Chapter : 5. COMPLEX NUMBERS AND QUADRATIC EQUATIONS

Exercise : 5A

Question: 1

Evaluate:

Solution:

We all know that i = √(-1) .

and i^{4n}= 1

i^{4n+1}= i (where n is any positive integer)

$i^{4n+2}=-1$

i^{4n+3}= -1

So,

(i) L.H.S = i^{19}

= $i^{4\times4+3}$

= i^{4n+3}

Since it is of the form i^{4n+3} so the solution would be simply $-i$

Hence the value of i^{19} is $-$ i.

(ii) L.H.S = i^{62}

$\Rightarrow i^{4\times15+2}$

$\Rightarrow i^{4n+2} \Rightarrow i^2=-1$

so it is of the form i^{4n+2} so its solution would be -1

(iii) L.H.S. = i^{373}

$\Rightarrow i^{4\times 93+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow$i

So, it is of the form of i^{4n+1} so the solution would be i.

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Hence the value of i^{19} is $-i$.

(ii) L.H.S = i^{62}

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$\Rightarrow i^{4n+2} \Rightarrow i^2 = -1$

so it is of the form i^{4n+2} so its solution would be -1

(iii) L.H.S. = i^{373}

$\Rightarrow i^{4\times 93+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i$

So, it is of the form of i^{4n+1} so the solution would be i.

Question: 2

Evaluate:

Solution:

Since $i = \sqrt{-1}$ so

(i) L.H.S. $= \left(\sqrt{-1}\right)^{192}$

$\Rightarrow i^{192}$

$\Rightarrow i^{4\times48} = 1$

Since it is of the form $i^{4n} = 1$ so the solution would be 1

(ii) L.H.S. $= \left(\sqrt{-1}\right)^{93}$

$\Rightarrow i^{4\times23+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i^1 = i$

Since it is of the form of $i^{4n+1} = i$ so the solution would be simply i.

(iii) L.H.S = $\left(\sqrt{-1}\right)^{30}$

$\Rightarrow i^{4\times7+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow i^2 = -1$

Since it is of the form i^{4n+2} so the solution would be -1

Question: 2

Evaluate:

Solution:

Since $i = \sqrt{-1}$ so

(i) L.H.S. $= \left(\sqrt{-1}\right)^{192}$

$\Rightarrow i^{192}$

$\Rightarrow i^{4\times 48} = 1$

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$\Rightarrow i^{4\times 7+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow i^{2} = -1$

Since it is of the form i^{4n+2} so the solution would be -1

Question: 3

Evaluate:

Solution:

(i) $\text{L.H.S.} = i^{-50}$

$\Rightarrow i^{-4\times 13+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow -1$

Since it is of the form i^{4n+2} so the solution would be -1

(ii) $\text{L.H.S.} = i^{-9}$

$\Rightarrow i^{-4\times 3+3}$

$\Rightarrow i^{4n+3}$

$\Rightarrow i^{3} = -i$

Since it is of the form of i^{4n+3} so the solution would be simply -i.

(iii) $\text{L.H.S.} = i^{-131}$

$\Rightarrow i^{-4\times 33+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i^1 = i$

Since it is of the form i^{4n+1}. so the solution would be i

Question: 3

Evaluate:

Solution:

(i) $\mathrm{L.H.S.} = i^{-50}$

$\Rightarrow i^{-4\times13+2}$

$\Rightarrow i^{4n+2}$

$\Rightarrow -1$

Since it is of the form i^{4n+2} so the solution would be -1

(ii) $\mathrm{L.H.S.} = i^{-9}$

$\Rightarrow i^{-4\times3+3}$

$\Rightarrow i^{4n+3}$

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Since it is of the form of i^{4n+3} so the solution would be simply -i.

(iii) $\mathrm{L.H.S.} = i^{-131}$

$\Rightarrow i^{-4\times33+1}$

$\Rightarrow i^{4n+1}$

$\Rightarrow i^1 = i$

Since it is of the form i^{4n+1}. so the solution would be i

Question: 4

Evaluate:

Solution:

(i) $\left(i^{41} + \frac{1}{i^{71}}\right) = i^{41} + i^{-71}$

$\Rightarrow i^{4\times10+1} + i^{-4\times18+1}$ (Since $i^{4n+1} = i$)

$\Rightarrow i^1 + i^1$

$\Rightarrow 2i$

Hence, $\left(i^{41} + \frac{1}{i^{71}}\right) = 2i$

(ii) $\left(i^{53} + \frac{1}{i^{53}}\right)$

$\Rightarrow i^{53} + i^{-53}$

$\Rightarrow i^{4\times13+1} + i^{-4\times14+3}$ (since $i^{4n+1} = i$

$\Rightarrow\ i^{1}+i^{3}\ i^{4n+3}=$ -1)

$\Rightarrow$ 0

Hence, $\left(i^{53}+\dfrac{1}{i^{53}}\right)$ = 0

Question: 4

Evaluate:

Solution:

(i) $\left(i^{41}+\dfrac{1}{i^{71}}\right)=i^{41}+i^{-71}$

$\Rightarrow\ i^{4\times10+1}+i^{-4\times18+1}$ (Since $i^{4n+1}=i$)

$\Rightarrow\ i^{1}+i^{1}$

$\Rightarrow$ 2i

Hence, $\left(i^{41}+\dfrac{1}{i^{71}}\right)$ = 2i

(ii) $\left(i^{53}+\dfrac{1}{i^{53}}\right)$

$\Rightarrow\ i^{53}+i^{-53}$

$\Rightarrow\ i^{4\times13+1}+i^{-4\times14+3}$ (since $i^{4n+1}=i$

$\Rightarrow\ i^{1}+i^{3}\ i^{4n+3}=$ -1)

$\Rightarrow$ 0

Hence, $\left(i^{53}+\dfrac{1}{i^{53}}\right)$ = 0

Question: 5

Prove that 1 + i<

Solution:

L.H.S.= $1 + i^2 + i^4 + i^6$

ToProve: $1 + i^2 + i^4 + i^6 = 0$

$\Rightarrow$ 1 + (-1) +1 + i^2

Since, $i^{4n} = 1$

(where n is any positive integer)

$\Rightarrow i^{4n+2}$

$\Rightarrow i^{2}=-1$

$\Rightarrow$1 + -1 + 1 + -1=0

⇒L.H.S = R.H.S

Hence proved.

Question: 5

Prove that 1 + i<

Solution:

L.H.S.= $1 + i^2 + i^4 + i^6$

ToProve: $1 + i^2 + i^4 + i^6 = 0$

$\Rightarrow 1 + (-1) + 1 + i^2$

Since, $i^{4n} = 1$

(where n is any positive integer)

$\Rightarrow i^{4n+2}$

$\Rightarrow i^2 = -1$

$\Rightarrow 1 + -1 + 1 + -1 = 0$

$\Rightarrow$L.H.S = R.H.S

Hence proved.

Question: 6

Prove that 6i

Solution:

Given: $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48}$

To prove: $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} = 7i$

$\Rightarrow 6i^{4\times12+2} + 5i^{4\times8+1} - 2i^{4\times3+3} + 6i^{4\times12}$

$\Rightarrow 6i^2 + 5i^1 - 2i^3 + 6i^0$

$\Rightarrow -6+5i+2i+6$

$\Rightarrow 7i$

$\Rightarrow \text{L.H.S} = \text{R.H.S}$

Hence proved.

Question: 6

Prove that 6i

Solution:

Given: $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48}$

To prove: $6i^{50} + 5i^{33} - 2i^{15} + 6i^{48} = 7i$

$\Rightarrow 6i^{4\times12+2} + 5i^{4\times8+1} - 2i^{4\times3+3} + 6i^{4\times12}$

$\Rightarrow 6i^2 + 5i^1 - 2i^3 + 6i^0$

$\Rightarrow -6+5i+2i+6$

$\Rightarrow 7i$

$\Rightarrow \text{L.H.S} = \text{R.H.S}$

Hence proved.

Question: 7

Prove that

Solution:

Given: $\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}$

To prove : $\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}=0.$

$\Rightarrow$ L.H.S.= $i^{-1} - i^{-2} + i^{-3} - i^{-4}$

$\Rightarrow$ $i^{-4\times1+3} - i^{-4\times1+2} + i^{-4\times1+3} - i^{-4\times1}$

since $i^{4n} = 1$

$\Rightarrow$ $i^{4n+1} = i$

$\Rightarrow$ $i^{4n+2} = -1$

$\Rightarrow$ $i^{4n+3} = -1$

So,

$\Rightarrow$ $i^1 - i^2 + i^3 - 1$

$\Rightarrow$ i +1 – i -1

$\Rightarrow$ 0

⇒L.H.S = R.H.S

Hence Proved

Question: 7

Prove that

Solution:

Given: $\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}$

To prove : $\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}=0.$

$\Rightarrow$ L.H.S.= $i^{-1} - i^{-2} + i^{-3} - i^{-4}$

$\Rightarrow$ $i^{-4\times1+3} - i^{-4\times1+2} + i^{-4\times1+3} - i^{-4\times1}$

since $i^{4n} = 1$

$\Rightarrow$ $i^{4n+1} = i$

$\Rightarrow$ $i^{4n+2} = -1$

$\Rightarrow$ $i^{4n+3} = -1$

So,

$\Rightarrow$ $i^1 - i^2 + i^3 - 1$

$\Rightarrow$ i +1 – i -1

$\Rightarrow$ 0

⇒L.H.S = R.H.S

Hence Proved

Question: 8

Prove that (1 + i

Solution:

L.H.S = $(1 + i^{10} + i^{20} + i^{30})$

= $(1 + i^{4\times2+2} + i^{4\times5} + i^{4\times7+2})$

since $\Rightarrow i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

= $1 + i^2 + 1 + i^2$

= 1 + -1 +1 + -1

= 0, which is a real no.

Hence, $(1 + i^{10} + i^{20} + i^{30})$ is a real number.

Question: 8

Prove that (1 + i

Solution:

L.H.S = $(1 + i^{10} + i^{20} + i^{30})$

= $(1 + i^{4\times2+2} + i^{4\times5} + i^{4\times7+2})$

since $\Rightarrow i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

= $1 + i^2 + 1 + i^2$

= 1 + -1 +1 + -1

= 0, which is a real no.

Hence, $(1 + i^{10} + i^{20} + i^{30})$ is a real number.

Question: 9

Prove that

Solution:

$$\text{L.H.S.} = \left\{ i^{21} - \left(\frac{1}{i}\right)^{46} \right\}^2$$

$$= \left\{ i^{4\times5+1} - i^{-4\times12+2} \right\}^2$$

since $i^{4n} = 1$

$i^{4n+1} = i$

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3 = -1$

$= \left\{i^1 - i^2\right\}^2$

$= \left\{i + 1\right\}^2$

Now, applying the formula $(a+b)^2 = a^2 + b^2 + 2ab$

$= i^2 + 1 + 2i \cdot$

= -1 + 1 + 2i

= 2i

L.H.S = R.H.S

Hence proved.

Question: 9

Prove that

Solution:

L.H.S.= $\left\{i^{21} - \left(\frac{1}{i}\right)^{46}\right\}^2$

$= \left\{i^{4\times5+1} - i^{-4\times12+2}\right\}^2$

since $i^{4n} = 1$

$i^{4n+1} = i$

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3 = -1$

$= \left\{i^1 - i^2\right\}^2$

$= \left\{i + 1\right\}^2$

Now, applying the formula $(a+b)^2 = a^2 + b^2 + 2ab$

$= i^2 + 1 + 2i \cdot$

= -1 + 1 + 2i

= 2i

L.H.S = R.H.S

Hence proved.

Question: 10

L.H.S = $\left\{i^{18} + \frac{1}{i^{25}}\right\}^3$

$\Rightarrow \left\{i^{4\times4+2} + i^{-4\times7+3}\right\}^3$

since $i^{4n} = 1$

$i^{4n+1} = i$

$i^{4n+2} = -1$

$i^{4n+3} = -1$

$= \left\{i^2 + i^3\right\}^3.$

$= (-1-i)^3.$

Applying the formula $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We have,

$+ 3i^2 + 3i + 1)$

i + 3 - 3i - 1

= 2(1-i)

L.H.S = R.H.S

Hence proved .

Question: 10

L.H.S = $\left\{i^{18} + \frac{1}{i^{25}}\right\}^3$

$\Rightarrow \left\{i^{4\times4+2} + i^{-4\times7+3}\right\}^3$

since $i^{4n} = 1$

$i^{4n+1} = i$

$i^{4n+2} = -1$

$i^{4n+3} = -1$

$= \left\{i^2 + i^3\right\}^3.$

$= (-1-i)^3.$

Applying the formula $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

We have,

$+ 3i^2 + 3i + 1)$

i + 3 - 3i - 1

= 2(1-i)

L.H.S = R.H.S

Hence proved .

Question: 11

Prove that (1 – i

Solution:

L.H.S = $(1 - i)^n \left(1 - \frac{1}{i}\right)^n$

= $(1-i)^n \left(1-i^{-4*1+3}\right)^n$

= $(1-i)^n \left(1-i^3\right)^n$

Since, i^{4n+3}. = -1

= $(1-i)^n (1+i)^n$.

Applying $a^n b^n = (ab)^n$

= $((1-i)(1+i))^n$.

= $\left(1-i^2\right)^n$

=2^n

L.H.S = R.H.S

Hce proved.

Question: 11

Prove that (1 – i

Solution:

L.H.S = $(1 - i)^n \left(1 - \frac{1}{i}\right)^n$

= $(1-i)^n \left(1-i^{-4*1+3}\right)^n$

= $(1-i)^n \left(1-i^3\right)^n$

Since, i^{4n+3}. = -1

= $(1-i)^n (1+i)^n$.

Applying $a^n b^n = (ab)^n$

= $((1-i)(1+i))^n$.

$= \left(1-i^2\right)^n$

$= 2^n$

L.H.S = R.H.S

Hce proved.

Question: 12

Prove tha

Solution:

L.H.S = $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$

Since we know that i = $\sqrt{-1}$.

So,

$= \sqrt{16}$ i + 3$\sqrt{25}$ i+ $\sqrt{36}$ i $-\sqrt{625}$ i

=4i + 15i + 6i - 25i

= 0

L.H.S = R.H.S

Hence proved.

Question: 12

Prove tha

Solution:

L.H.S = $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$

Since we know that i = $\sqrt{-1}$.

So,

$= \sqrt{16}$ i + 3$\sqrt{25}$ i+ $\sqrt{36}$ i $-\sqrt{625}$ i

=4i + 15i + 6i - 25i

= 0

L.H.S = R.H.S

Hence proved.

Question: 13

Prove that (1 + i

Solution:

L.H.S = (1 + i^2 + i^4 + i^6 + i^8 + ... + i^{20})

$$= \sum_{n=0}^{n=20} i^n$$

= 1 + -1 +1 + -1 + ... + 1

As there are 11 times 1 and 6 times it is with positive sign as i^0 =1 as this is the extra term and there are 5 times 1 with negative sign.

So, these 5 cancel out the positive one leaving one positive value i.e. 1

$= \sum_{n=0}^{20} i^n = 1$

L.H.S = R.H.S

Hence proved.

Question: 13

Prove that (1 + i

Solution:

L.H.S = $(1 + i^2 + i^4 + i^6 + i^8 + ... + i^{20})$

$= \sum_{n=0}^{n=20} i^n$

= 1 + -1 +1 + -1 + ... + 1

As there are 11 times 1 and 6 times it is with positive sign as i^0 =1 as this is the extra term and there are 5 times 1 with negative sign.

So, these 5 cancel out the positive one leaving one positive value i.e. 1

$= \sum_{n=0}^{20} i^n = 1$

L.H.S = R.H.S

Hence proved.

Question: 14

Prove that i

Solution:

L.H.S = $i^{53} + i^{72} + i^{93} + i^{102}$

$= i^{4\times13+1} + i^{4\times18} + i^{4\times23+1} + i^{4\times25+2}$

Since $i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$ (where n is any positive integer)

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

$= i + 1 + i + i^2$

= i+1+i-1

=2i

L.H.S = R.H.S

Hence proved.

Question: 14

Prove that i

Solution:

L.H.S = $i^{53} + i^{72} + i^{93} + i^{102}$

$= i^{4\times13+1} + i^{4\times18} + i^{4\times23+1} + i^{4\times25+2}$

Since $i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$ (where n is any positive integer)

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

$= i + 1 + i + i^2$

= i+1+i-1

=2i

L.H.S = R.H.S

Hence proved.

Question: 15

Prove that

Solution:

$$\text{L.H.S} = \sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$$

$= i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + \ldots + i^{13} + i^{14}$

since $i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

= i – 1 – i + 1 + i - 1......+i-1

As, all terms will get cancel out consecutively except the first two terms. so that will get remained will be the answer.

= i - 1

L.H.S = R.H.S

Hence proved.

Question: 15

Prove that

Solution:

$$\text{L.H.S} = \sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$$

$= i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + \ldots + i^{13} + i^{14}$

since $i^{4n} = 1$

$\Rightarrow i^{4n+1} = i$

$\Rightarrow i^{4n+2} = -1$

$\Rightarrow i^{4n+3} = -1$

= i – 1 – i + 1 + i - 1……+i-1

As, all terms will get cancel out consecutively except the first two terms. so that will get remained will be the answer.

= i - 1

L.H.S = R.H.S

Hence proved.

Exercise : 5B

Question: 1 A

Simplify each of

Solution:

Given: 2(3 + 4i) + i(5 – 6i)

Firstly, we open the brackets

2 × 3 + 2 × 4i + i × 5 – i × 6i

$= 6 + 8i + 5i – 6i^2$

$= 6 + 13i – 6(-1)$ [$\because$, $i^2 = -1$]

= 6 + 13i + 6

$= \underbrace{12}_{\text{Real part}} + \underbrace{13i}_{\text{Imaginary part}}$

Question: 1 B

Simplify each of

Solution:

Given: $\left(3+\sqrt{-16}\right)-\left(4-\sqrt{-9}\right)$

We re – write the above equation

$\left(3+\sqrt{(-1)\times 16}\right)(-1)\left(4-\sqrt{(-1)\times 9}\right)$

$= \left(3+\sqrt{16i^2}\right)-\left(4-\sqrt{9i^2}\right)$ [$\because$ $i^2 = -1$]

= (3 + 4i) – (4 – 3i)

Now, we open the brackets, we get

3 + 4i – 4 + 3i

$= \underbrace{-1}_{\text{Real part}} + \underbrace{7i}_{\text{Imaginary part}}$

Question: 1 C

Simplify each of

Solution:

Given: (–5 + 6i) – (–2 + i)

Firstly, we open the brackets

-5 + 6i + 2 – i

$= \underbrace{-3}_{\text{Real part}} + \underbrace{5i}_{\text{Imaginary part}}$

Question: 1 D

Simplify each of

Solution:

Given: (8 – 4i) – (- 3 + 5i)

Firstly, we open the brackets

8 – 4i + 3 – 5i

$= \underbrace{11}_{\text{Real part}} - \underbrace{9i}_{\text{Imaginary part}}$

Question: 1 E

Simplify each of

Solution:

Given: $(1 - i)^2 (1 + i) - (3 - 4i)^2$

$= (1 + i^2 - 2i)(1 + i) - (9 + 16i^2 - 24i)$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$= (1 - 1 - 2i)(1 + i) - (9 - 16 - 24i)$ $[\because i^2 = -1]$

$= (-2i)(1 + i) - (- 7 - 24i)$

Now, we open the brackets

$-2i \times 1 - 2i \times i + 7 + 24i$

$= -2i - 2i^2 + 7 + 24i$

$= -2(-1) + 7 + 22i$ $[\because, i^2 = -1]$

$= 2 + 7 + 22i$

$= \underbrace{9}_{\text{Real part}} + \underbrace{22i}_{\text{Imaginary part}}$

Question: 1 F

Simplify each of

Solution:

Given: $\left(5 + \sqrt{-3}\right)\left(5 - \sqrt{-3}\right)$

We re – write the above equation

$\left(5+\sqrt{(-1)\times 3}\right)\left(5-\sqrt{(-1)\times 3}\right)$

$= \left(5+\sqrt{3i^2}\right)\left(5-\sqrt{3i^2}\right)$ [∵, i^2 = -1]

$= (5+i\sqrt{3})(5-i\sqrt{3})$

Now, we know that,

(a + b)(a – b) = (a^2 – b^2)

Here, a = 5 and b = i√3

= $(5)^2$ – $(i\sqrt{3})^2$

= 25 – ($3i^2$)

= 25 – [3 × (-1)]

= 25 + 3

= 28 + 0

= $\underbrace{28}_{\text{Real part}} + \underbrace{0i}_{\text{Imaginary part}}$

Question: 1 G

Simplify each of

Solution:

Given: (3 + 4i) (2 – 3i)

Firstly, we open the brackets

3 × 2 + 3 × (-3i) + 4i × 2 – 4i × 3i

= 6 – 9i + 8i – $12i^2$

= 6 – i – 12(-1) [∵, i^2 = -1]

= 6 – i + 12

= $\underbrace{18}_{\text{Real part}} - \underbrace{i}_{\text{Imaginary part}}$

Question: 1 H

Simplify each of

Solution:

Given: $\left(-2+\sqrt{-3}\right)(-3+2\sqrt{-3})$

We re – write the above equation

$\left(-2+\sqrt{(-1)\times 3}\right)\left(-3+2\sqrt{(-1)\times 3}\right)$

$= \left(-2+\sqrt{3i^2}\right)(-3+2\sqrt{3i^2})$ [∵, i^2 = -1]

$= (-2+i\sqrt{3})(-3+2i\sqrt{3})$

Now, open the brackets,

$= -2 \times (-3) + (-2) \times 2i\sqrt{3} + i\sqrt{3} \times (-3) + i\sqrt{3} \times 2i\sqrt{3}$

$= 6 - 4i\sqrt{3} - 3i\sqrt{3} + 6i^2$

$= 6 - 7i\sqrt{3} + [6 \times (-1)]$ [$\because$, $i^2 = -1$]

$= 6 - 7i\sqrt{3} - 6$

$= \underbrace{0}_{\text{Real part}} \underbrace{- 7i\sqrt{3}}_{\text{Imaginary part}}$

Question: 2 A

Simplify each of

Solution:

Given: $(2 - \sqrt{-3})^2$

We know that,

$(a - b)^2 = a^2 + b^2 - 2ab$ ...(i)

