

Chapter : 10. AREA

Exercise : 10A

Question: 1

In the given figure consider $\triangle ABD$ and $\triangle BCD$

$$\text{Area of } \triangle ABD = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AB \times BD$$

$$= \frac{1}{2} \times 5 \times 7 = \frac{35}{2} \text{ ----- 1}$$

$$\text{Area of } \triangle BCD = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times DC \times DB$$

$$= \frac{1}{2} \times 5 \times 7 = \frac{35}{2} \text{ ----- 2}$$

From 1 and 2 we can tell that area of two triangle that is $\triangle ABD$ and $\triangle BCD$ are equal

Since the diagonal BD divides ABCD into two triangles of equal area and opp sides AB = DC

\therefore ABCD is a parallelogram

$$\therefore \text{Area of parallelogram ABCD} = \text{Area of } \triangle ABD + \text{Area of } \triangle BCD$$

$$= \left(\frac{35}{2} + \frac{35}{2} \right) = \frac{70}{2} \text{ cm}^2 = 35 \text{ cm}^2$$

$$\therefore \text{Area of parallelogram ABCD} = 35 \text{ cm}^2$$

Question: 2

Given

$$AB = 10 \text{ cm}$$

$$DL = 6 \text{ cm}$$

$$BM = 8 \text{ cm}$$

$$AD = ? \text{ (To find)}$$

Here, Area of parallelogram = base x height

In the given figure if we consider AB as base Area = AB x DL

If we consider DM as base Area = AD x BM

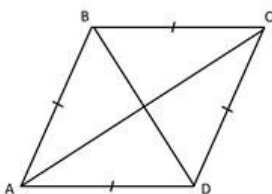
$$\therefore \text{Area} = AB \times DL = AD \times BM$$

$$= 10 \times 6 = AD \times 8$$

$$= 60 = 8 \times AD$$

$$\Rightarrow AD = \frac{60}{8} = 7.5 \text{ cm}$$

Question: 3



Here, Let ABCD be Rhombus with diagonals AC and BD

Here let AC = 24 and BD = 16

We know that, in a Rhombus, diagonals are perpendicular bisectors to each other

∴ if we consider $\triangle ABC$ AC is base and OB is height

Similarly, in $\triangle ADC$ AC is base and OD is height

Now, Area of Rhombus = Area of $\triangle ABC$ + Area of $\triangle ADC$

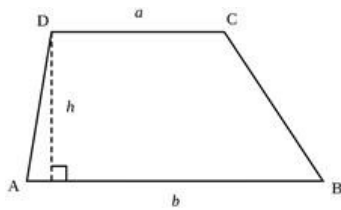
$$= \frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$$

$$= \frac{1}{2} \times 24 \times \frac{BD}{2} + \frac{1}{2} \times 24 \times \frac{BD}{2} \text{ (Since AC and BC are perpendicular bisectors } \therefore OB = OD = \frac{BD}{2})$$

$$= \frac{1}{2} \times 24 \times \frac{16}{2} + \frac{1}{2} \times 24 \times \frac{16}{2} = 96 + 96 = 192 \text{ cm}^2$$

∴ Area of Rhombus ABCD is 192cm^2

Question: 4



Given

$$AB = a = 9 \text{ cm}$$

$$DC = b = 6 \text{ cm}$$

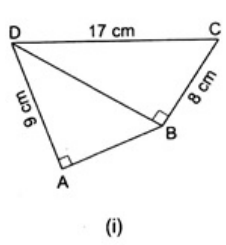
$$\text{Height (h)} = 8 \text{ cm}$$

We know that area of trapezium is $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$$\text{Therefore, Area of trapezium ABCD} = \frac{1}{2} \times (AB + DC) \times h = \frac{1}{2} \times (9 + 6) \times 8 = 60 \text{ cm}^2$$

∴ Area of Trapezium ABCD = 60cm^2

Question: 5A



Given

$$AD = 9 \text{ cm}$$

$$BC = 8 \text{ cm}$$

$$DC = 17 \text{ cm}$$

Here Area of Quad ABCD = Area of $\triangle ABD$ + Area of $\triangle BCD$

$$= \frac{1}{2} \times AB \times AD + \frac{1}{2} \times BC \times BD$$

By Pythagoras theorem in $\triangle BCD$

$$DC^2 = BD^2 + BC^2$$

$$17^2 = BD^2 + 8^2$$

$$BD^2 = 17^2 - 8^2 = 289 - 64 = 225$$

$$\therefore BD = 15 \text{ cm}$$

Similarly in $\triangle ABD$ using Pythagoras theorem

$$BD^2 = AD^2 + AB^2$$

$$15^2 = 9^2 + AB^2$$

$$AB^2 = 15^2 - 9^2 = 225 - 81 = 144$$

$$\therefore AB = 12 \text{ cm}$$

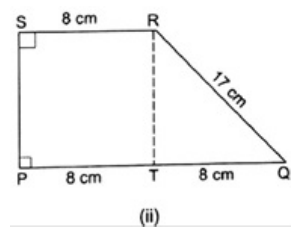
Now, Area of Quad ABCD = Area of $\triangle ABD$ + Area of $\triangle BCD$

$$= \frac{1}{2} \times AB \times AD + \frac{1}{2} \times BC \times BD$$

$$= \frac{1}{2} \times 12 \times 9 + \frac{1}{2} \times 8 \times 15 = 54 + 60 = 114 \text{ cm}^2$$

$$\therefore \text{Area of Quadrilateral ABCD} = 114 \text{ cm}^2$$

Question: 5B



Given :- Right trapezium

$$RS = 8 \text{ cm}$$

$$PT = 8 \text{ cm}$$

$$TQ = 8 \text{ cm}$$

$$RQ = 17 \text{ cm}$$

$$\text{Here } PQ = PT + TQ = 8 + 8 = 16$$

We know that area of trapezium is $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$$\text{That is } \frac{1}{2} \times (AB + DC) \times RT$$

Consider $\triangle TQR$

By Pythagoras theorem

$$RQ^2 = TQ^2 + RT^2$$

$$17^2 = 8^2 + RT^2$$

$$RT^2 = 17^2 - 8^2 = 289 - 64 = 225$$

$$\therefore RT = 15 \text{ cm}$$

$$\therefore \text{Area of trapezium} = \frac{1}{2} \times (RS + PQ) \times RT$$

$$= \frac{1}{2} \times (8 + 16) \times 15 = 180 \text{ cm}^2$$

$$\therefore \text{Area of trapezium PQRS} = 180 \text{ cm}^2$$

Question: 6

Given

$$AB = 7 \text{ cm}$$

$$AD = BC = 5 \text{ cm}$$

$$AL = BM = 4 \text{ cm (height)}$$

$$DC = ?$$

Here in the given figure $AB = LM$

$$\therefore LM = 7 \text{ cm} \text{ -----1}$$

Now Consider $\triangle ALD$

By Pythagoras theorem

$$AD^2 = AL^2 + DL^2$$

$$5^2 = 4^2 + DL^2$$

$$DL^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$\therefore DL = 3 \text{ cm} \text{ -----2}$$

Similarly in $\triangle BMC$

By Pythagoras theorem

$$BC^2 = BM^2 + MC^2$$

$$5^2 = 4^2 + MC^2$$

$$MC^2 = 5^2 - 4^2 = 25 - 16 = 9$$

$$\therefore MC = 3 \text{ cm} \text{ --3}$$

$$\therefore \text{from 1, 2 and 3}$$

$$DC = DL + LM + MC = 3 + 7 + 3 = 13 \text{ cm}$$

We know that area of trapezium is $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$

$$\therefore \text{Area of trapezium} = \frac{1}{2} \times (AB + DC) \times AL$$

$$= \frac{1}{2} \times (7 + 13) \times 4 = 40 \text{ cm}^2$$

$$\therefore \text{Area of trapezium ABCD} = 180 \text{ cm}^2$$

Question: 7

Given :

$$AL \perp BD \text{ and } CM \perp BD$$

$$\text{To prove : ar (quad. ABCD)} = \frac{1}{2} \times BD \times (AL + CM)$$

Proof:

$$\text{Area of } \triangle ABD = \frac{1}{2} \times BD \times AL$$

$$\text{Area of } \triangle CBD = \frac{1}{2} \times BD \times CM$$

$$\text{Now area of Quad ABCD} = \text{Area of } \triangle ABD + \text{Area of } \triangle CBD$$

$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

$$= \frac{1}{2} \times BD \times (AL + CM)$$

Hence proved

Question: 8

Given

$AL \perp BD$ and $CM \perp BD$

$BD = 14 \text{ cm}$

$AL = 8 \text{ cm}$

$CM = 6 \text{ cm}$

Here,

$$\text{Area of } \triangle ABD = \frac{1}{2} \times BD \times AL$$

$$\text{Area of } \triangle CBD = \frac{1}{2} \times BD \times CM$$

Now area of Quad ABCD = Area of $\triangle ABD$ + Area of $\triangle CBD$

$$= \frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CM$$

$$= \frac{1}{2} \times BD \times (AL + CM)$$

$$\therefore \text{Area of quad ABCD} = \frac{1}{2} \times BD \times (AL + CM) = \frac{1}{2} \times 14 \times (8 + 6) = 98\text{cm}^2$$

$$\therefore \text{Area of quad ABCD} = 98\text{cm}^2$$

Question: 9

Given

$AB \parallel DC$

To prove that: $\text{area}(\triangle AOD) = \text{area}(\triangle BOC)$

Here in the given figure Consider $\triangle ABD$ and $\triangle ABC$,

we find that they have same base AB and lie between two parallel lines AB and CD

According to the theorem: triangles on the same base and between same parallel lines have equal areas.

$$\therefore \text{Area of } \triangle ABD = \text{Area of } \triangle ABC$$

Now,

$$\text{Area of } \triangle AOD = \text{Area of } \triangle ABD - \text{Area of } \triangle AOB \text{ ---1}$$

$$\text{Area of } \triangle COB = \text{Area of } \triangle ABC - \text{Area of } \triangle AOB \text{ ---2}$$

\therefore From 1 and 2

We can conclude that $\text{area}(\triangle AOD) = \text{area}(\triangle BOC)$ (Since Area of $\triangle AOB$ is common)

Hence proved

Question: 10

Given

$AB \parallel DC$

To prove that : (i) $\text{area}(\triangle ACD) = \text{area}(\triangle ABE)$

(ii) $\text{area}(\triangle OCE) = \text{area}(\triangle OBD)$

(i)

Here in the given figure Consider $\triangle BDE$ and $\triangle ECD$,

we find that they have same base DE and lie between two parallel lines BC and DE

According to the theorem: triangles on the same base and between same parallel lines have equal areas.

