\cos\theta}{\sin\theta} \times \cos\theta$$

$$= \frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}$$

~~= R.H.S.~~

$$\left[\text{since},\ \sin\theta = \frac{1}{\text{cosec}\theta}\ \ and\ \cos\theta = \frac{1}{\sec\theta}\right]$$

$$= \frac{\sec\theta}{\text{cosec}^2\theta} - \frac{\text{cosec}\theta}{\sec^2\theta}$$

~~Hence, proved.~~

~~**Question: 32**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\cot^2\theta(\sec\theta - 1)}{(1+\sin\theta)} + \frac{\sec^2\theta(\sin\theta - 1)}{(1+\sec\theta)}$

$$= \frac{\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\left(\frac{1}{\cos\theta} - 1\right)}{(1+\sin\theta)} + \frac{\left(\frac{1}{\cos^2\theta}\right)(\sin\theta - 1)}{\left(1+\frac{1}{\cos\theta}\right)}$$

$$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)\sin^2\theta} + \frac{(\sin\theta-1)\cos\theta}{\cos^2\theta(1+\cos\theta)}$$

$$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)(1-\cos^2\theta)} + \frac{(\sin\theta-1)\cos\theta}{(1-\sin^2\theta)(1+\cos\theta)}$$

$$= \frac{\cos\theta(1-\cos\theta)}{(1+\sin\theta)(1-\cos\theta)(1+\cos\theta)} + \frac{(\sin\theta-1)\cos\theta}{(1-\sin\theta)(1+\sin\theta)(1+\cos\theta)}$$

$$= \frac{\cos\theta}{(1+\sin\theta)(1+\cos\theta)} - \frac{\cos\theta}{(1+\sin\theta)(1+\cos\theta)}$$

~~= 0~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 33**~~

~~Prove each of the~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\left\{\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{\text{cosec}^2\theta-\sin^2\theta}\right\}$ ~~$(\sin^2\theta \cos^2\theta)$~~

$$= \left\{\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right\}(\sin^2\theta \cos^2\theta)$$

$$= \left\{\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right\}(\sin^2\theta \cos^2\theta)$$

$$= \left\{\frac{\cos^2\theta(1-\sin^4\theta)+\sin^2\theta\left(1-\cos^4\theta\right)}{(1-\cos^4\theta)(1-\sin^4\theta)}\right\}(\sin^2\theta \cos^2\theta)$$

$$= \frac{\cos^2\theta+\sin^2\theta-\cos^2\theta\sin^4\theta-\cos^4\theta\sin^2\theta}{(1-\cos^2\theta)(1+\cos^2\theta)(1-\sin^2\theta)(1+\sin^2\theta)}(\sin^2\theta \cos^2\theta)$$

$$= \frac{1-\cos^2\theta\sin^2\theta(\cos^2\theta+\sin^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}(\sin^2\theta \cos^2\theta)$$

$$= \frac{1-\sin^2\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

~~=~~ $\frac{1-\sin^2\theta\cos^2\theta}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$

~~=~~ $\frac{1-\sin^2\theta\cos^2\theta}{1+1+\sin^2\theta\cos^2\theta}$

~~=~~ $\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

~~= R.H.S.~~

~~Question: 34~~

~~Prove each of the~~

~~Solution:~~

~~Consider the left – hand side =~~ $\frac{\sin A-\sin B}{\cos A+\cos B}+\frac{\cos A-\cos B}{\sin A+\sin B}$

~~=~~ $\frac{(\sin A-\sin B)\,(\sin A+\sin B)+(\cos A-\cos B)(\cos A+\cos B)}{(\cos A+\cos B)(\sin A+\sin B)}$

~~=~~ $\frac{\sin^2 A-\sin^2 B+\cos^2 A-\cos^2 B}{(\cos A+\cos B)(\sin A+\sin B)}$

~~=~~ $\frac{\sin^2 A+\cos^2 A-(\sin^2 B+\cos^2 B)}{(\cos A+\cos B)(\sin A+\sin B)}$

~~=~~ $\frac{1-1}{(\cos A+\cos B)(\sin A+\sin B)}$

~~= 0~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 35~~

~~Prove each of the~~

~~Solution:~~

~~Consider the L.H.S. =~~ $\frac{\tan A+\tan B}{\cot A+\cot B}$

~~=~~ $\frac{\tan A+\tan B}{\frac{1}{\tan A}+\frac{1}{\tan B}}$

~~=~~ $\frac{\tan A+\tan B}{\frac{\tan A+\tan B}{\tan A\tan B}}$

~~=~~ $\frac{(\tan A+\tan B)(\tan A\tan B)}{(\tan A+\tan B)}$

~~= tan A tan B~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 36 A~~

~~Show that none of~~

~~Solution:~~

~~If the given equation is an identity, then it is true for every value of θ.~~

~~So, let θ = 60°~~

~~So, for θ = 60°, consider the L.H.S. =~~ ~~$\cos^2 60° + \cos 60°$~~

~~=~~ ~~$(1/2)^2 + (1/2)$~~

~~= (1/4) + (1/2)~~

~~= 3/4 ≠ 1~~

~~Therefore, L.H.S. ≠ R.H.S.~~

~~Thus, the given equation is not an identity.~~

~~Question: 36 B~~

~~Show that none of~~

~~Solution:~~

~~If the given equation is an identity, then it is true for every value of θ.~~

~~So, let θ = 30°~~

~~So, for θ = 30°, consider the L.H.S. = $\sin^2 30° + \sin 30°$~~

~~= $(1/2)^2 + (1/2)$~~

~~= (1/4) + (1/2)~~

~~= 3/4 ≠ 2~~

~~Therefore, L.H.S. ≠ R.H.S.~~

~~Thus, the given equation is not an identity.~~

~~Question: 36 C~~

~~Show that none of~~

~~Solution:~~

~~If the given equation is an identity, then it is true for every value of θ.~~

~~So, let θ = 30°~~

~~So, for θ = 30°, consider the L.H.S. = $\tan^2 30° + \sin 30°$~~

~~= $(1/\sqrt{3})^2 + (1/2)$~~

~~= (1/3) + (1/2)~~

~~= 5/6~~

~~Consider the R.H.S. = $\cos^2 30° = (\sqrt{3}/2)^2$~~

~~= 3/4~~

~~Therefore, L.H.S. ≠ R.H.S.~~

~~Thus, the given equation is not an identity.~~

~~Question: 37~~

~~Prove that: (sin~~

~~Solution:~~

~~Consider R.H.S. = $(2\cos^3 \theta - \cos \theta) \tan \theta$~~

~~= $\cos \theta(2\cos^2 \theta - 1)\left(\frac{\sin \theta}{\cos \theta}\right)$~~

~~= $(2\cos^2 \theta - 1)\sin \theta$~~

~~Consider L.H.S. = $(\sin \theta - 2 \sin^3 \theta)$~~

~~= $\sin \theta(1 - 2 \sin^2 \theta)$~~

~~= $\sin \theta[1 - 2(1 - \cos^2 \theta)]$~~

~~= $\sin \theta [1 - 2 + 2\cos^2 \theta]$~~

~~$= \sin \theta\ (2\cos^2 \theta - 1)$~~

~~Therefore, L.H.S. = R.H.S.~~

~~Hence, proved.~~

~~Exercise : 8B~~

~~**Question: 1**~~

~~If a cos θ + b si~~

~~**Solution:**~~

~~Given: $a \cos \theta + b \sin \theta = m$ …….(1)~~

~~$a \sin \theta - b \cos \theta = n$ …….(2)~~

~~Square equation (1) and (2) on both sides:~~

~~$a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta = m^2$ …….(3)~~

~~$a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \cos \theta \sin \theta = n^2$ ……..(4)~~

~~Add equation (3) and (4):~~

~~$[a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \cos \theta \sin \theta] + [a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \cos \theta \sin \theta] = m^2 + n^2$~~

~~$\Rightarrow a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta) = m^2 + n^2$~~

~~$\Rightarrow a^2 + b^2 = m^2 + n^2$~~

~~Hence, proved.~~

~~**Question: 2**~~

~~If x = a sec θ +~~

~~**Solution:**~~

~~Given: $a \sec \theta + b \tan \theta = x$ …….(1)~~

~~$a \tan \theta + b \sec \theta = y$ …….(2)~~

~~Square equation (1) and (2) on both sides:~~

~~$a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta = x^2$ …….(3)~~

~~$a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \sec \theta \tan \theta = y^2$ ……..(4)~~

~~Subtract equation (4) from (3):~~

~~$[a^2 \sec^2 \theta + b^2 \tan^2 \theta + 2ab \sec \theta \tan \theta] - [a^2 \tan^2 \theta + b^2 \sec^2 \theta + 2ab \sec \theta \tan \theta] = x^2 - y^2$~~

~~$\Rightarrow a^2 (\sec^2 \theta - \tan^2 \theta) + b^2 (\tan^2 \theta - \sec^2 \theta) = x^2 - y^2$~~

~~$\Rightarrow a^2 - b^2 = x^2 - y^2$ ($\because \sec^2 \theta = 1 + \tan^2 \theta$)~~

~~Hence, proved.~~

~~**Question: 3**~~

~~If Given: $\frac{x}{a} \sin \theta - \frac{y}{b} \cos \theta = 1$ …….(1)~~

~~$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ …….(2)~~

~~Square equation (1) and (2) on both sides:~~

~~$\frac{x^2}{a^2} \sin^2 \theta + \frac{y^2}{b^2} \cos^2 \theta - 2\frac{xy}{ab} \cos \theta \sin \theta = 1$ …….(3)~~

~~$\frac{x^2}{a^2} \cos^2 \theta + \frac{y^2}{b^2} \sin^2 \theta + 2\frac{xy}{ab} \cos \theta \sin \theta = 1$ ……..(4)~~

~~Add equation (3) and (4):~~

~~$\frac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 1+1$~~

~~$\Rightarrow \frac{x^2}{a^2}(1) + \frac{y^2}{b^2}(1) = 2$~~

~~$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$~~

~~Hence, proved.~~

~~Question: 4~~

~~If (sec θ + tan θ~~

~~Solution:~~

~~Given: $(\sec\theta + \tan\theta) = m$ ....(1)~~

~~$(\sec\theta - \tan\theta) = n$ ....(2)~~

~~Multiply equation (1) and (2):~~

~~$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = mn$~~

~~$(\sec^2\theta - \tan^2\theta) = mn$~~

~~$1 = mn\ (\because 1 + \tan^2\theta = \sec^2\theta)$~~

~~Therefore, $mn = 1$.~~

~~Hence, proved.~~

~~Question: 5~~

~~If (cosec θ + cot~~

~~Solution:~~

~~Given: $(\operatorname{cosec}\theta + \cot\theta) = m$ ....(1)~~

~~$(\operatorname{cosec}\theta - \cot\theta) = n$ ....(2)~~

~~Multiply equation (1) and (2):~~

~~$(\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta) = mn$~~

~~$(\operatorname{cosec}^2\theta - \cot^2\theta) = mn$~~

~~$1 = mn\ (\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta)$~~

~~Therefore, $mn = 1$.~~

~~Hence, proved.~~

~~Question: 6~~

~~If x = a cos~~

~~Solution:~~

~~Given: $x = a\cos^3\theta$~~

~~$y = b\sin^3\theta$~~

~~Consider L.H.S. = $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3}$~~

~~$= \left(\frac{a\cos^3\theta}{a}\right)^{2/3} + \left(\frac{b\sin^3\theta}{b}\right)^{2/3}$~~

