Chapter : 21. CIRCLE

Exercise : 21A

Question: 1

Find the equation

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

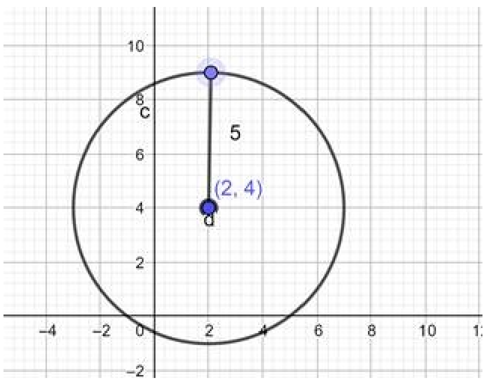
Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 5^2$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 25$



Ans; equation of a circle with Centre (2, 4) and radius 5 is:

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 25$

Question: 2

Find the equation

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

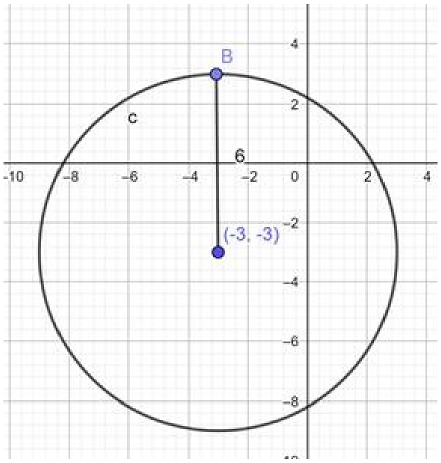
Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - (- 3))^2 + (y - (- 2))^2 = 6^2$

$\Rightarrow (x + 3)^2 + (y + 2)^2 = 36$

Ans; equation of a circle with Centre (- 3, - 2) and radius 6 is:

$\Rightarrow (x + 3)^2 + (y + 2)^2 = 36$



Question: 3

Find the equation

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - a)^2 + (y - a)^2 = (\sqrt{2})^2$

$\Rightarrow (x - a)^2 + (y - a)^2 = 2$

Ans; equation of a circle with Centre (a, a) and radius $\sqrt{2}$

is:

$(x - a)^2 + (y - a)^2 = 2$

Question: 4

Find the equation

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$(x - (a \cos \propto))^2 + (y - (a \sin \propto))^2 = a^2$

$\Rightarrow (x - a \cos \propto)^2 + (y - a \sin \propto)^2 = a^2$

$\Rightarrow x^2 - 2xa\cos \alpha + a^2 \cos^2 \alpha + y^2 - 2ya\sin \alpha + a^2 \sin^2 \alpha = a^2$

$\Rightarrow x^2 + y^2 + a^2 (\cos^2 \alpha + \sin^2 \alpha) - 2a(x\cos \alpha + y\sin \alpha) = a^2$

$\Rightarrow x^2 + y^2 + a^2 - 2a(x\cos \alpha + y\sin \alpha) = a^2$ ...$((\cos^2\alpha + \sin^2 \alpha) = 1)$

$\Rightarrow x^2 + y^2 - 2a(x\cos \alpha + y\sin \alpha) = 0$

Ans: equation of a circle with Centre $(a \cos \propto, a \sin \propto)$ and radius a is:

$x^2 + y^2 - 2a(x\cos \alpha + y\sin \alpha) = 0$

Question: 5

Find the equation

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - (- a))^2 + (y - (- b))^2 = \surd(a^22 - b^22)^2$

$\Rightarrow (x + a)^2 + (y + b)^2 = a^2 - b^2$

$\Rightarrow x^2 + 2xa + a^2 + y^2 + 2y\alpha + b^2 = a^2 - b^2$

$\Rightarrow x^2 + 2xa + y^2 + 2y\alpha = a^2 - 2b^2$

$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$

$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$

Ans; equation of a circle with Centre (- a, - b) and radius

is:

$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$

Question: 6

Find the equation

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

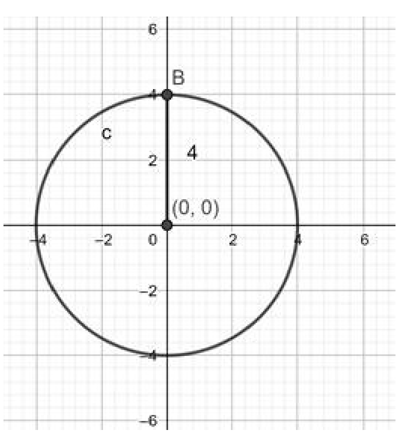
Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - 0)^2 + (y - 0)^2 = 4^2$

$\Rightarrow x^2 + y^2 = 16$



Ans; equation of a circle with . Centre at the origin and radius 4 is:

$x^2 + y^2 = 16$

Question: 7 A

Find the centre a

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$h = 3$, $k = 1$, $r^2 = 9$

⇒ centre = (3, 1) and radius = 3 units.

Ans: centre = (3, 1) and radius = 3 units.

Question: 7 B

Find the centre a

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$h = 1/2$, $k = - 1/3$, $r^2 = 1/16$

⇒ centre = (1/2, - 1/3) and radius = 1/4 units.

Ans: centre = (1/2, - 1/3) and radius = 1/4 units.

