Chapter : 1. REAL NUMBERS

Exercise : 1A

Question: 1

By Euclid's division lemma -

$x = bq + r$ ....(1)

where q is the quotient,

r is the remainder

and b is the divisor.

According to the question, $b = 61$, $r = 32$, $q = 27$.

Putting the values in equation (1) -

$\therefore x = 61(27) + 32$

$\Rightarrow x = 1679$.

Hence, the given number is 1679.

Question: 3

By what number sh

Solution:

By Euclid's division lemma -

$x = bq + r$ ....(1)

where q is the quotient,

r is the remainder

and b is the divisor.

According to the question, $x = 1365$, $r = 32$, $q = 31$, $b = ?$.

Putting the values in equation(1) -

$\therefore 1365 = b(31) + 32$

$\Rightarrow 31b = 1333$

$\Rightarrow b = 43$

Question: 4

Using Euclid's di

Solution:

i.

$\because$ here $405 < 2520$,

$\therefore b = 405$ and $a = 2520$.

By Euclid's division lemma -

$a = bq + r$ ....(1)

where q is the quotient, r is the remainder and b is the divisor.

Putting it in equation (1) -

$\Rightarrow 2520 = 405(6) + 90$.

Here 90 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 405; b = 90;

⇒ 405 = 90(4) + 45.

Here 45 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

a = 90; b = 45;

⇒ 90 = 45(2) + 0.

∵ remainder is zero.

∴ HCF is 45.

ii.

∵ here 504 < 1188,

∴ b = 504 and a = 1188.

By Euclid’s division lemma -

a = bq + r ....(1)

where q is the quotient, r is the remainder and b is the divisor.

Putting the values in equation (1) -

⇒ 1188 = 504(2) + 180.

Here 180 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 504; b = 180;

⇒ 504 = 180(2) + 144

Here 144 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now,a = 180; b = 144;

⇒ 180 = 144(1) + 36.

Here 36 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now,a = 144; b = 36;

⇒ 144 = 36(4) + 0.

∵ remainder is zero.

∴ HCF is 36.

iii.

∵ here 960 < 1575,

∴ b = 960 and a = 1575.

a = bq + r ....(1)

where q is the quotient, r is the remainder and b is the divisor.

Putting the values in equation (1) -

⇒ 1575 = 960(1) + 615.

Here 615 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 960; b = 615;

⇒ 960 = 615(1) + 345.

Here 345 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 615; b = 345;

$\Rightarrow 615 = 345(1) + 270.$

Here 270 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 345; b = 270;

$\Rightarrow 345 = 270(1) + 75.$

Here 75 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 270; b = 75;

$\Rightarrow 270 = 75(3) + 45.$

Here 45 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 75; b = 45;

$\Rightarrow 75 = 45(1) + 30.$

Here 30 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

Now, a = 45; b = 30;

$\Rightarrow 45 = 30(1) + 15.$

Here 15 is the remainder, which is not zero. Again applying the Euclid’s divison lemma -

a = 30; b = 15;

$\Rightarrow 30 = 15(2) + 0.$

$\because$ remainder is zero.

$\therefore$ HCF is 15.

Question: 5

Show that every p

Solution:

Using Euclid’s divison lemma -

a = bq + r;

We want to show that any even integer is of the form '2q' and any odd integer is of the form'2q + 1'

So, we take the two integers a and 2.

When we divide a with 2, the possible values of remainder are 0 and 1, which means

a = 2q + 0 or a = 2q + 1

Also, 2q is always an even integer for any integral value of q and so 2q + 1 will always be an odd integer. ($\because$ 'even integer + 1' is always an odd integer)

$\therefore$ when a is a positive even integer it will always be of the form 2q

And when a is positive odd integer it will always be of the form 2q + 1.

$\therefore$ for every positive integer it is either odd or even.

Question: 6

Show that any pos

Solution:

Let take a as any positive integer and b = 6. a > b

Then using Euclid’s algorithm, we get a = 6q + r here r is remainder and value of q is more than or equal to 0 and r = 0, 1, 2, 3, 4, 5 because $0 \le r < b$ and the value of b is 6

So total possible forms will be 6q + 0, 6q + 1 , 6q + 2,6q + 3,6q + 4,6q + 5.

$6q + 0 \rightarrow 6$ is divisible by 2 so it is an even number.

$6q + 1 \rightarrow 6$ is divisible by 2 but 1 is not divisible by 2 so it is an odd number.

$6q + 2 \rightarrow 6$ is divisible by 2 and 2 is also divisible by 2 so it is an even number.

$6q + 3 \rightarrow 6$ is divisible by 2 but 3 is not divisible by 2 so it is an odd number.

$6q + 4 \rightarrow 6$ is divisible by 2 and 4 is also divisible by 2 it is an even number.

$6q + 5 \rightarrow 6$ is divisible by 2 but 5 is not divisible by 2 so it is an odd number.

So odd numbers will in form of $6q + 1$, or $6q + 3$, or $6q + 5$.

Question: 7

Show that any pos

Solution:

Let a be any odd positive integer and b = 4. By division lemma there exist integer q and r such that

$a = 4q + r$, where $0 \le r \le 4$

so $a = 4q$ or, $a = 4q + 1$ or, $a = 4q + 2$ or, $a = 4q + 3$

$4q + 1 \rightarrow 4$ is divisible by 2 but 1 is not divisible by 2, so it is an odd number

$4q + 2 \rightarrow 4$ is divisible by 2 and 2 is also divisible by 2, so it is an even number

$4q + 3 \rightarrow 4$ is divisible by 2 but 3 is not divisible by 2, so it is an odd number

$4q + 4 \rightarrow 4$ is divisible by 2 and 4 is also divisible by 2, so it is an even number

$\therefore$ any odd integer is of the form $4q + 1$ or, $4q + 3$.

Exercise : 1B

Question: 1

Using prime facto

Solution:

i. HCF = 12, LCM = 252

Prime factorization of given numbers -

$36 = 2 \times 2 \times 3 \times 3$

$84 = 2 \times 2 \times 3 \times 7$

So HCF = product of common factors = $2 \times 2 \times 3 = 12$

And LCM = product of prime factors with highest powers = $2^2 \times 3^2 \times 7 = 252$

Verification -

HCF × LCM = product of numbers

$\Rightarrow$ LHS = $12 \times 252 = 3024$

$\Rightarrow$ RHS = $36 \times 84 = 3024$

$\because$ LHS = RHS(Hence verified)

ii. HCF = 1, LCM = 713

Prime factorization of given numbers -

$23 = 23 \times 1$

$31 = 31 \times 1$

So HCF = product of common factors = 1

And LCM = product of prime factors with highest powers = $23 \times 31 = 713$

Verification -

HCF × LCM = product of numbers

$\Rightarrow$ LHS = $1 \times 713 = 713$

$\Rightarrow$ RHS = $23 \times 31 = 713$

$\because$ LHS = RHS(Hence verified)

iii. HCF = 4, LCM = 9696

Prime factorization of given numbers -

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$404 = 2 \times 2 \times 101$

So HCF = product of common factors = $2 \times 2 = 4$

And LCM = product of prime factors with highest powers = $2^5 \times 3 \times 101 = 9696$.

