Chapter : 4. PRINCIPLE OF MATHEMATICAL INDUCTION

Exercise : 4

Question: 1

Using the princip

Solution:

To Prove:

$1 + 2 + 3 + 4 + \ldots + n = 1/2\ n(n + 1)$

Steps to prove by mathematical induction:

Let P(n) be a statement involving the natural number n such that

(i) P(1) is true

(ii) P(k + 1) is true, whenever P(k) is true

Then P(n) is true for all $n \in N$

Therefore,

Let P(n): $1 + 2 + 3 + 4 + \ldots + n = 1/2\ n(n + 1)$

Step 1:

$P(1) = 1/2\ 1(1 + 1) = 1/2 \times 2 = 1$

Therefore, P(1) is true

Step 2:

Let P(k) is true Then,

P(k): $1 + 2 + 3 + 4 + \ldots + k = 1/2\ k(k + 1)$

Now,

$1 + 2 + 3 + 4 + \ldots + k + (k + 1) = 1/2\ k(k + 1) + (k + 1)$

$= (k + 1)\{ 1/2\ k + 1\}$

$= 1/2\ (k + 1)\ (k + 2)$

$= P(k + 1)$

Hence, P(k + 1) is true whenever P(k) is true

Hence, by the principle of mathematical induction, we have

$1 + 2 + 3 + 4 + \ldots + n = 1/2\ n(n + 1)$ for all $n \in N$

Hence proved.

Question: 2

Using the princip

Solution:

To Prove:

$2 + 4 + 6 + 8 + \ldots. + 2n = n(n + 1)$

Steps to prove by mathematical induction:

Let P(n) be a statement involving the natural number n such that

(i) P(1) is true

(ii) P(k + 1) is true, whenever P(k) is true

Then P(n) is true for all n ϵ N

Therefore,

Let P(n): 2 + 4 + 6 + 8 + …. + 2n = n(n + 1)

Step 1:

P(1) = 1(1 + 1) = 1 × 2 = 2

Therefore, P(1) is true

Step 2:

Let P(k) is true Then,

P(k): 2 + 4 + 6 + 8 + …. + 2k = k(k + 1)

Now,

2 + 4 + 6 + 8 + …. + 2k + 2(k + 1) = k(k + 1) + 2(k + 1)

= k(k + 1) + 2(k + 1)

= (k + 1) (k + 2)

= P(k + 1)

Hence, P(k + 1) is true whenever P(k) is true

Hence, by the principle of mathematical induction, we have

2 + 4 + 6 + 8 + … + 2n = n(n + 1) for all n ϵ N

Question: 3

Using the princip

Solution:

To Prove:

$$1 + 3^1 + 3^2 + \dots + 3^{n-1} = \frac{3^n - 1}{2}$$

Steps to prove by mathematical induction:

Let P(n) be a statement involving the natural number n such that

(i) P(1) is true

(ii) P(k + 1) is true, whenever P(k) is true

Then P(n) is true for all n ϵ N

Therefore,

Let P(n): $1 + 3^1 + 3^2 + \dots + 3^{n-1} = \frac{3^n - 1}{2}$

Step 1:

P(1) = $\frac{3^1 - 1}{2} = \frac{2}{2}$ = 1

Therefore, P(1) is true

Step 2:

Let P(k) is true Then,

P(k): $1 + 3^1 + 3^2 + \dots + 3^{k-1} = \frac{3^k - 1}{2}$

Now,

$1 + 3^1 + 3^2 + ... + 3^{k-1} + 3^{(k+1)-1} = \frac{3^{(k)} - 1}{2} + 3^{(k+1)-1}$

$= \frac{3^k - 1}{2} + 3^{(k)}$

$= 3^{(k)}(\frac{1}{2} + 1) - \frac{1}{2}$

$= 3^{(k)}(\frac{3}{2}) - \frac{1}{2}$

$= 3^{(k+1)}(\frac{1}{2}) - \frac{1}{2}$

$= \frac{3^{(k+1)} - 1}{2}$

= P(k + 1)

Hence, P(k + 1) is true whenever P(k) is true

Hence, by the principle of mathematical induction, we have

$1 + 3^1 + 3^2 + ... + 3^{n-1} = \frac{3^n - 1}{2}$ for all n ϵ N

Question: 4

Using the princip

Solution:

To Prove:

$2 + 6 + 18 + ... + 2 \times 3^{n-1} = (3^n - 1)$

Steps to prove by mathematical induction:

Let P(n) be a statement involving the natural number n such that

(i) P(1) is true

(ii) P(k + 1) is true, whenever P(k) is true

Then P(n) is true for all n ϵ N

Therefore,

Let P(n): $2 + 6 + 18 + ... + 2 \times 3^{n-1} = (3^n - 1)$

Step 1:

$P(1) = 3^1 - 1 = 3 - 1 = 2$

Therefore, P(1) is true

Step 2:

Let P(k) is true Then,

P(k): $2 + 6 + 18 + ... + 2 \times 3^{k-1} = (3^k - 1)$

Now,

$2 + 6 + 18 + ... + 2 \times 3^{k-1} + 2 \times 3^{k+1-1} = (3^k - 1) + 2 \times 3^k$

