# Chapter : 12. GEOMETRICAL PROGRESSION

## Exercise : 12A

**Question: 1**

Find the $6^{th}$

**Solution:**

Given: GP is 2, 6, 18, 54….

The given GP is of the form, a, ar, $ar^2$, $ar^3$….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 2$

Second term in GP, $a_2 = 6$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$r = \frac{6}{2} = 3$

Now, $n^{th}$ term of GP is, $a_n = ar^{n-1}$

So, the $6^{th}$ term in the GP,

$a_6 = ar^5$

$= 2 \times 3^5$

$= 486$

$n^{th}$ term in the GP,

$a_n = ar^{n-1}$

$= 2.3^{n-1}$

Hence, $6^{th}$ term = 486 and $n^{th}$ term = $2.3^{n-1}$

**Question: 2**

Find the $17^{t}$

**Solution:**

Given GP is 2, 2√2, 4, 8√2 …..

The given GP is of the form, a, ar, $ar^2$, $ar^3$….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 2$

Second term in GP, $a_2 = 2\sqrt{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Now, $n^{th}$ term of GP is, $a_n = ar^{n-1}$

So, the $17^{th}$ term in the GP,

$a_{17} = ar^{16}$

$= 2 \times (\sqrt{2})^{16}$

$= 512$

$n^{th}$ term in the GP,

$a_n = ar^{n-1}$

$= 2(\sqrt{2})^{n-1}$

$= (\sqrt{2})^{n+1}$

Hence, $17^{th}$ term = 512 and $n^{th}$ term = $(\sqrt{2})^{n+1}$

**Question: 3**

Find the $7^{th}$

**Solution:**

Given GP is 0.4, 0.8, 1.6….

The given GP is of the form, a, ar, $ar^2$, $ar^3$….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 0.4$

Second term in GP, $a_2 = 0.8$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$r = \frac{0.8}{0.4} = 2$

Now, $n^{th}$ term of GP is, $a_n = ar^{n-1}$

So, the $7^{th}$ term in the GP,

$a_7 = ar^6$

$= 0.4 \times 2^6$

$= 25.6$

$n^{th}$ term in the GP,

$a_n = ar^{n-1}$

$= (0.4)(2)^{n-1}$

$= (0.2)2^n$

Hence, $7^{th}$ term = 25.6 and $n^{th}$ term = $(0.2)2^n$

**Question: 4**

Find the $10^t$

**Solution:**

Given GP is $-\frac{3}{4}, \frac{1}{2}, -\frac{1}{3}, \frac{2}{9}$.....

The given GP is of the form, a, ar, $ar^2$, $ar^3$….

Where r is the common ratio.

The first term in the given GP, $a = a_1 = -\frac{3}{4}$

The second term in GP, $a_2 = \frac{1}{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = -\frac{\frac{1}{2}}{\frac{3}{4}} = -\frac{2}{3}$$

Now, $n^{th}$ term of GP is, $a_n = ar^{n-1}$

So, the $10^{th}$ term, $a_{10} = ar^9$

$$a_{10} = ar^9 = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^9 = \frac{128}{6561}$$

Now, the required $n^{th}$ term, $a_n = ar^{n-1}$

$$a_n = \left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)^{n-1} = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n$$

Hence, the $10^{th}$ term, $a_{10} = \frac{128}{6561}$ and $n^{th}$ term, $a_n = \left(\frac{9}{8}\right)\left(-\frac{2}{3}\right)^n$.

**Question: 5**

Which term of the

**Solution:**

Given GP is 3, 6, 12, 24….

The given GP is of the form, a, ar, $ar^2$, $ar^3$….

Where r is the common ratio.

First term in the given GP, $a_1 = a = 3$

Second term in GP, $a_2 = 6$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{6}{3} = 2$$

Let us consider 3072 as the $n^{th}$ term of the GP.

Now, $n^{th}$ term of GP is, $a_n = ar^{n-1}$

$3072 = 3.2^{n-1}$

$\frac{3072 \times 2}{3} = 2^n$

$2^n = 2^{11}$

$n = 11$

So, 3072 is the $11^{th}$ term in GP.

**Question: 6**

Which term of the

**Solution:**

Given GP is $\frac{1}{4}, -\frac{1}{2}, 1$ …….

The given GP is of the form, a, ar, $ar^2$, $ar^3$….

Where r is the common ratio.

The first term in the given GP, $a = a_1 = \frac{1}{4}$

The second term in GP, $a_2 = -\frac{1}{2}$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = -\frac{4}{2} = -2$$

Let us consider -128 as the $n^{th}$ term of the GP.

Now, $n^{th}$ term of GP is, $a_n = ar^{n-1}$

$$-128 = \left(\frac{1}{4}\right)(-2)^{n-1}$$

$(-2)^n = 1024 = (-2)^{10}$

n = 10

So, -128 is the $10^{th}$ term in GP.

**Question: 7**

Which term of the

**Solution:**

Given GP is √3, 3, 3√3….

The given GP is of the form, a, ar, $ar^2$, $ar^3$….

Where r is the common ratio.

First term in the given GP, $a_1 = a = \sqrt{3}$

Second term in GP, $a_2 = 3$

Now, the common ratio, $r = \frac{a_2}{a_1}$

$$r = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Let us consider 729 as the $n^{th}$ term of the GP.

Now, $n^{th}$ term of GP is, $a_n = ar^{n-1}$

$729 = \sqrt{3}\,(\sqrt{3})^{n-1}$

$\sqrt{3}^n = \sqrt{3}^{12}$

n = 12

So, 729 is the $12^{th}$ term in GP.

**Question: 8**

Find the geometri

**Solution:**

The $n^{th}$ term of a GP is $a_n = ar^{n-1}$

It’s given in the question that $5^{th}$ term of the GP is 80 and $8^{th}$ term of GP is 640.

So, $a_5 = ar^4 = 80 \rightarrow (1)$

$a_8 = ar^7 = 640 \rightarrow (2)$

$\frac{(2)}{(1)} \rightarrow \frac{ar^7}{ar^4} = r^3 = \frac{640}{80} = 8$

Common ratio, r = 2,

$ar^4 = 80$

16a = 80

a = 5

The required GP is of the form a, ar, $ar^2$, $ar^3$, $ar^4$....

First term of GP, a = 5

Second term of GP, ar = 5 x 2 =10

Third term of GP, $ar^2 = 5 \times 2^2 = 20$

Fourth term of GP, $ar^3 = 5 \times 2^3 = 40$

Fifth term of GP, $ar^4 = 5 \times 2^4 = 80$

And so on...

The required GP is 5, 10, 20, 40, 80...

**Question: 9**

Find the GP whose

**Solution:**

The $n^{th}$ term of a GP is $a_n = ar^{n-1}$

It's given in the question that $4^{th}$ term of the GP is $\frac{1}{18}$ and $7^{th}$ term of GP is $-\frac{1}{486}$.

So, $a_4 = ar^3 = \frac{1}{18} \rightarrow (1)$

$a^7 = ar^6 = -\frac{1}{486} \rightarrow (2)$

$\frac{(2)}{(1)} \rightarrow \frac{ar^6}{ar^3} = r^3 = -\frac{1}{27}$

Common ratio, $r = -\frac{1}{3}$

$$ar^3 = \frac{1}{18}$$

$$a = -\frac{3}{2}$$

The required GP is of form a, ar, $ar^2$, $ar^3$, $ar^4$....

The first term of GP, $a = -\frac{3}{2}$

The second term of GP,$ar = -\frac{3}{2} \text{ x} - \frac{1}{3} = \frac{1}{2}$

The third term of GP, $ar^2 = \frac{1}{2} \text{ x} - \frac{1}{3} = -\frac{1}{6}$

The fourth term of GP, $ar^3 = -\frac{1}{6} \text{ x} - \frac{1}{3} = \frac{1}{18}$

The fifth term of GP, $ar^4 = \frac{1}{18} \text{ x} - \frac{1}{3} = -\frac{1}{54}$

And so on...

The required GP is $-\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, -\frac{1}{54}$ ........

**Question: 10**

The $5^{th}$

**Solution:**

It is given in the question that $5^{th}$, $8^{th}$ and $11^{th}$ terms of GP are a, b and c respectively.

Let us assume the GP is A, AR, $AR^2$, and AR3....

So, the $n^{th}$ term of this GP is $a_n = AR^{n-1}$

Now, $5^{th}$ term, $a_5 = AR^4 = a \rightarrow (1)$

$8^{th}$ term, $a_8 = AR^7 = b \rightarrow (2)$

$11^{th}$ term, $a_{11} = AR^{10} = c \rightarrow (3)$

Dividing equation (3) by (2) and (2) by (1),

$\frac{(3)}{(2)} \rightarrow \frac{AR^{10}}{AR^7} = R^3 = \frac{c}{b} \rightarrow (4)$

$\frac{(2)}{(1)} \rightarrow \frac{AR^7}{AR^4} = R^3 = \frac{b}{a} \rightarrow (5)$

So, both equation (4) and (5) gives the value of $R^3$. So we can equate them.

$\frac{c}{b} = \frac{b}{a} = R^3$ ,

$\therefore b^2 = ac$,

Hence proved.

**Question: 11**

The first term of

**Solution:**

It is given that the first term of GP is -3.

So, a = -3

It is also given that the square of the second term is equal to its $4^{th}$ term.

$\therefore (a_2)^2 = a_4$

$n^{th}$ term of GP, $a_n = ar^{n-1}$

So, $a_2 = ar$; $a_4 = ar^3$

$(ar)^2 = ar^3 \rightarrow a = r = -3$

Now, the $7^{th}$ term in the GP, $a_7 = ar^6$

$a_7 = (-3)^7 = -2187$

Hence, the $7^{th}$ term of GP is -2187.

**Question: 12**

Find the $6^{th}$

**Solution:**

The given GP is 8, 4, 2... $\frac{1}{1024}$ $. \rightarrow (1)$

First term in the GP, $a_1 = a = 8$

Second term in the GP, $a_2 = ar = 4$

The common ratio, $r = \frac{4}{8} = \frac{1}{2}$

The last term in the given GP is $\frac{1}{1024}$.

Second last term in the GP = $a_{n-1} = ar^{n-2}$

Starting from the end, the series forms another GP in the form,

$ar^{n-1}, ar^{n-2}, ar^{n-3}....ar^3, ar^2, ar, a \rightarrow (2)$

Common ratio of this GP is $\frac{1}{r}$.

So, common ratio = 2

$$a = \frac{1}{1024}$$

So, $6^{th}$ term of the GP (2),

$a_6 = ar^5$

$= \frac{1}{1024} \times 2^5 = \frac{1}{32}$

Hence, the $6^{th}$ term from the end of the given GP is $\frac{1}{32}$.

**Question: 13**

Find the $4^{th}$

**Solution:**

The given GP is $\frac{2}{27}, \frac{2}{9}, \frac{2}{3} ....162. \rightarrow (1)$

The first term in the GP, $a_1 = a = \frac{2}{27}$

The second term in the GP, $a_2 = \frac{2}{9}$

The common ratio, $r = 3$

The last term in the given GP is $a_n = 162$.

Second last term in the GP = $a_{n-1} = ar^{n-2}$

Starting from the end, the series forms another GP in the form,

$ar^{n-1}, ar^{n-2}, ar^{n-3}....ar^3, ar^2, ar, a \rightarrow (2)$

Common ratio of this GP is $r' = \frac{1}{r}$.

So, $r' = \frac{1}{3}$.

So, $4^{th}$ term of the GP (2),

$a_4 = ar^3$

$= 162 \times \frac{1}{3^3} = 6$

Hence, the $4^{th}$ term from the end of the given GP is 6.

**Question: 14**

If a, b, c are th

**Solution:**

As per the question, a, b and c are the $p^{th}$, $q^{th}$ and $r^{th}$ term of GP.

Let us assume the required GP as A, AR, $AR^2$, $AR^3$...

Now, the $n^{th}$ term in the GP, $a_n = AR^{n-1}$

$p^{th}$ term, $a_p = AR^{p-1} = a \rightarrow (1)$

$q^{th}$ term, $a_q = AR^{q-1} = b \rightarrow (2)$

$r^{th}$ term, $a_r = AR^{r-1} = c \rightarrow (3)$

$\frac{(1)}{(2)} \rightarrow \frac{R^{p-1}}{R^{q-1}} = R^{p-q} = \frac{a}{b} \rightarrow (i)$

$\frac{(2)}{(3)} \rightarrow \frac{R^{q-1}}{R^{r-1}} = R^{q-r} = \frac{b}{c} \rightarrow (ii)$

$\frac{(3)}{(1)} \rightarrow \frac{R^{r-1}}{R^{p-1}} = R^{r-p} = \frac{c}{a} \rightarrow (iii)$

Taking logarithm on both sides of equation (i), (ii) and (iii).

(p – q) log R = log a – log b,

$\therefore (p-q) = \frac{\log a - \log b}{\log R} \rightarrow (4)$

(q – r) log R = log b – log c

$\therefore (q-r) = \frac{\log b - \log c}{\log R} \rightarrow (5)$

(r – p) log R = log c – log a

$\therefore (r-p) = \frac{\log c - \log a}{\log R} \rightarrow (6)$

Now, multiply equation (4) with log c,

$(p-q)\log c = \left(\frac{\log a - \log b}{\log R}\right)\log c \rightarrow (7)$

Now, multiply equation (5) with log a,

$(q-r)\log a = \left(\frac{\log b - \log c}{\log R}\right)\log a \rightarrow (8)$

Now, multiply equation (6) with log b,

$(r-p)\log b = \left(\frac{\log c - \log a}{\log R}\right)\log b \rightarrow (9)$

Now, add equations (7), (8) and (9).

$$(p-q)\log c + (q-r)\log a + (r-p)\log b = \left(\frac{\log a - \log b}{\log R}\right)\log c + \left(\frac{\log b - \log c}{\log R}\right)\log a + \left(\frac{\log c - \log a}{\log R}\right)\log b$$

On solving the above equation, we will get,

(p – q) log c + (q – r) log a + (r – p) log b = 0

Hence proved.