Verification -

HCF × LCM = product of numbers

$\Rightarrow$ LHS = $4 \times 9696 = 38784$

$\Rightarrow$ RHS = $96 \times 404 = 38784$

$\because$ LHS = RHS(Hence verified)

iv. HCF = 18, LCM = 1584

Prime factorization of given numbers -

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$198 = 2 \times 3 \times 3 \times 11$

So HCF = product of common factors = $2 \times 3 \times 3 = 18$

And LCM = product of prime factors with highest powers = $2^4 \times 3^2 \times 11 = 1584$

Verification -

HCF × LCM = product of numbers

$\Rightarrow$ LHS = $18 \times 1584 = 28512$

$\Rightarrow$ RHS = $144 \times 198 = 28512$

$\because$ LHS = RHS(Hence verified)

v. HCF = 36, LCM = 11880

Prime factorization of given numbers -

$396 = 2 \times 2 \times 3 \times 3 \times 11$

$1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

So HCF = product of common factors = $2 \times 2 \times 3 \times 3 = 36$

And LCM = product of prime factors with highest powers = $2^3 \times 3^3 \times 5 \times 11 = 11880$

Verification -

HCF × LCM = product of numbers

$\Rightarrow$ LHS = $36 \times 11880 = 427680$

$\Rightarrow$ RHS = $396 \times 1080 = 427680$

$\because$ LHS = RHS(Hence verified)

vi. HCF = 128, LCM = 14976

Prime factorization of given numbers -

1152 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3

1664 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 13

So HCF = product of common factors = 2 × 2 × 2 × 2 × 2 × 2 × 2 = 128

And LCM = product of prime factors with highest powers = $2^7 \times 3^2 \times 13 = 14976$

Verification -

HCF × LCM = product of numbers

⇒ LHS = 128 × 14976 = 1916928

⇒ RHS = 1152 × 1664 = 1916928

∵ LHS = RHS(Hence verified)

Question: 2

Using prime facto

Solution:

i. HCF = 1, LCM = 1800

Prime factorization of given numbers -

8 = 2 × 2 × 2 × 1

9 = 3 × 3 × 1

25 = 5 × 5 × 1

So HCF = product of common factors = 1

And LCM = product of prime factors with highest powers = $2^3 \times 3^2 \times 5^2 = 1800$

ii. HCF = 3, LCM = 420

Prime factorization of given numbers -

12 = 2 × 2 × 3

15 = 3 × 5

21 = 3 × 7

So HCF = product of common factors = 3

And LCM = product of prime factors with highest powers = $2^2 \times 3 \times 5 \times 7 = 420$

iii. HCF = 1, LCM = 11339

Prime factorization of given numbers -

17 = 17 × 1

23 = 23 × 1

29 = 29 × 1

So HCF = 1

And LCM = product of prime factors with highest powers = 17 × 23 × 29 = 11339

iv. HCF = 4, LCM = 360

Prime factorization of given numbers -

36 = 2 × 2 × 3 × 3

24 = 2 × 2 × 2 × 3

$40 = 2 \times 2 \times 2 \times 5$

So HCF = product of common factors = $2 \times 2 = 4$

And LCM = product of prime factors with highest powers = $2^3 \times 3^2 \times 5 = 360$

v. HCF = 6, LCM = 2160

Prime factorization of given numbers -

$30 = 2 \times 3 \times 5$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

So HCF = product of common factors = $2 \times 3 = 6$

And LCM = product of prime factors with highest powers = $2^4 \times 3^3 \times 5 = 2160$

vi. HCF = 1, LCM = 1260

Prime factorization of given numbers -

$21 = 3 \times 7$

$28 = 2 \times 2 \times 7$

$36 = 2 \times 2 \times 3 \times 3$

$45 = 3 \times 3 \times 5$

So HCF = product of common factors = 1

And LCM = product of prime factors with highest powers = $2^2 \times 3^2 \times 5 \times 7 = 1260$

Question: 3

The HCF of two nu

Solution:

We know that LCM × HCF = product of numbers.

Let the other number be x.

So, $161 \times x = 23 \times 1449$

$\Rightarrow x = 207$

Question: 4

The HCF of two nu

Solution:

We know that LCM × HCF = product of numbers.

Let the other number be x.

So, $725 \times x = 145 \times 2175$

$\Rightarrow x = 435$

Question: 5

The HCF of two nu

Solution:

We know that LCM × HCF = product of numbers.

Let the LCM of numbers to be x.

So, $18 \times x = 12960$

$\Rightarrow x = 720$

Question: 6

Is it possible to

Solution:

No, ∵ HCF does not divided LCM exactly

Using Euclid's division lemma -

Take a = 760 and b = 18.

a = bq + r. where q is the quotient, r is the remainder and b is the divisor.

If HCF divides LCM completely, r = 0.

Putting the values -

Here 760 = 18(42) + 4

⇒ r = 4

∵ r is not equal to zero

∴ HCF does not divides LCM completely.

So this is not possible for two numbers to have HCF = 18 and LCM = 760.

Question: 7

Find the simplest

Solution:

(i) HCF for 69 and 92 is 23

So dividing both of them by 23 -

69 = 23 × 3

92 = 23 × 4

We get numerator as 3 and denominator as 4.

∴ simplest fraction is $\frac{3}{4}$.

(ii) HCF for 473 and 645 is 43

So dividing both of them by 43 -

473 = 43 × 11

645 = 43 × 15

We get numerator as 11 and denominator as 15

∴ simplest fraction is $\frac{11}{15}$.

(iii) HCF for 368 and 496 is 16

So, dividing both of them by 16 -

368 = 16 × 23

496 = 16 × 31

We get numerator as 23 and denominator as 31.

∴ simplest fraction is

(iv) HCF for 1095 and 1168 is 73

So, dividing both of them by 73 -

1095 = 73 × 15

1168 = 73 × 16

We get numerator as 15 and denominator as 16.

∴ simplest fraction is $\frac{23}{31}$.

Question: 8

Find the largest

Solution:

∵ we are getting 6 as remainder.

Subtracting 6 from the dividend will give us the exact division.

So, let’s subtract the remainder and find the HCF of numbers (largest number which divides both the number)

We need to find HCF of 438 - 6 = 432 and 606 - 6 = 600.

Prime factorization of numbers -

432 = 2 × 2 × 2 × 2 × 3 × 3 × 3

600 = 2 × 2 × 2 × 3 × 5 × 5

HCF of given numbers = 2 × 2 × 2 × 3 = 24.

Question: 9

Find the largest

Solution:

∵ we are getting 5 and 7 as remainder.

Subtracting 5 and 7 from the dividends will give us the exact division.

So, let’s subtract the remainder and find the HCF of numbers (largest number which divides both the number) -

We need to find HCF of 320 - 5 = 315 and 457 - 7 = 450.

Prime factorization of numbers -

315 = 3 × 3 × 5 × 7

450 = 2 × 3 × 3 × 5 × 5

HCF of given numbers = 3 × 3 × 5 = 45.

Question: 10

Find the least nu

Solution:

Find the LCM of the numbers (the least number which all the given numbers divides) -

Prime factorization of numbers -

35 = 5 × 7

56 = 2 × 2 × 2 × 7

91 = 7 × 13

LCM of given numbers = product of prime factors with highest powers = 2^3 × 5 × 7 × 13 = 3640.

∵ required remainder is 7.

We need to add 7 to the LCM of numbers.

∴ the least number that leaves remainder 7 will be 3640 + 7 = 3647.

Question: 11

Find the smallest

Solution:

Subtract the remainders from the given numbers:

If you look at the negative remainders -Remainder from 28 = 28 - 8 is - 20.Remainder from 32 = 32 - 12 is - 20

Find the LCM of the numbers (the least number which all the given numbers divide)

Prime factorization of numbers -

28 = 2 × 2 × 7

32 = 2 × 2 × 2 × 2 × 2

LCM of given numbers = product of prime factors with highest powers = $2^5 \times 7 = 224$.

