

Chapter : 24. HYPERBOLA

Exercise : 24

Question: 1

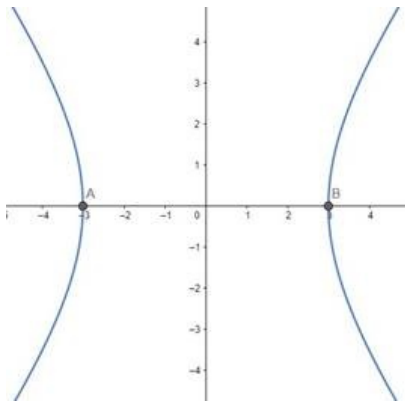
Find the (i) leng

Solution:

Given Equation: $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

$a = 3$ and $b = 4$



(i) Length of Transverse axis = $2a = 6$ units.

Length of Conjugate axis = $2b = 8$ units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 3, 0)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm 5, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{32}{3} = 10.67$ units.

Question: 2

Find the (i) leng

Solution:

Given Equation: $\frac{x^2}{25} - \frac{y^2}{4} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

$a = 5$ and $b = 2$



(i) Length of Transverse axis = $2a = 10$ units.

Length of Conjugate axis = $2b = 4$ units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 5, 0)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{25}} = \sqrt{\frac{29}{25}} = \frac{\sqrt{29}}{5}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm \sqrt{29}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{8}{5} = 1.6$ units.

Question: 3

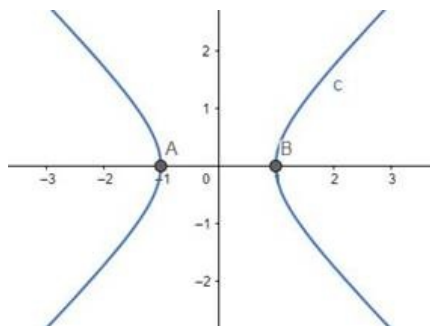
Find the (i) leng

Solution:

Given Equation: $x^2 - y^2 = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

$a = 1$ and $b = 1$



(i) Length of Transverse axis = $2a = 2$ units.

Length of Conjugate axis = $2b = 2$ units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 1, 0)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{1}} = \sqrt{2} = \sqrt{2}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm \sqrt{2}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = 2$ units.

Question: 4

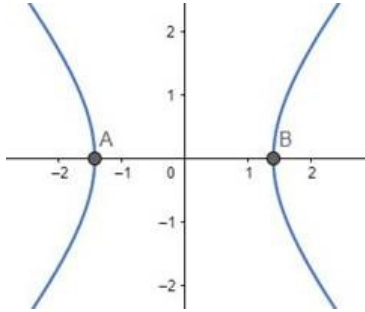
Find the (i) leng

Solution:

Given Equation: $3x^2 - 2y^2 = 6 \Rightarrow \frac{x^2}{2} - \frac{y^2}{3} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

$a = \sqrt{2}$ and $b = \sqrt{3}$



(i) Length of Transverse axis = $2a = 2\sqrt{2}$ units.

Length of Conjugate axis = $2b = 2\sqrt{3}$ units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm\sqrt{2}, 0)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{3}{2}} = \sqrt{\frac{5}{2}}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm\sqrt{5}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ units.

Question: 5

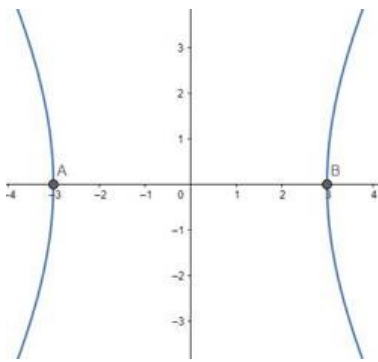
Find the (i) leng

Solution:

Given Equation: $25x^2 - 9y^2 = 225 \Rightarrow \frac{x^2}{9} - \frac{y^2}{25} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

$a = 3$ and $b = 5$



(i) Length of Transverse axis = $2a = 6$ units.

Length of Conjugate axis = $2b = 10$ units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 3, 0)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{25}{9}} = \sqrt{\frac{34}{9}} = \frac{\sqrt{34}}{3}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm \sqrt{34}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{50}{3} = 16.67$ units.