**Question: 15**

The third term of

**Solution:**

Given that the third term of the GP, $a_3 = 4$

Let us assume the GP mentioned in the question be,

$\frac{A}{R^2}, \frac{A}{R}, A, AR, AR^2 \ldots$

With the first term $\frac{A}{R^2}$ and common ratio R.

Now, the third term in the assumed GP is A.

So, A = 4 (given data)

Now,

Product of the first five terms of GP$=\frac{A}{R^2} \times \frac{A}{R} \times A \times AR \times AR^2 = A^5$

So, the required product $= A^5 = 4^5 = 1024$

∴ The product of first five terms of a GP with its third term 4 is 1024.

**Question: 16**

In a finite GP, p

**Solution:**

We need to prove that the product of the terms equidistant from the beginning and end is the product of first and last terms in a finite GP.

Let us first consider a finite GP.

$A, AR, AR^2 \ldots . AR^{n-1}, AR^n.$

Where n is finite.

Product of first and last terms in the given GP $= A.AR^n$

$= A^2R^n \rightarrow$ (a)

Now, $n^{th}$ term of the GP from the beginning $= AR^{n-1} \rightarrow (1)$

Now, starting from the end,

First term $= AR^n$

Last term = A

$\frac{1}{R} =$ Common Ratio

So, an $n^{th}$ term from the end of GP, $A_n = (AR^n)\left(\frac{1}{R^{n-1}}\right) = AR \rightarrow (2)$

So, the product of $n^{th}$ terms from the beginning and end of the considered GP from (1) and (2) = $(AR^{n-1})(AR)$

$= A^2R^n \rightarrow$ (b)

So, from (a) and (b) its proved that the product of the terms equidistant from the beginning and end is the product of first and last terms in a finite GP.

**Question: 17**

If $\frac{a+bx}{a-bx} = \frac{b+cx}{b-cx}$ (Given data in the question) $\rightarrow (1)$

Cross multiplying (1) and expanding,

$(a + bx)(b - cx) = (b + cx)(a-bx)$

$ab - acx + b^2x - bcx^2 = ba - b^2x + acx - bcx^2$

$2b^2x = 2acx$

$b^2 = ac \rightarrow$ (i)

If three terms are in GP, then the middle term is the Geometric Mean of first term and last term.

$\rightarrow b^2 = ac$

So, from (i) b, is the geometric mean of a and b.

So, a, b, c are in GP.

$\frac{b+cx}{b-cx} = \frac{c+dx}{c-dx}$ (Given data in the question) → (2)

Cross multiplying (2) and expanding,

$(b + cx)(c - dx) = (c + dx)(b - cx)$

$bc - bdx + c^2x - cdx^2 = cb - c^2x + bdx - dcx^2$

$2c^2x = 2bdx$

$c^2 = bd \rightarrow$ (ii)

So, from (ii), c is the geometric mean of b and d.

So, b, c, d is in GP.

∴a, b, c, d are in GP.

**Question: 18**

If a and b are th

**Solution:**

Given data is,

$x^2 - 3x + p = 0 \rightarrow (1)$

a and b are roots of (1)

So, $(x + a)(x + b) = 0$

$x^2 - (a + b)x + ab = 0$

So, $a + b = 3$ and $ab = p \rightarrow (2)$

Given data is,

$x^2 - 12x + q = 0 \rightarrow (3)$

c and d are roots of (1)

So, $(x + c)(x + d) = 0$

$x^2 - (c + d)x + cd = 0$

So, $c + d = 12$ and $cd = q \rightarrow (4)$

a, b, c, d are in GP.(Given data)

Similarly A, AR, $AR^2$, $AR^3$ also forms a GP, with common ratio R.

From (2),

$a + b = 3$

$A + AR = 3$

$\frac{3}{A} = 1 + R \rightarrow (5)$

From (4),

$c + d = 12$

$AR^2 + AR^3 = 12$

$AR^2 (1 + R) = 12 \rightarrow (6)$

Substituting value of $(1 + R)$ in (6).

$R = 2$

Now, substitute value of R in (5) to get value of A,

$A = 1$

Now, the GP required is A, AR, $AR^2$, and $AR^3$

1, 2, 4, 8…is the required GP.

So,

a = 1, b = 2, c = 4, d = 8

From (2) and (4),

ab = p and cd = q

So, p = 2, and q = 32.

$$\frac{q + p}{q - p} = \frac{cd + ab}{cd - ab} = \frac{34}{30} = \frac{17}{15}$$

So, (q + p): (q – p) = 17: 15.

## Exercise : 12C

**Question: 1 A**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r - 1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) 3 ÷ 1 = 3

n = 7 terms

$\therefore S_n = 1\frac{3^7 - 1}{3 - 1}$

$\Rightarrow S_n = \frac{2187 - 1}{3 - 1}$

$\Rightarrow S_n = \frac{2186}{2}$

$\Rightarrow S_n = 1093$

**Question: 1 B**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r - 1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $\sqrt{3} \div 1 = \sqrt{3}$ = 1.732

n = 10 terms

$\therefore S_n = 1.\frac{\sqrt{3}^{10}-1}{\sqrt{3}-1}$

$\Rightarrow S_n = \frac{1.732^{10}-1}{1.732-1}$

$\Rightarrow S_n = \frac{242.929-1}{0.732}$

$\Rightarrow S_n = \frac{241.929}{0.732}$

$\Rightarrow S_n = 330.504$

**Question: 1 C**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. ‘$S_n$’ represents the sum of the G.P. series upto $n^{th}$ terms, ‘a’ represents the first term, ‘r’ represents the common ratio and ‘n’ represents the number of terms.

Here,

a = 0.15

r = (ratio between the n term and n-1 term) 0.015÷ 0.15 = 0.1

n = 6 terms

$\Rightarrow S_n = 0.15 \times \frac{1-0.1^6}{1-0.1}$

$\Rightarrow S_n = 0.15 \times \frac{1-0.000001}{0.9}$

$\Rightarrow S_n = 0.15 \times \frac{0.999999}{0.9}$

$\therefore S_n = 16.67$

**Question: 1 D**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. ‘$S_n$’ represents the sum of the G.P. series upto $n^{th}$ terms, ‘a’ represents the first term, ‘r’ represents the common ratio and ‘n’ represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $-\frac{1}{2} \div 1 = -\frac{1}{2}$

n = 9 terms

$\therefore S_n = 1 \times \frac{1-\frac{-1}{2}^9}{1-(\frac{-1}{2})}$

$\Rightarrow S_n = \frac{1+\frac{1}{512}}{1+\frac{1}{2}}$

$\Rightarrow S_n = \frac{\frac{513}{2}}{\frac{3}{2}}$

$\therefore S_n = 171$

**Question: 1 E**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = $\sqrt{2}$

r = (ratio between the n term and n-1 term) $\frac{1}{\sqrt{2}} \div \sqrt{2} = \frac{1}{2}$

n = 8 terms

$\therefore S_n = \sqrt{2} \times \frac{1-\frac{1}{2}^8}{1-\frac{1}{2}}$

$\Rightarrow S_n = \sqrt{2} \times \frac{1-\frac{1}{256}}{\frac{1}{2}}$

$\Rightarrow S_n = \sqrt{2} \times \frac{\frac{255}{256}}{\frac{1}{2}}$

$\Rightarrow S_n = \sqrt{2} \times \frac{255}{128}$

$\therefore S_n = \frac{255\sqrt{2}}{128}$

**Question: 1 F**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = $\frac{2}{9}$

r = (ratio between the n term and n-1 term) $-\frac{1}{3} \div \frac{2}{9} = -\frac{3}{2} = 1.5$

n = 6 terms

$\therefore S_n = \frac{2}{9} \times \frac{1.5^6 - 1}{1.5-1}$

$\Rightarrow S_n = \frac{2}{9} \times \frac{10.39}{0.5}$

$\therefore S_n = 4.62$

**Question: 2 A**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = $\sqrt{7}$

r = (ratio between the n term and n-1 term) $\sqrt{7} \div \sqrt{21} = \sqrt{3}$

n terms

$\therefore S_n = \sqrt{7} \times \frac{\sqrt{3}^n - 1}{\sqrt{3}-1}$ [ Rationalizing the denominator]

$\Rightarrow S_n = \sqrt{7} \times \frac{\sqrt{3}^n - 1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow S_n = \sqrt{7} \times \frac{\left(\sqrt{3}^n - 1\right)\left(\sqrt{3}+1\right)}{3-1}$

$\therefore S_n = \frac{\sqrt{7}\left(\sqrt{3}^n - 1\right)\left(\sqrt{3}+1\right)}{2}$

**Question: 2 B**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $-\frac{1}{3} \div 1 = -\frac{1}{3}$

n terms

$\therefore S_n = 1 \times \frac{1-\frac{-1}{3}^n}{1-\frac{1}{3}}$

$\Rightarrow S_n = \frac{1-\frac{1}{3}^n}{\frac{2}{3}}$

$\therefore S_n = \frac{3-\frac{1}{3}^{n-1}}{2}$

**Question: 2 C**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r≠1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 1

r = (ratio between the n term and n-1 term) $-a \div 1 = -a$

n terms

$\therefore S_n = 1 \times \frac{(-a)^n - 1}{-a-1}$

[ Multiplying both numerator and denominator by -1 ]

$\Rightarrow S_n = \frac{1-(-a)^n}{1+a}$

**Question: 2 D**

Find the sum of t

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when $r \neq 1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$a = x^3$

r = (ratio between the n term and n-1 term) $x^5 \div x^3 = x^2$

n terms

$\therefore S_n = x^3 \times \frac{x^{2n} - 1}{x^2 - 1}$

$\Rightarrow S_n = \frac{x^3(x^n - 1)(x^n + 1)}{(x-1)(x+1)}$

**Question: 2 E**

Find the sum of t

**Solution:**

The given expression can be written as

$= (x^2 + xy) + (x^4 + x^2y^2) + (x^6 + x^3y^3) + \ldots$ To n terms

$= (x^2 + x^4 + x^6 + \ldots$ to n terms $) + (xy + x^2y^2 + x^3y^3 + \ldots$ to n terms $)$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when $r \neq 1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

a= $x^2$ first part and xy for the second part

r = (ratio between the n term and n-1 term) $x^2$ for the first part and xy for the second part

n terms

$\therefore S_n = x^2 \times \frac{x^{2n} - 1}{x^2 - 1} + xy \times \frac{x^n y^n - 1}{xy - 1}$

$\Rightarrow S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x+1)(x-1)} + \frac{x^{n+1}y^{n+1} - 1}{xy - 1}$

**Question: 3**

Find the sum to n

**Solution:**

This can also be written as

$= \left(x^2 + \frac{1}{x^2} + 2\right) + \left(x^4 + \frac{1}{x^4} + 2\right) + \left(x^6 + \frac{1}{x^6} + 2\right) + \ldots\ldots \text{to n term}$

$= (x^2 + x^4 + x^6 + \ldots.\text{to n terms}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \ldots.\text{to n terms}\right) + (2 + 2 + 2 + \ldots.\text{to n terms})$

$= (x^2 + x^4 + x^6 + \ldots \text{to n terms}) + \left(\frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6} + \ldots \text{to n terms}\right) + 2n$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r≠1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

$a = x^2, \frac{1}{x^2}$

r = (ratio between the n term and n-1 term) $x^2, \frac{1}{x^2}$

n terms

$$\therefore S_n = x^2 \times \frac{x^{2n} - 1}{x^2 - 1} + \frac{1}{x^2} \times \frac{(\frac{1}{x^2})^n - 1}{\frac{1}{x^2} - 1} + 2n$$

$$\Rightarrow S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x-1)(x+1)} + \frac{1}{x^2} \times \frac{\frac{1}{x^2}^n - 1}{\frac{x^2-1}{x^2}} + 2n$$

$$\Rightarrow S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x-1)(x+1)} + \frac{\frac{1}{x^2}^n - 1}{x^2 - 1} + 2n$$

$$\Rightarrow S_n = \frac{x^2(x^n - 1)(x^n + 1)}{(x-1)(x+1)} + \frac{\frac{1}{x^2}^n - 1}{(x-1)(x+1)} + 2n$$

$$\therefore S_n = \frac{x^2(x^n - 1)(x^n + 1) + \frac{1}{x^2}^n - 1}{(x-1)(x+1)} + 2n$$

(ii) If we divide and multiply the terms by (x-y)

$$= \frac{(x-y)(x+y) + (x-y)(x^2 + xy + y^2) + (x-y)(x^3 + x^2y + xy^2 + y^3) + \ldots \text{to n terms}}{(x-y)}$$

$$= \frac{(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4) + \ldots \text{to n terms}}{(x-y)}$$

$$= \frac{(x^2 + x^3 + x^4 + \ldots \text{to n terms}) + (y^2 + y^3 + y^4 + \ldots \text{to n terms})}{(x-y)}$$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r≠1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$a = x^2, y^2$

r = (ratio between the n term and n-1 term) x, y

n terms

$$\therefore S_n = \frac{x^2 \times \frac{x^n - 1}{x-1} + y^2 \times \frac{y^n - 1}{y-1}}{(x-y)}$$

$$\Rightarrow S_n = \frac{\frac{x^2(x^n - 1)}{x-1} + \frac{y^2(y^n - 1)}{y-1}}{(x-y)}$$

**Question: 4**

Find the sum :

**Solution:**

We can split the above expression into 2 parts. We will split 2n terms into 2 parts also which will leave it as n terms and another n terms .

