Chapter : 27. LIMITS

Exercise : 27A

Question: 1

Evaluate

Solution:

To evaluate: $\lim_{x\to 2}(5-x)$

Formula used:

We have,

$\lim_{x\to a} f(x) = f(a)$

As $x \to 2$, we have

$\lim_{x\to 2}(5-x) = 5-2$

$\lim_{x\to 2}(5-x) = 3$

Thus, the value of $\lim_{x\to 2}(5-x)$ is 3.

Question: 2

Evaluate

Solution:

To evaluate: $\lim_{x\to 1}(x^2-4x+3)$

Formula used:

We have,

$\lim_{x\to a} f(x) = f(a)$

As $x \to 1$, we have

$\lim_{x\to 1}(x^2-4x+3) = 1^2-4(1)+3$

$\lim_{x\to 1}(x^2-4x+3) = 0$

Thus, the value of $\lim_{x\to 1}(x^2-4x+3)$ is 0.

Question: 3

Evaluate

Solution:

To evaluate: $\lim_{x\to 3}\frac{x^2+9}{x+3}$

Formula used:

We have,

$\lim_{x\to a} f(x) = f(a)$

As $x \to 3$, we have

$$\lim_{x\to 3}\frac{x^2+9}{x+3}=\frac{3^2+9}{3+3}$$

$$\lim_{x\to 3}\frac{x^2+9}{x+3}=\frac{18}{6}$$

$$\lim_{x\to 3}\frac{x^2+9}{x+3}=3$$

Thus, the value of $\lim_{x\to 3}\frac{x^2+9}{x+3}$ is 3.

Question: 4

Evaluate

Solution:

To evaluate: $\lim_{x\to 3}\frac{x^2-4x}{x-2}$

Formula used:

We have,

$$\lim_{x\to a} f(x)=f(a)$$

As $x\to 3$, we have

$$\lim_{x\to 3}\frac{x^2-4x}{x-2}=\frac{3^2-4(3)}{3-2}$$

$$\lim_{x\to 3}\frac{x^2-4x}{x-2}=\frac{3^2-4(3)}{3-2}$$

$$\lim_{x\to 3}\frac{x^2-4x}{x-2}=-3$$

Thus, the value of $\lim_{x\to 3}\frac{x^2-4x}{x-2}$ is -3.

Question: 5

Evaluate

Solution:

To evaluate: $\lim_{x\to 5}\frac{x^2-25}{x-5}$

Formula used:

We have,

$$\lim_{x\to a} f(x)=f(a)$$

As $x\to 5$, we have

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=\lim_{x\to 5}\frac{(x+5)(x-5)}{x-5}$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=x+5$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=5+5$$

$$\lim_{x\to 5}\frac{x^2-25}{x-5}=10$$

Thus, the value of $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is -10.

Question: 6

Evaluate

Solution:

To evaluate: $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$

Formula used:

We have,

$\lim_{x \to a} f(x) = f(a)$ and

$x^3 - y^3 = (x-y)(x^2+xy+y^2)$

As $x \to 1$, we have

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1}$$

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = \lim_{x \to 1} (x^2 + x + 1)$$

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 1 + 1 + 1$$

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3$$

Thus, the value of $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$ is 3.

Question: 7

Evaluate

Solution:

To evaluate: $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$

Formula used:

We have,

$\lim_{x \to a} f(x) = f(a)$ and

$x^3 + y^3 = (x+y)(x^2-xy+y^2)$

As $x \to -2$, we have

$$\lim_{x \to -2} \frac{x^3 + 8}{x + 2} = \lim_{x \to -2} \frac{(x + 2)(x^2 - 2x + 4)}{x + 2}$$

$$\lim_{x \to -2} \frac{x^3 + 8}{x + 2} = \lim_{x \to -2} (x^2 - 2x + 4)$$

$$\lim_{x \to -2} \frac{x^3 + 8}{x + 2} = 4-4+4$$

$$\lim_{x \to -2} \frac{x^3 + 8}{x + 2} = 4$$

Thus, the value of $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$ is 4

Question: 8

Evaluate

Solution:

To evaluate: $\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$

Formula used:

We have,

$\lim_{x \to a} f(x) = f(a)$ and

As $x \to 3$, we have

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = \lim_{x \to 3} \frac{(x^2 + 9)(x^2 - 9)}{x - 3}$$

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = \lim_{x \to 3} \frac{(x^2 + 9)(x + 3)(x - 3)}{x - 3}$$

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = \lim_{x \to 3} (x^2 + 9)(x + 3)$$

$$\lim_{x \to 3} \frac{x^4 - 81}{x - 3} = (9 + 9)(3 + 3) = 18 \times 6 = 108$$

Thus, the value of $\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$ is 108.

Question: 9

Evaluate

Solution:

To evaluate: $\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$

Formula used:

We have,

$\lim_{x \to a} f(x) = f(a)$ and

As $x \to 3$, we have

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \lim_{x \to 3} \frac{(x - 3)(x - 1)}{(x - 3)(x + 2)}$$

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \lim_{x \to 3} \frac{(x - 1)}{(x + 2)}$$

$$\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3} = \frac{2}{5}$$

Thus, the value of $\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$ is $\frac{2}{5}$.

Question: 10

Evaluate

Solution:

To evaluate: $\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1}$

Formula used:

We have,

$$\lim_{x \to a} f(x) = f(a)$$

As $x \to \frac{1}{2}$, we have

$$\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \to \frac{1}{2}} \frac{(2x + 1)(2x - 1)}{2x - 1}$$

$$\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = \lim_{x \to \frac{1}{2}} (2x + 1)$$

$$\lim_{x \to \frac{1}{2}} \frac{4x^2 - 1}{2x - 1} = 2$$

Thus, the value of $\lim_{x \to \frac{1}{2}} \frac{4x^2-1}{2x-1}$ is 2.

Question: 11

Evaluate

Solution:

To evaluate: $\lim_{x \to 4} \frac{x^3-64}{x^2-16}$

Formula used:

We have,

$\lim_{x \to a} f(x) = f(a)$ and

$x^3-y^3=(x-y)(x^2+xy+y^2)$

As $x \to 4$, we have

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16} = \lim_{x \to 4} \frac{(x - 4)(x^2 + 4x + 16)}{(x + 4)(x - 4)}$$

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16} = \lim_{x \to 4} \frac{(x^2 + 4x + 16)}{(x + 4)}$$

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16} = \frac{48}{8}$$

$$\lim_{x \to 4} \frac{x^3 - 64}{x^2 - 16} = 6$$

Thus, the value of $\lim_{x \to 4} \frac{x^3-64}{x^2-16}$ is 6.

Question: 12

Evaluate

Solution:

To evaluate: $\lim_{x \to 2} \frac{x^5-32}{x^3-8}$

Formula used:

We have,

$\frac{x^m - y^m}{x - y}$=my$^{\text{m-1}}$

As $x \to 4$, we have

$$\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \to 2} \frac{x^5 - 2^5}{x^3 - 2^3}$$

$$\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \to 2} \frac{\frac{x^5 - 2^5}{x - 2}}{\frac{x^3 - 2^3}{x - 2}}$$

$$\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8} = \lim_{x \to 2} \frac{5(2)^4}{3(2)^2}$$

$$\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8} = \frac{20}{3}$$

Thus, the value of $\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8}$ is $\frac{20}{3}$

Question: 13

Evaluate

Solution:

To evaluate: $\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a}$

Formula used:

We have,

$\frac{x^m - y^m}{x - y}$=my$^{\text{m-1}}$

As $x \to a$, we have

$$\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{5}{2} - 1}$$

$$\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a} = \frac{5}{2} a^{\frac{3}{2}}$$

Thus, the value of $\lim_{x \to a} \frac{x^{\frac{5}{2}} - a^{\frac{5}{2}}}{x - a}$ is $\frac{5}{2} a^{\frac{3}{2}}$

Question: 14

Evaluate

Solution:

To evaluate: $\lim_{x \to a} \left\{ \frac{(x+2)^{\frac{5}{3}} - (a+2)^{\frac{5}{3}}}{x - a} \right\}$

Formula used:

We have,

$\frac{x^m - y^m}{x - y}$=my$^{\text{m-1}}$

As $x \to a$, we have

$$\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}\right\}=\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{(x+2)-(a+2)}\right\}$$

$$\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}\right\}=\frac{5}{3}(a+2)^{\frac{5}{3}-1}$$

$$\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}\right\}=\frac{5}{3}(a+2)^{\frac{2}{3}}$$

Thus, the value of $\lim_{x\to a}\left\{\frac{(x+2)^{\frac{5}{3}}-(a+2)^{\frac{5}{3}}}{x-a}\right\}$ is $\frac{5}{3}(a+2)^{\frac{2}{3}}$

Question: 15

Evaluate

Solution:

To evaluate: $\lim_{x\to 1}\frac{x^n-1}{x-1}$

Formula used:

We have,

$\frac{x^m-y^m}{x-y}$=my^{m-1}

As $x \to 1$, we have

$$\lim_{x\to a}\frac{x^n-1}{x-1}=n$$

Thus, the value of $\lim_{x\to a}\frac{x^n-1}{x-1}$ is n.

Question: 16

Evaluate

Solution:

To evaluate: $\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{x-a}$

Formula used:

We have,

$\frac{x^m-y^m}{x-y}$=my^{m-1}

As $x \to a$, we have

$$\lim_{x\to a}\frac{x^{\frac{1}{2}}-a^{\frac{1}{2}}}{x-a}=\frac{1}{2}a^{\frac{1}{2}-1}$$

$$\lim_{x\to a}\frac{x^{\frac{1}{2}}-a^{\frac{1}{2}}}{x-a}=\frac{1}{2\sqrt{a}}$$

Thus, the value of $\lim_{x\to a}\frac{x^{\frac{1}{2}}-a^{\frac{1}{2}}}{x-a}$ is $\frac{1}{2\sqrt{a}}$

Question: 17

Evaluate

Solution:

To evaluate: $\lim\limits_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{\frac{d}{dh}\left(\sqrt{x+h}-\sqrt{x}\right)}{\frac{d}{dh}(h)}$$

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0}\frac{\frac{1}{2\sqrt{x+h}}}{1}$$

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{1}{2\sqrt{x}}$$

Thus, the value of $\lim\limits_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$ is $\frac{1}{2\sqrt{x}}$

Question: 18

Evaluate

Solution:

To evaluate: $\lim\limits_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}=\lim_{h\to 0}\frac{\frac{d}{dh}\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)}{\frac{d}{dh}(h)}$$

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}=\lim_{h\to 0}\frac{\frac{-1}{2\sqrt{x+h}}+\frac{1}{2\sqrt{x}}}{1}$$

$$\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}=0$$

Thus, the value of $\lim_{h\to 0}\frac{1}{h}\left\{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right\}$ is 0.

