30. Linear Programming

Exercise 30.1

1. Question

A small manufacturing firm produces two types of gadgets A and B, which are first processed in the foundry, then sent to the machine shop for finishing. The number of man - hours of labour required in each shop for the production of each unit of A and B, and the number of man - hours the firm has available per week are as follows:

Gadget	Foundry	Machine - shop
A	10	5
B	6	4
Firm's capacity per week	1000	600

The profit on the sale of A is Rs 30 per unit as compared with ₹ 20 per unit of B. The problem is to determine the weekly production of gadgets A and B, so that the total profit is maximized. Formulate this problem as a LPP.

Answer

The given data can be shown in a table as follows:

Gadget	Foundry	Machine Shop	Profit
A	10	5	Rs 30
B	6	4	Rs 20
Firm's capacity per week	1000	600	

Now, let the required weekly production of gadgets A and B be x and y respectively

As it is given that profit on each gadget A is Rs. 30 and that on B is Rs. 20, so profit on x and y number of gadgets A and B respectively are 30x and 20y.

If z = total profit then we have,

$z = 30x + 20y$

Further, it is also given that the production of A and B requires 10 hours per week and 6 hours per week in the foundry. Also, the maximum capacity of the foundry is given as 1000 hours.

Now, x units of A and y units of B will require 10x + 6y hours

So, we have

$10x + 6y \leq 1000$

This is our first constraint.

Given, production of one unit gadget A requires 5x hours per week and y units of gadget B requires 4y hours per week, but the maximum capacity of the machine shop is 600 hours per week.

So, $5x + 4y \leq 600$

This is our second constraint.

Hence, the mathematical formulation of LPP is:

Find x and y which will maximize $z = 30x + 20y$

Subject to constraints,

$10x + 6y \leq 1000$

$5x + 4y \leq 600$

and also, as production cannot be less than zero, so x, y ≥ 0

2. Question

A company is making two products A and B. The cost of producing one unit of products A and B are Rs 60 and Rs 80 respectively. As per the agreement, the company has to supply at least 200 units of product B to its regular customers. One unit of product A requires one machine hour whereas product B has machine hours available abundantly within the company. Total machine hours available for product A are 400 hours. One unit of each product A and B requires one labour hour each and total of 500 labour hours are available. The company wants to minimize the cost of production by satisfying the given requirements. Formulate the problem as a LPP.

Answer

The given data can be shown in a table as follows:

Product	Machine Hours	Labour Hours	Profit
A	1	1	Rs 60
B	-	1	Rs 80
Total Capacity Minimum capacity of product B is 200 units	400 for A	500	

Let production of product A be x units and of be B y units.

Given,

Profit on 1 unit of product A = Rs.60

Profit on 1 unit of product B = Rs.80

So, profit on x units of A and y units of B is 60x and 80y respectively.

Let z = total profit,

So, we have

z = 60x + 80y

Given, a minimum supply of product B is 200

So, y ≥ 200 (First constraint)

Given that, production of one unit of product A requires 1 hour of machine hours, so x units of product A require x hours but total machine time available for product A is 400 hours

So, x ≤ 400 (Second constraint)

Given, each unit of product A and B requires one hour of labour hour, so x units of product A require x hours and y units of product B require y hours of labour hours, but total labour hours available is 500 so

x + y ≤ 500 (Third constraint)

Hence, mathematical formulation of LPP is,

Find x and y which

Minimize z = 60x + 80y

Subject to constraints,

y ≥ 200

x ≤ 400

x + y ≤ 500

and also, as production cannot be less than zero, so x, y ≥ 0

3. Question

A firm manufactures 3 products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 respectively. The firm has 2 machines and below is the required processing time in minutes for each machine on each product.

Machine	Products		
	A	B	C
M1	4	3	5
M2	2	2	4

Machines M_1 and M_2 have 2000 machine minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's. Set up a LPP to maximize the profit.

Answer

The given data can be formulated in a table as below.

Product	Machine (M_1)	Machine (M_2)	Profit
A	4	2	3
B	3	2	2
C	5	4	4
Capacity maximum	2000	2500	

Let, required production of product A, B and C be x, y and z units respectively.

Given, profit on one unit of product A, B and C are Rs 3, and Rs 2, Rs 4.

So, profit on x, y, z units of A, B, C Rs 3x, Rs 2y, Rs 4z.

Let U be the total profit, so

$U = 3x + 2y + 4z$

Given, one unit of product A, B and C requires 4, 3 and 5 minutes on machine M_1. So, x units of A, y units of B and z units of C need 4x, 3y and 5z minutes. Maximum capacity on machine M_1 is 2000 minutes, so,

$4x + 3y + 5z \leq 200\ 0$ (First constraint)

Given, one unit of product A, B and C requires 2, 2 and 4 minutes on machine M_2. So, x units of A, y units of B and z units of C require 2x, 2y and 4z minutes. Maximum capacity on machine M_2 is 2500 minutes, so,

$2x + 2y + 4z \leq 250\ 0$ (Second constraint)

Also, given that firm must manufacture more than 100 A's, 200 B's, 50 C's also not more than 150 A's, so,

$100 \leq x \leq 150$,

$y \geq 200$ (Other constraints)

$z \geq 50$

Hence, mathematical formulation of LPP is:

Find x, y and z which maximize $U = 3x + 2y + 4z$

Subject of constraints,

$4x + 3y + 5z \leq 2000$

$2x + 2y + 4z \leq 2500$

$100 \leq x \leq 150$,

$y \geq 200$

$z \geq 50$

and also, as production cannot be less than zero, so $x, y \geq 0$

4. Question

A firm manufactures two types of products A and B and sells them at a profit of Rs 2 on type A and Rs 3 on type B. Each product is processed on two machines M_1 and M_2. Type A requires one minute of processing time on M_1 and two minutes of M_2; type B requires one minute on M_1 and one minute on M_2. The machine M_1 is available for not more than 6 hours 40 minutes while machine M_2 is available for 10 hours during any working day. Formulate the problem as a LPP.

Answer

Given equation can be written in tabular form as

Product	M_1	M_2	Profit
A	1	2	2
B	1	1	3
Capacity	6hr. 40 min = 400 min	10 hr. = 600 min	

Let required production of product A be x units and product B be y units.

Given, profit on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B will be Rs 2x and Rs 3y respectively.