$= - 1 + 3 \times 3^k$

$= 3^{k+1-} 1$

= P(k + 1)

Hence, P(k + 1) is true whenever P(k) is true

Hence, by the principle of mathematical induction, we have

$2 + 6 + 18 + \ldots + 2\times3^{n-1} = (3^n - 1)$ for all n ϵ N

Question: 5

Using the princip

Solution:

To Prove:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=\left(1-\frac{1}{2^n}\right)$$

Steps to prove by mathematical induction:

Let P(n) be a statement involving the natural number n such that

(i) P(1) is true

(ii) P(k + 1) is true, whenever P(k) is true

Then P(n) is true for all n ϵ N

Therefore,

Let P(n): $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=\left(1-\frac{1}{2^n}\right)$

Step 1:

$P(1) = 1-\frac{1}{2^1} = 1-\frac{1}{2} = \frac{1}{2}$

Therefore, P(1) is true

Step 2:

Let P(k) is true Then,

P(k):$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^k} = 1-\frac{1}{2^k}$

Now,

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^{k+1}} = 1-\frac{1}{2^k}+\frac{1}{2^{k+1}}$$

$= 1-\frac{1}{2^k}+\frac{1}{2^{k+1}}$

$= 1+\frac{1}{2^k}\left(\frac{1}{2}-1\right)$

$= 1+\frac{1}{2^k}\left(-\frac{1}{2}\right)$

$= 1-\frac{1}{2^{k+1}}$

= P(k + 1)

Hence, P(k + 1) is true whenever P(k) is true

Hence, by the principle of mathematical induction, we have

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}=\left(1-\frac{1}{2^n}\right)$ for all n ϵ N

Question: 6

Using the princip

Solution:

To Prove:

$1^2 + 3^2 + 5^2 + 7^2 + \ldots + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Steps to prove by mathematical induction:

Let P(n) be a statement involving the natural number n such that

(i) P(1) is true

(ii) P(k + 1) is true, whenever P(k) is true

Then P(n) is true for all n ϵ N

Therefore,

Let P(n): $1^2 + 3^2 + 5^2 + 7^2 + \ldots + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Step 1:

$P(1) = \frac{1(2-1)(2+1)}{3} = \frac{3}{3} = 1$

Therefore, P(1) is true

Step 2:

Let P(k) is true Then,

P(k): $1^2 + 3^2 + 5^2 + 7^2 + \ldots + (2k - 1)^2 = \frac{k(2k-1)(2k+1)}{3}$

Now,

$1^2 + 3^2 + 5^2 + 7^2 + \ldots + (2(k + 1)-1)^2 = \frac{k(2k-1)(2k+1)}{3} + (2k + 2 - 1)^2$

$= \frac{k(2k-1)(2k+1)}{3} + (2k + 1)^2$

$= (2k + 1)[\frac{k(2k-1)}{3} + 2k + 1]$

$= (2k + 1)[\frac{2k^2-k+6k+3}{3}]$

$= (2k + 1)[\frac{2k^2+5k+3}{3}]$

$= (2k + 1)[\frac{(k+1)(2k+3)}{3}]$ (Splitting the middle term)

$= \frac{(k+1)(2k+1)(2k+3)}{3}$

= P(k + 1)

Hence, P(k + 1) is true whenever P(k) is true

Hence, by the principle of mathematical induction, we have

$1^2 + 3^2 + 5^2 + 7^2 + \ldots + (2n - 1)^2 = \frac{n(2n-1)(2n+1)}{3}$ for all n ϵ N

Question: 7

Using the princip

Solution:

To Prove:

$1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + n \times 2^n = (n - 1)2n + 1 + 2$

Let us prove this question by principle of mathematical induction (PMI)

Let P(n): $1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + \ldots\ldots + n \times 2^n$

For n = 1

LHS = 1 × 2 = 2

RHS = $(1 - 1) \times 2^{(1 + 1)} + 2$

= 0 + 2 = 2

Hence, LHS = RHS

P(n) is true for n 1

Assume P(k) is true

$1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + k \times 2^k = (k - 1) \times 2^{k+1} + 2$ ……(1)

We will prove that P(k + 1) is true

$1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + (k + 1) \times 2^{k+1} = ((k + 1) - 1) \times 2^{(k+1)+1} + 2$

$1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + (k + 1) \times 2^{k+1} = (k) \times 2^{k+2} + 2$

$1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + k2^k + (k + 1) \times 2^{k+1} = (k) \times 2^{k+2} + 2$ ……(2)

We have to prove P(k + 1) from P(k), i.e. (2) from (1)

From (1)

$1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + k \times 2^k = (k - 1) \times 2^{k+1} + 2$

Adding $(k + 1) \times 2^{k+1}$ both sides,

$(1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + k \times 2^k) + (k + 1) \times 2^{k+1} = (k - 1) \times 2^{k+1} + 2 + (k + 1) \times 2^{k+1}$

$= k \times 2^{k+1} - 2^{k+1} + 2 + k \times 2^{k+1} + 2^{k+1}$

$= 2k \times 2^{k+1} + 2$

$= k \times 2^{k+2} + 2$

$(1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3 + k \times 2^k) + (k + 1) \times 2^{k+1} = k \times 2^{k+2} + 2$

which is the same as P(k + 1)

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for

where n is a natural number

Put k = n - 1

$(1 \times 2^1 + 2 \times 2^2 + 3 \times 2^3) + n \times 2^n = (n - 1) \times 2^{n+1} + 2$

Hence proved.

