26. Scalar Triple Product

Exercise 26.1

1 A. Question

Evaluate the following :

$$\left[\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{j}\ \hat{k}\ \hat{i}\right]+\left[\hat{k}\ \hat{i}\ \hat{j}\right]$$

Answer

Formula: –

(i) $[\vec{a}\vec{b}\vec{c}] = \vec{a}(\vec{b}\times\vec{c}) = \vec{b}.(\vec{c}\times\vec{a}) = \vec{c}.(\vec{a}\times\vec{b})$

(ii) $\hat{\imath}.\hat{\imath} = 1, \hat{\jmath}.\hat{\jmath} = 1, \hat{k}.\hat{k} = 1$

(iii) $\vec{i}\times\vec{j} = \vec{k}, \vec{j}\times\vec{k} = \vec{i}, \vec{k}\times\vec{i} = \vec{j}$

we have

$$[\hat{\imath}\hat{\jmath}\hat{k}] + [\hat{\jmath}\hat{k}\hat{\imath}] + [\hat{k}\hat{\imath}\hat{\jmath}] = (\hat{\imath}\times\hat{\jmath}).\hat{k} + (\hat{\jmath}\times\hat{k}).\hat{\imath} + (\hat{k}\times\hat{\imath}).\hat{\jmath}$$

using Formula(i) and (iii)

$$\Rightarrow [\hat{\imath}\hat{\jmath}\hat{k}] + [\hat{\jmath}\hat{k}\hat{\imath}] + [\hat{k}\hat{\imath}\hat{\jmath}] = \hat{k}.\hat{k} + \hat{\imath}.\hat{\imath} + \hat{\jmath}.\hat{\jmath}$$

$$\Rightarrow [\hat{\imath}\hat{\jmath}\hat{k}] + [\hat{\jmath}\hat{k}\hat{\imath}] + [\hat{k}\hat{\imath}\hat{\jmath}] = 1 + 1 + 1 = 3$$

therefore, using Formula (ii)

$$[\hat{\imath}\hat{\jmath}\hat{k}] + [\hat{\jmath}\hat{k}\hat{\imath}] + [\hat{k}\hat{\imath}\hat{\jmath}] = 3$$

1 B. Question

Evaluate the following :

$$\left[2\hat{i}\ \hat{j}\ \hat{k}\right]+\left[\hat{i}\ \hat{k}\ \hat{j}\right]+\left[\hat{k}\ \hat{j}\ 2\hat{i}\right]$$

Answer

Formula: -

(i) $[\hat{a}\hat{b}\hat{c}] = (\hat{a}\times\hat{b}).\hat{c}$

(ii) $\hat{\imath}.\hat{\imath} = 1, \hat{\jmath}.\hat{\jmath} = 1, \hat{k}.\hat{k} = 1$

(iii) $\vec{i}\times\vec{j} = \vec{k}, \vec{j}\times\vec{k} = \vec{i}, \vec{k}\times\vec{i} = \vec{j}$

Given: -

we have

$$[2\hat{\imath}\hat{\jmath}\hat{k}] + [\hat{\imath}\hat{k}\hat{\jmath}] + [\hat{k}\hat{\imath}\hat{\jmath}] = (2\hat{\imath}\times\hat{\jmath}).\hat{k} + (\hat{\imath}\times\hat{\jmath}).\hat{\jmath} + (\hat{k}\times\hat{\jmath}).2\hat{\imath}$$

using Formula (i)

$$\Rightarrow [2\hat{\imath}\hat{\jmath}\hat{k}] + [\hat{\imath}\hat{k}\hat{\jmath}] + [\hat{k}\hat{\imath}\hat{\jmath}] = 2\hat{k}.\hat{k} + (-\hat{\jmath}).\hat{\jmath} + (-\hat{\imath}).2\hat{\imath}$$

using Formula (ii)

$$\Rightarrow [2\hat{\imath}\hat{\jmath}\hat{k}] + [\hat{\imath}\hat{k}\hat{\jmath}] + [\hat{k}\hat{\imath}\hat{\jmath}] = 2 - 1 - 2$$

$\Rightarrow \; [2\hat{i}\hat{j}\hat{k}] + [\hat{i}\hat{k}\hat{j}] + [\hat{k}\hat{i}\hat{j}] = -1$

therefore,

$[2\hat{i}\hat{j}\hat{k}] + [\hat{i}\hat{k}\hat{j}] + [\hat{k}\hat{i}\hat{j}] = -1$

2 A. Question

Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, when

$\vec{a} = 2\hat{i} - 3\hat{j}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$

Answer

Formula: -

(i) $\text{if}\ \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\ \text{and}\ \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\ \text{then}, [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Given: -

$\vec{a} = 2\hat{i} - 3\hat{j}, \vec{b} = \hat{i} + \hat{j} - \hat{k}\ \text{and}\ \vec{c} = 3\hat{i} - \hat{k}$

using Formula(i)

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 2(– 1 – 0) + 3(– 1 + 3)

= – 2 + 6

= 4

therefore,

$[\vec{a}\vec{b}\vec{c}] = 4$

2 B. Question

Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, when

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{j} + \hat{k}$

Answer

Formula: -

(i) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Given: -

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{j} + \hat{k}$

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 1(1 + 1) + 2(2 + 0) + 3(2 – 0)

= 2 + 4 + 6 = 12

therefore,

$$[\vec{a}\vec{b}\vec{c}] = 12$$

3 A. Question

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors:

$$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Given: -

$\vec{a} = 2\hat{i} + 3\hat{j}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}.$

we know that the volume of parallelepiped whose three adjacent edges are

$\vec{a}, \vec{b}\vec{c}$ is equal to $|[\vec{a}\vec{b}\vec{c}]|.$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 2(4 – 1) – 3(2 + 3) + 4(– 1 – 6)

= – 37

therefore, the volume of the parallelepiped is $[\vec{a}\vec{b}\vec{c}] = |-37| = 37\text{cubic unit.}$

3 B. Question

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors:

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \ \vec{c} = 3\hat{i} - \hat{j} - 2\hat{k}$$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Given: -

$\vec{a} = 2\hat{i} - 3\hat{j}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}.$

we know that the volume of parallelepiped whose three adjacent edges are

$\vec{a}, \vec{b}, \vec{c}$ is equal to $|[\vec{a}\vec{b}\vec{c}]|$.

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 2(– 4 – 1) – 3(– 2 + 3) + 4(– 1 – 6)

= – 35

therefore, the volume of the parallelepiped is $[\vec{a}\vec{b}\vec{c}] = |-35| = 35 \text{cubic unit.}$

3 C. Question

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors:

$\vec{a} = 11\hat{i},\ \vec{b} = 2\hat{j},\ \vec{c} = 13\hat{k}$

Answer

Formula : -

$$(\text{i})\text{if}\ \vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}\ \text{and}\ \vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}\text{then,}\ [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$(\text{ii}) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Given: -

$\vec{a} = 11\hat{\imath}, \vec{b} = 2\hat{\jmath}, \vec{c} = 3\hat{k}.$

we know that the volume of parallelepiped whose three adjacent edges are

$\vec{a}, b^{\to}, c^{\to}$ is equal to $|[\vec{a}\vec{b}\vec{c}]|$.