Let total profit be Z, so Z = 2x + 3y

Given, production of one unit of product A and B require 1 and 1 minute on machine M_1 respectively, so production of x units of product A and y units of product B require x minutes and y minutes on machine M_1 but total time available on machine M_1 is 600 minutes, so

$x + y \leq 400$ (First constraint)

Given, production of one unit of product A and B require 2 minutes and 1 minutes on machine M_2 respectively. So, production of x units of product A and y units of product B require 2x minutes and y minutes respectively on machine M_2, but machine M_2 is available for 600 minutes, so

$2x + y \leq 600$ (Second constraint)

Hence, the mathematical formulation of LPP is:

Find x and y which

maximize Z = 2x + 3y

Subject to constraints,

$x + y \leq 400$

$2x + y \leq 600$

and, $x, y \geq 0$ [Since production of the product cannot be less than zero]

5. Question

A rubber company is engaged in producing three types of tyres A, B and C. Each type requires processing in two plants, Plant I and Plant II. The capacities of the two plants, in the number of tyres per day, are as follows:

Plant	A	B	C
I	50	100	100
II	60	60	200

The monthly demand for tyre A, B and C is 2500, 3000 and 7000 respectively. If plant I costs Rs 2500 per day, and plant II costs Rs 3500 per day to operate, how many days should each be run per month to minimize cost while meeting the demand? Formulate the problem as LPP.

Answer

Plant	A	B	C	Cost
I	50	100	100	2500
II	60	60	200	3500
Monthly demand	2500	3000	7000	

Let plant 1 requires x days, and plant II requires y days per month to minimize cost.

Given, plant I and II costs Rs 2500 per day and Rs 3500 per day respectively, so cost to run plant I and II are Rs 2500x and Rs 3500y per month.

Let Z be the total cost per month,

So, Z = 2500x + 3500y

Given, production of tyre A from plant I and II is 50 and 60 respectively, so production of tyre A from plant I and II will be 50x and 60y respectively per month but the maximum demand of tyre A is 2500 per month so,

100x + 60y ≥ 2500 [First constraint]

Given, production of tyre B from plant I and II is 100 and 60 respectively, so production of tyre B from plant I and II will be 100x and 60y per month respectively but the maximum demand of tyre B is 3000 per month, so

100x + 200y ≥ 3000 [Second constraint]

Given, production of tyre C from plant I and II is 100 and 200 respectively. So, production of tyre C from plant I and II will be 100x and 200y per month respectively but the maximum demand of tyre C is 7000 per day, so

100x + 200y ≥ 7000 [Third constraint]

Hence, mathematical formulation of LPP is,

Find x and y which

Minimize Z = 2500x + 3500y

Subject to constraint,

50x + 60y ≥ 2500

100x + 60y ≥ 3000

100x + 200y ≥ 7000

And, x, y ≥ 0 [Since number of days cannot be less than zero]

6. Question

A company sells two different products A and B. The two products are produced in a common production process and are sold in two different markets. The production process has a total capacity of 45000 man - hours. It takes 5 hours to produce a unit of A and B hours to produce a unit of B. The market has been surveyed and company officials feel that the maximum number of units of A that can be sold is 7000 and that of B is 10, 000. If the profit is Rs 60 per unit for the product A and Rs 40 per unit for the product B, how many units of each product should be sold to maximize profit? Formulate the problem as LPP.

Answer

Product	Man Hours	Max. demand	Profit
A	5	7000	60
B	3	10000	40
Total Capacity	45000		

Let required production of product A be x units and production of product B be y units.

Given, profits on one unit of product A and B are Rs 60 and Rs 40 respectively, so profits on x units of product A and y units of product B are Rs 60x and Rs 40y.

Let Z be the total profit, so Z = 60x + 40y

Given, production of one unit of product A and B require 5 hours and 3 hours respectively man hours, so x unit of product A and y units of product B require 5x hours and 3y hours of man hours respectively but total man hours available are 45000 hours, so

$5x + 3y \le 45000$ (First constraint)

Given, demand for product A is maximum 7000, so

$x \le 7000$ (Second constraint)

Hence, mathematical formulation of LPP:

Find x and y which

maximize $Z = 60x + 40y$

Subject to constraints,

$5x + 3y \le 45000$

$x \le 7000$

$y \le 10000$

$x, y \le 0$ [Since production cannot be less than zero]

7. Question

To maintain his health a person must fulfil certain minimum daily requirements for several kinds of nutrients. Assuming that there are only three kinds of nutrients – calcium, protein and calories and the person's diet consists of only two food items, I and II, whose price and nutrient contents are shown in the table below:

	Food I (per lb)	Food II (per lb)	Minimum daily requirement for the nutrient
Calcium	10	5	20
Protein	5	4	20
Calories	2	6	13
Price (Rs)	60	100	

What combination of two food items will satisfy the daily requirement and entail the least cost? Formulate this as a LPP.

Answer

Let x and y be the packets of 25 gm of Food I and Food II purchased. Let Z be the price paid. Obviously, price has to be minimized.

Take a mass balance on the nutrients from Food I and II,

Calcium $10x + 4y \ge 20$

$5x + 2y \ge 10$ (i)

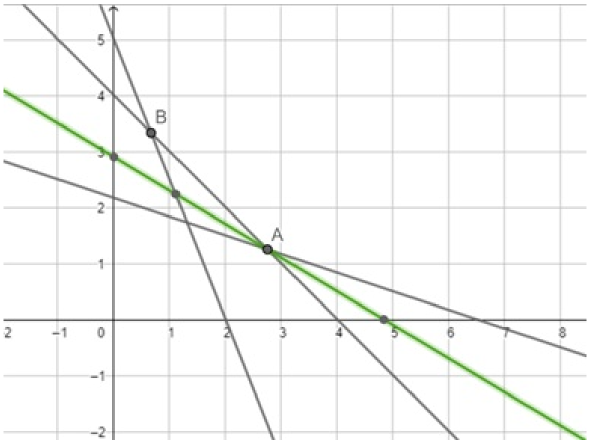
Protein $5x + 5y \ge 20$

$x + y \ge 4$ (ii)

Calories $2x + 6y > 13$ (iii)

These become the constraints for the cost function, Z to be minimized i.e., $0.6x + y = 7$, given cost of Food I is Rs 0.6/ - and Rs 1/ - per lb

From (i), (ii) & (iii) we get points on the X & Y - axis as [0, 5] & [2, 0] ; [0, 4] & [4, 0] ; [0, 13/6] & [6.5, 0] Plotting these



The smallest value of Z is 2.9 at the point (2.75, 1.25). We cannot say that the minimum value of z is 2.9 as the feasible region is unbounded.

Therefore, we have to draw the graph of the inequality $0.6x + y < 2.9$

Plotting this to see if the resulting line (in green) has any point common with the feasible region. Since there are no common points this is the minimum value of the function Z and the mix is

Food I = 2.75 lb; Food II = 1.25 lb; Price = Rs 2.9

When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities., this optimal value must occur at a corner point (vertex) of the feasible region.

Here the feasible region is the unbounded region A - B - C - D

Computing the value of 7 at the corner points of the feasible region ABHG

Point	Corner Point	Value of Z = 0.6x + y
A	2, 5	6.2
B	0.67, 3.33	3.73
C	2.75, 1.25	2.9
D	6.5, 2.16	6.06

8. Question

A manufacturer can produce two products, A and B, during a given time period. Each of these products requires four different manufacturing operations: grinding, turning, assembling and testing. The manufacturing requirements in hours per unit of products A and B are given below.