So, on replacing a by 2 and b by $\sqrt{-3}$ in eq. (i), we get

$(2)^2 + (\sqrt{-3})^2 - 2(2)(\sqrt{-3})$

$= 4 + (-3) - 4\sqrt{-3}$

$= 4 - 3 - 4\sqrt{-3}$

$= 1 - 4\sqrt{3}i^2$ [$\because$ $i^2 = -1$]

$= \underbrace{1}_{\text{Real part}} \underbrace{- 4i\sqrt{3}}_{\text{Imaginary part}}$

Question: 2 B

Simplify each of

Solution:

Given: $(5 - 2i)^2$

We know that,

$(a - b)^2 = a^2 + b^2 - 2ab$ ...(i)

So, on replacing a by 5 and b by 2i in eq. (i), we get

$(5)^2 + (2i)^2 - 2(5)(2i)$

$= 25 + 4i^2 - 20i$

$= 25 - 4 - 20i$ [$\because$ $i^2 = -1$]

$= \underbrace{21}_{\text{Real part}} \underbrace{- 20i}_{\text{Imaginary part}}$

Question: 2 C

Simplify each of

Solution:

Given: $(-3 + 5i)^3$

We know that,

$(-a + b)^3 = -a^3 + 3a^2b - 3ab^2 + b^3$ ...(i)

So, on replacing a by 3 and b by 5i in eq. (i), we get

$-(3)^3 + 3(3)^2(5i) - 3(3)(5i)^2 + (5i)^3$

$= -27 + 3(9)(5i) - 3(3)(25i^2) + 125i^3$

$= -27 + 135i - 225i^2 + 125i^3$

$= -27 + 135i - 225 \times (-1) + 125i \times i^2$

$= -27 + 135i + 225 - 125i$ [$\because i^2 = -1$]

$= \underbrace{198}_{\text{Real part}} + \underbrace{10i}_{\text{Imaginary part}}$

Question: 2 D

Simplify each of

Solution:

Given: $\left(-2 - \frac{1}{3}i\right)^3$

We know that,

$(-a - b)^3 = -a^3 - 3a^2b - 3ab^2 - b^3$ ...(i)

So, on replacing a by 2 and b by 1/3i in eq. (i), we get

$$-(2)^3 - 3(2)^2\left(\frac{1}{3}i\right) - 3(2)\left(\frac{1}{3}i\right)^2 - \left(\frac{1}{3}i\right)^3$$

$$= -8 - 4i - 6\left(\frac{1}{9}i^2\right) - \left(\frac{1}{27}i^3\right)$$

$$= -8 - 4i - \frac{2}{3}i^2 - \frac{1}{27}i(i^2)$$

$$= -8 - 4i - \frac{2}{3}(-1) - \frac{1}{27}i(-1) \quad [\because i^2 = -1]$$

$$= -8 - 4i + \frac{2}{3} + \frac{1}{27}i$$

$$= \left(-8 + \frac{2}{3}\right) + \left(-4i + \frac{1}{27}i\right)$$

$$= \left(\frac{-24 + 2}{3}\right) + \left(\frac{-108i + i}{27}\right)$$

$$= -\frac{22}{3} + \left(-\frac{107}{27}i\right)$$

$$= -\frac{22}{3} - \frac{107}{27}i$$

Real part Imaginary part

Question: 2 E

Simplify each of

Solution:

Given: $(4 – 3i)^{-1}$

We can re- write the above equation as

$$= \frac{1}{4 - 3i}$$

Now, rationalizing

$$= \frac{1}{4 - 3i} \times \frac{4 + 3i}{4 + 3i}$$

$$= \frac{4+3i}{(4-3i)(4+3i)} \text{ ...(i)}$$

Now, we know that,

$(a + b)(a – b) = (a^2 – b^2)$

So, eq. (i) become

$$= \frac{4 + 3i}{(4)^2 - (3i)^2}$$

$$= \frac{4 + 3i}{16 - 9i^2}$$

$$= \frac{4+3i}{16-9(-1)} \; [\because i^2 = -1]$$

$$= \frac{4 + 3i}{16 + 9}$$

$$= \frac{4 + 3i}{25}$$

$$= \underbrace{\frac{4}{25}}_{\text{Real part}} + \underbrace{\frac{3}{25}i}_{\text{Imaginary part}}$$

Question: 2 F

Simplify each of

Solution:

Given: $(-2 + \sqrt{-3})^{-1}$

We can re- write the above equation as

$$= \frac{1}{-2 + \sqrt{-3}}$$

$$= \frac{1}{-2+\sqrt{3i^2}} \; [\because i^2 = -1]$$

$$= \frac{1}{-2 + i\sqrt{3}}$$

Now, rationalizing

$$= \frac{1}{-2+i\sqrt{3}} \times \frac{-2-i\sqrt{3}}{-2-i\sqrt{3}}$$

$= \frac{-2-i\sqrt{3}}{(-2+i\sqrt{3})(-2-i\sqrt{3})}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{-2-i\sqrt{3}}{(-2)^2-\left(i\sqrt{3}\right)^2}$$

$$= \frac{-2-i\sqrt{3}}{4-(3i^2)}$$

$= \frac{-2-i\sqrt{3}}{4-3(-1)}$ [$\because$ $i^2 = -1$]

$$= \frac{-2-i\sqrt{3}}{4+3}$$

$$= \frac{-2-i\sqrt{3}}{7}$$

$$= -\frac{2+i\sqrt{3}}{7}$$

$$= \underbrace{-\frac{2}{7}}_{\text{Real part}} \underbrace{-\frac{\sqrt{3}}{7}i}_{\text{Imaginary part}}$$

Question: 2 G

Simplify each of

Solution:

Given: $(2 + i)^{-2}$

Above equation can be re – written as

$$= \frac{1}{(2+i)^2}$$

Now, rationalizing

$$= \frac{1}{(2+i)^2} \times \frac{(2-i)^2}{(2-i)^2}$$

$$= \frac{(2-i)^2}{(2+i)^2(2-i)^2}$$

$= \frac{4+i^2-4i}{(4+i^2+4i)(4+i^2-4i)}$ [$\because$ $(a - b)^2 = a^2 + b^2 - 2ab$]

$= \frac{4-1-4i}{(4-1+4i)(4-1-4i)}$ [$\because i^2 = -1$]

$= \frac{3-4i}{(3+4i)(3-4i)}$ ...(i)

Now, we know that,

(a + b)(a – b) = $(a^2 - b^2)$

So, eq. (i) become

$$= \frac{3-4i}{(3)^2-(4i)^2}$$

$$= \frac{3-4i}{9-16i^2}$$

$$= \frac{3-4i}{9-16(-1)}$$

$$= \frac{3-4i}{25}$$

$$= \underbrace{\frac{3}{25}}_{\text{Real part}} - \underbrace{\frac{4}{25}}_{\text{Imaginary part}} i$$

Question: 2 H

Simplify each of

Solution:

Given: $(1 + 2i)^{-3}$

Above equation can be re – written as

$$= \frac{1}{(1+2i)^3}$$

Now, rationalizing

$$= \frac{1}{(1+2i)^3} \times \frac{(1-2i)^3}{(1-2i)^3}$$

$$= \frac{(1-2i)^3}{(1+2i)^3(1-2i)^3}$$

We know that,

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$= \frac{(1)^3-3(1)^2(2i)+3(1)(2i)^2-(2i)^3}{[(1)^3+3(1)^2(2i)+3(1)(2i)^2+(2i)^3][(1)^3-3(1)^2(2i)+3(1)(2i)^2-(2i)^3]}$$

$$= \frac{1-6i+6i^2-8i^3}{[1+6i+6i^2+8i^3][1-6i+6i^2-8i^3]}$$

$$= \frac{1-6i+6(-1)-8i(-1)}{[1+6i+6(-1)+8i(-1)][1-6i+6(-1)-8i(-1)]} \quad [\because i^2 = -1]$$

$$= \frac{1-6i-6+8i}{[1+6i-6-8i][1-6i-6+8i]}$$

$$= \frac{-5+2i}{[-5-2i][-5+2i]}$$

$$= \frac{-5+2i}{-5(-5)-5(2i)-2i(-5)-2i(2i)}$$

$$= \frac{-5+2i}{25-10i+10i-4i^2}$$

$= \frac{-5+2i}{25-4(-1)}$ [$\because i^2 = -1$]

$$= \frac{-5+2i}{29}$$

$$= \underbrace{-\frac{5}{29}}_{\text{Real part}} + \underbrace{\frac{2}{29}i}_{\text{Imaginary part}}$$

Question: 2 I

Simplify each of

Solution:

Given: $(1 + i)^3 - (1 - i)^3$ ...(i)

We know that,

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

By applying the formulas in eq. (i), we get

$(1)^3 + 3(1)^2(i) + 3(1)(i)^2 + (i)^3 - [(1)^3 - 3(1)^2(i) + 3(1)(i)^2 - (i)^3]$

$= 1 + 3i + 3i^2 + i^3 - [1 - 3i + 3i^2 - i^3]$

$= 1 + 3i + 3i^2 + i^3 - 1 + 3i - 3i^2 + i^3$

$= 6i + 2i^3$

$= 6i + 2i(i^2)$

$= 6i + 2i(-1)$ [$\because i^2 = -1$]

$= 6i - 2i$

$= 4i$

$$= \underbrace{0}_{\text{Real part}} + \underbrace{4i}_{\text{Imaginary part}}$$

Question: 3 A

Express each of t

Solution:

Given: $\frac{1}{4+3i}$

Now, rationalizing

$$= \frac{1}{4+3i} \times \frac{4-3i}{4-3i}$$

$= \frac{4-3i}{(4+3i)(4-3i)}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{4-3i}{(4)^2-(3i)^2}$$

$$= \frac{4-3i}{16-9i^2}$$

$= \frac{4-3i}{16-9(-1)}$ [$\because$ i^2 = -1]

$$= \frac{4-3i}{16+9}$$

$$= \frac{4-3i}{25}$$

$$= \underbrace{\frac{4}{25}}_{\text{Real part}} - \underbrace{\frac{3}{25}i}_{\text{Imaginary part}}$$

Question: 3 B

Express each of t

Solution:

Given: $\frac{3+4i}{4+5i}$

Now, rationalizing

$$= \frac{3+4i}{4+5i} \times \frac{4-5i}{4-5i}$$

$= \frac{(3+4i)(4-5i)}{(4+5i)(4-5i)}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(3+4i)(4-5i)}{(4)^2-(5i)^2}$$

$$= \frac{3(4)+3(-5i)+4i(4)+4i(-5i)}{16-25i^2}$$

$= \frac{12-15i+16i-20i^2}{16-25(-1)}$ [$\because$ i^2 = -1]

$$= \frac{12+i-20(-1)}{16+25}$$

$$= \frac{12+i+20}{41}$$

$$= \frac{32+i}{41}$$

$$= \underbrace{\frac{32}{41}}_{\text{Real part}} + \underbrace{\frac{1}{41}i}_{\text{Imaginary part}}$$

Question: 3 C

Express each of t

Solution:

Given: $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$

Now, rationalizing

$$= \frac{5+\sqrt{2}i}{1-\sqrt{2}i} \times \frac{1+\sqrt{2}i}{1+\sqrt{2}i}$$

$$= \frac{(5+\sqrt{2}i)(1+\sqrt{2}i)}{(1-\sqrt{2}i)(1+\sqrt{2}i)} \ldots\text{(i)}$$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(5+\sqrt{2}i)(1+\sqrt{2}i)}{(1)^2-\left(\sqrt{2}i\right)^2}$$

$$= \frac{5(1)+5\left(\sqrt{2}i\right)+\sqrt{2}i(1)+\sqrt{2}i(\sqrt{2}i)}{1-2i^2}$$

$$= \frac{5+5\sqrt{2}i+\sqrt{2}i+2i^2}{1-2(-1)} \quad [\because i^2 = -1]$$

$$= \frac{5+6i\sqrt{2}+2(-1)}{1+2}$$

$$= \frac{3+6i\sqrt{2}}{3}$$

$$= \frac{3(1+2i\sqrt{2})}{3}$$

$$= \underbrace{1}_{\text{Real part}} + \underbrace{2i\sqrt{2}}_{\text{Imaginary part}}$$

Question: 3 D

Express each of t

Solution:

Given: $\frac{-2+5i}{3-5i}$

Now, rationalizing

$$= \frac{-2+5i}{3-5i} \times \frac{3+5i}{3+5i}$$

$$= \frac{(-2+5i)(3+5i)}{(3-5i)(3+5i)} \ldots\text{(i)}$$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(-2+5i)(3+5i)}{(3)^2-(5i)^2}$$

$$= \frac{-2(3)+(-2)(5i)+5i(3)+5i(5i)}{9-25i^2}$$

$$= \frac{-6-10i+15i+25i^2}{9-25(-1)} \quad [\because i^2 = -1]$$

$$= \frac{-6+5i+25(-1)}{9+25}$$

$$= \frac{-31+5i}{34}$$

$$= \underbrace{-\frac{31}{34}}_{\text{Real part}} + \underbrace{\frac{5}{34}i}_{\text{Imaginary part}}$$

Question: 3 E

Express each of t

Solution:

Given: $\frac{3-4i}{(4-2i)(1+i)}$

Solving the denominator, we get

$$\frac{3-4i}{(4-2i)(1+i)} = \frac{3-4i}{4(1)+4(i)-2i(1)-2i(i)}$$

$$= \frac{3-4i}{4+4i-2i-2i^2}$$

$$= \frac{3-4i}{4+2i-2(-1)}$$

$$= \frac{3-4i}{6+2i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 6 + 2i

$$= \frac{3-4i}{6+2i} \times \frac{6-2i}{6-2i}$$

$$= \frac{(3-4i)(6-2i)}{(6+2i)(6-2i)} \ldots\text{(i)}$$

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(3-4i)(6-2i)}{(6)^2-(2i)^2}$$

$$= \frac{3(6)+3(-2i)+(-4i)(6)+(-4i)(-2i)}{36-4i^2}$$

$$= \frac{18-6i-24i+8i^2}{36-4(-1)} \quad [\because i^2 = -1]$$

$$= \frac{18-30i+8(-1)}{36+4}$$

$$= \frac{18-30i-8}{40}$$

$$= \frac{10-30i}{40}$$

$$= \frac{10(1-3i)}{40}$$

$$= \frac{1-3i}{4}$$

$$= \frac{1}{4} - \frac{3}{4}i$$

Question: 3 F

Real part Imaginary part

Express each of t

Solution:

Given: $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$

Firstly, we solve the given equation

$$=\frac{3(2)+3(3i)-2i(2)+(-2i)(3i)}{(1)(2)+1(-i)+2i(2)+2i(-i)}$$

$$=\frac{6+9i-4i-6i^2}{2-i+4i-2i^2}$$

$$=\frac{6+5i-6(-1)}{2+3i-2(-1)}$$

$$=\frac{6+6+5i}{2+3i+2}$$

$$=\frac{12+5i}{4+3i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 4 + 3i

$$=\frac{12+5i}{4+3i}\times\frac{4-3i}{4-3i}$$

$=\frac{(12+5i)(4-3i)}{(4+3i)(4-3i)}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$=\frac{(12+5i)(4-3i)}{(4)^2-(3i)^2}$$

$$=\frac{12(4)+12(-3i)+5i(4)+5i(-3i)}{16-9i^2}$$

$=\frac{48-36i+20i-15i^2}{19-9(-1)}$ [$\because$ $i^2 = -1$]

$=\frac{48-16i-15(-1)}{16+9}$ [$\because$ $i^2 = -1$]

$$=\frac{48-16i+15}{25}$$

$$=\frac{63-16i}{25}$$

$=\frac{63}{25}-\frac{16}{25}i$

Real part Imaginary part

Question: 3 G

Express each of t

Solution:

Given: $\frac{(2+3i)^2}{(2-i)}$

Now, we rationalize the above equation by multiply and divide by the conjugate of (2 – i)

$$= \frac{(2+3i)^2}{(2-i)} \times \frac{(2+i)}{(2+i)}$$

$$= \frac{(2+3i)^2(2+i)}{(2-i)(2+i)}$$

$$= \frac{(4+9i^2+12i)(2+i)}{(2)^2-(i)^2}$$

[∵(a + b)(a – b) = (a^2 – b^2)]

$= \frac{[4+9(-1)+12i](2+i)}{4-i^2}$ [∵i^2 = -1]

$$= \frac{[4-9+12i](2+i)}{4-(-1)}$$

$$= \frac{(-5+12i)(2+i)}{5}$$

$$= \frac{-10-5i+24i+12i^2}{5}$$

$$= \frac{-10+19i+12(-1)}{5}$$

$$= \frac{-10-12+19i}{5}$$

$$= \frac{-22+19i}{5}$$

$$= -\frac{22}{5} + \frac{19}{5}i$$

Real part Imaginary part

Question: 3 H

Express each of t

Solution:

Given: $\frac{(1-i)^3}{(1-i^3)}$

The above equation can be re-written as

$$= \frac{(1)^3-(i)^3-3(1)^2(i)+3(1)(i)^2}{(1-i\times i^2)}$$

[∵(a – b)^3 = a^3 – b^3 – $3a^2b$ + $3ab^2$]

$= \frac{1-i^3-3i+3i^2}{[1-i(-1)]}$ [∵i^2 = -1]

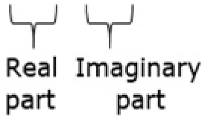
$$= \frac{1-i\times i^2-3i+3(-1)}{(1+i)}$$

$$= \frac{1-i(-1)-3i-3}{1+i}$$

$$= \frac{-2+i-3i}{1+i}$$

$$= \frac{-2-2i}{1+i}$$

$$= \frac{-2(1+i)}{1+i}$$



= -2 + 0i

Question: 3 I

Express each of t

Solution:

Given: $\frac{(1+2i)^3}{(1+i)(2-i)}$

We solve the above equation by using the formula

$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$$= \frac{(1)^3 + (2i)^3 + 3(1)^2(2i) + 3(1)(2i)^2}{1(2) + 1(-i) + i(2) + i(-i)}$$

$$= \frac{1 + 8i^3 + 6i + 12i^2}{2 - i + 2i - i^2}$$

$= \frac{1+8i\times i^2+6i+12(-1)}{2+i-(-1)}$ [∵ i^2 = -1]

$$= \frac{1 + 8i(-1) + 6i - 12}{2 + i + 1}$$

$$= \frac{1 - 8i + 6i - 12}{3 + i}$$

$$= \frac{-11 - 2i}{3 + i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 3 + i

$$= \frac{-11 - 2i}{3 + i} \times \frac{3 - i}{3 - i}$$

$= \frac{(-11-2i)(3-i)}{(3+i)(3-i)}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{(-11 - 2i)(3 - i)}{(3)^2 - (i)^2}$$

$$= \frac{-11(3) + (-11)(-i) + (-2i)(3) + (-2i)(-i)}{9 - i^2}$$

$= \frac{-33+11i-6i+2i^2}{9-(-1)}$ [∵ i^2 = -1]

$= \frac{-33+5i+2(-1)}{9+1}$ [∵ i^2 = -1]

$$= \frac{-33 + 5i - 2}{10}$$

$$= \frac{-35 + 5i}{10}$$

$$= \frac{5(-7 + i)}{10}$$

$$= \frac{-7 + i}{2}$$

Real part Imaginary part

$= \frac{-7}{2} + \frac{1}{2} i$

Question: 4

Simplify each of

Solution:

Given:

$$\left(\frac{5}{-3+2i} + \frac{2}{1-i}\right)\left(\frac{4-5i}{3+2i}\right)$$

$= \left[\frac{5(1-i)+2(-3+2i)}{(-3+2i)(1-i)}\right]\left(\frac{4-5i}{3+2i}\right)$ [Taking the LCM]

$$= \left[\frac{5-5i-6+4i}{(-3)(1-i)+2i(1-i)}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{-1-i}{-3+3i+2i-2i^2}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left[\frac{-(1+i)}{-3+5i-2(-1)}\right]\left(\frac{4-5i}{3+2i}\right)$$

$$= \left(\frac{-(1+i)}{-1+5i}\right)\left(\frac{4-5i}{3+2i}\right)$$

$$= \frac{-1(4-5i)-i(4-5i)}{-1(3+2i)+5i(3+2i)}$$

$$= \frac{-4+5i-4i+5i^2}{-3-2i+15i+10i^2}$$

$= \frac{-4+i+5(-1)}{-3+13i+10(-1)}$ [putting $i^2 = -1$]

$$= \frac{-9+i}{-13+13i}$$

$$= \frac{-(9-i)}{-(13-13i)}$$

$$= \frac{9-i}{13-13i}$$

Now, rationalizing by multiply and divide by the conjugate of (13 – 13i)

$$= \frac{9-i}{13-13i} \times \frac{13+13i}{13+13i}$$

$$= \frac{(9-i)(13+13i)}{(13-13i)(13+13i)}$$

$= \frac{117+117i-13i-13i^2}{(13)^2-(13i)^2}$ [$\because$ (a – b)(a + b) = (a^2 – b^2)]

$= \frac{117+104i-13(-1)}{169-169i^2}$ [$\because$ $i^2 = -1$]

$$= \frac{130+104i}{169(1-i^2)}$$

$= \frac{13(10+8i)}{169[1-(-1)]}$ [taking 13 common]

$$= \frac{10+8i}{13 \times 2}$$

$= \frac{5+4i}{13}$

$= \frac{5}{13} + \frac{4}{13}i$

(ii) Given:

$\left(\frac{1}{1+4i} - \frac{2}{1+i}\right)\left(\frac{1-i}{5+3i}\right)$

$= \left[\frac{1(1+i)-2(1+4i)}{(1+4i)(1+i)}\right]\left(\frac{1-i}{5+3i}\right)$ [Taking the LCM]

$= \left[\frac{1+i-2-8i}{(1)(1+i)+4i(1+i)}\right]\left(\frac{1-i}{5+3i}\right)$

$= \left[\frac{-1-7i}{1+i+4i+4i^2}\right]\left(\frac{1-i}{5+3i}\right)$

$= \left[\frac{-1-7i}{1+5i+4(-1)}\right]\left(\frac{1-i}{5+3i}\right)$

$= \left(\frac{-1-7i}{-3+5i}\right)\left(\frac{1-i}{5+3i}\right)$

$= \frac{-1(1-i)-7i(1-i)}{-3(5+3i)+5i(5+3i)}$

$= \frac{-1+i-7i+7i^2}{-15-9i+25i+15i^2}$

$= \frac{-1-6i+7(-1)}{-15+16i+15(-1)}$

$= \frac{-6i-8}{16i-30}$

$= \frac{-2(4+3i)}{-2(15-8i)}$

$= \frac{4+3i}{15-8i}$

Now, rationalizing by multiply and divide by the conjugate of (15 + 8i)

$= \frac{4+3i}{15-8i} \times \frac{15+8i}{15+8i}$

$= \frac{(4+3i)(15+8i)}{(15)^2-(8i)^2}$ [$\because$ (a – b)(a + b) = $(a^2 - b^2)$]

$= \frac{4(15+8i)+3i(15+8i)}{225-64i^2}$

$= \frac{60+32i+45i+24i^2}{225-64(-1)}$ [$\because$ i^2 = -1]

$= \frac{60+77i+24(-1)}{225+64}$

$= \frac{36+77i}{289}$

$= \frac{36}{289} + \frac{77}{289}i$

Question: 5

Show that

Solution:

Given: $\frac{3+2i}{2-3i}+\frac{3-2i}{2+3i}$

Taking the L.C.M, we get

$$=\frac{(3+2i)(2+3i)+(3-2i)(2-3i)}{(2-3i)(2+3i)}$$

$$=\frac{3(2)+3(3i)+2i(2)+2i(3i)+3(2)+3(-3i)-2i(2)+(-2i)(-3i)}{(2)^2-(3i)^2}$$

[$\because$(a + b)(a – b) = (a^2 – b^2)]

$$=\frac{6+9i+4i+6i^2+6-9i-4i+6i^2}{4-9i^2}$$

$$=\frac{12+12i^2}{4-9i^2}$$

Putting i^2 = -1

$$=\frac{12+12(-1)}{4-9(-1)}$$

$$=\frac{12-12}{4+9}$$

= 0 + 0i

Hence, the given equation is purely real as there is no imaginary part.

