Chapter : 7. SUMMATIVE ASSESSMENT I

Exercise : SAMPLE PAPER 1

Question: 1

Which of the foll

Solution:

A rational number is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q.

Since for option D numerator, p = -3 and denominator q = 5 both are integers.

-3/5 is a rational number.

Question: 2

The value of k fo

Solution:

If 3 is the solution for the equation. It must satisfy the expression.

So, putting x = 3 it must be zero.

33 - 4 × 32 + 2 × 3 + k = 0

27 – 4 × 9 + 6 + k = 0

k - 3 = 0

k = 3

Question: 3

Which of the foll

Solution:

We need to do hit and trial to find root of a cubic equation.

If it is a root of equation, it must satisfy the equation.

So, let's start with option A.

$(-2)^3 + 2(-2)^2 - 5(-2)-6 = -8 + 8 + 10 - 6 = 4$

Let's try option B

$(2)^3 + 2(2)^2 - 5(2)-6 = 8 + 8 – 10 - 6 = 0$

Let's try option C

$(-3)^3 + 2(-3)^2 - 5(-3)-6 = -27 + 18 + 15 - 6 = 0$

For option D

$(3)^3 + 2(3)^2 - 5(3)-6 = 27 + 18 – 15 - 6 = 24$

Hence Option B and C are correct

Verifying –

Factors of the given equation is $(x-2)(x + 3)(x + 1) = x^3 + 2x^2 - 5x - 6$.

Question: 4

The factorization

Solution:

$-x^2 + 7x - 12$ can be factorized as-

$-x^2 + 4x + 3x - 12$

$-x(x - 4) + 3(x - 4)$

$(x - 4)(3 - x)$

Also recheck by-

Sum of roots = 7 {-coefficient of x/ coefficient of x^2}

Product of roots = 12 {constant/ coefficient of x^2}

Question: 5

In the given figu

Solution:

Sum of angles in a straight line is 180°

So, ∠AOD + ∠DOC + ∠BOC = 180°

$3x + 5x + 4x = 180$

$12x = 180$

$x = 15$

∠BOC = 4x = 4×15 = 60°.

Question: 6

In the given figu

Solution:

Since we know all the angles in an equilateral triangle is of 60°.

So, ∠ABC = ∠ACB = ∠CAB = 60° ...(i)

Also for an isosceles triangle, the angles opposite to equal sides are equal.

So, ∠DBC = ∠DCB = x (let’s say)

Also sum of all angles in a triangle = 180°.

So, in ΔBDC,

∠DBC + ∠DCB + ∠BDC = 180°

x + x + 90 = 180 {since ∠BDC = 90°}

$2x = 90$

$x = 45°$

so ∠DCB = 45 ...(ii)

And ∠ACD = ∠ACB + ∠DCB = 60° + 45° = 105° {from (i) and (ii)}

Question: 7

Each of the equal

Solution:

Applying heron’s formula-

We know,

$s = \dfrac{a + b + c}{2}$ here a, b and c are sides of a triangle

So, $s = \frac{13 + 13 + 24}{2} = 25$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

So, Area = = $\sqrt{25(25-13)(25-13)(25-24)}$

Hence Area = $\sqrt{25(12)(12)(1)}$

= $\sqrt{3600}$

= 60 square units

Question: 8

In an isosceles r

Solution:

For a right-angled triangle,

Applying Pythagoras theorem,

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

Since triangle is isosceles.

So, base = perpendicular = x (let's say)

Hence $(\text{hypotenuse})^2 = (x)^2 + (x)^2$

$(4\sqrt{2})^2 = 2x^2$

$32 = 2x^2$

$x^2 = 16$

so, x = 4 cm.

Question: 9

If, $x = 7 + 4\sqrt{3}$ f

Solution:

Let $\sqrt{x} + \frac{1}{\sqrt{x}}$ to be y.

So y = $\sqrt{x} + \frac{1}{\sqrt{x}}$

Squaring both sides,

$$y^2 = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$$

$$= \left(\sqrt{x}\right)^2 + \left(\frac{1}{\sqrt{x}}\right)^2 + 2\left(\sqrt{x}\right)\left(\frac{1}{\sqrt{x}}\right) = x + \frac{1}{x} + 2$$

Also, $x = 7 + 4\sqrt{3}$

So $y^2 = 7 + 4\sqrt{3} + \frac{1}{7 + 4\sqrt{3}} + 2$

$$= 9 + 4\sqrt{3} + \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}} \text{ (on rationalizing)}$$

$$= 9 + 4\sqrt{3} + \frac{7 - 4\sqrt{3}}{(7)^2 - (4\sqrt{3})^2}$$

$$= 9 + 4\sqrt{3} + \frac{7 - 4\sqrt{3}}{49 - 48}$$

$$= 9 + 4\sqrt{3} + 7 - 4\sqrt{3}$$

= 16

So, y = √16 = 4

Hence $y = \sqrt{x} + \frac{1}{\sqrt{x}} = 4$

Question: 10

Factorize: (7a

Solution:

$(7a^3 + 56b^3)$

$= 7(a^3 + 8b^3)$

$= 7(a^3 + (2b)^3)$

$= 7(a + (2b))(a^2 + (2b)^2 - a(2b))$

[since $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$]

$= 7(a + 2b)(a^2 + 4b^2 - 2ab)$

Question: 11

Find the value of

Solution:

If (x - 1) is a factor of the polynomial $(a^2x^3 - 4ax + 4a - 1)$.

then it must satisfy it.

So, putting x = 1 the polynomial must be zero.

Putting x = 1 and equating to zero.

$= (a^2(1)^3 - 4a(1) + 4a - 1)$

$= a^2 - 4a + 4a - 1 = 0$

$= a^2 = 1$

So, a = �1.

