Chapter : 4. TRIANGLES

Exercise : 4A

Question: 1 A

D and E are point

Solution:

Given: AD = 3.6 cm, AB = 10 cm and AE = 4.5 cm.

Applying Thale's Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow EC = \frac{AE}{AD} \times DB$

$\Rightarrow EC = \frac{4.5}{3.6} \times DB$ [∵ DB = AB – AD ⇒ DB = 10 – 3.6 = 6.4]

$\Rightarrow EC = \frac{4.5}{3.6} \times 6.4$

⇒ EC = 8

Now, AC = AE + EC

⇒ AC = 4.5 + 8 = 12.5

Hence, EC = 8 cm and AC = 12.5 cm

Question: 1 B

D and E are point

Solution:

Given: AB = 13.3 cm, AC = 11.9 cm and EC = 5.1 cm.

Applying Thale's Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Since we need to find DB first, we add 1 on both sides

$$\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$\Rightarrow \frac{AD+DB}{DB} = \frac{AE+EC}{EC}$

$\Rightarrow \frac{AB}{DB} = \frac{AC}{EC}$

$\Rightarrow DB = \frac{AB \times EC}{AC}$

$\Rightarrow DB = \frac{13.3 \times 5.1}{11.9}$

⇒ DB = 5.7

AD is given by,

AD = AB – DB

⇒ AD = 13.3 – 5.7

⇒ AD = 7.6 cm

Hence, AD is 7.6

Question: 1 C

D and E are point

Solution:

Given: AD/DB = 4/7 or AD = 4 cm, DB = 7 cm, and AC = 6.6

Applying Thale's Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

We have AE at RHS but we need AC, as the value of AC is given. So by adding 1 to both sides of the equation, we can get the desired result

$\Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\Rightarrow \frac{AD+DB}{DB} = \frac{AE+EC}{EC}$

$\Rightarrow \frac{4+7}{7} = \frac{AC}{EC}$

$\Rightarrow \frac{11}{7} = \frac{6.6}{EC}$

$\Rightarrow EC = \frac{6.6\times7}{11}$

⇒ EC = 4.2

AE is given by,

AE = AC – EC

⇒ AE = 6.6 – 4.2

⇒ AE = 2.4

Hence, AE is 2.4 cm.

Question: 1 D

D and E are point

Solution:

Given: AD/AB = 8/15 or AD = 8 cm, AB = 15 cm, and EC = 3.5 cm

By applying Thale's Theorem,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AE+EC}$

$\Rightarrow \frac{8}{15} = \frac{AE}{AE+3.5}$

⇒ 8×(AE + 3.5) = 15×AE

⇒ 8×AE + 28 = 15×AE

⇒ 15×AE – 8×AE = 28

⇒ 7×AE = 28

⇒ AE = 28/7 = 4

Hence, AE is 4 cm.

Question: 2 A

D and E are point

Solution:

Given: AD = x cm,

DB = (x – 2) cm,

AE = (x + 2) cm and,

EC = (x – 1) cm

By applying Thale's Theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x - 1) = (x + 2)(x - 2)$

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow x = 4$

Thus, x = 4 cm

Question: 2 B

D and E are point

Solution:

Given: AD = 4 cm, DB = (x – 4) cm, AE = 8 cm and EC = (3x – 19) cm

By Thale's theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

$\Rightarrow 4(3x - 19) = 8(x - 4)$

$\Rightarrow 12x - 76 = 8x - 32$

$\Rightarrow 12x - 8x = 76 - 32$

$\Rightarrow 4x = 44$

$\Rightarrow x = 44/4 = 11$

Thus, x = 11 cm

Question: 2 C

D and E are point

Solution:

Given: AD = (7x – 4) cm, AE = (5x – 2), DB = (3x + 4) cm and EC = 3x cm

By Thale's theorem,

$\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$

$\Rightarrow 3x(7x - 4) = (5x - 2)(3x + 4)$

$\Rightarrow 21x^2 - 12x = 15x^2 + 20x - 6x - 8$

$\Rightarrow 21x^2 - 12x = 15x^2 + 14x - 8$

$\Rightarrow 21x^2 - 15x^2 - 12x - 14x + 8 = 0$

$\Rightarrow 6x^2 - 26x + 8 = 0$

$\Rightarrow 2\times(3x^2 - 13x + 4) = 0$ [Simplifying the equation]

$\Rightarrow 3x^2 - 13x + 4 = 0$

$\Rightarrow 3x^2 - 12x - x + 4 = 0$

$\Rightarrow 3x(x - 4) - (x - 4) = 0$

$\Rightarrow (3x - 1)(x - 4) = 0$

$\Rightarrow (3x - 1) = 0$ or $(x - 4) = 0$

$\Rightarrow x = 1/3$ or $x = 4$

Now since we've got two values of x, that is, 1/3 and 4. We shall check for its feasibility.

Substitute $x = 1/3$ in $AD = (7x - 4)$, we get

$AD = 7\times(1/3) - 4 = -1.67$, which is not possible since side of a triangle cannot be negative.

Hence, x = 4 cm.

Question: 3 A

D and E are point

Solution:

Here, by applying converse of Thale's theorem we can conclude whether or not DE ∥ BC.

By Thale's theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Solving for $\frac{AD}{DB}$,

$$\frac{AD}{DB} = \frac{5.7}{9.5} = \frac{57}{95} = 0.6 \ldots(i)$$

Solving for $\frac{AE}{EC}$,

$$\frac{AE}{EC} = \frac{4.8}{8} = 0.6 \ldots(ii)$$

As equation (i) is equal to equation (ii),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

it satisfies Thale's theorem.

Hence, we can say DE ∥ BC.

Question: 3 B

D and E are point

Solution:

Here, by applying converse of Thale's theorem we can conclude whether or not DE ∥ BC.

By Thale's theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Solving for $\frac{AD}{DB}$,

We need to find AD from given AB = 11.7 cm and BD = 6.5 cm.

AD = AB – BD

⇒ AD = 11.7 – 6.5

⇒ AD = 5.2

$\frac{AD}{DB} = \frac{5.2}{6.5} = \frac{52}{65} = 0.8$ …(i)

Solving for $\frac{AE}{EC}$,

We need to find EC from given AC = 11.2 cm and AE = 4.2 cm.

EC = AC – AE

⇒ EC = 11.2 – 4.2

⇒ EC = 7

$\frac{AE}{EC} = \frac{4.2}{7} = 0.6$ …(ii)

As equation (i) is not equal to equation (ii),

$$\frac{AD}{DB} \neq \frac{AE}{EC}$$

it doesn't satisfies Thale's theorem.

Hence, we can say DE not parallel to BC.

Question: 3 C

D and E are point

Solution:

Here, by applying converse of Thale's theorem we can conclude whether or not DE ∥ BC.

By Thale's theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Solving for $\frac{AD}{DB}$,

We need to find DB from given AB = 10.8 cm and AD = 6.3 cm.

DB = AB – AD

⇒ DB = 10.8 – 6.3

⇒ DB = 4.5

$\frac{AD}{DB} = \frac{6.3}{4.5} = \frac{63}{45} = 1.4$ …(i)

Solving for $\frac{AE}{EC}$,

We need to find AE from given AC = 9.6 cm and EC = 4 cm.

AE = AC – EC

⇒ AE = 9.6 – 4

⇒ AE = 5.6

$\frac{AE}{EC} = \frac{5.6}{4} = 1.4$ …(ii)

As equation (i) is equal to equation (ii),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

it satisfies Thale's theorem.

Hence, we can say DE ∥ BC.

Question: 3 D

D and E are point

Solution:

Here, by applying converse of Thale's theorem we can conclude whether or not DE ∥ BC.

By Thale's theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Solving for $\frac{AD}{DB}$,

We need to find DB from given AB = 12 cm and AD = 7.2 cm.

DB = AB – AD

⇒ DB = 12 – 7.2

⇒ DB = 4.8

$\frac{AD}{DB} = \frac{7.2}{4.8} = \frac{72}{48} = 1.5$ …(i)

Solving for $\frac{AE}{EC}$,

We need to find EC from given AC = 10 cm and AE = 6.4 cm.

EC = AC – AE

⇒ EC = 10 – 6.4

⇒ EC = 3.6

$\frac{AE}{EC} = \frac{6.4}{3.6} = \frac{64}{36} = 1.78$ …(ii)

As equation (i) is not equal to equation (ii),

$$\frac{AD}{DB} \neq \frac{AE}{EC}$$

it doesn't satisfies Thale's theorem.

Hence, we can say DE is not parallel to BC.

Question: 4 A

In a ΔABC, AD is

Solution:

Given: AB = 6.4 cm, AC = 8 cm and BD = 5.6 cm

Since AD bisects ∠A, we can apply angle-bisector theorem in ΔABC,

$$\frac{BD}{DC} = \frac{AB}{AC}$$

Substituting given values, we get

$$\frac{5.6}{DC} = \frac{6.4}{8}$$

$\Rightarrow DC = \frac{5.6 \times 8}{6.4}$

⇒ DC = 7

Thus, DC is 7 cm.

Question: 4 B

In a ΔABC, AD is

Solution:

Given: AB = 10 cm, AC = 14 cm and BC = 6 cm

Since AD bisects ∠A, we can apply angle-bisector theorem in ΔABC,

$\frac{BD}{DC} = \frac{AB}{AC}$

Substituting given values, we get

$\frac{BD}{DC} = \frac{10}{14}$

To find BD and DC,

Let BD = x cm, and it's given that BC = 6 cm, then DC = (6 – x) cm

Then

$\frac{x}{6 - x} = \frac{10}{14}$

⇒ 14x = 10(6 – x)

⇒ 14x = 60 – 10x

⇒ 14x + 10x = 60

⇒ 24x = 60

⇒ x = 60/24 = 2.5

⇒ BD = 2.5 cm

If BD = 2.5 cm and BC = 6 cm, then DC = (6 – x) = (6 – 2.5) = 3.5

Thus, BD is 2.5 cm and DC = 3.5 cm.

Question: 4 C

In a ΔABC, AD is

Solution:

Given: AB = 5.6 cm, BC = 6 cm and BD = 3.2 cm

Since AD bisects ∠A, we can apply angle-bisector theorem in ΔABC,

$\frac{BD}{DC} = \frac{AB}{AC}$

Substituting given values, we get

$\frac{3.2}{DC} = \frac{5.6}{AC}$

Here, DC is given by

DC = BC – BD

⇒ DC = 6 – 3.2 = 2.8

$\frac{3.2}{2.8} = \frac{5.6}{AC}$

⇒ $AC = \frac{5.6 \times 2.8}{3.2}$

⇒ AC = 4.9

Thus, AC is 4.9 cm.

Question: 4 D

In a ΔABC, AD is

Solution:

Given: AB = 5.6 cm, AC = 4 cm and DC = 3 cm

Since AD bisects ∠A, we can apply angle-bisector theorem in ΔABC,

$$\frac{BD}{DC} = \frac{AB}{AC}$$

Substituting given values, we get

$$\frac{BD}{3} = \frac{5.6}{4}$$

$\Rightarrow BD = \frac{5.6 \times 3}{4}$

⇒ BD = 4.2

Now, BC = BD + DC

⇒ BC = 4.2 + 3 = 7.2

Thus, BC is 7.2 cm.

Question: 5

M is a point on t

Solution:

(i). Given: ABCD is a parallelogram.

To Prove: $\frac{DM}{MN} = \frac{DC}{BN}$

Proof: In ΔDMC and ΔNMB,

∠DMC = ∠NMB [∵ they are vertically opposite angles]

∠DCM = ∠NBM [∵ they are alternate angles]

∠CDM = ∠MNB [∵ they are alternate angles]

By AAA-similarity, we can say

ΔDMC ~ ΔNMB

So, from similarity of the triangle, we can say

$$\frac{DM}{MN} = \frac{DC}{BN}$$

Hence, proved.

(ii). Given: ABCD is a parallelogram.

To Prove: $\frac{DN}{DM} = \frac{AN}{DC}$

Proof: As we have already derived

$$\frac{DM}{MN} = \frac{DC}{BN}$$

Add 1 on both sides of the equation, we get

$$\frac{DM}{MN} + 1 = \frac{DC}{BN} + 1$$

$\Rightarrow \frac{DM+MN}{MN} = \frac{DC+BN}{BN}$

$\Rightarrow \frac{DM+MN}{MN} = \frac{AB+BN}{BN}$ [∵ ABCD is a parallelogram and a parallelogram's opposite sides are always equal ⇒ DC = AB]

$\Rightarrow \frac{DN}{MN} = \frac{AN}{BN}$

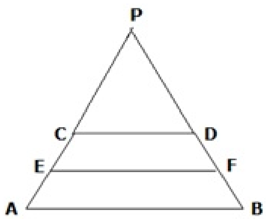
Hence, proved.

Question: 6

Show that the lin

Solution:

We can draw the trapezium as



Here, let EF be the line segment joining the oblique sides of the trapezium at midpoints E and F (say) correspondingly.

Construction: Extend AD and BC such that it meets at P.

To Prove: EF ∥ DC and EF ∥ AB

Proof: Given that, ABCD is trapezium which means DC ∥ AB. …(statement (i))

In ΔPAB,

DC ∥ AB (by statement (i))

So, this means we can apply Thale's theorem in ΔPAB. We get

$\frac{PD}{DA} = \frac{PC}{CB}$ …(ii)

∵ E and F are midpoints of AD and BC respectively, we can write

DA = DE + EA

Or DA = 2DE …(iii)

CB = CF + FB

Or CB = 2CF …(iv)

Substituting equation (iii) and (iv) in equation (ii), we get

$$\frac{PD}{2DE} = \frac{PC}{2CF}$$

$\Rightarrow \frac{PD}{DE} = \frac{PC}{CF}$

By applying converse of Thale's theorem, we can write DC ∥ EF.

Now if DC ∥ EF, and we already know that DC ∥ AB.

⇒ EF is also parallel to AB, that is, EF ∥ AB.

This means, DC ∥ EF ∥ AB.

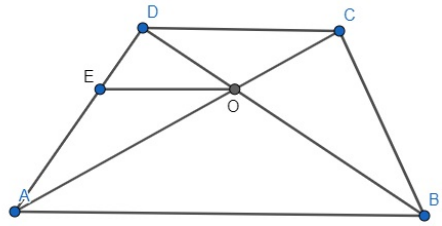
Hence, proved.

Question: 7

In the adjoining

Solution:

Given: In the adjoining figure, ABCD is a trapezium in which CD || AB and its diagonals intersect at O. If AO = (5x – 7) cm, OC = (2x + 1) cm, DO = (7x – 5) cm and OB = (7x + 1) cm.**To find:** the value of x.**Solution:**



In the trapezium ABCD, AB || DC and its diagonals intersect at O.Through O draw EO || AB meeting AD at E.Now In Δ ADCAs EO || AB || DCBy thales theorem which states that If a line is drawn parallel to one side of a triangle to intersect the othertwo sides in distinct points then the other two sides are divided in the same ratio. $\therefore \frac{AE}{ED} = \frac{AO}{OC}$ ...... (i)In Δ DAB,EO || ABBy thales theorem, $\therefore \frac{DE}{EA} = \frac{DO}{OB}$

$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD}$ ...... (ii)From (i) and (ii)$\frac{AO}{OC} = \frac{BO}{OD}$

Put the given values as:

$\Rightarrow \frac{5x-7}{2x+1} = \frac{7x-5}{7x+1}$

⇒ (5x – 7)(7x + 1) = (7x – 5)(2x + 1)

⇒ $35x^2 + 5x - 49x - 7 = 14x^2 - 10x + 7x - 5$

⇒ $35x^2 - 44x - 7 = 14x^2 - 3x - 5$

⇒ $35x^2 - 14x^2 - 44x + 3x - 7 + 5 = 0$

⇒ $21x^2 - 41x - 2 = 0$

⇒ $21x^2 - 42x + x - 2 = 0$

⇒ 21x(x – 2) + (x – 2) = 0

⇒ (21x + 1)(x – 2) = 0

⇒ (21x + 1) = 0 or (x – 2) = 0

⇒ x = -1/21 or x = 2

But x = -1/21 doesn’t satisfy the length of intersected lines.

So x ≠ -1/21

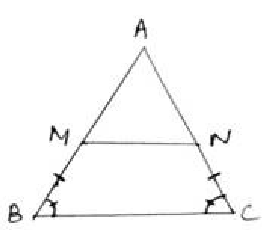
And thus, x = 2.

Question: 8

In a **&Del**

Solution:

We have



To show that, MN ∥ BC.

Given that, ∠B = ∠C and BM = CN.

So, AB = AC [sides opposite to equal angles (∠B = ∠C) are equal]

Subtract BM from both sides, we get

AB – BM = AC – BM

⇒ AB – BM = AC – CN

⇒ AM = AN

⇒ ∠AMN = ∠ANM

[angles opposite to equal sides (AM = AN) are equal] …(i)

We know in ΔABC,

∠A + ∠B + ∠C = 180° [∵ sum of angles of a triangle is 180°] …(ii)

And in ΔAMN,

∠A + ∠AMN + ∠ANM = 180° [∵ sum of angles of a triangle is 180°] …(iii)

Comparing equations (ii) and (iii), we get

∠A + ∠B + ∠C = ∠A + ∠AMN + ∠ANM

⇒ ∠B + ∠C = ∠AMN + ∠ANM

⇒ 2∠B = 2∠AMN [∵ from equation (i), and also ∠B = ∠C]

⇒ ∠B = ∠AMN

Thus, MN ∥ BC since the corresponding angles, ∠AMN = ∠B.

Question: 9

ΔABC and ΔDBC lie

Solution:

We can observe two triangles in the figure.

In ΔABC,

PQ ∥ AB

Applying Thale's theorem, we get

$\frac{CP}{PB} = \frac{CQ}{QA}$ **…(i)**

In ΔBDC,

PR ∥ BP

Applying Thale's theorem, we get

$\frac{CP}{QA} = \frac{CR}{RO}$ **…(ii)**

Comparing equations (i) and (ii),

$$\frac{CQ}{QA} = \frac{CR}{RO}$$

Now, applying converse of Thale's theorem, we get

QR ∥ AD

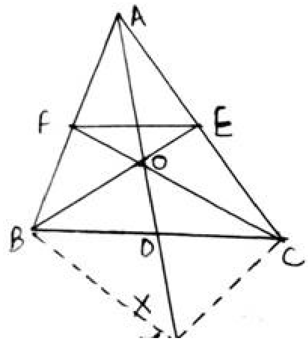
Hence, QR is parallel to the AD.

