Chapter : 17. TRIGONOMETRIC EQUATIONS

Exercise : 17

Question: 1

Find the principa

Solution:

To Find: Principal solution.

[NOTE: The solutions of a trigonometry equation for which $0 \le x < 2\pi$ is called principal solution]

(i) Given: $\sin x = \frac{\sqrt{3}}{2}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \implies x = n\pi + \frac{\pi}{3}(-1)^n$

Put n= 0 $\implies x = 0\times\pi + \frac{\pi}{3}(-1)^0 \implies x = \frac{\pi}{3}$

Put n= 1 $\implies x = 1\times\pi + \frac{\pi}{3}(-1)^1 \implies x = 1\times\pi + \frac{\pi}{3}(-1)^1 \implies x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

So principal solution is x= $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

(ii) Given: $\cos x = \frac{1}{2}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{1}{2} = \cos\frac{\pi}{3} \implies \theta = 2n\pi \pm \alpha$, $n \in I$

Put n= 0 $\implies x = 2n\pi \pm \frac{\pi}{3} \implies x = \frac{\pi}{3}$

Put n= 1 $\implies x = 2\pi \pm \frac{\pi}{3} \implies x = \frac{5\pi}{3}, \frac{7\pi}{3} \implies x = \frac{5\pi}{3}, \frac{7\pi}{3}$

[$\frac{7\pi}{3} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

(iii) Given: $\tan x = \sqrt{3}$

Formula used: $\tan\theta = \tan\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\tan x = \sqrt{3} = \tan\frac{\pi}{3} \implies x = n\pi + \alpha$, $n \in I$

Put n= 0 $\implies x = n\pi + \frac{\pi}{3} \implies x = \frac{\pi}{3}$

Put n= 1 $\Rightarrow$ x = $\pi + \frac{\pi}{3}$ $\Rightarrow$ x = $\frac{4\pi}{3}$ $\Rightarrow$ x = $\frac{4\pi}{3}$

So principal solution is x= $\frac{\pi}{3}$ and $\frac{4\pi}{3}$

(iv) Given: $\cot x = \sqrt{3}$

We know that $\tan\theta \times \cot\theta = 1$

So cotx = $\sqrt{3}$ $\Rightarrow$ tanx = $\frac{1}{\sqrt{3}}$

The formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

tanx = $\frac{1}{\sqrt{3}}$ = $\tan\frac{\pi}{6} \Rightarrow \theta = n\pi + \alpha$, $n \in I$

Put n= 0 $\Rightarrow$ x = $n\pi + \frac{\pi}{6}$ $\Rightarrow$ x = $\frac{\pi}{6}$

Put n= 1 $\Rightarrow$ x = $\pi + \frac{\pi}{6}$ $\Rightarrow$ x = $\frac{7\pi}{6}$

So principal solution is x= $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

(v) Given: cosec x = 2

We know that $\text{cosec}\theta \times \sin\theta = 1$

So sinx = $\frac{1}{2}$

Formula used: $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$, $n \in$

By using above formula, we have

sinx = $\frac{1}{2}$ = $\sin\frac{\pi}{6} \Rightarrow \theta = n\pi + \frac{\pi}{6}(-1)^n$

Put n= 0 $\Rightarrow \theta = 0\times\pi + \frac{\pi}{6}(-1)^0 \Rightarrow \theta = \frac{\pi}{6}$

Put n= 1 $\Rightarrow \theta = 1\times\pi + \frac{\pi}{6}(-1)^1 \Rightarrow \theta = 1\times\pi + \frac{\pi}{6}(-1)^1 \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

So principal solution is x= $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

(vi) Given: sec x = $\frac{2}{\sqrt{3}}$

We know that $\sec\theta \times \cos\theta = 1$

So cosx = $\frac{\sqrt{3}}{2}$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

cosx = $\frac{\sqrt{3}}{2}$ = $\cos\frac{\pi}{6} \Rightarrow x = 2n\pi \pm \alpha$, $n \in I$

Put n= 0 $\Rightarrow$ x = $2n\pi \pm \frac{\pi}{6}$ $\Rightarrow$ x = $\frac{\pi}{6}$

Put n= 1 $\Rightarrow$ x = $2\pi \pm \frac{\pi}{6}$ $\Rightarrow$ x = $\frac{11\pi}{6}, \frac{13\pi}{6}$ $\Rightarrow$ x = $\frac{11\pi}{6}, \frac{13\pi}{6}$

[$\frac{13\pi}{6} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

Question: 2

Find the principa

Solution:

To Find: Principal solution.

