

20. Mensuration-I (Area of a Trapezium and a Polygon)

Exercise 20.1

1. Question

A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m²?

Answer

Given: Base of parallelogram = 24cm and height of parallelogram = 10cm

Now, we know that area of parallelogram = base \times height
Therefore, Area of 1 tile = $24 \times 10 = 240\text{cm}^2$

And, Area of floor 1080m²

We know that 1m = 100cm
So, $1080\text{m}^2 = 1080 \times 100 \times 100 \text{ cm}^2$

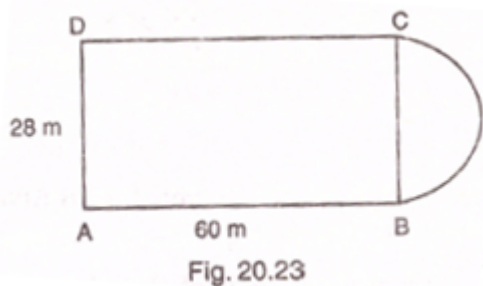
Now, Number of tiles = $\frac{\text{area of floor}}{\text{area of one tile}}$

Number of tiles = $(1080 \times 100 \times 100)/(24 \times 10) = 45000$

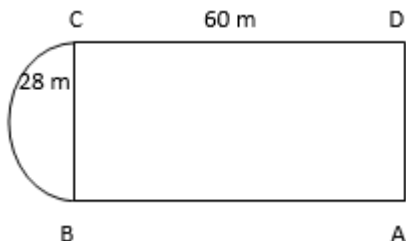
Therefore number of tiles = 45000

2. Question

A plot is in the form of a rectangle $ABCD$ having semi-circle on BC as shown in Fig. 20.23. If $AB = 60$ m and $BC = 28$ m, Find the area of the plot.



Answer



Area of the plot = area of the rectangle + area of semi-circle

$$\text{Radius of semi-circle} = \frac{BC}{2} = \frac{28}{2} = 14m$$

$$\text{Area of the rectangular plot} = \text{length} \times \text{breadth} = 60 \times 28 = 1680 \text{ m}^2$$

$$\text{Area of the semi-circular portion} = \frac{\pi r^2}{2}$$

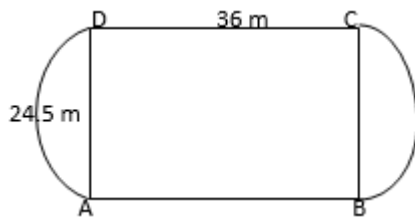
$$= \frac{1}{2} \times \frac{22}{7} \times 14 \times 14 = 308 \text{ m}^2$$

$$\text{Therefore, the total area of the plot} = 1680 + 308 = 1988 \text{ m}^2$$

3. Question

A playground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36 m and 24.5 m, find the area of the playground. (Take $\pi = 22/7$.)

Answer



Area of the plot = area of the rectangle + 2 × area of one semi-circle

$$\text{Radius of semi-circle} = \frac{BC}{2} = \frac{24.5}{2} = 12.25m$$

$$\text{Area of the plot} = \text{length} \times \text{breadth} + \frac{2 \times \pi r^2}{2}$$

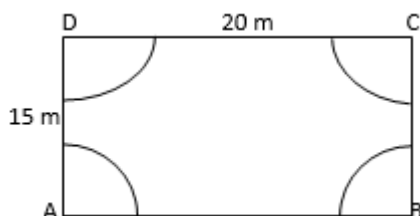
$$\text{Area of the plot} = 36 \times 24.5 + \frac{22}{7} \times 12.25 \times 12.25$$

$$\text{Area of the plot} = 882 + 471.625 = 1353.625 \text{ m}^2$$

4. Question

A rectangular piece is 20 m long and 15 m wide. From its four corners, quadrants of radii 3.5 m have been cut. Find the area of the remaining part.

Answer



Area of the plot = area of the rectangle - 4 × area of one quadrant

Radius of semi-circle = 3.5 m

Area of four quadrants = area of one circle

Area of the plot = length \times breadth + πr^2

$$\text{Area of the plot} = 20 \times 15 - \frac{22}{7} \times 3.5 \times 3.5$$

$$\text{Area of the plot} = 300 - 38.5 = 261.5 \text{ m}^2$$

5. Question

The inside perimeter of a running track (shown in Fig. 20.24) is 400 m. The length of each of the straight portion is 90 m and the ends are semi-circles. If track is everywhere 14 m wide, find the area of the track. Also, find the length of the outer running track.

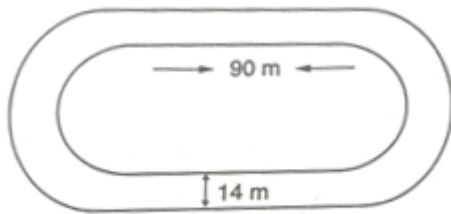


Fig. 20.24

Answer

Perimeter of the inner track = 2 \times Length of rectangle + perimeter of two semi-circular ends

Perimeter of the inner track = length + length + $2\pi r$

$$400 = 90 + 90 + \frac{2 \times 22 \times r}{7}$$

$$\frac{2 \times 22 \times r}{7} = 400 - 180$$

$$r = 35 \text{ m}$$

Hence, the radius of inner circle = 35 m

Radius of outer track = radius of inner track + width of the track

$$\text{Radius of outer track} = 35 + 14 = 49 \text{ m}$$

Length of outer track = 2 \times Length of rectangle + perimeter of two outer semi-circular ends

$$\text{Length of outer track} = 2 \times 90 + 2\pi r$$

$$\text{Length of outer track} = 180 + \frac{2 \times 22 \times 49}{7}$$

$$\text{Length of outer track} = 488 \text{ m}$$

Area of inner track = area of inner rectangle + area of two inner semi-circles

$$\text{Area of inner track} = \text{length} \times \text{breadth} + \pi r^2$$

$$\text{Area of inner track} = 90 \times 70 + \frac{22 \times 35 \times 35}{7}$$

$$\text{Area of inner track} = 6300 + 3850$$

$$\text{Area of inner track} = 10150 \text{ m}^2$$

$$\text{Area of outer track} = \text{area of outer rectangle} + \text{area of two outer semi-circles}$$

$$\text{Breadth of outer track} = 35 + 35 + 14 + 14 = 98 \text{ m}$$

$$\text{Area of outer track} = \text{length} \times \text{breadth} + \pi r^2$$

$$\text{Area of outer track} = 90 \times 98 + \frac{22 \times 49 \times 49}{7}$$

$$\text{Area of outer track} = 8820 + 7546$$

$$\text{Area of outer track} = 16366 \text{ m}^2$$

$$\text{Area of path} = \text{area of outer track} - \text{area of inner track}$$

$$\text{Area of path} = 16366 - 10150 = 6216 \text{ m}^2$$

6. Question

Find the area of Fig. 20.25, in square cm, correct to one place of decimal. (Take $\pi = 22/7$)