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 11 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 13 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 11(26 – 0) + 0 + 0 = 286

therefore, the volume of the parallelepiped is$[\vec{a}\ \vec{b}\ \vec{c}] = |286| = 286$ cubic unit.

3 D. Question

Find the volume of the parallelepiped whose coterminous edges are represented by the vectors:

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

Answer

Formula: -

$$(\mathrm{i})\text{if } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ and } \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k} \text{ then}, [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$(\mathrm{ii}) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Given: -

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}, \vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}.$

we know that the volume of parallelepiped whose three adjacent edges are

$\vec{a}, \vec{b}\vec{c}$ is equal to $|[\vec{a}\vec{b}\vec{c}]|$.

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 1(1 – 2) – 1(– 1 – 1) + 1(2 + 1)

= 4

therefore, the volume of the parallelepiped is $[\vec{a}\vec{b}\vec{c}] = |4|$

= 4 cubic unit.

4 A. Question

Show that each of the following triads of vectors is coplanar :

$\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} + 7\hat{k}, \vec{c} = 5\hat{i} + 6\hat{j} + 5\hat{k}$

Answer

Formula : –

$$(\mathrm{i})\text{if } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ and } \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k} \text{ then}, [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$(\mathrm{ii}) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii)Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar if and only if

$$\vec{a}.(\vec{b}\times\vec{c}) = 0$$

Given: -

$$\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} + 7\hat{k}, \vec{c} = 5\hat{i} + 6\hat{j} + 5\hat{k}$$

we know that three vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero

$$[\vec{a}\vec{b}\vec{c}] = 0.$$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & 2 & 7 \\ 5 & 6 & 5 \end{vmatrix}$$

using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 1(10 – 42) – 2(15 – 35) – 1(18 – 10)

= 0.

Hence, the Given vector are coplanar.

4 B. Question

Show that each of the following triads of vectors is coplanar :

$$\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}, \vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}, \vec{c} = -8\hat{i} - \hat{j} + 3\hat{k}$$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b}\times\vec{c}) = 0$

Given: -

$$\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}, \vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}, \vec{c} = -8\hat{i} - \hat{j} + 3\hat{k}$$

we know that three vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero

$$[\vec{a}\vec{b}\vec{c}] = 0.$$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= – 4(12 + 13) + 6(– 3 + 24) – 2(1 + 32)

= 0

hence, the Given vector are coplanar.

4 C. Question

Show that each of the following triads of vectors is coplanar :

$$\hat{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \hat{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}, \hat{c} = \hat{i} - 3\hat{j} + 5\hat{k}$$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b}\times\vec{c}) = 0$

Given: -

$$\hat{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \hat{b} = -2i + 3\hat{\jmath} - 4k, \hat{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$$

we know that three vector $\vec{a},\vec{b},\vec{c}$ are coplanar if their scalar triple product is zero

$$[\vec{a}\vec{b}\vec{c}] = 0.$$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 1(15 – 12) + 2(– 10 + 4) + 3(6 – 3)

= 3 – 12 + 9 = 0

5 A. Question

Find the value of λ so that the following vectors are coplanar.

$\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Given: -

$\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

we know that vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero

$[\vec{a}\vec{b}\vec{c}] = 0.$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

⇒ 0 = 1(λ – 1) + 1(2λ + λ) + 1(– 2 – λ)

⇒ 0 = λ – 1 + 3 λ – 2 – λ

⇒ 0 = 3 λ – 3

$\Rightarrow \lambda = 1$

5 B. Question

Find the value of λ so that the following vectors are coplanar.

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{c} = \lambda\hat{i} + \lambda\hat{j} + 5\hat{k}$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

(iii) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Given: -

$\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}, \vec{c} = \lambda\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

we know that vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero

$[\vec{a}\vec{b}\vec{c}] = 0.$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ \lambda & \lambda & 5 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$\Rightarrow 0 = 2(10 + 3\lambda) + 1(5 + 3\lambda) + 1(\lambda - 2\lambda)$

$\Rightarrow 0 = 8\lambda + 25$

$$\Rightarrow \lambda = \frac{-25}{8}$$

5 C. Question

Find the value of λ so that the following vectors are coplanar.

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 3\hat{i} + \lambda\hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c}$

$$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} \text{then}, [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Given: -

$\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 3\hat{i} + \lambda\hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$

we know that vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero

$[\vec{a}\vec{b}\vec{c}] = 0.$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & 2 & -3 \\ 3 & \lambda & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

now, using

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$\Rightarrow 0 = 1(2\lambda - 2) - 2(6 - 1) - 3(6 - \lambda)$

$\Rightarrow 0 = 5\lambda - 30$

$\Rightarrow \lambda = 6$

5 D. Question

Find the value of λ so that the following vectors are coplanar.

$\vec{a} = \hat{i} + 3\hat{j}, \vec{b} = 5\hat{k}, \vec{c} = \lambda\hat{i} - \hat{j}$

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Given: -

$\vec{a} = \hat{i} + 3\hat{j}, \vec{b} = 5\hat{k}, \vec{c} = \lambda\hat{i} - \hat{j}$

we know that vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if their scalar triple product is zero

$[\vec{a}\vec{b}\vec{c}] = 0.$

we have

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & 3 & 0 \\ 0 & 0 & 5 \\ \lambda & -1 & 0 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

⇒ 0 = 1(0 + 5) – 3(0 – 5λ) + 0

⇒ 0 = 5 + 15λ

$\Rightarrow \lambda = \frac{-1}{3}$

6. Question

Show that the four points having position vectors $6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{j} - 6\hat{k}, 2\hat{i} + 5\hat{j} + 10\hat{k}$ are not coplanar.

Answer

Formula : -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) if $\overrightarrow{OA} = a_{1\hat{i}} + a_2\hat{j} + a_3\hat{k}$ and $\overrightarrow{OB} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $OB - OA = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$

(iv) Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Given: -

$\overrightarrow{OA} = 6\hat{i} - 7\hat{j}, \overrightarrow{OB} = 16\hat{i} - 19\hat{j} - 4\hat{k}, \overrightarrow{OC} = 3\hat{j} - 6\hat{k}, \overrightarrow{OD} = 2\hat{i} + 5\hat{j} + 10\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 10\hat{i} - 12\hat{j} - 4\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -6\hat{i} + 10\hat{j} - 6\hat{k}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = -4\hat{i} + 12\hat{j} + 10\hat{k}$

The four points are coplaner if vector AB⃗,AC,AD⃗ are coplanar.

$$[\overrightarrow{AB}.\overrightarrow{AC}.\overrightarrow{AD}] = \begin{vmatrix} 10 & -12 & -4 \\ -6 & 10 & -6 \\ -4 & 12 & 10 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 10(100 + 72) + 12(– 60 – 24) – 4(– 72 + 40) = 840

≠0.

hence the point are not coplanar

7. Question

Show that the points A(– 1, 4, – 3), B(3, 2, – 5), C(– 3, 8, – 5) and D(– 3, 2, 1) are coplanar.