Question: 12

In the given figu

Solution:

Given- AC = BD

Subtracting BC on both sides-

(AC - BC) = (BD - BC)

AB = CD

Question: 13

In a ΔABC if 2

Solution:

In a triangle sum of all angles = 180°

So, ∠A + ∠B + ∠C = 180°

It is given that-

∠A = 3/2 ∠B

∠C = � ∠B

So, ∠A + ∠B + ∠C = (3/2) ∠B + ∠B + (1/2) ∠B = 180°

3∠B = 180°

∠B = 60°

Question: 14

In the given figu

Solution:

In ΔABC sum of all angles = 180°.

So, ∠BAC + ∠ABC + ∠ACB = 180°

30 + 50 + ∠ACB = 180

∠ACB = 100°

Since BCD represents a straight line ∠ACB + ∠ECD = 180°

So, ∠ECD = 80°

In ΔECD sum of all angles = 180°

So, ∠ECD + ∠EDC + ∠CED = 180°

60 + 40 + ∠CED = 180

∠CED = 80°

Since AEC represents a straight line, ∠CED + ∠AED = 180°

So, ∠AED = 120°

Question: 15

If $x = \dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} = \dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \dfrac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}}$ (on rationalizing we get)

$$= \frac{\left(\sqrt{5} - \sqrt{3}\right)^2}{\sqrt{5}^2 - \sqrt{3}^2} \quad \left\{\text{since } (a+b)(a-b) = a^2 - b^2\right\}$$

$$= \frac{\sqrt{5}^2 + \sqrt{3}^2 + 2 \times \sqrt{5} \times \sqrt{3}}{5 - 3}$$

$$= \frac{5 + 3 + 2(\sqrt{5})(\sqrt{3})}{2}$$

= 4 + (√5)(√3)

= 4 + √15

Similarly $y = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ (rationalising)

$$= \frac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\sqrt{5}^2-\sqrt{3}^2} \quad \left\{\text{since } (a+b)(a-b) = a^2 - b^2\right\}$$

$$= \frac{\sqrt{5}^2+\sqrt{3}^2-2\times\sqrt{5}\times\sqrt{3}}{5-3} = \frac{5+3-2\left(\sqrt{5}\right)\left(\sqrt{3}\right)}{2}$$

= (5 + 3-2(√5)(√3))/2

= 4- (√5) (√3)

= 4- √15

So, $x^2 + y^2 = \left(4+\sqrt{15}\right)^2 + \left(4-\sqrt{15}\right)^2$

$$= \left(4^2+\sqrt{15}^2+2\times 4\times\sqrt{15}\right) + \left(4^2+\sqrt{15}^2 - 2\times 4\times\sqrt{15}\right)$$

$$= \left(16+15+8\sqrt{15}\right) + \left(16+15-8\sqrt{15}\right)$$

= 32 + 30

= 62

(II) $\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}}$

Taking LCM as (3 + √5)(3-√5)

$$= \frac{\left(7+3\sqrt{5}\right)\left(3-\sqrt{5}\right)-\left(7-3\sqrt{5}\right)\left(3+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}$$

$$= \frac{\left(21-7\sqrt{5}+9\sqrt{5}-3\sqrt{5}\times\sqrt{5}\right)-\left(21-9\sqrt{5}+7\sqrt{5}-3\sqrt{5}\times\sqrt{5}\right)}{3^2-\sqrt{5}^2}$$

(since (a + b)(a - b) = a^2 - b^2)

$$= \frac{4\sqrt{5}}{(9-5)}$$

$$= \frac{4\sqrt{5}}{4} = \sqrt{5}$$

Question: 16

If 2 and -1/3 are

Solution:

We know for a cubic polynomial, sum of roots $= -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}$

Let the third root be x.

So, $x+2+\left(-\frac{1}{3}\right)=-\left(-\frac{2}{3}\right)$

$x+\frac{5}{3}=\frac{2}{3}$

$x=\frac{2}{3}-\frac{5}{3}$

$x=-1$

Question: 17

Find the remainde

Solution:

If we divide f(x) = $4x^2$ - $12x^2$ + 14x - 3 by (2x - 1) remainder can be find at value of –

(2x-1) = 0

Or x = 1/2

So, we will put x = 1/2 in f(x) = $4x^2$ - $12x^2$ + 14x – 3

$$f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^3-12\left(\frac{1}{2}\right)^2+\frac{14}{2}-3$$

$$=4\times\frac{1}{8}-12\times\frac{1}{4}+7-3$$

$$=\frac{1}{2}-3+7-3$$

$$=1+\frac{1}{2}$$

$$=\frac{3}{2}$$

Question: 18

Factorize: (p-q)<

Solution:

We know that –

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$.

here if a + b + c = 0

$a^3 + b^3 + c^3 = 3abc$.

So, $(p-q)^3 + (q-r)^3 + (r-p)^3 = 3(p-q)(q-r)(r-p)$ {since (p-q) + (q-r) + (r-p) = 0}

Question: 19

In the given figu

Solution:

Since DE || BC and AB acts as transversal.

So, ∠ADE = ∠ABC {corresponding angles}

since ∠ABC = 40°

So, ∠ADE = 40°

Since EF || AB and DN acts as transversal.

So, ∠ADE = ∠MEN {corresponding angles}

∠MEN = 40°

Hence, ∠ADE + ∠MEN = 80°

(ii) 140°

Since AB represents a straight line. Sum of angles in line AB = 180°

So, ∠BDE + ∠ADE = 180°

since, ∠ADE = 40°

So, ∠BDE = 140°

(iii) 140°

Since DE || BC and FM acts as transversal.

So, ∠EFC = ∠ MEN = 40°

And BC represents a straight line. Sum of angles in line BC = 180°

= ∠EFC + ∠BFE = 180°

= ∠BFE = 140°

Question: 20

In the given figu

Solution:

Taking ΔABC and ΔABD in consideration-

AD = BC

Since, it is given that

∠1 = ∠2 and ∠3 = ∠4

Adding them –

∠1 + ∠3 = ∠2 + ∠4.

= ∠DAB = ∠ABC

And AB is the common side on both triangle.

So, by side angle side(SAS) criteria-

Triangle ΔABC and ΔABD are congruent.

So, BD = AC (by congruency criteria).