$\therefore \text{Area of } \triangle BDE = \text{Area of } \triangle ECD$

Now,

$\text{Area of } \triangle ACD = \text{Area of } \triangle ECD + \text{Area of } \triangle ADE$ ---1

$\text{Area of } \triangle ABE = \text{Area of } \triangle BDE + \text{Area of } \triangle ADE$ ---2

\therefore From 1 and 2

We can conclude that $\text{area}(\triangle AOD) = \text{area}(\triangle BOC)$ (Since Area of $\triangle ADE$ is common)

Hence proved

(ii)

Here in the given figure Consider $\triangle BCD$ and $\triangle BCE$,

we find that they have same base BC and lie between two parallel lines BC and DE

According to the theorem : triangles on the same base and between same parallel lines have equal

areas.

$\therefore \text{Area of } \triangle BCD = \text{Area of } \triangle BCE$

Now,

$\text{Area of } \triangle OBD = \text{Area of } \triangle BCD - \text{Area of } \triangle BOC$ ---1

$\text{Area of } \triangle OCE = \text{Area of } \triangle BCE - \text{Area of } \triangle BOC$ ---2

\therefore From 1 and 2

We can conclude that $\text{area}(\triangle OCE) = \text{area}(\triangle OBD)$ (Since Area of $\triangle BOC$ is common)

Hence proved

Question: 11

Given

A triangle ABC in which points D and E lie on AB and AC of $\triangle ABC$ such that $\text{ar}(\triangle BCE) = \text{ar}(\triangle BCD)$.

To prove: $DE \parallel BC$

Proof:

Here, from the figure we know that $\triangle BCE$ and $\triangle BCD$ lie on same base BC and

It is given that $\text{area}(\triangle BCE) = \text{area}(\triangle BCD)$

Since two triangle have same base and same area they should equal altitude(height)

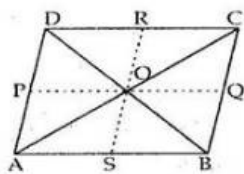
That means they lie between two parallel lines

That is $DE \parallel BC$

$\therefore DE \parallel BC$

Hence proved

Question: 12



Given : A parallelogram ABCD with a point 'O' inside it.

To prove : (i) $\text{area}(\triangle OAB) + \text{area}(\triangle OCD) = \frac{1}{2} \text{area}(\text{||gm ABCD})$,

(ii) $\text{area}(\triangle OAD) + \text{area}(\triangle OBC) = \frac{1}{2} \text{area}(\text{||gm ABCD})$.

Construction : Draw $PQ \parallel AB$ and $RS \parallel AD$

Proof:

(i)

$\triangle AOB$ and parallelogram ABQP have same base AB and lie between parallel lines AB and PQ.

According to theorem: If a triangle and parallelogram are on the same base and between the same

parallel lines, then the area of the triangle is equal to half of the area of the parallelogram.

$$\therefore \text{area}(\triangle AOB) = \frac{1}{2} \text{area}(\text{||gm ABQP}) \text{ ---1}$$

Similarly, we can prove that $\text{area}(\triangle COD) = \frac{1}{2} \text{area}(\text{||gm PQCD}) \text{ ---2}$

\therefore Adding -1 and -2 we get,

$$\text{area}(\triangle AOB) + \text{area}(\triangle COD) = \frac{1}{2} \text{area}(\text{||gm ABQP}) + \frac{1}{2} \text{area}(\text{||gm PQCD})$$

$$\text{area}(\triangle AOB) + \text{area}(\triangle COD) = \frac{1}{2} [\text{area}(\text{||gm ABQP}) + \text{area}(\text{||gm PQCD})] = \frac{1}{2} \text{area}(\text{||gm ABCD})$$

$$\therefore \text{area}(\triangle AOB) + \text{area}(\triangle COD) = \frac{1}{2} \text{area}(\text{||gm ABCD})$$

Hence proved

(ii)

$\triangle OAD$ and parallelogram ASRD have same base AD and lie between parallel lines AD and RS.

According to theorem: If a triangle and parallelogram are on the same base and between the same

parallel lines, then the area of the triangle is equal to half of the area of the parallelogram.

$$\therefore \text{area}(\triangle OAD) = \frac{1}{2} \text{area}(\text{||gm ASRD}) \text{ ---1}$$

Similarly, we can prove that $\text{area}(\triangle OBC) = \frac{1}{2} \text{area}(\text{||gm BCRS}) \text{ ---2}$

\therefore Adding -1 and -2 we get,

$$\text{area}(\triangle OAD) + \text{area}(\triangle OBC) = \frac{1}{2} \text{area}(\text{||gm ASRD}) + \frac{1}{2} \text{area}(\text{||gm BCRS})$$

$$\text{area}(\triangle OAD) + \text{area}(\triangle OBC) = \frac{1}{2} [\text{area}(\text{||gm ASRD}) + \text{area}(\text{||gm BCRS})] = \frac{1}{2} \text{area}(\text{||gm ABCD})$$

$$\therefore \text{area}(\triangle OAD) + \text{area}(\triangle OBC) = \frac{1}{2} \text{area}(\text{||gm } ABCD)$$

Hence proved

Question: 13

Given : ABCD is a quadrilateral in which a line through D drawn parallel to AC which meets BC produced in P.

To prove: area of $(\triangle ABP)$ = area of (quad ABCD)

Proof:

Here, in the given figure

$\triangle ACD$ and $\triangle ACP$ have same base and lie between same parallel line AC and DP.

According to the theorem : triangles on the same base and between same parallel lines have equal

areas.

$$\therefore \text{area of } (\triangle ACD) = \text{area of } (\triangle ACP) \text{ -----1}$$

Now, add area of $(\triangle ABC)$ on both side of (1)

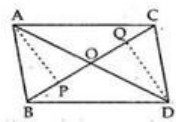
$$\therefore \text{area of } (\triangle ACD) + (\triangle ABC) = \text{area of } (\triangle ACP) + (\triangle ABC)$$

$$\text{Area of (quad ABCD)} = \text{area of } (\triangle ABP)$$

$$\therefore \text{area of } (\triangle ABP) = \text{Area of (quad ABCD)}$$

Hence proved

Question: 14



Given : $\triangle ABC$ and $\triangle DBC$ having same base BC and $\text{area}(\triangle ABC) = \text{area}(\triangle DBC)$.

To prove: $OA = OD$

Construction : Draw $AP \perp BC$ and $DQ \perp BC$

Proof :

$$\text{Here area of } \triangle ABC = \frac{1}{2} \times BC \times AP \text{ and area of } \triangle DBC = \frac{1}{2} \times BC \times DQ$$

$$\text{since, } \text{area}(\triangle ABC) = \text{area}(\triangle DBC)$$

$$\therefore \frac{1}{2} \times BC \times AP = \frac{1}{2} \times BC \times DQ$$

$$\therefore AP = DQ \text{ ----- 1}$$

Now in $\triangle AOP$ and $\triangle QOD$, we have

$$\angle APO = \angle DQO = 90^\circ \text{ and}$$

$$\angle AOP = \angle DOQ \text{ [Vertically opposite angles]}$$

$$AP = DQ \text{ [from 1]}$$

Thus by AAS congruency

$$\triangle AOP \cong \triangle QOD \text{ [AAS]}$$

Thus By corresponding parts of congruent triangles law [C.P.C.T]

$$\therefore OA = OD \text{ [C.P.C.T.]}$$

Hence BC bisects AD

Hence proved

Question: 15

Given : A $\triangle ABC$ in which AD is the median and P is a point on AD

To prove: (i) $\text{ar}(\triangle BDP) = \text{ar}(\triangle CDP)$,

(ii) $\text{ar}(\triangle ABP) = \text{ar}(\triangle ACP)$.

(i)

In $\triangle BPC$, PD is the median. Since median of a triangle divides the triangles into two equal areas

$$\text{So, } \text{area}(\triangle BDP) = \text{area}(\triangle CDP) \text{ ----1}$$

Hence proved

(ii)

In $\triangle ABC$ AD is the median

$$\text{So, } \text{area}(\triangle ABD) = \text{area}(\triangle ADC) \text{ ----2 and}$$

$$\text{area}(\triangle BDP) = \text{area}(\triangle CDP) \text{ [from 1]}$$

Now subtracting $\text{area}(\triangle BDP)$ from ----2 , we have

$$\text{area}(\triangle ABD) - \text{area}(\triangle BDP) = \text{area}(\triangle ADC) - \text{area}(\triangle BDP)$$

$$\text{area}(\triangle ABD) - \text{area}(\triangle BDP) = \text{area}(\triangle ADC) - \text{area}(\triangle CDP) \text{ [since } \text{area}(\triangle BDP) = \text{area}(\triangle CDP) \text{ from -1]}$$

$$\therefore \text{area}(\triangle ABP) = \text{area}(\triangle ACP)$$

Hence proved.