	A	B
Grinding	1	2
Turning	3	1
Assembling	6	3
Testing	5	4

The available capacities of these operations in hours for the given time period are: grinding 30; turning 60, assembling 200; testing 200. The contribution to profit is Rs 20 for each unit of A and Rs 30 for each unit of B. The firm can sell all that it produces at the prevailing market price. Determine the optimum amount of A and B to produce during the given time period. Formulate this as a LPP.

Answer

Product	Grinding	Turning	Assembling	Testing	Profit
A	1	3	6	5	2
B	2	1	3	4	3
Maximum Capacity	30 hours	60 hours	200 hours	200hours	

Let required production of product A and B be x and y respectively

Given, profits on one unit of product A and B are Rs 2 and Rs 3 respectively, so profits on x units of product A and y units of product B are given by 2x and 3y respectively. Let Z be total profit, so

$Z = 2x + 3y$

Given, production of 1 unit of product A and B require 1 hour and 2 hours of grinding respectively, so, production of x units of product A and y units of product B require x hours and 2y hours of grinding respectively but the maximum time available for grinding is 3 hours, so

$x + 2y \le 30$ (First constraint)

Given, production of 1 unit of product A and B require 3 hours and 1 hours of turning respectively, so x units of product A and y units of product B require 3x hours and y hours of turning respectively but total time available for turning is 60 hours, so

$3x + y \le 60$ (Second constraint)

Given, production of 1 unit of product A and B require 6 hour and 3 hours of assembling respectively, so production of x units of product A and y units of product B require 6x hours and 3y hours of assembling respectively but total time available for assembling is 200 hours, so

$6x + 3y \le 200$ (Third constraint)

Given, production of 1 unit of product A and B require 5 hours and 4 hours of testing respectively, so production of x units of product A and y units of product B require 5x hours and 4y hours of testing respectively but total time available for testing is 200 hours, so

$5x + 4y \le 200$ (Fourth constraint)

Hence, mathematical formulation of LPP is,

Find x and y which

maximize $Z = 2x + 3y$

Subject to constraints,

$x + 2y \le 30$

$3x + y \le 60$

$6x + 3y \le 200$

$5x + 4y \le 200$

and, $x, y \ge 0$ [Since production cannot be negative]

9. Question

Vitamins A and B are found in two different foods F_1 and F_2. One unit of food F1 contains 2 units of vitamin A and 3 units of vitamin B. One unit of food F_2 contains 4 units of vitamin A and 2 units of vitamin B. One unit of food F_1 and F_2 cost ₹ 50 and 25 respectively. The minimum daily requirements for a person of vitamin A and B is 40 and 50 units respectively. Assuming that anything in excess of daily minimum requirement of vitamin A and B is not harmful, find out the optimum mixture of food F_1 and F_2 at the minimum cost which meets the daily minimum requirement of vitamin A and B. Formulate this as a LPP.

Answer

Given information can be tabulated as below:

Foods	Vitamin A	Vitamin B	Cost
F_1	2	3	5
F_2	4	2	2.5
Minimum daily requirement	40	50	

Let required quantity of food F_1 be x units and quantity of food F_2 be y units.

Given, costs of one unit of food F_1 and F_2 are Rs 5 and Rs 2.5 respectively, so costs of x units of food F_1 and y units of food F_2 are Rs 5x and Rs 2.5y respectively.

Let Z be the total cost, so

Z = 5x + 2.5y

Given, one unit of food F_1 and food F_2 contain 2 and 4 units of vitamin A respectively, so x unit of Food F_1 and y units of food F_2 contain 2x and 4y units of vitamin A respectively, but minimum requirement of vitamin A is 40 unit, so

$2x + 4y \geq 40$ (First constraint)

Given, one unit of food F_1 and food F_2 contain 3 and 2 units of vitamin B respectively, so x unit of Food F_1 and y units of food F_2 contain 3x and 2y units of vitamin B respectively, but minimum daily requirement of vitamin B is 40 unit, so

$3x + 2y \geq 50$ (Second constraint)

Hence, mathematical formulation of LPP is,

Find x and y which

Minimize Z = 5x + 2.5y

Subject to constraint,

$2x + 4y \geq 40$

$3x + 2y \geq 50$

$x, y \geq 0$ [Since requirement of food F_1 and F_2 cannot be less zero.]

10. Question

An automobile manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop A, which performs the basic assembly operation, must work 5 man - days on each truck but only 2 man - days on each automobile. Shop B which performs finishing operations, must work 3 man - days for each automobile or truck that it produces. Because of men and machine limitations, shop A has 180 man - days per week available while shop B has 135 man - days per week. If the manufacturer makes a profit of Rs 30000 on each truck and Rs 2000 on each automobile, how many of each should he produce to maximize his profit? Formulate this as a LPP.

Answer

Let number of automobiles produces be x and let the number of trucks

Produced be y.

Let Z be the profit function to be maximized.

Z = 2000x + 30000y

The constraints are on the man hours worked

Shop A $2x + 5y \leq 180$ (i) assembly

Shop B $3x + 3y \leq 135$ (ii) finishing

$x, y \geq 0$

Corner points can ve obtained from

$2x = 3y + 5y = 180 \Rightarrow x = 0; y = 36$ and $x = 90; y = 0$

$3x + 3y \leq 135 \Rightarrow x = 0; y = 45$ and $x = 45; y = 0$

Solving (i) and (ii) gives x = 15 and y = 30

Corner point	Value of Z = 2000x + 30000y
0, 0	0
0, 36	10, 80, 000
15, 30	9, 30, 000
45, 0	90, 000

Thus 0 automobiles and 36 trucks give max. profit of Rs 10, 80, 000/ -

11. Question

Two tailors A and B earn Rs 150 and Rs 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. Form a linear programming problem to minimize the labour cost to produce at least 60 shirts and 32 pants.