(i) Given: $\sin x = \frac{-1}{2}$

Formula used: $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin x = \frac{-1}{2}$ = $-\sin\frac{\pi}{6}$ = $\sin(\pi + \frac{\pi}{6})$= $\sin\frac{7\pi}{6} \Rightarrow x = n\pi + \frac{7\pi}{6}(-1)^n$

Put n= 0 $\Rightarrow$ x = $0\times\pi + \frac{7\pi}{6}(-1)^0 \Rightarrow x = \frac{7\pi}{6}$

Put n= 1 $\Rightarrow$ x = $1\times\pi + \frac{7\pi}{6}(-1)^1 \Rightarrow x = 1\times\pi + \frac{7\pi}{6}(-1)^1 \Rightarrow x = \pi - \frac{7\pi}{6} = -\frac{\pi}{6}$

[NOTE: $-\frac{\pi}{6} = \frac{11\pi}{6}$]

So principal solution is x= $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

(ii) Given: $\sqrt{2}\cos x + 1 = 0 \Rightarrow \cos x = \frac{-1}{\sqrt{2}}$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4} \Rightarrow x = 2n\pi \pm \alpha$, $n \in I$

Put n= 0 $\Rightarrow$ x = $2\times 0\times\pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{4}$

Put n= 1 $\Rightarrow$ x = $2\pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{5\pi}{4}, \frac{11\pi}{4} \Rightarrow x = \frac{5\pi}{4}, \frac{11\pi}{4}$

[$\frac{11\pi}{4} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

(iii) Given: tan x = -1

Formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\tan x = -1 = \tan\frac{3\pi}{4} \Rightarrow x = n\pi + \alpha$, $n \in I$

Put n= 0 $\Rightarrow$ x = nπ + $\frac{3\pi}{4}$ $\Rightarrow$ x = $\frac{3\pi}{4}$

Put n= 1 $\Rightarrow$ x = π + $\frac{3\pi}{4}$ $\Rightarrow$ x = $\frac{7\pi}{4}$ $\Rightarrow$ x = $\frac{7\pi}{4}$

So principal solution is x= $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

(iv) Given: $\sqrt{3}\,\text{cosec}\,x + 2 = 0$ $\Rightarrow$ cosec x = $\frac{-2}{\sqrt{3}}$

We know that cosec$\theta \times$ sinθ = 1

So sinx = $\frac{-\sqrt{3}}{2}$

Formula used: sinθ = sinα $\Rightarrow$ θ = nπ + $(-1)^n\alpha$, n $\in$

By using above formula, we have

sinx = $\frac{-\sqrt{3}}{2}$ = sin$\frac{4\pi}{3}$ $\Rightarrow$ θ = nπ + $\frac{4\pi}{3}(-1)^n$

Put n= 0 $\Rightarrow$ x = 0$\times \pi$ + $\frac{4\pi}{3}(-1)^0$ $\Rightarrow$ x = $\frac{4\pi}{3}$

Put n= 1 $\Rightarrow$ x = 1$\times \pi$ + $\frac{4\pi}{3}(-1)^1$ $\Rightarrow$ x = $1 \times \pi + \frac{4\pi}{3}(-1)^1$ $\Rightarrow$ x = $\pi - \frac{4\pi}{3}$= $\frac{-\pi}{3}$

[NOTE: $\frac{-\pi}{3}$ =$\frac{5\pi}{3}$]

So principal solution is x= $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$

(v) Given: tan x = -$\sqrt{3}$

Formula used: tanθ = tanα $\Rightarrow$ θ = n$\pi \pm \alpha$, n $\in I$

By using above formula, we have

tan x = -$\sqrt{3}$ = tan$\frac{2\pi}{3}$ $\Rightarrow$ x = nπ + α, n $\in I$

Put n= 0 $\Rightarrow$ x = nπ + $\frac{2\pi}{3}$ $\Rightarrow$ x = $\frac{2\pi}{3}$

Put n= 1 $\Rightarrow$ x = π + $\frac{2\pi}{3}$ $\Rightarrow$ x = $\frac{5\pi}{3}$

So principal solution is x= $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