Answer

Formula: -

$$\text{(i)if}\ \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\ \text{and}\ \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}\ \text{then,}\ [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\text{(ii)}\ \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\text{(iii)Three vectors}\ \vec{a}, \vec{b}, \text{and}\ \vec{c} \text{are coplanar if and only if}\ \vec{a}.(\vec{b} \times \vec{c}) = 0$$

$$\text{(iv)if}\overrightarrow{OA} = a_{1\hat{i}} + a_2\hat{j} + a_3\hat{k}\ \text{and}\ \overrightarrow{OB} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, \text{then OB} - \text{OA} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

Given: -

AB = position vector of B - position vector of A

$= 4\hat{i} - 2\hat{j} - 2\hat{k}$

AC = position vector of c - position vector of A

$= -2\hat{i} + 4\hat{j} - 2\hat{k}$

AD = position vector of c - position vector of A

$= -2\hat{i} - 2\hat{j} + 4\hat{k}$

The four pint are coplanar if the vector are coplanar.

thus,

$$[\overrightarrow{AB}.\overrightarrow{AC}.\overrightarrow{AD}] = \begin{vmatrix} 4 & -2 & -2 \\ -2 & 4 & -2 \\ -2 & -2 & 4 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 4(16 - 4) + 2(- 8 - 4) - 2(- 4 + 8) = 0

hence proved.

8. Question

Show that four points whose position vectors are $6\hat{i} - 7\hat{j}, 16\hat{i} - 19\hat{j} - 4\hat{k}, 3\hat{i} - 6\hat{k}, 2\hat{i} - 5\hat{j} + 10\hat{k}$ are coplanar.

Answer

Formula : -

(i) if $\overrightarrow{OA} = a_{1\hat{i}} + a_2\hat{j} + a_3\hat{k}$ and $\overrightarrow{OB} = b_{1\hat{i}} + b_2\hat{j} + b_3\hat{k}$ then $\overrightarrow{OB} - \overrightarrow{OA} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$

(ii) $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

(iv) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

let

$\overrightarrow{OA} = 6\hat{i} - 7\hat{j}, \overrightarrow{OB} = 16\hat{i} - 19\hat{j} - 4\hat{k}, \overrightarrow{OC} = 3\hat{j} - 6\hat{k}, OD = 2\hat{i} - 5\hat{j} + 10\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 10\hat{i} - 12\hat{j} - 4\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -6\hat{i} + 10\hat{j} - 6\hat{k}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = -4\hat{i} + 2\hat{j} + 10\hat{k}$

The four points are coplanar if the vector $\overrightarrow{AB}, \overrightarrow{AD}, \overrightarrow{AC}$ are coplanar.

$$[\overrightarrow{AB}.\overrightarrow{AC}.\overrightarrow{AD}] = \begin{vmatrix} 10 & -12 & -4 \\ -6 & 10 & -6 \\ -4 & 2 & 10 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

= 10(100 + 12) + 12(– 60 – 24) – 4(– 12 + 40) = 0.

hence the point are coplanar

9. Question

Find the value of for which the four points with position vectors $-\hat{j}-\hat{k}, 4\hat{i}+5\hat{j}+\lambda\hat{k}, 3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

Answer

Formula : -

(i)if $\overrightarrow{OA} = a1\hat{i} + a2\hat{j} + a3\hat{k}$ and $\overrightarrow{OB} = b_{1i} + b_2\hat{j} + b_3\hat{k}$ then
$\overrightarrow{OB} - \overrightarrow{OA} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$
$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

(iv) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

Given: -

$OA = -\hat{j} - \hat{k}, OB = 4\hat{i} + 5\hat{j} - \lambda\hat{k}, OC = 3\hat{i} + 9\hat{j} + 4\hat{k}, OD = -4\hat{i} - 4\hat{j} + 4\hat{k}$

$AB = OB - OA = 4\hat{i} + 6\hat{j} + (\lambda + 1)\hat{k}$

$AC = OC - OA = 3\hat{i} + 10\hat{j} + 5\hat{k}$

$AD = OD - OA = -4\hat{i} + 5\hat{j} + 5\hat{k}$

The four points are coplaner if vector AB⃗,AC⃗,AD⃗ are coplanar.

$$[\vec{AB}, \vec{AC}, \vec{AD}] = \begin{vmatrix} 4 & 6 & (\lambda + 1) \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix}$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

⇒ 4(50 – 25) – 6(15 + 20) + (λ + 1)(15 + 40) = 0.

$\Rightarrow \lambda = 1$

hence the point are coplanar

10. Question

Prove that : -

$$\left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\}=0$$

Answer

Formula: -

$$\text{(i)}\left[\vec{a}\vec{b}\vec{c}\right] = \vec{a}\left(\vec{b}\times\vec{c}\right) = \vec{b}.\left(\vec{c}\times\vec{a}\right) = \vec{c}.\left(\vec{a}\times\vec{b}\right)$$

$$\text{(ii)}\vec{a}\times\vec{a} = \vec{b}\times\vec{b} = \vec{c}\times\vec{c} = 0$$

taking L.H.S

$$\left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = \left[\left(\vec{a}-\vec{b}\right)\left(\vec{b}-\vec{c}\right)\left(\vec{c}-\vec{a}\right)\right]$$

using Formula (i)

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = \left(\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-\vec{b}\times\vec{b}+\vec{b}\times\vec{c}\right).\left(\vec{c}-\vec{a}\right)$$

using Formula(ii)

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = \left(\vec{a}\times\vec{b}-\vec{a}\times\vec{c}-0+\vec{b}\times\vec{c}\right).\left(\vec{c}-\vec{a}\right)$$

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\}$$
$$= \left(\vec{a}\times\vec{b}\right).\vec{c}-\left(\vec{a}\times\vec{b}\right).\vec{a}+\left(\vec{c}\times a\right).\vec{c}-\left(\vec{c}\times\vec{a}\right).\vec{a}+\left(\vec{b}\times\vec{c}\right).\vec{c}$$
$$-\left(\vec{b}\times\vec{c}\right).\vec{a}$$

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = \left[\vec{a}\vec{b}\vec{c}\right]-\left[\vec{a}\vec{b}\vec{a}\right]+\left[\vec{c}\vec{a}\vec{c}\right]-\left[\vec{c}\vec{a}\vec{a}\right]+\left[\vec{b}\vec{c}\vec{c}\right]-\left[\vec{b}\vec{c}\vec{a}\right]$$

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = \left[\vec{a}\vec{b}\vec{c}\right]-\left[\vec{b}\vec{c}\vec{a}\right]$$

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = \left[\vec{a}\vec{b}\vec{c}\right]-\left[\vec{a}\vec{b}\vec{c}\right]$$

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = 0$$

$$\Rightarrow \left(\vec{a}-\vec{b}\right).\left\{\left(\vec{b}-\vec{c}\right)\times\left(\vec{c}-\vec{a}\right)\right\} = 0$$

L.H.S = R.H.S

11. Question

$\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of points A, B and C respectively, prove that : $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane of triangle ABC.