Answer

The data can be represented in a table below

	Taylor A		Taylor B	Limit
Variable	x		Y	
Shirts	6x	+	10y	≥ 60
Pants	4x	+	4y	≥ 32
Earn Rs.	150	+	200	Z

To minimize labour cost means to assume to minimize the earnings i.e,

Min Z = 150x + 200y

With constraints

x, y ≥ 0 at least 1 shirt and 1 pant is required

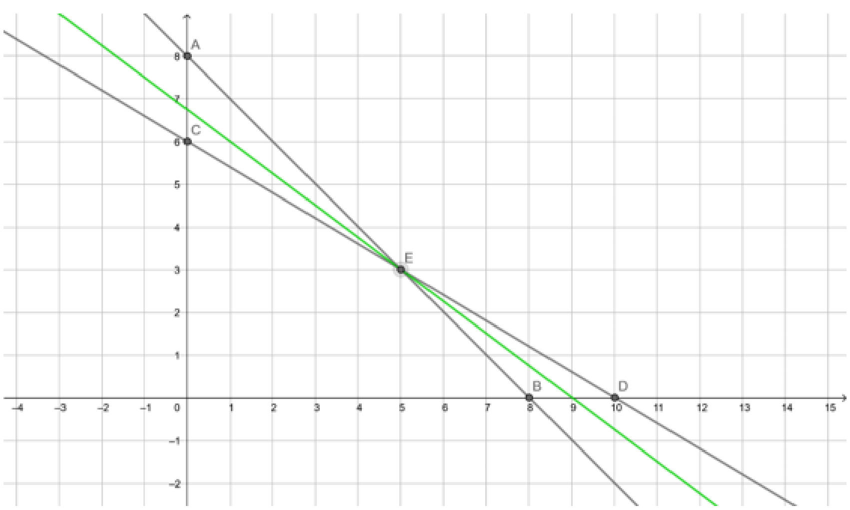
6x + 10y ≥ 60 require at least 60 shirts

4x + 4y ≥ 32 require at least 32 pants

On solving the above inequalities as equations, we get,

x = 5 and y = 3

other corner points obtained are [0, 6], [10, 0], [0, 8] and [8, 0]



The feasible region is the upper unbounded region A - E - D

Point E(5, 3) may not be minimal value. So, plot 150x + 200y < 1350 to see

If there is a common region with A - E - D.

The green line has no common point, therefore

Corner	Value of Z = 150x + 200y
0, 8	0
10, 0	1500
5, 3	1350

Thus, stitching 5 shirts and 3 pants minimizes labour cost to Rs 1350/ -

12. Question

An airline agrees to charter planes for a group. The group needs at least 160 first class seats and at least 300 tourist class seats. The airlines must use at least two of its model 314 planes which have 20 first class and 30 tourist class seats. The airline will also use some of its model 535 planes which have 20 first class seats and 60 tourist class seats. Each flight of a model 314 plane costs the company Rs 100, 000 and each flight of a model 535 plane costs Rs 150, 000. How many of each type of plane should be used to minimize the flight cost? Formulate this as a LPP.

Answer

	Model 314			
Variable	X		y	
F class	20x	+	20y	≥ 160
T class	30x	+	60y	≥ 300
Cost	1.x lakh	+	1.5y lakh	Z

The above LPP can be presented in a table above.

The flight cost is to be minimized i.e; Min Z = x + 1.5y

the constraints

x ≥ 2 at least 2 planes of model 314 must be used

y ≥ 0 at least 1 plane of model .53.5 must be used

20x + 20y ≥ 160 require at least 160 F class seats

30x + 60y ≥ 300 require at least 300 T class seats

Solving the above inequalities as equations we get,

When x = 0, y = 8 and when y = 0, x = 8

When x = 0, y = 5 and when y = 0, x = 10

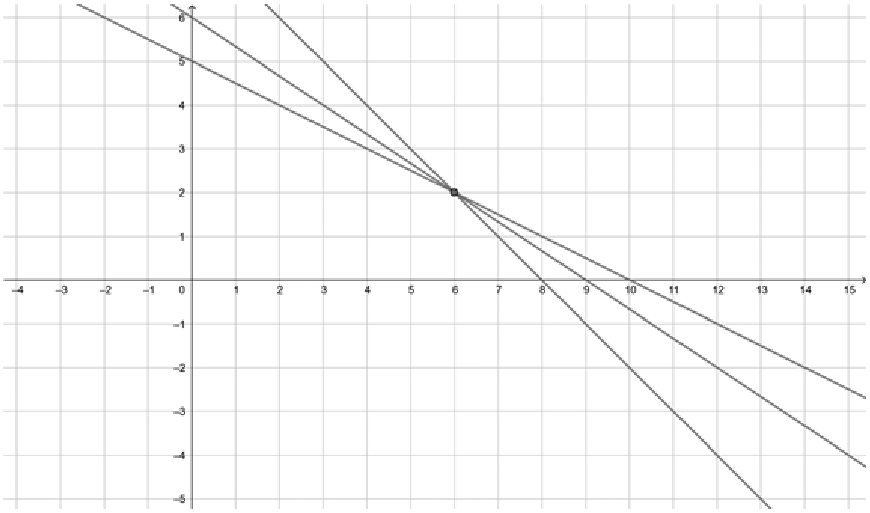
We get an unbounded region 8 - E - 10 as a feasible solution. Plotting the corner points and evaluating we have,

Corner point	Value of Z = x + 1.5y
10, 0	10
0, 8	12
6, 2	9

Since we obtained an unbounded region as the feasible solution a plot of Z (x + 1.5y< 9) is plotted.

Since there are no common points point E is the point that gives a minimum value.

Using 6 planes of model 314 & 2 of model 535 gives minimum cost of 9 lakh rupees.



13. Question

Amit's mathematics teacher has given him three long lists of problems with the instruction to submit not more than 100 of them (correctly solved) for credit. The problem in the first set are worth 5 points each, those in the second set are worth 4 points each, and those in the third set are worth 6 points each. Amit knows from experience that he requires on the average 3 minutes to solve a 5 point problem, 2 minutes to solve a 4 point problem, and 4 minutes to solve a 6 point problem. Because he has other subjects to worry about, he cannot afford to devote more than $3\frac{1}{2}$ hours altogether to his mathematics assignment. Moreover, the first two sets of problems involve numerical calculations and he knows that he cannot stand more than $2\frac{1}{2}$ hours work on this type of problem. Under these circumstances, how many problems in each of these categories shall he do in order to get maximum possible credit for his efforts? Formulate this as a LPP.

Answer

Sets	Time requirement	Points
I	3	5
II	2	4
III	4	6
Time for all three sets = $3\frac{1}{2}$ hours		
Time for set I and set II = $2\frac{1}{2}$ hours		
Maximum number of questions = 100		

Let he should solve x, y, z questions from set I, II and III respectively.

Given, each question from set I, II, III earn 5, 4, 6 points respectively, so x questions of set I, y questions of set II and z questions of set III earn 5x, 4y and 6z points, let total point credit be U

So, U = 5x + 4y + 6z

Given, each question of set I, II and III require 3, 2 and 4 minutes respectively, so, x questions of set I, y questions of set II and z questions of set III require 3x, 2y and 4z minutes respectively but given that total time to devote in all three sets is

$3\frac{1}{2}$ hours = 210 minutes and first two sets is $2\frac{1}{2}$ hours = 150 minutes

So,

3x + 2y + 4z ≤ 210 (First constraint)

3x + 2y ≤ 150 (Second constraint)

Given, total number of questions cannot exceed 100

So, x + y + z ≤ 100 (Third constraint)

Hence, mathematical formulation of LPP is

Find x and y which

maximize U = 5x + 4y + 6z

Subject to constraint,

3x + 2y + 4z ≤ 210

3x + 2y ≤ 150

x + y + z ≤ 100

x, y, z ≤ 0 [Since number of questions to solve from each set

cannot be less than zero.]

