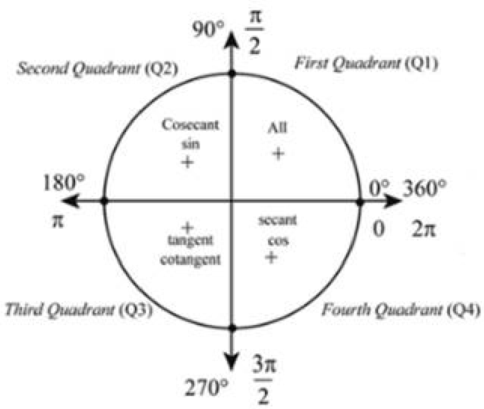
# Chapter : 15. TRIGONOMETRIC, OR CIRCULAR, FUNCTIONS

## Exercise : 15A

**Question: 1**

If $\cos\theta = -\frac{\sqrt{3}}{2}$




Since, θ is in III$^{rd}$ Quadrant. So, sin and cos will be negative but tan will be positive.

We know that,

$\cos^2\theta + \sin^2\theta = 1$

Putting the values, we get

$\left(-\frac{\sqrt{3}}{2}\right)^2 + \sin^2\theta = 1$ [given]

$\Rightarrow \frac{3}{4} + \sin^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \frac{3}{4}$

$\Rightarrow \sin^2\theta = \frac{4-3}{4}$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{4}}$

$\Rightarrow \sin\theta = \pm\frac{1}{2}$

Since, θ in III$^{rd}$ quadrant and sinθ is negative in III$^{rd}$ quadrant

$\therefore \sin\theta = -\frac{1}{2}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= -\frac{1}{2} \times \left(-\frac{2}{\sqrt{3}}\right)$

$= \frac{1}{\sqrt{3}}$

Now,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{2}}$

= -2

Now,

$\sec\theta = \frac{1}{\cos\theta}$

Putting the values, we get

$\sec\theta = \frac{1}{-\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{\frac{1}{\sqrt{3}}}$

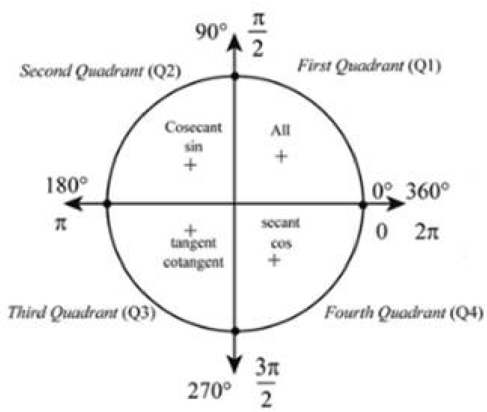
=√3

Hence, the values of other trigonometric Functions are:

| Cos θ | Sin θ | Tan θ | Cosec θ | Sec θ | Cot θ |
|---|---|---|---|---|---|
| $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $\frac{1}{\sqrt{3}}$ | -2 | $-\frac{2}{\sqrt{3}}$ | $\sqrt{3}$ |

**Question: 2**

If $\sin\theta = -\frac{1}{2}$



Since, $\theta$ is in IV$^{th}$ Quadrant. So, sin and tan will be negative but cos will be positive.

We know that,

$\sin^2\theta + \cos^2\theta = 1$

Putting the values, we get

$\left(-\frac{1}{2}\right)^2 + \cos^2\theta = 1$ [given]

$\Rightarrow \frac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{1}{4}$

$\Rightarrow \cos^2\theta = \frac{4-1}{4}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \sqrt{\frac{3}{4}}$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

Since, $\theta$ in IV$^{th}$ quadrant and $\cos\theta$ is positive in IV$^{th}$ quadrant

$\therefore \cos\theta = \frac{\sqrt{3}}{2}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= -\frac{1}{2} \times \left(\frac{2}{\sqrt{3}}\right)$

$= -\frac{1}{\sqrt{3}}$

Now,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{2}}$

= -2

Now,

$\sec\theta = \frac{1}{\cos\theta}$

Putting the values, we get

$\sec\theta = \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-\frac{1}{\sqrt{3}}}$

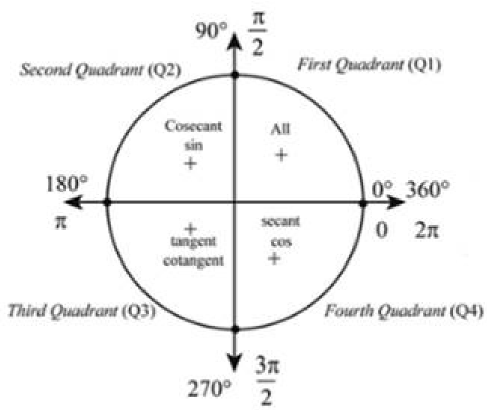
=-√3

Hence, the values of other trigonometric Functions are:

| Cos θ | Sin θ | Tan θ | Cosec θ | Sec θ | Cot θ |
|---|---|---|---|---|---|
| $\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{\sqrt{3}}$ | -2 | $\frac{2}{\sqrt{3}}$ | $-\sqrt{3}$ |

**Question: 3**

If $\operatorname{cosec}\theta = \frac{5}{3}$




Since, θ is in II$^{nd}$ Quadrant. So, cos and tan will be negative but sin will be positive.

Now, we know that

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

Putting the values, we get

$\sin\theta = \frac{1}{\frac{5}{3}}$

$\sin\theta = \frac{3}{5}$ ...(i)

We know that,

$\sin^2\theta + \cos^2\theta = 1$

Putting the values, we get

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$ [from (i)]

$\Rightarrow \frac{9}{25} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{9}{25}$

$\Rightarrow \cos^2\theta = \frac{25-9}{25}$

$\Rightarrow \cos^2\theta = \frac{16}{25}$

$\Rightarrow \cos\theta = \sqrt{\frac{16}{25}}$

$\Rightarrow \cos\theta = \pm\frac{4}{5}$

Since, θ in $\text{II}^{\text{nd}}$ quadrant and cosθ is negative in $\text{II}^{\text{nd}}$ quadrant

$\therefore \cos\theta = -\frac{4}{5}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{\frac{3}{5}}{-\frac{4}{5}}$

$= \frac{3}{5} \times \left(-\frac{5}{4}\right)$

$= -\frac{3}{4}$

Now,

$\sec\theta = \frac{1}{\cos\theta}$

Putting the values, we get

$\sec\theta = \frac{1}{-\frac{4}{5}}$

$= -\frac{5}{4}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-\frac{3}{4}}$

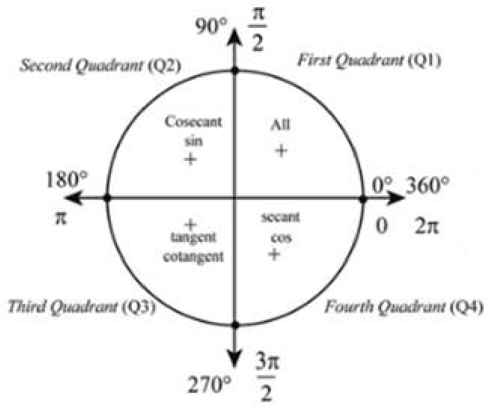
$= -\frac{4}{3}$

Hence, the values of other trigonometric Functions are:

| Cos θ | Sin θ | Tan θ | Cosec θ | Sec θ | Cot θ |
|---|---|---|---|---|---|
| $-\frac{4}{5}$ | $\frac{3}{5}$ | $-\frac{3}{4}$ | $\frac{5}{3}$ | $-\frac{5}{4}$ | $-\frac{4}{3}$ |

**Question: 4**

If sec θ = √2




Since, θ is in IV$^{th}$ Quadrant. So, sin and tan will be negative but cos will be positive.

Now, we know that

$\cos\theta = \frac{1}{\sec\theta}$

Putting the values, we get

$\cos\theta = \frac{1}{\sqrt{2}}$ ...(i)

We know that,

$\cos^2\theta + \sin^2\theta = 1$

Putting the values, we get

$\left(\frac{1}{\sqrt{2}}\right)^2 + \sin^2\theta = 1$ [given]

$\Rightarrow \frac{1}{2} + \sin^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \frac{1}{2}$

$\Rightarrow \sin^2\theta = \frac{2-1}{2}$

$\Rightarrow \sin^2\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{2}}$

$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}$

Since, θ in IV$^{th}$ quadrant and sinθ is negative in IV$^{th}$ quadrant

$\therefore \sin\theta = -\frac{1}{\sqrt{2}}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$= -\frac{1}{\sqrt{2}} \times (\sqrt{2})$

=-1

Now,

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\text{cosec}\,\theta = \frac{1}{-\frac{1}{\sqrt{2}}}$

= -√2

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-1}$

= -1

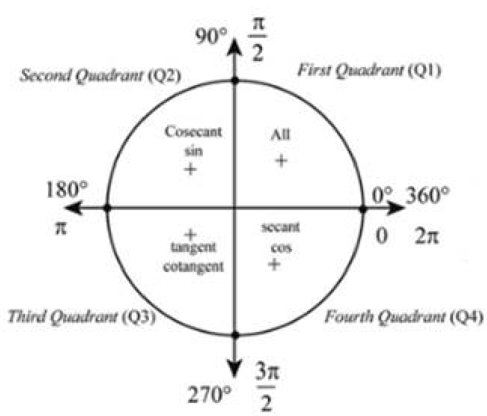
Hence, the values of other trigonometric Functions are:

| Cos θ | Sin θ | Tan θ | Cosec θ | Sec θ | Cot θ |
|---|---|---|---|---|---|
| $\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | -1 | -√2 | √2 | -1 |

**Question: 5**

If $\sin x = -\frac{2\sqrt{6}}{5}$

To find: cos x and cot x




Since, x is in III$^{rd}$ Quadrant. So, sin and cos will be negative but tan will be positive.

We know that,

$\sin^2 x + \cos^2 x = 1$

Putting the values, we get

$\left(-\frac{2\sqrt{6}}{5}\right)^2 + \cos^2 x = 1$ [given]

$\Rightarrow \frac{24}{25} + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \frac{24}{25}$

$\Rightarrow \cos^2 x = \frac{25-24}{25}$

$\Rightarrow \cos^2 x = \frac{1}{25}$

$\Rightarrow \cos x = \sqrt{\frac{1}{25}}$

$\Rightarrow \cos x = \pm\frac{1}{5}$

Since, x in III$^{rd}$ quadrant and cos x is negative in III$^{rd}$ quadrant

$\therefore \cos x = -\frac{1}{5}$

Now,

$\tan x = \frac{\sin x}{\cos x}$

Putting the values, we get

$\tan x = \frac{-\frac{2\sqrt{6}}{5}}{-\frac{1}{5}}$

$= -\frac{2\sqrt{6}}{5} \times (-5)$

$= 2\sqrt{6}$

Now,

$\cot x = \frac{1}{\tan x}$

Putting the values, we get

$\cot x = \frac{1}{2\sqrt{6}}$

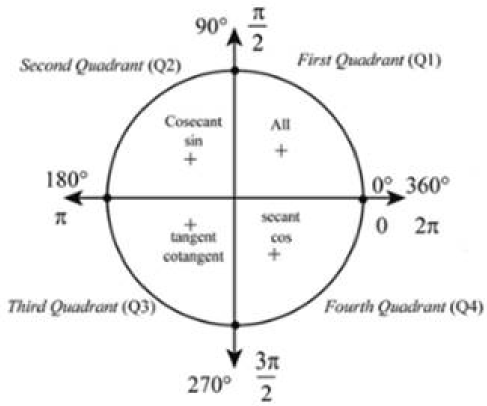
Hence, the values of other trigonometric Functions are:

| Cos x | Sin x | Cot x |
|---|---|---|
| $-\frac{1}{5}$ | $-\frac{2\sqrt{6}}{5}$ | $\frac{1}{2\sqrt{6}}$ |

**Question: 6**

If $\cos x = -\frac{\sqrt{15}}{4}$

To find: value of sinx




Given that: $\frac{\pi}{2} < x < \pi$

So, x lies in II$^{nd}$ quadrant and sin will be positive.

We know that,

$\cos^2 \theta + \sin^2 \theta = 1$

Putting the values, we get

$\left(-\frac{\sqrt{15}}{4}\right)^2 + \sin^2\theta = 1$ [given]

$\Rightarrow \frac{15}{16} + \sin^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \frac{15}{16}$

$\Rightarrow \sin^2\theta = \frac{16-15}{16}$

$\Rightarrow \sin^2\theta = \frac{1}{16}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{16}}$

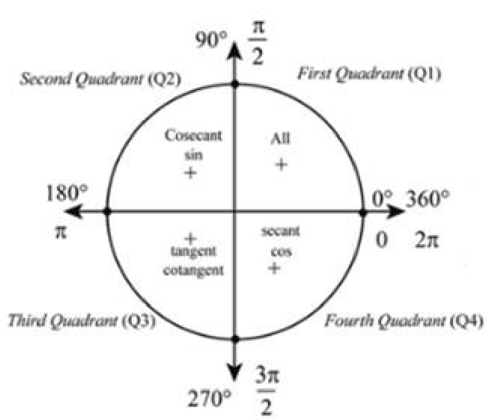
$\Rightarrow \sin\theta = \pm\frac{1}{4}$

Since, x in II$^{nd}$ quadrant and sinθ is positive in II$^{nd}$ quadrant

$\therefore \sin\theta = \frac{1}{4}$

**Question: 7**

If sec x = -2




Given that: $\pi < x < \frac{3\pi}{2}$

So, x lies in III$^{rd}$ Quadrant. So, sin and cos will be negative but tan will be positive.