Question: 6

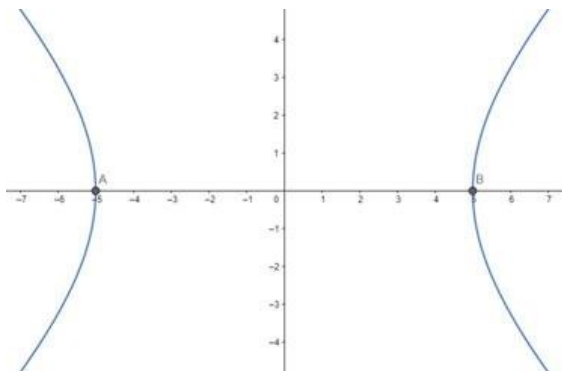
Find the (i) leng

Solution:

Given Equation: $24x^2 - 25y^2 = 600 \Rightarrow \frac{x^2}{25} - \frac{y^2}{24} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

$a = 5$ and $b = \sqrt{24} = 2\sqrt{6}$



(i) Length of Transverse axis = $2a = 10$ units.

Length of Conjugate axis = $2b = 4\sqrt{6}$ units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 5, 0)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{24}{25}} = \sqrt{\frac{49}{25}} = \frac{7}{5}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm 7, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{48}{5} = 9.6$ units.

Question: 7

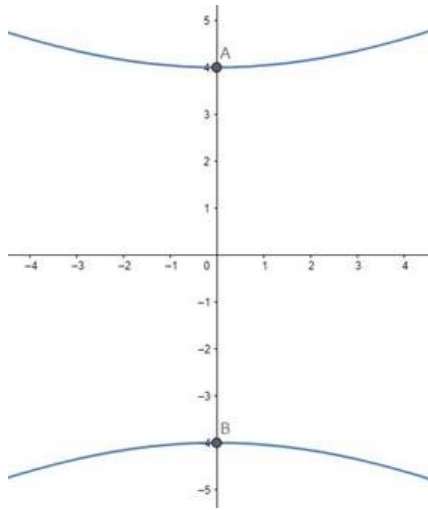
Find the (i) leng

Solution:

Given Equation: $\frac{y^2}{16} - \frac{x^2}{49} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

$a = 4$ and $b = 7$



(i) Length of Transverse axis = $2a = 8$ units.

Length of Conjugate axis = $2b = 14$ units.

(ii) Coordinates of the vertices = $(0, \pm a) = (0, \pm 4)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{49}{16}} = \sqrt{\frac{65}{16}} = \frac{\sqrt{65}}{4}$

(iii) Coordinates of the foci = $(0, \pm ae) = (0, \pm \sqrt{65})$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{98}{4} = 24.5$ units.

Question: 8

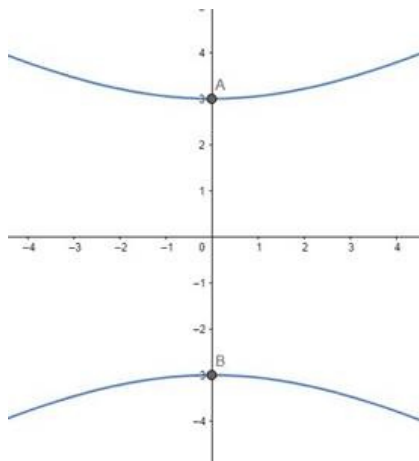
Find the (i) leng

Solution:

Given Equation: $\frac{y^2}{9} - \frac{x^2}{27} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

$a = 3$ and $b = \sqrt{27} = 3\sqrt{3}$



(i) Length of Transverse axis = $2a = 6$ units.

Length of Conjugate axis = $2b = 6\sqrt{3}$ units.

(ii) Coordinates of the vertices = $(0, \pm a) = (0, \pm 3)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{27}{9}} = \sqrt{\frac{36}{9}} = \frac{6}{3} = 2$

(iii) Coordinates of the foci = $(0, \pm ae) = (0, \pm 6)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{54}{3} = 18$ units.

Question: 9

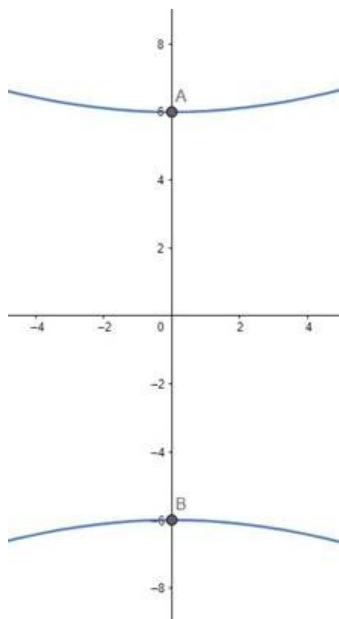
Find the (i) leng

Solution:

Given Equation: $3y^2 - x^2 = 108 \Rightarrow \frac{y^2}{36} - \frac{x^2}{108} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

$a = 6$ and $b = \sqrt{108} = 6\sqrt{3}$



(i) Length of Transverse axis = $2a = 12$ units.