Question: 16

Given : A quadrilateral ABCD with diagonals AC and BD and $BO = OD$

To prove: Area of $(\triangle ABC) = \text{area of } (\triangle ADC)$

Proof : $BO = OD$ [given]

Here AO is the median of $\triangle ABD$

$$\therefore \text{Area of } (\triangle AOD) = \text{Area of } (\triangle AOB) \text{ ----- 1}$$

And OC is the median of $\triangle BCD$

$$\therefore \text{Area of } (\triangle COD) = \text{Area of } (\triangle BOC) \text{ ----- 2}$$

Now by adding -1 and -2 we get

$$\text{Area of } (\triangle AOD) + \text{Area of } (\triangle COD) = \text{Area of } (\triangle AOB) + \text{Area of } (\triangle BOC)$$

$$\therefore \text{Area of } (\triangle ABC) = \text{Area of } (\triangle ADC)$$

Hence proved

Question: 17

Given : A $\triangle ABC$ in which AD is the median and E is the midpoint on line AD

$$\text{To prove: } \text{area}(\triangle BED) = \frac{1}{4} \text{area}(\triangle ABC)$$

Proof : here in $\triangle ABC$ AD is the median

$$\therefore \text{Area of } (\triangle ABD) = \text{Area of } (\triangle ADE)$$

$$\text{Hence Area of } (\triangle ABD) = \frac{1}{2} [\text{Area of } (\triangle ABC)] \text{ ----- 1}$$

No in $\triangle ABD$ E is the midpoint of AD and BE is the median

$$\therefore \text{Area of } (\triangle BDE) = \text{Area of } (\triangle ABE)$$

$$\text{Hence Area of } (\triangle BED) = \frac{1}{2} [\text{Area of } (\triangle ABD)] \text{ ----- 2}$$

Substituting (1) in (2), we get

$$\text{Hence Area of } (\triangle BED) = \frac{1}{2} \left[\frac{1}{2} \text{Area of } (\triangle ABC) \right]$$

$$\therefore \text{area}(\triangle BED) = \frac{1}{4} \text{area}(\triangle ABC)$$

Hence proved

Question: 18

Given : A $\triangle ABC$ in which AD is a line where D is a point on BC and E is the midpoint of AD

$$\text{To prove: } \text{ar}(\triangle BEC) = \frac{1}{2} \text{ar}(\triangle ABC)$$

Proof: In $\triangle ABD$ E is the midpoint of side AD

$$\therefore \text{Area of } (\triangle BDE) = \text{Area of } (\triangle ABE)$$

$$\text{Hence Area of } (\triangle BDE) = \frac{1}{2} [\text{Area of } (\triangle ABD)] \text{ -1}$$

Now, consider $\triangle ACD$ in which E is the midpoint of side AD

$$\therefore \text{Area of } (\triangle ECD) = \text{Area of } (\triangle AEC)$$

$$\text{Hence Area of } (\triangle ECD) = \frac{1}{2} [\text{Area of } (\triangle ACD)] \text{ -2}$$

Now, adding -1 and -2, we get

$$\text{Area of } (\triangle BDE) + \text{Area of } (\triangle ECD) = \frac{1}{2} [\text{Area of } (\triangle ABD)] + \frac{1}{2} [\text{Area of } (\triangle ACD)]$$

$$\therefore \text{area}(\triangle BEC) = \frac{1}{2} [\text{area}(\triangle ABD) + \text{area}(\triangle ACD)]$$

$$\therefore \text{Area}(\triangle BEC) = \frac{1}{2} \text{Area}(\triangle ABC)$$

Hence proved

Question: 19

Given : D is the midpoint of side BC of $\triangle ABC$ and E is the midpoint of BD and O is the midpoint of AE

$$\text{To prove : } \text{ar}(\triangle BOE) = \frac{1}{8} \text{ar}(\triangle ABC)$$

Proof : Consider $\triangle ABC$ here D is the midpoint of BC

$$\therefore \text{Area of } (\triangle ABD) = \text{Area of } (\triangle ACD)$$

$$\therefore \text{Area}(\triangle ABD) = \frac{1}{2} \text{Area}(\triangle ABC) \text{—1}$$

Now, consider $\triangle ABD$ here E is the midpoint of BD

$$\therefore \text{Area of } (\triangle ABE) = \text{Area of } (\triangle AED)$$

$$\therefore \text{Area}(\triangle ABE) = \frac{1}{2} \text{Area}(\triangle ABD) - 2$$

Substituting -1 in -2, we get

$$\therefore \text{Area}(\triangle ABE) = \frac{1}{2} \left(\frac{1}{2} \text{Area}(\triangle ABC) \right)$$

$$\text{Area}(\triangle ABE) = \frac{1}{4} \text{Area}(\triangle ABC) - 3$$

Now consider $\triangle ABE$ here O is the midpoint of AE

$$\therefore \text{Area of } (\triangle BOE) = \text{Area of } (\triangle AOB)$$

$$\therefore \text{Area}(\triangle BOE) = \frac{1}{2} \text{Area}(\triangle ABE) - 4$$

Now, substitute -3 in -4, we get

$$\text{Area}(\triangle BOE) = \frac{1}{2} \left(\frac{1}{4} \text{Area}(\triangle ABC) \right)$$

$$\therefore \text{area}(\triangle BOE) = \frac{1}{8} \text{area}(\triangle ABC)$$

Hence proved

Question: 20

Given : A parallelogram ABCD in which AC is the diagonal and O is some point on the diagonal AC

To prove: $\text{area}(\triangle AOB) = \text{area}(\triangle AOD)$

Construction : Draw a diagonal BD and mark the intersection as P

Proof:

We know that in a parallelogram diagonals bisect each other, hence P is the midpoint of $\triangle ABD$

$$\therefore \text{Area of } (\triangle APB) = \text{Area of } (\triangle APD) - 1$$

Now consider $\triangle BOD$ here OP is the median, since P is the midpoint of BD

$$\therefore \text{Area of } (\triangle OPB) = \text{Area of } (\triangle OPD) - 2$$

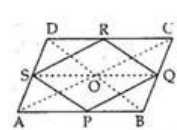
Adding -1 and -2 we get

$$\text{Area of } (\triangle APB) + \text{Area of } (\triangle OPB) = \text{Area of } (\triangle APD) + \text{Area of } (\triangle OPD)$$

$$\therefore \text{Area of } (\triangle AOB) = \text{Area of } (\triangle AOD)$$

Hence proved

Question: 21



Given : ABCD is a parallelogram and P,Q,R,S are the midpoints of AB,BC,CD,AD respectively

To prove: (i) PQRS is a parallelogram

$$(ii) \text{Area}(\text{||gm PQRS}) = \frac{1}{2} \times \text{area}(\text{||gm ABCD})$$

Construction : Join AC ,BD,SQ

Proof:

(i)

As S and R are midpoints of AD and CD respectively, in ΔACD

$SR \parallel AC$ [By midpoint theorem] ----- (1)

Similarly in ΔABC , P and Q are midpoints of AB and BC respectively

$PQ \parallel AC$ [By midpoint theorem] ----- (2)

From (1) and (2)

$SR \parallel AC \parallel PQ$

$\therefore SR \parallel PQ$ ----- (3)

Again in ΔACD as S and P are midpoints of AD and CB respectively

$SP \parallel BD$ [By midpoint theorem] ----- (4)

Similarly in ΔABC , R and Q are midpoints of CD and BC respectively

$RQ \parallel BD$ [By midpoint theorem] ----- (5)

From (4) and (5)

$SP \parallel BD \parallel RQ$

$\therefore SP \parallel RQ$ ----- (6)

From (3) and (6)

We can say that opposite sides are Parallel in PQRS

Hence we can conclude that PQRS is a parallelogram.

(ii)

Here ABCD is a parallelogram

S and Q are midpoints of AD and BC respectively

$\therefore SQ \parallel AB$

$\therefore SQAB$ is a parallelogram

Now, $\text{area}(\Delta SQP) = \frac{1}{2} \times \text{area of } (SQAB)$ ----- 1

[Since ΔSQP and $\parallel gm$ SQAB have same base and lie between same parallel lines]

Similarly

S and Q are midpoints of AD and BC respectively

$\therefore SQ \parallel CD$

$\therefore SQCD$ is a parallelogram

Now, $\text{area}(\Delta SQR) = \frac{1}{2} \times \text{area of } (SQCD)$ ----- 2

[Since ΔSQR and $\parallel gm$ SQCD have same base and lie between same parallel lines]

Adding (1) and (2) we get

$\text{area}(\Delta SQP) + \text{area}(\Delta SQR) = \frac{1}{2} \times \text{area of } (SQAB) + \frac{1}{2} \times \text{area of } (SQCD)$

$\therefore \text{area}(PQRS) = \frac{1}{2} (\text{area of } (SQAB) + \text{area of } (SQCD))$

$\therefore \text{Area}(\parallel gm PQRS) = \frac{1}{2} \times \text{area}(\parallel gm ABCD)$

Hence proved

Question: 22

Given : ABCDE is a pentagon EG is drawn parallel to DA which meets BA produced at G and CF is drawn parallel to DB which meets AB produced at F

To prove: $\text{area}(\text{pentagon } ABCDE) = \text{area}(\triangle DGF)$

Proof:

Consider quadrilateral ADEG. Here,

$$\text{area}(\triangle AED) = \text{area}(\triangle ADG) \text{ ----- (1)}$$

[since two triangles are on same base AD and lie between parallel line i.e, $AD \parallel EG$]

Similarly now, Consider quadrilateral BDCF. Here,

$$\text{area}(\triangle BCD) = \text{area}(\triangle BDF) \text{ ----- (2)}$$

[since two triangles are on same base AD and lie between parallel line i.e, $AD \parallel EG$]

Adding Eq (1) and (2) we get

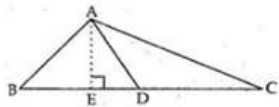
$$\text{area}(\triangle AED) + \text{area}(\triangle BCD) = \text{area}(\triangle ADG) + \text{area}(\triangle BDF) \text{ ----- (3)}$$

Now add $\text{area}(\triangle ABD)$ on both sides of Eq (3), we get

$$\therefore \text{area}(\triangle AED) + \text{area}(\triangle BCD) + \text{area}(\triangle ABD) = \text{area}(\triangle ADG) + \text{area}(\triangle BDF) + \text{area}(\triangle ABD)$$

$$\therefore \text{area}(\text{pentagon } ABCDE) = \text{area}(\triangle DGF)$$

Hence proved

Question: 23

Given : A $\triangle ABC$ with D as median

To prove : Median D divides a triangle into two triangles of equal areas.

Constructions: Drop a perpendicular AE onto BC

Proof: Consider $\triangle ABD$

$$\text{area}(\triangle ABD) = \frac{1}{2} \times BD \times AE$$

Now , Consider $\triangle ACD$

$$\text{area}(\triangle ACD) = \frac{1}{2} \times CD \times AE$$

since D is the median

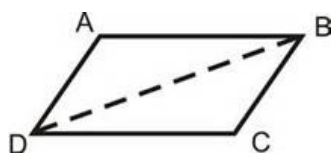
$$BD = CD$$

$$\therefore \frac{1}{2} \times BD \times AE = \frac{1}{2} \times CD \times AE$$

Hence , $\text{area}(\triangle ABD) = \text{area}(\triangle ACD)$

\therefore we can say that Median D divides a triangle into two triangles of equal areas.

