

Chapter : 3. FUNCTIONS

Exercise : 3A

Question: 1

Define a function

Solution:

Function as a set of ordered pairs: A function is a set of ordered pairs with the property that no two ordered pairs have the same first component and a different second component.

The domain of a function is the set of all first components, x , in the ordered pairs and the range of a function is the set of all second components, y , in the ordered pairs.

For. e.g. $\{(1,x), (2,y), (3,z)\}$ is a function, since there are no two pairs with the same first component.

Here, Domain is $\{1, 2, 3\}$ and Range is $\{x, y, z\}$

Question: 2

Define a function

Solution:

Function as a correspondence between two sets: Let A and B be two non - empty sets. Then, a function 'f' from set A to set B is a correspondence (rule) which associates elements of set A to elements of set B such that:

- (i) all elements of set A are associated with an element in set B .
- (ii) an element of set A is associated with a unique element in set B .

Question: 3

What is the funda

Solution:

Fundamental difference between Relation and Function:

Every function is a relation, but every relation need not be a function.

A relation f from A to B is called a function if

- (i) $\text{Dom}(f) = A$
- (ii) no two different ordered pairs in f have the same first component.

For. e.g.

Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4, 5\}$

Some relations f , g and h are defined as follows:

$f = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$

$g = \{(a, 1), (b, 3), (c, 5)\}$

$h = \{(a, 1), (b, 2), (b, 3), (c, 4), (d, 5)\}$

In the relation f ,

- $f = \{(\underline{a}, \underline{1}), (\underline{b}, \underline{2}), (\underline{c}, \underline{3}), (\underline{d}, \underline{4})\}$ (i) $\text{Dom}(f) = A$
- (ii) All first components are different.

So, f is a function.

In the relation g ,

- (i) $\text{Dom}(g) \neq A$

So, the condition is not satisfied. Thus, g is not a function.

In the relation h ,

$$h = \{(a, 1), (\underline{b}, 2), (\underline{b}, 3), (c, 4), (d, 5)\} \quad \text{(i) Dom}(h) = A$$

(i) Two first components are the same, i.e. b has

two different images.

So, h is not a function.

No, every relation is not a function.

Question: 4

Let $X = \{1, 2, 3,$

Solution:

$X = \{1, 2, 3, 4\}$ and $Y = \{1, 5, 9, 11, 15, 16\}$

and $F = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

(i) To show: F is a relation from X to Y

First elements in $F = 1, 2, 3, 4$

All the first elements are in Set X

So, the first element is from set X

Second elements in $F = 5, 9, 1, 11$

All the second elements are in Set Y

So, the second element is from set Y

Since the first element is from set X and the second element is from set Y

Hence, F is a relation from X to Y .

(ii) To show: F is a function from X to Y

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

$F = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

$F = \{(1, 5), (\textcircled{2}, 9), (3, 1), (4, 5), (\textcircled{2}, 11)\}$

Here, 2 is coming twice.

Hence, it does not have a unique (one) image.

So, it is not a function.

Question: 5

Let $X = \{-1, 0, 3$

Solution:

Given: $f: X \rightarrow \mathbb{R}, f(x) = x^3 + 1$

Here, $X = \{-1, 0, 3, 7, 9\}$

For $x = -1$

$$f(-1) = (-1)^3 + 1 = -1 + 1 = 0$$

For $x = 0$

$$f(0) = (0)^3 + 1 = 0 + 1 = 1$$

For $x = 3$

$$f(3) = (3)^3 + 1 = 27 + 1 = 28$$

For $x = 7$

$$f(7) = (7)^3 + 1 = 343 + 1 = 344$$

For $x = 9$

$$f(9) = (9)^3 + 1 = 729 + 1 = 730$$

\therefore the ordered pairs are $(-1, 0), (0, 1), (3, 28), (7, 344), (9, 730)$

Question: 6

Let $A = \{-1, 0, 1\}$

Solution:

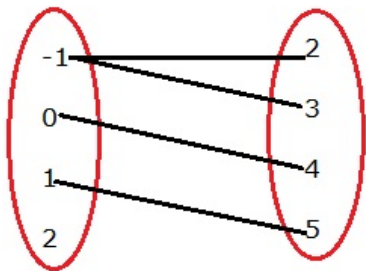
(i) Given: $A = \{-1, 0, 1, 2\}$ and $B = \{2, 3, 4, 5\}$

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

$$f = \{(-1, 2), (-1, 3), (0, 4), (1, 5)\}$$



$$f = \{(\textcircled{-1}, 2), (\textcircled{-1}, 3), (0, 4), (1, 5)\}$$

Here, -1 is coming twice.

Hence, it does not have a unique (one) image.

\therefore f is not a function

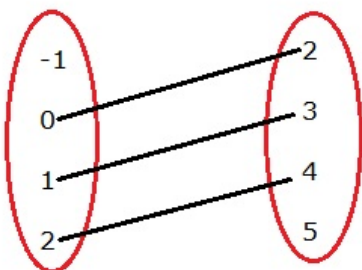
(ii) Given: $A = \{-1, 0, 1, 2\}$ and $B = \{2, 3, 4, 5\}$

Function:

(i) all elements of first set is associated with the elements of second set.

(ii) An element of first set has a unique image in second set.

$$g = \{(0, 2), (1, 3), (2, 4)\}$$



Here, -1 is not associated with any element of set B

Hence, it does not satisfy the condition of the function

$\therefore g$ is not a function.

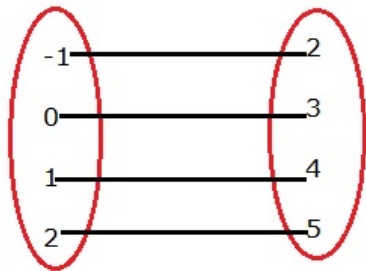
(iii) Given: $A = \{-1, 0, 1, 2\}$ and $B = \{2, 3, 4, 5\}$

Function:

(i) all elements of first set is associated with the elements of second set.

(ii) An element of first set has a unique image in second set.

$h = \{(-1, 2), (0, 3), (1, 4), (2, 5)\}$



Here, (i) all elements of set A are associated to element in set B .

(ii) an element of set A is associated to a unique element in set B .

$\therefore h$ is a function.

Question: 7

Let $A = \{1, 2\}$ and

Solution:

Given: $A = \{1, 2\}$ and $B = \{2, 4, 6\}$

$f = \{(x, y): x \in A, y \in B \text{ and } y > 2x + 1\}$

Putting $x = 1$ in $y > 2x + 1$, we get

$$y > 2(1) + 1$$

$$= y > 3$$

and $y \in B$

this means $y = 4, 6$ if $x = 1$ because it satisfies the condition $y > 3$

Putting $x = 2$ in $y > 2x + 1$, we get

$$y > 2(2) + 1$$

$$= y > 5$$

this means $y = 6$ if $x = 2$ because it satisfies the condition $y > 5$.

$\therefore f = \{(1, 4), (1, 6), (2, 6)\}$

$(1, 2), (2, 2), (2, 4)$ are not the members of ' f ' because they do not satisfy the given condition $y > 2x + 1$

Firstly, we have to show that f is a relation from A to B .

First elements = $1, 2$

All the first elements are in Set A

So, the first element is from set A

Second elements in $F = 4, 6$

All the second elements are in Set B

So, the second element is from set B

Since the first element is from set A and second element is from set B

Hence, F is a relation from A to B.

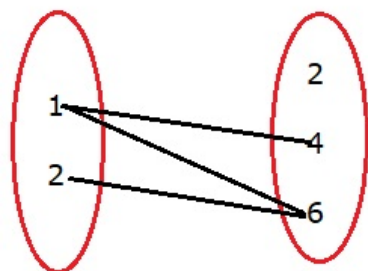
Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

Now, we have to show that f is not a function from A to B

$$f = \{(1, 4), (1, 6), (2, 6)\}$$



$$f = \{(\textcircled{1} \ 4), (\textcircled{1} \ 6), (2, 6)\}$$

Here, 1 is coming twice.

Hence, it does not have a unique (one) image.

So, it is not a function.

Question: 8

Let $A = \{0, 1, 2\}$

Solution:

Given: $A = \{0, 1, 2\}$ and $B = \{3, 5, 7, 9\}$

$$f = \{(x, y): x \in A, y \in B \text{ and } y = 2x + 3\}$$

For $x = 0$

$$y = 2x + 3$$

$$y = 2(0) + 3$$

$$y = 3 \in B$$

For $x = 1$

$$y = 2x + 3$$

$$y = 2(1) + 3$$

$$y = 5 \in B$$

For $x = 2$

$$y = 2x + 3$$

$$y = 2(2) + 3$$

$$y = 7 \in B$$

$$\therefore f = \{(0, 3), (1, 5), (2, 7)\}$$

$(0, 5), (0, 7), (0, 9), (1, 3), (1, 7), (1, 9), (2, 3), (2, 5), (2, 9)$ are not the members of 'f' because they are not satisfying the given condition $y = 2x + 3$

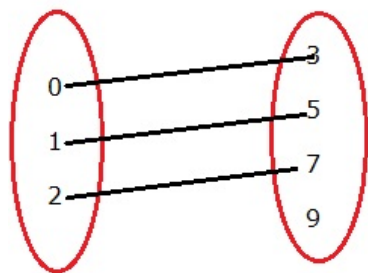
Now, we have to show that f is a function from A to B

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

$$f = \{(0, 3), (1, 5), (2, 7)\}$$



Here, (i) all elements of set A are associated with an element in set B.