Question: 21

In the given figu

Solution:

Since C is the mid-point of AB.

So, AC = BC.

Taking ΔACE and ΔBCD in consideration-

∠DBC = ∠EAC

AC = BC

Also ∠DCA = ∠ECB

Adding ∠DCE on both sides-

∠DCB = ∠ECA

So, by Angle side Angle(ASA) criteria ΔACE and ΔBCD are congruent.

And hence DC = EC (by congruency criteria).

Question: 22

In ΔABC if AL

Solution:

Sum of all angles in a triangle = 180°

∠A + ∠B + ∠C = 180°

∠A = 2 ∠CAM = 2 ∠MAB {since AM is bisector of ∠A}

= 2∠CAM + ∠B + ∠C = 180°

= 2∠CAM = 180 - (∠B + ∠C)

$$= \angle CAM = 90 - \frac{\angle B + \angle C}{2}$$

∠AML = ∠CAM + ∠C {Exterior Angle theorem}

$$= 90 - \frac{\angle B + \angle C}{2} + \angle C$$

$$= 90 + \frac{\angle C}{2} - \frac{\angle B}{2}$$

In Triangle ΔALM, Sum of all angles must be 180°

So, ∠LAM + ∠AML + 90 = 180

∠LAM + ∠AML = 90

∠LAM = 90 -∠AML

$$= 90 - \left(90 + \frac{\angle C}{2} - \frac{\angle B}{2}\right)$$

$$= \frac{\angle B}{2} - \frac{\angle C}{2}$$

Question: 23

In the given figu

Solution:

Since AH || EC

So, ∠GAE = ∠AEC = 30° {alternate angle}

Also ∠BAG = 100°-∠GAE

∠BAG = 70°

Here also, AB || DC and GH acts as transversal.

So, ∠BAG = ∠DHA = 70° {corresponding angles}

Similarly,

AH || EC and DC acts as transversal.

So, ∠DCE = ∠DHA = 70° {corresponding angles}

Question: 24

Factorize: a

Solution:

$a^3 - b^3 + 1 + 3ab$

$= a^3 + (-b)^3 + 13 - 3\{1 \times a \times (-b)\}$

$= \{a + (-b) + 1\}\ \{a^2 + (-b)^2 + 12 - a(-b) - (-b)1 - 1a\}$

using identity $\{a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)\} + (a-b + 1)(a^2 + b^2 + 1 + ab + b - a)$

Question: 25

$$x = \frac{1}{2-\sqrt{3}} = \frac{1}{2-\sqrt{3}} \times \frac{\left(2+\sqrt{3}\right)}{2+\sqrt{3}}\ \{\text{rationalizing}\}$$

If $x = \dfrac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$

$$x = \frac{2+\sqrt{3}}{4-3}$$

$$x = 2+\sqrt{3}$$

Now, $x^2 = (2 + \sqrt{3})^2 = 4 + 3 + 4\sqrt{3} = 7 + 4\sqrt{3}$

Also, $x^3 = x \times x^2 = (2 + \sqrt{3})(7 + 4\sqrt{3})$

$= 2(7) + 7(\sqrt{3}) + 2(4\sqrt{3}) + (\sqrt{3})(4\sqrt{3})$

$= 14 + 15\sqrt{3} + 12$

$= 26 + 15\sqrt{3}$

Put all the values in the expression: $x^3 - 2x^2 - 7x + 5$

$= (26 + 15\sqrt{3})-2(7 + 4\sqrt{3})-7(2 + \sqrt{3}) + 5$

$= 3$

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots.. + \frac{1}{\sqrt{8}+\sqrt{9}}$$

rationalize-

$$\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}} + \ldots.. + \frac{1}{\sqrt{8}+\sqrt{9}} \times \frac{\sqrt{8}-\sqrt{9}}{\sqrt{8}-\sqrt{9}}$$

$$= \frac{1-\sqrt{2}}{1^2-\sqrt{2}^2} + \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} + \ldots$$

$$= \frac{1-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \ldots$$

$= \sqrt{2}-1 + \sqrt{3}-\sqrt{2} + \ldots\sqrt{8}-\sqrt{7} + \sqrt{9}-\sqrt{8}$

$= \sqrt{9}-1$

$= 3-1$

$= 2$

Question: 26

If $x = \dfrac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$

$$x = \frac{\sqrt{a + 2b} + \sqrt{a - 2b}}{\sqrt{a + 2b} - \sqrt{a - 2b}} \times \frac{\sqrt{a + 2b} + \sqrt{a - 2b}}{\sqrt{a + 2b} + \sqrt{a - 2b}} \{\text{rationalizing}\}$$

$$x = \frac{\left(\sqrt{a + 2b} + \sqrt{a - 2b}\right)^2}{\sqrt{a + 2b}^2 - \sqrt{a - 2b}^2}$$

$$x = \frac{\sqrt{a + 2b}^2 + \sqrt{a - 2b}^2 + 2\left(\sqrt{a + 2b}\right)\left(\sqrt{a - 2b}\right)}{(a + 2b) - (a - 2b)}$$

$$\frac{a + 2b + a - 2b + 2\sqrt{(a + 2b)(a - 2b)}}{4b}$$

$$\frac{2a + 2\sqrt{(a + 2b)(a - 2b)}}{4b}$$

$$x = \frac{a + \sqrt{a^2 - (2b)^2}}{2b} \{\text{since } (a + b)(a - b) = a^2 - b^2\}$$

$$\text{So, } 2bx - a = \sqrt{a^2 - (2b)^2}$$

$$= (2bx - a)^2 = \sqrt{a^2 - (2b)^2}^2 \quad \{\text{squaring both sides}\}$$

$= 4b^2x^2 + a^2 - 4abx = a^2 - 4b^2$

$= 4b^2x^2 - 4abx + 4b^2 = 0$ {rearranging terms and cancelling a^2}

Dividing the expression by $4b - bx^2 - ax + b = 0$

Question: 27

If $(x^3$

Solution:

If $(x - 2)$ is a factor of the polynomial $(x^3 + mx^2 - x + 6)$ then it must satisfy it.