(ii) Given: $\frac{\sqrt{7}+i\sqrt{3}}{\sqrt{7}-i\sqrt{3}}+\frac{\sqrt{7}-i\sqrt{3}}{\sqrt{7}+i\sqrt{3}}$

Taking the L.C.M, we get

$$=\frac{(\sqrt{7}+i\sqrt{3})(\sqrt{7}+i\sqrt{3})+(\sqrt{7}-i\sqrt{3})(\sqrt{7}-i\sqrt{3})}{(\sqrt{7}-i\sqrt{3})(\sqrt{7}+i\sqrt{3})}$$

$=\frac{(\sqrt{7}+i\sqrt{3})^2+(\sqrt{7}-i\sqrt{3})^2}{(\sqrt{7})^2-(i\sqrt{3})^2}$ ...(i)

[$\because$(a + b)(a – b) = (a^2 – b^2)]

Now, we know that,

$(a + b)^2 + (a – b)^2 = 2(a^2 + b^2)$

So, by applying the formula in eq. (i), we get

$$=\frac{2\left[(\sqrt{7})^2+(i\sqrt{3})^2\right]}{7-3i^2}$$

$$=\frac{2[7+3i^2]}{7-3(-1)}$$

Putting i^2 = -1

$$=\frac{2[7+3(-1)]}{7+3}$$

$$=\frac{2[7-3]}{10}$$

$= \frac{8}{10} + 0i$

$= \frac{4}{5} + 0i$

Hence, the given equation is purely real as there is no imaginary part.

Question: 6

Find the real val

Solution:

Since $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real

Firstly, we need to solve the given equation and then take the imaginary part as 0

$\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$

We rationalize the above by multiply and divide by the conjugate of (1 -2i cos θ)

$= \frac{1 + i\cos\theta}{1 - 2i\cos\theta} \times \frac{1 + 2i\cos\theta}{1 + 2i\cos\theta}$

$= \frac{(1 + i\cos\theta)(1 + 2i\cos\theta)}{(1 - 2i\cos\theta)(1 + 2i\cos\theta)}$

We know that,

$(a - b)(a + b) = (a^2 - b^2)$

$= \frac{1(1) + 1(2i\cos\theta) + i\cos\theta(1) + i\cos\theta(2i\cos\theta)}{(1)^2 - (2i\cos\theta)^2}$

$= \frac{1 + 2i\cos\theta + i\cos\theta + 2i^2\cos^2\theta}{1 - 4i^2\cos^2\theta}$

$= \frac{1+3i\cos\theta+2(-1)\cos^2\theta}{1-4(-1)\cos^2\theta}$ [$\because i^2 = -1$]

$= \frac{1 + 3i\cos\theta - 2\cos^2\theta}{1 + 4\cos^2\theta}$

$= \frac{1 - 2\cos^2\theta}{1 + 4\cos^2\theta} + i\frac{3\cos\theta}{1 + 4\cos^2\theta}$

Since $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real [given]

Hence, imaginary part is equal to 0

i.e. $\frac{3\cos\theta}{1+4\cos^2\theta} = 0$

$\Rightarrow 3\cos\theta = 0 \times (1 + 4\cos^2\theta)$

$\Rightarrow 3\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \cos\theta = \cos 0$

Since, $\cos\theta = \cos y$

Then $\theta = (2n + 1)\frac{\pi}{2} \pm y$ where $n \in Z$

Putting $y = 0$

$\theta = (2n + 1)\frac{\pi}{2} \pm 0$

$\theta = (2n + 1)\frac{\pi}{2}$ where n $\in$ Z

Hence, for $\theta = (2n + 1)\frac{\pi}{2}$. where n $\in$ Z $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is purely real.

Question: 7

If |z + i| = |z –

Solution:

Let z = x + iy

Consider, |z + i| = |z – i|

⇒ |x + iy + i| = |x + iy – i|

⇒ |x + i(y +1)| = |x + i(y – 1)|

$\Rightarrow \sqrt{(x)^2 + (y+1)^2} = \sqrt{(x)^2 + (y-1)^2}$

$\left[\because |z| = modulus = \sqrt{a^2 + b^2}\right]$

$\Rightarrow \sqrt{x^2 + y^2 + 1 + 2y} = \sqrt{x^2 + y^2 + 1 - 2y}$

Squaring both the sides, we get

$\Rightarrow x^2 + y^2 + 1 + 2y = x^2 + y^2 + 1 - 2y$

$\Rightarrow x^2 + y^2 + 1 + 2y - x^2 - y^2 - 1 + 2y = 0$

⇒ 2y + 2y = 0

⇒ 4y = 0

⇒ y = 0

Putting the value of y in eq. (i), we get

z = x + i(0)

⇒ z = x

Hence, z is purely real.

Question: 8

Give an example o

Solution:

Let z_1 = 3 – 4i and z_2 = 4 – 3i

Here, $z_1 \neq z_2$

Now, calculating the modulus, we get,

$|z_1| = \sqrt{3^2 + (4)^2} = \sqrt{25} = 5$

$|z_2| = \sqrt{4^2 + (3)^2} = \sqrt{25} = 5$

Question: 9 A

Find the conjugat

Solution:

Given: z = (-5 – 2i)

Here, we have to find the conjugate of (-5 – 2i)

So, the conjugate of (- 5 – 2i) is (-5 + 2i)

Question: 9 B

Find the conjugat

Solution:

Given: $\frac{1}{4+3i}$

First, we calculate $\frac{1}{4+3i}$ and then find its conjugate

Now, rationalizing

$$=\frac{1}{4+3i}\times\frac{4-3i}{4-3i}$$

$=\frac{4-3i}{(4+3i)(4-3i)}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$=\frac{4-3i}{(4)^2-(3i)^2}$$

$$=\frac{4-3i}{16-9i^2}$$

$=\frac{4-3i}{16-9(-1)}$ [$\because i^2 = -1$]

$$=\frac{4-3i}{16+9}$$

$$=\frac{4-3i}{25}$$

$$=\frac{4}{25}-\frac{3}{25}i$$

Hence, $\frac{1}{4+3i}=\frac{4}{25}-\frac{3}{25}i$

So, a conjugate of $\frac{1}{4+3i}$ is $\frac{4}{25}+\frac{3}{25}i$

Question: 9 C

Find the conjugat

Solution:

Given: $\frac{(1+i)^2}{(3-i)}$

Firstly, we calculate $\frac{(1+i)^2}{(3-i)}$ and then find its conjugate

$\frac{(1+i)^2}{(3-i)}=\frac{1+i^2+2i}{(3-i)}$ [$\because (a + b)^2 = a^2 + b^2 + 2ab$]

$=\frac{1+(-1)+2i}{3-i}$ [$\because i^2 = -1$]

$$=\frac{2i}{3-i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 3 – i

$= \frac{2i}{3-i} \times \frac{3+i}{3+i}$

$= \frac{(2i)(3+i)}{(3+i)(3-i)}$ ...(i)

Now, we know that,

(a + b)(a – b) = (a^2 – b^2)

So, eq. (i) become

$= \frac{(2i)(3+i)}{(3)^2-(i)^2}$

$= \frac{2i(3)+2i(i)}{9-i^2}$

$= \frac{6i+2i^2}{9-(-1)}$ [∵ i^2 = -1]

$= \frac{6i+2(-1)}{9+1}$ [∵ i^2 = -1]

$= \frac{6i-2}{10}$

$= \frac{2(3i-1)}{10}$

$= \frac{(-1+3i)}{5}$

$= -\frac{1}{5} + \frac{3}{5}i$

Hence, $\frac{(1+i)^2}{(3-i)} = -\frac{1}{5} + \frac{3}{5}i$

So, the conjugate of $\frac{(1+i)^2}{(3-i)}$ is $-\frac{1}{5} - \frac{3}{5}i$

Question: 9 D

Find the conjugat

Solution:

Given: $\frac{(1+i)(2+i)}{(3+i)}$

Firstly, we calculate $\frac{(1+i)(2+i)}{(3+i)}$ and then find its conjugate

$\frac{(1+i)(2+i)}{(3+i)} = \frac{1(2)+1(i)+i(2)+i(i)}{(3+i)}$

$= \frac{2+i+2i+i^2}{3+i}$

$= \frac{2+3i-1}{3+i}$ [∵ i^2 = -1]

$= \frac{1+3i}{3+i}$

Now, we rationalize the above by multiplying and divide by the conjugate of 3 + i

$= \frac{1+3i}{3+i} \times \frac{3-i}{3-i}$

$= \frac{(1+3i)(3-i)}{(3+i)(3-i)}$ ...(i)

Now, we know that,

(a + b)(a – b) = $(a^2 - b^2)$

So, eq. (i) become

$$= \frac{(1+3i)(3-i)}{(3)^2 - (i)^2}$$

$$= \frac{1(3) + 1(-i) + 3i(3) + 3i(-i)}{9 - i^2}$$

$= \frac{3-i+9i-3i^2}{9-(-1)}$ [∵ i^2 = -1]

$= \frac{3+8i-3(-1)}{9+1}$ [∵ i^2 = -1]

$$= \frac{3 + 8i + 3}{10}$$

$$= \frac{6 + 8i}{10}$$

$$= \frac{2(3 + 4i)}{10}$$

$$= \frac{3 + 4i}{5}$$

$$= \frac{3}{5} + \frac{4}{5}i$$

Hence, $\frac{(1+i)(2+i)}{(3+i)} = \frac{3}{5} + \frac{4}{5}i$

So, the conjugate of $\frac{(1+i)^2}{(3-i)}$ is $\frac{3}{5} - \frac{4}{5}i$

Question: 9 E

Find the conjugat

Solution:

Given: z = √-3

The above can be re – written as

$z = \sqrt{(-1) \times 3}$

$z = \sqrt{3i^2}$ [∵ i^2 = -1]

z = 0 + i√3

So, the conjugate of z = 0 + i√3 is

$\bar{z} = 0 - i\sqrt{3}$

or $\bar{z} = -i\sqrt{3} = -\sqrt{-3}$

Question: 9 F

Find the conjugat

Solution:

Given: z = √2

The above can be re – written as

$z = \sqrt{2} + 0i$

Here, the imaginary part is zero

So, the conjugate of $z = \sqrt{2} + 0i$ is

$\bar{z} = \sqrt{2} - 0i$

or $\bar{z} = \sqrt{2}$

Question: 9 G

Find the conjugat

Solution:

Given: $z = -\sqrt{-1}$

The above can be re – written as

$z = -\sqrt{i^2}$ [$\because i^2 = -1$]

$z = 0 - i$

So, the conjugate of $z = (0 - i)$ is

$\bar{z} = 0 + i$

or $\bar{z} = i$

Question: 9 H

Find the conjugat

Solution:

Given: $z = (2 - 5i)^2$

First we calculate $(2 - 5i)^2$ and then we find the conjugate

$(2 - 5i)^2 = (2)^2 + (5i)^2 - 2(2)(5i)$

$= 4 + 25i^2 - 20i$

$= 4 + 25(-1) - 20i$ [$\because i^2 = -1$]

$= 4 - 25 - 20i$

$= -21 - 20i$

Now, we have to find the conjugate of $(-21 - 20i)$

So, the conjugate of $(- 21 - 20i)$ is $(-21 + 20i)$

Question: 10 A

Find the modulus

Solution:

Given: $z = (3 + \sqrt{-5})$

The above can be re – written as

$z = 3 + \sqrt{(-1) \times 5}$

$z = 3 + i\sqrt{5}$ [$\because i^2 = -1$]

Now, we have to find the modulus of $(3 + i\sqrt{5})$

So, $|z| = |3 + i\sqrt{5}| = \sqrt{(3)^2 + (\sqrt{5})^2} = \sqrt{9 + 5} = \sqrt{14}$

Hence, the modulus of (3 + √-5) is √14

Question: 10 B

Find the modulus

Solution:

Given: z = (-3 – 4i)

Now, we have to find the modulus of (-3 – 4i)

So, $|z| = |-3 - 4i| = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Hence, the modulus of (-3 – 4i) is 5

Question: 10 C

Find the modulus

Solution:

Given: z = (7 + 24i)

Now, we have to find the modulus of (7 + 24i)

So, $|z| = |7 + 24i| = \sqrt{(7)^2 + (24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$

Hence, the modulus of (7 + 24i) is 25

Question: 10 D

Find the modulus

Solution:

Given: z = 3i

The above equation can be re – written as

z = 0 + 3i

Now, we have to find the modulus of (0 + 3i)

So, $|z| = |0 + 3i| = \sqrt{(0)^2 + (3)^2} = \sqrt{9} = 3$

Hence, the modulus of (3i) is 3

Question: 10 E

Find the modulus

Solution:

Given: $\frac{(3+2i)^2}{(4-3i)}$

Firstly, we calculate $\frac{(3+2i)^2}{(4-3i)}$ and then find its modulus

$\frac{(3+2i)^2}{(4-3i)} = \frac{9+4i^2+12i}{(4-3i)}$ [∵ $(a + b)^2 = a^2 + b^2 + 2ab$]

$= \frac{9+4(-1)+12i}{4-3i}$ [∵ $i^2 = -1$]

$$= \frac{5 + 12i}{4 - 3i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of 4 + 3i

$= \frac{5+12i}{4-3i} \times \frac{4+3i}{4+3i}$

$= \frac{(5+12i)(4+3i)}{(4-3i)(4+3i)}$ ...(i)

Now, we know that,

(a + b)(a – b) = (a^2 – b^2)

So, eq. (i) become

$= \frac{5(4)+(5)(3i)+12i(4)+12i(3i)}{(4)^2-(3i)^2}$

$= \frac{20+15i+48i+36i^2}{16-9i^2}$

$= \frac{20+63i+36(-1)}{16-9(-1)}$ [∵ i^2 = -1]

$= \frac{20-36+63i}{16+9}$ [∵ i^2 = -1]

$= \frac{-16+63i}{25}$

$= -\frac{16}{25}+\frac{63}{25}i$

Now, we have to find the modulus of $\left(-\frac{16}{25}+\frac{63}{25}i\right)$

So, $|z| = \left|-\frac{16}{25}+\frac{63}{25}i\right| = \sqrt{\left(-\frac{16}{25}\right)^2+\left(\frac{63}{25}\right)^2}$

$= \sqrt{\frac{256}{625}+\frac{3969}{625}}$

$= \sqrt{\frac{4225}{625}}$

$= \frac{65}{25}$

$= \frac{13}{5}$

Hence, the modulus of $\frac{(3+2i)^2}{(4-3i)}$ is $\frac{13}{5}$

Question: 10 F

Find the modulus

Solution:

Given: $\frac{(2-i)(1+i)}{(1+i)}$

Firstly, we calculate $\frac{(2-i)(1+i)}{(1+i)}$ and then find its modulus

$\frac{(2-i)(1+i)}{(1+i)} = \frac{2(1)+2(i)+(-i)(1)+(-i)(i)}{(1+i)}$

$= \frac{2+2i-i-i^2}{1+i}$

$= \frac{2+i-(-1)}{1+i}$ [∵ $i^2 = -1$]

$= \frac{3+i}{1+i}$

Now, we rationalize the above by multiplying and divide by the conjugate of 1 + i

$= \frac{3+i}{1+i} \times \frac{1-i}{1-i}$

$= \frac{(3+i)(1-i)}{(1+i)(1-i)}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$= \frac{3(1-i)+i(1-i)}{(1)^2-(i)^2}$

$= \frac{3(1)+3(-i)+i(1)+i(-i)}{1-i^2}$

$= \frac{3-3i+i-i^2}{1-(-1)}$ [∵ $i^2 = -1$]

$= \frac{3-2i-(-1)}{1+1}$ [∵ $i^2 = -1$]

$= \frac{3-2i+1}{2}$

$= \frac{4-2i}{2}$

= 2 – i

Now, we have to find the modulus of (2 – i)

So, $|z| = |2-i| = |2+(-1)i| = \sqrt{(2)^2+(-1)^2} = \sqrt{4+1} = \sqrt{5}$

Question: 10 G

Find the modulus

Solution:

Given: z = 5

The above equation can be re – written as

z = 5 + 0i

Now, we have to find the modulus of (5 + 0i)

So, $|z| = |5+0i| = \sqrt{(5)^2+(0)^2} = 5$

Question: 10 H

Find the modulus

Solution:

Given: z = (1 + 2i)(i – 1)

Firstly, we calculate the (1 + 2i)(i – 1) and then find the modulus

So, we open the brackets,

1(i – 1) + 2i(i – 1)

= 1(i) + (1)(-1) + 2i(i) + 2i(-1)

= i – 1 + $2i^2$ – 2i

= - i – 1 + 2(-1) [∵ i^2 = - 1]

= - i – 1 – 2

= - i – 3

Now, we have to find the modulus of (-3 - i)

So, $|z| = |-3 - i| = |-3 + (-1)i| = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$

Question: 11 A

Find the multipli

Solution:

Given: (1 - i√3)

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$= \frac{1}{z}$

Putting z = 1 - i√3

So, Multiplicative inverse of $1 - i\sqrt{3} = \frac{1}{1 - i\sqrt{3}}$

Now, rationalizing by multiply and divide by the conjugate of (1 - i√3)

$= \frac{1}{1 - i\sqrt{3}} \times \frac{1 + i\sqrt{3}}{1 + i\sqrt{3}}$

$= \frac{1 + i\sqrt{3}}{(1 - i\sqrt{3})(1 + i\sqrt{3})}$

Using (a – b)(a + b) = (a^2 – b^2)

$= \frac{1 + i\sqrt{3}}{(1)^2 - \left(i\sqrt{3}\right)^2}$

$= \frac{1 + i\sqrt{3}}{1 - 3i^2}$

$= \frac{1+i\sqrt{3}}{1-3(-1)}$ [∵ i^2 = -1]

$= \frac{1 + i\sqrt{3}}{1 + 3}$

$= \frac{1 + i\sqrt{3}}{4}$

$= \frac{1}{4} + \frac{\sqrt{3}}{4}i$

Hence, Multiplicative Inverse of $(1 - i\sqrt{3})$ is $\frac{1}{4} + \frac{\sqrt{3}}{4}i$

Question: 11 B

Find the multipli

Solution:

Given: 2 + 5i

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$= \frac{1}{z}$

Putting z = 2 + 5i

So, Multiplicative inverse of $2 + 5\mathrm{i} = \frac{1}{2 + 5\mathrm{i}}$

Now, rationalizing by multiply and divide by the conjugate of (2+5i)

$= \frac{1}{2 + 5i} \times \frac{2 - 5i}{2 - 5i}$

$= \frac{2 - 5i}{(2 + 5i)(2 - 5i)}$

Using (a – b)(a + b) = (a^2 – b^2)

$= \frac{2 - 5i}{(2)^2 - (5i)^2}$

$= \frac{2 - 5i}{4 - 25i^2}$

$= \frac{2-5i}{4-25(-1)}$ [∵ i^2 = -1]

$= \frac{2 - 5i}{4 + 25}$

$= \frac{2 - 5i}{29}$

$= \frac{2}{29} - \frac{5}{29}i$

Hence, Multiplicative Inverse of (2+5i)is $\frac{2}{29} - \frac{5}{29}\mathrm{i}$

Question: 11 C

Find the multipli

Solution:

Given: $\frac{2+3i}{1+i}$

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$= \frac{1}{z}$

Putting z $= \frac{2 + 3i}{1 + i}$

So, Multiplicative inverse of $\frac{2+3i}{1+i} = \frac{1}{\frac{2+3i}{1+i}} = \frac{1+i}{2+3i}$

Now, rationalizing by multiply and divide by the conjugate of (2+3i)

$= \frac{1+i}{2+3i} \times \frac{2-3i}{2-3i}$

$= \frac{(1+i)(2-3i)}{(2+3i)(2-3i)}$

Using (a – b)(a + b) = (a^2 – b^2)

$= \frac{1(2-3i)+i(2-3i)}{(2)^2-(3i)^2}$

$= \frac{2-3i+2i-3i^2}{4-9i^2}$

$= \frac{2-i-3(-1)}{4-9(-1)}$ [∵ $i^2 = -1$]

$= \frac{5-i}{4+9}$

$= \frac{5-i}{13}$

$= \frac{5}{13} - \frac{1}{13}i$

Hence, Multiplicative Inverse of $\frac{(2+3i)}{1+i}$ is $\frac{5}{13} - \frac{1}{13}i$

Question: 11 D

Find the multipli

Solution:

Given: $\frac{(1+i)(1+2i)}{(1+3i)}$

To find: Multiplicative inverse

We know that,

Multiplicative Inverse of z = z^{-1}

$= \frac{1}{z}$

Putting $z = \frac{(1+i)(1+2i)}{(1+3i)}$

So, Multiplicative inverse of $\frac{(1+i)(1+2i)}{(1+3i)} = \frac{1}{\frac{(1+i)(1+2i)}{(1+3i)}}$