(ii) an element of set A is associated with a unique element in set B.

$\therefore f$ is a function.

$$\text{Dom}(f) = 0, 1, 2$$

$$\text{Range}(f) = 3, 5, 7$$

Question: 9

$$\text{Let } A = \{2, 3, 5,$$

Solution:

$$\text{Given: } A = \{2, 3, 5, 7\} \text{ and } B = \{3, 5, 9, 13, 15\}$$

$$f = \{(x, y): x \in A, y \in B \text{ and } y = 2x - 1\}$$

$$\text{For } x = 2$$

$$y = 2x - 1$$

$$y = 2(2) - 1$$

$$y = 3 \in B$$

$$\text{For } x = 3$$

$$y = 2x - 1$$

$$y = 2(3) - 1$$

$$y = 5 \in B$$

$$\text{For } x = 5$$

$$y = 2x - 1$$

$$y = 2(5) - 1$$

$$y = 9 \in B$$

$$\text{For } x = 7$$

$$y = 2x - 1$$

$$y = 2(7) - 1$$

$$y = 13 \in B$$

$$\therefore f = \{(2, 3), (3, 5), (5, 9), (7, 13)\}$$

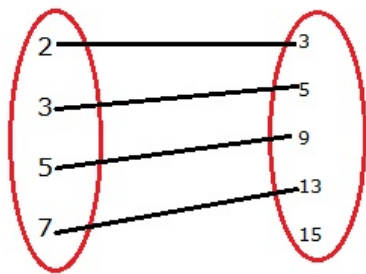
Now, we have to show that f is a function from A to B

Function:

(i) all elements of the first set are associated with the elements of the second set.

(ii) An element of the first set has a unique image in the second set.

$$f = \{(2, 3), (3, 5), (5, 9), (7, 13)\}$$



Here, (i) all elements of set A are associated with an element in set B.

(ii) an element of set A is associated with a unique element in set B.

$\therefore f$ is a function.

$$\text{Dom}(f) = 2, 3, 5, 7$$

$$\text{Range}(f) = 3, 5, 9, 13$$

Question: 10

$$\text{Let } g = \{(1, 2),$$

Solution:

Given:

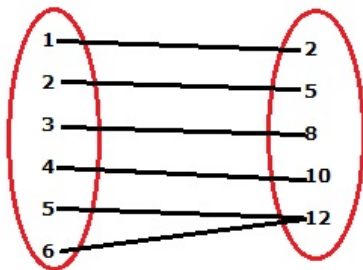
$$g = \{(1, 2), (2, 5), (3, 8), (4, 10), (5, 12), (6, 12)\}$$

We know that,

A function 'f' from set A to set B is a correspondence (rule) which associates elements of set A to elements of set B such that:

(i) all elements of set A are associated with an element in set B.

(ii) an element of set A is associated with a unique element in set B



Here, we observe that each element of the given set has appeared as the first component in one and only one ordered pair in 'g.' So, g is a function in the given set.

$$\text{Dom}(g) = 1, 2, 3, 4, 5, 6$$

$$\text{Range}(g) = 2, 5, 8, 10, 12$$

Question: 11

$$\text{Let } f = \{(0, -5),$$

Solution:

Given that: $f = \{(0, -5), (1, -2), (3, 4), (4, 7)\}$ be a function from \mathbb{Z} to \mathbb{Z} defined by linear function.

We know that, linear functions are of the form $y = mx + b$

$$\text{Let } f(x) = ax + b, \text{ for some integers } a, b$$

Here, $(0, -5) \in f$

$$\Rightarrow f(0) = -5$$

$$\Rightarrow a(0) + b = -5$$

$$\Rightarrow b = -5 \dots(i)$$

Similarly, $(1, -2) \in f$

$$\Rightarrow f(1) = -2$$

$$\Rightarrow a(1) + b = -2$$

$$\Rightarrow a + b = -2$$

$$\Rightarrow a + (-5) = -2 \text{ [from (i)]}$$

$$\Rightarrow a = -2 + 5$$

$$\Rightarrow a = 3$$

$$\therefore f(x) = ax + b$$

$$= 3x + (-5)$$

$$f(x) = 3x - 5$$

Question: 12

If $f(x) = x^2$

Solution:

Given: $f(x) = x^2$

To find: $\frac{f(5)-f(1)}{(5-1)} \dots(i)$

Firstly, we find the $f(5)$

Putting the value of $x = 5$ in the given eq., we get

$$f(5) = (5)^2$$

$$\Rightarrow f(5) = 25$$

Similarly,

$$f(1) = (1)^2$$

$$\Rightarrow f(1) = 1$$

Putting the value of $f(5)$ and $f(1)$ in eq. (i), we get

$$\frac{f(5) - f(1)}{(5 - 1)} = \frac{25 - 1}{5 - 1} = \frac{24}{4} = 6$$

Hence, the value of $\frac{f(5)-f(1)}{(5-1)} = 6$

Question: 13

If $f(x) = x^2$

Solution:

Given: $f(x) = x^2$

To find: $\frac{f(1.1)-f(1)}{(1.1-1)} \dots(i)$

Firstly, we find the $f(1.1)$

Putting the value of $x = 1.1$ in the given eq., we get

$$f(1.1) = (1.1)^2$$

$$\Rightarrow f(1.1) = 1.21$$

Similarly,

$$f(1) = (1)^2$$

$$\Rightarrow f(1) = 1$$

Putting the value of $f(1.1)$ and $f(1)$ in eq. (i), we get

$$\frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{1.21 - 1}{1.1 - 1} = \frac{0.21}{0.1} = 2.1$$

Hence, the value of $\frac{f(1.1) - f(1)}{(1.1 - 1)} = 2.1$

Question: 14

Let $X = \{12, 13,$

Solution:

Given: $f(x)$ = highest prime factor of x

and since $x \in A$, $A = \{12, 13, 14, 15, 16, 17\}$

Value of x can only be 12, 13, 14, 15, 16, 17

Doing prime factorization of the above, we get

2	14
7	7
	1

13	13
	1

2	12
2	6
3	3
	1

3	15
5	5
	1

2	16
2	8
2	4
2	2
	1

17	17
	1

Value of x	Highest Prime Factor of x
12	3
13	13
14	7
15	5
16	2
17	17

Hence, range of $f = \{2, 3, 5, 7, 13, 17\}$

Question: 15

Let \mathbb{R}^+

Solution:

Given that $f: \mathbb{R}^+ \rightarrow \mathbb{R}$ such that $f(x) = \log_e x$

To find: (i) Range of f

Here, $f(x) = \log_e x$

We know that the range of a function is the set of images of elements in the domain.

\therefore the image set of the domain of $f = \mathbb{R}$

Hence, the range of f is the set of all real numbers.

To find: (ii) $\{x : x \in \mathbb{R}^+ \text{ and } f(x) = -2\}$

We have, $f(x) = -2 \dots (a)$

and $f(x) = \log_e x \dots (b)$

From eq. (a) and (b), we get

$$\log_e x = -2$$

Taking exponential both the sides, we get

$$\Rightarrow e^{\log_e x} = e^{-2}$$

$[\because \text{Inverse property .i. } e^{\log_e x} = x]$

$$\Rightarrow x = e^{-2}$$

$$\therefore \{x : x \in \mathbb{R}^+ \text{ and } f(x) = -2\} = \{e^{-2}\}$$

To find: (iii) $f(xy) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$

We have,

$$f(xy) = \log_e(xy)$$

$$= \log_e(x) + \log_e(y)$$

[Product Rule for Logarithms]

$$= f(x) + f(y) \quad [\because f(x) = \log_e x]$$

$$\therefore f(xy) = f(x) + f(y) \text{ holds.}$$

Question: 16

Let $f : \mathbb{R} \rightarrow \mathbb{R}$

Solution:

Given that $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = 2^x$

To find: (i) Range of x

Here, $f(x) = 2^x$ is a positive real number for every $x \in \mathbb{R}$ because 2^x is positive for every $x \in \mathbb{R}$.

Moreover, for every positive real number x , $\exists \log_2 x \in \mathbb{R}$ such that

$$f(\log_2 x) = 2^{\log_2 x}$$

$$= x \quad [\because a^{\log_a x} = x]$$

Hence, the range of f is the set of all positive real numbers.

