Chapter : 21. SUMMATIVE ASSESSMENT II

Exercise : SAMPLE PAPER I

Question: 1

If the sum of the

Solution:

Let the roots of the given quadratic equation $3x^2 – (3k – 2)x – (k – 6)=0$ be α and β.

Now,

sum of roots = $\alpha + \beta = (3k – 2)/3$ and,

product of roots = $\alpha\beta = – (k – 6)/3$

[∵ If α and β are the roots of quadratic equation $ax^2 + bx + c=0$ then $\alpha + \beta = – b/a$ and $\alpha\beta = c/a$]

According to question –

sum of roots = product of roots

∴ $\alpha + \beta = \alpha\beta$

⇒ $(3k – 2)/3 = – (k – 6)/3$

⇒ $3k – 2 = – k + 6$

⇒ $4k = 8$

∴ $k = 2$

Hence, The value of k is 2.

Question: 2

The number of all

Solution:

All 2-digit numbers divisible by 6 are as follows: –

6, 12, ………., 96

The above series of numbers forms an arithmetic progression with

first term(a) = 6 and,

common difference(d) = (n + 1)th term – nth term = 12 – 6 = 6

last term or nth term(a_n) = 96

Let the number of terms in above series be n.

∵ $a_n = a + (n – 1) \times d$

⇒ $96 = 6 + (n – 1) \times 6$

⇒ $90 = 6n – 6$

⇒ $6n = 96$

∴ $n = 16$

Thus, total no. of all 2-digit numbers divisible by 6.

Question: 3

A fair die is thr

Solution:

Let P be the event of getting a composite number while throwing a dice.

Total no. of outcomes when n number of die are thrown = 6^n

∴ no. of total outcomes = n(S) = 6

Sample Space = {1, 2, 3, 4, 5, 6}

favourable elementary events = getting a composite number

= {4, 6}

∴ no. of favourable elementary events = n(P) = 2

Thus, the probability of getting a composite number = n(P)/n(S)

= 2/6

= 1/3

Question: 4

Which of the foll

Solution:

A **Tangent** is a line that intersects a **circle** at exactly one point.

The tangents drawn at the end points of a chord of a circle can be parallel only if that chord is the diameter of the circle. This will be clear from the fig.1 and fig.2 shown below.

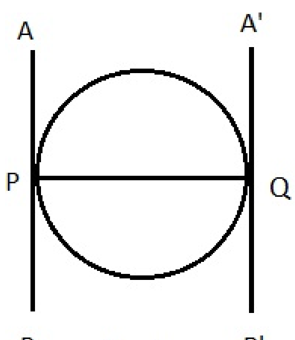


fig.1

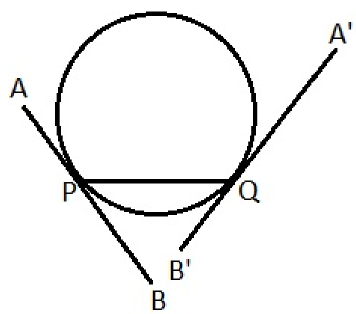


fig.2

Thus, statement (a) is incorrect.

A **secant** is a segment that intersects a **circle** twice.

From a point P in the exterior of a circle, infinite no. of secants can be drown through P to the circle. This can be shown in the fig.3 drawn below.

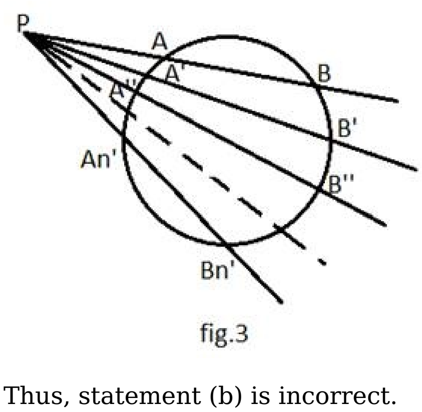


fig.3

Thus, statement (b) is incorrect.

A **Tangent** is a line that intersects a **circle** at exactly one point.

From a point P in the plane of a circle, two tangents can be drawn to the circle only if point P is exterior to the circle. This can be shown in the fig.4 drawn below.

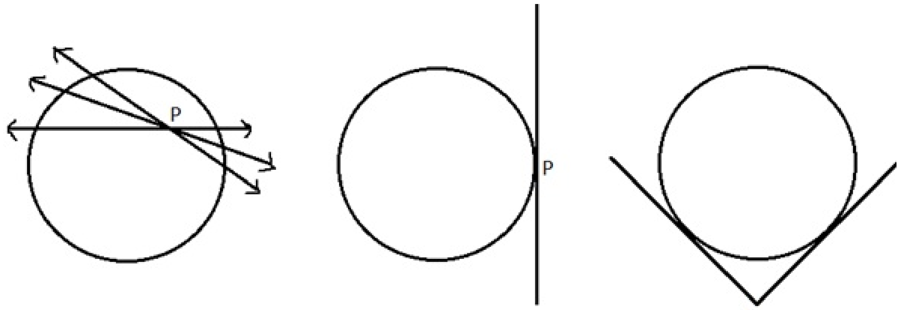


fig.4

Thus, statement (c) is incorrect.

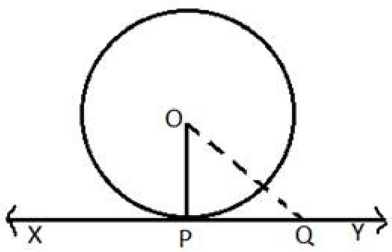


fig.5

In the above fig.5, we take a point Q on the tangent XY to the circle with centre O. Obviously, this point Q should lie outside to the circle otherwise XY will become secant. And, P is the point of contact. Clearly,

OQ > OP

Also, this is also true for all the points lying on the tangent XY except point P. And,

<u>we know that perpendicular distance is always the shortest distance</u>.

OP is shortest of all the distances b/w points O and any other points on XY i.e.

OP ⊥ XY

Hence, The perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

Thus, statement (d) is correct.

Question: 5

In the given figu

Solution:

In Δ PAB,

∠ APB = 60° and PA = 8 cm [given]

∴ PB = PA = 8 cm

[∵ tangents drawn from an exterior point to the circle are equal in length]

⇒ ∠ PAB = ∠ PBA = θ [LET]

Now, In Δ PAB

∠ APB + ∠ PAB + ∠ PBA = 180° [∵ Sum of all the angles of a Δ is 180°]

⇒ 60° + θ + θ = 180°

⇒ 60° + 2θ = 180°

⇒ 2θ = 120°

∴ θ = 60°

Thus, Δ PAB is an equilateral Triangle.

∴ Length of chord AB = 8 cm.

Question: 6

The angle of depr

Solution:

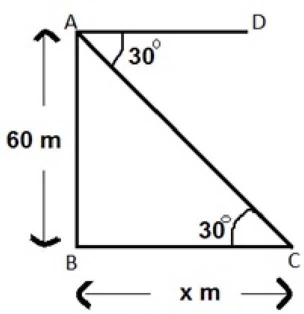


fig.7

Let the Distance of the object from the tower be x meters.

∴ BC = x m

Given –

height of tower = AB = 60 m

Angle of depression = ∠ DAC = 30°

∴ ∠ BCA = ∠ DAC = 30°

[∵ When two ∥ lines are intersected by a third line then the Alternate interior angles will be equal.]

Now, In Δ ABC

tan 30° = AB/BC = 60/x [∵ tan θ = perpendicular/base]

⇒ 1/√3 = 60/x

∴ x = 60√3 meters

Question: 7

In what ratio doe

Solution:

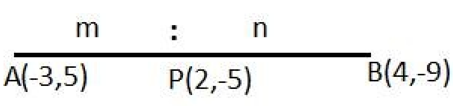


fig.8

Let the point P (2, – 5) divide the line segment joining A (– 3,5) and B(4, – 9) in the ratio m:n.

Let (x,y) ≡ (2, – 5)

$(x_1,y_1) \equiv (-3,5)$

and $(x_2,y_2) \equiv (4,-9)$

Using Section Formula,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\Rightarrow 2 = \frac{4m + (-3)n}{m + n}$$

⇒ 2 × (m + n) = 4m – 3n

⇒ 2m + 2n = 4m – 3n

$\Rightarrow 5n = 2m$

$\therefore m:n = 5:2$

Since the ratio is positive, Point P divides the line segment AB internally in the ratio 5:2.

Question: 8

Three solid spher

Solution:

Let the radius of the sphere so formed be r cm.

Given –

Radius of 1st sphere(r_1) = 6 cm

Radius of 2nd sphere(r_2) = 8 cm

Radius of 3rd sphere(r_3) = 10 cm

After Melting all these spheres, the volume will remain unchaged.

∴ Vol. of 1st sphere + Vol. of 2nd sphere + Vol. of 3rd sphere

= Vol. of new sphere so formed

$\Rightarrow (4/3)\pi(r_1)^3 + (4/3)\pi(r_2)^3 + (4/3)\pi(r_3)^3 = (4/3)\pi(r)^3$

Taking out $(4/3)\pi$ from both sides, we get –

$\Rightarrow (r_1)^3 + (r_2)^3 + (r_3)^3 = (r)^3$

$\Rightarrow (6)^3 + (8)^3 + (10)^3 = (r)^3$

$\Rightarrow 216 + 512 + 1000 = (r)^3$

$\Rightarrow (r)^3 = 1728$

$\therefore r = 12$ cm

Thus, the radius of new sphere is 12 cm.

Question: 9

Find the value of

Solution:

The given quadratic equation is $x^2 – 2px + 1 = 0$.

And, Discriminant D of the quadratic equation $ax^2 + bx + c = 0$ is given by –

$D = b^2 – 4ac$

Comparing the equation $ax^2 + bx + c = 0$ with given quadratic equation is $x^2 – 2px + 1 = 0$, we get –

a = 1, b = – 2p and, c = 1

$\therefore D = (– 2p)^2 – 4(1)(1) = 4p^2 – 4 = 4(p^2 – 1)$

For no real roots,

$D < 0$

$\Rightarrow 4(p^2 – 1) < 0$

$\Rightarrow p^2 – 1 < 0$

$\Rightarrow (p + 1)(p – 1) < 0$

$\therefore p \in (– 1,1)$

Thus, p can take any values between – 1 and 1 for no real roots of given quadratic equation.