Length of Conjugate axis = $2b = 12\sqrt{3}$ units.

(ii) Coordinates of the vertices = $(0, \pm a) = (0, \pm 6)$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{108}{36}} = \sqrt{1 + 3} = 2$

(iii) Coordinates of the foci = $(0, \pm ae) = (0, \pm 12)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{216}{6} = 36$ units.

Question: 10

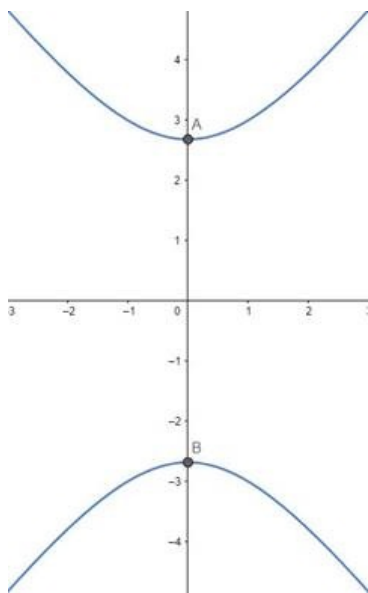
Find the (i) leng

Solution:

Given Equation: $5y^2 - 9x^2 = 36 \Rightarrow \frac{y^2}{36/5} - \frac{x^2}{4} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

$a = \sqrt{\frac{36}{5}} = \frac{6}{\sqrt{5}}$ and $b = 2$



(i) Length of Transverse axis = $2a = \frac{12}{\sqrt{5}}$ units.

Length of Conjugate axis = $2b = 4$ units.

(ii) Coordinates of the vertices = $(0, \pm a) = (0, \pm \frac{6}{\sqrt{5}})$

(iv) Here, eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{36/5}} = \sqrt{1 + \frac{20}{36}} = \frac{\sqrt{56}}{6} = \frac{\sqrt{14}}{3}$

(iii) Coordinates of the foci = $(0, \pm ae) = (0, \pm \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{14}}{3}) = (0, \frac{2\sqrt{14}}{\sqrt{5}})$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{8}{\frac{6}{\sqrt{5}}} = \frac{8\sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$ units.

Question: 11

Find the equation

Solution:

Given: Vertices at $(\pm 6, 0)$ and foci at $(\pm 8, 0)$

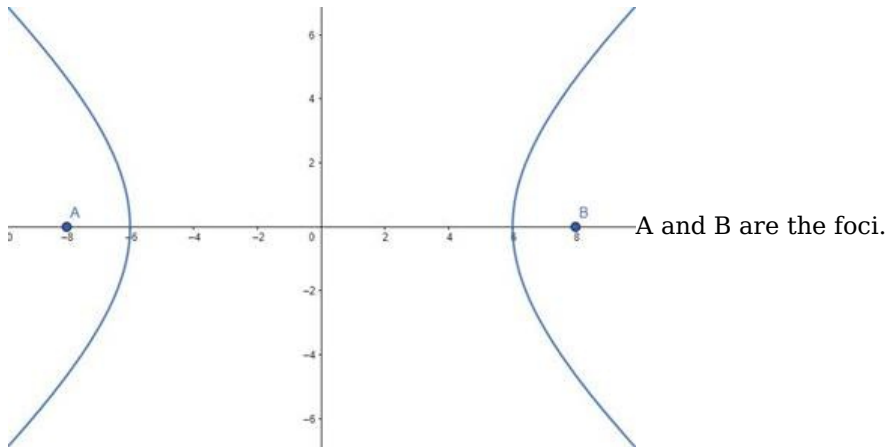
Need to find: The equation of the hyperbola.