Question: 7 C

Find the centre a

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$h = - 5$, $k = 3$, $r^2 = 20$

⇒ centre = (- 5, 3) and radius = $\sqrt{20} = 2\sqrt{5}$ units.

Ans: centre = (- 5, 3) and radius = $2\sqrt{5}$ units.

Question: 7 D

Find the centre a

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$h = 0$, $k = 1$, $r^2 = 2$

⇒ centre = (0, 1) and radius = √2units.

Ans: centre = (0, 1) and radius = √2units.

Question: 8

Find the equation

Solution:

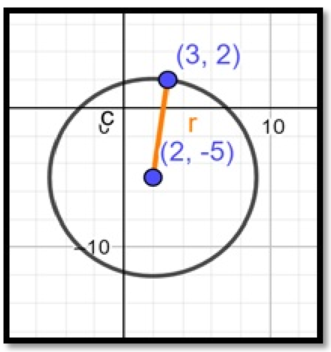
The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

In this question we know that (h, k) = (2, - 5), so for determining the equation of the circle we need to determine the radius of the circle.



Since the circle passes through (3, 2), that pair of values for x and y must satisfy the equation and we have:

$\Rightarrow (3 - 2)^2 + (2 - (- 5))^2 = r^2$

$\Rightarrow 1^2 + 7^2 = r^2$

$\Rightarrow r^2 = 49 + 1 = 50$

$\therefore r^2 = 50$

⇒ Equation of circle is:

$(x - 2)^2 + (y - (- 5))^2 = 50$

$\Rightarrow (x - 2)^2 + (y + 5)^2 = 50$

Ans: $(x - 2)^2 + (y + 5)^2 = 50$

Question: 9

Find the equation

Solution:

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Since, centre lies on Y - axis, ∴ it’s X - coordinate = 0, i.e.h = 0

Hence, (0, k) is the centre of the circle.

Substituting the given values in general form of the equation of a circle we get,

$\Rightarrow (3 - 0)^2 + (2 - k)^2 = 5^2$

$\Rightarrow (3)^2 + (2 - k)^2 = 25$

$\Rightarrow 9 + (2 - k)^2 = 25$

$\Rightarrow (2 - k)^2 = 25 - 9 = 16$

Taking square root on both sides we get,

$\Rightarrow 2 - k = \pm 4$

$\Rightarrow 2 - k = 4$ & $2 - k = -4$

$\Rightarrow k = 2 - 4$ & $k = 2 + 4$

$\Rightarrow k = -2$ & $k = 6$

$\therefore$ Equation of circle when k = - 2 is:

$x^2 + (y + 2)^2 = 25$

Equation of circle when k = 6 is:

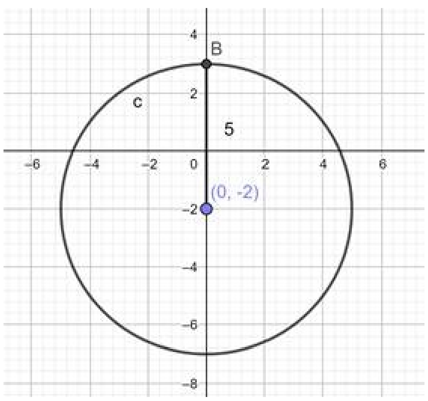
$x^2 + (y - 6)^2 = 25$

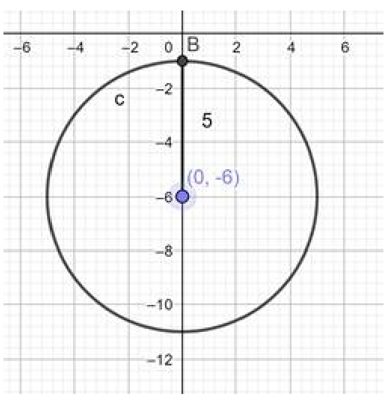
Ans: Equation of circle when k = - 2 is:

$x^2 + (y + 2)^2 = 25$

Equation of circle when k = 6 is:

$x^2 + (y - 6)^2 = 25$

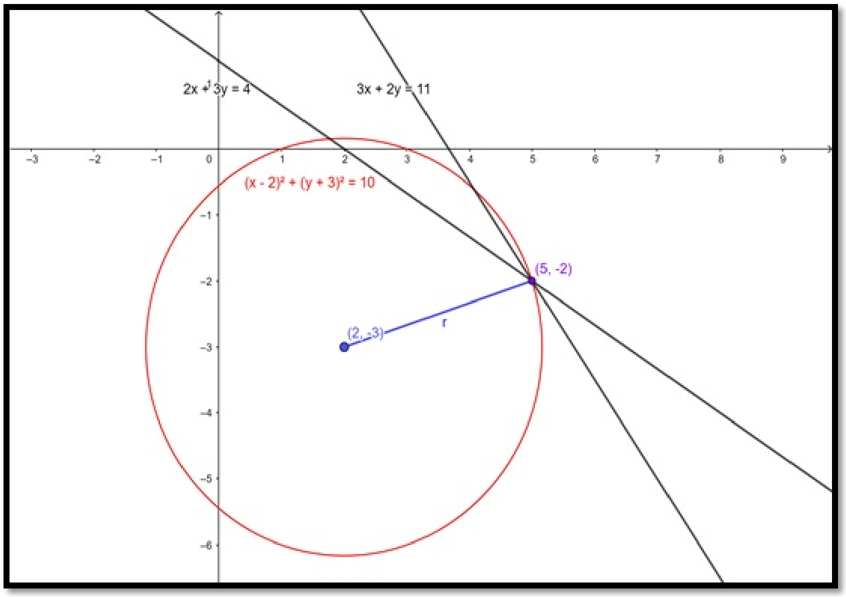




Question: 10

Find the equation

Solution:



The intersection of the lines: 3x + 2y = 11 and 2x + 3y = 4

Is (5, - 2)

∴ This problem is same as solving a circle equation with centre and point on the circle given.