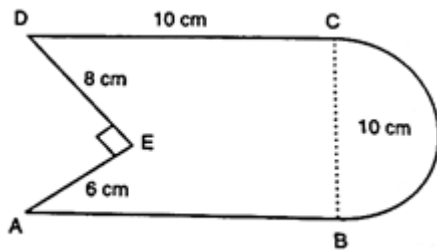


Fig. 20.25

Answer

$$\text{Area of the figure} = \text{area of square} + \text{area of semi circle} - \text{area of right angled triangle}$$

$$\text{Area of the figure} = \text{side} \times \text{side} + \frac{1}{2} \times \pi r^2 - \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\text{Area of the figure} = 10 \times 10 + \frac{1}{2} \times \frac{22}{7} \times 5 \times 5 - \frac{1}{2} \times 8 \times 6$$

$$\text{Area of the figure} = 100 + 78.57 - 24$$

$$\text{Area of the figure} = 100 + 78.57 - 24 = 154.57 \text{ cm}^2$$

$$\text{Area of the figure} = 154.57 \text{ cm}^2$$

7. Question

The diameter of a wheel of a bus is 90 cm which makes 315 revolutions per minute. Determine its speed in kilometres per hour. (Take $\pi = 22/7$)

Answer

Diameter of wheel = 90 cm

Perimeter of wheel = 2nd

$$\text{Perimeter of wheel} = \frac{22}{7} \times 90$$

Perimeter of wheel = 282.857 cm

Distance covered in 315 revolutions = $282.857 \times 315 = 89099.955 \text{ cm}$

One km = 100000 cm

$$\text{Distance covered} = \frac{89099.955}{100000} = 0.89 \text{ km}$$

Speed in km per hour = $0.89 \times 60 = 53.4 \text{ km per hour}$

8. Question

The area of a rhombus is 240 cm^2 and one of the diagonal is 16 cm. Find another diagonal.

Answer

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$240 = \frac{1}{2} \times 16 \times d_2$$

$$d_2 = \frac{240}{8} = 30$$

Hence, other diameter is 30 cm

9. Question

The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

Answer

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$\text{Area of rhombus} = \frac{1}{2} \times 7.5 \times 12$$

$$\text{Area of rhombus} = 6 \times 7.5 = 45.0$$

Hence, area of rhombus is 45 cm^2

10. Question

The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.

Answer

$$\text{Area of quadrilateral} = \frac{1}{2} \times d_1 \times (p_1 + p_2)$$

$$\text{Area of rhombus} = \frac{1}{2} \times 24 \times (8 + 13)$$

$$\text{Area of rhombus} = 12 \times 21 = 252$$

Hence, area of quadrilateral is 252 cm^2

11. Question

Find the area of a rhombus whose side is 6 cm and whose altitude is 4 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.

Answer

Side of rhombus = 6 cm

Altitude of rhombus = 4 cm

Since rhombus is a parallelogram, therefore area of parallelogram = base \times altitude

$$\text{Area of parallelogram} = 6 \times 4 = 24 \text{ cm}^2$$

Area of parallelogram = area of rhombus

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$24 = \frac{1}{2} \times 8 \times d_2$$

$$d_2 = \frac{24}{4} = 6$$

Hence, length of other diagonal of rhombus is 6 cm

12. Question

The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is Rs. 4.

Answer

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$\text{Area of rhombus} = \frac{1}{2} \times 45 \times 30$$

$$\text{Area of rhombus} = 675 \text{ cm}^2$$

$$\text{Area of one tile} = 675 \text{ cm}^2$$

$$\text{Area of 3000 tiles} = 675 \times 3000 = 2025000 \text{ cm}^2$$

$$\text{Area of tiles in m}^2 = \frac{2025000}{10000} = 202.5 \text{ m}^2$$

$$\text{Cost of tiles} = 202.5 \times 4 = \text{Rs } 810.0$$

13. Question

A rectangular grassy plot is 112 m long and 78 m broad. It has gravel path 2.5 m wide all around it on the side. Find the area of the path and the cost of constructing it at Rs. 4.50 per square metre.

Answer

Inner area of rectangle = length \times breadth

$$\text{Inner area of rectangle} = 112 \times 78 = 8736 \text{ m}^2$$

Width of path = 2.5 m

$$\text{Length of outer rectangle} = 112 + 2.5 + 2.5 = 117 \text{ m}$$

$$\text{Breadth of outer rectangle} = 78 + 2.5 + 2.5 = 83 \text{ m}$$

Outer area of rectangle = length \times breadth

$$= 117 \times 83 = 9711 \text{ m}^2$$

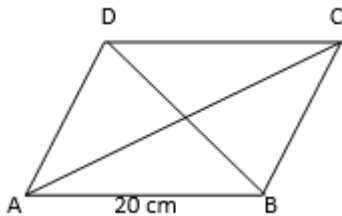
Area of path = outer area of rectangle – inner area of rectangle

$$\begin{aligned} \text{Area of path} &= 9711 - 8736 = 975 \text{ m}^2 \\ \text{Cost of construction for } 1 \text{ m}^2 &= 4.50 \text{ Rs} \\ \text{Cost of construction for } 975 \text{ m}^2 &= 975(4.50) = 4387.5 \text{ Rs} \end{aligned}$$

14. Question

Find the area of a rhombus, each side of which measures 20 cm and one of whose diagonals is 24 cm.

Answer



Length of side of rhombus = 20 cm

Length of one diagonal = 24 cm

In $\triangle AOB$

Using pythagoruous theorem:

$$AB^2 = OA^2 + OB^2$$

$$20^2 - 12^2 = OB^2$$

$$OB^2 = 400 - 144$$

$$OB^2 = 256$$

$$OB = 16$$

Hence, length of other diameter = $16 \times 2 = 32 \text{ cm}$

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$\text{Area of rhombus} = \frac{1}{2} \times 24 \times 32$$

$$\text{Area of rhombus} = 384 \text{ cm}^2$$

15. Question

The length of a side of a square field is 4 m. What will be the altitude of the rhombus, if the area of the rhombus is equal to the square field and one of its diagonal is 2 m?