(vi) Given: $\sqrt{3}\sec x + 2 = 0$ $\Rightarrow$ sec x = $\frac{-2}{\sqrt{3}}$

We know that sec$\theta \times$ cosθ = 1

So cosx = $\frac{-\sqrt{3}}{2}$

Formula used: cosθ = cosα $\Rightarrow$ θ = 2n$\pi \pm \alpha$, n $\in I$

By using the above formula, we have

cosx = $\frac{\sqrt{3}}{2}$ = cos$\frac{5\pi}{6}$ $\Rightarrow$ x = 2n$\pi \pm \alpha$, n $\in I$

Put n= 0 $\Rightarrow$ x = 2n$\pi \pm \frac{5\pi}{6}$ $\Rightarrow$ x = $\frac{5\pi}{6}$

Put n= 1 $\Rightarrow$ x = 2$\pi \pm \frac{5\pi}{6}$ $\Rightarrow$ x = $\frac{7\pi}{6}, \frac{17\pi}{6}$ $\Rightarrow$ x = $\frac{7\pi}{6}, \frac{17\pi}{6}$

[$\frac{17\pi}{6} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

Question: 3

Find the general

Solution:

To Find: General solution.

[NOTE: A solution of a trigonometry equation generalized by means of periodicity, is known as general solution]

(i) Given: sin 3x = 0

Formula used: sinθ = 0 $\Rightarrow$ θ = nπ , n $\in I$

By using above formula, we have

sin 3x = 0 $\Rightarrow$ 3x = nπ $\Rightarrow$ x = $\frac{n\pi}{3}$ where n $\in I$

So general solution is x= $\frac{n\pi}{3}$ where n $\in I$

(ii) Given: sin$\frac{3x}{2}$ = 0

Formula used: sinθ = 0 $\Rightarrow$ θ = nπ , n $\in I$

By using above formula, we have

sin $\frac{3x}{2}$ = 0 $\Rightarrow$ $\frac{3x}{2}$ = nπ $\Rightarrow$ x = $\frac{2n\pi}{3}$ where n $\in I$

So general solution is x= $\frac{2n\pi}{3}$ where n $\in I$

(iii) Given: sin$\left(x + \frac{\pi}{5}\right)$ = 0

Formula used: sinθ = 0 $\Rightarrow$ θ = nπ , n $\in I$

By using the above formula, we have

sin$\left(x + \frac{\pi}{5}\right)$ = 0 $\Rightarrow$ $x + \frac{\pi}{5}$ = nπ $\Rightarrow$ x = nπ - $\frac{\pi}{5}$ where n $\in I$

So general solution is x= nπ - $\frac{\pi}{5}$ where n $\in I$

(iv) Given: cos 2x = 0

Formula used: cosθ = 0 $\Rightarrow$ θ = (2n+1)$\frac{\pi}{2}$, n $\in I$

By using above formula, we have

cos 2x = 0 $\Rightarrow$ 2x = (2n+1)$\frac{\pi}{2}$ $\Rightarrow$ x = (2n+1)$\frac{\pi}{4}$ where n $\in I$

So general solution is x= $(2n+1)\frac{\pi}{4}$ where $n \in I$

(v) Given: $\cos\frac{5x}{2} = 0$

Formula used: $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$, $n \in I$

By using the above formula, we have

$\cos\frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{5}$ where $n \in I$

So general solution is x= $(2n+1)\frac{\pi}{5}$ where $n \in I$

(vi) Given: $\cos\left(x + \frac{\pi}{10}\right) = 0$

Formula used: $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$, $n \in I$

By using the above formula, we have

$\cos\left(x + \frac{\pi}{10}\right) = 0 \Rightarrow x + \frac{\pi}{10} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{2} - \frac{\pi}{10} \Rightarrow x = n\pi + \frac{2\pi}{5}$ where $n \in I$

So general solution is x= $n\pi + \frac{2\pi}{5}$ where $n \in I$

(vii) Given: tan 2x = 0

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

By using above formula, we have

$\tan 2x = 0 \Rightarrow 2x = n\pi \Rightarrow x = \frac{n\pi}{2}$ where $n \in I$

So general solution is x= $\frac{n\pi}{2}$ where $n \in I$

(viii) Given: $\tan\left(3x + \frac{\pi}{6}\right) = 0$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

By using above formula, we have

$\tan\left(3x + \frac{\pi}{6}\right) = 0 \Rightarrow 3x + \frac{\pi}{6} = n\pi \Rightarrow 3x = n\pi - \frac{\pi}{6} \Rightarrow x = \frac{n\pi}{3} - \frac{\pi}{18}$ where $n \in I$

So general solution is x = $\frac{n\pi}{3} - \frac{\pi}{18}$ where $n \in I$

(ix) Given: $\tan\left(2x - \frac{\pi}{4}\right) = 0$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

By using above formula, we have

$\tan\left(2x - \frac{\pi}{4}\right) = 0 \Rightarrow 2x - \frac{\pi}{4} = n\pi \Rightarrow 2x = n\pi - \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$ where $n \in I$

So general solution is x = $\frac{n\pi}{2} + \frac{\pi}{8}$ where $n \in I$

Question: 4

Find the general

Solution:

To Find: General solution.