Hence proved

Question: 24

Given: A parallelogram ABCD with a diagonal BD

To prove: $\text{area}(\triangle ABD) = \text{area}(\triangle BCD)$

Proof:

We know that in a parallelogram opposite sides are equal, that is

$AD = BC$ and $AB = CD$

Now, consider $\triangle ABD$ and $\triangle BCD$

Here $AD = BC$

$AB = CD$

$BD = BD$ (common)

Hence by SSS congruency

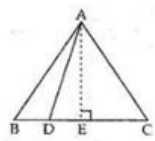
$\triangle ABD \cong \triangle BCD$

By this we can conclude that both the triangles are equal

$\therefore \text{area}(\triangle ABD) = \text{area}(\triangle BCD)$

Hence proved

Question: 25



Given: A $\triangle ABC$ with a point D on BC such that $BD = \frac{1}{2} DC$

To prove: $\text{area}(\triangle ABD) = \frac{1}{3} \times \text{area}(\triangle ABC)$

Construction: Drop a perpendicular onto BC

Proof: $\text{area}(\triangle ABC) = \frac{1}{2} \times BC \times AE$ -----(1)

and, $\text{area}(\triangle ABD) = \frac{1}{2} \times BD \times AE$ ----- (2)

given that $BD = \frac{1}{2} DC$ ----- (3)

so, $BC = BD + DC = BD + 2BD = 3BD$ [from 2]

$\therefore BD = \frac{1}{3} (BC)$

Sub BD in (1), we get

$\text{area}(\triangle ABD) = \frac{1}{2} \times \left(\frac{1}{3} (BC)\right) \times AE$

$\text{area}(\triangle ABD) = \frac{1}{3} \times \left(\frac{1}{2} BC \times AE\right)$

$\therefore \text{area}(\triangle ABD) = \frac{1}{3} \times \text{area}(\triangle ABC)$ [from 1]

Hence proved

Question: 26

Given : A $\triangle ABC$ in which a point D divides the Side BC in the ratio m:n.

To prove: $\text{area}(\triangle ABD) : \text{area}(\triangle ABC) = m:n$

Construction : Drop a perpendicular AL on BC

Proof:

$$\text{area}(\triangle ABD) = \frac{1}{2} \times BD \times AL \text{ ----- (1)}$$

$$\text{and, } \text{area}(\triangle ADC) = \frac{1}{2} \times DC \times AL \text{ ----- (2)}$$

$$BD:DC = m:n$$

$$\frac{BD}{DC} = \frac{m}{n}$$

$$\therefore BD = \frac{m}{n} \times DC \text{ -----(3)}$$

sub Eq (3) in eq (1)

$$\text{area}(\triangle ABD) = \frac{1}{2} \times \left(\frac{m}{n} \times DC\right) \times AL$$

$$\text{area}(\triangle ABD) = \frac{m}{n} \times \left(\frac{1}{2} \times DC \times AL\right)$$

$$\text{area}(\triangle ABD) = \frac{m}{n} \times \text{area}(\triangle ADC)$$

$$\therefore \frac{\text{area}(\triangle ABD)}{\text{area}(\triangle ADC)} = \frac{m}{n}$$

$$\therefore \text{Area}(\triangle ABD) : \text{Area}(\triangle ABC) = m:n$$

Hence proved

Exercise : CCE QUESTIONS

Question: 1

Out of the follow

Solution:

Here, $\triangle PQR$ and $\triangle SQR$ are on the same base QR but there is no parallel line to QR.

\therefore Here, Figure in option B is on the same base but not between the same parallels.

Question: 2

In which of the f

Solution:

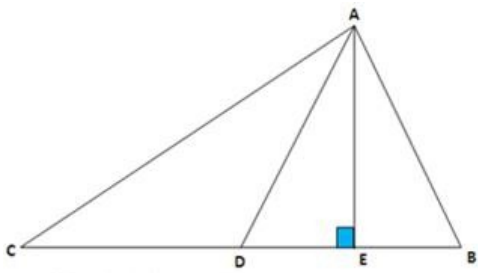
Here parallelogram ABCD and parallelogram ABQP lie on the same base AB and lie between the parallel line AB and DP.

\therefore Here, Figure in option C is on the same base and between the same parallels.

Question: 3

The median of a t

Solution:



In $\triangle ABC$, AD is the median Hence $BD = DC$
 Draw $AE \perp BC$
 Area of $\triangle ABD = \text{Area of } \triangle ADC$
 Thus median of a triangle divides it into two triangles of equal area.

Question: 4

The area of quadr

Solution:

Given:

$$\angle ABC = 90^\circ$$

$$\angle ACD = 90^\circ$$

$$CD = 8\text{cm}$$

$$AB = 9\text{cm}$$

$$AD = 17\text{cm}$$

Consider $\triangle ACD$

Here, By Pythagoras theorem : $AD^2 = CD^2 + AC^2$

$$17^2 = 8^2 + AC^2$$

$$= AC^2 = 17^2 - 8^2$$

$$= AC^2 = 289 - 64 = 225$$

$$= AC = 15$$

Now, Consider $\triangle ABC$

Here, By Pythagoras theorem : $AC^2 = AB^2 + BC^2$

$$15^2 = 9^2 + AC^2$$

$$= BC^2 = 15^2 - 9^2$$

$$= BC^2 = 225 - 81 = 144$$

$$= BC = 12$$

Here,

$$\text{Area (quad.ABCD)} = \text{Area } (\triangle ABC) + \text{Area } (\triangle ACD)$$

$$\text{Area (quad.ABCD)} = \frac{1}{2} \times AB \times BC + \frac{1}{2} \times AC \times CD$$

$$\text{Area (quad.ABCD)} = \frac{1}{2} \times 9 \times 12 + \frac{1}{2} \times 15 \times 8 = 54 + 60 = 114\text{cm}^2$$

$$\therefore \text{Area (quad.ABCD)} = 114\text{cm}^2$$

Question: 5

The area of trape

Solution:

Given:

$$\angle BEC = 90^\circ$$

$$\angle DAE = 90^\circ$$

$$CD = AE = 8\text{cm}$$

$$BE = 15\text{cm}$$

$$BC = 17\text{cm}$$

Consider $\triangle CEB$

Here, By Pythagoras theorem

$$BC^2 = CE^2 + EB^2$$

$$17^2 = CE^2 + 15^2$$

$$CE^2 = 17^2 - 15^2$$

$$CE^2 = 289 - 225 = 64$$

$$CE = 8$$

Here,

$$\angle AEC = 90^\circ$$

$$CD = CE = 8\text{cm}$$

\therefore AECD is a Square.

$$\therefore \text{Area (Trap. ABCD)} = \text{Area (Sq. AECD)} + \text{Area } (\triangle CEB)$$

$$\text{Area (Trap. ABCD)} = AE \times EC + \frac{1}{2} \times CE \times EB$$

$$\text{Area (Trap. ABCD)} = 8 \times 8 + \frac{1}{2} \times 8 \times 15 = 64 + 60 = 104\text{cm}^2$$

$$\therefore \text{Area (Trap. ABCD)} = 124\text{cm}^2$$

Question: 6

In the given figu

Solution:

Given:

$$AB = CD = 5\text{cm}$$

$$BD \perp DC$$

$$BD = 6.8\text{cm}$$

Now, consider the parallelogram ABCD

Here, let DC be the base of the parallelogram then BD becomes its altitude (height).

Area of the parallelogram is given by: Base \times Height

$$\therefore \text{area of } \parallel\text{gm ABCD} = CD \times BD = 5 \times 6.8 = 34\text{cm}^2$$

$$\therefore \text{area of } \parallel\text{gm ABCD} = 34\text{cm}^2.$$

Question: 7

In the given figu

Solution:

Given: ABCD is a $\parallel\text{gm}$ in which diagonals AC and BD intersect at O and $\text{ar}(\parallel\text{gm ABCD})$ is 52cm^2 .

Here,

$$\text{Ar } (\triangle ABD) = \text{ar}(\triangle ABC)$$

(\because $\triangle ABD$ and $\triangle ABC$ on same base AB and between same parallel lines AB and CD)

Here,

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle ABC) = 1/2 \times \text{ar}(\parallel\text{gm ABCD})$$

(\because $\triangle ABD$ and $\triangle ABC$ on same base AB and between same parallel lines AB and CD are half the area of the parallelogram)

$$\therefore \text{ar}(\triangle ABD) = \text{ar}(\triangle ABC) = 1/2 \times 52 = 26\text{cm}^2$$

Now, consider $\triangle ABC$

Here OB is the median of AC

(\because diagonals bisect each other in parallelogram)

$$\therefore \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC)$$

(\because median of a triangle divides it into two triangles of equal area)

$$\text{ar}(\triangle AOB) = 1/2 \times \text{ar}(\triangle ABC)$$

$$\text{ar}(\triangle AOB) = 1/2 \times 26 = 13\text{cm}^2$$

$$\therefore \text{ar}(\triangle AOB) = 13\text{cm}^2$$

Question: 8

In the given figu

Solution:

Area of parallelogram is: base \times height

Here,

$$\text{Base} = AB = 10\text{cm}$$

$$\text{Height} = DL = 4\text{cm}$$

$$\therefore \text{ar}(\parallel\text{gm ABCD}) = AB \times DL = 10 \times 4 = 40\text{cm}^2$$

$$\therefore \text{ar}(\parallel\text{gm ABCD}) = 40\text{cm}^2$$

Question: 9

In Given:

$$AB = 10\text{cm}$$

$$DL \perp AB$$

$$BM \perp AD$$

$$DL = 6\text{cm}$$

$$BM = 8\text{cm}$$

Now, consider the parallelogram ABCD

Here, let AB be the base of the parallelogram then DL becomes its altitude (height).

Area of the parallelogram is given by: Base \times Height

$$\therefore \text{area of } \parallel\text{gm ABCD} = AB \times DL = 10 \times 6 = 60\text{cm}^2$$

Now,

Consider AD as base of the parallelogram then BM becomes its altitude (height)

$$\therefore \text{area of } \parallel\text{gm ABCD} = AD \times BM = 60\text{cm}^2$$

$$AD \times 8 = 60\text{cm}^2$$

$$AD = 60/8 = 7.5\text{cm}$$

$$\therefore \text{length of AD} = 7.5\text{cm}.$$

Question: 10

The lengths of th

Solution:

Given:

Length of diagonals of rhombus: 12cm and 16cm.