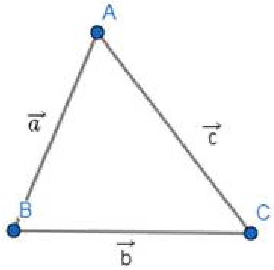
Answer

if $\vec{a}$ represents the sides AB,

if $\vec{b}$ represent the sides BC,

if $\vec{c}$ respresent the sidesAC of triangle ABC

$\vec{a}\times\vec{b}$ is perpendicular to plane of triangle ABC. ...... (i)



$\vec{b} \times \vec{c}$ is perpendicular to plane of triangle ABC. ...... (ii)

$\vec{c} \times \vec{a}$ is perpendicular to plane of triangle ABC. ...... (iii)

adding all the (i) + (ii) + (iii)

hence $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is a vector perpendicular to the plane of the triangle ABC

12 A. Question

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i}$ and $\hat{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$. Then,

If $c_1 = -1$ and $c_2 = 2$, find c_3 which makes $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

Answer

Formula: -

$$(i) \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}. \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}. \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}. \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$(ii) \text{if } \vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \text{ and } \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k} \text{ then, } [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(iii) Three vectors $\vec{a}, \vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Given: -

$\vec{a}, \vec{b}, \vec{c}$ are coplanar if

$$[\vec{a}\vec{b}\vec{c}] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 2 & c_3 \end{vmatrix} = 0$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

⇒ 0 – 1(c_3) + 1(2) = 0

⇒ c_3 = 2

12 B. Question

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i}$ and $\hat{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}.$ Then,

If $c_1 = -1$ and $c_3 = 1,$ show that no value of c_1 can make $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

Answer

Formula: -

(i)if $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

(ii) $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$
$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii)Three vectors $\vec{a}, \vec{b},$ and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

we know that $\vec{a}, \vec{b}, \vec{c}$ are coplanar if

$$[\vec{a}\vec{b}\vec{c}] = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ -1 & c_2 & 1 \end{vmatrix} = 0$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
$$= (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

⇒ 0 – 1(c_3) + 1(2) = 0

⇒ c_3 = 2

13. Question

Find for which the points A(3, 2, 1), B(4, λ, 5), C(4, 2, – 2) and D(6, 5, – 1) are coplanar.

Answer

Formula: -

(i)if $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}, \vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$ then, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$$\text{(ii)} \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii)Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b} \times \vec{c}) = 0$

(iv)if $\overrightarrow{OA} = a_{1\hat{\imath}} + a_2\hat{\jmath} + a_3\hat{k}$ and $\overrightarrow{OB} = b_{1\hat{\imath}} + b_2\hat{\jmath} + b_3\hat{k}$ then $OB - OA = (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

let position vector of

$OA = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

position vector of

$OB = 4\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

position vector of

$OC = 4\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

position vector of

$OD = 6\hat{\imath} + 5\hat{\jmath} - \hat{k}$

The four points are coplanar if the vector $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar

$\overrightarrow{AB} = \hat{\imath} + (\lambda - 2)\hat{\jmath} + 4\hat{k}$

$\overrightarrow{AC} = \hat{\imath} + 0\hat{\jmath} - 3\hat{k}$

$\overrightarrow{AD} = 3\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$$\begin{vmatrix} 1 & (\lambda - 2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

now, using

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$\Rightarrow 1(9) - (\lambda - 2)(-2 + 9) + 4(3 - 0) = 0$

$\Rightarrow 7\lambda = 35$

$\Rightarrow \lambda = 5$

14. Question

If four points A, B, C and D with position vectors $4\hat{i}+3\hat{j}+3\hat{k},\ 5\hat{i}+x\hat{j}+7\hat{k}, 5\hat{i}+3\hat{j}$ and $7\hat{i}+6\hat{j}+\hat{k}$ respectively are coplanar, then find the value of x.

Answer

Formula: -

(i) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c}$

$$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k} \text{ then, } [\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(ii) $$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

(iii) Three vectors $\vec{a}$,$\vec{b}$, and $\vec{c}$ are coplanar if and only if $\vec{a}.(\vec{b}\times\vec{c}) = 0$

(iv) if $\overrightarrow{OA} = a_{1\hat{i}} + a_2\hat{j} + a_3\hat{k}$ and $\overrightarrow{OB} = b_{1\hat{i}} + b_2\hat{j} + b_3\hat{k}$ then $OB - OA$

$$= (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$$

let position vector of

$OA = 4\hat{i} + 3\hat{j} + 3\hat{k}$

position vector of

$OB = 5\hat{i} + x\hat{j} + 7\hat{k}$

position vector of

$OC = 5\hat{i} + 3\hat{j}$

position vector of

$OD = 7\hat{i} + 6\hat{j} + \hat{k}$

The four points are coplanar if the vector $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar

$\overrightarrow{AB} = \hat{i} + (x-3)\hat{j} + 4k,$

$\overrightarrow{AC} = \hat{i} + 0\hat{j} - 3\hat{k},$

$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$

$$\begin{vmatrix} 1 & (x-2) & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1} a_{11}.\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12}.\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13}.\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

⇒ 1(9) – (x – 2)(– 2 + 9) + 4(3) = 0

⇒ 9 – 7x + 14 + 12 = 0

⇒ 35 = 7x

⇒ x = 5

Very short answer

1. Question

Write the value of $\left[2\hat{i}\ \ 3\hat{j}\ \ 4\hat{k}\right]$.

Answer

The meaning of the notation $[\vec{a}\ \vec{b}\ \vec{c}\ ]$ is the scalar triple product of the three vectors; which is computed as $\vec{a}.(\vec{b} \times \vec{c})$

So we have $2\hat{\imath}\,.(3\hat{\jmath} \times 4\hat{k}) = 2\hat{\imath}\,.12\hat{\imath} = 24$ $(\hat{\jmath} \times \hat{k} = \hat{\imath})$

2. Question

Write the value of $\left[\hat{i}+\hat{j}\ \hat{j}+\hat{k}\ \ \hat{k}-\hat{i}\right]$

Answer

Here we have $\vec{a} = \hat{\imath} + \hat{\jmath}, \vec{b} = \hat{\jmath} + \hat{k}, \vec{c} = \hat{k} - \hat{\imath}$

$\vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 0$

3. Question

Write the value of $\left[\hat{i}-\hat{j}\ \hat{j}-\hat{k}\ \ \hat{k}-\hat{i}\right]$.

Answer

The value of the above product is the value of the matrix $\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 0$

4. Question

Find the values of ‘a’ for which the vectors $\vec{\alpha} = \hat{i} + 2\hat{j} + \hat{k}, \vec{\beta} = a\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{\gamma} = \hat{i} + 2\hat{j} + a\hat{k}$ are coplanar.

Answer

Three vectors are coplanar iff (if and only if) $\vec{a}.(\vec{b} \times \vec{c}) = 0$

Hence we have value of the matrix $\begin{vmatrix} 1 & 2 & 1 \\ a & 1 & 2 \\ 1 & 2 & a \end{vmatrix} = 0$

We have $2a^2-3a+1=0$

$2a^2-2a-a+1=0$

Solving this quadratic equation we get $a = 1\,, a = \frac{1}{2}$

5. Question

Find the volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}$ and $\hat{i} + \hat{j} + \pi\hat{k}$.

Answer

Volume of the parallelepiped with its edges represented by the vectors $\vec{a}\,, \vec{b}, \vec{c}$ is $[\vec{a}\ \vec{b}\ \vec{c}\ ] = \vec{a}.(\vec{b} \times \vec{c})$

$=\begin{vmatrix}1 & 1 & 0\\ 1 & 2 & 0\\ 1 & 1 & \pi\end{vmatrix}=\pi$

6. Question

If $\vec{a}, \vec{b}$ are non-collinear vectors, then find the value of $\left[\vec{a}\,\vec{b}\,\hat{i}\right]\hat{i}+\left[\vec{a}\,\vec{b}\,\hat{j}\right]+\left[\vec{a}\,\vec{b}\,\hat{k}\right]\hat{k}$.

