Chapter : 5. CONGRUENCE OF TRIANGLES AND INEQUALITIES IN A TRIANGLE

Exercise : 5A

Question: 1

Given that

AB = AC and $\angle A = 70°$

To find: $\angle B$ and $\angle C$

AB = AC and also $\angle A = 70°$

As two sides of triangle are equal, we say that ΔABC is isosceles triangle.

Hence by the property of isosceles triangle, we know that base angles are also equal.

ie. we state that $\angle B = \angle C$. ...(1)

Now,

Sum of all angles in any triangle = 180°

$\therefore \angle A + \angle B + \angle C = 180°$

Hence,

$70° + \angle B + \angle C = 180°$

$2\angle B = 180° - 70°$ ...from (1)

$\therefore 2\angle B = 110°$

$\angle B = 55°$

Therefore, our base angles, $\angle B$ and $\angle C$, are 55° each.

Question: 2

Given: The given triangle is isosceles triangle. Also vertex angle is 100°

To find: Measure of base angles.

It is given that triangle is isosceles.

So let our given triangle be ΔABC.

And let $\angle A$ be the vertex angle, which is given as $\angle A = 100°$

By the property of isosceles triangle, we know that base angles are equal.

So,

$\angle B = \angle C$ ...(1)

We know that,

Sum of all angles in any triangle = 180°

$\therefore \angle A + \angle B + \angle C = 180°$

$100° + 2\angle B = 180°$ ...from (1)

$\therefore 2\angle B = 180° - 100°$

$2\angle B = 80°$

$\therefore \angle B = 40°$

Therefore, our base angles, $\angle B$ and $\angle C$, are 40° each.

Question: 3

Given: In ΔABC,

$AB=AC$ and $\angle B=65°$

To find : $\angle A$ and $\angle C$

It is given that $AB=AC$ and $\angle B=65°$

As two sides of the triangle are equal, we say that triangle is isosceles triangle, with vertex angle A.

Hence by the property of isosceles triangle we know that base angles are equal.

$\therefore \angle B = \angle C$

$\therefore \angle C = \angle B = 65°$

Also, We know that,

Sum of all angles in any triangle = 180°

$\therefore \angle A + \angle B + \angle C = 180°$

$\angle A + 65° + 65° = 180°$

$\angle A + 130° = 180°$

$\therefore \angle A = 180° - 130°$

$\angle A = 50°$

Hence, $\angle C = 65°$ and $\angle A = 50°$

Question: 4

Given: Our given triangle is isosceles triangle. Also, the vertex angle is twice the sum of the base angles

To find: Measures of angles of triangle.

It is given that that given triangle is isosceles triangle.

Let vertex angle be y and base angles be x each.

So by given condition,

$y = 2(x + x)$

$\therefore y = 4x$

Also, We know that,

Sum of all angles in any triangle = 180°

$\therefore y + x + x = 180°$

$y + 2x = 180°$

$4x + 2x = 180°$

$\therefore 6x = 180°$

$x = 30°$

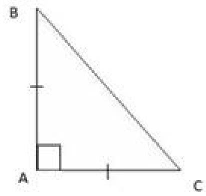
$\therefore y = 4 \times 30°$

$y = 120°$

Hence, vertex angle is 120° and base angles are 30° each.

Question: 5

Here given triangle is isosceles right angled triangle.



So let our triangle be ΔABC, right angled at A.

∴ ∠A = 90°

Here, AB = AC, as our given triangle is isosceles triangle.

Hence, base angles, ∠B and ∠C are equal.

Also, We know that,

Sum of all angles in any triangle = 180°

∴ ∠A + ∠B + ∠C = 180°

90° + 2 ∠B = 180°

2∠B = 90°

∠B = 45°

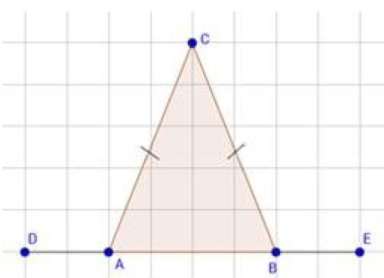
Hence the measure of each of the equal angles of a right-angled isosceles triangle is 45°

Question: 6

Given: ΔABC is isosceles triangle.

To prove: ∠CAD = ∠CBE

Let ΔABC be our isosceles triangle as shown in the figure.



We know that base angles of the isosceles triangle are equal.

Here, ∠CAB = ∠CBA ....(1)

Also here, ∠CAD and ∠CBE are exterior angles of the triangle.

So, we know that,

∠CAB +∠CAD = 180°... exterior angle theorem

And ∠CBA + ∠CBE = 180° ... exterior angle theorem

So from (1) and above statement, we conclude that,

∠CAB +∠CAD = 180°

And ∠CAB +∠CBE = 180°

Which implies that,

∠CAD = 180° - ∠CAB

And ∠CBE = 180° - ∠CAB

Hence we say that ∠CAD = ∠CBE

∴For the isosceles triangle, the exterior angles so formed are equal to each other.

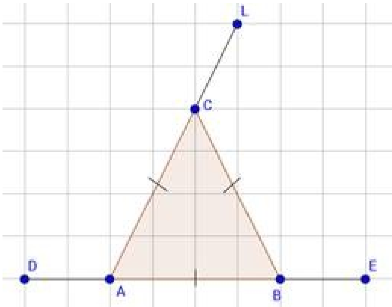
Question: 7

Given: ΔABC is equilateral triangle.

To prove: $\angle$CAD = $\angle$CBE = $\angle$BCL

Proof:

Let our triangle be ΔABC, which is equilateral triangle as shown in the figure.



Hence all angles are equal and measure 60° each.

$\therefore$ $\angle$CAB = $\angle$CBA = $\angle$BCA = 60° ...(1)

Also here, $\angle$CAD and $\angle$CBE are exterior angles of the triangle.

So, we know that,

$\angle$CAB +$\angle$CAD = 180° ... exterior angle theorem

$\angle$CBA + $\angle$CBE = 180° ... exterior angle theorem

$\angle$BCA + $\angle$BCL = 180° ... exterior angle theorem

From (1) and above statements, we state that,

60° +$\angle$CAD = 180°

60° + $\angle$CBE = 180°

60° + $\angle$BCL = 180°

Simplifying above statements,

$\angle$CAD = 180° - 60° = 120°

$\angle$CBE = 180° - 60° = 120°

$\angle$BCL = 180° - 60° = 120°

Hence, the measure of each exterior angle of an equilateral triangle is 120°

Question: 8

Given: AO = OB , DO = OC

To prove: AC=BD and AC||BD

Proof:

It is given that, O is the midpoint of each of the line segments AB and CD.

This implies that AO = OB and DO = OC

Here line segments AB and CD are concurrent.

So,

$\angle$AOC = $\angle$BOD .... As they are vertically opposite angles.

Now in ΔAOC and ΔBOD,

AO = OB,

OC = OD

Also, ∠AOC = ∠BOD

Hence, ΔAOC ≅ ΔBOD … by SAS property of congruency

So,

AC = BD … by cpct

∴ ∠ACO = ∠BDO … by cpct

But ∠ACO and ∠BDO are alternate angles.

∴ We conclude that AC is parallel to BD.

Hence we proved that AC=BD and AC||BD

Question: 9

Given: PA ⊥ AB, QB ⊥ AB and PA=QB

To prove: AO = OB and PO = OQ

It is given that PA ⊥ AB and QB ⊥ AB.

This means that ΔPAO and ΔQBO are right angled triangles.