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

In this question we know that (h, k) = (2, - 3), so for determining the equation of the circle we need to determine the radius of the circle.

Since the circle passes through (5, - 2), that pair of values for x and y must satisfy the equation and we have:

$\Rightarrow (5 - 2)^2 + (-2 - (-3))^2 = r^2$

$\Rightarrow 3^2 + 1^2 = r^2$

$\Rightarrow r^2 = 9 + 1 = 10$

$\therefore r^2 = 10$

⇒ Equation of circle is:

$(x - 2)^2 + (y - (-3))^2 = 10$

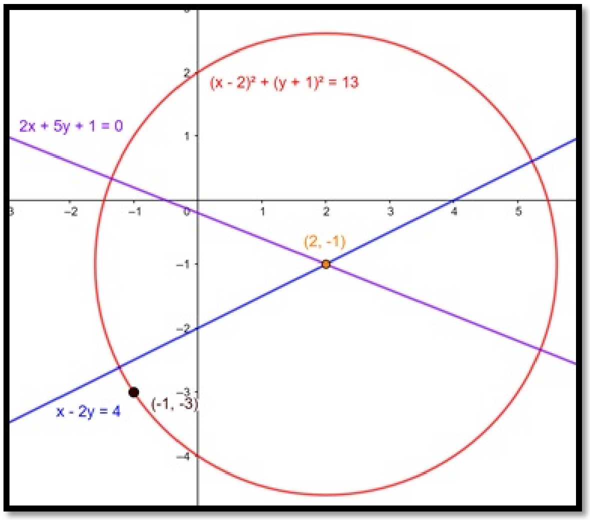
$\Rightarrow (x - 2)^2 + (y + 3)^2 = 10$

Ans:$(x - 2)^2 + (y + 5)^2 = 10$

Question: 11

Find the equation

Solution:



The intersection of the lines: x – 2y = 4 and 2x + 5y + 1 = 0.

is (2, - 1)

∴ This problem is same as solving a circle equation with centre and point on the circle given.

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

In this question we know that (h, k) = (2, - 1), so for determining the equation of the circle we need to determine the radius of the circle.

Since the circle passes through (- 1, - 3), that pair of values for x and y must satisfy the equation and we have:

$\Rightarrow (- 1 - 2)^2 + (- 3 - (- 1))^2 = r^2$

$\Rightarrow (- 3)^2 + (- 2)^2 = r^2$

$\Rightarrow r^2 = 9 + 4 = 13$

$\therefore r^2 = 13$

⇒ Equation of circle is:

$(x - 2)^2 + (y - (- 1))^2 = 13$

$\Rightarrow (x - 2)^2 + (y + 1)^2 = 13$

Ans: $(x - 2)^2 + (y + 1)^2 = 13$

Question: 12

If two diameters

Solution:

The point of intersection of two diameters is the centre of the circle.

∴ point of intersection of two diameters x – y = 9 and x – 2y = 7 is (11, 2).

∴ centre = (11, 2)

Area of a circle = πr^2

$38.5 = \pi r^2$

$\Rightarrow r^2 = \frac{38.5}{\pi}$

$\Rightarrow r^2 = 12.25$ sq.cm

the equation of the circle is:

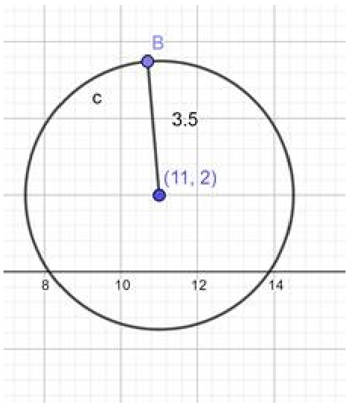
$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

$\Rightarrow (x - 11)^2 + (y - 2)^2 = 12.25$

Ans: $(x - 11)^2 + (y - 2)^2 = 12.25$



Question: 13 A

Find the equation

Solution:

The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (3, 2)$ & $(x_2, y_2) = (2, 5)$

We get:

$(x - 3)(x - 2) + (y - 2)(y - 5) = 0$

$\Rightarrow x^2 - 2x - 3x + 6 + y^2 - 5y - 2y + 10 = 0$

$\Rightarrow x^2 + y^2 - 5x - 7y + 16 = 0$

Ans: $x^2 + y^2 - 5x - 7y + 16 = 0$

Question: 13 B

Find the equation

Solution:

The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (5, - 3)$ & $(x_2, y_2) = (2, - 4)$

We get:

$(x - 5)(x - 2) + (y + 3)(y + 4) = 0$

$\Rightarrow x^2 - 2x - 5x + 10 + y^2 + 3y + 4y + 12 = 0$

$\Rightarrow x^2 + y^2 - 7x + 7y + 22 = 0$

Ans: $x^2 + y^2 - 7x + 7y + 22 = 0$

Question: 13 C

Find the equation

Solution:

The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (-2, -3)$ & $(x_2, y_2) = (-3, 5)$

We get:

$(x + 2)(x + 3) + (y + 3)(y - 5) = 0$

$\Rightarrow x^2 + 3x + 2x + 6 + y^2 - 5y + 3y - 15 = 0$

$\Rightarrow x^2 + y^2 + 5x - 2y - 9 = 0$

Ans: $x^2 + y^2 + 5x - 2y - 9 = 0$

Question: 13 D

Find the equation

Solution:

The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (p, q)$ & $(x_2, y_2) = (r, s)$

We get:

$(x - p)(x - r) + (y - q)(y - s) = 0$

$\Rightarrow x^2 - rx - px + pr + y^2 - sy - qy + qs = 0$

$\Rightarrow x^2 + y^2 - (r + p)x - (s + q)y + (pr + qs) = 0$

Ans: $x^2 + y^2 - (r + p)x - (s + q)y + (pr + qs) = 0$

Question: 14

The sides of a re

Solution:

The intersection points in clockwise fashion are:$(-2, 5), (4, 5), (4, -2), (-2, -2)$.

The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

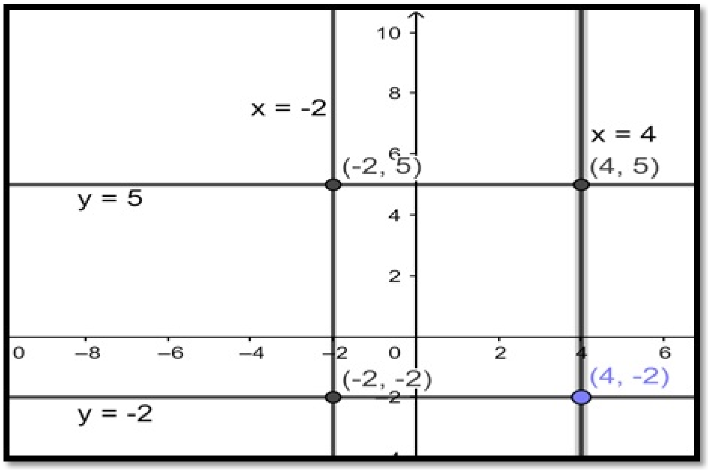
Substituting, values:$(x_1, y_1) = (-2, 5)$ & $(x_2, y_2) = (4, -2)$

We get:

$(x + 2)(x - 4) + (y - 5)(y + 2) = 0$

$\Rightarrow x^2 - 4x + 2x - 8 + y^2 + 2y - 5y - 10 = 0$

$\Rightarrow x^2 + y^2 - 2x - 3y - 18 = 0$



Ans:$x^2 + y^2 - 2x - 3y - 18 = 0$

Exercise : 21B

Question: 1

Show that the equ

Solution:

The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ where a, b, c, f, g, h are constants

For a circle, a = b and h = 0.

The equation becomes:

$x^2 + y^2 + 2gx + 2fy + c = 0$...(i)

Given, $x^2 + y^2 – 4x + 6y – 5 = 0$

Comparing with (i) we see that the equation represents a circle with $2g = - 4 \Rightarrow g = - 2$, $2f = 6 \Rightarrow f = 3$ and $c = - 5$.

Centre $(- g, - f) = \{ - (- 2), - 3\}$

$= (2, - 3)$.

Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-2)^2 + 3^2 - (-5)}$

$= \sqrt{4 + 9 + 5} = \sqrt{18} = 3\sqrt{2}$.

Question: 2

Show that the equ

Solution:

The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ where a, b, c, f, g, h are constants

For a circle, a = b and h = 0.

The equation becomes:

$x^2 + y^2 + 2gx + 2fy + c = 0$...(i)

Given, $x^2 + y^2 + x - y = 0$

Comparing with (i) we see that the equation represents a circle with $2g = 1 \Rightarrow g = \frac{1}{2}$, $2f = - 1$

$\Rightarrow f = -\frac{1}{2}$ and c = 0.

Centre (- g, - f) = $\{-\frac{1}{2}, -(-\frac{1}{2})\}$

= $(-\frac{1}{2}, \frac{1}{2})$.

Radius = $\sqrt{g^2 + f^2 - c}$

= $\sqrt{\frac{1}{2}^2 + (-\frac{1}{2}^2) - 0}$

= $\sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}$.

Question: 3

Show that the equ

Solution:

The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ where a, b, c, f, g, h are constants

For a circle, a = b and h = 0.

The equation becomes:

$x^2 + y^2 + 2gx + 2fy + c = 0$…(i)

Given, $3x^2 + 3y^2 + 6x - 4y - 1 = 0 \Rightarrow x^2 + y^2 + 2x - \frac{4}{3}y - \frac{1}{3} = 0$

Comparing with (i) we see that the equation represents a circle with 2g = 2 ⇒g = 1, 2f = $-\frac{4}{3} \Rightarrow f = -\frac{2}{3}$ and $c = -\frac{1}{3}$.