Now, we know that

$\cos x = \frac{1}{\sec x}$

Putting the values, we get

$\cos x = \frac{1}{-2}$ ...(i)

We know that,

$\cos^2 x + \sin^2 x = 1$

Putting the values, we get

$\left(-\frac{1}{2}\right)^2 + \sin^2 x = 1$ [given]

$\Rightarrow \frac{1}{4} + \sin^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$

$\Rightarrow \sin^2 x = \frac{4-1}{4}$

$\Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \sqrt{\frac{3}{4}}$

$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

Since, x in III$^{rd}$ quadrant and sinx is negative in III$^{rd}$ quadrant

$\therefore \sin x = -\frac{\sqrt{3}}{2}$

Now,

$\tan x = \frac{\sin x}{\cos x}$

Putting the values, we get

$\tan x = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$= -\frac{\sqrt{3}}{2} \times (-2\,)$

$=\sqrt{3}$

Now,

$\operatorname{cosec} x = \frac{1}{\sin x}$

Putting the values, we get

$\operatorname{cosec} x = \frac{1}{-\frac{\sqrt{3}}{2}}$

$= \ -\frac{2}{\sqrt{3}}$

Now,

$\cot x = \frac{1}{\tan x}$

Putting the values, we get

$\cot x = \frac{1}{\sqrt{3}}$

Hence, the values of other trigonometric Functions are:

| Cos x | Sin x | Tan x | Cosec x | Sec x | Cot x |
|---|---|---|---|---|---|
| $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | √3 | $-\frac{2}{\sqrt{3}}$ | -2 | $\frac{1}{\sqrt{3}}$ |

**Question: 8 A**

Find the value of

**Solution:**

$$\begin{array}{r|l} & 10 \\ 3 & 31 \\ & 30 \\ \hline & 1 \end{array}$$

To find: Value of $\sin\frac{31\pi}{3}$

$\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{1}{3}\pi\right)$

$= \sin\left(5 \times (2\pi) + \frac{1}{3}\pi\right)$

Value of sin x repeats after an interval of 2π, hence ignoring 5 × (2π)

$= \sin\left(\frac{1}{3}\pi\right)$

$= \sin\left(\frac{1}{3} \times 180°\right)$

= sin 60°

$= \frac{\sqrt{3}}{2}\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

**Question: 8 B**

Find the value of

**Solution:**

$$\begin{array}{r|l} & 8 \\ 2 & 17 \\ & 16 \\ \hline & 1 \end{array}$$

To find: Value of $\cos\frac{17\pi}{2}$

$\cos\frac{17\pi}{2} = \cos\left(8\pi + \frac{1}{2}\pi\right)$

$= \cos\left(4 \times (2\pi) + \frac{1}{2}\pi\right)$

Value of cos x repeats after an interval of 2π, hence ignoring 4 × (2π)

$= \cos\left(\frac{1}{2}\pi\right)$

$= \cos\left(\frac{1}{2} \times 180°\right)$

= cos 90°

= 0 [∵ cos 90° = 1]

**Question: 8 C**

Find the value of

**Solution:**

$$\begin{array}{r|l} & \ \ 8 \\ \hline 3 & 25 \\ & 24 \\ \hline & \ \ 1 \end{array}$$

To find: Value of $\tan\frac{-25\pi}{3}$

We know that,

tan(-θ) = - tan θ

$\therefore \tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right)$

$\tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right) = -\tan\left(8\pi + \frac{1}{3}\pi\right)$

$= -\tan\left(4 \times (2\pi) + \frac{1}{3}\pi\right)$

Value of tan x repeats after an interval of 2π, hence ignoring 4 × (2π)

$= -\tan\left(\frac{1}{3}\pi\right)$

$= -\tan\left(\frac{1}{3} \times 180°\right)$

= - tan 60°

= -√3

[∵ tan 60° = √3]

**Question: 8 D**

Find the value of

**Solution:**

To find: Value of $\cot\frac{13\pi}{4}$

We have,

$\cot\frac{13\pi}{4}$

Putting π = 180°

$= \cot\left(\frac{13 \times 180°}{4}\right)$

= cot (13 × 45°)

= cot (585°)

= cot [90° × 6 + 45°]

= cot 45°

[Clearly, 585° is in III$^{rd}$ Quadrant and the multiple of 90° is even]

= 1 [∵ cot 45° = 1]

**Question: 8 E**

Find the value of

**Solution:**

To find: Value of $\sec\left(-\frac{25\pi}{3}\right)$

We have,

$\sec\left(-\frac{25\pi}{3}\right) = \sec\frac{25\pi}{3}$

[∵ sec(-θ) = sec θ]

Putting π = 180°

$= \sec\frac{25\times180}{3}$

= sec[25 × 60°]

= sec[1500°]

= sec [90° × 16 + 60°]

Clearly, 1500° is in I$^{st}$ Quadrant and the multiple of 90° is even

= sec 60°

= 2 [∵ $\sec 60° = 2$]

**Question: 8 F**

Find the value of

**Solution:**

To find: Value of $\operatorname{cosec}\left(-\frac{41\pi}{4}\right)$

We have,

$\operatorname{cosec}\left(-\frac{41\pi}{4}\right) = -\operatorname{cosec}\frac{41\pi}{4}$

[∵ cosec(-θ) = -cosec θ]

Putting π = 180°

$= -\operatorname{cosec}\frac{41\times180}{4}$

= -cosec[41 × 45°]

= -cosec[1845°]

= -cosec [90° × 20 + 45°]

Clearly, 1845° is in I$^{st}$ Quadrant and the multiple of 90° is even

= -cosec 45°

= -√2 [∵ $\operatorname{cosec} 45° = \sqrt{2}$]

**Question: 9 A**

Find the value of

**Solution:**

To find: Value of sin 405°

We have,

sin 405° = sin [90° × 4 + 45°]

= sin 45°

[Clearly, 405° is in I$^{st}$ Quadrant and the multiple of 90° is even]

$= \frac{1}{\sqrt{2}}$ [$\because \sin 45° = \frac{1}{\sqrt{2}}$]

**Question: 9 B**

Find the value of

**Solution:**

To find: Value of sec (-1470°)

We have,

sec (-1470°) = sec (1470°)

[∵ sec(-θ) = sec θ]

= sec [90° × 16 + 30°]

Clearly, 1470° is in I$^{st}$ Quadrant and the multiple of 90° is even

= sec 30°

$= \frac{2}{\sqrt{3}}$ [$\because \sec 30° = \frac{2}{\sqrt{3}}$]

**Question: 9 C**

Find the value of

**Solution:**

To find: Value of tan (-300°)

We have,

tan (-300°) = - tan (300°)

[∵ tan(-θ) = -tan θ]

= - tan [90° × 3 + 30°]

Clearly, 300° is in IV$^{th}$ Quadrant and the multiple of 90° is odd

= - cot 30°

= -√3 [$\because \cot 30° = \sqrt{3}$]

**Question: 9 D**

Find the value of

**Solution:**

To find: Value of $\cot \frac{13\pi}{4}$

We have,

cot (585°) = cot [90° × 6 + 45°]

= cot 45°

[Clearly, 585° is in III$^{rd}$ Quadrant and the multiple of 90° is even]

= 1 [∵ cot 45° = 1]

**Question: 9 E**

Find the value of

**Solution:**

To find: Value of cosec (-750°)

We have,

cosec (-750°) = - cosec(750°)

[∵ cosec(-θ) = -cosec θ]

= - cosec [90° × 8 + 30°]

Clearly, 405° is in I$^{st}$ Quadrant and the multiple of 90° is even

= - cosec 30°

= -2 [∵ cosec 30° = 2]

**Question: 9 F**

Find the value of

**Solution:**

To find: Value of cos 2220°

We have,

cos (-2220°) = cos 2220°

[∵ cos(-θ) = cos θ]

= cos [2160 + 60°]

= cos [360° × 6 + 60°]

= cos 60°

[Clearly, 2220° is in I$^{st}$ Quadrant and the multiple of 360° is even]

$= \frac{1}{2}$ [∵ $\cos 60° = \frac{1}{2}$]

**Question: 10 A**

Prove that

**Solution:**

To prove: $\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2} = 8$

Taking LHS,

$= \tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2}$

Putting π = 180°

$= \tan^2\frac{180}{3} + 2\cos^2\frac{180}{4} + 3\sec^2\frac{180}{6} + 4\cos^2\frac{180}{2}$

= $\tan^2 60° + 2\cos^2 45° + 3\sec^2 30° + 4\cos^2 90°$

Now, we know that,

$\tan 60° = \sqrt{3}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\sec 30° = \frac{2}{\sqrt{3}}$

$\cos 90° = 0$

Putting the values, we get

$= \left(\sqrt{3}\right)^2 + 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 + 4(0)^2$

$= 3 + 2 \times \frac{1}{2} + 3 \times \frac{4}{3}$

= 3 + 1 + 4

= 8

= RHS

∴ LHS = RHS

Hence Proved

**Question: 10 B**

Prove that

**Solution:**

To prove: $\sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6} = \frac{7}{4}$

Taking LHS,

$= \sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6}$

Putting π = 180°

$= \sin\frac{180}{6}\cos 0 + \sin\frac{180}{4}\cos\frac{180}{4} + \sin\frac{180}{3}\cos\frac{180}{6}$

= sin 30° cos 0° + sin 45° cos 45° + sin 60° cos 30°

Now, we know that,

$\sin 30° = \frac{1}{2}$

$\cos 0° = 1$

$\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

Putting the values, we get

$= \frac{1}{2} \times 1 + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4}$

$= \frac{2+2+3}{4}$

$= \frac{7}{4}$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 10 C**

Prove that

**Solution:**

To prove: $4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3}+3\cos\frac{\pi}{3}\tan\frac{\pi}{4}+\operatorname{cosec}^2\frac{\pi}{2}=4$

Taking LHS,

$=4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3}+3\cos\frac{\pi}{3}\tan\frac{\pi}{4}+\operatorname{cosec}^2\frac{\pi}{2}$

Putting π = 180°

$=4\sin\frac{180}{6}\sin^2\frac{180}{3}+3\cos\frac{180}{3}\tan\frac{180}{4}+\operatorname{cosec}^2\frac{180}{2}$

$= 4 \sin 30° \sin^2 60° + 3 \cos 60° \tan 45° + \operatorname{cosec}^2 90°$

Now, we know that,

$\sin 30° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

tan 45° = 1

cosec 90° = 1

Putting the values, we get

$=4\times\frac{1}{2}\times\left(\frac{\sqrt{3}}{2}\right)^2+3\times\frac{1}{2}\times1+(1)^2$

$=2\times\frac{3}{4}+\frac{3}{2}+1$

$=\frac{3}{2}+\frac{3}{2}+1$

$=\frac{3+3+2}{2}$

= 4

= RHS

∴ LHS = RHS

Hence Proved

# Exercise : 15B

**Question: 1**

Find the value of

**Solution:**

(i)

Cos840° = Cos(2.360° + 120°) ...........(using Cos(2ϖ + x) = Cosx)

= Cos(120°)

= Cos(180° - 60°)

= - Cos60° ..............(using Cos(ϖ - x) = - Cosx)

$= -\frac{1}{2}$

(ii) sin870° = sin(2.360° + 150°) ............(using sin(2ϖ + x) = sinx)

= sin150°

= sin(180° - 30°) .........(using sin(ϖ - x) = sinx)

= sin30°

$= \frac{1}{2}$

(iii)tan( - 120°) = - tan12 .......(tan( - x) = tanx)

= - tan(180° - 60°) ....... (in II quadrant tanx is negative)

= - ( - tan60°)

= tan60°

$= \sqrt{3}$

(iv) $\sec\left(-420°\right) = \frac{1}{\cos\left(-420°\right)}$

$= \frac{1}{-\cos 420°}$ ...........(using cos( - x) = - cosx)

$= \frac{-1}{-\cos\left(360° + 60\right)}$ ............(using cos(2ϖ + x) = cosx)

$= \frac{-1}{\cos 60°} \Rightarrow \frac{-1}{1/2} = -2$

(v) $\operatorname{cosec}\left(690°\right) = \frac{1}{\sin\left(-690°\right)} \Rightarrow \frac{1}{-\sin\left(690°\right)} = \frac{1}{-\sin\left(2.360 - 30°\right)}$

........(IV quadrant sinx is negative)

$= \frac{1}{-\left(-\sin 30°\right)} \Rightarrow \frac{1}{1/2} = 2$

(vi)tan225° = tan(180° + 45°) ............(in III quadrant tanx is positive)

$\Rightarrow \tan 45° = 1$

(vii) $\cot\left(-315°\right) = \frac{1}{\tan\left(-315\right)°} \Rightarrow \frac{1}{-\tan\left(315°\right)} = \frac{1}{-\tan\left(360° - 45°\right)}$

.....(tan( - x) = - tanx)

$= \frac{1}{-\left(-\tan 45°\right)} \Rightarrow 1$.....(in IV quadrant tanx is negative)

(viii)sin( - 1230°) = sin1230° .............(using sin( - x) = sinx)

= sin(3.360° + 150°)

= sin150°

= sin(180° - 30°) ............(using sin(180° - x) = sinx)

= sin30°

$= \frac{1}{2}$

(ix)cos495° = cos(360° + 135°) …………(using cos(360° + x) = cosx)

= cos135°

= cos(180° - 45°) …………(using cos(180° - x) = - cosx)

= - cos45°

$$= -\frac{1}{\sqrt{2}}$$

**Question: 2**

Find the values o

**Solution:**

Sin135° = sin(180° - 45°) ………..(using sin(180° - x) = sinx)

$$= \sin 45° \Rightarrow \frac{1}{\sqrt{2}}$$

Cos135° = cos(180° - 45°) ……….(using cos(180° - x) = - cosx)

$$= \cos 45° \Rightarrow -\frac{1}{\sqrt{2}}$$

Tan135° = $\frac{\sin 135°}{\cos 135°} \Rightarrow \frac{1/\sqrt{2}}{-1/\sqrt{2}} = -1$

Cosec135° = $\frac{1}{\sin 135°} \Rightarrow \sqrt{2}$

Sec135° = $\frac{1}{\cos 135°} \Rightarrow -\sqrt{2}$

Cot135° = $\frac{1}{\tan 135°} \Rightarrow -1$

**Question: 3**

Prove that

**Solution:**

(i)sin80°cos20° - cos80°sin20° = sin(80° - 20°)

(using sin(A - B) = sinAcosB - cosAsinB)

= sin60°

$$= \frac{\sqrt{3}}{2}$$

(ii)cos45°cos15° - sin45°sin15° = cos(45° + 15°)

(using cos(A + B) = cosAcosB - sinAsinB)

= cos60°

$$= \frac{1}{2}$$

(iii)cos75°cos15° + sin75°sin15° = cos(75° - 15°)

(using cos(A - B) = cosAcosB + sinAsinB)

= cos60°

$= \frac{1}{2}$

(iv)sin40°cos20° + cos40°sin20° = sin(40° + 20°)

(using sin(A + B) = sinAcosB + cosAsinB)

= sin60°

$= \frac{\sqrt{3}}{2}$

(v)cos130°cos40° + sin130°sin40° = cos(130° - 40°)

(using cos(A - B) = cosAcosB + sinAsinB)

= cos90°

= 0

**Question: 4**

Prove that

**Solution:**

(i)sin(50° + θ)cos(20° + θ) - cos(50° + θ)sin(20° + θ)

= sin(50° + θ - (20° + θ))(using sin(A - B) = sinAcosB - cosAsinB)

= sin(50° + θ - 20° - θ)

= sin30°

$= \frac{1}{2}$

(ii)cos(70° + θ)cos(10° + θ) + sin(70° + θ)sin(10° + θ)

= cos(70° + θ - (10° + θ))(using cos(A - B) = cosAcosB + sinAsinB)

= cos(70° + θ - 10° - θ)

= cos60°

$= \frac{1}{2}$

**Question: 5**

Prove that

**Solution:**

(i)cos(n + 2)x.cos(n + 1)x + sin(n + 2)x.sin(n + 1)x

= sin((n + 2)x + (n + 1)x)(using cos(A - B) = cosAcosB + sinAsinB)

= cos(nx + 2x - (nx + x))

= cos(nx + 2x - nx - x)

= cosx

(ii)$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right)$

$= \cos\left(\frac{\pi}{4} - x + \frac{\pi}{4} - y\right)$(using cos(A + B) = cosAcosB - sinAsinB)

$$= \cos\left(\frac{2\pi}{4} - x - y\right)$$

$$= \cos\left(\frac{\pi}{2} - (x + y)\right) \left(\text{using} \cos\left(\frac{\pi}{2} - x\right) = \sin x\right)$$

= sin(x + y)

**Question: 6**

Prove that

**Solution:**

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}.\tan x}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}.\tan x}}$$

$$\Rightarrow \frac{\dfrac{1 + \tan x}{1 - 1.\tan x}}{\dfrac{1 - \tan x}{1 + 1.\tan x}} = \frac{1 + \tan x}{1 - \tan x}.\frac{1 + \tan x}{1 - \tan x}$$

$$\Rightarrow \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

Hence, Proved.

