Chapter : 6. LINEAR INEQUATIONS (IN ONE VARIABLE)

Exercise : 6A

Question: 1

Fill in the blank

Solution:

(i) 5x < 20 ⇒ x ……… 4

As, 5x < 20

Then,

Dividing both the sides by 5

$$\frac{x}{5} < \frac{20}{5}$$

x < 4

Therefore,

5x < 20 ⇒ x < 4

(ii) -3x > 9 ⇒ x ……… -3

As, -3x > 9

Then, Dividing both the sides by 3

$$\frac{x}{3} > -\left(\frac{9}{3}\right)$$

x > -3

Therefore,

-3x > 9 ⇒ x > -3

(iii) 4x > -16 ⇒ x ……… -4

As, 4x > -16

Then, Dividing both the sides by 4

$$\frac{x}{4} > -\left(\frac{16}{4}\right)$$

x > -4

Therefore,

4x > -16 ⇒ x > -4

(iv) -6x ≤ -18 ⇒ x ……… 3

As -6x ≤ -18

Then, Dividing both the sides by 6

$$\frac{-x}{6} \le \left(\frac{-18}{6}\right)$$

-x ≤ -3

Now multiplying by -1 on both sides

-x(-1) ≤ -3(-1)

x ≥ 3 (inequality sign reversed)

Therefore,

$-6x \leq -18 \Rightarrow x \geq 3$

(v) $x > -3 \Rightarrow -2x$ ……… 6

As, $x > -3$

Multiplying both sides by 2

Then,

$2x > -6$

Now multiplying both the sides by -1

$2x(-1) < 6(-1)$

$-2x > 6$

Therefore,

$x > -3 \Rightarrow -2x > 6$

(vi) $a < b$ and $c > 0 \Rightarrow \frac{a}{c}$ ……… $\frac{b}{c}$

As,

$a < b$ …(1)

$c > 0$

Dividing both sides by c in equation (1)

Then,

$$\frac{a}{c} < \frac{b}{c}$$

Therefore,

$a < b$ and $c > 0 \Rightarrow \frac{a}{c} < \frac{b}{c}$

(vii) $p - q = -3 \Rightarrow p$ ……… q

As,

$p - q = -3$

$p = q - 3$

From the above equation it is clear that p would always be less than q

Therefore,

$p - q = -3 \Rightarrow p < q$

(viii) $u - v = 2 \Rightarrow u$ ……… v

As,

$u - v = 2$

$u = v + 2$

From the above equation it is clear that u would always be greater than v

Therefore,

$u - v = 2 \Rightarrow u > v$

Question: 2

(i) $6x \leq 25, x \in N$

Dividing both the sides by 6 in the above equation,

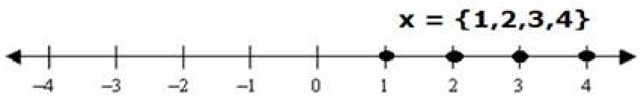
$\frac{6x}{6} \leq \frac{25}{6}$

x ≤ $\frac{25}{6}$

x ≤ 4.166

Since x is a natural number, therefore the value of x can be less than or equal to 4

Therefore, x = {1,2,3,4}



(ii) 6x ≤ 25, x ϵ Z

Dividing both the sides by 6 in the above equation,

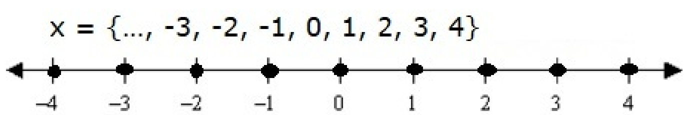
$\frac{6x}{6} \leq \frac{25}{6}$

x ≤ $\frac{25}{6}$

x ≤ 4.166

Since x is an integer so the possible values of x can be:

x = {..., -3, -2, -1, 0, 1, 2, 3, 4}



Question: 3

(i) -2x > 5, x ϵ Z

Multiply both the sides by -1 in above equation,

-2x(-1) > 5(-1)

2x < -5

Dividing both the sides by 2 in above equation,

$\frac{2x}{2} < \frac{-5}{2}$

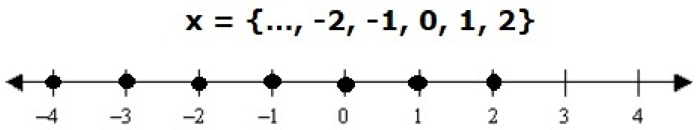
x < $\frac{-5}{2}$

x < 2.5

Since, x is an integer

Therefore, possible values of x can be

x = {..., -2, -1, 0, 1, 2}



(ii) -2x > 5, x ϵ R

Multiply both the sides by -1 in above equation,

-2x(-1) > 5(-1)

2x < -5

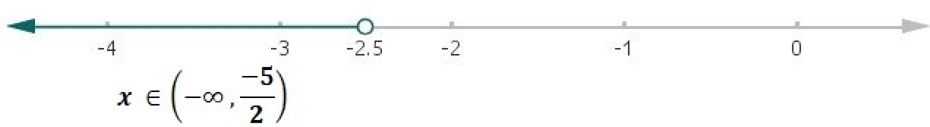
Dividing both the sides by 2 in above equation,

$\frac{2x}{2} < \frac{-5}{2}$

x < $\frac{-5}{2}$

Therefore,

x $\in \left(-\infty, \frac{-5}{2}\right)$



Question: 4

(i) 3x + 8 > 2, x ϵ Z

Subtracting 8 from both the sides in above equation

3x + 8 – 8 > 2 – 8

3x > - 6

Dividing both the sides by 3 in above equation

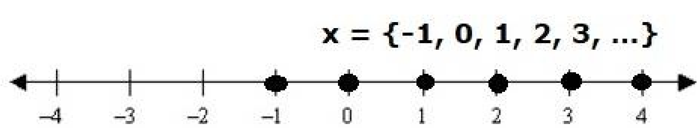
$\frac{3x}{3} > \frac{-6}{3}$

Thus, x > -2

Since x is an integer

Therefore, possible values of x can be

x = {-1, 0, 1, 2, 3, ...}



(ii) 3x + 8 > 2, x ϵ R

Subtracting 8 from both the sides in above equation

3x + 8 – 8 > 2 – 8

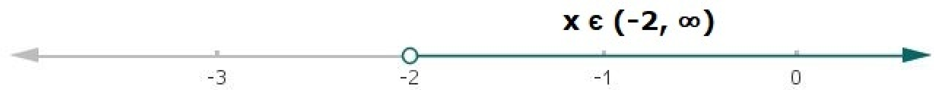
3x > - 6

Dividing both the sides by 3 in above equation

$\frac{3x}{3} > \frac{-6}{3}$

Thus, x > -2

x ϵ (-2, ∞)



Question: 5

(i) 5x + 2 < 17, x ϵ Z

Subtracting 2 from both the sides in the above equation,

5x + 2 – 2 < 17 – 2

5x < 15

Dividing both the sides by 5 in the above equation,

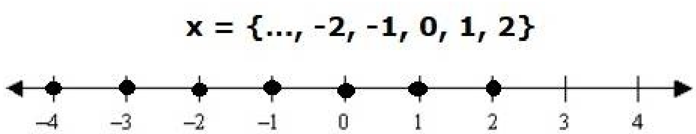
$\frac{5x}{5} < \frac{15}{5}$

x < 3

Since x is an integer

Therefore, possible values of x can be

x = {…, -2, -1, 0, 1, 2}



(ii) 5x + 2 < 17, x ϵ R

Subtracting 2 from both the sides in above equation

5x + 2 – 2 < 17 – 2

5x < 15

Dividing both the sides by 5 in above equation

$\frac{5x}{5} < \frac{15}{5}$

x < 3

Therefore, x ϵ (-∞, 3)

Question: 6

Solve each of the

Solution:

Given:

3x – 4 > x + 6, where x ϵ R.

3x – 4 > x + 6

Adding 4 to both sides in above equation

$3x - 4 + 4 > x + 6 + 4$

$3x > x + 10$

Now, subtracting x from both the sides in above equation

$3x - x > x + 10 - x$

$2x > 10$

Now, dividing both the sides by 2 in above equation

$\frac{2x}{2} > \frac{10}{2}$

$x > 5$

Therefore, $x \in (5, \infty)$

Question: 7

Given:

$3 - 2x \geq 4x - 9$, where $x \in R$.

$3 - 2x \geq 4x - 9$

Subtracting 3 from both the sides in the above equation,

$3 - 2x - 3 \geq 4x - 9 - 3$

$-2x \geq 4x - 12$

Now, subtracting 4x from both the sides in the above equation,

$-2x - 4x \geq 4x - 12 - 4x$

$-6x \geq -12$

Now, dividing both the sides by 6 in the above equation,

$\frac{-6x}{6} \geq \frac{-12}{6}$

$-x \geq -2$

Now, multiplying by (-1) on both the sides in above equation.

$(-x).(-1) \geq (-2).(-1)$

$x \leq 2$

Question: 8

Given:

$\frac{5x-8}{3} \geq \frac{4x-7}{2}$, where $x \in R$.