Centre (- g, - f) = $\{-1, -(-\frac{2}{3})\}$

= $(-1, \frac{2}{3})$.

Radius = $\sqrt{g^2 + f^2 - c}$

= $\sqrt{1^2 + (-\frac{2}{3})^2 - (-\frac{1}{3})}$

= $\sqrt{1 + \frac{4}{9} + \frac{1}{3}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$.

Question: 4

Show that the equ

Solution:

The general equation of a circle:

$x^2 + y^2 + 2gx + 2fy + c = 0$…(i) where c, g, f are constants.

Given, $x^2 + y^2 + 2x + 10y + 26 = 0$

Comparing with (i) we see that the equation represents a circle with 2g = 2⇒g = 1, 2f = 10⇒f = 5 and c = 26.

Centre (- g, - f) = (- 1, - 5).

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{1^2 + 5^2 - 26}$

$= \sqrt{26 - 26} = 0.$

Thus it is a point circle with radius 0.

Question: 5

Show that the equ

Solution:

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{(-\frac{3}{2})^2 + (-\frac{3}{2}^2) - 10}$

$= \sqrt{\frac{9}{2} - 10} = \sqrt{-\frac{11}{2}}$, which implies that the radius is negative. (not possible)

Therefore, $x^2 + y^2 - 3x + 3y + 10 = 0$ does not represent a circle.

Question: 6

Find the equation

Solution:

(i) The required circle equation

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 0^2+0^2 & 0 & 0 & 1 \\ 5^2+0^2 & 5 & 0 & 1 \\ 3^2+3^2 & 3 & 3 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2)\begin{vmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 3 & 3 & 1 \end{vmatrix} - x\begin{vmatrix} 0 & 0 & 1 \\ 25 & 0 & 1 \\ 18 & 3 & 1 \end{vmatrix}$$

$$+ y\begin{vmatrix} 0 & 0 & 1 \\ 25 & 5 & 1 \\ 18 & 3 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 25 & 5 & 0 \\ 18 & 3 & 3 \end{vmatrix} = 0$$

$\Rightarrow 15(x^2 + y^2) - 75x - 15y = 0$

$\Rightarrow x^2 + y^2 - 5x - y = 0$ is the equation with centre = (2.5, 0.5)

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-2.5^2) + (-0.5)^2 - 0} = 2.549$

(ii) The required circle equation

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 1^2+2^2 & 1 & 2 & 1 \\ 3^2+(-4)^2 & 3 & -4 & 1 \\ 5^2+(-6)^2 & 5 & -6 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2)\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 1 \\ 5 & -6 & 1 \end{vmatrix} - x\begin{vmatrix} 5 & 2 & 1 \\ 25 & -4 & 1 \\ 61 & -6 & 1 \end{vmatrix} + y\begin{vmatrix} 5 & 1 & 1 \\ 25 & 3 & 1 \\ 61 & 5 & 1 \end{vmatrix} - \begin{vmatrix} 5 & 1 & 2 \\ 25 & 3 & -4 \\ 61 & 5 & -6 \end{vmatrix} = 0$$

$\Rightarrow 8(x^2 + y^2) - 176x - 32y - 200 = 0$

$\Rightarrow x^2 + y^2 - 22x - 4y - 25 = 0$ is the equation with centre = (11, 2)

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-11)^2 + (-2)^2 - 25} = 10$

(iii) The required circle equation

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 20^2+3^2 & 20 & 3 & 1 \\ 19^2+8^2 & 19 & 8 & 1 \\ 2^2+(-9)^2 & 2 & -9 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2)\begin{vmatrix} 20 & 3 & 1 \\ 19 & 8 & 1 \\ 2 & -9 & 1 \end{vmatrix} - x\begin{vmatrix} 409 & 3 & 1 \\ 425 & 8 & 1 \\ 85 & -9 & 1 \end{vmatrix} + y\begin{vmatrix} 409 & 20 & 1 \\ 425 & 19 & 1 \\ 85 & 2 & 1 \end{vmatrix} - \begin{vmatrix} 409 & 20 & 3 \\ 425 & 19 & 8 \\ 85 & 2 & -9 \end{vmatrix} = 0$$

$\Rightarrow 102(x^2 + y^2) - 1428x - 612y - 11322 = 0$

$\Rightarrow x^2 + y^2 - 14x - 6y - 111 = 0$ is the equation with centre = (7, 3)

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-7)^2 + (-3)^2 - (-111)} = 13$

Question: 7

Find the equation

Solution:

The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

...(i), where (h, k) is the centre and r is the radius.