Let, the equation of the parabola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertices of the parabola is at $(\pm 6, 0)$

That means $a = 6$

The foci are given at $(\pm 8, 0)$



That means, $ae = 8$, where e is the eccentricity.

$$\Rightarrow 6e = 8 \text{ [As } a = 6]$$

$$\Rightarrow e = \frac{8}{6} = \frac{4}{3}$$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= \sqrt{1 + \frac{b^2}{a^2}} = \frac{4}{3}$$

$$= 1 + \frac{b^2}{a^2} = \frac{16}{9} \text{ [Squaring both sides]}$$

$$= \frac{b^2}{a^2} = \frac{16}{9} - 1 = \frac{7}{9}$$

$$= b^2 = a^2 \frac{7}{9}$$

$$= b^2 = 36 \times \frac{7}{9} = 4 \times 7 = 28 \text{ [As } a = 6]$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{36} - \frac{y^2}{28} = 1 \text{ [Answer]}$$

Question: 12

Find the equation

Solution:

Given: Vertices at $(0, \pm 5)$ and foci at $(0, \pm 8)$

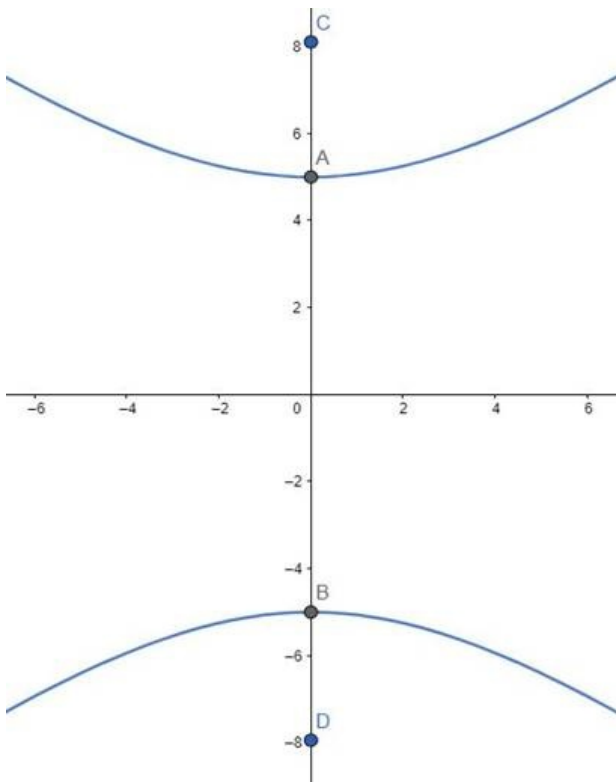
Need to find: The equation of the hyperbola.

Let, the equation of the parabola be: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Vertices of the parabola are at $(0, \pm 5)$

That means $a = 5$

The foci are given at $(0, \pm 8)$



A and B are the vertices. C and D are the foci.

That means, $ae = 8$, where e is the eccentricity.

$$= 5e = 8 \text{ [As } a = 5]$$

$$= e = \frac{8}{5}$$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= \sqrt{1 + \frac{b^2}{a^2}} = \frac{8}{5}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{64}{25} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{64}{25} - 1 = \frac{39}{25}$$

$$\Rightarrow b^2 = a^2 \frac{39}{25}$$

$$\Rightarrow b^2 = 25 \times \frac{39}{25} = 39 \text{ [As } a = 5]$$

So, the equation of the hyperbola is,

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{39} = 1 \text{ [Answer]}$$

Question: 13

Find the equation

Solution:

Given: Foci are $(\pm\sqrt{29}, 0)$, the transverse axis is of the length 10

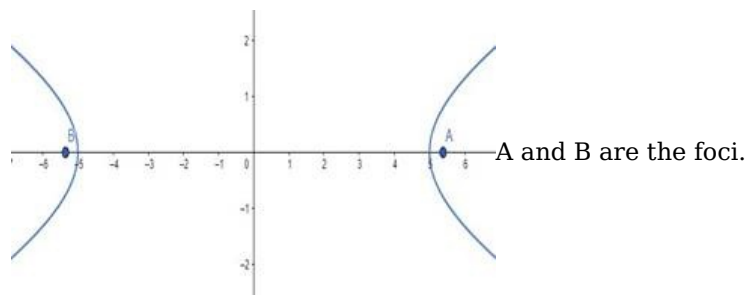
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The transverse axis is of the length 10, i.e., $2a = 10$

Therefore, $a = 5$

The foci are given at $(\pm\sqrt{29}, 0)$



That means, $ae = \pm\sqrt{29}$, where e is the eccentricity.

$$\Rightarrow 5e = \sqrt{29} \text{ [As } a = 5]$$

$$\Rightarrow e = \frac{\sqrt{29}}{5}$$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= \sqrt{1 + \frac{b^2}{a^2}} = \frac{\sqrt{29}}{5}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{29}{25} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{29}{25} - 1 = \frac{4}{25}$$

$$\Rightarrow b^2 = a^2 \frac{4}{25}$$

$$\Rightarrow b^2 = 25 \times \frac{4}{25} = 4 \text{ [As } a = 5]$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} - \frac{y^2}{4} = 1 \text{ [Answer]}$$