$= \frac{(1+3i)}{(1+i)(1+2i)}$

We solve the above equation

$= \frac{1+3i}{1(1)+1(2i)+i(1)+i(2i)}$

$= \frac{1+3i}{1+2i+i+2i^2}$

$= \frac{1+3i}{1+3i+2(-1)}$ [$\because i^2 = -1$]

$$= \frac{1+3i}{-1+3i}$$

Now, we rationalize the above by multiplying and divide by the conjugate of (-1 + 3i)

$$= \frac{1+3i}{-1+3i} \times \frac{-1-3i}{-1-3i}$$

$= \frac{(1+3i)(-1-3i)}{(-1+3i)(-1-3i)}$ ...(i)

Now, we know that,

$(a + b)(a - b) = (a^2 - b^2)$

So, eq. (i) become

$$= \frac{1(-1-3i)+3i(-1-3i)}{(-1)^2-(3i)^2}$$

$$= \frac{-1-3i-3i-9i^2}{1-9i^2}$$

$= \frac{-1-6i-9(-1)}{1-9(-1)}$ [$\because i^2 = -1$]

$$= \frac{-1-6i+9}{1+9}$$

$$= \frac{8-6i}{10}$$

$$= \frac{2(4-3i)}{10}$$

$$= \frac{4-3i}{5}$$

$$= \frac{4}{5} - \frac{3}{5}i$$

Hence, Multiplicative inverse of $\frac{(1+i)(1+2i)}{(1+3i)} = \frac{4}{5} - \frac{3}{5}i$

Question: 12

If $a+ib = \left(\frac{1-i}{1+i}\right)^{100}$

Consider the given equation,

$$a+ib = \left(\frac{1-i}{1+i}\right)^{100}$$

Now, we rationalize

$$= \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{100}$$

[Here, we multiply and divide by the conjugate of 1 + i]

$$= \left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^{100}$$

$$= \left(\frac{1+i^2-2i}{(1+i)(1-i)}\right)^{100}$$

Using $(a + b)(a - b) = (a^2 - b^2)$

$$= \left(\frac{1 + (-1) - 2i}{(1)^2 - (i)^2}\right)^{100}$$

$$= \left(\frac{-2i}{1 - i^2}\right)^{100}$$

$$= \left(\frac{-2i}{1-(-1)}\right)^{100} [\because i^2 = -1]$$

$$= \left(\frac{-2i}{2}\right)^{100}$$

$= (-i)^{100}$

$= [(-i)^4]^{25}$

$= (i^4)^{25}$

$= (1)^{25}$

$[\because i^4 = i^2 \times i^2 = -1 \times -1 = 1]$

$(a + ib) = 1 + 0i$

On comparing both the sides, we get

$a = 1$ and $b = 0$

hence, the value of a is 1 and b is 0

Question: 13

If $x + iy = \left(\frac{1+i}{1-i}\right)^{93} - \left(\frac{1-i}{1+i}\right)^{3}$

Now, rationalizing

$$x + iy = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^{93} - \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{3}$$

$$= \left(\frac{(1+i)^2}{(1-i)(1+i)}\right)^{93} - \left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^{3}$$

In denominator, we use the identity

$(a - b)(a + b) = a^2 - b^2$

$$= \left(\frac{1 + i^2 + 2i}{(1)^2 - (i)^2}\right)^{93} - \left(\frac{1 + i^2 - 2i}{(1)^2 - (i)^2}\right)^{3}$$

$$= \left(\frac{1 + (-1) + 2i}{1 - i^2}\right)^{93} - \left(\frac{1 + (-1) - 2i}{1 - i^2}\right)^{3}$$

$$= \left(\frac{2i}{1 - (-1)}\right)^{93} - \left(\frac{-2i}{1 - (-1)}\right)^{3}$$

$$= \left(\frac{2i}{2}\right)^{93} - \left(\frac{-2i}{2}\right)^{3}$$

$= (i)^{93} - (-i)^3$

$= (i)^{92+1} - [-(i)^3]$

$= [(i)^{92}(i)] - [-(i^2 \times i)]$

$= [(i^4)^{23}(i)] – [- (-i)]$

$= [(1)^{23}(i)] – i$

= i - i

x + iy = 0

∴ x = 0 and y = 0

Question: 14

If $x + iy = \frac{a+ib}{a-ib}$

Now, rationalizing

$$x + iy = \frac{a+ib}{a-ib} \times \frac{a+ib}{a+ib}$$

$$= \frac{(a+ib)(a+ib)}{(a-ib)(a+ib)}$$

$$= \frac{a(a+ib)+ib(a+ib)}{(a)^2-(ib)^2}$$

$[(a – b)(a + b) = a^2 – b^2]$

$$= \frac{a^2+iab+iab+i^2b^2}{a^2-i^2b^2}$$

$$= \frac{a^2+iab+iab+(-1)b^2}{a^2-(-1)b^2}\ [i^2 = -1]$$

$$x + iy = \frac{a^2+2iab-b^2}{a^2+b^2}$$

$$x + iy = \frac{(a^2-b^2)}{a^2+b^2} + i\frac{2ab}{a^2+b^2}$$

On comparing both the sides, we get

$$x = \frac{(a^2-b^2)}{a^2+b^2}\ \&\ y = \frac{2ab}{a^2+b^2}$$

Now, we have to prove that $x^2 + y^2 = 1$

Taking LHS,

$x^2 + y^2$

Putting the value of x and y, we get

$$\left[\frac{(a^2-b^2)}{a^2+b^2}\right]^2 + \left[\frac{2ab}{a^2+b^2}\right]^2$$

$$= \frac{1}{(a^2+b^2)^2}[(a^2-b^2)^2+(2ab)^2]$$

$$= \frac{1}{(a^2+b^2)^2}[a^4+b^4-2a^2b^2+4a^2b^2]$$

$$= \frac{1}{(a^2+b^2)^2}[a^4+b^4+2a^2b^2]$$

$$= \frac{1}{(a^2+b^2)^2}[(a^2+b^2)^2]$$

= 1

= RHS

Hence Proved

Question: 15

If $a + ib = \frac{c+i}{c-i}$

Now, rationalizing

$$a + ib = \frac{c+i}{c-i} \times \frac{c+i}{c+i}$$

$$= \frac{(c+i)(c+i)}{(c-i)(c+i)}$$

$$= \frac{(c+i)^2}{(c)^2 - (i)^2}$$

$[(a - b)(a + b) = a^2 - b^2]$

$$= \frac{c^2 + 2ic + i^2}{c^2 - i^2}$$

$a + ib = \frac{c^2+2ic+(-1)}{c^2-(-1)}$ $[i^2 = -1]$

$$a + ib = \frac{c^2 + 2ic - 1}{c^2 + 1}$$

$$a + ib = \frac{(c^2 - 1)}{c^2 + 1} + i\frac{2c}{c^2 + 1}$$

On comparing both the sides, we get

$$a = \frac{(c^2 - 1)}{c^2 + 1} \ \& \ b = \frac{2c}{c^2 + 1}$$

Now, we have to prove that $a^2 + b^2 = 1$

Taking LHS,

$a^2 + b^2$

Putting the value of a and b, we get

$$\left[\frac{(c^2 - 1)}{c^2 + 1}\right]^2 + \left[\frac{2c}{c^2 + 1}\right]^2$$

$$= \frac{1}{(c^2 + 1)^2}[(c^2 - 1)^2 + (2c)^2]$$

$$= \frac{1}{(c^2 + 1)^2}[c^4 + 1 - 2c^2 + 4c^2]$$

$$= \frac{1}{(c^2 + 1)^2}[c^4 + 1 + 2c^2]$$

$$= \frac{1}{(c^2 + 1)^2}[(c^2 + 1)^2]$$

= 1

= RHS

Now, we have to prove $\frac{b}{a} = \frac{2c}{c^2-1}$

Taking LHS, $\frac{b}{a}$

Putting the value of a and b, we get

$$\frac{b}{a}=\frac{\frac{2c}{c^2+1}}{\frac{(c^2-1)}{c^2+1}}=\frac{2c}{c^2+1}\times\frac{c^2+1}{c^2-1}=\frac{2c}{c^2-1}=RHS$$

Hence Proved

Question: 16

Show that

Solution:

To show: $(1-i)^n\left(1-\frac{1}{i}\right)^n=2^n$

Taking LHS,

$(1-i)^n\left(1-\frac{1}{i}\right)^n$

$=(1-i)^n\left(1-\frac{1}{i}\times\frac{i}{i}\right)^n$ [rationalize]

$=(1-i)^n\left(1-\frac{i}{i^2}\right)^n$

$=(1-i)^n\left(1-\frac{i}{-1}\right)^n$ [$\because i^2 = -1$]

$= (1 - i)^n(1 + i)^n$

$= [(1 - i)(1 + i)]^n$

$= [(1)^2 - (i)^2]^n$ [$(a + b)(a - b) = a^2 - b^2$]

$= (1 - i^2)^n$

$= [1 - (-1)]^n$[$\because i^2 = -1$]

$= (2)^n$

$= 2^n$

= RHS

Hence Proved

Question: 17

Find the smallest

Solution:

Given: $(1 + i)^{2n} = (1 - i)^{2n}$

Consider the given equation,

$(1 + i)^{2n} = (1 - i)^{2n}$

$\Rightarrow\frac{(1+i)^{2n}}{(1-i)^{2n}}=1$

$\Rightarrow\left(\frac{1+i}{1-i}\right)^{2n}=1$

Now, rationalizing by multiply and divide by the conjugate of $(1 - i)$

$\left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^{2n} = 1$

$\Rightarrow \left(\frac{(1+i)^2}{(1-i)(1+i)}\right)^{2n} = 1$

$\Rightarrow \left[\frac{1+i^2+2i}{(1)^2-(i)^2}\right]^{2n} = 1$

$[(a + b)^2 = a^2 + b^2 + 2ab$ & $(a - b)(a + b) = (a^2 - b^2)]$

$\Rightarrow \left[\frac{1+(-1)+2i}{1-(-1)}\right]^{2n} = 1$ $[i^2 = -1]$

$\Rightarrow \left[\frac{2i}{2}\right]^{2n} = 1$

$\Rightarrow (i)^{2n} = 1$

Now, $i^{2n} = 1$ is possible if n = 2 because $(i)^{2(2)} = i^4 = (-1)^4 = 1$

So, the smallest positive integer n = 2

Question: 18

Prove that (x + 1

Solution:

To Prove:

$(x + 1 + i) (x + 1 - i) (x - 1 + i) (x - 1 - i) = (x^4 + 4)$

Taking LHS

$(x + 1 + i) (x + 1 - i) (x - 1 + i) (x - 1 - i)$

$= [(x + 1) + i][(x + 1) - i][(x - 1) + i][(x - 1) - i]$

Using $(a - b)(a + b) = a^2 - b^2$

$$\underbrace{[(x + 1) + i][(x + 1) - i]}_{a = x + 1 \;\&\; b = i}\underbrace{[(x - 1) + i][(x - 1) - i]}_{a = x - 1 \;\&\; b = i}$$

$= [(x + 1)^2 - (i)^2] [(x - 1)^2 - (i)^2]$

$= [x^2 + 1 + 2x - i^2](x^2 + 1 - 2x - i^2]$

$= [x^2 + 1 + 2x - (-1)](x^2 + 1 - 2x - (-1)]$ $[\because i^2 = -1]$

$= [x^2 + 2 + 2x][x^2 + 2 - 2x]$

Again, using $(a - b)(a + b) = a^2 - b^2$

Now, $a = x^2 + 2$ and $b = 2x$

$= [(x^2 + 2)^2 - (2x)^2]$

$= [x^4 + 4 + 2(x^2)(2) - 4x^2]$ $[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$= [x^4 + 4 + 4x^2 - 4x^2]$

$= x^4 + 4$

= RHS

∴ LHS = RHS

Hence Proved

Question: 19

If a = (cosθ + i

Solution:

Given: a = cosθ + isinθ

To prove: $\frac{1+a}{1-a} = \left(\cot\frac{\theta}{2}\right) i$

Taking LHS,

$$\frac{1+a}{1-a}$$

Putting the value of a, we get

$$= \frac{1+\cos\theta + i\sin\theta}{1-(\cos\theta + i\sin\theta)}$$

$$= \frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta}$$

We know that,

1 + cos2θ = 2cos^{2}θ

or $1+\cos\theta = 2\cos^2\frac{\theta}{2}$

and $1-\cos\theta = 2\sin^2\frac{\theta}{2}$

Using the above two formulas

$$= \frac{2\cos^2\frac{\theta}{2} + i\sin\theta}{2\sin^2\frac{\theta}{2} - i\sin\theta}$$

Using, $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$= \frac{2\cos^2\frac{\theta}{2} + i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right]}{2\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right]}$$

$$= \cot\frac{\theta}{2}\left[\frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}}\right] \left[\because \frac{\cos\theta}{\sin\theta} = \cot\theta\right]$$

Rationalizing by multiply and divide by the conjugate of $\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}$

$$= \left(\cot\frac{\theta}{2}\right)\left[\frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}} \times \frac{\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}}{\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}}\right]$$

$$= \left(\cot\frac{\theta}{2}\right)\frac{\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)\left(\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right)}{\left(\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right)\left(\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right)}$$

$$= \left(\cot\frac{\theta}{2}\right)\frac{\left(\cos\frac{\theta}{2}\right)\left(\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right) + i\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} + i\cos\frac{\theta}{2}\right)}{\left(\sin\frac{\theta}{2}\right)^2 - \left(i\cos\frac{\theta}{2}\right)^2}$$

$$= \left(\cot\frac{\theta}{2}\right)\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2} + i\cos^2\frac{\theta}{2} + i\sin^2\frac{\theta}{2} + i^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} - i^2\cos^2\frac{\theta}{2}}$$

Putting $i^2 = -1$, we get

$$= \left(\cot\frac{\theta}{2}\right)\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2} + i\cos^2\frac{\theta}{2} + i\sin^2\frac{\theta}{2} + (-1)\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} - (-1)\cos^2\frac{\theta}{2}}$$

$$= \left(\cot\frac{\theta}{2}\right)\frac{\cos\frac{\theta}{2}\sin\frac{\theta}{2} + i\,(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}) - \sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}$$

We know that,

$\cos^2\theta + \sin^2\theta = 1$

$$= \left(\cot\frac{\theta}{2}\right)\left[\frac{i\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)}{1}\right]$$

$$= \cot\frac{\theta}{2}\,(i)$$

= RHS

Hence Proved

Question: 20

If z_1

Solution:

Given: $z_1 = (2 - i)$ and $z_2 = (1 + i)$

To find: $\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right|$

Consider,

$$\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$$

Putting the value of z_1 and z_2, we get

$$= \left|\frac{2 - i + 1 + i + 1}{2 - i - (1 + i) + i}\right|$$

$$= \left|\frac{4}{2 - i - 1 - i + i}\right|$$

$$= \left|\frac{4}{1 - i}\right|$$

Now, rationalizing by multiply and divide by the conjugate of 1 – i

$$= \left|\frac{4}{1 - i} \times \frac{1 + i}{1 + i}\right|$$

$$= \left|\frac{4(1 + i)}{(1 - i)(1 + i)}\right|$$

$= \left|\frac{4(1+i)}{(1)^2-(i)^2}\right|$ [(a – b)(a + b) = a^2 – b^2]

$= \left|\frac{4(1+i)}{1-i^2}\right|$

$= \left|\frac{4(1+i)}{1-(-1)}\right|$ [Putting i^2 = -1]

$= \left|\frac{4(1+i)}{2}\right|$

= |2(1 + i)|

= |2 + 2i|

Now, we have to find the modulus of (2 + 2i)

So, $|z| = |2 + 2i| = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Hence, the value of $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right| = 2\sqrt{2}$

Question: 21 A

Find the real val

Solution:

(1 – i) x + (1 + i) y = 1 – 3i

⇒ x – ix + y + iy = 1 – 3i

⇒ (x + y) – i(x – y) = 1 – 3i

Comparing the real parts, we get

x + y = 1 ...(i)

Comparing the imaginary parts, we get

x – y = -3 ...(ii)

Solving eq. (i) and (ii) to find the value of x and y

Adding eq. (i) and (ii), we get

x + y + x – y = 1 + (-3)

⇒ 2x = 1 – 3

⇒ 2x = -2

⇒ x = -1

Putting the value of x = -1 in eq. (i), we get

(-1) + y = 1

⇒ y = 1 + 1

⇒ y = 2

Question: 21 B

Find the real val

Solution:

x(3 – 2i) + iy(3 – 2i) = 12 + 5i

⇒ 3x – 2ix + 3iy – $2i^2$y = 12 + 5i

⇒ 3x + i(-2x + 3y) – 2(-1)y = 12 + 5i [∵ i^2 = -1]

⇒ $3x + i(-2x + 3y) + 2y = 12 + 5i$

⇒ $(3x + 2y) + i(-2x + 3y) = 12 + 5i$

Comparing the real parts, we get

$3x + 2y = 12$ …(i)

Comparing the imaginary parts, we get

$–2x + 3y = 5$ …(ii)

Solving eq. (i) and (ii) to find the value of x and y

Multiply eq. (i) by 2 and eq. (ii) by 3, we get

$6x + 4y = 24$ …(iii)

$–6x + 9y = 15$ …(iv)

Adding eq. (iii) and (iv), we get

$6x + 4y – 6x + 9y = 24 + 15$

⇒ $13y = 39$

⇒ $y = 3$

Putting the value of $y = 3$ in eq. (i), we get

$3x + 2(3) = 12$

⇒ $3x + 6 = 12$

⇒ $3x = 12 – 6$

⇒ $3x = 6$

⇒ $x = 2$

Hence, the value of $x = 2$ and $y = 3$

Question: 21 C

Find the real val

Solution:

Given: $x + 4yi = ix + y + 3$

or $x + 4yi = ix + (y + 3)$

Comparing the real parts, we get

$x = y + 3$

or $x – y = 3$ …(i)

Comparing the imaginary parts, we get

$4y = x$ …(ii)

Putting the value of $x = 4y$ in eq. (i), we get

$4y – y = 3$

⇒ $3y = 3$

⇒ $y = 1$

Putting the value of $y = 1$ in eq. (ii), we get

$x = 4(1) = 4$

Hence, the value of $x = 4$ and $y = 1$

Question: 21 D

Find the real val

Solution:

Given: $(1 + i) y^2 + (6 + i) = (2 + i)x$

Consider, $(1 + i) y^2 + (6 + i) = (2 + i)x$

$\Rightarrow y^2 + iy^2 + 6 + i = 2x + ix$

$\Rightarrow (y^2 + 6) + i(y^2 + 1) = 2x + ix$

Comparing the real parts, we get

$y^2 + 6 = 2x$

$\Rightarrow 2x - y^2 - 6 = 0$ …(i)

Comparing the imaginary parts, we get

$y^2 + 1 = x$

$\Rightarrow x - y^2 - 1 = 0$ …(ii)

Subtracting the eq. (ii) from (i), we get

$2x - y^2 - 6 - (x - y^2 - 1) = 0$

$\Rightarrow 2x - y^2 - 6 - x + y^2 + 1 = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

Putting the value of x = 5 in eq. (i), we get

$2(5) - y^2 - 6 = 0$

$\Rightarrow 10 - y^2 - 6 = 0$

$\Rightarrow -y^2 + 4 = 0$

$\Rightarrow - y^2 = -4$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \sqrt{4}$

$\Rightarrow y = \pm 2$

Hence, the value of $x = 5$ and $y = \pm 2$

Question: 21 E

Find the real val

Solution:

Given:

$$\frac{x + 3i}{2 + iy} = (1 - i)$$

$\Rightarrow x + 3i = (1 - i)(2 + iy)$

$\Rightarrow x + 3i = 1(2 + iy) - i(2 + iy)$

$\Rightarrow x + 3i = 2 + iy - 2i - i^2y$

$\Rightarrow x + 3i = 2 + i(y - 2) - (-1)y$ $[i^2 = -1]$

$\Rightarrow x + 3i = 2 + i(y - 2) + y$

$\Rightarrow x + 3i = (2 + y) + i(y - 2)$

Comparing the real parts, we get

x = 2 + y

⇒ x – y = 2 ...(i)

Comparing the imaginary parts, we get

3 = y – 2

⇒ y = 3 + 2

⇒ y = 5

Putting the value of y = 5 in eq. (i), we get

x – 5 = 2

⇒ x = 2 + 5

⇒ x = 7

Hence, the value of x = 7 and y = 5

Question: 21 F

Find the real val

Solution:

Consider,

$$\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$$

$$=\frac{x+xi-2i}{3+i}+\frac{2y-3iy+i}{3-i}=i$$

Taking LCM

$$\Rightarrow \frac{(x+xi-2i)(3-i)+(2y-3iy+i)(3+i)}{(3+i)(3-i)}=i$$

$$\Rightarrow \frac{3x+3xi-6i-xi-xi^2+2i^2+6y-9iy+3i+2iy-3i^2y+i^2}{(3)^2-(i)^2}=i$$

Putting i^2 = -1

$$\Rightarrow \frac{3x+2xi-6i-x(-1)+2(-1)+6y-7iy+3i-3(-1)y+(-1)}{9-(-1)}=i$$

$$\Rightarrow \frac{3x+2xi-6i+x-2+6y-7iy+3i+3y-1}{9+1}=i$$

$$\Rightarrow \frac{4x+2xi-3i-3+9y-7iy}{10}=i$$

⇒ 4x + 2xi – 3i – 3 + 9y – 7iy = 10i

⇒ (4x – 3 + 9y) + i(2x – 3 – 7y) = 10i

Comparing the real parts, we get

4x – 3 + 9y = 0

⇒ 4x + 9y = 3 ...(i)

Comparing the imaginary parts, we get

2x – 3 – 7y = 10

⇒ 2x – 7y = 10 + 3

⇒ 2x – 7y = 13 ...(ii)

Multiply the eq. (ii) by 2, we get

$4x - 14y = 26$ …(iii)

Subtracting eq. (i) from (iii), we get

$4x - 14y - (4x + 9y) = 26 - 3$

$\Rightarrow 4x - 14y - 4x - 9y = 23$

$\Rightarrow -23y = 23$

$\Rightarrow y = -1$

Putting the value of $y = -1$ in eq. (i), we get

$4x + 9(-1) = 3$

$\Rightarrow 4x - 9 = 3$

$\Rightarrow 4x = 12$

$\Rightarrow x = 3$

Hence, the value of $x = 3$ and $y = -1$

Question: 22

Find the real val

Solution:

Given: $(x - iy)(3 + 5i)$ is the conjugate of $(-6 - 24i)$

We know that,

Conjugate of $-6 - 24i = -6 + 24i$

$\therefore$ According to the given condition,

$(x - iy)(3 + 5i) = -6 + 24i$

$\Rightarrow x(3 + 5i) - iy(3 + 5i) = -6 + 24i$

$\Rightarrow 3x + 5ix - 3iy - 5i^2y = -6 + 24i$

$\Rightarrow 3x + i(5x - 3y) - 5(-1)y = -6 + 24i$ [$\because i^2 = -1$]

$\Rightarrow 3x + i(5x - 3y) + 5y = -6 + 24i$

$\Rightarrow (3x + 5y) + i(5x - 3y) = -6 + 24i$

Comparing the real parts, we get

$3x + 5y = -6$ …(i)

Comparing the imaginary parts, we get

$5x - 3y = 24$ …(ii)

Solving eq. (i) and (ii) to find the value of x and y

Multiply eq. (i) by 5 and eq. (ii) by 3, we get

$15x + 25y = -30$ …(iii)

$15x - 9y = 72$ …(iv)

Subtracting eq. (iii) from (iv), we get

$15x - 9y - 15x - 25y = 72 - (-30)$

$\Rightarrow -34y = 72 + 30$

$\Rightarrow -34y = 102$

$\Rightarrow y = -3$

Putting the value of y = -3 in eq. (i), we get

$3x + 5(-3) = -6$

$\Rightarrow 3x - 15 = -6$

$\Rightarrow 3x = -6 + 15$

$\Rightarrow 3x = 9$

$\Rightarrow x = 3$

Hence, the value of x = 3 and y = -3

Question: 23

Find the real val

Solution:

Let $z_1 = -3 + iyx^2$

So, the conjugate of z_1 is

$\bar{z}_1 = -3 - iyx^2$

and $z_2 = x^2 + y + 4i$

So, the conjugate of z_2 is

$\bar{z}_2 = x^2 + y - 4i$

Given that: $\bar{z}_1 = z_2$ & $z_1 = \bar{z}_2$

Firstly, consider $\bar{z}_1 = z_2$

$- 3 - iyx^2 = x^2 + y + 4i$

$\Rightarrow x^2 + y + 4i + iyx^2 = -3$

$\Rightarrow x^2 + y + i(4 + yx^2) = -3 + 0i$

Comparing the real parts, we get

$x^2 + y = -3$ ...(i)

Comparing the imaginary parts, we get

$4 + yx^2 = 0$

$\Rightarrow x^2y = -4$ ...(ii)

Now, consider $z_1 = \bar{z}_2$

$-3 + iyx^2 = x^2 + y - 4i$

$\Rightarrow x^2 + y - 4i - iyx^2 = - 3$

$\Rightarrow x^2 + y + i(-4i - yx^2) = - 3 + 0i$

Comparing the real parts, we get

$x^2 + y = -3$

Comparing the imaginary parts, we get

$-4 - yx^2 = 0$

$\Rightarrow x^2y = -4$

Now, we will solve the equations to find the value of x and y

From eq. (i), we get

$x^2 = -3 - y$

Putting the value of x^2 in eq. (ii), we get

$(-3 - y)(y) = -4$

$\Rightarrow -3y - y^2 = -4$

$\Rightarrow y^2 + 3y = 4$

$\Rightarrow y^2 + 3y - 4 = 0$

$\Rightarrow y^2 + 4y - y - 4 = 0$

$\Rightarrow y(y + 4) - 1(y + 4) = 0$

$\Rightarrow (y - 1)(y + 4) = 0$

$\Rightarrow y - 1 = 0$ or $y + 4 = 0$

$\Rightarrow y = 1$ or $y = -4$

When $y = 1$, then

$x^2 = -3 - 1$

$= -4$ [It is not possible]

When $y = -4$, then

$x^2 = -3 - (-4)$

$= -3 + 4$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \sqrt{1}$

$\Rightarrow x = \pm 1$

Hence, the values of $x = \pm 1$ and $y = -4$

Question: 24

If $z = (2 - 3i)$,

Solution:

Given: $z = 2 - 3i$

To Prove: $z^2 - 4z + 13 = 0$

Taking LHS, $z^2 - 4z + 13$

Putting the value of $z = 2 - 3i$, we get

$(2 - 3i)^2 - 4(2 - 3i) + 13$

$= 4 + 9i^2 - 12i - 8 + 12i + 13$

$= 9(-1) + 9$

$= -9 + 9$

$= 0$

$=$ RHS

Hence, $z^2 - 4z + 13 = 0$ …(i)

Now, we have to deduce $4z^3 - 3z^2 + 169$

Now, we will expand $4z^3 - 3z^2 + 169$ in this way so that we can use the above equation i.e. $z^2 - 4z + 13$

$= 4z^3 - 16z^2 + 13z^2 + 52z - 52z + 169$

Re – arrange the terms,

$= 4z^3 - 16z^2 + 52z + 13z^2 - 52z + 169$

$= 4z(z^2 - 4z + 13) + 13(z^2 - 4z + 13)$

= 4z(0) + 13(0) [from eq. (i)]

= 0

= RHS

Hence Proved

Question: 25

If (1 + i)z = (1

Solution:

Let z = x + iy

Then,

$\bar{z} = x - iy$

Now, Given: (1 + i)z = (1 – i) $\bar{z}$

Therefore,

(1 + i)(x + iy) = (1 – i)(x – iy)

$x + iy + xi + i^2y = x - iy - xi + i^2y$

We know that $i^2 = -1$, therefore,

x + iy + ix – y = x – iy – ix – y

2xi + 2yi = 0

x = -y

Now, as x = -y

z = - $\bar{z}$

Hence, Proved.

Question: 26

If $\frac{z-1}{z+1}$ is purely imaginary number

Let z = x + iy

So, $\frac{z-1}{z+1} = \frac{x+iy-1}{x+iy+1}$

$$= \frac{(x-1)+iy}{(x+1)+iy}$$

Now, rationalizing the above by multiply and divide by the conjugate of [(x + 1) + iy]

$$= \frac{(x-1)+iy}{(x+1)+iy} \times \frac{(x+1)-iy}{(x+1)-iy}$$

$$= \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]}$$

Using $(a - b)(a + b) = (a^2 - b^2)$

$$= \frac{(x-1)[(x+1)-iy]+iy[(x+1)-iy]}{(x+1)^2-(iy)^2}$$

$$= \frac{(x-1)(x+1)+(x-1)(-iy)+iy(x+1)+(iy)(-iy)}{x^2+1+2x-i^2y^2}$$

$$= \frac{x^2-1-ixy+iy+ixy+iy-i^2y^2}{x^2+1+2x-i^2y^2}$$

Putting $i^2 = -1$

$$= \frac{x^2-1+2iy-(-1)y^2}{x^2+1+2x-(-1)y^2}$$

$$= \frac{x^2-1+2iy+y^2}{x^2+1+2x+y^2}$$

$$= \frac{x^2-1+y^2}{x^2+1+2x+y^2} + i\frac{2y}{x^2+1+2x+y^2}$$

Since, the number is purely imaginary it means real part is 0

$$\therefore \frac{x^2-1+y^2}{x^2+1+2x+y^2} = 0$$

$\Rightarrow x^2 + y^2 - 1 = 0$

$\Rightarrow x^2 + y^2 = 1$

$\Rightarrow \sqrt{x^2+y^2} = \sqrt{1}$

$\Rightarrow \sqrt{x^2+y^2} = 1$

$\therefore |z| = 1$

Question: 27

Solve the system

Solution:

Given: $Re(z^2) = 0$ and $|z| = 2$

Let $z = x + iy$

$\therefore |z| = \sqrt{x^2+y^2}$

$\Rightarrow 2 = \sqrt{x^2+y^2}$ [given]

Squaring both the sides, we get

$x^2 + y^2 = 4$ …(i)

Since, $z = x + iy$

$\Rightarrow z^2 = (x + iy)^2$

$\Rightarrow z^2 = x^2 + i^2y^2 + 2ixy$

$\Rightarrow z^2 = x^2 + (-1)y^2 + 2ixy$

$\Rightarrow z^2 = x^2 - y^2 + 2ixy$

It is given that $Re(z^2) = 0$

$\Rightarrow x^2 - y^2 = 0$ …(ii)

Adding eq. (i) and (ii), we get

$x^2 + y^2 + x^2 – y^2 = 4 + 0$

$\Rightarrow 2x^2 = 4$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

Putting the value of $x^2 = 2$ in eq. (i), we get

$2 + y^2 = 4$

$\Rightarrow y^2 = 2$

$\Rightarrow y = \pm\sqrt{2}$

Hence, $z = \sqrt{2} \pm i\sqrt{2}, -\sqrt{2} \pm i\sqrt{2}$

Question: 28

Find the complex

Solution:

Given: $|z| = z + 1 + 2i$

Consider,

$|z| = (z + 1) + 2i$

Squaring both the sides, we get

$|z|^2 = [(z + 1) + (2i)]^2$

$\Rightarrow |z|^2 = |z + 1|^2 + 4i^2 + 2(2i)(z + 1)$

$\Rightarrow |z|^2 = |z|^2 + 1 + 2z + 4(-1) + 4i(z + 1)$

$\Rightarrow 0 = 1 + 2z – 4 + 4i(z + 1)$

$\Rightarrow 2z – 3 + 4i(z + 1) = 0$

Let $z = x + iy$

$\Rightarrow 2(x + iy) – 3 + 4i(x + iy + 1) = 0$

$\Rightarrow 2x + 2iy – 3 + 4ix + 4i^2y + 4i = 0$

$\Rightarrow 2x + 2iy – 3 + 4ix + 4(-1)y + 4i = 0$

$\Rightarrow 2x – 3 – 4y + i(4x + 2y + 4) = 0$

Comparing the real part, we get

$2x – 3 – 4y = 0$

$\Rightarrow 2x – 4y = 3$ …(i)

Comparing the imaginary part, we get

$4x + 2y + 4 = 0$

$\Rightarrow 2x + y + 2 = 0$

$\Rightarrow 2x + y = -2$ …(ii)

Subtracting eq. (ii) from (i), we get

$2x – 4y – (2x + y) = 3 – (-2)$

$\Rightarrow 2x – 4y – 2x – y = 3 + 2$

$\Rightarrow -5y = 5$

$\Rightarrow y = -1$

Putting the value of y = -1 in eq. (i), we get

2x – 4(-1) = 3

⇒ 2x + 4 = 3

⇒ 2x = 3 – 4

⇒ 2x = - 1

$\Rightarrow x = -\frac{1}{2}$

Hence, the value of z = x + iy

$= -\frac{1}{2} + i(-1)$

$z = -\frac{1}{2} - i$

Exercise : 5C

Question: 1

Express each of t

Solution:

(i) Let $z = \frac{1}{4+3i} = \frac{1}{4+3i} \times \frac{4-3i}{4-3i}$

$= \frac{4-3i}{16+9} = \frac{4}{25} - \frac{3}{25}i$

$\Rightarrow \bar{z} = \frac{4}{25} + \frac{3}{25}i$

(ii) Let z = $(2 + 3i)^2$ = (2 + 3i)(2 + 3i)

= 4 + 6i + 6i + $9i^2$

= 4 + 12i + $9i^2$

= 4 + 12i - 9

= - 5 + 12i

$\bar{z} = -5 - 12i$

(iii) Let $z = \frac{(2-i)}{(1-2i)^2} = \frac{(2-i)}{1+4i^2-4i}$

$= \frac{(2-i)}{1-4i-4} = \frac{2-i}{-3-4i}$

$= \frac{2-i}{-3-4i} \times \frac{-3+4i}{-3+4i} = \frac{(2-i)(-3+4i)}{9+16}$

$= \frac{-6+11i-4i^2}{25} = \frac{-2+11i}{25}$

$= \frac{-2}{25} + \frac{11}{25}i$

$\bar{z} = \frac{-2}{25} - \frac{11}{25}i$

(iv) Let $z = \frac{(1+i)(1+2i)}{(1+3i)} = \frac{1+i+2i+2i^2}{(1+3i)}$

$$= \frac{1 + 3i - 2}{1 + 3i} = \frac{-1 + 3i}{1 + 3i}$$

$$= \frac{-1 + 3i}{1 + 3i} \times \frac{1 - 3i}{1 - 3i} = \frac{-1 + 3i + 3i - 9i^2}{1 - 9i^2} = \frac{-1 + 6i + 9}{1 + 9} = \frac{8 + 6i}{10}$$

$$= \frac{8}{10} + \frac{6}{10}i$$

$$\bar{z} = \frac{8}{10} - \frac{6}{10}i$$

(v) Let $z = \left(\frac{1 + 2i}{2 + i}\right)^2 = \frac{1 + 4i^2 + 2i}{4 + i^2 + 4i} = \frac{1 - 4 + 2i}{4 - 1 + 4i} = \frac{-3 + 2i}{3 + 4i}$

$$= \frac{-3 + 2i}{3 + 4i} \times \frac{3 - 4i}{3 - 4i}$$

$$= \frac{-9 + 12i + 6i - 8i^2}{9 + 16} = \frac{-9 + 18i + 8}{25} = \frac{-1 + 18i}{25}$$

$$= \frac{-1}{25} + \frac{18}{25}i$$

$$\bar{z} = \frac{-1}{25} - \frac{18}{25}i$$

(vi) Let $z = \frac{(2 + i)}{(3 - i)(1 + 2i)} = \frac{2 + i}{3 + 6i - 1 - 2i^2}$

$$= \frac{2 + i}{3 + 6i - 1 + 2} = \frac{2 + i}{4 + 6i}$$

$$= \frac{2 + i}{4 + 6i} \times \frac{4 - 6i}{4 - 6i}$$

$$= \frac{8 - 12i + 4i - 6i^2}{16 + 36}$$

$$= \frac{8 - 8i + 6}{52}$$

$$= \frac{14 - 8i}{52}$$

$$= \frac{14}{52} - \frac{8}{52}i$$

$$\bar{z} = \frac{14}{52} + \frac{8}{52}i$$

Question: 2

Express each of t

Solution:

(i) Let $z = \frac{1 + 2i}{1 - 3i}$

$$= \frac{1 + 2i}{1 - 3i} \times \frac{1 + 3i}{1 + 3i} = \frac{1 + 3i + 2i + 6i^2}{1 - 9i^2}$$

$$= \frac{1 + 5i + 6i^2}{1 + 9} = \frac{-5 + 5i}{10}$$

$$z = \frac{-1}{2} + \frac{1}{2}i$$

$\Rightarrow \bar{z} = \frac{-1}{2} - \frac{1}{2}i$

$\Rightarrow |z|^2 = \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\therefore$ the multiplicative inverse of $\frac{1+2i}{1-3i}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\frac{-1}{2} - \frac{1}{2}i}{\frac{1}{2}} = -1 - i$$

(ii) Let $z = \frac{1+7i}{(2-i)^2}$

$$= \frac{1 + 7i}{4 + i^2 - 4i} = \frac{1 + 7i}{4 - 1 - 4i} = \frac{1 + 7i}{3 - 4i}$$

$$= \frac{1 + 7i}{3 - 4i} \times \frac{3 + 4i}{3 + 4i}$$

$$= \frac{3 + 4i + 21i + 28i^2}{9 + 16} = \frac{3 + 25i - 28}{25} = \frac{-25 + 25i}{25}$$

z = - 1 + i

$\Rightarrow \bar{z} = -1 - i$

$\Rightarrow |z|^2 = (-1)^2 + (1)^2 = 1 + 1 = 2$

$\therefore$ the multiplicative inverse of $\frac{1+7i}{(2-i)^2}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{-1 - i}{2} = \frac{-1}{2} - \frac{1}{2}i$$

(iii) Let $z = \frac{-4}{(1 + i\sqrt{3})}$

$$= \frac{-4}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$$

$$= \frac{-4 + i4\sqrt{3}}{1 + 3} = \frac{-4 + i4\sqrt{3}}{4}$$

$$= -1 + i\sqrt{3}$$

Z = - 1 + i√3

$\Rightarrow \bar{z} = -1 - i\sqrt{3}$

$\Rightarrow |z|^2 = (-1)^2 + (\sqrt{3})^2 = 1 + 3 = 4$

$\therefore$ the multiplicative inverse of $\frac{-4}{(1 + i\sqrt{3})}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{-1 + i\sqrt{3}}{4} = \frac{-1}{4} + \frac{i\sqrt{3}}{4}$$

Question: 3

If $(x + iy)^3$

Solution:

Given that, $(x + iy)^3 = (u + iv)$

$\Rightarrow x^3 + (iy)^3 + 3x^2iy + 3xi^2y^2 = u + iv$

$\Rightarrow x^3 - iy^3 + 3x^2iy - 3xy^2 = u + iv$

$\Rightarrow x^3 - 3xy^2 + i(3x^2y - y^3) = u + iv$

On equating real and imaginary parts, we get

$U = x^3 - 3xy^2$ and $v = 3x^2y - y^3$

Now , $\frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y}$

$= \frac{x(x^2 - 3y^2)}{x} + \frac{y(3x^2 - y^2)}{y}$

$= x^2 - 3y^2 + 3x^2 - y^2$

$= 4x^2 - 4y^2$

$= 4(x^2 - y^2)$

Hence, $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

Question: 4

If $(x + iy)^1$

Solution:

Given that, $(x + iy)^{1/3} = (a + ib)$

$\Rightarrow (x + iy) = (a + ib)^3$

$\Rightarrow (a + ib)^3 = x + iy$

$\Rightarrow a^3 + (ib)^3 + 3a^2ib + 3ai^2b^2 = x + iy$

$\Rightarrow a^3 - ib^3 + 3a^2ib - 3ab^2 = x + iy$

$\Rightarrow a^3 - 3ab^2 + i(3a^2b - b^3) = x + iy$

On equating real and imaginary parts, we get

$x = a^3 - 3ab^2$ and $y = 3a^2b - b^3$

Now , $\frac{x}{a} + \frac{y}{b} = \frac{a^3 - 3ab^2}{a} + \frac{3a^2b - b^3}{b}$

$= \frac{a(a^2 - 3b^2)}{a} + \frac{b(3a^2 - b^2)}{b}$

$= a^2 - 3b^2 + 3a^2 - b^2$

$= 4a^2 - 4b^2$

$= 4(a^2 - b^2)$

Hence, $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

Question: 5

Express (1 – 2i)<

Solution:

We have, $(1 - 2i)^{-3}$

$\Rightarrow \frac{1}{(1-2i)^3} = \frac{1}{1 - 8i^3 - 6i + 12i^2} = \frac{1}{1 + 8i - 6i - 12} = \frac{1}{2i - 11}$

$\Rightarrow \dfrac{1}{-11 + 2i}$

$= \dfrac{1}{-11 + 2i} \times \dfrac{-11-2i}{-11-2i}$

$= \dfrac{-11-2i}{(-11)^2-(2i)^2} = \dfrac{-11-2i}{121 + 4}$

$= \dfrac{-11-2i}{125}$

$= \dfrac{-11}{125} - \dfrac{2i}{125}$

Question: 6

Find real values

Solution:

We have, $(x^4 + 2xi) - (3x^2 + iy) = (3 - 5i) + (1 + 2iy)$.

$\Rightarrow x^4 + 2xi - 3x^2 + iy = 3 - 5i + 1 + 2iy$

$\Rightarrow (x^4 - 3x^2) + i(2x - y) = 4 + i(2y - 5)$

On equating real and imaginary parts, we get

$x^4 - 3x^2 = 4$ and $2x - y = 2y - 5$

$\Rightarrow x^4 - 3x^2 - 4 = 0$ eq(i) and $2x - y - 2y + 5 = 0$ eq(ii)

Now from eq (i), $x^4 - 3x^2 - 4 = 0$

$\Rightarrow x^4 - 4x^2 + x^2 - 4 = 0$

$\Rightarrow x^2 (x^2 - 4) + 1(x^2 - 4) = 0$

$\Rightarrow (x^2 - 4)(x^2 + 1) = 0$

$\Rightarrow x^2 - 4 = 0$ and $x^2 + 1 = 0$

$\Rightarrow x = \pm 2$ and $x = \sqrt{} - 1$

Real value of $x = \pm 2$

Putting $x = 2$ in eq (ii), we get

$2x - 3y + 5 = 0$

$\Rightarrow 2\times 2 - 3y + 5 = 0$

$\Rightarrow 4 - 3y + 5 = 0 = 9 - 3y = 0$

$\Rightarrow y = 3$

Putting $x = - 2$ in eq (ii), we get

$2x - 3y + 5 = 0$

$\Rightarrow 2\times - 2 - 3y + 5 = 0$

$\Rightarrow - 4 - 3y + 5 = 0 = 1 - 3y = 0$

$\Rightarrow y = \frac{1}{3}$

Question: 7

If z^2

Solution:

Let z= a + ib

⇒ $|z| = \sqrt{(a^2 + b^2)}$

Now , $z^2 + |z|^2 = 0$

⇒ $(a + ib)^2 + a^2 + b^2 = 0$

⇒ $a^2 + 2abi + i^2b^2 + a^2 + b^2 = 0$

⇒ $a^2 + 2abi - b^2 + a^2 + b^2 = 0$

⇒ $2a^2 + 2abi = 0$

⇒ $2a(a + ib) = 0$

Either a = 0 or z = 0

Since z≠ 0

a = 0 ⇒ z is purely imaginary.

Question: 8

If Let z= a + ib

Now, $\frac{z-1}{z+1} = \frac{a+ib-1}{a+ib+1}$

$$= \frac{(a-1) + ib}{(a+1) + ib}$$

$$\Rightarrow \frac{(a-1) + ib}{(a+1) + ib} \times \frac{(a+1) - ib}{(a+1) - ib}$$

$$= \frac{a^2 + a - iab - a - 1 + ib + iab + ib - i^2b^2}{(a+1)^2 + b^2}$$

$$= \frac{a^2 + -1 + ib + ib + b^2}{(a+1)^2 + b^2} = \frac{a^2 + b^2 - 1 + 2ib}{(a+1)^2 + b^2}$$

Given that $\frac{z-1}{z+1}$ is purely imaginary ⇒ real part = 0

$$\Rightarrow \frac{a^2 + b^2 - 1}{(a+1)^2 + b^2} = 0$$

⇒ $a^2 + b^2 - 1 = 0$

⇒ $a^2 + b^2 = 1$

⇒ $|z| = 1$

Hence proved.