Question: 10 A

Find the 10th ter

Solution:

The above series of numbers forms an arithmetic progression with

first term(a) = 4 and,

common difference(d) = (n + 1)th term – nth term = 9 – 4 = 5

last term or nth term(a_n) = 254

Let the total no. of terms in above A.P be n.

$\therefore a_n = a + (n - 1) \times d$

$\Rightarrow 254 = 4 + (n - 1) \times 5$

$\Rightarrow 250 = 5n - 5$

$\Rightarrow 5n = 255$

$\therefore n = 51$

∴ 10th term from the end of AP = 51 – 10 + 1 = 42th term from the beginning

∴ 42th term = $a_{42} = a + (42 - 1)d$

$= 4 + 41 \times 5$

$= 209$

Hence, 10th term from the end of AP is 209.

Question: 10 B

Which term of the

Solution:

Let the nth term of the AP be the first negative term.

In the given AP –

first term(a) = 24 and,

common difference(d) = (n + 1)th term – nth term = 21 – 24 = – 3

According to question –

$\therefore a_n < 0$

$\Rightarrow a + (n - 1) \times d < 0$

$\Rightarrow 24 + (n - 1) \times (-3) < 0$

$\Rightarrow -3n + 27 < 0$

$\Rightarrow 3n > 27$

$\therefore n > 9$

Thus, the first negative term of given AP is 10th term.

Question: 11

A circle is touch

Solution:

In the given figure,

AQ and AR are two tangents drawn from an exterior point A at contact points Q and R on the circle.

$\therefore$ AQ = AR

$\Rightarrow$ AQ = AC + CR.....(1)

Similarly,

BQ and BP are two tangents drawn from an exterior point B at contact points Q and P on the circle.

$\therefore$ BQ = BP.....(2)

And,

CR and CP are two tangents drawn from an exterior point C at contact points R and P on the circle.

$\therefore$ CR = CP.....(3)

Now, Equation (1) can be written as –

AQ = (AC + CR + AC + CR)/2

$\Rightarrow$ AQ = (AC + CP + AC + CR)/2 [using(3)]

$\Rightarrow$ AQ = (AC + CP + AR)/2

$\Rightarrow$ AQ = (AC + CP + AQ)/2

$\Rightarrow$ AQ = (AC + CP + AB + BQ)/2

$\Rightarrow$ AQ = (AC + CP + AB + BP)/2 [using(2)]

$\Rightarrow$ AQ = (AB + BC + AC)/2 [$\because$BP + CP=BC]

Thus, AQ = (1/2) × perimeter of Δ ABC

Question: 12

Two vertical of a

Solution:

Let the third vertex C ≡ (x_3,y_3)

In a Δ ABC,

Vertex A ≡ (x_1,y_1) ≡ (6,4)

Vertex B ≡ (x_2,y_2) ≡ (– 2,2)

Centroid(G) ≡ (x,y) ≡ (3,4)

Centroid of a Δ ABC is given by –

$x = (x_1 + x_2 + x_3)/3$

$\Rightarrow 3 = (6 – 2 + x_3)/3$

$\Rightarrow 9 = 4 + x_3$

$\therefore x_3 = 5$

And,

$y = (y_1 + y_2 + y_3)/3$

$\Rightarrow 4 = (4 + 2 + y_3)/3$

$\Rightarrow 12 = 6 + y_3$

$\therefore y_3 = 6$

Thus, the coordinates of third vertex C is (5,6).

Question: 13

A box contain 150

Solution:

Total no. of Oranges = 150

Probability of rotten oranges =0.06

∴ Probability of good oranges = 1 – 0.06 = 0.94

⇒ (no. of good oranges)/(no. of total oranges) = 0.94

⇒ no. of good oranges = 0.94 × 150 = 141

Thus, the number of good orange in the box = 141

Question: 14

A toy is in the f

Solution:

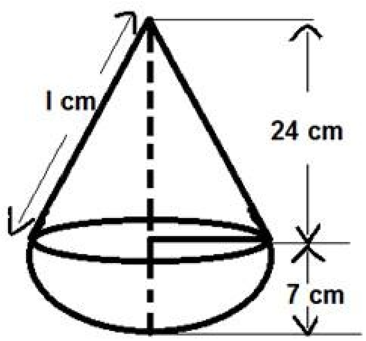


fig.10

Given –

Total Height of cone = 31 cm

Radius of hemisphere(r) = Base Radius of Cone

= Height of hemisphere

= 7 cm

∴ Height of cone(h) = 31 – 7 = 24 cm

Slant Height of Cone(l) = $\sqrt{(h^2 + r^2)} = \sqrt{(24^2 + 7^2)}$ = 25 cm

Now,

Total Surface Area of the Toy

= Curved Surface Area of Cone + Curved Surface Area of Hemisphere

$= \pi rl + 2\pi r^2$

$= \pi(rl + 2r^2)$

$= \pi(7 \times 25 + 2 \times (7)^2)$

$= \pi(175 + 98)$

$= \pi(273)$

$= 3.14 \times 273$

$= 857.22 \text{ cm}^2$

Question: 15

Solve: a^2

Solution:

The given quadratic equation is –

$a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$

Discriminant D of the quadratic equation $ax^2 + bx + c = 0$ is given by –

$D = b^2 - 4ac$

Comparing the equation $ax^2 + bx + c = 0$ with given quadratic equation is $a^2b^2x^2 - (4b^4 - 3a^4)x - 12a^2b^2 = 0$,we get –

$a = a^2b^2$, $b = -(4b^4 - 3a^4)$ and, $c = -12a^2b^2$

∴ The roots of the given quadratic equation is given by –

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{(4b^4 - 3a^4) \pm \sqrt{((4b^4 - 3a^4)^2 + 48a^4b^4)}}{2a^2b^2}$$

$$\Rightarrow x = \frac{(4b^4 - 3a^4) \pm \sqrt{(4b^4 + 3a^4)^2}}{2a^2b^2}$$

$$\Rightarrow x = \frac{(4b^4 - 3a^4) + (4b^4 + 3a^4)}{2a^2b^2}$$

$$\text{or } x = \frac{(4b^4 - 3a^4) - (4b^4 + 3a^4)}{2a^2b^2}$$

$$\Rightarrow x = \frac{8b^4}{2a^2b^2} \text{ or } x = \frac{-6a^4}{2a^2b^2}$$

$$\Rightarrow x = \frac{4b^2}{a^2} \text{ or } x = \frac{-3a^2}{b^2}$$

Thus, the roots of the given quadratic equation are $(4b^2/a^2)$ and $(-3a^2/b^2)$.

Question: 16 A

If the 8^{th}

Solution:

Let the first term and common difference of given AP be a and d respectively.

According to question –

8th term of AP = $a_8 = 31$ [Given]

$\Rightarrow a + (8 - 1)d = 31$

$\Rightarrow a + 7d = 31$.....(1)

15th term of AP = $a_{15} = 16 + a_{11}$

$\Rightarrow a + (15 - 1)d = 16 + a + (11 - 1)d$

$\Rightarrow 14d = 16 + 10d$

$\Rightarrow 4d = 16$

$\therefore d = 4$

Substituting the value of d in equation(1), we get –

$a = 31 - 7 \times 4 = 31 - 28 = 3$

Thus, the required AP is 3,7,11,15,.......

Question: 16 B

Find the sum of a

Solution:

All the two-digit odd positive numbers are –

11,13,15,17,……….,99

The above series of numbers forms an arithmetic progression with

first term(a) = 11 and,

common difference(d) = (n + 1)th term – nth term = 13 – 11 = 2

last term or nth term(a_n) = 99

Let the total no. of terms in above A.P be n.

$\therefore a_n = a + (n - 1) \times d$

$\Rightarrow 99 = 11 + (n - 1) \times 2$

$\Rightarrow 88 = 2n - 2$

$\Rightarrow 2n = 90$

$\therefore n = 45$

Sum of all the 45 terms of the AP is given by –

$S_{45} = (45/2)(11 + 99)$

[$\because S_n = (n/2)(a + l) = (n/2)[(2a + (n - 1)d]$

$= (45/2) \times 110$

$= 45 \times 55$

$= 2475$

Thus, the sum of all two-digit odd positive numbers = 2475.

Question: 17 A

In the adjoining

Solution:

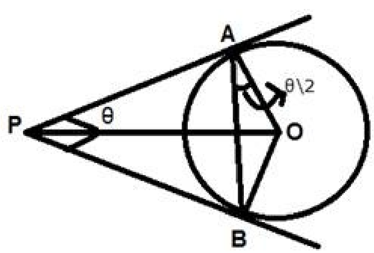


fig.11

Let ∠ APB = θ

In Δ APB,

PA = PB

[∵ <u>Tangents drawn from an exterior point to the circle are equal in length</u>]

⇒ Δ APB is an isoceles triangle.

∴ ∠ PAB = ∠ PBA = α [LET]

Now,

∠ APB + ∠ PAB + ∠ PBA = 180° [∵ sum of all the angles of Δ=180°]

$\Rightarrow \theta + \alpha + \alpha = 180°$

$\Rightarrow 2\alpha = 180° - \theta$

$\therefore \alpha = \angle PAB = 90° - (\theta/2)$

Also, $OA \perp AP$

[∵ radius of a circle is always ⊥ to the tangent at the point of contact.]

$\therefore \angle PAB + \angle OAB = 90°$

$\Rightarrow 90° - (\theta/2) + \angle OAB = 90°$

$\Rightarrow \angle OAB = (\theta/2) = (1/2)\angle APB$

$\therefore \angle APB = 2 \angle OAB$

Hence, Proved.

Question: 17 B

In the adjoining

Solution:

In the given figure,

DS and DR are the two tangents drawn from an external point D at the point of contacts S and R respectively. And,

$OS \perp DS$ and $OR \perp DR$

[∵ radius of a circle is always ⊥ to the tangent at the point of contact.]

⇒ OSDR is a square [∵ $AD \perp DC$ (Given)]

∴ DR = 10 cm

Similarly,

BP and BQ are the two tangents drawn from an external point B at the point of contacts A and Q respectively.

∴ BP = BQ = 27 cm

[∵ Tangents drawn from an exterior point to the circle are equal in length]

⇒ QC = BC – BQ = 38 – 27 = 11 cm

Also, CR and CQ are the two tangents drawn from an external point C at the point of contacts R and Q respectively.

∴ CR = CQ = 11 cm

[∵ Tangents drawn from an exterior point to the circle are equal in length]

∴ DC = x = DR + CR = 10 + 11 = 21 cm.

Thus, the value of x is 21 cm.