$(5x - 8).(2) \geq (4x-7).(3)$

$10x - 16 \geq 12x - 21$

Now, adding 16 to both the sides

10x -16 + 16 ≥ 12x -21 + 16

10x ≥ 12x -5

Now, subtracting 12x from both the sides of the above equation

10x -12x ≥ 12x – 5 – 12x

-2x ≥ -5

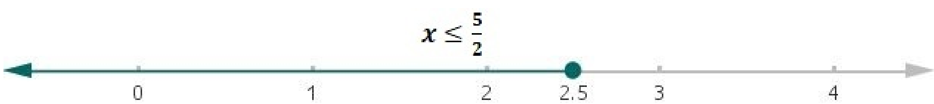
Now, multiplying by -1 on both the sides of above equation

(-2x)(-1) ≥ (-5)(-1)

2x ≤ 5 (inequality reversed)

Therefore,

$x \leq \frac{5}{2}$



Question: 9

Given:

$\frac{5x}{4} - \frac{4x-1}{3} > 1$, where x ϵ R.

$$\frac{3(5x) \ - 4(4x \ - 1)}{12} > 1$$

$$\frac{15x \ - 16x \ + \ 4}{12} > 1$$

$$\frac{-x \ + \ 4}{12} > 1$$

Now, multiplying by 12 on both the sides in the above equation,

$$\left(\frac{-x \ + \ 4}{12}\right).(12) > 1.(12)$$

-x + 4 > 12

Now, subtracting 4 from both the sides in above equation

-x + 4 – 4 > 12 – 4

-x > 8

Multiplying by -1 on both the sides of the above equation

x < - 8

Question: 10

Given:

$\frac{1}{2}\left(\frac{2}{3}x + 1\right) \geq \frac{1}{3}(x - 2)$, where x ϵ R.

$$\frac{1}{2}\left(\frac{2x}{3}\right)+\frac{1}{2}(1)\geq\frac{1}{3}(x)-\frac{1}{3}(2)$$

$$\frac{x}{3}+\frac{1}{2}\geq\frac{x}{3}-\frac{2}{3}$$

Now, subtracting $\frac{1}{2}$ from both the sides in the above equation

$$\frac{x}{3}+\frac{1}{2}-\frac{1}{2}\geq\frac{x}{3}-\frac{2}{3}-\frac{1}{2}$$

$$\frac{x}{3}\geq\frac{2x-4-3}{6}$$

$$\frac{x}{3}\geq\frac{2x-7}{6}$$

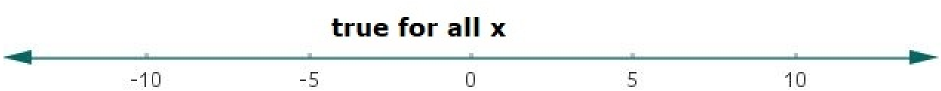
$$\frac{x}{3}\geq\frac{x}{3}-\frac{7}{6}$$

Now, subtracting $\frac{x}{3}$ from both the sides in athe bove equation,

$$\frac{x}{3}-\frac{x}{3}\geq\frac{x}{3}-\frac{7}{6}-\frac{x}{3}$$

$$0\geq-\frac{7}{6}$$

Therefore, the solution is: true for all values of x.



Question: 11

Solve each of the

Solution:

Given:

$\frac{2x-1}{12}-\frac{x-1}{3}<\frac{3x+1}{4}$, where x ϵ R.

Multiply by 12 on both sides in the above equation

$$12\left(\frac{2x-1}{12}\right)-12\left(\frac{x-1}{3}\right)<12\left(\frac{3x+1}{4}\right)$$

(2x - 1) – 4(x - 1) < 3(3x + 1)

2x – 1 – 4x + 4 < 9x + 3

3 – 2x < 9x + 3

Now, subtracting 3 on both sides in the above equation

3 – 2x – 3 < 9x + 3 - 3

-2x < 9x

Now, subtracting 9x from both the sides in the above equation

-2x – 9x < 9x – 9x

- 11x < 0

Multiplying -1 on both the sides in above equation

(-11x)(-1) < (0)(-1)

11x > 0

Dividing both sides by 11 in above equation

$$\frac{11x}{11} > \frac{0}{11}$$

Therefore,

x > 0

Question: 12

Solve each of the

Solution:

Given:

$\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$, where x ϵ R.

Multiplying 60 on both the sides in the above equation,

$$\frac{x}{4}(60) < \frac{(5x-2)}{3}(60) - \frac{(7x-3)}{5}(60)$$

15x < 20(5x - 2) – 12(7x - 3)

15x < 100x – 40 – 84x + 36

15x < 16x – 4

Now, subtracting 16x from both sides in above equation

15x – 16x < 16x – 4 – 16x

-x < -4

Now, multiplying by -1 on both sides in above equation

(-x)(-1) < (-4)(-1)

x > 4 (inequality sign reversed)

Question: 13

Solve each of the

Solution:

Given:

$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$, where x ϵ R.

Multiplying by 60 on both the sides in the above equation.

$$(60)\frac{(2x-1)}{3} \geq (60)\frac{(3x-2)}{4} - (60)\frac{(2-x)}{5}$$

20(2x – 1) ≥ 15(3x – 2) -12(2 – x)

40x – 20 ≥ 45x – 30 – 24 + 12x

40x – 20 ≥ 57x – 54

Now, Adding 20 on both the sides in the above equation

40x – 20 + 20 ≥ 57x – 54 + 20

40x ≥ 57x – 34

Now, subtracting 57x from both the sides in the above equation

40x – 57x ≥ 57x – 34 – 57x

-17x ≥ -34

Multiplying by -1 on both sides in the above equation

(-17x)(-1) ≥ (-34)(-1)

17x ≤ 34

Now, divide by 17 on both sides in the above equation

$\frac{17x}{17} \leq \frac{34}{17}$

Therefore,

x ≤ 2

Question: 14

Solve each of the

Solution:

Given:

$\frac{x-3}{x+1} < 0, x \in R$

Signs of x – 3

x – 3 = 0 → x = 0 (Adding both the sides by 3)

x – 3 < 0 → x < 3 (Adding both the sides by 3)

x – 3 > 0 → x > 3 (Adding both the sides by 3)

Signs of x + 1

x + 1 = 0 → x = -1 (Subtracting both the sides by 1)

x + 1 < 0 → x < -1 (Subtracting both the sides by 1)

x + 1 > 0 → x > -1 (Subtracting both the sides by 1)

$\frac{x-3}{x+1}$ is not defined when x = -1

The interval that satisfies the condition that $\frac{x-3}{x+1} < 0$ is -1 < x < 3

Therefore,

x ϵ (-1, 3)

Question: 15

Solve each of the

Solution:

Given:

$\frac{x-3}{x+4} < 0, x \in R$

Signs of x – 3

x – 3 = 0 → x =3 (Adding both the sides by 3)

x – 3 < 0 → x < 3 (Adding both the sides by 3)

x – 3 > 0 → x > 3 (Adding both the sides by 3)

Signs of x + 4

x + 4 = 0 → x = -4 (Subtracting both the sides by 4)

x + 4 < 0 → x < -4 (Subtracting both the sides by 4)

x + 4 > 0 → x > -4 (Subtracting both the sides by 4)

$\frac{x-3}{x+4}$ is not defined when x = -4

The interval that satisfies the condition that $\frac{x-3}{x+4} < 0$ is -4 < x < 3

Therefore,

x є (-4, 3)

Question: 16

Solve each of the

Solution:

Given:

$\frac{2x-3}{3x-7} < 0, x \in R.$

Signs of 2x – 3:

2x – 3 = 0 → $x = \frac{3}{2}$

(Adding 3 on both the sides and then dividing both sides by 2)

2x – 3 < 0 → $x < \frac{3}{2}$

(Adding 3 on both the sides and then dividing both sides by 2)

2x – 3 > 0 → $x > \frac{3}{2}$

(Adding 3 on both the sides and then dividing both sides by 2)

Signs of 3x – 7:

3x – 7 = 0 → $x = \frac{7}{3}$

(Adding 7 on both the sides and then dividing both sides by 3)

3x – 7 < 0 → $x < \frac{7}{3}$

(Adding 7 on both the sides and then dividing both sides by 3)

3x – 7 > 0 → $x > \frac{7}{3}$

(Adding 7 on both the sides and then dividing both sides by 3)

Zeroes of denominator:

3x – 7 = 0

$x = \frac{7}{3}$

(Adding 7 on both the sides and then dividing both sides by 3)

Interval that satisfies the required condition: < 0

$\frac{3}{2} < x < \frac{7}{3}$

Question: 17

Solve each of the

Solution:

Given:

$\frac{x-7}{x-2} \geq 0, x \in R.$

$\frac{x-7}{x-2} \geq 0$

Signs of x – 7:

x – 7 = 0 → x = 7(Adding 7 on both the sides)

x – 7 > 0 → x > 7 (Adding 7 on both the sides)

x – 7 < 0 → x < 7 (Adding 7 on both the sides)

Signs of x – 2:

x – 2 = 0 → x = 2 (Adding 2 on both the sides)

x – 2 > 0 → x > 2 (Adding 2 on both the sides)

x – 2 < 0 → x < 2 (Adding 2 on both the sides)

Zeroes of denominator:

x – 2 = 0 → at x = 2 $\frac{x-7}{x-2}$ will be undefined.