$= \left(\frac{3}{5} + \frac{3}{5^3} + \ldots \text{to n terms}\right) + \left(\frac{4}{5} + \frac{4}{5^2} + \ldots \text{to n terms}\right)$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$a = \frac{3}{5}, \frac{4}{5}$

r = (ratio between the n term and n-1 term) $\frac{3}{5^3} \div \frac{3}{5}, \frac{4}{5^2} \div \frac{4}{5} = \frac{1}{5^2}, \frac{1}{5}$

n terms

$$\therefore S_n = \frac{3}{5} \times \frac{1-\frac{1}{5^2}^n}{1-\frac{1}{5^2}} + \frac{4}{5} \times \frac{1-\frac{1}{5}^n}{1-\frac{1}{5}}$$

$$\Rightarrow S_n = \frac{3}{5} \times \frac{1-\frac{1}{5^2}^n}{\frac{24}{5^2}} + \frac{4}{5} \times \frac{1-\frac{1}{5}^n}{\frac{4}{5}}$$

$$\Rightarrow S_n = \frac{5\left(1-\frac{1}{5^2}^n\right)}{8} + \left(1 - \frac{1}{5}^n\right)$$

$$\Rightarrow S_n = \frac{\left(5-\frac{5}{5^{2n}}\right)}{8} + \left(1 - \frac{1}{5}^n\right)$$

$$\therefore S_n = \frac{\left(5-\frac{1}{5^{2n-1}}\right)}{8} + \left(1 - \frac{1}{5}^n\right)$$

**Question: 5**

Evaluate :

**Solution:**

We can write this as $(2 + 3^1)+(2+3^2) + (2 +3^3)+\ldots$ to 10 terms

$= ( 2+2+2+\ldots$ to 10 terms$) + ( 3+3^2+3^3+\ldots$ to 10 terms$)$

$= 2\times10 + (3+3^2+3^3+\ldots$ to 10 terms$)$

$= 20 + (3+3^2+3^3+\ldots$ to 10 terms$)$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when $r\neq1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 3

r =(ratio between the n term and n-1 term) 3

n = 10 terms

$$S_n = 3 \times \frac{3^{10} - 1}{3 - 1}$$

$$\Rightarrow S_n = 3 \times \frac{59049-1}{2}$$

$$\Rightarrow S_n = 3 \times \frac{59048}{2}$$

$$\Rightarrow S_n = 88572$$

Thus, sum of the given expression is

$= 20 + (3+3^2+3^3+\ldots$ to 10 terms$)$

$= 20 + 88572$

=88592

(ii) The given expression can be written as,

( $2^1$ + $3^{1-1}$) + ($2^2$ + $3^{2-1}$) + ...to n terms

= (2 + $3^0$) + ( $2^2$+ $3^1$) + ...to n terms

= (2 + 1) +($2^2$ + 3 ) + ...to n terms

= (2 + $2^2$ + ...to $\frac{n}{2}$ terms) + ( 1 + 3 + ... to $\frac{n}{2}$ terms)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r≠1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 2, 1

r = (ratio between the n term and n-1 term)2, 3

$\frac{n}{2}$ terms

$$S_n = 2 \times \frac{2^{\frac{n}{2}} - 1}{2-1} + 1 \times \frac{3^{\frac{n}{2}} - 1}{3-1}$$

$$\Rightarrow S_n = 2 \times \frac{2^{\frac{n}{2}} - 1}{1} + 1 \times \frac{3^{\frac{n}{2}} - 1}{2}$$

$$\Rightarrow S_n = 2^{\frac{n}{2}+1} - 2 + \frac{3^{\frac{n}{2}} - 1}{2}$$

(iii) We can rewrite the given expression as

( $5^1$ + $5^2$ + $5^3$+ ...to 8 terms)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 5

r =(ratio between the n term and n-1 term) 5

n = 8 terms

$$S_n = 5 \times \frac{5^8 - 1}{5-1}$$

$$\Rightarrow S_n = 5 \times \frac{390625-1}{4}$$

$$\Rightarrow S_n = 5 \times \frac{390624}{4}$$

$$\Rightarrow S_n = 488280$$

**Question: 6**

Find the sum of t

**Solution:**

The expression can be rewritten as

[ Taking 8 as a common factor ]

8( 1+ 11 + 111+ ... to n terms)

[Multiplying and dividing the expression by 9]

$= \frac{8}{9}$ ( 9 + 99+ 999 + … to n terms)

$= \frac{8}{9}$ ( (10-1) + (100-1) + (1000-1) + … to n terms )

$= \frac{8}{9}$ ( ( 10 + 100 + 1000 + … to n terms) – ( 1+1+1+ … to n terms)

$= \frac{8}{9}$ ( ( 10 + 100 + 1000 + … to n terms) – n)

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 10

r = (ratio between the n term and n-1 term) 10

n terms

$$S_n = 10 \times \frac{10^n - 1}{10 - 1}$$

$$\Rightarrow S_n = 10 \times \frac{10^n-1}{9}$$

$$\Rightarrow S_n = \frac{10^{n+1}-10}{9}$$

∴ The sum of the given expression is

$= \frac{8}{9}$ ( ( 10 + 100 + 1000 + … to n terms) – n)

$= \frac{8}{9}\left(\frac{10^{n+1}-10}{9} - n\right)$

(ii) The given expression can be rewritten as

[ taking 3 common ]

= 3( 1+11+111+ …to n terms)

[ multiplying and dividing the expression by 9 ]

$= \frac{3}{9}$ ( 9+99+999+ … to n terms )

$= \frac{3}{9}$ ( (10-1) + (100-1) + (1000-1) + … to n terms )

$= \frac{3}{9}$ ( ( 10+100+1000+ …to n terms ) – (1+1+1+ … to n terms) )

$= \frac{3}{9}$ ( (10+100+1000+ to n terms) – n )

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 10

r = (ratio between the n term and n-1 term) 10

n terms

$$S_n = 10 \times \frac{10^n - 1}{10 - 1}$$

$\Rightarrow S_n = 10 \times \frac{10^n - 1}{9}$

$\Rightarrow S_n = \frac{10^{n+1} - 10}{9}$

∴ The sum of the given expression is

$= \frac{3}{9}$ ( (10+100+1000+ to n terms) – n )

$= \frac{3}{9} \left( \frac{10^{n+1} - 10}{9} - n \right)$

(iii) We can rewrite the expression as

[ taking 7 as a common factor]

= 7(0.1+0.11+0.111+ … to n terms)

[ multiplying and dividing by 9 ]

$= \frac{7}{9}$ ( 0.9+0.99+0.999+ … to n terms )

$= \frac{7}{9}$ ( (1-0.1)+(1-0.01)+(1-0.001)+ … to n terms)

$= \frac{7}{9}$ ( (1+1+1+ … to n terms )–(0.1+0.01+0.001+… to n terms ))

$= \frac{7}{9}$ ( n – (0.1+0.01+0.001+ … to n terms ) )

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when |r|<1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 0.1

r = (ratio between the n term and n-1 term) 0.1

n terms

$$S_n = 0.1 \times \frac{1 - 0.1^n}{1 - 0.1}$$

$\Rightarrow S_n = 0.1 \times \frac{1-0.1^n}{0.9}$

[multiplying both numerator and denominator by 10]

$\Rightarrow S_n = \frac{1-0.1^n}{9}$

∴ The sum of the given expression is

$= \frac{7}{9}$ ( n – (0.1+0.01+0.001+ … to n terms ) )

$= \frac{7}{9} \left( n - \left( \frac{1-0.1^n}{9} \right) \right)$

**Question: 7**

The sum of n term

**Solution:**

In this question, we will try to rewrite the given sum of the progression like the formula for the sum a G.P. series.

It is given that $S_n = ( 2^n – 1)$

The formula for the sum of a G.P. series is,

$S_n = a\frac{r^n - 1}{r - 1}$

By solving the 2 equations together, we can say that

$(2^n - 1) = a\frac{r^n - 1}{r - 1}$

$\Rightarrow 1 \times \frac{(2^n - 1)}{2-1} = a\frac{r^n - 1}{r-1}$

By corresponding the numbers with the variables, we can conclude

a = 1

r = 2

The G.P. series will therefore look like ⇒ 1,2,4,8,16,......to n terms

∴ The given progression is a G.P. series with the common ration being 2.

**Question: 8**

In a GP, the rati

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when $r \neq 1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Sum of first 3 terms = $a \times \frac{r^3 - 1}{r-1}$

Sum of first 6 terms = $a \times \frac{r^6 - 1}{r-1}$

$\therefore \frac{a \times \frac{r^3-1}{r-1}}{a \times \frac{r^6-1}{r-1}} = \frac{125}{152}$

$\Rightarrow \frac{(r^3-1)}{(r^6-1)} = \frac{125}{152}$

$\Rightarrow 152r^3 - 152 = 125r^6 - 125$

$\Rightarrow 125r^6 - 152r^3 - 125 + 152 = 0$

$\Rightarrow 125r^6 - 152r^3 + 27 = 0$

$\Rightarrow 125r^6 - 125r^3 - 27r^3 + 27 = 0$

$\Rightarrow (125r^3 - 27)(r^3 - 1) = 0$

Either $125r^3 - 27 = 0$ or $r^3 - 1 = 0$

Either $125r^3 = 27$ or $r^3 = 1$

Either $r^3 = \frac{27}{125}$ or r=1

Either $r = \frac{3}{5}$ or r=1

Since $r \neq 1$ [ if r is 1, all the terms will be equal which destroys the purpose ]

$\therefore r = \frac{3}{5}$

**Question: 9**

Find the sum of t

**Solution:**

Tn represents the $n^{th}$ term of a G.P. series.

$r = 6 \div 3 = 2$

$T_n = ar^{n-1}$

$\Rightarrow 1536 = 3 \times 2^{n-1}$

$\Rightarrow 1536 \div 3 = 2^n \div 2$

$\Rightarrow 1536 \div 3 \times 2 = 2^n$

$\Rightarrow 1024 = 2^n$

$\Rightarrow 2^{10} = 2^n$

$\therefore$ n = 10

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 3

r = 2

n = 10 terms

$\therefore S_n = 3 \times \frac{2^{10}-1}{2-1}$

$\Rightarrow S_n = 3 \times (1024 - 1)$

$\Rightarrow S_n = 3 \times 1023$

$\therefore S_n = 3069$

**Question: 10**

How many terms of

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when r>1. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 2

r = (ratio between the n term and n-1 term) 6 ÷ 2 = 3

$S_n = 728$

$\therefore 728 = 2 \times \frac{3^n - 1}{3-1}$

$\Rightarrow 728 = 2 \times \frac{3^n - 1}{2}$

$\Rightarrow 728 = 3^n - 1$

$\Rightarrow 728 + 1 = 3^n$

$\Rightarrow 729 = 3^n$

$\Rightarrow 3^6 = 3^n$

$\therefore n = 6$

$\therefore$ 6 terms must be taken to reach the desired answer.

**Question: 11**

The common ratio

**Solution:**

'$T_n$' represents the $n^{th}$ term of a G.P. series.

$T_n = ar^{n-1}$

$\Rightarrow 486 = a(3)^{n-1}$

$\Rightarrow 486 = a(\ 3^n \div 3)\ )$

$\Rightarrow 486 \times 3 = a(3^n)$

$\Rightarrow 1458 = a(3^n\ )$ .........(i)

Sum of a G.P. series is represented by the formula, $Sn = a\frac{r^n-1}{r-1}$, when $r \neq 1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

$\therefore 728 = a \times \frac{3^n-1}{3-1}$

$\Rightarrow 728 = a \times \frac{3^n-1}{2}$

$\Rightarrow 728 \times 2 = a\ (3^n)$-a ...... [ Putting $a(3^n\ ) = 1458$ fromk (i) ]

$\Rightarrow 1456 = 1458$ -a

$\Rightarrow 1456-1458 = -a$

$\Rightarrow -2=-a$ [ Multipying both sides by -1]

$\Rightarrow a = 2$

**Question: 12**

The first term of

**Solution:**

'$T_n$' represents the $n^{th}$ term of a G.P. series.