It is also given that PA=QB

Now in ΔPAO and ΔQBO,

∠OAP = ∠OBQ = 90°

PO = OQ

Hence by hypotenuse-leg congruency,

ΔPAO ≅ ΔQBO

∴AO = OB and PO = OQ ….by cpct

Hence proved that AO = OB and PO = OQ

Question: 10

Given: AO = OD and CO = OB

To prove: AC = BD

Proof :

It is given that AO = OD and CO = OB

Here line segments AB and CD are concurrent.

So,

∠AOC = ∠BOD …. As they are vertically opposite angles.

Now in ΔAOC and ΔDOB,

AO = OD,

CO = OD

Also, ∠AOC = ∠BOD

Hence, ΔAOC ≅ ΔBOD … by SAS property of congruency

So,

AC = BD … by cpct

Here,

∠ACO ≠ ∠BDO or ∠OAC ≠ ∠OBD

Hence there are no alternate angles, unless both triangles are isosceles triangle.

Hence proved that AC=BD but AC may not be parallel to BD.

Question: 11

Here it is given that l ‖ m ie. AC ||DB.

Also given that AM = MB

Now in ΔAMC and ΔBMD,

∠CAM = ∠DBM ... Alternate angles

AM = MB

∠AMC = ∠BMD ... vertically opposite angles

Hence, ΔAMC ≅ ΔBMD ... by ASA property of congruency

∴ CM = MD ...cpct

Hence proved that M is also the midpoint of any line segment CD having its end points at l and m respectively.

Question: 12

ΔABC and ΔOBC are isosceles triangle.

∴ ∠ABC = ∠ACB and ∠OBC = ∠OCB ....(1)

Also,

∠ABC = ∠ABO + ∠OBC

And ∠ACB = ∠ACO + ∠OCB

From 1 and above equations, we state that,

∠ABC = ∠ABO + ∠OBC

And ∠ABC = ∠ACO + ∠OBC

This implies that,

∠ABO = ∠ABC - ∠OBC

And ∠ACO = ∠ABC - ∠OBC

Hence,

∠ABO = ∠ACO = ∠ABC - ∠OBC

Question: 13

Given that AB = AC and also DE || BC.

So by Basic proportionality theorem or Thales theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \frac{DB}{AD} = \frac{EC}{AE}$

Now adding 1 on both sides,

$\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

$\frac{DB+AD}{AD} = \frac{EC+AE}{AE}$

$\frac{AB}{AD} = \frac{AC}{AE}$ ... as AB = AD + DE and AC = AE + EC

But is given that AB = AC,

$\therefore \frac{AB}{AD} = \frac{AB}{AE}$

Hence,

AD = AE.

Question: 14

Here it is given that AX = AY.

Now in ΔCXA and ΔBYA,

AX = AY

∠XAC = ∠YAB … Same angle or common angle.

AC = AB … given condition Hence by SAS property of congruency,

ΔCXA ≅ ΔBYA

Hence by cpct, we conclude that,

CX = BY

Question: 15

It is given that AC = BC , ∠DCA = ∠ECB and ∠DBC = ∠EAC.

Adding angle ∠ECD both sides in ∠DCA = ∠ECB, we get,

∠DCA + ∠ECD = ∠ECB + ∠ECD

∴∠ECA = ∠DCB …addition property

Now in ΔDBC and ΔEAC,

∠ECA = ∠DCB

BC = AC

∠DBC = ∠EAC

Hence by ASA postulate, we conclude,

ΔDBC ≅ ΔEAC

Hence, by cpct, we get,

DC = EC

Question: 16

Given : BA ⊥ AC and DE ⊥ EF such that BA=DE and BF=DC

To prove: AC = EF

Proof:

In ΔABC, we have,

BC = BF + FC

And , in ΔDEF,

FD = FC + CD

But, BF = CD

So, BC = BF + FC

And, FD = FC +BF

∴ BC = FD

So, in ΔABC and ΔDEF, we have,

∠BAC = ∠DEF … given

BC = FD

AB = DE ...given

Thus by Right angle - Hypotenuse- Side property of congruence, we have,

ΔABC ≅ ΔDEF

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ AC = EF

Question: 17

Given: x=y and AB=CB

To prove: AE = CD

Proof:

In ΔABE, we have,

∠AEC = ∠EBA + ∠BAE ...Exterior angle theorem

y° = ∠EBA + ∠BAE

Now in ΔBCD, we have,

x° = ∠CBA + ∠BCD

Since, given that,

x = y ,

∠CBA + ∠BCD = ∠EBA + ∠BAE

∴ ∠BCD = ∠BAE ... as ∠CBA and ∠EBA and same angles.

Hence in ΔBCD and ΔBAE,

∠B = ∠B

BC = AB ...given

∠BCD = ∠BAE

Thus by ASA property of congruence, we have,

ΔBCD ≅ ΔBAE

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ CD = AE

Question: 18

Given: AB=AC and BD and AB are angle bisectors of ∠B and ∠C

To prove: BD = CE

Proof:

In ΔABD and ΔACE,

∠ABD = $\frac{1}{2}$ ∠B

And ∠ACE = $\frac{1}{2}$ ∠C

But ∠B = ∠C as AB = AC ... As in isosceles triangle, base angles are equal

∠ABD = ∠ACE

AB = AC

∠A = ∠A

Thus by ASA property of congruence,

$\Delta ABD \cong \Delta ACE$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore$ BD = CE

Question: 19

Given: BC = DC and BL ⊥ AD and DM ⊥ CM

To prove: BL=CM

Proof:

In ΔBLD and ΔCMD,

$\angle BLD = \angle CMD = 90°$ … given

$\angle BLD = \angle MDC$ … vertically opposite angles

BD = DC … given

Thus by AAS property of congruence,

$\Delta BLD \cong \Delta CMD$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore$ BL = CM

Question: 20

Given: BD = DC and DL ⊥ AB and DM ⊥ AC such that DL=DM

To prove: AB = AC

Proof:

In right angled triangles ΔBLD and ΔCMD,

$\angle BLD = \angle CMD = 90°$

BD = CD … given

DL = DM … given

Thus by right angled hypotenuse side property of congruence,

$\Delta BLD \cong \Delta CMD$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle ABD = \angle ACD$

In ΔABC, we have,

$\angle ABD = \angle ACD$

$\therefore$ AB= AC …. Sides opposite to equal angles are equal

Question: 21

Given: In ΔABC, AB=AC and the bisectors of $\angle B$ and $\angle C$ meet at a point O.

To prove: BO=CO and $\angle BAO = \angle CAO$

Proof:

In , ΔABC we have,

$\angle OBC = \frac{1}{2}\angle B$

$\angle OCB = \frac{1}{2}\angle C$

But $\angle B = \angle C$ … given

So, $\angle OBC = \angle OCB$

Since the base angles are equal, sides are equal

$\therefore$ OC = OB ...(1)

Since OB and OC are bisectors of angles $\angle B$ and $\angle C$ respectively, we have

$\angle ABO = \frac{1}{2} \angle B$

$\angle ACO = \frac{1}{2} \angle C$

$\therefore \angle ABO = \angle ACO$ ...(2)

Now in ΔABO and ΔACO

AB = AC ... given

$\angle ABO = \angle ACO$ ... from 2

BO = OC ... from 1

Thus by SAS property of congruence,

$\Delta ABO \cong \Delta ACO$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle BAO = \angle CAO$

ie. AO bisects $\angle A$

Question: 22

Given: PQR is an equilateral triangle and QRST is a square

To prove: PT=PS and $\angle PSR=15°$.