So, putting $x = 2$ the polynomial must be zero.

Putting $x = 2$ and equating to zero.

$= (23 + m2^2 -2 + 6)$

$= 4m + 12 = 0$

$= m = -3$

If we divide $f(x) = (x^3 + mx^2 -x + 6)$ by $(x - 3)$ remainder can be find at value of –

$(x - 3) = 0$

Or $x = 3$

So we will put $x = 3$ in $f(x) = (x^3 + mx^2 - x + 6)$

$f(3) = (3^3 + m3^2 - 3 + 6)$

$= 30 + 9m$

So remainder $= 30 + 9m$

$= 30 + 9(-3) = 30 - 27 = 3$

So, $r = 3$.

Question: 28

If r and s be the

Solution:

If we divide $f(x) = (x^3 + 2x^2 - 5ax - 7)$ by $(x + 1)$ remainder can be find at value of –

$(x + 1) = 0$

Or $x = -1$

So, we will put $x = -1$ in $f(x) = (x^3 + 2x^2 - 5ax - 7)$

$f(-1) = ((-1)^3 + 2(-1)^2 - 5a(-1)-7)$

$= -6 + 5a$

So, remainder $= r = -6 + 5a$

Also if we divide $f(x) = (x^3 + ax^2 - 12x + 6)$ by $(x - 2)$ remainder can be find at value of –

$(x - 2) = 0$

Or $x = 2$

So we will put $x = 2$ in $f(x) = (x^3 + ax^2 - 12x + 6)$

$f(2) = (2^3 + a2^2 - 12(2) + 6)$

$= 4a - 10$

So, remainder $= s = 4a - 10$

Also it is given that $2r + s = 6$

So putting r and s from above expressions-

$2(-6 + 5a) + (4a - 10) = 6$

$= 14a = 28$

$= a = 2$

Question: 29

Prove that: (a +

Solution:

We know that –

$a^3 + b^3 + c^3 - 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc - ca)$.

So applying the theorem here,

$(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a) = ((a + b) + (b + c) + (c + a))((a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c) - (b + c)(c + a)-(c + a)(a + b))$

$= (2(a + b + c))((a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c)-(b + c)(c + a)-(c + a)(a + b))$

{since $((a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c) - (b + c)(c + a) - (c + a)(a + b))$

$= (a^2 + b^2 + c^2 – ab – bc - ca)$}

$= 2 (a^3 + b^3 + c^3 - 3(a)(b)(c))$

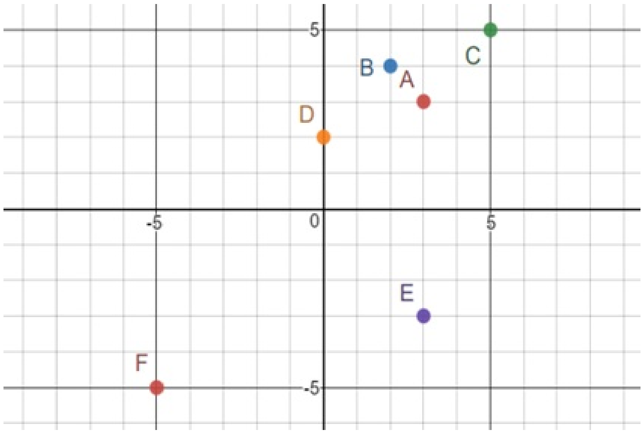
{using this theorem again: $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$}

Question: 30

On a graph paper

Solution:

It is clear from the graph A and E are mirror image wrt. x-axis and there is no mirror image points wrt. y-axis.



Question: 31

In the given figu

Solution:

We know that,

Sum of all angles in a triangle = 180°

So, in ΔBEC

= 40 + x + 90 = 180

So, x = 50°

Now, in ΔADC-

= 50 + 30 + ∠ADC = 180

= ∠ADC = 100°

Since BC represents a straight line, sum of angles = 180°.

So, ∠ADC + y = 180

hence y = 80° since ∠ADC = 100°

By exterior angle sum theorem of the smaller triangle formed-

z = ∠DAE + ∠BEA = 90° + 30° = 120°

Question: 32

In the given figu

Solution:

In ΔBDC ∠DBC = ∠DCB so

BD = DC ...(i)

{sides opposite to equal angles in a triangle are equal}

Now let’s consider that ΔABD and ΔADC –

AB = AC {given}

AD is a common side.

And BD = DC {from equation (i)}

Hence ΔABD and ΔADC are congruent.

So ∠BAD = ∠DAC (congruency criteria)

Hence AD bisects ∠BAC.

Question: 33

In the given figu

Solution:

Since diagonal of square bisects the angles.

So, ∠CBD = ∠CDB = 45° [Also all angles of square are right angles i.e. half of all is 45°] (1)

Also similarly ∠ABD = ∠ADB = 45°

Since lines EF || BD

By corresponding angles-

∠CEF = ∠CDB = 45°

Also ∠CFE = ∠CBD = 45°

So, CE = CF {since sides opposite to equal angles are equal} ...(i)

And CD = BC {sides of a square are equal} ...(ii)

Subtracting I from II

CD-CE = BC-CF

So, BF = DE

Also let's consider ΔADX and ΔABX {where X is intersection point of AM and BD}

∠ABD = ∠ADB = 45°

AX is a common side.

AD = AB {sides of a square are equal}

The triangles are congruent by SAS (side angle side) criteria.

So, ∠DAM = ∠MAB (congruency criteria)

Hence AM bisects ∠BAD.

Question: 34

In the given figu

Solution:

Draw one line EF || CD and AB.

Since EF || CD and CE is transversal.

∠FEC + ∠ECD = 180°

∠FEC = 60° {since ∠ECD = 120°}

Also, EF || AB and AE is transversal.

∠FEA + ∠BAE = 180°

∠FEA = 80° {since ∠BAE = 100°}

And x = ∠FEC + ∠FEA

= 60° + 80°

= 140°

Exercise : SAMPLE PAPER 2

Question: 1

An irrational num

Solution:

Irrational numbers are numbers which cannot be expressed as simple fraction or simple ratios of two integers. That leaves us with just two options A and C. So, only √5 comes in between 2 and

2.5.