Answer

$$\text{Length of square} = 4 \text{ m}$$

$$\text{Area of square} = \text{side}^2$$

$$\text{Area of square} = 4 \times 4 = 16 \text{ m}^2$$

$$\text{Area of square} = \text{area of rhombus}$$

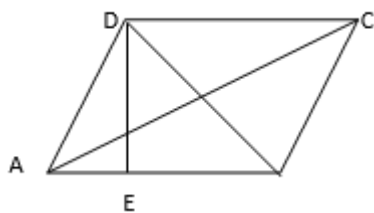
$$\text{Area of rhombus} = 16 \text{ m}^2$$

$$\text{Area of rhombus} = \frac{1}{2} \times d_1 \times d_2$$

$$16 = \frac{1}{2} \times 2 \times d_2$$

$$\text{Other diagonal of rhombus} = 16 \text{ m}$$

In $\triangle AOB$



Using pythagorous theorem:

$$AB^2 = OA^2 + OB^2$$

$$AB^2 = 8^2 + 1^2$$

$$AB^2 = 65$$

$$AB = \sqrt{65}$$

Rhombus is a parallelogram, area of parallelogram = base \times altitude

$$\text{Area of parallelogram} = AB \times DE$$

$$\text{Area of parallelogram} = \sqrt{65} \times DE$$

$$DE = \frac{16}{\sqrt{65}}$$

$$\text{Altitude of Rhombus} = \frac{16}{\sqrt{65}} \text{ cm}$$

16. Question

Find the area of the field in the form of a rhombus, if the length of each side be 14 cm and the altitude be 16 cm.

Answer

Side of rhombus = 14 cm

Altitude of rhombus = 16 cm

Rhombus is a type of parallelogram

Area of parallelogram = base \times altitude

$$\text{Area of parallelogram} = 14 \times 16 = 224 \text{ cm}^2$$

17. Question

The cost of fencing a square field at 60 paise per metre is Rs. 1200. Find the cost of reaping the field at the rate of 50 paise per 100 sq. metres.

Answer

$$\text{Perimeter of square field} = \frac{\text{cost of fencing}}{\text{rate of fencing}}$$

$$\text{Perimeter of square field} = \frac{1200}{0.6} = 2000 \text{ m}$$

$$\text{Perimeter of square} = 4 \times \text{side}$$

$$\text{Side of square} = \frac{\text{perimeter}}{4} = \frac{2000}{4} = 500 \text{ m}$$

$$\text{Area of square} = \text{side}^2$$

$$\text{Area of square} = 500 \times 500 = 250000 \text{ m}^2$$

$$\text{Cost of reaping} = \frac{250000 \times 0.5}{100} = 1250$$

Cost of reaping is Rs 1250

18. Question

In exchange of a square plot one of whose sides is 84 m, a man wants to buy a rectangular plot 144 m long and of the same area as of the square plot. Find the width of the rectangular plot.

Answer

$$\text{Area of square} = \text{side}^2$$

$$\text{Area of square} = 84 \times 84$$

Area of square = area of rectangle

$$84 \times 84 = 144 \times \text{width}$$

$$\text{Width} = \frac{84 \times 84}{144} = 49 \text{ m}$$

Therefore width of rectangle = 49 m

19. Question

The area of a rhombus is 84 m^2 . If its perimeter is 40 m, then find its altitude.

Answer

Perimeter of rhombus = $4 \times \text{side}$

$$\text{Side of rhombus} = \frac{\text{perimeter}}{4} = \frac{40}{4} = 10 \text{ m}$$

Rhombus is a type of parallelogram, Hence area of rhombus = area of parallelogram

Therefore area of parallelogram = $\text{base} \times \text{altitude}$

$\text{Base} \times \text{altitude} = \text{area of parallelogram}$

$$10 \times \text{altitude} = 84$$

$$\text{Altitude of rhombus} = 8.4 \text{ m}$$

20. Question

A garden is in the form of a rhombus whose side is 30 metres and the corresponding altitude is 16 m. Find the cost of levelling the garden at the rate of Rs. 2 per m^2 .

Answer

Side of rhombus = 30 m

Altitude of rhombus = 16 m

Rhombus is a type of parallelogram

Area of parallelogram = $\text{base} \times \text{altitude}$

$$\text{Area of parallelogram} = 30 \times 16 = 480 \text{ m}^2$$

Cost of levelling the garden = $\text{area} \times \text{rate}$

$$\text{Cost of levelling the garden} = 480 \times 2 = 960$$

Cost of levelling the garden is Rs 960

21. Question

A field in the form of a rhombus has each side of length 64 m and altitude 16 m. What is the side of a square field which has the same area as that of a rhombus?

Answer

Side of rhombus = 30 m

Altitude of rhombus = 16 m

Rhombus is a type of parallelogram

Area of parallelogram = base \times altitude

Area of parallelogram = $64 \times 16 = 1024 \text{ m}^2$

Since area of rhombus = area of square

Area of square = side²

Side of a square = $\sqrt{\text{area}}$

Side of square = $\sqrt{1024} = 32$

Therefore side of square = 32 m

22. Question

The area of a rhombus is equal to the area of a triangle whose base and the corresponding altitude are 24.8 cm and 16.5 cm respectively. If one of the diagonals of the rhombus is 22 cm, find the length of the other diagonal.