Question: 9

If z_1

Solution:

Let $z_1 = a + ib$ such that $|z_1| = \sqrt{(a^2 + b^2)} = 1$

Now, $z_2 = \frac{z_1-1}{z_1+1} = \frac{a+ib-1}{a+ib+1} = \frac{(a-1)+ib}{(a+1)+ib}$

$$\Rightarrow \frac{(a-1) + ib}{(a+1) + ib} \times \frac{(a+1) - ib}{(a+1) - ib}$$

$$= \frac{a^2 + a - iab - a - 1 + ib + iab + ib - i^2b^2}{(a+1)^2 + b^2}$$

$$= \frac{a^2 + -1 + ib + ib + b^2}{(a + 1)^2 + b^2} = \frac{a^2 + b^2 - 1 + 2ib}{(a + 1)^2 + b^2}$$

$$= \frac{(a^2 + b^2) - 1 + 2ib}{(a + 1)^2 + b^2} = \frac{1 - 1 + 2ib}{(a + 1)^2 + b^2} [\because a^2 + b^2 = 1]$$

$$= 0 + \frac{2ib}{(a + 1)^2 + b^2}$$

Thus, the real part of z_2 is 0 and z_2 is purely imaginary.

Question: 10

For all z

Solution:

Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$Now, \frac{z + \bar{z}}{2} = \frac{(a + ib) + (a - ib)}{2} = \frac{2a}{2} = a = Re(z)$

Hence Proved.

(ii) Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$w, \frac{z + \bar{z}}{2}$

$$= \frac{(a + ib) + (a - ib)}{2}$$

$$= \frac{2a}{2} = \frac{a}{1} = Re\,(z)$$

Hence, Proved.

(iii) Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$Now, z\bar{z} = (a + ib)(a - ib) = a^2 - (ib)^2 = a^2 + b^2 = |z|^2$

Hence Proved.

(iv) Let z = a + ib

$\Rightarrow \bar{z} = a - ib$

$Now, z + \bar{z} = (a + ib) + (a - ib) = 2a = 2Re(z)$

Hence , $z + \bar{z}$ is real.

(v) Case 1. Let z = a + 0i

$\Rightarrow \bar{z} = a - 0i$

$Now, z - \bar{z} = (a + 0\mathrm{i}) - (a - 0\mathrm{i}) = 0$

Case 2. Let z = 0 + bi

$\Rightarrow \bar{z} = 0 - bi$

$Now, z - \bar{z} = (0 + ib) - (0 - ib) = 2ib = 2iIm(z) = Imaginary$

Case 2. Let z = a + ib

$\Rightarrow\ \bar{z} = a - ib$

$Now\ , z - \bar{z} = (a + ib) - (a - ib) = 2ib = 2iIm(z) = Imaginary$

Thus, $\left(z - \bar{z}\right)$ is 0 or imaginary.

Question: 11

If z_1

Solution:

We have, $z_1 = (1 + i)$ and $z_2 = (-2 + 4i)$

Now, $\frac{z_1 z_2}{\bar{z_1}} = \frac{(1+i)(-2+4i)}{(1+i)}$

$$= \frac{-2 + 4i - 2i + 4i^2}{(1 - i)} = \frac{-2 + 4i - 2i - 4}{(1 - i)} = \frac{-6 + 2i}{(1 - i)}$$

$$= \frac{-6 + 2i}{(1 - i)} \times \frac{(1 + i)}{(1 + i)}$$

$$= \frac{-6 - 6i + 2i + 2i^2}{1 + 1}$$

$$= \frac{-6 - 4i - 2}{2} = \frac{-8 - 4i}{2}$$

= - 4 - 2i

Hence, $Im\left(\frac{z_1 z_2}{z_2}\right) = -2$

Question: 12

If a and b are re

Solution:

We have,

$$\frac{1 - ix}{1 + ix} = (a - ib) = \frac{a - ib}{1}$$

Applying componendo and dividendo , we get

$$\frac{(1 - ix) + (1 + ix)}{(1 - ix) - (1 + ix)} = \frac{a - ib + 1}{a - ib - 1}$$

$$\Rightarrow \frac{1 - ix + 1 + ix}{1 - ix - 1 + ix} = \frac{a - ib + 1}{a - ib - 1}$$

$$\Rightarrow \frac{2}{-2ix} = \frac{a - ib + 1}{-(-a + ib + 1)}$$

$$\Rightarrow ix = \frac{1 - a + ib}{1 + a - ib} \times \frac{1 + a + ib}{1 + a + ib}$$

$$= \frac{1 + a + ib - a - a^2 - aib + ib + aib + i^2b^2}{(1 + a)^2 - i^2b^2}$$

$$\Rightarrow ix = \frac{1 - a^2 - b^2 + 2ib}{(1 + a)^2 - i^2b^2} = \frac{1 - a^2 - b^2 + 2ib}{(1 + a)^2 + b^2} = \frac{1 - (a^2 + b^2) + 2ib}{1 + a^2 + 2a + b^2}$$

$$\Rightarrow ix = \frac{1 - (a^2 + b^2) + 2ib}{1 + 2a + (a^2 + b^2)}$$

$$\Rightarrow ix = \frac{1-1+2ib}{1+2a+1}\ [\because a^2+b^2=1]$$

$$\Rightarrow ix = \frac{2ib}{2+2a}$$

$$\Rightarrow x = \frac{2b}{2+2a} = Real\ value$$

Exercise : 5D

Question: 1

Find the modulus

Solution:

Let Z = 4 = r(cosθ + isinθ)

Now, separating real and complex part, we get

4 = rcosθ..........eq.1

0 = rsinθ............eq.2

Squaring and adding eq.1 and eq.2, we get

$16 = r^2$

Since r is always a positive no., therefore,

r = 4,

hence its modulus is 4.

now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{0}{4}$$

Tanθ = 0

Since cosθ = 1, sinθ = 0 and tanθ = 0. Therefore the θ lies in first quadrant.

Tanθ = 0, therefore θ = 0°

Representing the complex no. in its polar form will be

Z = 4(cos0° + isin0°)

Question: 2

Find the modulus

Solution:

Let Z = -2 = r(cosθ + isinθ)

Now, separating real and complex part, we get

-2 = rcosθ.......... eq.1

0 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no.,therefore,

r = 2,

hence its modulus is 2.

now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{0}{-2}$$

Tanθ = 0

Since cosθ = -1, sinθ = 0 and tanθ = 0. Therefore the θ lies in second quadrant.

Tanθ = 0, therefore θ = π

Representing the complex no. in its polar form will be

Z = 2(cosπ + isinπ)

Question: 3

Find the modulus

Solution:

Let Z = -i = r(cosθ + isinθ)

Now , separating real and complex part , we get

0 = rcosθ..........eq.1

-1 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

1 = r^2

Since r is always a positive no., therefore,

r = 1,

hence its modulus is 1.

now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-1}{0}$$

Tanθ = -∞

Since cosθ = 0 , sinθ = -1 and tanθ = -∞ . therefore the θ lies in fourth quadrant.

Tanθ = -∞, therefore $\theta = -\frac{\pi}{2}$

Representing the complex no. in its polar form will be

$$Z = 1\{\cos\left(-\frac{\pi}{2}\right) + i\sin(-\frac{\pi}{2})\}$$

Question: 4

Find the modulus

Solution:

Let Z = 2i = r(cosθ + isinθ)

Now , separating real and complex part , we get

0 = rcosθ eq.1

2 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

4 = r^2

Since r is always a positive no., therefore, r = 2,

hence its modulus is 2.

now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{2}{0}$$

Tanθ = ∞

Since cosθ = 0, sinθ = 1 and tanθ = ∞. Therefore the θ lies in first quadrant.

tanθ = ∞, therefore $\theta = \frac{\pi}{2}$

Representing the complex no. in its polar form will be

$$Z = 2\{\cos\left(\frac{\pi}{2}\right) + i\sin(\frac{\pi}{2})\}$$

Question: 5

Find the modulus

Solution:

Let Z = 1 - i = r(cosθ + isinθ)

Now , separating real and complex part , we get

1 = rcosθ ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

r = √2,

hence its modulus is √2.

now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-1}{1}$$

Tanθ = -1

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$ and tanθ = -1 . therefore the θ lies in fourth quadrant.

Tanθ = -1, therefore $\theta = -\frac{\pi}{4}$

Representing the complex no. in its polar form will be

$$Z = \sqrt{2}\{\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\}$$

Question: 6

Find the modulus

Solution:

Let Z = 1 - i = r(cosθ + isinθ)

Now , separating real and complex part , we get

-1 = rcosθ ……….eq.1

1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

$r = \sqrt{2},$

hence its modulus is √2.

now, dividing eq.2 by eq.1 , we get,

$$\frac{r sin\theta}{r cos\theta} = \frac{1}{-1}$$

Tanθ = -1

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ = -1. therefore the θ lies in second quadrant.

Tanθ = -1, therefore $\theta = \frac{3\pi}{4}$

Representing the complex no. in its polar form will be

$Z = \sqrt{2}\{\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4})\}$

Question: 7

Find the modulus

Solution:

Let Z = $\sqrt{3}$ + i = r(cosθ + isinθ)

Now , separating real and complex part , we get

$\sqrt{3}$ = rcosθ eq.1

1 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r =2,

hence its modulus is 2.

now, dividing eq.2 by eq.1, we get,

$$\frac{r sin\theta}{r cos\theta} = \frac{1}{\sqrt{3}}$$

$$\mathrm{Tan}\theta = \frac{1}{\sqrt{3}}$$

Since $\cos\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$. therefore the θ lies in first quadrant.

$\mathrm{Tan}\theta = \frac{1}{\sqrt{3}}$, therefore $\theta = \frac{\pi}{6}$

Representing the complex no. in its polar form will be

$Z = 2\{\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})\}$

Question: 8

Find the modulus

Solution:

Let Z = $\sqrt{3}i$ - 1 = r(cosθ + isinθ)

Now , separating real and complex part , we get

-1 = rcosθ eq.1

$\sqrt{3}$ = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

4 = r^2

Since r is always a positive no., therefore,

r = 2,

hence its modulus is 2.

now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{\sqrt{3}}{-1}$$

$$\text{Tan}\theta = -\frac{\sqrt{3}}{1}$$

Since $\cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = -\frac{\sqrt{3}}{1}$. therefore the θ lies in second quadrant.

Tanθ = $-\sqrt{3}$, therefore $\theta = \frac{2\pi}{3}$

Representing the complex no. in its polar form will be

$$Z = 2\{\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})\}$$

Question: 9

Find the modulus

Solution:

Let Z = $-\sqrt{3}i$ + 1 = r(cosθ + isinθ)

Now , separating real and complex part , we get

1 = rcosθ eq.1

$-\sqrt{3}$ = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

4 = r^2

Since r is always a positive no., therefore,

r = 2,

hence its modulus is 2.

now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-\sqrt{3}}{1}$$

$$\text{Tan}\theta = -\frac{\sqrt{3}}{1}$$

Since $\cos\theta = \frac{1}{2}$, $\sin\theta = -\frac{\sqrt{3}}{2}$ and $\tan\theta = -\frac{\sqrt{3}}{1}$. therefore the θ lies in the fourth quadrant.

Tanθ = $-\sqrt{3}$, therefore θ = $-\frac{\pi}{3}$

Representing the complex no. in its polar form will be

Z = $2\{\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})\}$

Question: 10

Find the modulus

Solution:

Let Z = 2 - 2i = r(cosθ + isinθ)

Now , separating real and complex part , we get

2 = rcosθ ……….eq.1

-2 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$8 = r^2$

Since r is always a positive no. therefore,

r = $2\sqrt{2}$,

hence its modulus is 2√2.

now, dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{-2}{2}$$

Tanθ = -1

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$ and tanθ = -1 . therefore the θ lies in the fourth quadrant.

Tanθ = -1, therefore $\theta = -\frac{\pi}{4}$

Representing the complex no. in its polar form will be

$Z = 2\sqrt{2}\{\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\}$

Question: 11

Find the modulus

Solution:

Let Z = 4√2i - 4 = r(cosθ + isinθ)

Now, separating real and complex part , we get

-4 = rcosθ ……….eq.1

4√3 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$64 = r^2$

Since r is always a positive no., therefore,

r = 8

hence its modulus is 8.

now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{4\sqrt{3}}{-4}$$

$$\text{Tan}\theta = -\frac{\sqrt{3}}{1}$$

Since $\cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = -\frac{\sqrt{3}}{1}$. therefore the θ lies in second the quadrant.

Tanθ = -√3, therefore θ= $\frac{2\pi}{3}$.

Representing the complex no. in its polar form will be

$$Z = 8\{\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})\}$$

Question: 12

Find the modulus

Solution:

Let Z = 3√2i - 3√2 = r(cosθ + isinθ)

Now, separating real and complex part , we get

-3√2 = rcosθ eq.1

3√2 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

$36 = r^2$

Since r is always a positive no., therefore,

r = 6

hence its modulus is 6.

now, dividing eq.2 by eq.1, we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{3\sqrt{2}}{-3\sqrt{2}}$$

$$\text{Tan}\theta = -\frac{1}{1}$$

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ = -1 . therefore the θ lies in secothe nd quadrant.

Tanθ = -1 , therefore θ = $\frac{3\pi}{4}$.

Representing the complex no. in its polar form will be

$$Z = 6\{\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4})\}$$

Question: 13

Find the modulus

Solution:

$$= \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{1+i^2+2i}{1-i^2}$$

$$= \frac{2i}{2}$$

= i

Let Z = i = r(cosθ + isinθ)

Now , separating real and complex part , we get

0 = rcosθ ……….eq.1

1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

hence its modulus is 1.

now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{1}{0}$$

tanθ = ∞

Since cosθ = 0, sinθ = 1 and tanθ = ∞. Therefore the θ lies in first quadrant.

tanθ = ∞, therefore $\theta = \frac{\pi}{2}$

Representing the complex no. in its polar form will be

$$Z = 1\{\cos(\frac{\pi}{2}) + i\sin(\frac{\pi}{2})\}$$

Question: 14

Find the modulus

Solution:

$$= \frac{1 - i}{1 + i} \times \frac{1 - i}{1 - i}$$

$$= \frac{1 + i^2 - 2i}{1 - i^2}$$

$$= -\frac{2i}{2}$$

= -i

Let Z = -i = r(cosθ + isinθ)

Now, separating real and complex part , we get

0 = rcosθ……….eq.1

-1 = rsinθ …………eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

hence its modulus is 1.

now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{-1}{0}$$

$Tan\theta = -\infty$

Since $\cos\theta = 0$, $\sin\theta = -1$ and $\tan\theta = -\infty$, therefore the θ lies in fourth quadrant.

$Tan\theta = -\infty$, therefore $\theta = -\frac{\pi}{2}$

Representing the complex no. in its polar form will be

$Z = 1\{\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})\}$

Question: 15

Find the modulus

Solution:

$$= \frac{1+3i}{1-\ 2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+6i^2+5i}{1-4i^2}$$

$$= \frac{5i-5}{5}$$

= i - 1

Let Z = 1 - i = $r(\cos\theta + i\sin\theta)$

Now , separating real and complex part , we get

$-1 = r\cos\theta$ eq.1

$1 = r\sin\theta$ eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

$r = \sqrt{2}$,

hence its modulus is $\sqrt{2}$.

now, dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{1}{-1}$$

$Tan\theta = -1$

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = -1$. therefore the θ lies in second quadrant.

$Tan\theta = -1$, therefore $\theta = \frac{3\pi}{4}$

Representing the complex no. in its polar form will be

$Z = \sqrt{2}\{\cos(\frac{3\pi}{4}) + i\sin(\frac{3\pi}{4})\}$

Question: 16

Find the modulus

Solution:

$$\frac{1-\ 3i}{1+\ 2i} \times \frac{1-\ 2i}{1-\ 2i}$$

$= \frac{1 + 6i^2 - 5i}{1 - 4i^2}$

$= \frac{-5i - 5}{5}$

= -i - 1

Let Z = -1 - i = r(cosθ + isinθ)

Now , separating real and complex part , we get

-1 = rcosθ eq.1

-1 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

r = √2,

hence its modulus is √2.

now , dividing eq.2 by eq.1 , we get,

$\frac{rsin\theta}{rcos\theta} = \frac{-1}{-1}$

tanθ = 1

Since $\cos\theta = -\frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$ and tanθ = 1 . therefore the θ lies in third quadrant.

Tanθ = 1, therefore $\theta = -\frac{3\pi}{4}$

Representing the complex no. in its polar form will be

$Z = \sqrt{2}\{\cos(-\frac{3\pi}{4}) + i\sin(-\frac{3\pi}{4})\}$

Question: 17

Find the modulus

Solution:

$= \frac{5 - i}{2 - 3i} \times \frac{2 + 3i}{2 + 3i}$

$= \frac{10 - 3i^2 + 13i}{4 - 9i^2}$

$= \frac{+13i + 13}{13}$

= i + 1

Let Z = 1 + i = r(cosθ + isinθ)

Now , separating real and complex part , we get

1 = rcosθ eq.1

1 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore, r = √2,

hence its modulus is √2.

now , dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{1}{1}$$

Tanθ = 1

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and tanθ = 1. therefore the θ lies in first quadrant.

Tanθ = 1, therefore $\theta = \frac{\pi}{4}$

Representing the complex no. in its polar form will be

$Z = \sqrt{2}\{\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\}$

Question: 18

Find the modulus

Solution:

$$= \frac{-16}{1 + \sqrt{3}i} \times \frac{1 - \sqrt{3}i}{1 - \sqrt{3}i}$$

$$= \frac{-16 + 16\sqrt{3}i}{1 - 3i^2}$$

$$= \frac{16\sqrt{3}i - 16}{4}$$

= $4\sqrt{3}$i - 4

Let Z = $4\sqrt{3}$i - 4 = r(cosθ + isinθ)

Now , separating real and complex part , we get

-4 = rcosθ eq.1

$4\sqrt{3}$ = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

64 = r^2

Since r is always a positive no., therefore,

r = 8,

hence its modulus is 8.

now, dividing eq.2 by eq.1 , we get,

$$\frac{rsin\theta}{rcos\theta} = \frac{4\sqrt{3}}{-4}$$

tanθ = -√3

Since $\cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and tanθ = -√3. therefore the θ lies in second quadrant.

Tanθ = -√3, therefore $\theta = \frac{2\pi}{3}$

Representing the complex no. in its polar form will be

$Z = 8\{\cos(\frac{2\pi}{3}) + i\sin(\frac{2\pi}{3})\}$

Question: 19

Find the modulus

Solution:

$$= \frac{2 + 6\sqrt{3}i}{5 + \sqrt{3}i} \times \frac{5 - \sqrt{3}i}{5 - \sqrt{3}i}$$

$$= \frac{10 + 28\sqrt{3}i - 18i^2}{25 - 3i^2}$$

$$= \frac{28\sqrt{3}i + 28}{28}$$

= √3i + 1

Let Z = √3i + 1 = r(cosθ + isinθ)

Now , separating real and complex part , we get

1 = rcosθ eq.1

√3 = rsinθ eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r = 2,

hence its modulus is 2.

now , dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{\sqrt{3}}{1}$$

tanθ = √3

Since $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ and tanθ = $\sqrt{3}$. therefore the θ lies in first quadrant.

Tanθ = √3, therefore $\theta = \frac{\pi}{3}$

Representing the complex no. in its polar form will be

$Z = 2\{\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})\}$

Question: 20

Find the modulus

Solution:

$$= \sqrt{\frac{1+i}{1-i}} \times \sqrt{\frac{1+i}{1+i}}$$

$$= \sqrt{\frac{(1+i)^2}{1-i^2}}$$

$$= \frac{1+i}{\sqrt{2}}$$

$= \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

Let $Z = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = r(\cos\theta + i\sin\theta)$

Now, separating real and complex part , we get

$\frac{1}{\sqrt{2}} = r\cos\theta$ eq.1

$\frac{1}{\sqrt{2}} = r\sin\theta$ eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

hence its modulus is 1.

now , dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{\frac{i}{\sqrt{2}}}{\frac{i}{\sqrt{2}}}$$

$\tan\theta = 1$

Since $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$ and $\tan\theta = 1$. therefore the θ lies in first quadrant.

$\tan\theta = 1$, therefore $\theta = \frac{\pi}{4}$

Representing the complex no. in its polar form will be

$Z = 1\{\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\}$

Question: 21

Find the modulus

Solution:

Let $Z = -i - \sqrt{3} = r(\cos\theta + i\sin\theta)$

Now , separating real and complex part , we get

$-\sqrt{3} = r\cos\theta$ eq.1

$-1 = r\sin\theta$ eq.2

Squaring and adding eq.1 and eq.2, we get

$4 = r^2$

Since r is always a positive no., therefore,

r = 2

hence its modulus is 2.

now, dividing eq.2 by eq.1, we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{-1}{-\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

Since $\cos\theta = -\frac{\sqrt{3}}{2}$, $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$. therefore the θ lies in third quadrant.

$\tan\theta = \frac{1}{\sqrt{3}}$, therefore $\theta = -\frac{5\pi}{6}$.

Representing the complex no. in its polar form will be

$Z = 2\{\cos(-\frac{5\pi}{6}) + i\sin(-\frac{5\pi}{6})\}$

Question: 22

Find the modulus

Solution:

$= i^{75}$

$= i^{4n+3}$ where n = 18

since $i^{4n+3} = -i$

$i^{75} = -i$

Let Z = -i = r(cosθ + isinθ)

Now , separating real and complex part , we get

0 = rcosθ ……….eq.1

-1 = rsinθ …........eq.2

Squaring and adding eq.1 and eq.2, we get

$1 = r^2$

Since r is always a positive no., therefore,

r = 1,

hence its modulus is 1.

now , dividing eq.2 by eq.1 , we get,

$$\frac{r\sin\theta}{r\cos\theta} = \frac{-1}{0}$$

tanθ = -∞

Since cosθ = 0 , sinθ = -1 and tanθ = -∞ . therefore the θ lies in fourth quadrant.