Area of the rhombus is given by: $\frac{\text{product of diagonals}}{2}$

$$\therefore \text{Area of the rhombus} = \frac{12 \times 16}{2} = 96\text{cm}^2$$

Question: 11

Two parallel side

Solution:

Given:

Lengths of parallel sides of trapezium: 12cm and 8cm

Distance between two parallel lines (height): 6.5cm

Area of the trapezium is given by: $\frac{(\text{sum of parallel sides}) \times \text{height}}{2}$

$$\therefore \text{Area of the trapezium} = \frac{(12 + 8) \times 6.5}{2} = 65\text{cm}^2$$

Question: 12

In the given figu

Solution:

Given:

$$AL \perp DC$$

$$BM \perp DC$$

$$AB = 7\text{cm}$$

$$BC = AD = 5\text{cm}$$

$$AL = BM = 4\text{cm}$$

Here,

$$MC = DL \text{ and } AB = LM = 7 \text{ cm}$$

Consider the $\triangle BMC$

Here, by Pythagoras theorem

$$BC^2 = BM^2 + MC^2$$

$$5^2 = 4^2 + MC^2$$

$$MC^2 = 25 - 16$$

$$MC^2 = 9$$

$$MC = 3\text{cm}$$

$$\therefore MC = DL = 3\text{cm}$$

$$CD = DL + LM + MC = 3 + 7 + 3 = 13\text{cm}$$

Now,

Area of the trapezium is given by: $\frac{(\text{sum of parallel sides}) \times \text{height}}{2}$

$$\therefore \text{Area of the rhombus} = \frac{(13+7) \times 4}{2} = 40\text{cm}^2$$

Question: 13

In a quadrilatera

Solution:

Given:

$$BD = 16\text{cm}$$

$$AL \perp BD$$

$$CM \perp BD$$

$$AL = 9\text{cm}$$

$$CM = 7\text{cm}$$

Here,

$$\text{Area of quadrilateral ABCD} = \text{area}(\triangle ABD) + \text{area}(\triangle BCD)$$

$$\text{Area of triangle} = 1/2 \times \text{base} \times \text{height}$$

$$\text{area}(\triangle ABD) = 1/2 \times \text{base} \times \text{height} = 1/2 \times BD \times CM = 1/2 \times 16 \times 7 = 56\text{cm}^2$$

$$\text{area}(\triangle BCD) = 1/2 \times \text{base} \times \text{height} = 1/2 \times BD \times AL = 1/2 \times 16 \times 9 = 72\text{cm}^2$$

$$\therefore \text{Area of quadrilateral ABCD} = \text{area}(\triangle ABD) + \text{area}(\triangle BCD) = 56 + 72 = 128\text{cm}^2$$

Question: 14

ABCD is a rhombus

Solution:

$$\text{Given: } \angle DCB = 60^\circ$$

Let the length of the side be x

Here, consider $\triangle BCD$

$$BC = DC \text{ (all sides of rhombus are equal)}$$

$$\therefore \angle CDB = \angle CBD \text{ (angles opposite to equal sides are equal)}$$

Now, by angle sum property

$$\angle CDB + \angle CBD + \angle BCD = 180^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$\therefore 2 \times \angle CBD = 120^\circ$$

$$\angle CBD = \frac{120}{2} = 60^\circ$$

$$\therefore \angle CDB = \angle CBD = 60^\circ$$

$$\therefore \triangle ADC \text{ is equilateral triangle}$$

$$\therefore BC = CD = BD = x \text{ cm}$$

In Rhombus diagonals bisect each other.

Consider $\triangle COD$

By Pythagoras theorem

$$CD^2 = OD^2 + OC^2$$

$$x^2 = \left[\frac{x}{2}\right]^2 + OC^2$$

$$OC^2 = x^2 - \left[\frac{x}{2}\right]^2$$

$$OC = \left[\frac{\sqrt{4x^2 - x^2}}{2}\right]$$

$$OC = \frac{\sqrt{3} \times x}{2} \text{ cm}$$

$$\therefore AC = 2 \times OC = 2 \times \frac{\sqrt{3} \times x}{2} = \sqrt{3}x$$

$$AC: BD = \sqrt{3}x : x = \sqrt{3} : 1$$

$$\therefore AC: BD = \sqrt{3} : 1$$

Question: 15

In the given figure

Solution:

Given: ar(quadr. EABC) = 17cm^2 and ar(||gm ABCD) = 25cm^2

We know that any two parallelogram having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{Area} (\text{||gm ABCD}) = \text{Area} (\text{||gm ABFE}) = 25\text{cm}^2$$

Here,

$$\text{Area} (\text{||gm ABFE}) = \text{Area} (\text{quad. EABC}) + \text{Area} (\triangle BCF)$$

$$25\text{cm}^2 = 17\text{cm}^2 + \text{Area} (\triangle BCF)$$

$$\text{Area} (\triangle BCF) = 25 - 17 = 8\text{cm}^2$$

$$\therefore \text{Area} (\triangle BCF) = 8\text{cm}^2$$

Question: 16

$\triangle ABC$ and $\triangle BDE$ are

Solution:

Given: $\triangle ABC$ and $\triangle BDE$ are two equilateral triangles, D is the midpoint of BC.

Consider $\triangle ABC$

Here, let $AB = BC = AC = x$ cm (equilateral triangle)

Now, consider $\triangle BED$

Here,

$$BD = \frac{1}{2} BC$$

$$\therefore BD = ED = EB = \frac{1}{2} BC = \frac{x}{2} \text{ (equilateral triangle)}$$

Area of the equilateral triangle is given by: $\frac{\sqrt{3}}{4} a^2$ (a is side length)

$$\therefore \text{ar}(\triangle BDE): \text{ar}(\triangle ABC) = \frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2: \frac{\sqrt{3}}{4} x^2 = \frac{1}{4}:1 = 1:4$$

Question: 17

In a Given:

P and Q are midpoints of AB and CD respectively

$$\text{ar}(\text{||gm ABCD}) = 16\text{cm}^2$$

Now, consider the (||gm ABCD)

Here,

Q is the midpoint of DC and P is the midpoint of AB.

∴ By joining P and Q (||gm ABCD) is divided into two equal parallelograms.

That is, $\text{ar}(\text{||gm ABCD}) = \text{ar}(\text{||gm APQD}) + \text{ar}(\text{||gm PQCB})$

$\text{ar}(\text{||gm ABCD}) = 2 \times \text{ar}(\text{||gm APQD})$ (∵ $\text{ar}(\text{||gm APQD}) = \text{ar}(\text{||gm PQCB})$)

$2 \times \text{ar}(\text{||gm APQD}) = 16\text{cm}^2$ (∵ $\text{ar}(\text{||gm ABCD}) = 16\text{cm}^2$)

$\text{ar}(\text{||gm APQD}) = 16/2 = 8\text{cm}^2$

∴ $\text{ar}(\text{||gm APQD}) = 8\text{cm}^2$

Question: 18

The figure formed

Solution:

Given: A rectangle with sides 8cm and 6cm.

Consider the Rectangle ABCD

Here $DR = RD = AP = PB = 8/2 = 4\text{cm}$ (∵ P and R are the midpoints of DC and AB respectively)

and $AS = SD = BQ = QC = 6/2 = 3\text{cm}$ (∵ S and Q are the midpoints of AD and BC respectively)

Now, consider the ΔRSD

By Pythagoras theorem

$$SR^2 = SD^2 + DR^2$$

$$SR^2 = 3^2 + 4^2$$

$$SR^2 = 9 + 16$$

$$SR^2 = 25$$

$$SR = 5\text{ cm}$$

Similarly using Pythagoras theorem in ΔQRC , ΔPBQ and ΔAPS

We get $RQ = QP = PS = 5\text{cm}$

∴ $SR = RQ = QP = PS = 5\text{cm}$

∴ PQSR is Rhombus of side length 5cm

Area of the rhombus is given by: $\frac{\text{product of diagonals}}{2}$

$$\therefore \text{Area of the rhombus} = \frac{PR \times SQ}{2} = \frac{8 \times 6}{2} = 24\text{cm}^2$$

$$\therefore \text{Area(PQRS)} = 24\text{cm}^2$$

Question: 19

In ΔABC , if D is

Solution:

Given: D is the midpoint of BC and E is the midpoint of AD

Here,

D is the midpoint of BC and AD is the median of ΔABC

$\text{Area}(\Delta ABD) = \text{Area}(\Delta ADC)$ (∵ median divides the triangle into two triangles of equal areas)

$$\therefore \text{Area}(\Delta ABD) = \text{Area}(\Delta ADC) = \frac{1}{2} \text{Area}(\Delta ABC)$$

Now, consider ΔABD

Here, BE is the median

$$\text{Area } (\Delta ABE) = \text{Area } (\Delta BED)$$

$$\therefore \text{Area } (\Delta ABE) = \text{Area } (\Delta BED) = \frac{1}{2} \text{Area } (\Delta ABD)$$

$$\text{Area } (\Delta BED) = \frac{1}{2} \text{Area } (\Delta ABD)$$

$$\text{Area } (\Delta BED) = \frac{1}{2} \times \left[\frac{1}{2} \text{Area } (\Delta ABC) \right] \left(\because \text{Area } (\Delta ABD) = \frac{1}{2} \text{Area } (\Delta ABC) \right)$$

$$\text{Area } (\Delta BED) = \frac{1}{4} \text{Area } (\Delta ABC)$$

$$\therefore \text{Area } (\Delta BED) = \frac{1}{4} \text{Area } (\Delta ABC)$$

Question: 20

The vertex A of Δ

Solution:

Given:

Here,

D is the midpoint of BC and AD is the median of ΔABC

$\text{Area } (\Delta ABD) = \text{Area } (\Delta ADC)$ (\because median divides the triangle into two triangles of equal areas)

$$\therefore \text{Area } (\Delta ABD) = \text{Area } (\Delta ADC) = \frac{1}{2} \text{Area } (\Delta ABC)$$

Now, consider ΔABD

Here, BE is the median

$$\text{Area } (\Delta ABE) = \text{Area } (\Delta BED)$$

$$\therefore \text{Area } (\Delta ABE) = \text{Area } (\Delta BED) = \frac{1}{2} \text{Area } (\Delta ABD)$$

$$\text{Area } (\Delta BED) = \frac{1}{2} \text{Area } (\Delta ABD)$$

$$\text{Area } (\Delta BED) = \frac{1}{2} \times \left[\frac{1}{2} \text{Area } (\Delta ABC) \right] \left(\because \text{Area } (\Delta ABD) = \frac{1}{2} \text{Area } (\Delta ABC) \right) - 1$$

$$\text{Area } (\Delta BED) = \frac{1}{4} \text{Area } (\Delta ABC)$$

Similarly,

$$\text{Area } (\Delta EDC) = \frac{1}{4} \text{Area } (\Delta ABC) - 2$$

Add -1 and -2

$$\text{Area } (\Delta BED) + \text{Area } (\Delta EDC) = \frac{1}{4} \text{Area } (\Delta ABC) + \frac{1}{4} \text{Area } (\Delta ABC) = \frac{1}{2} \text{Area } (\Delta ABC)$$

$$\therefore \text{Area } (\Delta \text{ BEC}) = \frac{1}{2} \text{Area } (\Delta \text{ ABC})$$

Question: 21

In $\Delta \text{ ABC}$, it given

Solution:

Given: D is the midpoint of BC; E is the midpoint of BD and O is the midpoint of AE.