$T_n = ar^{n-1}$

$\Rightarrow \frac{1}{81} = 27 \times r^{8-1}$

$\Rightarrow \frac{1}{81} = 27 \times r^7$

$\Rightarrow \frac{1}{81} \div \frac{1}{27} = r^7$

$\Rightarrow \frac{1}{2187} = r^7$

$\Rightarrow \left(\frac{1}{3}\right)^7 = r^7$

$\therefore r = \frac{1}{3}$

Sum of a G.P. series is represented by the formula, $Sn = a\frac{1-r^n}{1-r}$, when $|r|<1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

a = 27

r = (ratio between the n term and n-1 term) $\frac{1}{3}$

n = 10 terms

$\therefore S_n = 27 \times \frac{1-\frac{1}{3}^{10}}{1-\frac{1}{3}}$

$\Rightarrow S_n = 27 \times \frac{1-\frac{1}{59049}}{\frac{2}{3}}$

$\Rightarrow S_n = 27 \times \frac{\frac{59048}{59049}}{\frac{2}{3}}$

$\Rightarrow S_n = 27 \times \frac{39524}{19683}$

$\therefore S_n = \frac{39524}{729}$

**Question: 13**

The $2^{nd}$

**Solution:**

$2^{nd}$ term = $ar^{2-1} = ar^1$

$5^{th}$ term = $ar^{5-1} = ar^4$

Dividing the $5^{th}$ term using the $3^{rd}$ term

$\frac{ar^4}{ar} = \frac{\frac{1}{16}}{\frac{-1}{2}}$

$r^3 = -\frac{1}{8}$

$\therefore r = \frac{-1}{2}$

$\therefore a = 1$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when $|r|<1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

n = 8 terms

$S_n = 1 \times \frac{1-\frac{-1}{2}^8}{1-\frac{-1}{2}}$

$\Rightarrow S_n = \frac{1-\frac{1}{256}}{\frac{3}{2}}$

$\Rightarrow S_n = \frac{\frac{255}{256}}{\frac{3}{2}}$

$\therefore S_n = \frac{170}{256}$

**Question: 14**

The $4^{th}$

**Solution:**

$4^{th}$ term = $ar^{4-1} = ar^3 = \frac{1}{27}$

$7^{th}$ term = $ar^{7-1} = ar^6 = \frac{1}{729}$

Dividing the $7^{th}$ term by the $4^{th}$ term,

$$\frac{ar^6}{ar^3} = \frac{\frac{1}{729}}{\frac{1}{27}}$$

$\Rightarrow r^3 = \frac{1}{27}$ ......(i)

$\therefore r = \frac{1}{3}$

$ar^3 = \frac{1}{27}$ [putting from eqn (i) ]

$a\frac{1}{27} = \frac{1}{27}$

$\therefore a = 1$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{1-r^n}{1-r}$, when $|r|<1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Here,

$a = 1$

$r = \frac{1}{3}$

n terms

$\therefore S_n = 1 \times \frac{1-\frac{1}{3}^n}{1-\frac{1}{3}}$

$\Rightarrow S_n = \frac{1-\frac{1}{3}^n}{\frac{2}{3}}$

$\Rightarrow S_n = \frac{3\left(1-\frac{1}{3^n}\right)}{2}$

$\therefore S_n = \frac{3-\frac{1}{3^{n-1}}}{2}$

**Question: 15**

A GP consists of

**Solution:**

Let the terms of the G.P. be a, ar, $ar^2$, $ar^3$, ... , $ar^{n-2}$, $ar^{n-1}$

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n-1}{r-1}$, when $r\neq1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Thus, the sum of this G.P. series is $S_n = a\frac{r^n-1}{r-1}$

The odd terms of this series are a, $ar^2$, $ar^4$, ... , $ar^{n-2}$

{since the number of terms of the G.P. series is even; the $2^{nd}$ last term will be an odd term.}

Here,

No. of terms will be $\frac{n}{2}$ as we are splitting up the n terms into 2 equal parts of odd and even terms.
{ since the no. of terms is even, we have 2 equal groups of odd and even terms }

Sum of the odd terms $\Rightarrow$

$$S_n = a \times \frac{r^{2(\frac{n}{2})} - 1}{r^2 - 1}$$

$$\Rightarrow S_n = a \times \frac{r^n - 1}{(r-1)(r+1)}$$

By the problem,

$$a\frac{r^n - 1}{r - 1} = 5 \times a \times \frac{r^n - 1}{(r-1)(r+1)}$$

$$\Rightarrow 1 = \frac{5}{(r+1)}$$

$\Rightarrow r + 1 = 5$

$\Rightarrow \therefore r = 4$

Thus, the common ratio (r) = 4

**Question: 16**

Show that the rat

**Solution:**

Sum of a G.P. series is represented by the formula, $S_n = a\frac{r^n - 1}{r-1}$, when $r \neq 1$. '$S_n$' represents the sum of the G.P. series upto $n^{th}$ terms, 'a' represents the first term, 'r' represents the common ratio and 'n' represents the number of terms.

Thus, the sum of the first n terms of the G.P. series is, $S_n = a\frac{r^n - 1}{r-1}$

Sum of $(n+1)^{th}$ term to $2n^{th}$ term

= Sum of the first $2n^{th}$ term – the sum of $1^{st}$ term to $n^{th}$ term

$$= a\frac{r^{2n}-1}{r-1} - a\frac{r^n - 1}{r-1}$$

$$= \frac{(ar^{2n} - a) - (ar^n - a)}{r-1}$$

$$= \frac{ar^{2n} - a - ar^n + a}{r-1}$$

$$= \frac{ar^n(r^n - 1)}{r-1}$$

The ratio of the sum of first n terms of the G.P. to the sum of the terms from $(n + 1)^{th}$ to $(2n)^{th}$ term

$$= \frac{a\frac{r^n - 1}{r-1}}{\frac{ar^n(r^n - 1)}{r-1}}$$

[Cancelling out the common factors from the numerator and denominator ⇒ a, (r-1), ($r^n$ – 1) ]

$$= \frac{1}{r^n}$$

Hence Proved.

# Exercise : 12D

**Question: 1**

What will 15625 a

**Solution:**

To find: The amount after three years

Given: (i) Principal – 15625

(ii) Time – 3 years

(iii) Rate – 8% per annum

Formula used: $A = P\left(1+ \frac{r}{100}\right)^t$

$$A = 15625\left(1+ \frac{8}{100}\right)^3$$

$$A=15625\left(\frac{108}{100}\right)^3$$

A = 19683

Ans) 19683

**Question: 2**

The value of a ma

**Solution:**

To find: The amount after three days

Given: (i) Principal – 80000

(ii) Time – 3 days

(iii) Rate – 15% per annum

Deduction = P × R × T

$$= 80000 \times \frac{15}{100} \times \frac{3}{365}$$

= 98.63

The final amount after deduction = 80000 – 98.63

= 79901.37

The value of the machine after 3 days is Rs. 79901.37

**Question: 3**

Three years befor

**Solution:**

To find: Present population of the village

Given: (i) Three years back population - 10000

(ii) Time – 3 years

(iii) Rate – 20% per annum

Number of people migrated on the very first year is 20% of 10000

$\Rightarrow \frac{10000\times 20}{100} = 2000$

People left after migration in the very first year = 10000 – 2000

= 8000

Number of people migrated in the second year is 20% of 8000

$\Rightarrow \frac{8000\times 20}{100} = 1600$

People left after migration in the second year = 8000 – 1600

= 6400

Number of people migrated in the third year is 20% of 6400

$\Rightarrow \frac{6400\times20}{100}=1280$

People left after migration in the third year = 6400 – 1280

= 5120

Ans) The present population is 5120

**Question: 4**

What will 5000 am

**Solution:**

To find: The amount after ten years

Given: (i) Principal – 5000

(ii) Time – 10 years

(iii) Rate – 10% per annum

Formula used: $A=P\left(1+\frac{r}{100}\right)^t$

$\Rightarrow A=5000\left(1+\frac{10}{100}\right)^{10}$

$\Rightarrow A=5000\left(\frac{110}{100}\right)^{10}$

$\Rightarrow A=5000(1.1)^{10}$

$\Rightarrow A=5000\times2.594$

$\Rightarrow A=12970$

Ans) The amount after years will be Rs.12970

**Question: 5**

A manufacturer re

**Solution:**

To find: The amount after five years

Given: (i) Principal – 156250

(ii) Time – 5 years

(iii) Rate – 20% per annum

Formula used: $A=P\left(1-\frac{r}{100}\right)^t$

$\Rightarrow A=156250\left(1-\frac{20}{100}\right)^5$

$\Rightarrow A=156250\left(\frac{80}{100}\right)^5$

$\Rightarrow A=156250(0.8)^5$

$\Rightarrow A=156250\times0.32768$

$\Rightarrow A=51200$

Ans) The amount after five years will be Rs.51200

**Question: 6**

The number of bac

**Solution:**

To find: The number of bacteria after

(i) $2^{nd}$ hour

(ii) $5^{th}$ hour

(iii) nth hour

Given: (i) Initially, there were 50 bacteria

(ii) Rate – 100% per hour

The formula used: $A=P\left(1+\frac{r}{100}\right)^t$

(i) For $2^{nd}$ hour

$\Rightarrow$ No. of bacteria $=50\left(1+\frac{100}{100}\right)^2$

$\Rightarrow$ No. of bacteria $=50(1+1)^2$

$\Rightarrow$ No. of bacteria $=50(2)^2$

$\Rightarrow$ No. of bacteria $=50\times4$

$\Rightarrow$ No. of bacteria $=200$

(ii) For $5^{th}$ hour

$\Rightarrow$ No. of bacteria $=50\left(1+\frac{100}{100}\right)^5$

$\Rightarrow$ No. of bacteria $=50(1+1)^5$

$\Rightarrow$ No. of bacteria $=50(2)^5$

$\Rightarrow$ No. of bacteria $=50\times32$

$\Rightarrow$ No. of bacteria $=1600$

(iii) For $n^{th}$ hour

$\Rightarrow$ No. of bacteria $=50\left(1+\frac{100}{100}\right)^n$

$\Rightarrow$ No. of bacteria $=50(1+1)^n$

$\Rightarrow$ No. of bacteria $=50(2)^n$

$\Rightarrow$ No. of bacteria $=2^n50$

Ans) Number of bacteria in a $2^{nd}$ hour will be 200, the number of bacteria in a $5^{th}$ hour will be 1600 and number of bacteria in an $n^{th}$ hour will be $2^n50$

## Exercise : 12E

**Question: 1**

If p, q, r are in

**Solution:**

To prove: $p^{th}$, $q^{th}$ and $r^{th}$ terms of any GP are in GP.

Given: (i) p, q and r are in AP

The formula used: (i) General term of GP, $T_n = ar^{n-1}$

As p, q, r are in A.P.

⇒ q – p = r – q = d = common difference … (i)

Consider a G.P. with the first term as a and common difference R

Then, the $p^{th}$ term will be $ar^{p-1}$

the $q^{th}$ term will be $ar^{q-1}$

the $r^{th}$ term will be $ar^{r-1}$

Considering $p^{th}$ term and $q^{th}$ term

$$\Rightarrow \frac{q^{th} \text{ term}}{p^{th} \text{ term}} = \frac{ar^{q-1}}{ar^{p-1}}$$

$$\Rightarrow \frac{q^{th} \text{ term}}{p^{th} \text{ term}} = r^{q-1-p+1}$$

$$\Rightarrow \frac{q^{th} \text{ term}}{p^{th} \text{ term}} = r^{q-p}$$

From eqn. (i) q – p = d

$$\Rightarrow \frac{q^{th} \text{ term}}{p^{th} \text{ term}} = r^{d}$$

Considering $q^{th}$ term and $r^{th}$ term

$$\Rightarrow \frac{r^{th} \text{ term}}{q^{th} \text{ term}} = \frac{ar^{r-1}}{ar^{q-1}}$$

$$\Rightarrow \frac{r^{th} \text{ term}}{q^{th} \text{ term}} = r^{r-1-q+1}$$

$$\Rightarrow \frac{r^{th} \text{ term}}{q^{th} \text{ term}} = r^{r-q}$$

From eqn. (i) r – q = d

$$\Rightarrow \frac{r^{th} \text{ term}}{q^{th} \text{ term}} = r^{d}$$

We can see that $p^{th}$, $q^{th}$ and $r^{th}$ terms have common ration i.e $r^d$

Hence they are in G.P.

Hence Proved

**Question: 2**

If a, b, c are in

**Solution:**

To prove: log $a^n$, log $b^n$, log $c^n$ are in AP.

Given: a, b, c are in GP

Formula used: (i) log ab = log a + log b

As a, b, c are in GP

⇒ $b^2$ = ac

Taking power n on both sides

$\Rightarrow b^{2n} = (ac)^n$

Taking log both side

$\Rightarrow \log b^{2n} = \log(ac)^n$

$\Rightarrow \log b^{2n} = \log(a^n c^n)$

$\Rightarrow 2\log b^n = \log(a^n) + \log(c^n)$

Whenever a,b,c are in AP then 2b = a+c, considering this and the above equation we can say that $\log a^n$, $\log b^n$, $\log c^n$ are in AP.

Hence Proved

**Question: 3**

If a, b, c are GP

**Solution:**

To prove: $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in AP.

Given: a, b, c are in GP

Formula used: (i) $\frac{1}{\log_a m} = \log_m a = \frac{\log a}{\log m}$

As, a, b, c are in GP

$\Rightarrow \frac{b}{a} = \frac{c}{b}$

Taking log both side $\log\frac{b}{a} = \log\frac{c}{b}$

$\Rightarrow \log b - \log a = \log c - \log b$

$\Rightarrow 2\log b = \log a + \log c$

Dividing by log m

$\Rightarrow 2\left(\frac{\log b}{\log m}\right) = \frac{\log a}{\log m} + \frac{\log c}{\log m}$

$\Rightarrow 2\log_m b = \log_m a + \log_m c \quad \left(\text{As, } \log_m a = \frac{\log a}{\log m}\right)$

$\Rightarrow 2\left(\frac{1}{\log_b m}\right) = \frac{1}{\log_a m} + \frac{1}{\log_c m} \quad \left(\text{As } \frac{1}{\log_a m} = \log_m a\right)$

Whenever any number a,b,c are in AP then 2b = a+c, considering this and the above equation we can say that $\frac{1}{\log_a m}, \frac{1}{\log_b m}, \frac{1}{\log_c m}$ are in AP

Hence proved

**Question: 4**

Find the values o

**Solution:**

To find: Value of k

Given: k + 12, k – 6 and 3 are in GP

Formula used: (i) when a,b,c are in GP $b^2 = ac$

As, k + 12, k – 6 and 3 are in GP

$\Rightarrow (k - 6)^2 = (k + 12)(3)$

$\Rightarrow k^2 - 12k + 36 = 3k + 36$

⇒ $k^2 - 15k = 0$

⇒ $k (k - 15) = 0$

⇒ $k = 0$ , Or $k = 15$

Ans) We have two values of k as 0 or 15

**Question: 5**

Three numbers are

**Solution:**

To find: The numbers

Given: Three numbers are in A.P. Their sum is 15

Formula used: When a,b,c are in GP, $b^2 = ac$

Let the numbers be a - d, a, a + d

According to first condition

$a + d + a + a - d = 15$

⇒ $3a = 15$

⇒ $a = 5$

Hence numbers are 5 - d, 5, 5 + d

When 1, 4, 19 be added to them respectively then the numbers become –

$5 - d + 1, 5 + 4, 5 + d + 19$

⇒ $6 - d, 9, 24 + d$

The above numbers are in GP

Therefore, $9^2 = (6 - d) (24 + d)$

⇒ $81 = 144 - 24d + 6d - d^2$

⇒ $81 = 144 - 18d - d^2$

⇒ $d^2 + 18d - 63 = 0$

⇒ $d^2 + 21d - 3d - 63 = 0$

⇒ $d (d + 21) -3 (d + 21) = 0$

⇒ $(d - 3) (d + 21) = 0$

⇒ $d = 3$, Or $d = -21$

Taking d = 3, the numbers are

$5 - d, 5, 5 + d = 5 - 3, 5, 5 + 3$

$= 2, 5, 8$

Taking d = -21, the numbers are

$5 - d, 5, 5 + d = 5 - (-21), 5, 5 + (-21)$

$= 26, 5, -16$

Ans) We have two sets of triplet as 2, 5, 8 and 26, 5, -16.