**Question: 7**

Prove that

**Solution:**

(i)sin75° = sin(90° - 15°) ........(using sin(A - B) = sinAcosB - cosAsinB)

= sin90°cos15° - cos90°sin15°

= 1.cos15° - 0.sin15°

= cos15°

Cos15° = cos(45° - 30°) ............(using cos(A - B) = cosAcosB + sinAsinB)

= cos45°.cos30° + sin45°.sin30°

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2\sqrt{2}}.1 \Rightarrow \frac{\sqrt{3} + 1}{2\sqrt{2}}.\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\sin 75° = \cos 15° = \frac{\sqrt{6} + \sqrt{2}}{4}$$

(ii) $\frac{\cos 135° - \cos 120°}{\cos 135° + \cos 120°} = \frac{\cos(180° - 45°) - \cos(180° - 60°)}{\cos(180° - 45°) + \cos(180° - 60°)}$ (using sin(180° - x) = sinx) (using cos(180° - x) = - cosx)

$$= \frac{-\cos 45° - (-\cos 60°)}{-\cos 45° + (-\cos 60°)}$$

$$= \frac{\cos 60° - \cos 45°}{-(\cos 60° + \cos 45°)}$$

$$= -\frac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + \frac{1}{2}} \Rightarrow -\frac{\frac{1-\sqrt{2}}{2}}{\frac{\sqrt{2}+1}{2}} = -\frac{1-\sqrt{2}}{\sqrt{2}+1}.\frac{(-\sqrt{2}+1)}{(-\sqrt{2}+1)}$$

$$= -\frac{-\sqrt{2}+1+2-\sqrt{2}}{-2+\sqrt{2}-\sqrt{2}+1} \Rightarrow -\frac{-2\sqrt{2}+3}{-1} = 3-2\sqrt{2}$$

(iii)tan15° + cot15° =

First, we will calculate tan15°,

$$\tan 15° = \frac{\sin 15°}{\cos 15°} \text{......................(1)}$$

$$[\cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}, \sin 15° = \sin(45° - 30°)$$

$$= \sin 45°.\cos 30° - \cos 45°.\sin 30° = \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\tan 15° = \frac{\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} \Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} \text{ and } \cot 15° = \frac{1}{\tan 15°}\frac{1}{\frac{\sqrt{3}-1}{\sqrt{3}+1}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Putting in eq(1),

$$\tan 15° + \cot 15° = \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{3-1}\frac{3+1-2\sqrt{3}+3+1+2\sqrt{3}}{2}$$

$$= \frac{8}{2} = 4$$

**Question: 8**

Prove that

**Solution:**

(i) $\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\text{Sin}15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\text{Cos}15^0 - \sin 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{2}{2\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

(ii)cot105° - tan105° = cot(180° - 75°) - tan(180° - 75°)

(II quadrant tanx is negative and cotx as well)

= - cot75° - ( - tan75°)

= tan75° - cot75°

$\text{Tan}75° = \frac{\sin 75°}{\cos 75°} \Rightarrow \frac{\sin(90°-15°)}{\cos(90°-15°)} = \frac{-\cos 15°}{\sin 15°}$

(using sin(90° - x) = - cosx and cos(90° - x) = sinx)

$$= -\frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} \Rightarrow \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

$\text{Cot}75° = \frac{1}{\tan 75°} \Rightarrow \frac{\sqrt{3}-1}{-\sqrt{3}-1}$

$$\text{Cot}105° - \tan 105° = \frac{\sqrt{3}-1}{-\sqrt{3}-1} - \frac{-\sqrt{3}-1}{\sqrt{3}-1} \Rightarrow \frac{(\sqrt{3}-1)-(-\sqrt{3}-1)}{(-\sqrt{3}-1)(\sqrt{3}-1)} = \frac{3+1-2\sqrt{3}-(3+1+2\sqrt{3})}{(-3+1-\sqrt{3}+\sqrt{3})}$$

$= \frac{-4\sqrt{3}}{-2} \Rightarrow 2\sqrt{3}$

(iii)$\frac{\tan 69° + \tan 66°}{1-\tan 69°.\tan 66°} = \tan(69° + 66°) \Rightarrow \tan 135° = \tan(180° - 45°)$

(II quadrant tanx negative)

$\Rightarrow$ - tan45° = - 1

**Question: 9**

Prove that

**Solution:**

First we will take out cos9°common from both numerator and denominator,

$$\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} = \frac{\cos 9(1+\tan 9°)}{\cos 9°(1-\tan 9°)} \Rightarrow \frac{\tan 45° + \tan 9°}{1-\tan 45°.\tan 9°} = \tan(45° + 9°) \Rightarrow \tan 54°$$

$$\left(\text{using} \tan(x+y) = \frac{\tan x + \tan y}{1-\tan x.\tan y} \text{and} \tan 45° = 1\right)$$

**Question: 10**

Prove that

**Solution:**

First we will take out cos8° common from both numerator and denominator,

$$\frac{\cos 8° - \sin 8°}{\cos 8° + \sin 8°} = \frac{\cos 8°(1-\tan 8°)}{\cos 8°(1+\tan 8°)} \Rightarrow \frac{\tan 45° - \tan 8°}{1+\tan 45°.\tan 8°} = \tan(45°-8°) \Rightarrow \tan 37°$$

$$[\text{using} \tan(x-y) = \frac{\tan x - \tan y}{1+\tan x.\tan y} \text{and} \tan 45° = 1$$

**Question: 11**

Prove that

**Solution:**

$$\frac{\cos(\pi+\theta).\cos(-\theta)}{\cos(\pi-\theta).\cos\left(\frac{\pi}{2}+\theta\right)} = \frac{-\cos\theta.\cos\theta}{-\cos\theta.-\sin\theta}$$

$$\Rightarrow \frac{\cos\theta}{-\sin\theta} = -\cot\theta$$

$$\left(\text{Using} \cos(\pi-\theta) = -\cos\theta \text{and} \cos\left(\frac{\pi}{2}-\theta\right) = -\sin\theta, \cos(-\theta) = -\cos\theta\right)$$

$$\left(\text{In III quadrant} \cos x \text{ is negative}, \cos(\pi+\theta) = -\cos\theta\right)$$

**Question: 12**

Prove that

**Solution:**

Using sin(90° + θ) = cosθ and sin( - θ) = sinθ,tan(90° + θ) = - cotθ

Sin(180° + θ) = - sinθ(III quadrant sinx is negative)

$$\frac{\cos\theta}{\sin(90°+\theta)} + \frac{\sin(-\theta)}{\sin(180°+\theta)} - \frac{\tan(90°+\theta)}{\cot\theta} = \frac{\cos\theta}{\cos\theta} + \frac{-\sin\theta}{-\sin\theta} - \frac{-\cot\theta}{\cot\theta}$$

$$= 1 + (1) - (-1) \Rightarrow 1 + 1 + 1 = 3$$

**Question: 13**

Prove that

**Solution:**

Using cos(90° + θ) = - sinθ(I quadrant cosx is positive

cosec( - θ) = - cosecθ

tan(270° - θ) = tan(180° + 90° - θ) = tan(90° - θ) = cotθ

(III quadrant tanx is positive)

Similarly sin(270° + θ) = - cosθ (IV quadrant sinx is negative cot(360° - θ) = cotθ(IV quadrant cotx is negative)

$$= \frac{\sin(180°+\theta).\cos(90°+\theta).\tan(270°-\theta).\cot(360°-\theta)}{\sin(360°-\theta).\cos(360°-\theta).\operatorname{cosec}(-\theta).\sin(270°+\theta)}$$

$$= \frac{-\sin\theta.-\sin\theta.\cot\theta.-\cot\theta}{-\sin\theta.\cos\theta.-\operatorname{cosec}\theta.-\cos\theta}$$

$$= \cot\theta.\tan\theta.\cot\theta.\tan\theta \Rightarrow 1$$

**Question: 14**

If θ and Φ lie in

**Solution:**

$$\text{Given } \sin\theta = \frac{8}{17} \text{and } \cos\phi = \frac{12}{13}$$

$$\cos\theta = \sqrt{}\left(1-\sin^2\theta\right) \Rightarrow \sqrt{}(1-\left(\frac{8}{17}\right)^2 = \sqrt{\left(\frac{289-84}{289}\right)} \Rightarrow \sqrt{\left(\frac{225}{289}\right)} = \frac{15}{17}$$

$$\sin\phi = \sqrt{\left(1-\left(\frac{12}{13}\right)^2\right)} \Rightarrow \sqrt{\left(\frac{169-144}{169}\right)} = \sqrt{}\left(\frac{25}{169}\right) \Rightarrow \frac{5}{13}$$

(i)sin(θ - Φ) = sinθcosΦ + cosθsinΦ

$$= \frac{8}{17}.\frac{12}{13} + \frac{15}{17}.\frac{5}{13} \Rightarrow \frac{96+75}{221} = \frac{171}{221}$$

(ii) cos(θ - Φ) = cosθ.cosΦ + sinθ.sinΦ

$$= \frac{15}{17}.\frac{12}{13} + \frac{8}{17}.\frac{5}{13} \Rightarrow \frac{180+40}{221} = \frac{220}{221}$$

(iii)We will first find out the Values of tanθ and tanΦ,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{8/17}{15/17} = \frac{8}{15}\text{an}\tan\phi = \frac{\sin\phi}{\cos\phi} \Rightarrow \frac{5/13}{12/13} = \frac{5}{12}$$

$$\tan(\theta - \Phi) = \tan(\theta-\phi) = \frac{\tan\theta - \tan\phi}{1+\tan\theta.\tan\phi} \Rightarrow \frac{\frac{8}{15}-\frac{5}{12}}{1+\frac{8}{15}.\frac{5}{12}}$$

**Question: 15**

$$\text{Given}\sin x = \frac{1}{\sqrt{5}}\text{and}\sin y = \frac{1}{\sqrt{10}},$$

Now we will calculate value of cos x and cosy

$$\cos x = \sqrt{\left(1-\sin^2 x\right)} \Rightarrow \sqrt{\left(1-\left(\frac{1}{\sqrt{5}}\right)^2\right)} = \sqrt{\left(\frac{5-1}{5}\right)} \Rightarrow \sqrt{\left(\frac{4}{5}\right)} = \frac{2}{\sqrt{5}}$$

$$\cos y = \sqrt{\left(1-\sin x^2\right)} \Rightarrow \sqrt{\left(1-\left(\frac{1}{\sqrt{10}}\right)^2\right)} = \sqrt{\left(\frac{10-1}{10}\right)} \Rightarrow \sqrt{\left(\frac{9}{10}\right)} = \frac{3}{\sqrt{10}}$$

Sin(x + y) = sinx.cosy + cosx.siny

$$= \frac{1}{\sqrt{5}}.\frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}}.\frac{1}{\sqrt{10}} \Rightarrow \frac{3+2}{\sqrt{50}} = \frac{5}{5\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin(x + y) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x + y = \frac{\pi}{4}$$

**Question: 16**

If x and y are ac

**Solution:**

$$\text{Given } \cos x = \frac{13}{14} \text{ and } \cos y = \frac{1}{7}$$

Now we will calculate value of sinx and siny

$$\sin x = \sqrt{(1-\cos^2 x)} \Rightarrow \sqrt{\left(1-\left(\frac{13}{14}\right)^2\right)} = \sqrt{\left(\frac{196-169}{196}\right)} \Rightarrow \sqrt{\left(\frac{27}{196}\right)} = \frac{3\sqrt{3}}{14}$$

$$\sin y = \sqrt{(1-\cos^2 y)} \Rightarrow \sqrt{\left(1-\left(\frac{1}{7}\right)^2\right)} = \sqrt{\left(\frac{49-1}{49}\right)} \Rightarrow \sqrt{\left(\frac{48}{49}\right)} = \frac{4\sqrt{3}}{7}$$