Question: 18

Draw a circle of

Solution:

Steps of Construction:

1. Draw a circle with centre O with radius OL and a point P outside it. Join PO and bisect it. Let M be the midpoint of PO.

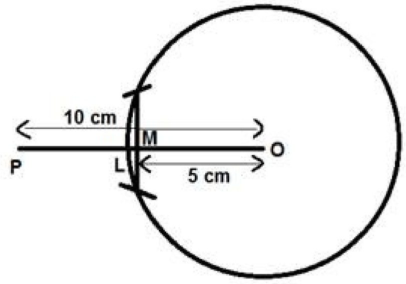


fig.13

2. Taking M as centre and MO as radius, we will draw a circle.

Let it intersect the given circle at the points Q and R.

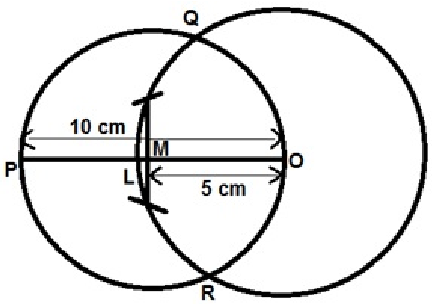


fig.14

3. Join PQ and PR.

Then PQ and PR are the required two tangents.

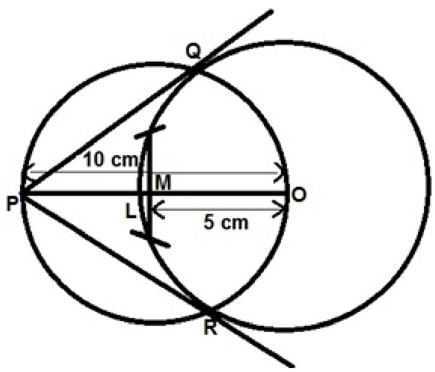


fig.15

4. Join OQ. Then ∠ PQO is an angle in the semicircle and,

∴ ∠ PQO = 90°

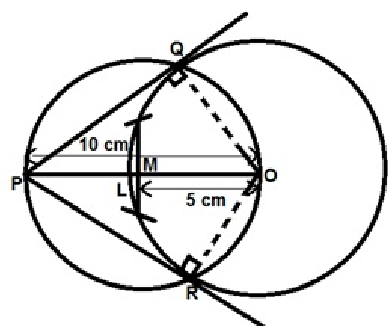


fig.16

Since, OQ is a radius of the given circle, PQ has to be a tangent to the circle.

Similarly,

PR is also a tangent to the circle.

After measuring the lenghts of tangents using scale, we find that both the tangents are equal in length which concludes that all the measurements and steps done correctly.

Length of Each Tangent = 8 cm

Question: 19

The three vertice

Solution:

Let the coordinates of the fourth vertex D be (x_4, y_4).

We know that –

Diagonals of a ∥gm bisect each other.

∴ Mid – point of diagonal AC ≡ Mid – point of diagonal BD

$$\Rightarrow \left(\frac{1+5}{2}, \frac{-2+10}{2}\right) \equiv \left(\frac{3+x_4}{2}, \frac{6+y_4}{2}\right)$$

$\Rightarrow 6 = 3 + x_4$ and $8 = 6 + y_4$

$\Rightarrow x_4 = 3$ and $y_4 = 2$

Thus, the coordinates of the fourth vertex D is (3,2).

Question: 20

Find the third ve

Solution:

Let the third vertex $A \equiv (x_1, y_1)$

In a Δ ABC,

Vertex $B \equiv (x_2, y_2) \equiv (-3, 1)$

Vertex $C \equiv (x_3, y_3) \equiv (0, -2)$

Centroid(G) $\equiv (x, y) \equiv (0, 0)$

Centroid of a Δ ABC is given by –

$x = (x_1 + x_2 + x_3)/3$

$\Rightarrow 0 = (x_1 - 3 + 0)/3$

$\Rightarrow 0 = x_1 - 3$

$\therefore x_1 = 3$

And,

$y = (y_1 + y_2 + y_3)/3$

$\Rightarrow 0 = (y_1 + 1 - 2)/3$

$\Rightarrow 0 = y_1 - 1$

$\therefore y_1 = 1$

Thus, the coordinates of third vertex A is (3,1).

Question: 21

Cards marked with

Solution:

Sample Space = Cards marked with 2-digit numbers

= {10,11,12,........,99)

No. of Sample Space = n(S) = 90

(a) Let P be the event of getting a card marked with 2-digit numbers which is divisible by 10.

∴ favourable elementary events = {10,20,30,…………,90}

no. of favourable elementary events = n(P) = 9

Thus, Probability of getting a card marked with number divisible by 10 = n(P)/n(S) = 9/90 = 1/10

(b) Let P be the event of getting a card marked with 2-digit square numbers.

∴ favourable elementary events = {16,25,36,…….,81}

no. of favourable elementary events = n(P) = 6

Thus, Probability of getting a card marked with number divisible by 10 = n(P)/n(S) = 6/90 = 1/15

(c) Let P be the event of getting a card marked with 2-digit prime numbers less than 25.

∴ favourable elementary events = {11,13,17,19,23}

no. of favourable elementary events = n(P) = 5

Thus, Probability of getting a card marked with number divisible by 10 = n(P)/n(S) = 5/90 = 1/18

Question: 22

A road which is 7

Solution:

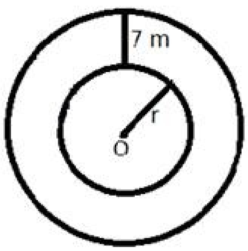


fig.17

Let the radius of the circular park be r meters.

Given –

Circumference of circular park = 352 m

⇒ 2 × π × r = 352

⇒ 2 × (22/7) × r = 352

⇒ r = (7/44) × 352

∴ r = 7 × 8 = 56 m

⇒ outer radius = 56 + 7 = 63 m

∴ Area of the road = $\pi(63^2 – 56^2)$

= (22/7)(63 + 56)(63 – 56)

= (22/7)(119)(7)

= 22 × 119

= 2618 m^2

Question: 23 A

A round table cov

Solution:

In the given figure, all the six desings covering equal area of the circle, therefore each design will subtend equal angles at the centre which is equal to $(360°/6)$ i.e. $60°$.

Also, the six triangles will be equal in area which is obtained by joining vertices of hexagon to the centre.

The triangle obtained will be equilateral because adjacent sides will be equal to the radius i.e. base angles will be equal and angle b/w them is $60°$ which concludes that other two angles will also be equal to $60°$ each.

∴ Area of six equilateral $\Delta = 6 \times (\sqrt{3}/4) \times (\text{radius})^2$

$= (3\sqrt{3}/2) \times (28)^2$

$= 1.5 \times 1.73 \times 784$

$= 2034.48\ \text{cm}^2$

Area of Circle $= \pi \times (\text{radius})^2 = (22/7) \times (28)^2 = (22/7) \times 784$

$= 22 \times 112$

$= 2464\ \text{cm}^2$

Area of the designs = Area of Circle – Area of six equilateral Δ

$= (2464 - 2034.48)\ \text{cm}^2$

$= 429.52\ \text{cm}^2$

∴ Cost of making designs =Rs. (0.50×429.52) = Rs. 214.76

Question: 23 B

In an equilateral

Solution:

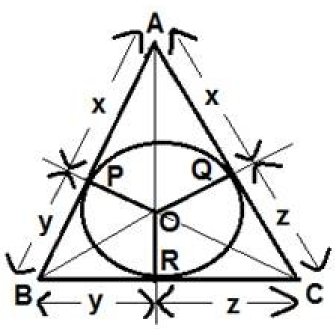


fig.19

Let the radius of the circle be r cm.

In the fig.19,

AR and AQ are making a pair of tangents drawn from vertex A of Δ ABC on the circle.

∴ AR = AQ = x [LET]

BR and BP are making a pair of tangents drawn from vertex B of Δ ABC on the circle.

∴ BR = BP = y [LET]

CP and CQ are making a pair of tangents drawn from vertex C of Δ ABC on the circle.

∴ CP = CQ = z [LET]

Given –

Δ ABC is an equilateral triangle.

∴ AB = BC = AC = 12 cm

⇒ AR + BR = BP + CP = AQ + CQ = 12

$\Rightarrow x + y = y + z = x + z = 12$.....(1)

Now,

$(x + y + y + z + x + z) = (12 + 12 + 12)$

$\Rightarrow 2 \times (x + y + z) = 36$

$\Rightarrow x + y + z = 18$.....(2)

Subtracting equation(1) from equation(2), we get –

$x = y = z = 6$ cm

Also, <u>the line joining the centre the circle to the vertices of Δ which circumscribes the circle bisects the angles of a Δ.</u>

$\therefore \angle OBP = 30°$

In Δ BOP,

$\tan \angle OBP = OP/BP = r/6$

$\Rightarrow \tan 30° = r/6$

$\Rightarrow 1/\sqrt{3} = r/6$

$\therefore r = 6/\sqrt{3} = 2\sqrt{3} = 3.46$ cm

Area of Δ ABC = $(\sqrt{3}/4) \times (\text{side})^2$

$=(1.73/4) \times (12)^2$

$= 1.73 \times 36$

$= 62.28$ cm^2

Area of circle = $\pi \times (\text{radius})^2$

$= 3.14 \times (3.46)^2$

$= 37.59$ cm^2

Thus, Area of the triangle which is not included in the circle

= Area of Δ ABC – Area of circle

$= (62.28 – 37.59)$ cm^2

$= 24.69$ cm^2

Question: 24

If a sphere has t

Solution:

Let the radius of the sphere be r cm.

Given –

Height of cone(h) = 40 cm

Radius of cone(r) = 30 cm

$\therefore$ Slant height of cone(l) = $\sqrt{(h^2 + r^2)} = \sqrt{(40^2 + 30^2)} = 50$ cm

According to question –

Surface Area of Sphere = Total Surface Area of Circular Cone

$\Rightarrow 4 \times \pi \times r^2 = \pi \times r \times (r + l)$

$\Rightarrow 4r = r + l$

$\Rightarrow 3r = l$

∴ r = (l/3) = (50/3) cm

Thus, the radius of the Sphere = (50/3) cm

Question: 25 A

A two–digit numbe

Solution:

Let the two-digit number be xy(i.e. 10x + y).

After reversing the digits of the number xy, the new number becomes yx (i.e. 10y + x).