~~$= (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3}$~~

~~$= (\cos^2\theta + (\sin^2\theta)$~~

~~= 1 = R.H.S.~~

~~Hence, proved.~~

~~Question: 7~~

~~If (tan θ + sin θ~~

~~Solution:~~

~~Given: tan θ + sin θ = m …….(1)~~

~~tan θ – sin θ = n …….(2)~~

~~Square equation (1) and (2) on both sides:~~

~~$\tan^2\theta + \sin^2\theta + 2\sin\theta\tan\theta = m^2$ …….(3)~~

~~$\tan^2\theta + \sin^2\theta - 2\sin\theta\tan\theta = n^2$ ……..(4)~~

~~Subtract equation (4) from (3):~~

~~$[\tan^2\theta + \sin^2\theta + 2\sin\theta\tan\theta] - [\tan^2\theta + \sin^2\theta - 2\sin\theta\tan\theta] = m^2 - n^2$~~

~~$\Rightarrow 4\sin\theta\tan\theta = m^2 - n^2$~~

~~Square both sides:~~

~~$\Rightarrow 16\sin^2\theta\tan^2\theta = (m^2 - n^2)^2$~~

~~Therefore, $(m^2 - n^2)^2 = 16\sin^2\theta\tan^2\theta$~~

~~Also, 16mn = 16 × (tan θ + sin θ) × (tan θ – sin θ)~~

~~$= 16(\tan^2\theta - \sin^2\theta)$~~

~~$= 16[(\sin^2\theta/\cos^2\theta) - \sin^2\theta]$~~

~~$= 16[\sin^2\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)]$~~

~~$= 16\sin^2\theta(\sin^2\theta/\cos^2\theta)$~~

~~$= 16\sin^2\theta\tan^2\theta$~~

~~Therefore, $(m^2 - n^2)^2 = 16mn$~~

~~Hence, proved.~~

~~Question: 8~~

~~If (cot θ + tan θ~~

~~Solution:~~

~~Given: (cot θ + tan θ) = m~~

~~(sec θ – cos θ) = n~~

~~Since, m = cot θ + tan θ~~

~~= (1/tan θ) + tan θ~~

~~$= \frac{1+\tan^2\theta}{\tan\theta}$~~

~~$= \sec^2\theta/\tan\theta$~~

~~= 1/(sin θ cos θ)~~

~~Also, n = sec θ – cos θ~~

~~= (1/cos θ) – cos θ~~

~~$= (1 - \cos^2\theta)/\cos\theta$~~

~~= $\sin^2\theta/\cos\theta$~~

~~Now, consider the left – hand side:~~

~~$(m^2n)^{2/3} – (mn^2)^{2/3}$ =~~ $\left[\left(\frac{1}{\sin\theta\cos\theta}\right)^2 \times \frac{\sin^2\theta}{\cos\theta}\right]^{2/3} - \left[\left(\frac{1}{\sin\theta\cos\theta}\right) \times \left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right]^{2/3}$

~~=~~ $\left[\frac{\sin^2\theta}{\sin^2\theta\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^4\theta}{\sin\theta\cos^3\theta}\right]^{2/3}$

~~=~~ $\left[\frac{1}{\cos^3\theta}\right]^{2/3} - \left[\frac{\sin^3\theta}{\cos^3\theta}\right]^{2/3}$

~~=~~ $\frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

~~= $(1 – \sin^2\theta)\cos^2\theta$~~

~~= $\cos^2\theta/ \cos^2\theta$~~

~~= 1~~

~~Question: 9~~

~~If (cosec θ – sin~~

~~Solution:~~

~~Given: (cosec θ – sin θ) = a^3~~

~~(sec θ – cos θ) = b^3~~

~~Since, a^3 = (cosec θ – sin θ)~~

~~= (1/sin θ) – sin θ~~

~~=~~ $\frac{1-\sin^2\theta}{\sin\theta}$

~~= $\cos^2\theta/\sin\theta$~~

~~Therefore, $a^2 = (a^3)^{2/3} = (\cos^2\theta/\sin\theta)^{2/3}$~~

~~Also, b^3 = sec θ – cos θ~~

~~= (1/cos θ) – cos θ~~

~~= $(1 – \cos^2\theta)/\cos\theta$~~

~~= $\sin^2\theta/\cos\theta$~~

~~Therefore, $b^2 = (b^3)^{2/3} = (\sin^2\theta/\cos\theta)^{2/3}$~~

~~Now, consider the left – hand side:~~

~~$a^2 b^2 (a^2 + b^2)$ =~~ $\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{2/3} \times \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$

~~=~~ $\left(\frac{\cos^2\theta\sin^2\theta}{\cos\theta\sin\theta}\right)^{2/3} \times \left(\left[\left(\frac{\cos^2\theta}{\sin\theta}\right)\right]^{\frac{2}{3}} + \left[\left(\frac{\sin^2\theta}{\cos\theta}\right)\right]^{\frac{2}{3}}\right)$

~~=~~ $[\cos^3\theta]^{2/3} + [\sin^3\theta]^{2/3}$

~~= $\sin^2\theta + \cos^2\theta$~~

~~= 1 = R.H.S.~~

~~Hence, proved.~~

~~Question: 10~~

~~If (2 sin θ~~

~~Solution:~~

~~Given: $2 \sin \theta + 3 \cos \theta = 2$~~

~~Consider $(2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 4\sin^2 \theta + 9 \cos^2 \theta + 12\sin \theta \cos \theta + 9 \sin^2 \theta + 4 \cos^2 \theta - 12 \sin \theta \cos \theta$~~

~~$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13\sin^2 \theta + 13 \cos^2 \theta$~~

~~$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13(\sin^2 \theta + \cos^2 \theta)$~~

~~$\Rightarrow (2 \sin \theta + 3 \cos \theta)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13$~~

~~$\Rightarrow (2)^2 + (3 \sin \theta - 2 \cos \theta)^2 = 13$~~

~~$\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 13 - 4$~~

~~$\Rightarrow (3 \sin \theta - 2 \cos \theta)^2 = 9$~~

~~$\Rightarrow (3 \sin \theta - 2 \cos \theta) = \pm 3$~~

~~Hence, proved.~~

~~Question: 11~~

~~If (sin θ +~~

~~Solution:~~

~~Given:~~ ~~(sin θ + cos θ) = √2 cos θ~~**~~To show:~~** ~~cot θ = (√2 + 1)~~**~~Solution:~~**~~(sin θ + cos θ) = √2 cos θ~~

~~Divide both sides by sin θ,~~ $\Rightarrow \frac{sin\theta + cos\theta}{sin\theta} = \frac{\sqrt{2}cos\theta}{sin\theta}$

$\Rightarrow \frac{sin\theta}{sin\theta} + \frac{\cos\theta}{sin\theta} = \frac{\sqrt{2}cos\theta}{sin\theta}$ ~~Since~~ $\cot\theta = \frac{cos\theta}{sin\theta}$ ~~⇒ 1 + cot θ = √2 cot θ~~

~~⇒ 1 = √2 cot θ - cot θ~~

~~⇒ (√2 – 1)cot θ = 1~~

~~$\Rightarrow \cot \theta = \frac{1}{\sqrt{2}-1}$~~

~~$\Rightarrow \cot \theta = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$~~

~~$\Rightarrow \cot \theta = \frac{\sqrt{2}+1}{2-1}$~~

~~⇒ cot θ = √2 + 1~~

~~Question: 12~~

~~If (cos θ + sin θ~~

~~Solution:~~

~~Given: cos θ + sin θ = √2sin θ~~

~~Consider $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2\sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta -$~~

~~$2 \sin \theta \cos \theta$~~

~~$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2\sin^2 \theta + 2 \cos^2 \theta$~~

~~$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2(\sin^2 \theta + \cos^2 \theta)$~~

~~$\Rightarrow (\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$~~

~~$\Rightarrow (\sqrt{2}\sin \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$~~

~~$\Rightarrow (\sin \theta - \cos \theta)^2 = 2 - 2\sin^2 \theta$~~

~~$\Rightarrow (\sin\theta - \cos\theta)^2 = 2(1 - \sin^2\theta)$~~

~~$\Rightarrow (\sin\theta - \cos\theta)^2 = 2(\cos^2\theta)$~~

~~$\Rightarrow (\sin\theta - \cos\theta) = \pm\sqrt{2}\cos\theta$~~

~~Hence, proved.~~

~~**Question: 13**~~

~~If sec θ + tan θ~~

~~**Solution:**~~

~~(i) Given: $\sec\theta + \tan\theta = p$ ……(1)~~

~~Then, $(\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = p$~~

~~$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = p$~~

~~$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = p$~~

~~$\Rightarrow \sec\theta - \tan\theta = (1/p)$ ……(2)~~

~~Adding equation (1) and (2), we get:~~

~~$2\sec\theta = p + (1/p)$~~

~~$\Rightarrow \sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$~~

~~Therefore, $\sec\theta = \frac{1}{2}\left(p + \frac{1}{p}\right)$~~

~~(ii) Given: $\sec\theta + \tan\theta = p$ ……(1)~~

~~Then, $(\sec\theta + \tan\theta) \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = p$~~

~~$\Rightarrow \frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta} = p$~~

~~$\Rightarrow \frac{1}{\sec\theta - \tan\theta} = p$~~

~~$\Rightarrow \sec\theta - \tan\theta = (1/p)$ ……(2)~~

~~Subtracting equation (2) from (1), we get:~~

~~$2\tan\theta = p - (1/p)$~~

~~$\Rightarrow \tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$~~

~~(iii) Since $\sin\theta = \tan\theta/\sec\theta$~~

~~$= \frac{\frac{1}{2}\left(p - \frac{1}{p}\right)}{\frac{1}{2}\left(p + \frac{1}{p}\right)}$~~