Proof:

Since ΔPQR is equilateral triangle,

$\angle PQR = \angle PRQ = 60°$

Since QRTS is a square,

$\angle RQT = \angle QRS = 90°$

In ΔPQT,

$\angle PQT = \angle PQR + \angle RQT$

$= 60° + 90°$

$= 150°$

In ΔPRS,

$\angle PRS = \angle PRQ + \angle QRS$

$= 60° + 90°$

$= 150°$

$\therefore \angle PQT = \angle PRS$

Thus in ΔPQT and ΔPRS,

PQ = PR ... sides of equilateral triangle

$\angle PQT = \angle PRS$

QT = RS ... side of square

Thus by SAS property of congruence,

$\Delta PQT \cong \Delta PRS$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore$ PT = PS

Now in ΔPRS, we have,

PR = RS

$\therefore \angle PRS = \angle PSR$

But $\angle PRS = 150°$

SO, by angle sum property,

$\angle PRS + \angle PSR + \angle SPR = 180°$

$150° + \angle PSR + \angle SPR = 180°$

$2\angle PSR = 180° - 150°$

$2\angle PSR = 30°$

$\angle PSR = 15°$

Question: 23

Given: $\angle ABC = 90°$, BCDE is a square on side BC and ACFG is a square on AC

To prove: AD = EF

Proof:

Since BCDE is square,

$\angle BCD = 90°$ ...(1)

In ΔACD,

$\angle ACD = \angle ACB + \angle BCD$

$= \angle ACB + 90°$ ...(2)

In ΔBCF,

$\angle BCF = \angle BCA + \angle ACF$

Since ACFG is square,

$\angle ACF = 90°$ ...(3)

From 2 and 3, we have,

$\angle ACD = \angle BCF$ ....(4)

Thus in ΔACD and ΔBCF, we have,

AC = CF ...sides of square

$\angle ACD = \angle BCF$ ...from 4

CD = BC ... sides of square

Thus by SAS property of congruence,

$\Delta ACD \cong \Delta BCF$

Hence, we know that, corresponding parts of the congruent triangles are equal

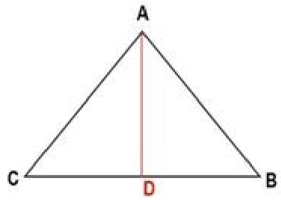
$\therefore$ AD = BF

Question: 24

Given: ΔABC is isosceles triangle where AB = AC and BD = DC

To prove: $\angle BAD = \angle DAC$

Proof:



In ΔABD and ΔADC

AB = AC …given

BD = DC …given

AD = AD … common side

Thus by SSS property of congruence,

ΔABD ≅ ΔADC

Hence, we know that, corresponding parts of the congruent triangles are equal

∠BAD = ∠DAC

Question: 25

Given: ABCD is a quadrilateral in which AB∥DC and BP = PC

To prove: AB=CQ and DQ=DC+AB

Proof:

In ΔABP and ΔPCQ we have,

∠PAB = ∠PQC …alternate angles

∠APB = ∠CPQ … vertically opposite angles

BP = PC … given

Thus by AAS property of congruence,

ΔABP ≅ ΔPCQ

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ AB = CQ …(1)

But, DQ = DC + CQ

= DC + AB …from 1

Question: 26

Given: OA=OB and OP=OQ

To prove: PX=QX and AX=BX

Proof:

In ΔOAQ and ΔOPB, we have

OA = OB …given

∠O = ∠O …common angle

OQ = OP … given

Thus by SAS property of congruence,

ΔOAP ≅ ΔOPB

Hence, we know that, corresponding parts of the congruent triangles are equal

∠OBP = ∠OAQ …(1)

Thus, in ΔBXQ and ΔPXA, we have,

BQ = OB – OQ

And PA = OA – OP

But OP = OQ

And OA = OB …given

Hence, we have, BQ = PA …(2)

Now consider ΔBXQ and ΔPXA,

∠BXQ = ∠PXA … vertically opposite angles

∠OBP = ∠OAQ …from 1

BQ = PA … from 2

Thus by AAS property of congruence,

ΔBXQ ≅ ΔPXA

Hence, we know that, corresponding parts of the congruent triangles are equal

∴ PX = QX

And AX = BX

Question: 27

Given: ABCD is a square and PB=PD

To prove: CPA is a straight line

Proof:

ΔAPD and ΔAPB,

DA = AB …as ABCD is square

AP = AP … common side

PB = PD … given

Thus by SSS property of congruence,

ΔAPD ≅ ΔAPB

Hence, we know that, corresponding parts of the congruent triangles are equal

∠APD = ∠APB …(1)

Now consider ΔCPD and ΔCPB,

CD = CB … ABCD is square

CP = CP … common side

PB = PD … given

Thus by SSS property of congruence,

ΔCPD ≅ ΔCPB

Hence, we know that, corresponding parts of the congruent triangles are equal

∠CPD = ∠CPB … (2)

Now,

Adding both sides of 1 and 2,

∠CPD +∠APD = ∠APB + ∠CPB …(3)

Angels around the point P add upto 360°

$\therefore \angle CPD + \angle APD + \angle APB + \angle CPB = 360°$

From 4,

$2(\angle CPD + \angle APD) = 360°$

$\angle CPD + \angle APD = \frac{360°}{2} = 180°$

This proves that CPA is a straight line.

Question: 28

Given: ABC is an equilateral triangle, PQ $\parallel$AC and CR=BP

To prove: QR bisects PC or PM = MC

Proof:

Since, ΔABC is equilateral triangle,

$\angle A = \angle ACB = 60°$

Since, PQ $\parallel$AC and corresponding angles are equal,

$\angle BPQ = \angle ACB = 60°$

In ΔBPQ,

$\angle B = \angle ACB = 60°$

$\angle BPQ = 60°$

Hence, ΔBPQ is an equilateral triangle.

$\therefore$ PQ = BP = BQ

Since we have BP = CR,

We say that PQ = CR ...(1)

Consider the triangles ΔPMQ and ΔCMR,

$\angle PQM = \angle CRM$ ...alternate angles

$\angle PMQ = \angle CMR$ ... vertically opposite angles

PQ = CR ... from 1

Thus by AAS property of congruence,

$\Delta PMQ \cong \Delta CMR$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore$ PM = MC

Question: 29

Given: ABCD is a quadrilateral in which AB=AD and BC=DC

To prove: AC bisects $\angle A$ and $\angle C$, and AC is the perpendicular bisector of BD

Proof:

In ΔABC and ΔADC, we have

AB = AD ...given

BC = DC ... given

AC = AC ... common side

Thus by SSS property of congruence,

$\Delta ABC \cong \Delta ADC$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle BAC = \angle DAC$

$\therefore \angle BAO = \angle DAO$ ...(1)

It means that AC bisects $\angle BAD$ ie $\angle A$

Also, $\angle BCA = \angle DCA$ ... cpct

It means that AC bisects $\angle BCD$, ie $\angle C$

Now in ΔABO and ΔADO

AB = AD ...given

$\angle BAO = \angle DAO$ ... from 1

AO = AO ... common side

Thus by SAS property of congruence,

$\Delta ABO \cong \Delta ADO$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\angle BOA = \angle DAO$

But $\angle BOA + \angle DAO = 180°$

$2\angle BOA = 180°$

$\therefore \angle BOA = \frac{180°}{2} = 90°$

Also $\Delta ABO \cong \Delta ADO$

So, BO = OD

Which means that AC = BD

Question: 30

Given: IP $\perp$BC, IQ $\perp$CA and IR $\perp$ AB and the bisectors of $\angle B$ and $\angle C$ of ΔABC meet at I