According to question –

xy = 35.....(1)

And,

(10x + y) + 18 = (10y + x)

⇒ 9x – 9y = – 18

⇒ x – y = – 2.....(2)

From equation(2), we get –

x = y – 2.....(3)

Substitute the value of x in equation(1), we get –

y(y – 2) = 35

⇒ y^2 – 2y – 35 = 0

⇒ y^2 – 7y + 5y – 35 = 0

⇒ y(y – 7) + 5(y – 7) = 0

⇒ (y – 7)(y + 5) = 0

∴ y = 7 [∵ y = – 5 is invalid because digit of a number can't be – ve.]

Substituting the value of y in equation (3), we get –

x = 5

Thus, the required number is 57.

Question: 25 B

Two water taps to

Solution:

Let the tap of the smaller diameter and larger diameter fills the tank alone in x and (x – 10) hours respectively.

In 1 hour, the tap of the smaller diameter can fill 1/x part of the tank.

In 1 hour, the tap of the larger diameter can fill 1/(x – 10) part of the tank.

Two water taps together can fill a tank in $9\frac{3}{8}$ hours = 75/ 8 hours.

But in 1 hour the taps fill 8/75 part of the tank.

$$\frac{1}{x} + 1\frac{1}{x-10} = 8/75$$

$$\Rightarrow \frac{x-10+x}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow \frac{2(x-5)}{x^2-10x} = \frac{8}{75}$$

$\Rightarrow 4x^2 – 40x = 75x – 375$

$\Rightarrow 4x^2 – 115x + 375 = 0$

$\Rightarrow 4x^2 – 100x – 15x + 375 = 0$

$\Rightarrow 4x(x – 25) – 15(x – 25) = 0$

$\Rightarrow (4x -15)(x – 25) = 0$

$\Rightarrow x = 25, 15/4$

Taking $x = 15 / 4$

$\Rightarrow x – 10 = -25 /4$ (But, time cannot be negative)

Now, taking $x = 25$

$\Rightarrow x – 10 = 15$

Larger diameter of the tap can the tank 15 hours and smaller diameter of the tank can fill

the tank in 25 hours.

Question: 26

Prove that the an

Solution:

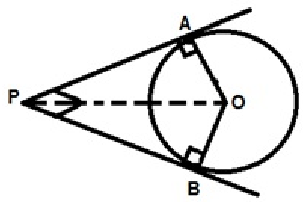


fig.20

In the fig.20, PA and PB are the two tangents drawn from an external point P at the point of contacts A and B on the circle with centre O respectively.

$\therefore$ OA $\perp$ PA and OB $\perp$ PB

[$\because$ radius of a circle is always $\perp$ to the tangent at the point of contact.]

$\therefore \angle OAP = \angle OBP = 90°$

we know that –

Sum of all the angles of a quadrilateral = 360°

In quadrilateral OAPB,

$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$

$\Rightarrow 180° + \angle APB + \angle AOB = 360°$

$\therefore \angle APB + \angle AOB = 180°$

Hence, the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact to the centre.

Question: 27

From the top of a

Solution:

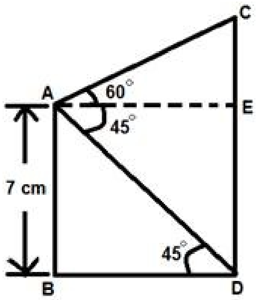


fig.21

Given –

Angle of Elevation = ∠ EAC = 60°

Angle of Depression = ∠ EAD = ∠ BDA = 45°

Height of Building = AB = ED = 7 m

In Δ ABD,

tan 45° = AB/BD

⇒ 1 = 7/BD

⇒ BD = 7 m

∴ AE = BD = 7 m [from fig.21]

And, In Δ ACE

tan ∠ CAE = CE/AE

⇒ tan 60° = CE/7

⇒ √3 = CE/7

⇒ CE = 7√3 m

Thus, Height of Tower = CE + ED = 7√3 + 7

= 7(1.73 + 1)

= 7 × 2.73

= 19.11 m

Question: 28

Puja works in a b

Solution:

Given –

Monthly Salary = Rs. 35000

∴ Annual Salary = Rs. (12 × 35000) = Rs. 420000

Annual Increment = Rs. 1500

Let us consider this situation as an AP with

first term = a = Rs. 420000

and, Common Difference = d = Rs. 1500

∴ Salary in 10th year is given by –

a_{10} = a + (10 – 1)d = 420000 + 9 × 1500 = Rs. 433500

Thus, Monthly Salary in 10th year = Rs. (433500/12)

= Rs. 36125

Question: 29

In the given figu

Solution:

Area of quadrant CAB = $(\pi/4) \times (\text{radius})^2$

$= (22/28) \times (7)^2$

$= 37.5\ cm^2$

Area of Δ EAB = (1/2) × base × height

= (1/2) × 7 × 2

$= 7\ cm^2$

Thus, Area of shaded Region

= Area of quadrant CAB – Area of Δ EAB

$= (37.5 – 7)\ cm^2$

$= 30.5\ cm^2$

Question: 30

The radii of the

Solution:

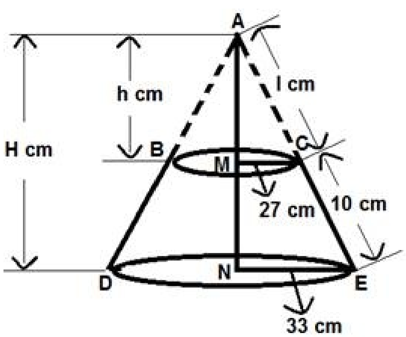


fig.23

Given –

MC = 27 cm, NE = 33 cm and CE = 10 cm

Let AM = h cm, AN = H cm and AC = l cm

∴ AE = AC + CE = (l + 10) cm

In the above fig.19,

Δ AMC and Δ ANE are similar triangles because their corresponding angles are equal.

$$\therefore \frac{AM}{AN} = \frac{AC}{AE} = \frac{MC}{NE}$$

$$\Rightarrow \frac{h}{H} = \frac{l}{l+10} = \frac{27}{33} \ldots..(1)$$

On cross multiplying last two fractional parts of equation(1), we get –

33l = 27l + 270

⇒ 6l = 270

∴ l = 45 cm

∴ AE = 45 + 10 = 55 cm

In Δ ANE,

$AN^2 + NE^2 = AE^2$ [by using pythagoras theorem]

$\Rightarrow H^2 + (33)^2 = (55)^2$

$\Rightarrow H^2 + 1089 = 3025$

$\Rightarrow H^2 = 1936$

$\therefore$ H = 44 cm

From first and last fractional parts of equation(1), we get –

h = (27/33) × 44 = 36 cm

$\therefore$ Height of frustum = H – h = 44 – 36 = 8 cm

Now,

Capacity of Frustum = Vol. of Cone ADE – Vol. of cone ABC

$= (1/3)\pi \times (NE)^2 \times (AN) – (1/3)\pi \times (MC)^2 \times (AM)$

$= (1/3)\pi \times [(33)^2 \times (44) – (27)^2 \times (36)]$

= (22/21) × [47916 – 26244]

= (22/21) × 21672

= 22 × 1032

$= 22704 \text{ cm}^3$

Total Surface Area of Frustum

= Area of Curved Part(Trapezium)

+ Area of Upper Circular Part

+ Area of lower Circular Part

= [(1/2) × (sum of parallel sides) × (height of frustum)]

$+ [\pi \times (MC)^2] + [\pi \times (NE)^2]$

$= [(1/2) \times 2\pi(27 + 33) \times 8] + [(22/7) \times (27)^2] + [(22/7) \times (33)^2]$

$= 480(22/7) + (22/7) \times [(27)^2 + (33)^2]$

= 480(22/7) + (22/7) × 1818

= (22/7) × 2298

= 22 × 328.28

$= 7222.16 \text{ cm}^2$

Thus, Capacity of Frustum $= 22704 \text{ cm}^3$

and, Total Surface Area of Frustum $= 7222.16 \text{ cm}^2$

Question: 31

From an external

Solution:

In the given fig.,

CA and CE are the two tangents drawn from an external point C at the point of contacts A and E respectively.

$\therefore$ CA = CE

[$\because$ Tangents drawn from an exterior point to the circle are equal in length]

Similarly, DE and DB are the two tangents drawn from an external point D at the point of contacts E and B respectively.

∴ DE = DB

[∵ Tangents drawn from an exterior point to the circle are equal in length]

Perimeter of Δ PCD = PC + CD + PD

= PC + CE + DE + PD

= PC + CA + BD + PD

= PA + PB [∵ PA = PC + CA and PB = PD + BD]

= 14 + 14 [∵ PA=PB]

= 28 cm

Question: 32

Construct a ΔABC

Solution:

Steps of Construction :

1. Draw a line Segment BC = 5.4 cm and draw an angle of 60°

at point B and mark a length of AB = 4.5 cm on the line passing through B. Then join AC.

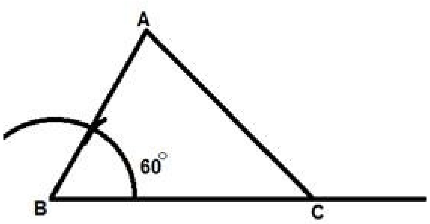


fig.25

2. Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

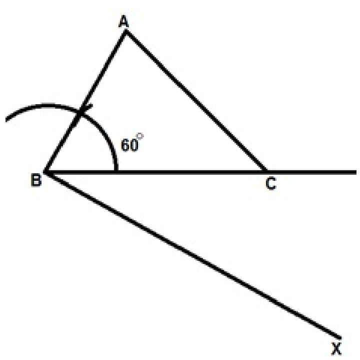


fig.26

3. Locate 4 points [the greater of 4 and 3 in (3/4)] B_1, B_2, B_3, B_4 on BX so that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

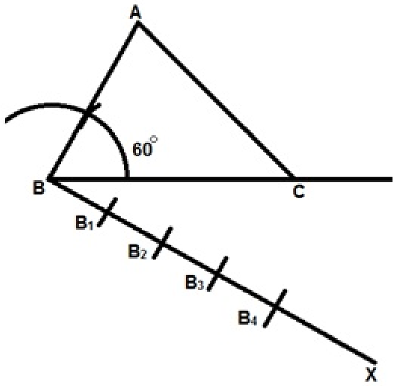


fig.27

4. Join B_3 [the 3rd point, 3 being smaller of 3 and 4 in (3/4)] to C and draw a line through B_4 parallel to B_3C, intersecting the extended line segment BC at C′.