Hence,

Cos(x - y) = cosx.cosy + sinx.siny

$$= \frac{13}{14}.\frac{1}{7} + \frac{3\sqrt{3}}{14}.\frac{4\sqrt{3}}{7} \Rightarrow \frac{13+36}{98} = \frac{49}{98}$$

$$\cos(x-y) = \frac{1}{2}$$

$$x - y = \frac{\pi}{3}$$

**Question: 17**

$$\text{IfGiven } \sin x = \frac{12}{13} \text{ and } \sin y = \frac{4}{5},$$

Here we will find values of cosx and cosy

$$\cos x = \sqrt{(1-\sin^2 x)} \Rightarrow \sqrt{\left(1-\left(\frac{12}{13}\right)^2\right)} = \sqrt{\left(\frac{169-144}{169}\right)} \Rightarrow \sqrt{\left(\frac{25}{169}\right)} = \frac{5}{13}$$

$$\cos y = \sqrt{(1-\sin^2 y)} \Rightarrow \sqrt{\left(1-\left(\frac{4}{5}\right)^2\right)} = \sqrt{\left(\frac{25-16}{25}\right)} \Rightarrow \sqrt{\left(\frac{9}{25}\right)} = \frac{3}{5}$$

(i) sin(x + y) = sinx.cosy + cosx.siny

$$\Rightarrow \frac{12}{13}.\frac{3}{5} + \frac{5}{13}.\frac{4}{5} \Rightarrow \frac{36+20}{65} = \frac{56}{65}$$

(ii)cos(x + y) = cosx.cosy + sinx.siny

$$= \frac{5}{13}.\frac{3}{5} + \frac{12}{13}.\frac{4}{5} \Rightarrow \frac{15+48}{65} = \frac{63}{65}$$

(iii)Here first we will calculate value of tanx and tany,

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \frac{12/13}{5/13} = \frac{5}{12} \text{ and } \tan y = \frac{\sin y}{\cos y} \Rightarrow \frac{4/5}{3/5} = \frac{4}{3}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x.\tan y} \Rightarrow \frac{\frac{5}{12}-\frac{4}{3}}{1+\frac{5}{12.}.\frac{4}{3}} = \frac{\frac{5-16}{12}}{\frac{36+20}{36}} \Rightarrow \frac{\frac{-11}{12}}{\frac{56}{36}} = \frac{-33}{56}$$

**Question: 18**

$$\text{Given } \cos x = \frac{3}{5} \text{ and } \cos y = \frac{-24}{25}$$

We will first find out value of sinx and siny,

$$\sin x = \sqrt{\left(1-\cos^2 x\right)} \Rightarrow \sqrt{\left(1-\left(\frac{3}{5}\right)^2\right)} = \sqrt{\left(\frac{25-9}{25}\right)} \Rightarrow \sqrt{\left(\frac{16}{25}\right)} = \frac{4}{5}$$

$$\sin y = \sqrt{\left(1-\cos^2 y\right)} \Rightarrow \sqrt{\left(1-\left(\frac{-24}{25}\right)^2\right)} = \sqrt{\left(\frac{625-576}{625}\right)} \Rightarrow \sqrt{\left(\frac{49}{625}\right)} = \frac{7}{25}$$

(i)sin(x + y) = sinx.cosy + cosx.siny

$$= \frac{4}{5}.\frac{-24}{25} + \frac{3}{5}.\frac{7}{25} \Rightarrow \frac{-96+21}{125} = \frac{-75}{125}$$

$$= \frac{-3}{5}$$

(ii)cos(x - y) = cosx.cosy + sinx.siny

$$= = \frac{3}{5}.\frac{-24}{25} + \frac{4}{5}.\frac{7}{25} \Rightarrow \frac{-72+28}{125} = \frac{-44}{125}$$

(iii)Here first we will calculate value of tanx and tany,

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \frac{4/5}{3/5} = \frac{4}{3} \text{ and } \tan y = \frac{\sin y}{\cos y} \Rightarrow \frac{7/25}{-24/25} = \frac{7}{-24}$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x.\tan y} \Rightarrow \frac{\frac{4}{3}+\frac{-7}{24}}{1+\frac{4}{3.}.\frac{-7}{24}} = \frac{\frac{32-7}{24}}{\frac{72-28}{72}} \Rightarrow \frac{\frac{25}{24}}{\frac{44}{72}} = \frac{75}{44}$$

**Question: 19**

Prove that

**Solution:**

$$\text{(i)}\cos\left(\frac{\pi}{3} + x\right) = \cos\frac{\pi}{3}.\cos x - \sin\frac{\pi}{3}.\sin x$$

$$\Rightarrow \frac{1}{2}.\cos x - \frac{\sqrt{3}}{2}.\sin x = \frac{1}{2}\left(\cos x - \sqrt{3}\sin x\right)$$

$$\text{(ii)}\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right)$$

$$= \sin\frac{\pi}{4}.\cos x + \cos\frac{\pi}{4}.\sin x + \sin\frac{\pi}{4}.\cos x - \cos\frac{\pi}{4}.\sin x$$

$$= 2.\sin\frac{\pi}{4}.\cos x \Rightarrow 2.\frac{1}{\sqrt{2}}.\cos x = \sqrt{2}.\cos x$$

$$\text{(iii)}\frac{1}{\sqrt{2}}.\cos\left(\frac{\pi}{4} + x\right) = \frac{1}{\sqrt{2}}.\left(\cos\frac{\pi}{4}.\cos x - \sin\frac{\pi}{4}.\sin x\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}.\cos x - \frac{1}{\sqrt{2}}.\sin x\right) = \frac{1}{2}\left(\cos x - \sin x\right)$$

$$\text{(iv)}\cos x + \cos\left(\frac{2\pi}{3} + x\right) + \cos\left(\frac{2\pi}{3} - x\right)$$

$$= \cos x + \cos\frac{2\pi}{3}.\cos x - \sin\frac{2\pi}{3}.\sin x + \cos\frac{2\pi}{3}.\cos x + \sin\frac{2\pi}{3}.\sin x$$

$$= \cos x + 2.\cos\left(\pi - \frac{\pi}{3}\right).\cos x$$

$$= \cos x + 2.\left(-\frac{1}{2}\right).\cos x$$

$$= \cos x - \cos x \Rightarrow 0$$

**Question: 20**

Prove that

**Solution:**

$$\text{(i)}2\sin\frac{5\pi}{12}.\sin\frac{\pi}{12} = -\left(\cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) - \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)\right)$$

.........[Using –2sinx.siny = cos(x + y)–cos (x–y)]

$$= -\left(\cos\frac{6\pi}{12} - \cos\frac{4\pi}{12}\right)$$

$$= -\left(\cos\frac{\pi}{2} - \cos\frac{\pi}{3}\right) \Rightarrow -\left(0 - \frac{1}{2}\right) = \frac{1}{2}$$

$$\text{(ii)}2\cos\frac{5\pi}{12}.\cos\frac{\pi}{12} = \cos\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \cos\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

...........[using 2cosx.cosy = cos(x + y) + cos(x–y)]

$$= \cos\frac{6\pi}{12} + \cos\frac{4\pi}{12} \Rightarrow \cos\frac{\pi}{2} + \cos\frac{\pi}{3} = 0 + \frac{1}{2}$$

$$= \frac{1}{2}$$

(iii) $$2\sin\frac{5\pi}{12}.\cos\frac{\pi}{12} = \sin\left(\frac{5\pi}{12} + \frac{\pi}{12}\right) + \sin\left(\frac{5\pi}{12} - \frac{\pi}{12}\right)$$

...[Using2sinx.cosy = sin(x + y) + sin(x–y)]

$$= \sin\frac{6\pi}{12} + \sin\frac{4\pi}{12} \Rightarrow \sin\frac{\pi}{2} + \sin\frac{\pi}{3}$$

$$= 1 + \frac{\sqrt{3}}{2} \Rightarrow \frac{2+\sqrt{3}}{2}$$

# Exercise : 15C

**Question: 1**

Prove that

**Solution:**

In this question the following formula will be used:

Sin( A +B)= sinA cos B + cosA sinB

Sin( A - B)= sinA cos B - cosA sinB

= sin150° cosx + cos 150° sinx + sin150° cosx – cos150° sinx

=2sin150°cosx

=2sin(90° + 60°)cosx

=2cos60°cosx

$=2\times \frac{1}{2}$cosx

=cosx

**Question: 2**

Prove that

**Solution:**

In this question the following formulas will be used:

cos (A + B) = cosAcosB – sinAsinB

cos (A - B) = cosAcosB+ sinAsinB

= cos x + cos $120^0$ cosx – sin120sinx + cos $120^0$cosx+sin120sinx

= cosx + 2cos120 cosx

=cosx + 2cos (90 + 30) cosx

= cosx + 2 (-sin30) cosx

=cosx - 2 $\times \frac{1}{2}$cosx

$=\cos x - \cos x$

=0.

**Question: 3**

In this question the following formulas will be used:

sin (A - B) = sinA cos B - cosA sinB

cos (A - B) = cosAcosB+ sinAsinB

$= \sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6} + \cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}$

$= \sin x \times \frac{\sqrt{3}}{2} - \cos x \times \frac{1}{2} + \cos x \times \frac{1}{2} + \sin x \times \frac{\sqrt{3}}{2}$

$= \sin x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{\sqrt{3}}{2}$

$= (\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}) \sin x$

$= \sqrt{3} \sin x.$

**Question: 4**

In this question the following formulas will be used:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(\frac{\pi}{4} + x) = \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \tan x}$$

$$= \frac{1+\tan x}{1-\tan x} \because \tan\frac{\pi}{4} = 1$$

**Question: 5**

In this question the following formulas will be used:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\tan(\frac{\pi}{4} - x) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \tan x}$$

$$= \frac{1-\tan x}{1+\tan x} \because \tan\frac{\pi}{4} = 1$$

**Question: 6**

Express each of t

**Solution:**

1. $\sin 10x + \sin 6x = 2\sin\frac{10x+6x}{2} \cos\frac{10x-sinx}{2}$

$=2\sin\frac{18x}{2} \cos\frac{4x}{2}$

$=2\sin 9x \cos 2x$

Using,

sin( A +B)= sinA cos B + cosA sinB

2. $\sin 7x - \sin 3x = 2\cos\frac{7x+3x}{2} \sin\frac{7x-3x}{2}$

$= 2\cos\frac{10x}{2} \sin\frac{4x}{2}$

= 2cos5x sin2x

Using,

sin( A - B)= sinA cos B - cosA sinB

$3. \cos 7x + \cos 5x = 2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}$

$= 2\cos\frac{12x}{2}\cos\frac{2x}{2}$

$= 2\cos 6x \cos x$

Using,

cos (A + B) = cosAcosB – sinAsinB

$4. \cos 2x - \cos 4x$= -2sin $\frac{2x+4x}{2}\sin\frac{2x-4x}{2}$

= -2sin $\frac{6x}{2}\sin\frac{-2x}{2}$

= 2sin3x sinx

Using,

cos (A - B) = cosAcosB+ sinAsinB

**Question: 7**

Express each of t

**Solution:**

i) 2sin 6x cos 4x = sin (6x+4x) + sin (6x-4x)

= sin 10x + sin 2x

Using,

2sinAcosB = sin (A+ B) + sin (A - B)

ii) 2cos 5x sin3x = sin (5x + 3x) – sin (5x – 3x)

= sin8x – sin2x

Using,

2cosAsinB = sin(A + B) – sin (A - B)

iii) 2cos7xcos3x = cos (7x+3x) + cos (7x – 3x)

=cos10x + cos 4x

Using,

2cosAcosB = cos (A+ B) + cos (A - B)

iv)2sin8xsin2x = cos (8x - 2x) – cos (8x + 2x)

= cos6x – cos10x

Using,

2sinAsinB = cos (A - B) – cos (A+ B)

**Question: 8**

Prove that

**Solution:**

$$\frac{\sin x + \sin 3x}{\cos x - \cos 3x}$$

$$=\frac{2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}}{-2\sin\frac{x+3x}{2}\sin\frac{x-3x}{2}}$$

$= \frac{2\sin\frac{4x}{2}\cos\frac{2x}{2}}{2\sin\frac{4x}{2}\sin\frac{2x}{2}}$

$= \frac{\cos x}{\sin x}$

= cotx

Using the formula,

sinA + sinB = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

cosA - cosB = -2sin$\frac{A+B}{2}$sin$\frac{A-B}{2}$

**Question: 9**

Prove that

**Solution:**

$\frac{\sin 7x - \sin 5x}{\cos 7x + \cos 5x}$

$= \frac{2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}}$

$= \frac{2\cos 6x \sin x}{2\cos 6x \cos x}$

$= \frac{\sin x}{\cos x}$

= tanx

Using the formula,

sinA - sinB = 2cos$\frac{A+B}{2}$ sin$\frac{A-B}{2}$

cosA + cosB = 2cos$\frac{A+B}{2}$cos$\frac{A-B}{2}$

**Question: 10**

Prove that

**Solution:**

$\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$

$= \frac{2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}$

$= \frac{2\sin 4x \cos x}{2\cos 4x \cos x}$

= tan4x

Using the formula,

sinA + sinB = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

cosA + cosB = 2cos$\frac{A+B}{2}$cos$\frac{A-B}{2}$

**Question: 11**

$= \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$=\frac{-2\sin\frac{9x+5x}{2}\sin\frac{9x-5x}{2}}{2\cos\frac{17x+3x}{2}\sin\frac{17x-3x}{2}}$$

$$=\frac{-2\sin 7x\sin 2x}{2\cos 10x\sin 7x}$$

$$=\frac{-\sin 2x}{\cos 10x}$$

Using the formula,

cosA - cosB = -2sin$\frac{A+B}{2}$sin$\frac{A-B}{2}$

sinA - sinB = 2cos$\frac{A+B}{2}$ sin$\frac{A-B}{2}$

**Question: 12**

Prove that

**Solution:**

$$=\frac{\sin x+\sin 3x+\sin 5x}{\cos x+\cos 3x+\cos 5x}$$

$$=\frac{(\sin 5x+\sin x)+\sin 3x}{(\cos 5x+\cos x)+\cos 3x}$$

$$=\frac{2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2}+\sin 3x}{2\cos\frac{5x+x}{2}\cos\frac{5x-x}{2}+\cos 3x}$$

$$=\frac{2\sin 3x\cos x+\sin 3x}{2\cos 3x\cos x+\cos 3x}$$

$$=\frac{\sin 3x(2\cos x+1)}{\cos 3x(2\cos x+1)}$$

= tan3x.