Here,

D is the midpoint of BC and AD is the median of $\Delta \text{ ABC}$

Area $(\Delta \text{ ABD}) = \text{Area } (\Delta \text{ ADC})$ (\because median divides the triangle into two triangles of equal areas)

$$\therefore \text{Area } (\Delta \text{ ABD}) = \text{Area } (\Delta \text{ ADC}) = \frac{1}{2} \text{Area } (\Delta \text{ ABC})$$

Now, consider $\Delta \text{ ABD}$

Here, AE is the median

Area $(\Delta \text{ ABE}) = \text{Area } (\Delta \text{ BED})$

$$\therefore \text{Area } (\Delta \text{ ABE}) = \text{Area } (\Delta \text{ BED}) = \frac{1}{2} \text{Area } (\Delta \text{ ABD})$$

$$\text{Area } (\Delta \text{ ABE}) = \frac{1}{2} \text{Area } (\Delta \text{ ABD})$$

$$\text{Area } (\Delta \text{ ABE}) = \frac{1}{2} \times \left[\frac{1}{2} \text{Area } (\Delta \text{ ABC}) \right] \left(\because \text{Area } (\Delta \text{ ABD}) = \frac{1}{2} \text{Area } (\Delta \text{ ABC}) \right) - 1$$

$$\text{Area } (\Delta \text{ ABE}) = \frac{1}{4} \text{Area } (\Delta \text{ ABC})$$

Consider $\Delta \text{ ABE}$

Here, BO is the median

Area $(\Delta \text{ BOE}) = \text{Area } (\Delta \text{ BOA})$

$$\therefore \text{Area } (\Delta \text{ BOE}) = \text{Area } (\Delta \text{ BOA}) = \frac{1}{2} \text{Area } (\Delta \text{ ABE})$$

$$\text{Area } (\Delta \text{ BOE}) = \frac{1}{2} \times \left[\frac{1}{4} \text{Area } (\Delta \text{ ABC}) \right] \left(\because \text{Area } (\Delta \text{ ABE}) = \frac{1}{4} \text{Area } (\Delta \text{ ABC}) \right)$$

$$\text{Area } (\Delta \text{ BOE}) = \frac{1}{8} \text{Area } (\Delta \text{ ABC})$$

$$\therefore \text{Area } (\Delta \text{ BOE}) = \frac{1}{8} \text{Area } (\Delta \text{ ABC})$$

Question: 22

If a triangle and

Solution:

Given:

We know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

$$\text{Area}(\triangle ABF) = 1/2 \text{Area}(\text{||gm ABCD}) - 1$$

$$\text{Area}(\triangle ABF) : \text{Area}(\text{||gm ABCD}) = 1/2 \text{Area}(\text{||gm ABCD}) : \text{Area}(\text{||gm ABCD}) \text{ (from -1)}$$

$$\text{Area}(\triangle ABF) : \text{Area}(\text{||gm ABCD}) = 1/2 : 1 = 1:2$$

$$\therefore \text{Area}(\triangle ABF) : \text{Area}(\text{||gm ABCD}) = 1:2$$

Question: 23

In the given figu

Solution:

Given: ABCD is a trapezium, $AB \parallel DC$, $AB = a$ cm and $DC = b$ cm, E and F are the midpoints of AD and BC.

Since E and F are midpoints of AD and BC, EF will be parallel to both AB and CD.

$$EF = \frac{a+b}{2}$$

Height between EF and DC and height between EF and AB are equal, because E and F are midpoints OF AD and BC and $EF \parallel AB \parallel DC$.

Let height between EF and DC and height between EF and AB be h cm.

$$\text{Area of trapezium} = 1/2 \times (\text{sum of parallel lines}) \times \text{height}$$

Now,

$$\text{Area (Trap.ABFE)} = 1/2 \times (a + \frac{a+b}{2}) \times h.$$

and

$$\text{Area (Trap.ABFE)} = 1/2 \times (b + \frac{a+b}{2}) \times h.$$

$$\text{Area (Trap.ABFE)} : \text{Area (Trap.ABFE)} = 1/2 \times (a + \frac{a+b}{2}) \times h : 1/2 \times (b + \frac{a+b}{2}) \times h$$

$$\text{Area (Trap.ABFE)} : \text{Area (Trap.ABFE)} = \frac{2a+a+b}{2} : \frac{2b+a+b}{2} = 3a + b : a + 3b$$

$$\therefore \text{Area (Trap.ABFE)} : \text{Area (Trap.ABFE)} = 3a + b : a + 3b$$

Question: 24

ABCD is a quadril

Solution:

Given: a quadrilateral whose diagonal AC divides it into two parts, equal in area.

Here,

A quadrilateral is any shape having four sides, it is given that diagonal AC of the quadrilateral divides it into two equal parts.

We know that the rectangle, parallelogram and rhombus are all quadrilaterals, in these quadrilaterals if a diagonal is drawn say AC it divides it into equal areas.

\therefore This diagonal divide the quadrilateral into two equal or congruent triangles.

Question: 25

In the given figu

Solution:

Given: $\text{Area}(\text{||gm ABCD}) = \text{Area}(\text{rectangle ABEF})$

Consider $\triangle AFD$

Clearly AD is the hypotenuse

$$\therefore AD > AF$$

$$\text{Perimeter of Rectangle ABEF} = 2 \times (AB + AF) - 1$$

$$\text{Perimeter of Parallelogram ABCD} = 2 \times (AB + AD) - 2$$

On comparing -1 and -2, we can see that

$$\text{Perimeter of ABCD} > \text{perimeter of ABEF} (\because AD > AF)$$

Question: 26

In the given figure

Solution:

Given: ABCD is a rectangle inscribed in a quadrant of a circle of radius 10cm and AD = 25cm

Consider ΔADC

By Pythagoras theorem

$$AC^2 = AD^2 + DC^2$$

$$10^2 = (25)^2 + AC^2$$

$$AC^2 = 10^2 - (25)^2$$

$$AC^2 = 100 - 20 = 80$$

$$AC = 45$$

$$\text{Area of rectangle} = \text{length} \times \text{breadth} = DC \times AD$$

$$\text{Area of rectangle} = 45 \times 25 = 40\text{cm}^2$$

$$\therefore \text{Area of rectangle} = 40\text{cm}^2$$

Question: 27

Look at the state

Solution:

Consider Statement (I) :

Two or more parallelograms on the same base and between the same parallels are equal in area.
Rectangle is also a parallelogram.

\therefore It is true.

Consider Statement (II) :

Here, let AB be the base of the parallelogram then DE becomes its altitude (height).

Area of the parallelogram is given by: Base \times Height

$$\therefore \text{Area of } \parallel\text{gm ABCD} = AB \times DE = 10 \times 6 = 60\text{cm}^2$$

Now,

Consider AD as base of the parallelogram then BF becomes its altitude (height)

$$\therefore \text{area of } \parallel\text{gm ABCD} = AD \times BF = 60\text{cm}^2$$

$$AD \times 8 = 60\text{cm}^2$$

$$AD = \frac{60}{8} = 7.5\text{cm}$$

\therefore length of AD = 7.5cm.

\therefore Statement (II) is correct.

Consider Statement (III)

Area of parallelogram is base \times height

\therefore Statement (III) is false

\therefore Statement (I) and (II) are true and statement (III) is false

Question: 28

The question consists of two parts.

Solution:

Assertion:

Here, Area ($\triangle ABD$) = Area($\triangle ABC$) (\because Triangles on same base and between same parallel lines) -1

Subtract Area ($\triangle AOB$) on both sides of -1

$$\text{Area}(\triangle ABD) - \text{Area}(\triangle AOB) = \text{Area}(\triangle ABC) - \text{Area}(\triangle AOB)$$

$$\text{Area}(\triangle AOD) = \text{Area}(\triangle BOC)$$

\therefore Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

Question: 29

The question consists of two parts.

Solution:

Given: $\angle DCB = 60^\circ$

Let the length of the side be x

Here, consider $\triangle BCD$

$BC = DC$ (all sides of rhombus are equal)

$\therefore \angle CDB = \angle CBD$ (angles opposite to equal sides are equal)

Now, by angle sum property

$$\angle CDB + \angle CBD + \angle BCD = 180^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$2 \times \angle CBD = 180^\circ - 60^\circ$$

$$\therefore 2 \times \angle CBD = 120^\circ$$

$$\angle CBD = \frac{120}{2} = 60^\circ$$

$$\therefore \angle CDB = \angle CBD = 60^\circ$$

$\therefore \triangle ADC$ is equilateral triangle

$$\therefore BC = CD = BD = x \text{ cm}$$

In Rhombus diagonals bisect each other.

Consider $\triangle COD$

By Pythagoras theorem

$$CD^2 = OD^2 + OC^2$$

$$x^2 = \left[\frac{x}{2}\right]^2 + OC^2$$

$$OC^2 = x^2 - \left[\frac{x}{2}\right]^2$$

$$OC = \left[\frac{\sqrt{4x^2 - x^2}}{2}\right]$$

$$OC = \frac{\sqrt{3} \times x}{2} \text{ cm}$$

$$\therefore AC = 2 \times OC = 2 \times \frac{\sqrt{3} \times x}{2} = \sqrt{3}x$$

$$AC: BD = \sqrt{3}x : x = \sqrt{3} : 1$$

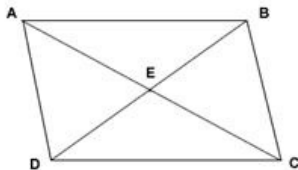
$$\therefore AC: BD = \sqrt{3} : 1$$

\therefore Both Assertion but Reason are true and Reason is not a correct explanation of Assertion.

Question: 30

The question cons

Solution:



Consider ΔABD

We know that diagonals in a parallelogram bisect each other

\therefore E is the midpoint of BD, AE is median of ΔABD

\therefore Area (ΔADE) = Area (ΔAEB) (\because Median divides the triangle into two triangles of equal areas)

Similarly we can prove

$$\text{Area } (\Delta ADE) = \text{Area } (\Delta DEC)$$

$$\text{Area } (\Delta DEC) = \text{Area } (\Delta CEB)$$

$$\text{Area } (\Delta CEB) = \text{Area } (\Delta AEB)$$

\therefore Diagonals of a \parallel gm divide into four triangles of equal area.