**Question: 6**

Three numbers are

**Solution:**

To find: Three numbers

Given: Three numbers are in A.P. Their sum is 21

Formula used: When a,b,c are in GP, $b^2 = ac$

Let the numbers be a - d, a, a + d

According to first condition

a + d + a +a – d = 21

⇒ 3a = 21

⇒ a = 7

Hence numbers are 7 - d, 7, 7 + d

When second number is reduced by 1 and third is increased by 1 then the numbers become –

7 – d , 7 – 1 , 7 + d + 1

⇒ 7 – d , 6 , 8 + d

The above numbers are in GP

Therefore, $6^2 = (7 – d) (8 + d)$

⇒ $36 = 56 + 7d – 8d – d^2$

⇒ $d^2 + d – 20 = 0$

⇒ $d^2 + 5d – 4d – 20 = 0$

⇒ d (d + 5) – 4 (d + 5) = 0

⇒ (d – 4) (d + 5) = 0

⇒ d = 4, Or d = -5

Taking d = 4, the numbers are

7 - d, 7, 7 + d = 7 - 4, 7, 7 + 4

= 3, 7, 11

Taking d = -5, the numbers are

7 - d, 7, 7 + d = 7 – (-5), 7, 7 + (-5)

= 12, 7, 2

Ans) We have two sets of triplet as 3, 7, 11 and 12, 7, 2.

**Question: 7**

The sum of three

**Solution:**

To find: Three numbers

Given: Three numbers are in G.P. Their sum is 56

Formula used: When a,b,c are in GP, $b^2 = ac$

Let the three numbers in GP be a, ar, $ar^2$

According to condition :-

$a + ar + ar^2 = 56$

$a(1 + r + r^2) = 56$ … (i)

1, 7, 21 be subtracted from them respectively, we obtain the numbers as :-

$a - 1, ar - 7, ar^2 - 21$

According to question the above numbers are in AP

$\Rightarrow ar - 7 - (a - 1) = ar^2 - 21 - (ar - 7)$

$\Rightarrow ar - 7 - a + 1 = ar^2 - 21 - ar + 7$

$\Rightarrow ar - a - 6 = ar^2 - ar - 14$

$\Rightarrow 8 = ar^2 - 2ar + a$

$\Rightarrow 8 = a(r^2 - 2r + 1)$

Multiplying the above eqn. with 7

$\Rightarrow 56 = 7a(r^2 - 2r + 1)$

$\Rightarrow a(1 + r + r^2) = 7a(r^2 - 2r + 1)$

$\Rightarrow 1 + r + r^2 = 7r^2 - 14r + 7$

$\Rightarrow 6r^2 - 15r + 6 = 0$

$\Rightarrow 6r^2 - 12r - 3r + 6 = 0$

$\Rightarrow 6r(r - 2) -3 (r - 2) = 0$

$\Rightarrow (6r - 3) (r - 2) = 0$

$\Rightarrow r = \frac{3}{6} = \frac{1}{2}$ Or $r = 2$

Putting $r = \frac{1}{2}$ in eqn. (i)

$a(1 + r + r^2) = 56$

$$a\left(1+\frac{1}{2}+ \frac{1}{2^2}\right)=56$$

$$a\left(\frac{4+2+1}{4}\right)=56$$

$$a\left(\frac{7}{4}\right)=56$$

$a = 32$

The numbers are $a, ar, ar^2$

$\Rightarrow 32, 32\times\frac{1}{2}, 32\times\frac{1}{2^2}$

$\Rightarrow 32, 16, 8$

Putting $r = 2$ in eqn. (i)

$a(1 + r + r^2) = 56$

$$a\left(1+2 + 2^2\right)=56$$

$$a(1+2+4)=56$$

$$a(7)=56$$

$a = 8$

The numbers are $a, ar, ar^2$

$\Rightarrow 8, 8\times2, 8\times2^2$

⇒ 8, 16, 32

Ans) We have two sets of triplet as 32, 16, 8 and 8, 16, 32.

**Question: 8**

If a, b, c are in

**Solution:**

To prove: $\frac{a^2+ab+b^2}{ab+bc+ca} = \frac{b+a}{c+b}$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

a, b, c are in GP,

⇒ $b^2 = ac$ ... (i)

⇒ $b = \sqrt{ac}$ ... (ii)

Taking LHS = $\frac{a^2+ab+b^2}{ab+bc+ca}$

Substituting the value $b^2 = ac$ from eqn. (i)

LHS = $\frac{a^2+ab+ac}{ab+bc+b^2}$

⇒ $\frac{a(a+b+c)}{b(a+b+c)}$

⇒ $\frac{a}{b}$

Substituting the value $b = \sqrt{ac}$ from eqn. (ii)

⇒ $\frac{a}{\sqrt{ac}}$

⇒ $\frac{\sqrt{a}}{\sqrt{c}}$

Multiplying and dividing with $\left(\sqrt{a}+\sqrt{c}\right)$

⇒ $\frac{\sqrt{a}\left(\sqrt{a}+\sqrt{c}\right)}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\right)}$

⇒ $\frac{(a+\sqrt{ac})}{(\sqrt{ac}+c)}$

⇒ $\frac{a+b}{b+c}$ = RHS

Hence Proved

**Question: 9**

If (a – b), (b –

**Solution:**

To prove: $(a + b + c)^2 = 3(ab + bc + ca)$.

Given: (a – b), (b – c), (c – a) are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

As, (a – b), (b – c), (c – a) are in GP

$\Rightarrow (b - c)^2 = (a - b)(c - a)$

$\Rightarrow b^2 - 2cb + c^2 = ac - a^2 - bc + ab$

$\Rightarrow a^2 + b^2 + c^2 - bc - ac - ab = 0$

Adding 3(ab + bc + ac) both side

$\Rightarrow a^2 + b^2 + c^2 - bc - ac - ab + 3(ab + bc + ac) = 3(ab + bc + ac)$

$\Rightarrow a^2 + b^2 + c^2 + 2bc + 2ac + 2ab = 3(ab + bc + ac)$

$\Rightarrow (a + b + c)^2 = 3(ab + bc + ac)$

Hence Proved

**Question: 10**

If a, b, c are in

**Solution:**

(i) $a(b^2 + c^2) = c(a^2 + b^2)$

To prove: $a(b^2 + c^2) = c(a^2 + b^2)$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

When a,b,c are in GP, $b^2 = ac$

Taking LHS = $a(b^2 + c^2)$

$= a(ac + c^2)$ $[b^2 = ac]$

$= (a^2c + ac^2)$

$= c(a^2 + ac)$

$= c(a^2 + b^2)$ $[b^2 = ac]$

= RHS

Hence Proved

(ii) $\frac{1}{\left(a^2-b^2\right)} + \frac{1}{b^2} = \frac{1}{\left(b^2-c^2\right)}$

To prove: $a(b^2 + c^2) = c(a^2 + b^2)$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c are in GP, $b^2 = ac$

Taking LHS = $\frac{1}{\left(a^2-b^2\right)} + \frac{1}{b^2}$

$\Rightarrow \frac{b^2+a^2-b^2}{\left(a^2-b^2\right)\left(b^2\right)}$

$\Rightarrow \frac{a^2}{\left(a^2-b^2\right)(ac)}$

$\Rightarrow \frac{a^2}{\left(a^3c-a^2c^2\right)}$

$\Rightarrow \frac{a^2}{a^2(ac-c^2)}$

$\Rightarrow \frac{1}{(b^2-c^2)}$ [$b^2$ = ac]

Hence Proved

(iii) (a + 2b + 2c)(a – 2b + 2c) = $a^2 + 4c^2$

To prove: (a + 2b + 2c)(a – 2b + 2c) = $a^2 + 4c^2$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2$ = ac

Proof:When a,b,c are in GP, $b^2$ = ac

Taking LHS = (a + 2b + 2c)(a – 2b + 2c)

⇒ [(a + 2c) + 2b] [(a + 2c) – 2b]

⇒ [$(a + 2c)^2 - (2b)^2$] [(a + b) (a – b) = $a^2 - b^2$]

⇒ [$(a^2 + 4ac + 4c^2) - 4b^2$]

⇒ [$(a^2 + 4ac + 4c^2) - 4b^2$] [ $b^2$ = ac]

⇒ [$(a^2 + 4ac + 4c^2 - 4ac$]

⇒ $a^2 + 4c^2$ = RHS

Hence Proved

(iv)$a^2b^2c^2 \left(\frac{1}{a^3}+ \frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

To prove: $a^2b^2c^2 \left(\frac{1}{a^3}+ \frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2$ = ac

Proof: When a,b,c are in GP, $b^2$ = ac

Taking LHS = $a^2b^2c^2 \left(\frac{b^3c^3+a^3c^3+a^3b^3}{a^3b^3c^3}\right)$

$\Rightarrow \left(\frac{b^3c^3+a^3c^3+a^3b^3}{abc}\right)$

$\Rightarrow \left(\frac{b^2bc^3+(ac)^2ac+a^3b^2b}{abc}\right)$

$\Rightarrow \left(\frac{acbc^3+(b^2)^2ac+a^3acb}{abc}\right)$ [$b^2$ = ac]

$\Rightarrow \left(\frac{acbc^3+b^3abc+a^3acb}{abc}\right)$

$\Rightarrow \left(a^3+b^3+c^3\right)$ = RHS

Hence Proved

**Question: 11**

If a, b, c, d are

**Solution:**

(i) (b + c)(b + d) = (c + a)(c + a)

To prove: (b + c)(b + d) = (c + a)(c + a)

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ ... (i)

$\Rightarrow b^2 = ac$ ... (ii)

$\Rightarrow c^2 = bd$ ... (iii)

Taking LHS = (b + c)(b + d)

$= b^2 + bd + bc + cd$

Using eqn. (i) , (ii) and (iii)

$= ac + c^2 + ad + cd$

$= c(a + c) + d(a + c)$

$= (a + c)(c + d)$

Hence Proved

(ii) $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

To prove: $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ ... (i)

$\Rightarrow b^2 = ac$ ... (ii)

$\Rightarrow c^2 = bd$

$\Rightarrow d = \frac{c^2}{b}$ ... (iii)

Taking LHS = $\frac{ab-cd}{b^2-c^2}$

$= \frac{ab-c\frac{c^2}{b}}{b^2-c^2}$ [From eqn. (iii)]

$= \frac{ab-\frac{c^3}{b}}{b^2-c^2}$

$= \frac{\frac{ab^2-c^3}{b}}{b^2-c^2}$

$= \frac{ab^2-c^3}{b(b^2-c^2)}$

$= \frac{a^2c - c^3}{bac - bc^2}$ [From eqn. (ii)]

$= \frac{c(a^2 - c^2)}{b(ac - c^2)}$

$= \frac{c(a-c)(a+c)}{b(ac - c^2)}$

$= \frac{(ac - c^2)(a+c)}{b(ac - c^2)}$

$= \frac{(a+c)}{b}$

= RHS

Hence Proved

(iii) $(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$

To prove: $(a + b + c + d)^2 = (a + b)^2 + 2(b + c)^2 + (c + d)^2$

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$ … (iii)

Taking LHS $= (a + b + c + d)^2$

$\Rightarrow (a + b + c + d)(a + b + c + d)$

$\Rightarrow a^2 + ab + ac + ad + ba + b^2 + bc + bd + ca + cb + c^2 + cd + da + db + dc + d^2$

On rearranging

$\Rightarrow a^2 + ab + ba + b^2 + ac + ad + bc + bd + ca + cb + c^2 + cd + da + db + dc + d^2$

On rearranging

$\Rightarrow (a + b)^2 + ac + ad + bc + bd + ca + cb + da + db + c^2 + cd + dc + d^2$

On rearranging

$\Rightarrow (a + b)^2 + ac + ad + bc + bd + ca + cb + da + db + (c + d)^2$

On rearranging

$\Rightarrow (a + b)^2 + ac + ca + ad + bc + cb + da + bd + db + (c + d)^2$

Using eqn. (i)

$\Rightarrow (a + b)^2 + ac + ca + bc + bc + bc + bc + bd + db + (c + d)^2$

Using eqn. (ii)

$\Rightarrow (a + b)^2 + b^2 + b^2 + bc + bc + bc + bc + bd + db + (c + d)^2$

Using eqn. (iii)

$\Rightarrow (a + b)^2 + 2b^2 + 4bc + c^2 + c^2 + (c + d)^2$

On rearranging

$\Rightarrow (a + b)^2 + 2b^2 + 4bc + 2c^2 + (c + d)^2$

$\Rightarrow (a + b)^2 + 2[b^2 + 2bc + c^2 ] + (c + d)^2$

$\Rightarrow (a + b)^2 + 2(b + c)^2 + (c + d)^2$

= RHS

Hence proved

**Question: 12**

If a, b, c are in

**Solution:**

To prove: $\frac{1}{(a+b)}$ , $\frac{1}{(2b)}$, $\frac{1}{(b+c)}$ are in AP

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

When a,b,c are in GP, $b^2 = ac$

Taking $\frac{1}{(a+b)}$ and $\frac{1}{(b+c)}$

$$\frac{1}{(a+b)} + \frac{1}{(b+c)}$$

$\Rightarrow \frac{b+c+a+b}{(a+b)(b+c)}$

$\Rightarrow \frac{a+c+2b}{ab+ac+b^2+bc}$

$\Rightarrow \frac{a+c+2b}{ab+b^2+b^2+bc}$ [ $b^2 = ac$ ]

$\Rightarrow \frac{a+c+2b}{ab+2b^2+bc}$

$\Rightarrow \frac{a+c+2b}{b(a+c+2b)}$

$\Rightarrow \frac{1}{b}$

$\Rightarrow 2\times \frac{1}{2b}$

We can see that $\frac{1}{(a+b)} + \frac{1}{(b+c)} = 2\times \frac{1}{2b}$

Hence we can say that $\frac{1}{(a+b)}$ , $\frac{1}{(2b)}$, $\frac{1}{(b+c)}$ are in AP.