Smallest no, which leaves remainder 8 and 12 when divided by 28 and 32

= LCM - 20 = 224 - 20 = 204

Question: 12

Find the smallest

Solution:

Find the LCM of the numbers (the least number which all the given numbers divide) -

Prime factorization of numbers -

468 = 2 × 2 × 3 × 3 × 13

520 = 2 × 2 × 2 × 5 × 13

LCM of given numbers = product of prime factors with highest powers = $2^3 \times 3^2 \times 5 \times 13 = 4680$

∵ the number is increased by 17 to get perfect division.

We need to subtract 17 to the LCM of numbers.

∴ the least number when added by 17 gives exact division will be 4680 - 17 = 4663.

Question: 13

Find the greatest

Solution:

Find the LCM of the numbers (the least number which all the given numbers divide) -

Prime factorization of numbers -

15 = 3 × 5

24 = 2 × 2 × 2 × 3

36 = 2 × 2 × 3 × 3

LCM of given numbers = product of prime factors with highest powers = $2^3 \times 3^2 \times 5 = 360$

We know that the greatest four-digit number is 9999.

So, the greatest value closest to 9999 which 360 can divide can be found by Euclid’s division lemma -

Putting the values in the equation a = bq + r -

9999 = 360(27) + 279

So, highest four - digit number 360 can completely divide = 9999 - remainder = 9999 - 279 = 9720.

Question: 14

In a seminar, the

Solution:

The number of room will be minimum if each room accommodates maximum number of participants.

∵ in each room the same number of participants are to be seated and all of them must be of the same subject.

Therefore, the number of participants in each room must be the HCF of 60, 84 and 108

So HCF of 60, 84 and 108 -

Prime factors of numbers are -

$60 = 2 \times 2 \times 3 \times 5$

$84 = 2 \times 2 \times 3 \times 7$

$108 = 2 \times 2 \times 3 \times 3 \times 3$

$\therefore$ HCF $= 2 \times 2 \times 3 = 12$

Therefore, in each room 12 participants can be seated.

Total number of students = 60 + 84 + 108 = 252

So minimum number of room required to accommodate all students = 252/12 = 21

Question: 15

Three sets of Eng

Solution:

The number of stacks will be minimum if each stack accommodates maximum number of books.

∵ height of each stack is the same and all of them are of the same subject.

Therefore, the number of books in each stack must be the HCF of 336,240 and 96.

So HCF of 336,240 and 96 -

Prime factors of numbers are -

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$\therefore$ HCF $= 2 \times 2 \times 2 \times 2 \times 3 = 48$

Therefore, in each stack 48 books can be placed.

Total number of books = 336 + 240 + 96 = 672

So minimum number of stacks required to accommodate all books = 672/48 = 14

Question: 16

Three pieces of t

Solution:

∵ length of each plank is the same and we are supposed to find plank of greatest possible length.

Therefore, the greatest possible length of each plank must be the HCF of 42, 49 and 63.

So HCF of 42, 49 and 63 -

Prime factors of numbers are -

$42 = 2 \times 3 \times 7$

$49 = 7 \times 7$

$63 = 3 \times 3 \times 7$

$\therefore$ HCF = 7

Therefore, maximum plank length of each plank = 7m

Total available length of plank = 42 + 49 + 63 = 154m

So number of planks of this maximum possible length = 154/7 = 22.

Question: 17

Find the greatest

Solution:

We know 1m = 100cm.

So,

7m = 700cm,

3m 85cm = 300 + 85 = 385cm,

and 12m 95cm = 1200 + 95 = 1295cm.

The greatest possible length that can measure all the three given lengths will be HCF of the lengths -

So HCF of 700, 385 and 1295 -

Prime factors of numbers are -

$700 = 2 \times 2 \times 5 \times 5 \times 7$

$385 = 5 \times 7 \times 11$

$1295 = 5 \times 7 \times 37$

$\therefore$ HCF = $5 \times 7 = 35$.

So the greatest possible length that can be used = 35cm.

Question: 18

Find the maximum

Solution:

$\because$ each student gets the same number of pens and pencils.

Maximum number of students among which it can be distributed will be equal to HCF of number of pencils and number of pens.

So HCF of 1001 and 910 -

Prime factors of the numbers are -

$1001 = 7 \times 11 \times 13$

$910 = 2 \times 5 \times 7 \times 13$

$\therefore$ HCF = $7 \times 13 = 91$.

Question: 19

Find the least nu

Solution:

We know 1m = 100cm.

So,

15m 17cm = 1500 + 17 = 1517cm

And 9m 2cm = 900 + 2 = 902 cm

Least number of square tiles required to pave the ceiling of this area if largest tiles are used.

Length of largest tile = H.C.F. of 1517 cm and 902 cm

Prime factors of 1517 and 902 -

$1517 = 41 \times 37$

$902 = 2 \times 11 \times 41$

So HCF = 41

∴ Area of tiles of largest length and width = $(41 \times 41)\ cm^2$

And area of room = length × width = $(1517 \times 902)\ cm^2$

So minimum number of tiles required = $(1517 \times 902)\ cm^2 / (41 \times 41)\ cm^2 = 814$.

Question: 20

Three measuring r

Solution:

Least length of cloth that can be measured using this rod will be LCM of the three lengths.

So LCM of 64, 80 and 96 -

Prime factors of the given numbers are -

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$80 = 2 \times 2 \times 2 \times 2 \times 5$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

So LCM = product of prime factors with highest powers = $2^6 \times 3 \times 5 = 960$

∴ least length of cloth that can be measured using the rods = 960 cm = 9.6m

Question: 21

An electronic dev

Solution:

The devices will beep simultaneously at the LCM of intervals of beeps

So LCM of 60 and 62 -

Prime factors of the numbers are -

$60 = 2 \times 2 \times 3 \times 5$

$62 = 2 \times 31$

So LCM = product of prime factors with highest powers = $2^2 \times 3 \times 5 \times 31 = 1860$ seconds

∴ devices will beep simultaneously after 1860 seconds.

We know 1 min = 60 seconds,

So 1860 seconds = 1860/60 minutes = 31 minutes.

∴ the next time they will beep simultaneously at 10:31 hrs.

Question: 22

The traffic light

Solution:

The traffic lights will change at the LCM of intervals of all the three lights.

So LCM of 48, 72 and 108 -

Prime factors of the numbers are -

48 = 2 × 2 × 2 × 2 × 3

72 = 2 × 2 × 2 × 3 × 3

108 = 2 × 2 × 3 × 3 × 3

So LCM = product of prime factors with highest powers = $2^4 \times 3^3$ = 432

∴ traffic light will change after 432 seconds.

We know 1 min = 60 seconds,

So 432 seconds = 432/60 minutes = 7 minutes and 12 seconds.

∴ the next time lights will simultaneously change at 8:7:12 hrs.

Question: 23

Six bells commenc

Solution:

All the clocks will toll together at the LCM of their intervals.

So LCM of the intervals = 2,4,6,8,10,12 = 120.

∴ all clocks will toll together after every 120 minutes.

Given interval = 30 hrs = 30 × 60 minutes = 1800 minutes {∵ 1hr = 60 minutes}

∴ number of times clock will tol = 1800/120 = 15.

plus one time when they commenced together

So the answer will be 16 times.

Question: 24

Find the missing

Solution:

Let’s start from the bottom -

The factors of last numbers are 11 and 5.