\therefore Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

Question: 31

The question cons

Solution:

$$\text{Area of trapezium} = 1/2 \times (\text{sum of parallel sides}) \times \text{height} = 1/2 \times (25 + 15) \times 6 = 120\text{cm}^2$$

$$\therefore \text{Area of trapezium} = 120\text{cm}^2$$

\therefore Assertion is correct.

Area of an equilateral triangle is given by: $\frac{\sqrt{3}}{4} \times a^2$ (here 'a' is length of the side)

$$\therefore \text{Area of an equilateral triangle with side length 8 cm} = \frac{\sqrt{3}}{4} \times 8^2 = 16\sqrt{3}$$

\therefore Reason is correct

\therefore Both Assertion but Reason are true and Reason is not a correct explanation of Assertion.

Question: 32

The question cons

Solution:

Here, let AB be the base of the parallelogram then DE becomes its altitude (height).

Area of the parallelogram is given by: Base \times Height

$$\therefore \text{Area of } \parallel\text{gm } ABCD = AB \times DE = 16 \times 8 = 128\text{cm}^2$$

Now,

Consider AD as base of the parallelogram then BF becomes its altitude (height)

$$\therefore \text{area of } \parallel\text{gm ABCD} = AD \times BF = 128\text{cm}^2$$

$$AD \times 10 = 128\text{cm}^2$$

$$AD = \frac{128}{10} = 12.8\text{cm}$$

$$\therefore \text{length of AD} = 12.8\text{cm}$$

\therefore Assertion is false and Reason is true

Question: 33

Which of the foll

Solution:

The correct answer is Option (D)

Δ ABC and Δ BCD does not lie between parallel lines and also Δ AOB and Δ COD are not congruent.

Question: 34

Which of the foll

Solution:

The correct answer is Option (B)

Area of parallelogram = base \times corresponding height.

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

The area of

Solution:

Area of the $\parallel\text{gm}$ is Base \times Height

Here, height is distance between the Base and its corresponding parallel side.

$$\therefore \text{Area} (\parallel\text{gm ABCD}) = \text{Base} \times \text{Height} = DC \times DL$$

(\because Here DC is taken as length and DL is the distance between DC and its corresponding parallel side AB).

Question: 2

Two parallelogram

Solution:

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

Consider two $\parallel\text{gms}$ ABCD and PQRS which are on same base and lie between same parallel lines.

$$\therefore \text{ar}(\parallel\text{gm ABCD}) = \text{ar}(\parallel\text{gm PQRS}) - 1$$

$$\therefore \text{ar}(\parallel\text{gm ABCD}) : \text{ar}(\parallel\text{gm PQRS}) = 1:1 \quad (\because \text{eq -1})$$

Question: 3

ABCD is a quadril

Solution:

Quadrilateral is any closed figure which has four sides.

Rhombus, Rectangle, Parallelograms are few Quadrilaterals.

When a Diagonal AC of a quadrilateral divides it into two parts of equal areas, it is not necessary for the figure to be a Rhombus or a Rectangle or a Parallelogram, it can be any Quadrilateral.

Question: 4

In the given figu

Solution:

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{ar}(\parallel\text{gm ABCD}) = \text{ar}(\parallel\text{gm ABPQ}) - 1$$

We also know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

$$\therefore \text{ar}(\triangle BMP) = \frac{1}{2} \text{ar}(\parallel\text{gm ABPQ})$$

But, from -1

$$\text{ar}(\parallel\text{gm ABCD}) = \text{ar}(\parallel\text{gm ABPQ})$$

$$\therefore \text{ar}(\triangle BMP) = \frac{1}{2} \text{ar}(\parallel\text{gm ABCD})$$

Question: 5

The midpoints of

Solution:

Join EF

$$\text{Here Area } (\triangle AEF) = \text{Area } (\triangle BDF) = \text{Area } (\triangle DEF) = \text{Area } (\triangle DEC) = \frac{1}{4} \text{Area } (\triangle ABC) - 1$$

Consider any vertex of the triangle.

Let us consider Vertex B

Here, BDEF form a parallelogram.

$$\text{Area } (\parallel\text{gm BDEF}) = \text{Area } (\triangle BDF) + \text{Area } (\triangle DEF)$$

$$\text{Area } (\parallel\text{gm BDEF}) = \frac{1}{4} \text{Area } (\triangle ABC) + \frac{1}{4} \text{Area } (\triangle ABC) = \frac{1}{2} \text{Area } (\triangle ABC) \text{ (from -1)}$$

$$\therefore \text{Area } (\parallel\text{gm BDEF}) = \frac{1}{2} \text{Area } (\triangle ABC)$$

Similarly, we can prove for other vertices.

Question: 6

Let ABCD be a

Solution:

Given:

$$AD = 6\text{cm}$$

$$DL \perp AB$$

$$BM \perp AD$$

$$DL = 8\text{cm}$$

$$BM = 10\text{cm}$$

Now, consider the parallelogram ABCD

Here, let AD be the base of the parallelogram then BM becomes its altitude (height).

Area of the parallelogram is given by: Base \times Height

$$\therefore \text{area of } \parallel\text{gm ABCD} = AD \times BM = 6 \times 10 = 60\text{cm}^2$$

Now,

Consider AB as base of the parallelogram then DL becomes its altitude (height)

$$\therefore \text{area of } \parallel\text{gm ABCD} = AB \times DL = 60\text{cm}^2$$

$$AB \times 8 = 60\text{cm}^2$$

$$AB = \frac{60}{8} = 7.5\text{cm}$$

$$\therefore \text{length of AB} = 7.5\text{cm}.$$

Question: 7

Find the area of

Solution:

Given: Length of parallel sides 14 cm and 10 cm, height is 6cm

We know that area of trapezium is given by: $\frac{1}{2}$ (sum of parallel sides) \times height

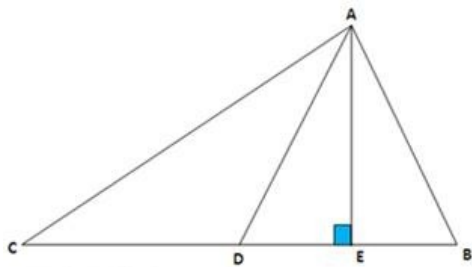
$$\therefore \text{Area of trapezium} = \frac{1}{2} (14 + 10) \times 6 = 72\text{cm}^2$$

$$\therefore \text{Area of trapezium} = 72\text{cm}^2$$

Question: 8

Show that the med

Solution:



Consider the Figure

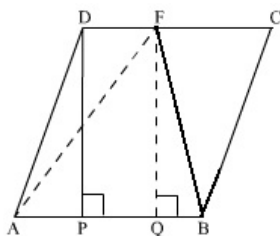
Here,

In $\triangle ABC$, AD is the median Hence $BD = DC$ Draw $AE \perp BC$ Area of $\triangle ABD = \text{Area of } \triangle ADC$ Thus median of a triangle divides it into two triangles of equal area.

Question: 9

Prove that area o

Solution:



We know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

Consider the figure,

Here,

$$\text{Area}(\triangle ABF) = \frac{1}{2} \text{Area}(\text{||gm ABCD}) \text{ (From above statement) } -1$$

$$\text{Area}(\text{||gm ABCD}) = \text{Base} \times \text{Height} -2$$

Sub -2 in -1

$$\text{Area}(\triangle ABF) = \frac{1}{2} \times \text{Base} \times \text{Height}$$

Question: 10

In the adjoining

Solution:

Given: $BD = 14\text{cm}$, $AL = 8\text{ cm}$, $CM = 6\text{ cm}$ and also, $AL \perp BD$ and $CM \perp BD$.

Here,

$$\text{Area (Quad.ABCD)} = \text{Area } (\triangle ABD) + \text{Area } (\triangle ABC)$$

$$\text{Area } (\triangle ABD) = \frac{1}{2} \text{ base} \times \text{height} = \frac{1}{2} \times BD \times AL = \frac{1}{2} \times 14 \times 8 = 56\text{cm}^2$$

$$\text{Area } (\triangle ABC) = \frac{1}{2} \text{ base} \times \text{height} = \frac{1}{2} \times BD \times CM = \frac{1}{2} \times 14 \times 6 = 42\text{cm}^2$$

$$\therefore \text{Area (Quad.ABCD)} = \text{Area } (\triangle ABD) + \text{Area } (\triangle ABC) = 56 + 42 = 98\text{ cm}^2$$

$$\therefore \text{Area (Quad.ABCD)} = 98\text{ cm}^2$$

Question: 11

In the adjoining

Solution:

Given: $AC \parallel DP$

We know that any two or Triangles having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{Area } (\triangle ACD) = \text{Area } (\triangle ACP) -1$$

Add Area $(\triangle ABC)$ on both sides of eq -1

We get,

$$\text{Area } (\triangle ACD) + \text{Area } (\triangle ABC) = \text{Area } (\triangle ACP) + \text{Area } (\triangle ABC)$$

That is,

$$\text{Area (quad.ABCD)} = \text{Area } (\triangle ABP)$$

Question: 12

In the given figu

Solution:

Given: $BE \parallel AC$

We know that any two or more Triangles having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{Area } (\triangle ACE) = \text{Area } (\triangle ACB) -1$$

Add Area $(\triangle ADC)$ on both sides of eq -1

We get,

$$\text{Area } (\triangle ACE) + \text{Area } (\triangle ADC) = \text{Area } (\triangle ACB) + \text{Area } (\triangle ADC)$$

That is,

$$\text{Area } (\triangle ADE) = \text{Area (quad. ABCD)}$$

Question: 13

In the given figure

Solution:

Given: area of \parallel gm ABCD is 80 cm^2

We know that any two \parallel gm having the same base and lying between the same parallel lines are equal in area.

$$\therefore \text{ar}(\parallel \text{gm ABCD}) = \text{ar}(\parallel \text{gm ABEF}) - 1$$

We also know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

$$\therefore \text{ar}(\triangle ABD) = 1/2 \times \text{ar}(\parallel \text{gm ABCD}) \text{ and,}$$

$$\text{ar}(\triangle BEF) = 1/2 \times \text{ar}(\parallel \text{gm ABEF})$$

(i)

$$\text{ar}(\parallel \text{gm ABCD}) = \text{ar}(\parallel \text{gm ABEF})$$

$$\therefore \text{ar}(\parallel \text{gm ABEF}) = 80 \text{ cm}^2 (\because \text{ar}(\parallel \text{gm ABCD}) = 80 \text{ cm}^2)$$