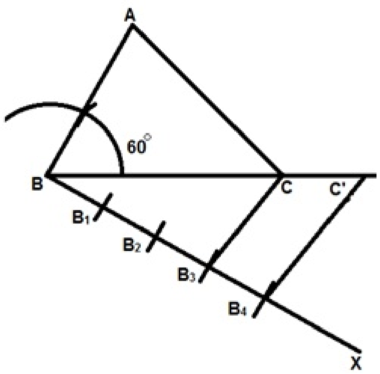


fig.28

5. Draw a line through C′ parallel to CA intersecting the extended line segment BA at A′.

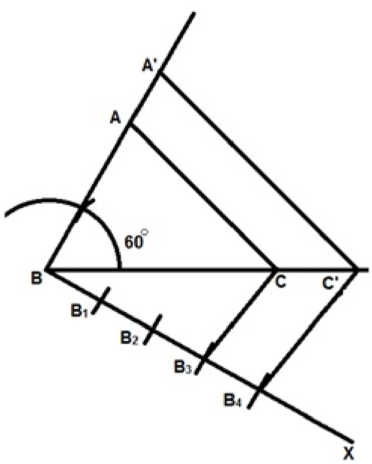


fig.29

Then A′BC′ is the required triangle.

Question: 33

A bag contain 5 r

Solution:

Let the no. of blue balls in the bag be x.

Let B and R be the event of drawing a blue and red ball respectively.

∴ total no. of balls in the bag = x + 5

According to question –

Probability of drawing blue ball

= 3 × Probability of drawing blue ball

⇒ [no. of blue balls/total no. of balls]

= 3 × [no. of red balls/total no. of balls]

⇒ (x/x + 5) =3 × (5/x + 5)

∴ x = 15

Thus, the no. of blue balls in the bag is 15.

Question: 34

In what ratio is

Solution:

Let the point on the y-axis which divides the line segment joining the points A(– 2, – 3) and B(3, 7) be C(0,y).

Let the ratio in which y-axis divides AB line segment be m:n.

Let (x,y) ≡ (0,y)

$(x_1,y_1) \equiv (-2, -3)$

and $(x_2,y_2) \equiv (3,7)$

Using Section Formula,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\Rightarrow 0 = \frac{3m + (-2)n}{m + n}$$

⇒ 3m = 2n

∴ m:n = 2:3

Now,

$$y = \frac{my_2 + ny_1}{m + n}$$

$$\Rightarrow y = \frac{2 \times (7) + 3 \times (-3)}{3 + 2}$$

⇒ y = (5/5) = 1

Thus, the line segment joining the points (– 2, – 3) and (3, 7) divided by the y-axis in the ratio 2:3 internally and the coordinates of the point of division is (0,1).

Exercise : SAMPLE PAPER II

Question: 1

The values of k f

Solution:

Any quadratic equation in the form $ax^2 + bx + c = 0$ has equal roots if and only if Discriminant, D = 0

Where, $D = b^2 - 4ac$

In the given equation,

a = 2

b = k

$c = 3$

Now, above equation will have equal roots if

$D = 0$

i.e.

$(k)^2 - 4(2)(3) = 0$

$\Rightarrow k^2 = 24$

$\Rightarrow k = \pm 2\sqrt{6}$

Question: 2

How many terms ar

Solution:

In the given AP,

First term, $a = 7$

Common difference, $a_2 - a_1 = (11 - 7) = 4$

Let the no of terms be n

Nth term, $a_n = 139$

We know that, For any AP

$a_n = a + (n - 1)d$

where,

a_n = nth term

d = common difference

n = no of terms

using the above formula for given AP, we have

$139 = 7 + (n - 1)(4)$

$\Rightarrow 4(n - 1) = 132$

$\Rightarrow n - 1 = 33$

$\Rightarrow n = 34$

Hence, there are 34 terms in given AP.

Question: 3

One card is drawn

Solution:

We know that,

Probability $= \frac{\text{no of Favourable outcomes}}{\text{no of total outcomes}}$

Now,

No of total outcomes i.e. total no of cards = 52

No of favourable outcomes i.e. no of black suits of 10 = 2

Probability (Getting a 10 of black suit) $= \frac{2}{52} = \frac{1}{26}$

Question: 4

In a circle of ra

Solution:

Given, A circle with center O and radius, OT = 7 cm and PT = 24 cm

Now, we know that

Tangent at a point on the circle is perpendicular to the radius through the point of contact.

i.e.

$OT \perp OP$

By Pythagoras Theorem in ΔOTP [i.e. $\text{Hypotenuse}^2 = \text{Base}^2 + \text{Height}^2$]

$(OP)^2 = (OT)^2 + (PT)^2$

$\Rightarrow (OP)^2 = (7)^2 + (24)^2$

$\Rightarrow (OP)^2 = 49 + 576 = 625$

$\Rightarrow OP = 25$ cm

Question: 5

The ratio in whic

Solution:

We know that any point on y axis is in the form (0, x) where x is any real number, let y axis intersect the line segment AB at point P with coordinates (0, c)

And we have

Coordinates of A = (– 3, 2)

Coordinates of B = (6, 1)

Let P divides AB in K:1

Now, By using section formula i.e.

The coordinates of Point P which divides line AB in a ration m : n is

$$= \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

Where, (x_1, y_1) and (x_2, y_2) are the coordinates of points A and B respectively.

So,

Coordinates of P= $\left(\frac{k(-3) + 1(6)}{k+1}, \frac{k(1) + 1(2)}{k+1}\right)$

$$(0, c) = \left(\frac{-3k + 6}{k + 1}, \frac{k + 2}{k + 1}\right)$$

$$\frac{-3k + 6}{k + 1} = 0 \Rightarrow -3k = -6 \Rightarrow k = 2$$

So,

P divides AB in 2:1

Question: 6

The distance of t

Solution:

Coordinates of Given Point (say P) = (6, – 6)

Coordinates of Origin (say O) = (0, 0)

By using distance formula i.e

Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Where, (x_1, y_1) and (x_2, y_2) are the coordinates of points A and B respectively.

So, we have

$$OP = \sqrt{(0-6)^2 + \left(0-(-6)\right)^2}$$

$\Rightarrow$ OP $=\sqrt{(36 + 36)}$

$\Rightarrow$ OP $=6\sqrt{2}$ units

Question: 7

A kite is flowing

Solution:

Consider, the situation in the form of a triangle ABC where A is the kite and AC shows the height of kite i.e.

AC = 75 cm

And

AB be the string with angle of inclination i.e.

$\angle CAB = \theta = 60°$

We have to find length of string i.e. AB

Clearly, ABC is a right – angled triangle

So, we have

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AC}{AB}$$

$$\Rightarrow \sin 60° = \frac{75}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{75}{AB}$$

On cross multiplying we get,

$$\Longrightarrow \sqrt{3} \times AB = 75 \times 2$$

$$\Rightarrow AB = \frac{150}{\sqrt{3}}$$

On rationalizing we get,

$$\Rightarrow AB = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{150}{3}\sqrt{3} = 50\sqrt{3}\ \text{cm}$$

Question: 8

A solid metal con

Solution:

For solid metal cone,

Height, h = 24 cm

Base radius, b = 12 cm

We know,

Volume of a solid cone $= \frac{1}{3}\pi r^2 h$

Where r is base radius and h is the height of cone.

Putting the values,

Volume of given cone $= \frac{1}{3}\pi(12)^2 24 = \pi \times 12 \times 12 \times 8 \text{ cm}^3$

For a solid spherical ball,

Diameter = 6 cm

Radius, r = 3 cm $\left[\text{As Radius} = \frac{\text{Diameter}}{2}\right]$

We know,

Volume of Solid sphere $= \frac{4}{3}\pi r^3$

Where, r is radius of sphere.

Putting the values, we have

Volume of ball $= \frac{4}{3} \times \pi \times (3)^3 = \pi \times 12 \times 3 \text{ cm}^3$

No of balls can be made $= \frac{\text{Total Volume of cone}}{\text{Volume of one spherical ball}} = \frac{\pi \times 12 \times 12 \times 8}{\pi \times 12 \times 3}$

On solving, we get

No of balls = 32

Question: 9

If the roots of t

Solution:

As the equation is in the form $Ax^2 + Bx + C = 0$ with non-zero A.

In which,

A = a - b

B = b - c

C = c - a

And we know that if the roots of a equation are equal then we have

Discriminant, D = 0

Where, $D = b^2 - 4ac$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (b - c)^2 - 4(a - b)(c - a) = 0$

$\Rightarrow (b - c)^2 + 4(a - b)(a - c) = 0$

$\Rightarrow b^2 + c^2 - 2bc + 4(a^2 - ac - ab + bc) = 0$

$\Rightarrow b^2 + c^2 - 2bc + 4a^2 - 4ac - 4ab + 4bc = 0$

$\Rightarrow 4a^2 + b^2 + c^2 - 4ac + 2bc - 4ac = 0$

$\Rightarrow (-2a)^2 + b^2 + c^2 + 2(-2a)c + 2bc + 2(-2a)c = 0$

$\Rightarrow (-2a + b + c)^2 = 0$

[using $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2za$]

⇒ $-2a + b + c = 0$

⇒ $b + c = 2a$

Hence Proved.

Question: 10 A

Find the 10^t

Solution:

First term, a = 4

Common difference, $a_2 - a_1 = 14 - (4) = 10$

Let the no of terms be n

We know, that nth term of an AP is

$a_n = a + (n - 1)d$

where a is first term and d is common difference.

$254 = 4 + (n - 1)10$

⇒ $250 = (n - 1)10$

⇒ $n - 1 = 25$

⇒ $n = 26$

10^{th} term from last will be 17^{th} term from starting

And $a_{10} = a + 16d$

$= 4 + 16(10)$

$= 164$

Question: 10 B

Or, which term of

Solution:

Given AP = 3, 15, 27, 39, ...

First term, a = 3

Common difference, $a_2 - a_1 = 15 - 3 = 12$

And we know

Nth term of an AP, $a_n = a + (n - 1)d$

Where a is first term and d is common difference.