Answer

Length of base of triangle = 24.8 cm

Length of altitude of triangle = 16.5 cm

Area of triangle = $\frac{1}{2} \times \text{base} \times \text{altitude}$

Area of triangle = $\frac{1}{2} \times 24.8 \times 16.5 = 204.6$

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

$204.6 = \frac{1}{2} \times 22 \times d_2$

$d_2 = \frac{204.6}{11} = 18.6$

Hence, the length of other diagonal is 18.6 cm

Exercise 20.2

1. Question

Find the area, in square metres, of the trapezium whose bases and altitudes are as under:

(i) bases = 12 dm and 20 dm, altitude = 10 dm

(ii) bases = 28 cm and 3 dm, altitude = 25 cm

(iii) bases = 8 m and 60 dm, altitude = 40 dm

(iv) bases = 150 cm and 30 dm, altitude = 9 dm

Answer

(i) Area of trapezium = $\frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$

Length of bases of trapezium = 12 dm and 20 dm

10 dm = 1 m

Therefore length of bases in m = 1.2 m and 2 m

Similarly length of altitude in m = 1 m

$$\text{Area of trapezium} = \frac{1}{2}(1.2 + 2.0) \times 1$$

$$\text{Area of trapezium} = \frac{1}{2} \times 3.2 = 1.6 \text{ m}^2$$

(ii) Area of trapezium = $\frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$

Length of bases of trapezium = 28 cm and 3 dm

10 dm = 1 m

Therefore length of bases in m = 0.28 m and 0.3 m

Similarly length of altitude in m = 0.25 m

$$\text{Area of trapezium} = \frac{1}{2}(0.28 + 0.3) \times 0.25$$

$$\text{Area of trapezium} = \frac{1}{2} \times 0.58 \times 0.25 = 0.0725 \text{ m}^2$$

(iii) Area of trapezium = $\frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$

Length of bases of trapezium = 8 m and 60 dm

10 dm = 1 m

Therefore length of bases in m = 8 m and 6 m

Similarly length of altitude in m = 4 m

$$\text{Area of trapezium} = \frac{1}{2}(8 + 6) \times 4$$

$$\text{Area of trapezium} = \frac{1}{2} \times 14 \times 4 = 28 \text{ m}^2$$

(iv) Area of trapezium = $\frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$

Length of bases of trapezium = 150 cm and 30 dm

10 dm = 1 m

Therefore length of bases in m = 1.5 m and 3 m

Similarly length of altitude in m = 0.9 m

$$\text{Area of trapezium} = \frac{1}{2}(1.5 + 3) \times 0.9$$

$$\text{Area of trapezium} = \frac{1}{2} \times 4.5 \times 0.9 = 2.025 \text{ m}^2$$

2. Question

Find the area of trapezium with base 15 cm and height 8 cm, if the side parallel to the given base is 9 cm long.

Answer

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$$

Length of bases of trapezium = 15 cm and 9 cm

Similarly length of altitude in m = 8 cm

$$\text{Area of trapezium} = \frac{1}{2}(15 + 9) \times 8$$

$$\text{Area of trapezium} = \frac{1}{2} \times 24 \times 8 = 96 \text{ m}^2$$

3. Question

Find the area of a trapezium whose parallel sides are of length 16 dm and 22 dm and whose height is 12 dm.

Answer

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$$

Length of bases of trapezium = 15 cm and 9 cm

Similarly length of altitude in m = 8 cm

$$\text{Area of trapezium} = \frac{1}{2}(15 + 9) \times 8$$

$$\text{Area of trapezium} = \frac{1}{2} \times 24 \times 8 = 96 \text{ m}^2$$

4. Question

Find the height of a trapezium, the sum of the lengths of whose bases (parallel sides) is 60 cm and whose area is 600 cm².

Answer

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$$

Length of bases of trapezium = 60 cm and 60 cm

$$\text{Area of trapezium} = \frac{1}{2}(60 + 60) \times \text{altitude}$$

$$\text{Length of altitude} = \frac{2 \times \text{area}}{120} = \frac{2 \times 600}{120} = 10$$

Therefore length of altitude = 10 cm

5. Question

Find the altitude of a trapezium whose area is 65 cm^2 and whose base are 13 cm and 26 cm.

Answer

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$$

Length of bases of trapezium = 13 cm and 26 cm

$$\text{Area of trapezium} = \frac{1}{2}(13 + 26) \times \text{altitude}$$

$$\text{Length of altitude} = \frac{2 \times \text{area}}{39} = \frac{2 \times 65}{39} = 3.33$$

Therefore length of altitude = 3.33 cm

6. Question

Find the sum of the lengths of the bases of a trapezium whose area is 4.2 m^2 and whose height is 280 cm.

Answer

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of lengths of parallel sides}) \times \text{altitude}$$

$$\text{Area of trapezium} = \frac{1}{2}(\text{sum of sides}) \times \text{altitude}$$

$$\text{Sum of parallel sides} = \frac{2 \times \text{area}}{2.8} = \frac{2 \times 4.2}{2.8} = 3$$

Therefore sum of length of parallel sides = 3 m

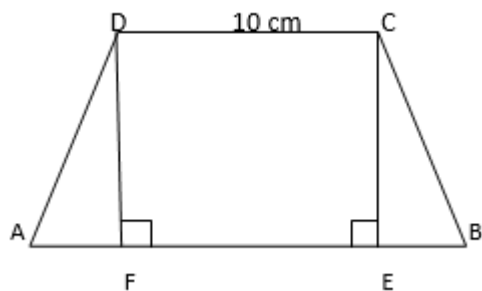
7. Question

Find the area of a trapezium whose parallel sides of lengths 10 cm and 15 cm are at a distance of 6 cm from each other. Calculate this area as

(i) the sum of the areas of two triangles and one rectangle.

(ii) the difference of the area of a rectangle and the sum of the areas of two triangles.

Answer



Area of a trapezium ABCD = area ($\triangle DFA$) + area (rectangle DFEC) + area ($\triangle CEB$) = $(\frac{1}{2} \times AF \times DF) + (FE \times DF) + (\frac{1}{2} \times EB \times CE)$ = $(\frac{1}{2} \times AF \times h) + (FE \times h) + (\frac{1}{2} \times EB \times h)$

$$= \frac{1}{2} \times h \times (AF + 2FE + EB) = \frac{1}{2} \times h \times (AF + FE + EB + FE) = \frac{1}{2} \times h \times (AB + FE)$$

$$= \frac{1}{2} \times h \times (AB + CD) \text{ [Opposite sides of rectangle are equal]}$$

$$= \frac{1}{2} \times 6 \times (15 + 10)$$

$$= \frac{1}{2} \times 6 \times 25 = 75$$

Area of trapezium = 75 cm^2

8. Question

The area of a trapezium is 960 cm^2 . If the parallel sides are 34 cm and 46 cm, find the distance between them.