(i) Given: $\sin x = \frac{\sqrt{3}}{2}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \implies x = n\pi + (-1)^n . \frac{\pi}{3}$

So general solution is $x = n\pi + (-1)^n . \frac{\pi}{3}$ where $n \in I$

(ii) Given: $\cos x = 1$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = 1 = \cos(0°) \implies x = 2n\pi$, $n \in I$

So general solution is $x = 2n\pi$ where $n \in I$

(iii) Given: $\sec x = \sqrt{2}$

We know that $\sec\theta \times \cos\theta = 1$

So $\cos x = \frac{1}{\sqrt{2}}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} \implies x = 2n\pi \pm \frac{\pi}{4}$, $n \in I$

So general solution is $x = 2n\pi \pm \frac{\pi}{4}$ where $n \in I$

Question: 5

Find the general

Solution:

To Find: General solution.

(i) Given: $\cos x = \frac{-1}{2}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{-1}{2} = -\cos(\frac{\pi}{3}) = \cos(\pi - \frac{\pi}{3}) = \cos(\frac{2\pi}{3}) \implies x = 2n\pi \pm \frac{2\pi}{3}$, $n \in I$

So general solution is $x = 2n\pi \pm \frac{2\pi}{3}$ where $n \in I$

(ii) Given: $\operatorname{cosec} x = -\sqrt{2}$

We know that $\operatorname{cosec}\theta \times \sin\theta = 1$

So $\sin x = \frac{-1}{\sqrt{2}}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in$

By using above formula, we have

$\sin x = \frac{-1}{\sqrt{2}} = \sin\frac{5\pi}{4} \implies x = n\pi + (-1)^n.\frac{5\pi}{4}$

So general solution is $x = n\pi + (-1)^n.\frac{5\pi}{4}$ where $n \in I$

(iii) Given: tan x = -1

Formula used: $\tan\theta = \tan\alpha \implies \theta = n\pi + \alpha$, $n \in I$

By using above formula, we have

$\tan x = -1 = \tan\frac{3\pi}{4} \implies x = n\pi + \frac{3\pi}{4}$, $n \in I$

So the general solution is $x = n\pi + \frac{3\pi}{4}$ where $n \in I$

Question: 6

Find the general

Solution:

To Find: General solution.

(i) Given: $\sin 2x = \frac{1}{2}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin 2x = \frac{1}{2} = \sin\frac{\pi}{6} \implies 2x = n\pi + (-1)^n.\frac{\pi}{6} \implies x = \frac{n\pi}{2} + (-1)^n.\frac{\pi}{12}$, $n \in I$

So general solution is $x = \frac{n\pi}{2} + (-1)^n.\frac{\pi}{12}$ where $n \in I$

(ii) Given: $\cos 3x = \frac{1}{\sqrt{2}}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos 3x = \frac{1}{\sqrt{2}} = \cos(\frac{\pi}{4}) \implies 3x = 2n\pi \pm \frac{\pi}{4} \implies x = \frac{2n\pi}{3} \pm \frac{\pi}{12}$, $n \in I$

So the general solution is $x = \frac{2n\pi}{3} \pm \frac{\pi}{12}$ where $n \in I$

(iii) Given: $\tan\frac{2x}{3} = \sqrt{3}$

Formula used: $\tan\theta = \tan\alpha \implies \theta = n\pi + \alpha$, $n \in I$

By using above formula, we have

$\tan\frac{2x}{3} = \sqrt{3} = \tan\frac{\pi}{3} \implies \frac{2x}{3} = n\pi + \frac{\pi}{3} \implies x = \frac{3n\pi}{2} + \frac{\pi}{2}$, $n \in I$

So general solution is x =(3n+1)$\frac{\pi}{2}$,where n$\in I$

Question: 7

Find the general

Solution:

To Find: General solution.