Question: 19

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{1+x}-1\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{\frac{1}{2\sqrt{x+1}}}{1}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\frac{1}{2}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}$ is $\frac{1}{2}$

Question: 20

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{2-x}-\sqrt{2+x}\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x} = \lim_{x\to 0}\frac{-\frac{1}{2\sqrt{2-x}}-\frac{1}{2\sqrt{2+x}}}{1}$$

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x} = \frac{-2}{2\sqrt{2}}$$

$$\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x} = \frac{-1}{\sqrt{2}}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{2-x}-\sqrt{2+x}}{x}$ is $\frac{-1}{\sqrt{2}}$

Question: 21

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{1+x+x^2}-1\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x} = \lim_{x\to 0}\frac{\frac{1+2x}{2\sqrt{1+x+x^2}}}{1}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}=\frac{1}{2}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}$ is $\frac{1}{2}$

Question: 22

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{\sqrt{3-x}-1}{2-x}$

Formula used:

We have,

$\lim_{x\to a} f(x) = f(a)$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{3-x}-1}{2-x}=\frac{\sqrt{3}-1}{2}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{3-x}-1}{2-x}$ is $\frac{\sqrt{3}-1}{2}$.

Question: 23

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$

Formula used:

Multiplying numerator and denominator by $\sqrt{a+x}+\sqrt{a-x}$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}=\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}\left(\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}\right)$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}=\lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{a+x-a+x}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}=\lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{2x}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}=\lim_{x\to 0}\sqrt{a+x}+\sqrt{a-x}$$

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}=2\sqrt{a}$$

Thus, the value of $\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$ is $2\sqrt{a}$.

Question: 24

Evaluate

Solution:

To evaluate: $\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} = \lim_{x\to 1}\frac{\frac{d}{dx}\left(\sqrt{3+x}-\sqrt{5-x}\right)}{\frac{d}{dx}(x^2-1)}$$

$$\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} = \lim_{x\to 1}\frac{\frac{1}{2\sqrt{3+x}}+\frac{1}{2\sqrt{5-x}}}{2x}$$

$$\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1} = \frac{1}{4}$$

Thus, the value of $\lim_{x\to 1}\frac{\sqrt{3+x}-\sqrt{5-x}}{x^2-1}$ is $\frac{1}{4}$

Question: 25

Evaluate

Solution:

To evaluate: $\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = \lim_{x\to 2}\frac{\frac{d}{dx}(x^2-4)}{\frac{d}{dx}\left(\sqrt{x+2}-\sqrt{3x-2}\right)}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}} = \lim_{x\to 2}\frac{2x}{\frac{1}{2\sqrt{x+2}}-\frac{3}{2\sqrt{3x-2}}}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}=\frac{4}{\frac{1}{2\sqrt{2+2}}-\frac{3}{2\sqrt{6-2}}}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}=\frac{8}{\frac{1}{2}-\frac{3}{2}}$$

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}=-8$$

Thus, the value of $\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$ is -8.

Question: 26

Evaluate

Solution:

To evaluate: $\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$

Formula used:

Multiplying numerator and denominator with conjugates of numerator and denominator i.e $\left(1+\sqrt{5-x}\right)\left(3+\sqrt{5+x}\right)$

$$\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)=\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)\left(\frac{1+\sqrt{5-x}}{1+\sqrt{5-x}}\right)\left(\frac{3+\sqrt{5+x}}{3+\sqrt{5+x}}\right)$$

$$\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)=\lim_{x\to 4}\left(\frac{4-x}{x-4}\right)\left(\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right)$$

$$\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)=\lim_{x\to 4}-\left(\frac{1+\sqrt{5-x}}{3+\sqrt{5+x}}\right)$$

$$\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)=-\frac{1}{3}$$

Thus, the value of $\lim_{x\to 4}\left(\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}\right)$ is $-\frac{1}{3}$

Question: 27

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{a+x}-\sqrt{a}\right)}{\frac{d}{dx}\left(x\sqrt{a(a+x)}\right)}$$

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}=\lim_{x\to 0}\frac{\frac{1}{2\sqrt{a+x}}}{x\left(\frac{a}{2\sqrt{a(a+x)}}\right)+\sqrt{a(a+x)}}$$

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}=\frac{\frac{1}{2\sqrt{a}}}{a}$$

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}=\frac{1}{2a\sqrt{a}}$$

Thus, the value of $\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x\sqrt{a(a+x)}}$ is $\frac{1}{2a\sqrt{a}}$

Question: 28

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a}f(x)=\lim_{x\to a}g(x)=0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to 0$, we have

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\lim_{x\to 0}\frac{\frac{d}{dx}\left(\sqrt{1+x^2}-\sqrt{1+x}\right)}{\frac{d}{dx}\left(\sqrt{1+x^3}-\sqrt{1+x}\right)}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\lim_{x\to 0}\frac{\frac{2x}{2\sqrt{1+x^2}}-\frac{1}{2\sqrt{1+x}}}{\frac{3x^2}{2\sqrt{1+x^3}}-\frac{1}{2\sqrt{1-x}}}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}=\lim_{x\to 0}\frac{-\frac{1}{2}}{-\frac{1}{2}}$$

$$\lim_{x\to0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}} = \text{-1}$$

Thus, the value of $\lim_{x\to0}\frac{\sqrt{1+x^2}-\sqrt{1+x}}{\sqrt{1+x^3}-\sqrt{1+x}}$ is -1.

Question: 29

Evaluate

Solution:

To Evaluate: $\lim_{x\to1}\left(\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right)$

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} \mathrm{f(x)} = \lim_{x\to a} \mathrm{g(x)} = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{\mathrm{f(x)}}{\mathrm{g(x)}} = \lim_{x\to a}\frac{\mathrm{f'(x)}}{\mathrm{g'(x)}}$$

As $\mathrm{x} \to 0$, we have

$$\lim_{x\to1}\left(\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right) = \frac{0}{0}$$

Therefore,

$$\lim_{x\to1}\left(\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right) = \lim_{x\to1}\frac{4x^3-6x}{3x^2-10x+3} = \frac{4-6}{3-10+3} = -\frac{2}{-4} = -\frac{1}{2}$$

Hence,

$$\lim_{x\to1}\left(\frac{x^4-3x^2+2}{x^3-5x^2+3x+1}\right) = -\frac{1}{2}$$

Question: 30

Evaluate

Solution:

To evaluate: $\lim_{x\to2}\left(\frac{3^x-3^{3-x}-12}{3^{3-x}-3^{\frac{x}{2}}}\right)$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} \mathrm{f(x)} = \lim_{x\to a} \mathrm{g(x)} = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{\mathrm{f(x)}}{\mathrm{g(x)}} = \lim_{x\to a}\frac{\mathrm{f'(x)}}{\mathrm{g'(x)}}$$

As $\mathrm{x} \to 0$, we have

$$\lim_{\mathrm{x}\to0}\frac{2^{\mathrm{x}}-1}{\mathrm{x}} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to2}\left(\frac{3^x-3^{3-x}-12}{3^{3-x}-3^{\frac{x}{2}}}\right)= \lim_{x\to2}\frac{\frac{d}{dx}(3^x-3^{3-x}-12)}{\frac{d}{dx}\left(3^{3-x}-3^{\frac{x}{2}}\right)}$$

$$\lim_{x\to2}\left(\frac{3^x-3^{3-x}-12}{3^{3-x}-3^{\frac{x}{2}}}\right)= \lim_{x\to0}\frac{3^x\ln 3+3^{3-x}\ln 3}{-3^{3-x}\ln 3+3^{\frac{x}{2}}\left(\frac{1}{2}\right)\ln 3}$$

$$\lim_{x\to2}\left(\frac{3^x-3^{3-x}-12}{3^{3-x}-3^{\frac{x}{2}}}\right)=\frac{\ln 3+27\ln 3}{-27\ln 3+\left(\frac{1}{2}\right)\ln 3}$$

$$\lim_{x\to2}\left(\frac{3^x-3^{3-x}-12}{3^{3-x}-3^{\frac{x}{2}}}\right)=\frac{28\ln 3}{-26.5\ln 3}$$

Thus, the value of $\lim_{x\to2}\left(\frac{3^x-3^{3-x}-12}{3^{3-x}-3^{\frac{x}{2}}}\right)$ is $\frac{28\ln 3}{-26.5\ln 3}$

Question: 31

Evaluate

Solution:

To evaluate: $\lim_{x\to0}\frac{e^{4x}-1}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x)=\lim_{x\to a} g(x)=0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x\to0$, we have

$$\lim_{x\to0}\frac{e^{4x}-1}{x}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to0}\frac{e^{4x}-1}{x}= \lim_{x\to0}\frac{\frac{d}{dx}(e^{4x}-1)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to0}\frac{e^{4x}-1}{x}= \lim_{x\to0}\frac{4e^{4x}}{1}$$

$$\lim_{x\to0}\frac{e^{4x}-1}{x}=4$$

Thus, the value of $\lim_{x\to0}\frac{e^{4x}-1}{x}$ is 4.

Question: 32

Evaluate

Solution:

To evaluate: $\lim_{x\to0}\frac{e^{2+x}-e^2}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(e^{2+x} - e^2)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \lim_{x \to 0} \frac{e^{2+x}}{1}$$

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = e^2$$

Thus, the value of $\lim_{x \to 0} \frac{e^{2+x} - e^2}{x}$ is e^2.