Question: 8

Using the princip

Solution:

To Prove:

$$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^n \times 2^{n+1} = \frac{12}{5}(6^n - 1)$$

Let us prove this question by principle of mathematical induction (PMI)

Let P(n): $3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^n \times 2^{n+1}$

For n = 1

LHS = $3 \times 2^2 = 12$

RHS = $\left(\frac{12}{5}\right) \times (6^1 - 1)$

$= \frac{12}{5} \times 5 = 12$

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^k \times 2^{k+1} = \frac{12}{5}(6^k - 1)$ ......(1)

We will prove that P(k + 1) is true

$$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^{k+1} \times 2^{k+2} = \frac{12}{5}(6^{k+1} - 1)$$

$$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^{k+1} \times 2^{k+2} = \frac{12}{5}(6^{k+1}) - \frac{12}{5}$$

$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^k \times 2^{k+1} + 3^{k+1} \times 2^{k+2} =$
$\frac{12}{5}(6^{k+1}) - \frac{12}{5}$ ...(2)

We have to prove P(k + 1) from P(k) ie (2) from (1)

From (1)

$$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^k \times 2^{k+1} = \frac{12}{5}(6^k - 1)$$

Adding $3^{k+1} \times 2^{k+2}$ both sides

$$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots + 3^k \times 2^{k+1} + 3^{k+1} \times 2^{k+2}$$
$$= \frac{12}{5}(6^k - 1) + 3^{k+1} \times 2^{k+2}$$

$$= \frac{12}{5}(6^k - 1) + 3^k \times 2^k \times 12$$

$$= \frac{12}{5}(6^k - 1) + 6^k \times 12$$

$$= \left(6^k\left(\frac{12}{5} + 12\right) - \frac{12}{5}\right)$$

$$= \left(\frac{72}{5}\right) - \frac{12}{5}$$

$$= \frac{12}{5}(6^{k+1}) - \frac{12}{5}$$

$$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots + 3^k \times 2^{k+1} + 3^{k+1} \times 2^{k+2}$$
$$= \frac{12}{5}(6^{k+1}) - \frac{12}{5}$$

which is the same as P(k + 1)

Therefore, P (k + 1) is true whenever P(k) is true.

By the principle of mathematical induction, P(n) is true for×

where n is a natural number

Put k = n - 1

$$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^n \times 2^{n+1} = \frac{12}{5}(6^n) - \frac{12}{5}$$

$3 \times 2^2 + 3^2 \times 2^3 + 3^3 \times 2^4 + \ldots\ldots \times + 3^n \times 2^{n+1} = \frac{12}{5}(6^n - 1)$

Hence proved

Question: 9

Using the princip

Solution:

To Prove:

$$\frac{1}{1} + \frac{1}{(1 + 2)} + \frac{1}{(1 + 2 + 3 + \ldots\ldots + n)} = \frac{2n}{(n + 1)}$$

Let us prove this question by principle of mathematical induction (PMI)

Let P(n): $\frac{1}{1} + \frac{1}{(1+2)} + \frac{1}{(1+2+3+\ldots\ldots+n)} = \frac{2n}{(n+1)}$

For n = 1

LHS = 1

RHS = $\frac{2\times1}{(1+1)}$ = 1

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

$\frac{1}{1} + \frac{1}{(1+2)} + \ldots\ldots + \frac{1}{(1+2+3+\ldots\ldots\times+k)} = \frac{2k}{(k+1)}$ ……(1)

We will prove that P(k + 1) is true

RHS = $\frac{2(k+1)}{(k+1+1)} = \frac{2k+2}{k+2}$

LHS = $\frac{1}{1} + \frac{1}{(1+2)} + \ldots\ldots + \frac{1}{(1+2+3+\ldots\ldots+(k+1))}$

= $\frac{1}{1} + \frac{1}{(1+2)} + \ldots\ldots + \frac{1}{(1+2+3+\ldots\ldots+k)} + \frac{1}{(1+2+3+\ldots\ldots+(k+1))}$ [Writing the last

Second term]

= $\frac{2k}{(k+1)} + \frac{1}{(1+2+3+\ldots\ldots+(k+1))}$ [From 1]

$$= \frac{2k}{(k + 1)} + \frac{1}{\frac{(k + 1) \times (k + 2)}{2}}$$

{ 1 + 2 + 3 + 4 + … + n = [n(n + 1)]/2 put n = k + 1 }

$$= \frac{2k}{(k + 1)} + \frac{2}{(k + 1) \times (k + 2)}$$

$$= \frac{2}{(k + 1)}\left(\frac{k}{1} + \frac{1}{k + 2}\right)$$

= $\frac{2}{k+1}\left(\frac{(k+1)\times(k+1)}{k+2}\right)$

[Taking LCM and simplifying]

= $\frac{2(k+1)}{(k+2)}$

= RHS

Therefore , $\frac{1}{1} + \frac{1}{(1+2)} + \ldots\ldots + \frac{1}{(1+2+3+\ldots\ldots\times+(k+1))} = \frac{2k+2}{k+2}$

LHS = RHS

Therefore, P (k + 1) is true whenever P(k) is true.