Using the formula,

sinA + sinB = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

cosA + cosB = 2cos$\frac{A+B}{2}$cos$\frac{A-B}{2}$

**Question: 13**

Prove that

**Solution:**

$$=\frac{(\sin 7x+\sin 5x)+(\sin 9x+\sin 3x)}{(\cos 7x+\cos 5x)+(\cos 9x+\cos 3x)}$$

$$=\frac{2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2}+2\sin\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}+2\cos\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}$$

$$=\frac{2\sin 6x\cos x+2\sin 6x\cos 3x}{2\cos 6x\cos x+2\cos 6x\cos 3x}$$

$$=\frac{2\sin 6x(\cos x+\cos 3x)}{2\cos 6x(\cos x+\cos 3x)}$$

$$=\frac{\sin 6x}{\cos 6x}$$

$=\tan 6x$

Using the formula,

sinA + sinB = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

**Question: 14**

Prove that

**Solution:**

L.H.S

cot 4x (sin 5x + sin3x)

$= \cot 4x \left(2\sin\frac{5x+3x}{2}\cos\frac{5x-3x}{2}\right)$

= cot 4x (2 sin4x cosx)

$= \frac{\cos 4x}{\sin 4x}(2 \sin 4x \cos x)$

= 2cos4xcosx

R.H.S

cot x (sin 5x - sin3x)

$= \cot x \left(2\cos\frac{5x+3x}{2}\sin\frac{5x-3x}{2}\right)$

= cot x (2 cos4x sinx)

$= \frac{\cos x}{\sin x}(2 \cos 4x \sin x)$

= 2cos4xcosx

L.H.S=R.H.S

Hence, proved.

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

**Question: 15**

Prove that

**Solution:**

= (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

$= \left(2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}\right)\sin x + \left(-2\sin\frac{3x+x}{2}\sin\frac{3x-x}{2}\right)\cos x$

= (2sin2x cosx) sinx-(2sin2x sinx) cosx

= 0.

Using the formula,

$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$

$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

**Question: 16**

Prove that

**Solution:**

$= (\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$=(-2\sin\frac{x+y}{2}\sin\frac{x-y}{2})^{2}+(2\cos\frac{x+y}{2}\sin\frac{x-y}{2})^{2}$

$=4\sin^2\left(\frac{x-y}{2}\right)(\sin^2\left(\frac{x+y}{2}\right)+\cos^2\left(\frac{x+y}{2}\right))$

$=4\sin^2\left(\frac{x-y}{2}\right)$

Using the formula,

$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

**Question: 17**

Prove that

**Solution:**

$=\frac{\sin 2x-\sin 2y}{\cos 2y-\cos 2x}$

$=\frac{2\cos\frac{2x+2y}{2}\sin\frac{2x-2y}{2}}{-2\sin\frac{2x+2y}{2}\sin\frac{2y-2x}{2}}$

$=\frac{\cos(x+y)\sin(x-y)}{\sin(x+y)\sin(x-y)}$

$=\frac{\cos(x+y)}{\sin(x+y)}$

$=\cot(x+y)$

Using the formula,

$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\sin A-\sin B=2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$

**Question: 18**

Prove that

**Solution:**

$=\frac{\cos x-\cos y}{\cos y-\cos x}$

$=\frac{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{-2\sin\frac{x+y}{2}\sin\frac{y-x}{2}}$

$=\frac{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{2\sin\frac{x+y}{2}\sin\frac{x-y}{2}}$

$=\frac{\cos\frac{x+y}{2}\cos\frac{x-y}{2}}{\sin\frac{x+y}{2}\sin\frac{x-y}{2}}$

$=\cot\frac{x+y}{2}\cot\frac{x-y}{2}$

Using the formula,

$\cos A-\cos B=-2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

$\cos A+\cos B=2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

**Question: 19**

Prove that

**Solution:**

$=\frac{sinx+\sin y}{sinx-\sin y}$

$=\frac{2sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2cos\frac{x+y}{2}\sin\frac{x-y}{2}}$

$=\tan\frac{x+y}{2}\cot\frac{x-y}{2}$

Using the formula,

sinA + sinB = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

sinA - sinB = 2cos$\frac{A+B}{2}$ sin$\frac{A-B}{2}$

**Question: 20**

Prove that

**Solution:**

=Sin3x+sin2x-sinx

= (sin3x- sinx)+sin2x

= ( 2cos$\frac{3x+x}{2}$sin$\frac{3x-2x}{2}$) + sin2x

= 2cos2xsinx +sin2x

= 2cos2xsinx + 2sinxcosx

= 2sinx (cos2x + cosx )

= 2sinx (2cos$\frac{2x+x}{2}$ cos$\frac{2x-x}{2}$ )

= 4sinxcos$\frac{x}{2}$ cos$\frac{3x}{2}$

Using the formula,

sinA - sinB = 2cos$\frac{A+B}{2}$ sin$\frac{A-B}{2}$

cosA + cosB = 2cos$\frac{A+B}{2}$cos$\frac{A-B}{2}$

**Question: 21**

$=\frac{\cos 4x\sin 3x-\cos 2x\sin x}{\sin 4x\sin x+\cos 6x\cos x}$

$=\frac{2\cos 4x\sin 3x-2\cos 2x\sin x}{2\sin 4x\sin x+2\cos 6x\cos x}$

$=\frac{\sin(4x+3x)-\sin(4x-3x)-\{\sin(2x+x)-\sin(2x-x)\}}{\cos(4x-x)-\cos(4x+x)+\cos(6x+x)+\cos(6x-x)}$

$=\frac{\sin 7x+\sin x-\sin 3x+\sin x}{\cos 3x-\cos 5x+\cos 7x+\cos 5x}$

$=\frac{\sin 7x-\sin 3x}{\cos 3x+\cos 7x}$

$=\frac{2\cos\frac{7x+3x}{2}\sin\frac{7x-3x}{2}}{2\cos\frac{7x+3x}{2}\cos\frac{7x-3x}{2}}$

$=\frac{\cos 5x\sin 2x}{\cos 5x\cos 2x}$

$=\tan 2x$

Using the formulas,

2cosAsinB = sin (A + B) – sin (A - B)

2cosAcosB = cos (A + B) + cos (A - B)

2sinAsinB = cos (A - B) – cos (A + B)

**Question: 22**

Prove that

**Solution:**

$= \frac{\cos 2x \sin x + \cos 6x \sin 3x}{\sin 2x \sin x + \sin 6x \sin 3x}$

$= \frac{2\cos 2x \sin x + 2\cos 6x \sin 3x}{2\sin 2x \sin x + 2\sin 6x \sin 3x}$

$= \frac{\sin(2x+x) - \sin(2x-x) + \{\sin(6x+3x) - \sin(6x-3x)\}}{\cos(2x-x) - \cos(2x+x) + \cos(6x-3x) - \cos(6x+3x)}$

$= \frac{\sin 3x - \sin x + \sin 9x - \sin 3x}{\cos x - \cos 3x + \cos 3x - \cos 9x}$

$= \frac{\sin 9x - \sin x}{\cos x - \cos 9x}$

$= \frac{2\cos\frac{9x+x}{2}\sin\frac{9x-x}{2}}{-2\sin\frac{x+9x}{2}\sin\frac{x-9x}{2}}$

$= \frac{2\cos\frac{9x+x}{2}\sin\frac{9x-x}{2}}{2\sin\frac{x+9x}{2}\sin\frac{9x-x}{2}}$

$= \frac{\cos 5x \sin 4x}{\sin 5x \cos 4x}$

$=\cot 5x$

Using the formulas,

2cosAsinB = sin (A + B) – sin (A - B)

2sinAsinB = cos (A - B) – cos (A + B)

**Question: 23**

Prove that

**Solution:**

L.H.S

$=\sin 10° \sin 30° \sin 50° \sin 70°$

$=\frac{1}{2}(2\sin 70° \sin 10°) \sin 50° \frac{1}{2}$

$=\frac{1}{4}\{\cos(70° - 10°) - \cos(70° + 10°)\} \sin 50°$

$=\frac{1}{4}\{\cos 60° \sin 50° - \cos 80° \sin 50°\}$

$=\frac{1}{4}\{\frac{1}{2} \sin 50° - \cos 80° \sin 50°\}\}$

$=\frac{1}{8}\{\sin 50° - 2\cos 80° \sin 50°\}$

$=\frac{1}{8}\{\sin 50° - (\sin(80° + 50°) - \sin(80° - 50°)\}$

$=\frac{1}{8}\{\sin 50° - \sin 130° + \sin 30°\}$

$=\frac{1}{8}\{\sin 50° - \sin 130° + \frac{1}{2}\}$

$=\frac{1}{8}\{\sin 50° - \sin(180° - 50°) + \frac{1}{2}\}$

$=\frac{1}{8}\{\sin 50° - \sin 50° + \frac{1}{2}\}$

$=\frac{1}{16}$

=R.H.S

**Question: 24**

Prove that

**Solution:**

L.H.S

$=\sin 20° \sin 40° \sin 60° \sin 80°$

$=\frac{1}{2}(2\sin 80° \sin 20°) \sin 40° \frac{\sqrt{3}}{2}$

$=\frac{\sqrt{3}}{4}\{\cos(80° - 20°) - \cos(80° + 20°)\} \sin 40°$

$=\frac{\sqrt{3}}{4}\{\cos 60° \sin 40° - \cos 100° \sin 40°\}$

$=\frac{\sqrt{3}}{4}\{\frac{1}{2}\sin 40° - \cos 100° \sin 40°\}\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - 2\cos 100° \sin 40°\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - (\sin(100° + 40°) - \sin(100° - 40°)\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 140° + \sin 60°\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 140° + \frac{\sqrt{3}}{2}\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin(180° - 40°) + \frac{\sqrt{3}}{2}\}$

$=\frac{\sqrt{3}}{8}\{\sin 40° - \sin 40° + \frac{\sqrt{3}}{2}\}$

$=\frac{3}{16}$

=R.H.S

**Question: 25**

Prove that

**Solution:**

L.H.S

$=\cos 10° \cos 30° \cos 50° \cos 70°$

$=\frac{1}{2}(2\cos 70° \cos 10°) \cos 50° \frac{\sqrt{3}}{2}$

$=\frac{\sqrt{3}}{4}\{\cos(70° + 10°) + \cos(70° - 10°)\} \cos 50°$

$=\frac{\sqrt{3}}{4}\{\cos 80° \cos 50° + \cos 60° \cos 50°\}$

$=\frac{\sqrt{3}}{4}\{cos 80° \cos 50° + \frac{1}{2}\cos 50°\}\}$

$=\frac{\sqrt{3}}{8}\{2 \cos 80° \cos 50° + \cos 50°\}$

$=\frac{\sqrt{3}}{8}\{(\cos(80° + 50°) - \cos(80° - 50°) + \cos 50°\}$

$=\frac{\sqrt{3}}{8}\{\cos 130° - \cos 30° + \cos 50°\}$

$=\frac{\sqrt{3}}{8}\{\cos 130° - \cos 50° + cos 30°\}$

$=\frac{\sqrt{3}}{8}\{\cos(180° - 50°) - \cos( 50°) + \frac{\sqrt{3}}{2}\}$

$=\frac{\sqrt{3}}{8}\{\cos 50° - \cos 50° + \frac{\sqrt{3}}{2}\}$

$=\frac{3}{16}$

**Question: 26**

If cosx+cosy = $\frac{1}{3}$ --------- i

sinx+siny = $\frac{1}{4}$ ----------ii

dividing ii by I we get,

$\Rightarrow \frac{\sin x+\sin y}{\cos x+\cos y} = \frac{\frac{1}{4}}{\frac{1}{3}}$

$\Rightarrow \frac{\sin x+\sin y}{\cos x+\cos y} = \frac{3}{4}$

$\Rightarrow \frac{2 \sin\frac{x+y}{2}\cos\frac{x-y}{2}}{2 \cos\frac{x+y}{2}\cos\frac{x-y}{2}} = \frac{3}{4}$

$\Rightarrow \tan(\frac{x+y}{2}) = \frac{3}{4}$

Using the formula,

sinA + sinB = $2\sin\frac{A+B}{2} \cos\frac{A-B}{2}$

cosA + cosB = $2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$

**Question: 27 A**

Prove that

**Solution:**

L.H.S

$= 2\cos 45° \cos 15°$

$=2\cos 45° \cos(45° - 30°)$

$=2\frac{1}{\sqrt{2}}\ (\cos 45° \cos 30° + \sin 45° \sin 30°)$

$=\sqrt{2}(\ \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\ \frac{1}{\sqrt{2}}\times\frac{1}{2})$

$=\sqrt{2}(\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}})$

$=\sqrt{2}(\frac{\sqrt{3}+1}{2\sqrt{2}})$

$=\frac{\sqrt{3}+1}{\sqrt{2}}$

**Question: 27 B**

Prove that

**Solution:**

L.H.S

$= 2\sin 75° \sin 15°$

$=2\sin(45°+30°)\sin(45°-30°)$

$=\cos(45°-30°-45°-30°)-\cos(45°+30°+45°-30°)$

$=\cos(-60°)$ - $\cos 90°$

$=\cos 60°-0$

$=\frac{1}{2}$

**Question: 27 C**

Prove that

**Solution:**

L.H.S

$\Rightarrow \cos 15°-\sin 15°$

$\Rightarrow \cos(45°-30°)-\sin(45°-30°)$

$\Rightarrow (\cos 45° \cos 30°+\sin 45° \sin 30°)$ - $(\sin 45° \cos 30°-\cos 45° \sin 30°)$

$\Rightarrow (\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2})-(\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2})$

$\Rightarrow \frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}-\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}$

$\Rightarrow \frac{2}{2\sqrt{2}}$

$\Rightarrow \frac{1}{\sqrt{2}}$

# Exercise : 15D

**Question: 1 A**

If $\sin x=\frac{\sqrt{5}}{3}$

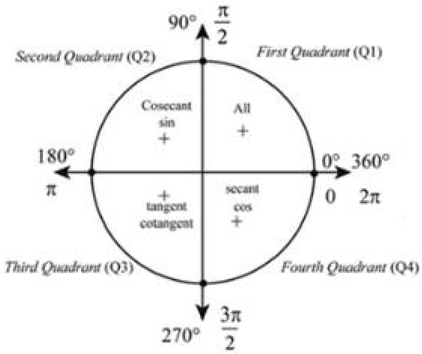
To find: sin2x

We know that,

sin2x = 2 sinx cosx ...(i)

Here, we don’t have the value of cos x. So, firstly we have to find the value of cosx

We know that,




$\sin^2 x + \cos^2 x = 1$

Putting the values, we get

$\left(\frac{\sqrt{5}}{3}\right)^2 + \cos^2 x = 1$

$\Rightarrow \frac{5}{9} + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \frac{5}{9}$