Question: 33

Evaluate

Solution:

To evaluate: $\lim_{x \to 4} \frac{e^x - e^4}{x - 4}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{e^{2+x} - e^2}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 4} \frac{e^x - e^4}{x - 4} = \lim_{x \to 4} \frac{\frac{d}{dx}(e^x - e^4)}{\frac{d}{dx}(x - 4)}$$

$$\lim_{x \to 4} \frac{e^x - e^4}{x - 4} = \lim_{x \to 4} \frac{e^x}{1}$$

$$\lim_{x \to 4} \frac{e^x - e^4}{x - 4} = e^4$$

Thus, the value of $\lim_{x \to 4} \frac{e^x - e^4}{x-4}$ is e^4.

Question: 34

Evaluate

Solution:

To evaluate: $\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(e^{3x} - e^{2x})}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x} = \lim_{x \to 0} \frac{3e^{3x} - 2e^{2x}}{1}$$

$$\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x} = 3-2$$

$$\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x} = 1$$

Thus, the value of $\lim_{x \to 0} \frac{e^{3x} - e^{2x}}{x}$ is 1.

Question: 35

Evaluate

Solution:

To evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{e^x - x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{e^x - x - 1}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}(e^x - x - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{e^x - x - 1}{x} = \lim_{x\to 0}\frac{e^x - 1}{1}$$

$\lim_{x\to 0}\frac{e^x-x-1}{x}$ =1-1

$\lim_{x\to 0}\frac{e^x-x-1}{x}$ =0

Thus, the value of $\lim_{x\to 0}\lim_{x\to 0}\frac{e^x-x-1}{x}$ is 0.

Question: 36

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{e^{bx} - e^{ax}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{e^{bx} - e^{ax}}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}\left(e^{bx} - e^{ax}\right)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{e^{bx} - e^{ax}}{x} = \lim_{x\to 0}\frac{be^{bx} - ae^{ax}}{1}$$

$\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}$ = b-a

Thus, the value of $\lim_{x\to 0}\frac{e^{bx}-e^{ax}}{x}$ is b-a.

Question: 37

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{a^x-b^x}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(a^x - b^x)}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \lim_{x \to 0} \frac{a^x \ln a - b^x \ln b}{1}$$

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \ln a - \ln b$$

$$\lim_{x \to 0} \frac{a^x - b^x}{x} = \ln \frac{a}{b}$$

Thus, the value of $\lim_{x \to 0} \frac{a^x - b^x}{x}$ is $\ln \frac{a}{b}$.

Question: 38

Evaluate

Solution:

To evaluate: $\lim_{x \to 0} \frac{a^x - a^{-x}}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x \to 0} \frac{a^x - a^{-x}}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x \to 0} \frac{a^x - a^{-x}}{x} = \lim_{x \to 0} \frac{\frac{d}{dx}(a^x - a^{-x})}{\frac{d}{dx}(x)}$$

$$\lim_{x \to 0} \frac{a^x - a^{-x}}{x} = \lim_{x \to 0} \frac{a^x \ln a + a^{-x} \ln a}{1}$$

$$\lim_{x\to 0}\frac{a^x - a^{-x}}{x} = 2\ln a$$

Thus, the value of $\lim_{x\to 0}\frac{a^x-a^{-x}}{x}$ is $2\ln a$.

Question: 39

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{2^x-1}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{2^x - 1}{x} = \frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{2^x - 1}{x} = \lim_{x\to 0}\frac{\frac{d}{dx}(2^x - 1)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{2^x - 1}{x} = \lim_{x\to 0}\frac{2^x\ln 2}{1}$$

$$\lim_{x\to 0}\frac{2^x - 1}{x} = \ln 2$$

Thus, the value of $\lim_{x\to 0}\frac{2^x-1}{x}$ is $\ln 2$.

Question: 40

Evaluate

Solution:

To evaluate: $\lim_{x\to 0}\frac{3^{2+x}-9}{x}$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$ or $\pm\infty$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

As $x \to 0$, we have

$$\lim_{x\to 0}\frac{3^{2+x}-9}{x}=\frac{0}{0}$$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$$\lim_{x\to 0}\frac{3^{2+x}-9}{x}=\lim_{x\to 0}\frac{\frac{d}{dx}(3^{2+x}-9)}{\frac{d}{dx}(x)}$$

$$\lim_{x\to 0}\frac{3^{2+x}-9}{x}=\lim_{x\to 0}\frac{(2+x)\ln 3}{1}$$

$$\lim_{x\to 0}\frac{3^{2+x}-9}{x}=2\ln 3$$

Thus, the value of $\lim_{x\to 0}\frac{3^{2+x}-9}{x}$ is $2\ln 3$

Exercise : 27B

Question: 1

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate forms.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x}=1$

So $\lim_{x\to 0}\frac{\sin 4x}{6x}=\lim_{x\to 0}(\frac{\sin 4x}{4x})\times\frac{4}{6}=\frac{4}{6}=\frac{2}{3}$

Therefore, $\lim_{x\to 0}\frac{\sin 4x}{6x}=\frac{2}{3}$

Question: 2

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x}=1$

So $\lim_{x\to 0}\frac{\sin 5x}{\sin 8x}=\lim_{x\to 0}(\frac{\sin 5x}{5x})\times\frac{8x}{\sin 8x}\times\frac{5x}{8x}=\frac{5x}{8x}=\frac{5}{8}$

Therefore, $\lim_{x\to 0}\frac{\sin 5x}{\sin 8x}=\frac{5}{8}$

Question: 3

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to0}\frac{\tan x}{x} = 1$

So $\lim_{x\to0}\frac{\tan 3x}{\tan 5x} = \lim_{x\to0}(\frac{\tan 3x}{3x})\times\frac{5x}{\sin 5x}\times\frac{3x}{5x} = \frac{3x}{5x} = \frac{3}{5}$

Therefore, $\lim_{x\to0}\frac{\tan 3x}{\tan 5x} = \frac{3}{5}$

Question: 4

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfied any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to0}\frac{\tan x}{x} = 1$ '

So $\lim_{x\to0}\frac{\tan\alpha x}{\tan\beta x} = \lim_{x\to0}(\frac{\tan\alpha x}{\alpha x})\times\frac{\beta x}{\sin\beta x}\times\frac{\alpha x}{\beta x} = \frac{\alpha x}{\beta x}= \frac{\alpha}{\beta}$

Therefore, $\lim_{x\to0}\frac{\tan\alpha x}{\tan\beta x} = \frac{\alpha}{\beta}$

Question: 5

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to0}\frac{\sin x}{x} = 1$ and $\lim_{x\to0}\frac{\tan x}{x} = 1$

So $\lim_{x\to0}\frac{\sin 4x}{\tan 7x} = \lim_{x\to0}(\frac{\sin 4x}{4x})\times\frac{7x}{\sin 7x}\times\frac{4x}{7x} = \frac{4x}{7x}= \frac{4}{7}$

Therefore, $\lim_{x\to0}\frac{\sin 4x}{\tan 7x} = \frac{4}{7}$

Question: 6

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to0}\frac{\tan x}{x} = 1$ and $\lim_{x\to0}\frac{\sin x}{x} = 1$

So $\lim_{x\to 0}\frac{\tan 3x}{\sin 4x} = \lim_{x\to 0}(\frac{\tan 3x}{3x})\times \frac{4x}{\sin 4x}\times \frac{3x}{4x} = \frac{3x}{4x} = \frac{3}{4}$

Therefore, $\lim_{x\to 0}\frac{\tan 3x}{\sin 4x} = \frac{3}{4}$

Question: 7

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim_{x\to 0}\frac{\tan x}{x} = 1$

So $\lim_{x\to 0}\frac{\sin mx}{\tan nx} = \lim_{x\to 0}(\frac{\sin mx}{mx})\times \frac{nx}{\tan nx}\times \frac{mx}{nx} = \frac{mx}{nx} = \frac{m}{n}$

Therefore, $\lim_{x\to 0}\frac{\sin mx}{\tan nx} = \frac{m}{n}$

Question: 8

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, inderterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$

So $\lim_{x\to 0}\frac{\sin x - 2\sin 3x + \sin 5x}{x} = \lim_{x\to 0}(\frac{\sin x}{x} - \frac{2\sin 3x}{x} + \frac{\sin 5x}{x}) = \lim_{x\to 0}(\frac{\sin x}{x} - \frac{2\sin 3x}{3x}\times 3 + \frac{5\sin 5x}{5x})$

By using the above formula, we have

$\therefore \lim_{x\to 0}(\frac{\sin x}{x} - \frac{2\sin 3x}{3x}\times 3 + \frac{5\sin 5x}{5x}) = 1 - 2\times 3 + 5 = 0$

Therefore, $\lim_{x\to 0}\frac{\sin x - 2\sin 3x + \sin 5x}{x} = 0$

Question: 9

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$ or we can used L hospital Rule,

So, by using the rule, Differentiate numerator and denominator

$\lim_{x\to \frac{\pi}{6}}\frac{4\sin x\cos x + \cos x}{4\sin x\cos x - 3\cos x} = \lim_{x\to \frac{\pi}{6}}\frac{4\sin x + 1}{4\sin x - 3} = \frac{2+1}{2-3} = -3$

Therefore, $\lim_{x\to 0} \frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$ = -3

Question: 10

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0} \frac{\sin x}{x}$ = 1 or we can used L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to 0} \frac{\sin 2x+3x}{2x+\sin 3x} = \lim_{x\to 0} \frac{\frac{\sin 2x+3x}{x}}{\frac{2x+\sin 3x}{x}} = \lim_{x\to 0} \frac{\frac{\sin 2x}{x}+3}{2+\frac{\sin 3x}{x}} = \lim_{x\to 0} \frac{\frac{2\sin 2x}{2x}+3}{2+\frac{3\sin 3x}{3x}} = \frac{2+3}{2+3} = 1$$

ALTER:by using the rule, Differentiate numerator and denominator

$$\lim_{x\to 0} \frac{2\cos 2x+3}{2+3\cos 3x} = \frac{5}{5} = 1$$

Therefore, $\lim_{x\to 0} \frac{\sin 2x+3x}{2x+\sin 3x}$ = 1

Question: 11

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0} \frac{\tan x}{x}$ = 1 or we can used L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to 0} \frac{\tan 2x-x}{3x-\tan x} = \lim_{x\to 0} \frac{\frac{\tan 2x-x}{x}}{\frac{3x-\tan x}{x}} = \lim_{x\to 0} \frac{\frac{\tan 2x}{x}-1}{3-\frac{\tan x}{x}} = \lim_{x\to 0} \frac{\frac{2\tan 2x}{2x}-1}{3-\frac{\tan x}{x}} = \frac{2-1}{3-1} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0} \frac{\tan 2x-x}{3x-\tan x} = \frac{1}{2}$