Intervals that satisfy the required condition: ≥ 0

x < 2 or x = 7 or x >7

Therefore,

x є (-∞, -2) υ [7, ∞)

Question: 18

Given:

$\frac{3}{x-2} < 2, x \in R.$

Subtracting 2 from both the sides in the above equation,

$\frac{3}{x-2} - 2 < 2 - 2$

$\frac{3-2(x-2)}{x-2} < 0$

$\frac{3-2x+4}{x-2} < 0$

$\frac{7\ -\ 2x}{x-2} < 0$

Signs of 7 – 2x:

$7 - 2x = 0 \to x = \frac{7}{2}$

(Subtracting by 7 on both the sides, then multiplying by -1 on both the sides and then dividing both the sides by 2)

$7 - 2x < 0 \to x > \frac{7}{2}$

(Subtracting by 7 on both the sides, then multiplying by -1 on both the sides and then dividing both the sides by 2)

$7 - 2x > 0 \to x < \frac{7}{2}$

(Subtracting by 7 on both the sides, then multiplying by -1 on both the sides and then dividing both the sides by 2)

Signs of $x - 2$:

$x - 2 = 0 \to x = 2$ (Adding 2 on both the sides)

$x - 2 < 0 \to x < 2$ (Adding 2 on both the sides)

$x - 2 > 0 \to x > 2$ (Adding 2 on both the sides)

Zeroes of denominator:

$x - 2 = 0 \to x = 2$

At $x = 2$, $\frac{7-2x}{x-2}$ is not defined

Intervals satisfying the condition: < 0

$x < 2$ and $x > \frac{7}{2}$

Therefore,

$$x \in (-\infty, 2) \cup \left(\frac{7}{2}, \infty\right)$$

Question: 19

Given:

$\frac{1}{x-1} \leq 2, x \in R.$

Subtracting 2 from both the sides in the above equation

$$\frac{1}{x-1} - 2 \leq 2 - 2$$

$$\frac{1-2(x-1)}{x-1} \leq 0$$

$$\frac{1 - 2x + 2}{x-1} \leq 0$$

$$\frac{3 - 2x}{x-1} \leq 0$$

Signs of $3 - 2x$:

$3 - 2x = 0 \to x = \frac{3}{2}$

(Subtracting by 3 on both the sides, then multiplying by -1 on both the sides and then dividing both the sides by 2)

$3 - 2x < 0 \to x > \frac{3}{2}$

(Subtracting by 3 on both the sides, then multiplying by -1 on both the sides and then dividing both the sides by 2)

$3 - 2x > 0 \rightarrow x < \frac{3}{2}$

(Subtracting by 3 on both the sides, then multiplying by -1 on both the sides and then dividing both the sides by 2)

Signs of x – 1:

$x - 1 = 0 \rightarrow x = 1$ (Adding 1 on both the sides)

$x - 1 < 0 \rightarrow x < 1$ (Adding 1 on both the sides)

$x - 1 > 0 \rightarrow x > 1$ (Adding 1 on both the sides)

Zeroes of denominator:

$x - 1 = 0 \rightarrow x = 1$

At x = 1, $\frac{3-2x}{x-1}$ is not defined

Intervals satisfying the condition: ≤ 0

$x < 1$ and $x \geq \frac{3}{2}$

Therefore,

$$x \in (-\infty, 1) \cup \left[\frac{3}{2}, \infty\right)$$

Question: 20

Given:

$\frac{5x+8}{4-x} < 2, x \in R.$

Subtracting both the sides by 2

$$\frac{5x+8}{4-x} - 2 < 2 - 2$$

$$\frac{5x+8 - 2(4-x)}{4-x} < 0$$

$$\frac{5x+8-8+2x}{4-x} < 0$$

$$\frac{7x}{4-x} < 0$$

Now dividing both the sides by 7

$$\frac{7x}{7(4-x)} < \frac{0}{7}$$

$$\frac{x}{4-x} < 0$$

Signs of x:

$x = 0$

$x < 0$

$x > 0$

Signs of 4 – x:

$4 - x = 0 \rightarrow x = 4$

(Subtracting 4 from both the sides, then dividing by -1 on both the sides)

$4 - x < 0 \rightarrow x > 4$

(Subtracting 4 from both the sides, then multiplying by -1 on both the sides)

$4 - x > 0 \rightarrow x < 4$

(Subtracting 4 from both the sides, then multiplying by -1 on both the sides)

At x = 4, $\frac{x}{4-x}$ is not defined

Intervals satisfying the condition: < 0

x < 0 or x > 4

Therefore,

$x \in (-\infty, 0) \cup (4, \infty)$

Question: 21

Given:

$|3x - 7| > 4, x \in R.$

$3x - 7 < -4$ or $3x - 7 > 4$

(Because $|x| > a$, $a>0$ then $x < -a$ and $x > a$)

$3x - 7 < -4$

Now, adding 7 to both the sides in the above equation

$3x - 7 + 7 < -4 + 7$

$3x < 3$

Now, dividing by 3 on both the sides of above equation

$$\frac{3x}{3} < \frac{3}{3}$$

$x < 1$

Now,

$3x - 7 > 4$

Adding 7 on both the sides in above equation

$3x - 7 + 7 > 4 + 7$

$3x > 11$

Now, dividing by 3 on both the sides in the above equation

$$\frac{3x}{3} > \frac{11}{3}$$

$$x > \frac{11}{3}$$

Therefore,

$$x \in (-\infty, 1) \cup \left(\frac{11}{3}, \infty\right)$$

Question: 22

Given:

$|5 - 2x| \le 3, x \in R.$

$5 - 2x \ge -3$ or $5 - 2x \le 3$

$5 - 2x \ge -3$

Subtracting 5 from both the sides in the above equation

$5 - 2x - 5 \geq -3 - 5$

$-2x \geq -8$

Now, multiplying by -1 on both the sides in the above equation

$-2x(-1) \geq -8(-1)$

$2x \leq 8$

Now dividing by 2 on both the sides in the above equation

$$\frac{2x}{2} \leq \frac{8}{2}$$

$x \leq 4$

$5 - 2x \leq 3$

Subtracting 5 from both the sides in the above equation

$5 - 2x - 5 \leq 3 - 5$

$-2x \leq -2$

Now, multiplying by -1 on both the sides in the above equation

$-2x(-1) \leq -2(-1)$

$2x \geq 2$

Now dividing by 2 on both the sides in the above equation

$$\frac{2x}{2} \geq \frac{2}{2}$$

$x \geq 1$

Therefore,

$x \in [1, 4]$

Question: 23

Given:

$|4x - 5| \leq \frac{1}{3}, x \in R.$

$$4x - 5 \leq \frac{1}{3} \text{ or } 4x - 5 \geq -\frac{1}{3}$$

$$4x - 5 \leq \frac{1}{3}$$

Adding 5 to both the sides in the above equation

$$4x - 5 + 5 \leq \frac{1}{3} + 5$$

$$4x \leq \frac{1 + 15}{3}$$

$$4x \leq \frac{16}{3}$$

Now, dividing both the sides by 4 in the above equation

$$\frac{4x}{4} \leq \frac{16}{3.(4)}$$

$$x \leq \frac{4}{3}$$

Now,

$4x - 5 \geq -\frac{1}{3}$

Adding 5 to both the sides in the above equation

$4x - 5 + 5 \geq -\frac{1}{3} + 5$

$4x \geq \frac{-1 + 15}{3}$

$4x \geq \frac{14}{3}$

Now, dividing both the sides by 4 in the above equation

$\frac{4x}{4} \geq \frac{14}{3.(4)}$

$x \geq \frac{7}{6}$

Therefore,

$x \in \left[\frac{7}{6}, \frac{4}{3}\right]$

Question: 24

Given:

$\frac{1}{|x|-3} \leq \frac{1}{2}$, x ϵ R.

Intervals of |x|:

|x| = -x, x<0

|x| = x, x ≥ 0

Domain of $\frac{1}{|x|-3} \leq \frac{1}{2}$

|x| + 3 = 0

X = -3 or x = 3

Therefore,

-3 < x < 3

Now, combining intervals with domain:

x < -3, -3<x<0, 0 ≤ x <3, x >3

For x < -3

$\frac{1}{|x|-3} \leq \frac{1}{2} \rightarrow \frac{1}{-x-3} \leq \frac{1}{2}$

Now, subtracting $\frac{1}{2}$ from both the sides

$\frac{1}{-x - 3} - \frac{1}{2} \leq \frac{1}{2} - \frac{1}{2}$

$\frac{2 - (-x - 3)}{2(-x - 3)} \leq 0$

$\frac{x + 5}{-2x - 6} \leq 0$

Signs of x + 5:

x + 5 = 0 → x = -5 (Subtracting 5 from both the sides)

$x + 5 > 0 \rightarrow x > -5$ (Subtracting 5 from both the sides)

$x + 5 < 0 \rightarrow x < -5$ (Subtracting 5 from both the sides)

Signs of $-2x - 6$:

$-2x - 6 = 0 \rightarrow x = -3$

(Adding 6 on both the sides, then multiplying both the sides by -1 and then dividing both the sides by 2)

$-2x - 6 > 0 \rightarrow x < -3$

(Adding 6 on both the sides, then multiplying both the sides by -1 and then dividing both the sides by 2)

$-2x - 6 < 0 \rightarrow x > -3$

(Adding 6 on both the sides, then multiplying both the sides by -1 and then dividing both the sides by 2)

Intervals satisfying the required condition: ≤ 0

$x < -5$, $x = -5$, $x > -3$

or

$x \leq -5$ or $x > -3$

Similarly, for $-3 < x < 0$:

$x \leq -5$ or $x > -3$

Merging overlapping intervals:

$-3 < x < 0$

For, $0 \leq x < 3$:

$\frac{1}{|x|-3} \leq \frac{1}{2} \rightarrow \frac{1}{x-3} \leq \frac{1}{2}$

Subtracting $\frac{1}{2}$ from both the sides

$$\frac{1}{x-3} - \frac{1}{2} \leq \frac{1}{2} - \frac{1}{2}$$

$$\frac{2-(x-3)}{2(x-3)} \leq 0$$

$$\frac{5-x}{2(x-3)} \leq 0$$

Multiplying both the sides by 2

$$\frac{2(5-x)}{2(x-3)} \leq 0(2)$$

$$\frac{(5-x)}{(x-3)} \leq 0$$

Signs of $5 - x$:

$5 - x = 0 \rightarrow x = 5$

(Subtracting 5 from both the sides and then dividing both sides

by -1)

$5 - x > 0 \rightarrow x < 5$

(Subtracting 5 from both the sides and then multiplying both sides by -1)

$5 – x < 0 \rightarrow x > 5$

(Subtracting 5 from both the sides and then multiplying both sides by -1)

Signs of $x – 3$:

$x – 3 = 0 \rightarrow x = 3$ (Adding 3 to both the sides)

$5 – x > 0 \rightarrow x > 3$ (Adding 3 to both the sides)

$5 – x < 0 \rightarrow x < 3$ (Adding 3 to both the sides)

Intervals satisfying the condition: $x \le 0$

$x < 3$ or $x = 5$ or $x > 5$

or

$x <3$ and $x \ge 5$

Similarly, for $0 \le x < 3$:

$x <3$ and $x \ge 5$

Merging overlapping intervals:

$0 \le x < 3$

Now, combining all the intervals satisfying condition: ≤ 0

$x \le -5$ or $-3 < x < 0$ or $0 \le x < 3$ or $x \ge 5$

Therefore

$x \in (-\infty, -5] \cup (-3, 3) \cup [5, \infty)$

Question: 25

Given:

$\frac{|x-3|-x}{x} < 2$, $x \in R$.

Intervals of $|x - 3|$

$|x – 3| = -(x – 3)$ or $(x – 3)$

When $|x - 3| = x – 3$

$x – 3 \ge 0$

Therefore, $x \ge 3$

When $|x - 3| = -(x – 3)$

$(x – 3) < 0$

Therefore, $x < 3$

Intervals: $x \ge 3$ or $x < 3$

Domain of $\frac{|x-3|-x}{x} < 2$:

$\frac{|x-3|-x}{x}$ is not defined for $x = 0$

Therefore, $x > 0$ or $x < 0$

Now, combining intervals and domain:

$x < 0$ or $0 < x < 3$ or $x \ge 3$

For $x = 0$

$\frac{|x-3|-x}{x} < 2 \rightarrow \frac{-(x-3)-x}{x} < 2$

Now, subtracting 2 from both the sides

$$\frac{-(x-3)-x}{x}-2<2-2$$

$$\frac{-x+3-x-2x}{x}<2-2$$

$$\frac{3-4x}{x}<0$$

Signs of 3 – 4x:

3 – 4x = 0 → $x = \frac{3}{4}$

(Subtracting 3 from both the sides and then dividing both sides by -1)

3 – 4x > 0 → $x < \frac{3}{4}$

(Subtracting 3 from both the sides and then multiplying both sides by -1)

3 – 4x < 0 → $x > \frac{3}{4}$

(Subtracting 3 from both the sides and then multiplying both sides by -1)

Signs of x:

x = 0

x < 0

x > 0

Intervals satisfying the required condition: < 0

x < 0 or $x > \frac{3}{4}$

Combining the intervals:

x < 0 or $x > \frac{3}{4}$ and x < 0

Merging the overlapping intervals:

x < 0

Similarly, for 0 < x < 3

x < 0 or $x > \frac{3}{4}$ and 0 < x < 3

Merging the overlapping intervals:

$\frac{3}{4} < x < 3$

For, x ≥ 3

$\frac{|x-3|-x}{x} < 2 \rightarrow \frac{(x-3)-x}{x} < 2$

Now, subtracting 2 from both the sides

$$\frac{(x-3)-x}{x}-2<2-2$$

$$\frac{x-3-x-2x}{x}<2-2$$

$$\frac{-3-2x}{x}<0$$

Signs of -3 – 2x:

$-3 - 2x = 0 \rightarrow x = \frac{-3}{2}$

(Adding 3 to both the sides and then dividing both sides by -2)

$-3 - 2x > 0 \rightarrow x < \frac{-3}{2}$

(Adding 3 to both the sides and then multiplying both sides by -1)

$-3 - 2x < 0 \rightarrow x > \frac{-3}{2}$

(Adding 3 to both the sides and then multiplying both sides by -1)

Signs of x:

$x = 0$

$x < 0$

$x > 0$

Intervals satisfying the required condition: < 0

$x < \frac{-3}{2}$ or $x > 0$

Combining the intervals:

$x < \frac{-3}{2}$ or $x > 0$ and $x \geq 3$

Merging the overlapping intervals:

$x \geq 3$

Combining all the intervals:

$x < 0$ or $\frac{3}{4} < x < 3$ or $x \geq 3$

Merging overlapping intervals:

$x < 0$ and $x > \frac{3}{4}$

Therefore,

$x \in (-\infty, 0) \cup \left(\frac{3}{4}, \infty\right)$

Question: 26

Given:

$\left|\frac{2x-1}{x-1}\right| < 2$, x ϵ R.

$-2 < \left|\frac{2x-1}{x-1}\right| < 2$

$\frac{2x-1}{x-1} > -2$ and $\frac{2x-1}{x-1} < 2$

When,

$\frac{2x-1}{x-1} > -2$

Adding 2 to both sides in the above equation

$\frac{2x-1}{x-1} + 2 > -2 + 2$

$\frac{2x-1+2(x-1)}{x-1} > 0$

$\frac{2x-1+2x-2}{x-1} > 0$

$\frac{4x-3}{x-1} > 0$

Signs of 4x – 3:

$4x - 3 = 0 \rightarrow x = \frac{3}{4}$

(Adding 3 to both sides and then dividing both sides by 4)

$4x - 3 > 0 \rightarrow x > \frac{3}{4}$

(Adding 3 to both sides and then dividing both sides by 4)

$4x - 3 < 0 \rightarrow x < \frac{3}{4}$

(Adding 3 to both sides and then dividing both sides by 4)

Signs of x – 1:

$x - 1 = 0 \rightarrow x = 1$ (Adding 1 to both the sides)

$x - 1 > 0 \rightarrow x > 1$ (Adding 1 to both the sides)

$x - 1 < 0 \rightarrow x < 1$ (Adding 1 to both the sides)

At x = 1, $\frac{4x-3}{x-1}$ is not defined.

Intervals that satisfy the required condition: > 0

$x < \frac{3}{4}$ or $x > 1$

Now, when $\frac{2x-1}{x-1} < 2$

Subtracting 2 from both the sides

$\frac{2x-1}{x-1} - 2 < 2 - 2$

$\frac{2x-1-2(x-1)}{x-1} < 0$

$\frac{2x-1-2x+2)}{x-1} < 0$

$\frac{1}{x-1} < 0$

Signs of x – 1:

$x - 1 = 0 \rightarrow x = 1$ (Adding 1 on both the sides)

$x - 1 < 0 \rightarrow x < 1$ (Adding 1 on both the sides)

$x - 1 > 0 \rightarrow x > 1$ (Adding 1 on both the sides)

At x = 1, $\frac{1}{x-1}$ is not defined

Interval which satisfy the required condition: < 0

$x < 1$

Now, combining the intervals:

$x < \frac{3}{4}$ or $x > 1$ and $x < 1$

Merging the overlapping intervals:

$x < \frac{3}{4}$

Therefore,

$x \in \left(-\infty, \frac{3}{4}\right)$

Question: 27

Given:

$\frac{|x-3|}{x-3} < 0$, x ϵ R.

|x - 3| < 0

The above condition can’t be true because the absolute value cannot be less than 0

Therefore,

There is no solution for x ϵ R .

