Chapter : 26. THREE-DIMENSIONAL GEOMETRY

Exercise : 26A

Question: 1

If a point lies o

Solution:

X and y coordinates of a point are its distance from the origin along or parallel to the horizontal x-axis and y-axis. To measure the x and y coordinates, you must move either to the left of the origin or to its right. In case of a point on the z-axis, you do not move to the right or to the left of the origin. Hence x and y coordinates are 0 for a point on the z-axis.

Question: 2

If a point lies o

Solution:

x-coordinate is the distance of a point from the origin parallel or along the x-axis. To measure the x coordinate, you must move either to the left of the origin or to its right. In case of a point lying on the yz-plane, you do not move to the right or to the left of the origin. Hence x coordinate is 0 for a point on the yz-plane.

Question: 3

In which plane do

Solution:

Here the x, y, z coordinates of the point are 4, -3, 0. As the distance of point along the z-axis is 0, the plane in which the point lies is the xy-plane.

Question: 4

In which octant d

Solution:

The position of a point in a octant is signified by the signs of the x, y, z coordinates.

Here is a table showing signs of the x, y, z coordinates in all the octants.

Number	x sign	y sign	z sign
I	+	+	+
II	-	+	+
III	-	-	+
IV	+	-	+
V	+	+	-
VI	-	+	-
VII	-	-	-
VIII	+	-	-

According to the table

(i) (-4, -1, -6) lies in octant VII

(ii) (2, 3, -4) lies in octant V

(iii) (-6, 5, -1) lies in octant VI

(iv) (4, -3, -2) lies in octant VIII

(v) (-1, -6, 5) lies in octant III

(vi) (4, 6, 8) lies in octant I

Exercise : 26B

Question: 1

Find the distance

Solution:

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

(i) A(5, 1, 2) and B(4, 6, -1)

Here, (x_1,y_1,z_1) = (5, 1, 2)

(x_2,y_2,z_2)= (4, 6, -1)

Therefore,

D = $\sqrt{(4\ -5)^2+(6-1)^2+(-1-2)^2}$

= $\sqrt{(-1)^2+(5)^2+(-3)^2}$

= $\sqrt{1+25+9}$

=$\sqrt{35}$

Distance between points A and B is $\sqrt{35}$.

(ii) P(1, -1, 3) and Q(2, 3, -5)

Here, (x_1,y_1,z_1)= (1, -1, 3)

(x_2,y_2,z_2)= (2, 3, -5)

Therefore,

D = $\sqrt{(2-1)^2+(3-(-1))^2+(-5-3)^2}$

= $\sqrt{(1)^2+(4)^2+(-8)^2}$

= $\sqrt{1+16+64}$

=$\sqrt{81}$ = 9

Distance between points P and Q are 9 units.

(iii) R(1, -3, 4) and S(4, -2, -3)

Here, (x_1,y_1,z_1)= (1, -3, 4)

(x_2,y_2,z_2)= (4, -2, -3)

Therefore,

D = $\sqrt{(4-1)^2+(-2-(-3))^2+(-3-4)^2}$

= $\sqrt{(3)^2+(1)^2+(-7)^2}$

= $\sqrt{9+1+49}$

=$\sqrt{59}$

Distance between points R and S is $\sqrt{59}$ units.

(iv) C(9, -12, -8) and the origin

Coordinates of origin are (0, 0, 0)

Here, (x_1,y_1,z_1)= (9, -12, -8)

(x_2,y_2,z_2)= (0, 0, 0)

Therefore,

D = $\sqrt{(0-9)^2+(0-(-12))^2+(0-(-8))^2}$

= $\sqrt{(-9)^2+(12)^2+(8)^2}$

$= \sqrt{81 + 144 + 64}$

$= \sqrt{289} = 17$

Distance between points C and origin is 17 units.

Question: 2

Show that the poi

Solution:

To prove: Points A, B, C form equilateral triangle.

Formula:

The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$(x_1, y_1, z_1) = (1, -1, -5)$

$(x_2, y_2, z_2) = (3, 1, 3)$

$(x_3, y_3, z_3) = (9, 1, -3)$

Length AB $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(3-1)^2 + (1-(-1))^2 + (3-(-5))^2}$

$= \sqrt{(2)^2 + (2)^2 + (8)^2}$

$= \sqrt{4 + 4 + 64}$

$= \sqrt{72} = 6\sqrt{2}$

Length BC $= \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(9-3)^2 + (1-1)^2 + (-3-3)^2}$

$= \sqrt{(6)^2 + (0)^2 + (-6)^2}$

$= \sqrt{36 + 0 + 36}$

$= \sqrt{72} = 6\sqrt{2}$

Length AC $= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(9-1)^2 + (1-(-1))^2 + (-3-(-5))^2}$

$= \sqrt{(8)^2 + (2)^2 + (2)^2}$

$= \sqrt{64 + 4 + 4}$

$= \sqrt{72} = 6\sqrt{2}$

Hence, AB = BC = AC

Therefore, Points A, B, C make an equilateral triangle.