To prove: IP=IQ=IR and IA bisects $\angle A$

Proof:

In ΔBIP and ΔBIR we have,

$\angle PBI = \angle RBI$ ...given

$\angle IRB = \angle IPB = 90°$ ...Given

IB = IB ...common side

Thus by AAS property of congruence,

$\Delta BIP \cong \Delta BIR$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore$ IP = IR

Similarly,

IP = IQ

Hence, IP = IQ = IR

Now in ΔAIR and ΔAIQ

IR = IQ ...proved above

IA = IA ... Common side

$\angle IRA = \angle IQA = 90°$

Thus by SAS property of congruence,

$\Delta AIR \cong \Delta AIQ$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore \angle IAR = \angle IAQ$

This means that IA bisects $\angle A$

Question: 31

Given: P is a point in the interior of $\angle AOB$ and $PL \perp OA$ and $PM \perp OB$ such that PL=PM

To prove: $\angle POL = \angle POM$

Proof:

In ΔOPL and ΔOPM, we have

$\angle OPM = \angle OPL = 90°$ …given

OP = OP …common side

PL = PM … given

Thus by Right angle hypotenuse side property of congruence,

$\Delta OPL \cong \Delta OPM$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore \angle POL = \angle POM$

Ie. OP is the bisector of $\angle AOB$

Question: 32

Given: ABCD is a square, AM = MB and $PQ \perp CM$

To prove: PA=BQ and CP=AB+PA

Proof:

In ΔAMP and ΔBMQ, we have

$\angle AMP = BMQ$ …vertically opposite angle

$\angle PAM = \angle MBQ = 90°$ …as ABCD is square

AM = MB …given

Thus by AAS property of congruence,

$\Delta AMP \cong \Delta BMQ$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore$ PA = BQ and MP = MQ …(1)

Now in ΔPCM and ΔQCM

PM = QM … from 1

$\angle PMC = \angle QMC$ … given

CM = CM … common side

Thus by AAS property of congruence,

$\Delta PCM \cong \Delta QCM$

Hence, we know that, corresponding parts of the congruent triangles are equal

$\therefore$ PC = QC

PC = QB + CB

PC = AB + PA …as AB = CB and PA = QB

Question: 33

Given: AB ⊥ BO and NM ⊥ OM

In ΔABO and ΔNMO,

∠OBA = ∠OMN

OB = OM …O is mid point of BM

∠BOA = ∠MON …vertically opposite angles

Thus by AAS property of congruence,

ΔABO ≅ ΔNMO

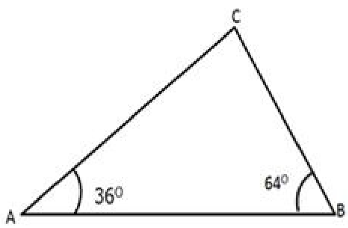
Hence, we know that, corresponding parts of the congruent triangles are equal

∴ AB = MN

Hence, we can calculate the width of the river by calculating MN

Question: 34

Given: ∠A=36° and ∠B=64°



To find: The longest and shortest sides of the triangle

Given that ∠A=36° and ∠B=64°

Hence, by the angle sum property in ΔABC, we have

∠A + ∠B + ∠C = 180°

36° + 64°+ ∠C = 180°

100° + ∠C = 180°

∠C = 80°

So, we have ∠A=36°, ∠B=64° and ∠C = 80°

∴∠C is largest and ∠A is shortest

Hence,

Side opposite to ∠C is longest.

∴AB is longest

Side opposite to ∠A is shortest.

∴ BC is shortest

Question: 35

It is given that ∠A=90°.

In right angled triangle at 90°

Sum of all angles in triangle is 180°, so other two angles must be less that 90°

So, other angles are smaller than ∠A.

Hence ∠A is largest angle.

We know that side opposite to largest angle is largest.

$\therefore$BC is longest side, which is opposite to $\angle A$.

Question: 36

In ΔABC given that $\angle A = \angle B = 45°$

So, by the angle sum property in ΔABC, we have

$\angle A + \angle B + \angle C = 180°$

$45° + 45° + \angle C = 180°$

$90° + \angle C = 180°$

$\therefore \angle C = 180° - 90°$

$\angle C = 90°$

Hence, largest angle is $\angle C$

We know that side opposite to largest angle is longest, which is AB

Hence our longest side is AB

Question: 37

Given: In ΔABC, BD=BC and $\angle B = 60°$ and $\angle A = 70°$

To prove: AD>CD and AD>AC

Proof:

In ΔABC, by the angle sum property, we have

$\angle A + \angle B + \angle C = 180°$

$70° + 60° + \angle C = 180°$

$130° + \angle C = 180°$

$\angle C = 50°$

Now in ΔBCD we have,

$\angle CBD = \angle DAC + \angle ACB$ ... as $\angle CBD$ is the exterior angle of $\angle ABC$

$= 70° + 50°$

Since BC = BD ...given

So, $\angle BCD = \angle BDC$

$\therefore \angle BCD + \angle BDC = 180° - \angle CBD$

$= 180° - 120° = 60°$

$2\angle BCD = 60°$

$\angle BCD = \angle BDC = 30°$

Now in ΔACD we have

$\angle A = 70°$, $\angle D = 30°$

And $\angle ACD = \angle ACB + \angle BCD$

$= 50° + 30° = 80°$

$\therefore \angle ACD$ is greatest angle

So, the side opposite to largest angle is longest, ie AD is longest side.

$\therefore$AD > CD

Since, $\angle BDC$ is smallest angle,

The side opposite to $\angle BDC$, ie AC, is the shortest side in ΔACD.

$\therefore AD > AC$

Question: 38

Given: In ΔABC, $\angle B=35°, \angle C=65°$ and $\angle BAX = \angle XAC$

To find: Relation between AX, BX and CX in descending order.

In ΔABC, by the angle sum property, we have

$\angle A + \angle B + \angle C = 180°$

$\angle A + 35° + 65° = 180°$

$\angle A + 100° = 180°$

$\therefore \angle A = 80°$

But $\angle BAX = \frac{1}{2} \angle A$

$= \frac{1}{2} \times 80° = 40°$

Now in ΔABX,

$\angle B = 35°$

$\angle BAX = 40$

And $\angle BXA = 180° - 35° - 40°$

$= 105°$

So, in ΔABX,

$\angle B$ is smallest, so the side opposite is smallest, ie AX is smallest side.