Question: 28

Given:

$\frac{|x|-1}{|x|-2} \geq 0$, x ϵ R. – {–2, 2}

Intervals of |x|:

x ≥ 0, |x| = x and x < 0, |x| = -x

Domain of $\frac{|x|-1}{|x|-2} \geq 0$

$\frac{|x|-1}{|x|-2}$ is not defined for x = -2 and x = 2

Therefore, Domain: x < -2 or -2 < x < 2 or x > 2

Combining intervals with domain:

x<-2, -2<x<0, 0 ≤ x<2, x>2

For x < -2:

$$\frac{|x|-1}{|x|-2} = \frac{-x-1}{-x-2}$$

$\frac{-x-1}{-x-2} \geq 0$

Signs of – x – 1:

-x -1 = 0 → x = -1

(Adding 1 to both the sides and then dividing by -1 on both the sides)

-x – 1> 0 → x < -1

(Adding 1 to both the sides and then multiplying by -1 on both the sides)

-x – 1 < 0 → x > -1

(Adding 1 to both the sides and then multiplying by -1 on both the sides)

Signs of – x – 2:

-x -2 = 0 → x = -2

(Adding 2 to both the sides and then dividing by -1 on both the sides)

-x – 2> 0 → x < -2

(Adding 2 to both the sides and then multiplying by -1 on both the sides)

-x – 2 < 0 → x > -2

(Adding 2 to both the sides and then multiplying by -1 on both the sides)

Intervals satisfying the required condition: ≥ 0

x < - 2 or x = -1 or x > -1

Merging overlapping intervals:

x < -2 or x ≥ -1

Combining the intervals:

x < -2 or x ≥ -1 and x < -2

Merging overlapping intervals:

x < -2

Similarly, for -2 < x < 0:

$$\frac{|x| - 1}{|x| - 2} = \frac{-x - 1}{-x - 2}$$

$\frac{-x-1}{-x-2} \geq 0$

Therefore,

Intervals satisfying the required condition: ≥ 0

x < - 2 or x = -1 or x > -1

Merging overlapping intervals:

x < -2 or x ≥ -1

Combining the intervals:

x < -2 or x ≥ -1 and -2 < x < 0

Merging overlapping intervals:

-1 ≤ x < 0

For 0 ≤ x < 2,

$$\frac{|x| - 1}{|x| - 2} = \frac{x - 1}{x - 2}$$

$\frac{x-1}{x-2} \geq 0$

Signs of x – 1:

x – 1 = 0 → x = 1(Adding 1 to both the sides)

x – 1 > 0 → x > 1(Adding 1 to both the sides)

x – 1 < 0 → x < 1(Adding 1 to both the sides)

Signs of x – 2:

x – 2 = 0 → x = 2(Adding 2 to both the sides)

x – 2 < 0 → x < 2(Adding 2 to both the sides)

x – 2 > 0 → x > 2(Adding 2 to both the sides)

At x = 2, $\frac{x-1}{x-2}$ is not defined

Intervals satisfying the required condition: ≥ 0

x < 1 or x = 1 or x > 2

Merging overlapping intervals:

x ≤ 1 or x > 2

Combining the intervals:

x ≤ 1 or x > 2 and 0 ≤ x < 2

Merging overlapping intervals:

0 ≤ x ≤ 1

Similarly, for x > 2:

$$\frac{|x| - 1}{|x| - 2} = \frac{x - 1}{x - 2}$$

$\frac{x-1}{x-2} \geq 0$

Therefore,

Intervals satisfying the required condition: ≥ 0

x < 1 or x = 1 or x > 2

Merging overlapping intervals:

x ≤ 1 or x > 2

Combining the intervals:

x ≤ 1 or x > 2 and x > 2

Merging overlapping intervals:

x > 2

Combining all the intervals:

x < -2 or -1 ≤ x < 0 or 0 ≤ x ≤ 1 or x >2

Merging the overlapping intervals:

x < -2 or -1 ≤ x ≤ 1 or x > 2

Therefore,

x є (-∞, -2) U [-1,1] U (2, ∞)

Question: 29

Solve each of the

Solution:

Given:

$\frac{1}{2-|x|} \geq 1$, x є R. – {–2, 2}

Intervals of |x|:

x ≥ 0, |x| = x and x < 0, |x| = -x

Domain of $\frac{1}{2-|x|} \geq 1$

$\frac{1}{2-|x|} \geq 1$ is undefined at x = -2 and x = 2

Therefore, Domain: x < -2 or x > 2

Combining intervals with domain:

x<-2, -2<x<0, 0 ≤ x<2, x>2

For x < - 2

$\frac{1}{2-(-x)} \geq 1$

Subtracting 1 from both the sides

$\frac{1}{2+x} - 1 \geq 1$ -1

$\frac{1-(2+x)}{2+x} \geq 0$

$\frac{1-2-x}{2+x} \geq 0$

$\frac{-1-x}{2+x} \geq 0$

Signs of -1 -x:

-1 – x = 0 → x = -1

(Adding 1 to both the sides and then dividing by -1 on both the sides)

-1 – x > 0 → x < -1

(Adding 1 to both the sides and then multiplying by -1 on both the sides)

-1 – x < 0 → x > -1

(Adding 1 to both the sides and then multiplying by -1 on both the sides)

Signs of 2 + x:

2 + x = 0 → x = -2 (Subtracting 2 from both the sides)

2 + x > 0 →x > -2 (Subtracting 2 from both the sides)

2 + x < 0 →x < -2 (Subtracting 2 from both the sides)

Intervals satisfying the required condition: ≥ 0

-2 < x < 1 or x = -1

Merging overlapping intervals:

-2 < x ≤ 1

Combining the intervals:

-2 < x ≤ 1 and x < -2

Merging the overlapping intervals:

No solution.

Similarly, for -2<x<0:

$\frac{1}{2-(-x)} \geq 1$

Therefore,

Intervals satisfying the required condition: ≥ 0

-2 < x ≤ 1 and x < -2

Merging overlapping intervals:

-2 < x ≤ 1

Combining the intervals:

-2 < x ≤ 1 and -2 < x < 0

Merging the overlapping intervals:

-2 < x ≤ 1

For 0 ≤ x < 2

$\frac{1}{2-x} \geq 1$

Subtracting 1 from both the sides

$\frac{1}{2-x} - 1 \geq 1$ -1

$\frac{1-(2-x)}{2-x} \geq 0$

$\frac{1-2+x}{2+x} \geq 0$

$\frac{x-1}{2+x} \geq 0$

Signs of x -1:

x - 1 = 0 → x = 1(Adding 1 to both the sides)

x - 1 > 0 → x > 1(Adding 1 to both the sides)

x - 1 < 0 → x < 1(Adding 1 to both the sides)

Signs of 2 + x:

2 + x = 0 → x = -2 (Subtracting 2 from both the sides)

2 + x > 0 →x > -2 (Subtracting 2 from both the sides)

2 + x < 0 →x < -2 (Subtracting 2 from both the sides)

Intervals satisfying the required condition: ≥ 0

1 < x < 2 or x = 1

Merging overlapping intervals:

1 ≤ x < 2

Combining the intervals:

1 ≤ x < 2 and 0 ≤ x < 2

Merging the overlapping intervals:

1 ≤ x < 2

Similarly, for x >2:

$\frac{1}{2-x} \geq 1$

Therefore,

Intervals satisfying the required condition: ≥ 0

1 < x < 2 or x = 1

Merging overlapping intervals:

1 ≤ x < 2

Combining the intervals:

1 ≤ x < 2 and x > 2

Merging the overlapping intervals:

No solution.

Now, combining all the intervals:

No solution or -2 < x ≤ 1 or 1 ≤ x < 2

Merging the overlapping intervals:

-2 < x ≤ 1 or 1 ≤ x < 2

Thus, x є (-2, -1] υ [1,2)

Question: 30

Given:

$|x + a| + |x| > 3$, $x \in R$.

$|x + a| = -(x + a)$ or $(x + a)$

$|x| = -x$ or x

When $|x + a| = -(x + a)$ and $|x| = -x$

Then,

$|x + a| + |x| > 3 \rightarrow -(x + a) + (-x) > 3$

$-x - a - x > 3$

$-2x - a > 3$

Adding a on both the sides in above equation

$-2x - a + a > 3 + a$

$-2x > 3 + a$

Dividing both the sides by 2 in above equation

$$\frac{-2x}{2} > \frac{3 + a}{2}$$

$$-x > \frac{3 + a}{2}$$

Multiplying both the sides by -1 in the above equation

$$-x(-1) > \left(\frac{3 + a}{2}\right)(-1)$$

$$x < -\left(\frac{3+a}{2}\right)$$

Now when, $|x + a| = -(x + a)$ and $|x| = x$

Then,

$|x + a| + |x| > 3 \rightarrow -(x + a) + x > 3$

$-x - a + x > 3$

$-a > 3$

In this case no solution for x.

Now when, $|x + a| = (x + a)$ and $|x| = -x$

Then,

$|x + a| + |x| > 3 \rightarrow (x + a) + (-x) > 3$

$x + a - x > 3$

$a > 3$

In this case no solution for x.

Now when,

$|x + a| = (x + a)$ and $|x| = x$

Then,

$|x + a| + |x| > 3 \rightarrow (x + a) + (x) > 3$

$x + a + x > 3$

$2x + a > 3$

Subtracting a from both the sides in above equation

$2x + a - a > 3 - a$

$2x > 3 - a$

Dividing both the sides by 2 in above equation

$$\frac{2x}{2} > \frac{3-a}{2}$$

$x > \frac{3-a}{2}$

Therefore,

$x < -\left(\frac{3+a}{2}\right)$ or $x > \left(\frac{3-a}{2}\right)$

Question: 31

Given:

$x - 4 > 1$, $x \neq 4$.