Question: 3

Show that the poi

Solution:

To prove: Points A, B, C form isosceles triangle.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

$(x_1,y_1,z_1)=(4, 6, -3)$

$(x_2,y_2,z_2)=(0, 2, 3)$

$(x_3,y_3,z_3)=(-4, -4, -1)$

Length AB $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$=\sqrt{(0-4)^2+(2-6)^2+(3-(-3))^2}$

$=\sqrt{(-4)^2+(-4)^2+(6)^2}$

$=\sqrt{16+16+36}$

$=\sqrt{68}=2\sqrt{17}$

Length BC $=\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

$=\sqrt{(-4-0)^2+(-4-2)^2+(-1-3)^2}$

$=\sqrt{(-4)^2+(-6)^2+(-4)^2}$

$=\sqrt{16+36+16}$

$=\sqrt{68}=2\sqrt{17}$

Length AC $=\sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2}$

$=\sqrt{(-4-4)^2+(-4-6)^2+(-1-(-5))^2}$

$=\sqrt{(-8)^2+(-10)^2+(2)^2}$

$=\sqrt{64+100+4}$

$=\sqrt{168}$

Here, AB = BC

∴ vertices A, B, C forms an isosceles triangle.

Question: 4

Show that the poi

Solution:

To prove: Points A, B, C form isosceles triangle.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

$(x_1,y_1,z_1)=(0, 1, 2)$

(x_2,y_2,z_2)= (2, -1, 3)

(x_3,y_3,z_3)= (1, -3, 1)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(2-0)^2+(-1-1)^2+(3-2)^2}$

= $\sqrt{(2)^2+(-2)^2+(1)^2}$

= $\sqrt{4+4+1}$

= $\sqrt{9}$

Length BC = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

= $\sqrt{(1-2)^2+(-3+1)^2+(1-3)^2}$

= $\sqrt{(-1)^2+(-2)^2+(-2)^2}$

= $\sqrt{1+4+4}$

= $\sqrt{9}$

Length AC = $\sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2}$

= $\sqrt{(1-0)^2+(-3-1)^2+(1-2)^2}$

= $\sqrt{(1)^2+(-4)^2+(-1)^2}$

= $\sqrt{1+16+1}$

= $\sqrt{18}$

Also, $AB^2 + BC^2 = 9 + 9 = 18 = AC^2$

Therefore, points A, B, C forms an isosceles right-angled triangle.

Question: 5

Show that the poi

Solution:

To prove: Points A, B, C, D form square.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (1, 1, 1)

(x_2,y_2,z_2)= (-2, 4, 1)

(x_3,y_3,z_3)= (-1, 5, 5)

(x_4,y_4,z_4)= (2, 2, 5)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(-2-1)^2+(4-1)^2+(1-1)^2}$

$= \sqrt{(-3)^2 + (3)^2 + (0)^2}$

$= \sqrt{9 + 9 + 0}$

$= \sqrt{18}$

Length BC $= \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(-1 + 2)^2 + (5 - 4)^2 + (5 - 1)^2}$

$= \sqrt{(1)^2 + (1)^2 + (4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

Length CD $= \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2}$

$= \sqrt{(2 + 1)^2 + (2 - 5)^2 + (5 - 5)^2}$

$= \sqrt{(3)^2 + (3)^2 + (0)^2}$

$= \sqrt{9 + 9 + 0}$

$= \sqrt{18}$

Length AD $= \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2}$

$= \sqrt{(2 - 1)^2 + (2 - 1)^2 + (5 - 1)^2}$

$= \sqrt{(1)^2 + (1)^2 + (4)^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

Length AC $= \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(-1 - 1)^2 + (5 - 1)^2 + (5 - 1)^2}$

$= \sqrt{(-2)^2 + (4)^2 + (4)^2}$

$= \sqrt{4 + 16 + 16}$

$= \sqrt{36}$

Length BD $= \sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

$= \sqrt{(2 + 2)^2 + (2 - 4)^2 + (5 - 1)^2}$

$= \sqrt{(4)^2 + (-2)^2 + (4)^2}$

$= \sqrt{16 + 4 + 16}$

$= \sqrt{36}$

Here, AB = BC = CD = AD

Also, AC = BD

This means all the sides are the same and diagonals are also equal.

Hence vertices A, B, C, D form a square.

Question: 6

Show that the poi

Solution:

To prove: Points A, B, C, D form parallelogram.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (1, 2, 3)

(x_2,y_2,z_2)= (-1, -2, -1)

(x_3,y_3,z_3)= (2, 3, 2)

(x_4,y_4,z_4)= (4, 7, 6)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(-1-1)^2+(-2-2)^2+(-1-3)^2}$

$= \sqrt{(-2)^2+(-4)^2+(-4)^2}$

$= \sqrt{4+16+16}$

$= \sqrt{36}$

Length BC = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

$= \sqrt{(2+1)^2+(3+2)^2+(2+1)^2}$

$= \sqrt{(3)^2+(5)^2+(3)^2}$

$= \sqrt{9+25+9}$

$= \sqrt{43}$

Length CD = $\sqrt{(x_4-x_3)^2+(y_4-y_3)^2+(z_4-z_3)^2}$

$= \sqrt{(4-2)^2+(7-3)^2+(6-2)^2}$

$= \sqrt{(2)^2+(4)^2+(4)^2}$

$= \sqrt{4+16+16}$

$= \sqrt{36}$

Length AD = $\sqrt{(x_4-x_1)^2+(y_4-y_1)^2+(z_4-z_1)^2}$

$= \sqrt{(4-1)^2+(7-2)^2+(6-3)^2}$

$= \sqrt{(3)^2+(5)^2+(3)^2}$

$= \sqrt{9+25+9}$

$= \sqrt{43}$

Length AC = $\sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2}$

$= \sqrt{(2-1)^2+(3-2)^2+(2-3)^2}$

$= \sqrt{(1)^2+(1)^2+(-1)^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

Length BD $= \sqrt{(x_4-x_2)^2+(y_4-y_2)^2+(z_4-z_2)^2}$

$= \sqrt{(4+1)^2+(7+2)^2+(6+1)^2}$

$= \sqrt{(5)^2+(9)^2+(7)^2}$

$= \sqrt{25+81+49}$

$= \sqrt{155}$

Here, AB = CD which are opposite sides of polygon.