By the principle of mathematical induction, P(n) is true for×

where n is a natural number

Put k = n - 1

$$\frac{1}{1} + \frac{1}{(1\ +\ 2)} + \ldots\ldots + \frac{1}{(1\ +\ 2\ +\ 3\ +\ \ldots\ldots\times\ +n)} = \frac{2n}{n\ +\ 1}$$

Hence proved

Question: 10

Using the princip

Solution:

To Prove:

$$\frac{1}{2\times 5} + \frac{1}{(5\times 8)} + \ldots\ldots + \frac{1}{(3n-1)\times(3n\ +\ 2)} = \frac{n}{(6n\ +\ 4)}$$

For n = 1

LHS = $\frac{1}{2\times5} = \frac{1}{10}$

RHS = $\frac{1\times1}{(6+4)} = \frac{1}{10}$

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

$\frac{1}{2\times5} + \frac{1}{(5\times8)} + \ldots\ldots + \frac{1}{(3k-1)\times(3k+2)} = \frac{k}{(6k+4)}$ ......(1)

We will prove that P(k + 1) is true

RHS = $\frac{k+1}{(6(k+1)+4)} = \frac{k+1}{(6k+10)}$

LHS = $\frac{1}{2\times5} + \frac{1}{(5\times8)} + \ldots\ldots + \frac{1}{(3k-1)\times(3k+2)} + \frac{1}{(3(k+1)-1)\times(3(k+1)+2)}$

[Writing the Last second term]

$= \frac{1}{2\times5} + \frac{1}{(5\times8)} + \ldots\ldots + \frac{1}{(3k-1)\times(3k+2)} + \frac{1}{(3(k+1)-1)\times(3(k+1)+2)}$

$= \frac{k}{(6k+4)} + \frac{1}{(3(k+1)-1)\times(3(k+1)+2)}$ [Using 1]

$= \frac{k}{(6k+4)} + \frac{1}{(3k+2)\times(3k+5)}$

$= \frac{k}{(6k+4)} + \frac{1}{(3k+2)\times(3k+5)}$

$= \frac{1}{(3k+2)}\times[\frac{(3k+2)\times(k+1)}{2\times(3k+5)}]$ (Taking LCM and simplifying)

$= \frac{k+1}{(6k+10)}$

= RHS

Therefore, $\frac{1}{2\times5} + \frac{1}{(5\times8)} + \ldots\ldots + \frac{1}{(3k-1)\times(3k+2)} + \frac{1}{(3(k+1)-1)\times(3(k+1)+2)} = \frac{k+1}{(6k+10)}$

LHS = RHS

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for

where n is a natural number

Put k = n - 1

$\frac{1}{2\times5} + \frac{1}{(5\times8)} + \dots\dots\times + \frac{1}{(3n-1)\times(3n+2)} = \frac{n}{(6n+4)}$

Hence proved.

Question: 11

Using the princip

Solution:

To Prove:

$$\frac{1}{1\times4} + \frac{1}{(4\times7)} + \dots\dots + \frac{1}{(3n-2)\times(3n+1)} = \frac{n}{(3n+1)}$$

Let us prove this question by principle of mathematical induction (PMI)

Let P(n): $\frac{1}{1\times4} + \frac{1}{(4\times7)} + \dots\dots + \frac{1}{(3n-2)\times(3n+1)} = \frac{n}{(3n+1)}$

For n = 1

LHS = $\frac{1}{1\times4} = \frac{1}{4}$

RHS = $\frac{1}{(3+1)} = \frac{1}{4}$

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

$= \frac{1}{1\times4} + \frac{1}{(4\times7)} + \dots\dots + \frac{1}{(3k-2)\times(3k+1)} = \frac{k}{(3k+1)}$ ......(1)

We will prove that P(k + 1) is true

RHS = $\frac{k+1}{(3(k+1)+1)} = \frac{k+1}{(3k+4)}$

LHS = $\frac{1}{1\times4} + \frac{1}{(4\times7)} + \dots\dots + \frac{1}{(3(k+1)-2)\times(3(k+1)+1)}$

$= \frac{1}{1\times4} + \frac{1}{(4\times7)} + \dots\dots + \frac{1}{(3k-2)\times(3k+1)} + \frac{1}{(3k+1)\times(3k+4)}$

[Writing the second last term]

$= \frac{k}{(3k+1)} + \frac{1}{(3k+1)\times(3k+4)}$ [Using 1]

$= \frac{1}{(3k+1)}(k + \frac{1}{(3k+4)})$

$= \frac{1}{(3k+1)}(\frac{(3k^2+4k+1)}{(3k+4)})$

$= \frac{k+1}{(3k+4)}$

(Splitting the numerator and cancelling the common factor)

= RHS

LHS = RHS

Therefore, P (k + 1) is true whenever P(k) is true.