Question: 10

In the given figu

Solution:

We have the diagram as,



Given: BD = DC & OD = DX

To Prove: $\frac{AO}{AX} = \frac{AF}{AB}$ **and also, EF ∥ BC**

Proof: Since, from the diagram we can see that diagonals OX and BC bisect each other in quadrilateral BOCX. Thus, BOCX is a parallelogram.

If BOCX is a parallelogram, BX ∥ OC, and BO ∥ CX.

⇒ BX ∥ FC (as OC extends to FC) and CX ∥ BE (BO extends to BE)

⇒ BX ∥ OF and CX ∥ OE

∵ BX ∥ OF, applying Thale's theorem in ΔABX, we get

$\frac{AO}{AX} = \frac{AF}{AB}$ **…(i)**

Now since CX ∥ OE, applying Thale's theorem in ΔACX, we get

$\frac{AO}{AX} = \frac{AE}{AC}$ **…(ii)**

By equations (i) and (ii), we get

$$\frac{AF}{AB} = \frac{AE}{AC}$$

By applying converse of Thale's theorem in the above equation, we can write

EF ∥ BC

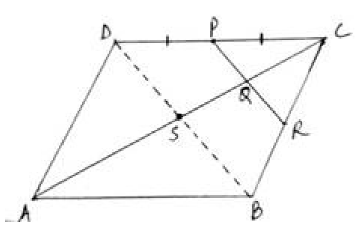
Hence, proved.

Question: 11

ABCD is a paralle

Solution:

We have the diagram as



Given: DP = PC &

CQ = (1/4)AC …(i)

To Prove: CR = RB

Proof: Join B to D

As diagonals of a parallelogram bisect each other at S.

$CS = \frac{1}{2}AC$ **…(ii)**

Dividing equation (i) by (ii), we get

$$\frac{CQ}{CS} = \frac{AC}{4} \times \frac{2}{AC}$$

⇒ $\frac{CQ}{CS} = \frac{1}{2}$

⇒ CQ = CS/2

⇒ Q is the midpoint of CS.

According to midpoint theorem in ΔCSD, we have

PQ ∥ DS

Similarly, in ΔCSB, we have

QR ∥ SB

Also, given that CQ = QS

We can conclude that, by the converse of midpoint theorem, CR = RB.

That is, R is the midpoint of CB.

Hence, proved.

Question: 12

In the adjoining

Solution:

Given: AD = AE …(i)

& AB = AC …(ii)

Subtracting AD from both sides of equation (ii), we get

AB – AD = AC – AD

⇒ AB – AD = AC – AE [from equation (i)]

⇒ DB = EC [∵ AB – AD = DB & AC – AE = EC] …(iii)

Now, divide equation (i) by (iii), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

By converse of Thale's theorem, we can conclude by this equation that DE ∥ BC.

So, ∠DEC + ∠ECB = 180° [∵ sum of interior angles on the same transversal line is 180°]

Or ∠DEC + ∠DBC = 180° [∵ AB = AC ⇒ ∠C = ∠B]

Hence, we can write DEBC is cyclic and points D, E, B and C are concyclic.

Question: 13

In ΔABC, the bise

Solution:

Given: ∠PBR = ∠QBR & PQ ∥ AC.

In ΔBQP,

BR bisects ∠B such that ∠PBR = ∠QBR.

Since angle-bisector theorem says that, if two angles are bisected in a triangle then it equates their relative lengths to the relative lengths of the other two sides of the triangles.

So by applying angle-bisector theorem, we get

$$\frac{QR}{PR} = \frac{BQ}{BP}$$

⇒ QR × BP = PR × BQ

Hence, proved.

Exercise : 4B

Question: 1 A

In each of the gi

Solution:

In these triangles ABC and PQR, observe that

∠BAC = ∠PQR = 50°

∠ABC = ∠QPR = 60°

∠ACB = ∠PRQ = 70°

Thus, by angle-angle-angle similarity, i.e., AAA similarity,

ΔABC ~ ΔPQR

Question: 1 B

In each of the gi

Solution:

In triangles ABC & EFD,

∠ABC ≠ ∠EDF

$$\frac{AB}{DF} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{BC}{DE} = \frac{4.5}{9} = \frac{1}{2}$$

So, clearly, since no criteria satisfies, ΔABC is not similar to ΔEFD.

Question: 1 C

In each of the gi

Solution:

In triangles ABC & PQR,

∠ACB = ∠PQR

$$\frac{CA}{QR} = \frac{8}{6} = \frac{4}{3}$$

$$\frac{BC}{PQ} = \frac{6}{4.5} = \frac{4}{3}$$

By SAS criteria, we can say

ΔABC ~ ΔPQR

Question: 1 D

In each of the gi

Solution:

In triangles DEF & PQR,

$\frac{DE}{QR} = \frac{2.5}{5} = \frac{1}{2}$

$\frac{EF}{PQ} = \frac{2}{4} = \frac{1}{2}$

$\frac{DF}{PR} = \frac{3}{6} = \frac{1}{2}$

By SSS criteria, we can write

ΔDEF ~ ΔPQR

Question: 1 E

In each of the gi

Solution:

In ΔABC, we can find ∠ABC.

∠ABC + ∠BCA + ∠CAB = 180° [∵ sum of all the angles of a triangle is 180°]

⇒ ∠ABC + 70° + 80° = 180°

⇒ ∠ABC + 150° = 180°

⇒ ∠ABC = 180° - 150°

⇒ ∠ABC = 30°

We can observe from triangles ABC & MNR,

∠ABC = ∠MNR

∠CAB = ∠RMN

Hence, by AA similarity we can say, ΔABC ~ ΔMNR

Question: 2

In the given figu

Solution:

(i) To find ∠DOC, we can observe the straight line DB.

∠DOC + ∠COB = 180° [∵ sum of all angles in a straight line is 180°]

⇒ ∠DOC + 115° = 180°

⇒ ∠DOC = 180° - 115°

⇒ ∠DOC = 65°

(ii) In ΔDOC,

And given that, ∠CDO = 70°, ∠DOC = 65° (from (i))

∠DOC + ∠DCO + ∠CDO = 180°

⇒ 65° + ∠DCO + 70° = 180°

⇒ ∠DCO + 135° = 180°

⇒ ∠DCO = 180° - 135°

⇒ ∠DCO = 45°

(iii) We have derived ∠DCO from (ii), ∠DCO = 45°

Thus, ∠OAB = 45° [∵ ∠OAB = ∠DCO as ΔODC ~ ΔOBA]

(iv) It’s given that, ∠CDO = 70°

Thus, ∠OBA = 70° [∵ ∠OBA = ∠CDO as ΔODC ~ ΔOBA]

Question: 3

In the given figu

Solution:

(i). Given that, AB = 8 cm

BO = 6.4 cm,

OC = 3.5 cm

& CD = 5 cm

ΔOAB ~ ΔOCD

When two triangles are similar, they can be written in the ratio as

$$\frac{OA}{OC} = \frac{AB}{CD}$$

Substitute gave values in the above equations,

$$\frac{OA}{3.5} = \frac{8}{5}$$

$\Rightarrow OA = \frac{8\times3.5}{5}$

⇒ OA = 5.6

Thus, OA = 5.6 cm

(ii). Given that, AB = 8 cm

BO = 6.4 cm,

OC = 3.5 cm

& CD = 5 cm

ΔOAB ~ ΔOCD

When two triangles are similar, they can be written in the ratio as

$$\frac{BO}{DO} = \frac{AB}{CD}$$

Substitute gave values in the above equations,

$$\frac{6.4}{DO} = \frac{8}{5}$$

$\Rightarrow DO = \frac{5\times6.4}{8}$

⇒ DO = 4

Thus, DO = 4 cm

Question: 4

In the given figu

Solution:

Given is that ∠ADE = ∠B

From the diagram clearly, ∠EAD = ∠BAC [∵ they are common angles]

Now, since two of the angles are correspondingly equal. Then by AA similarity criteria, we can say

ΔADE ~ ΔABC

Further, it's given that

AD = 3.8 cm

AE = 3.6 cm

BE = 2.1 cm

BC = 4.2 cm

DE =?

To find AB, we can express it in the form AB = AE + BE = 3.6 + 2.1

⇒ AB = 5.7

So for the condition that ΔADE ~ ΔABC,

$$\frac{DE}{BC} = \frac{AD}{AB}$$

Substituting given values in the above equation,

$$\Rightarrow \frac{DE}{4.2} = \frac{3.8}{5.7}$$

$$\Rightarrow DE = \frac{3.8 \times 4.2}{5.7}$$

⇒ DE = 2.8

Thus, DE = 2.8 cm

Question: 5

The perimeters of

Solution:

Given that, ΔABC ~ ΔPQR

And perimeter of ΔABC = 32 cm & perimeter of ΔPQR = 24 cm

We can write relationship as,

$$\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta PQR} = \frac{AB}{PQ}$$

$$\Rightarrow \frac{32}{24} = \frac{AB}{12}$$

$$\Rightarrow AB = \frac{32 \times 12}{24}$$

⇒ AB = 16

Thus, AB = 16 cm.

Question: 6

The corresponding

Solution:

Given that, ΔABC ~ ΔDEF

Also, BC = 9.1 cm & EF = 6.5 cm

And perimeter of ΔDEF = 25 cm

We need to find perimeter of ΔABC = ?

We can write relationship as,

$$\frac{\text{the perimeter of } \Delta ABC}{\text{the perimeter of } \Delta DEF} = \frac{BC}{EF}$$

$$\Rightarrow \frac{\text{the perimeter of } \Delta ABC}{25} = \frac{9.1}{6.5}$$

$\Rightarrow$ perimeter of $\Delta ABC = \frac{9.1 \times 25}{6.5}$

⇒ perimeter of ΔABC = 35

Thus, perimeter of ΔABC = 35 cm

Question: 7

In the given figu

Solution:

Given that, ∠CAB = 90°

AC = 75 cm

AB = 1 m

BC = 1.25 m

To show that, ΔBDA ~ ΔBAC

In the diagram, we can see

∠BDA = ∠BAC = 90°

∠DBA = ∠CBA [They are common angles]

So by AA-similarity theorem,

ΔBDA ~ ΔBAC

Thus, now since ΔBDA ~ ΔBAC, we can write as

$\frac{AD}{AC} = \frac{AB}{BC}$

$\Rightarrow \frac{AD}{75} = \frac{100}{125}$ **[∵ AC = 75 cm, AB = 1 m = 100 cm & BC = 1.25 m = 125 cm]**

$\Rightarrow AD = \frac{100 \times 75}{125}$

⇒ AD = 60 cm

Hence, AD = 60 cm or 0.6 m

Question: 8

In the given figu

Solution:

Given that, ∠ABC = 90°

AB = 5.7 cm

BD = 3.8 cm

CD = 5.4 cm

In order to find BC, we need to prove that ΔBDC and ΔABC are similar.

∠BDC = ∠ABC = 90°

∠ACB = ∠DCB [They are common angles]

By this we have proved ΔBDC ~ ΔABC, by AA-similarity criteria.

So we can write,

$\frac{BD}{AB} = \frac{DC}{BC}$

$\Rightarrow \frac{3.8}{5.7} = \frac{5.4}{BC}$

$\Rightarrow BC = \frac{5.4\times5.7}{3.8}$

⇒ BC = 8.1

Hence, BC = 8.1 cm.

Question: 9

In the given figu

Solution:

Given that, ∠ABC = 90°

AD = 4 cm

BD = 8 cm

In order to find CD, we need to prove that ΔBDC and ΔABC are similar.

∠BDC = ∠ADB = 90°

∠DBA = ∠DCB

We have proved ΔDBA ~ ΔDCB, by AA-similarity criteria.

So we can write,

$$\frac{BD}{CD} = \frac{AD}{BD}$$

$\Rightarrow \frac{8}{CD} = \frac{4}{8}$

$\Rightarrow CD = \frac{8\times8}{4}$

⇒ CD = 16

Hence, CD = 16 cm.

Question: 10

P and Q are point

Solution:

There are two triangles here, ΔAPQ and ΔABC. We shall prove these triangles to be similar.

$$\frac{AP}{AB} = \frac{2}{4+2} = \frac{2}{6} = \frac{1}{3}$$

& $\frac{AQ}{AC} = \frac{3}{6+3} = \frac{3}{9} = \frac{1}{3}$

$\Rightarrow \frac{AP}{AB} = \frac{AQ}{AC}$

Also, ∠A = ∠A [common angle]

So by AA-similarity criteria,

ΔAPQ ~ ΔABC

Thus,

$$\frac{PQ}{BC} = \frac{AQ}{AC}$$

And we know $\frac{PQ}{BC} = \frac{1}{3}$

⇒ BC = 3×PQ

Hence, proved.

Question: 11

ABCD is a paralle

Solution:

Given that, AB ∥ DC & AD ∥ BC

To Prove: AF × FB = EF × FD

Proof: In ΔDAF & ΔBEF

∠DAF = ∠BEF [∵ they are alternate angles]

∠AFD = ∠EFB [∵ they are vertically opposite angles]

This implies that ΔDAF ~ ΔBEF by AA-similarity criteria.

⇒ $\frac{AF}{EF} = \frac{FD}{FB}$

Now cross-multiply them,

AF × FB = FD × EF

Hence, proved.

Question: 12

In the given figu

Solution:

Observe in ΔBED & ΔACB, we have

∠BED = ∠ACB = 90°

Now according to what's given, DB ⊥ BC and AC ⊥ BC we can write,

∠B + ∠C = 180°

This clearly means BD ∥ CA

⇒ ∠EBD = ∠CAB [They are alternate angles]

AA Similarity theorem: The postulate states that two triangles are similar if they have two corresponding angles that are congruent or equal in measure.

Thus, by AA-similarity theorem, ΔBED ~ ΔACBNow, by property of similarity of triangles,

So, $\frac{BE}{AC} = \frac{DE}{BC}$

Cross-multiplying, we get,

⇒ $\frac{BE}{DE} = \frac{AC}{BC}$

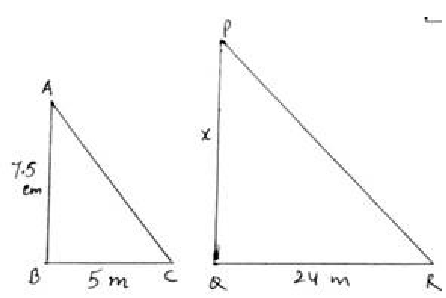
Hence, proved.

Question: 13

A vertical pole o

Solution:

We have



Let the two triangles be ΔABC and ΔPQR.

Given that, AB = 7.5 cm

BC = 5 m = 500 cm

QR = 24 m = 2400 cm

We have to find PQ = x (say).

We need to prove ΔABC is similar to ΔPQR.

We can observe that,

∠ABC = ∠PQR = 90°

∠ACB = ∠PRQ [∵ the sum castes same angle at all places at the same time]

Thus, by AA-similarity criteria, we can say

ΔABC ~ ΔPQR

So,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

Substitute the given values in this equation,

$$\frac{7.5}{x} = \frac{500}{2400}$$

⇒ $x = \frac{7.5 \times 2400}{500}$

⇒ x = 36 cm

Thus, height of the tower is 36 cm.

Question: 14

In an isosceles Δ

Solution:

To prove: ΔACP ~ ΔBCQ

Proof:

Given that, ΔABC is an isosceles triangle. ⇒ AC = BC

Also, if ΔABC is an isosceles triangle,

then ∠CAB = ∠CBA ...(i)

Subtracting it by 180° from both sides, we get

180° - ∠CAB = 180° - ∠CBA

⇒ ∠CAP = ∠CBQ ...(ii)

Also, given that AP × BQ = AC × AC

Or $\frac{AP}{AC} = \frac{AC}{BQ}$

Or $\frac{AP}{AC} = \frac{BC}{BQ}$ **[∵ AC = BC] ...(iii)**

Recollecting equations (i), (ii) and (iii),

By SAS-similarity criteria, we get

ΔACP ~ ΔBCQ

Hence, proved.

Question: 15

In the given figu

Solution:

To Prove: ΔACB ~ ΔDCE

Proof:

Given that, ∠1 = ∠2

⇒ ∠DBC = ∠DCE

Also in ΔABC & ΔDCE, we get

∠DCE = ∠ACB [they are common angles to both triangles]

And $\frac{AC}{BD} = \frac{CB}{CE}$

Or $\frac{AC}{CB} = \frac{BD}{CE}$

Or $\frac{AC}{CB} = \frac{DC}{CE}$ **[∵ BD = DC as ∠1 = ∠2]**

Thus by SAS-similarity criteria, we get

ΔACB ~ ΔDCE

Hence, proved.

Question: 16

ABCD is a quadril

Solution:

Given: AD = BC

P, Q, R and S are the midpoints of AB, AC, CD and BD respectively.

So in ΔABC, if P and Q are midpoints of AB and A respectively ⇒ PQ ∥ BC

And PQ = (1/2)BC ...(i)

Similarly in ΔADC,

QR = (1/2)AD ...(ii)

In ΔBCD,

SR = (1/2)BC ...(iii)

In ΔABD,

PS = (1/2)AD = (1/2)BC [∵ AD = BC]

Using equations (i), (ii), (iii) & (iv), we get

PQ = QR = SR = PS

All these sides are equal.

⇒ PQRS is a rhombus.

Hence, shown that PQRS is a rhombus.

Question: 17

In a circle, two

Solution:

Given: AB and CD are chords of the circle, intersecting at point P.

(a). To Prove: ΔPAC ~ ΔPDB

Proof: In ΔPAC and ΔPDB,

∠APC = ∠DPB [∵ they are vertically opposite angles]

∠CAP = ∠PDB [∵ angles in the same segment are equal]

Thus, by AA-similarity criteria, we can say that,

ΔPAC ~ ΔPDB

Hence, proved.

(b). To Prove: PA × PB = PC × PD

Proof: As already proved that ΔPAC ~ ΔPDB

We can write as,

$$\frac{PA}{PD} = \frac{PC}{PB}$$

By cross-multiplying, we get

PA × PB = PC × PD

Hence, proved.

Question: 18

Two chords AB and

Solution:

Given: AB and CD are chords of a circle intersecting at point P outside the circle.