BC = AD which are opposite sides of polygon.

Also the diagonals AC and BD are not equal in length.

Hence, the polygon is not a rectangle.

Question: 7

Show that the poi

Solution:

To prove: Points P, Q, R, S forms rectangle.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D$=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (2, 3, 5)

(x_2,y_2,z_2)= (-4, 7, -7)

(x_3,y_3,z_3)= (-2, 1, -10)

(x_4,y_4,z_4)= (4, -3, 2)

Length PQ $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(-4-2)^2+(7-3)^2+(-7-5)^2}$

$= \sqrt{(-6)^2+(4)^2+(-12)^2}$

$= \sqrt{36+16+144}$

$= \sqrt{196}$

Length QR $= \sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

$= \sqrt{(-2+4)^2+(1-7)^2+(-10+7)^2}$

$= \sqrt{(2)^2+(-6)^2+(-3)^2}$

$= \sqrt{4+36+9}$

$= \sqrt{49}$

Length RS = $\sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2}$

$= \sqrt{(4+2)^2 + (-3-1)^2 + (2+10)^2}$

$= \sqrt{(6)^2 + (-4)^2 + (12)^2}$

$= \sqrt{36 + 16 + 144}$

$= \sqrt{196}$

Length PS = $\sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2}$

$= \sqrt{(4-2)^2 + (-3-3)^2 + (2-5)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

$= \sqrt{4 + 36 + 9}$

$= \sqrt{49}$

Length PR = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(-2-2)^2 + (1-3)^2 + (-10-5)^2}$

$= \sqrt{(-4)^2 + (-2)^2 + (-15)^2}$

$= \sqrt{16 + 4 + 225}$

$= \sqrt{245}$

Length QS = $\sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

$= \sqrt{(4+4)^2 + (-3-7)^2 + (2+7)^2}$

$= \sqrt{(8)^2 + (-10)^2 + (9)^2}$

$= \sqrt{64 + 100 + 81}$

$= \sqrt{245}$

Here, PQ = RS which are opposite sides of polygon.

QR = PS which are opposite sides of polygon.

Also the diagonals PR = QS.

Hence, the polygon is a rectangle.

Question: 8

Show that the poi

Solution:

To prove: Points P, Q, R, S forms rhombus.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

$(x_1,y_1,z_1) = (1, 3, 4)$

$(x_2,y_2,z_2) = (-1, 6, 10)$

$(x_3,y_3,z_3) = (-7, 4, 7)$

$(x_4,y_4,z_4) = (-5, 1, 1)$

Length PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(-1-1)^2 + (6-3)^2 + (10-4)^2}$

$= \sqrt{(-2)^2 + (3)^2 + (6)^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

Length QR = $\sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2 + (z_3 - z_2)^2}$

$= \sqrt{(-7+1)^2 + (4-6)^2 + (7-10)^2}$

$= \sqrt{(-6)^2 + (-2)^2 + (-3)^2}$

$= \sqrt{36 + 34 + 9}$

$= \sqrt{49}$

Length RS = $\sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2}$

$= \sqrt{(-5+7)^2 + (1-4)^2 + (1-7)^2}$

$= \sqrt{(2)^2 + (-3)^2 + (-6)^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

Length PS = $\sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2}$

$= \sqrt{(-5-1)^2 + (1-3)^2 + (1-4)^2}$

$= \sqrt{(-6)^2 + (-2)^2 + (-3)^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

Length PR = $\sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$

$= \sqrt{(-7-1)^2 + (4-3)^2 + (7-4)^2}$

$= \sqrt{(-8)^2 + (1)^2 + (3)^2}$

$= \sqrt{64 + 1 + 9}$

$= \sqrt{74}$

Length QS = $\sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

$= \sqrt{(-5+1)^2 + (1-6)^2 + (1-10)^2}$

$= \sqrt{(-4)^2 + (-5)^2 + (-9)^2}$

$= \sqrt{16 + 25 + 81}$

$= \sqrt{122}$

Here, PQ = RS =QR = PS .

Also the diagonals PR ≠ QS.

Hence, the polygon is a rhombus as all sides are equal and diagonals are not equal.

Question: 9

Show that D(-1, 4

Solution:

To prove: D is circumcenter of triangle ABC

Let us consider D as circumcenter of triangle ABC.

∴ AD = BC = CD.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$D=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

(x_1,y_1,z_1)= (3, 2, -5)

(x_2,y_2,z_2)= (-3. 8, -5)

(x_3,y_3,z_3)= (-3, 2, 1)

(x_4,y_4,z_4) = (-1, 4, -3)

Length AD = $\sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2 + (z_4 - z_1)^2}$

$= \sqrt{(-1 - 3)^2 + (4 - 2)^2 + (-3 + 5)^2}$

$= \sqrt{(-4)^2 + (2)^2 + (2)^2}$

$= \sqrt{16 + 4 + 4}$

$= \sqrt{24}$

Length BD = $\sqrt{(x_4 - x_2)^2 + (y_4 - y_2)^2 + (z_4 - z_2)^2}$

$= \sqrt{(-1 + 3)^2 + (4 - 8)^2 + (-3 + 5)^2}$

$= \sqrt{(2)^2 + (-4)^2 + (2)^2}$

$= \sqrt{4 + 16 + 4}$

$= \sqrt{24}$

Length CD = $\sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2 + (z_4 - z_3)^2}$

$= \sqrt{(-1 + 3)^2 + (4 - 2)^2 + (-3 - 1)^2}$

$= \sqrt{(2)^2 + (2)^2 + (-4)^2}$

$= \sqrt{4 + 4 + 16}$

$= \sqrt{24}$

Hence, the condition is consistent.