By the principle of mathematical induction, P(n) is true for

where n is a natural number

Hence proved.

Question: 12

Using the princip

Solution:

To Prove:

$$\frac{1}{1\times3}+\frac{1}{(3\times5)}+\dots\dots+\frac{1}{(2n-1)\times(2n+1)}=\frac{n}{(2n+1)}$$

Let us prove this question by principle of mathematical induction (PMI)

Let P(n): $\frac{1}{1\times3}+\frac{1}{(3\times5)}+\dots\dots+\frac{1}{(2n-1)\times(2n+1)}=\frac{n}{(2n+1)}$

For n = 1

LHS = $\frac{1}{1\times3}=\frac{1}{3}$

RHS = $\frac{1}{(2+1)}=\frac{1}{3}$

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

$=\frac{1}{1\times3}+\frac{1}{(3\times5)}+\dots\dots+\frac{1}{(2k-1)\times(2k+1)}=\frac{k}{(2k+1)}$ ……(1)

We will prove that P(k + 1) is true

RHS = $\frac{k+1}{(2(k+1)+1)}=\frac{k+1}{(2k+3)}$

LHS = $\frac{1}{1\times3}+\frac{1}{(3\times5)}+\dots\dots+\frac{1}{(2(k+1)-1)\times(2(k+1)+1)}$

$=\frac{1}{1\times3}+\frac{1}{(3\times5)}+\dots\dots+\frac{1}{(2k-1)\times(2k+1)}+\frac{1}{(2k+1)\times(2k+3)}$

[Writing the second last term]

$=\frac{k}{(2k+1)}+\frac{1}{(2k+1)\times(2k+3)}$ [Using 1]

$=\frac{1}{(2k+1)}(k+\frac{1}{(2k+3)})$

$=\frac{1}{(2k+1)}(\frac{(2k^2+3k+1)}{(2k+3)})$

$=\frac{k+1}{(2k+3)}$

(Splitting the numerator and cancelling the common factor)

= RHS

LHS = RHS

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for×

where n is a natural number

Hence proved.

Question: 13

Using the princip

Solution:

To Prove:

$$\frac{1}{2\times 5}+\frac{1}{(5\times 8)}+\ldots\ldots+\frac{1}{(3n-1)\times(3n+2)}=\frac{n}{(6n+4)}$$

For n = 1

LHS = $\frac{1}{2\times 5}=\frac{1}{10}$

RHS = $\frac{1\times 1}{(6+4)}=\frac{1}{10}$

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

$\frac{1}{2\times 5}+\frac{1}{(5\times 8)}+\ldots\ldots+\frac{1}{(3k-1)\times(3k+2)}=\frac{k}{(6k+4)}$ ......(1)

We will prove that P(k + 1) is true

RHS = $\frac{k+1}{(6(k+1)+4)}=\frac{k+1}{(6k+10)}$

LHS = $\frac{1}{2\times 5}+\frac{1}{(5\times 8)}+\ldots\ldots+\frac{1}{(3k-1)\times(3k+2)}+\frac{1}{(3(k+1)-1)\times(3(k+1)+2)}$[Writing the Last second term]

$=\frac{1}{2\times 5}+\frac{1}{(5\times 8)}+\ldots\ldots+\frac{1}{(3k-1)\times(3k+2)}+\frac{1}{(3(k+1)-1)\times(3(k+1)+2)}$

$=\frac{k}{(6k+4)}+\frac{1}{(3(k+1)-1)\times(3(k+1)+2)}$ [Using 1]

$=\frac{k}{(6k+4)}+\frac{1}{(3k+2)\times(3k+5)}$

$=\frac{k}{(6k+4)}+\frac{1}{(3k+2)\times(3k+5)}$

$=\frac{1}{(3k+2)}\times\left[\frac{(3k+2)\times(k+1)}{2\times(3k+5)}\right]$ (Taking LCM and simplifying)

$=\frac{k+1}{(6k+10)}$

= RHS

Therefore, $\frac{1}{2\times 5}+\frac{1}{(5\times 8)}+\ldots\ldots+\frac{1}{(3k-1)\times(3k+2)}+\frac{1}{(3(k+1)-1)\times(3(k+1)+2)}=\frac{k+1}{(6k+10)}$

LHS = RHS

Therefore, P (k + 1) is true whenever P(k) is true.

By the principle of mathematical induction, P(n) is true for

where n is a natural number

Put k = n - 1

$\frac{1}{2\times 5}+\frac{1}{(5\times 8)}+\ldots\ldots\times+\frac{1}{(3n-1)\times(3n+2)}=\frac{n}{(6n+4)}$

Hence proved.