Now, let the m^{th} term be 132 more than 54^{th} term

In that case,

$a_m = a_{54} + 132$

⇒ $a + (m - 1)d = a + 53d + 132$

⇒ $(m - 1)12 = 53(12) + 132$

⇒ $12m - 12 = 636 + 132$

⇒ $12m = 768 + 12$

⇒ $12m = 780$

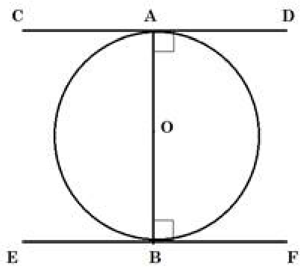
$\Rightarrow$ m = 65

hence, 65^{th} term will be 132 more than its 54^{th} term.

Question: 11

Prove that the ta

Solution:



Let AB be the diameter of a circle with center O.

CD and EF are two tangents at ends A and B respectively.

To Prove : CD || EF

Proof :

OA $\perp$ CD and OB $\perp$ EF [Tangents drawn at a point on circle is perpendicular to the radius through point of contact]

$\angle$OAD = $\angle$OBE = 90°

$\angle$OAD + $\angle$OBE = 90° + 90° = 180°

Considering AB as a transversal

$\Rightarrow$ CD || EF

[Two sides are parallel, if any pair of the interior angles on the same sides of transversal is supplementary]

Question: 12

From an external

Solution:

Given: From an external point P, two tangents, PA and PB are drawn to a circle with center O. At a point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. And PA = 14 cm

To Find: Perimeter of ΔPCD

As we know that, Tangents drawn from an external point to a circle are equal.

So we have

AC = CE ...[1] [Tangents from point C]

ED = DB ...[2] [Tangents from point D]

Now Perimeter of Triangle PCD

= PC + CD + DP

= PC + CE + ED + DP

= PC + AC + DB + DP [From 1 and 2]

= PA + PB

Now,

PA = PB = 14 cm as tangents drawn from an external point to a circle are equal

So we have

Perimeter = PA + PB = 14 + 14 = 28 cm

Question: 13

The area of the c

Solution:

Area of circular base = 616 cm^2

We know that,

Area of circle = πr^2

Where r is the radius of circle

Let the radius of circular base be r

We have,

$\pi r^2 = 616\ cm^2$

$\Rightarrow \frac{22}{7} \times r^2 = 616$

$\Rightarrow r^2 = 196$

$\Rightarrow r = 14$ cm

Now, Height = 48 cm [Given]

And we know,

Slant height, $l = \sqrt{(r^2 + h^2)}$

Where r is radius and h is the height of the cone

$l = \sqrt{(14^2 + 48^2)} = \sqrt{(196 + 2304)} = \sqrt{2500} = 50$ cm

Now,

Total surface area of a cone = $\pi r(l + r)$

Where r is radius and l is slant height.

So, Putting values we have

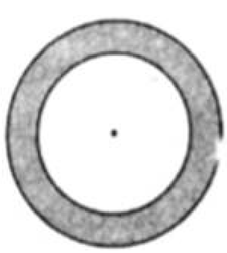
Total surface of cone = $\pi(14)(50 + 14)$

$= \frac{22}{7} \times 14 \times 64 = 2816\ cm^2$

Question: 14

In the adjoining

Solution:



Given,

Outer radius of circle, R = 21 cm

Area of enclosed region = 770 cm^2

Let the radius of inner circle be r.

Area of enclosed region = Area of outer circle – Area of inner circle

⇒ $770 = \pi R^2 - \pi r^2$

⇒ $\frac{770}{\pi} = R^2 - r^2$

⇒ $770 \times \frac{7}{22} = R^2 - r^2$

⇒ $35(7) = (21)^2 - r^2$

⇒ $r^2 = 441 - 245$

⇒ $r^2 = 196$

⇒ r = 14 cm

Question: 15

Solve for x: 12ab

Solution:

$12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$3ax(4bx - 3a) + 2b(4bx - 3a) = 0$

$(3ax + 2b)(4bx - 3a) = 0$

So, we have

$3ax + 2b = 0$ or $4bx - 3a = 0$

$x = -\frac{2b}{3a}$ or $x = \frac{3a}{4b}$

Question: 16

If the 8^{th}

Solution:

Let the a be first term and d be common difference

As we know

$a_n = a + (n - 1)d$

Given,

⇒ $a_8 = 31$

⇒ $a + 7d = 31$

⇒ $a = 31 - 7d$ ...[1]

Also, As 15^{th} term is 16 more than 11^{th} term

⇒ $a_{15} = a_{11} + 16$

⇒ $a + 14d = a + 10d + 16$

⇒ $4d = 16$

⇒ $d = 4$

Using this value in equation [1]

$a = 31 - 7(4) = 3$

So, AP is

a, a + d, a + 2d, ...

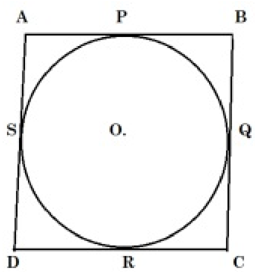
3, 3 + 4, 3 + 2(4), ...

3, 7, 11, ...

Question: 17 A

Prove that the pa

Solution:



Consider a circle circumscribed by a parallelogram ABCD, Let side AB, BC, CD and AD touch circles at P, Q, R and S respectively.

To Proof : ABCD is a rhombus.

As ABCD is a parallelogram

AB = CD and BC = AD [opposite sides of a parallelogram are equal] ...[1]

Now, As tangents drawn from an external point are equal.

We have

AP = AS [tangents from point A]

BP = BQ [tangents from point B]

CR = CQ [tangents from point C]

DR = DS [tangents from point D]

Add the above equations

AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ AB + CD = AS + DS + BQ + CQ

⇒ AB + CD = AD + BC

⇒ AB + AB = BC + BC [From 1]

⇒ AB = BC ...[2]

From [1] and [2]

AB = BC = CD = AD

And we know,

A parallelogram with all sides equal is a rhombus

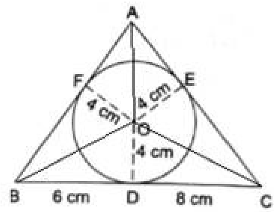
So, ABCD is a rhombus.

Hence Proved !

Question: 17 B

A ΔABC is drawn t

Solution:



Given: ΔABC that is drawn to circumscribe a circle with radius r = 4 cm and BD = 6 cm DC = 8 cm

To Find: AB and AC

Now,

As we know tangents drawn from an external point to a circle are equal.

Then,

FB = BD = 6 cm [Tangents from same external point B]

DC = EC = 8 cm [Tangents from same external point C]

AF = EA = x (let) [Tangents from same external point A]

Using the above data we get

AB = AF + FB = x + 6 cm

AC = AE + EC = x + 8 cm

BC = BD + DC = 6 + 8 = 14 cm

Now we have heron's formula for area of triangles if its three sides a, b and c are given

ar = √(s(s – a)(s – b)(s – c))

Where,

$$s = \frac{a + b + c}{2}$$

So for ΔABC

a = AB = x + 6

b = AC = x + 8

c = BC = 14 cm

$$s = \frac{x + 6 + x + 8 + 14}{2} = x + 14$$

And

ar(ΔABC) = √((x + 14)(x + 14 – (x + 6))(x + 14 – (x + 8))(x + 14 – 14))

= √((x + 14)(8)(6)(x)) [1]

ar(ABC) = ar(AOB) + ar(BOC) + ar(AOC)

at, tangent at a point on the circle is perpendicular to the radius through point of contact,

So, we have

OF ⊥ AB, OE ⊥ AC and OD ⊥ BC

Therefore, AOB, BOC and AOC are right – angled triangles.

And area of right angled triangle =1/2 × Base × Height

Using the formula,

$$ar(ABC) = \frac{1}{2} \times OF \times AB + \frac{1}{2} \times OD \times BC + 2 \times \frac{1}{2} \times OE \times AC$$

Using [1] we have,

$$\sqrt{(x + 14)(8)(6)(x)} = \frac{1}{2}(4)(x + 6) + \frac{1}{2}(4)(14) + \frac{1}{2}(4)(x + 8)$$

Squaring both side

$\Rightarrow 48x(x + 14) = (2x + 6 + 28 + 2x + 16)^2$

$\Rightarrow 48x^2 + 672x = (56 + 4x)^2$

$\Rightarrow 48x^2 + 672x = (4(14 + x))^2$

$\Rightarrow 48x^2 + 672x = 16(196 + x^2 + 28x)$

$\Rightarrow 3x^2 + 42x = 196 + x^2 + 28x$

$\Rightarrow 2x^2 + 14x - 196 = 0$

$\Rightarrow x^2 + 7x - 98= 0$

$\Rightarrow x^2 + 14x - 7x - 98 = 0$

$\Rightarrow x(x + 14) - 7(x + 14) = 0$

$\Rightarrow (x - 7)(x + 14) = 0$

$\Rightarrow x = 7$ or $x = -14$ cm

Negative value of x is not possible, as length can't be negative

Therefore,

$x = 7$ cm

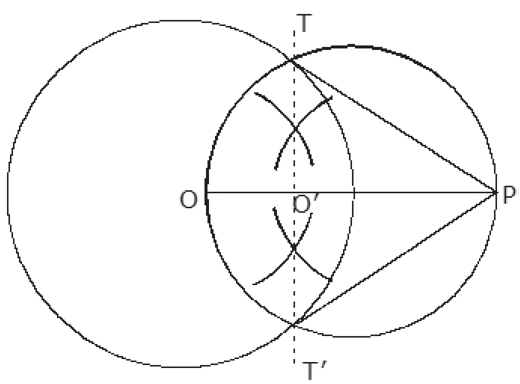
$\Rightarrow AB = x + 6 = 7 + 6 = 13$ cm

$\Rightarrow AC = x + 8 = 7 + 8 = 15$ cm

Question: 18

Draw a circle of

Solution:



Steps of Construction:

1. Take a point O and draw a circle of radius 6 cm [i.e. diameter 12 cm]

2. Mark a point P at a distance of 10 cm from O in any direction. Join OP

3. Draw right bisector of OP, intersecting OP at O'

4. Taking O' as center and O'O=O'P as radius, draw a circle to intersect the previous circle at T and T'.

5. Join PT and PT', which are required tangents.

6. Measured PT and PT' by a ruler and we get PT = PT' = 8 cm

Question: 19

Show that the poi

Solution:

For the points A, B and C to be vertices of an equilateral triangle,

AB = BC = CA and we have distance formula,

For two point P(x_1, y_1) and Q(x_2, y_2)

PQ= $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Using the above formula, and coordinates we have

AB=$\sqrt{(- a - a)^2 + (- a - a)^2}$

⇒ AB= $\sqrt{4a^2 + 4a^2} = \sqrt{8}\, a$

$$BC = \sqrt{\left(-a\sqrt{3} - (-a)\right)^2 + \left(a\sqrt{3} - (-a)\right)^2}$$

$$\Rightarrow BC = \sqrt{\left(-a\sqrt{3} + a\right)^2 + \left(a\sqrt{3} + a\right)^2}$$

$$\Rightarrow BC = \sqrt{a^2 + 3a^2 - 2\sqrt{3}a^2 + a^2 + 3a^2 + 2\sqrt{3}a^2} = \sqrt{8}a$$

$$AC = \sqrt{\left(-a\sqrt{3} - a\right)^2 + \left(a\sqrt{3} - (a)\right)^2}$$

$$\Rightarrow AC = \sqrt{\left(-a\sqrt{3} - a\right)^2 + \left(a\sqrt{3} - a\right)^2}$$

$$\Rightarrow AC = \sqrt{a^2 + 3a^2 + 2\sqrt{3}a^2 + a^2 + 3a^2 - 2\sqrt{3}a^2} = \sqrt{8}a$$

As AB = BC = AC

ABC is an equilateral triangle.