14. Question

A farmer has a 100 - acre farm. He can sell the tomatoes, lettuce, or radishes he can raise. The price he can obtain is Rs 1 per kilogram for tomatoes, Rs 0.75 a head for lettuce and Rs 2 per kilogram for radishes. The average yield per acre is 2000 kgs for radishes, 3000 heads of lettuce and 1000 kilograms of radishes. Fertilizer is available at Rs 0.50 per kg and the amount required per acre is 100 kgs each for tomatoes and lettuce and 50 kilograms for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man - days for tomatoes and radishes and 6 man – days for lettuce. A total of 400 man - days of labour are available at ₹ 20 per man - day. Formulate this problem as a LPP to maximize the farmer's total profit.

Answer

Given information can be tabulated below

Product	Yield	Cultivation	Price	Fertilizers
Tomatoes	2000 kg	5 days	1	100 kg
Lettuce	3000 kg	6 days	0.75	100 kg
Radishes	1000 kg	5 days	2	50 kg
Average 200 kg/per acre Total land = 100 Acre Cost of fertilizers = Rs. 0.50 per kg A total of 400 days of cultivation labour with Rs 20 per day				

Let required quantity of field for tomatoes, lettuce and radishes be x, y and z

acre respectively.

Given, costs of cultivation and harvesting of tomatoes, lettuce and radishes are 5 x20 = Rs 100, 6 x20 = Rs 120, 5 x20 = Rs 100 respectively per acre. Cost of fertilizers for tomatoes, lettuce and radishes 100 x 0.50 = Rs 50, 100 x 0.50 = Rs 50 and 50 x0.50 = Rs 25 respectively per acre.

So, total costs of production of tomatoes, lettuce and radishes are (Rs 100 + 50)x = Rs 150x, (Rs 120 + 50)y = Rs 170y and radishes are (Rs 100 + 25)z = Rs 125z respectively total selling price of tomatoes, lettuce and radishes, according to yield are (Rs2000x1)x = Rs 2000x, (Rs3000 x0.75)y = Rs 2250y and (Rs1000x2)z = Rs 2000z respectively.

Let U be the total profit,

So,

U = (2000x - 150x) + (2250y - 170y) + (2000z - 125z)

U = 1850x + 2080y + 1875z

Given, farmer has 100 - acre farm

So, x + y + z ≤ 100 (First constraint)

Number of cultivation and harvesting days are 400 So, 5x + 6y + 5z ≤ 400

Hence, mathematical formulation of LPP is,

Find x, y, z which

maximize U = 1850x + 2080y + 1875z

Subject to constraint,

x + y + z ≤ 100

5x + 6y + 5z ≤ 400

x, y, z ≥ 0 [Since farm used for cultivation cannot be less than zero.]

15. Question

A firm has to transport at least 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost of engaging each large van is Rs 400 and each small van is Rs 200. Not more than Rs 3000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost.

Answer

Given information can be tabulated as below:

Product	Dept 1	Dept 2	Selling Price	Labour Cost	Raw material cost
A	3	4	25	16	4
B	2	6	30	20	4
Capacity	130	260			

Let the required product of product A and B be x and y units respectively.

Given, labour cost and raw material cost of one unit of product A is Rs 16 and Rs 4, so total cost of product A is Rs 16 + Rs 4 = Rs 20

And given selling price of 1 unit of product A is Rs 25,

So, profit on one unit of product

A = 25 - 20 = Rs 5

Again, given labour cost and raw material cost of one unit of product B is Rs 20 and Rs 4 So, that cost of product B is Rs 20 + Rs 4 = Rs 24

And given selling price of 1 unit of product B is Rs 30

So, profit on one unit of product B = 30 - 24 = Rs 6

Hence, profits on x unit of product A and y units of product B are Rs 5x and Rs 6y respectively.

Let Z be the total profit, so,

$Z = 5x + 6y$

Given, production of one unit of product A and B need to process for 3 and 4 hours respectively in department 1, so production of x units of product A and y units of product B need to process for 3x and 4y hours respectively in Department 1. But total capacity of Department 1 is 130 hours,

So, $3x + 2y \le 130$ (First constraint)

Given, production of one unit of product. A and B need to process for 4 and 6 hours respectively in department 2, so production of x units of product A and y units of product B need to process for 4x and 6y hours respectively in Department 2 but total capacity of Department 2 is 260 hours

So, $4x + 6y \le 260$ (Second constraint)

Hence, mathematical formulation of LPP is,

Find x and y which

Maximize $Z = 5x + 6y$

Subject to constraint,

$3x + 2y \le 130$,

$4x + 6y \le 260$

$x, y \ge 0$ [Since production cannot be less than zero]

16. Question

A firm manufactures two products, each of which must be processed through two departments, 1 and 2. The hourly requirements per unit for each product in each department, the weekly capacities in each department, selling price per unit, labour cost per unit, and raw material cost per unit are summarized as follows :

	Product A	Product B	Weekly capacity
Department 1	3	2	130
Department 2	4	6	260
Selling price per unit	Rs. 25	Rs. 30	
Labour cost per unit	Rs. 16	Rs. 20	
Raw material cost per unit	Rs. 4	Rs. 4	

The problem is to determine the number of units to produce each product so as to maximize total contribution to profit. Formulate this as a LPP.

Answer

We have to maximize the profit by calculating the number of units needed to produce each product.

For Product A,

Cost Price per unit = Labour Cost + Raw material cost per unit

Cost Price per unit = 16 + 4 = Rs. 20

Selling Price per unit = Rs. 25

Profit per unit = S.P – C.P = 25 – 20 = Rs. 5

For Product B,

Cost Price per unit = Labour Cost + Raw material cost per unit

Cost Price per unit = 20 + 4 = Rs. 24

Selling Price per unit = Rs. 30

Profit per unit = S.P – C.P = 30 – 24 = Rs. 6

Let number of units produced of Product A be x and number of units produced of Product B be y.

Hence, Total Profit = $5x + 6y$

To Maximize : $z = 5x + 6y$

For Department 1,

$3x + 2y \leq 130$

For Department 2,

$4x + 6y \leq 260$

Hence,

$Z = 5x + 6y$

$3x + 2y \leq 130$

$4x + 6y \leq 260$

$x, y \geq 0$ [Since production cannot be less than zero]

Exercise 30.2

1. Question

Solve each of the following linear programming problems by graphical method.

Maximize $Z = 5x + 3y$

Subject to :

3x + 5y ≤ 15

5x + 2y ≤ 10

x, y ≥ 0

Answer

Given,

Objective function is: Z = 5x + 3y

Constraints are:

3x + 5y ≤ 15

5x + 2y ≤ 10

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

3x + 5y≤ 15 → 3x + 5y = 15 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 3 (0,3) - - - - first coordinate.