(i) Given: sec 3x = -2

We know that sec$\theta \times$ cosθ = 1

So cos 3x = $\frac{-1}{2}$

Formula used: cosθ = cos$\alpha \implies \theta$ = 2n$\pi \pm \alpha$, n$\in I$

By using above formula, we have

cos 3x = $\frac{-1}{2}$= -cos$\frac{\pi}{3}$= cos$\left(\pi - \frac{\pi}{3}\right)$= cos$\frac{2\pi}{3}$ $\implies$ 3x = 2n$\pi \pm \frac{2\pi}{3}$ $\implies$ x = $\frac{2n\pi}{3} \pm \frac{2\pi}{9}$, n$\in I$

So the general solution is x = $\frac{2n\pi}{3} \pm \frac{2\pi}{9}$, ,where n$\in I$

(ii) Given: cot 4x = -1

We know that tan$\theta \times$ cotθ = 1

So tan 4x = -1

Formula used: tanθ = tan$\alpha \implies \theta$ = n$\pi \pm \alpha$, n$\in I$

By using above formula, we have

tan 4x = -1= tan$\frac{3\pi}{4}$ $\implies$ 4x = n$\pi + \frac{3\pi}{4}$ $\implies$ x = $\frac{n\pi}{4} + \frac{3\pi}{16}$, n$\in I$

So general solution is x = (4n+3)$\frac{\pi}{16}$,where n$\in I$

(iii) Given: cosec 3x = $\frac{-2}{\sqrt{3}}$

We know that cosec$\theta \times$ sinθ = 1

So sin 3x = $\frac{-\sqrt{3}}{2}$

Formula used: sinθ = sin$\alpha \implies \theta$ = nπ + $(-1)^n.\alpha$, n$\in I$

By using above formula, we have

sin 3x = $\frac{-\sqrt{3}}{2}$ = sin$\frac{4\pi}{3}$ $\implies$ 3x= nπ + $(-1)^n$. $\frac{4\pi}{3}$ $\implies$ x= $\frac{n\pi}{3}$ + $(-1)^n$. $\frac{4\pi}{9}$, n$\in I$

So general solution is x = $\frac{n\pi}{3}$ + $(-1)^n$. $\frac{4\pi}{9}$, where n$\in I$

Question: 8

Find the general

Solution:

To Find: General solution.

(i) Given: $4\cos^2 x = 1 \Longrightarrow \cos^2 x = \left(\frac{1}{4}\right)$

$\therefore \cos^2 x = \cos^2\frac{\pi}{3}$

Formula used: $\cos^2\theta = \cos^2\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$x = n\pi \pm \frac{\pi}{3}$, $n \in I$

So the general solution is $x = n\pi \pm \frac{\pi}{3}$ where $n \in I$

(ii) Given: $4\sin^2 x - 3 = 0 \Longrightarrow \sin^2 x = \frac{3}{4} = \sin^2\frac{\pi}{3}$

$\therefore \sin^2 x = \sin^2\frac{\pi}{3}$

Formula used: $\sin^2\theta = \sin^2\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$x = n\pi \pm \frac{\pi}{3}$, $n \in I$

So the general solution is $x = n\pi \pm \frac{\pi}{3}$ where $n \in I$

(ii) Given: $\tan^2 x = 1 \Longrightarrow \tan^2 x = \tan^2 \frac{\pi}{4}$

$\therefore \tan^2 x = \tan^2 \frac{\pi}{4}$

The formula used: $\tan^2\theta = \tan^2\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$x = n\pi \pm \frac{\pi}{4}$, $n \in I$

So the general solution is $x = n\pi \pm \frac{\pi}{4}$ where $n \in I$

Question: 9

Find the general

Solution:

To Find: General solution.

(i) Given: $\cos 3x = \cos 2x \Longrightarrow \cos 3x - \cos 2x = 0 \Longrightarrow -2\sin\frac{(5x)}{2}\sin\frac{(x)}{2} = 0$

[NOTE: $\cos C - \cos D = -2\sin\frac{(C+D)}{2}\sin\frac{(C-D)}{2}$]

So, $\sin\frac{(5x)}{2} = 0$ or $\sin\frac{(x)}{2} = 0$

Formula used: $\sin\theta = 0 \Longrightarrow \theta = n\pi$, $n \in I$

$\frac{(5x)}{2} = n\pi$ or $\frac{(x)}{2} = m\pi$ where n, m $\in I$

$x = 2n\pi/5$ or $x = 2m\pi$ where n, m $\in I$

So general solution is $x = 2n\pi/5$ or $x = 2m\pi$ where n, m $\in I$

(ii) Given: cos 5x = sin 3x $\Longrightarrow$ cos 5x = $\cos\left(\frac{\pi}{2} - 3\text{x}\right)$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2\text{n}\pi \pm \alpha$, n $\in I$