Question: 2

Which of the foll

Solution:

A polynomial in one variable is an algebraic expression that consists of terms in the form of ax^n, where n is either zero or positive only. Given the options all expressions except D has the value of n as negative.

Question: 3

Solve the e

Solution:

Given, $\dfrac{1}{\sqrt{18}-\sqrt{32}}$

Rationalising the above term,

$$\therefore \frac{1}{\sqrt{18}-\sqrt{32}} \times \frac{\sqrt{18}+\sqrt{32}}{\sqrt{18}+\sqrt{32}} = \frac{\sqrt{18}+\sqrt{32}}{\left(\sqrt{18}-\sqrt{32}\right)\left(\sqrt{18}+\sqrt{32}\right)}$$

Using the formula (a + b) (a – b) = a^2 – b^2 for the denominator,

$$\Rightarrow \frac{3\sqrt{2}+4\sqrt{2}}{18-32} = \frac{\sqrt{2}(3+4)}{-14}$$

$$\Rightarrow \frac{7\sqrt{2}}{-14} = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$$

Question: 4

If p(x) = (x

Solution:

Given, p(x) = (x^4 – x^2 + x)

Substituting the value of 1/2 in place of will give,

$$\Rightarrow p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)$$

$$\Rightarrow p\left(\frac{1}{2}\right) = \frac{1}{16} - \frac{1}{4} + \frac{1}{2}$$

$$\Rightarrow p\left(\frac{1}{2}\right) = \frac{1-4+8}{16}$$

$$\therefore p\left(\frac{1}{2}\right) = \frac{5}{16}$$

Question: 5

If p(x) = x^3

Solution:

Given, p(x) = x^3 + x^2 + ax + 115

(x^3 + x^2 + ax + 115) is exactly divisible by (x + 5)

Hence, substituting x = -5 will give us the value of a

⇒ $(-5)^3 + (-5)^2 + a(-5) + 115 = 0$

⇒ $-125 + 25 - 5a + 115 = 0$

⇒ $5a = 15$

∵ $a = 3$

Question: 6

The equation of y

Solution:

We know that, the value of x is always zero on the y-axis.

Question: 7

In the given figu

Solution:

According to the figure,

⇒ $4x + 5x = 180°$ [Angle on a straight line]

⇒ $9x = 180°$

∵ $x = 20°$

Question: 8

In the given figu

Solution:

Given,

$\angle BAC = 40°$

$\angle ACB = 65°$

According to figure,

∵ $\angle ACE = 40°$ [Alternate angles]

∵ $\angle ACB + \angle ACE = x°$ [Alternate angles]

⇒ $x° = 65° + 40°$

∵ $x = 105°$

Question: 9

Factorize: √2x

Solution:

Given, $\sqrt{2}x^2 + 3x + \sqrt{2}$

By splitting the middle term,

⇒ $\sqrt{2}\, x^2 + 2x + x + \sqrt{2}$

⇒ $\sqrt{2}\, x(x + \sqrt{2}) + 1(x + \sqrt{2})$

∵ $(x + \sqrt{2})(\sqrt{2}\, x + 1)$

Question: 10

Prove that &radic

Solution:

Let’s assume that √5 is a rational number.

Hence, √5 can be written in the form a/b [where a and b (b ≠ 0) are co-prime (i.e. no common factor other than 1)]

∵ $\sqrt{5} = a/b$

⇒ $\sqrt{5}\, b = a$

Squaring both sides,

⇒ $(\sqrt{5}\, b)^2 = a^2$

⇒ $5b^2 = a^2$

⇒ $a^2/5 = b^2$

Hence, 5 divides a^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number

So, 5 divides a too

Hence, we can say a/5 = c where, c is some integer

So, a = 5c

Now we know that,

$5b^2 = a^2$

Putting a = 5c,

⇒ $5b^2 = (5c)^2$

⇒ $5b^2 = 25c^2$

⇒ $b^2 = 5c^2$

∵ $b^2/5 = c^2$

Hence, 5 divides b^2

By theorem, if p is a prime number and p divides a^2, then p divides a, where a is a positive number

So, 5 divides b too

By earlier deductions, 5 divides both a and b

Hence, 5 is a factor of a and b

∵ a and b are not co-prime.

Hence, the assumption is wrong.

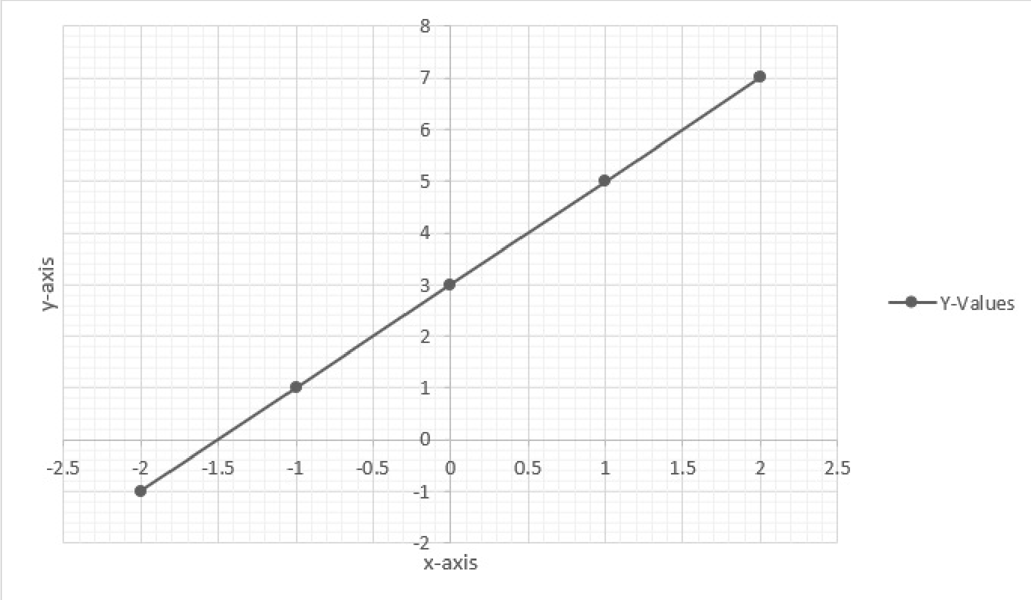
∵ By contradiction,

∵ $\sqrt{5}$ is irrational

Question: 11

Draw the graph of

Solution:



Question: 12

If x = (3 + √8),

Solution:

Given, x = (3 + √8)

Let us calculate 1/x,

$$\Rightarrow \frac{1}{x} = \frac{1}{3+\sqrt{8}}$$

Rationalising the above term,

$$\Rightarrow \frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$$

Using the formula (a + b) (a – b) = (a^2 – b^2),

$$\Rightarrow \frac{1}{x} = \frac{3-\sqrt{8}}{9-8}$$

$$\therefore \frac{1}{x} = 3-\sqrt{8}$$

Now,

$$\left(x+\frac{1}{x}\right) = 3+\sqrt{8}+3-\sqrt{8}$$

$$\therefore \left(x+\frac{1}{x}\right) = 6$$

On squaring both sides, we get

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 6^2$$

$$\Rightarrow x^2+\frac{1}{x^2}+2 = 36$$

$$\therefore \left(x^2+\frac{1}{x^2}\right) = 34$$

Question: 13

Find the area of

Solution:

Given, three sides of a triangle 52 cm, 56 cm, 60cm

Area of a triangle is given by,

$$\sqrt{s(s-a)(s-b)(s-c)}$$

where,

$s = \frac{a+b+c}{2}$ and a, b, c are the sides of the triangle

$$\Rightarrow s = \frac{52+56+60}{2}$$

$$\therefore s = \frac{168}{2} = 84$$

$\because$ Area of triangle = $\sqrt{84(84-52)(84-56)(84-60)}$

$$= \sqrt{84*32*28*24}$$

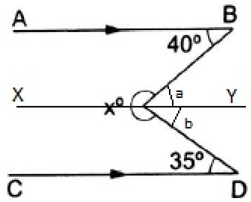
$$= \sqrt{1806336} = 1344 \text{ cm}^2$$

Question: 14

In the given figu

Solution:

Lets draw another line XY || AB and CD.



According to the figure,

⇒ ∠a = 40° [Alternate angles]

⇒ ∠b = 35° [Alternate angles]

∵ ∠x + ∠a + ∠b = 360° [Angle at a point = 360°]

∵ ∠x = 360° - 40° - 35° = 285°

Question: 15

Find the values o

Solution:

Given, $x^4 + ax^3 - 7x^2 - 8x + b = 0$

∵ x = -2, -3 are a root of the above equation (∵ they are exactly divisible)

Substituting the value -2 and -3 in place of x will give,

⇒ $(-2)^4 + a(-2)^3 - 7(-2)^2 - 8(-2) + b = 0$

⇒ 16 – 8a – 28 + 16 + b = 0

$\because$ 8a – b = 4 …. (i)

$\Rightarrow (-3)^4 + a(-3)^3 - 7(-3)^2 - 8(-3) + b = 0$

$\Rightarrow 81 - 27a - 63 + 24 + b = 0$

$\because$ 27a – b = 42 …. (ii)

Simultaneously solving eq(i) and eq(ii) we get,

$\because$ a = 2

$\because$ b = 12

Question: 16

Using remainder t

Solution:

Given, $p(x) = x^3 - 3x^2 + 4x + 50$

Divisor, (x + 3)

$\because$ x = -3

Substituting -3 in place of x gives us,

$\Rightarrow (-3)^3 - 3(-3)^2 + 4(-3) + 50$

= -27 – 27 – 12 + 50 = -16

Question: 17

Factorize: (2x

Solution:

Given, $(2x^3 + 54)$

Taking common terms out,

$\Rightarrow 2(x^3 + 27)$

Using the formula, $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

$\Rightarrow 2(x + 3)(x^2 - 3x + 3^2)$

$\because 2(x + 3)(x^2 - 3x + 9)$

Question: 18

Find the product

Solution:

Given, $(a - b - c)(a2 + b^2 + c^2 + ab + ac - bc)$

$= a^3 + ab^2 + ac^2 + a^2b + a^2c - abc - a^2b - b^3 - bc^2 - ab^2 - abc + b^2c - a^2c - b^2c - c^3 - abc - ac^2 - bc^2$

Cancelling the terms with opposite signs,

$= a^3 - b^3 - c^3 - 3abc$

Question: 19

In a ΔABC, if

Solution:

Let the three angles of a triangle be ∠A, ∠B, ∠C

Given, ∠A - ∠B = 33°

$\Rightarrow \angle A = \angle B + 33°$

$\angle B - \angle C = 18°$

$\Rightarrow \angle C = \angle B - 18°$

Now,

$\angle A + \angle B + \angle C = 180°$ [Sum of all angles of a triangle = 180°]

$\Rightarrow \angle B + 33° + \angle B + \angle B - 18° = 180°$

$\Rightarrow 3\angle B = 180° - 15°$

$\because \angle B = 55°$

$\because \angle A = \angle B + 33° = 88°$

$\because \angle C = \angle B - 18° = 37°$

Question: 20

In the given figu

Solution:

Given, $\angle A = 70°$

Let the two angles $\angle B = 2x$ and $\angle C = 2y$.