(a). To Prove: ΔPAC ~ ΔPDB

Proof: We know

∠ABD + ∠ACD = 180° [∵ opposite angles of cyclic quadrilateral are supplementary] …(i)

∠PCA + ∠ACD = 180° [∵ they are linear pair angle] …(ii)

Comparing equations (i) & (ii), we get

∠ABD + ∠ACD = ∠PCA + ∠ACD

⇒ ∠ABD = ∠PCA

Also, ∠APC = ∠BPD [∵ they are common angles]

Thus, by AA-similarity criteria, ΔPAC ~ ΔPDB

Hence, proved.

(b). To Prove: PA × PB = PC × PD

Proof: We have already proved that, ΔPAC ~ ΔPDB

Thus the ratios can be written as,

$$\frac{PA}{PD} = \frac{PC}{PB}$$

By cross-multiplication, we get

PA × PB = PC × PD

Hence, proved.

Question: 19

In a right triang

Solution:

By the property that says, if a perpendicular is drawn from the vertex of a right triangle

to the hypotenuse then the triangles on both the sides of the perpendicular are similar to the whole triangle and also to each other.

We can conclude by the property in ΔBDC,

ΔCQD ~ ΔDQB

(a). To Prove: DQ^2 = DP × QC

Proof: As already proved, ΔCQD ~ ΔDQB

We can write the ratios as,

$$\frac{CQ}{DQ} = \frac{DQ}{QB}$$

By cross-multiplication, we get

DQ^2 = QB × QC ...(i)

Now since, quadrilateral PDQB forms a rectangle as all angles are 90° in PDQB.

⇒ DP = QB & PB = DQ

And thus replacing QB by DP in equation (i), we get

DQ^2 = DP × QC

Hence, proved.

(b). To Prove: DP^2 = DQ × AP

Prof: Similarly using same property, we get

ΔAPD ~ ΔDPB

We can write the ratios as,

$$\frac{AP}{DP} = \frac{PD}{PB}$$

By cross-multiplication, we get

DP^2 = PB × AP

⇒ DP^2 = DQ × AP [∵ PB = DQ]

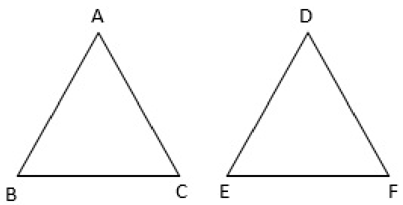
Hence, proved.

Exercise : 4C

Question: 1

ΔABC ~ ΔDEF and t

Solution:



We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{64}{121} = \frac{BC^2}{EF^2} = \frac{BC^2}{(15.4)^2}$$

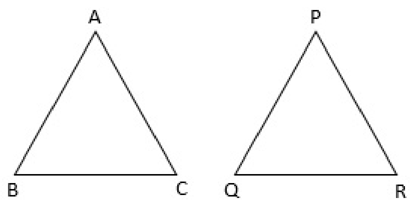
$$\Rightarrow BC^2 = \frac{64}{121} \times (15.4)^2$$

$$\Rightarrow BC = \sqrt{\frac{64}{121} \times (15.4)^2} = \frac{8}{11} \times 15.4 = 8 \times 1.4 = 11.2 \text{ cm}$$

Question: 2

The areas of two

Solution:



We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{16} = \frac{BC^2}{QR^2} = \frac{(4.5)^2}{QR^2}$$

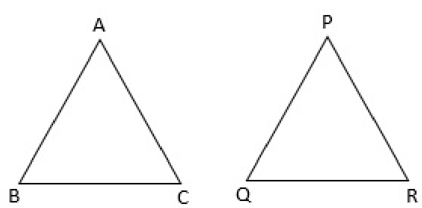
$$\Rightarrow QR^2 = \frac{16}{9} \times (4.5)^2$$

$$\Rightarrow QR = \sqrt{\frac{16}{9} \times (4.5)^2} = \frac{4}{3} \times 4.5 = 1.5 \times 4 = 6 \text{ cm}$$

Question: 3

ΔABC ~ ΔPQR and a

Solution:



Given that $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{4}{1}$

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{4}{1} = \frac{BC^2}{QR^2} = \frac{(12)^2}{QR^2}$$

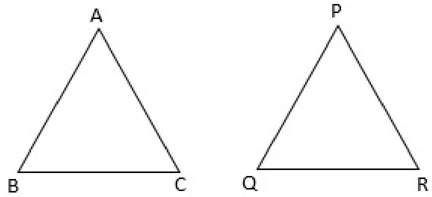
$$\Rightarrow QR^2 = \frac{1}{4} \times (12)^2$$

$$\Rightarrow QR = \sqrt{\frac{1}{4} \times (12)^2} = \frac{1}{2} \times 12 = 6 \text{ cm}$$

Question: 4

The areas of two

Solution:



Let the two triangles be ABC and PQR and their longest sides are BC and QR.

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their longest sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{169}{121} = \frac{BC^2}{QR^2} = \frac{(26)^2}{QR^2}$$

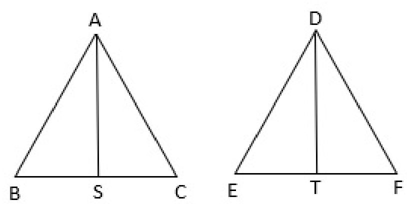
$$\Rightarrow QR^2 = \frac{121}{169} \times (26)^2$$

$$\Rightarrow QR = \sqrt{\frac{121}{169} \times (26)^2} = \frac{11}{13} \times 26 = 22 \text{ cm}$$

Question: 5

ΔABC ~ΔDEF and th

Solution:



Let the two triangles ABC and DEF have their altitudes as AS and DT.

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding altitudes.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{100}{49} = \frac{AS^2}{DT^2} = \frac{5^2}{DT^2}$$

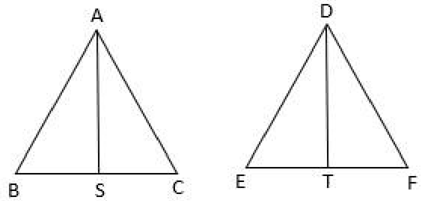
$$\Rightarrow DT^2 = \frac{49}{100} \times (5)^2$$

$$\Rightarrow DT = \sqrt{\frac{49}{100} \times (5)^2} = \frac{7}{10} \times 5 = 3.5 \text{ cm}$$

Question: 6

The corresponding

Solution:



Let the two triangles ABC and DEF have their altitudes as AS and DT.

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding altitudes.

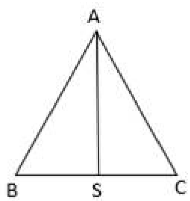
$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AS^2}{DT^2} = \frac{6^2}{9^2}$$

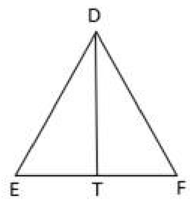
$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{36}{81} = \frac{4}{9}$$

Question: 7

The areas of two

Solution:





Let the two triangles ABC and DEF have their altitudes as AS and DT.

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding altitudes.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{81}{49} = \frac{AS^2}{DT^2} = \frac{(6.3)^2}{DT^2}$$

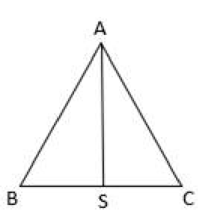
$$\Rightarrow DT^2 = \frac{49}{81} \times (6.3)^2$$

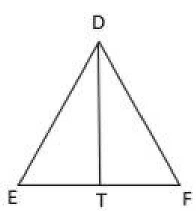
$$\Rightarrow DT = \sqrt{\frac{49}{81} \times (6.3)^2} = \frac{7}{9} \times 6.3 = 4.9 \text{ cm}$$

Question: 8

The areas of two

Solution:





Let the two triangles ABC and DEF have their medians as AS and DT.

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding medians.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{100}{64} = \frac{AS^2}{DT^2} = \frac{AS^2}{(5.6)^2}$$

$$\Rightarrow AS^2 = \frac{100}{64} \times (5.6)^2$$

$$\Rightarrow DT = \sqrt{\frac{100}{64} \times (5.6)^2} = \frac{10}{8} \times 5.6 = 7 \text{ cm}$$

Question: 9

In the given figu

Solution:

We have

$\frac{AP}{AB} = \frac{1}{4}$ and $\frac{AQ}{AC} = \frac{1.5}{6} = \frac{1}{4}$

Also ∠ A = ∠ A

So, by SAS similarity criterion ΔAPQ ~ ΔABC

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{1^2}{4^2} = \frac{1}{16}$$

$$\Rightarrow ar(\Delta APQ) = \frac{1}{16} \times ar(\Delta ABC)$$

Hence, proved.

Question: 10

In the given figu

Solution:

It is given that DE || BC

∴∠ ADE = ∠ ABC (Corresponding angles)

∠ AED = ∠ ACB (Corresponding angles)

So, by AA similarity criterion ΔADE ~ ΔABC

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow ar(\Delta ABC) = \frac{6^2}{3^2} \times ar(\Delta ADE)$$

$$\Rightarrow ar(\Delta ABC) = 4 \times 15 = 60cm^2$$

Hence, proved.

Question: 11

ΔABC is right-ang

Solution:

In ΔABC and ΔADC

∴∠ BAC = ∠ ADC (90° angle)

∠ ACB = ∠ ACD (Common)

So, by AA similarity criterion ΔADC ~ ΔABC

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADC)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ADC)} = \frac{13^2}{5^2} = \frac{169}{25} = 169:25$$

Question: 12

In the given figu

Solution:

It is given that DE || BC

∴∠ ADE = ∠ ABC (Corresponding angles)

∠ AED = ∠ ACB (Corresponding angles)

So, by AA similarity criterion ΔADE ~ ΔABC

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{3^2}{5^2} = \frac{9}{25}$$

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(BCED)} = \frac{9}{25-9} = \frac{9}{16} = 9:16$$

Hence, proved.

Question: 13

In ΔABC, D and E

Solution:

In ΔABC and ΔADE

It is given that AD = DB and AE = EC

$$\therefore \frac{AD}{AB} = \frac{1}{2} \text{ and } \frac{AE}{AC} = \frac{1}{2}$$

Also ∠ A = ∠ A

So, by SAS similarity criterion ΔADE ~ ΔABC

We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding sides.

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{AE^2}{AC^2}$$

$$\Rightarrow \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{1^2}{2^2} = \frac{1}{4} = 1:4$$

Exercise : 4D

Question: 1

The sides of cert

Solution:

In a right angled triangle

(Hypotenuse) ^{2} = (Base)^2 + (Height)^2

where hypotenuse is the longest side.

(i) L.H.S. = (Hypotenuse)^2 = (18)^2 = 324

R.H.S. = (Base)^2 + (Height)^2 = (9)^2 + (16)^2 = 81 + 256 = 337

⇒L.H.S. ≠ R.H.S.

∴It is not a right triangle.

(ii) L.H.S. = (Hypotenuse)^2 = (27)^2 = 729

R.H.S. = (Base)^2 + (Height)^2 = $(7)^2 + (25)^2 = 49 + 625 = 674$

⇒ L.H.S. ≠ R.H.S.

∴It is not a right triangle.

(iii) L.H.S. = (Hypotenuse)^2 = $(5)^2 = 25$

R.H.S. = (Base)^2 + (Height)^2 = $(1.4)^2 + (4.8)^2 = 1.96 + 23.04 = 25$

⇒ L.H.S. = R.H.S.

∴It is a right triangle.

(iv) L.H.S. = (Hypotenuse)^2 = $(4)^2 = 16$

R.H.S. = (Base)^2 + (Height)^2 = $(1.6)^2 + (3.8)^2 = 2.56 + 14.44 = 17$

⇒ L.H.S. ≠ R.H.S.

∴It is not a right triangle.

(v) L.H.S. = (Hypotenuse)^2 = $(a + 1)^2$

R.H.S. = (Base)^2 + (Height)^2 = $(a-1)^2 + (2\sqrt{a})^2 = a^2 + 1 - 2a + 4a = a^2 + 1 + 2a = (a + 1)^2$

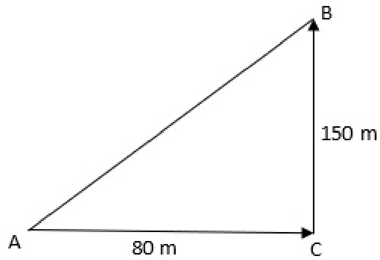
⇒ L.H.S. = R.H.S.

∴It is a right triangle.

Question: 2

A man goes 80 m d

Solution:



The starting point of the man is A and the last point is B so we need to find AB. From the figure, ΔABC is a right triangle.

In a right angled triangle

(Hypotenuse)^2 = (Base)^2 + (Height)^2

where hypotenuse is the longest side.

$(AB)^2 = (AC)^2 + (BC)^2$

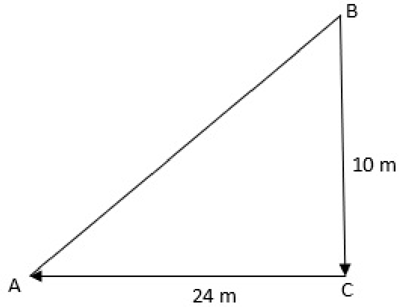
⇒ $AB^2 = (80)^2 + (150)^2 = 6400 + 22500 = 28900$

⇒ AB = 170 m

Question: 3

A man goes 10 m d

Solution:



The starting point of the man is B and the last point is A so we need to find AB. From the figure, ΔABC is a right triangle.

In a right angled triangle

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

where hypotenuse is the longest side.

$(AB)^2 = (AC)^2 + (BC)^2$

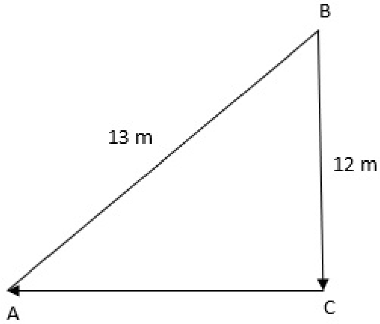
$\Rightarrow AB^2 = (24)^2 + (10)^2 = 576 + 100 = 676$

$\Rightarrow AB = 26$ m

Question: 4

A 13-m-long ladde

Solution:



Ladder AB = 13 m and distance from the window BC = 12 m.

AC is the distance of the ladder from the building.

From the figure, ΔABC is a right triangle.

In a right angled triangle

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$

where hypotenuse is the longest side.

$(AB)^2 = (AC)^2 + (BC)^2$

$\Rightarrow 13^2 = (AC)^2 + (12)^2$

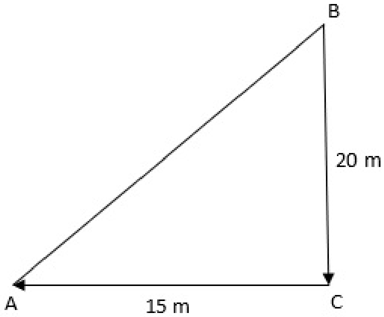
$\Rightarrow AC^2 = 169 - 144 = 25$

$\Rightarrow AC = 5$ m

Question: 5

A ladder is place

Solution:



Ladder AB and distance from the window BC = 20 m.

AC is the distance of the ladder from the building = 15 m.

From the figure, ΔABC is a right triangle.

In a right angled triangle

(Hypotenuse) ^{2} = (Base)^2 + (Height)^2

where hypotenuse is the longest side.

$(AB)^2 = (AC)^2 + (BC)^2$

$\Rightarrow AB^2 = (20)^2 + (15)^2$

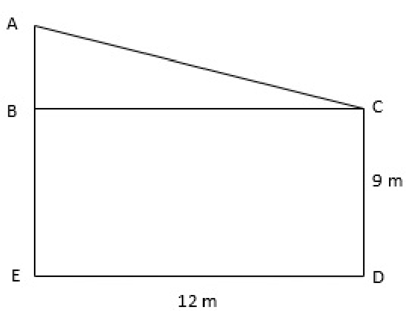
$\Rightarrow AB^2 = 400 + 225 = 625$

$\Rightarrow AB = 25$ m

Question: 6

Two vertical pole

Solution:



AE(height of the first building) = 14 m , CD(height of the second building) = 9 m , ED(distance between their feet) = BC = 12 m

AE – AB = 14 m – 9 m = 5 m

From the figure, ΔABC is a right triangle.

In a right angled triangle

(Hypotenuse) ^{2} = (Base)^2 + (Height)^2

where hypotenuse is the longest side.

$(AC)^2 = (AB)^2 + (BC)^2$

$\Rightarrow AC^2 = (5)^2 + (12)^2$

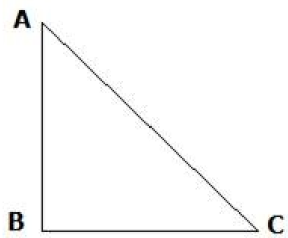
$\Rightarrow AB^2 = 25 + 144 = 169$

$\Rightarrow AB = 13$ m

Question: 7

A guy wire attach

Solution:



Pole AB = 18 m and distance from the window BC.

AC is the length of the wire = 24 m.

From the figure, ΔABC is a right triangle.

In a right angled triangle

(Hypotenuse) ^{2} = (Base)^2 + (Height)^2

where hypotenuse is the longest side.

(AC)^2 = (AB)^2 + (BC)^2

⇒ 24^2 = (18) ^{2} + (BC) ^{2}

⇒ BC^2 = 576 - 324 = 252

⇒ BC = 6√7 m

Question: 8

In the given figu

Solution:

ΔPOR is a right triangle because ∠O = 90°.

In a right angled triangle

(Hypotenuse)^2 = (Base)^2 + (Height)^2

where hypotenuse is the longest side.

(PR)^2 = (OP)^2 + (OR)^2

⇒ PR^2 = (6) ^{2} + (8) ^{2}

⇒ PR^2 = 36 + 64 = 100

⇒ PR = 10 m

Now, $PR^2 + PQ^2 = 10^2 + 24^2$ = 100 + 576 = 676

Also, $QR^2 = 26^2$ = 676

⇒ $PR^2 + PQ^2 = QR^2$

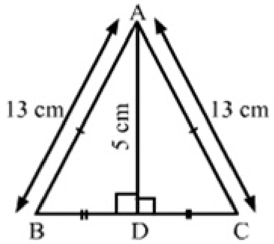
which satisfies Pythagoras theorem.

Hence, ΔPQR is right angled triangle.

Question: 9

ΔABC is an isosce

Solution:



Δ ABC is an isosceles triangle.