Answer

for any vector $\vec{r}$

We have $\vec{r} = (\vec{r}.\hat{\imath})\hat{\imath} + (\vec{r}.\hat{\jmath})\hat{\jmath} + (\vec{r}.\hat{k})\hat{k}$

Replacing $(\vec{r})$=$\vec{a} \times \vec{b}$

$\vec{a} \times \vec{b} = (\vec{a} \times \vec{b}.\hat{\imath})\hat{\imath} + (\vec{a} \times \vec{b}.\hat{\jmath})\hat{\jmath} + (\vec{a} \times \vec{b}.\hat{k})\hat{k}$

$\vec{a} \times \vec{b} = [\vec{a}\,\vec{b}\,\hat{\imath}\,]\hat{\imath} + [\vec{a}\,\vec{b}\,\hat{\jmath}\,]\hat{\jmath} + [\vec{a}\,\vec{b}\,\hat{k}\,]\hat{k}$

7. Question

If the vectors (sec^2 A) $\hat{i}+\hat{j}+\hat{k}, \hat{i}+\left(\sec^2 B\right)\hat{j}+\hat{k}, \hat{i}+\hat{j}+\left(\sec^2 C\right)\hat{k}$ are coplanar, then find the value of cosec^{2}A A + cosec^{2}B + cosec^{2}C.

Answer

For three vectors to be coplanar we have $\begin{vmatrix}sec^2\,A & 1 & 1\\ 1 & sec^2\,B & 1\\ 1 & 1 & sec^2\,C\end{vmatrix} = 0$

Which gives $sec^2 A\, sec^2 B\, sec^2 C - sec^2 A - sec^2 B - sec^2 C + 2 = 0$ ....(1)

$sec^2\theta = \frac{cosec^2\theta - 2}{cosec^2\theta - 1}$ ....(2)

Substituting equation 2 in 1 we have

$$\frac{(cosec^2 A - 2)(cosec^2 B - 2)(cosec^2 C - 2)}{(cosec^2 A - 1)(cosec^2 B - 1)(cosec^2 C - 1)} - \frac{cosec^2 A - 2}{cosec^2 A - 1} - \frac{cosec^2 B - 2}{cosec^2 B - 1} - \frac{cosec^2 C - 2}{cosec^2 B - 1} + 2 = 0$$

Let $cosec^2 A = x\ cosec^2 B = y\ and\ cosec^2 C = z$

So we have $\frac{(x-2)(y-2)(z-2)}{(x-1)(y-1)(z-1)} - \frac{x-2}{x-1} - \frac{y-2}{y-1} - \frac{z-2}{z-1} + 2 = 0$

=(x-2)(y-2)(z-2)-(x-2)(y-1)(z-1)-(x-1)(y-2)(z-1)-(x-1)(y-1)(z-2)+2(x-1)(y-1)(z-1)=0

Solving we have x+y+z=4

Hence cosec^{2}A + cosec^{2}B + cosec^{2}C = 4

8. Question

For any two vectors of $\vec{a}$ and $\vec{b}$ of magnitudes 3 and 4 respectively, write the value of $\left[\vec{a}\,\vec{b}\,\vec{a}\times\vec{b}\right]+\left(\vec{a}\,.\,\vec{b}\right)^2$.

Answer

$[\vec{a}\,\vec{b}\,\vec{c}\,] = \vec{a}.(\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b}.\vec{c})$ the dot and cross can be interchanged in scalar triple product.

Let the angle between $\vec{a}$ and $\vec{b}$ vector be θ

$\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 12\cos\theta$

$\left[\vec{a}\ \vec{b}\ \vec{a}\times\vec{b}\right]+\left(\vec{a}\,.\,\vec{b}\right)^2$. $=\vec{a}.\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$= \vec{a} \times \left(\vec{b}.(\vec{a} \times \vec{b})\right)$

$= (\vec{a} \times \vec{b}).(\vec{a} \times \vec{b})$

$= |(\vec{a} \times \vec{b})||(\vec{a} \times \vec{b})| \cos 0$

$= (|\vec{a}||\vec{b}| \sin \theta)^2$

$=144 \sin^2 \theta+144 \cos^2 \theta$

=144(1)

=144

9. Question

If $\left[3\vec{a}\ 7\vec{b}\ \vec{c}\ \vec{d}\right]=\lambda\left[\vec{a}\ \vec{b}\ \vec{c}\ \right]+\mu\left[\vec{b}\ \vec{c}\ \vec{d}\right]$, then find the value of λ + μ.

Answer

$[3\vec{a}\ 7\vec{b}\ \vec{c}\ \vec{d}] = \lambda[\vec{a}\ \vec{b}\ \vec{c}\ ] + \mu\ [\vec{b}\ \vec{c}\ \vec{d}]$

$3\vec{a} = \lambda\vec{a}$

λ = 3

$\vec{c} = \mu\vec{c}$

μ = 1

So, λ + μ = 3 + 1

= 4

10. Question

If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, then find the value of $\dfrac{\vec{a}\,.\left(\vec{b}\times\vec{c}\right)}{\left(\vec{c}\times\vec{a}\right).\,\vec{b}}+\dfrac{\vec{b}.\left(\vec{a}\times\vec{c}\right)}{\vec{c}.\left(\vec{a}\times\vec{b}\right)}$.

Answer

$[\vec{a}\ \vec{b}\ \vec{c}\ ] = \vec{a}.(\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b}.\vec{c})$ the dot and cross can be interchanged in scalar triple product.

Also $[\vec{a}\ \vec{b}\ \vec{c}\ ] = [\vec{c}\ \vec{a}\ \vec{b}\ ] = [\vec{b}\ \vec{c}\ \vec{a}\ ]$ (cyclic permutation of three vectors does not change the value of the scalar triple product)

$[\vec{a}\ \vec{b}\ \vec{c}\ ] = -[\vec{a}\ \vec{c}\ \vec{b}\ ]$

Using these results $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{\vec{b}.(\vec{a}\times\vec{c})}{\vec{c}.(\vec{a}\times\vec{b})} = \frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{-\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} = 0$

11. Question

Find $\vec{a}\,.\left(\vec{b}\times\vec{c}\right)$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$.

Answer

$\vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = -10$

MCQ

1. Question

Mark the correct alternative in each of the following:

If $\bar{a}$ lies in the plane of vectors $\bar{b}$ and $\bar{c}$, then which of the following is correct?

A. $[\bar{a}\,\bar{b}\,\bar{c}]=0$

B. $[\bar{a}\,\bar{b}\,\bar{c}]=1$

C. $[\bar{a}\,\bar{b}\,\bar{c}]=3$

D. $[\bar{b}\,\bar{c}\,\bar{a}]=1$

Answer

Here, $\vec{a}$ lies in the plane of vectors $\vec{b}$ and $\vec{c}$, which means $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

We know that $\vec{b}\ X\ \vec{c}$ is perpendicular to $\vec{b}$ and $\vec{c}$.

Also dot product of two perpendicular vector is zero.

Since, $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar, $\vec{b}\ X\ \vec{c}$ is perpendicular to $\vec{a}$.