~~$= \frac{\left(p - \frac{1}{p}\right)}{\left(p + \frac{1}{p}\right)}$~~

~~$= \frac{p^2 - 1}{p^2 + 1}$~~

~~**Question: 14**~~

~~If tan A = n tan~~

~~**Solution:**~~

~~Given: tan A = n tan B~~

~~Therefore,~~ $\tan B = \frac{\tan A}{n}$ -

~~Thus,~~ $\cot B = \frac{n}{\tan A}$ ~~Squaring both sides, we get,~~

~~$\Rightarrow \cot^2 B = n^2/\tan^2 A$ ......(1)~~

~~Also, sin A = m sin B~~

~~Therefore, sin B = sin A/m~~

~~Thus, cosec B = m/sin A~~

~~$\Rightarrow \text{cosec}^2 B = m^2/\sin^2 A$ ......(2)~~

~~Now, subtract equation (2) from (1):~~

~~$\text{cosec}^2 B - \cot^2 B =$~~ $\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A}$

~~$\Rightarrow 1 =$~~ $\frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A}$

~~$\Rightarrow 1 =$~~ $\frac{m^2 - n^2 \cos^2 A}{\sin^2 A}$

~~$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$~~

~~$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$~~

~~$\Rightarrow m^2 - 1 = n^2 \cos^2 A - \cos^2 A$~~

~~$\Rightarrow (n^2 - 1)\cos^2 A = m^2 - 1$~~

~~$\Rightarrow \cos^2 A = (m^2 - 1)/(n^2 - 1)$~~

~~Hence, proved.~~

~~Question: 15~~

~~If m = (cos θ – s~~

~~Solution:~~

~~Given: m = (cos θ – sin θ)~~

~~n = (cos θ + sin θ)~~

~~Now,~~ $\frac{m}{n} = \frac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)}$

~~Multiply numerator and denominator by cos θ – sin θ :~~

~~Therefore,~~ $\frac{m}{n} = \frac{(\cos\theta - \sin\theta)}{(\cos\theta + \sin\theta)} \times \frac{\cos\theta - \sin\theta}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta - \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}$

~~Now,~~ $\frac{n}{m} = \frac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$

~~Multiply numerator and denominator by cos θ + sin θ :~~

~~Therefore,~~ $\frac{n}{m} = \frac{(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)} \times \frac{\cos\theta + \sin\theta}{\cos\theta + \sin\theta}$

$= \frac{(\cos\theta + \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}$

~~Now, consider~~ $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \sqrt{\frac{(\cos\theta - \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}} + \sqrt{\frac{(\cos\theta + \sin\theta)^2}{(\cos^2\theta - \sin^2\theta)}}$

$= \frac{\cos\theta - \sin\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}} + \frac{\cos\theta + \sin\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}}$

$$= \frac{1}{\sqrt{(\cos^2\theta - \sin^2\theta)}}(\cos\theta - \sin\theta + \cos\theta + \sin\theta)$$

$$= \frac{2\cos\theta}{\sqrt{(\cos^2\theta - \sin^2\theta)}}$$

~~Divide numerator and denominator by cos θ:~~

$$= \frac{2}{\sqrt{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}}$$

$$= \frac{2}{\sqrt{(1 - \tan^2\theta)}}$$

~~Therefore,~~ $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{2}{\sqrt{(1 - \tan^2\theta)}}$

~~Hence, proved.~~

~~Exercise : 8C~~

~~Question: 1~~

~~Write the value o~~

~~Solution:~~

~~Consider $(1 - \sin^2\theta) \sec^2 \theta = (\cos^2 \theta) \times \sec^2 \theta$~~

~~$(\because \sin^2 \theta + \cos^2 \theta = 1)$~~

~~$= 1$~~

~~Question: 2~~

~~Write the value o~~

~~Solution:~~

~~Consider $(1 - \cos^2\theta) \operatorname{cosec}^2 \theta = (\sin^2 \theta) \times \operatorname{cosec}^2 \theta$~~

~~$(\because \sin^2 \theta + \cos^2 \theta = 1)$~~

~~$= 1$~~

~~Question: 3~~

~~Write the value o~~

~~Solution:~~

~~Consider $(1 + \tan^2\theta) \cos^2 \theta = (\sec^2 \theta) \times \cos^2 \theta$~~

~~$(\because 1 + \tan^2 \theta = \sec^2 \theta)$~~

~~$= 1$~~

~~Question: 4~~

~~Write the value o~~

~~Solution:~~

~~Consider $(1 + \cot^2\theta) \times \sin^2 \theta = (\operatorname{cosec}^2 \theta) \times \sin^2 \theta$~~

~~$(\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta)$~~

~~$= 1$~~

~~Question: 5~~

~~Write the value o~~

~~Solution:~~

~~Consider $\sin^2 \theta + \frac{1}{1 + \tan^2 \theta}$~~

~~$= (\sin^2\theta) + (1/\sec^2 \theta)$~~

~~$(\because 1 + \tan^2 \theta = \sec^2 \theta)$~~

~~$= (\sin^2\theta) + (\cos^2\theta)$~~

~~$(\because \sin^2 \theta + \cos^2 \theta = 1)$~~

~~$= 1$~~

~~**Question: 6**~~

~~Write the value o~~

~~**Solution:**~~

~~Consider $\cot^2 \theta - \frac{1}{\sin^2 \theta}$~~

~~$= (\cot^2 \theta) - (\mathrm{cosec}^2 \theta)$~~

~~$= - (\mathrm{cosec}^2 \theta - \cot^2 \theta)$~~

~~$(\because 1 + \cot^2 \theta = \mathrm{cosec}^2 \theta)$~~

~~$= - 1$~~

~~**Question: 7**~~

~~Write the value o~~

~~**Solution:**~~

~~Consider $\sin \theta \cos (90° - \theta) + \cos \theta \sin (90° - \theta) = \sin \theta \sin \theta + \cos \theta \cos \theta$~~

~~$= \sin^2 \theta + \cos^2 \theta$~~

~~$(\because \sin^2 \theta + \cos^2 \theta = 1)$~~

~~$= 1$~~

~~**Question: 8**~~

~~Write the value o~~

~~**Solution:**~~

~~Consider $\mathrm{cosec}^2 (90° - \theta) - \tan^2 \theta = \sec^2 \theta - \tan^2 \theta$~~

~~$(\because 1 + \tan^2 \theta = \sec^2 \theta)$~~

~~$= 1$~~

~~**Question: 9**~~

~~Write the value o~~

~~**Solution:**~~

~~Consider $\sec^2 \theta (1 + \sin \theta)(1 - \sin \theta) = \sec^2 \theta (1 - \sin^2 \theta)$~~

~~$= \sec^2 \theta \cos^2 \theta$~~

~~$(\because \sin^2 \theta + \cos^2 \theta = 1)$~~

~~$= 1$~~

~~**Question: 10**~~

~~Write the value o~~

~~**Solution:**~~

~~Consider $\text{cosec}^2\,\theta\,(1 + \cos\theta)(1 - \cos\theta) = \text{cosec}^2\,\theta\,(1 - \cos^2\theta)$~~

~~$= \text{cosec}^2\,\theta\,\sin^2\theta$~~

~~$(\because \sin^2\theta + \cos^2\theta = 1)$~~

~~$= 1$~~

~~Question: 11~~

~~Write the value o~~

~~Solution:~~

~~Consider $\sin^2\theta\,\cos^2\theta\,(1 + \tan^2\theta)(1 + \cot^2\theta)$~~

~~$= \sin^2\theta\,\cos^2\theta\,(\sec^2\theta)(\text{cosec}^2\,\theta)$ $(\because 1 + \cot^2\theta = \text{cosec}^2\,\theta$ and $1 + \tan^2\theta = \sec^2\theta)$~~

~~$= \sin^2\theta\,(\text{cosec}^2\,\theta)\,\cos^2\theta\,(\sec^2\theta)$~~

~~$= 1 \times 1$~~

~~$= 1$~~

~~Question: 12~~

~~Write the value o~~

~~Solution:~~

~~Consider $(1 + \tan^2\theta)(1 + \sin\theta)(1 - \sin\theta)$~~

~~$= (1 + \tan^2\theta)(1 - \sin^2\theta)$ $(\because \sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta)$~~

~~$= (\sec^2\theta)\,(\cos^2\theta)$~~

~~$= 1$~~

~~Question: 13~~

~~Write the value o~~

~~Solution:~~

~~Consider $3\cot^2\theta - 3\,\text{cosec}^2\,\theta = -3(\text{cosec}^2\,\theta - \cot^2\theta)$~~

~~$= -3(1)$ $(\because 1 + \cot^2\theta = \text{cosec}^2\,\theta)$~~

~~$= -3$~~

~~Question: 14~~

~~Write the value o~~

~~Solution:~~

~~Consider $4\tan^2\theta - \frac{4}{\cos^2\theta} = 4\tan^2\theta - 4\sec^2\theta$~~

~~$= 4(\tan^2\theta - \sec^2\theta)$~~

~~$= 4(-1)$ $(\because 1 + \tan^2\theta = \sec^2\theta)$~~

~~$= -4$~~

~~Question: 15~~

~~Write the value o~~

~~Solution:~~

~~Consider $\frac{\tan^2\theta - \sec^2\theta}{\cot^2\theta - \text{cosec}^2\theta} = \frac{-1}{-1}$ $(\because 1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\,\theta)$~~

~~$= 1$~~

~~**Question: 16**~~

~~If sin θ = 1/2, w~~

~~**Solution:**~~

~~Give: sin θ = 1/2~~

~~Therefore cosec θ = 1/sin θ~~

~~= 2~~

~~Consider 3 $\cot^2 \theta + 3 = 3 (\cot^2 \theta + 1)$~~

~~= 3 $\operatorname{cosec}^2 \theta$ (∵ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$)~~

~~= 3 $(2)^2$~~

~~= 3 × 4~~

~~= 12~~

~~**Question: 17**~~

~~If cos θ = 2/3, w~~

~~**Solution:**~~

~~Give: cos θ = 2/3~~

~~Therefore sec θ = 1/cos θ~~

~~= 3/2~~

~~Consider 4 $\tan^2 \theta + 4 = 4 (\tan^2 \theta + 1)$~~

~~= 4 $\sec^2 \theta$ (∵ $1 + \tan^2 \theta = \sec^2 \theta$)~~

~~= 4 $(3/2)^2$~~

~~= 4 × (9/4)~~

~~= 9~~

~~**Question: 18**~~

~~If cos θ = 7/25,~~

~~**Solution:**~~

~~Given: cos θ = 7/25~~

~~Therefore sin θ = $\sqrt{(1 - \cos^2 \theta)}$~~

~~= √(1 –(49/625))~~

~~= √[(625 – 49)/625]~~

~~= √(576/625)~~

~~= 24/25~~

~~Thus, tan θ = sin θ/cos θ = (24/25)/(7/25)~~

~~= 24/7~~

~~Also, cot θ = 1/tan θ = 7/24~~

~~Therefore, tan θ + cot /θ = (24/7) + (7/24)~~

~~= (576 + 49)/(24 × 7)~~

~~= 625/168~~

~~**Question: 19**~~

~~If cos θ = 2/3, w~~

~~**Solution:**~~

~~Given: cos θ = 2/3~~

~~Thus, sec θ = 1/cos θ~~

~~= 3/2~~

~~Now, consider $\frac{\sec\theta-1}{\sec\theta+1}=\frac{\frac{3}{2}-1}{\frac{3}{2}+1}$~~