Putting A(- 2, 3), B(5, 2) and c(6, - 1) in (i) we get

$h^2 + k^2 + 4h - 6k + 13 = r^2$ ...(ii)

$h^2 + k^2 - 10h - 4k + 29 = r^2$ ...(iii)and

$h^2 + k^2 - 12h + 2k + 37 = r^2$ ...(iv)

subtracting (ii) from (iii) and also from (iv),

$- 14h + 2k + 16 = 0 \Rightarrow - 7h + k + 8 = 0$

$- 16h + 8k + 24 = 0 \Rightarrow - 2h + k + 3 = 0$

Subtracting,

$5h - 5 = 0 \Rightarrow h = 1$

$k = - 1$

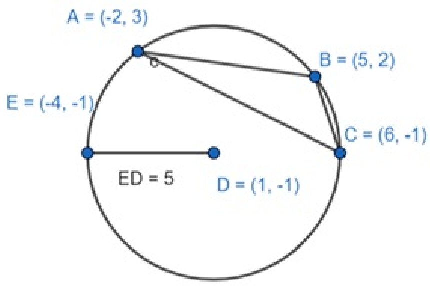
Centre = (1, - 1)

Putting these values in (ii) we get, radius = $\sqrt{1 + 1 + 4 + 6 + 13} = \sqrt{25} = 5$

Equation of the circle is

$(x - 1)^2 + \{y - (- 1)\}^2 = 5^2$

$(x - 1)^2 + (y + 1)^2 = 25.$

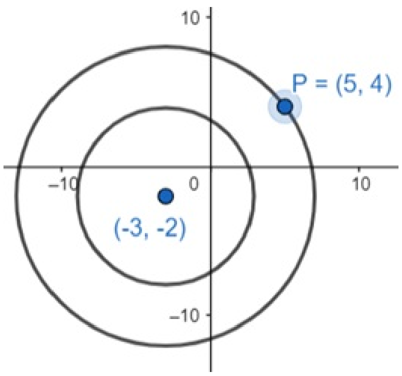


Question: 8

Find the equation

Solution:

2 or more circles are said to be concentric if they have the same centre and different radii.



Given, $x^2 + y^2 + 4x + 6y + 11 = 0$

The concentric circle will have the equation

$x^2 + y^2 + 4x + 6y + c' = 0$

As it passes through P(5, 4), putting this in the equation

$5^2 + 4^2 + 4 \times 5 + 6 \times 4 + c' = 0$

$\Rightarrow 25 + 16 + 20 + 24 + c' = 0$

$\Rightarrow c' = - 85$

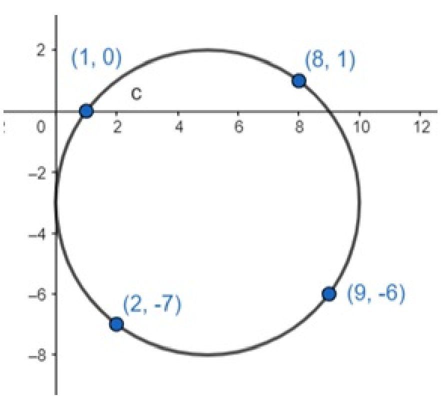
The required equation is

$x^2 + y^2 + 4x + 6y - 85 = 0$

Question: 9

Show that the poi

Solution:



The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

...(i), where (h, k) is the centre and r is the radius.

Putting (1, 0) in (i)

$(1 - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + 1 - 2h = r^2$ ..(ii)

Putting (2, - 7) in (i)

$(2 - h)^2 + (- 7 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + 53 - 4h + 14k = r^2$

$\Rightarrow (h^2 + k^2 + 1 - 2h) + 52 - 2h + 14k = r^2$

h - 7k - 26 = 0..(iii) [from (ii)]

Similarly putting (8, 1)

7h + k - 32 = 0..(iv)

Solving (iii)&(iv)

h = 5 and k = - 3

centre(5, - 3)

Radius = 25

To check if (9, - 6) lies on the circle, $(9 - 5)^2 + (- 6 + 3)^2 = 5^2$

Hence, proved.

Question: 10

Find the equation

Solution:

The equation of a circle:

$x^2 + y^2 + 2gx + 2fy + c = 0$...(i)

Putting (1, 3) & (2, - 1)in (i)

2g + 6f + c = - 10..(ii)

4g - 2f + c = - 5..(iii)

Since the centre lies on the given straight line, (- g, - f) must satisfy the equation as

- 2g –f - 4 = 0...(iv)

Solving, f = - 1, g = - 1.5, c = - 1

The equation is

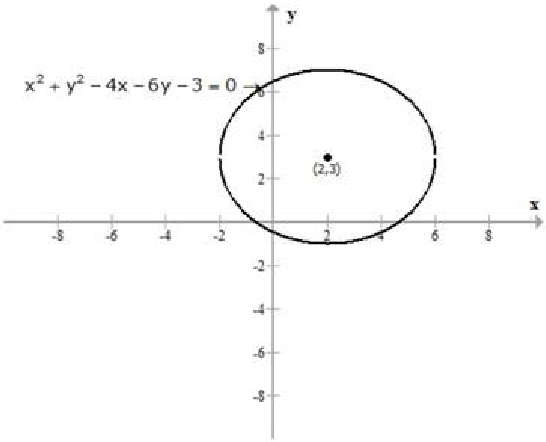
$x^2 + y^2 - 3x - 2y - 1 = 0$

Question: 11

Find the equation

Solution:

The given image of the circle is:



We know that the general equation of the circle is given by:

$x^2 + y^2 + 2gx + 2fy + c = 0$

Also,

Radius r = $\sqrt{g^2 + f^2 - c}$

Now,

$r = \sqrt{(2)^2 + (3)^2 - (-3)}$

$r = \sqrt{4 + 9 + 3}$

r = 4 units.