Also, AB = AC = 13 cm

Suppose the altitude from A on BC meets BC at D. Therefore, D is the midpoint of BC.

AD = 5 cm

ΔADB and ΔADC are right-angled triangles.

Applying Pythagoras theorem,

$AB^2 = BD^2 + AD^2$

$\Rightarrow BD^2 = 13^2 - 5^2$

$\Rightarrow BD^2 = 169 - 25 = 144$

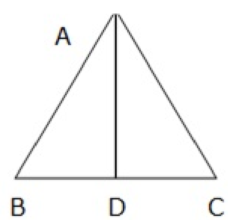
$\Rightarrow BD = 12$ cm

So, BC = $2\times 12 = 24$ cm

Question: 10

Find the length o

Solution:



Δ ABC is an isosceles triangle.

Also, AB = AC = 2a

The AD is the altitude. Therefore, D is the midpoint of BC.

$$BD = \frac{a}{2}$$

ΔADB and ΔADC are right-angled triangles.

Applying Pythagoras theorem,

$AB^2 = BD^2 + AD^2$

$$\Rightarrow (2a)^2 = \frac{a^2}{4} + AD^2$$

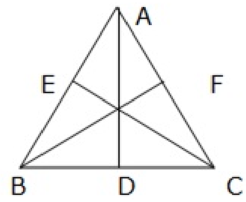
$$\Rightarrow AD^2 = \frac{16a^2 - a^2}{4} = \frac{15a^2}{4}$$

$$\Rightarrow AD = \frac{a\sqrt{15}}{2}$$

Question: 11

ΔABC is an equila

Solution:



Δ ABC is an equilateral triangle.

Also, BC = AB = AC = 2a

The AD, CE, and BF are the altitude at BC, AB and AC respectively. Therefore, D, E, and F are the midpoint of BC, AB and AC respectively.

Now, ΔADB and ΔADC are right-angled triangles.

Applying Pythagoras theorem,

$AB^2 = BD^2 + AD^2$

$\Rightarrow (2a)^2 = a^2 + AD^2$

$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = a\sqrt{3}$ **units**

Similarly ΔACE and ΔBEC are right-angled triangles.

Applying Pythagoras theorem,

$CE = a\sqrt{3}$ **units**

And ΔABF and ΔBFC are right-angled triangles.

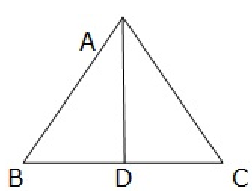
Applying Pythagoras theorem,

$BF = a\sqrt{3}$ **units**

Question: 12

Find the height o

Solution:



Δ ABC is an equilateral triangle.

Also, BC = AB = AC = 12 cm

The AD is the altitude at BC. Therefore, D is the midpoint of BC.

Now, ΔADB and ΔADC are right-angled triangles.

Applying Pythagoras theorem,

$AB^2 = BD^2 + AD^2$

$\Rightarrow (12)^2 = 6^2 + AD^2$

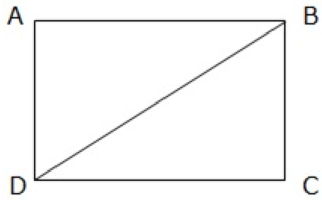
$\Rightarrow AD^2 = 144 - 36 = 108$

$\Rightarrow AD = 6\sqrt{3}$ **cm**

Question: 13

Find the length o

Solution:



Given that AB = 30cm and AD = 16 cm

∵ ∠ A = 90°

∴ ΔADB is a right-angled triangle.

Applying Pythagoras theorem,

$BD^2 = BA^2 + AD^2$

⇒ $BD^2 = 30^2 + 16^2$

⇒ $BD^2 = 900 + 256 = 1156$

⇒ BD = 34 cm

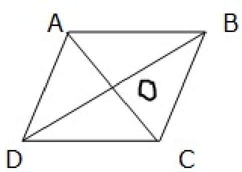
∵ Diagonals of a rectangle are equal

∴ AC = 34 cm

Question: 14

Find the length o

Solution:



ABCD is a rhombus where AC = 24 cm and BD = 10 cm.

We know that diagonals of a rhombus bisect each other at 90°.

⇒ ∠AOB = 90°, OA = 12 cm and OB = 5 cm

∴ ΔAOB is a right-angled triangle.

Applying Pythagoras theorem,

$BA^2 = BO^2 + AO^2$

⇒ $BA^2 = 5^2 + 12^2$

⇒ $BA^2 = 25 + 144 = 169$

⇒ BA = AD = CD = BC = 13 cm

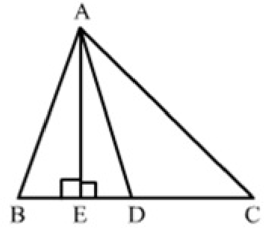
∵Sides of a rhombus are equal.

Question: 15

In ΔABC, D is the

Solution:

In right-angled triangle AED, applying Pythagoras theorem,



$AB^2 = AE^2 + BE^2$

$\Rightarrow AE^2 = AB^2 - BE^2$ ….(i)

In right-angled triangle AED, applying Pythagoras theorem,

$AD^2 = AE^2 + ED^2$

$\Rightarrow AE^2 = AD^2 - ED^2$ ….(ii)

Therefore,

$AB^2 - BE^2 = AD^2 - ED^2$

$$AB^2 = AD^2 - ED^2 + (\frac{1}{2}BC - DE)^2$$

$$\Rightarrow AB^2 = AD^2 - ED^2 + \frac{1}{4}BC^2 + DE^2 - BC \times DE$$

$$\Rightarrow AB^2 = AD^2 + \frac{1}{4}BC^2 - BC \times DE$$

Question: 16

In the given figu

Solution:

In ΔACB and ΔCDB,

∠ABC = ∠CBD (Common)

∠ACB = ∠CDB (90°)

So, by AA similarity criterion ΔACB ~ ΔCDB

Similarly, In ΔACB and ΔADC,

∠ABC = ∠ADC (Common)

∠ACB = ∠ADC (90°)

So, by AA similarity criterion ΔACB ~ ΔADC

We know that if two triangles are similar then the ratio of their corresponding sides is equal.

$$\Rightarrow \frac{BC}{BD} = \frac{AB}{BC} \text{ and } \frac{AC}{AD} = \frac{AB}{AC}$$

$\Rightarrow BC^2 = AB \times BD$ ….(i)

And $AC^2 = AB \times AD$ …..(ii)

Dividing (i) and (ii), we get

$$\frac{BC^2}{AC^2} = \frac{AB \times BD}{AB \times AD} = \frac{BD}{AD}$$

Hence, proved.

Question: 17

In the given figu

Solution:

(i) ΔAEC and ΔAED are right triangles.

Applying Pythagoras theorem we get,

$AC^2 = EC^2 + AE^2$

And $AD^2 = ED^2 + AE^2$

$\Rightarrow\ b^2 = h^2 + (\frac{a}{2} + x)^2$

$\Rightarrow\ b^2 = h^2 + (\frac{a}{2})^2 + x^2 + xa$ **....(i)**

And $p^2 = h^2 + x^2$ ....(ii)

Putting (ii) in (i),

$\Rightarrow\ b^2 = p^2 + (\frac{a}{2})^2 + xa$

$\Rightarrow\ b^2 = p^2 + \frac{a^2}{4} + xa$ **.....(iii)**

Hence, proved.

(ii) ΔAEB is a right triangle.

Applying Pythagoras theorem we get,

$AB^2 = EB^2 + AE^2$

$\Rightarrow\ c^2 = h^2 + (a - \frac{a}{2} - x)^2$

$\Rightarrow\ c^2 = h^2 + (\frac{a}{2} - x)^2$

$\Rightarrow\ c^2 = h^2 + (\frac{a}{2})^2 + x^2 - xa$ **....(iv)**

Putting (ii) in (iv),

$\Rightarrow\ c^2 = p^2 + (\frac{a}{2})^2 - xa$

$\Rightarrow\ c^2 = p^2 + \frac{a^2}{4} - xa$ **.....(v)**

Hence, proved.

(iii) Adding (iii) and (v),

$c^2 + b^2 = p^2 + \frac{a^2}{4} + xa + p^2 + \frac{a^2}{4} - xa$

$\Rightarrow\ c^2 + b^2 = 2p^2 + \frac{2a^2}{4}$

$\Rightarrow\ c^2 + b^2 = 2p^2 + \frac{a^2}{2}$

Hence, proved.

(iv) Subtracting (iii) and (v),

$b^2 - c^2 = p^2 + \frac{a^2}{4} + xa - p^2 - \frac{a^2}{4} + xa$

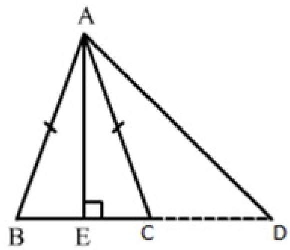
$\Rightarrow\ b^2 - c^2 = 2xa$

Hence, proved.

Question: 18

In ΔABC, AB = AC.

Solution:



Draw AE⊥BC. Applying Pythagoras theorem in right-angled triangle AED,

Since, ABC is an isosceles triangle and AE is the altitude and we know that the altitude is also the median of the isosceles triangle.

So, BE = CE

And DE + CE = DE + BE = BD

$AD^2 = AE^2 + ED^2$

$\Rightarrow AE^2 = AD^2 - ED^2$ ...(i)

In ΔACE,

$AC^2 = AE^2 + EC^2$

$\Rightarrow AE^2 = AC^2 - EC^2$ ...(ii)

Using (i) and (ii),

$\Rightarrow AD^2 - ED^2 = AC^2 - EC^2$

$\Rightarrow AD^2 - AC^2 = ED^2 - EC^2$

$\Rightarrow AD^2 - AC^2 = (DE + CE)(DE - CE)$

$\Rightarrow AD^2 - AC^2 = (DE + BE)\, CD$

$\Rightarrow AD^2 - AC^2 = BD.CD$

Question: 19

ABC is an isoscel

Solution:

ΔABC is right triangle.

Applying Pythagoras theorem we get,

$AC^2 = AB^2 + BC^2$ {∵ AB = BC}

$\Rightarrow AC^2 = 2AB^2$

Given that the two triangles ΔACD and ΔABE are similar.

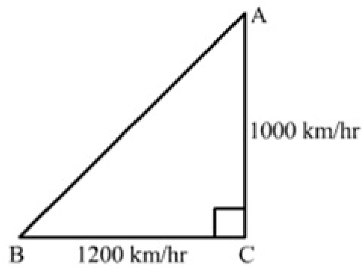
We know that if two triangles are similar then the ratio of their areas is equal to the ratio of the squares of their corresponding altitudes.

$$\Rightarrow \frac{ar(\Delta ACD)}{ar(\Delta ABE)} = \frac{AC^2}{AB^2} = \frac{AB^2}{2AB^2} = \frac{1}{2}$$

Question: 20

An aeroplane leav

Solution:



Let A be the first aeroplane flied due north at a speed of 1000 km/hr and B be the second aeroplane flied due west at a speed of 1200 km/hr

Distance covered by plane A in 1.5 hrs = 1000 × 32 = 1500km

Distance covered by plane B in 1.5 hrs = 1200 × 32 = 1800km

Now, in right triangle ABC

By using Pythagoras theorem, we have

$AB^2 = BC^2 + AC^2$

$\Rightarrow AB^2 = (1800)^2 + (1500)^2$

$\Rightarrow AB^2 = 3240000 + 2250000$

$\Rightarrow AB^2 = 5490000$

$\Rightarrow AB = 300\sqrt{61}$ km

Question: 21

In a ΔABC, AD is

Solution:

(a) In right triangle ALC

Using Pythagoras theorem, we have

$AC^2 = AL^2 + LC^2$

$\Rightarrow AC^2 = AD^2 - DL^2 + (DL + DC)^2$

$\Rightarrow AC^2 = AD^2 - DL^2 + (DL + \frac{BC}{2})^2$

$\Rightarrow AC^2 = AD^2 - DL^2 + DL^2 + \frac{BC^2}{4} + DL \times BC$

$\Rightarrow AC^2 = AD^2 + \frac{BC^2}{4} + DL \times BC$ ....(1)

(b) In right triangle ALD

Using Pythagoras theorem, we have

$AL^2 = AD^2 - LD^2$

Again, in ΔABL

Using Pythagoras theorem, we have

$AB^2 = AL^2 + LB^2$

$\Rightarrow AB^2 = AD^2 - DL^2 + LB^2$

$\Rightarrow AB^2 = AD^2 - DL^2 + (BD - DL)^2$

$\Rightarrow AB^2 = AD^2 - DL^2 + (\frac{BC}{2} - DL)^2$

$$\Rightarrow AB^2 = AD^2 - DL^2 + DL^2 + \frac{BC^2}{4} - DL \times BC$$

$$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4} - DL \times BC \text{ ......(2)}$$

(c) Adding (1) and (2)

$$AC^2 + AB^2 = AD^2 + \frac{BC^2}{4} - DL \times BC + AD^2 + \frac{BC^2}{4} + DL \times BC$$

$$\Rightarrow AC^2 + AB^2 = 2AD^2 + \frac{BC^2}{2}$$

Question: 22

Naman is doing fl

Solution:

Naman pulls in the string at the rate of 5 cm per second.

Hence, after 12 seconds the length of the string he will pull is given by

12 × 5 = 60 cm or 0.6 m

Now, in ΔBMC

By using Pythagoras theorem, we have

$BC^2 = CM^2 + MB^2$

$\Rightarrow BC^2 = (2.4)^2 + (1.8)^2 = 9$

$\therefore$ **BC = 3 m**

Now, BC′ = BC – 0.6 = 3 – 0.6 = 2.4 m

Now, in ΔBC′M

By using Pythagoras theorem, we have

$C'M^2 = BC'^2 - MB^2$

$\Rightarrow C'M^2 = (2.4)^2 - (1.8)^2 = 2.52$

$\therefore$ **C′M = 1.6 m**

The horizontal distance of the fly from him after 12 seconds is given by

C′A = C′M + MA = 1.6 + 1.2 = 2.8 m

Exercise : 4E

Question: 1

State the two pro

Solution:

Two triangles are similar, if

(i) their corresponding angles are equal and

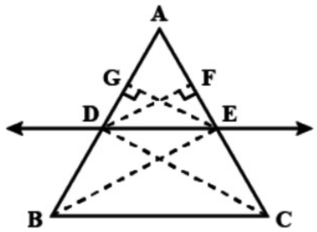
(ii) their corresponding sides are in the same ratio (or proportion).

Question: 2

State the basic p

Solution:

Basic Proportionality Theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.



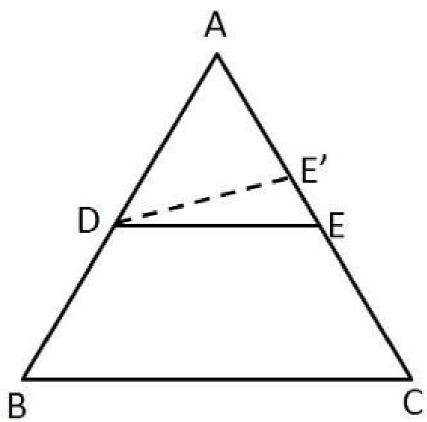
According to the theorem: $\frac{AD}{BD} = \frac{AE}{CE}$

Question: 3

State the convers

Solution:

Converse of Thales' Theorem: If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.



According to figure above,DE || BC.

Question: 4

State the midpoin

Solution:

Midpoint Theorem: The line joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Question: 5

State the AAA-sim

Solution:

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar. This criterion is referred to as the AAA (Angle–Angle–Angle) criterion of similarity of two triangles.

Question: 6

State the AA-simi

Solution:

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This is referred to as the AA-similarity criterion for two triangles.

Question: 7

State the SSS-cri

Solution:

If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar. This criterion is referred to as the SSS (Side–Side–Side)-similarity criterion for two triangles.

Question: 8

State the SAS-sim

Solution:

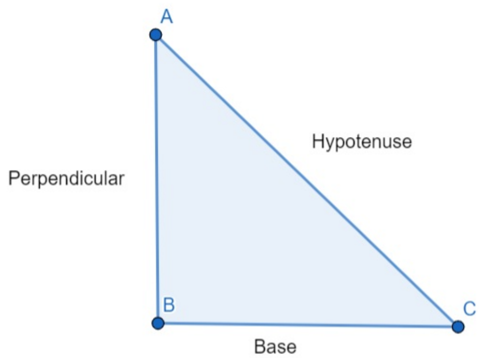
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. This criterion is referred to as the SAS (Side–Angle–Side) similarity criterion for two triangles.

Question: 9

State Pythagoras'

Solution:

In a right angled trianglPythagoras' Theorem: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



As shown in a right angled triangle ABC above,

(Hypotenuse)^2 = (Base)^2 + (Perpendicular)^2(AC)^2 = (BC)^2 + (AB)^2

Question: 10

State the convers

Solution:

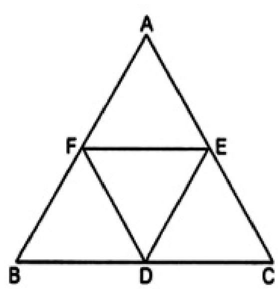
Converse of Pythagoras' Theorem: In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

Question: 11

If D, E and F are

Solution:

<u>The figure is shown below:</u>



<u>We know that the midpoint theorem states that the line joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.</u>

Since D, E and F are respectively the midpoints of sides AB, BC and CA of ΔABC,

DE = AB/2; EF = BC/2; DF = AC/2

⇒ DE/AB = 1/2; EF/BC = 1/2; DF/AC = 1/2

⇒ DE/AB = EF/BC = DF/AC = 1/2

<u>We know that if in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar (SSS criteria).</u>

So Δ ABC ~ Δ DEF.

<u>We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.</u>

∴ ar(Δ ABC)/ar(Δ DEF) = (AB/DE)^2

⇒ ar(ΔABC)/ar(ΔDEF) = (2DE/DE)^2

⇒ ar(ΔABC)/ar(ΔDEF) = (2/1)^2

⇒ ar(ΔABC)/ar(ΔDEF) = (4/1)

But we need to find the ratio of the areas of ΔDEF and ΔABC.

∴ ar(ΔDEF)/ar(ΔABC) = (1/4)

∴ ar(ΔABC):ar(ΔDEF) = 1:4

<u>1: 4</u>

Question: 12

Two triangles ABC

Solution:

<u>We know that if one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar (SAS criteria).</u>

Here in the given triangles, ∠A = ∠P = 70° .