To find: (ii) $\{x : f(x) = 1\}$

We have, $f(x) = 1$... (a)

and $f(x) = 2^x$... (b)

From eq. (a) and (b), we get

$$2^x = 1$$

$$= 2^x = 2^0 \quad [\because 2^0 = 1]$$

Comparing the powers of 2, we get

$$\Rightarrow x = 0$$

$$\therefore \{x : f(x) = 1\} = \{0\}$$

To find: (iii) $f(x + y) = f(x) \cdot f(y)$ for all $x, y \in \mathbb{R}$

We have,

$$f(x + y) = 2^{x + y}$$

$$= 2^x \cdot 2^y$$

[The exponent "product rule" tells us that, when multiplying two powers that have the same base, you can add the exponents or vice - versa]

$$= f(x) \cdot f(y) \quad [\because f(x) = 2^x]$$

$$\therefore f(x + y) = f(x) \cdot f(y) \text{ holds for all } x, y \in \mathbb{R}$$

Question: 17

Let $f : \mathbb{R} \rightarrow \mathbb{R}$

Solution:

It is given that $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{C} \rightarrow \mathbb{C}$

Thus, Domain (f) = \mathbb{R} and Domain (g) = \mathbb{C}

We know that, Real numbers \neq Complex Number

\therefore , Domain (f) \neq Domain (g)

\therefore f(x) and g(x) are not equal functions

\therefore f \neq g

Question: 18

f, g and h are th

Solution:

(i) $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = x^2$

Since the value of x is squared, f(x) will always be equal or greater than 0.

\therefore the range is $[0, \infty)$

(ii) $g: \mathbb{R} \rightarrow \mathbb{R}$ such that $g(x) = x^2 + 1$

Since, the value of x is squared and also adding with 1, g(x) will always be equal or greater than 1.

\therefore Range of g(x) = $[1, \infty)$

(iii) $h: \mathbb{R} \rightarrow \mathbb{R}$ such that $h(x) = \sin x$

We know that, $\sin(x)$ always lies between -1 to 1

\therefore Range of h(x) = $(-1, 1)$

Question: 19

Let $f : \mathbb{R}$

Solution:

Given: $f(x) = x^2 + 1$

To find: (i) $f^{-1}\{10\}$

We know that, if $f: X \rightarrow Y$ such that $y \in Y$. Then $f^{-1}(y) = \{x \in X: f(x) = y\}$.

In other words, $f^{-1}(y)$ is the set of pre - images of y

Let $f^{-1}\{10\} = x$. Then, $f(x) = 10$...(i)

and it is given that $f(x) = x^2 + 1$...(ii)

So, from (i) and (ii), we get

$$x^2 + 1 = 10$$

$$\Rightarrow x^2 = 10 - 1$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \sqrt{9}$$

$$\Rightarrow x = \pm 3$$

$$\therefore f^{-1}\{10\} = \{-3, 3\}$$

To find: (ii) $f^{-1}\{-3\}$

Let $f^{-1}\{-3\} = x$. Then, $f(x) = -3$...(iii)

and it is given that $f(x) = x^2 + 1$...(iv)

So, from (iii) and (iv), we get

$$x^2 + 1 = -3$$

$$\Rightarrow x^2 = -3 - 1$$

$$\Rightarrow x^2 = -4$$

Clearly, this equation is not solvable in \mathbb{R}

$$\therefore f^{-1}\{-3\} = \emptyset$$

Question: 20

The function

Solution:

$$\text{Given: } F(x) = \frac{9}{5}x + 32 \dots (i)$$

To find: (i) $F(0)$

Substituting the value of $x = 0$ in eq. (i), we get

$$F(x) = \frac{9}{5}x + 32$$

$$\Rightarrow F(0) = \frac{9}{5} \times 0 + 32$$

$$\Rightarrow F(0) = 32$$

It means $0^\circ \text{ C} = 32^\circ \text{ F}$

To find: (ii) $F(-10)$

Substituting the value of $x = -10$ in eq. (i), we get

$$F(x) = \frac{9}{5}x + 32$$

$$\Rightarrow F(-10) = \frac{9}{5} \times (-10) + 32$$

$$\Rightarrow F(-10) = 9 \times (-2) + 32$$

$$\Rightarrow F(-10) = -18 + 32$$

$$\Rightarrow f(-10) = 14$$

It means $-10^\circ \text{ C} = 14^\circ \text{ F}$

To find: (iii) the value of x when $F(x) = 212$

$$\text{It is given that } F(x) = \frac{9}{5}x + 32$$

Substituting the value of $F(x) = 212$ in the above equation, we get

$$212 = \frac{9}{5}x + 32$$

$$\Rightarrow 212 - 32 = \frac{9}{5}x$$

$$\Rightarrow 180 = \frac{9}{5}x$$

$$\Rightarrow x = 180 \times \frac{5}{9}$$

$$\Rightarrow x = 20 \times 5$$

$$= x = 100$$

It means $212^{\circ}\text{F} = 100^{\circ}\text{C}$

Exercise : 3B

Question: 1

If $f(x) = x^2$

Solution:

Given: $f(x) = x^2 - 3x + 4$ ----- (1)

and $f(x) = f(2x + 1)$

Need to Find: Value of x

Replacing x by $(2x + 1)$ in equation (1) we get,

$$f(2x + 1) = (2x + 1)^2 - 3(2x + 1) + 4$$
 ----- (2)

According to the given problem, $f(x) = f(2x + 1)$

Comparing (1) and (2) we get,

$$x^2 - 3x + 4 = (2x + 1)^2 - 3(2x + 1) + 4$$

$$= x^2 - 3x + 4 = 4x^2 + 4x + 1 - 6x - 3 + 4$$

$$= 4x^2 + 4x + 1 - 6x - 3 + 4 - x^2 + 3x - 4 = 0$$

$$= 3x^2 + x - 2 = 0$$

$$= 3x^2 + 3x - 2x - 2 = 0$$

$$= 3x(x + 1) - 2(x + 1) = 0$$

$$= (3x - 2)(x + 1) = 0$$

So, either $(3x - 2) = 0$ or $(x + 1) = 0$

Therefore, the value of x is either $\frac{2}{3}$ or -1 [Answer]

Question: 2

If Given: $f(x) = \frac{x-1}{x+1}$

(i)

Need to prove: $f\left(\frac{1}{x}\right) = -f(x)$

Now replacing x by $\frac{1}{x}$ we get,

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 1}{\frac{1}{x} + 1}$$

$$= f\left(\frac{1}{x}\right) = \frac{1-x}{1+x}$$

$$= f\left(\frac{1}{x}\right) = \frac{-(x-1)}{(x+1)} = -f(x) \text{ [Proved]}$$

(ii)

Need to prove: $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

Now replacing x by $-\frac{1}{x}$ we get,

$$f\left(\frac{-1}{x}\right) = \frac{\frac{-1}{x} - 1}{\frac{-1}{x} + 1}$$

$$= f\left(\frac{-1}{x}\right) = \frac{-1-x}{-1+x}$$

$$= f\left(\frac{-1}{x}\right) = \frac{-(x+1)}{x-1}$$

$$= f\left(\frac{-1}{x}\right) = \frac{-1}{\frac{x-1}{x+1}} = \frac{-1}{f(x)} \text{ [Proved]}$$

Question: 3

If Given: $f(x) = x^3 - \frac{1}{x^3}$

Need to prove: $f(x) + f\left(\frac{1}{x}\right) = 0$

Replacing x by $\frac{1}{x}$ we get,

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3$$

Now according to the problem,

$$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3$$

$$= f(x) + f\left(\frac{1}{x}\right) = 0 \text{ [Proved]}$$

Question: 4

If Given: $f(x) = \frac{x+1}{x-1}$

Need to prove: $f\{f(x)\} = x$

Now replacing x by f(x) we get,

$$f\{f(x)\} = \frac{f(x) + 1}{f(x) - 1}$$

$$= f\{f(x)\} = \frac{\frac{x+1}{x-1} + 1}{\frac{x+1}{x-1} - 1}$$

$$= f\{f(x)\} = \frac{x+1+x-1}{x+1-x+1}$$

$$= f\{f(x)\} = \frac{2x}{2}$$

$$= f\{f(x)\} = x \text{ [Proved]}$$

Question: 5

If $f(x) =$

Solution:

Given: $f(x) = \frac{1}{(2x+1)}$, where $x \neq \frac{-1}{2}$

Need to prove: $f\{f(x)\} = \frac{2x+1}{2x+3}$ when $x \neq \frac{-3}{2}$

Now placing f(x) in place of x

$$= f\{f(x)\} = \frac{1}{2f(x)+1}$$

$$= f\{f(x)\} = \frac{1}{2\frac{1}{2x+1}+1}$$

$$= f\{f(x)\} = \frac{1}{\frac{2+2x+1}{2x+1}} = \frac{2x+1}{2x+3}, \text{ where } x \neq \frac{-3}{2} \text{ [Proved]}$$

Question: 6

If Given: $f(x) = \frac{1}{(1-x)}$

Need to prove: $f\{f\{f(x)\}\} = x$

Replacing x by f(x),

$$f\{f(x)\} = \frac{1}{1-f(x)}$$

$$= f\{f(x)\} = \frac{1}{1-\frac{1}{1-x}} = \frac{1}{\frac{1-x-1}{1-x}} = \frac{1-x}{-x}$$