Hence, D is circumcenter of triangle ABC.

Question: 10 A

Show that the fol

Solution:

To prove: the 3 points are collinear.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

$D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

$(x_1,y_1,z_1)=(-2, 3, 5)$

$(x_2,y_2,z_2)=(1, 2, 3)$

$(x_3,y_3,z_3)=(7, 0, -1)$

Length AB $= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

$= \sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$

$= \sqrt{(3)^2+(-1)^2+(-2)^2}$

$= \sqrt{9+1+4}$

$= \sqrt{14}$

Length BC $= \sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

$= \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$

$= \sqrt{(6)^2+(-2)^2+(-4)^2}$

$= \sqrt{36+4+16}$

$= \sqrt{56} = 2\sqrt{14}$

Length AC $= \sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2}$

$= \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$

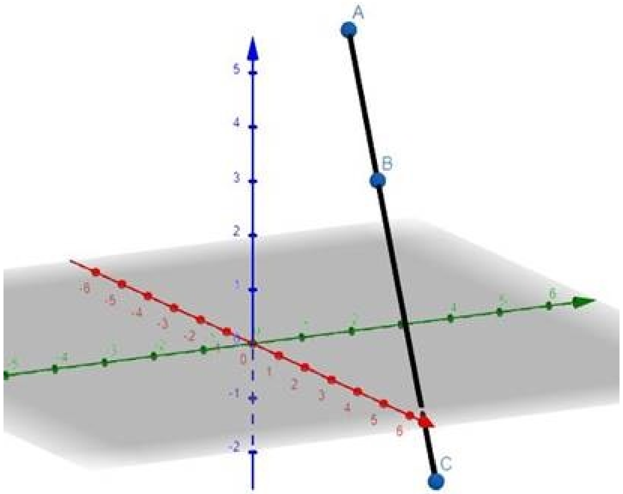
$= \sqrt{(9)^2+(-3)^2+(-6)^2}$

$= \sqrt{81+9+36}$

$= \sqrt{126} = 3\sqrt{14}$

AB + BC $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = AC

Therefore A, B, C are collinear.



Question: 10 B

Show that the fol

Solution:

To prove: the 3 points are collinear.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Here,

(x_1,y_1,z_1)= (3, -5, 1)

(x_2,y_2,z_2)= (-1, 0, 8)

(x_3,y_3,z_3)= (7, -10, -6)

Length AB = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

= $\sqrt{(-1-3)^2+(0+5)^2+(8-1)^2}$

= $\sqrt{(-4)^2+(5)^2+(7)^2}$

= $\sqrt{16+25+49}$

= $\sqrt{90}$ = $3\sqrt{10}$

Length BC = $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

= $\sqrt{(7+1)^2+(-10-0)^2+(-6-8)^2}$

= $\sqrt{(8)^2+(-10)^2+(-14)^2}$

= $\sqrt{64+100+196}$

= $\sqrt{360}$ = $6\sqrt{10}$

Length AC = $\sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2}$

= $\sqrt{(7-3)^2+(-10+5)^2+(-6-1)^2}$

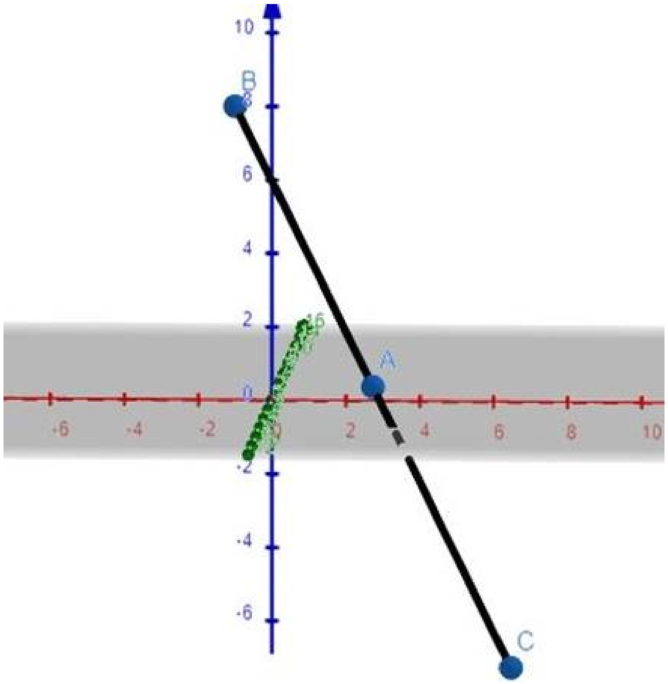
$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90} = 3\sqrt{10}$

BA + BC = $3\sqrt{10}$ + 3 $\sqrt{10}$ =6 $\sqrt{10}$ = BC

Therefore A, B, C are collinear.



Question: 10 C

Show that the fol

Solution:

To prove: the 3 points are collinear.