And AB/PQ = AC/PR

i.e. 6/4.5 = 6/9

⇒ 2/3 = 2/3

Hence ΔABC ~ ΔPQR.

<u>SAS-similarity</u>

Question: 13

If ΔABC ~ΔDEF suc

Solution:

Given: △ABC ~△DEF such that 2AB = DE and BC = 6 cm.

<u>From SSS-similarity criterion,</u>

<u>We get</u>

<u>AB/DE = BC/EF</u>

Substituting the given values,

AB/2AB = 6cm/EF

1/2 = 6cm/EF

EF = 2 × 6cm

EF = 12cm

<u>12cm</u>

Question: 14

In the given figu

Solution:

<u>We know that the basic proportionality theorem states that</u>

“If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.”

So if DE || BC,

Then AD/DB = AE/EC

By substituting the given values,

⇒ x cm/(3x + 4)cm = (x + 3)cm/(3x + 19)cm

Cross multiplying, we get

⇒ $3x^2 + 19x = 3x^2 + 9x + 4x + 12$

⇒ $3x^2 + 19x - 3x^2 - 9x - 4x = 12$

⇒ 6x = 12

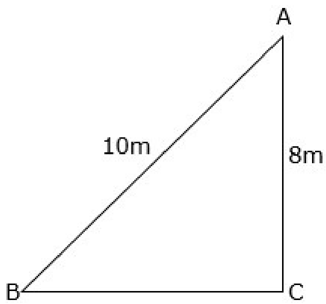
⇒ x = 2

x = 2

Question: 15

A ladder 10 m lon

Solution:



Let AB be the ladder and CA be the wall with the window at A.

Let the distance of foot of ladder from base of wall BC be x.

Also, AB = 10m and CA = 8m

From Pythagoras Theorem,

we have: $AB^2 = BC^2 + CA^2$

⇒ $(10)^2 = x^2 + 8^2$

⇒ $x^2 = 100 – 64$

⇒ $x^2 = 36$

⇒ x = 6m

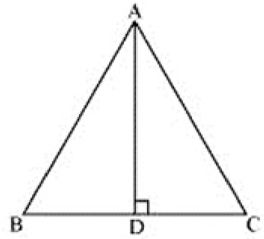
So, BC = 6m.

Length of the ladder is 6m.

Question: 16

Find the length o

Solution:



Let △ABC be the equilateral triangle whose side is 2a cm.

Let us draw altitude AD such that AD ⊥ BC.

We know that altitude bisects the opposite side.

So, BD = DC = a cm.

In △ADC, ∠ADC = 90°.

We know that the Pythagoras Theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

So, by applying Pythagoras Theorem,

$AC^2 = AD^2 + DC^2$

$(2a\ cm)^2 = AD^2 + (a\ cm)^2$

$4a^2\ cm^2 = AD^2 + a^2\ cm^2$

$AD^2 = 3a^2\ cm^2$

$AD = \sqrt{3}\ a\ cm$

The length of altitude is √3 a cm.

Question: 17

ΔABC ~ ΔDEF

Solution:

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

i.e. $ar(\triangle ABC)/ar(\triangle DEF) = (BC/EF)^2$

Substituting the given values, we get

$\Rightarrow 64cm^2/169cm^2 = (4cm/EF\ cm)^2$

$\Rightarrow 64/169 = 16/EF^2$

$\Rightarrow EF^2 = 42.25$

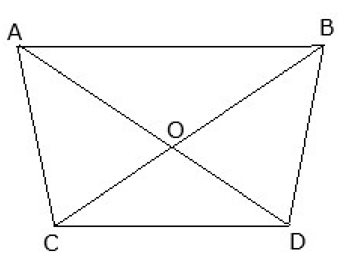
$\Rightarrow EF = 6.5cm$

6.5 cm

Question: 18

In a trapezium AB

Solution:



Let us consider $\triangle$AOB and $\triangle$COD.

∠AOB = ∠COD (∵ vertically opposite angles)

∠OBA = ∠ODC (∵ alternate interior angles)

∠OAB = ∠OCD (∵ alternate interior angles)

We know that if in two triangles, corresponding angles are equal,

then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar (AAA criteria).

So, ΔAOB ≅ ΔCOD.

Given, AB = 2CD and ar(ΔAOB) = 84 cm^2

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

∴ ar($\triangle$AOB)/ar($\triangle$COD) = $(AB/CD)^2$

⇒ 84cm^2/ar(ΔCOD) = $(2CD/CD)^2$

⇒ 84cm^2/ar(ΔCOD) = 4

⇒ ar(ΔCOD) = 84cm^2/4

⇒ ar(ΔCOD) = 21cm^2

ar(ΔCOD) = 21cm^2

Question: 19

The corresponding

Solution:

Let the smaller triangle be $\triangle$ABC and larger triangle be $\triangle$DEF.

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

i.e. ar($\triangle$ABC)/ar($\triangle$DEF) = $(AB/DE)^2$

Substituting the given values, we get

⇒ 48cm^2/ ar($\triangle$DEF) = $(2/3)^2$

⇒ 48cm^2/ ar($\triangle$DEF) = 4/9

⇒ ar($\triangle$DEF)= (48 × 9)/4 cm^2

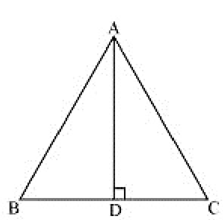
⇒ ar($\triangle$DEF) = 108cm^2

108cm^2

Question: 20

In an equilateral

Solution:



Let $\triangle$ABC be the equilateral triangle whose side is a cm.

Let us draw altitude AD(h) such that AD ⊥ BC.

We know that altitude bisects the opposite side.

So, BD = DC = a cm.

In △ ADC, ∠ADC = 90°.

We know that the Pythagoras Theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

So, by applying Pythagoras Theorem,

$AC^2 = AD^2 + DC^2$

$(a\ cm)^2 = AD^2 + (a/2\ cm)^2$

$a^2\ cm^2 = AD^2 + a^2/4\ cm^2$

$AD^2 = 3a^2/4\ cm^2$

$AD = \sqrt{3}\ a/2\ cm = h$

We know that area of a triangle = 1/2 × base × height

$Ar(\Delta ABC) = 1/2 \times a\ cm \times \sqrt{3}\ a/2\ cm$

$\Rightarrow ar(\Delta ABC) = \sqrt{3}\ a^2/4\ cm^2$

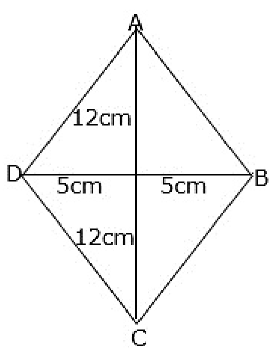
Hence proved.

$ar(\Delta ABC) = \sqrt{3}\ a^2/4\ cm^2$

Question: 21

Find the length o

Solution:



The diagonals of a rhombus bisect each other at right angles.

Let the intersecting point be O.

So, we get right angled triangles.

We know that the Pythagoras Theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Let us consider ΔAOB.

By Pythagoras Theorem,

$AB^2 = AO^2 + OB^2$

$AB^2 = 12^2 + 5^2$

$AB^2 = 144 + 25$

$AB^2 = 169$

$AB = 13cm$

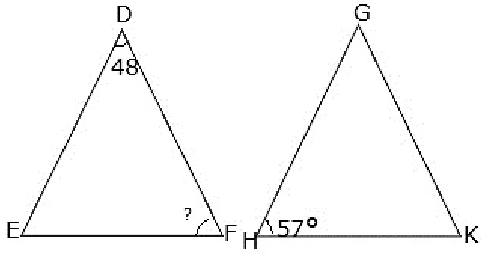
The length of side of the rhombus is 13cm.

Question: 22

Two triangles DEF

Solution:

Given that ΔDEF ≅ ΔGHK.



We know that if in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar (AAA criteria).

$\therefore$ ∠D = 48° = ∠G

∠H = 57° = ∠E

∠F = ∠K = x°

We know that the sum of angles in a triangle = 180°.

So, in ΔDEF,

⇒ 48° + 57° + x° = 180°

⇒ 105° + x° = 180°

⇒ x° = 180° - 105°

⇒ x° = 75° = ∠F

Ans. ∠F = 75º

Question: 23

In the given figu

Solution:

We have MN || BC,

So, ∠AMN = ∠B and ∠ANM = ∠C (Corresponding angles)

We know that if two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar (AA criteria).

$\therefore$ ΔAMN ~ ΔABC.

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

i.e. ar(ΔAMN)/ ar(ΔABC) = $(AM/AB)^2$

Given that AM: MB = 1: 2.

Since AB = AM + MB,

AB = 1 + 2 = 3.

⇒ ar(ΔAMN)/ ar(ΔABC) = $(1/3)^2$

⇒ ar(ΔAMN)/ ar(ΔABC) = 1/9

area(ΔAMN)/ area(ΔABC) = 1/9

Question: 24

In triangles BMP

Solution:

Given: PB = 5 cm,

MP = 6 cm,

BM = 9 cm and,

NR = 9 cm

Now, it is also given that: ΔBMP ~ ΔCNR

When two triangles are similar, then the ratios of the lengths of their corresponding sides are proportional.

$\Rightarrow \frac{BM}{CN} = \frac{BP}{CR} = \frac{MP}{NR}$**...(i)**

$\Rightarrow \frac{BM}{CN} = \frac{MP}{NR}$

$\Rightarrow CN = \frac{BM \times NR}{MP}$

$\Rightarrow CN = \frac{9\text{ cm} \times 9\text{ cm}}{6\text{ cm}}$

⇒ CN = 54/6 = 13.5 cm.

Similarly,

$\Rightarrow \frac{BM}{CN} = \frac{BP}{CR}$

$\Rightarrow CR = \frac{BP \times CN}{BM}$

$\Rightarrow CR = \frac{5\text{ cm} \times 13.5\text{ cm}}{9\text{ cm}}$

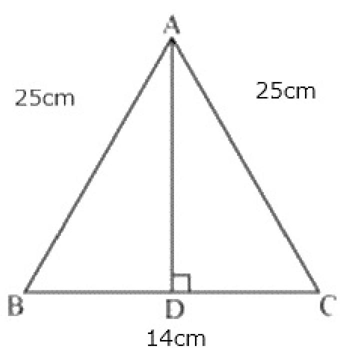
⇒ CR = 7.5 cm

∴ Perimeter of ΔCNR = CN + NR+ CR = 13.5+9+7.5=30 cm

Question: 25

Each of the equal

Solution:



Let △ABC be the isosceles triangle whose sides are AB = AC = 25cm, BC = 14cm.

Let us draw altitude AD such that AD ⊥ BC.

We know that altitude bisects the opposite side.

So, BD = DC = 7cm.

In △ADC, ∠ADC = 90°.

We know that the Pythagoras Theorem states that in a right triangle, the square of the

hypotenuse is equal to the sum of the squares of the other two sides.

So, by applying Pythagoras Theorem,

$AC^2 = AD^2 + DC^2$

$(25\text{ cm})^2 = AD^2 + (7\text{ cm})^2$

$625\text{ cm}^2 = AD^2 + 49\text{ cm}^2$

$AD^2 = 576\text{ cm}^2$

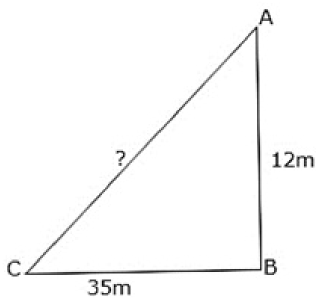
AD = 24 cm

The length of altitude is 24 cm.

Question: 26

A man goes 12 m d

Solution:



From ΔABC, we note that

A is the starting point.

AB = 12m, BC = 35m

CA = distance from starting point = x m

We know that the Pythagoras Theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

By Pythagoras Theorem,

$CA^2 = AB^2 + BC^2$

$CA^2 = 12^2 + 35^2$

$CA^2 = 144 + 1225$

$CA^2 = 1369$

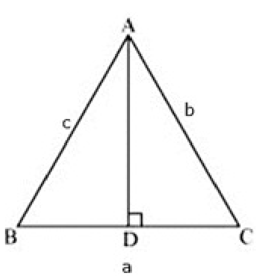
CA = 37m

The man is 37 m far from the starting point.

Question: 27

If the lengths of

Solution:



Given that ΔABC is the triangle whose sides are AB = c, AC = b, BC = a

And AD is the bisector of ∠A.

We know that altitude bisects the opposite side.

So, let BD = DC = x.

Since AD bisects ∠A,

AC/AB = CD/DB

Substituting the given values,

b/c = CD/(a-CD)

Cross multiplying,

⇒ b(a – CD) = c (CD)

⇒ ba – b(CD) = c (CD)

⇒ ba = CD (b + c)

⇒ CD = ba/ (b + c)

Since CD = BD,

BD = ba/ (b + c)

BD = ba/(b + c) and DC = ba/(b + c)

Question: 28

In the given figu

Solution:

In ΔAMN and ΔABC

∠AMN = ∠ABC = 76° (Given)

∠A = ∠A (common)

By AA Similarity criterion, ΔAMN ~ Δ ABC

If two triangles are similar, then the ratio or the their corresponding sides are proportional

$$\therefore \frac{AM}{AB} = \frac{MN}{BC}$$

$$\Rightarrow \frac{AM}{AM + MB} = \frac{MN}{BC}$$

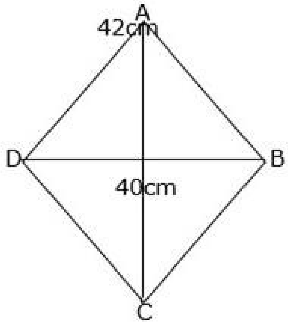
$$\Rightarrow \frac{a}{a + b} = \frac{MN}{c}$$

$$\Rightarrow MN = \frac{ac}{a + b}$$

Question: 29

The lengths of th

Solution:



The diagonals of a rhombus bisect each other at right angles.

Let the intersecting point be O.

So, we get right angled triangles.

We know that the Pythagoras Theorem states that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Let us consider ΔAOB.

By Pythagoras Theorem,

$AB^2 = AO^2 + OB^2$

$AB^2 = 21^2 + 20^2$

$AB^2 = 441 + 400$

$AB^2 = 841$

AB = 29cm

The length of each side of the rhombus is 29cm.

Question: 30

For each of the f

Solution:

(i) T

Two similar figures have the same shape but not necessarily the same size. Therefore, all circles are similar.

(ii) F

Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (or proportion).

Consider an example,

Let a rectangle have sides 2cm and 3cm and another rectangle have sides 2cm and 5cm.

Here, the corresponding angles are equal but the corresponding sides are not in the same ratio.

(iii) F

Two triangles are similar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (or proportion).

(iv) T

Midpoint Theorem states that the line joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

(v) F

Two triangles are similar, if

(i) their corresponding angles are equal and

(ii) their corresponding sides are in the same ratio (or proportion).

But here, the corresponding sides are

AB/DE = 6/12 = 1/2 and AC/DF = 8/9

AB/DE ≠ AC/DF

(vi) F

The polygon formed by joining the midpoints of sides of any quadrilateral is a parallelogram.

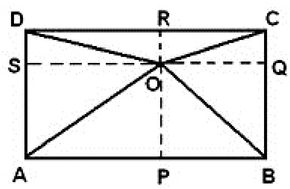
(vii) T

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

(viii) T

The perimeters of the two triangles are in the same ratio as the sides. The corresponding medians also will be in this same ratio.

(ix) T



Let us construct perpendiculars OP, OQ, OR and OS from point O.

Let us take LHS = $OA^2 + OC^2$

From Pythagoras theorem,

= $(AS^2 + OS^2) + (OQ^2 + QC^2)$

As also AS = BQ, QC = DS and OQ = BP = OS,

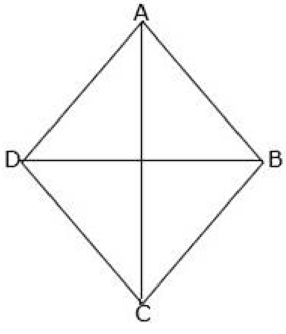
= $(BQ^2 + OQ^2) + (OS^2 + DC^2)$

Again by Pythagoras theorem,

= $OB^2 + OD^2$ = RHS

As LHS = RHS, hence proved.

(x) T



In rhombus ABCD, AB = BC = CD = DA.We know that diagonals of a rhombus bisect each other perpendicularly.i.e. AC ⊥ BD, ∠AOB = ∠BOC = ∠COD = ∠AOD = 90° andOA = OC = AC/2, OB = OD = BD/2Let us consider right angled triangle AOB.

By Pythagoras theorem,$AB^2 = OA^2 + OB^2$

⇒ $AB^2 = (AC/2)^2 + (BD/2)^2$

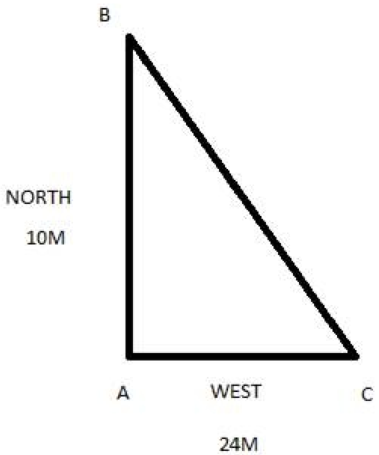
⇒ $AB^2 = AC^2/4 + BD^2/4$ ⇒ $4AB^2 = AC^2 + BD^2$ ⇒ $AB^2 + AB^2 + AB^2 + AB^2 = AC^2 + BD^2$ ∴ $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

A man goes 24 m d

Solution:



Since the man goes C to A = 24 m west and then A to B = 10 m north, he is forming a right angle triangle with respect to starting point C.