Answer

$$\text{Area of trapezium} = \frac{1}{2} (\text{sum of lengths of parallel sides}) \times \text{distance between parallel sides}$$

$$\text{Area of trapezium} = \frac{1}{2} (\text{sum of sides}) \times \text{distance between parallel sides}$$

$$\text{Distance between parallel sides} = \frac{2 \times \text{area}}{\text{sum of sides}} = \frac{2 \times 960}{80} = 24$$

Therefore Distance between parallel sides = 24 cm

9. Question

Find the area of Fig. 20.35 as the sum of the areas of two trapezium and a rectangle.

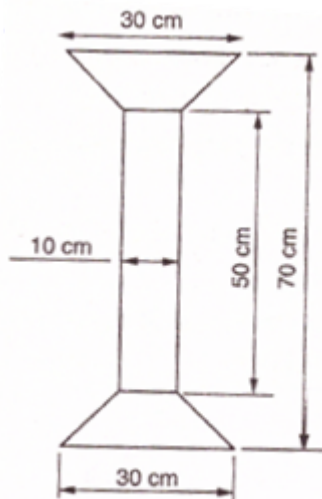


Fig. 20.35

Answer

Area of figure = Area of two trapeziums + area of rectangle

Length of rectangle = 50 cm

Breadth of rectangle = 10 cm

Length of parallel sides of trapezium = 30 cm and 10 cm

Distance between parallel sides of trapezium = $\frac{70-50}{2} = \frac{20}{2} = 10 \text{ cm}$

Area of figure = $2 \times \frac{1}{2} (\text{sum of sides}) \times \text{distance between parallel sides} + \text{length} \times \text{breadth}$

Area of figure = $2 \times \frac{1}{2} (30 + 10) \times 10 + 50 \times 10$

Area of figure = $400 + 500 = 900 \text{ cm}^2$

10. Question

Top surface of a table is trapezium in shape. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.

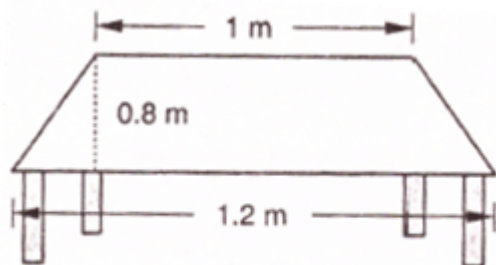


Fig. 20.36

Answer

Length of parallel sides of trapezium = 1.2m and 1m

Distance between parallel sides of trapezium = $0.8m$

Area of trapezium = $\frac{1}{2}(\text{sum of sides}) \times \text{distance between parallel sides}$

$$\text{Area of trapezium} = \frac{1}{2}(1.2 + 1) \times 0.8$$

$$\text{Area of trapezium} = 0.88 \text{ m}^2$$

11. Question

The cross-section of a canal is a trapezium in shape. If the canal is 10 m wide at the top 6 m wide at the bottom and the area of cross-section is 72 m^2 determine its depth.

Answer

Length of parallel sides of trapezium = 10m and 6m

Distance between parallel sides of trapezium = x meter

Area of trapezium = $\frac{1}{2}(\text{sum of sides}) \times \text{distance between parallel sides}$

$$72 = \frac{1}{2}(10 + 6) \times x$$

$$x = \frac{72}{8} = 9$$

Therefore depth of river is 9m

12. Question

The area of a trapezium is 91 cm^2 and its height is 7 cm. If one of the parallel sides is longer than the other by 8 cm, find the two parallel sides.

Answer

Let the length of one parallel side of trapezium = x meter

Length of other parallel side of trapezium = $(x + 8)m$

$$\text{Area of trapezium} = 91 \text{ cm}^2$$

Area of trapezium = $\frac{1}{2}(\text{sum of sides}) \times \text{distance between parallel sides}$

$$91 = \frac{1}{2}(x + x + 8) \times 7$$

$$x + 4 = \frac{91}{7}$$

$$x + 4 = 13$$

$$x = 13 - 4 = 9$$

Therefore length of one parallel side of trapezium = 9 cm

Length of other parallel side of trapezium = $9+8 = 17$ cm

13. Question

The area of a trapezium is 384 cm^2 . Its parallel sides are in the ratio $3 : 5$ and the perpendicular distance between them is 12 cm. Find the length of each one of the parallel sides.

Answer

Let the length of one parallel side of trapezium = $3x$ meter

Length of other parallel side of trapezium = $5x$ meter

Area of trapezium = 384 cm^2

Distance between parallel sides = 12 cm

Area of trapezium = $\frac{1}{2}(\text{sum of sides}) \times \text{distance between parallel sides}$

$$384 = \frac{1}{2}(3x + 5x) \times 12$$

$$4x = \frac{384}{12}$$

$$4x = 32$$

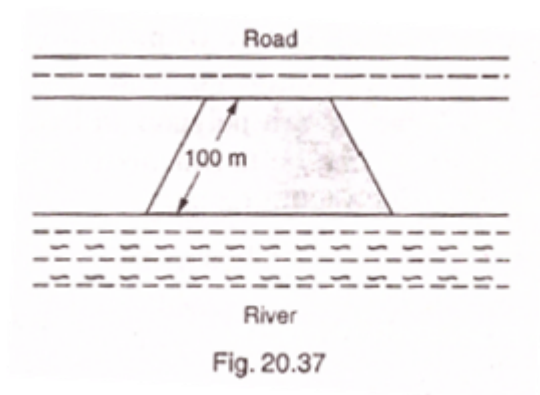
$$x = \frac{32}{4} = 8$$

Therefore length of one parallel side of trapezium = $3x = 3 \times 8 = 24$ cm

Length of other parallel side of trapezium = $5x = 5 \times 8 = 40$ cm

14. Question

Mohan wants to buy a trapezium shaped field. Its side along the river is parallel and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.



Answer

Let the length of side of trapezium shaped field along road = x meter

Length of other side of trapezium shaped field along road = $2x$ meter

Area of trapezium = 10500 cm^2

Distance between parallel sides = 100 m

Area of trapezium = $\frac{1}{2}(\text{sum of sides}) \times \text{distance between parallel sides}$

$$10500 = \frac{1}{2}(x + 2x) \times 100$$

$$3x = \frac{10500}{50}$$

$$x = \frac{210}{3}$$

$$x = 70$$

Therefore length of side along road = 70 m

Length of side along river = $70 \times 2 = 140 \text{ m}$

15. Question

The area of a trapezium is 1586 cm^2 and the distance between the parallel sides is 26 cm . If one of the parallel sides is 38 cm , find the other.