Question: 14

Find the equation

Solution:

Given: Foci are $(\pm 5, 0)$, the conjugate axis is of the length 8

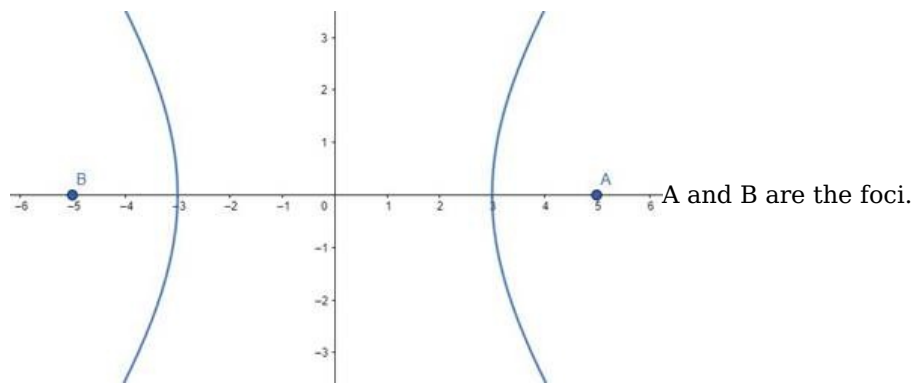
Need to find: The equation of the hyperbola and eccentricity.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The conjugate axis is of the length 8, i.e., $2b = 8$

Therefore, $b = 4$

The foci are given at $(\pm 5, 0)$



That means, $ae = 5$, where e is the eccentricity.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= a \sqrt{1 + \frac{b^2}{a^2}} = 5$$

$$= \sqrt{1 + \frac{b^2}{a^2}} = \frac{5}{a}$$

$$= 1 + \frac{b^2}{a^2} = \frac{25}{a^2} \text{ [Squaring both sides]}$$

$$= \frac{b^2}{a^2} = \frac{25}{a^2} - 1 = \frac{25 - a^2}{a^2}$$

$$= b^2 = 25 - a^2$$

$$= a^2 = 25 - b^2 = 25 - 16 = 9$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$$\text{Eccentricity, } e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3} \text{ [Answer]}$$

Question: 15

Find the equation

Solution:

Given: Foci are $(\pm 3\sqrt{5}, 0)$ the length of the latus rectum is 8 units

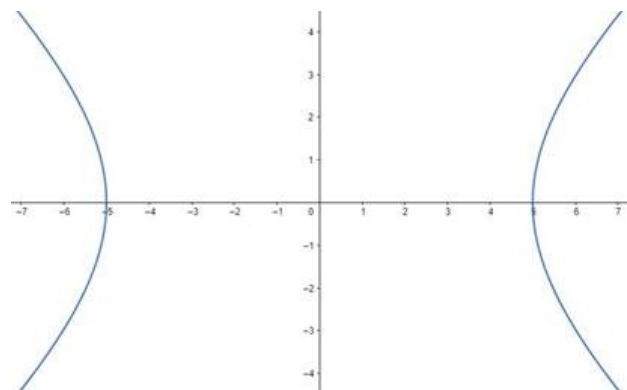
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The length of the latus rectum is 8 units.

$$\text{Therefore, } \frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a \text{ ---- (1)}$$

The foci are given at $(\pm 3\sqrt{5}, 0)$



That means, $ae = 3\sqrt{5}$, where e is the eccentricity.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= a \sqrt{1 + \frac{b^2}{a^2}} = 3\sqrt{5}$$

$$= a \frac{\sqrt{a^2 + b^2}}{a} = 3\sqrt{5}$$

$$= a^2 + b^2 = 45 \text{ [Squaring both sides]}$$

$$= a^2 + 4a = 45 \text{ [From (1)]}$$

$$= a^2 + 4a - 45 = 0$$

$$= a^2 + 9a - 5a - 45 = 0$$

$$= (a + 9)(a - 5) = 0$$

So, either $a = 5$ or, $a = -9$

That means, either $b = 2\sqrt{5}$ or, $b = \sqrt{-36}$

The value of $b = \sqrt{-36}$ is not a valid one. So, the b value and its corresponding a value is not acceptable.