So last number is 11 × 5 = 55

Now factors of the second number from bottom is 55 and 3

So second last number is 55 × 3 = 165

Also factors for the third number from bottom is 165 and 2.

So third last number is 165 × 2 = 330

Similarly factors for the first number is 330 and 2.

So first number is 330 × 2 = 660

Exercise : 1C

Question: 1

Without actual di

Solution:

(i) $\frac{23}{(2^3 \times 5^2)}$

∵ denominator is of the form, $2^n \times 5^m$,

where n = 3 and m = 2.

∴ it is terminating in nature.

$$\frac{23}{(2^3 \times 5^2)} = \frac{23}{(2^3 \times 5^2)} \times \frac{5}{5} = \frac{23 \times 5}{(2^3 \times 5^3)} = \frac{115}{10^3} = 0.0115$$

ii. $\frac{24}{125}$

∵ denominator is of the form, $2^n \times 5^m$,

125 = 5 × 5 × 5.

where n = 0 and m = 3.

∴ it is terminating in nature.

$$\frac{24}{125} = \frac{24}{5^3} = \frac{24}{5^3} \times \frac{2^3}{2^3} = \frac{24 \times 8}{10^3} = \frac{192}{1000} = 0.192$$

iii. $\frac{171}{800}$

∵ denominator is of the form, $2^n \times 5^m$,

800 = 2 × 2 × 2 × 2 × 2 × 2 × 5 × 5

where n = 5 and m = 2.

∴ it is terminating in nature.

$$\frac{171}{800} = \frac{171}{2^6 \times 5^2} \times \frac{5^4}{5^4} = \frac{21375}{10^6} = 0.21375$$

iv. $\frac{15}{1600}$

∵ denominator is of the form, $2^n \times 5^m$,

1600 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 5 × 5

where n = 6 and m = 2.

∴ it is terminating in nature.

$$\frac{15}{1600} = \frac{15}{2^6 \times 5^2} \times \frac{5^4}{5^4} = \frac{9375}{10^6} = 0.009375$$

v. $\frac{17}{320}$

∵ denominator is of the form, $2^n \times 5^m$,

320 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 5

where n = 6 and m = 1.

∴ it is terminating in nature.

$$\frac{17}{320} = \frac{17}{2^6 \times 5} \times \frac{5^5}{5^5} = \frac{53125}{10^6} = 0.053125$$

vi. $\frac{19}{3125}$

∵ denominator is of the form, $2^n \times 5^m$,

3125 = 5 × 5 × 5 × 5 × 5

where n = 0 and m = 5.

∴ it is terminating in nature.

$$\frac{19}{3125} = \frac{19}{2^0 \times 5^5} \times \frac{2^5}{2^5} = \frac{608}{10^5} = 0.00608$$

Question: 2

Without actual di

Solution:

(i) Denominator is $2^3 \times 3$ which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

(ii) Denominator is $2^2 \times 3^3 \times 5$ which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

(iii) Denominator is $2^2 \times 5^3 \times 7^2$ which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

(iv) Denominator is $35 = 5 \times 7$ which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

(v) Denominator is $210 = 2 \times 3 \times 5 \times 7$ which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

(vi) Denominator is $147 = 3 \times 7 \times 7$ which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

(vii) Denominator is $343 = 7 \times 7 \times 7$ which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

(viii) Denominator is $455 = 5 \times 91$, which is not in the form $2^n \times 5^m$

∴ the fraction will not be a terminating decimal.

Question: 3

Express each of t

Solution:

i) Let $x = 0.\overline{8}$ - (i)

Multiply 10 on both sides -

$10x = 8.\overline{8}$ - (ii)

Subtract the equations (i) from (ii) -

$9x = 8$

So $x = \frac{8}{9}$

ii) Let $x = 2.\overline{4}$ - (i)

Multiply 10 on both sides -

$10x = 24.\overline{4}$ - (ii)

Subtract the equations (i) from (ii) -

$9x = 22$

So $x = \frac{22}{9}$

iii) Let $x = 0.\overline{24}$ - (i)

Multiply 100 on both sides -

$100x = 24.\overline{24}$ - (ii)

Subtract the equations (i) from (ii) -

$99x = 24$

So x = $\frac{24}{99} = \frac{8}{33}$

iv) Let x = $0.1\overline{2}$ - (i)

Multiply 10 on both sides -

10x = $1.\overline{2}$ - (ii)

Multiply 10 on both sides -

100x = $12.\overline{2}$ - (iii)

Subtract the equations (ii) from (iii) -

90x = 11

So x = $\frac{11}{90}$

v) Let x = $2.2\overline{4}$ - (i)

Multiply 10 on both sides -

10x = $22.\overline{4}$ - (ii)

Multiply 10 on both sides -

100x = $224.\overline{4}$ - (iii)

Subtract the equations (ii) from (iii) -

90x = 202

So x = $\frac{202}{90} = \frac{101}{45}$

vi) Let x = $0.\overline{365}$ - (i)

Multiply 1000 on both sides -

1000x = $365.\overline{365}$ - (ii)

Subtract the equations (i) from (ii) -

999x = 365

So x = $\frac{365}{999}$

Exercise : 1D

Question: 1

Define (i) ration

Solution:

A rational number is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non - zero denominator q.

(ii) irrational numbers

An irrational number is a number that cannot be expressed as a fraction p/q for any integers p and q. Irrational numbers have decimal expansions that neither terminate nor become periodic

(iii) real numbers.

Real numbers are numbers that can be found on the number line. This includes both the rational and irrational numbers.

Question: 2

Classify the foll

Solution:

A rational number is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non - zero denominator q. While, an irrational number is a number that cannot be expressed as a fraction p/q for any integers p and q. Irrational numbers have decimal expansions that neither terminate nor become periodic.

{π,3.$\overline{142857}$, 5.636363 ..., 2.040040004 ..., $\sqrt{21}$, 1.535335333 ..., 3.121221222 ..., $\sqrt[3]{3}$ are irrational}

i. rational ii. rational

iii. irrational iv. rational

v. rational vi. irrational

vii. irrational viii. irrational

ix. irrational x. irrational

Question: 3

Prove that each o

Solution:

(i) let's assume that √6 is rational.

By definition, that means there are two integers a and b with no common divisors where:

⇒ a/b = √6

So let's take square of both the sides -

⇒ (a/b)(a/b) = (√6)(√6)⇒ $a^2/b^2 = 6$⇒ $a^2 = 6b^2$

Last statement means the RHS (right hand side) is even, because it is a product of integers and one of those integers (at least 6) is even. So a^2 must be even. But any odd number times itself is odd, so if a^2 is even, then a is even.

Since a is even, there is some integer c such that:

⇒ 2c = a.

Now let's replace a with 2c:

⇒ $a^2 = 6b^2$⇒ $(2c)^2 = (2)(3)b^2$⇒ $2c^2 = 3b^2$

But now we can argue the same thing for b, because the LHS is even, so the RHS must be even and that means b is even.

Now this is the contradiction: if a is even and b is even, then they have a common divisor (2). Then our initial assumption must be false, so the square root of 6 cannot be rational.

Thus, proved that √6 is irrational.