His distance from the starting point can be calculated by using Pythagoras theorem

$(AC)^2 = (AB)^2 + (BC)^2$

⇒ $(AC)^2 = (24)^2 + (10)^2$

⇒ $(AC)^2 = 576 + 100$

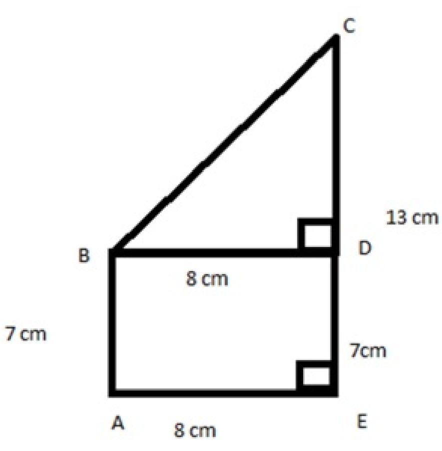
⇒ $(AC)^2 = 676$

⇒ AC = 26

Question: 2

Two poles of heig

Solution:



Let AB and CE be the two poles of the height 13 cm and 7 cm each which are perpendicular to the ground. The distance between them is 8 cm.

Now since CE and AB are ⊥ ground AE

BD ⊥ to CE and BD = 8 cm

Top of pole AB is B and top of pole CE is C

Now Δ BDC is right angled at D and BC, the hypotenuse is the distance between the top of the poles and CD = 13 – 7 = 6

$(BC)^2 = (BD)^2 + (CD)^2$

$\Rightarrow (BC)^2 = 64 + 36$

$\Rightarrow (BC)^2 = 100$

$\Rightarrow (BC) = 10$ **cm**

The distance between the top of the poles is 10 cm

Question: 3

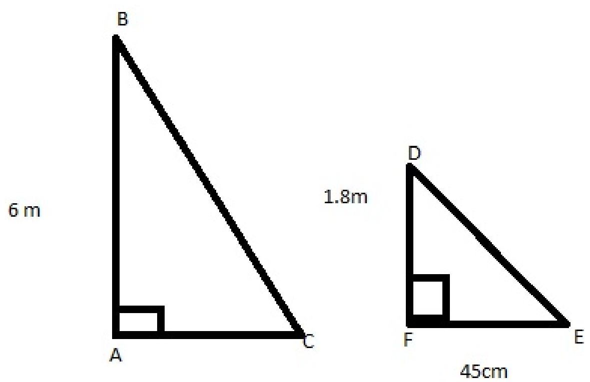
A vertical stick

Solution:

Let DF be the stick of 1.8 m height and AB be the pole of 6 m height.

AC and FE are the shadows of the pole and stick respectively.

FE = 45cm = .45 m



Since the shadows are formed at the same time, the two Δs are similar by AA similarity criterion

So $\frac{AB}{DF} = \frac{AC}{FE} = \frac{BC}{DE}$

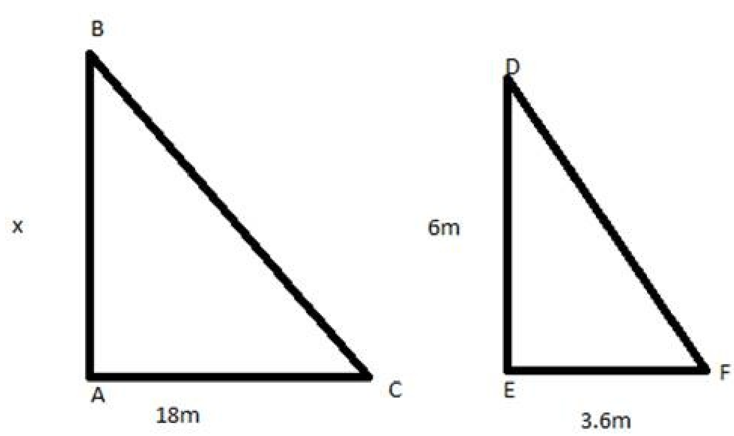
$\Rightarrow \frac{6}{1.8} = \frac{x}{45}$

$\Rightarrow x = 1.5$ **m**

Question: 4

A vertical pole 6

Solution:



Let DE be the pole of 6 m length casting shadow of 3.6 m . Let AB be the tower x meter height casting shadow of 18mat the same time.

Since pole and tower stands vertical to the ground, they form right angled triangle with ground.

Δ ABC and Δ EDF are similar by AA similarity criterion

∴ x/6 = 18 /3.6

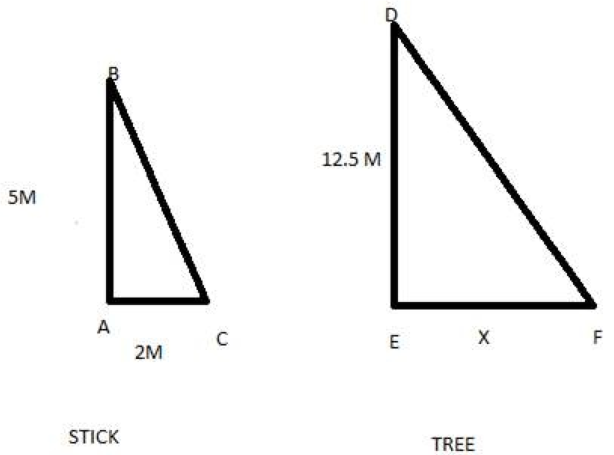
⇒ x = 30

The height of the tower is 30 m

Question: 5

The shadow of a 5

Solution:



SINCE BOTH the tree and the stick are forming shadows at the same time the sides of the triangles so formed, would be in same ration ∵ of AA similarity criterian

12.5 /5 = x / 2

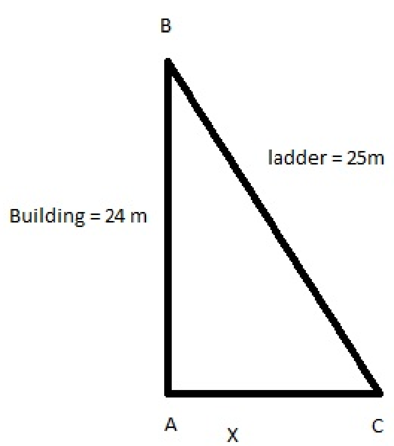
⇒ x = 5

Shadow of the tree would be 5 m long.

Question: 6

A ladder 25 m lon

Solution:



Let BC be the ladder placed against the wall AB. The distance of the ladder from the wall is the base of the right angled triangle as building stands vertically straight to the ground.

By Pythagoras theorem

$(BC)^2 = (AB)^2 + (AC)^2$

$(25)^2 = (24)^2 + (x)^2$

$x = 7$

the distance of ladder from the wall is 7m

Question: 7

In the given figu

Solution:

The ΔMOP is right angled at O so MP is hypotenuse

$(MP)^2 = (OM)^2 + (OP)^2$

$(MP)^2 = (16)^2 + (12)^2$

$(MP)^2 = 400$

MP = 20 cm

Δ NMP is right angled at M so NP is the hypotenuse so

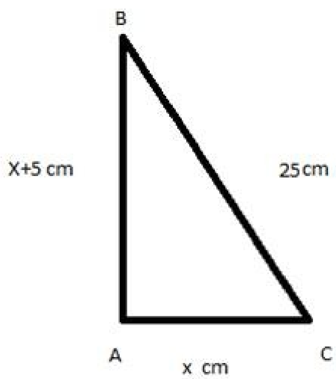
$(NP)^2 = (21)^2 + (20)^2$

NP = 29

Question: 8

The hypotenuse of

Solution:



Given (BC) = 25 cm

By Pythagoras theorem

$(BC)^2 = (AB)^2 + (AC)^2$

$(25)^2 = (x + 5)^2 + x^2$

$625 = x^2 + 25 + 10x + x^2 \because (a + b)^2 = a^2 + b^2 + 2ab$

$x^2 + 5x - 300 = 0$

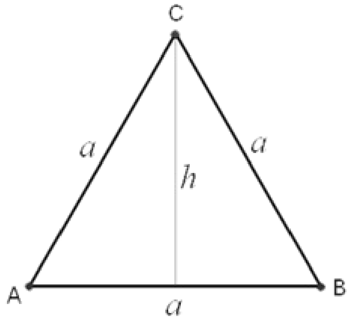
$x (x + 15) - 10(x + 15) = 0$

Since x = -15 is not possible so side of the triangle is 15 cm and 20 cm

Question: 9

The height of an

Solution:



Since Δ ABC is an equilateral triangle so the altitude (Height = h) from the C is the median for AB dividing AB into two equal halves of 6 cm each

Now there are two right angled Δs

$h^2 = a^2$ - 1/2 (AB) ^{2}

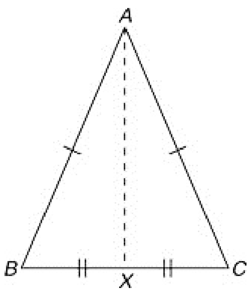
$h^2 = (12)^2 - 6^2$

h = 6 √3

Question: 10

ΔABC is an isosce

Solution:



The given triangle is isosceles so the altitude from the one of the vertex is median for the side opposite to it.

AB = AC = 13 cm

h = 5 cm (altitude)

Δ ABX is a right angled triangle, right angled at X

$(AB)^2 = h^2 + (BX)^2$ (BX = 1/2 BC)

169 = 25 + (BX) ^{2}

BX = 12

⇒ BC = 24

Question: 11

In a ΔABC it is g

Solution:

By internal angle bisector theorem, the bisector of vertical angle of a triangle divides the base in the ratio of the other two sides.

Hence in ΔABC, we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

Here AB = 6 cm, AC = 8 cm

So $\frac{AB}{AC} = \frac{6}{8} = \frac{3}{4} = \frac{BD}{DC}$

Question: 12

In a ΔABC i

Solution:

By internal angle bisector theorem, the bisector of vertical angle of a triangle divides the base in the ratio of the other two sides"Hence in ΔABC, we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{6}{x} = \frac{4}{5}$$

⇒ x = 7.5cm

Question: 13

In a ΔABC, it is

Solution:

By internal angle bisector theorem, the bisector of vertical angle of a triangle divides the base in the ratio of the other two sides"Hence in Δ ABC

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{10}{14} = \frac{(6-x)}{x}$$

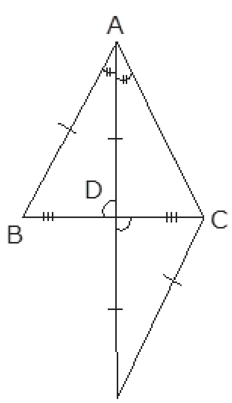
⇒ 10x -84 + 14x = 0

⇒ x = CD = 3.5 cm

Question: 14

In a triangle, th

Solution:



In Δ ABC, AD bisects ∠ A and meets BC in D such that BD = DC

Extend AD to E and join C to E such that CE is ∥ to AB

∠ BAD = ∠ CAD

Now AB ∥ CE and AE is transversal

∠ BAD = ∠ CED (alternate interior ∠s)

But ∠ BAD = ∠ CED = ∠CAD

In Δ AEC

∠ CEA = ∠CAE

∴ AC = CE……………… 1

In Δ ABD and Δ DCE

∠BAD = ∠CED (alternate interior ∠s)

∠ADB = ∠ CDE (vertically opposite ∠s)

BD = BC (given)

Δ ABD ≅ Δ DCE

AB = EC (CPCT)

AC = EC (from 1)

⇒AB = AC

⇒ ABC is an isosceles Δ with AB = AC

Question: 15

In an equilateral

Solution:

Δ ABC is an equilateral triangle

By Pythagoras theorem in triangle ABD

$AB^2 = AD^2 + BD^2$

but BD = 1/2 BC (∵ In a triangle, the perpendicular from the vertex to the base bisects the base)

thus $AB^2 = AD^2 + \{1/2\ BC\}^2$

$AB^2 = AD^2 + 1/4\ BC^2$

$4\ AB^2 = 4AD^2 + BC^2$

$4\ AB^2 - BC^2 = 4\ AD^2$

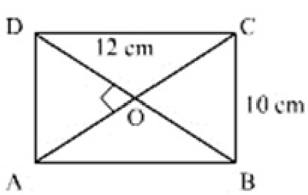
(as AB = BC we can subtract them)

Thus $3AB^2 = 4AD^2$

Question: 16

In a rhombus of s

Solution:



Since the diagonals of the rhombus bisects each other at 90°

∴ DO = OB = 6 cm

∠ AOD = ∠ DOC = ∠ COB = ∠BOA = 90°

Δ AOD is right angled Δ with

AD = 10 cm (given)

OD = 6 cm

∠ AOD = 90°

So DA = 10 = hypotenuse

$(DA)^2 = (DO)^2 + (AO)^2$

100 – 36 = $(AO)^2$

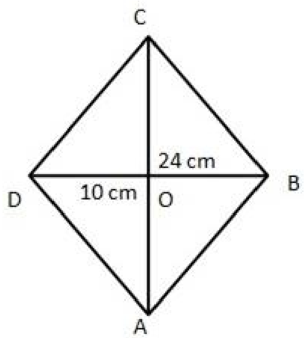
8 = AO

∴ AC = 16

Question: 17

The lengths of th

Solution:



In a rhombus the diagonals bisect each other at 90°

AC = 24cm (given)

∴ AD = 12cm

BD = 10cm (given)

∴ BO = 5cm

In right angled Δ AOB

$AB^2 = AO^2 + OB^2$

$AB^2 = (12)^2 + 5^2$

$AB^2 = 144 + 25$

$AB^2 = 169$

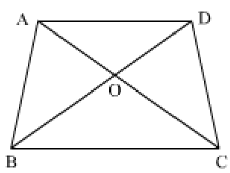
AB = 13cm

Hence the length of the sides of the rhombus is 13 cm

Question: 18

If the diagonals

Solution:



Given that ABCD is a quadrilateral and diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{OB}{OD}$

IN Δ AOD and ΔBOC

$$\frac{AO}{OC} = \frac{BO}{OD}$$

∠ AOD = ∠COB

Thus Δ AOC ~ ΔBOC (SAS similarity criterion)

⇒ ∠OAD = ∠ OCB ……………..1

Now transversal AC intersect AD and BC such the ∠CAD = ∠ACB

(from …..1) (alternate opposite angles)

So AD ∥ BC

Hence ABCD is a trapezium

Question: 19

In the given figu

Solution:

ABCD is a Trapezium with AC and BD as diagonals and AB ∥ DC

In Δ AOB and Δ DOC

∠AOB = ∠DOC (vertically opposite angles)

∠CDO = ∠ OBA (alternate interior angles) (AB ∥ DC and BD is transversal)

Δ AOB ~ Δ DOC (AA similarity criterion)

$$\frac{AO}{OC} = \frac{OB}{OD}$$

$$\frac{3x - 1}{5x - 3} = \frac{2x + 1}{6x - 5}$$

$$(3x - 1)(6x - 5) = (5x - 3)(2x + 1)$$

$$18x^2 - 21x + 5 = 10x^2 - x - 3$$

$$8x^2 - 20x + 8 = 0$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$2x(x - 2) - 1(x - 2) = 0$$

$$(2x - 1)(x - 2) = 0$$

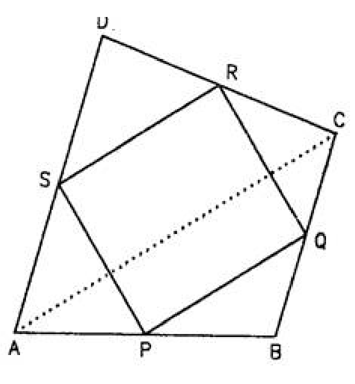
$$x = 1/2, 2$$

x = 1/2 is not possible so x = 2 cm

Question: 20

The line segments

Solution:



In the given quadrilateral ABCD

P, Q, R, S are the midpoints of the sides AB, BC, CD and AD respectively.

Construction: - Join AC

In Δ ABC and Δ ADC

P and Q are midpoints of AB and CB

S and R are midpoints of AD and DC

So by Mid Point Theorem

PQ ∥ AC and PQ = 1/2 AC………….1

And SR ∥ AC and SR = 1/2 AC……………..2

From 1 and 2

PQ ∥ SR and PQ = SR

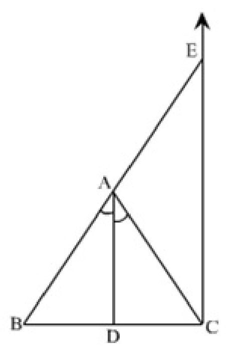
Since a pair of opposite side is equal (=) and parallel (∥)

PQRS is a parallelogram

Question: 21

If the bisector o

Solution:



Given in Δ ABC, AD bisects the ∠A meeting BC at D

BD = DC and ∠BAD = ∠ CAD………… 1

Construction:- Extend BA to E and join C to E such CE ∥ AD……… 4

∠BAD = ∠AEC (corresponding ∠s)…………….. 2

∠CAD = ∠ ACE (alternate interior ∠s)………………. 3

From 1 , 2 and 3

∠ ACE = ∠AEC

In Δ AEC

∠ ACE = ∠AEC

∴ AC = AE (sides opposite to equal angles are equal)……….. 5

In Δ BEC

AD ∥ CE (From ….4)

And D is midpoint of BC (given)

By converse of midpoint theorem

A line drawn from the midpoint of a side, parallel to the opposite side of the triangle meets the third side in its middle and is half of it

∴ A is midpoint of BE

BA = AE……… 6

From 5 and 6

AB = BC

⇒ ΔABC is an isosceles triangle

Question: 22

In ΔABC it is giv

Solution:

It is given that in Δ ABC,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

∠B = 70° and ∠C = 50°

∠A = 180° – (70° + 50°) (∠ sum property of triangle)

= 180° - 120°

= 60°

Since, $\frac{AB}{AC} = \frac{BD}{DC}$

∴ AD is the bisector of ∠A

Hence, ∠BAD = 60°/2 = 30°

Question: 23

In ΔABC, DE || BC

Solution:

By Basic *Proportionality Theorem*

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. ..