Then, angle bisector of B, $\angle OBC = x$ and angle bisector of C, $\angle OCB = y$

$\because \angle A + \angle B + \angle C = 180°$ [Sum of all angles of a triangle = 180°]

$\Rightarrow 70° + 2x + 2y = 180°$

$\Rightarrow 2x + 2y = 110°$

$\because x + y = 55°$ .... (i)

Now,

$\angle BOC + x + y = 180°$ [Sum of all angles of a triangle = 180°]

$\Rightarrow \angle BOC = 180° - (x + y)$

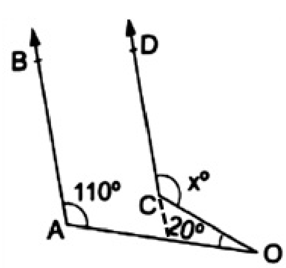
$\Rightarrow \angle BOC = 180° - 55°$ [from eq. (i)]

$\because \angle BOC = 125°$

Question: 21

In the given figu

Solution:



Given, $\angle BAO = 110°$, $\angle AOC = 20°$

$\angle CEO = 110°$ [Corresponding angles]

$\because x° = 110° + 20°$ [Exterior angle = Sum of two opposite interior angles]

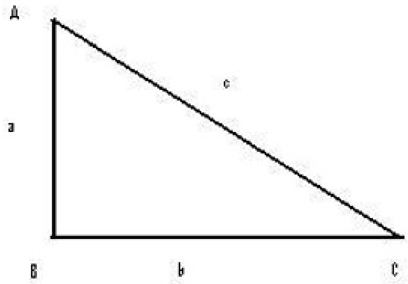
$\because x° = 130°$

Question: 22

In a right-angled

Solution:

Given, ΔABC is a right- angled triangle at B i.e. $\angle B = 90°$



To prove AC is the longest side of ΔABC

Proof:

In ΔABC,

$\angle A + \angle B + \angle C = 180°$ [Sum of all angles of a triangle = 180°]

$\angle A + 90° + \angle C = 180°$ [Given $\angle B = 90°$]

$\angle A + \angle C = 180° - 90°$

$\because \angle A + \angle C = 90°$

Hence, $\angle A < 90°$

$\angle A < \angle B$

$BC < AC$ [Side opposite to a larger angle is longer]

Similarly,

$\angle C < 90°$

$\angle C < \angle B$

$AB < AC$ [Side opposite to a larger angle is longer]

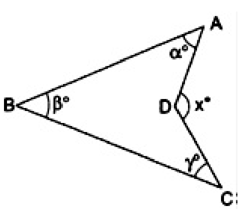
Hence,

$\because$ AC is the longest side of ΔABC i.e. the hypotenuse.

Question: 23

In the given figu

Solution:



In ΔABC,

$\angle A + \angle B + \angle C = 180°$ [Sum of all angles of a triangle = 180°]

According to the figure,

$\Rightarrow \angle B + (\alpha + \angle DAC) + (\gamma + \angle DCA) = 180°$

$\Rightarrow \angle DAC + \angle DCA + \alpha + \beta + \gamma = 180°$

$\Rightarrow \angle DAC + \angle DCA = 180° - (\alpha + \beta + \gamma)$ .... (i)

In ΔADC,

$\Rightarrow x + \angle DAC + \angle DCA = 180°$ [Sum of all angles of a triangle = 180°]

⇒ x = 180° - ∠DAC - ∠DCA

⇒ x = 180° - 180° + (α + β + γ)

∵ x = (α + β + γ)

Hence proved.

Question: 24

Find six rational

Solution:

Since, we want six numbers, we write 1 and 2 as rational numbers with denominator 6 + 1 = 7

So, multiply in numerator and denominator by 7, we get

$3 = \frac{3\times7}{1\times7} = \frac{21}{7}$ and $4 = \frac{4\times7}{1\times7} = \frac{28}{7}$

We know that, 21 < 22 < 23 < 24 < 25 < 26 < 27 < 28

$$\frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$$

Hence, six rational numbers between $3 = \frac{21}{7}$ and $4 = \frac{28}{7}$ are

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$$

Question: 25

If

Rationalising the above term,

$$\Rightarrow \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} * \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

Using the formula (a + b) (a - b) = $(a^2 – b^2)$

$$\Rightarrow \frac{5+3+2\sqrt{15}}{5-3} = \frac{8+2\sqrt{15}}{2}$$

∵ 4 + √15

Comparing with a + √15 b,

∵ a = 4, b = 1

OR

Solution: Given, $(5a – 7b)^3 + (9c – 5a)^3 + (7b – 9c)^3$

Using the formula, $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b) (b + c) (c + a)$

⇒ $a^3 + b^3 + c^3 = (a + b + c)^3 - 3(a + b) (b + c) (c + a)$

⇒ $(5a – 7b)^3 + (9c – 5a)^3 + (7b – 9c)^3 = (5a - 7b + 9c – 5a + 7b – 9c)^3 – 3(5a – 7b + 9c – 5a) (9c – 5a + 7b – 9c) (7b - 9c + 5a – 7b)$

⇒ $(5a – 7b)^3 + (9c – 5a)^3 + (7b – 9c)^3 = 0^3 – 3(-7b + 9c) (-5a + 7b) (-9c + 5a)$

∵ $(5a – 7b)^3 + (9c – 5a)^3 + (7b – 9c)^3 = 3(5a – 7b) (7b – 9c) (9c – 5a)$

Question: 26

Factorize:

Solution:

Given, $12(x^2 + 7x)^2 - 8(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$

By splitting the middle term i.e. $8(x^2 + 7x)(2x - 1)$, we get

$= 12(x^2 + 7x)^2 - 18(x^2 + 7x)(2x - 1) + 10(x^2 + 7x)(2x - 1) - 15(2x - 1)^2$

$= 6(x^2 + 7x)[2(x^2 + 7x) - 3(2x - 1)] + 5(2x - 1)[2(x^2 + 7x) - 3(2x - 1)]$

$= [2(x^2 + 7x) - 3(2x - 1)][6(x^2 + 7x) + 5(2x - 1)]$

$= (2x^2 + 14x - 6x + 3)(6x^2 + 42x + 10x - 5)$

$= (2x^2 + 8x + 3)(6x^2 + 52x - 5)$

Question: 27

If $(x^3$

Solution:

Given, $(x^3 + ax^2 + bx + 6)$ exactly divisible by $(x - 2)$

$\because$ x = 2 is a root of the above equation.