So, $\vec{a}\cdot(\vec{b}\times\vec{c}) = 0$

$\Rightarrow [\vec{a}\,\vec{b}\,\vec{c}] = 0$

2. Question

Mark the correct alternative in each of the following:

The value of $[\bar{a}-\bar{b}\;\bar{b}-\bar{c}\;\bar{c}-\bar{a}]$, where $|\bar{a}|=1, |\bar{b}|=5, |\bar{c}|=3$ is

A. 0

B. 1

C. 6

D. none of these

Answer

$[\vec{a}-\vec{b}\;\vec{b}-\vec{c}\;\vec{c}-\vec{a}] = [\vec{a}\;\vec{b}-\vec{c}\;\vec{c}-\vec{a}] - [\vec{b}\;\vec{b}-\vec{c}\;\vec{c}-\vec{a}]$

$= [\vec{a}\,\vec{b}\;\vec{c}-\vec{a}] - [\vec{b}\,\vec{c}\;\vec{c}-\vec{a}] - [\vec{b}\,\vec{b}\;\vec{c}-\vec{a}] + [\vec{b}\,\vec{b}\;\vec{c}-\vec{a}]$

$= [\vec{a}\,\vec{b}\,\vec{c}] - [\vec{a}\,\vec{b}\,\vec{a}] - [\vec{b}\,\vec{c}\,\vec{c}] - [\vec{b}\,\vec{c}\,\vec{a}] - 0 + 0$

$= [\vec{a}\,\vec{b}\,\vec{c}] - 0 - 0 - [\vec{b}\,\vec{c}\,\vec{a}]$

$= [\vec{a}\,\vec{b}\,\vec{c}] - [\vec{a}\,\vec{b}\,\vec{c}]$

= 0

3. Question

Mark the correct alternative in each of the following:

If $\bar{a}, \bar{b}, \bar{c}$ are three non-coplanar mutually perpendicular unit vectors, then $[\bar{a}\,\bar{b}\,\bar{c}]$ is

A. ±1

B. 0

C. –2

D. 2

Answer

Here, $\vec{a} \perp \vec{b} \perp \vec{c}$ and $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$.

$\Rightarrow \vec{a}\, X\, \vec{b} \,||\, \vec{c}$

⇒ angle between $\vec{a}\, X\, \vec{b}$ and $\vec{c}$ is $\theta = 0°$ or $\theta = 180°$.

$[\vec{a}\, \vec{b}\, \vec{c}] = (\vec{a}\, X\, \vec{b}) \cdot \vec{c}$

$= |\, \vec{a}\,|\,|\, \vec{b}\,|\, sin\theta\, \hat{n} \cdot \vec{c}$

$= 1 \cdot 1 \cdot 1\, \hat{n} \cdot \vec{c}$

$= |\, \hat{n}\,|\,|\, \vec{c}\,|\, cos\theta$

$= 1 \cdot 1\, cos\theta$

= ±1

4. Question

Mark the correct alternative in each of the following:

If $\bar{r}\, .\bar{a} = \bar{r}\, .\bar{b} = \bar{r}\, .\bar{c} = 0$ for some non-zero vector $\bar{r}$, then the value of $\left[\bar{a}\, \bar{b}\, \bar{c}\right]$, is

A. 2

B. 3

C. 0

D. none of these

Answer

Here, $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$

$\Rightarrow \vec{r} \perp \vec{a}, \vec{r} \perp \vec{b}, \vec{r} \perp \vec{c}$

$\Rightarrow \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.

$\Rightarrow [\vec{a}\, \vec{b}\, \vec{c}] = 0$

5. Question

Mark the correct alternative in each of the following:

For any three vector $\bar{a}, \bar{b}, \bar{c}$ the expression $\left(\bar{a} - \bar{b}\right) . \left\{\left(\bar{b} - \bar{c}\right) \times \left(\bar{c} - \bar{a}\right)\right\}$ equals

A. $\left[\bar{a}\, \bar{b}\, \bar{c}\right]$

B. $2\left[\bar{a}\, \bar{b}\, \bar{c}\right]$

C. $\left[\bar{a}\, \bar{b}\, \bar{c}\right]^2$

D. none of these

Answer

$(\vec{a} - \vec{b}) \cdot \{(\vec{b} - \vec{c})\, X\, (\vec{c} - \vec{a})\} = [\vec{a} - \vec{b}\; \vec{b} - \vec{c}\; \vec{c} - \vec{a}]$

$= [\vec{a}\; \vec{b} - \vec{c}\; \vec{c} - \vec{a}] - [\vec{b}\; \vec{b} - \vec{c}\; \vec{c} - \vec{a}]$

$= [\vec{a}\; \vec{b}\; \vec{c} - \vec{a}] - [\vec{b}\; \vec{c}\; \vec{c} - \vec{a}] - [\vec{b}\; \vec{b}\; \vec{c} - \vec{a}] + [\vec{b}\; \vec{b}\; \vec{c} - \vec{a}]$

$= [\vec{a}\; \vec{b}\; \vec{c}] - [\vec{a}\; \vec{b}\; \vec{a}] - [\vec{b}\; \vec{c}\; \vec{c}] - [\vec{b}\; \vec{c}\; \vec{a}] - 0 + 0$

$= [\vec{a}\; \vec{b}\; \vec{c}] - 0 - 0 - [\vec{b}\; \vec{c}\; \vec{a}]$

$= [\vec{a}\; \vec{b}\; \vec{c}] - [\vec{a}\; \vec{b}\; \vec{c}]$

= 0

6. Question

Mark the correct alternative in each of the following:

If $\bar{a}$, $\bar{b}$, $\bar{c}$ are non-coplanar vectors, then $\frac{\bar{a}.(b\_X\,\overline{c})}{\overline{(c\,X\,\bar{a})}.\bar{b}} + \frac{\bar{b}.(\bar{a}X\bar{c})}{\bar{c}.(\bar{a}X\bar{b})}$ is

A. 0

B. 2

C. 1

D. none of these

Answer

$$\frac{\vec{a}\cdot(b^{\rightarrow} X\,\overrightarrow{c)}}{\overline{(c\,X\,\vec{a})}\cdot b^{\rightarrow}} + \frac{\vec{b}\cdot(\vec{a}X\vec{c})}{\vec{c}\cdot(\vec{a}X\vec{b})} = \frac{[\vec{a}\;\vec{b}\;\vec{c}]}{[\vec{c}\;\vec{a}\;\vec{b}]} + \frac{[\vec{b}\;\vec{a}\;\vec{c}]}{[\vec{c}\;\vec{a}\;\vec{b}]}$$

$$= \frac{[\vec{a}\;\vec{b}\;\vec{c}] + [\vec{b}\;\vec{a}\;\vec{c}]}{[\vec{c}\;\vec{a}\;\vec{b}]}$$

$$= \frac{[\vec{a}\;\vec{b}\;\vec{c}] - [\vec{a}\;\vec{b}\;\vec{c}]}{[\vec{c}\;\vec{a}\;\vec{b}]}$$

=0

7. Question

Mark the correct alternative in each of the following:

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is equal to

A. 0

B. 1

C. $(\frac{1}{4})|\vec{a}|^2|\vec{b}|^2$

D. $(\frac{3}{4})|\vec{a}|^2|\vec{b}|^2$

Answer

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}^2 = [\vec{a}\,\vec{b}\,\vec{c}]^2$$

$$= [(\overrightarrow{aXb}) \cdot \vec{c}]^2$$

$$= \left[|\vec{a}||\vec{b}| \sin\left(\frac{\pi}{6}\right) \cdot \vec{c}\right]^2$$

$$= |\vec{a}|^2 |\vec{b}|^2 \left(\frac{1}{4}\right) \cdot \vec{c}^2$$

$= |\vec{a}|^2 |\vec{b}|^2 \left(\frac{1}{4}\right) \cdot |\overrightarrow{c}|^2 \cos 0$ (∵ $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$ ⇒ angle is 0)

$$= |\vec{a}|^2 |\vec{b}|^2 \left(\frac{1}{4}\right) \cdot |\overrightarrow{c}|^2$$

$= |\vec{a}|^2 |\vec{b}|^2 \left(\frac{1}{4}\right)$ (∵ $\vec{c}$ is unit vector)

$$= \left(\frac{1}{4}\right) |\vec{a}|^2 |\vec{b}|^2$$

8. Question

Mark the correct alternative in each of the following:

If $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} - 2\hat{k}$, then the volume of the parallelepiped with conterminous edges $\bar{a} + \bar{b}$, $\bar{b} + \bar{c}$, $\bar{c} + \bar{a}$ is

A. 2

B. 1

C. –1

D. 0

Answer

Let $\vec{e} = \vec{a} + \vec{b} = 5\hat{\imath} - 7\hat{\jmath} + 10\hat{k}$

$\vec{f} = \vec{b} + \vec{c} = 8\hat{\imath} - 7\hat{\jmath} + 3\hat{k}$

$\vec{g} = \vec{c} + \vec{a} = 7\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$

Now,the volume of the parallelepiped with conterminous edges $\vec{e}$, $\vec{f}$,$\vec{g}$ is given by

$V = [\vec{e}\,\vec{f}\,\vec{g}\,]$

$$= \begin{bmatrix} e_1 & e_2 & e_3 \\ f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \end{bmatrix} = \begin{bmatrix} 5 & -7 & 10 \\ 8 & -7 & 3 \\ 7 & -6 & 3 \end{bmatrix}$$

=5× (-21+18)+7× (24-21)+10× (-48+49) ×

=5× (-3)+7× 3+10× 1

=-15+21+10

=16

9. Question

Mark the correct alternative in each of the following:

If $\left[2\bar{a} + 4\bar{b}\,\bar{c}\,\bar{d}\right] = \lambda\left[\bar{a}\,\bar{c}\,\bar{d}\right] + \mu\left[\bar{b}\,\bar{c}\,\bar{d}\right]$ then $\lambda + \mu =$

A. 6

B. –6

C. 10

D. 8

Answer

$\lambda[\vec{a}\,\vec{c}\,\vec{d}] + \mu[\vec{b}\,\vec{c}\,\vec{d}] = [2\vec{a} + 4\vec{b}\,\vec{c}\,\vec{d}]$

$= [2\vec{a}\,\vec{c}\,\vec{d}] + [4\vec{b}\,\vec{c}\,\vec{d}]$

$= 2[\vec{a}\,\vec{c}\,\vec{d}] + 4[\vec{b}\,\vec{c}\,\vec{d}]$

Now, comparing the coefficient of lhs and rhs we get, λ=2 and μ=4

∴ λ + μ = 2+4

=6

10. Question

Mark the correct alternative in each of the following:

$\left[\bar{a}\,\bar{b}\,\bar{a}\,X\,\bar{b}\right] + \left(\bar{a}\,.\bar{b}\right)^2$

A. $|\bar{a}|^2 |\bar{b}|^2$

B. $|\,\bar{a}+\bar{b}\,|^2$

C. $|\bar{a}|^2 + |\bar{b}|^2$

D. $2|\bar{a}|^2 + |\bar{b}|^2$

Answer

$[\vec{a}\,\vec{b}\,\vec{a}\,X\,\vec{b}] + (\vec{a}\cdot\vec{b})^2 = (\vec{a}\,X\,\vec{b})\cdot(\vec{a}\,X\,\vec{b}) + (\vec{a}\cdot\vec{b})^2$

$= (\vec{a}\,X\,\vec{b})^2 + (\vec{a}\cdot\vec{b})^2$

$= |a|^2|b|^2\sin^2\theta + |a|^2|b|^2\cos^2\theta$

$= |a|^2|b|^2(\sin^2\theta + \cos^2\theta)$

$= |a|^2|b|^2$

11. Question

Mark the correct alternative in each of the following:

If the vectors $4\hat{i} + 11\hat{j} + m\hat{k}$, $7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then m =

A. 0

B. 38

C. –10

D. 10

Answer

$\vec{a} = (4\;11\;m)$

$\vec{b} = (7\ 02\ 6)$

$\vec{c} = (1\ 05\ 4)$

Here, vector a, b, and c are coplanar. So, $[\vec{a}\ \vec{b}\ \vec{c}\ ] = 0$.

$\therefore \begin{bmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{bmatrix} = 0$

∴ 4(8-30)-11(28-6)+m(35-2)= 0

∴ 4(-22)-11(22)+33m = 0

∴ -88 -242 +33m = 0

∴ 33m = 330

∴ m = 10

12. Question

Mark the correct alternative in each of the following:

For non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ the relation $|(\vec{a} \times \vec{b}).\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds good, if

A. $\vec{a}.\vec{b} = \vec{b}.\vec{c} = 0$

B. $\vec{a}.\vec{b} = 0 = \vec{c}.\vec{a}$

C. $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

D. $\vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

Answer

Let $\vec{e} = \vec{a} X \vec{b}$

$|\vec{e}| = |\vec{a}||\vec{b}|sin\alpha$ --------(1) (∵ α is angle between $\vec{a}$ and $\vec{b}$)

Then $|(\vec{a} X \vec{b}) \cdot \vec{c}| = |\vec{e} \cdot \vec{c}|$

$= |\vec{e}||\vec{c}|cos\theta$ (∵θ is angle between $\vec{e}$ and $\vec{c}$ ⇒ θ is angle between $\vec{a}$ X$\vec{b}$ and $\vec{c}$)

$= |\vec{a}||\vec{b}||\vec{c}|cos\theta sin\alpha$ (∵ using (1))

Hence, $|(\overrightarrow{a X b}) \cdot \vec{c} = |\vec{a}||\vec{b}||\vec{c}|$ if and only if $cos\theta sin\alpha = 1$

if and only if $cos\theta = 1$ and $\sin\alpha = 1$

if and only if $\theta = 0$ and $\alpha = \frac{\pi}{2}$

$\alpha = \frac{\pi}{2}$⇒ $\vec{a}$ and $\vec{b}$ are perpendicular.

Also $\vec{e}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

θ=0⇒ $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

∴ $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular.