(ii) assume that 2 - √3 is rational2 - √3 = a/b , where a and b are integers . ⇒ - √3 = a/b - 2⇒ √3 = 2 - a/b ⇒ √3 = 2b/b - a/b⇒ √3 = (2b – a) / bwe know that a, b and 2 are integers and they are also rational {i.e RHS is rational}therefore √3 will be rational.but we know that √3 is irrational.there is a contradictionso, 2 - √3 is an irrational number

(iii) assume that 3 + √2 is rational3 + √2 = a/b , where a and b are integers . ⇒ √2 = a/b - 3 ⇒ √2 = (a - 3b)/bwe know that a, b and 3 are integers and they are also rational {i.e RHS is rational}therefore √2 will be rational.but we know that √2 is irrational.there is a contradictionso, 3 + √2 is an irrational number

(iv) assume that 2 + √5 is rational2 + √5 = a/b , where a and b are integers . ⇒ √5 = a/b - 2 ⇒ √5 = a/b - 2b/b⇒ √5 = (a - 2b)/ bwe know that a, b and 2 are integers and they are also rational {i.e RHS is rational}therefore √5 will be rational.but we know that √5 is irrational.there is a contradictionso, 2 + √5 is an irrational number

(v) assume that 5 + 3√2 is rational5 + 3√2 = a/b , where a and b are integers . ⇒ 3√2 = a/b - 5 ⇒ 3√2 = a/b - 5b/b⇒ √2 = (a - 5b)/3bwe know that a, b, 3 and 5 are integers and they are also rational {i.e RHS is rational}therefore √2 will be rational.but we know that √2 is irrational.there is a contradictionso, 5 + 3√2 is an irrational number.

(vi) assume that $3\sqrt{7}$ is rational⇒ $3\sqrt{7} = a/b$, where a and b are integers . ⇒ $\sqrt{7} = a/3b$we know that a, b and 3 are integers and they are also rational {i.e RHS is rational}therefore $\sqrt{7}$ will be rational.but we know that $\sqrt{7}$ is irrational.there is a contradictionso, $3\sqrt{7}$ is an irrational number

(vii) $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3}{5}.\sqrt{5}$ {Rationalising}

Now, let us assume that $3\sqrt{5} / 5$ is a rational number

⇒ $3\sqrt{5}/5 = p/q$ (where, p & q are integers, q not equal to 0)

⇒ $3\sqrt{5} = 5p/q$

⇒ $\sqrt{5} = 5p/3q$

Here, LHS is an irrational number whereas RHS is a rational number.

So by contradiction $3\sqrt{5} / 5$ is an irrational number

(viii) assume that $2 - 3\sqrt{5}$ is rational$2 - 3\sqrt{5} = a/b$, where a and b are integers . ⇒ $- 3\sqrt{5} = a/b - 2$⇒ $3\sqrt{5} = 2 - a/b$ ⇒ $3\sqrt{5} = 2b/b - a/b$⇒ $\sqrt{5} = (2b – a) / 3b$we know that a, b, 2 and 3 are integers and they are also rational {i.e RHS is rational}therefore $\sqrt{5}$ will be rational.but we know that $\sqrt{5}$ is irrational.there is a contradictionso, $2 - 3\sqrt{5}$ is an irrational number

(ix) Multiplying both sides by $(\sqrt{5} - \sqrt{3})$.$(\sqrt{5} - \sqrt{3}) (\sqrt{5} + \sqrt{3}) = 5 - 3 = 2$

⇒ $2 = p/q \times (\sqrt{5} - \sqrt{3})$ ⇒ $(\sqrt{5} - \sqrt{3}) = 2q/p$,

∴ $\sqrt{5} - \sqrt{3}$ is rational $= 2q/p$ ⇒ $\sqrt{5} + \sqrt{3} = p/q$ ⇒ $\sqrt{5} - \sqrt{3} = 2q/p$

Adding the equations -

⇒ $\sqrt{5} = \frac{\left[\left(\frac{p}{q}\right) + \left(\frac{2q}{p}\right)\right]}{2}$, which is a rational number.

But we know that $\sqrt{5}$ is IRRATIONAL.

Therefore, the assumption is wrong and $\sqrt{3} + \sqrt{5}$ is irrational.

Question: 4

Prove that

Solution:

Assume $\frac{1}{\sqrt{3}}$ to be rational. So we can write it in the form of a/b where a and b are co - prime.

So , $a/b = \frac{1}{\sqrt{3}}$

And ∴ $b/a = \sqrt{3}$

Since b/a is rational but $\sqrt{3}$ is irrational.

By contradiction $\frac{1}{\sqrt{3}}$ is irrational.

.

Question: 5

i. Give an exampl

Solution:

Let's take $(2 + \sqrt{3})$ and $(2 – \sqrt{3})$.

These are irrational numbers.

Their sum $= (2 + \sqrt{3}) + (2 – \sqrt{3}) = 4$ {which is rational}.

ii) Let's take $(3 + \sqrt{2})$ and $(3 – \sqrt{2})$.

These are irrational numbers.

Their product $= (3 + \sqrt{2}) \times (3 – \sqrt{2}) = 9 + 3\sqrt{2} – 3\sqrt{2} – (\sqrt{2})^2$

= 9 – 4 = 5 {which is rational}

Question: 6

State whether the

Solution:

i. True ii. True

iii. False iv. False

v. True vi. True

(iii) Let’s take (2 + $\sqrt{3}$) and (2 – $\sqrt{3}$).

These are irrational numbers.

Their sum = (2 + $\sqrt{3}$) + (2 – $\sqrt{3}$) = 4 {which is rational}.

∴ it is false.

(iv) Let’s take (3 + $\sqrt{2}$) and (3 – $\sqrt{2}$).

These are irrational numbers.

Their product = (3 + $\sqrt{2}$) × (3 – $\sqrt{2}$) = 9 + 3√2 – 3√2 – $(\sqrt{2})^2$

= 9 – 4 = 5 {which is rational}

∴ it is false.

Question: 7

Prove that

Solution:

Assume that 2√3 - 1 is rational2√3 = a/b , where a and b are integers . ⇒ 2√3 = a/b + 1 ⇒ 2√3 = a/b + b/b⇒ √3 = (a + b) / 2bwe know that a, b, and 2 are integers and they are also rational {i.e RHS is rational}therefore √3 will be rational.but we know that √3 is irrational.there is a contradictionso, 2√3 - 1 is an irrational number.

Question: 8

Prove that

Solution:

5√2 is an irrational number and subtraction of a rational number and an irrational number is an irrational number.

∴, 4 - 5√2 is irrational, where 5√2 is an irrational number while 4 is a rational number.

Alternative - Assume that 4 - 5√2 is rational4 - 5√2 = a/b , where a and b are integers . ⇒ - 5√2 = a/b - 4⇒ 5√2 = 4 - a/b ⇒ 5√2 = 4b/b - a/b⇒ √2 = (4b – a) / 5bwe know that a, b, 4 and 5 are integers and they are also rational {i.e RHS is rational}therefore √2 will be rational.but we know that √2 is irrational.there is a contradictionso, 4 - 5√2 is an irrational number

Question: 9

Prove that

Solution:

2√3 is an irrational number and subtraction of a rational number and an irrational number is an irrational number.

∴, 5 - 2√3 is irrational, where 2√3 is an irrational number while 5 is a rational number.

Alternative - Assume that 5 - 2√3 is rational5 - 2√3 = a/b , where a and b are integers . ⇒ - 2√3 = a/b - 5⇒ 2√3 = 5 - a/b ⇒ 2√3 = 5b/b - a/b⇒ √3 = (5b – a) / 2bwe know that a, b, 2 and 5 are integers and they are also rational {i.e RHS is rational}therefore √3 will be rational.but we know that √3 is irrational.there is a contradictionso, 5 - 2√3 is an irrational number

Question: 10

Prove that

Solution:

assume that 5√2 is rational $\Rightarrow 5\sqrt{2} = \frac{a}{b}$ where a and b are integers . $\Rightarrow \sqrt{2} = \frac{a}{5b}$ we know that a, b and 5 are integers and they are also rational {i.e RHS is rational}therefore √2 will be rational.but we know that √2 is irrational.there is a contradictionso, 5√2 is an irrational number

∴ it is proved that 5√2 is irrational.