Tanθ = -∞ , therefore $\theta = -\frac{\pi}{2}$

Representing the complex no. in its polar form will be

$Z = 1\{\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2})\}$

Question: 23

Find the modulus

Solution:

$$= \frac{1 - i}{\frac{1}{2} + i\frac{\sqrt{3}}{2}}$$

$$= \frac{2 - 2i}{1 + i\sqrt{3}}$$

$$= \frac{2 - 2i}{1 + \sqrt{3}i} \times \frac{1 - \sqrt{3}i}{1 - \sqrt{3}i}$$

$= \frac{2 - 2\sqrt{3}i - 2i + 2\sqrt{3}i^2}{1 - 3i^2}$

$= \frac{(2 - 2\sqrt{3}) + i(2\sqrt{3} + 2)}{4}$

$= \frac{(1 - \sqrt{3}) + i(\sqrt{3} + 1)}{2}$

Let $Z = \frac{(1-\sqrt{3}) + i(\sqrt{3}+1)}{2} = r(\cos\theta + i\sin\theta)$

Now , separating real and complex part , we get

$\frac{1-\sqrt{3}}{2} = r\cos\theta$ eq.1

$\frac{1+\sqrt{3}}{2} = r\sin\theta$ eq.2

Squaring and adding eq.1 and eq.2, we get

$2 = r^2$

Since r is always a positive no., therefore,

$r = \sqrt{2}$,

hence its modulus is $\sqrt{2}$.

now, dividing eq.2 by eq.1 , we get,

$\frac{r\sin\theta}{r\cos\theta} = \frac{\frac{1+\sqrt{3}}{2}}{\frac{1-\sqrt{3}}{2}}$

$\tan\theta = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$

Since $\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}}$, $\sin\theta = \frac{1+\sqrt{3}}{2\sqrt{2}}$ and $\tan\theta = \frac{1+\sqrt{3}}{1-\sqrt{3}}$. therefore the θ lies in second quadrant. As

$\mathrm{Tan}\theta = \frac{1+\sqrt{3}}{1-\sqrt{3}}$, therefore $\theta = \frac{7\pi}{12}$

Representing the complex no. in its polar form will be

$Z = \sqrt{2}\{\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12})\}$

Question: 24

Find the modulus

Solution:

= sin(90° + 30°) - icos(90° + 30°)

= cos30° + isin30°

Since, sin(90°+ α) = cosα

And cos(90° + α) = -sinα

$= \frac{\sqrt{3}}{2} + i\frac{1}{2}$

hence it is of the form

$Z = \frac{\sqrt{3}}{2} + i\frac{1}{2} = r(\cos\theta + i\sin\theta)$

Therefore r = 1

Hence its modulus is 1 and argument is $\frac{\pi}{6}$.

Exercise : 5E

Question: 1

$x^2 + 2$

Solution:

This equation is a quadratic equation.

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Given:

$\Rightarrow x^2 + 2 = 0$

$\Rightarrow x^2 = -2$

$\Rightarrow x = \pm \sqrt{(-2)}$

But we know that $\sqrt{(-1)} = i$

$\Rightarrow x = \pm\sqrt{2}\, i$

Ans: $x = \pm\sqrt{2}\, i$

Question: 2

Given:

$x^2 + 5 = 0$

$\Rightarrow x^2 = -5$

$\Rightarrow x = \pm\sqrt{(-5)}$

$\Rightarrow x = \pm\sqrt{5}\, i$

Ans: $x = \pm\sqrt{5}\, i$

Question: 3

$2x^2 +$

Solution:

$2x^2 + 1 = 0$

$\Rightarrow 2x^2 = -1$

$\Rightarrow x^2 = -\frac{1}{2}$

$\Rightarrow x = \pm\sqrt{-\frac{1}{2}}$

$\Rightarrow x = \pm\sqrt{\frac{1}{2}}\, i$

$\Rightarrow x = \pm\frac{i}{\sqrt{2}}$

Ans: $x = \pm\frac{i}{\sqrt{2}}$

Question: 4

$x^2 + x$

Solution:

Given:

$x^2 + x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1^2 - (4 \times 1 \times 1)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-3}}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{3}i}{2}$$

$$\Rightarrow x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

Ans: $x = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $x = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

Question: 5

$x^2 - x$

Solution:

Given:

$x^2 - x + 2 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - (4 \times 1 \times 2)}}{2 \times 1}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1-8}}{2}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{-7}}{2}$$

$$\Rightarrow x = \frac{1 \pm \sqrt{7}i}{2}$$

$$\Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

Ans: $x = \frac{1}{2} + \frac{\sqrt{7}}{2}i$ and $x = \frac{1}{2} - \frac{\sqrt{7}}{2}i$

Question: 6

$x^2 + 2$

Solution:

Given:

$x^2 + 2x + 2 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{(2)^2 - (4 \times 1 \times 2)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{-4}}{2}$$

$$\Rightarrow x = \frac{-2 \pm 2i}{2}$$

$$\Rightarrow x = -\frac{2}{2} \pm \frac{2}{2} i$$

$$\Rightarrow x = -1 \pm i$$

Ans: x = -1 + i and x = -1-i

Question: 7

$2x^2$ –

Solution:

Given:

$2x^2 - 4x + 3 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4 \times 2 \times 3)}}{2 \times 2}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{16-24}}{4}$$

$$\Rightarrow x = \frac{4 \pm \sqrt{-8}}{4}$$

$$\Rightarrow x = \frac{4 \pm 2\sqrt{2} i}{4}$$

$$\Rightarrow x = \frac{4}{4} \pm \frac{2\sqrt{2}}{4} i$$

$$\Rightarrow x = 1 \pm \frac{i}{\sqrt{2}}$$

Ans: $x = 1 + \frac{i}{\sqrt{2}}$ and $x = 1 - \frac{i}{\sqrt{2}}$

Question: 8

$x^2 + 3$

Solution:

Given:

$x^2 + 3x + 5 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2-(4\times1\times5)}}{2\times1}$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{9-20}}{2}$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{-11}}{2}$$

$$\Rightarrow x = \frac{-3 \pm \sqrt{11}i}{2}$$

$$\Rightarrow x = -\frac{3}{2} \pm \frac{\sqrt{11}}{2}i$$

Ans: $x = -\frac{3}{2} + \frac{\sqrt{11}}{2}i$ and $x = -\frac{3}{2} - \frac{\sqrt{11}}{2}i$

Question: 9

Given:

$$\sqrt{5}x^2 + x + \sqrt{5} = 0$$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{(1)^2-(4\times\sqrt{5}\times\sqrt{5})}}{2\times\sqrt{5}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-20}}{2\sqrt{5}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-19}}{2\sqrt{5}}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{19}i}{2\sqrt{5}}$$

$$\Rightarrow x = -\frac{1}{2\sqrt{5}} \pm \frac{\sqrt{19}}{2\sqrt{5}}i$$

Ans: $x = -\frac{\sqrt{5}}{10} + \frac{\sqrt{\frac{19}{5}}}{2}i$ and $x = -\frac{\sqrt{5}}{10} - \frac{\sqrt{\frac{19}{5}}}{2}i$

Question: 10

$25x^2$ –

Solution:

Given:

$25x^2 - 30x + 11 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-30) \pm \sqrt{(-30)^2-(4\times25\times11)}}{2\times25}$$

$$\Rightarrow x = \frac{30 \pm \sqrt{900-1100}}{50}$$

$$\Rightarrow x = \frac{30 \pm \sqrt{-200}}{50}$$

$\Rightarrow x = \frac{30 \pm 10\sqrt{2}i}{50}$

$\Rightarrow x = -\frac{30}{50} \pm \frac{10\sqrt{2}}{50}i$

Ans: $x = -\frac{3}{5} + \frac{\sqrt{2}}{5}i$ and $x = -\frac{3}{5} - \frac{\sqrt{2}}{5}i$

Question: 11

$8x^2$ +

Solution:

Given:

$8x^2 + 2x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-2 \pm \sqrt{(2)^2 - (4 \times 8 \times 1)}}{2 \times 8}$

$\Rightarrow x = \frac{-2 \pm \sqrt{4-32}}{16}$

$\Rightarrow x = \frac{-2 \pm \sqrt{-28}}{16}$

$\Rightarrow x = \frac{-2 \pm 2\sqrt{7}i}{16}$

$\Rightarrow x = -\frac{2}{16} \pm \frac{2\sqrt{7}}{16}i$

Ans: $x = -\frac{1}{8} + \frac{\sqrt{7}}{8}i$ and $x = -\frac{1}{8} - \frac{\sqrt{7}}{8}i$

Question: 12

$27x^2$ +

Solution:

Given:

$27x^2 + 10x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-10 \pm \sqrt{(10)^2 - (4 \times 27 \times 1)}}{2 \times 27}$

$\Rightarrow x = \frac{-10 \pm \sqrt{100-108}}{54}$

$\Rightarrow x = \frac{-10 \pm \sqrt{-8}}{54}$

$\Rightarrow x = \frac{-10 \pm 2\sqrt{2}i}{54}$

$\Rightarrow x = -\frac{10}{54} \pm \frac{2\sqrt{2}}{54}i$

Ans: $x = -\frac{5}{27} + \frac{\sqrt{2}}{27}i$ and $x = -\frac{5}{27} - \frac{\sqrt{2}}{27}i$

Question: 13

Solution:

Given:

$2x^2 - \sqrt{3}x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-\sqrt{3}) \pm \sqrt{(-\sqrt{3})^2 - (4\times 2\times 1)}}{2\times 2}$$

$$\Rightarrow x = \frac{\sqrt{3} \pm \sqrt{3-8}}{4}$$

$$\Rightarrow x = \frac{\sqrt{3} \pm \sqrt{-5}}{4}$$

$$\Rightarrow x = \frac{\sqrt{3} \pm \sqrt{5}i}{4}$$

$$\Rightarrow x = \frac{\sqrt{3}}{4} \pm \frac{\sqrt{5}}{4}i$$

Ans: $x = \frac{\sqrt{3}}{4} + \frac{\sqrt{5}}{4}i$ and $x = \frac{\sqrt{3}}{4} - \frac{\sqrt{5}}{4}i$

Question: 14

$17x^2$ –

Solution:

Given:

$17x^2 - 8x + 1 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - (4\times 17\times 1)}}{2\times 17}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{64-68}}{34}$$

$$\Rightarrow x = \frac{8 \pm \sqrt{-4}}{34}$$

$$\Rightarrow x = \frac{8 \pm 2i}{34}$$

$$\Rightarrow x = \frac{8}{34} \pm \frac{2}{34}i$$

Ans: $x = \frac{4}{17} + \frac{1}{17}i$ and $x = \frac{4}{17} - \frac{1}{17}i$

Question: 15

$3x^2$ +

Solution:

Given:

$3x^2 + 5 = 7x$

$\Rightarrow 3x^2 - 7x + 5 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-7) \pm \sqrt{(-7)^2 - (4 \times 3 \times 5)}}{2 \times 3}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 60}}{6}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{-11}}{6}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{11}i}{6}$$

$$\Rightarrow x = \frac{7}{6} \pm \frac{\sqrt{11}}{6}i$$

Ans: $x = \frac{7}{6} + \frac{\sqrt{11}}{6}i$ and $x = \frac{7}{6} - \frac{\sqrt{11}}{6}i$

Question: 16

Solution:

Given:

$$3x^2 - 4x + \frac{20}{3} = 0$$

Multiplying both the sides by 3 we get,

$$9x^2 - 12x + 20 = 0$$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\Rightarrow x = \frac{-(-12) \pm \sqrt{(-12)^2 - (4 \times 9 \times 20)}}{2 \times 9}$$

$$\Rightarrow x = \frac{12 \pm \sqrt{144 - 720}}{18}$$

$$\Rightarrow x = \frac{12 \pm \sqrt{-576}}{18}$$

$$\Rightarrow x = \frac{12 \pm 24i}{18}$$

$$\Rightarrow x = \frac{12}{18} \pm \frac{24}{18}i$$

$$\Rightarrow x = \frac{2}{3} \pm \frac{4}{3}i$$

Ans: $x = \frac{2}{3} + \frac{4}{3}i$ and $x = \frac{2}{3} - \frac{4}{3}i$

Question: 17

$3x^2 +$

Solution:

Given:

$3x^2 + 7ix + 6 = 0$

$\Rightarrow 3x^2 + 9ix - 2ix + 6 = 0$

$\Rightarrow 3x(x + 3i) - 2i\left(x - \frac{6}{2i}\right) = 0$

$\Rightarrow 3x(x + 3i) - 2i\left(x - \frac{3\times i}{i\times i}\right) = 0$ ...($i^2 = -1$)

$\Rightarrow 3x(x + 3i) - 2i(x - \frac{3\times i}{-1}) = 0$

$\Rightarrow 3x(x + 3i) - 2i(x + 3i) = 0$

$\Rightarrow (x + 3i)(3x - 2i) = 0$

$\Rightarrow x + 3i = 0$ & $3x - 2i = 0$

$\Rightarrow x = 3i$ & $x = \frac{2}{3}i$

Ans: $x = 3i$ and $x = \frac{2}{3}i$

Question: 18

$21x^2$ –

Solution:

Given:

$21x^2 - 28x + 10 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-28) \pm \sqrt{(-28)^2 - (4\times 21\times 10)}}{2\times 21}$

$\Rightarrow x = \frac{28 \pm \sqrt{784 - 840}}{42}$

$\Rightarrow x = \frac{28 \pm \sqrt{-56}}{42}$

$\Rightarrow x = \frac{28 \pm 2\sqrt{14}i}{42}$

$\Rightarrow x = \frac{28}{42} \pm \frac{2\sqrt{14}}{42}i$

Ans: $x = \frac{2}{3} + \frac{\sqrt{14}}{21}i$ and $x = \frac{2}{3} - \frac{\sqrt{14}}{21}i$

Question: 19

$x^2 + 1$

Solution:

Given:

$x^2 + 13 = 4x$

$\Rightarrow x^2 - 4x + 13 = 0$

Solution of a general quadratic equation $ax^2 + bx + c = 0$ is given by:

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - (4\times 1\times 13)}}{2\times 1}$

$\Rightarrow x = \frac{4 \pm \sqrt{16-52}}{2}$

$\Rightarrow x = \frac{4 \pm \sqrt{-36}}{2}$

$\Rightarrow x = \frac{4 \pm 6i}{2}$

$\Rightarrow x = \frac{4}{2} \pm \frac{6}{2} i$

$\Rightarrow x = 2 \pm 3i$

Ans: x = 2 + 3i & x = 2-3i

Question: 20

$x^2 + 3$

Solution:

Given:

$x^2 + 3ix + 10 = 0$

$\Rightarrow x^2 + 5ix - 2ix + 10 = 0$

$\Rightarrow x(x + 5i) - 2i\left(x - \frac{10}{2i}\right) = 0$

$\Rightarrow x(x + 5i) - 2i\left(x - \frac{5 \times i}{i \times i}\right) = 0$

$\Rightarrow x(x + 5i) - 2i(x - \frac{5 \times i}{-1}) = 0$

$\Rightarrow x(x + 5i) - 2i(x + 5i) = 0$

$\Rightarrow (x + 5i)(x - 2i) = 0$

$\Rightarrow x + 5i = 0$ & $x - 2i = 0$

$\Rightarrow x = -5i$ & $x = 2i$

Ans: x = -5i & x = 2i

Question: 21

Given:

$2x^2 + 3ix + 2 = 0$

$\Rightarrow 2x^2 + 4ix - ix + 2 = 0$

$\Rightarrow 2x(x + 2i) - i\left(x - \frac{2}{i}\right) = 0$

$\Rightarrow 2x(x + 2i) - i\left(x - \frac{2 \times i}{i \times i}\right) = 0$

$\Rightarrow 2x(x + 2i) - i(x - \frac{2 \times i}{-1}) = 0$

$\Rightarrow 2x(x + 2i) - i(x + 2i) = 0$

$\Rightarrow (x + 2i)(2x - i) = 0$

$\Rightarrow x + 2i = 0$ & $2x - i = 0$

$\Rightarrow x = -2i$ & $x = \frac{i}{2}$

Ans: x = -2i and $x = \frac{i}{2}$

Exercise : 5F

Question: 1

Solution:

Let, $(a + ib)^2 = 5 + 12i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 5 + 12i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 5 + 12i$

now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 5$………….eq.1

$\Rightarrow 2ab = 12$……..eq.2

$\Rightarrow a = \frac{6}{b}$

now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{6}{b}\right)^2 - b^2 = 5$

$\Rightarrow 36 - b^4 = 5b^2$

$\Rightarrow b^4 + 5b^2 - 36 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -9$ or $b^2 = 4$

As b is real no. so, $b^2 = 4$

$b = 2$ or $b = -2$

Therefore, $a = 3$ or $a = -3$

Hence the square root of the complex no. is $3 + 2i$ and $-3 - 2i$.

Question: 2

Solution:

Let, $(a + ib)^2 = -7 + 24i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -7 + 24i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -7 + 24i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -7$………….eq.1

$\Rightarrow 2ab = 24$…….. eq.2

$\Rightarrow a = \frac{12}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{12}{b}\right)^2 - b^2 = -7$

$\Rightarrow 144 - b^4 = -7b^2$

$\Rightarrow\ b^4 - 7b^2 - 144 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = -9$ or $b^2 = 16$

As b is real no. so, $b^2 = 16$

b= 4 or b= -4

Therefore, a= 3 or a= -3

Hence the square root of the complex no. is 3 + 4i and -3 -4i.

Question: 3

Solution:

Let, $(a + ib)^2 = -2 + 2\sqrt{3}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -2 + 2\sqrt{3}i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -2 + 2\sqrt{3}i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -2$..............eq.1

$\Rightarrow 2ab = 2\sqrt{3}$........eq.2

$\Rightarrow a = \frac{\sqrt{3}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow\ \left(\frac{\sqrt{3}}{b}\right)^2 - b^2 = -2$

$\Rightarrow\ 3 - b^4 = -2b^2$

$\Rightarrow\ b^4 - 2b^2 - 3 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -1$ or $b^2 = 3$

As b is real no. so, $b^2 = 3$

b= $\sqrt{3}$ or b = $-\sqrt{3}$

Therefore, a= 1 or a= -1

Hence the square root of the complex no. is 1 + $\sqrt{3}$i and -1 -$\sqrt{3}$i.

Question: 4

Solution:

Let, $(a + ib)^2 = 1 + 4\sqrt{3}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 1 + 4\sqrt{3}i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 1 + 4\sqrt{3}i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 1$.............eq.1

$\Rightarrow 2ab = 4\sqrt{3}$.......eq.2

$\Rightarrow a = \frac{2\sqrt{3}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{2\sqrt{3}}{b}\right)^2 - b^2 = 1$

$\Rightarrow 12 - b^4 = b^2$

$\Rightarrow b^4 + b^2 - 12 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = -4$ or $b^2 = 3$

as b is real no. so, $b^2 = 3$

$b = \sqrt{3}$ or $b = -\sqrt{3}$

Therefore, a= 2 or a= -2

Hence the square root of the complex no. is $2 + \sqrt{3}i$ and $-2 -\sqrt{3}i$.

Question: 5

Solution:

Let, $(a + ib)^2 = 0 + i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 0 + i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 0 + i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 0$ eq.1

$\Rightarrow 2ab = 1$........ eq.2

$\Rightarrow a = \frac{1}{2b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{1}{2b}\right)^2 - b^2 = 0$

$\Rightarrow 1 - 4b^4 = 0$

$\Rightarrow 4b^2 = 1$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = -\frac{1}{2}$ or $b^2 = \frac{1}{2}$

As b is real no. so, $b^2 = 3$

b= $\frac{1}{\sqrt{2}}$ or b= $-\frac{1}{\sqrt{2}}$

Therefore , a= $\frac{1}{\sqrt{2}}$ or a= $-\frac{1}{\sqrt{2}}$

Hence the square root of the complex no. is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$ and $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$.

Question: 6

Solution:

Let, $(a + ib)^2 = 0 + 4i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 0 + 4i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 0 + 4i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 0$ eq.1

$\Rightarrow 2ab = 4$........ eq.2

$\Rightarrow a = \frac{2}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{2}{b}\right)^2 - b^2 = 0$

$\Rightarrow 4 - b^4 = 0$

$\Rightarrow b^4 = 4$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = -2$ or $b^2 = 2$

As b is real no. so, $b^2 = 2$

b= $\sqrt{2}$ or b= $-\sqrt{2}$

Therefore , a= $\sqrt{2}$ or a= $-\sqrt{2}$

Hence the square root of the complex no. is $\sqrt{2} + \sqrt{2}i$ and $-\sqrt{2} - \sqrt{2}i$.

Question: 7

Solution:

Let, $(a + ib)^2 = 3 + 4\sqrt{7}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 3 + 4\sqrt{7}i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 3 + 4\sqrt{7}i$

now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 3$ ………….eq.1

$\Rightarrow 2ab = 4\sqrt{7}$…….. eq.2

$\Rightarrow a = \frac{2\sqrt{7}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{2\sqrt{7}}{b}\right)^2 - b^2 = 3$

$\Rightarrow 12 - b^4 = 3b^2$

$\Rightarrow b^4 + 3b^2 - 28 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -7$ or $b^2 = 4$

as b is real no. so, $b^2 = 4$

$b = 2$ or $b = -2$

Therefore , $a = \sqrt{7}$ or $a = -\sqrt{7}$

Hence the square root of the complex no. is $\sqrt{7} + 2i$ and $-\sqrt{7} - 2i$.

Question: 8

Solution:

Let, $(a + ib)^2 = 16 - 30i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 16 - 30i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 16 - 30i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 16$………….eq.1

$\Rightarrow 2ab = -30$…….. eq.2

$\Rightarrow a = -\frac{15}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(-\frac{15}{b}\right)^2 - b^2 = 16$

$\Rightarrow 225 - b^4 = 16b^2$

$\Rightarrow b^4 + 16b^2 - 225 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = -25$ or $b^2 = 9$

as b is real no. so, $b^2 = 9$

b= 3 or b= -3

Therefore , a= -5 or a= 5

Hence the square root of the complex no. is -5 + 3i and 5 - 3i.

Question: 9

Solution:

Let, $(a + ib)^2 = -4 - 3i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -4 -3i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -4 - 3i$

now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = -4$.............eq.1

$\Rightarrow 2ab = -3$........ eq.2

$\Rightarrow a = -\frac{3}{2b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(-\frac{3}{2b}\right)^2 - b^2 = -4$

$\Rightarrow 9 - 4b^4 = -16b^2$

$\Rightarrow 4b^4 - 16b^2 - 9= 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = \frac{9}{2}$ or $b^2 = -2$

As b is real no. so, $b^2 = \frac{9}{2}$

b= $\frac{3}{\sqrt{2}}$ or b= -$\frac{3}{\sqrt{2}}$

Therefore , a= -$\frac{1}{\sqrt{2}}$ or a= $\frac{1}{\sqrt{2}}$

Hence the square root of the complex no. is $-\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}}i$ and $\frac{1}{\sqrt{2}} - \frac{3}{\sqrt{2}}i$.