$\therefore AX < BX$ ...(1)

Now consider ΔAXC,

$\angle CAX = \frac{1}{2} \times \angle A$

$= \frac{1}{2} \times 80° = 40°$

$\angle AXC = 180° - 40° - 65°$

$= 180° - 105° = 75°$

Hence, in ΔAXC we have,

$\angle CAX = 40°, \angle C = 65°, \angle AXC = 75°$

$\therefore \angle CAX$ is smallest in ΔAXC

So the side opposite to $\angle CAX$ is shortest

Ie CX is shortest

$\therefore CX < AX$ .... (2)

From 1 and 2 ,

$BX > AX > CX$

This is required descending order

Question: 39

Given: $\angle BAD = \angle DAC$

To prove: $AB>BD$ and $AC>DC$

Proof:

In ΔACD,

$\angle ADB = \angle DAC + \angle ACD$ … exterior angle theorem

$= \angle BAD + \angle ACD$ … given that $\angle BAD = \angle DAC$

$\angle ADB > \angle BAD$

The side opposite to angle $\angle ADB$ is the longest side in ΔADB

So, $AB > BD$

Similarly in ΔABD

$\angle ADC = \angle ABD + \angle BAD$ … exterior angle theorem

$= \angle ABD + \angle CAD$ … given that $\angle BAD = \angle DAC$

$\angle ADC > \angle CAD$

The side opposite to angle $\angle ADC$ is the longest side in ΔACD

So, $AC > DC$

Question: 40

Given: $AB=AC$

To prove: $AD>AC$

Proof:

In ΔABC,

$\angle ACD = \angle B + \angle BAC$

$= \angle ACB + \angle BAC$ …as $\angle C = \angle B$ as $AB = AC$

$= \angle CAD + \angle CDA + \angle BAC$ …as $\angle ACB = \angle CAD + \angle CDA$

$\therefore \angle ACD > \angle CDA$

So the side opposite to $\angle ACD$ is the longest

$\therefore AD > AC$

Question: 41

Given: $AC>AB$ and $\angle BAD = \angle DAC$

To prove: $\angle ADC>\angle ADB$

Proof:

Since $AC > AB$

$\angle ABC > \angle ACB$

Adding $\frac{1}{2}\angle A$ on both sides

$\angle ABC + \frac{1}{2}\angle A > \angle ACB + \frac{1}{2}\angle A$

$\angle ABC + \angle BAD > \angle ACB + \angle DAC$ … As AD is a bisector of $\angle A$

$\therefore \angle ADC > \angle ADB$

Question: 42

Given: S is any point on the side QR

To prove: $PQ+QR+RP>2PS$.

Proof:

Since in a triangle, sum of any two sides is always greater than the third side.

So in ΔPQS, we have,

PQ + QS > PS ...(1)

Similarly, ΔPSR, we have,

PR + SR > PS ...(2)

Adding 1 and 2

PQ + QS + PR + SR > 2PS

PQ + PR + QR > 2PS ...as PR = QS +SR

Question: 43

Given: XOY is a diameter and XZ is any chord of the circle.

To prove: XY>XZ

Proof:

In ΔXOZ,

OX + OZ > XZ ... sum of any sides in a triangle is a greater than its third side

∴ OX + OY > XZ ... As OZ = OY, radius of circle

Hence, XY > XZ ...As OX + OY = XY

Question: 44

Given: O is a point within ΔABC

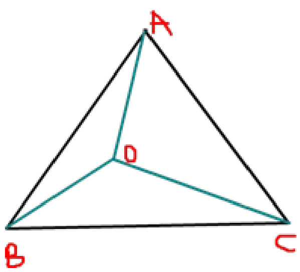
To prove:

(i) AB+AC>OB+OC

(ii) AB+BC+CA>OA+OB+OC

(iii) OA+OB+OC>$\frac{1}{2}$(AB+BC+CA)

Proof:



In ΔABC,

AB +AC >BC ....(1)

And in ΔOBC,

OB + OC > BC ...(2)

Subtracting 1 from 2 we get,

(AB + AC) – (OB + OC) > (BC – BC)

Ie AB + AC > OB + OC

From I, AB + AC > OB + OC

Similarly, AB + BC > OA + OC

And AC + BC > OA + OB

Adding both sides of these three inequalities, we get,

(AB + AC) + (AB + BC) + (AC + BC) > (OB + OC) + (OA + OC) + (OA + OB)

Ie. 2(AB + BC + AC) > 2(OA + OB + OC)

∴ AB + BC + OA > OA + OB + OC

In ΔOAB,

OA + OB > AB ...(1)

In ΔOBC,

OB + OC > BC ...(2)

In ΔOCA

OC + OA > CA ...(3)

Adding 1,2 and 3,

(OA + OB) + (OB + OC) + (OC+ OA) >AB + BC +CA

Ie. 2(OA + OB + OC) > AB + BC + CA ∴ OA + OB + OC > $\frac{1}{2}$ (AB + BC + CA)

Question: 45

Our given lengths are AB=3cm, BC=3.5cm and CA=6.5cm.

∴ AB + BC = 3 + 3.5 = 6.5 cm

But CA = 6.5 cm

So, AB + BC = CA

A triangle can be drawn only when the sum of two sides is greater than the third side

So, a triangle cannot be drawn with such lengths

Exercise : CCE QUESTIONS

Question: 1

Which of the foll

Solution:

From the above given four options, SSA is not a criterion for the congruence of triangles

∴ Option (A) is correct

Question: 2

If AB=QR, BC=RP a

Solution:

It is given in the question that,

AB = QR

BC = RP

And, CA = PQ

∴ By SSS congruence criterion

$\Delta CBA \cong \Delta PQR$

Hence, option (B) is correct

Question: 3

If ΔABC≅

Solution:

According to the condition given in the question, If $\Delta ABC \cong \Delta PQR$ and ΔABC is not congruent to ΔRPQ

Then, clearly BC ≠ PQ

∴ It is false

Hence, option (A) is correct

Question: 4

It is given that

Solution:

It is given in the question that,

$\Delta ABC \cong \Delta FDE$ where,

AB = 5 cm

FD = 5 cm

∠ B = 40°

∠ A = 80°

We know that sum of all angles of a triangle is equal to 180^0

∴ ∠ A + ∠ B + ∠ C = 180^o

$80^o + 40^o$ + ∠ C = 180^o

∠ C = $180^o - 120^0$

= 60^o

As, Angle C = Angle E

∴ Angle E = 60^o

Hence, option (B) is correct

Question: 5

In ΔABC, AB=2.5cm

Solution:

It is given in the question that,

In ΔABC

AB = 2.5 cm

BC = 6 cm

We know that, the length of a side must be less than the sum of the other two sides

Let us assume the side of AC be x cm

∴ x < 2.5 + 6

x < 8.5

Also, we know that the length of a side must be greater then the difference between the other two sides

∴ x > 6 – 2.5

x > 3.5

Hence, the limits of the value of x is

3.5 < x < 8.5

∴ It is clear the length of AC cannot be 3.4 cm

Hence, option (A) is correct

Question: 6

In ΔABC, ∠<

Solution:

It is given in the question that,

In ΔABC, ∠A = 40^o

∠B = 60^o

We know that, sum of all angles of a triangle is equal to 180^0

∴ ∠A + ∠B + ∠C = 180^o

60^o + 40^o + ∠C = 180^o

∠C = 180^o – 100^o

∠C = 80^o

Hence, the side which is opposite to ∠C is the longest side of the triangle

∴ Option (C) is correct

Question: 7

In ΔABC, ∠<

Solution:

It is given in the question that,

In ΔABC, we have

∠B = 35^o

∠C = 65^o

Also the bisector AD of ∠BAC meets at D

∴ ∠A + ∠B + ∠C = 180^o

∠A + 35^o + 65^o = 180^o

∠A = 180^o – 100^o

∠A = 80^o

As, AD is the bisector of ∠BAC

∴ ∠BAD = ∠CAD = 40^o

In ΔABD, we have

∠BAD > ∠ABD

BD > AD

Also, in ΔACD

∠ACD > ∠CAD

AD > CD

Hence, BD > AD > CD

∴ Option (B) is correct

Question: 8

In the given figu

Solution:

From the given figure, we have

AB > AC

∴ ∠ACB > ∠ABC

Also, ∠ADB > ACD

∠ADB > ACB > ∠ABC

∠ADB > ∠ABD

∴ AB > AD

Hence, option (C) is correct

Question: 9

In the given figu

Solution:

From the given figure, we have

AB > AC

Also, ∠C > ∠B

$\frac{1}{2}\angle C > \frac{1}{2}\angle B$

∠OCB > ∠OBC (Given)

∴ OB > OC

Hence, option (C) is correct

Question: 10

In the given figu

Solution:

It is given in the question that,

In ΔOAB and ΔOAC, we have

AB = AC

OB = OC

OA = OA (Common)

∴ By SSS congruence criterion

$\Delta OAB \cong \Delta OAC$

∴ ∠ABO = ∠ACO

So, ∠ABO: ∠ACO = 1: 1

Hence, option (A) is correct

Question: 11

In ΔABC, IF

Solution:

It is given in the question that,

In ΔABC, we have

$\angle C > \angle B$

We know that, side opposite to the greater angle is larger

$\therefore$ AB >AC

Hence, option (B) is correct

Question: 12

O is any point in

Solution:

From the given question, we have

In ΔOAB, ΔOBC and ΔOCA we have:

OA + OB > AB

OB + OC > BC

And, OC + OA > AC

Adding all these, we get:

2 (OA + OB + OC) > (AB + BC + CA)

(OA + OB + OC > $\frac{1}{2}$ $(AB + BC + CA)$

$\therefore$ Option (C) is correct

Question: 13

If the altitudes

Solution:

It is given in the question that,

In ΔABC, BL is parallel to AC

Also, CM is parallel AB such that BL = CM

We have to prove that: AB = AC

Now, in ΔABL and ΔACM we have:

BL = CM (Given)

$\angle BAL = \angle CAM$ (Common)

$\angle ALB = \angle AMC$ (Each angle equal to 90^o)

$\therefore$ By AAS congruence criterion

$\Delta ABL \cong \Delta ACM$

AB = AC (By Congruent parts of congruent triangles)

As opposite sides of the triangle are equal, so it is an isosceles triangle

Hence, option (B) is correct

Question: 14

In the given figu

Solution:

From the given figure, we have

AE = DB

And, CB = EF

Now, AB = (AD – DB)

= (AD – AE)

DE = (AD – AE)

Now, in ΔABC and ΔDEF we have:

AB = DE

CB = EF

∠ABC = ∠FED

∴ By SAS congruence criterion

ΔABC ≅ ΔDEF

Hence, option (A) is correct

Question: 15

In the given figu

Solution:

From the given figure, we have

BE is perpendicular to CA

Also, CF is perpendicular to BA

And, BE = CF

Now, in ΔABE and ΔACF we have:

BE = CF (Given)

∠BEA = ∠CFA = 90^{o}

∠A = ∠A (Common)

∴ By AAS congruence criterion

ΔABE ≅ ΔACF

Hence, option (A) is correct

Question: 16

In the given figu

Solution:

From the given figure, we have

D is the mid-point of BC

Also, DE is perpendicular to AB

DF is perpendicular to AC

And, DE = DF

Now, in ΔBED and ΔCFD we have:

DE = DF

BD = CD

∠E = ∠F = 90^{o}

∴ By RHS congruence rule

ΔBED ≅ ΔCFD

Thus, ∠B = ∠C

AC = AB

Hence, option (A) is correct

Question: 17

In ΔABC and ΔDEF,

Solution:

From the question, we have:

In ΔABC and ΔDEF

AB = DE (Given)

BC = EF (Given)

So, in order to have ΔABC ≅ ΔDEF

∠B must be equal to ∠E

∴ Option (B) is correct

Question: 18

In ΔABC and ΔDEF,

Solution:

From the question, we have:

In ΔABC and ΔDEF

∠B = ∠E (Given)

∠C = ∠F (Given)

So, in order to have ΔABC ≅ ΔDEF

BE must be equal to EF

∴ Option (C) is correct

Question: 19

In ΔABC and ΔPQR,

Solution:

It is given in the question that,

In ΔABC and ΔPQR, we have

AB = AC

Also, ∠C = ∠B

As, ∠C = ∠P and, ∠B = ∠Q

∴ ∠P = ∠Q

So, both triangles are isosceles but not congruent

Hence, option (A) is correct

Question: 20

Which is true?

Solution:

We know that,

Sum of all angles of a triangle is equal to 180^o

∴ A triangle can have two acute angles because sum of two acute angles of a triangle is always less than 180^o

Thus, it satisfies the angle sum property of a triangle

Hence, option (C) is correct

Question: 21

Three statements

Solution:

Here we can clearly see that the true statements are as follows:

(I) In a ΔABC in which $AB=AC$, the altitude AD bisects BC.

(II) If the altitudes AD, BE and CF of ΔABC are equal, then ΔABC is equilateral.

$\therefore$ Option C is correct

Question: 22

The question cons

Solution:

According to the question,

In ΔABD and ΔACD,

Since, sum of any two sides of a triangle is greater than the third side.

$AB + DB > AD$ (i)

$AC + DC > AD$ (ii)

Adding (i) and (ii)

$AB + AC + DB + DC > 2AD$

$AB + AC + BC > 2AD$

Hence, the assertion and the reason are both true, but Reason does not explain the assertion.

$\therefore$ Option B is correct

Question: 23

The question cons

Solution:

Since, sum of two sides is greater than the third side

$\therefore AB + BC > AC$ (i)

$CD + DA > AC$ (ii)

Adding (i) and (ii),

$AB + BC + CD + DA > 2AC$

Hence, the assertion is true and also the reason gives the right explanation of the assertion.

$\therefore$ Option A is correct

Question: 24

The question cons

Solution:

Since, angles opposite to equal sides are equal

$AB = AC$

$\angle ABC = \angle ACB$ (i)

$DB = DC$

∠DBC = ∠DCB (ii)

Subtracting (ii) from (i),

∠ABC - ∠DBC = ∠ACB - ∠DCB

Hence, the assertion is true and also the reason gives the right explanation of the assertion.

∴ Option A is correct

Question: 25

The question cons

Solution:

In ΔBDC and ΔCEB,

∠DCB = ∠EBC (Given)

BC = CB (Common)

∠B = ∠C (AC = AB)

$\frac{1}{2}\angle B = \frac{1}{2}\angle C$

∠CEB = ∠BCE

∴ ΔBDC ≅ ΔCEB

BD = CE (By c.p.c.t.)

And, we know that the sum of two sides is always greater than the third side in any triangle.

But, (5 + 4) < 10

Hence, the reason is true, but the assertion is false.

∴ Option D is true

Question: 26

The question cons

Solution:

According to the question,

AB = AC

∠ACB = ∠ABC (i)

Now, ∠ACD >∠ACB =∠ABC (Side BC is produced to D)

And, In ΔADC, side DC is produced to B

∠ACB>∠ADC (ii)

∠ABC>∠ADC

Now, using (i) and (ii),

AD >AB

Hence, the reason is wrong but the assertion is true.

∴ Option C is correct

Question: 27

Match the followi

Solution:

The parts of the question are solved below:

a. Given: In △ABC, AB = AC and ∠A=50°

Thus, $\angle B = \angle C$

Now, $\angle A + \angle B + \angle C = 180°$ (The angle sum property of triangle)

$50 + 2\angle B = 180°$

$2\angle B = 130°$

$\angle C = \angle B = 65°$

b. As per the question,

Let the vertical angle be A and $\angle B = \angle C$

Now, $\angle A + \angle B + \angle C = 180°$ (The angle sum property of triangle)

$130 + 2\angle B = 180°$

$2\angle B = 50°$

$\angle C = \angle B = 25°$

c. We know that, the sum of three altitudes of a triangle ABC is less than its perimeter.

d. Here, ABCD is a square and EDC is a equilateral triangle.