**Question: 13**

If a, b, c are in

**Solution:**

To prove: $a^2$, $b^2$, $c^2$ are in GP

Given: a, b, c are in GP

Proof: As a, b, c are in GP

$\Rightarrow b^2 = ac$ ... (i)

Considering $b^2$, $c^2$

$\frac{c^2}{b^2}$ = common ratio = r

$\Rightarrow \frac{c^2}{ac}$ [From eqn. (i)]

$\Rightarrow \frac{c}{a} = r$

Considering $a^2$, $b^2$

$\frac{b^2}{a^2}$ = common ratio = r

$\Rightarrow \frac{ac}{a^2}$ [From eqn. (i)]

$\Rightarrow \frac{c}{a} = r$

We can see that in both the cases we have obtained a common ratio.

Hence $a^2$, $b^2$, $c^2$ are in GP.

**Question: 14**

If a, b, c are in

**Solution:**

To prove: $a^3$, $b^3$, $c^3$ are in GP

Given: a, b, c are in GP

Proof: As a, b, c are in GP

$\Rightarrow b^2 = ac$

Cubing both sides

$\Rightarrow \left(b^2\right)^3 = (ac)^3$

$\Rightarrow b^6 = a^3c^3$

$\Rightarrow \frac{b^3}{a^3} = \frac{c^3}{b^3}$ = common ratio = r

From the above equation, we can say that $a^3$, $b^3$, $c^3$ are in GP

**Question: 15**

If a, b, c are in

**Solution:**

To prove: $(a^2 + b^2)$, $(ab + bc)$, $(b^2 + c^2)$ are in GP

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c are in GP,

$b^2 = ac$ ... (i)

Considering $(a^2 + b^2)$, $(ab + bc)$, $(b^2 + c^2)$

$(ab + bc)^2 = (a^2b^2 + 2ab^2c + b^2c^2)$

$= (a^2b^2 + ab^2c + ab^2c + b^2c^2)$

$= (a^2b^2 + b^4 + a^2c^2 + b^2c^2)$ [From eqn. (i)]

$= [b^2 (a^2 + b^2) + c^2 (a^2 + b^2)]$

$(ab + bc)^2 = [(b^2 + c^2) (a^2 + b^2)]$

From the above equation we can say that $(a^2 + b^2)$, $(ab + bc)$, $(b^2 + c^2)$ are in GP

**Question: 16**

If a, b, c, d are

**Solution:**

To prove: $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in GP.

Given: a, b, c are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When a,b,c,d are in GP then

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$ … (iii)

Considering $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$

$(a^2 - b^2)(c^2 - d^2) = a^2c^2 - a^2d^2 - b^2c^2 + b^2d^2$

$= (ac)^2 - (ad)^2 - (bc)^2 + (bd)^2$

From eqn. (i) , (ii) and (iii)

$= (b^2)^2 - (bc)^2 - (bc)^2 + (c^2)^2$

$= b^4 - 2b^2c^2 + c^4$

$(a^2 - b^2)(c^2 - d^2) = (b^2 - c^2)^2$

From the above equation we can say that $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in GP

**Question: 17**

If a, b, c, d are

**Solution:**

To prove: $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$ are in GP.

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$ … (iii)

Considering $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$

$$\frac{1}{(a^2+b^2)} \times \frac{1}{(c^2+d^2)} = \frac{1}{a^2c^2+a^2d^2+b^2c^2+b^2d^2}$$

$= \frac{1}{(ac)^2+(ad)^2+(bc)^2+(bd)^2}$

From eqn. (i) , (ii) and (iii)

$= \frac{1}{\left(b^2\right)^2+(bc)^2+(bc)^2+\left(c^2\right)^2}$

$= \frac{1}{b^4+2b^2c^2+c^4}$

$\frac{1}{\left(a^2+b^2\right)} \times \frac{1}{\left(c^2+d^2\right)} = \frac{1}{\left(b^2+c^2\right)^2}$

From the above equation, we can say that $\frac{1}{\left(a^2+b^2\right)}$ , $\frac{1}{\left(b^2+c^2\right)}$ , $\frac{1}{\left(c^2+d^2\right)}$ are in GP.

**Question: 18**

If $(p^2$

**Solution:**

To prove: p, q, r are in GP

Given: $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP,

$(pq + qr)^2 = (p^2 + q^2)(q^2 + r^2)$

$p^2q^2 + 2pq^2r + q^2r^2 = p^2q^2 + p^2r^2 + q^4 + q^2r^2$

$2pq^2r = p^2r^2 + q^4$

$pq^2r + pq^2r = p^2r^2 + q^4$

$pq^2r - q^4 = p^2r^2 - pq^2r$

$q^2(pr - q^2) = pr(pr - q^2)$

$q^2 = pr$

From the above equation we can say that p, q and r are in G.P.

**Question: 19**

If a, b, c are in

**Solution:**

To prove: a, (a – b) and (d – c) are in GP.

Given: a, b, c are in AP, and a, b, d are in GP

Proof: As a,b,d are in GP then

$b^2 = ad$ … (i)

As a, b, c are in AP

$2b = (a + c)$ … (ii)

Considering a, (a – b) and (d – c)

$(a - b)^2 = a^2 - 2ab + b^2$

$= a^2 - (2b)a + b^2$

From eqn. (i) and (ii)

$= a^2 - (a+c)a + ad$

$= a^2 – a^2 - ac + ad$

$= ad – ac$

$(a – b)^2 = a (d – c)$

From the above equation we can say that a, (a – b) and (d – c) are in GP.

**Question: 20**

If a, b, c are in

**Solution:**

To prove: $x^2$, $b^2$, $y^2$ are in AP.

Given: a, b, c are in AP, and a, x, b and b, y, c are in GP

Proof: As, a,b,c are in AP

$\Rightarrow 2b = a + c$ ... (i)

As, a,x,b are in GP

$\Rightarrow x^2 = ab$ ... (ii)

As, b,y,c are in GP

$\Rightarrow y^2= bc$ ... (iii)

Considering $x^2$, $b^2$, $y^2$

$x^2 + y^2 = ab + bc$ [From eqn. (ii) and (iii)]

$= b (a + c)$

$= b(2b)$ [From eqn. (i)]

$x^2 + y^2 = 2b^2$

From the above equation we can say that $x^2$, $b^2$, $y^2$ are in AP.

## Exercise : 12F

**Question: 1**

Find two positive

**Solution:**

(i) AM = 25 and GM = 20

To find: Two positive numbers a and b

Given: AM = 25 and GM = 20

Formula used: (i) Arithmetic mean between $a \text{ and } b = \frac{a+b}{2}$

(ii) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Arithmetic mean of two numbers $= \frac{a+b}{2}$

$$\frac{a+b}{2} = 25$$

$\Rightarrow a + b = 50$

$\Rightarrow b = 50 – a$ ... (i)

Geometric mean of two numbers $= \sqrt{ab}$

$\Rightarrow \sqrt{ab} = 20$

$\Rightarrow ab=400$

Substituting value of b from eqn. (i)

a(50 – a) = 400

$\Rightarrow 50a - a^2 = 400$

On rearranging

$\Rightarrow a^2 - 50a + 400 = 0$

$\Rightarrow a^2 - 40a - 10a + 400$

⇒ a(a – 40) – 10(a – 40) = 0

⇒ (a – 10) (a – 40) = 0

⇒ a = 10, 40

Substituting, a = 10 Or a = 40 in eqn. (i)

b = 40 Or b = 10

Therefore two numbers are 10 and 40

(ii) AM = 10 and GM = 8

To find: Two positive numbers a and b

Given: AM = 10 and GM = 8

Formula used: (i) Arithmetic mean between $a \text{ and } b = \frac{a+b}{2}$

(ii) Geometric mean between $a \text{ and } b=\sqrt{ab}$

Arithmetic mean of two numbers $= \frac{a+b}{2}$

$$\frac{a+b}{2}=10$$

⇒ a + b = 20

⇒ a = 20 – b ... (i)

Geometric mean of two numbers $=\sqrt{ab}$

$\Rightarrow \sqrt{ab}=8$

$\Rightarrow ab=64$

Substituting value of a from eqn. (i)

b(20 – b) = 64

$\Rightarrow 20b - b^2 = 64$

On rearranging

$\Rightarrow b^2 - 20b + 64 = 0$

$\Rightarrow b^2 - 16b - 4b + 64$

⇒ b(b – 16) – 4(b – 16) = 0

⇒ (b – 16) (b – 4) = 0

⇒ b = 16, 4

Substituting, b = 16 Or b = 4 in eqn. (i)

a = 4 Or b = 16

Therefore two numbers are 16 and 4

**Question: 2**

Find the GM betwe

**Solution:**

(i) 5 and 125

To find: Geometric Mean

Given: The numbers are 5 and 125

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{5 \times 25}$

$= \sqrt{625}$

= 25

The geometric mean between 5 and 125 is 25

(ii) 1 and $\frac{9}{16}$

To find: Geometric Mean

Given: The numbers are 1 and $\frac{9}{16}$

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$$= \sqrt{1 \times \frac{9}{16}}$$

$$= \sqrt{\frac{9}{16}}$$

$$= \frac{3}{4}$$

The geometric mean between 1 and $\frac{9}{16}$ is $\frac{3}{4}$.

(iii) 0.15 and 0.0015

To find: Geometric Mean

Given: The numbers are 0.15 and 0.0015

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{0.15 \times 0.0015}$

$= \sqrt{0.000225}$

= 0.015

The geometric mean between 0.15 and 0.0015 is 0.015.

(iv) -8 and -2

To find: Geometric Mean

Given: The numbers are -8 and -2

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{-8 \times -2}$

$= \sqrt{16}$

$= \pm 4$

Mean is a number which has to fall between two numbers.

Therefore we will take -4 as our answer as +4 doesn't lie between -8 and -2

The geometric mean between -8 and -2 is -4.

(v) -6.3 and -2.8

To find: Geometric Mean

Given: The numbers are -6.3 and -2.8

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{-6.3 \times -2.8}$

$= \sqrt{17.64}$

$= \pm 4.2$

Mean is a number which has to fall between two numbers.

Therefore we will take -4.2 as our answer as +4.2 doesn't lie between -6.3 and -2.8

The geometric mean between -6.3 and -2.8 is -4.2.

(vi) $a^3b$ and $ab^3$

To find: Geometric Mean

Given: The numbers are $a^3b$ and $ab^3$

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{a^3b \times ab^3}$

$= \sqrt{a^4b^4}$

$= a^2b^2$

The geometric mean between $a^3b$ and $ab^3$ is $a^2b^2$.

**Question: 13**

Insert two geomet

**Solution:**

To find: Two geometric Mean

Given: The numbers are 9 and 243

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of geometric mean

Let $G_1$ and $G_2$ be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{243}{9}\right)^{\frac{1}{2+1}}$

⇒ r $= 27^{\frac{1}{3}}$

⇒ r = 3

$G_1$ = ar = 9×3 = 27

$G_2$ = ar$^2$= 9×3$^2$ = 9×9 = 81

Two geometric mean between 9 and 243 are 27 and 81.

**Question: 4**

Insert three geom

**Solution:**

To find: Three geometric Mean

Given: The numbers $\frac{1}{3}$ and 432

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of geometric mean

Let $G_1$, $G_2$ and $G_3$ be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{3+1}}$

⇒ r $= \left(\frac{432}{\left(\frac{1}{3}\right)}\right)^{\frac{1}{2+1}}$

⇒ r $= \left(\frac{432\times 3}{1}\right)^{\frac{1}{3+1}}$

⇒ r $= (1296)^{\frac{1}{4}}$

⇒ r = 6

$G_1$ = ar = $\left(\frac{1}{3}\right)$×6 = 2

$G_2$ = ar$^2$= $\left(\frac{1}{3}\right)$×6$^2$ = $\left(\frac{1}{3}\right)$×36 = 12

$G_3$ = ar$^3$= $\left(\frac{1}{3}\right)$×6$^3$ = $\left(\frac{1}{3}\right)$×216 = 72

Three geometric mean between $\frac{1}{3}$ and 432 are 2, 12 and 72.