Question: 12

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\tan x}{x} = 1$ or we can used L hospital Rule,

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to 0}\frac{x^2 - \tan 2x}{\tan x} = \lim_{x\to 0}\frac{\frac{x^2 - \tan 2x}{x}}{\frac{\tan x}{x}} = \lim_{x\to 0}\frac{x - \frac{\tan 2x}{x}}{\frac{\tan x}{x}} = \lim_{x\to 0}\frac{x - \frac{2\tan 2x}{2x}}{\frac{\tan x}{x}} = \frac{0-2}{1} = -2$$

Therefore, $\lim_{x\to 0}\frac{x^2 - \tan 2x}{\tan x} = -2$

Question: 13

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim_{x\to 0}\frac{\tan x}{x} = 1$

So, by using the above formula, we have

Divide numerator and denominator by x,

$$\lim_{x\to 0}\frac{x\cos x + \sin x}{x^2 + \tan x} = \lim_{x\to 0}\frac{\frac{x\cos x + \sin x}{x}}{\frac{x^2 + \tan x}{x}} = \lim_{x\to 0}\frac{\cos x + \frac{\sin x}{x}}{x + \frac{\tan x}{x}} = \frac{1+1}{0+1} = 2$$

Therefore, $\lim_{x\to 0}\frac{x\cos x + \sin x}{x^2 + \tan x} = 2$

Question: 14

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

NOTE : $\tan x - \sin x = \frac{\sin x}{\cos x} - \sin x = \frac{\sin x - \sin x\cos x}{\cos x} = \sin x\,(\frac{1-\cos x}{\cos x})$

$$\lim_{x\to 0}\frac{\tan x - \sin x}{\sin^3 x} = \lim_{x\to 0}\frac{\frac{1-\cos x}{\cos x}}{\sin^2 x} = \lim_{x\to 0}\frac{1-\cos x}{\sin^2 x\cos x}$$

Divide numerator and denominator by x^2,

$$= \lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin^2 x\cos x}{x^2}}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = 1/2$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$ or we can used L hospital Rule,

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin^2 x\cos x}{x^2}} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{\tan x - \sin x}{\sin^3 x} = \frac{1}{2}$

Question: 15

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form are $0 \times \infty$

Formula used: $\lim_{x\to 0}\frac{x}{\sin x} = 1$

So, by using the above formula, we have

$\lim_{x\to 0} x \ \operatorname{cosec} x = \lim_{x\to 0}\frac{x}{\sin x} = 1$

Therefore, $\lim_{x\to 0} x \ \operatorname{cosec} x = 1$

Question: 16

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $0 \times \infty$

Formula used: $\lim_{x\to 0}\frac{x}{\tan x} = 1$

So, by using the above formula, we have

$\lim_{x\to 0} x \ \cot 2x = \lim_{x\to 0}\frac{2x}{2\tan 2x} = \frac{1}{2}$

Therefore, $\lim_{x\to 0} x \ \cot x = \frac{1}{2}$

Question: 17

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{\sin x}{x} = 1$

So, by using the above formula, we have

$\lim_{x\to 0}\frac{\sin x \cos x}{3x} = \lim_{x\to 0}\frac{\sin x}{x} \times \frac{\cos x}{3} = \frac{1}{3}$

Therefore, $\lim_{x\to 0}\frac{\sin x \cos x}{3x} = \frac{1}{3}$

Question: 18

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

So, by using the above formula, we have

$\lim_{x \to 0} \frac{\sin(x/4)}{x} = \lim_{x \to 0} \frac{\sin(x/4)}{4(x/4)} = \frac{1}{4}$

Therefore, $\lim_{x \to 0} \frac{\sin(x/4)}{x} = \frac{1}{4}$

Question: 19

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x \to 0} \frac{\tan x}{x} = 1$

So, by using the above formula, we have

$\lim_{x \to 0} \frac{\tan(x/2)}{3x} = \lim_{x \to 0} \frac{\tan(x/2)}{6(x/2)} = \frac{1}{6}$ [Divide and multiply with 2 on denominator]

Therefore, $\lim_{x \to 0} \frac{\tan(x/2)}{3x} = \frac{1}{6}$

Question: 20

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

[NOTE: 1 – cos x = 2 $\sin^2(x/2)$]

Formula used: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

So, by using the above formula, we have

$\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x} = \lim_{x \to 0} \frac{2 \sin 2(x/2)}{\sin^2 x}$

Divide numerator and denominator by x^2, we have

$$\lim_{x\to 0}\frac{2\sin^2(x/2)}{\sin^2 x} = \lim_{x\to 0}\frac{\frac{2\sin^2(\frac{x}{2})}{x^2}}{\frac{\sin^2 x}{x^2}} = \lim_{x\to 0}\frac{\frac{2\sin^2(\frac{x}{2})}{4\frac{x^2}{4}}}{\frac{\sin^2 x}{x^2}} = \frac{\frac{2}{4}}{1} = \frac{2}{4} = \frac{1}{2}$$

[NOTE: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$]

Therefore, $\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x} = \frac{1}{2}$

Question: 21

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos 3x}{x^2} = \lim_{x\to 0}\frac{9[1-\cos 3x]}{(3x)^2} = \frac{9}{2}$$

Therefore, $\lim_{x\to 0}\frac{1-\cos 3x}{x^2} = \frac{9}{2}$

Question: 22

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$

Divide numerator and denominator by x^2, we have

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos x}{\sin^2 2x} = \lim_{x\to 0}\frac{\frac{[1-\cos x]}{x^2}}{\frac{\sin^2 2x}{x^2}} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{1-\cos x}{\sin^2 2x} = \frac{1}{2}$

Question: 23

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$ and $\lim\limits_{x\to 0}\frac{\tan x}{x}=1$

Divide numerator and denominator by x^2, we have

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos 2x}{3\tan^2 x}=\lim_{x\to 0}\frac{\frac{4[1-\cos 2x]}{(4)x^2}}{\frac{3\tan^2 x}{x^2}}=\frac{4}{6}=\frac{2}{3}$$

Therefore, $\lim\limits_{x\to 0}\frac{1-\cos 2x}{3\tan^2 x}=\frac{1}{6}$

Question: 24

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

Divide numerator and denominator by x^2, we have

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos 4x}{1-\cos 6x}=\lim_{x\to 0}\frac{\frac{16[1-\cos 4x]}{(4x)^2}}{\frac{36[1-\cos 6x]}{(6x)^2}}=\frac{\frac{16}{2}}{\frac{36}{2}}=\frac{8}{18}=\frac{4}{9}$$

Therefore, $\lim\limits_{x\to 0}\frac{1-\cos 4x}{1-\cos 6x}=\frac{4}{9}$

Question: 25

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

Formula used: $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$

Divide numerator and denominator by m^2 and n^2, we have

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}=\lim_{x\to 0}\frac{\frac{m^2[1-\cos mx]}{(mx)^2}}{\frac{n^2[1-\cos nx]}{(nx)^2}}=\frac{m^2}{n^2}$$

Therefore, $\lim\limits_{x\to 0}\frac{1-\cos mx}{1-\cos nx}=\frac{m^2}{n^2}$

Question: 26

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

We know that sin 2x = 2 sin x cosx

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$ and $\lim_{x\to 0}\frac{\sin x}{x} = 1$

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3} = \lim_{x\to 0}\frac{2\sin x-2\sin x\cos x}{x^3} = \lim_{x\to 0}\frac{2\sin x[1-\cos x]}{x^3} = \lim_{x\to 0}\frac{2\sin x}{x}\times\frac{[1-\cos x]}{x^2} = \frac{2}{2} = 1$$

Therefore, $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3} = 1$

Question: 27

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

NOTE : tan x – sin x = $\frac{\sin x}{\cos x}$ – sin x = $\frac{\sin x - \sin x\cos x}{\cos x}$ = sin x $\left(\frac{1-\cos x}{\cos x}\right)$

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3} = \lim_{x\to 0}\frac{\left(\frac{1-\cos x}{\cos x}\right)\sin x}{x^3} = \lim_{x\to 0}\frac{1-\cos x}{x^2\ \cos x}\times\frac{\sin x}{x}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ = 1/2 and $\lim_{x\to 0}\frac{\sin x}{x}$ = 1 or we can used L hospital Rule,

So, by using the above formula, we have

$$\lim_{x\to 0}\frac{1-\cos x}{x^2\ \cos x}\times\frac{\sin x}{x} = \frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{\tan x-\sin x}{x^3} = \frac{1}{2}$

Question: 28

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\tan 2x-\sin 2x}{x^3} = \lim_{x\to 0}\frac{\sin 2x+\sin 2x\cos 2x}{x^3} = \lim_{x\to 0}\frac{\sin 2x(1-\cos 2x)}{x^3} = \lim_{x\to 0}\frac{2\sin 2x}{2x}\times\frac{4(1-\cos 2x)}{(2x)^2} = 4$$

Therefore, $\lim_{x\to 0}\frac{\tan 2x-\sin\ 2x}{x^3} = 4$

Question: 29

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\infty \times \infty$

cosec x – cot x = (1 – cos x)/sinx

$$\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}=\lim_{x\to 0}\frac{1-\cos x}{x\sin x}=\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{x\sin x}{x^2}}=\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin x}{x}}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2}=1/2$ and $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}=\lim_{x\to 0}\frac{\frac{1-\cos x}{x^2}}{\frac{\sin x}{x}}=\frac{1}{2}$$