Put, y = 0 ⇒ x = 5 (5,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin the just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

5x + 2y≤ 10 → 5x + 2y = 10 (corresponding equation)

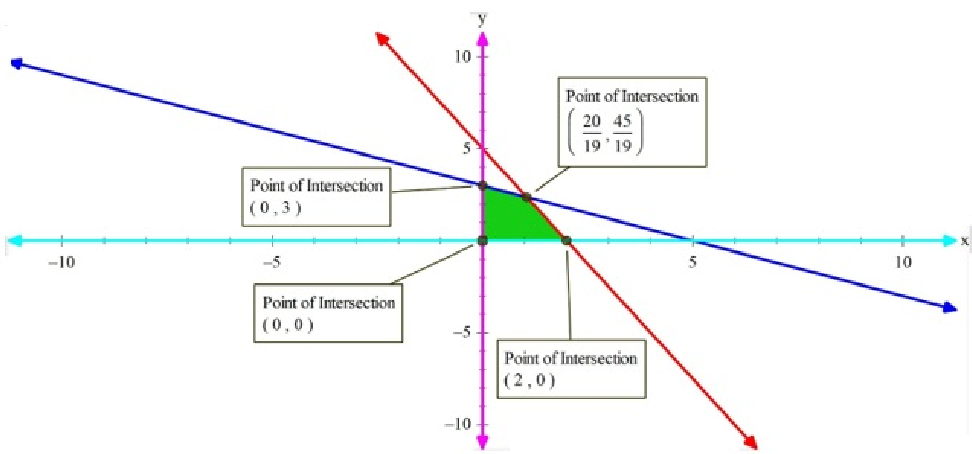
Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 5 (0,5) - - - - first coordinate.

Put, y = 0 ⇒ x = 2 (2,0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x - axis

Hence we obtain a plot as shown in figure:



The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations.

But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving 3x + 5y = 15 and 5x + 2y = 10 gives $\left(\frac{20}{19}, \frac{45}{19}\right)$

Similarly solving 3x + 5y = 15 and x = 0 gives (0, 3)

And solving 5x + 2y = 10 and y = 0 gives (2,0)

And solving x = 0 and y = 0 gives (0,0)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because$ Z = 5x + 3y

$\therefore$ Z at $\left(\frac{20}{19}, \frac{45}{19}\right) = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} = \frac{235}{19}$

Z at (0,3) = 5× 0 + 3× 3 = 9

Z at (2,0) = 5× 2 + 3× 0 = 10

Z at (0,0) = 0

We can see that Z is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ and max. value is $\frac{235}{19}$

$\therefore$ x = $\frac{20}{19}$; y = $\frac{45}{19}$ and maximum the value of Z is $\frac{235}{19}$

2. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 9x + 3y

Subject to :

2x + 3y ≤ 13

3x + y ≤ 5

x, y ≥ 0

Answer

Given,

Objective function is: Z = 9x + 3y

Constraints are:

2x + 3y ≤ 13

3x + y ≤ 5

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

2x + 3y ≤ 13 → 2x + 3y = 13 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 3 (0,13/3) - - - - first coordinate.

Put, y = 0 ⇒ x = 13/2 (13/2,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin the just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

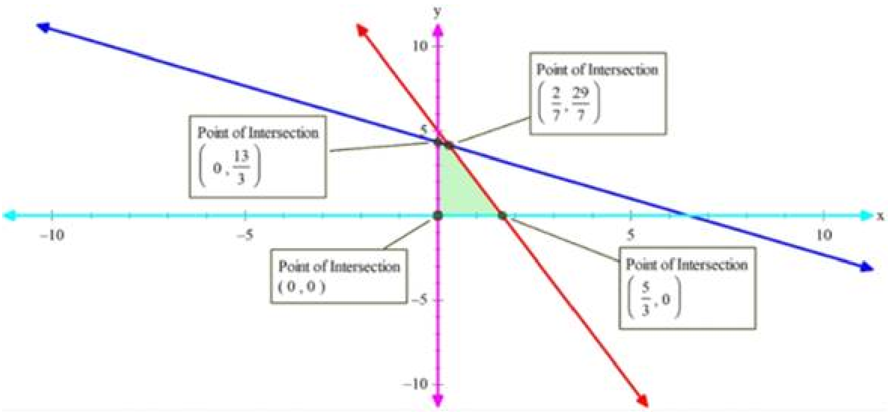
$3x + y \leq 5 \rightarrow 3x + y = 5$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 5$ (0,5) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 5/3$ (5/3,0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x – axis



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $2x + 3y = 13$ and $3x + y = 5$ gives $\left(\frac{2}{7}, \frac{29}{7}\right)$

Similarly solving $2x + 3y = 13$ and $x = 0$ gives (0, 13/3)

And solving $3x + y = 5$ and $y = 0$ gives (5/3,0)

And solving $x = 0$ and $y = 0$ gives (0,0)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 9x + 3y$

$\therefore$ Z at $\left(\frac{2}{7}, \frac{29}{7}\right) = 9 \times \frac{2}{7} + 3 \times \frac{29}{7} = \frac{105}{7} = 15$

Z at (0,13/3) = 9× 0 + 3× 13/3 = 13

Z at (5/3,0) = 9× (5/3) + 3× 0 = 15

Z at (0,0) = 0

We can see that Z is maximum at $\left(\frac{2}{7}, \frac{29}{7}\right)$ And max. value is 15

Z is also maximum at (5/3,0) with value = 15

This illustrates that $9x + 3y$ overlaps with $3x + y = 5$.

$\therefore$ Z is maximum at all the points on $3x + y = 5$, and max value is 15.

3. Question

Solve each of the following linear programming problems by graphical method.

Minimize Z = 18x + 10y

Subject to :

4x + y ≥ 20

2x + 3y ≥ 30

x, y ≥ 0

Answer

Given,

Objective function is: Z = 18x + 10y

Constraints are:

4x + y ≥ 20

2x + 3y ≥ 30

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

4x + y ≥ 20 → 4x + y = 20 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 20 (0,20) - - - - first coordinate.