Hence, the acceptable value of a is 5 and b is $2\sqrt{5}$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} - \frac{y^2}{20} = 1 \text{ [Answer]}$$

Question: 16

Find the equation

Solution:

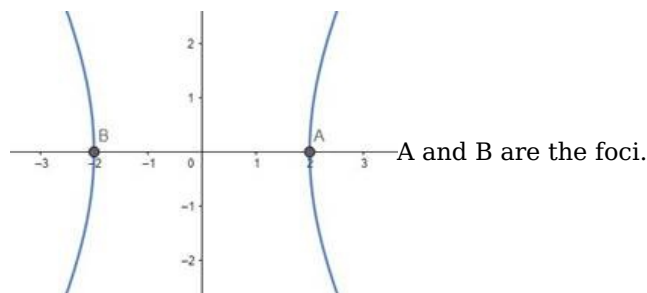
Given: Vertices are $(\pm 2, 0)$ and the eccentricity is 2

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Vertices are $(\pm 2, 0)$, that means, $a = 2$

And also given, the eccentricity, $e = 2$



We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= \sqrt{1 + \frac{b^2}{a^2}} = 2$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = 4 \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = 3$$

$$\Rightarrow b^2 = 3a^2 = 3 \times 4 = 12 \text{ [As } a = 2]$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{4} - \frac{y^2}{12} = 1 \text{ [Answer]}$$

Question: 17

Find the equation

Solution:

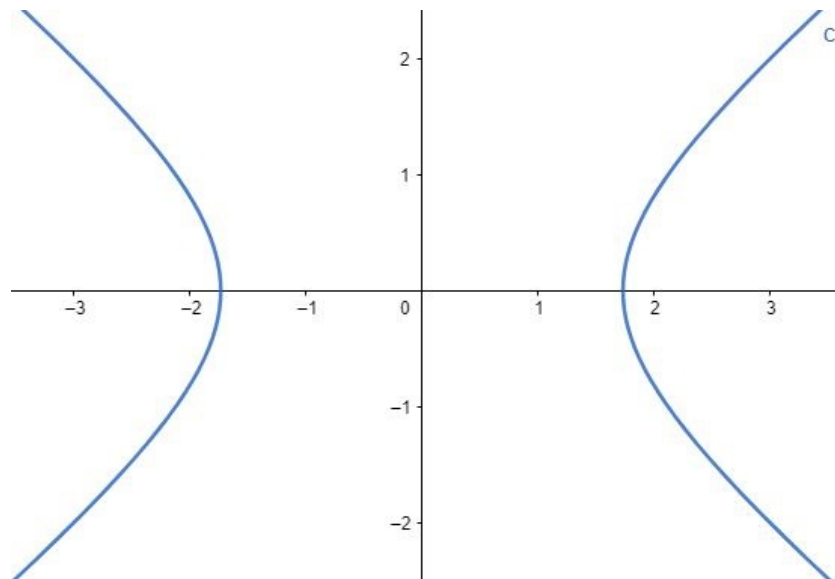
Given: Foci are $(\pm\sqrt{5}, 0)$, and the eccentricity is $\sqrt{\frac{5}{3}}$

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The eccentricity, $e = \sqrt{\frac{5}{3}}$

And also given, foci are $(\pm\sqrt{5}, 0)$



That means, $ae = \sqrt{5}$

$$\Rightarrow a = \frac{\sqrt{5}}{e}$$

$$\Rightarrow a = \frac{\sqrt{5}}{\sqrt{\frac{5}{3}}} \quad [\text{As } e = \sqrt{\frac{5}{3}}]$$

$$\Rightarrow a = \sqrt{3}$$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{5}{3}}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{5}{3} \quad [\text{Squaring both sides}]$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$\Rightarrow b^2 = \frac{2}{3}a^2 = \frac{2}{3} \times 3 = 2 \quad [\text{As } a = \sqrt{3}]$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{3} - \frac{y^2}{2} = 1 \quad [\text{Answer}]$$

Question: 18

Find the equation

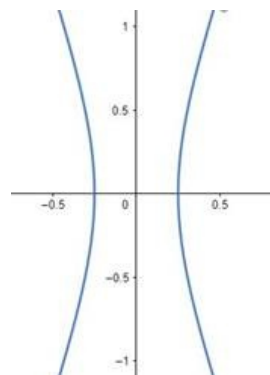
Solution:

Given: The length of latus rectum is 4, and the eccentricity is 3

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

The length of the latus rectum is 4 units.