Formula:

The distance between two points (x_1,y_1,z_1) and (x_2,y_2,z_2) is given by

D=$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Here,

Vertices should be R(-1, 4, -2)

The solution according is

(x_1,y_1,z_1)= (3, -2, 4)

(x_2,y_2,z_2)= (1, 1, 1)

(x_3,y_3,z_3)= (-1, 4,- 2)

Length PQ= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$= \sqrt{(1-3)^2 + (1+2)^2 + (1-4)^2}$

$= \sqrt{(-2)^2 + (3)^2 + (-3)^2}$

$= \sqrt{4 + 9 + 9}$

$= \sqrt{22}$

Length QR= $\sqrt{(x_3-x_2)^2+(y_3-y_2)^2+(z_3-z_2)^2}$

$= \sqrt{(-1-1)^2+(4-1)^2+(-2-1)^2}$

$= \sqrt{(-2)^2+(3)^2+(-3)^2}$

$= \sqrt{4+9+9}$

$= \sqrt{22}$

Length PR = $\sqrt{(x_3-x_1)^2+(y_3-y_1)^2+(z_3-z_1)^2}$

$= \sqrt{(-1-3)^2+(4+2)^2+(-2-4)^2}$

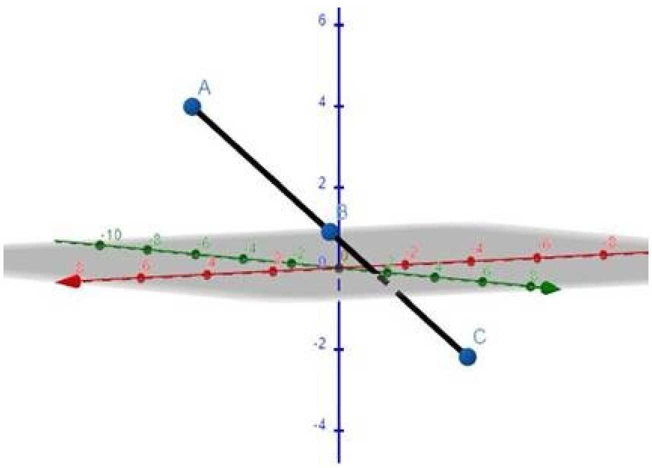
$= \sqrt{(-4)^2+(6)^2+(-6)^2}$

$= \sqrt{16+36+36}$

$= \sqrt{88} = 2\sqrt{22}$

PQ + QR= $\sqrt{22}$ + $\sqrt{22}$ =2 $\sqrt{22}$ = PR

Therefore P, Q, R are collinear.



Question: 11

Find the equation

Solution:

Consider, C(x,y,z) point equidistant from points A(-1, 2, 3) and B(3, 2, 1).

∴ AC = BC

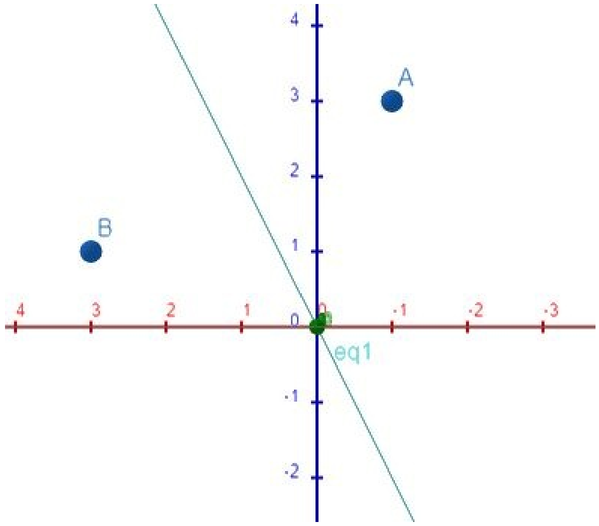
$\sqrt{(x+1)^2+(y-2)^2+(z-3)^2} = \sqrt{(x-3)^2+(y-2)^2+(z-1)^2}$

Squaring both sides,

$(x+1)^2+(y-2)^2+(z-3)^2 = (x-3)^2+(y-2)^2+(z-1)^2$

$x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 - 2z + 1$

8x -4z = 0



Equation of curve is 8x-4z = 0

Question: 12

Find the point on

Solution:

Consider, C(0,y,0) point which lies on y axis and is equidistant from points A(3, 1, 2) and B(5, 5, 2).

∴ AC = BC

$$\sqrt{(0-3)^2+(y-1)^2+(0-2)^2}=\sqrt{(0-5)^2+(y-5)^2+(0-2)^2}$$

Squaring both sides,

$$(0-3)^2+(y-1)^2+(0-2)^2=(0-5)^2+(y-5)^2+(0-2)^2$$

9+ y^2 -2y +1 + 4 = 25 + y^2 -10y +25 + 4

8y = 40

Y = 5

The point C is (0,5,0).

Question: 13

Find the point on

Solution:

Consider, C(0,0,z) point which lies on z axis and is equidistant from points A(1, 5, 7) and B(5, 1, -4).

∴ AC = BC

$$\sqrt{(0-1)^2+(0-5)^2+(z-7)^2}=\sqrt{(0-5)^2+(0-1)^2+(z+4)^2}$$

Squaring both sides,

$$(0-1)^2+(0-5)^2+(z-7)^2=(0-5)^2+(0-1)^2+(z+4)^2$$

1 +25+ z^2 -14z +49 = 25 + 1 + z^2 +8z +16

-22z = -33

Z = 1.5

The point C is (0,0,1.5).

Question: 14

Find the coordina

Solution:

Consider, D(x,y,z) point equidistant from points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0).