Adding 4 to both the sides in above equation

$x - 4 + 4 > 1 + 4$

$x > 5$

Therefore,

$x \in (5, \infty)$

Question: 32

$\frac{4}{x+1} \leq 3$ and $3 \leq \frac{6}{x+1}$

When,

$$\frac{4}{x+1} \leq 3$$

Subtracting 3 from both the sides

$$\frac{4}{x+1} - 3 \leq 3 - 3$$

$$\frac{4 - 3\,(x+1)}{x+1} \leq 0$$

$$\frac{4 - 3x - 3}{x+1} \leq 0$$

$$\frac{1 - 3x}{x+1} \leq 0$$

Signs of $1 - 3x$:

$1 - 3x = 0 \rightarrow x = \frac{1}{3}$

(Subtract 1 from both the sides and then divide both sides by -3)

$1 - 3x > 0 \rightarrow x < \frac{1}{3}$

(Subtract 1 from both the sides, then multiply by -1 on both sides and then divide both sides by 3)

$1 - 3x < 0 \rightarrow x > \frac{1}{3}$

(Subtract 1 from both the sides, then multiply by -1 on both sides and then divide both sides by 3)

Interval satisfying the required condition ≤ 0 , $x > 0$

$x = \frac{1}{3}$ or $x > \frac{1}{3}$

Or

$x \geq \frac{1}{3}$

Now when,

$3 \leq \frac{6}{x+1}$

Subtracting 3 from both the sides

$3 - 3 \leq \frac{6}{x+1} - 3$

$0 \leq \frac{6 - 3(x+1)}{x+1}$

$0 \leq \frac{6 - 3x - 3}{x+1}$

$0 \leq \frac{3 - 3x}{x+1}$

Dividing both sides by 3

$0 \leq \frac{1 - x}{x+1}$

Multiplying by -1 on both sides

$0 \geq \frac{x - 1}{x+1}$

Signs of x - 1:

$x - 1 = 0 \rightarrow x = 1$ (Adding 1 to both the sides)

$x - 1 > 0 \rightarrow x > 1$ (Adding 1 to both the sides)

$x - 1 < 0 \rightarrow x < 1$ (Adding 1 to both the sides)

Interval satisfying the required condition: ≤ 0

$x \leq 1$

Combining the intervals:

$\frac{1}{3} \leq x < 1$ such that $x > 0$

Question: 33

$-11 \leq 4x - 3$ and $4x - 3 \leq 13$

When,

$-11 \leq 4x - 3$

$4x - 3 \geq -11$

Adding 3 to both the sides

$4x - 3 + 3 \geq -11 + 3$

$4x \geq -8$

Divide both the sides by 4 in above equation

$\frac{4x}{4} \geq \frac{-8}{4}$

$x \geq -2$

Now when,

$4x – 3 \leq 13$

Adding 3 to both the sides in the above equation

$4x – 3 + 3 \leq 13 + 3$

$4x \leq 16$

Dividing both the sides by 4 in the above question

$\frac{4x}{4} \leq \frac{16}{4}$

$x \leq 4$

Combining the intervals:$x \geq -2$ and $x \leq 4$

Therefore,

$x \in [-2, 4]$

Question: 34

When,

$5x – 7 < x + 3$

Adding 7 to both the sides in the above equation

$5x – 7 + 7 < x + 3 + 7$

$5x < x + 10$

Now, subtracting x from both the sides

$5x – x < x + 10 -x$

$4x < 10$

Dividing both the sides by 4 in above equation

$\frac{4x}{4} < \frac{10}{4}$

$x < \frac{5}{2}$

Now when,

$1 – \frac{3x}{2} \geq x – 4$

Subtracting 1 from both the sides in the above equation

$1 - \frac{3x}{2} -1 \geq x – 4 – 1$

$\frac{-3x}{2} \geq x - 5$

Now multiplying both the sides by 2 in the above equation

$2.\left(\frac{-3x}{2}\right) \geq 2x – 10$

$-3x \geq 2x – 10$

Now subtracting 2x from both the sides in the above equation

$-3x – 2x \geq 2x -10 – 2x$

$-5x \geq -10$

Now, multiplying both the sides by -1 in the above equation

-5x(-1) ≥ -10(-1)

5x ≤ 10

Now, dividing both the sides by 5 in the above equation

$\frac{5x}{5} \le \frac{10}{5}$

x ≤ 2

Therefore,

$x < \frac{5}{2}$ and x ≤ 2

Question: 35

-2 < $\frac{6-5x}{4}$ and $\frac{6-5x}{4}$ < 7

$\frac{6-5x}{4}$ > -2

Multiplying both the sides by 4 in the above equation

$\left(\frac{6-5x}{4}\right)$(4) > -2(4)

6 – 5x > -8

Now subtracting 6 from both the sides

6 – 5x – 6 > - 8 – 6

-5x > -14

Multiplying both the sides by -1 in above equation

-5x(-1) > -14(-1)

5x < 14

Now, dividing both the sides by 5 in above equation

$\frac{5x}{5} < \frac{14}{5}$

$x < \frac{14}{5}$

Now when,

$\frac{6-5x}{4}$ < 7

Multiplying both the sides by 4 in the above equation

$\left(\frac{6-5x}{4}\right)(4)$ < 7(4)

6 – 5x < 28

Now, subtracting 6 from both sides in above equation

6 – 5x – 6 < 28 – 6

-5x < 22

Multiplying both the sides by -1 in above equation

-5x(-1) < 22(-1)

5x > -22

Dividing both the sides by 5 in above equation

$\frac{5x}{5} > \frac{-22}{5}$

$x > \frac{-22}{5}$

Therefore,

$x \in \left(\frac{-22}{5}, \frac{14}{5}\right)$

Question: 36

$3x - x > x + \frac{4-x}{3}$ and $x + \frac{4-x}{3} > 3$

When,

$3x - x > x + \frac{4-x}{3}$

$2x > x + \frac{4-x}{3}$

Subtracting x from both the sides in above equation

$2x - x > x + \frac{4-x}{3} - x$

$x > \frac{4-x}{3}$

Multiplying both the sides by 3 in the above equation

$3x > 3\left(\frac{4-x}{3}\right)$

$3x > 4 - x$

Adding x on both the sides in above equation

$3x + x > 4 - x + x$

$4x > 4$

Dividing both the sides by 4 in above equation

$\frac{4x}{4} > \frac{4}{4}$

$x > 1$

Now when,

$x + \frac{4-x}{3} > 3$

Multiplying both the sides by 3 in above equation

$3x + 3\left(\frac{4-x}{3}\right) > 3(3)$

$3x + 4 - x > 9$

$2x + 4 > 9$

Subtracting 4 from both the sides in above equation

$2x + 4 - 4 > 9 - 4$

$2x > 5$

Dividing both the sides by 2 in above equation

$\frac{2x}{2} > \frac{5}{2}$

$x > \frac{5}{2}$

Merging overlapping intervals

$x > \frac{5}{2}$

Therefore,

$x \in \left(\frac{5}{2}, \infty\right)$

Question: 37

Solve the followi

Solution:

When,

$\frac{7x-1}{2} < -3$

Multiplying both the sides by 2

$\left(\frac{7x-1}{2}\right)(2) < -3(2)$

7x -1 < -6

Adding 6 to both the sides in above equation

7x – 1 + 6 < -6 + 6

7x + 5 < 0

Subtracting 5 from both the sides in above equation

7x + 5 – 5 < 0 – 5

7x < - 5

Dividing both the sides by 7 in above equation

$\frac{7x}{7} < \frac{-5}{7}$

Therefore,

$x < \frac{-5}{7}$

Now when,

$\frac{3x+8}{5} + 11 < 0$

Subtracting both the sides by 11 in the above equation

$\frac{3x+8}{5} + 11 - 11 < 0 - 11$

$\frac{3x+8}{5} < -11$

Multiplying both the sides by 5 in the above equation

$\left(\frac{3x+8}{5}\right)(5) < -11(5)$

3x + 8 < -55

Subtracting 8 from both the sides in above equation

3x + 8 – 8 < -55 – 8

$3x < -63$

Dividing both the sides by 3 in above equation

$\frac{3x}{3} < \frac{-63}{3}$

Therefore,

$x < -21$

Question: 38

Solve the followi

Solution:

$-12 < 4 - \frac{3x}{-5}$ and $4 - \frac{3x}{-5} \leq 2$

When,

$-12 < 4 - \frac{3x}{-5}$

$4 - \frac{3x}{-5} > -12$

Subtracting 4 from both the sides in above equation

$4 - \frac{3x}{-5} -4 > 12 - 4$

$- \frac{3x}{-5} > -16$

$\frac{3x}{5} > -16$

Multiplying both the sides by 5 in the above equation

$\left(\frac{3x}{5}\right)(5) > -16(5)$

$3x > -80$

Dividing both the sides by 3 in above equation

$\left(\frac{3x}{3}\right) > \frac{-80}{3}$

Therefore,

$x > \frac{-80}{3}$

Now when,

$4 - \frac{3x}{-5} \leq 2$

Subtracting both the sides by 4 in the above equation

$4 - \frac{3x}{-5} - 4 \leq 2 - 4$

$- \frac{3x}{-5} \leq - 2$

$\frac{3x}{5} \leq -2$

Multiplying both the sides by 5 in the above equation

$3x \leq -10$

Dividing both the sides by 3 in the above equation

$\frac{3x}{3} \leq \frac{-10}{3}$

Therefore,

$x \le \frac{-10}{3}$

Therefore: $x \in \left(\frac{-80}{3}, \frac{-10}{3}\right]$

Question: 39

Solve the followi

Solution:

1 ≤ |x - 2| and |x - 2| ≤ 3

When,

|x - 2| ≥ 1

Then,

x – 2 ≤ -1 and x -2 ≥ 1

Now when,

x – 2 ≤ - 1

Adding 2 to both the sides in above equation

x – 2 + 2 ≤ -1 + 2

x ≤ 1

Now when,

x – 2 ≥ 1

Adding 2 to both the sides in above equation

x – 2 + 2 ≥ 1 + 2

x ≥ 3

For |x – 2| ≥ 1: x ≤ 1 or x ≥ 3

When,

|x - 2| ≤ 3

Then,

x – 2 ≥ - 3 and x – 2 ≤ 3

Now when,

x – 2 ≥ -3

Adding 2 to both the sides in above equation

x – 2 + 2 ≥ -3 + 2

x ≥ -1

Now when,

x – 2 ≤ 3

Adding 2 to both the sides in above equation

x – 2 + 2 ≤ 3 + 2

x ≤ 5

For |x – 2| ≤ 3: x ≥ -1 or x ≤ 5

Combining the intervals:

$x \le 1$ or $x \ge 3$ and $x \ge -1$ or $x \le 5$

Merging the overlapping intervals:

$-1 \le x \le 1$ and $3 \le x \le 5$

Therefore,

$x \in [-1, 1] \cup [3, 5]$

Question: 40

Find all pairs of

Solution:

Let the pair of consecutive even positive integers be x and x + 2.