~~= [(1/2)/(5/2)]~~

~~= 1/5~~

~~**Question: 20**~~

~~If 5 tan θ = 4, w~~

~~**Solution:**~~

~~Given: 5 tan θ = 4~~

~~Therefore, tan θ = 4/5~~

~~Now, consider $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$ and divide numerator and denominator by cos θ:~~

~~$=\frac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}$~~

~~$=\frac{1-\tan\theta}{1+\tan\theta}$~~

~~$=\frac{1-\frac{4}{5}}{1+\frac{4}{5}}$~~

~~= (1/5)/(9/5)~~

~~= 1/9~~

~~**Question: 21**~~

~~If 3 cot θ = 4, w~~

~~**Solution:**~~

~~Given: 3 cot θ = 4~~

~~Therefore, cot θ = 4/3~~

~~Therefore, tan θ = 3/4~~

~~Now, consider $\frac{2\cos\theta+\sin\theta}{4\cos\theta-\sin\theta}$ and divide numerator and denominator by cos θ:~~

~~$=\frac{2\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{4\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}$~~

~~$=\frac{2+\tan\theta}{4-\tan\theta}$~~

~~$=\frac{2+\frac{3}{4}}{4-\frac{3}{4}}$~~

~~= (11/4)/(13/4)~~

~~= 11/13~~

~~**Question: 22**~~

~~If cot θ = 1/√3 w~~

~~**Solution:**~~

~~Given: cot θ = 1/√3~~

~~Thus, tan θ = 1/cot θ = √3~~

~~Therefore sec θ = √(1 + tan^{2} θ)~~

~~= √(1 + 3)~~

~~= √(4)~~

~~= 2~~

~~Therefore sec^{2} θ = 4~~

~~Now, cos^{2} θ = 1/sec^{2} θ = 1/4~~

~~So, consider $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{1-\cos^2\theta}{1+1-\sin^2\theta}$~~

~~$= \frac{1-\cos^2\theta}{1+\cos^2\theta}$~~

~~$= \frac{1-\frac{1}{4}}{1+\frac{1}{4}}$~~

~~= (3/4)/(5/4)~~

~~= 3/5~~

~~**Question: 23**~~

~~If tan θ = 1/√5wr~~

~~**Solution:**~~

~~Given: tan θ = 1/√5~~

~~∴ tan^{2} θ = 1/5~~

~~Consider $\frac{(\operatorname{cosec}^2\theta-\sec^2\theta)}{(\operatorname{cosec}^2\theta+\sec^2\theta)} = \frac{\left(\frac{1}{\sin^2\theta}-\frac{1}{\cos^2\theta}\right)}{\left(\frac{1}{\sin^2\theta}+\frac{1}{\cos^2\theta}\right)}$~~

~~Multiply numerator and denominator by sin θ:~~

~~$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$~~

~~$= \frac{1-\frac{1}{5}}{1+\frac{1}{5}}$~~

~~= 4/6~~

~~= 2/3~~

~~**Question: 24**~~

~~If cot A = 4/3 an~~

~~**Solution:**~~

~~We are given that: cot A = 4/3~~

~~⇒ tan (90° – A) = 4/3~~

~~Since A + B = 90°, therefore B = 90° – A~~

~~Therefore, tan (90° – A) = tan B = 4/3~~

~~**Question: 25**~~

~~If cos B = 3/5 an~~

~~Solution:~~

~~We are given that: cos B = 3/5~~

~~⇒ sin (90° – B) = 3/5~~

~~Since A + B = 90°, therefore A = 90° – B~~

~~Therefore, sin (90° – B) = sin A = 3/5~~

~~Question: 26~~

~~If √3sin θ = cos~~

~~Solution:~~

~~We are given that: √3sin θ = cos θ~~

~~∴sin θ /cos θ = 1/√3~~

~~⇒ tan θ = 1/√3~~

~~⇒ tan θ = tan 30°~~

~~On comparing both sides, we get,~~

~~θ = 30°~~

~~Question: 27~~

~~Write the value o~~

~~Solution:~~

~~Consider tan 10° tan 20° tan 70° tan 80°~~

~~= tan 10° tan 20° tan (90° – 20°) tan (90° – 10°)~~

~~= tan 10° tan 20° cot 0° cot 10°~~

~~= tan 10° cot 10° tan 20° cot 20°~~

~~= 1 × 1~~

~~= 1~~

~~Question: 28~~

~~Write the value o~~

~~Solution:~~

~~Consider tan 1° tan 2°... tan 88° tan 89°~~

~~= tan 1° tan 2°... tan 44° tan 45° tan 46° ...tan 88° tan 89°~~

~~= tan 1° tan 2°...tan 44° tan 45° tan (90° – 44°) ...tan (90° – 2°) tan (90° – 1°)~~

~~= tan 1° tan 2° ... tan 44 ° tan 45° cot 44° ...cot 2° cot 1°~~

~~= tan 1° cot 1° tan 2° cot 2°... tan 44 ° cot 44 ° tan 45°~~

~~= 1 × 1 × ...× 1~~

~~= 1~~

~~Question: 29~~

~~Write the value o~~

~~Solution:~~

~~Consider cos 1° cos 2° cos 3° ... cos 180°~~

~~= cos 1° cos 2° cos 3° ...× cos 90° × ... × cos 180°~~

~~= cos 1° cos 2° cos 3° ... × 0 × ... × cos 180°~~

~~= 0 (∵ cos 90° = 0)~~

~~**Question: 30**~~

~~If tan A = 5/12,~~

~~**Solution:**~~

~~Given: tan A = 5/12~~

~~Consider (sin A + cos A) sec A = (sin A + cos A)(1/cos A)~~

~~= (sin A/cos A) + (cos A/cos A)~~

~~= tan A + 1~~

~~= (5/12) + 1~~

~~= 17/12~~

~~**Question: 31**~~

~~If sin θ = cos (θ~~

~~**Solution:**~~

~~We are given that: sin θ = cos (θ – 45°)~~

~~∴We can rewrite it as: cos (90° – θ) = cos (θ – 45°)~~

~~On comparing both sides, we get,~~

~~90° – θ = θ – 45°~~

~~⇒ θ + θ = 90° + 45°~~

~~⇒ 2θ = 135°~~

~~⇒ θ = 65.5°~~

~~**Question: 32**~~

~~Find the value of~~

~~**Solution:**~~

~~Consider $\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\operatorname{cosec} 40^\circ}{\sec 50^\circ}$ – 4 cos 50° cosec 40°~~

~~= $\frac{\sin 50^\circ}{\cos(90^\circ - 50^\circ)} + \frac{\operatorname{cosec} 40^\circ}{\sec(90^\circ - 40^\circ)}$ – 4 cos 50° cosec (90° – 40°)~~

~~= $\frac{\sin 50^\circ}{\sin 50^\circ} + \frac{\operatorname{cosec} 40^\circ}{\operatorname{cosec} 40^\circ}$ – 4 cos 50° sec 50°~~

~~= 1 + 1 – 4~~

~~= – 2~~

~~**Question: 33**~~

~~Find the value of~~

~~**Solution:**~~

~~Consider sin 48° sec 42° + cos 48° cosec 42°~~

~~= sin 48° sec (90° – 48°) + cos 48° cosec (90° – 48°)~~

~~= sin 48° cosec 48° + cos 48° sec 48°~~

~~= 1 + 1~~

~~= 2~~

~~**Question: 34**~~

~~If x = a sin θ an~~

~~**Solution:**~~

~~Given: $x = a \sin \theta$~~

~~$y = b \cos \theta$~~

~~Then $b^2x^2 + a^2y^2 = b^2(a \sin \theta)^2 + a^2(b \cos \theta)^2$~~

~~$= a^2b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta$~~

~~$= a^2b^2 (\sin^2 \theta + \cos^2 \theta)$~~

~~$= (a^2b^2) \times 1$~~

~~$= a^2b^2$~~

~~**Question: 35**~~

~~If 5x = sec θ and~~

~~**Solution:**~~

~~Given: 5x = sec θ, and 5/x = tan θ~~

~~Consider $5(x^2 – (1/ x^2))$~~ $= \frac{5}{5}\left(5x^2 - \frac{5}{x^2}\right)$

~~=~~ $\frac{1}{5}\left(25x^2 - \frac{25}{x^2}\right)$

~~=~~ $\frac{1}{5}\left((5x)^2 - \left(\frac{5}{x}\right)^2\right)$

~~$= (1/5)\ [\sec^2 \theta – \tan^2 \theta]$~~

~~= (1/5)[1]~~

~~$= 1/5$ (∵ $\sec^2 x – \tan^2 x = 1$)~~

~~**Question: 36**~~

~~If cosec θ = 2x a~~

~~**Solution:**~~

~~Given: 2x = cosec θ , and 2/x = cot θ~~

~~Consider $2(x^2 – (1/ x^2))$ =~~ $\frac{2}{2}\left(2x^2 - \frac{2}{x^2}\right)$

$= \frac{1}{2}\left(4x^2 - \frac{4}{x^2}\right)$

~~=~~ $\frac{1}{2}\left((2x)^2 - \left(\frac{2}{x}\right)^2\right)$

~~$= (1/2)(\text{cosec}^2 \theta – \cot^2 \theta)$~~

~~$= 1/2$ (∵ $\text{cosec}^2 x – \cot^2 x = 1$)~~

~~**Question: 37**~~

~~If sec θ + tan θ~~

~~**Solution:**~~

~~Given: sec θ + tan θ = x ……(1)~~

~~Then, (sec θ + tan θ) ×~~ $\frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$ ~~= x~~

~~⇒~~ $\frac{\sec^2\theta - \tan^2\theta}{\sec\theta - \tan\theta}$ ~~= x~~

~~⇒~~ $\frac{1}{\sec\theta - \tan\theta}$ ~~= x~~

~~⇒ sec θ – tan θ = (1/x) ……(2)~~

~~Adding equation (1) and (2), we get:~~

~~2 sec θ = x + (1/x)~~

~~= (x^2 + 1)/x~~

~~⇒ sec θ = (x^2 + 1)/2x~~

~~Therefore, sec θ = (x^2 + 1)/2x~~

~~Question: 38~~

~~Find the value of~~

~~Solution:~~

~~Consider~~ $\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$

~~=~~ $\frac{\cos 38^\circ \operatorname{cosec} (90^\circ - 38^\circ)}{\tan 18^\circ \tan(90^\circ - 55^\circ) \tan 60^\circ \tan(90^\circ - 18^\circ) \tan 55^\circ}$