∴ AD = 0D

$$\sqrt{(x-a)^2+(y-0)^2+(z-0)^2}=\sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$

Squaring both sides,

$$(x-a)^2+(y-0)^2+(z-0)^2=(x-0)^2+(y-0)^2+(z-0)^2$$

x^2 +2ax + a^2 + y^2 + z^2 = x^2 + y^2 + z^2

a(2x-a) = 0

as a ≠ 0.

X = a/2

∴ BD = 0D

$$\sqrt{(x-a)^2+(y-0)^2+(z-0)^2}=\sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$

Squaring both sides,

$$(x-0)^2+(y-b)^2+(z-0)^2=(x-0)^2+(y-0)^2+(z-0)^2$$

x^2 + $y^{2\,+}$ 2by + b^2 + z^2 = x^2 + y^2 + z^2

b(2y-b) = 0

as b ≠ 0.

y= b/2

∴ CD = 0D

$$\sqrt{(x-0)^2+(y-0)^2+(z-c)^2}=\sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$$

Squaring both sides,

$$(x-0)^2+(y-0)^2+(z-c)^2=(x-0)^2+(y-0)^2+(z-0)^2$$

x^2 + $y^{2\,+}$ z^2 + 2cz + c^2 = x^2 + y^2 + z^2

c(2z-c) = 0

as c ≠ 0.

z= c/2

Therefore, the pint D(a/2,b/2,c/2) is equidistant to points A(a, 0, 0), B(0, b, 0), C(0, 0, c) and O(0, 0, 0).

Question: 15

Find the point in

Solution:

The general point on yz plane is D(0, y, z). Consider this point is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2).

∴ AD = BD

$$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2}=\sqrt{(0-1)^2+(y+1)^2+(z-0)^2}$$

Squaring both sides,

$$(0-3)^2+(y-2)^2+(z+1)^2=(0-1)^2+(y+1)^2+(z-0)^2$$

9+ y^2 -4y + 4 + z^2 + 2z + 1 = 1+ y^2 +2y + 1+ z^2

-6y + 2z + 12 = 0 ….(1)

Also, AD = CD

$$\sqrt{(0-3)^2+(y-2)^2+(z+1)^2} = \sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$$

Squaring both sides,

$$(0-3)^2+(y-2)^2+(z+1)^2 = (0-2)^2+(y-1)^2+(z-2)^2$$

$9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 4 + y^2 - 2y + 1 + z^2 - 4z + 4$

-2y + 6z + 5 = 0 ….(2)

Simultaneously solving equation (1) and (2) we get

Y= 31/16, z = -3/16

The point which is equidistant to the points A(3, 2, -1), B(1, -1, 0) and C(2, 1, 2) is (0, 31/16, -3/16).

Question: 16

Find the point in

Solution:

The general point on xy plane is D(x, y, 0). Consider this point is equidistant to the points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

∴ AD = BD

$$\sqrt{(x-2)^2+(y-0)^2+(0-3)^2} = \sqrt{(x-0)^2+(y-3)^2+(0-2)^2}$$

Squaring both sides,

$$(x-2)^2+(y-0)^2+(0-3)^2 = (x-0)^2+(y-3)^2+(0-2)^2$$

$X^2 - 4x + 4 + y^2 + 9 = X^2 + y^2 - 6y + 9 + 4$

-4x = -6y ….(1)

Also, AD = CD

$$\sqrt{(x-2)^2+(y-0)^2+(0-3)^2} = \sqrt{(x-0)^2+(y-0)^2+(0-1)^2}$$

Squaring both sides,

$$(x-2)^2+(y-0)^2+(0-3)^2 = (x-0)^2+(y-0)^2+(0-1)^2$$

$X^2 - 4x + 4 + y^2 + 9 = X^2 + y^2 + 1$

-4x = -12 ….(2)

Simultaneously solving equation (1) and (2) we get

X = 3, y = 2.

The point which is equidistant to the points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1) is (3, 2, 0).

Exercise : 26C

Question: 1

Find the coordina

Solution:

The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1)

and Q (x_2, y_2, z_2) in the ratio m: n are

$$(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n})$$

Point A(3, 2, 5) and B(-4, 2, -2), m and n are 4 and 3 respectively.

Using the above formula, we get,

$$= (\frac{4 \times -4 + 3 \times 3}{4+3}, \frac{4 \times 2 + 3 \times 2}{4+3}, \frac{4 \times -2 + 3 \times 5}{4+3})$$

$$= (\frac{-7}{7}, \frac{14}{7}, \frac{7}{7})$$

(-1,2,1), is the point which divides the two points in ratio 4 : 3.

Question: 2

Let A(2, 1, -3) a

Solution:

The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1)

and Q (x_2, y_2, z_2) in the ratio m: n are

$$(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n})$$

Point A(2, 1, -3) and B(5, -8, 3), m and n are 2 and 1 respectively.

Using the above formula, we get,

$$(\frac{2 \times 5 + 1 \times 2}{2+1}, \frac{2 \times -8 + 1 \times 1}{2+1}, \frac{2 \times 3 + 1 \times -3}{2+1})$$

$$(\frac{12}{3}, \frac{-15}{3}, \frac{3}{3})$$

(4,-5, 1), is the point of trisection of the segment AB.

Question: 3

Find the coordina

Solution:

The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1)

and Q (x_2, y_2, z_2) externally in the ratio m: n are

$$(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n})$$

Point A(-2, 4, 7) and B(3, -5, 8), m and n are 2 and 1 respectively.