(ii)

$$\text{ar}(\triangle ABD) = 1/2 \times \text{ar}(\parallel \text{gm ABCD})$$

$$\text{ar}(\triangle ABD) = 1/2 \times 80 = 40 \text{ cm}^2 (\because \text{ar}(\parallel \text{gm ABCD}) = 80 \text{ cm}^2)$$

$$\therefore \text{ar}(\triangle ABD) = 40 \text{ cm}^2$$

(iii)

$$\text{ar}(\triangle BEF) = 1/2 \times \text{ar}(\parallel \text{gm ABEF})$$

$$\text{ar}(\triangle BEF) = 1/2 \times 80 = 40 \text{ cm}^2 (\because \text{ar}(\parallel \text{gm ABEF}) = 80 \text{ cm}^2)$$

$$\therefore \text{ar}(\triangle BEF) = 40 \text{ cm}^2$$

Question: 14

In trapezium ABCD

Solution:

Given: $AB \parallel DC$ and L is the midpoint of BC, $PQ \parallel AD$

Construction: Drop a perpendicular DM from D onto AP

Consider $\triangle PBL$ and $\triangle CQL$

Here,

$$\angle LPB = \angle LQC \text{ (Alternate interior angles, } AB \parallel DC \text{)}$$

$$BL = LC \text{ (L is midpoint of BC)}$$

$$\angle PLB = \angle QLC \text{ (vertically opposite angles)}$$

\therefore By AAS congruency

$$\triangle PBL \cong \triangle CQL$$

$$\therefore PB = CQ \text{ (C.P.C.T)}$$

$$\text{Area}(\parallel \text{gm APQD}) = \text{base} \times \text{height} = AP \times DM - 1$$

$$\text{Area}(\text{Trap. ABCD}) = 1/2 \times (\text{sum of parallel sides}) \times \text{height} = 1/2 \times (AB + DC) \times DM$$

$$\text{Area}(\text{Trap. ABCD}) = 1/2 \times (AB + DC) \times DM = 1/2 \times (AP + PB + DC) \times DM (\because AB = AP + PB)$$

$$\text{Area}(\text{Trap. ABCD}) = 1/2 \times (AP + CQ + DC) \times DM (\because PB = CQ)$$

$$\text{Area (Trap.ABCD)} = 1/2 \times (\text{AP} + \text{DQ}) \times \text{DM} (\because \text{DC} + \text{CQ} = \text{DQ})$$

$$\text{Area (Trap.ABCD)} = 1/2 \times (2 \times \text{AP}) \times \text{DM} (\because \text{AP} = \text{DQ})$$

$$\text{Area (Trap.ABCD)} = \text{AP} \times \text{DM} \text{---2}$$

From -1 and -2

$$\text{Area (Trap.ABCD)} = \text{Area (||gm APQD)}$$

Question: 15

In the adjoining

Solution:

Given: ABCD is a || gm and O is a point on the diagonal AC.

Construction: Drop perpendiculars DM and BN onto diagonal AC.

Here,

DM = BN (perpendiculars drawn from opposite vertices of a ||gm to the diagonal are equal)

Now,

$$\text{Area } (\Delta AOB) = 1/2 \times \text{base} \times \text{height} = 1/2 \times \text{AO} \times \text{BN} \text{---1}$$

$$\text{Area } (\Delta AOD) = 1/2 \times \text{base} \times \text{height} = 1/2 \times \text{AO} \times \text{DM} \text{---2}$$

From -1 and -2

$$\text{Area } (\Delta AOB) = \text{Area } (\Delta AOD) (\because \text{BN} = \text{DM})$$

Question: 16

ΔABC and ΔBDE are

Solution:

Given: ΔABC and ΔBDE are two equilateral triangles, D is the midpoint of BC.

Consider ΔABC

Here, let $\text{AB} = \text{BC} = \text{AC} = x \text{ cm}$ (equilateral triangle)

Now, consider ΔBED

Here,

$$\text{BD} = 1/2 \text{ BC}$$

$$\therefore \text{BD} = \text{ED} = \text{EB} = 1/2 \text{ BC} = x/2 \text{ (equilateral triangle)}$$

Area of the equilateral triangle is given by: $\frac{\sqrt{3}}{4} a^2$ (a is side length)

$$\therefore \text{ar}(\Delta BDE) : \text{ar}(\Delta ABC) = \frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2 : \frac{\sqrt{3}}{4} x^2 = \frac{1}{4} : 1 = 1 : 4$$

$$\text{That is } \frac{\text{ar}(\Delta BDE)}{\text{ar}(\Delta ABC)} = \frac{1}{4}$$

$$\therefore \text{ar}(\Delta BDE) = \frac{1}{4} \text{ar}(\Delta ABC)$$

Hence Proved

Question: 17

In ΔABC , D is the

Solution:

Given: D is the midpoint of AB and P Point is any point on BC, $\text{CQ} \parallel \text{PD}$

In Quadrilateral DPQC

$$\text{Area } (\Delta DPQ) = \text{Area } (\Delta DPC)$$

Add Area (ΔBDP) on both sides

We get,

$$\text{Area } (\Delta DPQ) + \text{Area } (\Delta BDP) = \text{Area } (\Delta DPC) + \text{Area } (\Delta BDP)$$

$$\text{Area } (\Delta BPQ) = \text{Area } (\Delta BCD) \text{ -1}$$

D is the midpoint BC, and CD is the median

$$\therefore \text{Area } (\Delta BCD) = \text{Area } (\Delta ACD) = \frac{1}{2} \times \text{Area } (\Delta ABC) \text{ -2}$$

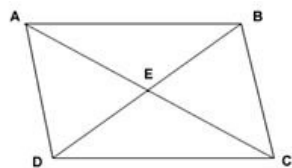
Sub -2 in -1

$$\text{Area } (\Delta BPQ) = \frac{1}{2} \times \text{Area } (\Delta ABC) \quad (\because \text{Area } (\Delta BCD) = \frac{1}{2} \times \text{Area } (\Delta ABC))$$

Question: 18

Show that the dia

Solution:



Consider ΔABD

We know that diagonals in a parallelogram bisect each other

\therefore E is the midpoint of BD, AE is median of ΔABD

$\therefore \text{Area } (\Delta ADE) = \text{Area } (\Delta AEB)$ (\because Median divides the triangle into two triangles of equal areas)

Similarly we can prove

$$\text{Area } (\Delta ADE) = \text{Area } (\Delta DEC)$$

$$\text{Area } (\Delta DEC) = \text{Area } (\Delta CEB)$$

$$\text{Area } (\Delta CEB) = \text{Area } (\Delta AEB)$$

\therefore Diagonals of a \parallel gm divide into four triangles of equal area.

Question: 19

In the given figu

Solution:

Given: $BD \parallel CA$, E is the midpoint of CA and $BD = \frac{1}{2} CA$

Consider ΔBCD and ΔDEC

Here,

$$BD = EC \quad (\because \text{E is the midpoint of AC that is } CE = \frac{1}{2} CA, BD = \frac{1}{2} CA)$$

$$CD = CD \text{ (common)}$$

$$\angle BDC = \angle ECD \text{ (alternate interior angles, } DB \parallel AC)$$

\therefore By SAS congruency

$$\Delta BCD \cong \Delta DEC$$

$$\therefore \text{Area } (\Delta BCD) = \text{Area } (\Delta DEC) \text{ -1}$$

Here,

Area (ΔBCE) = Area (ΔDEC) (triangles on same base CE and between same parallel lines) -2

E is the midpoint of AC, BE is the median of ΔABC

$$\therefore \text{Area} (\Delta BCE) = \text{Area} (\Delta ABE) = 1/2 \times \text{Area} (\Delta ABC)$$

$$\therefore \text{Area} (\Delta DEC) = 1/2 \times \text{Area} (\Delta ABC) (\because \text{Area} (\Delta BCE) = \text{Area} (\Delta DEC))$$

$$\therefore \text{Area} (\Delta BCD) = 1/2 \times \text{Area} (\Delta ABC) (\because \text{Area} (\Delta DEC) = \text{Area} (\Delta BCD))$$

Question: 20

The given figure

Solution:

Given: $EG \parallel DA$, $CF \parallel DB$

Here, in Quadrilateral ADEG

$$\text{Area} (\Delta AED) = \text{Area} (\Delta ADG) \text{ -1}$$

In Quadrilateral CFBD

$$\text{Area} (\Delta CBD) = \text{Area} (\Delta BCF) \text{ -2}$$

Add -1 and -2

$$\text{Area} (\Delta AED) + \text{Area} (\Delta CBD) = \text{Area} (\Delta ADG) + \text{Area} (\Delta BCF) \text{ -3}$$

Add Area (ΔABD) to -3

$$\text{Area} (\Delta AED) + \text{Area} (\Delta CBD) + \text{Area} (\Delta ABD) = \text{Area} (\Delta ADG) + \text{Area} (\Delta BCF) + \text{Area} (\Delta ABD)$$

$$\text{Area} (\text{pentagon } ABCDE) = \text{Area} (\Delta DGF)$$

Question: 21

In the adjoining

Solution:

Given: D divides the side BC of ΔABC in the ratio m:n

$$\text{Area} (\Delta ABD) = 1/2 \times BD \times AL$$

$$\text{Area} (\Delta ADC) = 1/2 \times CD \times AL$$

$$\text{Area} (\Delta ABD): \text{Area} (\Delta ADC) = 1/2 \times BD \times AL: 1/2 \times CD \times AL$$

$$\text{Area} (\Delta ABD): \text{Area} (\Delta ADC) = BD: CD$$

$$\text{Area} (\Delta ABD): \text{Area} (\Delta ADC) = m: n (\because BD:CD = m:n)$$

Question: 22

In the give figur

Solution:

Given: X and Y are the midpoints of AC and AB respectively, $QP \parallel BC$ and CYQ and BXP are straight lines.

Construction: Join QB and PC

In Quadrilateral BCQP

Area (ΔQBC) = Area (ΔBCP) (Triangles on same base BC and between same parallel lines are equal in area) -1 and,

Area ($\parallel gm ACBQ$) = Area ($\parallel gm ABQP$) (parallelograms on same base BC and between same parallel lines are equal in area) -2

Subtract -1 from -2

$$\text{Area}(\text{||gm ACBQ}) - \text{Area}(\Delta \text{ QBC}) = \text{Area}(\text{||gm ABCP}) - \text{Area}(\Delta \text{ BCP})$$

$$\text{Area}(\Delta \text{ ACQ}) = \text{Area}(\Delta \text{ ABP})$$

$$\therefore \text{Area}(\Delta \text{ ABP}) = \text{Area}(\Delta \text{ ACQ})$$