Therefore, $\frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$ ---- (1)

And also given, the eccentricity, $e = 3$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= \sqrt{1 + \frac{b^2}{a^2}} = 3$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = 9 \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = 8$$

$$\Rightarrow b^2 = 8a^2$$

$$\Rightarrow 2a = 8a^2 \text{ [From (1)]}$$

$$\Rightarrow a = \frac{1}{4}$$

Therefore,

$$b^2 = 2a = 2 \times \frac{1}{4} = \frac{1}{2}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{\cancel{1}/16} - \frac{y^2}{\cancel{1}/2} = 1 \Rightarrow 16x^2 - 2y^2 = 1 \text{ [Answer]}$$

Question: 19

Find the equation

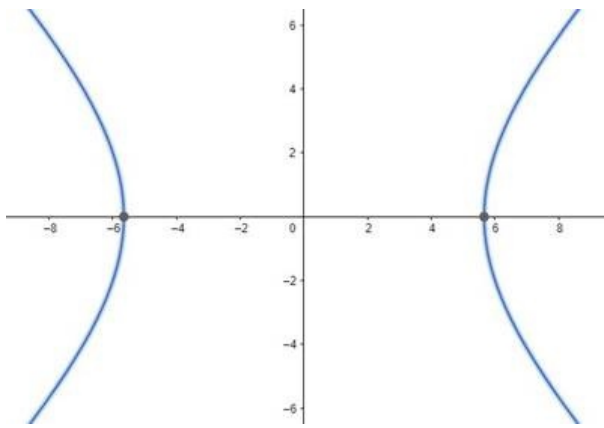
Solution:

Given: Eccentricity is $\sqrt{2}$, and the distance between foci is 16

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Distance between the foci is 16, i.e., $2ae = 16$



And also given, the eccentricity, $e = \sqrt{2}$

Therefore,

$$2a\sqrt{2} = 16$$

$$a = \frac{16}{2\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2} \text{ ---- (1)}$$

$$\text{We know that, } e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$$= \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{2}$$

$$= 1 + \frac{b^2}{a^2} = 2 \text{ [Squaring both sides]}$$

$$= \frac{b^2}{a^2} = 1$$

$$= b^2 = a^2 = 32 \text{ [From (1)]}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1 \Rightarrow x^2 - y^2 = 32 \text{ [Answer]}$$

Question: 20

Find the equation

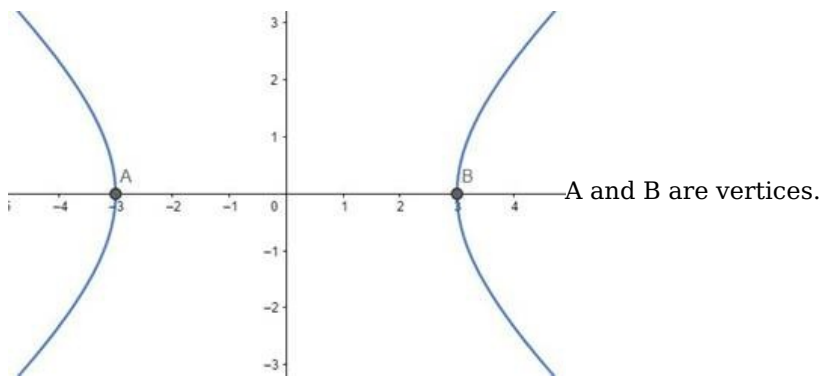
Solution:

Given: Vertices are $(0, \pm 3)$ and the eccentricity is $\frac{4}{3}$

Need to find: The equation of the hyperbola and coordinates of foci.

$$\text{Let, the equation of the hyperbola be: } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices are $(\pm 3, 0)$, that means, $a = 3$



And also given, the eccentricity, $e = \frac{4}{3}$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= \sqrt{1 + \frac{b^2}{a^2}} = \frac{4}{3}$$

$$= 1 + \frac{b^2}{a^2} = \frac{16}{9} \text{ [Squaring both sides]}$$

$$= \frac{b^2}{a^2} = \frac{16}{9} - 1 = \frac{7}{9}$$

$$= b^2 = \frac{7}{9}a^2 = \frac{7}{9} \times 9 = 7 \text{ [As } a = 3]$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{7} = 1$$

Coordinates of the foci = $(\pm ae, 0) = (\pm 4, 0)$ [Answer]

Question: 21

Find the equation

Solution:

Given: Foci are $(0, \pm 13)$, the conjugate axis is of the length 24

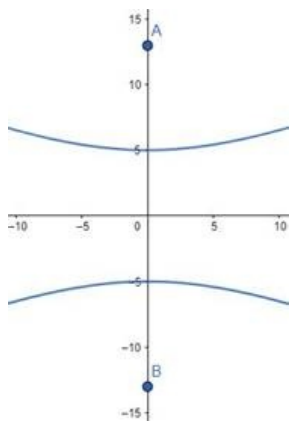
Need to find: The equation of the hyperbola.

$$\text{Let, the equation of the hyperbola be: } \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

The conjugate axis is of the length 24, i.e., $2b = 24$

Therefore, $b = 12$

The foci are given at $(0, \pm 13)$



A and B are the foci.