Using the above formula, we get,

$$(\frac{2 \times 3 - 1 \times -2}{2-1}, \frac{2 \times -5 - 1 \times 4}{2-1}, \frac{2 \times 8 - 1 \times 7}{2-1})$$

= (8,-14,9), is the point that divides the two point A and B externally in the ratio 2:1.

Question: 4

Find the ratio in

Solution:

Let the ratio be k:1 in which point R divides point P and point Q.

Using $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n})$, we get,

Here m and n are k and 1. The point which this formula gives is already given, i.e. R(5,4,-6) and the joining points are P(3, 2, -4) and Q(9, 8, -10).

$$(5,4,-6) = \frac{k \times 9 + 1 \times 3}{k+1}, \frac{k \times 8 + 1 \times 2}{k+1}, \frac{k \times -10 + 1 \times -4}{k+1})$$

Taking any point and finding the value of k, we get

$$5 = \frac{k \times 9 + 1 \times 3}{k+1}$$

5k + 5 = 9k + 3

4k = 2

$$K = \frac{1}{2}$$

Therefore, the ratio be 1:2.

Question: 5

Find the ratio in

Solution:

Let the ratio be k:1 in which point R divides point P and point Q.

Using $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n})$, we get,

Here m and n are k and 1. The point which this formula gives is already given, i.e. R(5,9,-14) and the joining points are P(2, -3, 4) and Q(3, 1, -2).

$$(5,9,-14) = (\frac{k \times 3 + 1 \times 2}{k+1}, \frac{k \times 1 + 1 \times -3}{k+1}, \frac{k \times -2 + 1 \times 4}{k+1})$$

Taking any point and finding the value of k, we get

$$5 = \frac{k \times 3 + 1 \times 2}{k+1}$$

5k + 5 = 3k + 2

2k = -3

$$K = -\frac{3}{2}$$

Since, the ratio is -3:2. hence the division is external division.

The external division ratio is 3:2.

Question: 6

Find the ratio in

Solution:

Let the plane XZ divides the points A(-1, -3, 4) and B(4, 2, -1) in ratio k:1.

Hence, using section formula $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n})$, we get

$$= (\frac{k \times 4 + 1 \times -1}{k+1}, \frac{k \times 2 + 1 \times -3}{k+1}, \frac{k \times -1 + 1 \times 4}{k+1})$$

On XZ plane, Y co- ordinate of every point be zero, therefore

$$\frac{k \times 2 + 1 \times -3}{k+1} = 0$$

2k – 3 = 0

$K = \frac{3}{2}$

The ratio is 3:2 in XZ plane which divides the line joined from points A and B.

Question: 7

Find the coordina

Solution:

Let the plane XY divides the points A(3,4,1) and B(5, 1, 6) in ratio k:1.

Hence, using section formula $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n})$, we get

$$= (\frac{k \times 5 + 1 \times 3}{k+1}, \frac{k \times 1 + 1 \times 4}{k+1}, \frac{k \times 6 + 1 \times 1}{k+1})$$

On XY plane, Z co- ordinate of every point be zero, therefore

$$\frac{k \times 6 + 1 \times 1}{k+1} = 0$$

6k + 1 = 0

$K = -\frac{1}{6}$

The ratio is 1:6 externally in XZ plane which divides the line joined from points A and B.

Question: 8

Find the ratio in

Solution:

Let the plane x – 2y + 3z = 5 divides the join of A(3, -5, 4) and B(2, 3, -7) in ratio k:1.

The point which will come by section formula will be in the plane. Putting that in the plane equation will give the point coordinates. The points are A(3, -5, 4) and B(2, 3, -7).

Using section formula, $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n})$, we get

$$= \left(\frac{k \times 2 + 1 \times 3}{k+1}, \frac{k \times 3 + 1 \times -5}{k+1}, \frac{k \times -7 + 1 \times 4}{k+1}\right)$$

Putting this point in the plane equation, we get

$$\frac{2k + 3}{k+1} - 2\left(\frac{3k-5}{k+1}\right) + 3\left(\frac{-7k + 4}{k+1}\right) = 5$$

2k + 3 – 6k + 10 – 21k + 12 = 5k + 5

-25k + 25 = 5k + 5

-30k = -20

$k = \frac{2}{3}$

the ratio is 2:3. And the point of intersection of the plane and the line is $\left(\frac{13}{5}, -\frac{9}{5}, -\frac{2}{5}\right)$.

Question: 9

The vertices of a

Solution:

The given co-ordinates: A(3, 2, 0), B(5, 3, 2) and C(-9, 6, -3)

Now, AB = $\sqrt{(5-3)^2 + (3-2)^2 + (2-0)^2} = \sqrt{4+1+4} = 3$

Also, AC = $\sqrt{(-9-3)^2+(6-2)^2+(-3-0)^2} = \sqrt{144+16+9} = 13$

Now, we have, $\frac{AB}{AC} = \frac{3}{13}$

By the property of internal angle bisector,

$$\frac{AB}{AC} = \frac{BD}{CD}$$

Therefore, $\frac{BD}{CD} = \frac{3}{13}$

Applying the section formula, we get,

D(x, y, z) = $\left(\frac{3\times5-9\times13}{3+13}, \frac{3\times3+6\times13}{3+13}, \frac{3\times2-3\times13}{3+13}\right)$

D(x, y, z) = $(-\frac{102}{16}, \frac{87}{16}, \frac{33}{16})$

Answer.