We need to the find the equation of the circle which is concentric to the given circle and touches y-axis.

The centre of the circle remains the same.

Now, y-axis will be tangent to the circle.

Point of contact will be (0, 3)

Therefore, radius = 2

Now,

Equation of the circle:

$(x - 2)^2 + (y - 3)^2 = (2)^2$

$x^2 + 4 - 4x + y^2 + 9 - 6y = 4$

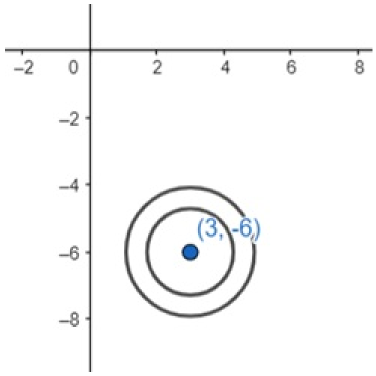
$x^2 + y^2 - 4x - 6y + 9 = 0$

Question: 12

Find the equation

Solution:

2 or more circles are said to be concentric if they have the same centre and different radii.



Given, $x^2 + y^2 - 6x + 12y + 15 = 0$

Radius $r = \sqrt{g^2 + f^2 - c} = \sqrt{(-3^2) + 6^2 - 15} = \sqrt{30}$

The concentric circle will have the equation

$x^2 + y^2 - 6x + 12y + c' = 0$

Also given area of circle = 2×area of the given circle.

$\Rightarrow r'^2 = 2 \times r^2 = 2 \times 30 = 60$

We can get $c' = 45 - 60 = -15$

The required equation is $x^2 + y^2 - 6x + 12y - 15 = 0$.

Question: 13

Prove that the ce

Solution:

Given,

$x^2 + y^2 - 4x - 6y - 12 = 0$

centre $(-g_1, -f_1) = (2, 3)$

$x^2 + y^2 + 2x + 4y - 5 = 0$

centre $(-g_2, -f_2) = (-1, -2)$

$x^2 + y^2 - 10x - 16y + 7 = 0$

centre $(-g_3, -f_3) = (5, 8)$

to prove that the centres are collinear,

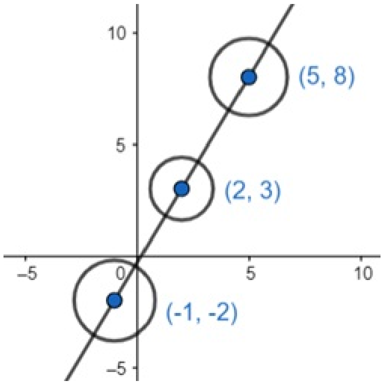
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Where x_1, y_1 are the coordinates of the ist centre and so on.

$$\Rightarrow \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

$= 2(-2 - 8) - 3(-1 - 5) + 1(-8 + 10)$

$= -20 + 18 + 2 = 0$



The centres are collinear.

Question: 14

Find the equation

Solution:

The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

…(i), where (h, k) is the centre and r is the radius.

Putting A(1, 1) in (i)

$(1 - h)^2 + (1 - k)^2 = 1^2$

$\Rightarrow h^2 + k^2 + 2 - 2h - 2k = 1$

$\Rightarrow h^2 + k^2 - 2h - 2k = - 1$..(ii)

Putting B(2, 2) in (i)

$(2 - h)^2 + (2 - k)^2 = 1^2$

$\Rightarrow h^2 + k^2 + 8 - 4h - 4k = 1$

$\Rightarrow h^2 + k^2 - 4h - 4k = - 7$

$\Rightarrow (h^2 + k^2 - 2h - 2k) - 2h - 2k = - 7$

$\Rightarrow - 1 - 2h - 2k = - 7$ [from (ii)]

$\Rightarrow - 2h - 2k = - 6$

$\Rightarrow h + k = 3 \Rightarrow h= 3 - k$

Putting it in (ii)

$\Rightarrow (3 - k)^2 + k^2 - 2(3 - k) - 2k = - 1$

$\Rightarrow 9 + 2k^2 - 6k - 6 + 2k - 2k = - 1$

$\Rightarrow 2k^2 + 4 - 6k = 0$

$\Rightarrow k^2 - 3k + 2 = 0$

$\Rightarrow k = 2$ or $k = 1$

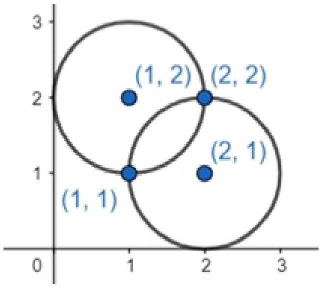
When k = 2, h = 3 - 2 = 1

Equation of 1 circle

$(x - 1)^2 + (y - 2)^2 = 1$

When k = 1, h = 3 - 1 = 2

$(x - 2)^2 + (y - 1)^2 = 1$



Question: 15

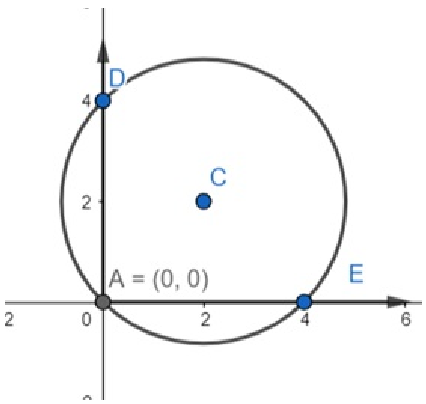
Find the equation

Solution:

From the figure

AD = b units and AE = a units.