Question: 14

Using the princip

Solution:

To Prove:

$$\left(1 + \frac{3}{1}\right) \times \left(1 + \frac{5}{4}\right) \times \left(1 + \frac{7}{9}\right) \times \ldots\ldots \times \left\{1 + \frac{2n + 1}{n^2}\right\} = (n + 1)^2$$

Let us prove this question by principle of mathematical induction (PMI)

Let P(n): $\left(1 + \frac{3}{1}\right) \times \left(1 + \frac{5}{4}\right) \times \left(1 + \frac{7}{9}\right) \times \ldots\ldots \times \{1 + \frac{2n+1}{n^2}\} = (n + 1)^2$

For n = 1

LHS = $1 + \frac{3}{1} = 4$

RHS = $(1 + 1)^2$ = 4

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

= $\left(1 + \frac{3}{1}\right) \times \left(1 + \frac{5}{4}\right) \times \left(1 + \frac{7}{9}\right) \times \ldots\ldots \times \{1 + \frac{2k+1}{k^2}\} = (k + 1)^2$ ...(1)

We will prove that P(k + 1) is true

RHS = $((k + 1) + 1)^2 = (k + 2)^2$

LHS = $\left(1 + \frac{3}{1}\right) \times \left(1 + \frac{5}{4}\right) \times \left(1 + \frac{7}{9}\right) \times \ldots\ldots \times \{1 + \frac{2(k+1)+1}{(k+1)^2}\}$

[Now writing the second last term]

= $\left(1 + \frac{3}{1}\right) \times \left(1 + \frac{5}{4}\right) \times \left(1 + \frac{7}{9}\right) \times \ldots\ldots \times \left\{1 + \frac{2k+1}{k^2}\right\} \times \{1 + \frac{2(k+1)+1}{(k+1)^2}\}$

= $(k + 1)^2 \times \left\{1 + \frac{2(k+1)+1}{(k+1)^2}\right\}$ [Using 1]

= $(k + 1)^2 \times \left\{1 + \frac{(2k+3)}{(k+1)^2}\right\}$

= $(k + 1)^2 \times \left\{\frac{(k+1)^2 + (2k+3)}{(k+1)^2}\right\}$

= $(k + 1)^2 + (2k + 3)$

= $k^2 + 2k + 1 + 2k + 3$

= $(k + 2)^2$

= RHS

LHS = RHS

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for

where n is a natural number

Hence proved.

Question: 15

Using the princip

Solution:

To Prove:

$\left(1 + \frac{1}{1}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 + \frac{1}{3}\right) \times \ldots\ldots \times \{1 + \frac{1}{n^1}\} = (n + 1)^1$

Let us prove this question by principle of mathematical induction (PMI)

Let P(n): $\left(1 + \frac{1}{1}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 + \frac{1}{3}\right) \times \ldots\ldots \times \{1 + \frac{1}{n^1}\} = (n + 1)^1$

For n = 1

LHS = $1 + \frac{1}{1} = 2$

RHS = $(1 + 1)^1$ = 2

Hence, LHS = RHS

P(n) is true for n = 1

Assume P(k) is true

= $\left(1 + \frac{1}{1}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 + \frac{1}{3}\right) \times \ldots\ldots \times \{1 + \frac{1}{k^1}\} = (k + 1)^1$ ...(1)

We will prove that P(k + 1) is true

RHS = $((k + 1) + 1)^1 = (k + 2)^1$

LHS = $\left(1 + \frac{1}{1}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 + \frac{1}{3}\right) \times \ldots\ldots \times \{1 + \frac{1}{(k+1)^1}\}$

[Now writing the second last term]

= $\left(1 + \frac{1}{1}\right) \times \left(1 + \frac{1}{2}\right) \times \left(1 + \frac{1}{3}\right) \times \ldots\ldots \times \left\{1 + \frac{1}{k^1}\right\} \times \{1 + \frac{1}{(k+1)^1}\}$

= $(k + 1)^1 \times \left\{1 + \frac{1}{(k+1)^1}\right\}$ [Using 1]

= $(k + 1)^1 \times \left\{\frac{(k+1)+1}{(k+1)^1}\right\}$

= $(k + 1)^2 \times \left\{\frac{(k+2)^1}{(k+1)^2}\right\}$

= k + 2

= RHS

LHS = RHS

Therefore, P (k + 1) is true whenever P(k) is true.

By the principle of mathematical induction, P(n) is true for

where n is a natural number

Hence proved.

Question: 16

Using the princip

Solution:

To Prove:

n × (n + 1) × (n + 2) is multiple of 6

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

n × (n + 1) × (n + 2) is multiple of 6

Let P(n): $\mathrm{n} \times (\mathrm{n} + 1) \times (\mathrm{n} + 2)$, which is multiple of 6

For n = 1 P(n) is true since $1 \times (1 + 1) \times (1 + 2) = 6$, which is multiple of 6

Assume P(k) is true for some positive integer k , ie,

= k × (k + 1) × (k + 2) = 6m , where m ∈ N ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true

Consider ,

= (k + 1) × ((k + 1) + 1)× ((k + 1) + 2)

= (k + 1)×{ k + 2 }×{ (k + 2) + 1 }

= [(k + 1)×(k + 2)× (k + 2)] + (k + 1)×(k + 2)

= [k×(k + 1)×(k + 2) + 2×(k + 1)×(k + 2)] + (k + 1)×(k + 2)

= [6m + 2×(k + 1)×(k + 2)] + (k + 1)×(k + 2)

= 6m + 3×(k + 1)×(k + 2)

Now , (k + 1) & (k + 2) are consecutive integers , so their product is even

Then , (k + 1)×(k + 2) = 2×w (even)

Therefore ,

= 6m + 3×[2×w]

= 6m + 6×w

= 6(m + w)

= 6×q where q = (m + w) is some natural number

Therefore

$(k + 1) \times ((k + 1) + 1) \times ((k + 1) + 2)$ is multiple of 6

Therefore, P (k + 1) is true whenever P(k) is true.