Therefore, $\lim_{x\to 0}\frac{\operatorname{cosec} x-\cot x}{x}=\frac{1}{2}$

Question: 30

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form are $\infty \times \infty$

cosec 2x – cot 2x = (1 – cos 2x)/sin 2x

$$\lim_{x\to 0}\frac{\cot 2x-\operatorname{cosec} 2x}{x}=\lim_{x\to 0}\frac{\cos 2x-1}{x\sin 2x}=\lim_{x\to 0}\frac{\frac{\cos 2x-1}{x^2}}{\frac{x\sin 2x}{x^2}}=\lim_{x\to 0}\frac{\frac{4[\cos 2x-1]}{(2x)^2}}{\frac{2\sin 2x}{2x}}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2}=1/2$ and $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$\lim_{x\to 0}\frac{\cot 2x-\operatorname{cosec} 2x}{x}=\lim_{x\to 0}\frac{\frac{4[\cos 2x-1]}{(2x)^2}}{\frac{2\sin 2x}{2x}}=\frac{-4}{2}=-2$$

Therefore, $\lim_{x\to 0}\frac{\cot 2x-\operatorname{cosec} 2x}{x}=-2$

Question: 31

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

$$\lim_{x\to 0}\frac{\sin 2x(1-\cos 2x)}{x^3}=\lim_{x\to 0}\frac{\sin 2x}{x}\times\frac{(1-\cos 2x)}{x^2}=\lim_{x\to 0}\frac{2\sin 2x}{2x}\times\frac{4(1-\cos 2x)}{(2x)^2}$$

Formula used: $\lim_{x\to 0}\frac{1-\cos x}{x^2}=1/2$ and $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$\lim_{x\to 0}\frac{\sin 2x(1-\cos 2x)}{x^3}=4$$

Therefore, $\lim_{x\to 0}\frac{\sin 2x(1-\cos 2x)}{x^3}=4$

Question: 32

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x-2}{\tan x-1}$ = So $\lim_{x\to\frac{\pi}{4}}\frac{2\sec x(\sec x\tan x)-0}{\sec^2 x-0} = \lim_{x\to\frac{\pi}{4}}\frac{2\sec x(\sec x\tan x)}{\sec^2 x} = \lim_{x\to\frac{\pi}{4}} 2\tan x = 2$

Therefore, $\lim_{x\to\frac{\pi}{4}}\frac{\sec^2 x-2}{\tan x-1} = 2$

Question: 33

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x\to\frac{\pi}{4}}\frac{\mathrm{cosec}^2 x-2}{\cot x-1} = \lim_{x\to\frac{\pi}{4}}\frac{2\mathrm{cosec}\, x(-\mathrm{cosec}\, x\cot x)-0}{-\mathrm{cosec}^2 x-0} = \lim_{x\to\frac{\pi}{4}}\frac{2\mathrm{cosec}\, x(-\mathrm{cosec}\, x\cot x)}{-\mathrm{cosec}^2 x} = \lim_{x\to\frac{\pi}{4}} 2\cot x = 2$

Therefore, $\lim_{x\to\frac{\pi}{4}}\frac{\mathrm{cosec}^2 x-2}{\cot x-1} = 2$

Question: 34

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Formis $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{x-\frac{\pi}{4}} = \lim_{x\to\frac{\pi}{4}}\frac{0-\sec^2 x}{1-0} = \lim_{x\to\frac{\pi}{4}}\frac{-\sec^2 x}{1}$ = -2

Therefore, $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{x-\frac{\pi}{4}} = -2$

Question: 35

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}=\lim_{x\to\pi}\frac{3\cos 3x-3\cos x}{-3(\pi-x)^2}$

Again, indeterminate Form is $\frac{0}{0}$

So, Differtiate both sides w.r.t x again, we have

$\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}=\lim_{x\to\pi}\frac{-9\sin 3x+3\sin x}{6(\pi-x)}$

Again, indeterminate Form is $\frac{0}{0}$

So, Differtiate both sides w.r.t x again, we have

$\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}=\lim_{x\to\pi}\frac{-27\cos 3x+3\cos x}{-6}=\frac{-27\cos 3\pi+3\cos\pi}{-6}=\frac{27-3}{-6}= -4$

Therefore, $\lim_{x\to\pi}\frac{\sin 3x-3\sin x}{(\pi-x)^3}=-4$

Question: 36

Evaluate the foll

Solution:

To Find: Limits

NOTE: First Check the form of imit. Used this method if the limit is satisfying any one from 7 indeterminate form.

In this Case, indeterminate Form is $\frac{0}{0}$

By using L hospital Rule,

Differtiate both sides w.r.t x

So $\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}=\lim_{x\to\frac{\pi}{2}}\frac{0+(-2\sin 2x)}{2(\pi-2x)(-2)}=\lim_{x\to\frac{\pi}{2}}\frac{-2\sin 2x}{-4(\pi-2x)}=\lim_{x\to\frac{\pi}{2}}\frac{2\sin 2x}{4(\pi-2x)}$

Again, indeterminate Form is $\frac{0}{0}$

So, Differtiate both sides w.r.t x again, we have

$\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}=\lim_{x\to\frac{\pi}{2}}\frac{4\cos 2x}{4(-2)}=\lim_{x\to\frac{\pi}{2}}\frac{\cos 2x}{(-2)}=\frac{\cos\pi}{(-2)}=\frac{-1}{(-2)}=\frac{1}{2}$

Therefore, $\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}=\frac{1}{2}$

Question: 37

Evaluate the foll

Solution:

$$=\lim_{x\to a}\frac{(\cos x-\cos a)}{(x-a)}$$

$= \lim_{x\to a} \frac{-2\times\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{x-a}$ [$\because \cos x - \cos a = -2\times\sin\frac{x+a}{2}\sin\frac{x-a}{2}$]

$$= \lim_{x\to a} \sin\frac{x+a}{2} \times -\frac{\sin\left(\frac{x-a}{2}\right)}{\frac{(x-a)}{2}}$$

$$= -1\times\lim_{x\to a}\sin\frac{x+a}{2}\ \left[\because \lim_{x\to a}\frac{\sin\theta}{\theta} = 1\right]$$

$$= -1\times\sin\frac{(a+a)}{2}$$

$$= -1\times\sin\frac{2a}{2}$$

= -sin (a)

$$\therefore \lim_{x\to a}\frac{(\cos x - \cos a)}{(x-a)} = -\sin a$$

Question: 38

Evaluate the foll

Solution:

$$= \lim_{x\to a}\frac{(\sin x - \sin a)}{(x-a)}$$

$$= \lim_{x\to a}\frac{\left(2\times\cos\frac{x+a}{2}\sin\frac{x-a}{2}\right)}{(x-a)}\ \left[\because \sin x - \sin a = 2\times\cos\frac{x+a}{2}\sin\frac{x-a}{2}\right]$$

$$= 1\times\lim_{x\to a}\cos\frac{x+a}{2}\ \left[\because \lim_{x\to a}\frac{\sin\theta}{\theta} = 1\right]$$

$$= \cos\frac{a+a}{2}$$

=cosa

$$\therefore \lim_{x\to a}\frac{\sin x - \sin a}{x-a} = \cos a$$

Question: 39

Evaluate the foll

Solution:

$$= \lim_{x\to a}\frac{(\sin x - \sin a)}{\left(\sqrt{x}-\sqrt{a}\right)}$$

$= \lim_{x\to a}\frac{(\sin x - \sin a)}{\left(\sqrt{x}-\sqrt{a}\right)}\times\frac{\left(\sqrt{x}+\sqrt{a}\right)}{\left(\sqrt{x}+\sqrt{a}\right)}$ [Multiply and divide by √x-√a]

$$= \lim_{x\to a}\frac{(\sin x - \sin a)\times\left(\sqrt{x}+\sqrt{a}\right)}{(x-a)}$$

$$= \cos a\times\lim_{x\to a}\left(\sqrt{x}+\sqrt{a}\right)\ \left[\because \lim_{x\to a}\frac{\sin x - \sin a}{x-a} = \cos a\right]$$

=2√a×cosa

=2√acosa

$$\therefore \lim_{x\to a}\frac{(\sin x-\sin a)}{(\sqrt{x}-\sqrt{a})}=2\sqrt{a}\cos a$$

Question: 40

Evaluate the foll

Solution:

$$=\lim_{x\to 0}\frac{\left(2\sin\frac{5x-3x}{2}\cos\frac{5x+3x}{2}\right)}{\sin x}\left[Applying\ \sin C-\sin D = 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}\right]$$

$$=\lim_{x\to 0}2\cos 4x$$

$$=2\times 1$$

$$\therefore \lim_{x\to 0}\frac{\sin 5x-\sin 3x}{\sin x}=2$$

Question: 41

Evaluate the foll

Solution:

$$=\lim_{x\to 0}\frac{\left(1-\cos 5x-(1-\cos 3x)\right)}{x^2}$$

$$=\lim_{x\to 0}\left(\frac{1-\cos 5x}{x^2}-\frac{1-\cos 3x}{x^2}\right)$$

$$=\left(\lim_{x\to 0}\frac{1-\cos 5x}{x^2}\times\frac{25}{25}\right)-\left(\lim_{x\to 0}\frac{1-\cos 3x}{x^2}\times\frac{9}{9}\right)$$

$$=\frac{25}{2}-\frac{9}{2}\left[\because \lim_{x\to 0}\frac{1-\cos ax}{(ax)^2}=\frac{1}{2}\right]$$

$$=\frac{16}{2}$$

=8

$$\therefore \lim_{x\to 0}\frac{(\cos 3x-\cos 5x)}{x^2}=8$$

Question: 42

Evaluate the foll

Solution:

$$=\lim_{x\to 0}\frac{\left(2\times\sin\frac{3x+5x}{2}\times\cos\frac{3x-5x}{2}\right)}{\left(2\times\cos\frac{6x+4x}{2}\sin\frac{6x-4x}{2}\right)}$$

$$=\lim_{x\to 0}\frac{\sin 4x\cos x}{\cos 5x\sin x}$$

$$=\lim_{x\to 0}\frac{\sin 4x}{\cos 5x\times\frac{\sin x}{\cos x}}\times\frac{4x}{4x}$$

$$= 4 \times \lim_{x\to 0} \frac{\sin 4x}{4x} \times \frac{1}{\cos 5x} \times \frac{x}{\tan x} \left[\begin{array}{l} \because \lim_{x\to 0} \frac{\sin\theta}{\theta} = 1 \\ \lim_{x\to 0} \frac{\theta}{\tan\theta} = 1 \end{array}\right]$$