Put, y = 0 ⇒ x = 5 (5,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin the just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

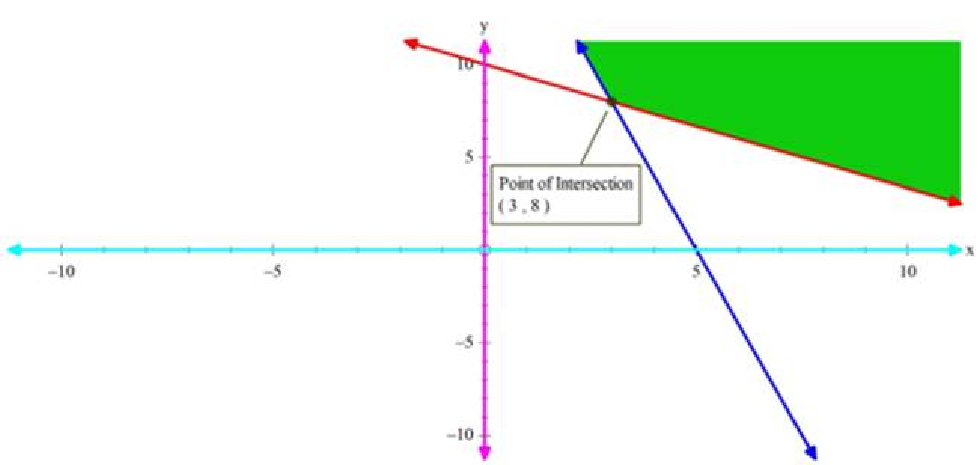
2x + 3y ≥ 30 → 2x + 3y = 30 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 10 (0,10) - - - - first coordinate.

Put, y = 0 ⇒ x = 15 (15,0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x – axis



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving 2x + 3y = 30 and 4x + y = 20 gives (3, 8)

Similarly solving 4x + y = 20 and x = 0 gives (0, 20)

And solving 2x + 3y = 30 and y = 0 gives (15,0)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 18x + 10y

∴ Z at (3, 8) = 18 × 3 + 8 × 10 = 134

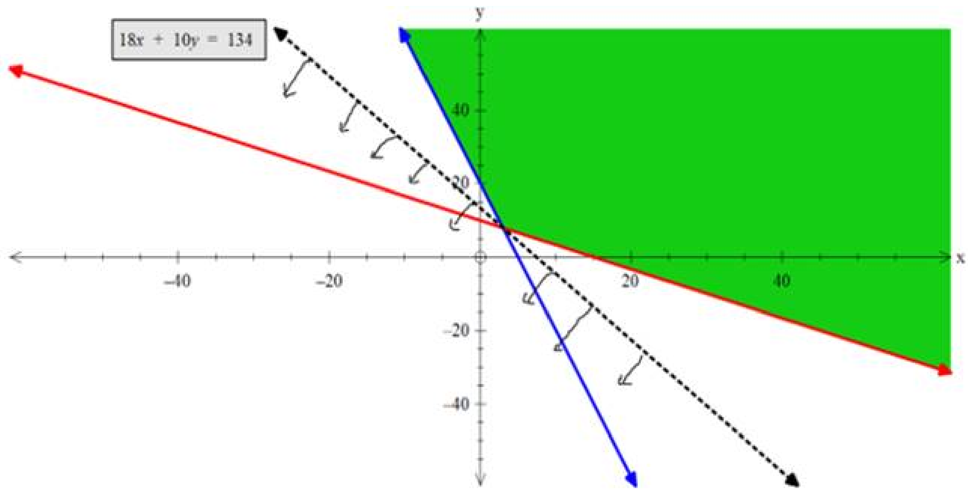
Z at (0,20) = 18 × 0 + 10 × 20 = 200

Z at (15,0) = 18 × (15) + 10 × 0 = 270

Note: It is unbounded so we can’t say directly by seeing that z = 134 is minimum because there might be possibility that some other points from feasible region may result a smaller number.

In such a case minima will not be possible.

So we check this by setting inequation corresponding to the minimum value of Z.



∴ inequation is 18x + 10y < 134

As (0,0) satisfies the inequation, so it will bound its region towards origin and hence will not overlap with the feasible region.

∴ We can say that minima are possible.

We can see that Z is minimum at (3, 8) and min. value is 134

∴ Z is minimum at x = 3 and y = 8 ; and min value is 134.

4. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 50x + 30y

Subject to :

2x + y ≤ 18

3x + 2y ≤ 34

x, y ≥ 0

Answer

Given,

Objective function is: Z = 50x + 30y

Constraints are:

2x + y ≤ 18

3x + 2y ≤ 34

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

2x + y ≤ 18 → 2x + y = 18 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 18 (0,18) - - - - first coordinate.

Put, y = 0 ⇒ x = 9 (9,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin the just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

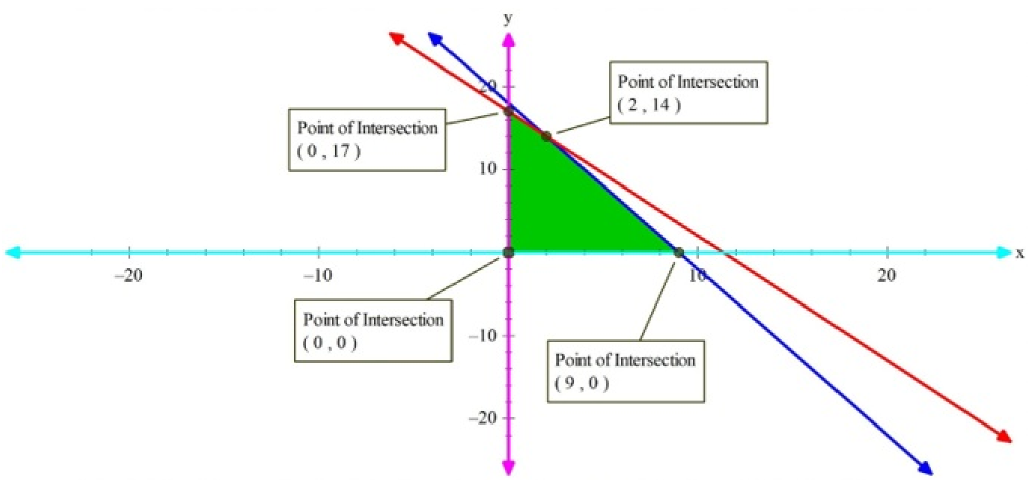
3x + 2y ≤ 34 → 3x + 2y = 34 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 17 (0, 17) - - - - first coordinate.

Put, y = 0 ⇒ x = 34/3 (34/3, 0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x – axis



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $3x + 2y = 34$ and $2x + y = 18$ gives (2,14)

Similarly solving $3x + 2y = 34$ and $x = 0$ gives (0, 17)

And solving $2x + y = 18$ and $y = 0$ gives (9,0)

And solving $x = 0$ and $y = 0$ gives (0,0)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 50x + 30y$

$\therefore$ Z at (2,14) $= 50 \times 2 + 30 \times 14 = 520$

Z at (0,17) $= 50 \times 0 + 30 \times 17 = 510$

Z at (9,0) $= 50 \times (9) + 30 \times 0 = 450$

Z at (0,0) = 0

We can see that Z is maximum at (2, 14) and max. value is 520

$\therefore$ Z is maximum at $x = 2$ and $y = 14$; and max value is 520.

5. Question

Solve each of the following linear programming problems by graphical method.