~~=~~ $\frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \cot 55^\circ \tan 60^\circ \cot 18^\circ \tan 55^\circ}$

~~=~~ $\frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \cot 18^\circ \cot 55^\circ \tan 55^\circ \tan 60^\circ}$

~~=~~ $\frac{1}{1 \times 1 \times \tan 60^\circ}$

~~= cot 60°~~

~~= 1/√3~~

~~Question: 39~~

~~If sin θ = x, wri~~

~~Solution:~~

~~Given: sin θ = x~~

~~Therefore, cosec θ = 1/x~~

~~Using the identity 1 + $\cot^2$ θ = cosec^2 θ, we get~~

~~cot θ = √(cosec^2 θ – 1)~~

~~= √($(1/x)^2$ – 1)~~

~~=~~ $\sqrt{\frac{x^2-1}{x^2}}$

~~=~~ $\frac{\sqrt{x^2-1}}{x}$

~~Question: 40~~

~~If sec θ = x, wri~~

~~Solution:~~

~~Given: sec θ = x~~

~~Using the identity 1 + $\tan^2$ θ = $\sec^2$ θ, we get~~

~~tan θ = √($\sec^2$ θ – 1)~~

~~= √(x^2 – 1)~~

Exercise : ~~MULTIPLE CHOICE QUESTIONS (MCQ)~~

~~**Question: 1**~~

~~Choos~~

~~**Solution:**~~

~~sec 30° = 1/cos 30°~~

~~= 1/(√3/2)~~

~~= 2/√3~~

~~cosec 60° = 1/sin 60°~~

~~= 1/(√3/2)~~

~~= 2/√3~~

~~Therefore, sec 30° /cosec 30° = (2/√3)/(2/√3)~~

~~= 1~~

~~**Question: 2**~~

~~Choos~~

~~**Solution:**~~

$$\frac{\tan 35^\circ}{\cot 55^\circ}+\frac{\cot 78^\circ}{\tan 12^\circ}=\frac{\tan 35^\circ}{\cot(90-35)^\circ}+\frac{\cot(90-12)^\circ}{\tan 12^\circ}$$

$$=\frac{\tan 35^\circ}{\tan 35^\circ}+\frac{\tan 12^\circ}{\tan 12^\circ}$$

~~= 1 + 1~~

~~= 2~~

~~**Question: 3**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider tan 10° tan 15° tan 75° tan 80°~~

~~= tan 10° tan 80° tan 15° tan 75°~~

~~= tan 10° tan (90 – 10)° tan 15° tan (90 – 15)°~~

~~= (tan 10° cot 10°) (tan 15° cot 15°)~~

~~= (1) × (1) = 1~~

~~**Question: 4**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider tan 5° tan 25° tan 30° tan 65° tan 85°~~

~~= tan 5° tan 85° tan 25° tan 65° tan 30°~~

~~= tan 5° tan (90 – 5)° tan 25° tan (90 – 25)° tan 30°~~

~~= (tan 5° cot 5°) (tan 25° cot 25°) tan 30°~~

~~= (1) × (1) × (1/√3)~~

~~= 1/√3~~

~~**Question: 5**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider cos 1° cos 2° cos 3° … cos 180°~~

~~= cos 1° cos 2° cos 3° …× cos 90° × … cos 180°~~

~~= cos 1° cos 2° cos 3° … × 0 × cos 180°~~

~~= 0 (∵ cos 90° = 0)~~

~~Question: 6~~

~~Choos~~

~~Solution:~~

~~Consider~~ $\frac{2\sin^2 63°+1+2\sin^2 27°}{3\cos^2 17°-2+3\cos^2 73°}$ ~~=~~ $\frac{2\sin^2 63°+1+2\sin^2(90°-63°)}{3\cos^2 17°-2+3\cos^2(90°-17°)}$

~~=~~ $\frac{2\sin^2 63°+1+2\cos^2 63°}{3\cos^2 17°-2+3\sin^2 17°}$

~~=~~ $\frac{2\left(\cos^2 63°+\sin^2 63°\right)+1}{3\left(\cos^2 17°+\sin^2 17°\right)-2}$

~~= (2 + 1)/(3 – 1)~~

~~= 3~~

~~Question: 7~~

~~Choose the correc~~

~~Solution:~~

~~Consider (sin 47° cos 43°) + (cos 47° sin 43°)~~

~~= (sin 47° cos (90 – 47)°) + (cos 47° sin (90 – 47)°)~~

~~= $\sin^2$ 47° + $\cos^2$ 47°~~

~~= 1~~

~~Question: 8~~

~~Choose the correc~~

~~Solution:~~

~~Consider (sec 70° sin 20°) + (cos 20° cosec 70°)~~

~~= (sec (90 – 20)° sin 20°) + (cos 20° cosec (90 – 20)°)~~

~~= (cosec 70° sin70°) + (cos 20° sec 20°)~~

~~= 1 + 1 (∵ cosec θ = 1/sin θ and sec θ = 1/cos θ)~~

~~= 2~~

~~Question: 9~~

~~Choose the correc~~

~~Solution:~~

~~We are given that: sin 3A = cos (A – 10°)~~

~~∴ We can rewrite it as: cos (90° – 3A) = cos (A – 10°)~~

~~On comparing both sides, we get,~~

~~90° – 3A = A – 10°~~

~~⇒ A + 3A = 90° + 10°~~

~~⇒ 4A = 100°~~

~~⇒ A = 25°~~

~~Question: 10~~

~~Choose the correc~~

~~Solution:~~

~~We are given that: sec 4A = cosec (A – 10°)~~

~~∴We can rewrite it as: cosec (90° – 4A) = cosec (A – 10°)~~

~~On comparing both sides, we get,~~

~~90° – 4A = A – 10 °~~

~~⇒ A + 4A = 90° + 10°~~

~~⇒ 5A = 100°~~

~~⇒ A = 20°~~

~~Question: 11~~

~~Choose the correc~~

~~Solution:~~

~~We are given that: sin A = cos B~~

~~∴We can rewrite it as: sin A = sin(90° – B)~~

~~On comparing both sides, we get,~~

~~90° – B = A~~

~~⇒ A + B = 90°~~

~~Question: 12~~

~~Choose the correc~~

~~Solution:~~

~~We are given that: cos (α + β) = 0~~

~~∴We can rewrite it as: cos (α + β) = cos (90° – 0°)~~

~~On comparing both sides, we get,~~

~~(α + β) = 0 = 90°~~

~~⇒ α = 90° – β~~

~~Therefore, sin (α – β) = sin (90° – β – β)~~

~~= sin (90° – 2β)~~

~~= cos 2β~~

~~Question: 13~~

~~Choose the correc~~

~~Solution:~~

~~Consider sin (45° + θ) – cos (45° – θ) = sin (45° + θ) – sin (90° – (45° – θ))~~

~~= sin (45° + θ) – sin (45° + θ)~~

~~= 0~~

~~Question: 14~~

~~Choose the correc~~

~~Solution:~~

~~sec^2 10° – cot^2 80° = sec^2 10° – tan^2 (90° – 80°)~~

~~= $\sec^2 10° - \tan^2 10°$~~

~~= 1~~

~~**Question: 15**~~

~~Choose the correc~~

~~**Solution:**~~

~~$\operatorname{cosec}^2 57° - \tan^2 33° = \operatorname{cosec}^2 57° - \cot^2 (90° - 33°)$~~

~~= $\operatorname{cosec}^2 57° - \cot^2 57°$~~

~~= 1~~

~~**Question: 16**~~

~~Choos~~

~~**Solution:**~~

~~Consider $\frac{2\tan^2 30° \sec^2 52° \sin^2 38°}{\operatorname{cosec}^2 70° - \tan^2 20°} = \frac{2\tan^2 30° \sec^2 52° \cos^2(90° - 38°)}{\operatorname{cosec}^2 70° - \cot^2(90° - 20°)}$~~

~~= $\frac{2\tan^2 30° \sec^2 52° \cos^2 52°}{\operatorname{cosec}^2 70° - \cot^2 70°}$~~

~~= $(2\tan^2 30° \times 1)/1$~~

~~= $2\tan^2 30°$~~

~~= $2(1/\sqrt{3})^2$~~

~~= 2/3~~

~~**Question: 17**~~

~~Choose the~~

~~**Solution:**~~

~~Consider $\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$~~

~~= $\frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2 22° + \cos^2(90° - 22°)} + \sin^2 63° + \cos 63° \sin (90° - 63°)$~~

~~= $\frac{\sin^2 22° + \cos^2 22°}{\cos^2 22° + \sin^2 22°} + \sin^2 63° + \cos 63° \cos 63°$~~

~~= $(1/1) + (\sin^2 63° + \cos^2 63°)$~~

~~= 1 + 1~~

~~= 2~~

~~**Question: 18**~~

~~Choos~~

~~**Solution:**~~

~~Consider $\frac{\cot(90° - \theta) \sin(90° - \theta)}{\sin \theta} + \frac{\cot 40°}{\tan 50°} - (\cos^2 20° + \cos^2 70°)$~~

~~= $\frac{\tan \theta \cos \theta}{\sin \theta} + \frac{\cot(90° - 50°)}{\tan 50°} - (\cos^2 20° + \cos^2 (90° - 70°))$~~

~~= $(\tan \theta \times \cot \theta) + (\tan 50°/\tan 50°) - (\cos^2 20° + \sin^2 20°)$~~

~~= 1 + 1 – 1~~

~~= 1~~

~~**Question: 19**~~

~~Choos~~

~~**Solution:**~~

~~Consider~~ $\frac{\cos 38° \operatorname{cosec} 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°}$

~~=~~ $\frac{\cos 38° \operatorname{cosec} (90° - 38°)}{\tan 18° \tan 35° \tan 60° \tan (90 - 18)° \tan (90° - 35°)}$

~~=~~ $\frac{\cos 38° \sec 38°}{= \tan 18° \tan 35° \tan 60° \cot 18° \cot 35°}$

~~=~~ $\frac{1}{\tan 60°}$

~~= 1/√3~~

~~**Question: 20**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: 2 sin 2θ = √3~~

~~Therefore, sin 2θ = √3/2~~

~~⇒ sin 2θ = sin 60°~~

~~On comparing both sides, we get:~~

~~2θ = 60°~~

~~⇒ θ = 60°/2~~

~~⇒ θ = 30°~~

~~**Question: 21**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: 2 cos 3θ = 1~~