$$= 4$$

$$\therefore \lim_{x\to 0} \frac{(\sin 3x + \sin 5x)}{(\sin 6x - \sin 4x)} = 4$$

Question: 43

Evaluate the foll

Solution:

$$= \lim_{x\to 0} \frac{[\sin(2+x) - \sin(2-x)]}{x}$$

$$= \lim_{x\to 0} \frac{\left[2 \times \cos\frac{(2+x+2-x)}{2} \times \sin\frac{(2+x-2+x)}{2}\right]}{x}$$

$$= \lim_{x\to 0} \frac{(2 \times \cos 2 \times \sin x)}{x}$$

$$= 2\cos 2 \lim_{x\to 0} \frac{\sin x}{x}$$

$$=2\cos 2$$

$$\lim_{x\to 0} \frac{[\sin(2+x) - \sin(2-x)]}{x} = 2\cos 2$$

Question: 44

Evaluate the foll

Solution:

$$= \lim_{x\to 0} \frac{1 - \cos 2x}{(\cos 2x - \cos 8x)}$$

$$= \lim_{x\to 0} \frac{2 \times \sin x \times \sin x}{2 \times \sin 3x \times \sin 5x} \times \frac{5x \times 3x}{x \times x} \times \frac{1}{15}$$

$$= \frac{1}{15} \times 1 \times 1 \times 1 \times 1 \left[\because \lim_{x\to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{1}{15}$$

$$\lim_{x\to 0} \frac{1 - \cos 2x}{(\cos 2x - \cos 8x)} = \frac{1}{15}$$

Question: 45

Evaluate the foll

Solution:

$$= \lim_{x\to\frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x$$

$$= -1 \times \lim_{y\to 0} y \tan\left(y + \frac{\pi}{2}\right) \left[x - \frac{\pi}{2} = y\right]$$

$$= -1 \times \lim_{y\to 0} y \cot y \times -1$$

$= \lim_{y \to 0} \frac{y}{\tan y}$

$=1$

$\therefore \lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) \tan x = 1$

Question: 46

Evaluate the foll

Solution:

$= \lim_{x \to 0} \frac{\left(\sqrt{1+2x} - \sqrt{1-2x}\right)}{\sin x}$

$= \lim_{x \to 0} \frac{\left(\sqrt{1+2x} - \sqrt{1-2x}\right)}{\sin x} \times \frac{\sqrt{1+2x} + \sqrt{1-2x}}{\left(\sqrt{1+2x} + \sqrt{1-2x}\right)}$

$= \lim_{x \to 0} \frac{1 + 2x - 1 + 2x}{\sin x} \times \frac{1}{\sqrt{1+2x} + \sqrt{1-2x}}$

$= 4 \times \lim_{x \to 0} \frac{x}{\sin x} \times \frac{1}{\sqrt{1+2x} + \sqrt{1-2x}}$

$= 4 \times \frac{1}{2} \times 1$

$=2$

$\therefore \lim_{x \to 0} \frac{\left(\sqrt{1+2x} - \sqrt{1-2x}\right)}{\sin x} = 2$

Question: 47

Evaluate the foll

Solution:

$= \lim_{h \to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$

$= \lim_{h \to 0} \frac{a^2 (\sin(a+h) - \sin a) + 2ah \sin(a+h) + h^2 \sin(a+h)}{h}$

$= 2a \sin a + 0 + \lim_{h \to 0} \frac{a^2 \times 2 \times \cos\left(a + \frac{h}{2}\right) \times \sin h}{h}$

$= 2a \sin a + 2a^2 \cos a$

$=2a^2\cos a+2a\sin a$

$\therefore \lim_{h \to 0} \frac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h} = 2a^2 \cos a + 2a \sin a$

Question: 48

Evaluate the foll

Solution:

$= \lim_{x \to 0} \frac{(e^{3+x} - \sin x - e^3)}{x}$

$= -\lim_{x \to 0} \frac{\sin x}{x} + \lim_{x \to 0} \frac{e^{3+x} - e^3}{x}$

$$= -1 + \lim_{x \to 0} \frac{e^3(e^x - 1)}{x}$$

$=-1+e^3$

$$\therefore \lim_{x \to 0} \frac{(e^{3+x} - \sin x - e^3)}{x} = e^3 - 1$$

Question: 49

Evaluate the foll

Solution:

$$= \lim_{x \to 0} \frac{(e^{\tan x} - 1)}{\tan x}$$

As x tends to 0, tan(x) also tends to zero,

So,

$$\lim_{x \to 0} \frac{(e^{\tan x} - 1)}{\tan x} = \lim_{\tan x \to 0} \frac{(e^{\tan x} - 1)}{\tan x}$$

$=1$

$$\therefore \lim_{x \to 0} \frac{(e^{\tan x} - 1)}{\tan x} = 1$$

Question: 50

Evaluate the foll

Solution:

$$= \lim_{x \to 0} \frac{e^{\tan x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{e^{\tan x} - 1}{x} \times \frac{\tan x}{\tan x}$$

$$= \lim_{x \to 0} \frac{e^{\tan x} - 1}{\tan x} \times \frac{\tan x}{x}$$

$=1\times1$

$=1$

$$\therefore \lim_{x \to 0} \frac{e^{\tan x} - 1}{x} = 1$$

Question: 51

Evaluate the foll

Solution:

$$= \lim_{x \to 0} \frac{ax + x\cos x}{b \sin x}$$

$$= \lim_{x \to 0} \frac{ax}{b \sin x} + \frac{x\cos x}{b \sin x}$$

$$= \lim_{x \to 0} \frac{ax}{b \sin x} + \lim_{x \to 0} \frac{x\cos x}{b \sin x}$$

$$= \frac{a}{b} + \frac{1}{b} \left[\begin{array}{l} \because \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \\ \lim_{x \to 0} \cos \theta = 1 \end{array} \right]$$

$= \frac{a+1}{b}$

$\therefore \lim_{x \to 0} \frac{ax + x\cos x}{b\sin x} = \frac{a+1}{b}$

Question: 52

Evaluate the foll

Solution:

$= \lim_{x \to 0} \frac{\sin(ax) + bx}{ax + \sin(bx)}$

$= \lim_{x \to 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} \times \frac{bx}{ax} \times \frac{a}{b}$

$= \lim_{x \to 0} \frac{\frac{\sin ax + bx}{ax}}{\frac{ax + \sin bx}{bx}} \times \frac{a}{b}$

$= \frac{a}{b} \times \frac{\lim_{x \to 0} \frac{\sin ax + bx}{ax}}{\lim_{x \to 0} \frac{ax + \sin bx}{bx}}$

$= \frac{a}{b} \times \frac{1 + \frac{b}{a}}{1 + \frac{a}{b}}$

$= \frac{a}{b} \times \frac{b}{a}$

=1

$\therefore \lim_{x \to 0} \frac{\sin(ax) + bx}{ax + \sin(bx)} = 1$

Question: 53

Evaluate the foll

Solution:

$= \lim_{x \to 0} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

$= \lim_{x \to 0} \frac{\sin x}{\pi(\pi - x)} \times \frac{x}{x}$

$= \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \frac{\pi - (\pi - x)}{\pi(\pi - x)}$

$= 1 \times \lim_{x \to 0} \left(\frac{1}{\pi - x} - \frac{1}{\pi}\right)$

$= \frac{1}{\pi} - \frac{1}{\pi}$

$= 0$

$\therefore \lim_{x \to 0} \frac{\sin(\pi - x)}{\pi(\pi - x)} = 0$

Question: 54

Evaluate the foll

Solution:

$$= \lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$$

As x tends to$\pi/2$, $x - \pi/2$ tends to zero.

Let, $y = x - \frac{\pi}{2}$

$$= \lim_{y \to 0} \frac{\tan\left(2y + \frac{\pi}{2} \times 2\right)}{y}$$

$$= \lim_{y \to 0} \frac{\tan(\pi + 2y)}{y}$$

$$= \lim_{y \to 0} \frac{\tan 2y}{2y} \times 2$$

=2

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = 2$$

Question: 55

Evaluate the foll

Solution:

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$$

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} \times \frac{4 \times x \times x}{2x \times 2x}$$

$$= 4 \times \lim_{x \to 0} \frac{\frac{\cos 2x - 1}{2x \times 2x}}{\frac{\cos x - 1}{x \times x}}$$

$$= 4 \times \frac{\lim_{x \to 0} \frac{\cos 2x - 1}{(2x)^2}}{\lim_{x \to 0} \frac{\cos x - 1}{x^2}}$$

$$= 4 \times \frac{\frac{1}{2}}{\frac{1}{2}} = 4$$

$$\therefore \lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} = 4$$

Question: 56

Evaluate the foll

Solution:

$$= \lim_{x \to 0} (\csc x - \cot x)$$

$$= \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right)$$

$$= \lim_{x \to 0} \left(\frac{1 - \cos x}{\sin x}\right)$$

$$= \lim_{x\to 0}\left(\frac{2\times \sin\frac{x}{2}\times \sin\frac{x}{2}}{2\times \sin\frac{x}{2}\times \cos\frac{x}{2}}\right) [\because 1-\cos\theta = 2\sin\theta\times\sin\theta]$$

$$= \lim_{x\to 0}\left(\tan\frac{x}{2}\right)$$

=0

$$\therefore \lim_{x\to 0}(\csc x - \cot x) = 0$$

Question: 57

Evaluate the foll

Solution:

$$= \lim_{x\to 0}\frac{1-\cos 2mx}{1-\cos 2nx}$$

$$= \lim_{x\to 0}\frac{\frac{1-\cos 2mx}{(2mx)^2}\times(2mx)^2}{\frac{1-\cos 2nx}{(2nx)^2}\times(2nx)^2}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}}\times\frac{m\times m}{n\times n}\left[\because \frac{1-\cos\theta}{\theta\times\theta} = \frac{1}{2}\right]$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x\to 0}\frac{1-\cos 2mx}{1-\cos 2nx} = \frac{m^2}{n^2}$$