$\therefore$ ED = CD = AB = BC = AD = EC

In ΔECB,

EC = BC

$\angle C = \angle B = x$

$\angle ECD = 60°$ and $\angle DCB = 90°$

$\angle ECB = 60° + 90°$

$= 150°$

Now, $x + x + 150° = 180°$

$2x = 30°$

$x = 15°$

$\therefore \angle EBC = 15°$

$\therefore$ a = r, b = s, c = p, d = q

Question: 28

Fill in the blank

Solution:

a) Sum of any two sides of a triangle > the third side

b) Difference of any two sides of a triangle < the third side

c) Sum of three altitudes of a triangle < sum of its three side

d) Sum of any two sides of a triangle > twice the median to the 3rd side

e) Perimeter of a triangle > sum of its three medians

Question: 29

Fill in the blank

Solution:

a) Each angle of an equilateral triangle measures **<u>60°</u>**

b) Medians of an equilateral triangle are **<u>equal</u>**

c) In a right triangle, the hypotenuse is the **<u>longest</u>** side

d) Drawing a $\triangle ABC$ with AB = 3cm, BC = 4cm and CA = 7cm is **not possible**.

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

In an equilateral

Solution:

We know that,

In any equilateral triangle all the angles are equal

Let the three angles of the triangle $\angle A$, $\angle B$ and $\angle C$ be x

$\therefore x + x + x = 180°$

$3x = 180°$

$x = \frac{180}{3}$

$x = 60$

Hence, $\angle A = 60^{o}$

Question: 2

In a ΔABC, if AB=

Solution:

It is given in the question that,

In triangle ABC, AB = AC

$\angle B = 65°$

As ABC is an isosceles triangle

$\therefore \angle C = \angle B$

$\angle C = 65°$

Now, we now that sum of all angles of a triangle is 180^{o}

$\therefore \angle A + \angle B + \angle C = 180°$

$\angle A + 65° + 65° = 180°$

$\angle A + 130° = 180°$

$\angle A = 180° - 130°$

$\angle A = 50°$

Question: 3

In a right ΔABC,

Solution:

It is given in the question that,

In right triangle ABC,

$\angle B = 90^{o}$

So, $\angle A + \angle C = 90^{o}$

$\therefore \angle A, \angle C < \angle B$

Hence, the side opposite to $\angle B$ is longest

Thus, AC is the longest side

Question: 4

In a ΔABC,

Solution:

It is given in the question that,

In triangle ABC, ∠B > ∠C

We know that, in a triangle side opposite to greater angle is longer

∴ AC is longer than AB

Question: 5

Can we construct

Solution:

We know that,

The sum of two sides must be greater than the third side

In this case, we have

AB + BC = 5 + 4 = 9 cm

AC = 9 cm

∴ AC must be greater than the sum of AB and BC

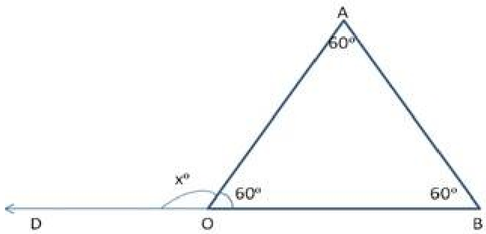
Hence, the sum of two sides is not greater than the third side. So, ΔABC cannot be constructed

Question: 6

Find the measure

Solution:

From the figure, we have



∠AOD is the exterior angle

∴ ∠AOD + ∠AOB = 180°

60° + ∠AOB = 180°

∠AOB = 180° – 60°

∠AOB = 120°

Hence, the measure of each of the exterior angle of an equilateral triangle is 120°

Question: 7

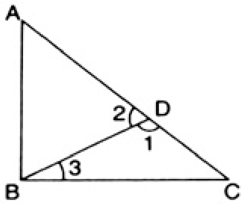
Show that the dif

Solution:

In a triangle let AC > AB

Then, along AC draw AD = AB and join BD

Proof: In Δ ABD,



$\angle$ ABD = $\angle$ ADB (AB = AD) ….(i)

$\angle$ ABD = $\angle$ 2 (angles opposite to equal sides) ….(ii)

Now, we know that the exterior angle of a triangle is greater than either of its opposite interior angles.

$\therefore \angle 1 > \angle ABD$

$\angle 1 > \angle 2$ ….(iii)

Now, from (ii)

$\angle 2 > \angle 3$ ….(iv) ($\angle 2$ is an exterior angle)

Using (iii) and (iv),

$\angle 1 > \angle 3$

BC > DC (side opposite to greater angle is longer)

BC > AC – AD

BC > AC – AB (since, AB = AD)

Hence, the difference of two sides is less than the third side of a triangle

Question: 8

In a right ΔABC,

Solution:

It is given in the question that,

In right triangle ABC, $\angle B = 90^o$

Also D is the mid-point of AC

$\therefore$ AD = DC

$\angle ADB = \angle BDC$ (BD is the altitude)

BD = BD (Common)

So, by SAS congruence criterion

$\therefore \Delta ADB \cong \Delta CDB$

$\angle A = \angle C$ (CPCT)

As, $\angle B = 90^o$

So, by using angle sum property

$\angle A = \angle ABD = 45^o$

Similarly, $\angle BDC = 90^o$ (BD is the altitude)

$\angle C = 45^o$

$\angle DBC = 45^o$

$\angle ABD = 45^o$

Now, by isosceles triangle property we have:

BD = CD and

BD = AD

AS, AD + DC = AC

BD + BD = AC

2BD = AC

BD = $\frac{1}{2}AC$

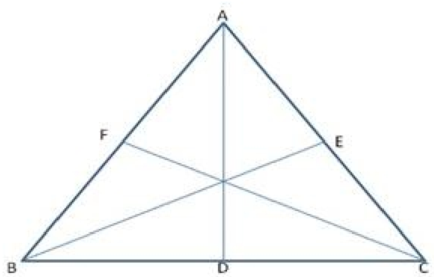
Hence, proved

Question: 9

Prove that the pe

Solution:

Let ABC be the triangle where D, E and F are the mid-points of BC, CA and AB respectively



As, we know that the sum of two sides of the triangle is greater than twice the median bisecting the third side

∴ AB + AC > 2AD

Similarly, BC + AC > 2CF

Also, BC + AB > 2BE

Now, by adding all these we get:

(AB + BC) + (BC + AC) + (BC + AB) > 2AD + 2CD + 2BE

2 (AB + BC + AC) > 2(AD + BE + CF)

∴ AB + BC + AC > AD + BE + CF

Hence, the perimeter of the triangle is greater than the sum of its medians

Question: 10

Which is true?