Question: 10

If the three cons

Solution:

the vertices of the parallelogram be A(3, 4, -3), B(7, 10, -3) and C(5, -2, 7), and the fourth coordinate be D(a,b,c).

the property of parallelogram is the diagonal bisect each other. Therefore,

diagonal AC and BD will bisect each other, and the bisecting point will be equal to the two diagnals. By using section formula, we get

$(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n})$, where m and n are 1 and 1.

Finding the coordinate of mid point of diagonal AC,

$$= (\frac{1\times5+1\times3}{1+1}, \frac{1\times-2+1\times4}{1+1}, \frac{1\times7+1\times-3}{1+1})$$

= (4,1,2)

Now, Finding the coordinate of mid point of diagonal BD,

$$= \left(\frac{1\times a+1\times7}{1+1}, \frac{1\times b+1\times10}{1+1}, \frac{1\times c+1\times-3}{1+1}\right)$$

$$= \left(\frac{a+7}{2}, \frac{b+10}{2}, \frac{c-3}{2}\right)$$

Equating the two mid points, we get

$(4,1,2) = (\frac{a+7}{2}, \frac{b+10}{2}, \frac{c-3}{2})$. Thus,

$$4 = \frac{a+7}{2}$$

$$1 = \frac{b+10}{2}$$

$$2 = \frac{c-3}{2}$$

Therefore,

8 = a + 7

a = 1

b + 10 = 2

b = -8

and

c = 7

therefore the point is (1, -8, 7).

Question: 11

Two vertices of a

Solution:

Since the centroid of a triangle = $\left(\frac{x_2+x_1+x_1}{3}, \frac{y_2+y_1+y_2}{3}, \frac{z_2+z_1+z_3}{3}\right)$

The points are A(2, -4, 3) and B(3, -1, -2), and its centroid is (1, 0, 3). And let its third vertex C(a,b,c).

Using the formula, we get

$$= \left(\frac{2+3+\ a}{3}, \frac{-4-1+b}{3}, \frac{3-1+\ c}{3}\right)$$

$$= \left(\frac{5+\ a}{3}, \frac{-5+b}{3}, \frac{2+\ c}{3}\right)$$

Equating it with the coordinates of centroid, we get

$$1\ = \frac{5+\ a}{3}$$

a = -2

$$0\ = \frac{-5+b}{3}$$

b = -5

and, $\frac{2+c}{3} = 3$

c = 7

therefore, the point is (-2,-5,7)

Question: 12

If the origin is

Solution:

Since, centroid of a triangle is found by $\left(\frac{x_2+x_1+x_1}{3}, \frac{y_2+y_1+y_2}{3}, \frac{z_2+z_1+z_3}{3}\right)$

The points are A(a,1,3) and B(-2, b, -5), and its centroid is (0, 0, 0) and its third vertex C(4,7,c).

Using the formula, we get

$$= \left(\frac{-2+4+\ a}{3}, \frac{1+7+b}{3}, \frac{3-5+\ c}{3}\right)$$

$$= \left(\frac{2+\ a}{3}, \frac{8+b}{3}, \frac{-2+\ c}{3}\right)$$

Equating it with the coordinates of centroid, we get

$$0\ = \frac{2+\ a}{3}$$

a = -2

$$0 = \frac{8+b}{3}$$

b = -8

and, $\frac{-2+c}{3} = 0$

c = 2

therefore, a= -2, b= -8, c=2.

Question: 13

The midpoints of

Solution:

The midpoints of the sides of a triangle are (1, 5, -1), (0, 4, -2) and (2, 3, 4).

Let its vertices be $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$.

The mid point of AB is (1,5,-1), therefore

$$\frac{x_2 + x_1}{2} = 1$$

$x_1 + x_2 = 2$..........eq.1

$$\frac{y_2 + y_1}{2} = 5$$

$y_1 + y_2 = 10$......eq.2

$$\frac{z_2 + z_1}{2} = -1$$

$z_1 + z_2 = -2$........eq.3

Mid point of AC is (2,3,4), therefore

$$\frac{x_3 + x_1}{2} = 2$$

$x_1 + x_3 = 4$..........eq.4

$$\frac{y_3 + y_1}{2} = 3$$

$y_1 + y_3 = 6$......eq.5

$$\frac{z_3 + z_1}{2} = 4$$

$z_1 + z_3 = 8$........eq.6

Mid point of BC is (0,4,-2), therefore

$$\frac{x_2 + x_3}{2} = 0$$

$x_2 + x_3 = 0$..........eq.7

$$\frac{y_2 + y_3}{2} = 4$$

$y_3 + y_2 = 8$......eq.8

$$\frac{z_2 + z_3}{2} = -2$$

$z_3 + z_2 = -4$........eq.9

now, adding the equations 1,4 and 7, and divide it by two we get,

$x_1 + x_2 + x_3 = 3$

now subtracting 1, 4, 7 individually, we get

$x_1 = 3$, $x_2 = -1$ and $x_3 = 1$

now, adding the equations 2,5 and 8, and divide it by two we get,

$y_1 + y_2 + y_3 = 12$

now subtracting 1, 4, 7 individually, we get

$y_1 = 4$, $y_2 = 6$ and $y_3 = 2$

now, adding the equations 3,6 and 9, and divide it by two we get,

$z_1 + z_2 + z_3 = 1$

now subtracting 1, 4, 7 individually, we get

$z_1 = 5$, $z_2 = -7$ and $z_3 = 3$

therefore, the coordinates are A(3,4,5), B(-1,6,-7) and C(1,2,3).