Question: 10

Solution:

Let, $(a + ib)^2 = -15 - 8i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = -15 -8i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = -15 - 8i$

Now, separating real and complex parts, we get

⇒ $a^2 - b^2 = -15$.............eq.1

⇒ $2ab = -8$........ eq.2

⇒ $a = -\frac{4}{b}$

Now, using the value of a in eq.1, we get

⇒ $\left(-\frac{4}{b}\right)^2 - b^2 = -15$

⇒ $16 - b^4 = -15b^2$

⇒ $b^4 - 15b^2 - 16 = 0$

Simplify and get the value of b^2 , we get,

⇒ $b^2 = 16$ or $b^2 = -1$

As b is real no. so, $b^2 = 16$

b= 4 or b= -4

Therefore , a= -1 or a= 1

Hence the square root of the complex no. is -1 + 4i and 1 - 4i.

Question: 11

Solution:

Let, $(a + ib)^2 = -11 - 60i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

⇒ $a^2 + (bi)^2 + 2abi = -11 - 60i$

Since $i^2 = -1$

⇒ $a^2 - b^2 + 2abi = -11 - 60i$

now, separating real and complex parts, we get

⇒ $a^2 - b^2 = -11$.............eq.1

⇒ $2ab = -60$........ eq.2

⇒ $a = -\frac{30}{b}$

now, using the value of a in eq.1, we get

⇒ $\left(-\frac{30}{b}\right)^2 - b^2 = -11$

⇒ $900 - b^4 = -11b^2$

⇒ $b^4 - 11b^2 - 900 = 0$

Simplify and get the value of b^2, we get,

⇒ $b^2 = 36$ or $b^2 = -25$

as b is real no. so, $b^2 = 36$

b= 6 or b= -6

Therefore , a= -5 or a= 5

Hence the square root of the complex no. is -5 + 6i and 5 – 6i.

Question: 12

Solution:

Let, $(a + ib)^2 = 7 - 30\sqrt{2}i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 7 - 30\sqrt{2}i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 7 - 30\sqrt{2}i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 7$ eq.1

$\Rightarrow 2ab = 30\sqrt{2}$........ eq.2

$\Rightarrow a = \frac{15\sqrt{2}}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(\frac{15\sqrt{2}}{b}\right)^2 - b^2 = 7$

$\Rightarrow 450 - b^4 = 7b^2$

$\Rightarrow b^4 + 7b^2 - 450 = 0$

Simplify and get the value of b^2 , we get,

$\Rightarrow b^2 = -25$ or $b^2 = 18$

As b is real no. so, $b^2 = 18$

$b = 3\sqrt{2}$ or $b = -3\sqrt{2}$

Therefore , a= 5 or a= - 5

Hence the square root of the complex no. is $5 + 3\sqrt{2}i$ and $- 5 - 3\sqrt{2}i$.

Question: 1

Solution:

Let, $(a + ib)^2 = 0 - 8i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 0 - 8i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 0 - 8i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 0$ eq.1

$\Rightarrow 2ab = -8$........ eq.2

$\Rightarrow a = -\frac{4}{b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(-\frac{4}{b}\right)^2 - b^2 = 0$

$\Rightarrow 16 - b^4 = 0$

$\Rightarrow b^4 = 16$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = -4$ or $b^2 = 4$

As b is real no. so, $b^2 = 4$

b= 2 or b= -2

Therefore , a= -2 or a= 2

Hence the square root of the complex no. is -2 + 2i and 2 - 2i.

Question: 14

Solution:

Let, $(a + ib)^2 = 1 - i$

Now using, $(a + b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow a^2 + (bi)^2 + 2abi = 1 - i$

Since $i^2 = -1$

$\Rightarrow a^2 - b^2 + 2abi = 1 - i$

Now, separating real and complex parts, we get

$\Rightarrow a^2 - b^2 = 1$.............eq.1

$\Rightarrow 2ab = -1$........ eq.2

$\Rightarrow a = -\frac{1}{2b}$

Now, using the value of a in eq.1, we get

$\Rightarrow \left(-\frac{1}{2b}\right)^2 - b^2 = 1$

$\Rightarrow 1 - 4b^4 = 4b^2$

$\Rightarrow 4b^4 + 4b^2 - 1 = 0$

Simplify and get the value of b^2, we get,

$\Rightarrow b^2 = \frac{-4 \pm \sqrt{32}}{8}$

As b is real no. so, $b^2 = \frac{-4 + 4\sqrt{2}}{8}$

$b^2 = \frac{-1 + \sqrt{2}}{2}$

b= $\sqrt{\frac{-1 + \sqrt{2}}{2}}$ or b= -$\sqrt{\frac{-1 + \sqrt{2}}{2}}$

Therefore , a= -$\sqrt{\frac{1 + \sqrt{2}}{2}}$ or a= $\sqrt{\frac{1 + \sqrt{2}}{2}}$

Hence the square root of the complex no. is $-\sqrt{\frac{1 + \sqrt{2}}{2}} + \sqrt{\frac{-1 + \sqrt{2}}{2}}i$ and $\sqrt{\frac{1 + \sqrt{2}}{2}} - \sqrt{\frac{-1 + \sqrt{2}}{2}}i$.

Exercise : 5G

Question: 1

Evaluate

Solution:

we have, $\frac{1}{i^{78}}$

$= \frac{1}{(i^4)^{19}.i^2}$

We know that, $i^4 = 1$

$\Rightarrow \frac{1}{1^{19}.i^2}$

$\Rightarrow \frac{1}{i^2} = \frac{1}{-1}$

$\Rightarrow \frac{1}{i^{78}} = -1$

Question: 2

Evaluate (i^5

Solution:

we have, $i^{57} + i^{70} + i^{91} + i^{101} + i^{104}$

$= (i^4)^{14}.i + (i^4)^{17}.i^2 + (i^4)^{22}.i^3 + (i^4)^{25}.i + (i^4)^{26}$

We know that, $i^4 = 1$

$\Rightarrow (1)^{14}.i + (1)^{17}.i^2 + (1)^{22}.i^3 + (1)^{25}.i + (1)^{26}$

$= i + i^2 + i^3 + i + 1$

$= i - 1 - i + i + 1$

$= i$

Question: 3

Evaluate

<

Solution:

we have, $\left(\frac{i^{180}+i^{178}+i^{176}+i^{174}+i^{172}}{i^{170}+i^{168}+i^{166}+i^{164}+i^{162}}\right)$

$= \left(\frac{i^{180}+i^{178}+i^{176}+i^{174}+i^{172}}{i^{170}+i^{168}+i^{166}+i^{164}+i^{162}}\right)$

$= \left(\frac{(i^4)^{45} + (i^4)^{44}.i^2 + (i^4)^{44} + (i^4)^{43}.i^2 + (i^4)^{43}}{(i^4)^{42}.i^2 + (i^4)^{42} + (i^4)^{41}.i^2 + (i^4)^{41} + (i^4)^{40}.i^2}\right)$

$= \left(\frac{(1)^{45} + (1)^{44}.i^2 + (1)^{44} + (1)^{43}.i^2 + (1)^{43}}{(1)^{42}.i^2 + (1)^{42} + (1)^{41}.i^2 + (1)^{41} + (1)^{40}.i^2}\right)$

$= \left(\frac{1 + i^2 + 1 + .i^2 + 1}{i^2 + 1 + i^2 + 1 + i^2}\right)$

$= \left(\frac{1 - 1 + 1 - 1 + 1}{-1 + 1 - 1 + 1 - 1}\right)$

$= \left(\frac{1}{-1}\right)$

= -1

Question: 4

Evaluate (i^4

Solution:

we have, $i^{4n+1} - i^{4n-1}$

$= i^{4n}.i - i^{4n}.i^{-1}$

$= (i^4)^n.i - (i^4)^n.\ i^{-1}$

$= (1)^n.i - (1)^n.\ i^{-1}$

$= i - i^{-1}$

$= i - \frac{1}{i}$

$= \frac{i^2 - 1}{i}$

$= \frac{-1-1}{i}$

$= \frac{-2}{i} \times \frac{i}{i}$

$= \frac{-2i}{i^2} = \frac{-2i}{-1}$

= 2i

Question: 5

Evaluate

Solution:

we have, $\left(\sqrt{36} \times \sqrt{-25}\right)$

$= 6 \times \sqrt{-1 \times 25}$

$= 6 \times \left(\sqrt{-1} \times \sqrt{25}\right)$

$= 6 \times \left(\sqrt{-1} \times 5\right)$

= 6×5i = 30i

Question: 6

Find the sum (i

~~**Solution:**~~

~~We have~~ $\cancel{i^n + i^{n+1} + i^{n+2} + i^{n+3}}$

$\cancel{= i^n + i^n.i + i^n.i^2 + i^n.i^3}$

$\cancel{= i^n (1 + i + i^2 + i^3)}$

$\cancel{= i^n (1 + i - 1 - i)}$

$\cancel{= i^n (0) = 0}$

~~**Question: 7**~~

~~Find the sum (i +~~

~~**Solution:**~~

~~we have, i + i^2 + i^3 + i^4 +.... up to 400 terms~~

~~We know that given series is GP where a=i , r = i and n = 400~~

~~Thus,~~ $S = \frac{a(1-r^n)}{1-r}$

$$= \frac{i(1-(i)^{400})}{1-i}$$

$$= \frac{i(1-(i^4)^{100})}{1-i}$$

$$= \frac{i(1-1^{100})}{1-i} \ [\because i^4 = 1]$$

$$= \frac{i(1-1)}{1-i} = 0$$

~~**Question: 8**~~

~~Evaluate (1 + i~~

~~**Solution:**~~

~~we have, 1 + i^10 + i^20 + i^30~~

~~= 1 + (i^4)^2.i^2 + (i^4)^5 + (i^4)^7.i^2~~

~~We know that, i^4 = 1~~

~~= 1 + (1)^2.i^2 + (1)^5 + (1)^7.i^2~~

~~= 1 + i^2 + 1 + i^2~~

~~= 1-1 + 1-1~~

~~= 0~~

~~**Question: 9**~~

~~Evaluate:~~

~~**Solution:**~~

~~we have,~~ $\left(i^{41} + \frac{1}{i^{71}}\right)$

~~i^41 = i^40 . i = i~~

~~i^71 = i^68 . i^3 = -i~~

~~Therefore,~~

$$\left(i^{41} + \frac{1}{i^{71}}\right) = i - \frac{1}{i} = \frac{i^2 - 1}{i}$$

$$\left(i^{41} + \frac{1}{i^{71}}\right) = -\frac{2}{i} \times \frac{i}{i}$$

$$\left(i^{41} + \frac{1}{i^{71}}\right) = -\frac{2i}{i^2} = 2i$$

~~Hence,~~ $\left(i^{41} + \frac{1}{i^{71}}\right) = 2i$

~~**Question: 10**~~

~~Find the least po~~

~~**Solution:**~~

~~We have,~~ $\left(\frac{1+i}{1-i}\right)^n = 1$

~~Now,~~ $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{(1+i)^2}{1^2 - i^2}$

$= \frac{1^2 + 2i + i^2}{1-(-1)}$

$= \frac{1 + 2i - 1}{2}$

$= i$

$\therefore \left(\frac{1+i}{1-i}\right)^n = (i)^n = 1 \Rightarrow$ n is multiple of 4

~~∴ the least positive integer n is 4~~

~~**Question: 11**~~

~~Express (2 – 3i)<~~

~~**Solution:**~~

~~we have, $(2 - 3i)^3$~~

~~$= 2^3 - 3 \times 2^2 \times 3i - 3 \times 2 \times (3i)^2 - (3i)^3$~~

~~$= 8 - 36i + 54 + 27i$~~

~~$= 46 - 9i.$~~

~~**Question: 12**~~

~~Express~~

~~**Solution:**~~

~~we have,~~ $\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

$= \frac{(3)^2 - \left(i\sqrt{5}\right)^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i}$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= \frac{9+5}{2\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i}$

$= \frac{14\sqrt{2}i}{2\left(\sqrt{2}i\right)^2}$

$= \frac{7\sqrt{2}i}{-2}$

$= \frac{-7\sqrt{2}i}{2}$

~~**Question: 13**~~

~~Express~~

~~**Solution:**~~

~~we have,~~ $\frac{3-\sqrt{-16}}{1-\sqrt{-9}}$

~~We know that $\sqrt{-1} = i$~~

~~Therefore,~~

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3-4i}{1-3i}$$

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3-4i}{1-3i} \times \frac{1+3i}{1+3i}$$

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3+9i-4i-12\,i^2}{(1)^2-(3i)^2}$$

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{15+5i}{1+9} = \frac{15}{10} + \frac{5i}{10} = \frac{3}{2} + \frac{1}{2}i$$

~~Hence,~~

$$\frac{3-\sqrt{-16}}{1-\sqrt{-9}} = \frac{3}{2} + \frac{i}{2}$$

~~**Question: 14**~~

~~Solve for x: (1 –~~

~~**Solution:**~~

~~we have, $(1 - i)x + (1 + i)y = 1 - 3i$~~

~~$\Rightarrow x - ix + y + iy = 1 - 3i$~~

~~$\Rightarrow (x+y) + i(-x+y) = 1 - 3i$~~

~~On equating the real and imaginary coefficients we get,~~

~~$\Rightarrow x+y = 1$ (i) and $-x+y = -3$ (ii)~~

~~From (i) we get~~

~~$x = 1 - y$~~

~~substituting the value of x in (ii), we get~~

~~$-(1-y)+y = -3$~~

~~$\Rightarrow -1+y+y = -3$~~

~~$\Rightarrow 2y = -3+1$~~

~~$\Rightarrow y = -1$~~

~~$\Rightarrow x = 1-y = 1-(-1) = 2$~~

~~Hence, $x = 2$ and $y = -1$~~

~~**Question: 15**~~

~~Solve for x: x~~

~~**Solution:**~~

~~We have, $x^2 - 5ix - 6 = 0$~~

~~Here, $b^2 - 4ac = (-5i)^2 - 4 \times 1 \times -6$~~

~~$= 25i^2 + 24 = -25 + 24 = -1$~~

~~Therefore, the solutions are given by~~ $x = \frac{-(-5i) \pm \sqrt{-1}}{2 \times 1}$

$$x = \frac{5i \pm i}{2 \times 1}$$

$x = \frac{5i \pm i}{2}$

~~Hence, x= 3i and x = 2i~~

~~Question: 16~~

~~Find the conjugat~~

~~Solution:~~

~~Let~~ $z = \frac{1}{3+4i}$

$= \frac{1}{3+4i} \times \frac{3-4i}{3-4i} = \frac{3-4i}{9+16}$

$= \frac{3}{25} - \frac{4}{25}i$

$\Rightarrow \bar{z} = \frac{3}{25} + \frac{4}{25}i$

~~Question: 17~~

~~If z = (1 – i), f~~

~~Solution:~~

~~we have, z = (1 – i)~~

$\Rightarrow \bar{z} = 1 + i$

$\Rightarrow |z|^2 = (1)^2 + (-1)^2 = 2$

~~∴ the multiplicative inverse of (1 – i),~~

$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{1+i}{2}$

$z^{-1} = \frac{1}{2} + \frac{1}{2}i$

~~Question: 18~~

~~If z =~~

~~Solution:~~

~~we have,~~ $z = (\sqrt{5} + 3i)$

$\Rightarrow \bar{z} = (\sqrt{5} - 3i)$

$\Rightarrow |z|^2 = \left(\sqrt{5}\right)^2 + (3)^2$

$= 5 + 9 = 14$

~~∴ the multiplicative inverse of~~ $(\sqrt{5} + 3i)$~~,~~

$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14}$

$z^{-1} = \frac{\sqrt{5}}{14} + \frac{3}{14}i$

~~Question: 19~~

~~Prove that arg (z~~

~~Solution:~~

Let $z = r(\cos\theta + i\sin\theta)$

$\Rightarrow \arg(z) = \theta$

Now, $\bar{z} = r(\cos\theta - i\sin\theta) = r(\cos(-\theta) + i\sin(-\theta))$

$\Rightarrow \arg(\bar{z}) = -\theta$

Thus, $\arg(z) + \arg(\bar{z}) = \theta - \theta = 0$

Hence proved.

Question: 20

If $|z| = 6$ and ar

Solution:

we have, $|z| = 6$ and $\arg(z) = \frac{3\pi}{4}$

Let $z = r(\cos\theta + i\sin\theta)$

We know that, $|z| = r = 6$

And $\arg(z) = \theta = \frac{3\pi}{4}$

Thus, $z = r(\cos\theta + i\sin\theta) = 6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

Question: 21

Find the principa

Solution:

Let, $z = -2i$

Let $0 = r\cos\theta$ and $-2 = r\sin\theta$

By squaring and adding, we get

$(0)^2 + (-2)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$

$\Rightarrow 0+4 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 4 = r^2$

$\Rightarrow r = 2$

$\therefore \cos\theta = 0$ and $\sin\theta = -1$

Since, θ lies in fourth quadrant, we have

$$\theta = -\frac{\pi}{2}$$

Since, $\theta \in (-\pi, \pi]$ it is principal argument.

Question: 22

Write the princip

Solution:

Let, $z = \left(1 + i\sqrt{3}\right)^2$

$= (1)^2 + \left(i\sqrt{3}\right)^2 + 2\sqrt{3}i$

$= 1 - 1 + 2\sqrt{3}i$

$z = 0 + 2\sqrt{3}i$

~~Let $0 = r\cos\theta$ and $2\sqrt{3} = r\sin\theta$~~

~~By squaring and adding, we get~~

~~$(0)^2 + (2\sqrt{3})^2 = (r\cos\theta)^2 + (r\sin\theta)^2$~~

~~$\Rightarrow 0+(2\sqrt{3})^2 = r^2(\cos^2\theta + \sin^2\theta)$~~

~~$\Rightarrow (2\sqrt{3})^2 = r^2$~~

~~$\Rightarrow r = 2\sqrt{3}$~~

~~$\therefore \cos\theta = 0$ and $\sin\theta = 1$~~

~~Since, θ lies in first quadrant, we have~~

$$\theta = \frac{\pi}{2}$$

~~Since, $\theta \in (-\pi, \pi]$ it is principal argument.~~

~~Question: 23~~

~~Write -9 in polar~~

~~Solution:~~

~~we have, $z = -9$~~

~~Let $-9 = r\cos\theta$ and $0 = r\sin\theta$~~

~~By squaring and adding, we get~~

~~$(-9)^2 + (0)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$~~

~~$\Rightarrow 81 = r^2(\cos^2\theta + \sin^2\theta)$~~

~~$\Rightarrow 81 = r^2$~~

~~$\Rightarrow r = 9$~~

$\therefore \cos\theta = -1$ and $\sin\theta = 0$

$\Rightarrow \theta = \pi$

~~Thus, the required polar form is $9(\cos\pi + i\sin\pi)$~~

~~Question: 24~~

~~Write 2i in polar~~

~~Solution:~~

~~Let, $z = 2i$~~

~~Let $0 = r\cos\theta$ and $2 = r\sin\theta$~~

~~By squaring and adding, we get~~

~~$(0)^2 + (2)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$~~

~~$\Rightarrow 0+4 = r^2(\cos^2\theta + \sin^2\theta)$~~

~~$\Rightarrow 4 = r^2$~~

~~$\Rightarrow r = 2$~~

~~$\therefore \cos\theta = 0$ and $\sin\theta = 1$~~

~~Since, θ lies in first quadrant, we have~~

$$\theta = \frac{\pi}{2}$$

~~Thus, the required polar form is~~ $2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

~~Question: 25~~

~~Write -3i in pola~~

~~Solution:~~

~~Let, z = -3i~~

~~Let 0 = rcosθ and -3 = rsinθ~~

~~By squaring and adding, we get~~

~~$(0)^2 + (-3)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$~~

~~$\Rightarrow 0+9 = r^2(\cos^2\theta + \sin^2\theta)$~~

~~$\Rightarrow 9 = r^2$~~

~~$\Rightarrow r = 3$~~

~~$\therefore \cos\theta = 0$ and $\sin\theta = -1$~~

~~Since, θ lies in fourth quadrant, we have~~

$\theta = \frac{3\pi}{2}$

~~Thus, the required polar form is~~ $3\left(\cos\left(\frac{3\pi}{2}\right) + i\sin\left(\frac{3\pi}{2}\right)\right)$

~~Question: 26~~

~~Write z = (1 – i)~~

~~Solution:~~

~~we have, z = (1 – i)~~

~~Let 1 = rcosθ and -1 = rsinθ~~

~~By squaring and adding, we get~~

~~$(1)^2 + (-1)^2 = (r\cos\theta)^2 + (r\sin\theta)^2$~~

~~$\Rightarrow 1+1 = r^2(\cos^2\theta + \sin^2\theta)$~~

~~$\Rightarrow 2 = r^2$~~

~~$\Rightarrow r = \sqrt{2}$~~

$\therefore \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{-1}{\sqrt{2}}$

~~Since, θ lies in fourth quadrant, we have~~

$\theta = -\frac{\pi}{4}$

~~Thus, the required polar form is~~ $\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

~~Question: 27~~

~~Write z = (-1 + i~~

~~Solution:~~

~~we have, z = (-1 + i~~$\sqrt{3}$)

~~Let -1 = rcosθ and √3 = rsinθ~~

~~By squaring and adding, we get~~

~~$(-1)^2 + (\sqrt{3})^2 = (r\cos\theta)^2 + (r\sin\theta)^2$~~

~~$\Rightarrow 1+3 = r^2(\cos^2\theta + \sin^2\theta)$~~

~~$\Rightarrow 4 = r^2$~~

~~$\Rightarrow r = 2$~~

$\therefore \cos\theta = \dfrac{-1}{2}$ and $\sin\theta = \dfrac{\sqrt{3}}{2}$

~~Since, θ lies in second quadrant, we have~~

$\theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

~~Thus, the required polar form is~~ $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

~~Question: 28~~

~~If |z| = 2 and ar~~

~~Solution:~~

~~we have, $|z| = 2$ and $\arg(z) =$~~ $\dfrac{\pi}{4}$,

~~Let $z = r(\cos\theta + i\sin\theta)$~~

~~We know that, $|z| = r = 2$~~

~~And $\arg(z) = \theta =$~~ $\dfrac{\pi}{4}$

~~Thus, $z = r(\cos\theta + i\sin\theta) =$~~ $2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$