Solution:

We know that,

A triangle can have two acute angles because the sum of two acute angles is always less than 180^{o} which satisfies the angle sum property of a triangle

Hence, option (A) is correct

Question: 11

In ΔABC, BD

Solution:

It is given that,

BD is perpendicular to AC and CE is perpendicular to AB

Now, in ΔBDC and ΔCEB we have:

BE = CD (Given)

$\angle BEC = \angle CDB = 90^o$

And, BC = BC (Common)

∴ By RHS congruence rule

$\Delta BDC \cong \Delta CEB$

BD = CE (By CPCT)

Hence, proved

Question: 12

In ΔABC, AB=AC. S

Solution:

It is given in the question that,

In ΔABC,

AB = AC

We know that, angles opposite to equal sides are equal

∴ $\angle ACB = \angle ABC$

Now, in ΔACD we have:

AC = AD

$\angle ADC = \angle ACD$ (The Angles opposite to equal sides are equal)

By using angle sum property in triangle BCD, we get:

$\angle ABC + \angle BCD + \angle ADC = 180^o$

$\angle ACB + \angle ACB + \angle ACD + \angle ACD = 180^o$

$2 (\angle ACB + \angle ACD) = 180^o$

$2 (\angle BCD) = 180^o$

$\angle BCD = \frac{180}{2}$

$\angle BCD = 90^o$

Hence, proved

Question: 13

In the given figu

Solution:

From the given figure,

In triangles DAC and CBD, we have:

AD = BC

AC = BD

DC = DC

So, by SSS congruence rule

$\Delta ADC \cong \Delta BCD$

∴ By Congruent parts of congruent triangles we have:

$\angle CAD = \angle CBD$

∠ADC = ∠BCD

∠ACD = ∠BDC

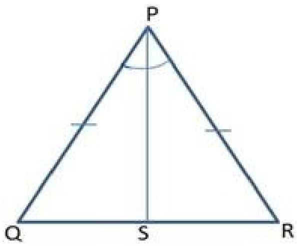
Hence, proved

Question: 14

Prove that the an

Solution:

We have a triangle PQR where PS is the bisector of ∠ P



Now in ΔPQS and ΔPSR, we have:

PQ = PR (Given)

PS = PS (Common)

∠ QPS = ∠ PRS (As PS is the bisector of ∠ P)

∴ By SAS congruence rule

ΔPQS ≅ ΔPSR

∠ Q = ∠ R (By Congruent parts of congruent triangles)

Hence, it is proved that the angles opposite to equal sides of a triangle are equal

Question: 15

In an isosceles Δ

Solution:

From the given figure, we have:

(i) In ΔABO and ΔACO

AB = AC (Given)

AO = AO (Common)

∠ ABO = ∠ ACO

∴ By SAS congruence rule

$\Delta ABO \cong \Delta ACO$

OB = OB (By CPCT)

(ii) As, By SAS congruence rule

ΔABO ≅ ΔACO

∴ ∠ OAB = ∠ OAC (By Congruent parts of congruent triangles)

Hence, proved

Question: 16

Prove that, of al

Solution:

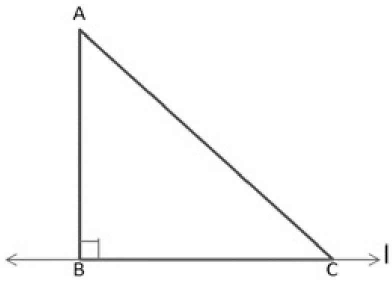
It is given in the question that,

l is the straight line and A is a point that is not lying on l

AB is perpendicular to line l and C is the point on l

As, $\angle B = 90^o$

So in ΔABC, we have:



$\angle A + \angle B + \angle C = 180^o$

$\angle A + \angle B = 90^o$

$\therefore \angle C < 90^o$

$\angle C < \angle B$

AB < AC

As C is that point which can lie anywhere on l

$\therefore$ AB is the shortest line segment drawn from A to l

Hence, proved

Question: 1

Each question con

Solution:

We know that,

Each angle of an equilateral triangle is equal to 60^o also angles opposite to equal sides of a triangle are equal to each other

$\therefore$ Both assertion and reason are true and reason is the correct explanation of the assertion

Hence, option (A) is correct

Question: 18

Each question con

Solution:

From the given figure in the question, we have

In ΔABD, we have:

AB + BD > AD

Similarly, in ΔADC

AC + CD > AD

Adding both expressions, we get:

AB + AC + BD + CD > AD + AD

AB + AC + BD + DC > 2AD

AB + AC + BC > 2AD

$\therefore$ Assertion and reason both are true and reason is the correct explanation of the assertion

Hence, option (A) is correct

Question: 19

Math the followin

Solution:

a) In ΔABC, ∠ A=70°

As AB = AC and we know that angles opposite to equal sides are equal

∴ In triangle ABC,

∠ A + ∠ B + ∠ C = 180°

70° + 2∠ C = 180°

2∠ C = 180° – 70°

∠ C = $\frac{110}{2}$

∴ ∠ C = 55°

(b) We know that,

Angles opposite to equal sides are equal

It is given that, vertical angle of the isosceles triangle = 120°

Let the base angle be x

∴ 120° + x + x = 180°

120° + 2x = 180°

2x = 180° – 120°

2x = 60°

x = $\frac{60}{2}$

x = 30°

Hence, each base angle of the isosceles triangle is equal to 30°

(c) We know that,

The sum of the three medians of the triangle is always less than the perimeter

(d) We know that,

In a triangle the sum of any two sides is always greater than the third side

Hence, the correct match is as follows:

(a) – (s)

(b) – (r)

(c) – (p)

(d) – (q)

Question: 20

In the given figu

Solution:

It is given in the question that,

PQ > PR

And, QS and RS are the bisectors of ∠ Q and ∠ R

We have, angle opposite to the longer side is greater

∴ PQ > PR

∠ R > ∠ Q

$\frac{1}{2}\angle R > \frac{1}{2}\angle Q$

∠ SRQ > ∠ RQS

SQ > SR

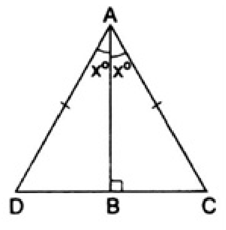
Hence, proved

Question: 21

In the given figu

Solution:

We will have to make the following construction in the given figure:



Produce CB to D in such a way that BD=BC and join AD.

Now, in ΔABC and ΔABD,

BC=BD (constructed)

AB=AB (common)

∠ABC=∠ABD (each 90°)

∴ by S.A.S.

ΔABC ≅ ΔABD

∠CAB=∠DAB and AC=AD (by c.p.c.t.)

∴∠CAD=∠CAB+∠BAD

=x°+x°

=2x°

But, AC=AD

∠ACD=∠ADB=2x°

∴ ΔACD is equilateral triangle.

AC=CD

AC=2BC

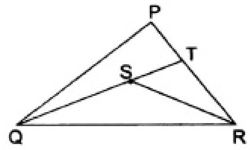
Hence, proved

Question: 22

S is any point in

Solution:

Following construction is to be made in the given figure.



Extend QS to meet PR at T.

Now, in Δ PQT,

PQ+PT>QT (sum of two sides is greater than the third side in a triangle)

PQ+PT>SQ+ST (i)

Now, In Δ STR,

ST+TR>SR (ii)(sum of two sides is greater than the third side in a triangle)

Now, adding (i) and (ii),

PQ+PT+ST+TR>SQ+ST+SR

PQ+PT+TR>SQ+SR

PQ+PR>SQ+SR

SQ+SR<PQ+PR

Hence, proved

Question: 23

Show that in a qu

Solution:

Here, ABCD is a quadrilateral and AC and BD are its diagonals.

Now, As we that, sum of two sides of a triangle is greater than the third side.

∴ In Δ ACB,

AB + BC > AC (i)

In Δ BDC,

CD + BC > BD (ii)

In Δ BAD,

AB + AD>BD (iii)

In Δ ACD,

AD + DC > AC (iv)

Now, adding (i), (ii), (iii) and (iv):

AB + BC + CD + BC + AB + AD + AD + DC> AC + BD + BD + AC

2AB + 2BC + 2CD + 2AD > 2AC + 2BD

Thus, AB + BC + CD + AD > AC + BD

Hence, proved