That means, $ae = 13$, where e is the eccentricity.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= a \sqrt{1 + \frac{b^2}{a^2}} = 13$$

$$= a \frac{\sqrt{a^2 + b^2}}{a} = 13$$

$$= a^2 + b^2 = 169 \text{ [Squaring both sides]}$$

$$= a^2 = 169 - b^2 = 169 - 144 = 25 \text{ [As } b = 12\text{]}$$

So, the equation of the hyperbola is,

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{144} = 1 \text{ [Answer]}$$

Question: 22

Find the equation

Solution:

Given: Foci are $(0, \pm 10)$ and the length of latus rectum is 9 units

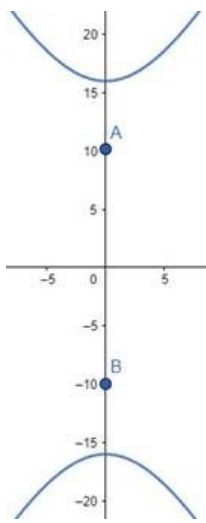
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

The length of the latus rectum is 9 units.

$$\text{Therefore, } \frac{2b^2}{a} = 9 \Rightarrow b^2 = \frac{9}{2}a \text{ ---- (1)}$$

The foci are given at $(0, \pm 10)$



That means, $ae = 10$, where e is the eccentricity.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$= a \sqrt{1 + \frac{b^2}{a^2}} = 10$$

$$= a \frac{\sqrt{a^2 + b^2}}{a} = 10$$

$$= a^2 + b^2 = 100 \text{ [Squaring both sides]}$$

$$= a^2 + \frac{9}{2}a = 100 \text{ [From (1)]}$$

$$= 2a^2 + 9a - 200 = 0$$

$$= 2a^2 + 25a - 16a - 200 = 0$$

$$= (2a + 25)(a - 16) = 0$$

$$\text{So, either } a = 16 \text{ or, } a = -\frac{25}{2}$$

$$\text{That means, either } b = \sqrt{\frac{9}{2} \times 16} = 6\sqrt{2} \text{ or, } b = \sqrt{-\frac{9 \times 25}{2 \times 2}}$$

The value of $b = \sqrt{-\frac{9 \times 25}{2 \times 2}}$ is not a valid one. So, the b value and its corresponding a value is not acceptable.

Hence, the acceptable value of a is 16 and b is $6\sqrt{2}$

So, the equation of the hyperbola is,

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{256} - \frac{x^2}{72} = 1 \text{ [Answer]}$$

Question: 23

Find the equation

Solution:

Given: Foci at $(0, \pm\sqrt{14})$ and passing through the point P(3, 4)

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

It passes through the point P(3, 4)

So putting the values of (x, y) we get,

$$\frac{3^2}{a^2} - \frac{4^2}{b^2} = 1 \Rightarrow \frac{9}{a^2} - \frac{16}{b^2} = 1 \text{ ---- (1)}$$

Foci at $(0, \pm\sqrt{14})$

So, $ae = \sqrt{14}$

We know, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$= a \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{14}$$

$$= \sqrt{a^2 + b^2} = \sqrt{14}$$

$$\Rightarrow a^2 + b^2 = 14 \text{ [Squaring on both sides]}$$

$$\Rightarrow a^2 = 14 - b^2 \text{ ---- (2)}$$

Comparing (1) and (2) we get,

$$\frac{9}{14 - b^2} - \frac{16}{b^2} = 1$$

$$\frac{9}{14 - b^2} = 1 + \frac{16}{b^2} = \frac{b^2 + 16}{b^2}$$

$$9b^2 = 14b^2 - b^4 + 224 - 16b^2$$

$$b^4 + 11b^2 - 224 = 0$$

Solving the equations we get,

$$b_1 = \sqrt{\frac{1}{2}(-11 + 3\sqrt{113})}$$

$$b_2 = -\sqrt{\frac{1}{2}(-11 + 3\sqrt{113})}$$

$$b_3 = (-i)\sqrt{\frac{1}{2}(11 + 3\sqrt{113})}$$

$$b_4 = i\sqrt{\frac{1}{2}(11 + 3\sqrt{113})}$$

With the help of any of these values of b we can't find out the equation of the hyperbola.

* This is the only process we can apply in this standard.