$\Rightarrow 2^3 + a(2)^2 + b(2) + 6 = 0$

$\Rightarrow 8 + 4a + 2b + 6 = 0$

$\because 4a + 2b = -14 b = \frac{-14-4a}{2} \quad ..... (i)$

Given, $(x^3 + ax^2 + bx + 6)$ divided by $(x - 3)$ leaves a remainder 3

$\because 3^3 + a(3)^2 + b(3) + 6 = 3$

$\Rightarrow 27 + 9a + 3b + 6 = 3$

$\because 9a + 3b = -30$ .... (ii)

Put value of b from (i) in this equation to get, $9a + 3\left(\frac{-14-4a}{2}\right) = -30$ 18a - 42 - 12 a

= - 606a - 42 = -606a = -60 + 426a = -18a = -3Put the value of a in (i) to get:

$b = \frac{-14-4(-3)}{2} \quad b = \frac{-14+12}{2} \quad b = \frac{-2}{2}$

Solving simultaneously eq (i) and eq (ii), we get

a = -3, b = -1

Question: 28

Without actual di

Solution:

Let’s find the roots of the equation $(x^2 + 2x - 3)$

$\Rightarrow x^2 + 3x - x - 3 = 0$

$\Rightarrow x(x + 3) - 1(x + 3) = 0$

$\because (x + 3)(x - 1)$

Hence, if (x + 3) and (x – 1) satisfies the equation $x^3 - 3x^2 - 13x + 15 = 0$, then $(x^3 - 3x^2 - 13x + 15)$ will be exactly divisible by $(x^2 + 2x - 3)$.

For x = -3,

$\Rightarrow (-3)^3 - 3(-3)^2 - 13(-3) + 15$

$\Rightarrow$ -27 – 27 + 39 + 15 = 0

For x = 1,

$\Rightarrow 1^3 - 3(1)^2 - 13(1) + 15$

$\Rightarrow$ 1 – 3 – 13 + 15 = 0

Hence proved.

Question: 29

Factorize: a

Solution:

Given, $a^3 - b^3 + 1 + 3ab$

$\Rightarrow a^3 + (-b)^3 + 1^3 - 3(1 * a * (-b))$

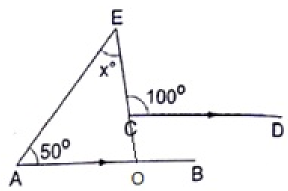
$\Rightarrow [a + (-b) + 1] [a^2 + (-b)^2 + 1^2 - a(-b) - (-b)1 - 1a]$

$\because (a - b + 1) (a^2 + b^2 + 1 + ab + b - a)$

Question: 30

In the given figu

Solution:



Given, ∠ECD = 100°, ∠EAB = 50°

∠COB = 100°

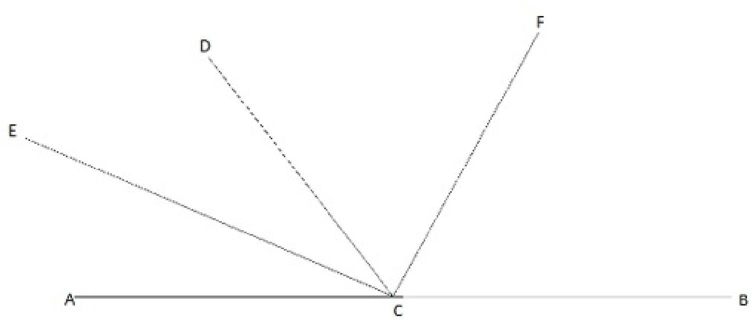
$\because$ x = 100° - 50° [Exterior angle = Sum of two opposite interior angles of a triangle]

$\because$ x = 50°

Question: 31

Prove that the bi

Solution:



Given, ∠ACD and ∠BCD are linear pairs

CE and CF bisect ∠ACD and ∠BCD respectively

To prove:

∠ECF = 90°

$\because$ ∠ACD + ∠BCD = 180° [Angle on a straight line]

$\Rightarrow$ ∠ACD/2 + ∠BCD/2 = 180°/2 = 90°

⇒ ∠ECD + ∠DCF = 90° [∵ CE and CF bisect ∠ACD and ∠BCD respectively]

∵ ∠ECD + ∠DCF = ∠ECF = 90°

Hence Proved.

Question: 32

In the given figu

Solution:

Let the ratio be y

∵ ∠DAB = y

∵ ∠DAC = 3y

∵ y + 3y + 108° = 180° [Angle on a straight line]

⇒ 4y = 72°

∵ y = 18°

∵ ∠DAC = 3y = 54°

∠ABD = 18° [∵ AD = DB, ΔABD is an isosceles triangle]

In ΔABC,

⇒ x + ∠A + ∠B = 180° [Sum of all angles of a triangle = 180°]

⇒ x = 180° - 72° - 18°

∵ x = 90°

Question: 33

In the given figu

Solution:

In ΔABC,

∠A = 180° - 70° - 20° [Sum of all angles of a triangle = 180°]

∵ ∠A = 90°

∵ ∠BAN = 45° [∵ AN is the bisector of ∠A]

In ΔABN,

∠N = 180° - 70° - 45° [Sum of all angles of a triangle = 180°]

∵ ∠N = 65°

In ΔAMN,

∠MAN = 180° - 90° - 65° [Sum of all angles of a triangle = 180°]

∵ ∠MAN = 25°

Question: 34

If the bisector o

Solution:

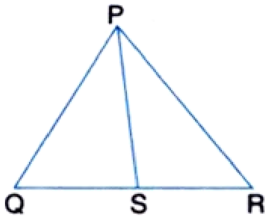
Given,

In ΔPQR,

PS bisects ∠QPR and QS = SR

To prove:

PQ = PR



In ΔPQS and ΔPRS

$QS = SR$ [Given]

$\angle QPS = \angle RPS$ [Given]

$PS = PS$ [Common]

ΔPQS is congruent to ΔPRS [S.A.S]

$\because PQ = PR$ [C.P.C.T.C]

Hence Proved.