In Δ ABC, DE ∥ BC

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{2.4}{DB} = \frac{3.2}{4.8}$$

DB = 3.6 cm

AB = AD + DB

AB = 6 cm

Question: 24

In a ΔABC, if DE

Solution:

BY Basic Proportionality Theorem:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

$$\frac{AD}{BD} = \frac{AE}{EC}$$

Or $\frac{BD}{AD} = \frac{EC}{AE}$

Adding 1 to both sides

$$\frac{BD}{AD} + 1 = \frac{EC}{AE} + 1$$

$\frac{BD + AD}{AD} = \frac{EC + AE}{AE}$

$\frac{AB}{AD} = \frac{AC}{AE}$

$7.2/\ 4.5 = 6.4\ /\ AE$

AE = 4 cm

Question: 25

In ΔABC, DE || BC

Solution:

By Basic Proportionality Theorem:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$(7x - 4)/\ (3x + 4) = (5x - 2)/\ 3x$

$3x\ (7x - 4) = (5x - 2)\ (3x + 4)$

$21\ x^2 - 12x = 15x^2 + 14x - 8$

$6\ x^2 - 26x + 8 = 0$

$3\ x^2 - 13x + 4 = 0$

$3\ x^2 - 12x - x + 4 = 0$

$3x(x - 4) - 1(x - 4) = 0$

X = 1/3, 4

Since x cannot be 1/3 so x = 4

Question: 26

In ΔABC, DE || BC

Solution:

BY Basic Proportionality Theorem:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

$$\frac{AD}{BD} = \frac{AE}{EC}$$

$\frac{3}{5} = \frac{AE}{AC - AE}$

$\frac{3}{5} = \frac{AE}{5.6 - AE}$

$3\ (5.6 - AE) = 5\ AE$

16.8 = 8AE

AE = 2.1 cm

Question: 27

ΔABC ~ ΔDEF and t

Solution:

Since the $\triangle$ ABC ~ $\triangle$ DEF

Their sides will be same ratios. Let the ratio be K

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ **= K............1**

AB + BC + AC = K (DE + EF + DF)

$\frac{30}{18}$ **= K**

1.67 = k

From...... 1

$\frac{BC}{EF}$ **= 1.67**

EF = 5.4 cm

Question: 28

ΔABC ~ ΔDEF such

Solution:

Since the Δ ABC ~ Δ DEF

So the sides of the triangles are in the same ratio be k

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ **= K............1**

$\frac{AB}{DE} = \frac{9.1}{6.5}$ **= K**

K = 1.4

$\frac{\text{Perimeter of } \Delta \text{ABC}}{\text{Perimeter of } \Delta \text{DEF}}$ **= K**

$\frac{\text{Perimeter of } \Delta \text{ABC}}{25}$ **= 1.4**

Perimeter of Δ ABC **= 1.4 × 25**

Perimeter of Δ ABC **= 35 cm**

Question: 29

In ΔABC, it is gi

Solution:

Perimeter of Δ ABC = AB + BC + CA

= 9 + 6 + 7.5

= 22.5 cm

Since the Δ ABC ~ Δ DEF

$\frac{\text{Perimeter of } \Delta \text{ABC}}{\text{Perimeter of } \Delta \text{DEF}} = \frac{BC}{EF}$ **(ratio of perimeter of triangles is equal to the ratio of the sides of the triangle)**

$\frac{22.5}{\text{Perimeter of } \Delta \text{DEF}} = \frac{6}{8}$

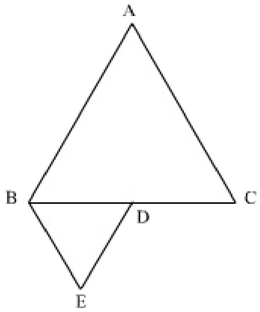
Perimeter of Δ DEF **=** $\frac{22.5 \times 8}{6}$

Perimeter of Δ DEF **= 30 cm**

Question: 30

ABC and BDE are t

Solution:



Δ ABC and Δ BDE are two equilateral triangles

Let a be the side of Δ ABC

Since D is midpoint of BC

So the side of equilateral ΔBDE = $\frac{a}{2}$

Area of equilateral Δ = $\frac{\sqrt{3}}{4}$ **(side)2**

Area of Δ ABC = $\frac{\sqrt{3}}{4}$ a^2 **............1**

Area of Δ BDE = $\frac{\sqrt{3}}{4}$ $(\frac{a}{2})^2$

= $\frac{\sqrt{3}}{4}$ $\frac{a^2}{4}$

Putting value of $\frac{\sqrt{3}}{4} \times a^2$ **...........from 1**

Area of Δ BDE = $\frac{1}{4}$ **Area of Δ ABC**

$\frac{\text{Area of } \Delta \text{ ABC}}{\text{Area of } \Delta \text{ BDE}} = \frac{4}{1}$

Question: 31

It is given that

Solution:

In ΔABC,∠ A + ∠B + ∠ C = 180°

30° + ∠B + 50° = 180°

∠ B = 100° Given that Δ ABC~ΔDEF∠D = ∠A = 30°

∠ E = ∠ B = 100° ∠ F = ∠ C = 50°Also, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}\frac{AB}{DE} = \frac{AC}{DF}\frac{5}{DE} = \frac{8}{7.5}$**DE = 4.6875And as neither BC nor EF is given we can not find either of them. So, none of the given options is correct.Now, Δ ABC~ΔDFE,Then,∠D = ∠ A = 30° ∠ F = ∠ B = 100° ∠ E = ∠ C = 50°Also,** $\frac{AB}{DF} = \frac{BC}{FE} = \frac{AC}{DE}\frac{AB}{DF} = \frac{AC}{DE}\frac{5}{7.5} = \frac{8}{DE}$**DE = 12cmTherefore, the correct option is (b).**

Question: 32

In the given figu

Solution:

∠ABD + ∠ BAD = 90°

∠ ABD + (90-∠CAD) = 90° ∠ ABD = ∠ DACIn ΔBDA and Δ ADC,∠ ABD = ∠ CAD∠ BDA = ∠ ADC = 90°Therefore, ΔBDA and Δ ADC are similar by AAA.$\frac{BD}{AD} = \frac{AD}{CD}$**BD.CD = AD**^{2}**Therefore the correct option is (c).**

Question: 33

In ΔABC, AB = 6√3

Solution:

AB = 6√3cm.In ΔABC,$AB^2 + BC^2 = AC^2$ $\left(6\sqrt{3}\right)^2 + (6)^2 = 12^2$

Since the square of the longest side is equal to the sum of the squares of the remaining two sides of Δ ABC. Therefore ABC is right angled at B.

∠ B = 90°

Question: 34

In ΔABC and Δ DEF

Solution:

$\frac{AB}{DE} = \frac{BC}{FD} = \frac{AC}{EF}$**With the ratio given, we can observe that Δ ABC ~ Δ EDF,∠ A = ∠E,∠ B = ∠D,∠ C = ∠ F**

Question: 35

In ΔDEF and ΔPQR,

Solution:

Given ∠ D = ∠ Q and ∠E = ∠RBy AA similarity, ΔDEF~ΔQRP$\frac{DE}{QR} = \frac{EF}{RP} = \frac{DF}{QP}$**We have to find the option which is not true.**

Question: 36

If ΔABC ~ ΔEDF an

Solution:

ΔABC~ΔEDF,Therefore,$\frac{AB}{DE} = \frac{BC}{DF} = \frac{AC}{EF}$

We have to find the option which is not true.∴ The correct option is (c) .

Question: 37

In ΔABC and ΔDEF,

Solution:

Δ ABC~Δ DEFBy AA similarity, the triangles are similar

For triangles to be congruent, AB = DE, but given that AB = 3DE.

Question: 38

If in ΔABC and ΔP

Solution:

Given $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$**Therefore ΔABC~ΔQRPor ΔPQR~ΔCAB.**

Question: 39

In the given figu

Solution:

In ΔAPB and Δ DPC,∠APB = ∠DPC = 50° $\frac{AP}{PB} = \frac{6}{3} = 2$

$\frac{PD}{PC} = \frac{5}{2.5} = 2$**By SAS property, Δ APB~ΔDPC∠PBA = ∠DPCIn Δ DPC,∠D + ∠ P + ∠C = 180°**

$\angle$ C = 100° ∴ $\angle$PBA = $\angle$DPC = 100°

Question: 40

Corresponding sid

Solution:

If two triangles are similar, the ratio of the area of triangle is equal to the square of the ratio of the sides.Ratio of Area = (Ratio of Side)^2 = $(\frac{4}{9})^2$ = 16:81∴ The correct option is (d).

Question: 41

It is given that

Solution:

If two triangles are similar, the ratio of the area of triangle is equal to the square of the ratio of the sides.$\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)} = (\frac{QR}{BC})^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Question: 42

In an equilateral

Solution:

Given that D and E are of AB and AC respectively,Therefore, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} = \frac{1}{2}$

Δ ABC~Δ ADEIf two triangles are similar, the ratio of the area of triangle is equal to the square of the ratio of the sides.$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta ADE)} = (\frac{AB}{AD})^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$**∴ The correct option is (b).**

Question: 43

In ΔABC and ΔDEF,

Solution:

$\frac{AD}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$**Therefore Δ ABC~Δ DEFIf two triangles are similar, the ratio of the area of triangle is equal to the square of the ratio of the sides.**

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = (\frac{AB}{DE})^2 = \left(\frac{5}{7}\right)^2 = \frac{25}{49}$$

Question: 44

ΔABC ~ Δ DEF such

Solution:

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DEF)} = \frac{36}{49} = \left(\frac{AB}{DE}\right)^2 \frac{AB}{DE} = \sqrt{\frac{36}{49}} = \frac{6}{7}$$

Question: 45

Two isosceles tri

Solution:

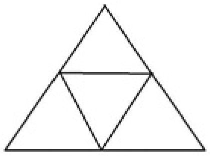
It is given that the corresponding angles are equal, that implies that the triangles are similar.$\frac{\text{Area}(\Delta 1)}{\text{Area}(\Delta 2)} = \frac{25}{36} = \left(\frac{h1}{h2}\right)^2$

$$\frac{h1}{h2} = \sqrt{\frac{25}{36}} = \frac{5}{6}$$

Question: 46

The line segments

Solution:



In this figure,As given that the inner triangle is formed by joining the midpoints of the sides.Therefore the outer three triangles are similar to bigger triangle.By Basic Proportionality Theorem,The inner triangle is also similar to the bigger triangle.

Question: 47

If ΔABC ~ &

Solution:

ΔABC~ΔQRP$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta QRP)} = \frac{9}{4} = \left(\frac{AC}{PR}\right)^2 \frac{BC}{PR} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

PR = $\frac{2}{3}BC$**PR =** $\frac{2 \times 15}{3}$ **= 10cm**

Question: 48

In the given figu

Solution:

In Δ DOB and Δ AOC,∠DOB = ∠AOC = 45° (vertically opposite angle)∠OAC = ∠ODB (angles in the same segment)

∠OCA = ∠OBD (angles in the same segment)Therefore, Δ DOB~ Δ AOC by AA similarity, $\frac{OD}{OA} = \frac{OBOC}{OCOA} = \frac{OB}{OD} = 1$**Therefore, OC = OA.**

Question: 49

In an isosceles Δ

Solution:

$AB^2 = 2AC^2$

$AB^2 = AC^2 + AC^2$

$AB^2 = AC^2 + BC^2$

Therefore, it is an isosceles triangle right angled at C. ∠ C = 90°

Question: 50

In ΔABC, if AB =

Solution:

$AB^2 + BC^2 = 16^2 + 12^2 = 256 + 144 = 400 = 20^2 = AC^2$

Therefore, ABC is a right angled triangle.

Question: 51

Which of the foll

Solution:

If two triangles ABC and PQR are similar,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

That is their corresponding sides are proportional.

Question: 52

Which of the foll

Solution:

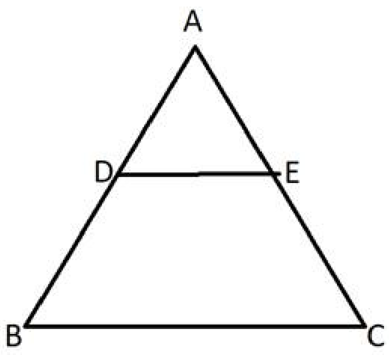
The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

Question: 53

Match the followi

Solution:

A)



Given that DE||BC,by B.P.T.,$\frac{AD}{DB} = \frac{AE}{EC}$

Let AE = x

Then, from the figure, EC = 5.6-x

$$\frac{AD}{DB} = \frac{x}{5.6 - x} = \frac{3}{5}$$

5x = 3(5.6-x)

5x = 16.8-3x

8x = 16.8

x = 2.1cm

Therefore, (A)-(s)

B)As Δ **ABC~ΔDEF,**$\frac{AB}{DE} = \frac{BC}{EF} = \frac{3}{2}$

3EF = 2BC

3EF = 2 × 6

EF = 4cm

Therefore,(B)-(q)

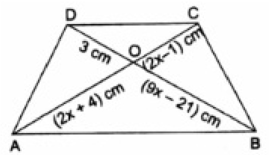
C)

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{9}{16} = \left(\frac{BC}{QR}\right)^2 \frac{BC}{QR} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

QR = $\frac{4}{3}$BC**QR =** $\frac{4 \times 4.5}{3}$ **= 6cm**

Therefore (C)-(p)

D)



$\frac{OA}{OB} = \frac{OC}{OD}$**(BPT)**

$$\frac{2x + 4}{9x - 21} = \frac{2x - 1}{3}$$

⇒ 3(2x + 4) = (2x-1)(9x-21)

⇒ 6x + 12 = $18x^2$-42x-9x + 21

⇒ $18x^2$-57x + 9 = 0

⇒18x2-54x-3x + 9 = 0

⇒ 18x(x-3)-3(x-3) = 0

⇒ (18x-3)(x-3) = 0

So, x = 3 or x = $\frac{1}{6}$

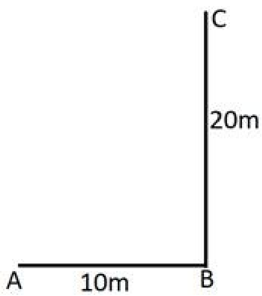
But for x = $\frac{1}{6}$, 2x-1<0 which is not possible.Therefore, (D)-(r)

Question: 54

Match the followi

Solution:

A)



The man starts from A, goes east 10m to B. From B, he goes 20m to C.

$AC^2 = AB^2 + BC^2$

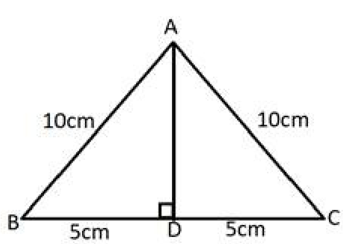
$AC^2 = 10^2 + 20^2$

AC^2 = 100 + 400 = 500

AC = √ 500 = 10√5

Therefore, (A)-(R)

B)



In ΔABD,

$AB^2 = AD^2 + BD^2$

$10^2 = AD^2 + 5^2$

$AD^2 = 100-25 = 75$

$AD = \sqrt{75} = 5\sqrt{3}$

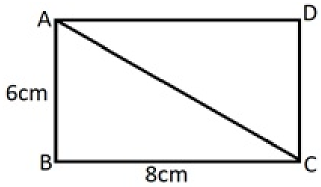
Therefore, (B)-(Q)

C)

Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{Side})^2 = \frac{\sqrt{3}}{4} \times 10^2 = 25\sqrt{3}$**cm**

Therefore,(C)-(P)

D)



In ΔABC,

$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8^2$

$AD^2 = 36 + 64 = 100$

$AD = \sqrt{100} = 10$

Therefore, (D)-(S)

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

ΔABC ~ ΔDEF and t

Solution:

Given: ΔABC ~ ΔDEF

Perimeter of ΔABC = 32 cm

Perimeter of ΔDEF = 24 cm

AB = 10 cm

To find: DE

∵ ΔABC ~ ΔDEF

∴ The ratio of the corresponding sides of Δ ABC and Δ DEF are equal to the ratio of the perimeter of the corresponding triangles.

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{32}{24}$

$\Rightarrow \frac{AB}{DE} = \frac{32}{24} \Rightarrow \frac{10}{DE} = \frac{4}{3} \Rightarrow DE = 10 \times \frac{3}{4} = \frac{30}{4} = 7.5\text{ cm}$

Question: 2

In the given figu

Solution:

Given: DE ∥ BC

DE = 5 cm

BC = 8 cm

AD = 3.5 cm

To find: AB

∵ DE ∥ BC

∴ By Basic proportionality theorem, we have

$\frac{AD}{AB} = \frac{AE}{AC}$ **...........(i)**

Now, in Δ ADE and Δ ABC, we have

$\frac{AD}{AB} = \frac{AE}{AC}$ **[By (i)]**

∠DAE = ∠BAC [Common angle]

∴ By SAS criterion,

Δ ADE ~ Δ ABC

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$

$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \frac{3.5}{AB} = \frac{5}{8} \Rightarrow AB = 8 \times \frac{3.5}{5} = 8 \times 0.7 = 5.6cm$

Question: 3

Two poles of heig

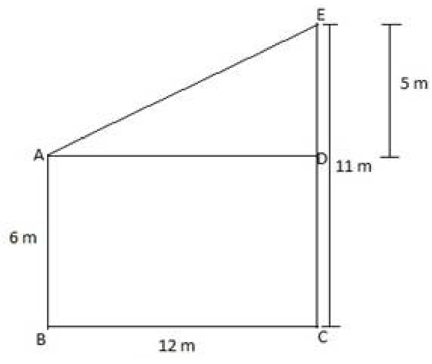
Solution:

Given: Height of pole 1 = 6 m

Height of pole 2 = 11 m

Distance between the feet of pole 1 and pole 2 = 12 m

To find: Distance between the tops of both the poles



Clearly, In Δ ADE,

DE = 5 m

AD = 12 m

Also, ∠ADE = 90° [∵ Both the poles stand vertically upright]

∴ By applying Pythagoras theorem, we have

$AE^2 = AD^2 + DE^2$

$\Rightarrow AE^2 = (12)^2 + (5)^2 = 144 + 25 = 169$

⇒ AE = √169 = 13 m

Question: 4

The areas of two

Solution:

Given: Area of triangle 1 = 25 cm^2

Area of triangle 2 = 36 cm^2

Altitude of triangle 1 = 3.5 cm

To find: Altitude of triangle 2

Let the altitude of triangle 2 be x.

∵ The areas of two similar triangles are in the ratio of the squares of the corresponding altitudes.