$\Rightarrow \cos^2 x = \frac{9-5}{9}$

$\Rightarrow \cos^2 x = \frac{4}{9}$

$\Rightarrow \cos x = \sqrt{\frac{4}{9}}$

$\Rightarrow \cos x = \pm\frac{2}{3}$

It is given that $0 < x < \frac{\pi}{2}$

$\Rightarrow \cos x = \frac{2}{3}$

Putting the value of sinx and cosx in eq. (i), we get

sin2x = 2sinx cosx

$\sin 2x = 2 \times \frac{\sqrt{5}}{3} \times \frac{2}{3}$

$\therefore \sin 2x = \frac{4\sqrt{5}}{9}$

**Question: 1 B**

If $\sin x = \frac{\sqrt{5}}{3}$

To find: cos2x

We know that,

$\cos 2x = 1 - 2\sin^2 x$

Putting the value, we get

$\cos 2x = 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2$

$\cos 2x = 1 - 2 \times \frac{5}{9}$

$\cos 2x = 1 - \frac{10}{9}$

$\cos 2x = \frac{9-10}{9}$

$\therefore \cos 2x = -\frac{1}{9}$

**Question: 1 C**

If From part (i) and (ii), we have

$\sin 2x = \frac{4\sqrt{5}}{9}$

and $\cos 2x = -\frac{1}{9}$

We know that,

$\tan x = \frac{\sin x}{\cos x}$

Replacing x by 2x, we get

$\tan 2x = \frac{\sin 2x}{\cos 2x}$

Putting the values of sin 2x and cos 2x, we get

$\tan 2x = \frac{\frac{4\sqrt{5}}{9}}{-\frac{1}{9}}$

$\tan 2x = \frac{4\sqrt{5}}{9} \times (-9)$

$\therefore \tan 2x = -4\sqrt{5}$

**Question: 2 A**

If $\cos x = \frac{-3}{5}$

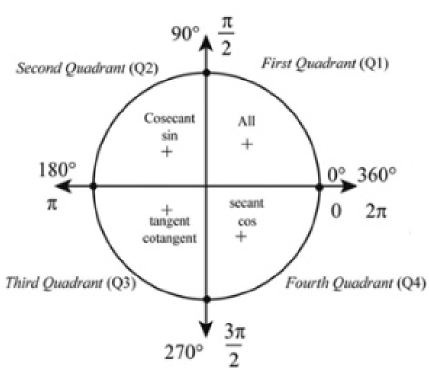
To find: sin2x

We know that,

sin2x = 2 sinx cosx ...(i)

Here, we don’t have the value of sin x. So, firstly we have to find the value of sinx

We know that,




$\cos^2 x + \sin^2 x = 1$

Putting the values, we get

$\left(-\frac{3}{5}\right)^2 + \sin^2 x = 1$

$\Rightarrow \frac{9}{25} + \sin^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \frac{9}{25}$

$\Rightarrow \sin^2 x = \frac{25-9}{25}$

$\Rightarrow \sin^2 x = \frac{16}{25}$

$\Rightarrow \sin x = \sqrt{\frac{16}{25}}$

$\Rightarrow \sin x = \pm\frac{4}{5}$

It is given that $\pi < x < \frac{3\pi}{2}$

$\Rightarrow \sin x = -\frac{4}{5}$

Putting the value of sinx and cosx in eq. (i), we get

sin2x = 2sinx cosx

$\sin 2x = 2 \times \left(-\frac{4}{5}\right) \times \left(-\frac{3}{5}\right)$

$\therefore \sin 2x = \frac{24}{25}$

**Question: 2 B**

If $\cos x = \frac{-3}{5}$

To find: cos2x

We know that,

$\cos 2x = 2\cos^2 x - 1$

Putting the value, we get

$\cos 2x = 2\left(-\frac{3}{5}\right)^2 - 1$

$\cos 2x = 2 \times \frac{9}{25} - 1$

$\cos 2x = \frac{18}{25} - 1$

$\cos 2x = \frac{18-25}{25}$

$\therefore \cos 2x = -\frac{7}{25}$

**Question: 2 C**

If From part (i) and (ii), we have

$\sin 2x = \frac{24}{25}$

and $\cos 2x = -\frac{7}{25}$

We know that,

$\tan x = \frac{\sin x}{\cos x}$

Replacing x by 2x, we get

$\tan 2x = \frac{\sin 2x}{\cos 2x}$

Putting the values of sin 2x and cos 2x, we get

$\tan 2x = \frac{\frac{24}{25}}{-\frac{7}{25}}$

$\tan 2x = \frac{24}{25} \times (-\frac{25}{7})$

$\therefore \tan 2x = -\frac{24}{7}$

**Question: 3 A**

If $\tan x = -\frac{5}{12}$

To find: sin 2x

We know that,

$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

Putting the values, we get

$\sin 2x = \frac{2\times\left(-\frac{5}{12}\right)}{1+\left(-\frac{5}{12}\right)^2}$

$\sin 2x = \frac{-\frac{5}{6}}{1+\frac{25}{144}}$

$\sin 2x = \frac{-5}{6\left(\frac{144+25}{144}\right)}$

$\sin 2x = \frac{-5\times144}{6\times169}$

$\sin 2x = \frac{-5\times24}{169}$

$\sin 2x = -\frac{120}{169}$

**Question: 3 B**

If $\tan x = -\frac{5}{12}$

To find: cos 2x

We know that,

$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

Putting the values, we get

$\cos 2x = \frac{1-\left(-\frac{5}{12}\right)^2}{1+\left(-\frac{5}{12}\right)^2}$

$\cos 2x = \frac{1-\frac{25}{144}}{1+\frac{25}{144}}$

$\cos 2x = \frac{\frac{144-25}{144}}{\left(\frac{144+25}{144}\right)}$

$\cos 2x = \frac{\frac{119}{144}}{\frac{169}{144}}$

$\cos 2x = \frac{119}{169}$

**Question: 3 C**

If $\tan x = -\frac{5}{12}$

To find: tan 2x

We know that,

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

Putting the values, we get

$\tan 2x = \frac{2\times\left(-\frac{5}{12}\right)}{1-\left(-\frac{5}{12}\right)^2}$

$\tan 2x = \frac{-\frac{5}{6}}{1-\frac{25}{144}}$

$\tan 2x = \frac{-5}{6\left(\frac{144-25}{144}\right)}$

$\tan 2x = \frac{-5\times144}{6\times119}$

$\tan 2x = \frac{-5\times24}{119}$

$\tan 2x = -\frac{120}{119}$

**Question: 4 A**

If $\sin x = \frac{1}{6}$

To find: sin 3x

We know that,

$\sin 3x = 3\sin x - \sin^3 x$

Putting the values, we get

$\sin 3x = 3\times\left(\frac{1}{6}\right)-\left(\frac{1}{6}\right)^3$

$\sin 3x = \frac{1}{6}\left[3-\left(\frac{1}{6}\right)^2\right]$

$\sin 3x = \frac{1}{6}\left[3-\frac{1}{36}\right]$

$\sin 3x = \frac{1}{6}\left[\frac{108-1}{36}\right]$

$\sin 3x = \frac{107}{216}$

**Question: 4 B**

If $\cos x = -\frac{1}{2}$

To find: cos 3x

We know that,

$\cos 3x = 4\cos^3 x - 3\cos x$

Putting the values, we get

$\cos 3x = 4 \times \left(-\frac{1}{2}\right)^3 - 3 \times \left(-\frac{1}{2}\right)$

$\cos 3x = 4 \times \left(-\frac{1}{8}\right) + \frac{3}{2}$

$\cos 3x = \left[-\frac{1}{2} + \frac{3}{2}\right]$

$\cos 3x = \left[\frac{-1+3}{2}\right]$

$\cos 3x = \frac{2}{2}$

cos 3x = 1

**Question: 5**

Prove that

**Solution:**

To Prove: $\frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$

Taking LHS,

$= \frac{\cos 2x}{\cos x - \sin x}$

$= \frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$ [$\because$ cos 2x = $\cos^2$x – $\sin^2$x]

Using, ($a^2$ – $b^2$) = (a – b)(a + b)

$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x - \sin x)}$

= cos x + sin x

= RHS

$\therefore$ LHS = RHS

Hence Proved

**Question: 6**

Prove that

**Solution:**

To Prove: $\frac{\sin 2x}{1 + \cos 2x} = \tan x$

Taking LHS,

$= \frac{\sin 2x}{1 + \cos 2x}$

$= \frac{2 \sin x \cos x}{1 + \cos 2x}$ [$\because$ sin 2x = 2 sinx cosx]

$= \frac{2 \sin x \cos x}{2 \cos^2 x}$ [$\because$ 1 + cos 2x = 2 $\cos^2$x]

$= \frac{\sin x}{\cos x}$

= tan x $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

$\therefore$ LHS = RHS

Hence Proved

**Question: 7**

$\frac{\sin 2x}{1-\cos 2x} = \tan x$

Taking LHS,

$= \frac{\sin 2x}{1-\cos 2x}$

$= \frac{2\sin x \cos x}{1-\cos 2x}$ [$\because$ sin 2x = 2 sinx cosx]

$= \frac{2\sin x \cos x}{2\sin^2 x}$ [$\because$ 1 – cos 2x = 2 $\sin^2$x]

$= \frac{\cos x}{\sin x}$

= cot x $\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

= RHS

$\therefore$ LHS = RHS

Hence Proved

**Question: 8**

Prove that

**Solution:**

To Prove: $\frac{\tan 2x}{1+\sec 2x} = \tan x$

Taking LHS,

$= \frac{\frac{\sin 2x}{\cos 2x}}{1+\frac{1}{\cos 2x}}$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta} \;\&\sec\theta = \frac{1}{\cos\theta}\right]$

$= \frac{\sin 2x}{\cos 2x\left(\frac{\cos 2x+1}{\cos 2x}\right)}$

$= \frac{\sin 2x}{1+\cos 2x}$

$= \frac{2\sin x \cos x}{1+\cos 2x}$ [$\because$ sin 2x = 2 sinx cosx]

$= \frac{2\sin x \cos x}{2\cos^2 x}$ [$\because$ 1 + cos 2x = 2 $\cos^2$x]

$= \frac{\sin x}{\cos x}$

= tan x $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

$\therefore$ LHS = RHS

Hence Proved

**Question: 9**

Prove that

**Solution:**

To Prove: sin 2x(tan x + cot x) = 2

Taking LHS,

sin 2x(tan x + cot x)

We know that,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ & $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$= \sin 2x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$

$= \sin 2x\left(\frac{\sin x(\sin x)+\cos x(\cos x)}{\cos x \sin x}\right)$

$= \sin 2x\left(\frac{\sin^2 x+\cos^2 x}{\cos x \sin x}\right)$

We know that,

sin 2x = 2 sinx cosx

$= 2\sin x\cos x\left(\frac{\sin^2 x+\cos^2 x}{\cos x \sin x}\right)$

= $2(\sin^2 x + \cos^2 x)$

= 2 × 1 [∵ $\cos^2\theta + \sin^2\theta = 1$]

= 2

= RHS

∴ LHS = RHS

Hence Proved

**Question: 10**

Prove that

**Solution:**

To Prove: cosec 2x + cot 2x = cot x

Taking LHS,

= cosec 2x + cot 2x ...(i)

We know that,

$\operatorname{cosec} x = \frac{1}{\sin x}$ & $\cot x = \frac{\cos x}{\sin x}$

Replacing x by 2x, we get

$\operatorname{cosec} 2x = \frac{1}{\sin 2x}$ & $\cot 2x = \frac{\cos 2x}{\sin 2x}$

So, eq. (i) becomes

$= \frac{1}{\sin 2x} + \frac{\cos 2x}{\sin 2x}$

$= \frac{1+\cos 2x}{\sin 2x}$

$= \frac{2\cos^2 x}{\sin 2x}$ [∵ 1 + cos 2x = $2\cos^2 x$]

$= \frac{2\cos^2 x}{2\sin x\cos x}$ [∵ sin 2x = 2 sinx cosx]

$= \frac{\cos x}{\sin x}$

= cot x $\left[\because \cot x = \frac{\cos x}{\sin x}\right]$

= RHS

Hence Proved

**Question: 11**

Prove that

**Solution:**

To Prove: $\cos 2x + 2\sin^2 x = 1$

Taking LHS,

$= \cos 2x + 2\sin^2 x$

$= (2\cos^2 x - 1) + 2\sin^2 x$ [$\because 1 + \cos 2x = 2\cos^2 x$]

$= 2(\cos^2 x + \sin^2 x) - 1$

$= 2(1) - 1$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= 2 - 1$

$= 1$

= RHS

$\therefore$ LHS = RHS

Hence Proved

**Question: 12**

Prove that

**Solution:**

To Prove: $(\sin x - \cos x)^2 = 1 - \sin 2x$

Taking LHS,

$= (\sin x - \cos x)^2$

Using,

$(a - b)^2 = (a^2 + b^2 - 2ab)$

$= \sin^2 x + \cos^2 x - 2\sin x \cos x$

$= (\sin^2 x + \cos^2 x) - 2\sin x \cos x$

$= 1 - 2\sin x \cos x$ [$\because \cos^2\theta + \sin^2\theta = 1$]

$= 1 - \sin 2x$ [$\because \sin 2x = 2\sin x \cos x$]

= RHS

$\therefore$ LHS = RHS

Hence Proved

**Question: 13**

Prove that

**Solution:**

To Prove: $\cot x - 2\cot 2x = \tan x$

Taking LHS,

$= \cot x - 2\cot 2x$ ...(i)

We know that,

$$\cot x = \frac{\cos x}{\sin x}$$

Replacing x by 2x, we get

$\cot 2x = \frac{\cos 2x}{\sin 2x}$

So, eq. (i) becomes

$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{\sin 2x}\right)$

$= \frac{\cos x}{\sin x} - 2\left(\frac{\cos 2x}{2\sin x\cos x}\right)$ [∵ sin 2x = 2 sinx cosx]

$= \frac{\cos x}{\sin x} - \left(\frac{\cos 2x}{\sin x\cos x}\right)$

$= \frac{\cos x(\cos x) - \cos 2x}{\sin x\cos x}$

$= \frac{\cos^2 x - \cos 2x}{\sin x\cos x}$

$= \frac{\cos^2 x - [2\cos^2 x - 1]}{\sin x\cos x}$ [∵ 1 + cos 2x = 2 $\cos^2$x]