Answer

Let the length of other parallel side of trapezium = $x \text{ cm}$

Length of one parallel side of trapezium = 38 cm

Area of trapezium = 1586 cm^2

Distance between parallel sides = 26 cm

Area of trapezium = $\frac{1}{2}(\text{sum of sides}) \times \text{distance between parallel sides}$

$$1586 = \frac{1}{2}(38 + x) \times 26$$

$$38 + x = \frac{1586}{13}$$

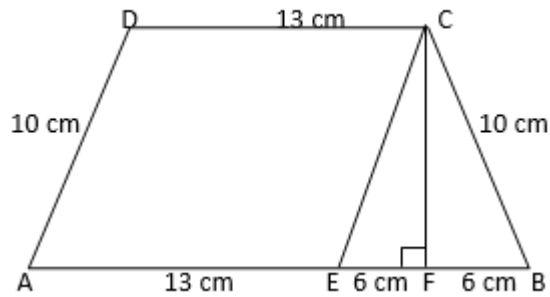
$$x = 122 - 38 = 84$$

Therefore the length of other parallel side of trapezium = 84 cm

16. Question

The parallel sides of a trapezium are 25 cm and 13 cm ; its nonparallel sides are equal, each being 10 cm , find the area of the trapezium.

Answer



In $\triangle CEF$

$CE = 10$ cm and $EF = 6$ cm

Using pythagorouss theorem:

$$CE^2 = CF^2 + EF^2$$

$$CF^2 = CE^2 - EF^2$$

$$CF^2 = 10^2 - 6^2$$

$$CF^2 = 100 - 36$$

$$CF^2 = 64$$

$$CF = 8$$
 cm

Area of trapezium = area of parallelogram AECD + area of area of triangle CEF

Area of trapezium = *base* \times *height* + $\frac{1}{2}$ (*base* \times *hight*)

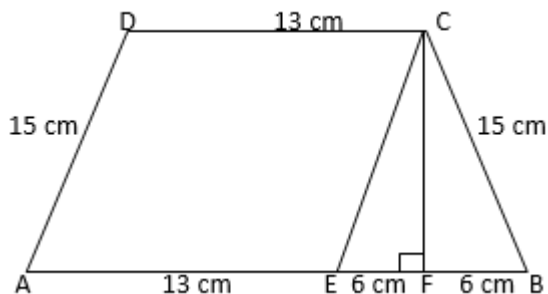
$$\text{Area of trapezium} = 13 \times 8 + \frac{1}{2} (12 \times 8)$$

$$\text{Area of trapezium} = 104 + 48 = 152 \text{ cm}^2$$

17. Question

Find the area of a trapezium whose parallel sides are 25 cm, 13 cm and the other sides are 15 cm each.

Answer



In $\triangle CEF$

$$CE = 10 \text{ cm and } EF = 6 \text{ cm}$$

Using pythagoruous theorem:

$$CE^2 = CF^2 + EF^2$$

$$CF^2 = CE^2 - EF^2$$

$$CF^2 = 15^2 - 6^2$$

$$CF^2 = 225 - 36$$

$$CF^2 = 189$$

$$CF = 17 \text{ cm}$$

Area of trapezium = area of parallelogram AECD + area of area of triangle CEF

$$\text{Area of trapezium} = \text{base} \times \text{height} + \frac{1}{2} (\text{base} \times \text{hight})$$

$$\text{Area of trapezium} = 13 \times 17 + \frac{1}{2} (12 \times 17)$$

$$\text{Area of trapezium} = 221 + 102 = 323 \text{ cm}^2$$

18. Question

If the area of a trapezium is 28 cm^2 and one of its parallel sides is 6 cm, find the other parallel side if its altitude is 4 cm.

Answer

Let the length of other parallel side of trapezium = $x \text{ cm}$

Length of one parallel side of trapezium = 6 cm

Area of trapezium = 28 cm^2

Length of altitude of trapezium = 4 cm

$$\text{Area of trapezium} = \frac{1}{2} (\text{sum of sides}) \times \text{distance between parallel sides}$$

$$28 = \frac{1}{2} (6 + x) \times 4$$

$$6 + x = \frac{28}{2}$$

$$x = 14 - 6 = 8$$

Therefore the length of other parallel side of trapezium = 8 cm

19. Question

In Fig. 20.38, a parallelogram is drawn in a trapezium, the area of the parallelogram is 80 cm^2 , find the area of the trapezium.

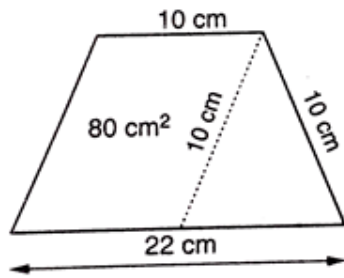
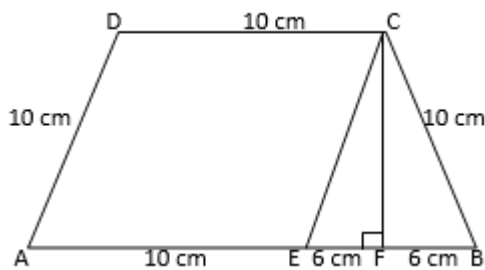


Fig. 20.38

Answer



In $\triangle CEF$

$$CE = 10 \text{ cm and } EF = 6 \text{ cm}$$

Using pythagoruous theorem:

$$CE^2 = CF^2 + EF^2$$

$$CF^2 = CE^2 - EF^2$$

$$CF^2 = 10^2 - 6^2$$

$$CF^2 = 100 - 36$$

$$CF^2 = 64$$

$$CF = 8 \text{ cm}$$

$$\text{Area of parallelogram} = 80 \text{ cm}^2$$

$$\text{Area of trapezium} = \text{area of parallelogram AECD} + \text{area of area of triangle CEF}$$

$$\text{Area of trapezium} = \text{base} \times \text{height} + \frac{1}{2} (\text{base} \times \text{hight})$$

$$\text{Area of trapezium} = 80 + \frac{1}{2} (12 \times 8)$$

$$\text{Area of trapezium} = 80 + 48 = 128 \text{ cm}^2$$

20. Question

Find the area of the field shown in Fig. 20.39 by dividing it into a square, a rectangle and a trapezium.