Question: 20

Find the area of

Solution:

As the diagonal of rhombus divides it into two parts, it is sufficient to calculate the area of one part and double it.

Consider, the Diagonal AC,

Then,

ar(ABCD) = 2× ar(ΔABC)

Now,

A = (3,0); B = (4, 5); C = (– 1, 4)

As we know area of triangle formed by three points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\Rightarrow ar(\Delta ABC) = \frac{1}{2}[3(5-4) + 4(4-3) + (-1)(0-5)]$$

$$= \frac{1}{2}[3 + 4 + 5]$$

=6 square units

⇒ ar(ABCD) = 2× ar(ΔABC) = 2(6) = 12 square units

Question: 21

Cards marked with

Solution:

Total no of numbers = 60 – 13 + 1 = 48

[As total no's from a to b are (b – a + 1)]

(a) No Divisible by 5 in the given sequence = {15, 20, 25, 30, 35, 40, 45, 50, 55, 60}

So, we have

No of favourable outcomes = 10

No of total outcomes = 48

And,

Probability of an event= $\frac{\text{No of favourable outcomes}}{\text{No of Total outcomes}}$

Therefore,

P(Getting a card having no divisible by 5)= $\frac{10}{48} = \frac{5}{24}$

(b) Perfect squares in the given sequence = {16, 25, 36, 49}

So, we have

No of favourable outcomes = 4

No of total outcomes = 48

And,

Probability of an event= $\frac{\text{No of favourable outcomes}}{\text{No of Total outcomes}}$

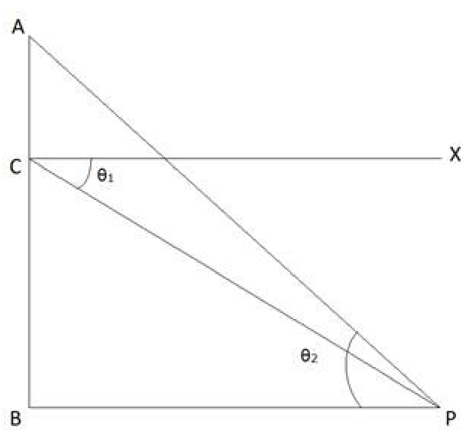
Therefore,

P(Getting a card having a perfect square)= $\frac{4}{48} = \frac{1}{12}$

Question: 22 A

A window in a bui

Solution:



Let us consider this situation by a diagram as shown, in which AB is a building and C depicts the window and A be the top.

Now Given,

Height of window from the ground, BC = 10 m

Angle of depression of point P from window, ∠XCP = 30°

⇒ ∠XCP = ∠CPB = θ_1 = 30° [Alternate Angles]

Angle of elevation of top of the building from point P, ∠APB = 60°

⇒ ∠ APB = θ_2 = 60°

Now, In Δ BCP

$$\tan\theta_1 = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{BP}$$

$$\Rightarrow\ \tan 30° = \frac{10}{BP}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{BP}$$

Cross – Multiplying we get,

BP=10√3 meters

Now, In ΔABP

$$\tan\theta_2 = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BP}$$

$$\Rightarrow \tan 60° = \frac{AB}{10\sqrt{3}}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{10\sqrt{3}}$$

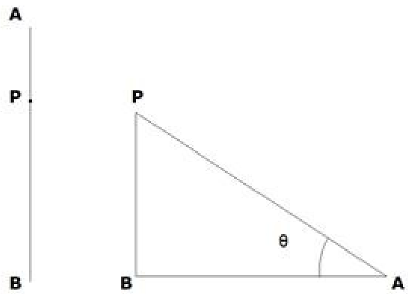
⇒ AB = 10√3 × √3 = 30 meters

So, Height of building is 30 meters.

Question: 22 B

In a violent stor

Solution:



Let AB be a tree, and P be the point of break,

And As tree falls, we can consider the situation as a right angled triangle at B

Given,

Angle of broken tree with ground, θ = 30°

Distance of top of broken tree from root, AB = 30 m

In ΔAPB

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BP}{AB}$$

$$\Rightarrow \tan 30° = \frac{BP}{30}$$

$\Rightarrow$ $BP = \dfrac{30}{\sqrt{3}} = 10\sqrt{3}$ meters

So, tree bents at a height of $10\sqrt{3}$ meters from the ground.

Also, In Δ APB

$\Rightarrow \sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BP}{AP}$

$\Rightarrow \sin 30° = \dfrac{10\sqrt{3}}{AP}$

$\Rightarrow \dfrac{1}{2} = \dfrac{10\sqrt{3}}{AP}$

On cross – multiplying

$AP = 10\sqrt{3} \times 2 = 20\sqrt{3}$ meters

Original height of tree = AP + BP

$= 20\sqrt{3} + 10\sqrt{3}$

$= 30\sqrt{3}$ meters

Question: 23

A wire bent in th

Solution:

Given,

Radius of circle made by wire, r = 42 cm

Circumference of circle of radius r = $2\pi r$

Circumference of circle made by wire $= 2 \times \frac{22}{7} \times 42 = 264\ \text{cm}^2$

As, the same wire is bent to make a square the perimeter of square will be equal to circumference of circle.

Let the side of square be a.

Perimeter of square of side 'a' = 4a

We have,

4a = 264

a = 66 cm

Now,

Ratio of areas$= \frac{\text{Area of circle}}{\text{Area of Square}}$

Area of circle of radius r = πr^2

Area of square of radius a = a^2

Putting value, we get

Ratio of areas$= \frac{\left(\frac{22}{7}\times 42\times 42\right)}{66\times 66} = \frac{22\times 6\times 42}{66\times 66} = \frac{14}{11}$

Required ratio is 14 : 11

Question: 24

A metallic sphere

Solution:

We know volume of sphere of radius R is $\frac{4}{3}\pi R^3$

And

Volume of cone of radius r and height h is $\frac{1}{3}\pi r^2 h$

So, Given,

Radius of sphere, R = 10.5 cm

Radius of cone, r = 3.5 cm

Height of cone, h = 3 cm

No of cones can be made by melting sphere $= \frac{\text{Volume of sphere}}{\text{Volume of one cone}}$

Using formulas, and putting values

No of cones $= \frac{\left(\frac{4}{3}\pi(10.5)^3\right)}{\frac{1}{3}\pi(3.5)^2(3)} = \frac{4\times10.5\times10.5\times10.5}{3.5\times3.5\times3} = 126$

Hence, 126 cones can be made.

Question: 25

In the given figu

Solution:

Let semicircle I, II and III are semicircles with diameters AB, AC and BC respectively

Area of shaded region =

Area of semicircle I + Area of semicircle II + Area of triangle ABC – Area of semicircle III

As, ∠BAC is in semicircle,

∠BAC = 90° [Angle in a semicircle is right angle]

And ABC is a right – angled triangle at A

By Pythagoras Theorem

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$(BC)^2 = (AB)^2 + (AC)^2$

$\Rightarrow (BC)^2 = 3^2 + 4^2 = 9 + 16 = 25$

$\Rightarrow$ BC = 5 cm

Now, For semicircle I

Diameter = AB = 3 cm

Radius, $r_1 = \frac{\text{Diameter}}{2} = 1.5 \text{ cm}$

Area of semicircle of radius r $= \frac{\pi r^2}{2}$

Area of semicircle I$= \frac{22}{7} \times \frac{(1.5)^2}{2} = \frac{11\times1.5\times1.5}{7} = \frac{99}{28} \text{ cm}^2$

For semicircle II

Diameter = AC = 4 cm

Radius, $r_2 = \frac{\text{Diameter}}{2} = 2 \text{ cm}$

Area of semicircle of radius r $= \frac{\pi r^2}{2}$

Area of semicircle II$= \frac{22}{7} \times \frac{(2)^2}{2} = \frac{11\times4}{7} = \frac{44}{7}$ cm^2

For semicircle III

Diameter = BC = 5 cm

Radius,$r_3 = \frac{\text{Diameter}}{2} = \frac{5}{2}$ cm

Area of semicircle of radius r $= \frac{\pi r^2}{2}$

Area of semicircle I $= \frac{22}{7} \times \frac{\left(\frac{5}{2}\right)^2}{2} = \frac{11\times5\times5}{7\times4} = \frac{275}{28}$ cm^2

Area of a right – angled triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$

Area of ΔABC$= \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 3 \times 4 = 6$ cm^2

Required area (From eqn [1])$= \frac{99}{28} + \frac{44}{7} + \frac{275}{28} + 6 = \frac{718}{28} = \frac{359}{14}$

Question: 26 A

₹ 250 is divided

Solution:

Let the no of children is x and amount given to each child is y

As the total amount is 250 ₹

We have,

xy = 250

$\Rightarrow y = \frac{250}{x}$ ...[1]

Also, given if no of children is increased by 25, the amount to each get less by 50 paise i.e. 0.5 ₹

So, we have

(x + 25)(y – 0.5) = 250

$\Rightarrow (x + 25)\left(\frac{250}{x} - \frac{1}{2}\right) = 250$ [By 1]

$$\Rightarrow \frac{(x + 25)(500 - x)}{2x} = 250$$

$\Rightarrow 500x - x^2 + 12500 - 25x = 500x$

$\Rightarrow x^2 + 25x - 12500 = 0$

$\Rightarrow x^2 + 125x - 100x - 12500 = 0$

$\Rightarrow x(x + 125) - 100(x + 125) = 0$

$\Rightarrow (x - 100)(x + 125) = 0$

so,

$\Rightarrow x - 100 = 0$ or $x + 125 = 0$

$\Rightarrow x = 100$ or $- 125$

However, no. of students can't be negative

Hence, x = 100

So, there were 100 students.