$= \frac{\cos^2 x - 2\cos^2 x + 1}{\sin x\cos x}$

$= \frac{-\cos^2 x + 1}{\sin x\cos x}$

$= \frac{1 - \cos^2 x}{\sin x\cos x}$

$= \frac{\cos^2 x + \sin^2 x - \cos^2 x}{\sin x\cos x}$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

$= \frac{\sin^2 x}{\sin x\cos x}$

$= \frac{\sin x}{\cos x}$

= tan x $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 14**

Prove that

**Solution:**

To Prove: $\cos^4 x + \sin^4 x = \frac{1}{2}(2 - \sin^2 2x)$

Taking LHS,

= $\cos^4$x + $\sin^4$x

Adding and subtracting 2$\sin^2$x $\cos^2$x, we get

= $\cos^4$x + $\sin^4$x + 2$\sin^2$x $\cos^2$x – 2$\sin^2$x $\cos^2$x

We know that,

$a^2 + b^2 + 2ab = (a + b)^2$

= ($\cos^2$x + $\sin^2$x) – 2$\sin^2$x $\cos^2$x

= (1) – 2$\sin^2$x $\cos^2$x [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

= 1 – 2$\sin^2$x $\cos^2$x

Multiply and divide by 2, we get

$= \frac{1}{2}[2 \times (1 - 2\sin^2 x \cos^2 x)]$

$= \frac{1}{2}[2 - 4\sin^2 x \cos^2 x]$

$= \frac{1}{2}[2 - (2\sin x \cos x)^2]$

$= \frac{1}{2}[2 - (\sin 2x)^2]$ [∵ sin 2x = 2 sinx cosx]

$= \frac{1}{2}(2 - \sin^2 2x)$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 15**

Prove that

**Solution:**

To Prove: $\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{1}{2}(2 + \sin 2x)$

Taking LHS,

$= \frac{\cos^3 x - \sin^3 x}{\cos x - \sin x}$ ...(i)

We know that,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

So, $\cos^3 x - \sin^3 x = (\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x)$

So, eq. (i) becomes

$= \frac{(\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x)}{\cos x - \sin x}$

$= \cos^2 x + \cos x \sin x + \sin^2 x$

$= (\cos^2 x + \sin^2 x) + \cos x \sin x$

$= (1) + \cos x \sin x$ [∵ $\cos^2 \theta + \sin^2 \theta = 1$]

$= 1 + \cos x \sin x$

Multiply and Divide by 2, we get

$= \frac{1}{2}[2 \times (1 + \cos x \sin x)]$

$= \frac{1}{2}[2 + 2\sin x \cos x]$

$= \frac{1}{2}[2 + \sin 2x]$ [∵ sin 2x = 2 sinx cosx]

= RHS

∴ LHS = RHS

Hence Proved

**Question: 16**

Prove that

**Solution:**

To prove: $\frac{1-\cos 2x+\sin x}{\sin 2x+\cos x} = \tan x$

Taking LHS,

$= \frac{1-\cos 2x+\sin x}{\sin 2x+\cos x}$

$= \frac{(1-\cos 2x)+\sin x}{\sin 2x+\cos x}$

We know that,

1 – cos 2x = 2 $\sin^2$x & sin 2x = 2 sinx cosx

$= \frac{2\sin^2 x+\sin x}{2\sin x\cos x+\cos x}$

Taking sinx common from the numerator and cosx from the denominator

$= \frac{\sin x(2\sin x+1)}{\cos x(2\sin x+1)}$

$= \frac{\sin x}{\cos x}$

= tan x $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 17**

Prove that

**Solution:**

To Prove: $\cos x\cos 2x\cos 4x\cos 8x = \frac{\sin 16x}{16\sin x}$

Taking LHS,

= cosx cos2x cos4x cos8x

Multiply and divide by 2sinx, we get

$= \frac{1}{2\sin x}[2\sin x\cos x\cos 2x\cos 4x\cos 8x]$

$= \frac{1}{2\sin x}[(2\sin x\cos x)\cos 2x\cos 4x\cos 8x]$

$= \frac{1}{2\sin x}[\sin 2x\cos 2x\cos 4x\cos 8x]$ [∵ sin 2x = 2 sinx cosx]

Multiply and divide by 2, we get

$= \frac{1}{2\times 2\sin x}[(2\sin 2x\cos 2x)\cos 4x\cos 8x]$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 2x, we get

sin 2(2x) = 2 sin(2x) cos(2x)

or sin 4x = 2 sin 2x cos 2x

$= \frac{1}{4\sin x}[\sin 4x\cos 4x\cos 8x]$

Multiply and divide by 2, we get

$= \frac{1}{2\times 4\sin x}[2\sin 4x\cos 4x\cos 8x]$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 4x, we get

sin 2(4x) = 2 sin(4x) cos(4x)

or sin 8x = 2 sin 4x cos 4x

$= \frac{1}{8\sin x}[\sin 8x\cos 8x]$

Multiply and divide by 2, we get

$= \frac{1}{2\times 8\sin x}[2\sin 8x\cos 8x]$

We know that,

sin 2x = 2 sinx cosx

Replacing x by 8x, we get

sin 2(8x) = 2 sin(8x) cos(8x)

or sin 16x = 2 sin 8x cos 8x

$= \frac{1}{16\sin x}[\sin 16x]$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 18 A**

Prove that

**Solution:**

To Prove: $2\sin 22\frac{1}{2}^{\circ}\cos 22\frac{1}{2}^{\circ} = \frac{1}{\sqrt{2}}$

Taking LHS,

$= 2\sin 22\frac{1}{2}^{\circ}\cos 22\frac{1}{2}^{\circ}$ ...(i)

We know that,

2sinx cosx = sin 2x

Here, $x = 22\frac{1}{2} = \frac{45}{2}$

So, eq. (i) become

$= \sin 2\left(\frac{45}{2}\right)$

= sin 45°

$= \frac{1}{\sqrt{2}}\left[\because \sin(45^{\circ}) = \frac{1}{\sqrt{2}}\right]$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 18 B**

Prove that

**Solution:**

To Prove: $2\cos^2 15° - 1 = \frac{\sqrt{3}}{2}$

Taking LHS,

= 2 $\cos^2$ 15° - 1 ...(i)

We know that,

1 + cos 2x = 2 $\cos^2 x$

Here, x = 15°

So, eq. (i) become

= [1 + cos 2(15°)] – 1

= 1 + cos 30° - 1

= cos 30° $\left[\because \cos(30°) = \frac{\sqrt{3}}{2}\right]$

$= \frac{\sqrt{3}}{2}$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 18 C**

Prove that

**Solution:**

To Prove: 8 $\cos^3$ 20° - 6 cos 20° = 1

Taking LHS,

= 8 $\cos^3$ 20° - 6 cos 20°

Taking 2 common, we get

= 2(4 $\cos^3$ 20° - 3 cos 20°) ...(i)

We know that,

cos 3x = 4$\cos^3 x$ – 3 cosx

Here, x = 20°

So, eq. (i) becomes

= 2[cos 3(20°)]

= 2[cos 60°]

$= 2 \times \frac{1}{2}$ $\left[\because \cos(60°) = \frac{1}{2}\right]$

= 1

= RHS

∴ LHS = RHS

Hence Proved

**Question: 18 D**

Prove that

**Solution:**

To prove: $3\sin 40° - \sin^3 40° = \frac{\sqrt{3}}{2}$

Taking LHS,

= 3 sin 40° - $\sin^3$ 40° …(i)

We know that,

sin 3x = 3 sinx – $\sin^3$x

Here, x = 40°

So, eq. (i) becomes

= sin 3(40°)

= sin 120°

= sin (180° - 60°)

= sin 60° [∵sin (180° - θ) = sin θ]

$= \frac{\sqrt{3}}{2}\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 19 A**

Prove that

**Solution:**

To Prove: $\sin^2 24° - \sin^2 6° = \frac{\sqrt{5}-1}{8}$

Taking LHS,

= $\sin^2$24° - $\sin^2$6°

We know that,

$\sin^2$A – $\sin^2$B = sin(A + B) sin(A – B)

= sin(24°+ 6°) sin(24° - 6°)

= sin 30° sin 18° …(i)

Now, we will find the value of sin 18°

Let x = 18°

so, 5x = 90°

Now, we can write

2x + 3x = 90°

so 2x = 90° - 3x

Now taking sin both the sides, we get

sin2x = sin(90° - 3x)

sin2x = cos3x [as we know, sin(90°- 3x) = Cos3x ]

We know that,

sin2x = 2sinxcosx

Cos3x = $4\cos^3 x$ - 3cosx

2sinxcosx = $4\cos^3 x$ - 3cosx

⇒ 2sinxcosx - $4\cos^3 x$ + 3cosx = 0

⇒ cosx (2sinx - $4\cos^2 x$ + 3) = 0

Now dividing both side by cosx we get,

2sinx - $4\cos^2 x$ + 3 = 0

We know that,

$\cos^2 x + \sin^2 x = 1$

or $\cos^2 x = 1 - \sin^2 x$

⇒ 2sinx – 4(1 – $\sin^2 x$) + 3 = 0

⇒ 2sinx – 4 + $4\sin^2 x$ + 3 = 0

⇒ 2sinx + $4\sin^2 x$ – 1 = 0

We can write it as,

$4\sin^2 x$ + 2sinx - 1 = 0

Now applying formula

Here, $ax^2 + bx + c = 0$

So, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

now applying it in the equation

$$\sin x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2}$$

$$\sin x = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$\sin x = \frac{(-2 \pm 2.\sqrt{5})}{8}$$

$$\sin x = \frac{2(-1 \pm \sqrt{5})}{8}$$

$$\sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$$

Now sin 18° is positive, as 18° lies in first quadrant.

$\therefore \sin 18° = \frac{\sqrt{5} - 1}{4}$

Putting the value in eq. (i), we get

= sin 30° sin 18°

$= \frac{1}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{5}-1}{8}$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 19 B**

Prove that

**Solution:**

To Prove: $\sin^2 72° - \cos^2 30° = \frac{\sqrt{5}-1}{8}$

Taking LHS,

$= \sin^2 72° - \cos^2 30°$

$= \sin^2(90° - 18°) - \cos^2 30°$

$= \cos^2 18° - \cos^2 30°$ …(i)

Here, we don’t know the value of cos 18°. So, we have to find the value of cos 18°

Let x = 18°

so, 5x = 90°

Now, we can write

2x + 3x = 90°

so 2x = 90° - 3x

Now taking sin both the sides, we get

sin2x = sin(90° - 3x)

sin2x = cos3x [as we know, sin(90°- 3x) = Cos3x ]

We know that,

sin2x = 2sinxcosx

$\text{Cos}3x = 4\cos^3 x - 3\cos x$

$2\sin x\cos x = 4\cos^3 x - 3\cos x$

$\Rightarrow 2\sin x\cos x - 4\cos^3 x + 3\cos x = 0$

$\Rightarrow \cos x\,(2\sin x - 4\cos^2 x + 3) = 0$

Now dividing both side by cosx we get,

$2\sin x - 4\cos^2 x + 3 = 0$

We know that,

$\cos^2 x + \sin^2 x = 1$

or $\cos^2 x = 1 - \sin^2 x$

$\Rightarrow 2\sin x - 4(1 - \sin^2 x) + 3 = 0$

$\Rightarrow 2\sin x - 4 + 4\sin^2 x + 3 = 0$

$\Rightarrow 2\sin x + 4\sin^2 x - 1 = 0$

We can write it as,

$4\sin^2x + 2\sin x - 1 = 0$

Now applying formula

Here, $ax^2 + bx + c = 0$

So, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

now applying it in the equation

$\sin x = \frac{-2 \pm\sqrt{2^2 - 4(4)(-1)}}{2}$

$\sin x = \frac{-2 \pm\sqrt{4+16}}{8}$

$\sin x = \frac{-2 \pm\sqrt{20}}{8}$

$\sin x = \frac{(-2 \pm 2.\sqrt{5})}{8}$

$\sin x = \frac{2(-1 \pm \sqrt{5})}{8}$

$\sin x = \frac{-1 \pm \sqrt{5}}{4}$

$\sin 18° = \frac{-1 \pm \sqrt{5}}{4}$

Now sin 18° is positive, as 18° lies in first quadrant.