~~Therefore, cos 3θ = 1/2~~

~~⇒ cos 3θ = cos 60°~~

~~On comparing both sides, we get:~~

~~3θ = 60°~~

~~⇒ θ = 60°/3~~

~~⇒ θ = 20°~~

~~**Question: 22**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given√3tan 2θ – 3 = 0~~

~~Therefore, √3tan 2θ = 3~~

~~⇒ tan 2θ = 3/√3~~

~~⇒ tan 2θ = √3~~

~~⇒ tan 2θ = tan 60°~~

~~On comparing both sides, we get:~~

~~2θ = 60°~~

~~⇒ θ = 60°/2~~

~~⇒ θ = 30°~~

~~**Question: 23**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: tan x = 3 cot x~~

~~⇒ tan x/cot x = 3~~

~~Since, cot x = 1/tan x~~

~~Therefore, tan x/cot x = 3 ⇒ tan^2 x = 3~~

~~Taking square root on both sides:~~

~~⇒ tan x = √3~~

~~⇒ tan x = tan 60°~~

~~Comparing both sides:~~

~~⇒ x = 60°~~

~~**Question: 24**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: x tan 45° cos 60° = sin 60° cot 60°~~

~~⇒ x × 1 × (1/2) = (√3/2) × (1/√3)~~

~~⇒ x/2 = 1/2~~

~~⇒ x = 1~~

~~**Question: 25**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: tan^{2}45° – cos^{2}30° = x sin 45° cos 45°~~

~~⇒ (1)^2 – (√3/2)^2 = x × (1/√2) × (1/√2)~~

~~⇒ 1 – (3/4) = x × (1/2)~~

~~⇒ x/2 = 1 – (3/4)~~

~~⇒ x/2 = 1/4~~

~~⇒ x = 2/4~~

~~⇒ x = 1/2~~

~~**Question: 26**~~

~~Choose the correc~~

~~**Solution:**~~

~~sec^2 60° – 1 = (1/cos^2 60°) – 1~~

~~= [1/(1/2)^2] – 1~~

~~= [1/(1/4)] – 1~~

~~= 4 – 1~~

~~= 3~~

~~**Question: 27**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider (cos 0° + sin 30° + sin 45°) (sin 90° + cos 60° – cos 45°)~~

~~= [1 + (1/2) + (1/√2)] × [1 + (1/2) – (1/√2)]~~

~~= [(3/2) + (1/√2)] × [(3/2) – (1/√2)]~~

~~= $(3/2)^2 – (1/\sqrt{2})^2$~~

~~= (9/4) – (1/2)~~

~~= (9 – 2)/4~~

~~= 7/4~~

~~**Question: 28**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider $\sin^2 30° + 4 \cot^2 45° – \sec^2 60° = (1/2)^2 + 4(1)^2 – (2)^2$~~

~~= (1/4) + 4 – 4~~

~~= 1/4~~

~~**Question: 29**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider $3 \cos^2 60° + 2 \cot^2 30° – 5 \sin^2 45° = 3 (1/2)^2 + 2(\sqrt{3})^2 – 5(1/\sqrt{2})^2$~~

~~= 3(1/4) + 2(3) – 5(1/2)~~

~~= (3/4) + 6 – (5/2)~~

~~= (3 + 24 – 10)/4~~

~~= 17/4~~

~~**Question: 30**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider $\cos^2 30° \cos^2 45° + 4 \sec^2 60° + 2\cos^2 90° – 2 \tan^2 60°$~~

$$= 3\left(\frac{1}{2}\right)^2 + 2(\sqrt{3})^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 3\left(\frac{1}{4}\right) + 2(3) - 5\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + 6 - \frac{5}{2}$$

$$= \frac{17}{4}$$

~~**Question: 31**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: cosec θ = √10~~

~~Therefore, sin θ = 1/√10~~

~~Since, sin^2 θ + cos^2 θ = 1~~

~~Therefore, cos θ = √(1 – sin^2 θ)~~

~~=~~ $\sqrt{\left(1-\frac{1}{10}\right)}$

~~=~~ $\sqrt{\frac{(10-1)}{10}}$

~~=~~ $\sqrt{\frac{9}{10}}$

~~=~~ $\frac{3}{\sqrt{10}}$

~~Therefore, sec θ = 1/cos θ =~~ $\frac{\sqrt{10}}{3}$

~~**Question: 32**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: tan θ = 8/15 = Perpendicular/Base~~

~~On comparing, we get:~~

~~Perpendicular = 8~~

~~Base = 15~~

~~Therefore, (Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2~~

~~= 64 + 225~~

~~= 289~~

~~Therefore, hypotenuse = √289~~

~~= 17~~

~~Therefore cosec θ = Hypotenuse/Perpendicular~~

~~= 17/8~~

~~**Question: 33**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: sin θ = a/b~~

~~Since, sin^2 θ + cos^2 θ = 1~~

~~Therefore, cos θ = √(1 – sin^2 θ)~~

~~=~~ $\sqrt{\left(1-\frac{a^2}{b^2}\right)}$

~~=~~ $\frac{\sqrt{(b^2-a^2)}}{b}$

~~**Question: 34**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: tan θ = √3 = Perpendicular/Base~~

~~On comparing, we get:~~

~~Perpendicular = √3~~

~~Base = 1~~

~~Therefore, (Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2~~

~~= 3 + 1~~

~~= 4~~

~~Therefore, hypotenuse = √4~~

~~= 2~~

~~Therefore sec θ = Hypotenuse/Base~~

~~= 2/1 = 2~~

~~Question: 35~~

~~Choose the correc~~

~~Solution:~~

~~Given: sec θ = 25/7~~

~~Therefore, cos θ = 1/sec θ = 7/25 = Base/Hypotenuse~~

~~Therefore, on comparing, Base = 7 and Hypotenuse = 25~~

~~In a right – angled triangle, (Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2~~

~~625 = (Perpendicular)^2 + 49~~

~~Therefore, Perpendicular = √(625 – 49) = √(576)~~

~~= 24~~

~~Therefore sin θ = Perpendicular/Hypotenuse~~

~~= 24/25~~

~~Question: 36~~

~~Choose the correc~~

~~Solution:~~

~~Given: sin θ = 1/2 = Perpendicular/Hypotenuse~~

~~Therefore, on comparing, Perpendicular = 1 and Hypotenuse = 2~~

~~In a right – angled triangle, (Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2~~

~~4 = 1+ (Base)^2~~

~~Therefore, Base = √(4 – 1)~~

~~= √(3)~~

~~Therefore cot θ = Base/ Perpendicular~~

~~= √3/1~~

~~= √3~~

~~Question: 37~~

~~Choose the correc~~

~~Solution:~~

~~Given: cos θ = 4/5 = Base/Hypotenuse~~

~~Therefore, on comparing, Base = 4 and Hypotenuse = 5~~

~~In a right – angled triangle, (Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2~~

~~25 = (Perpendicular)^2 + 16~~

~~Therefore, Perpendicular = √(25 – 16) = √(9)~~

~~= 3~~

~~Therefore tan θ = Perpendicular/Base~~

~~= 3/4~~

~~**Question: 38**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: 3x = cosec θ , and 3/x = cot θ~~

~~Consider 3(x^2 – (1/ x^2)) =~~ $\frac{3}{3}\left(3x^2 - \frac{3}{x^2}\right)$

= $\frac{1}{3}\left(9x^2 - \frac{9}{x^2}\right)$

~~=~~ $\frac{1}{3}\left((3x)^2 - \left(\frac{3}{x}\right)^2\right)$

~~= (1/3)(cosec^2 θ – cot^2 θ)~~

~~= 1/3 (∵ cosec^2 x – cot^2 x = 1)~~

~~**Question: 39**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: 2x = sec A, and 2/x = tan A~~

~~Consider 2(x^2 – (1/ x^2))~~ = $\frac{2}{2}\left(2x^2 - \frac{2}{x^2}\right)$

~~=~~ $\frac{1}{2}\left(4x^2 - \frac{4}{x^2}\right)$

~~=~~ $\frac{1}{2}\left((2x)^2 - \left(\frac{2}{x}\right)^2\right)$

~~= (1/2) [sec^2 A – tan^2 A]~~

~~= (1/2)[1]~~

~~= 1/2 (∵ sec^2 x – tan^2 x = 1)~~

~~**Question: 40**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: tan θ = 4/3 = Perpendicular/Base~~

~~Therefore, (Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2~~

~~= 16 + 9~~

~~= 25~~

~~Therefore, hypotenuse = √25~~

~~= 5~~

~~Therefore sin θ = Perpendicular/Hypotenuse~~

~~= 4/5~~

~~Also, cos θ = Base/Hypotenuse~~

~~= 3/5~~

~~Thus, sin θ + cos θ = (4/5) + (3/5)~~

~~= 7/5~~

~~**Question: 41**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: tan θ + cot θ = 5~~

~~Squaring both sides, we get:~~

~~$\tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 25$~~

~~$\tan^2\theta + \cot^2\theta = 25 - 2\tan\theta\cot\theta$~~

~~= 25 – 2~~

~~= 23~~

~~**Question: 42**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: cos θ + sec θ = 5/2~~

~~Squaring both sides, we get:~~

~~$\cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta = 25/4$~~

~~$\cos^2\theta + \sec^2\theta = (25/4) - 2\cos\theta\sec\theta$~~

~~= 25/4 – 2~~

~~= (25 – 8)/4~~

~~= 17/4~~

~~**Question: 43**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: tan θ = 1/√7~~

~~$\therefore \tan^2\theta = 1/7$~~

~~Consider $\frac{(\mathrm{cosec}^2\theta - \sec^2\theta)}{(\mathrm{cosec}^2\theta + \sec^2\theta)} = \frac{\left(\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}\right)}{\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)}$~~

~~Multiply numerator and denominator by sin θ:~~

~~$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$~~

~~$= \frac{1-\frac{1}{7}}{1+\frac{1}{7}}$~~

~~= 6/8~~

~~= 3/4~~

~~**Question: 44**~~

~~Choose the correc~~

~~Solution:~~

~~Given: 7tan θ = 4~~

~~Therefore tan θ = 4/7~~

~~Consider $\frac{7\sin\theta-3\cos\theta}{7\sin\theta+3\cos\theta}$ and divide numerator and denominator by cos θ:~~

~~$\frac{\frac{(7\sin\theta-3\cos\theta)}{\cos\theta}}{\frac{7\sin\theta+3\cos\theta}{\cos\theta}}=\frac{7\tan\theta-3}{7\tan\theta+3}$~~