Now again replacing x by f(x) we get,

$$f\{f\{f(x)\}\} = \frac{1-f(x)}{-f(x)}$$

$$= f\{f\{f(x)\}\} = \frac{1-\frac{1-x}{-x}}{\frac{1-x}{-x}}$$

$$= f\{f\{f(x)\}\} = \frac{\frac{1-x-1}{-x}}{\frac{1-x}{-x}}$$

$$= f\{f\{f(x)\}\} = \frac{-x}{-1} = x \text{ [Proved]}$$

Question: 7

If Given: $f(x) = \frac{2x}{(1+x^2)}$

Need to prove: $f(\tan\theta) = \sin 2\theta$

$$f(\tan\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$= f(\tan\theta) = \frac{2\tan\theta}{\sec^2\theta} \text{ [as } 1 + \tan^2\theta = \sec^2\theta]$$

$$= f(\tan\theta) = 2\frac{\sin\theta}{\cos\theta}\cos^2\theta \text{ [as } \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \sec\theta = \frac{1}{\cos\theta}]$$

$$= f(\tan\theta) = 2\sin\theta\cos\theta = \sin 2\theta \text{ [Proved]}$$

Question: 8

If Given: $y = f(x) = \frac{3x+1}{5x-3}$

Need to prove: $x = f(y)$

Replacing x by y in the function,

$$f(y) = \frac{3y+1}{5y-3}$$

Now, given in the problem that $y = f(x)$

$$f(y) = \frac{3f(x)+1}{5f(x)-3}$$

$$= f(y) = \frac{3\frac{3x+1}{5x-3}+1}{5\frac{3x+1}{5x-3}-3}$$

$$\Rightarrow f(y) = \frac{9x+3+5x-3}{15x+5-15x+9}$$

$$\Rightarrow f(y) = \frac{14x}{14} = x$$

$$\Rightarrow x = f(y) \text{ [Proved]}$$

Exercise : 3C

Question: 1

Find the domain o

Solution:

(i)

$$\text{Given: } f(x) = \frac{3x+5}{x^2-9}$$

Need to find: Where the functions are defined.

To find the domain of the function $f(x)$ we need to equate the denominator to 0.

Therefore,

$$x^2 - 9 = 0$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm 3$$

It means that the denominator is zero when $x = 3$ and $x = -3$

So, the domain of the function is the set of all the real numbers except +3 and -3.

The domain of the function, $D_{f(x)} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

(ii)

$$\text{Given: } f(x) = \frac{2x-3}{x^2+x-2}$$

Need to find: Where the functions are defined.

To find the domain of the function $f(x)$ we need to equate the denominator to 0.

Therefore,

$$x^2 + x - 2 = 0$$

$$\Rightarrow x^2 + 2x - x - 2 = 0$$

$$\Rightarrow x(x+2) - 1(x+2) = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow x = -2 \text{ \& } x = 1$$

It means that the denominator is zero when $x = 1$ and $x = -2$

So, the domain of the function is the set of all the real numbers except 1 and -2.

The domain of the function, $D_{f(x)} = (-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

(iii)

$$\text{Given: } f(x) = \frac{x^2-2x+1}{x^2-8x+12}$$

Need to find: Where the functions are defined.

To find the domain of the function $f(x)$ we need to equate the denominator to 0.

Therefore,

$$x^2 - 8x + 12 = 0$$

$$\Rightarrow x^2 - 2x - 6x + 12 = 0$$

$$\Rightarrow x(x - 2) - 6(x - 2) = 0$$

$$\Rightarrow (x - 2)(x - 6) = 0$$

$$\Rightarrow x = 2 \text{ \& } x = 6$$

It means that the denominator is zero when $x = 2$ and $x = 6$

So, the domain of the function is the set of all the real numbers except 2 and 6.

The domain of the function, $D_{f(x)} = (-\infty, 2) \cup (2, 6) \cup (6, \infty)$.

(iv)

$$\text{Given: } f(x) = \frac{x^2 - 8}{x^2 - 1}$$

Need to find: Where the functions are defined.

To find the domain of the function $f(x)$ we need to equate the denominator to 0.

Therefore,

$$x^2 - 1 = 0$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

It means that the denominator is zero when $x = -1$ and $x = 1$

So, the domain of the function is the set of all the real numbers except -1 and +1.

The domain of the function, $D_{f(x)} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

Question: 2

Find the domain a

Solution:

$$\text{Given: } f(x) = \frac{1}{x}$$

Need to find: Where the functions are defined.

$$\text{Let, } f(x) = \frac{1}{x} = y \text{ ---- (1)}$$

To find the domain of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$x = 0$$

It means that the denominator is zero when $x = 0$

So, the domain of the function is the set of all the real numbers except 0.

The domain of the function, $D_{f(x)} = (-\infty, 0) \cup (0, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$$\frac{1}{y} = x$$

$$\Rightarrow y = \frac{1}{x} = f(x_1)$$

To find the range of the function $f(x_1)$ we need to equate the denominator of the function to 0.

Therefore,

$$x = 0$$

It means that the denominator is zero when $x = 0$

So, the range of the function is the set of all the real numbers except 0.

The range of the function, $R_{f(x)} = (-\infty, 0) \cup (0, \infty)$.

Question: 3

Find the domain a

Solution:

Given: $f(x) = \frac{1}{(x-5)}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{1}{x-5} = y$ ---- (1)

To find the domain of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$x - 5 = 0$$

$$\Rightarrow x = 5$$

It means that the denominator is zero when $x = 5$

So, the domain of the function is the set of all the real numbers except 5.

The domain of the function, $D_{f(x)} = (-\infty, 5) \cup (5, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$$\frac{1}{y-5} = x$$

$$\Rightarrow y - 5 = \frac{1}{x}$$

$$\Rightarrow y = \frac{1}{x} + 5 = \frac{1+5x}{x} = f(x_1)$$

To find the range of the function $f(x_1)$ we need to equate the denominator of the function to 0.

Therefore,

$$x = 0$$

It means that the denominator is zero when $x = 0$

So, the range of the function is the set of all the real numbers except 0.

The range of the function, $R_{f(x)} = (-\infty, 0) \cup (0, \infty)$.

Question: 4

Find the domain a

Solution:

Given: $f(x) = \frac{x-3}{2-x}$

Need to find: Where the functions are defined.

Let, $f(x) = \frac{x-3}{2-x} = y$ ---- (1)

To find the domain of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$2 - x = 0$$

$$\Rightarrow x = 2$$

It means that the denominator is zero when $x = 2$

So, the domain of the function is the set of all the real numbers except 2.

The domain of the function, $D_{f(x)} = (-\infty, 2) \cup (2, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$$\frac{y-3}{2-y} = x$$

$$\Rightarrow y - 3 = 2x - xy$$

$$\Rightarrow y + xy = 2x + 3$$

$$\Rightarrow y(1 + x) = 2x + 3$$

$$\Rightarrow y = \frac{2x+3}{1+x} = f(x_1)$$

To find the range of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$x + 1 = 0$$

$$\Rightarrow x = -1$$

It means that the denominator is zero when $x = -1$

So, the range of the function is the set of all the real numbers except -1.

The range of the function, $R_{f(x)} = (-\infty, -1) \cup (-1, \infty)$.

Question: 5

Find the domain a

Solution:

$$\text{Given: } f(x) = \frac{3x-2}{x+2}$$

Need to find: Where the functions are defined.

$$\text{Let, } f(x) = \frac{3x-2}{x+2} = y \text{ ---- (1)}$$

To find the domain of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$x + 2 = 0$$

$$\Rightarrow x = -2$$

It means that the denominator is zero when $x = -2$

So, the domain of the function is the set of all the real numbers except -2.

The domain of the function, $D_{f(x)} = (-\infty, -2) \cup (-2, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$$\frac{3y-2}{2+y} = x$$

$$= 3y - 2 = 2x + xy$$

$$= 3y - xy = 2x + 2$$

$$= y = \frac{2x+2}{3-x} = f(x_1)$$

To find the range of the function $f(x_1)$ we need to equate the denominator of the function to 0.

Therefore,

$$3 - x = 0$$

$$= x = 3$$

It means that the denominator is zero when $x = 3$

So, the range of the function is the set of all the real numbers except 3.

The range of the function, $R_{f(x)} = (-\infty, 3) \cup (3, \infty)$.

Question: 6

Find the domain a

Solution:

$$\text{Given: } f(x) = \frac{x^2-16}{x-4}$$

Need to find: Where the functions are defined.

To find the domain of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$x - 4 = 0$$

$$= x = 4$$

It means that the denominator is zero when $x = 4$

So, the domain of the function is the set of all the real numbers except 4.

The domain of the function, $D_{f(x)} = (-\infty, 4) \cup (4, \infty)$.

Now if we put any value of x from the domain set the output value will be either (-ve) or (+ve), but the value will never be 8

So, the range of the function is the set of all the real numbers except 8.

The range of the function, $R_{f(x)} = (-\infty, 8) \cup (8, \infty)$.