Question: 11

Prove that

Solution:

$$\frac{2}{\sqrt{7}} = \left(\frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}\right) = \frac{2}{7}.\sqrt{7}$$

Now, let us assume that 2√7 / 7 is a rational number

⇒ 2√7/7 = p/q (where, p & q are integers, q not equal to 0)

⇒ 2√7 = 7p/q

⇒ √7 = 7p/2q

Here, LHS is an irrational number whereas RHS is a rational number.

So by contradiction 2√7 / 7 is an irrational number

Exercise : 1E

Question: 1

State Euclid’s di

Solution:

According to Euclid’s Division Lemma if we have two positive integers a and b, then there exists unique integers *q* and *r* which satisfies the condition $a = bq + r$ where $0 \leq r \leq b$.

Question: 2

State fundamental

Solution:

The fundamental theorem of arithmetic, also called the unique factorization theorem or the unique - prime - factorization theorem, states that every integer greater than 1 either is prime itself or is the product of prime numbers, and that this product is unique, up to the order of the factors.

Question: 3

Express 360 as pr

Solution:

Factorization of 360 = 2 × 2 × 2 × 3 × 3 × 5 = $(2^3 \times 3^2 \times 5)$

Question: 4

If a and b are tw

Solution:

Two prime numbers don’t have a common factor other than 1. So HCF = 1.

Question: 5

If a and b are tw

Solution:

∵ HCF of two prime numbers is 1 and HCF × LCF = product of numbers.

So LCM of numbers = product of numbers = a × b = ab.

Question: 6

If the product of

Solution:

∵ HCF × LCM = product of numbers

∴ 25 × LCM = 1050

And LCM = 1050/25 = 42.

Question: 7

What is a composi

Solution:

A whole number that can be divided evenly by numbers other than 1 or itself is a composite number. We can also say, "A non - prime number is a composite number".

Question: 8

If a and b are re

Solution:

Two relatively prime numbers don't have a common factor other than 1. So HCF = 1.

Question: 9

If the rational n

Solution:

b = $(2^{m} \times 5^{m})$, where m and n are some non - negative integers.

Question: 10

Simplify:

Solution:

2√45 = 2√(3*3*5)= 2*3√5=6√5

Also 3√20 = 3√2*2*5= 3*2√5= 6√5

So numerator = 6√5 + 6√5 = 12√5

And denominator is 2√5.

∴ expression becomes $\frac{12\sqrt{5}}{2\sqrt{5}}$ = 6

Question: 11

Write the decimal

Solution:

$\frac{73}{2^4 \times 5^3} \times \frac{5^1}{5^1} = \frac{365}{10^4} = 0.0365.$

Question: 12

Show that there i

Solution:

$(2^n \times 5^n) = 10^n$ {using rules of exponent}

And 10^n either ends up with 1 for n = 0 and for n>0 and integer, ends up with zero.

Also for negative value of n it ends with fractions not ending with 5.

∴ this will never end with 5.

Question: 13

Is it possible to

Solution:

No, ∵ HCF does not divided LCM exactly

Using Euclid's division lemma -

Take a = 520 and b = 25.

a = bq + r. where q is the quotient, r is the remainder and b is the divisor.

If HCF divides LCM completely, r = 0.

Here 520 = 25(20) + 20

So, r = 20

∵ r is not equal to zero

∴ HCF does not divides LCM completely.

So this is not possible for two numbers to have HCF = 25 and LCM = 520.

Question: 14

Give an example o

Solution:

Let's take $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$.

These are irrational numbers.

Their sum = $(2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$ {which is rational}.

Question: 15

Give an example o

Solution:

Let's take $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$.

These are irrational numbers.

Their product = $(3 + \sqrt{2}) \times (3 - \sqrt{2}) = 9 + 3\sqrt{2} - 3\sqrt{2} - (\sqrt{2})^2$

$= 9 - 4 = 5$ {which is rational}

Question: 16

If a and b are re

Solution:

Two relatively prime numbers don't have a common factor other than 1.

i.e, HCF = 1 and we know HCF × LCM = product of numbers.

So LCM = product of numbers = a × b = ab.

Question: 17

The LCM of two nu

Solution:

No, ∵ HCF should divide LCM exactly.

Using Euclid's division lemma -

Take a = 1200 and b = 500.

a = bq + r. where q is the quotient, r is the remainder and b is the divisor.

If HCF divides LCM completely, r = 0.

Here 1200 = 500(2) + 200

r = 200

∵ r is not equal to zero.

∴ HCF does not divides LCM completely.

So this is not possible for two numbers to have HCF = 500 and LCM = 1200.

Question: 18

Express

Solution:

Let $x = 0.\overline{4}$ - (i)

Multiply 10 on both sides -

$10x = 4.\overline{4}$ - (ii)

Subtract the equations (i) from (ii) -

⇒ 9x = 4

So $x = \frac{4}{9}$

Question: 19

Express

Solution:

Let $x = 0.\overline{23}$ - (i)

Multiply 100 on both sides -

$100x = 23.\overline{23}$ - (ii)

Subtract the equations (i) from (ii) -

⇒ 99x = 23

So $x = \frac{23}{99}$

Question: 20

Express why 0.150

Solution:

Irrational numbers are non - terminating non - recurring decimals.

Thus, 0.15015001500015 … is an irrational number.

Question: 21

Show that

Solution:

assume that $\frac{\sqrt{2}}{3}$ is rational⇒ $\frac{\sqrt{2}}{3}$ = a/b, where a and b are integers. ⇒ √2 = 3a/bwe know that a, b and 3 are integers and they are also rational {i.e RHS is rational}therefore √2 will be rational.but we know that √2 is irrational.there is a contradictionso, $\frac{\sqrt{2}}{3}$ is an irrational number.

Question: 22

Write a rational

Solution:

∵, √3 = 1.732...

So, we may take 1.8 as the required rational number between √3 and 2.

Thus, the required rational number is 1.8 or $\frac{18}{10} = \frac{9}{5}$

Question: 23

Explain why

Solution:

∵, $3.\overline{1416}$ is a non - terminating repeating decimal.

∴, is a rational number.

Exercise : MULTIPLE CHOICE QUESTIONS (MCQ)

Question: 1

Which of the foll

Solution:

Co - prime numbers are the numbers which have only one common divisor = 1 or their HCF = 1.

Here for option **(A)** 14 = 2 × 7 35 = 5 × 7∴HCF(14,25) = 7.

(B)18 = 2 × 3 × 3 25 =5 × 5∴ HCF(18,25) = 1.

(C) 31 = 1 × 31 93 = 3 × 31∴ HCF(31,93) = 31.

(D) 32 = 2 × 2 × 2 × 2 × 2 62 = 2 × 31∴ HCF(32,62) = 2.

So answers will be (B).

Question: 2

If a = $(2^2$

Solution:

a = $(2^2 \times 3^3 \times 5^4)$

b = $(2^3 \times 3^2 \times 5)$

Here HCF (a, b) = product of common factors = $2^2 \times 3^2 \times 5 = 4 \times 9 \times 5 = 180$.

Question: 3

HCF of $(2^3$

Solution:

HCF is product of common factors,

Here HCF = $2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60$.