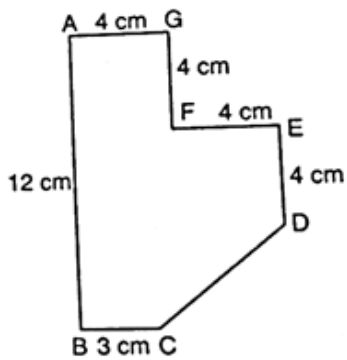
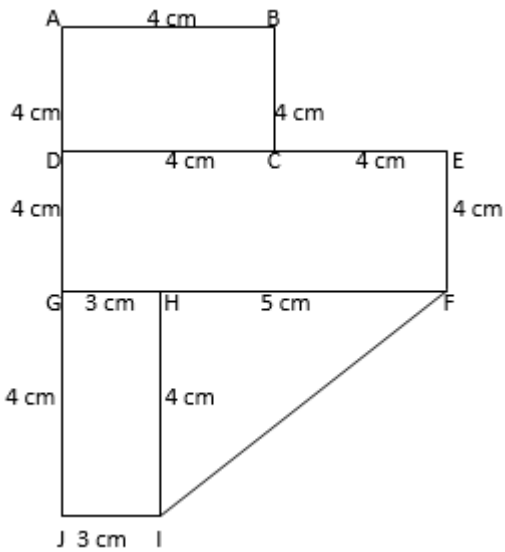


Fig. 20.39

Answer



Area of given figure = area of square ABCD + area of rectangle DEFG + area of rectangle GHIJ + area of triangle FHI

Area of given figure = $Side \times side + length \times breadth + length \times breadth + \frac{1}{2} \times base \times altitude$

Area of given figure = $4 \times 4 + 8 \times 4 + 3 \times 4 + \frac{1}{2} \times 5 \times 4$

Area of given figure = $16 + 32 + 12 + 10 = 70 \text{ cm}^2$

Exercise 20.3

1. Question

Find the area of the pentagon shown in fig. 20.48, if $AD = 10 \text{ cm}$, $AG = 8 \text{ cm}$, $AH = 6 \text{ cm}$, $AF = 5 \text{ cm}$, $BF = 5 \text{ cm}$, $CG = 7 \text{ cm}$ and $EH = 3 \text{ cm}$.

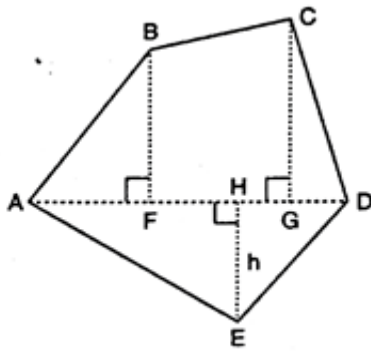


Fig. 20.48

Answer

$$GH = AG - AH = 8 - 6 = 2 \text{ cm}$$

$$HF = AH - AF = 6 - 5 = 1 \text{ cm}$$

$$GD = AD - AG = 10 - 8 = 2 \text{ cm}$$

Area of given figure = area of triangle AFB + area of trapezium BCGF + area of triangle CGD + area of triangle AHE + area of triangle EGD

$$\text{Area of right angled triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\text{Area of trapezium} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{altitude}$$

Area of given pentagon =

$$\frac{1}{2} \times AF \times BF + \frac{1}{2} \times (CG + BF) \times FG + \frac{1}{2} \times GD \times CG + \frac{1}{2} \times AH \times EH + \frac{1}{2} \times HD \times EH$$

$$\text{Area of given pentagon} = \frac{1}{2} \times 5 \times 5 + \frac{1}{2} \times (7 + 5) \times 3 + \frac{1}{2} \times 2 \times 7 + \frac{1}{2} \times 6 \times 3 + \frac{1}{2} \times 4 \times 3$$

$$\text{Area of given pentagon} = 12.5 + 18 + 7 + 9 + 6 = 52.5 \text{ cm}^2$$

Therefore area of given pentagon is 70 cm²

2. Question

Find the area enclosed by each of the following figures [fig. 20.49 (i)-(ii)] as the sum of the areas of a rectangle and a trapezium.

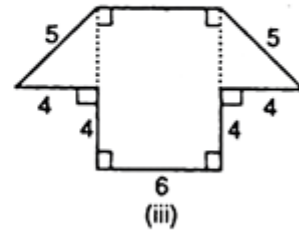
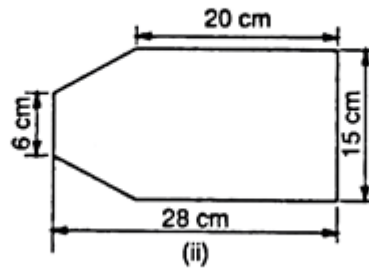
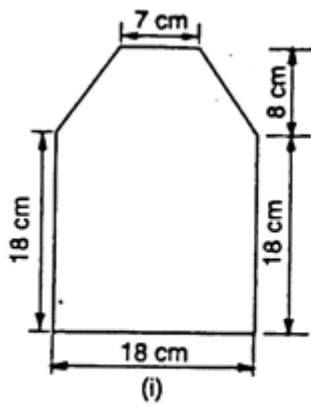


Fig. 20.49

Answer

Figure (i)

Area of given figure = Area of trapezium + area of rectangle

$$\text{Area of given figure} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{altitude} + \text{length} \times \text{breadth}$$

$$\text{Area of given figure} = \frac{1}{2} \times (18 + 7) \times 8 + 18 \times 18$$

$$\text{Area of given figure} = 100 + 324 = 424$$

Therefore area of given figure is 424 cm^2

Figure (ii)

Area of given figure = Area of trapezium + area of rectangle

$$\text{Area of given figure} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{altitude} + \text{length} \times \text{breadth}$$

$$\text{Area of given figure} = \frac{1}{2} \times (15 + 6) \times 8 + 15 \times 20$$

$$\text{Area of given figure} = 84 + 300 = 384$$

Therefore area of given figure is 384 cm^2

Figure (iii)

Using pythagorous theorem in the right angled triangle

$$5^2 = 4^2 + x^2$$

$$x^2 = 25 - 16$$

$$x^2 = 9$$

$$x = 3 \text{ cm}$$

Area of given figure = Area of trapezium + area of rectangle

$$\text{Area of given figure} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{altitude} + \text{length} \times \text{breadth}$$

$$\text{Area of given figure} = \frac{1}{2} \times (14 + 6) \times 3 + 4 \times 6$$

$$\text{Area of given figure} = 30 + 24 = 54$$

Therefore area of given figure is 54 cm^2

3. Question

There is a pentagonal shaped park as shown in Fig. 20.50. Jyoti and Kavita divided it in two different ways.