By using the above formula, we have

5x = $2\text{n}\pi + \left(\frac{\pi}{2} - 3\text{x}\right)$ or 5x = $2\text{n}\pi - \left(\frac{\pi}{2} - 3\text{x}\right)$

8x = $2\text{n}\pi + \frac{\pi}{2}$ or 2x = $2\text{n}\pi - \frac{\pi}{2}$

x = $\frac{n\pi}{4} + \frac{\pi}{16}$ or x = $\text{n}\pi - \frac{\pi}{4}$ where n $\in I$

So general solution is $x = \frac{n\pi}{4} + \frac{\pi}{16}$ or x = $\text{n}\pi - \frac{\pi}{4}$ where n $\in I$

(iii) Given: cos mx = sin nx $\Longrightarrow$ cos mx = $\cos\left(\frac{\pi}{2} - \text{nx}\right)$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2\text{k}\pi \pm \alpha$, k $\in I$

By using the above formula, we have

mx = $2\text{k}\pi + \left(\frac{\pi}{2} - \text{nx}\right)$ or 5x = $2\text{k}\pi - \left(\frac{\pi}{2} - \text{nx}\right)$

(m+n)x = $2\text{k}\pi + \frac{\pi}{2}$ or (m-n)x = $2\text{k}\pi - \frac{\pi}{2}$

x = $\frac{2k\pi}{(\text{m+n})} + \frac{\pi}{2(\text{m+n})}$ or x = $\frac{2k\pi}{(\text{m}-\text{n})} + \frac{\pi}{2(\text{m}-\text{n})}$ where k $\in I$

x = $\frac{(4k+1)\pi}{2(\text{m+n})}$ or x = $\frac{(4k-1)\pi}{2(\text{m}-\text{n})}$ where k $\in I$

So the general solution is $x = \frac{(4k+1)\pi}{2(\text{m+n})}$ or x = $\frac{(4k-1)\pi}{2(\text{m}-\text{n})}$ where k $\in I$

Question: 10

Find the general

Solution:

To Find: General solution.

Given: sin x = tan x $\Longrightarrow$ sin x = sin x $\div$ cos x

So sin x = 0 or cos x = 1 = cos(0)

Formula used: $\sin\theta = 0 \Longrightarrow \theta = \text{n}\pi$, n $\in I$ *and* $\cos\theta = \cos\alpha \implies \theta = 2\text{k}\pi \pm \alpha$, k $\in I$

x = nπ or x = 2kπ where n, k $\in I$

So general solution is x =nπ or x = 2kπ where n, k $\in I$

Question: 11

Find the general

Solution:

To Find: General solution.

Given: 4sin x cos x + 2sin x + 2cos x + 1 = 0 $\Longrightarrow$ 2sin x(2cos x + 1) + 2cos x + 1 = 0

So (2cos x + 1)(2sin x + 1) = 0

cos x = $\frac{-1}{2}$ = $\cos(\frac{2\pi}{3})$ or sin x = $\frac{-1}{2}$ = $\sin\frac{7\pi}{6}$

Formula used: cosθ = cosα $\Rightarrow$ θ = 2n$\pi \pm \alpha$ or sinθ = sinα $\Rightarrow$ θ = mπ + $(-1)^m\alpha$ where n,m $\in I$

x = 2n$\pi \pm \frac{2\pi}{3}$ or x = mπ + $(-1)^m \cdot \frac{7\pi}{6}$ where n, m $\in I$

So the general solution is x =2n$\pi \pm \frac{2\pi}{3}$ or x = mπ + $(-1)^m \cdot \frac{7\pi}{6}$ where n, m $\in I$

Question: 12

Find the general

Solution:

To Find: General solution.

Given: $\sec^2 2x = 1 - \tan 2x \Rightarrow 1 + \tan^2 2x + \tan 2x = 1 \Rightarrow \tan 2x\,(1+\tan 2x) = 0$

So, tan 2x = 0 or tan 2x = -1 = tan $(\frac{3\pi}{4})$

Formula used: : tanθ= 0 $\Rightarrow$ θ = nπ , n $\in I$ and tanθ = tanα $\Rightarrow$ θ = k$\pi \pm \alpha$, k $\in I$

By using above formula, we have

2x = nπ or 2x = k$\pi \pm \frac{3\pi}{4}$ $\Rightarrow$ x = $\frac{n\pi}{2}$ or x = $\frac{k\pi}{2} \pm \frac{3\pi}{8}$

So the general solution is x =$\frac{n\pi}{2}$ or x = $\frac{k\pi}{2} \pm \frac{3\pi}{8}$ where n, k $\in I$

Question: 13

Find the general

Solution:

To Find: General solution.