Question: 4

LCM of $(2^3$

Solution:

LCM of the given numbers = product of prime factors with highest powers = $2^4 \times 5 \times 7 \times 3$ = 1680.

Question: 5

The HCF of two nu

Solution:

We know that product of numbers = LCM × HCF.

Let the other number be x.

So, $x \times 54 = 27 \times 162$

Or $x = 81$.

Question: 6

The product of tw

Solution:

We know that product of numbers = LCM × HCF

So, 1600 = 5 × LCM

And ∴ LCM = 320.

Question: 7

What is the large

Solution:

The largest number that can divide both the numbers will be the HCF of the two numbers -

So prime factors of the numbers –

$1152 = 2^7 \times 3^2$

$1664 = 2^7 \times 13$.

∴ HCF of number = 2^7 = 128.

Question: 8

What is the large

Solution:

∵ we are getting 5 and 8 as remainder.

Subtracting 5 and 8 from the dividends will give us the exact division.

So let's subtract the remainder and find the HCF of numbers (largest number which divides both the number) -

We need to find HCF of 70 - 5 = 65 and 125 - 8 = 117.

Prime factorization of numbers -

$65 = 5 \times 13$

$117 = 3 \times 3 \times 13$

HCF of given numbers = 13.

Question: 9

What is the large

Solution:

∵ we are getting 5 as remainder.

Subtracting 5 from the dividend will give us the exact division.

So let’s subtract the remainder and find the HCF of numbers (largest number which divides both the number) -

We need to find HCF of 245 - 5 = 240 and 1029 - 5 = 1024.

Prime factorization of numbers -

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

HCF of given numbers = $2 \times 2 \times 2 \times 2 = 16$.

Question: 10

The simplest form

Solution:

Prime factors of the numbers –

$1095 = 3 \times 5 \times 73$

$1168 = 2^4 \times 73$

Dividing them -

We get numerator as $3 \times 5 = 15$ and denominator as $2^4 = 16$.

Question: 11

Euclid’s division

Solution:

The theorem itself.

Question: 12

A number when div

Solution:

Given: A number when divided by 143 leaves 31 as remainder.**To find:** the remainder when the same number is divided by 13.**Solution:**Let the number be a.

By Euclid’s division lemma -

Putting the values of divisor as 143 and remainder as 31.

a = 143(q) + 31, where q is the quotient when divided by 143.

a = {13(11)}q + 31

a = 13(11)q + 13(2) + 5

a = 13(11q + 2) + 5 ..... (i)

Again when it is divided by 13, let the remainder be r.

So, a = 13(p) + r ......(ii)where p is the quotient when divided by 13.

Comparing (i) and (ii) -

11q + 2 = p and r = 5.

So remainder = 5.

Question: 13

Which of the foll

Solution:

An irrational number is non - terminating non-repeating decimal and can’t be expressed in p/q form. So here option D is neither terminating non-repeating decimal.

Question: 14

π is

Solution:

∵ π is non - terminating non-periodic in nature and does not satisfy the rational number definition. It is irrational.

Question: 15

∵ the given number is non - terminating but repeating decimal. ∴ it’s a rational number.

Question: 16

2.13113111311113...

Solution:

∵ the given number is neither terminating nor repeating decimal, it’s an irrational number.

Question: 17

The number 3.2463

Solution:

∵ the number is non - terminating but repeating decimal as after 2 places of decimal it repeats. ∴ it’s a rational number.

Question: 18

Which of the foll

Solution:

For the terminating decimal the denominator must be in the form $2^n \times 5^m$ where m and n are non - negative integers.

For option C denominator is 625 which can be written as $2^0 \times 5^4$.

∴ option C is the answer.

Question: 19

The decimal expan

Solution:

Denominator = $2^2 \times 5 = 2 \times 10$.

So, expression is equal to $\frac{37}{2 \times 10} = \frac{18.5}{10} = 1.85$

∴ it terminates after two decimal places.

Question: 20

The decimal of th

Solution:

The expression $\frac{14753}{1250} = 11.8024$, which terminates after 4 places of decimal.

Question: 21

The number 1.732

Solution:

∵ the given number is terminating decimal.

∴ it’s a rational number.

Question: 22

a and b are two p

Solution:

Least prime factor being 3 or 5 means that 2 is not a factor.

So the numbers are both odd.

a + b will be even no. as a is odd and b is odd {∵ odd + odd = even}

Least prime factor of an even no is 2.

Question: 23

$\sqrt{2}$ is a non - terminating non - repeating decimal. So, it's an irrational number.

Question: 24

$\frac{1}{\sqrt{2}}$ to be rational. So we can write it in the form of a/b where a and b are co - prime.

So , a/b = $\frac{1}{\sqrt{2}}$

And ∴ b/a = √2

Since b/a is rational but √2 is irrational.

By contradiction $\frac{1}{\sqrt{2}}$ is irrational.

Question: 25

Here 2 is rational and $\sqrt{2}$ is irrational. So their sum is irrational.

Question: 26

What is the least

Solution:

Least number that is divisible by all the natural numbers from 1 to 10 will be LCM of the numbers -

LCM of numbers from 1 to 10 = product of prime factors with highest powers = $2^3 \times 3^2 \times 5 \times 7$ = 2520.

Exercise : FORMATIVE ASSESSMENT (UNIT TEST)

Question: 1

The decimal repre

Solution:

Here the denominator 150 = $2 \times 3 \times 5^2$ which is not of the form $2^n \times 5^m$.Hence the given fraction is non - terminating in nature.

Also $\frac{71}{150} = \frac{71 \times 2}{150 \times 2} = \frac{142}{300} = 0.47\overline{3}$ (multiply and divide by 2)

Question: 2

Which of the foll

Solution:

For a terminating decimal, the denominator must be in the form $2^n \times 5^m$ where m and n are non negative integers.

For option B denominator is 80 which can be written as $2^4 \times 5$.

∴ option B is the answer.

Question: 3

One dividing a po

Solution:

Using Euclid’s division lemma –

$n = 9(q) + 7$, where q is the quotient when divided by 9.

Using this, we have

$3n - 1 = 3(9q + 7) - 1 \Rightarrow 3n - 1 = 27q + 21 - 1 \Rightarrow 3n - 1 = 27q + 20$Now, we have to set the above equation in a way, such that$3n - 1 = 9q' + r'$, where $0 \leq r < 9 \Rightarrow 3n - 1 = 27q + 18 + 2 \Rightarrow 3n - 1 = 9(3q + 2) + 2$According to Euclid's divison lemma, we have$r = 2$, when $3n - 1$ is divided by 9

Question: 4

Solve

Solution:

$0.\overline{68} = \frac{68}{99}$ and $0.\overline{73} = \frac{73}{99}$.

$\therefore, 0.\overline{68} + 0.\overline{73} = \frac{68+73}{99} = \frac{141}{99} = 1.\overline{42}$

Question: 5

Show that any num

Solution:

For a number to end with zero, it must contain 5 as one of it’s prime factor.

In case of 4^n, 5 can’t be a prime factor. So it can’t end with zero.

Question: 6

The HCF of two nu

Solution:

Let the other number be x.

We know product of numbers = LCM × HCF.

So, $x \times 81 = 27 \times 162$

And $x = 54$.

Question: 7

Examine whether <

Solution:

The denominator 30 can be written as $2 \times 3 \times 5$, which is not in the form $2^n \times 5^m$.

$\therefore$ it is not a terminating decimal.

Question: 8

First the simples

Solution:

Prime factors of the numbers are -

$148 = 2 \times 2 \times 37$

$185 = 5 \times 37$

So HCF = 37

Dividing the numbers we get numerator as 4 and denominator as 5.