By the principle of mathematical induction, P(n) is true for all natural numbers, ie, N

Hence proved.

Question: 17

Using the princip

Solution:

To Prove:

$x^{2n} - y^{2n}$ *is divisible by* $x + y$

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

Let P(n): $x^{2n} - y^{2n}$ *is divisible by* $x + y$

For n = 1 P(n) is true since $x^{2n} - y^{2n} = x^2 - y^2 = (x + y) \times (x - y)$

which is divisible by x + y

Assume P(k) is true for some positive integer k , ie,

= $x^{2k} - y^{2k}$ *is divisible by* $x + y$

Let $x^{2k} - y^{2k} = m \times (x + y)$, where m ∈ N ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true

Consider ,

$= x^{2(k+1)} - y^{2(k+1)}$

$= x^{2k} \times x^2 - y^{2k} \times y^2$

$= x^2(x^{2k} - y^{2k} + y^{2k}) - y^{2k} \times y^2$ [Adding and subtracting y^{2k}]

$= x^2(\mathrm{m} \times (\mathrm{x} + \mathrm{y}) + y^{2k}) - y^{2k} \times y^2$ [Using 1]

$= m \times (x + y)x^2 + y^{2k}x^2 - y^{2k}y^2$

$= m \times (x + y)x^2 + y^{2k}(x^2 - y^2)$

$= m \times (x + y)x^2 + y^{2k}(x - y)(x + y)$

$= (x + y)\{mx^2 + y^{2k}(x - y)\}$, which is factor of (x + y)

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for all natural numbers ie, N

Hence proved

Question: 18

Using the princip

Solution:

To Prove:

$x^{2n-1} - 1\ is\ divisible\ by\ x - 1$

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

Let P(n): $x^{2n-1} - 1\ is\ divisible\ by\ x - 1$

For n = 1

P(n) is true since $x^{2n-1} - 1 = x^{2-1} - 1 = (x - 1)$

which is divisible by x - 1

Assume P(k) is true for some positive integer k , ie,

$= x^{2k-1} - 1\ is\ divisible\ by\ x - 1$

$Let\ x^{2k-1} - 1 = m \times (x - 1)$, where m ∈ N ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true

Consider ,

$= x^{2(k+1)-1} - 1$

$= x^{2k-1} \times x^2 - 1$

$= x^2(x^{2k-1}) - 1$

$= x^2(x^{2k-1} - 1 + 1) - 1$ [Adding and subtracting 1]

$= x^2(\mathrm{m} \times (\mathrm{x} - 1) + 1) - 1$ [Using 1]

$= x^2(\mathrm{m} \times (\mathrm{x} - 1)) + x^2 \times 1 - 1$

$= x^2(\mathrm{m} \times (\mathrm{x} - 1)) + x^2 - 1$

$= x^2(\mathrm{m} \times (\mathrm{x} - 1)) + (x^1 - 1)(\mathrm{x} + 1)$

$= (x - 1)\{mx^2 + (x + 1)\}$, which is factor of (x - 1)

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for all natural numbers, ie, N.

Hence proved.

Question: 19

Using the princip

Solution:

To Prove:

$41^n - 14^n$ *is a divisible of* 27

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

Let P(n): $41^n - 14^n$ *is a divisible of* 27

For n = 1 P(n) is true since $41^n - 14^n = 41^1 - 14^1 = 27$

which is multiple of 27

Assume P(k) is true for some positive integer k , ie,

= $41^n - 14^n$ *is a divisible of* 27

$\therefore\ 41^k - 14^k = m \times 27$, where m ∈ N ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true

Consider ,

$= 41^{k+1} - 14^{k+1}$

$= 41^k \times 41 - 14^k \times 14$

= $41(41^k - 14^k + 14^k) - 14^k \times 14$ [Adding and subtracting 14^k]

$= 41(41^k - 14^k) + 41 \times 14^k - 14^k \times 14$

$= 41(27\mathrm{m}) + 14^k(41 - 14)$ [Using 1]

$= 41(27\mathrm{m}) + 14^k(27)$

$= 27(41m + 14^k)$

$= 27 \times r$, where r = $(41m + 14^k)$ is a natural number

Therefore $41^{k+1} - 14^{k+1}$ is divisible of 27

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for all natural numbers, ie, N.

Hence proved.

Question: 20

Using the princip

Solution:

To Prove:

$4^n + 15n - 1$ *is a divisible of* 9

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

Let P(n): $4^n + 15n - 1$ *is a divisible of* 9

For n = 1 P(n) is true since $4^n + 15n - 1 = 4^1 + 15 \times 1 - 1 = 18$

which is divisible of 9

Assume P(k) is true for some positive integer k , ie,

= $4^k + 15k - 1$ *is a divisible of* 9

$\therefore\ 4^k + 15k - 1 = m \times 9$, where m ∈ N ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true.