Question: 58

Evaluate the foll

Solution:

$$= \lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx}$$

$$= \lim_{x\to 0}\frac{\frac{1-\cos mx}{mx\times mx}}{\frac{1-\cos nx}{nx\times nx}}\times\frac{m\times m}{n\times n}$$

$$= \frac{m^2}{n^2}$$

$$\therefore \lim_{x\to 0}\frac{1-\cos mx}{1-\cos nx} = \frac{m^2}{n^2}$$

Question: 59

Evaluate the foll

Solution:

$$= \lim_{x\to 0}\frac{\sin mx\times\sin mx}{\sin nx\times\sin nx}$$

$$= \lim_{x\to 0}\frac{\frac{\sin mx\times\sin mx}{mx\times mx}}{\frac{\sin nx\times\sin nx}{nx\times nx}}\times\frac{m^2}{n^2}$$

$= \frac{m^2}{n^2}$

$\therefore \lim_{x\to 0} \frac{\sin mx \times \sin mx}{\sin nx \times \sin nx} = \frac{m^2}{n^2}$

Question: 60

Evaluate the foll

Solution:

$= \lim_{x\to 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x} \times \frac{3x}{3x}$

$= \lim_{x\to 0} \frac{\frac{\sin 2x + \sin 3x}{3x}}{\frac{2x + \sin 3x}{3x}}$

$= \frac{\frac{2}{3} + 1}{\frac{2}{3} + 1}$

=1

$\therefore \lim_{x\to 0} \frac{\sin 2x + \sin 3x}{2x + \sin 3x} = 1$

Question: 61

Evaluate the foll

Solution:

$= \lim_{x\to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x}$

$= \lim_{x\to 0} \frac{\frac{1}{\cos 4x} - \frac{1}{\cos 2x}}{\frac{1}{\cos 3x} - \frac{1}{\cos x}}$

$= \lim_{x\to 0} \frac{(\cos 2x - \cos 4x) \times \cos x \times \cos 3x}{(\cos x - \cos 3x) \times \cos 2x \times \cos 4x}$

$= \lim_{x\to 0} \frac{2 \times \sin 3x \times \sin x \times \cos x \times \cos 3x}{2 \times \sin 2x \times \sin x \times \cos 2x \times \cos 4x}$

$= \lim_{x\to 0} \frac{\sin 6x \times \cos x}{\sin 4x \times \cos 4x} \times \frac{2}{2}$

$= 2 \times \lim_{x\to 0} \frac{\sin 6x \times \cos x}{\sin 8x} \times \frac{8x}{6x} \times \frac{6}{8}$

$= \frac{3}{2} \times \lim_{x\to 0} \frac{\frac{\sin 6x}{6x} \times \cos x}{\frac{\sin 8x}{8x}}$

$= \frac{3}{2} \times \frac{1 \times 1}{1}$

$= \frac{3}{2}$

$\therefore \lim_{x\to 0} \frac{\sec 4x - \sec 2x}{\sec 3x - \sec x} = \frac{3}{2}$

Question: 62

Evaluate the foll

Solution:

$$= \lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin x\times\sin x}$$

$$= \lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin x\times\sin x}\times\frac{\sqrt{2}+\sqrt{1+\cos x}}{\sqrt{2}+\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\frac{2-(1-\cos x)}{\sin x\times\sin x\times\sqrt{2}+\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0}\frac{1-\cos x}{2\times\sin\frac{x}{2}\cos\frac{x}{2}\sin x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$= \lim_{x\to 0}\frac{2\times\sin\frac{x}{2}\times\sin\frac{x}{2}}{2\times\sin\frac{x}{2}\cos\frac{x}{2}\sin x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$= \lim_{x\to 0}\frac{\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}}{\frac{\sin x}{x}}\times\lim_{x\to 0}\frac{1}{\cos\frac{x}{2}\times\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$= \frac{1}{2}\times\frac{1}{\left(\sqrt{2}+\sqrt{2}\right)}$$

$$= \frac{1}{4\sqrt{2}}$$

$$\therefore \lim_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin x\times\sin x} = \frac{1}{4\sqrt{2}}$$

Question: 63

Evaluate the foll

Solution:

$$= \lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$$

$$= \lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}\times\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$$

$$= \lim_{x\to 0}\frac{1+\sin x-(1-\sin x)}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= \lim_{x\to 0}\frac{2\times\sin x}{x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= 2\times\lim_{x\to 0}\frac{\sin x}{x}\times\lim_{x\to 0}\frac{1}{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$= 2\times 1\times\frac{1}{2}$$

=1

$$\therefore \lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} = 1$$

Question: 64

Evaluate the foll

Solution:

$$= \lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2}$$

$$= \lim_{y \to 0} \frac{2 - \sqrt{3}\cos\left(y + \frac{\pi}{6}\right) - \sin\left(y + \frac{\pi}{6}\right)}{y^2 \times 36}$$

$$= \frac{1}{36} \times \lim_{y \to 0} \frac{2 - \frac{3}{2}\cos y + \frac{\sqrt{3}}{2}\sin y - \frac{1}{2}\cos y - \frac{\sqrt{3}}{2}\sin y}{y^2}$$

$$= \frac{1}{36} \times \lim_{y \to 0} \frac{2(1 - \cos y)}{y^2}$$

$$= 2 \times \frac{1}{2} \times \frac{1}{36}$$

$$= \frac{1}{36}$$

$$\therefore \lim_{x \to \frac{\pi}{6}} \frac{2 - \sqrt{3}\cos x - \sin x}{(6x - \pi)^2} = \frac{1}{36}$$

Question: 65

Evaluate the foll

Solution:

$$= \lim_{x \to 0} \frac{\cos ax - \cos bx}{\cos cx - 1}$$

$$= \lim_{x \to 0} \frac{1 - \cos bx - (1 - \cos ax)}{\cos cx - 1}$$

$$= \lim_{x \to 0} \frac{\frac{(1 - \cos bx)}{(bx)^2} \times b^2 - \frac{(1 - \cos ax)}{(ax)^2} \times a^2}{\frac{-(1 - \cos cx)}{(cx)^2} \times c^2}$$

$$= \frac{a^2 - b^2}{c^2}$$

$$\therefore \lim_{x \to 0} \frac{\cos ax - \cos bx}{\cos cx - 1} = \frac{a^2 - b^2}{c^2}$$

Question: 66

Evaluate the foll

Solution:

$$= \lim_{x \to a} \frac{\cos x - \cos a}{\cot x - \cot a}$$

$$= \lim_{x \to a} \frac{(\cos x - \cos a)}{\frac{\sin(a - x)}{\sin x \sin a}}$$

$$= \sin a \times \lim_{x \to a} \frac{\sin\left(\frac{x + a}{2}\right) \times \sin x}{\cos\left(\frac{x - a}{2}\right)}$$

$=\sin^3 a$

$\therefore \lim_{x \to a} \frac{\cos x - \cos a}{\cot x - \cot a} = \sin a \times \sin a \times \sin a$

Question: 67

Evaluate the foll

Solution:

$$= \lim_{x \to \frac{\pi}{4}} \frac{\left(\frac{-\sin x\,(\cos 2x)}{\cos x \cos x \cos x}\right)}{\left(\frac{\cos x - \sin x}{\sqrt{2}}\right)}$$

$$= -\sqrt{2} \times \lim_{x \to \frac{\pi}{4}} \frac{\sin x\,(\cos x + \sin x)}{\cos x \times \cos x \times \cos x}$$

$$= -\sqrt{2} \times \frac{\frac{1}{\sqrt{2}} \times \sqrt{2}}{\frac{1}{\sqrt{2}}^{3}}$$

$$= -4$$

Question: 68

Evaluate the foll

Solution:

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} \times \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\sqrt{2} + \sqrt{1 + \sin x}\left(\sqrt{2} \cos x \cos x\right)}$$

$\text{Let,} y = x - \frac{\pi}{2}$

$$= \lim_{y \to 0} \frac{1 - \cos y}{\sqrt{2} + \sqrt{1 + \cos y}\left(\sqrt{2} \sin y \sin y\right)}$$

$$= \frac{1}{2\sqrt{2}} \times \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{8}$$

$$\therefore \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\sqrt{2} \cos x \cos x} = \frac{1}{8}$$

Question: 69

Evaluate the foll

Solution:

$$= \lim_{x \to \frac{\pi}{6}} \frac{\cot x \times \cot x - 3}{\csc x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\cos x \times \cos x) - 3 \times \sin x \times \sin x}{\sin x\,(1 - 2\sin x)}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{1 - 4 \times \sin x \times \sin x}{\sin x\,(1 - 2\sin x)}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(1 - 2\sin x) \times (1 + 2\sin x)}{\sin x\,(1 - 2\sin x)}$$

=4

$$\therefore \lim_{x \to \frac{\pi}{6}} \frac{\cot x \times \cot x - 3}{\csc x - 2} = 4$$

Question: 70

Evaluate the foll

Solution:

$$= \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

$$= \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} \times \frac{\sqrt{2 + \cos x} + 1}{\sqrt{2 + \cos x} + 1}$$

$$= \lim_{x \to \pi} \frac{1 + \cos x}{(\pi - x)^2} \times \frac{1}{\sqrt{2 + \cos x} + 1}$$

Let,$y = x - \pi$

$$= \lim_{y \to 0} \frac{1 - \cos y}{x^2 \times \sqrt{2 - \cos y} + 1}$$

$$= \frac{1}{4}$$

$$\therefore \lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2} = \frac{1}{4}$$

Question: 71

Evaluate the foll

Solution:

$$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x}$$

Let,$y = x - \frac{\pi}{4}$

$$= \lim_{y \to 0} \frac{2\tan x}{1 - \cos x + \sin x}$$

$$= \lim_{y \to 0} \frac{\frac{2\cos\frac{x}{2}}{\cos x}}{\sin\frac{x}{2} + \cos\frac{x}{2}}$$

$= 2$

$$\therefore \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{1 - \sqrt{2}\sin x} = 2$$

Question: 72

Evaluate the foll

Solution:

$$= \lim_{x \to \frac{\pi}{6}} \frac{2 \times \sin x \times \sin x + \sin x - 1}{2 \times \sin x \times \sin x - 3 \sin x + 1}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(2 \sin x - 1) \times (\sin x + 1)}{(2 \sin x - 1)(\sin x - 1)}$$