So, it is given that both the integers are greater than 8

Therefore,

$x > 8$ and $x + 2 > 8$

When,

$x + 2 > 8$

Subtracting 2 from both the sides in above equation

$x + 2 – 2 > 8 – 2$

$x > 6$

Since $x > 8$ and $x > 6$

Therefore,

$x > 8$

It is also given that sum of both the integers is less than 25

Therefore,

$x + (x + 2) < 25$

$x + x + 2 < 25$

$2x + 2 < 25$

Subtracting 2 from both the sides in above equation

$2x + 2 – 2 < 25 – 2$

$2x < 23$

Dividing both the sides by 2 in above equation

$$\frac{2x}{2} < \frac{23}{2}$$

$x < 11.5$

Since $x > 8$ and $x < 11.5$

So, the only possible value of x can be 10

Therefore, $x + 2 = 10 + 2 = 12$

Thus, the required possible pair is (10, 12).

Question: 41

Find all pairs of

Solution:

Let the pair of consecutive even positive integers be x and x + 2.

So, it is given that both the integers are greater than 8

Therefore,

$x > 8$ and $x + 2 > 8$

When,

$x + 2 > 8$

Subtracting 2 from both the sides in above equation

$x + 2 – 2 > 8 – 2$

$x > 6$

Since $x > 8$ and $x > 6$

Therefore,

$x > 8$

It is also given that sum of both the integers is less than 25

Therefore,

$x + (x + 2) < 25$

$x + x + 2 < 25$

$2x + 2 < 25$

Subtracting 2 from both the sides in above equation

$2x + 2 – 2 < 25 – 2$

$2x < 23$

Dividing both the sides by 2 in above equation

$$\frac{2x}{2} < \frac{23}{2}$$

$x < 11.5$

Since $x > 8$ and $x < 11.5$

So, the only possible value of x can be 10

Therefore, $x + 2 = 10 + 2 = 12$

Thus, the required possible pair is (10, 12).

Question: 42

A company manufac

Solution:

Given:

Cost function $C(x) = 25000 + 30x$

Revenue function $R(x) = 43x$

To Find:

Number of cassettes to be sold to realize some profit

In order, to gain profit: $R(x) > C(x)$

Therefore,

$43x > 25000 + 30x$

25000 + 30x < 43x

Subtracting 30x from both the sides in above equation

25000 + 30x – 30x < 43x – 30x

25000 < 13x

Dividing both the sides by 13 in above equation

$$\frac{25000}{13} < \frac{13x}{13}$$

1923.07 < x

Thus, we can say that 1923 cassettes must be sold by the company in order to realize some profit.

Question: 43

The watering acid

Solution:

Let x be the third pH value.

Now, it is given that the average pH reading of three daily measurements is between 8.2 and 8.5

Also, the first two pH readings are 8.48 and 8.35

Therefore,

$8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$

Multiplying throughout by 3 in the above equation

$3(8.2) < \left(\frac{8.48 + 8.35 + x}{3}\right)(3) < 3(8.5)$

24.6 < 8.48 + 8.35 + x < 25.5

24.6 < 16.83 + x < 25.5

Subtracting throughout by 16.83 in above equation

24.6 – 16.83 < 16.83 + x – 16.83 < 25.5 – 16.83

7.77 < x < 8.67

Therefore,

The value of third pH reading ranges from 7.77 to 8.67

Question: 44

A manufacturer ha

Solution:

Let x litres of 2% boric and acid solution be added to 640 litres of 8% solution of boric acid.

%Strength = $\frac{\frac{8}{100} \times 640 + \frac{2}{100} \times x}{640+x}$

$= \frac{5120+2x}{100(640+x)}$

It is given that boric acid content in the resulting mixture ranges from 4% to 6%

Therefore,

$$\frac{4}{100} < \frac{5120 + 2x}{100(640 + x)} < \frac{6}{100}$$

Multiplying throughout by 100 in the above equation

$$\frac{4}{100}(100) < \frac{5120 + 2x}{100(640 + x)}(100) < \frac{6}{100}(100)$$

$4 < \frac{5120+2x}{640+x} < 6$

$\frac{5120+2x}{640+x} > 4$ and $\frac{5120+2x}{640+x} < 6$

When,

$\frac{5120+2x}{640+x} > 4$

Multiplying both the sides by (640 + x) in the above equation

$\frac{5120+2x}{640+x}(640 + x) > 4(640 + x)$

$5120 + 2x > 2560 + 4x$

Subtracting 2x from both the sides in above equation

$5120 + 2x - 2x > 2560 + 4x - 2x$

$5120 > 2560 + 2x$

Subtracting 2560 from both the sides in above equation

$5120 - 2560 > 2560 + 2x - 2560$

$2560 > 2x$

Dividing both the sides by 2 in above equation

$$\frac{2560}{2} > \frac{2x}{2}$$

$1280 > x$

Now when,

$\frac{5120+2x}{640+x} < 6$

Multiplying both the sides by (640 + x) in the above equation

$\frac{5120+2x}{640+x}(640 + x) < 6(640 + x)$

$5120 + 2x < 3840 + 6x$

Subtracting 2x from both the sides in above equation

$5120 + 2x - 2x < 3840 + 6x - 2x$

$5120 < 3840 + 4x$

Subtracting 3840 from both the sides in above equation

$5120 - 3840 < 3840 + 4x - 3840$

$1280 < 4x$

Dividing both the sides by 4 in above equation

$$\frac{1280}{4} < \frac{4x}{4}$$

$320 < x$

Thus, the value of 2% boric acid solution to be added ranges from:

320 to 1280 litres

Question: 45

How many litres o

Solution:

Let x litres of water be added.

Then total mixture = x + 600

Amount of acid contained in the resulting mixture is 45% of 600 litres.

It is given that the resulting mixture contains more than 25% and less than 30% acid content.

Therefore,

45% of 600 > 25% of (x + 600)

And

30% of (x+600) > 45% of 600

When,

45% of 600 > 25% of (x+600)

Multiplying both the sides by 100 in above equation

$\frac{45}{100} \times 600 > \frac{25}{100} \times (x + 600)$

45 × 600 > 25(x + 600)

27000 > 25x + 15500

Subtracting 15500 from both the sides in above equation

27000 – 15500 > 25x + 15500 – 15500

11500 > 25x

Dividing both the sides by 25 in above equation

$\frac{11500}{25} > \frac{25x}{25}$

460 > x

Now when,

45% of 600 < 30% of (x+600)

Multiplying both the sides by 100 in the above equation

$\frac{45}{100} \times 600 < \frac{30}{100} \times (x + 600)$

45 × 600 < 30(x + 600)

27000 < 30x + 18000

Subtracting 18000 from both the sides in above equation

27000 – 18000 < 30x + 18000 – 18000

9000 < 30x

Dividing both the sides by 30 in above equation

$\frac{9000}{30} > \frac{30x}{30}$

300 < x

Thus, the amount of water required to be added ranges from 300 litres to 460 litres.

Question: 46

To receive grade

Solution:

Let x marks be scored by Tanvy in her last paper.

It is given that Tanvy scored 89, 93, 95 and 91 marks in first 4 papers.

To receive grade A, she must obtain an average of 90 marks or more.

Therefore,

$\frac{89+93+95+91+x}{5} \geq 90$

Multiplying both the sides by 5 in the above equation

$\left(\frac{89+93+95+91+x}{5}\right)(5) \geq 90(5)$

368 + x ≥ 450

Subtracting 368 from both the sides in the above equation

368 + x – 368 ≥ 450 – 368

x ≥ 82

Therefore, Tanvy should score minimum of 82 marks in her last paper to get grade A in the course.