Given: $\tan^3 x - 3\tan x = 0 \Rightarrow \tan x(\tan^2 x - 3) = 0 \Rightarrow$ tan x = 0 or tanx = $\pm\sqrt{3}$

$\Rightarrow$ tan x = 0 or tanx = tan$(\frac{\pi}{3})$ or tan x = tan$(\frac{2\pi}{3})$

$\Rightarrow$ Formula used: tanθ= 0 $\Rightarrow$ θ = nπ , n $\in I$, tanθ = tanα $\Rightarrow$ θ = k$\pi \pm \alpha$, k $\in I$

So x = nπ or x = kπ + $\frac{\pi}{3}$ or x = pπ + $\frac{2\pi}{3}$ where n, k, p $\in I$

So general solution is x = nπ or x = kπ + $\frac{\pi}{3}$ or x = pπ + $\frac{2\pi}{3}$ where n, k, p $\in I$

Question: 14

Find the general

Solution:

To Find: General solution.

Given: sin x + sin 3x + sin 5x = 0 $\Rightarrow$ sin 3x + 2sin 3x cos 2x= 0 $\Rightarrow$ sin 3x (1 + 2cos 2x) = 0

[NOTE: sin C + sin D = 2sin (C+D)/2 × cos (C-D)/2]

$\Rightarrow$ sin 3x = 0 or cos 2x = $\frac{-1}{2}$ = cos$(\frac{2\pi}{3})$

Formula used: sinθ= 0 $\Rightarrow$ θ = nπ , n $\in I$, cosθ = cosα $\Rightarrow$ θ = 2k$\pi \pm \alpha$, k $\in I$

$\Rightarrow$ 3x = nπ or 2x = 2k$\pi \pm \frac{2\pi}{3}$ $\Rightarrow$ x = $\frac{n\pi}{3}$ or x = k$\pi \pm \frac{\pi}{3}$ where n,k $\in I$

So general solution is $x = \frac{n\pi}{3}$ or $x = k\pi \pm \frac{\pi}{3}$ where n, k, $\in I$

Question: 15

Find the general

Solution:

To Find: General solution.

Given: sin x tan x – 1 = tan x – sin x $\Longrightarrow$ sin x(tan x + 1) = tan x + 1

So $\sin x = 1 = \sin(\frac{\pi}{2})$ or $\tan x = -1 = \tan(\frac{3\pi}{4})$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$ *and* $\tan\theta = \tan\alpha \implies \theta = k\pi \pm \alpha$, $k \in I$

$\Longrightarrow x = n\pi + (-1)^n \frac{\pi}{2}$ or $x = k\pi \pm \frac{3\pi}{4}$ where n, $k \in I$

So general solution is $x = n\pi + (-1)^n \frac{\pi}{2}$ or $x = k\pi \pm \frac{3\pi}{4}$ where n, k, $\in I$

Question: 16

Find the general

Solution:

To Find: General solution.

Given: cos x + sin x = 1 $\Longrightarrow \cos(x - \frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

[divide $\sqrt{2}$ on both sides and cos(x-y) = cos x cos y - sin x sin y]

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2k\pi \pm \alpha$, $k \in I$

$\Longrightarrow x - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{4} \Longrightarrow x = 2k\pi \pm \frac{\pi}{4} + \frac{\pi}{4} \Longrightarrow x = 2k\pi + \frac{\pi}{4} + \frac{\pi}{4}$ or $\Longrightarrow x = 2k\pi - \frac{\pi}{4} + \frac{\pi}{4}$

$\Longrightarrow x = 2k\pi + \frac{\pi}{2}$ or $x = 2k\pi$

So general solution is $x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi$ where $n \in I$

Question: 17

Find the general

Solution:

To Find: General solution.