$\therefore \sin 18° = \frac{\sqrt{5}-1}{4}$

Now, we know that

$\cos^2x + \sin^2x = 1$

or $\cos x = \sqrt{1} - \sin^2x$

$\therefore \cos 18° = \sqrt{1} - \sin^2 18°$

$\Rightarrow \cos 18° = \sqrt{1 - \left(\frac{\sqrt{5}-1}{4}\right)^2}$

$\Rightarrow \cos 18° = \sqrt{\frac{16-(5+1-2\sqrt{5})}{16}}$

$\Rightarrow \cos 18° = \sqrt{\frac{16-6+2\sqrt{5}}{16}}$

$\Rightarrow \cos 18° = \frac{1}{4}\sqrt{10+2\sqrt{5}}$

Putting the value in eq. (i), we get

$= \cos^2 18° - \cos^2 30°$

$= \left(\frac{1}{4}\sqrt{10+2\sqrt{5}}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \quad \left[\because \cos 30° = \frac{\sqrt{3}}{2}\right]$

$= \frac{1}{16}\left(10 + 2\sqrt{5}\right) - \frac{3}{4}$

$= \frac{10+2\sqrt{5}-12}{16}$

$= \frac{2\sqrt{5}-2}{16}$

$= \frac{2(\sqrt{5}-1)}{16}$

$= \frac{\sqrt{5}-1}{8}$

= RHS

∴ LHS = RHS

Hence Proved

**Question: 20**

Prove that tan 6<

**Solution:**

To Prove: $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ = 1$

Taking LHS,

$= \tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ$

Multiply and divide by $\tan 54^\circ \tan 18^\circ$

$= \frac{\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ}{\tan 54^\circ \tan 18^\circ} \times \tan 54^\circ \tan 18^\circ$

$= \frac{(\tan 6^\circ \tan 54^\circ \tan 66^\circ)(\tan 18^\circ \tan 42^\circ \tan 72^\circ)}{\tan 54^\circ \tan 18^\circ}$ ...(i)

We know that,

$\tan x \tan(60^\circ - x) \tan(60^\circ + x) = \tan 3x$

In first $x = 6^\circ$

$\tan 6^\circ \tan(60^\circ - 6^\circ) \tan(60^\circ + 6^\circ)$

$= \tan 6^\circ \tan 54^\circ \tan 66^\circ$

and

In second $x = 18^\circ$

$\tan 18^\circ \tan(60^\circ - 18^\circ) \tan(60^\circ + 18^\circ)$

$= \tan 18^\circ \tan 42^\circ \tan 78^\circ$

So, eq. (i) becomes

$= \frac{[\tan 3(6^\circ)][\tan 3(18^\circ)]}{\tan 54^\circ \tan 18^\circ}$

$= \frac{\tan 18^\circ \tan 54^\circ}{\tan 54^\circ \tan 18^\circ}$

= 1

= RHS

∴ LHS = RHS

Hence Proved

**Question: 21**

If $\tan\theta = \frac{a}{b}$

To Prove: $a \sin 2\theta + b \cos 2\theta = b$

Given: $\tan\theta = \frac{a}{b}$

We know that,

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a}{b}$

By Pythagoras Theorem,

$(\text{Perpendicular})^2 + (\text{Base})^2 = (\text{Hypotenuse})^2$

$\Rightarrow (a)^2 + (b)^2 = (H)^2$

$\Rightarrow a^2 + b^2 = (H)^2$

$\Rightarrow H = \sqrt{a^2 + b^2}$

So,

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{\sqrt{a^2+b^2}}$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{\sqrt{a^2+b^2}}$

Taking LHS,

$= a \sin 2\theta + b \cos 2\theta$

We know that,

$\sin 2\theta = 2 \sin\theta \cos\theta$

and $\cos 2\theta = 1 - 2\sin^2\theta$

$= a(2 \sin\theta \cos\theta) + b(1 - 2\sin^2\theta)$

Putting the values of $\sin\theta$ and $\cos\theta$, we get

$= a \times 2 \times \frac{a}{\sqrt{a^2+b^2}} \times \frac{b}{\sqrt{a^2+b^2}} + b\left[1 - 2 \times \left(\frac{a}{\sqrt{a^2+b^2}}\right)^2\right]$

$= \frac{2a^2b}{a^2+b^2} + b\left[1 - 2 \times \frac{a^2}{a^2+b^2}\right]$

$= \frac{2a^2b}{a^2+b^2} + b - \frac{2a^2b}{a^2+b^2}$

$= b$

= RHS

$\therefore$ LHS = RHS

Hence Proved

# Exercise : 15E

**Question: 1**

If Given: $\sin x = \frac{\sqrt{5}}{3}$ and $\frac{\pi}{2} < x < \pi$ i.e, x lies in the Quadrant II .

To Find: i)$\sin\frac{x}{2}$ ii)$\cos\frac{x}{2}$ iii)$\tan\frac{x}{2}$

Now, since $\sin x = \frac{\sqrt{5}}{3}$

We know that $\cos x = \pm\sqrt{1 - \sin^2 x}$

$\cos x = \pm\sqrt{1 - (\frac{\sqrt{5}}{3})^2}$

$\cos x = \pm\sqrt{1 - \frac{5}{9}}$

$\cos x = \pm\sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

since cos x is negative in II quadrant, hence cos x = - $\frac{2}{3}$

i) sin $\frac{x}{2}$

Formula used:

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{-2}{3})}{2}} = \pm\sqrt{\frac{\frac{5}{3}}{2}} = \pm\sqrt{\frac{5}{6}}$

Since sinx is positive in II quadrant, hence $\sin\frac{x}{2} = \sqrt{\frac{5}{6}}$

ii)cos $\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+(\frac{-2}{3})}{2}} = = \pm\sqrt{\frac{\frac{1}{3}}{2}} = = \pm\sqrt{\frac{1}{6}}$

since cosx is negative in II quadrant, hence $\cos\frac{x}{2} = -\frac{1}{\sqrt{6}}$

iii)tan $\frac{x}{2}$

Formula used:

$\tan x = \frac{\sin x}{\cos x}$

hence, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{\frac{5}{6}}}{-\frac{1}{\sqrt{6}}} = \frac{\sqrt{5}}{\sqrt{6}} \times \frac{\sqrt{6}}{-1} = -\sqrt{5}$

Here, tanx is negative in II quadrant.

**Question: 2**

Given: cos x = = $-\frac{3}{5}$ and $\frac{\pi}{2} < x < \pi$.i.e, x lies in II quadrant

To Find: i)sin $\frac{x}{2}$ ii)cos $\frac{x}{2}$ iii)tan $\frac{x}{2}$

i) sin $\frac{x}{2}$

Formula used:

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{-3}{5})}{2}} = \pm\sqrt{\frac{\frac{8}{5}}{2}} = \pm\frac{2}{\sqrt{5}}$

Since sinx is positive in II quadrant, hence $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

ii)cos $\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+(\frac{-3}{5})}{2}} = = \pm\sqrt{\frac{\frac{2}{5}}{2}} = = \pm\sqrt{\frac{1}{5}}$

since cosx is negative in II quadrant, hence $\cos\frac{x}{2} = -\frac{1}{\sqrt{5}}$

iii)tan $\frac{x}{2}$

Formula used:

$\tan x = \frac{\sin x}{\cos x}$

hence, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\frac{2}{\sqrt{5}}}{-\frac{1}{\sqrt{5}}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{-1}$ = -2

Here, tanx is negative in II quadrant.

**Question: 3**

Given: $\sin x = \frac{-1}{2}$ and x lies in Quadrant IV.

To Find: i)sin $\frac{x}{2}$ ii)cos $\frac{x}{2}$ iii)tan $\frac{x}{2}$

Now, since $\sin x = \frac{-1}{2}$

We know that $\cos x = \pm\sqrt{1-\sin^2 x}$

$\cos x = \pm\sqrt{1-(\frac{-1}{2})^2}$

$\cos x = \pm\sqrt{1-\frac{1}{4}}$

$\cos x = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

since cos x is positive in IV quadrant, hence $\cos x = \frac{\sqrt{3}}{2}$

i) sin $\frac{x}{2}$

Formula used:

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Now, $\sin\frac{x}{2} = \pm\sqrt{\frac{1-(\frac{\sqrt{3}}{2})}{2}} = \pm\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{2-\sqrt{3}}{4}} = = \pm\frac{\sqrt{2-\sqrt{3}}}{2}$

Since sinx is negative in IV quadrant, hence $\sin\frac{x}{2} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

ii)cos $\frac{x}{2}$

Formula used:

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

now, $\cos\frac{x}{2} = \pm\sqrt{\frac{1+(\frac{\sqrt{3}}{2})}{2}} = = \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = = \pm\frac{\sqrt{2+\sqrt{3}}}{2}$

since cosx is positive in IV quadrant, hence $\cos\frac{x}{2} = \frac{\sqrt{2+\sqrt{3}}}{2}$

iii)$\tan\frac{x}{2}$

Formula used:

$\tan x = \frac{\sin x}{\cos x}$

hence, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{-\frac{\sqrt{2-\sqrt{3}}}{2}}{\frac{\sqrt{2+\sqrt{3}}}{2}} = -\frac{\sqrt{2-\sqrt{3}}}{2} \times \frac{2}{\sqrt{2+\sqrt{3}}}$ = -1

Here, tanx is negative in IV quadrant.

**Question: 4**

Given: $\cos\frac{x}{2} = \frac{12}{13}$ and x lies in Quadrant I i.e, All the trigonometric ratios are positive in I quadrant

To Find: i)sin x ii)cos x iii)cot x

i)sin x

Formula used:

We have, Sin x = $\sqrt{1-\cos^2 x}$

We know that, $\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$ (∵cos x is positive in I quadrant)

⇒ $2\cos^2\frac{x}{2} - 1 = \cos x$

⇒ $2\times(\frac{12}{13})^2 - 1 = \cos x$

⇒ $2\times(\frac{144}{169}) - 1 = \cos x$

⇒ $\cos x = \frac{119}{169}$

Since, Sin x = $\sqrt{1-\cos^2 x}$

⇒ Sin x = $\sqrt{1-(\frac{119}{169})^2}$

⇒ Sin x = $\frac{120}{169}$

Hence, we have Sin x = $\frac{120}{169}$ .

ii)cos x

Formula used:

We know that, $\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$ (∵cos x is positive in I quadrant)

⇒ $2\cos^2\frac{x}{2} - 1 = \cos x$

⇒ $2\times(\frac{12}{13})^2 - 1 = \cos x$

$\Rightarrow 2\times(\frac{144}{169}) - 1 = \cos x$

$\Rightarrow \cos x = \frac{119}{169}$

iii) cot x

Formula used:

$\cot x = \frac{\cos x}{\sin x}$

$\cot x = \frac{\frac{119}{169}}{\frac{120}{169}} = \frac{119}{169}\times\frac{169}{120} = \frac{119}{120}$

Hence, we have $\cot x = \frac{119}{120}$

**Question: 5**

Given: $\sin x = \frac{3}{5}$ and $0< x<\frac{\pi}{2}$ i.e, x lies in Quadrant I and all the trignometric ratios are positive in quadrant I.

To Find: $\tan\frac{x}{2}$

Formula used:

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

Now, $\cos x = \sqrt{1-\sin^2 x}$ (∵cos x is positive in I quadrant)

$\Rightarrow \cos x = \sqrt{1-(\frac{3}{5})^2} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

Since, $\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{\frac{3}{5}}{1+\frac{4}{5}} = \frac{3}{5}\times\frac{5}{9} = \frac{1}{3}$

Hence, $\tan\frac{x}{2} = \frac{1}{3}$

**Question: 6**

To Prove: $\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$

Proof: Consider L.H.S,

$\cot\frac{x}{2} - \tan\frac{x}{2} = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} - \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$

$= \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{\cos x}{\sin\frac{x}{2}\cos\frac{x}{2}}$ ($\because \cos^2 x - \sin^2 x = \cos 2x$)

$\Rightarrow (\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x)$

Here multiply and divide L.H.S by 2

$= \frac{2\cos x}{2\sin\frac{x}{2}\cos\frac{x}{2}}$

$= \frac{2\cos x}{\sin x}$ (∵2sinxcosx = sin2x)

$\Rightarrow (2\sin\frac{x}{2}\cos\frac{x}{2} = \sin x)$

$\cot\frac{x}{2} - \tan\frac{x}{2} = 2\cot x$ = R.H.S

$\therefore$L.H.S = R.H.S, Hence proved

**Question: 7**

Prove that

**Solution:**

To Prove: $\tan(\frac{\pi}{4}+\frac{x}{2})$ = tan x + sec x

Proof: Consider L.H.S,

$\tan(\frac{\pi}{4}+\frac{x}{2}) = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}$ ($\because$ this is of the form $\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$)

$= \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} = \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$

$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$

Multiply and divide L.H.S by $\cos\frac{x}{2}+\sin\frac{x}{2}$

$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} \times \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$

$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$

$= \frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$ ($\because \cos^2\frac{x}{2}-\sin^2\frac{x}{2} = \cos x$)

$= \frac{1+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$

$= \frac{1+\sin x}{\cos x}$ ($\because 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x$)

$= \frac{1}{\cos x}+\frac{\sin x}{\cos x}$

$\tan(\frac{\pi}{4}+\frac{x}{2})$ = sec x + tan x = R.H.S

$\because$ L.H.S = R.H.S, Hence proved

**Question: 8**

Prove that

**Solution:**

To Prove: $\sqrt{\frac{1+\sin x}{1-\sin x}} = \tan(\frac{\pi}{4}+\frac{x}{2})$

Proof: Consider, L.H.S = $\sqrt{\frac{1+\sin x}{1-\sin x}}$

Multiply and divide L.H.S by $\sqrt{1+\sin x}$

$= \sqrt{\frac{1+\sin x}{1-\sin x}} \times \frac{\sqrt{1+\sin x}}{\sqrt{1+\sin x}} = \frac{1+\sin x}{\sqrt{1-\sin^2 x}}$

$= \frac{1+\sin x}{\cos x} = \frac{1+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$ ($\because 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x$)

$= \frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos x}$ $(\because \cos^2 x + \sin^2 x = 1)$

$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$

$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} \times \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$ $(\because x^2 + y^2 = (x+y)(x-y))$

$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$

Multiply and divide the above with cos $\frac{x}{2}$

$= \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$

Here, since $\tan\frac{\pi}{4} = 1$

$\sqrt{\frac{1+\sin x}{1-\sin x}} = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}} = \tan(\frac{\pi}{4} + \frac{x}{2})$ = R.H.S

Since, L.H.S = R.H.S, Hence proved.

**Question: 9**

Prove that

**Solution:**

To prove: $\tan(\frac{\pi}{4} + \frac{x}{2}) + \tan(\frac{\pi}{4} - \frac{x}{2}) = 2\sec x$

Proof: Consider, L.H.S = $\tan(\frac{\pi}{4} + \frac{x}{2}) + \tan(\frac{\pi}{4} - \frac{x}{2})$

$\tan(\frac{\pi}{4} + \frac{x}{2}) + \tan(\frac{\pi}{4} - \frac{x}{2}) = \frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}} + \frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}\tan\frac{x}{2}}$

$(\because \tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$ and $\tan(x-y) = \frac{\tan x-\tan y}{1+\tan x\tan y})$

$= \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} + \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

$= \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}} + \frac{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$

$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} + \frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$

$= \frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2+(\cos\frac{x}{2}-\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$

By Expanding the numerator we get,

$= \frac{2}{\cos x}$ $(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x)$

$\tan(\frac{\pi}{4} + \frac{x}{2}) + \tan(\frac{\pi}{4} - \frac{x}{2}) = 2\sec x$ = R.H.S

since L.H.S = R.H.S, Hence proved.

**Question: 10**

Prove that

**Solution:**

To Prove: $\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$

Proof: consider, L.H.S = $\frac{\sin x}{1+\cos x}$

$\frac{\sin x}{1+\cos x} = \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{1+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$ $(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x \text{ and } 2\cos\frac{x}{2}\sin\frac{x}{2} = \sin x)$

$= \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$ $(\because \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = 1)$

$= \frac{2\cos\frac{x}{2}\sin\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \tan\frac{x}{2}$

$\frac{\sin x}{1+\cos x} = \tan\frac{x}{2}$ = R.H.S

Since L.H.S = R.H.S, Hence proved.