D(0, b), E(a, 0) and A(0, 0) lies on the circle. C is the centre.



The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

...(i), where (h, k) is the centre and r is the radius.

Putting A(0, 0) in (i)

$(0 - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 = r^2$ ...(ii)

Similarly putting D(0, b) in (i)

$(0 - h)^2 + (b - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + b^2 - 2kb = r^2$

$\Rightarrow r^2 + b^2 - 2kb = r^2$

$\Rightarrow b^2 - 2kb = 0$

$\Rightarrow$ (b- 2k)b = 0

Either b = 0ork = $\frac{b}{2}$

Similarly putting E(a, 0) in (i)

$(a - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + a^2 - 2ha = r^2$

$\Rightarrow r^2 + a^2 - 2ha = r^2$

$\Rightarrow a^2 - 2ha = 0$

$\Rightarrow$ (a- 2h)a = 0

Either a = 0orh = $\frac{a}{2}$

Centre = $C\left(\frac{a}{2}, \frac{b}{2}\right)$

$r^2 = h^2 + k^2$

$$\Rightarrow r^2 = \frac{a^2 + b^2}{4}$$

Putting the value of r^2 , h and k in equation (i)

$(x - h)^2 + (y - k)^2 = r^2$

$$\left(x-\frac{a}{2}\right)^2 + \left(y-\frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

$$\Rightarrow x^2 + y^2 + \frac{a^2}{4} + \frac{b^2}{4} - xa - yb = \frac{a^2 + b^2}{4}$$

$$\Rightarrow x^2 + y^2 - xa - yb = 0$$

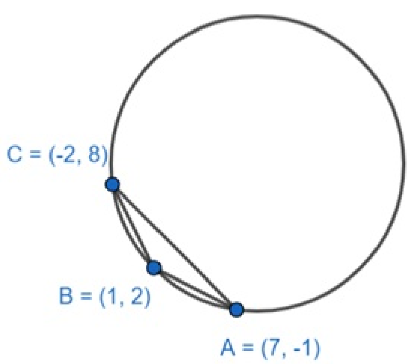
which is the required equation.

Question: 16

Find the equation

Solution:

Solving the equations we get the coordinates of the triangle:



The required circle equation

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ (-2)^2+8^2 & -2 & 8 & 1 \\ 1^2+2^2 & 1 & 2 & 1 \\ 7^2+(-1)^2 & 7 & -1 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2)\begin{vmatrix} -2 & 8 & 1 \\ 1 & 2 & 1 \\ 7 & -1 & 1 \end{vmatrix} - x\begin{vmatrix} 68 & 8 & 1 \\ 5 & 2 & 1 \\ 50 & -1 & 1 \end{vmatrix} + y\begin{vmatrix} 68 & -2 & 1 \\ 5 & 1 & 1 \\ 50 & 7 & 1 \end{vmatrix} - \begin{vmatrix} 68 & -2 & 8 \\ 5 & 1 & 2 \\ 50 & 7 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 27(x^2 + y^2) - 459x - 513y + 1350 = 0$$

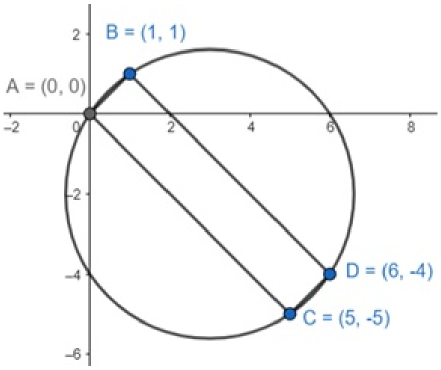
$\Rightarrow x^2 + y^2 - 17x - 19 + 50 = 0.$

Question: 17

Show that the qua

Solution:

Solving the euations we get the coordinates of the quadrilateral.



Slope of AB $= \frac{1-0}{1-0} = 1$

Slope of CD = 1

AB||CD

Slope of BD = AC = - 1

AC||BD

So they form a rectangle with all sides = 90°

The quadrilateral is cyclic as sum of opposite angles = 180°.

Now, AD = diameter of the circle equation of the circle with extremities A(0, 0)&D(6, - 4) is

(x - 0)(x - 6) + (y - 0)(y + 4) = 0

$x^2 + y^2 - 6x + 4y = 0$

Question: 18

If (- 1, 3) and

Solution:

Given $x^2 + y^2 - 6x + 5y - 7 = 0$

Centre(3, $-\frac{5}{2}$)

As (- 1, 3) & (∝, β) are the 2 extremities of the diameter, using mid - point formula we can write

$$\frac{\propto -1}{2} = 3$$

$$\Rightarrow \propto = 7$$

and $\frac{\beta + 3}{2} = -\frac{5}{2}$

$$\Rightarrow \beta = -8$$

(∝, β) = (7, - 8)