**Question: 5**

Insert four geome

**Solution:**

To find: Four geometric Mean

Given: The numbers 6 and 192

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of

geometric mean

Let $G_1$, $G_2$, $G_3$ and $G_4$ be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{4+1}}$

⇒ r $= \left(\frac{192}{6}\right)^{\frac{1}{4+1}}$

⇒ r $= (32)^{\frac{1}{5}}$

⇒ r = 2

$G_1$ = ar = 6×2 = 12

$G_2 = ar^2 = 6\times2^2 = 24$

$G_3 = ar^3 = 6\times2^3 = 48$

$G_4 = ar^4 = 6\times2^4 = 96$

Four geometric mean between 6 and 192 are 12, 24, 48 and 96.

**Question: 6**

The AM between tw

**Solution:**

To prove: Prove that a:b $=(2+\sqrt{3}):(2-\sqrt{3})$

Given: Arithmetic mean is twice of Geometric mean.

Formula used: (i) Arithmetic mean between a and b = $\frac{a+b}{2}$

(ii) Geometric mean between a and b $=\sqrt{ab}$

AM = 2(GM)

$\frac{a+b}{2} = 2\left(\sqrt{ab}\right)$

⇒ a + b = $4(\sqrt{ab})$

Squaring both side

⇒ $(a + b)^2 = 16ab$ … (i)

We know that $(a - b)^2 = (a + b)^2 - 4ab$

From eqn. (i)

⇒ $(a - b)^2 = 16ab - 4ab$

$\Rightarrow (a - b)^2 = 12ab$ … (ii)

Dividing eqn. (i) and (ii)

$$\frac{(a+b)^2}{(a-b)^2} = \frac{16ab}{12ab}$$

$$\Rightarrow \left(\frac{a+b}{a-b}\right)^2 = \frac{16}{12}$$

Taking square root both side

$$\Rightarrow \frac{a+b}{a-b} = \frac{4}{2\sqrt{3}}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{2}{\sqrt{3}}$$

Applying componendo and dividend

$$\Rightarrow \frac{a+b+a-b}{a+b-a+b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{2a}{2b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Hence Proved

**Question: 7**

If a, b, c are in

**Solution:**

To prove: $b^2$ is the AM between $x^2$ and $y^2$.

Given: (i) a, b, c are in AP

(ii) x is the GM between a and b

(iii) y is the GM between b and c

Formula used: (i) Arithmetic mean between $a \text{ and } b = \frac{a+b}{2}$

(ii) Geometric mean between $a \text{ and } b = \sqrt{ab}$

As a, b, c are in A.P.

$\Rightarrow 2b = a + c$ … (i)

As x is the GM between a and b

$\Rightarrow x = \left(\sqrt{ab}\right)$

$\Rightarrow x^2 = ab$ … (ii)

As y is the GM between b and c

$\Rightarrow y = \left(\sqrt{bc}\right)$

$\Rightarrow y^2 = bc$ … (iii)

Arithmetic mean of $x^2$ and $y^2$ is $\left(\frac{x^2+y^2}{2}\right)$

Substituting the value from (ii) and (iii)

$\left(\frac{x^2+y^2}{2}\right)=\left(\frac{ab+bc}{2}\right)$

$= \frac{b(a+c)}{2}$

Substituting the value from eqn. (i)

$= \frac{b(2b)}{2}$

$= b^2$

Hence Proved

**Question: 8**

Show that the pro

**Solution:**

To prove: Product of n geometric means between a and b is equal to the nth power of the single GM between a and b.

Formula used:(i) Geometric mean between $a \text{ and } b=\sqrt{ab}$

(ii) Sum of n terms of A.P. $= \frac{(n)(n+1)}{2}$

Let the n geometric means between and b be $G_1, G_2, G_3, \ldots G_n$

Hence a, $G_1, G_2, G_3, \ldots G_n$, b are in GP

$\Rightarrow G_1 = ar, G_2 = ar^2$ and so on ...

Now, we have n+2 term

$\Rightarrow b = ar^{n+2-1}$

$\Rightarrow b = ar^{n+1}$

$\Rightarrow r=\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ ... (i)

The product of n geometric means is $G_1 \times G_2 \times G_3 \times \ldots G_n$

$= ar \times ar^2 \times ar^3 \times \ldots ar^n$

$= a^n \times r^{(1+2+3\ldots + n)}$

$= a^n \times r^{(n)\left(\frac{n+1}{2}\right)}$ $\left[\text{Sum of n terms of A.P.} = \frac{(n)(n+1)}{2}\right]$

Substituting the value of r from eqn. (i)

$= a^n \times \left(\frac{b}{a}\right)^{\left(\frac{1}{n+1}\right)(n)\left(\frac{n+1}{2}\right)}$

$= a^n \times \left(\frac{b}{a}\right)^{\left(\frac{n}{2}\right)}$

$= a^n \times \frac{b^{\frac{n}{2}}}{a^{\frac{n}{2}}}$

$= a^{\frac{n}{2}} \times b^{\frac{n}{2}}$

$= (ab)^{\frac{n}{2}}$

$= \left(\sqrt{ab}\right)^n$ ... (ii)

Single geometric mean between a and b $=\sqrt{ab}$

$n^{th}$ power of single geometric mean between a and b $=\left(\sqrt{ab}\right)^n$

Hence Proved

**Question: 9**

If AM and GM of t

**Solution:**

To find: The quadratic equation.

Given: (i) AM of roots of quadratic equation is 10

(ii) GM of roots of quadratic equation is 8

Formula used: (i) Arithmetic mean between a and b = $\frac{a+b}{2}$

(ii) Geometric mean between a and b$=\sqrt{ab}$

Let the roots be p and q

Arithmetic mean of roots p and q = $\frac{p+q}{2}$ = 10

$\Rightarrow \frac{p+q}{2} = 10$

$\Rightarrow$ p + q = 20 = sum of roots … (i)

Geometric mean of roots p and q = $\sqrt{pq}$ = 8

$\Rightarrow$ pq = 64 = product of roots … (ii)

Quadratic equation = $x^2$ – (sum of roots)x + (product of roots)

From equation (i) and (ii)

Quadratic equation = $x^2$ – (20)x + (64)

= $x^2$ –20x + 64

$x^2$ –20x + 64

# Exercise : 12G

**Question: 1**

Find the sum of e

**Solution:**

It is Infinite Geometric Series.

Here, a=8,

$r = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

The formula used:Sum of an infinite Geometric series= $\frac{a}{1-r}$

$\therefore$Sum= $\frac{8}{1-\frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2}-1}$

Sum= $\frac{8\sqrt{2}}{\sqrt{2}-1}$

**Question: 2**

Find the sum of e

**Solution:**

It is Infinite Geometric Series.

Here, a=6,

$r = \frac{1.2}{6} = \frac{2}{10} = 0.2$

The formula used:Sum of an infinite Geometric series= $\frac{a}{1-r}$

$\therefore$Sum=$\frac{6}{1-0.2} = \frac{6}{0.8} = \frac{15}{2}$

Sum= $\frac{15}{2}$

**Question: 3**

Find the sum of e

**Solution:**

It is Infinite Geometric Series

Here, a=√2

$r = \frac{\frac{-1}{\sqrt{2}}}{\sqrt{2}} = \frac{-1}{2}$

$\therefore$Sum= $\frac{a}{1-r} = \frac{\sqrt{2}}{1-\frac{-1}{2}} = \frac{\sqrt{2}}{1+\frac{1}{2}} = \frac{2\sqrt{2}}{3}$

Sum= $\frac{2\sqrt{2}}{3}$

**Question: 4**

Find the sum of e

**Solution:**

It is Infinite Geometric Series

Here, a=10

r= $\frac{-9}{10} = -0.9$

$\therefore$Sum $= \frac{a}{1-r} = \frac{10}{1-(-0.9)} = \frac{10}{1+0.9} = \frac{10}{1.9} = \frac{100}{19}$

Sum= $\frac{100}{19}$

**Question: 5**

Find the sum of e

**Solution:**

This geometric series is the sum of two geometric series:

$\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \cdots \infty$ & $\frac{3}{5^2} + \frac{3}{5^4} + \frac{4}{5^6} + \cdots \infty$

Sum of geometric series: $\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \cdots \infty$ Here, a= $\frac{2}{5}$

$r = \frac{\frac{2}{5^3}}{\frac{2}{5}} = \frac{1}{5^2} = \frac{1}{25}$

$\therefore$ Sum $= \frac{a}{1-r} = \frac{\frac{2}{5}}{1-\frac{1}{25}} = \frac{\frac{2}{5}}{\frac{25-1}{25}} = \frac{2 \times 25}{24 \times 5} = \frac{5}{12}$

Sum of geometric series: $\frac{3}{5^2} + \frac{3}{5^4} + \frac{4}{5^6} + \cdots \infty$ Here, a= $\frac{3}{5^2}$

$$r = \frac{\frac{3}{5^4}}{\frac{3}{5^2}} = \frac{1}{5^2} = \frac{1}{25}$$

$\therefore$ Sum $= \frac{a}{1-r} = \frac{\frac{3}{5^2}}{1-\frac{1}{25}} = \frac{\frac{3}{5^2}}{\frac{25-1}{25}} = \frac{3 \times 25}{25 \times 24} = \frac{1}{8}$

$\therefore$Sum of the given infinite series=sum of both the series= $\frac{5}{12} + \frac{1}{8} = \frac{(5\times2)+(1\times3)}{24}$

$$= \frac{10+3}{24} = \frac{13}{24}$$

Sum= $\frac{13}{24}$

**Question: 6**

Prove that 9

**Solution:**

L.H.S=$9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty$

$=9^{(1/3)+(1/9)+(1/27)+\ldots\infty}$

The series in the exponent is an infinite geometric series

Whose, a= $\frac{1}{3}$

$$r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1 \times 3}{1 \times 9} = \frac{1}{3}$$

$\therefore$Sum of the series in the exponent= $\frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1\times3}{3\times2} = \frac{1}{2}$

$\therefore$L.H.S=$9^{1/2}$

=3=R.H.S

Hence, Proved that $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty = 3$

**Question: 7**

Find the rational

**Solution:**

(i) Let, x=0.3333…

⇒ x=0.3+0.03+0.003+…

⇒ x=3(0.1+0.01+0.001+0.0001+…∞)

⇒ x=3($\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots \infty$)

This is an infinite geometric series.

Here, a=1/10 and r=1/10

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{1 \times 10}{9 \times 10} = \frac{1}{9}$$

$\therefore x = 3 \times \frac{1}{9} = \frac{1}{3}$

$0.\overline{3} = \frac{1}{3}$

(ii) Let, x=0.231231231....

⇒ x=0.231+0.000231+0.000000231+...∞

⇒ x=231(0.001+0.000001+0.000000001+...∞)

⇒ $x=231\left(\frac{1}{10^3}+\frac{1}{10^6}+\frac{1}{10^9}+\frac{1}{10^{12}}+\ldots\infty\right)$

This is an infinite geometric series.

Here, $a=\frac{1}{10^3}$ and $r=\frac{1}{10^3}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = \frac{1 \times 1000}{999 \times 1000} = \frac{1}{999}$$

⇒ $x = 231 \times \frac{1}{999} = \frac{231}{999}$

$0.\overline{231} = \frac{231}{999}$

(iii) Let, x=3.525252552...

⇒ x=3+0.52+0.0052+0.000052+...∞

⇒ x=3+52(0.01+0.0001+...∞)

⇒ $x=3+52\left(\frac{1}{10^2}+\frac{1}{10^4}+\frac{1}{10^6}+\frac{1}{10^8}+\ldots\infty\right)$

Here, $a=\frac{1}{10^2}$ and $r=\frac{1}{10^2}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{1 \times 100}{99 \times 100} = \frac{1}{99}$$

⇒ $x = 3 + \left(52 \times \frac{1}{99}\right) = \frac{297+52}{999} = \frac{349}{999}$

$3.\overline{52} = \frac{349}{999}$

**Question: 8**

Express the recur

**Solution:**

Let, x=0.125125125... ...(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=125.125125125... ...(ii)

Equation (ii)-(i),

⇒ 1000x-x=125.125125125-0.125125125=125

⇒ 999x=125

$\Rightarrow x = \frac{125}{999}$

$0.\overline{125} = \frac{125}{999}$

**Question: 9**

Write the value o

**Solution:**

Let, x=0.423423423... ...(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=423.423423423... ...(ii)

Equation (ii)-(i),

⇒ 1000x-x=423.423423423-0.423423423=423

⇒ 999x=423

$\Rightarrow x = \frac{423}{999} = \frac{47}{111}$

$0.\overline{423} = \frac{47}{111}$

**Question: 10**

Write the value o

**Solution:**

Let, x=2.134134134... ...(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=2134.134134134... ...(ii)

Equation (ii)-(i),

⇒ 1000x-x=2134.134134134-2.134134134=2132

⇒ 999x=2132

$\Rightarrow x = \frac{2132}{999}$

$2.\overline{134} = \frac{2132}{999}$

**Question: 11**

The sum of an inf

**Solution:**

Given: $\frac{a}{1-r} = 6$, a=2

To find:r=?