Question: 7

Find the domain a

Solution:

$$\text{Given: } f(x) = \frac{1}{\sqrt{2x-3}}$$

Need to find: Where the functions are defined.

$$\text{Let, } f(x) = \frac{1}{\sqrt{2x-3}} = y \text{ ---- (1)}$$

The condition for the function to be defined,

$$2x - 3 > 0$$

$$= x > \frac{3}{2}$$

So, the domain of the function is the set of all the real numbers greater than $\frac{3}{2}$.

The domain of the function, $D_{f(x)} = (\frac{3}{2}, \infty)$.

Now putting any value of x within the domain set we get the value of the function always a fraction whose denominator is not equals to 0.

The range of the function, $R_{f(x)} = (0, 1)$.

Question: 8

Find the domain a

Solution:

$$\text{Given: } f(x) = \frac{ax-b}{cx-d}$$

Need to find: Where the functions are defined.

$$\text{Let, } f(x) = \frac{ax-b}{cx-d} = y \text{ ---- (1)}$$

To find the domain of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

$$cx - d = 0$$

$$\Rightarrow x = \frac{d}{c}$$

It means that the denominator is zero when $x = \frac{d}{c}$

So, the domain of the function is the set of all the real numbers except d/c.

The domain of the function, $D_{f(x)} = (-\infty, \frac{d}{c}) \cup (\frac{d}{c}, \infty)$.

Now, to find the range of the function we need to interchange x and y in the equation no. (1)

So the equation becomes,

$$\frac{ay-b}{cy-d} = x$$

$$\Rightarrow ay - b = cxy - dx$$

$$\Rightarrow ay - cxy = b - dx$$

$$\Rightarrow y = \frac{b-dx}{a-cx}$$

To find the range of the function f(x) we need to equate the denominator of the function to 0.

Therefore,

$$a - cx = 0$$

$$\Rightarrow x = \frac{a}{c}$$

It means that the denominator is zero when $x = \frac{a}{c}$

So, the range of the function is the set of all the real numbers except a/c.

The range of the function, $R_{f(x)} = (-\infty, \frac{a}{c}) \cup (\frac{a}{c}, \infty)$.

Question: 9

Find the domain a

Solution:

$$\text{Given: } f(x) = \sqrt{3x-5}$$

Need to find: Where the functions are defined.

The condition for the function to be defined,

$$3x - 5 \geq 0$$

$$\Rightarrow x \geq \frac{5}{3}$$

So, the domain of the function is the set of all the real numbers greater than equals to $\frac{5}{3}$.

The domain of the function, $D_{f(x)} = [\frac{5}{3}, \infty)$.

Putting $\frac{5}{3}$ in the function we get, $f(x) = 0$

It means the range of the function is defined for all the values greater than equals to 0.

The range of the function, $R_{f(x)} = [0, \infty)$.

Question: 10

Find the domain a

Solution:

$$\text{Given: } f(x) = \sqrt{\frac{x-5}{3-x}}$$

Need to find: Where the functions are defined.

The condition for the function to be defined,

$$3 - x > 0$$

$$\Rightarrow x < 3$$

So, the domain of the function is the set of all the real numbers lesser than 3.

The domain of the function, $D_{f(x)} = (-\infty, 3)$.

The condition for the range of the function to be defined,

$$x - 5 \geq 0 \text{ \& } 3 - x > 0$$

$$\Rightarrow x \geq 5 \text{ \& } x < 3$$

Both the conditions can't be satisfied simultaneously. That means there is no range for the function $f(x)$.

Question: 11

Find the domain a

Solution:

$$\text{Given: } f(x) = \frac{1}{\sqrt{x^2-1}}$$

Need to find: Where the functions are defined.

The condition for the function to be defined,

$$x^2 - 1 > 0$$

$$\Rightarrow x^2 > 1$$

$$\Rightarrow x > 1$$

So, the domain of the function is the set of all the real numbers greater than 1.

The domain of the function, $D_{f(x)} = (1, \infty)$.

Now putting any value of x within the domain set we get the value of the function always a fraction whose denominator is not equals to 0.

The range of the function, $R_{f(x)} = (0, 1)$

Question: 12

Find the domain a

Solution:

Given: $f(x) = 1 - |x - 2|$

Need to find: Where the functions are defined.

Since $|x - 2|$ gives real no. for all values of x , the domain set can possess any real numbers.

So, the domain of the function, $D_{f(x)} = (-\infty, \infty)$.

Now the given function is $f(x) = 1 - |x - 2|$, where $|x - 2|$ is always positive. So, the maximum value of the function is 1.

Therefore, the range of the function, $R_{f(x)} = (-\infty, 1)$

Question: 13

Find the domain a

Solution:

Given: $f(x) = \frac{|x-4|}{x-4}$

Need to find: Where the functions are defined.

To find the domain of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$x - 4 = 0$$

$$\Rightarrow x = 4$$

It means that the denominator is zero when $x = 4$

So, the domain of the function is the set of all the real numbers except 4.

The domain of the function, $D_{f(x)} = (-\infty, 4) \cup (4, \infty)$.

The numerator is an absolute function of the denominator. So, for any value of x from the domain set, we always get either +1 or -1 as the output. So, the range of the function is a set containing -1 and +1

Therefore, the range of the function, $R_{f(x)} = \{-1, 1\}$

Question: 14

Find the domain a

Solution:

Given: $f(x) = \frac{x^2-9}{x-3}$

Need to find: Where the functions are defined.

To find the domain of the function $f(x)$ we need to equate the denominator of the function to 0.

Therefore,

$$x - 3 = 0$$

$$\Rightarrow x = 3$$

It means that the denominator is zero when $x = 3$

So, the domain of the function is the set of all the real numbers except 3.

The domain of the function, $D_{f(x)} = (-\infty, 3) \cup (3, \infty)$.

Now if we put any value of x from the domain set the output value will be either (-ve) or (+ve), but the value will never be 6

So, the range of the function is the set of all the real numbers except 6.

The range of the function, $R_{f(x)} = (-\infty, 6) \cup (6, \infty)$.

Question: 15

Find the domain a

Solution:

Given: $f(x) = \frac{1}{2 - \sin 3x}$

Need to find: Where the functions are defined.

The maximum value of an angle is 2π

So, the maximum value of $x = 2\pi/3$.

Whereas, the minimum value of x is 0

Therefore, the domain of the function, $D_{f(x)} = (0, 2\pi/3)$.

Now, the minimum value of $\sin\theta = 0$ and the maximum value of $\sin\theta = 1$. So, the minimum value of the denominator is 1, and the maximum value of the denominator is 2.

Therefore, the range of the function, $R_{f(x)} = (1/2, 1)$.

Exercise : 3D

Question: 1

Consider the real

Solution:

Given: $f(x) = x + 5 \forall x \in \mathbb{R}$

To Find: Domain and Range of $f(x)$.

The domain of the given function is all real numbers except where the expression is undefined. In this case, there is no real number which makes the expression undefined.

As $f(x)$ is a polynomial function, we can have any value of x .

Therefore,

$$\text{Domain}(f) = (-\infty, \infty) \{x \mid x \in \mathbb{R}\}$$

Now,

$$\text{Let } y = f(x)$$

$$y = x + 5$$

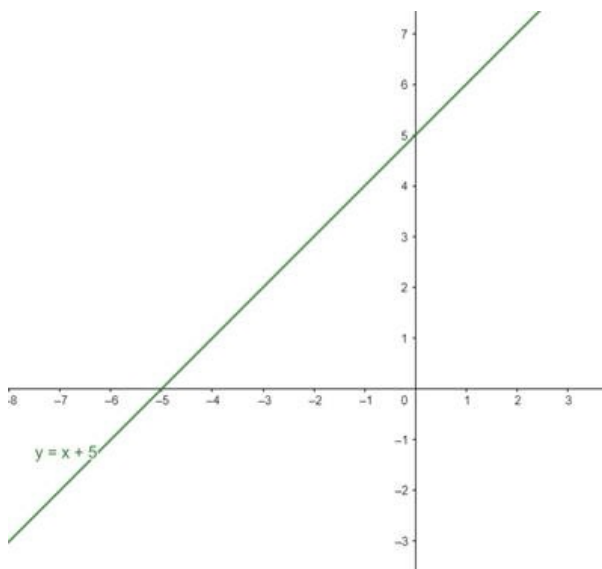
$$x = y - 5$$

The range is set of all valid values of y

Therefore,

$$\text{Range}(f) = (-\infty, \infty) \{y \mid y \in \mathbb{R}\}$$

Graph:



Question: 2

Consider th

Solution:

Given:

$$f(x) = \begin{cases} 1 - x & \text{when } x < 0 \\ 1x, & \text{when } x = 0 \\ x + 1, & \text{when } x > 0 \end{cases}$$

To Find:

Domain and Range of $f(x)$

When $f(x) = 1 - x \mid x < 0$

In this case there is no value of x ($x < 0$) which makes the above expression undefined.

Therefore,

$$\text{Domain}(f) = (-\infty, 0) \dots(1)$$

When $f(x) = x \mid x = 0$

In this case there is no value other than 0 which makes the above expression undefined.