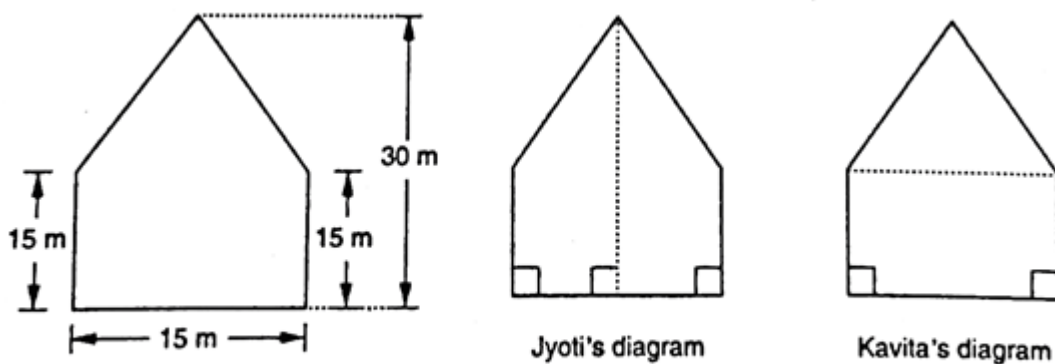


Fig. 20.50

Find the area of this park using both ways. Can you suggest some another way of finding its area?

Answer

Area of given figure = Area of trapezium + area of rectangle

$$\text{Area of Jyoti's diagram} = 2 \times \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{altitude}$$

$$\text{Area of given figure} = 2 \times \frac{1}{2} \times (15 + 30) \times 7.5$$

$$\text{Area of given figure} = 337.5 \text{ cm}^2$$

Therefore area of given figure is 54 cm^2

Area of given pentagon = Area of triangle + area of rectangle

$$\text{Area of given pentagon} = \frac{1}{2} \times \text{base} \times \text{altitude} + \text{length} \times \text{breadth}$$

$$\text{Area of given pentagon} = \frac{1}{2} \times 15 \times 15 + 15 \times 15$$

$$\text{Area of given pentagon} = 112.5 + 225 = 337.5$$

Therefore area of given pentagon is 337.5 cm^2

4. Question

Find the area of the following polygon, if $AL = 10$ cm, $AM = 20$ cm, $AN = 50$ cm. $AO = 60$ cm and $AD = 90$ cm.

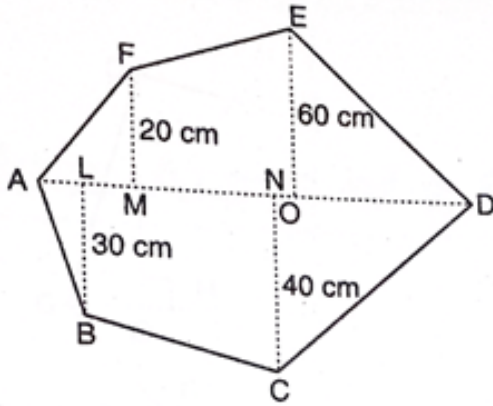


Fig. 20.51

Answer

$AL = 10$ cm; $AM = 20$ cm; $AN = 50$ cm; $AO = 60$ cm; $AD = 90$ cm given

$$LM = AM - AL = 20 - 10 = 10 \text{ cm}$$

$$MN = AN - AM = 50 - 20 = 30 \text{ cm}$$

$$OD = AD - AO = 90 - 60 = 30 \text{ cm}$$

$$ON = AO - AN = 60 - 50 = 10 \text{ cm}$$

$$DN = OD + ON = 30 + 10 = 40 \text{ cm}$$

$$OM = MN + ON = 30 + 10 = 40 \text{ cm}$$

$$LN = LM + MN = 10 + 30 = 40 \text{ cm}$$

Area of given figure = area of triangle AMF + area of trapezium FMNE + area of triangle END + area of triangle ALB + area of trapezium LBCN + area of triangle DNC

$$\text{Area of right angled triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\text{Area of trapezium} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{altitude}$$

Area of given hexagon =

$$\frac{1}{2} \times AM \times FM + \frac{1}{2} \times (MF + OE) \times OM + \frac{1}{2} \times OD \times OE + \frac{1}{2} \times AL \times BL + \frac{1}{2} \times (BL + CN) \times LN + \frac{1}{2} \times DN \times CN$$

Area of given hexagon =

$$\frac{1}{2} \times 20 \times 20 + \frac{1}{2} \times (20 + 60) \times 40 + \frac{1}{2} \times 30 \times 60 + \frac{1}{2} \times 10 \times 30 + \frac{1}{2} \times (30 + 40) \times 40 + \frac{1}{2} \times 40 \times 40$$

$$\text{Area of given hexagon} = 200 + 1600 + 900 + 150 + 1400 + 800 = 5050 \text{ cm}^2$$

Therefore area of given hexagon is 5050 cm^2

5. Question

Find the area of the following regular hexagon.

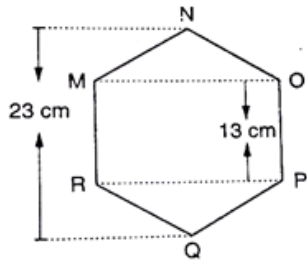


Fig. 20.52

Answer

$NQ = 23$ cm given

$$ND + QC = 23 - 13 = 10$$

$$ND = QC = 5 \text{ cm}$$

$$OM = 24 \text{ cm}$$

Area of given hexagon = 2 times area of triangle MNO + area of rectangle MOPR

$$\text{Area of right angled triangle} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\text{Area of rectangle} = \text{length} \times \text{breadth}$$

$$\text{Area of regular hexagon} = \frac{1}{2} \times OM \times ND + MO \times OP + \frac{1}{2} \times RP \times QC$$

$$\text{Area of given hexagon} = \frac{1}{2} \times 24 \times 5 + 24 \times 13 + \frac{1}{2} \times 24 \times 5 = 432 \text{ cm}^2$$

Therefore area of given hexagon is 432 cm^2