$\therefore \frac{2}{1-r} = 6$

$\Rightarrow 1 - r = \frac{2}{6} = \frac{1}{3}$

⇒ 3(1-r)=1

⇒ 3-3r=1

⇒ 3r=3-1

$\Rightarrow$ r= $\frac{2}{3}$

Common ratio r= $\frac{2}{3}$

**Question: 12**

The sum of an inf

**Solution:**

Given: $\frac{a}{1-r} = 20$ & $\frac{a^2}{1-r^2} = 100$

(because on squaring both first term a and common ratio r will be squared.)

To find: the series

a=20(1-r)...(i)

$\Rightarrow \frac{a^2}{1-r^2} = 100 = \frac{(20\times(1-r))^2}{(1-r)(1+r)}$ ...(from (i))

$\Rightarrow 100 = 400 \times \frac{1-r}{1+r}$

$\Rightarrow$ 100(1+r)=400(1-r)

$\Rightarrow$ 100+100r=400-400r

$\Rightarrow$ 100r+400r=400-100

$\Rightarrow$ 500r=300

$\Rightarrow$ 5r=3

$\Rightarrow r = \frac{3}{5}$

Put this value of r in equation (i) we get

$$a = 20\left(1-\frac{3}{5}\right) = \frac{20\times 2}{5} = 8$$

$\therefore$The infinite geometric series is:8, $\frac{24}{5}, \frac{72}{25}, \frac{216}{125}, \frac{648}{625}, \ldots \infty$

**Question: 13**

The sum of an inf

**Solution:**

Let the first term Of G.P. be a, and common ratio be r.

$\therefore \frac{a}{1-r} = 57 \ldots (1)$

On cubing each term will become,

$a^3, a^3r^3$, ....

$\therefore$This sum= $\frac{a^3}{1-r^3} = 9747 \ldots (2)$

a=57(1-r) put this in equation 2 we get

$$\frac{(57\times(1-r))^3}{1-r^3} = 9747$$

$\Rightarrow \frac{57^3\times(1-r)^3}{1-r^3} = 9747$

$\Rightarrow \frac{(1-r)\times(1-r)^2}{(1-r)(1+r+r^2)} = \frac{9747}{57\times57\times57} = \frac{1}{19}$

$\Rightarrow 19(1-2r+r^2)=1+r+r^2$

$\Rightarrow 19r^2-r^2-38r-r+19-1=0$

$\Rightarrow 18r^2-39r+18=0$

$\Rightarrow 6r^2-13r+6=0$

$\Rightarrow (2r-3)(3r-2)=0$

$\Rightarrow r= 2/3, 3/2$

But $-1<r<1$

$\Rightarrow r=2/3$

Substitute this value of r in equation 1 we get

$$a = 57 \times \left(1 - \frac{2}{3}\right) = 19$$

Thus the first term of G.P. is 19, and the common ratio is 2/3

$\therefore$G.P=19, $\frac{38}{3}, \frac{76}{9}$, ....

19, $\frac{38}{3}, \frac{76}{9}$, ....

# Exercise : 12H

**Question: 1**

If the $5^{th}$

**Solution:**

Given: $5^{th}$ term of a GP is 2.

To find: the product of its first nine terms.

First term is denoted by a, the common ratio is denote by r.

$\therefore ar^4 = 2$

We have to find the value of: $a \times ar^1 \times ar^2 \times ar^3 \times \ldots \times ar^8$

$= a^9r^{1 + 2 + 3 + 4 + \ldots + 8}$

$= a^9r^{36}$

$= (ar^4)^9$

$= (2)^9$

$= 512$

Ans:512.

**Question: 2**

If the (p + q)th

**Solution:**

Let,

$t_{p + q} = m = Ar^{p + q - 1} = Ar^{p - 1}r^q$

and

$t_{p - q} = n = Ar^{p - q - 1} = Ar^{p - 1}r^{- q}$

We know that $p^{th}$ term $= Ar^{p - 1}$

$\therefore m \times n = A^2r^{2p - 2}$

$\Rightarrow Ar^{p-1} = (mn)^{1/2}$

$\Rightarrow$ $p^{th}$ term = $(mn)^{1/2}$

Ans: $p^{th}$ term = $(mn)^{1/2}$

**Question: 3**

If $2^{nd}$

**Solution:**

We have been given that $2^{nd}$, $3^{rd}$ and $6^{th}$ terms of an AP are the three consecutive terms of a GP.

Let the three consecutive terms of the G.P. be $a, ar, ar^2$.

Where a is the first consecutive term and r is the common ratio.

$2^{nd}$, $3^{rd}$ terms of the A.P. are a and ar respectively as per the question.

$\therefore$ The common difference of the A.P. = ar - a

And the sixth term of the A.P. = $ar^2$

Since the second term is a and the sixth term is $ar^2$(In A.P.)

We use the formula:$t = a + (n - 1)d$

$\therefore$ $ar^2 = a + 4(ar - a)$…(the difference between $2^{nd}$ and $6^{th}$ term is 4(ar - a))

$\Rightarrow ar^2 = a + 4ar - 4a$

$\Rightarrow ar^2 + 3a - 4ar = 0$

$\Rightarrow a(r^2 - 4r + 3) = 0$

$\Rightarrow a(r - 1)(r - 3) = 0$

Here, we have 3 possible options:

1)a = 0 which is not expected because all the terms of A.P. and G.P. will be 0.

2)r = 1,which is also not expected because all th terms would be equal to first term.

3)r = 3,which is the required answer.

Ans:common ratio = 3

**Question: 4**

Write the quadrat

**Solution:**

Let the roots of the required quadratic equation be a and b.

The arithmetic and geometric means of roots are A and G respectively.

$\Rightarrow A = (a + b)/2$…(i)

And $G = \sqrt{ab}$ …(ii)

We know that the equation whose roots are given is =

$x^2 - (a + b)x + ab = 0$

From (i) and (ii) we get:

$x^2 - 2A + G^2 = 0$

Thus, $x^2 - 2A + G^2 = 0$ is the required quadratic equation.

Ans: $x^2 - 2A + G^2 = 0$ is the required quadratic equation.

**Question: 5**

If a, b, c are in

**Solution:**

It is given that:

$a^{1/x} = b^{1/y} = c^{1/z}$

Let $a^{1/x} = b^{1/y} = c^{1/z} = k$

$\Rightarrow a^{1/x} = k$

$\Rightarrow (a^{1/x})^{x} = k^{x}$...(Taking power of x on both sides.)

$\Rightarrow a^{1/x \times x} = k^{x}$

$\Rightarrow a = k^{x}$

Similarly $b = k^{y}$

And $c = k^{z}$

It is given that a,b,c are in G.P.

$\Rightarrow b^{2} = ac$

Substituting values of a,b,c calculated above,we get:

$\Rightarrow (k^{y})^{2} = k^{x}k^{z}$

$\Rightarrow k^{2y} = k^{x + z}$

Comparing the powers we get,

$2y = x + z$

Which is the required condition for x,y,z to be in A.P.

Hence, proved that x,y,z, are in A.P.

**Question: 6**

If a, b, c are in

**Solution:**

To prove: $x^{b - c}. y^{c - a}. z^{a - b} = 1$....(i)

It is given that a,b,c are in A.P.

$\Rightarrow 2b = a + c$...(ii)

And x,y,z, are in G.P.

$\Rightarrow y^{2} = xz$

$\Rightarrow x = y^{2}/z$

Substitute this value of x in equation (i),we get

L.H.S =

$\Rightarrow (\frac{y^2}{z})^{b-c} \times y^{c-a} \times z^{a-b}$

$\Rightarrow y^{2(b-c) + c-a}.z^{a-b-(b-c)}$

$\Rightarrow y^{2b-2c + c-a}.z^{a + c-b-b}$

$\Rightarrow y^{2b-c-a}.z^{a + c-2b}$

$\Rightarrow y^{0}.z^{0}$...(Using equation (i))

= 1 = R.H.S

Hence, proved that . If a, b, c are in AP and x, y, z are in GP then $x^{b-c}$. $y^{c-a}$. $z^{a-b}$ = 1

**Question: 7**

Prove that

**Solution:**

It is Infinite Geometric Series.

Here,a = 1,

$$r = \frac{\frac{-1}{3}}{1} = \frac{-1}{3}$$

Formula used:Sum of an infinite Geometric series = $\frac{a}{1-r}$

$\therefore$ Sum = $\frac{1}{1-\frac{-1}{3}} = \frac{1\times 3}{3+1} = \frac{3}{4}$ = R.H.S.

Hence, Proved that $\left(1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\frac{1}{3^4}\ldots\infty\right)=\frac{3}{4}$

**Question: 8**

Express

**Solution:**

Let,x = 0.123123123....

$\Rightarrow$ x = 0.123 + 0.000123 + 0.000000123 + ...∞

$\Rightarrow$ x = 123(0.001 + 0.000001 + 0.000000001 + ...∞)

$\Rightarrow$ x = $123(\frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \frac{1}{10^{12}} + \ldots\infty)$

This is an infinite geometric series.

Here,a = $\frac{1}{10^3}$ and r = $\frac{1}{10^3}$

$$\therefore \text{ Sum } = \frac{a}{1-r} = \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = \frac{1 \times 1000}{999 \times 1000} = \frac{1}{999}$$

$\Rightarrow x = 123 \times \frac{1}{999} = \frac{123}{999}$

Ans : $0.\overline{123} = \frac{123}{999}$

**Question: 9**

Express

**Solution:**

Let ,x = 0.6666...

$\Rightarrow$ x = 0.6 + 0.06 + 0.006 + ...

$\Rightarrow$ x = 6(0.1 + 0.01 + 0.001 + 0.0001 + ...∞)

$\Rightarrow$ x = $6(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \ldots\infty)$

This is an infinite geometric series.

Here,a = 1/10 and r = 1/10

$$\therefore \text{ Sum } = \frac{a}{1-r} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{1 \times 10}{9 \times 10} = \frac{1}{9}$$

$\therefore x = 6 \times \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$

Ans: $0.\overline{6} = \frac{2}{-}$

**Question: 10**

Express

**Solution:**

Let,x = 0.68686868...

$\Rightarrow x = 0.68 + 0.0068 + 0.000068 + \ldots\infty$

$\Rightarrow x = 68(0.01 + 0.0001 + \ldots\infty)$

$\Rightarrow x = 68(\frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^6} + \frac{1}{10^8} + \ldots\infty)$

Here,a = $\frac{1}{10^2}$ and r = $\frac{1}{10^2}$

$$\therefore \text{ Sum } = \frac{a}{1-r} = \frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{1 \times 100}{99 \times 100} = \frac{1}{99}$$

$\Rightarrow x = \left(68 \times \frac{1}{99}\right) = \frac{68}{999} = \frac{68}{999}$

Ans: $0.\overline{68} = \frac{68}{999}$

**Question: 11**

The second term o

**Solution:**

Given: second term of a GP is 24 and its fifth term is 81.

To find: sum of first five terms of the G.P.

$ar = 24$ & $ar^4 = 81$

dividing these two terms we get:

$\Rightarrow \frac{ar^4}{ar} = \frac{81}{24}$

$\Rightarrow r^3 = \frac{27}{8}$

Taking cube root on both the sides we get,

$\Rightarrow r = \frac{3}{2}$

Substituting this value of r in ar = 24 we get

a = 24/(3/2) = (24 × 2)/3 = 16

$\therefore$ Sum of first Five terms of a G.P. = $a(r^n - 1)/(r - 1)$

$= 16 \times \frac{(\frac{3}{2})^5 - 1}{\frac{3}{2} - 1} = 16 \times \frac{\frac{243}{32} - 1}{\frac{3}{2} - 1}$

$= 16 \times \frac{242 \times 2}{32 \times 1} = 242$

Ans:242

**Question: 12**

The ratio of the

**Solution:**

The first three terms of a G.P. are:a,ar,$ar^2$

The first six terms of a G.P. are:a,ar,$ar^2$,$ar^3$,$ar^4$,$ar^5$

It is given that the ratio of the sum of first three terms is to that of first six terms of a GP is 125 : 152.

$\Rightarrow a + ar + ar^2 = 125x$ & $a + ar + ar^2 + ar^3 + ar^4 + ar^5 = 152x$

$\Rightarrow a + ar + ar^2 + r^3( a + ar + ar^2) = 152x$

$\Rightarrow 125x + r^3(125x) = 152x$

$\Rightarrow r^3(125x) = 152x - 125x = 27x$

$\Rightarrow r^3 = \frac{27}{125} = \left(\frac{3}{5}\right)^3$

$\Rightarrow r = 3/5$

Ans:common ratio = $\frac{3}{5}$

**Question: 13**

The sum of first

**Solution:**

Let the first three terms of G.P. be $\frac{a}{r}, a, ar$

It is given that $\frac{a}{r} \times a \times ar = 1$

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

And

$$\frac{a}{r} + a + ar = \frac{39}{10}$$

$\Rightarrow a\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$

$\Rightarrow \left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$ ...(a = 1)

$\Rightarrow \left(\frac{1}{r} + r\right) = \frac{39}{10} - 1 = \frac{29}{10}$

$\Rightarrow 10(1 + r^2) = 29r$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (2r - 5)(5r - 2) = 0$

$\Rightarrow r = \frac{5}{2}, \frac{2}{5}$

Therefore the first three terms are:

i)if $r = \frac{5}{2}$ then

$\frac{2}{5}, 1, \frac{5}{2}$

ii)if r = $\frac{2}{5}$ then

$\frac{5}{2}, 1, \frac{2}{5}$

Ans:Common ratio r = $\frac{5}{2}, \frac{2}{5}$ and the first three terms are:

i)if r = $\frac{5}{2}$ then

$\frac{2}{5}, 1, \frac{5}{2}$

ii)if r = $\frac{2}{5}$ then

$\frac{5}{2}, 1, \frac{2}{5}$