∴ $\vec{a}$• $\vec{b}$=$\vec{b}$• $\vec{c}$=$\vec{c}$• $\vec{a}$=0

13. Question

Mark the correct alternative in each of the following:

$\left(\bar{a}+\bar{b}\right).\left(\bar{b}+\bar{c}\right)X\left(\bar{a}+\bar{b}+\bar{c}\right)=$

A. 0

B. $-\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

C. $2\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

D. $\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

Answer

$(\vec{a}+\vec{b})\cdot(\vec{b}+\vec{c})\,X\,(\vec{a}+\vec{b}+\vec{c})=[(\vec{a}+\vec{b})\,(\vec{b}+\vec{c})\,(\vec{a}+\vec{b}+\vec{c})]$

$=[\vec{a}\ \overrightarrow{b}+\overrightarrow{c}\ \vec{a}+\vec{b}+\vec{c}]+[\vec{b}\ \overrightarrow{b}+\overrightarrow{c}\ \vec{a}+\vec{b}+\vec{c}]$

$=[\vec{a}\ \overrightarrow{b}\ \vec{a}+\vec{b}+\vec{c}]+[\vec{a}\ \overrightarrow{c}\ \vec{a}+\vec{b}+\vec{c}]+[\vec{b}\ \overrightarrow{b}\ \vec{a}+\vec{b}+\vec{c}]+[\vec{b}\ \overrightarrow{c}\ \vec{a}+\vec{b}+\vec{c}]$

$=[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{a}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{b}\,\vec{b}]+[\vec{a}\,\vec{c}\,\vec{c}]+[\vec{a}\,\vec{c}\,\vec{b}]+[\vec{a}\,\vec{c}\,\vec{a}]+0+[\vec{b}\,\vec{c}\,\vec{a}]$
$\quad+[\vec{b}\,\vec{c}\,\vec{b}]+[\vec{b}\,\vec{c}\,\vec{c}]$

$=[\vec{a}\,\vec{b}\,\vec{c}]+0+0+0+[\vec{a}\,\vec{c}\,\vec{b}]+0+[\vec{b}\,\vec{c}\,\vec{a}]+0+0$

$=[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{a}\,\vec{c}\,\vec{b}]-[\vec{a}\,\vec{c}\,\vec{b}]$

$=[\vec{a}\,\vec{b}\,\vec{c}]$

14. Question

Mark the correct alternative in each of the following:

If a⃗ ,b⃗,c⃗ are three non-coplanar vectors, then $\left(\bar{a}+\bar{b}+\bar{c}\right).\left[\left(\bar{a}+\bar{b}\right)X\left(\bar{a}+\bar{c}\right)\right]$equal.

A. 0

B. $\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

C. $2\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

D. $-\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

Answer

$(\vec{a}+\vec{b}+\vec{c})\cdot[(\vec{a}+\vec{b})\,X\,(\vec{a}+\vec{c})]=[(\vec{a}+\vec{b}+\vec{c})\,(\vec{a}+\vec{b})\,(\vec{a}+\vec{c})]$

$=[\vec{a}+\vec{b}+\vec{c}\ \vec{a}\ \overrightarrow{a}+\overrightarrow{c}]+[\vec{a}+\vec{b}+\vec{c}\ \vec{b}\ \overrightarrow{a}+\overrightarrow{c}]$

$=[\vec{a}+\vec{b}+\vec{c}\ \vec{a}\ \overrightarrow{a}]+[\vec{a}+\vec{b}+\vec{c}\ \vec{a}\ \overrightarrow{c}]+[\vec{a}+\vec{b}+\vec{c}\ \vec{b}\ \overrightarrow{a}]+[\vec{a}+\vec{b}+\vec{c}\ \vec{b}\ \overrightarrow{c}]$

$=[\vec{a}\ \overrightarrow{a}\ \vec{a}]+[\vec{b}\,\vec{a}\,\vec{a}]+[\vec{c}\,\vec{a}\,\vec{a}]+[\vec{a}\,\vec{a}\,\vec{c}]+[\vec{b}\,\vec{a}\,\vec{c}]+[\vec{c}\,\vec{a}\,\vec{c}]+[\vec{a}\,\vec{b}\,\vec{a}]+$ $+[\vec{c}\,\vec{b}\,\vec{c}]$
$[\vec{b}\,\vec{b}\,\vec{a}]+[\vec{c}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{b}\,\vec{b}\,\vec{c}]$

$=0+0+0+0+[\vec{b}\,\vec{a}\,\vec{c}]+0+0+0+[\vec{c}\,\vec{b}\,\vec{a}]+[\vec{a}\,\vec{b}\,\vec{c}]+0+0$

$=2[\vec{a}\,\vec{b}\,\vec{c}]$

15. Question

Mark the correct alternative in each of the following:

$\left(\bar{a}+2\bar{b}-\bar{c}\right).\left\{\left(\bar{a}-\bar{b}\right)X\left(\bar{a}-\bar{b}-\bar{c}\right)\right\}$ is equal to

A. $\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

B. $2\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

C. $3\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

D. 0

Answer

$(\vec{a}+2\vec{b}-\vec{c})\cdot\{(\vec{a}-\vec{b})\,X\,(\vec{a}-\vec{b}-\vec{c})\}=[\vec{a}+2\vec{b}-\vec{c}\;\vec{a}-\vec{b}\;\vec{a}-\vec{b}-\vec{c}]$

$=[\vec{a}\;\vec{a}-\vec{b}\;\vec{a}-\vec{b}-\vec{c}]+[2\vec{b}\;\vec{a}-\vec{b}\;\vec{a}-\vec{b}-\vec{c}]-[\vec{c}\;\vec{a}-\vec{b}\;\vec{a}-\vec{b}-\vec{c}]$

$=[\vec{a}\;\vec{a}\;\vec{a}-\vec{b}-\vec{c}]-[\vec{a}\;\vec{b}\;\vec{a}-\vec{b}-\vec{c}]+[2\vec{b}\;\vec{a}\;\vec{a}-\vec{b}-\vec{c}]-[2\vec{b}\;\vec{b}\;\vec{a}-\vec{b}-\vec{c}]$
$-[\vec{c}\;\vec{a}\;\vec{a}-\vec{b}-\vec{c}]+[\vec{c}\;\vec{b}\;\vec{a}-\vec{b}-\vec{c}]$

$=0-[\vec{a}\;\vec{b}\;\vec{a}]-[\vec{a}\;\vec{b}\;\vec{b}]-[\vec{a}\;\vec{b}\;\vec{c}]+[2\vec{b}\;\vec{a}\;\vec{a}]-[2\vec{b}\;\vec{a}\;\vec{b}]-[2\vec{b}\;\vec{a}\;\vec{c}]-[2\vec{b}\;\vec{b}\;\vec{a}]$
$+[2\vec{b}\;\vec{b}\;\vec{b}]+[2\vec{b}\;\vec{b}\;\vec{c}]-[\vec{c}\;\vec{a}\;\vec{a}]+[\vec{c}\;\vec{a}\;\vec{b}]+[\vec{c}\;\vec{a}\;\vec{c}]+[\vec{c}\;\vec{b}\;\vec{a}]$
$-[\vec{c}\;\vec{b}\;\vec{b}]-[\vec{c}\;\vec{b}\;\vec{c}]$

$=0-0-0-[\vec{a}\;\vec{b}\;\vec{c}]+0-0-2[\vec{b}\;\vec{a}\;\vec{c}]-0+0+0-0+[\vec{c}\;\vec{a}\;\vec{b}]+0+[\vec{c}\;\vec{b}\;\vec{a}]$
$-0-0$

$=-[\vec{a}\;\vec{b}\;\vec{c}]+2[\vec{a}\;\vec{b}\;\vec{c}]+[\vec{a}\;\vec{b}\;\vec{c}]-[\vec{a}\;\vec{b}\;\vec{c}]$

$=[\vec{a}\;\vec{b}\;\vec{c}]$