Therefore,

$$\text{Domain}(f) = 0 \dots(2)$$

When $f(x) = x + 1 \mid x > 0$

In this case there is no value of x ($x > 0$) which makes the above expression undefined.

Therefore,

$$\text{Domain}(f) = (0, \infty) \dots(3)$$

From equations (1),(2) & (3) We can say that the domain of $f(x)$ as a whole :

$$\text{Domain}(f) = (-\infty, \infty)$$

Now when, $f(x) = 1 - x$

$$x = 1 - f(x)$$

As x ranges from $-\infty$ to 0, then $f(x)$ ranges from 1 to ∞

Therefore,

$$\text{Range}(f) = (1, \infty) \dots(4)$$

Now when, $f(x) = x$

As $x = 0$

Therefore,

$$\text{Range}(f) = 0 \dots (5)$$

Now when, $f(x) = x + 1$

$$x = f(x) - 1$$

As x ranges from 0 to ∞ , then $f(x)$ ranges from 1 to ∞

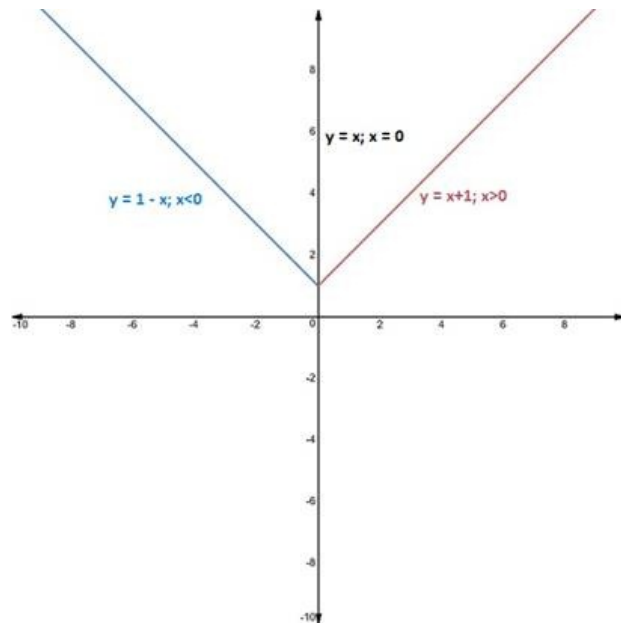
Therefore,

$$\text{Range}(f) = (1, \infty) \dots (6)$$

From (4), (5) & (6) the range of $f(x)$ as whole:

$$\text{Range}(f) = 0 \cup (1, \infty)$$

Graph:



Question: 3

Find the do

Solution:

Given:

$$f(x) = \sqrt{x}$$

To Find: Domain and Range of $f(x)$.

The domain of the given function is set of all positive real numbers including 0. In this case, if the value of x is a negative the number then it makes the expression undefined.

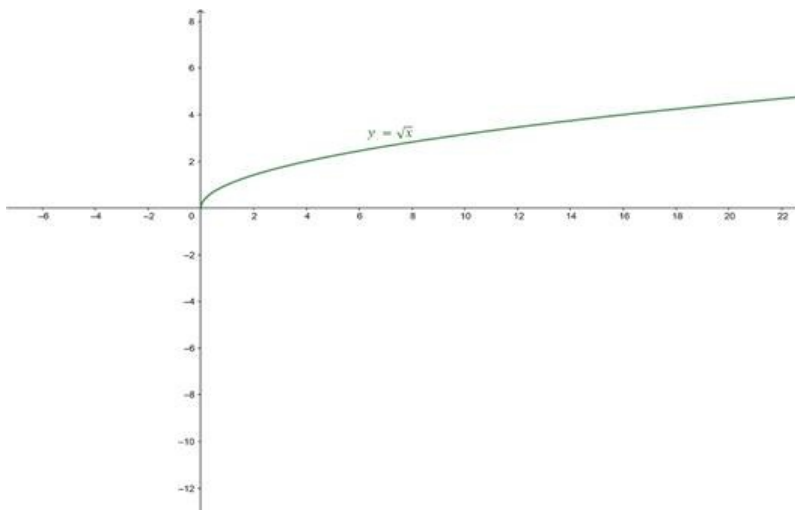
Therefore,

$$\text{Domain}(f) = [0, \infty) \forall x \in \mathbb{R}^+ \cup \{0\}$$

As the value of x varies from 0 to ∞ , value of \sqrt{x} varies from $\sqrt{0}$ to $\sqrt{\infty}$, Hence,

$$\text{Range}(f) = [0, \infty) \forall x \in \mathbb{R}^+ \cup \{0\}$$

Graph:



Question: 4

Find the do

Solution:

Given:

$$f(x) = x^{1/3} \forall x \in \mathbb{R}$$

To Find: Domain and range of the given function.

$$\text{Here, } f(x) = x^{1/3}$$

The domain of the above function would be,

$$\text{Domain}(f) = (-\infty, \infty) \{x \mid x \in \mathbb{R}\}$$

Because all real numbers have a cube root. There is no value of x which makes the function undefined.

Now, to find the range

$$\text{Consider } f(x) = y$$

$$\text{Then, } y = x^{1/3}$$

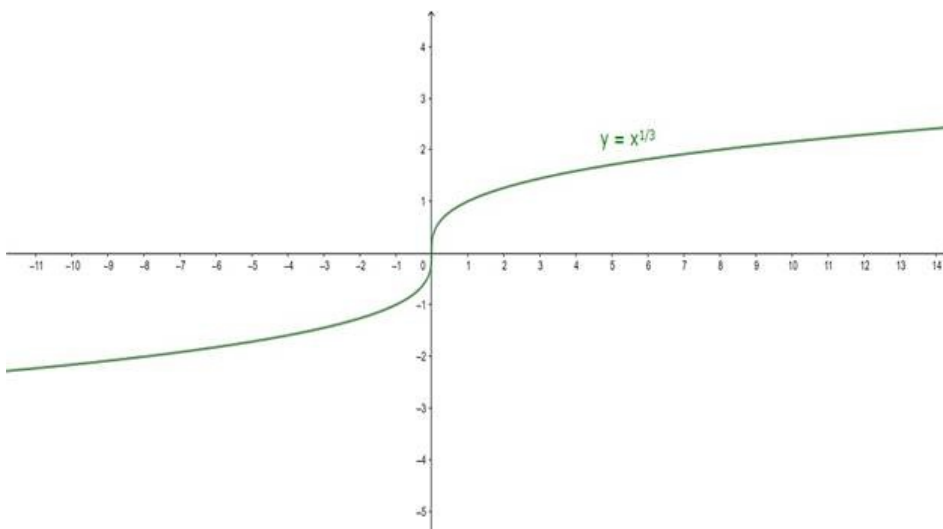
$$y^3 = x$$

Since $f(x)$ is continuous, it follows that

$$\text{Range}(f) = (-\infty, \infty) \{y \mid y \in \mathbb{R}\}$$

Because for every value of y there would be a cube of that value.

Graph:



Exercise : 3E

Question: 1

Let $f : \mathbb{R}$

Solution:

(i) Given:

$$f(x) = x + 1 \text{ and } g(x) = 2x - 3$$

(i) To find: $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

$$= (x + 1) + (2x - 3)$$

$$= x + 1 + 2x - 3$$

$$= 3x - 2$$

Therefore,

$$(f + g)(x) = 3x - 2$$

(ii) To find: $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

$$= (x + 1) - (2x - 3)$$

$$= x + 1 - 2x + 3$$

$$= 4 - x$$

Therefore,

$$(f - g)(x) = 4 - x$$

(iii) To find: $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (x + 1)(2x - 3)$$

$$= x(2x) - 3(x) + 1(2x) - 1(3)$$

$$= 2x^2 - 3x + 2x - 3$$

$$= 2x^2 - x - 3$$

Therefore,

$$(fg)(x) = 2x^2 - x - 3$$

(iv) To find: $\left(\frac{f}{g}\right)(x)$

$$\text{Sol. } \left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$$

$$= \left(\frac{x + 1}{2x - 3}\right)$$

Therefore,

$$\left(\frac{f}{g}\right)(x) = \left(\frac{x + 1}{2x - 3}\right)$$

Question: 2

Let $f : \mathbb{R}$

Solution:

(i) Given:

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + x$$

(i) To find: $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

$$= (2x + 5) + (x^2 + x)$$

$$= 2x + 5 + x^2 + x$$

$$= x^2 + 3x + 5$$

Therefore,

$$(f + g)(x) = x^2 + 3x + 5$$

(ii) To find: $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

$$= (2x + 5) - (x^2 + x)$$

$$= 2x + 5 - x^2 - x$$

$$= -x^2 + x + 5$$

Therefore,

$$(f - g)(x) = -x^2 + x + 5$$

(iii) To find: $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (2x + 5) \cdot (x^2 + x)$$

$$= 2x(x^2) + 2x(x) + 5(x^2) + 5x$$

$$= 2x^3 + 2x^2 + 5x^2 + 5x$$

$$= 2x^3 + 7x^2 + 5x$$

Therefore,

$$(fg)(x) = 2x^3 + 7x^2 + 5x$$

(iv) To find: $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$$

$$= \left(\frac{2x + 5}{x^2 + x}\right)$$

Therefore,

$$\left(\frac{f}{g}\right)(x) = \left(\frac{2x + 5}{x^2 + x}\right)$$

Question: 3

. Let $f: \mathbb{R}$

Solution:

(i) Given:

$$f(x) = x^3 + 1 \text{ and } g(x) = x + 1$$

(i) To find: $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$

$$= (x^3 + 1) + (x + 1)$$

$$= x^3 + 1 + x + 1$$

$$= x^3 + x + 2$$

Therefore,

$$(f + g)(x) = x^3 + x + 2$$

(ii) To find: $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$

$$= (x^3 + 1) - (x + 1)$$

$$= x^3 + 1 - x - 1$$

$$= x^3 - x$$

Therefore,

$$(f - g)(x) = x^3 - x$$

(iii) To find: $\left(\frac{1}{f}\right)(x)$

$$\left(\frac{1}{f}\right)(x) = \left(\frac{1}{f(x)}\right)$$

$$= \left(\frac{1}{x^3 + 1}\right)$$

Therefore,

$$\left(\frac{1}{f}\right)(x) = \left(\frac{1}{x^3 + 1}\right)$$

(iv) To find: $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \left(\frac{f(x)}{g(x)}\right)$$

$$= \left(\frac{x^3 + 1}{x + 1}\right)$$

$$= \left(\frac{x^3 + 1^3}{x + 1}\right)$$

$$= \left(\frac{(x+1)(x^2-x+1)}{x+1}\right) \text{ (Because } a^3 + b^3 = (a + b)(a^2 - ab + b^2))$$

Therefore,

$$\left(\frac{f}{g}\right)(x) = x^2 - x + 1$$

Question: 4

. Let $f: \mathbb{R}$

Solution:

Given:

$$f(x) = \frac{x}{e}$$

(i) To find: $(cf)(x)$

$$(cf)(x) = c.f(x)$$

$$= c.\left(\frac{x}{c}\right)$$

$$= x$$

Therefore,

$$(cf)(x) = x$$

(ii) To find: $(c^2f)(x)$

$$(c^2f)(x) = c^2.f(x)$$

$$= c.\left(\frac{x}{c}\right)$$

$$= cx$$

Therefore,

$$(c^2f)(x) = cx$$

(iii) To find: $\left(\frac{1}{c}f\right)(x)$

$$\left(\frac{1}{c}f\right) = \frac{1}{c}.f(x)$$

$$= \frac{1}{c}\left(\frac{x}{c}\right)$$

Therefore,

$$\left(\frac{1}{c}f\right)(x) = \frac{x}{c^2}$$

Question: 5

. Let $f:(2,$

Solution:

Given:

$$f(x)=\sqrt{x-2}: x > 2 \text{ and } g(x)=\sqrt{x+2}: x > 2$$

(i) To find: $(f + g)(x)$

$$\text{Domain}(f) = (2, \infty)$$

$$\text{Range}(f) = (0, \infty)$$

$$\text{Domain}(g) = (2, \infty)$$

$$\text{Range}(g) = (2, \infty)$$

$$(f + g)(x) = f(x) + g(x)$$

$$= \sqrt{x-2} + \sqrt{x+2}$$

Therefore,

$$(f + g)(x) = \sqrt{x-2} + \sqrt{x+2}$$

(ii) To find: $(f - g)(x)$

$$\text{Range}(g) \subseteq \text{Domain}(f)$$

Therefore,

$$(f - g)(x) \text{ exists.}$$

$$(f - g)(x) = f(x) - g(x)$$

$$= \sqrt{x-2} + \sqrt{x+2}$$

Therefore,

$$(f - g)(x) = \sqrt{x-2} - \sqrt{x+2}$$

(iii) To find: $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$

$$= (\sqrt{x-2}) \cdot (\sqrt{x+2})$$

$$= \sqrt{(x-2)(x+2)}$$

$$= \sqrt{x^2 - 2^2} \quad (\because a^2 - b^2 = (a-b)(a+b))$$

$$= \sqrt{x^2 - 4}$$

Therefore,

$$(fg)(x) = \sqrt{x^2 - 4}$$

Exercise : 3F

Question: 1

Find the set of v

Solution:

$$f(x) = 1 - 3x, \quad g(x) = 2x^2 - 1$$

To find:- Set of values of x for which $f(x) = g(x)$

Consider,

$$f(x) = g(x)$$

$$1 - 3x = 2x^2 - 1$$

$$2x^2 + 3x - 2 = 0$$

$$2x^2 + 4x - x - 2 = 0$$

$$2x(x+2) - (x+2) = 0$$

$$(x+2)(2x-1) = 0$$

$$x = -2 \text{ or } x = \frac{1}{2}$$

The set values for which $f(x)$ and $g(x)$ have same value is $\{-2, \frac{1}{2}\}$.

Question: 2

Find the set of v

Solution:

$$f(x) = x + 3, \quad g(x) = 3x^2 - 1$$

To find:- Set of values of x for which $f(x) = g(x)$

Consider,

$$f(x) = g(x)$$

$$x+3 = 3x^2 - 1$$

$$3x^2 - x - 4 = 0$$

$$3x^2 - 4x + 3x - 4 = 0$$

$$x(3x-4) + (3x-4) = 0$$

$$(3x - 4)(x + 1) = 0$$

$$x = 4/3 \text{ or } x = -1$$

The set values for which $f(x)$ and $g(x)$ have same value is $\{ 4/3, -1 \}$.

Question: 3

Let $X = \{$

Solution:

Given, $X = \{-1, 0, 2, 5\}$

$$f : X \rightarrow \mathbb{R} : f(x) = x^3 + 1$$

Finding $f(x)$ for each value of x ,

$$(1) f(-1) = (-1)^3 + 1 = -1 + 1 = 0$$

$$(2) f(0) = (0)^3 + 1 = 0 + 1 = 1$$

$$(3) f(2) = (2)^3 + 1 = 8 + 1 = 9$$

$$(4) f(5) = (5)^3 + 1 = 125 + 1 = 126$$

f in ordered pair is represented as

$$f = \{(-1,0), (0,1), (2,9), (5,126)\}$$

Question: 4

Given :

$$A = \{-2, -1, 0, 2\}$$

$$f : A \rightarrow \mathbb{Z} : f(x) = x^2 - 2x - 3$$

Finding $f(x)$ for each value of x ,

$$f(-2) = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$$

$$f(-1) = (-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = 0$$

$$f(0) = (0)^2 - 2(0) - 3 = 0 + 0 - 3 = -3$$

$$f(2) = (2)^2 - 2(2) - 3 = 4 - 4 - 3 = -3$$

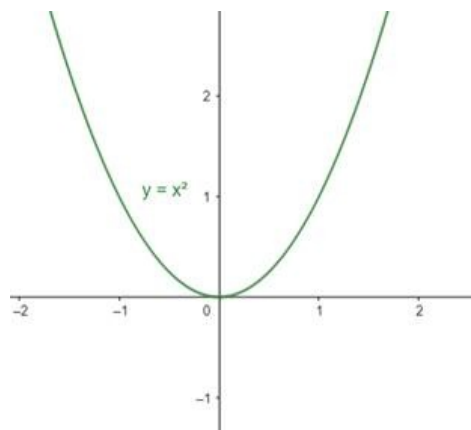
f in ordered pair is represented as

$$f = \{(-2,5), (-1,0), (0,-3), (2,-3)\}$$

Question: 5

$$\text{Given: } f(x) = x^2$$

The graph for the given function is



(i) Range(f):

For finding the range of the given function, let $y = f(x)$

Therefore,

$$y = x^2$$

$$x = \sqrt{y}$$

The value of $y \geq 0$.

Hence, Range(f) is $[0, \infty)$.

(ii) Let $y = f(x) = x^2$

Given $y = 4$.

Therefore, $x^2 = 4$

$$x = 2 \text{ or } x = -2$$

The set of values for which $y = 4$ is $x = \{2, -2\}$.

Question: 6

Let $f : \mathbb{R}$

Solution:

Given:

$$f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = x^2 + 1$$

To find inverse of $f(x)$

Let $y = f(x)$

$$y = x^2 + 1$$

$$y - 1 = x^2$$

$$x = \sqrt{y - 1}$$

$$f^{-1}(x) = \sqrt{x - 1}$$

Substituting $x = 10$,

$$f^{-1}(10) = \sqrt{10 - 1} = \sqrt{9} = 3$$

Question: 7

Given, $f : \mathbb{R}^+ \rightarrow \mathbb{R} : f(x) = \log_e x$

$$f(x) = -2$$

$$\log_e x = -2$$

Taking antilog on both sides

$$x = e^{-2}$$

Hence, the value of x for which $f(x) = -2$ is e^{-2} .

Question: 8

Given, $A = \{6, 10, 11, 15, 12\}$

$f : A \rightarrow \mathbb{N} : f(n)$ is the highest prime factor of n

(1) When $n = 6$, the highest prime factor of 6 is 3.