~~$=\frac{7\left(\frac{4}{7}\right)-3}{7\left(\frac{4}{7}\right)+3}$~~

~~$=\frac{4-3}{4+3}$~~

~~= 1/7~~

~~Question: 45~~

~~Choose the correc~~

~~Solution:~~

~~Given: 3cot θ = 4~~

~~Therefore cot θ = 4/3~~

~~Consider $\frac{5\sin\theta+3\cos\theta}{5\sin\theta-3\cos\theta}$ and divide numerator and denominator by sin θ:~~

~~$\frac{\frac{(5\sin\theta+3\cos\theta)}{\sin\theta}}{\frac{5\sin\theta-3\cos\theta}{\sin\theta}}=\frac{5+3\cot\theta}{5-3\cot\theta}$~~

~~$=\frac{5+3\left(\frac{4}{3}\right)}{5-3\left(\frac{4}{3}\right)}$~~

~~$=\frac{5+4}{5-4}$~~

~~= 9~~

~~Question: 46~~

~~Choose the correc~~

~~Solution:~~

~~Given: tan θ = a/b~~

~~Consider $\frac{a\sin\theta-b\cos\theta}{a\sin\theta+b\cos\theta}$ and divide numerator and denominator by cos θ:~~

~~$\frac{\frac{(a\sin\theta-b\cos\theta)}{\cos\theta}}{\frac{a\sin\theta+b\cos\theta}{\cos\theta}}=\frac{a\tan\theta-b}{a\tan\theta+b}$~~

~~$=\frac{a\left(\frac{a}{b}\right)-b}{a\left(\frac{a}{b}\right)+b}$~~

~~$=\frac{a^2-b^2}{a^2+b^2}$~~

~~Question: 47~~

~~Choose the correc~~

~~Solution:~~

~~Given: sin A + $\sin^2$ A = 1~~

~~Therefore sin A = 1 – $\sin^2$ A = $\cos^2$ A ……(1)~~

~~Now, consider $\cos^2A + \cos^4A = \cos^2 A(1 + \cos^2A)$~~

~~Put the value of $\cos^2A$ in the above equation:~~

~~Therefore, $\cos^2A + \cos^4A = \cos^2 A(1 + \cos^2A)$~~

~~$= (1 - \sin^2 A)(1+1 - \sin^2 A)$~~

~~Again, from equation (1), we have $1 - \sin^2 A = \sin A$. So, put the value of sin A in the above equation:~~

~~Therefore, $\cos^2A + \cos^4A = (\sin A)(1+ \sin A)$~~

~~$= \sin A + \sin^2 A$~~

~~= 1 (given)~~

~~Therefore, $\cos^2A + s\cos^4A = 1$~~

~~**Question: 48**~~

~~Choose the correc~~

~~**Solution:**~~

~~Given: $\cos A + \cos^2 A = 1$~~

~~Therefore $\cos A = 1 - \cos^2 A = \sin^2 A$ ......(1)~~

~~Now, consider $\sin^2A + \sin^4A = \sin^2 A(1 + \sin^2A)$~~

~~Put the value of $\sin^2A$ in the above equation:~~

~~Therefore, $\sin^2A + \sin^4A = \sin^2 A(1 + \sin^2A)$~~

~~$= (1 - \cos^2 A)(1+1 - \cos^2 A)$~~

~~Again, from equation (1), we have $1 - \cos^2 A = \cos A$. So put the value of cos A in the above equation:~~

~~Therefore, $\sin^2A + \sin^4A = (\cos A)(1+ \cos A)$~~

~~$= \cos A + \cos^2 A$~~

~~= 1 (given)~~

~~Therefore, $\sin^2A + \sin^4A = 1$~~

~~**Question: 49**~~

~~Choos~~

~~**Solution:**~~

~~Consider~~ $\sqrt{\frac{1-\sin A}{1+\sin A}}$ ~~and rationalize:~~

$\sqrt{\frac{1-\sin A}{1+\sin A}}$ ~~=~~ $\sqrt{\frac{1-\sin A}{1+\sin A} \times \frac{1-\sin A}{1-\sin A}}$

~~=~~ $\sqrt{\frac{(1-\sin A)^2}{1-\sin^2 A}}$

~~=~~ $\sqrt{\frac{(1-\sin A)^2}{\cos^2 A}}$

~~=~~ $\frac{1-\sin A}{\cos A}$

~~=~~ $\frac{1}{\cos A} - \frac{\sin A}{\cos A}$

~~= sec A – tan A~~

~~Question: 50~~

~~Choos~~

~~Solution:~~

~~Consider~~ $\sqrt{\frac{1+\cos A}{1-\cos A}}$ ~~and rationalize:~~

$\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}}$

$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$

$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$

$= \frac{1+\cos A}{\sin A}$

$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$

~~= cosec A + cot A~~

~~Question: 51~~

~~Choose the correc~~

~~Solution:~~

~~Given: tan θ = a/b~~

~~Consider~~ $\frac{\cos\theta+\sin\theta}{\cos\theta-\text{siin}\theta}$ ~~and divide numerator and denominator by cos θ:~~

$\frac{\frac{(\cos\theta+\sin\theta)}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}} = \frac{1+\tan\theta}{1-\tan\theta}$

$= \frac{1+\frac{a}{b}}{1-\frac{a}{b}}$

$= \frac{b+a}{b-a}$

~~Question: 52~~

~~Choose the correc~~

~~Solution:~~

~~Consider (cosec θ – cot θ)~~^{2} $= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)\,(1-\cos\theta)}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta}$

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 53**~~

~~Choose the correc~~

~~**Solution:**~~

~~Consider (sec A + tan A)(1 – sin A) = $\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$~~

~~= $\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$~~

~~= $\frac{1-\sin^2 A}{\cos A}$~~

~~= $\frac{\cos^2 A}{\cos A}$~~

~~= cos A~~

~~Exercise : FORMATIVE ASSESSMENT (UNIT TEST)~~

~~**Question: 1**~~

~~Consider $\frac{\cos^2 56^\circ+\cos^2 34^\circ}{\sin^2 56^\circ+\sin^2 34^\circ}$ + 3 tan^2 56° tan^2 34°~~

~~= $\frac{\cos^2 56^\circ+\cos^2(90^\circ-56^\circ)}{\sin^2 56^\circ+\sin^2(90^\circ-56^\circ)}$ + 3 tan^2 56° tan^2 (90° – 56°)~~

~~= $\frac{\cos^2 56^\circ+\sin^2 56^\circ}{\sin^2 56^\circ+\cos^2 56^\circ}$ + 3 tan^2 56° cot^2 56°~~

~~= (1/1) + 3(1)~~

~~= 1 + 3~~

~~= 4~~

~~**Question: 2**~~

~~The value of (sin~~

~~**Solution:**~~

~~Consider (sin^2 30° cos^2 45° + 4 tan^2 30°+ (1/2) sin^2 90° + (1/8) cot^2 60°)~~

~~= [(1/2)^2 × (1/√2)^2 + 4 (1/√3)^2 + (1/2) × (1)^2 + (1/8)(1/√3)^2]~~

~~= [(1/4) × (1/2)] + [(4/3)] + (1/2) + (1/24)~~

~~= (1/8) + (4/3) + (1/2) + (1/24)~~

~~= (3 + 32 + 12 + 1)/24~~

~~= 48/24~~

~~= 2~~

~~**Question: 3**~~

~~If cos A + cos~~

~~**Solution:**~~

~~Given: cos A + cos^2 A = 1~~

~~Therefore cos A = 1 – cos^2 A = sin^2 A ......(1)~~

~~Now, consider sin^{2}A + sin^{4}A = sin^2 A(1 + sin^{2}A)~~

~~Put the value of sin^{2}A in the above equation:~~

~~Therefore, $\sin^2 A + \sin^4 A = \sin^2 A(1 + \sin^2 A)$~~

~~$= (1 – \cos^2 A)(1+1 – \cos^2 A)$~~

~~Again, from equation (1), we have $1 – \cos^2 A = \cos A$. So put the value of cos A in the above equation:~~

~~Therefore, $\sin^2 A + \sin^4 A = (\cos A)(1+ \cos A)$~~

~~$= \cos A + \cos^2 A$~~

~~= 1 (given)~~

~~Therefore, $\sin^2 A + \sin^4 A = 1$~~

~~**Question: 4**~~

~~If $\sin \theta = \sqrt{3}/2$,~~

~~**Solution:**~~

~~Given: $\sin \theta = \sqrt{3}/2$~~

~~Therefore, $\operatorname{cosec} \theta = 1/\sin \theta = 2/\sqrt{3}$~~

~~$\cos \theta = \sqrt{(1 – \sin^2 \theta)}$~~

~~$= \sqrt{(1 – (3/4))}$~~

~~$= \sqrt{(1/4)}$~~

~~$= 1/2$~~

~~$\cot \theta = \cos \theta/\sin \theta = (1/2)/\sqrt{3}/2$~~

~~$= 1/\sqrt{3}$~~

~~Therefore, $(\operatorname{cosec} \theta + \cot \theta) = (2/\sqrt{3}) + (1/\sqrt{3})$~~

~~$= 3/\sqrt{3}$~~

~~$= \sqrt{3}$~~

~~**Question: 5**~~

~~If cot A = 4/5, p~~

~~**Solution:**~~

~~Given: cot A = 4/5~~

~~Consider $\frac{\sin A+\cos A}{\sin A-\cos A}$ and divide numerator and denominator by sin A:~~

~~$\frac{\frac{(\sin A+\cos A)}{\sin A}}{\frac{\sin A- \cos A}{\sin A}} = \frac{1+\cot A}{1-\cot A}$~~

~~$= \frac{1+\left(\frac{4}{5}\right)}{1-\left(\frac{4}{5}\right)}$~~

~~$= \frac{5+4}{5-4}$~~

~~= 9~~

~~**Question: 6**~~

~~If 2x = sec A and~~

~~**Solution:**~~

~~Given: 2x = sec A, and 2/x = tan A~~

~~Therefore, $(2x)^2 = \sec^2 A$~~

~~⇒ $4x^2 = \sec^2 A$ [1]and~~ $\left(\frac{2}{x}\right)^2 = \tan^2 A$ $\Rightarrow \frac{4}{x^2} = \tan^2 A$ ~~[2]On subtracting [2] from [1], we get~~

$$4x^2 - \frac{4}{x^2} = \sec^2 A - \tan^2 A$$

$\Rightarrow 4\left(x^2 - \frac{1}{x^2}\right) = 1$ ~~(∵ $\sec^2 x - \tan^2 x = 1$)~~

$$\Rightarrow x^2 - \frac{1}{x^2} = \frac{1}{4}$$

~~**Question: 7**~~

~~If √3 tan θ = 3 s~~

~~**Solution:**~~

~~Given: √3 tanθ = 3 sinθ~~

~~⇒~~ $\frac{\sin\theta}{\cos\theta} = \frac{3\sin\theta}{\sqrt{3}}$

~~⇒~~ $\frac{1}{\cos\theta} = \sqrt{3}$

~~⇒~~ $\frac{1}{\cos\theta} = \sqrt{3}$

~~⇒~~ $\cos\theta = \frac{1}{\sqrt{3}}$

~~Also, we know,~~ $\sin\theta = \sqrt{1 - \cos^2\theta}$

~~⇒~~ $\sin\theta = \sqrt{1 - \cos^2\left(\frac{1}{3}\right)}$

~~⇒~~ $\sin\theta = \sqrt{\frac{2}{3}}$

~~Now, we need to prove that:~~

~~$\sin^2\theta - \cos^2\theta = 1/3$~~

~~⇒ L.H.S =~~ $\left(\sqrt{\frac{2}{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