∴ We have,

$$\frac{\text{Area of triangle 1}}{\text{Area of triangle 2}} = \frac{(\text{Altitude of triangle 1})^2}{(\text{Atitude of triangle 2})^2}$$

$$\Rightarrow \frac{25}{36} = \frac{(3.5)^2}{x^2} \Rightarrow x^2 = 12.25 \times \frac{36}{25} = 17.64$$

⇒ x = √17.64 = 42 cm

Question: 5

If ΔABC ~ ΔDEF su

Solution:

Given: ΔABC ~ ΔDEF

2AB = DE ....(i)

BC = 6 cm

To find: EF

∵ ΔABC ~ ΔDEF

∴ Ratio of all the corresponding sides of Δ ABC and Δ DEF are equal.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$ **..........(ii)**

Also, from (i), we have

2AB = DE

$\Rightarrow \frac{AB}{DE} = \frac{1}{2}$ **..........(iii)**

$\Rightarrow \frac{BC}{EF} = \frac{1}{2}$ **[By (ii) and (iii)]**

$$\Rightarrow \frac{6}{EF} = \frac{1}{2} \Rightarrow EF = 6 \times 2 = 12 \text{ cm}$$

Question: 6

In the given figu

Solution:

Given: DE ∥ BC

AD = x cm

DB = (3x + 4) cm

AE = (x + 3) cm

EC = (3x + 19) cm

To find: x

∵ DE ∥ BC

∴ By Basic Proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{x}{3x+4} = \frac{x+3}{3x+19}$$

⇒ x (3x + 19) = (x + 3) (3x + 4)

⇒ $3x^2 + 19x = 3x^2 + 13x + 12$

⇒ $19x - 13x = 3x^2 + 12 - 3x^2$

⇒ 6x = 12 or x = 2

Question: 7

A ladder 10 m lon

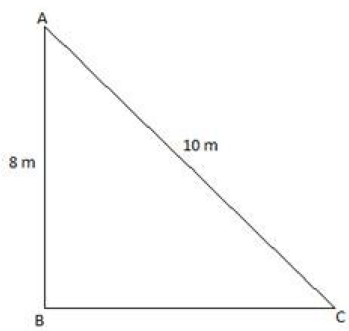
Solution:

Given: Height of the window from the ground = 8 m

Length of the ladder = 10 m

To find: Distance of the foot of the ladder from the base of the wall.

Consider the following diagram corresponding to the question.



Here, AB = Height of the window form the ground = 8 m

AC = Length of the ladder = 10 m

BC = Distance of the foot of the ladder from the base of the wall

Now, in Δ ABC,

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

⇒ $BC^2 = AC^2 - AB^2$

⇒ $BC^2 = (10)^2 - (8)^2 = 100 - 64 = 36$

⇒ BC = √36 = 6 m

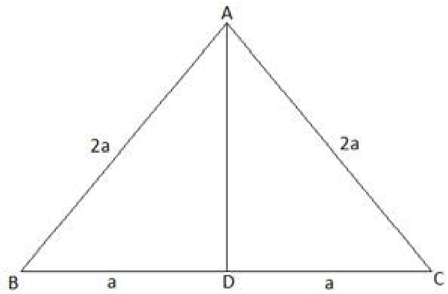
Question: 8

Find the length o

Solution:

Given: Side of equilateral triangle = 2a cm

To find: Length of altitude



Let Δ ABC be an equilateral triangle with side 2a cm.

Let AD be the altitude of Δ ABC.

Here, BD = DC = a

In Δ ABD,

Using Pythagoras theorem, we have

$AB^2 = AD^2 + BD^2 \Rightarrow AD^2 = AB^2 - BD^2 \Rightarrow AD^2 = (2a)^2 - (a)^2 = 4a^2 - a^2 = 3a^2$

$\Rightarrow AD^2 = 3a^2$

$\Rightarrow AD = \sqrt{3a^2} = \sqrt{3}\ a$ **cm**

Question: 9

ΔABC ~ ΔDEF such

Solution:

Given: Δ ABC ~ Δ DEF

ar (Δ ABC) = 64 cm^2, ar (Δ DEF) = 169 cm^2

BC = 4 cm

To find: EF

∵ The ratios of the areas of two similar triangles are equal to the ratio of squares of any two corresponding sides.

∴ We have

$$\frac{\text{ar }(\Delta ABC)}{\text{ar }(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{64}{169} = \frac{4^2}{EF^2} \Rightarrow EF^2 = 16 \times \frac{169}{64} = \frac{169}{4} \Rightarrow EF = \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5 \text{ cm}$$

Question: 10

In a trapezium AB

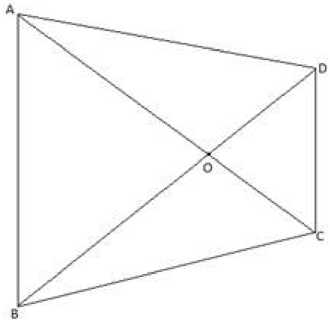
Solution:

Given: AB ∥ CD

AB = 2CD ....(i)

ar (Δ AOB) = 84 cm^2

To find: ar (Δ COD)



In Δ AOB and Δ COD,

∠ AOB = ∠ COD [Vertically Opposite angles]

∠ OAB = ∠ OCD [Alternate interior angles (AB ∥ CD)]

∠ OBA = ∠ ODC [Alternate interior angles (AB ∥ CD)]

⇒ Δ AOB ~ Δ COD [By AAA criterion]

Now,

∵ The ratios of the areas of two similar triangles are equal to the ratio of squares of any two corresponding sides.

∴ We have

$$\frac{\text{ar}(\Delta AOB)}{\text{ar}(\Delta COD)} = \frac{AB^2}{CD^2} = \left(\frac{AB}{CD}\right)^2 \Rightarrow \frac{84}{\text{ar}(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

Also, from (i), we have

$$\frac{AB}{CD} = 2$$

$$\Rightarrow \frac{84}{\text{ar}(\Delta COD)} = 2^2 = 4 \Rightarrow \text{ar}(\Delta COD) = \frac{84}{4} = 21\ \text{cm}^2$$

Question: 11

The corresponding

Solution:

Given: Let the smaller triangle be Δ ABC and the larger triangle be Δ DEF.

The ratio of AB and DE = 2 : 3

$\Rightarrow \frac{AB}{DE} = \frac{2}{3}$ **..........(i)**

ar (Δ ABC) = 48 cm^2

To find: ar (Δ DEF)

∵ The ratios of the areas of two similar triangles are equal to the ratio of squares of any two corresponding sides.

∴ We have

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{AB^2}{DE^2} \Rightarrow \frac{48}{\text{ar}(\Delta DEF)} = \frac{2^2}{3^2} = \frac{4}{9} \Rightarrow \text{ar}(\Delta DEF) = 48 \times \frac{9}{4} = 12 \times 9 = 108\ \text{cm}^2$$

Question: 12

In the given figu

Solution:

Given: LM ∥ CB and LN ∥ CD

To prove: $\frac{AM}{AB} = \frac{AN}{AD}$

In Δ AML, LM ∥ CB

∴ By Basic proportionality theorem, we have

$\frac{AM}{AB} = \frac{AL}{AC}$ **..........(i)**

In Δ ALN, LN ∥ CD

∴ By Basic proportionality theorem, we have

$\frac{AL}{AC} = \frac{AN}{AD}$ **..........(ii)**

By (i) and (ii), we have

$$\frac{AM}{AB} = \frac{AL}{AC} = \frac{AN}{AD} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$$

Question: 13

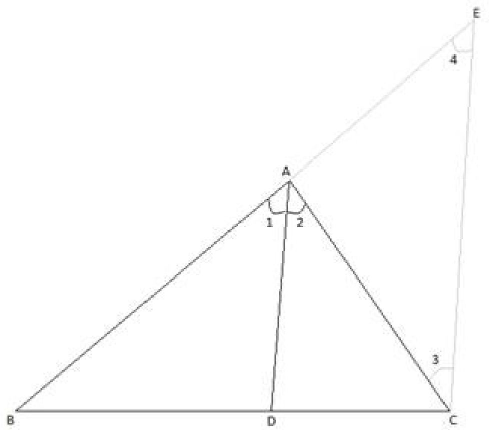
Prove that the in

Solution:

Given: Δ ABC with the internal bisector AD of ∠A which intersects BC at D.

To prove: $\frac{BD}{DC} = \frac{AB}{AC}$

First, we construct a line EC ∥ AD which meets BA produced in E.



Now, we have

CE ∥ DA ⇒ ∠2 = ∠3 [Alternate interior angles are equal (transversal AC)]

Also, ∠1 = ∠4 [Corresponding angles are equal (transversal AE)]

We know that AD bisects ∠A ⇒ ∠1 = ∠2

⇒ ∠4 = ∠1 = ∠2 = ∠3

⇒ ∠3 = ∠4

⇒ AE = AC [Sides opposite to equal angles are equal] ....(i)

Now, consider Δ BCE,

AD ∥ EC

⇒ $\frac{BD}{DC} = \frac{BA}{AE}$ **[By Basic Proportionality theorem]**

⇒ $\frac{BD}{DC} = \frac{AB}{AC}$ **[∵ BA = AB and AE = AC (From (i))]**

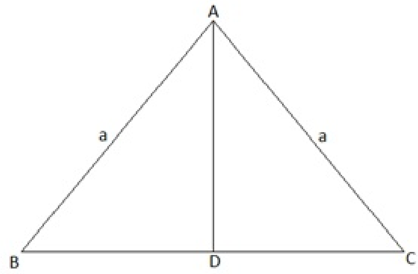
Question: 14

In an equilateral

Solution:

Let Δ ABC be an equilateral triangle with side a.

To prove: Area of Δ ABC = $\frac{\sqrt{3}}{4}a^2$



In Δ ABC, AD bisects BC

$\Rightarrow BD = DC = \frac{a}{2}$

Now, in Δ ACD

Using Pythagoras theorem,

$AC^2 = AD^2 + DC^2$

$\Rightarrow AD^2 = AC^2 - DC^2$

$\Rightarrow AD^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4} = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

$\Rightarrow AD = \sqrt{\frac{3a^2}{4}} = \frac{\sqrt{3}a}{2}$

Now, in Δ ABC

Area of Δ ABC = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times a \times \frac{\sqrt{3}a}{2} = \frac{\sqrt{3}a^2}{4}$

Question: 15

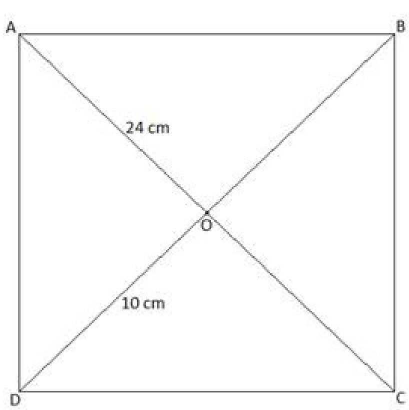
Find the length o

Solution:

Given: Length of one of the diagonals = 24 cm

Length of the other diagonal = 10 cm

To find: Length of the side of the rhombus



∵ The length of all sides of rhombus is equal.

∴ Let side of rhombus ABCD be x cm.

Also, we know that the diagonals of a rhombus are perpendicular bisectors of each other.

⇒ AO = OC = 12 cm and BO = OD = 5 cm

Also, ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°

Now, consider Δ AOD

AO = 12 cm and OD = 5 cm

∠AOD = 90°

So, using Pythagoras theorem, we have

$AD^2 = AO^2 + OD^2 = 12^2 + 5^2 = 144 + 25 = 169$

⇒ AD = √169 = 13 cm

Question: 16

Prove that the ra

Solution:

Let Δ ABC and Δ DEF be two similar triangles, i.e., Δ ABC ~ Δ DEF.

⇒ Ratio of all the corresponding sides of Δ ABC and Δ DEF are equal.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Let these ratios be equal to some number α.

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \alpha$$

⇒ AB = α DE, BC = α EF, AC = α DF ....(i)

Now, perimeter of Δ ABC = AB + BC + AC

= α DE + α EF + α DF [From (i)]

= α (DE + EF + DF)

= α (perimeter of Δ DEF)

$$\Rightarrow \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \alpha$$

$$\Rightarrow \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Question: 17

In the given figu

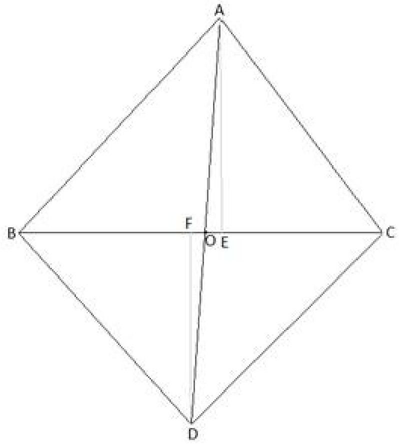
Solution:

Given: Δ ABC and Δ DBC have the same base BC.

AD and BC intersect at O.

To show: $\frac{ar\,(\Delta ABC)}{ar\,(\Delta DBC)} = \frac{AO}{DO}$

First, we construct the altitudes, AE and DF, of Δ ABC and Δ DBC, respectively.



Consider, Δ AOE and Δ DOF,

∠DFO = ∠AEO [Right angles]

∠DOF = ∠AOE [Vertically Opposite angles]

So, by AA criterion,

ΔAOE ~ ΔDOF

⇒ Ratio of all the corresponding sides of Δ AOE and Δ DOF are equal.

⇒ $\frac{AO}{DO} = \frac{AE}{DF}$ **……….(i)**

Now, we know that

$$\text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

⇒ Area of ΔABC = ar (ΔABC) = $\frac{1}{2} \times BC \times AE$ **……….(ii)**

Similarly, Area of ΔDBC = ar (ΔDBC) = $\frac{1}{2} \times BC \times DF$ **……….(iii)**

Dividing (ii) by (iii),

$$\frac{ar\ (\Delta ABC)}{ar\ (\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF} = \frac{AE}{DF}$$

$$\Rightarrow \frac{ar\ (\Delta ABC)}{ar\ (\Delta DBC)} = \frac{AE}{DF}$$

⇒ $\frac{ar\ (\Delta ABC)}{ar\ (\Delta DBC)} = \frac{AO}{DO}$ **[From (i)]**

Question: 18

In the given figu

Solution:

Given: XY ∥ AC

ar (Δ XBY) = ar (XACY) ……….(i)

To show: $\frac{AX}{AB} = \frac{2-\sqrt{2}}{2}$

Consider Δ ABC, XY ∥ AC

So, Using Basic Proportionality theorem, we have

$\frac{XB}{AB} = \frac{YB}{CB}$ **……….(ii)**

Now, in Δ XBY and Δ ABC,

∠XBY = ∠ABC [common angle]

$\frac{XB}{AB} = \frac{YB}{CB}$ **[Using (ii)]**

⇒ Δ XBY ~ Δ ABC [By SAS criterion]

Now, we know that the ratios of the areas of two similar triangles are equal to the ratio of squares of any two corresponding sides.

$$\Rightarrow \frac{ar\,(\Delta XBY)}{ar(\Delta ABC)} = \frac{XB^2}{AB^2}$$

From (i), we have

ar (Δ XBY) = ar (XACY)

Let ar (Δ XBY) = x = ar (XACY) ⇒ ar (Δ ABC) = ar (Δ XBY) + ar (XACY) = x + x = 2x

$$\Rightarrow \frac{ar(\Delta XBY)}{ar(\Delta ABC)} = \frac{x}{2x} = \frac{1}{2}$$

$$\Rightarrow \frac{ar\,(\Delta XBY)}{ar(\Delta ABC)} = \frac{XB^2}{AB^2}$$

$$\Rightarrow \frac{XB^2}{AB^2} = \frac{1}{2} \Rightarrow \frac{XB}{AB} = \sqrt{\frac{1}{2}} \Rightarrow \frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

Now, we know that

XB = AB – AX

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{XB}{AB} = \frac{AB - AX}{AB} \Rightarrow \frac{AB - AX}{AB} = \frac{1}{\sqrt{2}} \Rightarrow \frac{AB}{AB} - \frac{AX}{AB} = \frac{1}{\sqrt{2}} \Rightarrow 1 - \frac{AX}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{AX}{AB} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$

Rationalizing the denominator, we have

$$\frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2 - \sqrt{2}}{2}$$

$$\Rightarrow \frac{AX}{AB} = \frac{2 - \sqrt{2}}{2}$$

Question: 19

In the given figu

Solution:

Given: AD ⊥ CB (produced)

To prove: $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$

In Δ ADC, DC = DB + BC ....(i)

First, in Δ ADB,

Using Pythagoras theorem, we have

$AB^2 = AD^2 + DB^2 \Rightarrow AD^2 = AB^2 - DB^2$ ....(ii)

Now, applying Pythagoras theorem in Δ ADC, we have

$AC^2 = AD^2 + DC^2$

$= (AB^2 - DB^2) + DC^2$ [Using (ii)]

$= AB^2 - DB^2 + (DB + BC)^2$ [Using (i)]

Now, ∵ $(a + b)^2 = a^2 + b^2 + 2ab$

∴ $AC^2 = AB^2 - DB^2 + DB^2 + BC^2 + 2DB \cdot BC$

⇒ $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$

Question: 20

In the given figu

Solution:

Given: PA ⊥ AC, QB ⊥ AC and RC ⊥ AC

AP = x, QB = z, RC = y, AB = a and BC = b

To show: $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

In Δ PAC, we have

QB ∥ PA

So, by Basic Proportionality theorem, we have

$\frac{PC}{QC} = \frac{AC}{BC}$ **..........(i)**

In Δ ARC, we have

QB ∥ RC

So, by Basic Proportionality theorem, we have

$\frac{AR}{AQ} = \frac{AC}{AB}$ **..........(ii)**

Now, Consider Δ PAC and Δ QBC,

∠PCA = ∠QCB [Common angle]

$\frac{PC}{QC} = \frac{AC}{BC}$ **[By (i)]**

So, by SAS criterion,

Δ PAC ~ Δ QBC

⇒ Ratio of all the corresponding sides of Δ ABC and Δ DEF are equal.

$$\Rightarrow \frac{QB}{PA} = \frac{BC}{AC}$$

$\Rightarrow \frac{z}{x} = \frac{b}{a+b}$ **..........(iii)**

Now, consider Δ ARC and Δ AQB,

∠RAC = ∠QAB [Common angle]

$\frac{AR}{AQ} = \frac{AC}{AB}$ **[By (ii)]**

So, by SAS criterion,

Δ ARC ~ Δ AQB

⇒ Ratio of all the corresponding sides of Δ ARC and Δ AQB are equal.

$$\Rightarrow \frac{QB}{RC} = \frac{AB}{AC}$$

$\Rightarrow \frac{z}{y} = \frac{a}{a+b}$ **..........(iv)**

Now, adding (iii) and (iv), we get

$$\frac{z}{x}+\frac{z}{y}=\frac{b}{a+b}+\frac{a}{a+b}$$

$$\Rightarrow z\left(\frac{1}{x}+\frac{1}{y}\right)=\frac{b+a}{a+b}=1$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}=\frac{1}{z}$$