Question: 9

Which of the foll

Solution:

$\sqrt{2}$, $\sqrt[3]{6}$, π, 0.232332333...

An irrational number is non - terminating, non - repeating and cannot be represented in a fractional form. So the decimals which are non - terminating and non - repeating are irrationals.

Question: 10

Prove that (2 +√3

Solution:

√3 is an irrational number and addition of a rational number and an irrational number is an irrational number.

∴, 2 + √3 is irrational, where √3 is an irrational number while 2 is a rational number.

Alternative - Assume that 2 + √3 is rational $\Rightarrow 2+\sqrt{3}=\frac{a}{b}$, where a and b are integers.

$\Rightarrow \sqrt{3}=\frac{a}{b}-2$

$\Rightarrow \sqrt{3}=\frac{a-2b}{b}$ we know that a, b and 2 are integers, therefor a - 2b is also an integertherefore

√3 will be rational.but we know that √3 is irrational.there is a contradictionso, 2 + √3 is an irrational number

Question: 11

Find the HCF and

Solution:

12 = 2 × 2 × 3

15 = 3 × 5

18 = 2 × 3 × 3

27 = 3 × 3 × 3

So HCF = 3

And LCM = product of prime factors with highest powers = $2^2 \times 3^3 \times 5 = 540$

Question: 12

Give an example o

Solution:

Let’s take (2 + $\sqrt{3}$) and (2 – $\sqrt{3}$).

These are irrational numbers.

Their sum = (2 + $\sqrt{3}$) + (2 – $\sqrt{3}$) = 4 {which is rational}.

Question: 13

Give prime factor

Solution:

4620 = 2 × 2 × 3 × 5 × 7 × 11 = 2^2× 3 × 5 × 7 × 11

Question: 14

Find the HCF of 1

Solution:

Prime factors of numbers are -

$1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

$1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$HCF = 2 \times 2 \times 2 \times 3 \times 3 = 72$

Question: 15

Find the HCF and

Solution:

HCF of numerators (8,10,16) = 2 and HCF of denominators (9,27,81) = 9.

LCM of numerators(8,10,16) = 80 and LCM of denominators(9,27,81) = 81.

So HCF of fraction $= \frac{\text{HCF of numerator}}{\text{LCM of denominator}} = \frac{2}{81}$.

and LCM of fraction $= \frac{\text{LCM of numerator}}{\text{HCF of denominator}} = \frac{80}{9}$.

Question: 16

Find the largest

Solution:

∵ we are getting 6 and 8 as remainder.

Subtracting 6 and 8 from the dividends will give us the exact division.

So let's subtract the remainder and find the HCF of numbers (largest number which divides both the number) -

We need to find HCF of 546 - 6 = 540 and 764 - 8 = 756.

Prime factorization of numbers -

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7$

HCF of given numbers $= 2 \times 2 \times 3 \times 3 \times 3 = 108$

Question: 17

Prove that

Solution:

Let us assume that $\sqrt{3}$ is a rational number.

That is, we can find integers a and b ($\neq 0$) such that $\sqrt{3} = (a/b)$

Suppose, a and b have a common factor other than 1, then we can divide by the common factor, and assume that a and b are co - prime.

So, $\sqrt{3}b = a$

$\Rightarrow 3b^2 = a^2$ (Squaring on both sides)

Therefore, a^2 is divisible by 3.

∴ 'a' is also divisible by 3.

So, we can write a = 3c for some integer c.

Equation (1) becomes,

$3b^2 = (3c)^2$

$3b^2 = 9c^2$

$b^2 = 3c^2$

This means that b^2 is divisible by 3, and so b is also divisible by 3.

Therefore, a and b have at least 3 as a common factor.

But this contradicts the fact that a and b are co - prime.

This contradiction has arisen because of our incorrect assumption that √3 is rational.

So, we conclude that √3 is irrational.

Question: 18

Show that every p

Solution:

Let a be any odd positive integer and b = 4.

By division lemma there exist integer q and r such that

$a = 4q + r$, where $0 \leq r \leq 4$

so $a = 4q$ or, $a = 4q + 1$ or, $a = 4q + 2$ or, $a = 4q + 3$

$4q + 1$ 4 is divisible by 2 but 1 is not divisible by 2,so it is an odd number

$4q + 2$ 4 is divisible by 2 and 2 is also divisible by 2,so it is an even number

$4q + 3$ 4 is divisible by 2 but 3 is not divisible by 2,so it is an odd number

$4q + 4$ 4 is divisible by 2 and 4 is also divisible by 2,so it is an even number

∴, any odd integer is of the form $4q + 1$ or, $4q + 3$.

Question: 19

Show that one and

Solution:

We applied Euclid Division algorithm on n and 3.

$a = bq + r$ on putting $a = n$ and $b = 3$

$n = 3q + r$, $0 < r < 3$

So,

$n = 3q$ (I)

$n = 3q + 1$ (II)

$n = 3q + 2$ (III)

Case - I: When $n = 3q$

In this case, we have

$n = 3q$, which is divisible by 3

Now, $n = 3q$

$n + 2 = 3q + 2$

$n + 2$ leaves remainder 2 when divided by 3

Again, $n = 3q$

$n + 4 = 3q + 4 = 3(q + 1) + 1$

$n + 4$ leaves remainder 1 when divided by 3

$n + 4$ is not divisible by 3.

Thus, n is divisible by 3 but $n + 2$ and $n + 4$ are not divisible by 3.

Case - II: when n = 3q + 1

In this case, we have

n = 3q + 1,

n leaves remainder 1 when divided by 3.

n is divisible by 3

Now, n = 3q + 1

n + 2 = (3q + 1) + 2 = 3(q + 1)

n + 2 is divisible by 3.

Again, n = 3q + 1

n + 4 = 3q + 1 + 4 = 3q + 5 = 3(q + 1) + 2

n + 4 leaves remainder 2 when divided by 3

n + 4 is not divisible by 3.

Thus, n + 2 is divisible by 3 but n and n + 4 are not divisible by 3.

Case - III: When n + 3q + 2

In this case, we have

n = 3q + 2

n leaves remainder 2 when divided by 3.

n is not divisible by 3.

Now, n = 3q + 2

n + 2 = 3q + 2 + 2 = 3(q + 1) + 1

n + 2 leaves remainder 1 when divided by 3

n + 2 is not divisible by 3.

Again, n = 3q + 2

n + 4 = 3q + 2 + 4 = 3(q + 2)

n + 4 is divisible by 3.

Thus, n + 4 is divisible by 3 but n and n + 2 are not divisible by 3.

Question: 20

Show that

Solution:

$3\sqrt{2}$ is an irrational number and subtraction of a rational number and an irrational number is an irrational number.

∴ $4 + 3\sqrt{2}$ is irrational, where $3\sqrt{2}$ is an irrational number while 4 is a rational number.

Alternative - Assume that $4 + 3\sqrt{2}$ is rational$4 + 3\sqrt{2} = a/b$, where a and b are integers. $\Rightarrow 3\sqrt{2} = a/b - 4 \Rightarrow 3\sqrt{2} = a/b - 4b/b \Rightarrow \sqrt{2} = (a - 4b) / 3b$we know that a, b, 3 and 4 are integers and they are also rational {i.e RHS is rational}therefore $\sqrt{2}$ will be rational.but we know that $\sqrt{2}$ is irrational.there is a contradictionso, $4 + 3\sqrt{2}$ is an irrational number