~~⇒ L.H.S =~~ $\frac{2}{3} - \frac{1}{3}$

~~⇒ L.H.S = 1/3~~

~~**Question: 8**~~

~~Prove that~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\sin^2 73° + \sin^2 17°}{\cos^2 28° + \cos^2 62°}$

~~=~~ $\frac{\sin^2 73° + \sin^2(90° - 73°)}{\cos^2 28° + \cos^2(90° - 28°)}$

~~=~~ $\frac{\sin^2 73° + \cos^2 73°}{\cos^2 28° + \sin^2 28°}$

~~= 1/1~~

~~= 1~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 9~~

~~If 2 sin 2θ = √3,~~

~~Solution:~~

~~Given: 2 sin 2θ = √3~~

~~Therefore, sin 2θ = √3/2~~

~~⇒ sin 2θ = sin 60°~~

~~⇒ 2θ = 60°/2 = 30°~~

~~Therefore, θ = 30°~~

~~Question: 10~~

~~Prove that~~

~~Solution:~~

~~Consider $\sqrt{\frac{1+\cos A}{1-\cos A}}$ and rationalize:~~

~~$\sqrt{\frac{1+\cos A}{1-\cos A}} = \sqrt{\frac{1+\cos A}{1-\cos A} \times \frac{1+\cos A}{1+\cos A}}$~~

~~$= \sqrt{\frac{(1+\cos A)^2}{1-\cos^2 A}}$~~

~~$= \sqrt{\frac{(1+\cos A)^2}{\sin^2 A}}$~~

~~$= \frac{1+\cos A}{\sin A}$~~

~~$= \frac{1}{\sin A} + \frac{\cos A}{\sin A}$~~

~~= cosec A + cot A~~

~~Question: 11~~

~~If cosec θ + cot~~

~~Solution:~~

~~Given: cosec θ + cot θ = p~~

~~$p^2 - 1 = (\text{cosec}\,\theta + \cot\theta)^2 - 1$~~

~~$= \text{cosec}^2\,\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta - 1$~~

~~$= \text{cosec}^2\,\theta - 1 + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$~~

~~$= \cot^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$~~

~~= 2 cot θ (cot θ + cosec θ)~~

~~Also, $p^2 + 1 = (\text{cosec}\,\theta + \cot\theta)^2 + 1$~~

~~$= \text{cosec}^2\,\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta + 1$~~

~~$= \text{cosec}^2\,\theta + 1 + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$~~

~~$= \text{cosec}^2\,\theta + \text{cosec}^2\,\theta + 2\,\text{cosec}\,\theta\cot\theta$~~

~~= 2 cosec θ (cosec θ + cot θ)~~

~~Now, consider L.H.S. =~~ $\frac{(p^2-1)}{(p^2+1)}$

~~=~~ $\frac{2\cot\theta\,(\cot\theta+\operatorname{cosec}\theta)}{2\operatorname{cosec}\theta\,(\operatorname{cosec}\theta+\cot\theta)}$

~~= cot θ/cosec θ~~

~~= cos θ~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 12**~~

~~Prove that~~

~~**Solution:**~~

~~Consider R.H.S. =~~ $\frac{1-\cos A}{1+\cos A}$ ~~and rationalize:~~

$\frac{1-\cos A}{1+\cos A} = \frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$

~~=~~ $\frac{(1-\cos A)^2}{1-\cos^2 A}$

~~=~~ $\frac{(1-\cos A)^2}{\sin^2 A}$

~~=~~ $\left(\frac{1-\cos A}{\sin A}\right)^2$

~~=~~ $\left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$

~~= (cosec A – cot A)^2~~

~~= L.H.S.~~

~~Hence, proved.~~

~~**Question: 13**~~

~~If 5 cot θ = 3, s~~

~~**Solution:**~~

~~Given: 5cot θ = 3~~

~~Therefore cot θ = 3/5~~

~~Consider~~ $\frac{(5\sin\theta-3\cos\theta)}{(4\sin\theta+3\cos\theta)}$ ~~and divide numerator and denominator by sin θ:~~

$\frac{\frac{(5\sin\theta-3\cos\theta)}{\sin\theta}}{\frac{4\sin\theta+3\cos\theta}{\sin\theta}} = \frac{5-3\cot\theta}{4+3\cot\theta}$

~~=~~ $\frac{5-3\left(\frac{3}{5}\right)}{4+3\left(\frac{3}{5}\right)}$

~~=~~ $\frac{25-9}{20+9}$

~~= 16/29~~

~~Hence, showed.~~

~~**Question: 14**~~

~~Prove that (sin 3~~

~~**Solution:**~~

~~Consider L.H.S. = (sin 32° cos 58° + cos 32° sin 58°)~~

~~= sin 32° cos (90° – 32°) + cos 32° sin (90° – 32°)~~

~~= $\sin^2 32° + \cos^2 32°$~~

~~= 1~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 15**~~

~~If x = a sin θ +~~

~~**Solution:**~~

~~Given: a sin θ + b cos θ = x …….(1)~~

~~a cos θ – b sin θ = y …….(2)~~

~~Square equation (1) and (2) on both sides:~~

~~$a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \cos \theta \sin \theta = x^2$ …….(3)~~

~~$a^2 \cos^2 \theta + b^2 \sin^2 \theta – 2ab \cos \theta \sin \theta = y^2$ ……..(4)~~

~~Add equation (3) and (4):~~

~~$[a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \cos \theta \sin \theta] + [a^2 \cos^2 \theta + b^2 \sin^2 \theta – 2ab \cos \theta \sin \theta] = x^2 + y^2$~~

~~$\Rightarrow a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta) = x^2 + y^2$~~

~~$\Rightarrow a^2 + b^2 = x^2 + y^2$~~

~~Hence, proved.~~

~~**Question: 16**~~

~~Prove that~~

~~**Solution:**~~

~~Consider L.H.S. = $\frac{(1+\sin\theta)}{(1-\sin\theta)}$~~

~~Multiply numerator and denominator by (1 + sin θ):~~

~~$= \frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}$~~

~~$= \frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}$~~

~~$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$~~

~~$= \left(\frac{1+\sin\theta}{\cos\theta}\right)^2$~~

~~$= [(1/\cos \theta) + (\sin \theta/\cos \theta)]^2$~~

~~$= (\sec \theta + \tan \theta)^2$~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 17**~~

~~Prove that~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{1}{\sec\theta-\tan\theta}-\frac{1}{\cos\theta}$

~~Multiply and divide the first term by (sec θ + tan θ):~~

~~=~~ $\left(\frac{1}{\sec\theta-\tan\theta}\times\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\right)-\frac{1}{\cos\theta}$

~~=~~ $\frac{\sec\theta+\tan\theta}{\sec^2\theta-\tan^2\theta}$ ~~– sec θ~~

~~= sec θ + tan θ – sec θ (∵1 + tan~~^{2} ~~θ = sec~~^{2} ~~θ)~~

~~= tan θ~~

~~Consider R.H.S. =~~ $\frac{1}{\cos\theta}-\frac{1}{\sec\theta+\tan\theta}$

~~Multiply and divide the second term by (sec θ – tan θ):~~

~~=~~ $\frac{1}{\cos\theta}-\left(\frac{1}{\sec\theta+\tan\theta}\times\frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}\right)$

~~= sec θ –~~ $\frac{\sec\theta-\tan\theta}{\sec^2\theta-\tan^2\theta}$

~~= sec θ – sec θ + tan θ (∵1 + tan~~^{2} ~~θ = sec~~^{2} ~~θ)~~

~~= tan θ~~

~~Therefore, L.H.S. = R.H.S.~~

~~Hence, proved.~~

~~**Question: 18**~~

~~Prove that~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\sin A\left(1-2\sin^2 A\right)}{\cos A(2\cos^2 A-1)}$

~~=~~ $\frac{\sin A\left(\sin^2 A+\cos^2 A-2\sin^2 A\right)}{\cos A\,(2\cos^2 A-\sin^2 A-\cos^2 A)}$

~~=~~ $\frac{\sin A\left(\cos^2 A-\sin^2 A\right)}{\cos A(\cos^2 A-\sin^2 A)}$

~~= sin A/cos A~~

~~= tan A~~

~~= R.H.S.~~

~~Hence, proved.~~

~~**Question: 19**~~

~~Prove that~~

~~**Solution:**~~

~~Consider L.H.S. =~~ $\frac{\tan A}{(1-\cot A)}+\frac{\cot A}{(1-\tan A)}$

~~=~~ $\frac{\tan A}{\left(1-\frac{1}{\tan A}\right)}+\frac{\cot A}{(1-\tan A)}$

~~=~~ $\frac{\tan^2 A}{\tan A-1}+\frac{\cot A}{(1-\tan A)}$

~~=~~ $\frac{\tan^2 A}{\tan A-1}-\frac{\cot A}{(\tan A-1)}$

~~=~~ $\frac{\tan^2 A-\cot A}{\tan A-1}$

~~=~~ $\frac{\tan^2 A-\frac{1}{\tan A}}{\tan A-1}$

~~=~~ $\frac{\tan^3 A-1}{\tan A\ (\tan A-1)}$

~~=~~ $\frac{(\tan A-1)(\tan^2 A+1+\tan A)}{\tan A(\tan A-1)}$

~~=~~ $\frac{(\tan^2 A+1+\tan A)}{\tan A}$

~~= tan A + (1/tan A) + 1~~

~~= 1 + tan A + cot A~~

~~= R.H.S.~~

~~Hence, proved.~~

~~Question: 20~~

~~If sec 5A = cosec~~

~~Solution:~~

~~We are given that: sec 5A = cosec (A – 36°)~~

~~∴We can rewrite it as: cosec (90° – 5A) = cosec (A – 36°)~~

~~On comparing both sides, we get,~~

~~90° – 5A = A – 36°~~

~~⇒ A + 5A = 90° + 36°~~

~~⇒ 6A = 126°~~

~~⇒ A = 21°~~

~~Hence, proved.~~