Given: cos x - sin x = 1 $\Longrightarrow \cos(x + \frac{\pi}{4}) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

[divide $\sqrt{2}$ on both sides and cos(x-y) = cos x cos y - sin x sin y]

So sin x = 0 or cos x = 0

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2k\pi \pm \alpha$, $k \in I$

$\Longrightarrow x + \frac{\pi}{4} = 2k\pi \pm \frac{3\pi}{4} \Longrightarrow x = 2k\pi \pm \frac{3\pi}{4} - \frac{\pi}{4} \Longrightarrow x = 2k\pi + \frac{3\pi}{4} - \frac{\pi}{4}$ or $\Longrightarrow x = 2k\pi - \frac{3\pi}{4} - \frac{\pi}{4}$

$\Longrightarrow x = 2k\pi - \pi$ or $x = 2k\pi + \frac{\pi}{2}$

So general solution is $x = 2n\pi + \frac{\pi}{2}$ or $x = (2n-1)\pi$ where $n \in I$

Question: 18

Find the general

Solution:

To Find: General solution.

Given: $\sqrt{3}$ cos x + sin x = 1 $\Rightarrow$ cos (x - $\frac{\pi}{6}$) = $\frac{1}{2}$ = cos($\frac{\pi}{3}$) or cos($\frac{5\pi}{3}$)

[Divide $\sqrt{2}$ on both sides and cos(x-y) = cos x cos y - sin x sin y]

Formula used: cosθ = cosα $\Rightarrow$ $\theta = 2n\pi \pm \alpha$

By using above formula, we have

$\Rightarrow$ x - $\frac{\pi}{6}$ = 2nπ $\pm$ $\frac{\pi}{3}$ $\Rightarrow$ x = 2nπ $\pm$ $\frac{\pi}{3}$ + $\frac{\pi}{6}$

$\Rightarrow$ x = 2nπ + $\frac{\pi}{2}$ or x = 2nπ - $\frac{\pi}{6}$ where n $\in I$

So general solution is x = 2nπ + $\frac{\pi}{2}$ or x = 2nπ - $\frac{\pi}{6}$ where n $\in I$

Question: 19

Find the general

Solution:

To Find: General solution.

Given: 2 tan x – cot x + 1 = 0 $\Rightarrow$ 2tan^{2}x – 1 + tan x = 0 $\Rightarrow$ 2tan^{2}x – 1 + 2tan x – tanx = 0 $\Rightarrow$ 2tanx(tanx +1) – (1+ tanx) = 0

$\Rightarrow$ (2tanx-1) (1+ tanx) = 0 $\Rightarrow$ tan x = $\frac{1}{2}$ = $\tan^{-1}\frac{1}{2}$ or tan x = -1 = tan$\frac{3\pi}{4}$

Formula used: tanθ = tanα $\Rightarrow$ $\theta = n\pi + \alpha$, n $\in I$

x = nπ + $\tan^{-1}\frac{1}{2}$ or x = nπ + $\frac{3\pi}{4}$

So the general solution is x = nπ + $\tan^{-1}\frac{1}{2}$ or x = nπ + $\frac{3\pi}{4}$ where n $\in I$

Question: 20

Find the general

Solution:

To Find: General solution.

Given: sin x tan x – 1 = tan x – sin x $\Rightarrow$ sin x(tan x + 1) = tan x + 1

So sin x = 1 = sin ($\frac{\pi}{2}$) or tan x = -1 = tan($\frac{3\pi}{4}$)

Formula used: sinθ = sinα $\Rightarrow$ $\theta = n\pi + (-1)^n\alpha$, n $\in I$ *and* tanθ = tanα $\Rightarrow$ $\theta = k\pi + \alpha$, k $\in I$

$\Rightarrow$ x = nπ + $(-1)^n\frac{\pi}{2}$ or x = kπ + $\frac{3\pi}{4}$ where n, k $\in I$

So general solution is x = nπ + $(-1)^n\frac{\pi}{2}$ or x = kπ + $\frac{3\pi}{4}$ where n, k $\in I$

Question: 21

Find the general

Solution:

To Find: General solution.

Given: cot x + tan x = 2 cosec x $\Rightarrow$ $\cos^2x + \sin^2x$ = 2 sinx cosx cosec x $\Rightarrow$ 1 = sin 2x cosec x

$\Rightarrow$ cosec 2x = cosecx $\Rightarrow$ sin x = sin 2x $\Rightarrow$ sin x = 2 sin x cos x $\Rightarrow$ sin x = 0 or cos x = $\frac{1}{2}$ = $\cos(\frac{\pi}{3})$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi$, $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$

By using above formula , we have

$x = n\pi$ or $x = 2m\pi \pm \frac{\pi}{3}$ where n, m$\in I$

So general solution is $x = n\pi$ or $x = 2m\pi \pm \frac{\pi}{3}$ where n, m$\in I$