Hence, $f(6) = 3$.

(2) When $n = 10$, the highest prime factor of 10 is 5.

Hence, $f(10) = 5$.

(3) When $n = 11$, the highest prime factor of 11 is 11 as 11 itself is a prime number. Hence, $f(11) = 11$.

(4) When $n = 15$, the highest prime factor of 15 is 5.

Hence, $f(15) = 5$.

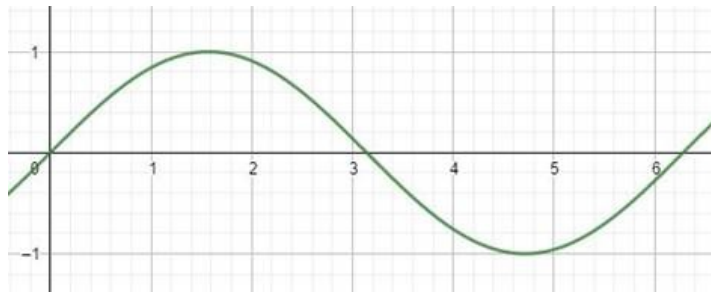
(5) When $n = 12$, the highest prime factor of 12 is 3.

Hence, $f(12) = 3$.

Hence range of f is $\{3, 5, 11\}$.

Question: 9

The graph of $\sin(x)$ is



$\sin(x)$ is a periodic function whose values always lie between -1 to +1. The maximum value is attained at $n\frac{\pi}{2}$ where n is odd and minimum when n is even. Hence, Range is $[-1, +1]$.

Question: 10

$|x|$ is defined as

$$|x| = x; x \geq 0$$

$$-x; x < 0$$

The value of $|x|$ is never a negative value.

Hence range of $|x|$ is $[0, \infty)$.

Question: 11

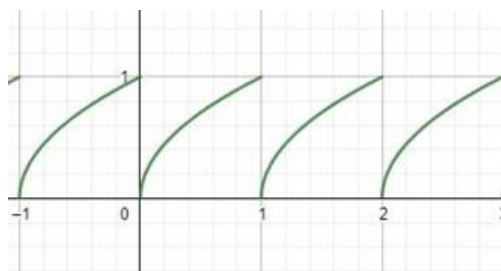
$$\text{Given, } f(x) = \sqrt{x - [x]}$$

Where $[x]$ is the Greatest Integer Function of x .

$$f(x) = \sqrt{\{x\}}$$

where $\{x\}$ is fractional part of x .

The graph of $f(x)$ is



(i) $\text{dom}(f)$

Domain of $\{x\}$ is \mathbb{R} .

The value of the fractional part of x is always either positive or zero.

Hence domain of $f(x)$ is \mathbb{R} .

(ii) range(f)

Range of $\{x\}$ is $[0,1)$.

As the root value $[0,1)$ between interval lies between $[0,1)$.

Hence range of $f(x)$ is $[0,1)$.

Question: 12

Given, $f(x) = \frac{x-5}{5-x}$

(i) dom(f)

Here $f(x)$ is a polynomial function whose domain is \mathbb{R} except for points at which denominator becomes zero.

Hence $x \neq 5$

The domain is $(-\infty, \infty) - \{5\}$

(ii) range(f)

Let $y = \frac{x-5}{5-x}$

For the specified domain

$$y = -1$$

Range is $\{-1\}$.

Question: 13

Given, $f = \{(1, 6), (2, 5), (4, 3), (5, 2), (8, -1), (10, -3)\}$

$g = \{(2, 0), (3, 2), (5, 6), (7, 10), (8, 12), (10, 16)\}$

(1) Domain of $f = \{1, 2, 4, 5, 8, 10\}$

Domain of $g = \{2, 3, 5, 7, 8, 10\}$

Domain of $(f+g) = \{x : x \in D_f \cap D_g\}$

where D_f = Domain of function f , D_g = Domain of function g

Domain of $(f+g) = \{2, 5, 8, 10\}$.

(2) Domain of quotient function $f/g = \{x : x \in D_f \cap D_g \text{ and } g(x) \neq 0\}$

Domain of $(f/g) = \{2, 5, 8, 10\}$.

Question: 14

Given, $f(x) = \frac{x-1}{x}$

$$F(x) = 1 - 1/x$$

To find $f(1/x)$ replacing x by $1/x$

$$F(1/x) = 1 - 1/(1/x)$$

$$F(1/x) = 1 - x$$

Question: 15

Given. $f(x) = \frac{kx}{x+1}$, $x \neq -1$

$$F(f(x)) = f\left(\frac{kx}{x+1}\right)$$

$$= \frac{k \frac{kx}{x+1}}{\frac{kx}{x+1} + 1}$$

$$= \frac{k^2 x}{kx+x+1}$$

Given that $f(f(x)) = x$

$$x = \frac{k^2 x}{kx+x+1}$$

Dividing both sides by x

$$1 = \frac{k^2}{kx+x+1}$$

$$kx + x + 1 = k^2$$

$$1 k^2 - kx - (x+1) = 0$$

$$k = \frac{-(-x) + \sqrt{(-x)^2 - 4(1)(-(x+1))}}{2(1)} \text{ or } k = \frac{-(-x) - \sqrt{(-x)^2 - 4(1)(-(x+1))}}{2(1)}$$

$$k = \frac{x + \sqrt{x^2 + 4x + 4}}{2} \text{ or } k = \frac{x - \sqrt{x^2 + 4x + 4}}{2}$$

$$k = \frac{x+x+2}{2} \text{ or } k = \frac{x-x-2}{2}$$

$$k = x + 1 \text{ or } k = -1$$

As value of x is variable we take $k = -1$.

Therefore, $k = -1$

Question: 16

$|x|$ is defined as

$$|x| = x; x \geq 0$$

$$-x; x < 0$$

$$\frac{1}{|x|} = \frac{1}{x}; x > 0$$

$$= \frac{-1}{x}; x < 0$$

$$\frac{x}{|x|} = 1; x > 0$$

$$= -1; x < 0$$

Hence $f(x)$ gives output values 1 and -1 only.

Range is $\{1, -1\}$.

Question: 17

$\log x$ function has domain \mathbb{R}^+ .

When x is replaced by $|x|$, the function f shows value as

$$f(x) = \log(x); x > 0$$

$$= \log(-x); x < 0$$

Hence in the function x cannot be zero as log function is not defined for $x=0$.

Domain of $f(x)$ is $\mathbb{R} - \{0\}$

Question: 18

$$\left(x + \frac{1}{x}\right) = t$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = t^2$$

Let's assume that,

$$\Rightarrow x^2 + \frac{1}{x^2} - 2\left(x\right)\left(\frac{1}{x}\right) = t^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = t^2 - 2$$

$$= f(t) = t^2 - 2$$

$$= f(x) = x^2 - 2$$

Question: 19

(i) domain

$$f(x) = \frac{ax + b}{bx - a}$$

As f(x) is a polynomial function whose domain is R except for the points where the denominator becomes 0.

$$\text{Hence } x \neq \frac{a}{b}$$

$$\text{Domain is } \mathbb{R} - \left\{\frac{a}{b}\right\}$$

(ii) range

$$\text{Let } y = \frac{ax + b}{bx - a}$$

$$Y(bx - a) = ax + b$$

$$byx - ay = ax + b$$

$$byx - ax = ay + b$$

$$x(by - a) = ay + b$$

$$x = \frac{ay + b}{by - a}$$

x is not defined when denominator is zero.

$$by - a \neq 0$$

$$y \neq a/b$$

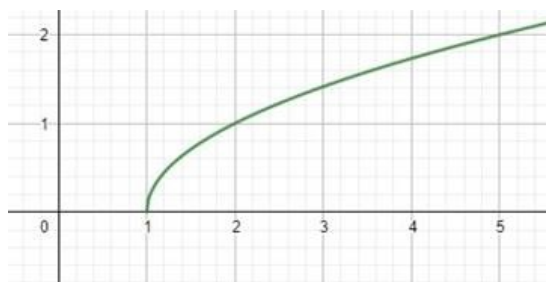
$$\text{Range is } \mathbb{R} - \{a/b\}.$$

Question: 20

Write the domain

Solution:

The graph of f(x) is



(i) domain

Domain for \sqrt{x} is $[0, \infty)$.

Hence, domain for $\sqrt{x-1}$ is $[1, \infty)$.

(ii) Range

As the range of function $f(x) = \sqrt{x}$ is given by the interval $[0, +\infty)$.

The graph of the given function $f(x) = \sqrt{x-1}$ is the graph of \sqrt{x} shifted 1 unit to the right. A shift to the right does not affect the range.

Hence the range of $f(x) = \sqrt{x-1}$ is also given by the interval: $[0, +\infty)$.

Question: 21

(i) Domain

$|x|$ is defined for all real values.

Hence $-|x|$ is also defined for all real values.

The domain is \mathbb{R} .

(ii) Range

Range for $|x|$ is $[0, \infty)$

Therefore, range for $-|x|$ is $(-\infty, 0]$.