Consider ,

$= 4^{k+1} + 15(k + 1) - 1$

$= 4^{k} \times 4 + 15k + 15 - 1$

$= 4^{k} \times 4 + 15k + 14 + (60k + 4) - (60k + 4)$ [Adding and subtracting $60k + 4$]

$= (4^{k+1} + 60k - 4) + 15k + 14 - (60k - 4)$

$= 4(4^{k} + 15k - 1) + 15k + 14 - (60k - 4)$

$= 4(9\mathrm{m}) - 45k + 18$ [Using 1]

$= 4(9\mathrm{m}) - 9(5k - 2)$

$= 9\,[\,(4m) - (5k - 2)]$

$= 9 \times r$, where r = $[\,(4m) - (5k - 2)]$ is a natural number

Therefore $4^{k} + 15k - 1\ is\ a\ divisible\ of\ 9$

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for all natural numbers, ie, N.

Hence proved.

Question: 21

Using the princip

Solution:

To Prove:

$3^{2n+2} - 8n - 9\ is\ a\ divisible\ of\ 8$

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

Let P(n): $3^{2n+2} - 8n - 9\ is\ a\ divisible\ of\ 8$

For n = 1 P(n) is true since

$3^{2n+2} - 8n - 9 = 3^{2+2} - 8 \times 1 - 9 = 81 - 17 = 64$

which is divisible of 8

Assume P(k) is true for some positive integer k , ie,

$= 3^{2k+2} - 8k - 9\ is\ a\ divisible\ of\ 8$

$\therefore\ 3^{2k+2} - 8k - 9 = m \times 8$, where m ∈ N ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true

Consider ,

$= 3^{2(k+1)+2} - 8(k + 1) - 9$

$= 3^{2(k+1)} \times 3^{2} - 8k - 8 - 9$

$= 3^{2}(3^{2(k+1)} - 8k - 9 + 8k + 9) - 8k - 8 - 9$

[Adding and subtracting 8k + 9]

$= 3^{2}(3^{2(k+1)} - 8k - 9) + 3^{2}(8k + 9) - 8k - 17$

$= 9(3^{2k+2} - 8k - 9) + 9(8k + 9) - 8k - 17$

= 9(8m) + 72k + 81 -8k-17 [Using 1]

= 9(8m) + 64k + 64

= 8(9m + 8k + 8)

= 8×r , where r = 9m + 8k + 8 is a natural number

Therefore $3^{2k+2} - 8k - 9$ is a divisible of 8

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for all natural numbers, ie, N.

Hence proved.

Question: 22

Using the princip

Solution:

To Prove:

$2^{3n} - 1$, which is multiple of 7

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

$2^{3n} - 1$ is multiple of 7

Let P(n): $2^{3n} - 1$, which is multiple of 7

For n = 1 P(n) is true since $2^3 - 1 = 8 - 1 = 7$, which is multiple of 7

Assume P(k) is true for some positive integer k , ie,

= $2^{3k} - 1 = 7m$, where m ∈ N ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true

Consider ,

$= 2^{3(k+1)} - 1$

$= 2^{3k} \times 2^3 - 1$

$= 2^{3k} \times 2^3 + 2^3 - 2^3 - 1$ [Adding and subtracting 2^3]

$= 2^3(2^{3k} - 1) + 2^3 - 1$

$= 2^3(7m) + 2^3 - 1$ [Using 1]

$= 2^3(7m) + 7$

$= 7 (2^3m + 1)$

$= 7 \times r$, where r = $2^3m + 1$ is a natural number

Therefore $2^{3n} - 1$ is multiple of 7

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for all natural numbers ie, N

Hence proved

Question: 23

Using the princip

Solution:

To Prove:

$3^n \geq 2^n$

Let us prove this question by principle of mathematical induction (PMI) for all natural numbers

Let P(n): $3^n \geq 2^n$

For n = 1 P(n) is true since $3^n \geq 2^n\ i \times e \times\ 3 \geq 2$, which is true

Assume P(k) is true for some positive integer k , ie,

= $3^k \geq 2^k$ ...(1)

We will now prove that P(k + 1) is true whenever P(k) is true

Consider ,

$= \ 3^{(k\,+\,1)}$

$\therefore\ 3^{(k\,+\,1)}\ =\ 3^k \times 3 >\ 2^k \times 3$ [Using 1]

$=\ 3^k \times 3 >\ 2^k \times 2 \times \frac{3}{2}$ [Multiplying and dividing by 2 on RHS]

$$=\ 3^{k\,+\,1}\ >\ 2^{k\,+\,1} \times \frac{3}{2}$$

Now , $2^{k\,+\,1} \times \frac{3}{2}\ >\ 2^{k\,+\,1}$

$\therefore 3^{k\,+\,1}\ >\ 2^{k\,+\,1}$

Therefore, P (k + 1) is true whenever P(k) is true

By the principle of mathematical induction, P(n) is true for all natural numbers, ie, N.

Hence proved.