Question: 26 B

The hypotenuse of

Solution:

Let the shortest side be x cm [Let it be base]

Length of hypotenuse = 2x + 6 [in cm]

Length of other side = Length of hypotenuse – 2 = 2x + 6 – 2 = 2x + 4 [in cm] [Let it be perpendicular]

As we know, By Pythagoras Theorem

$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

$\Rightarrow (2x + 6)^2 = x^2 + (2x + 4)^2$

$\Rightarrow 4x^2 + 36 + 24x = x^2 + 4x^2 + 16 + 16x$

$[(a + b)^2 = a^2 + b^2 + 2ab]$

$\Rightarrow x^2 - 8x - 20 = 0$

$\Rightarrow x^2 - 10x + 2x - 20 = 0$

$\Rightarrow x(x - 10) + 2(x - 10) = 0$

$\Rightarrow (x + 2)(x - 10) = 0$

$\Rightarrow x = -2$ or $x = 10$ cm

However, Length can't be negative hence $x = -2$ is not possible

Therefore,

x = 10 cm

we have,

Shortest Side = x = 10 cm

Hypotenuse = 2x + 6 = 2(10) + 6 = 26 cm

Third side = 2x + 4 = 2(10) + 4 = 24 cm

Question: 27

If the sum of fir

Solution:

We know that sum of first n terms of an AP is

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Where a is first term and d is common difference

So,

Sum of first 2n terms

$$S_{2n} = \frac{2n}{2}(2a + (2n-1)d)$$

Sum of first 3n terms

$$S_{3n} = \frac{3n}{2}(2a + (3n-1)d)$$

Now, Taking RHS

$$3(S_2 - S_1) = 3(S_{2n} - S_n)$$

$$= 3\left[\frac{2n}{2}(2a + (2n-1)d) - \frac{n}{2}(2a + (n-1)d)\right]$$

$$= \frac{3n}{2}(4a + 4nd - 2d - 2a - nd + d)$$

$$= \frac{3n}{2}(2a + 3nd - d)$$

$$= \frac{3n}{2}(2a + (3n-1)d) = S_{3n} = S_3 = LHS$$

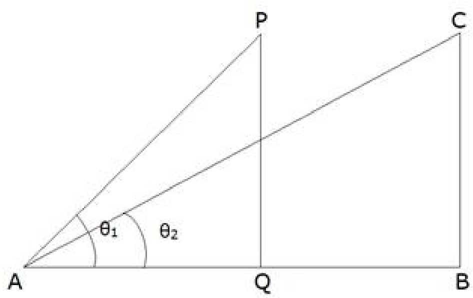
RHS = LHS

Hence Proved

Question: 28

The angle of elev

Solution:



Let the jet plane goes from point P to point C and we have given,

Initially angle of elevation from point A, $\angle PAQ = \theta_1 = 60°$

After 15 seconds,

Angle of elevation from point A, $\angle CAB = \theta_2 = 30°$

As the plane is flying at a constant height,

BC = PQ = $1500\sqrt{3}$ m

Now,

In ΔABC

$$\tan\theta_2 = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB}$$

$$\Rightarrow \tan 30° = \frac{1500\sqrt{3}}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AB} \Longrightarrow AB = 1500 \times 3 = 4500 \text{ m}$$

In ΔAPQ

$$\tan\theta_1 = \frac{\text{Perpendicular}}{\text{Base}} = \frac{PQ}{AQ}$$

$$\Rightarrow \tan 60° = \frac{1500\sqrt{3}}{AQ}$$

$$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{AQ} \Longrightarrow AQ = 1500 \text{ m}$$

So, we have

QB = AB – AQ = 4500 – 1500 = 3000 m

And

QB = PC

So, jet plane travels 3000 m in 15 seconds

And we know,

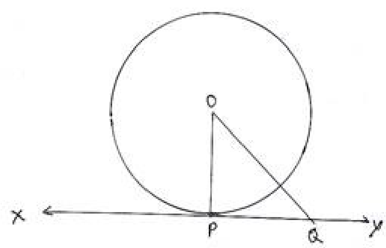
$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Speed of plane} = \frac{3000}{15} = 200\ \text{ms}^{-1}$$

Question: 29

Prove that the ta

Solution:



Given: A circle with center O and P be any point on a circle and XY is a tangent on circle passing through point P.

To prove : OP⊥XY

Proof :

Take a point Q on XY other than P and join OQ .

The point Q must lie outside the circle. (because if Q lies inside the circle, XY will become a secant and not a tangent to the circle).

Therefore, OQ is longer than the radius OP of the circle. That is, OQ > OP.

Since this happens for every point on the line XY except the point P, OP is the shortest of all the distances of the point O to the points of XY.

So OP is perpendicular to XY.

[As Out of all the line segments, drawn from a point to points of a line not passing through the point, the smallest is the perpendicular to the line.]

Question: 30

A quadrilateral A

Solution:

Given: A quadrilateral ABCD, And a circle is circumscribed by ABCD

Also, Sides AB, BC, CD and DA touch circle at P, Q, R and S respectively.

To Prove: AB + CD = AD + BC

Proof:

In the Figure,

As tangents drawn from an external point are equal.

We have

AP = AS [tangents from point A]

BP = BQ [tangents from point B]

CR = CQ [tangents from point C]

DR = DS [tangents from point D]

Add the above equations

⇒ AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ AB + CD = AS + DS + BQ + CQ

⇒ AB + CD = AD + BC

Hence Proved.

Question: 31 A

A solid is made u

Solution:

Total area to be painted = TSA of cube + CSA of hemisphere – Base area of hemisphere

[TSA = Total surface area & CSA = Curved surface area]

Given,

Diameter of hemisphere = 4.2 cm

Radius of hemisphere, r = 2.1 cm $\left[\text{Radius} = \frac{\text{Diameter}}{2}\right]$

Side of cube, a = 5 cm

And

TSA of cube = $6a^2$, where a is the side of cube

CSA of hemisphere = $3\pi r^2$, where r is the base radius

Base area = πr^2 [As base is circular]

Therefore,

Total area to be painted = $6a^2 + 3\pi r^2 - \pi r^2 = 6a^2 + 2\pi r^2$

$= 6(5)^2 + \left(2 \times \frac{22}{7} \times 2.1 \times 2.1\right)$

= 6(25) + (2 × 22 × 0.3 × 2.1)

= 177.72 cm^2

Question: 31 B

The diameter of t

Solution:

Given,

The diameter of lower end = 10 cmAs Radius = Diameter/2 Radius of lower end, r_2 = 5 cm

The diameter of upper end = 30 cmRadius of upper end, r_1 = 15 cm

Height of bucket, h = 24 cm

(i) As we know

volume of frustum of a cone = $\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

Where, h = height, r_1 and r_2 are radii of two ends ($r_1 > r_2$)

Capacity of bucket $= \frac{1}{3}\pi(24)(5^2 + 15^2 + 5(15))$

$= 3.14 \times 8 \times (25 + 225 + 75)$

$= 3.14 \times 8 \times 325 = 8164$ cm^3

(ii) Area of metal used to make bucket = CSA of frustum + base area

We know that,

Curved surface area of frustum = $\pi l(r_1 + r_2)$

Where, r_1 and r_2 are the radii of two ends ($r_1 > r_2$)

And l = slant height and

$l = \sqrt{(h^2 + (r_1 - r_2)^2)}$

So, we have

Slant height, $l = \sqrt{(24^2 + (15 - 5)^2)}$

$\Rightarrow l = \sqrt{(576 + 100)}$

$\Rightarrow l = \sqrt{676}$

$\Rightarrow l = 26$ cm

And as the base has lower end,

Base area = πr_2^2, where r_2 is the radius of lower end

Therefore,

Area of metal sheet used = $\pi l(r_1 + r_2) + \pi r_2^2$

$= \pi(26)[15 + 5] + \pi(5)^2$

$= 520\pi + 25\pi$

$= 545\pi = 545(3.14) = 1711.3$ cm^2

Question: 32

Find the value of

Solution:

Three points A, B and C are collinear if and only if

Area(ΔABC) = 0

As we know area of triangle formed by three points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$ar(\Delta ABC) = \frac{1}{2}[-1(k - (-1)) + 2(-1 - 3) + 5(3 - k)]$

$\Rightarrow 0 = \frac{1}{2}[-k - 1 - 8 + 15 - 5k]$

$\Rightarrow 0 = -6k + 6$

$\Rightarrow 6k = 6$

$\Rightarrow k = 1$

So, For k = 1, A, B and C are collinear.

Question: 33

Two dice are thro

Solution:

When two dice are thrown, the possible outcomes are

{(1,1) (1,2) (1,3) (1,4) (1,5) (1,6), (2,1) (2,2) (2,3) (2,4) (2,5) (2,6), (3,1) (3,2) (3,3) (3,4) (3,5) (3,6), (4,1) (4,2) (4,3) (4,4) (4,5) (4,6), (5,1) (5,2) (5,3) (5,4) (5,5) (5,6), (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

The outcomes in which sum of no's is 9 are ={(3,6) (4,5) (5,4) (6,3)}

No of Total possible outcomes = 36

No of favourable outcomes = 4

And, Probability of an event = $\frac{\text{No of favourable outcomes}}{\text{No of Total outcomes}}$

Therefore,

P(Getting sum 9)= $\frac{4}{36} = \frac{1}{9}$

Question: 34

A circus tent is

Solution:

Total area of canvas required = CSA of cylindrical part + CSA of conical part [CSA = Curved surface area]

Now,

Radius of cone = Radius of cylinder = r = 52.5 m

Height of cylindrical part, h = 3 cm = 0.03 m [As 1 m = 100 cm]

Lateral height of conical part, l = 53 m

Now, we know

CSA of cylinder = 2πrh

Where, r is base radius and h is height of cylinder and

CSA of cone = πrl

Where, r is base radius and l is slant height.

Area of canvas required = 2πrh + πrl

= πr(2h + l)

$$= \frac{22}{7} \times 52.5 \times [2(0.03) + 53] = 22 \times 7.5 \times 53.06$$

= 8754.9 m^2