Exercise : 6B

Question: 1

Find the solution

Solution:

$$\frac{1}{x-2} < 0$$

We have to find values of x for which $\frac{1}{x-2}$ is less than zero that is negative

Now for $\frac{1}{x-2}$ to be negative x – 2 should be negative that is x – 2 < 0

⇒ x – 2 < 0

⇒ x < 2

Hence x should be less than 2 for $\frac{1}{x-2} < 0$

x < 2 means x can take values from -∞ to 2 hence x ∈ (-∞, 2)

Hence the solution set for $\frac{1}{x-2} < 0$ is (-∞, 2)

Question: 2

Find the solution

Solution:

|x – 1| < 2

Square both sides

⇒ $(x - 1)^2 < 4$

⇒ $x^2 - 2x + 1 < 4$

⇒ $x^2 - 2x - 3 < 0$

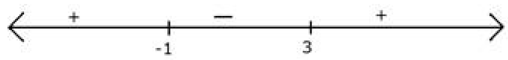
⇒ $x^2 - 3x + x - 3 < 0$

⇒ x(x – 3) + 1(x – 3) < 0

⇒ (x + 1)(x – 3) < 0

Observe that when x > 3 (x – 3)(x + 1) is positive

And for each root the sign changes hence



We want less than 0 that is negative part

Hence x should be between -1 and 3 for (x – 3)(x + 1) to be negative

Hence x ∈ (-1, 3)

Hence solution set for |x – 1| < 2 is (-1, 3)

Question: 3

Find the solution

Solution:

|2x – 3| < 1

Square both sides

⇒ $(2x - 3)^2 < 1^2$

⇒ $4x^2 - 12x + 9 < 1$

⇒ $4x^2 - 12x + 8 < 0$

Divide throughout by 4

⇒ $x^2 - 3x + 2 < 0$

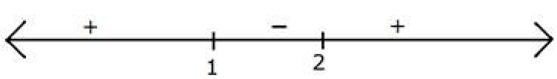
⇒ $x^2 - 2x - x + 2 < 0$

⇒ x(x – 2) – 1(x – 2) < 0

⇒ (x – 1)(x – 2) < 0

Observe that when x is greater than 2 (x – 1)(x – 2) is positive

And for each root the sign changes hence



We want less than 0 that is negative part

Hence x should be between 1 and 2 for (x – 1)(x – 2) to be negative

Hence x ∈ (1, 2)

Hence the solution set of |2x – 3| < 1 is (1, 2)

Question: 4

Find the solution

Solution:

$\frac{|x-2|}{(x-2)} < 0$ means we have to find values of x for which $\frac{|x-2|}{(x-2)}$ is negative

Observe that the numerator |x – 2| is always positive because of mod, hence for $\frac{|x-2|}{(x-2)}$ to be a negative quantity the denominator (x – 2) has to be negative

That is x – 2 should be less than 0

⇒ x – 2 < 0

$\Rightarrow$ x < 2

Hence x should be less than 2 for $\frac{|x-2|}{(x-2)} < 0$

x < 2 means x can take values from -∞ to 2 hence x ∈ (-∞, 2)

Hence the solution set for $\frac{|x-2|}{(x-2)} < 0$ is (-∞, 2)

Question: 5

Find the solution

Solution:

$$\frac{x+1}{x+2} < 1$$

$$\Rightarrow \frac{x+1}{x+2} - 1 < 0$$

$$\Rightarrow \frac{x+1-x-2}{x+2} < 0$$

$$\Rightarrow \frac{-1}{x+2} < 0$$

We have to find values of x for which $\frac{-1}{x+2} < 0$ that is $\frac{-1}{x+2}$ is negative

The numerator of $\frac{-1}{x+2}$ is -1 which is negative hence for $\frac{-1}{x+2}$ to be negative x + 2 must be positive (otherwise if x + 2 is negative then negative upon negative will be positive)

That is x + 2 should be greater than 0

$\Rightarrow$ x + 2 > 0

$\Rightarrow$ x > -2

Hence x should be greater than -2 for $\frac{-1}{x+2} < 0$

x > -2 means x can take values from -2 to ∞ hence x ∈ (-2, ∞)

Hence the solution set for $\frac{x+1}{x+2} < 0$ is (-2, ∞)

Question: 6

Solve the system

Solution:

We have to find values of x for which both the equations hold true

x – 2 ≥ 0 and 2x – 5 ≤ 3

we will solve both the equations separately and then their intersection set will be solution of the system

x – 2 ≥ 0

$\Rightarrow$ x ≥ 2

Hence x ∈ (2, ∞)

Now, 2x – 5 ≤ 3

$\Rightarrow$ 2x ≤ 3 + 5

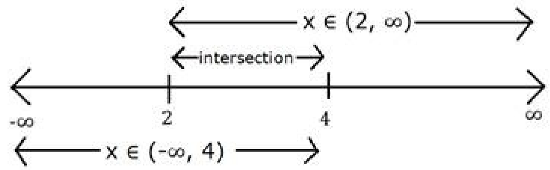
$\Rightarrow$ 2x ≤ 8

$\Rightarrow$ x ≤ 4

Hence x ∈ (-∞, 4)

The intersection of set (2, ∞) and (-∞, 4) is (2, 4)

Representing on number line



Hence solution set for given system of equation is x ∈ (2, 4)

Question: 7

Solve –4x > 16

Solution:

We have to find integer values of x for which -4x > 16 (why only integer values because it is given that x ∈ Z that is set of integers)

-4x > 16

⇒ -x > 4

⇒ x < -4

The integers less than -4 are -5, -6, -7, -8, ...

Generalizing the solution in terms of n

x = -(4 + n) where n is integers from 1 to infinity

Hence solution of -4x > 16 is x = -(4 + n) ∀ n = {1, 2, 3, 4...}

Question: 8

Solve x + 5 >

Solution:

x + 5 > 4x – 10

⇒ 5 + 10 > 4x – x

⇒ 15 > 3x

Divide by 3

⇒ 5 > x

⇒ x < 5

x < 5 means x is from -∞ to 5 that is x ∈ (-∞, 5)

Hence solution of x + 5 > 4x – 10 is x ∈ (-∞, 5)

Question: 9

Solve

Solution:

$$\frac{3}{x-2} < 1$$

$$\Rightarrow \frac{3}{x-2} - 1 < 0$$

$$\Rightarrow \frac{3-x+2}{x-2} < 0$$

$$\Rightarrow \frac{5-x}{x-2} < 0$$

Observe that $\frac{5-x}{x-2}$ is zero at x = 5 and not defined at x = 2

Hence plotting these two points on number line

Now for x > 5, $\frac{5-x}{x-2}$ is negative

For every root and not defined value of $\frac{5-x}{x-2}$ the sign will change

We want the negative part hence x < 2 and x > 5

x < 2 means x is from negative infinity to 2 and x > 5 means x is from 5 to infinity

Hence x ∈ (-∞, 2) U (5, ∞)

Hence solution of $\frac{3}{x-2} < 1$ is x ∈ (-∞, 2) U (5, ∞)

Question: 10

Solve

Solution:

$$\frac{x}{x-5} > \frac{1}{2}$$

$$\Rightarrow \frac{x}{x-5} - \frac{1}{2} > 0$$

$$\Rightarrow \frac{2x - x + 5}{2(x-5)} > 0$$

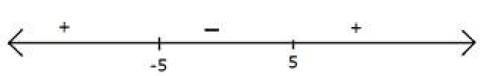
$$\Rightarrow \frac{x+5}{x-5} > 0$$

Observe that $\frac{x+5}{x-5}$ is zero at x = -5 and not defined at x = 5

Hence plotting these two points on number line

Now for x > 5, $\frac{x+5}{x-5}$ is positive

For every root and not defined value of $\frac{x+5}{x-5}$ the sign will change



We want greater than zero that is the positive part hence x < -5 and x > 5

x < -5 means x is from negative infinity to -5 and x > 5 means x is from 5 to infinity

Hence x ∈ (-∞, -5) U (5, ∞)

Hence solution of $\frac{x}{x-5} > \frac{1}{2}$ is x ∈ (-∞, 2) U (5, ∞)

Question: 11

Solve |x| < 4,

Solution:

|x| < 4

Square

$\Rightarrow x^2 < 16$

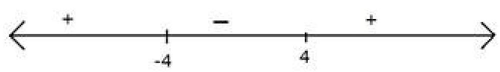
$\Rightarrow x^2 - 16 < 0$

$\Rightarrow x^2 - 4^2 < 0$

$\Rightarrow (x + 4)(x - 4) < 0$

Observe that when x is greater than 4, $(x + 4)(x - 4)$ is positive

And for each root the sign changes hence



We want less than 0 that is negative part

Hence x should be between -4 and 4 for $(x + 4)(x - 4)$ to be negative

Hence $x \in (-4, 4)$

Hence the solution set of $|x| < 4$ is $(-4, 4)$

Question: 12

Solve $|x| > 4$,

Solution:

$|x| > 4$

Square

$\Rightarrow x^2 > 16$

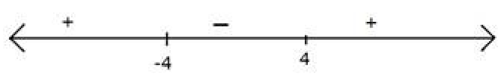
$\Rightarrow x^2 - 16 > 0$

$\Rightarrow x^2 - 4^2 > 0$

$\Rightarrow (x + 4)(x - 4) > 0$

Observe that when x is greater than 4, $(x + 4)(x - 4)$ is positive

And for each root the sign changes hence



We want greater than 0 that is positive part

Hence x should be less than -4 and greater than 4 for $(x + 4)(x - 4)$ to be positive

x less than -4 means x is from negative infinity to -4 and x greater than 4 means x is from 4 to infinity

Hence $x \in (-\infty, -4)$ and $x \in (4, \infty)$

Hence the solution set of $|x| > 4$ is $(-\infty, -4) \cup (4, \infty)$