= -3

$$\therefore \lim_{x \to \frac{\pi}{6}} \frac{2 \times \sin x \times \sin x + \sin x - 1}{2 \times \sin x \times \sin x - 3 \sin x + 1} = -3$$

Exercise : 27C

Question: 1

If Left Hand Limit(L.H.L.):

$$\lim_{x \to 3^-} f(x)$$

$$= \lim_{x \to 3^-} |x| - 3$$

$$= \lim_{x \to 3^-} -(x - 3)$$

= - (3 - 3)

= 0

Right Hand Limit(R.H.L.):

$$\lim_{x \to 3^+} f(x)$$

$$= \lim_{x \to 3^+} |x| - 3$$

$$= \lim_{x \to 3^+} (x - 3)$$

= 3-3

= 0

Since,

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

We can say that the limit exists and

$$\lim_{x \to 3} f(x) = 0$$

Question: 2

Let Left Hand Limit(L.H.L.):

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x}{|x|}$$

$$= \lim_{x \to 0^-} \frac{x}{(-x)}$$

$$= \lim_{x \to 0^-} -1$$

= -1

Right Hand Limit(R.H.L.):

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{x}{|x|}$

$= \lim\limits_{x\to 0^+} \frac{x}{(+x)}$

$= \lim\limits_{x\to 0^+} 1$

=1

Since $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x), \lim\limits_{x\to 0} f(x)$ does not exist

Question: 3

Let Left Hand Limit(L.H.L.):

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} \frac{|x-3|}{x-3}$

$= \lim\limits_{x\to 3^-} \frac{-(x-3)}{x-3}$

$= \lim\limits_{x\to 3^-} -1$

= -1

Right Hand Limit(R.H.L.):

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} \frac{|x-3|}{x-3}$

$= \lim\limits_{x\to 3^+} \frac{(x-3)}{x-3}$

$= \lim\limits_{x\to 3^+} 1$

= 1

$\lim\limits_{x\to 3^-} f(x) \neq \lim\limits_{x\to 3^+} f(x)$

Thus, $\lim\limits_{x\to 3} f(x)$ does not exist.

Question: 4

Let Left Hand Limit(L.H.L.):

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} 1+x^2$

= 1 + $(1)^2$

= 1 + 1

= 2

Right Hand Limit(R.H.L.):

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} 2-x$

= 2 – (1)

= 2 – 1

=1

$\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x)$

Thus, $\lim\limits_{x\to 1} f(x)$ does not exist.

Question: 5

Let Left Hand Limit(L.H.L.):

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{x - |x|}{x}$$

$$= \lim_{x \to 0^-} \frac{x - (-x)}{x}$$

$$= \lim_{x \to 0^-} \frac{x + x}{x}$$

$$= \lim_{x \to 0^-} \frac{2x}{x}$$

$$= \lim_{x \to 0^-} 2$$

= 2

Right Hand Limit(R.H.L.):

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x - |x|}{x}$$

$$= \lim_{x \to 0^+} \frac{x - (x)}{x}$$

$$= \lim_{x \to 0^+} \frac{0}{x}$$

$$= \lim_{x \to 0^+} 0$$

= 0

$$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

Thus, $\lim_{x \to 0} f(x)$ does not exist.

Question: 6

Let Left Hand Limit(L.H.L.):

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 5x - 4$$

= 5(1) – 4

= 5 – 4

= 1

Right Hand Limit(R.H.L.):

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 4x^3 - 3x$$

$= 4\,(1)^3 - 3\,(1)$

= 4 – 3

= 1

$$\therefore \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

Thus, $\lim_{x \to 1} f(x) = 1$

Question: 7

Let Left Hand Limit(L.H.L.):

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 4x - 5$

= 4 (2) – 5

= 8 – 5

= 3

Right Hand Limit(R.H.L.):

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x - a$

= 2 – a

Since $\lim_{x\to 2} f(x)$ it exists,

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

→ 3 = 2 – a

→ a = 2 – 3

→ a = -1

Question: 8

Let Left Hand Limit(L.H.L.):

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{3x}{|x| + 2x}$

$= \lim_{x\to 0^-} \frac{3x}{(-x) + 2x}$

$= \lim_{x\to 0^-} \frac{3x}{x}$

$= \lim_{x\to 0^-} 3$

= 3

Right Hand Limit(R.H.L.):

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{3x}{|x| + 2x}$

$= \lim_{x\to 0^+} \frac{3x}{(x) + 2x}$

$= \lim_{x\to 0^+} \frac{3x}{3x}$

$= \lim_{x\to 0^+} 1$

= 1

Since

$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Thus, $\lim_{x\to 0} f(x)$ does not exist.

Question: 9

Let Left Hand Limit(L.H.L.):

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x + k$

= 0 + k

= k

Right Hand Limit(R.H.L.):

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \cos x$$

= cos (0)

= 1

It is given that $\lim_{x\to 0} f(x)$ exists. Therefore,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

→ k = 1

Question: 10

Show that

Solution:

Let x = 0+h for x tending to 0^+

Since x→ 0, h also tends to 0

Right Hand Limit(R.H.L.):

$$\lim_{x\to 0^+} f(x)$$

$$= \lim_{x\to 0^+} \frac{1}{x}$$

$$= \lim_{h\to 0^+} \frac{1}{0+h}$$

$$= \lim_{h\to 0^+} \frac{1}{+h}$$

$$= +\frac{1}{0}$$

$$= +\infty$$

Let x=0 -h for x tending to 0^-

Since x→0, h also tends to 0.

Left Hand Limit(L.H.L.):

$$= \lim_{x\to 0^-} f(x)$$

$$= \lim_{x\to 0^-} \frac{1}{x}$$

$$= \lim_{h\to 0^-} \frac{1}{0-h}$$

$$= \lim_{h\to 0^-} \frac{1}{-h}$$

$$= -\frac{1}{0}$$

= - ∞

Since,

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

Thus, $\lim_{x \to 0} \frac{1}{x}$ does not exist.

Question: 11

Show that

Solution:

Let x = 0 + h, when x is tends to 0^+

Since x tends to 0, h will also tend to 0.

Right Hand Limit(R.H.L):

$\lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0^+} \frac{1}{|x|}$

$= \lim_{x \to 0^+} \frac{1}{(x)}$

$= \lim_{h \to 0^+} \frac{1}{(0 + h)}$

$= \frac{1}{0}$

$= \infty$

Let x = 0 - h, when x is tends to 0^-

Since x tends to 0, h will also tend to 0.

Left Hand Limit(L.H.L.):

$\lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0^-} \frac{1}{|x|}$

$= \lim_{x \to 0^-} \frac{1}{(-x)}$

$= \lim_{h \to 0^-} \frac{1}{-(0 - h)}$

$= \lim_{h \to 0^-} \frac{1}{h}$

$= \frac{1}{0}$

$= \infty$

Thus,

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} \frac{1}{|x|} = \infty.$

Question: 12

Show that

Solution:

Left Hand Limit(L.H.L.):

$\lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0^-} e^{\frac{-1}{(-x)}}$

$= \lim_{x \to 0^-} e^{\frac{1}{x}}$

$= e^{\frac{1}{0}}$

$= e^{\infty}$

$\lim_{x \to 0^+} f(x)$

Right Hand Limit(R.H.L.):

$= \lim_{x \to 0^+} e^{\frac{-1}{x}}$

$= e^{\frac{-1}{0}}$

$= e^{-\infty}$

$= \frac{1}{e^{\infty}}$

[Formula $\frac{1}{\infty} = 0$, anything to the power infinity is also infinity. Thus $\frac{1}{e\infty} = \frac{1}{\infty} = 0$]

=0

Since

$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} e^{-1/x}$ does not exist.

Question: 13

Show that

Solution:

Let x = 0 + h, when x is tends to 0^+

Since x tends to 0, h will also tend to 0.

Right Hand Limit(R.H.L.):

$\lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0^+} \sin\frac{1}{x}$

$= \lim_{h \to 0^+} \sin\frac{1}{0+h}$

$= \sin\frac{1}{0}$

$= \sin \infty$

$= \infty$

Let x = 0 - h, when x is tends to 0^-

Since x tends to 0, h will also tend to 0.

Left Hand Limit(L.H.L.):

$$\lim_{x\to 0^-} f(x)$$

$$= \lim_{x\to 0^-} \sin\frac{1}{x}$$

$$= \lim_{h\to 0^-} \sin\frac{1}{0-h}$$

$$= \sin\frac{1}{-0}$$

$$= -\sin\frac{1}{0}$$

= - sin ∞

= -∞

Since,

$$\lim_{x-0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

$\therefore \lim_{x\to 0} \sin\frac{1}{x}$ does not exist.

Question: 14

Show that

Solution:

Left Hand Limit(L.H.L.):

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{|x|}$$

$$= \lim_{x\to 0^-} \frac{x}{(-x)}$$

$$= \lim_{x\to 0^-} -1$$

= -1

Right Hand Limit(R.H.L.):

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x}{|x|}$$

$$= \lim_{x\to 0^+} \frac{x}{(x)}$$

$$= \lim_{x\to 0^+} 1$$

= 1

Since

$$\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$$

Thus, $\lim_{x\to 0} \frac{x}{|x|}$ does not exist.

Question: 15

Let $\lim_{x \to \frac{\pi}{2}} f(x) = \lim_{x \to \frac{\pi}{2}} \frac{k\cos x}{\pi - 2x}$

Let $h = x - \frac{\pi}{2}$

$\rightarrow x = h + \frac{\pi}{2}$

$x \rightarrow \frac{\pi}{2}$

or, $h + \frac{\pi}{2} \rightarrow \frac{\pi}{2}$

or , h → 0

Putting this in the original sum,

$$= \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$$

$$= \lim_{h \to 0} \frac{-k\sin h}{\pi - \pi + h}$$

$$= \lim_{h \to 0} \frac{-k\sin h}{h}$$

$$= -k \lim_{h \to 0} \frac{\sin h}{h}$$

[Applying formula $\lim_{x \to 0} \frac{\sin x}{x} = 1$]

= -k × 1

= -k

$f\left(\frac{\pi}{2}\right) = 3$

It is given that $\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

∴ -k = 3

→ k = -3