Maximize $Z = 4x + 3y$

Subject to :

$8x + 6y \leq 48$

$3x + 4y \leq 24$

$x \leq 5$, $y \leq 6$

$x, y \geq 0$

Answer

Given,

Objective function is: $Z = 4x + 3y$

Constraints are:

$8x + 6y \leq 48$

$3x + 4y \leq 24$

$x \leq 5$

$y \leq 6$

$x, y \geq 0$

First convert the given inequations into corresponding equations and plot them:

$8x + 6y \leq 48 \rightarrow 8x + 6y = 48$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 8$ (0,8) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 6$ (6,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$3x + 4y \leq 24 \rightarrow 3x + 4y = 24$ (corresponding equation)

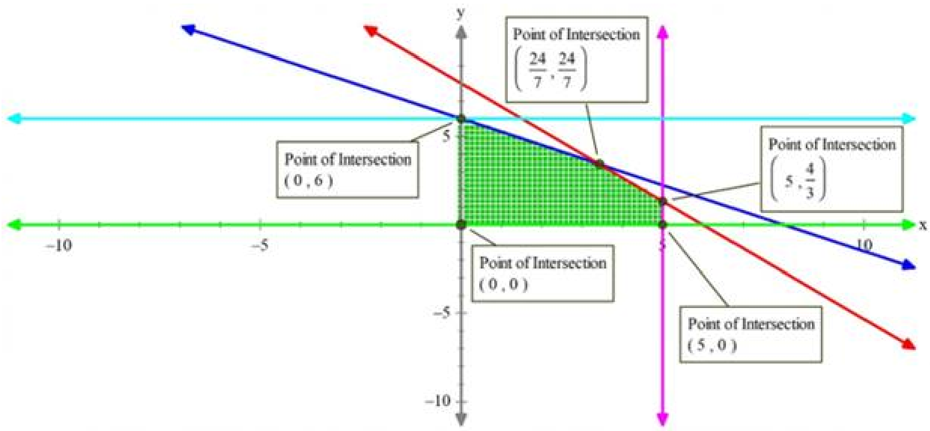
Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 6$ (0,6) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 8$ (8,0) - - - - second coordinate

$x = 0$ is the y - axis and $y = 0$ is the x - axis

$x = 5$ and $y = 6$ are lines parallel to y - axis and x - axis respectively.



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $3x + 4y = 24$ and $8x + 6y = 48$ gives $\left(\frac{24}{7}, \frac{24}{7}\right)$

Similarly, solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(0,0) (5,0) (0,6), $\left(\frac{24}{7}, \frac{24}{7}\right)$, $\left(5, \frac{4}{3}\right)$

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 4x + 3y$

$\therefore$ Z at $\left(\frac{24}{7}, \frac{24}{7}\right) = 4 \times \left(\frac{24}{7}\right) + 3 \times \left(\frac{24}{7}\right) = 24$

Z at (0,6) = $4 \times 0 + 3 \times 6 = 18$

Z at (5,0) = $4 \times (5) + 3 \times 0 = 20$

Z at (0,0) = 0

Z at $\left(5, \frac{4}{3}\right) = 4 \times 5 + 3 \times \left(\frac{4}{3}\right) = 24$

We can see that Z is maximum at $\left(\frac{24}{7}, \frac{24}{7}\right)$ and max. value is 24

∴ Z is maximum at x = 24/7 and y = 24/7 ; and max value is 24

Also it has a maximum at x = 5 and y = 4/3 with max value = 24

6. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 15x + 10y

Subject to :

2x + 3y ≤ 70

3x + 2y ≤ 80

x, y ≥ 0

Answer

Given,

Objective function is: Z = 15x + 10y

Constraints are:

2x + 3y ≤ 70

3x + 2y ≤ 80

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

2x + 3y ≤ 70 → 2x + 3y = 70 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 70/3 (0,70/3) - - - - first coordinate.

Put, y = 0 ⇒ x = 35 (35,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

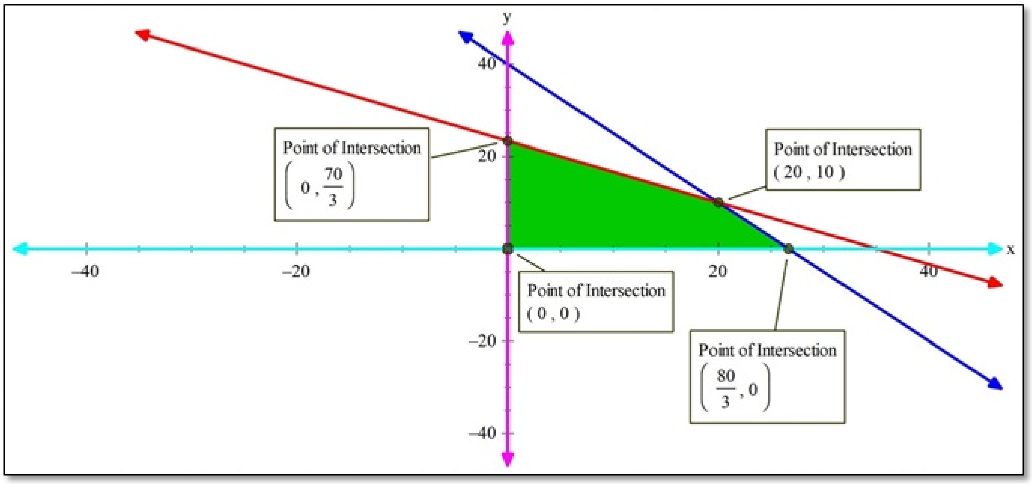
3x + 2y ≤ 80 → 3x + 2y = 80 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 40 (0,40) - - - - first coordinate.

Put, y = 0 ⇒ x = 80/3 (80/3,0) - - - - second coordinate

x = 0 is the y-axis and y = 0 is the x-axis



Hence, we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving 3x + 2y = 80 and 2x + 3y = 70 gives (20, 10)

Similarly solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

(0,0) (80/3 ,0) (0,70/3),(20,10)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 15x + 10y

∴ Z at (20, 10) = 15 × (20) + 10 × (10) = 400

Z at (0,70/3) = 15 × (70/3) + 10 × 0 = 350

Z at (80/3,0) = 15 × (80/3) + 10 × 0 = 400

Z at (0,0) = 0

Clearly, we can see that Z is maximum at $(20,10)$ and max. value is 400

∴ Z is maximum at x = 20 and y = 10 ; and max value is 400

Also it has a maximum at x = 80/3 and y = 0 with max value = 400

7. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 10x + 6y

Subject to :

2x + 5y ≤ 34

3x + y ≤ 12

x, y ≥ 0

Answer

Given,

Objective function is: Z = 10x + 6y

Constraints are:

2x + 5y ≤ 34

3x + y ≤ 12

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

2x + 5y ≤ 34 → 2x + 5y = 34 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 34/5 (0,34/5) - - - - first coordinate.

Put, y = 0 ⇒ x = 17 (17,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

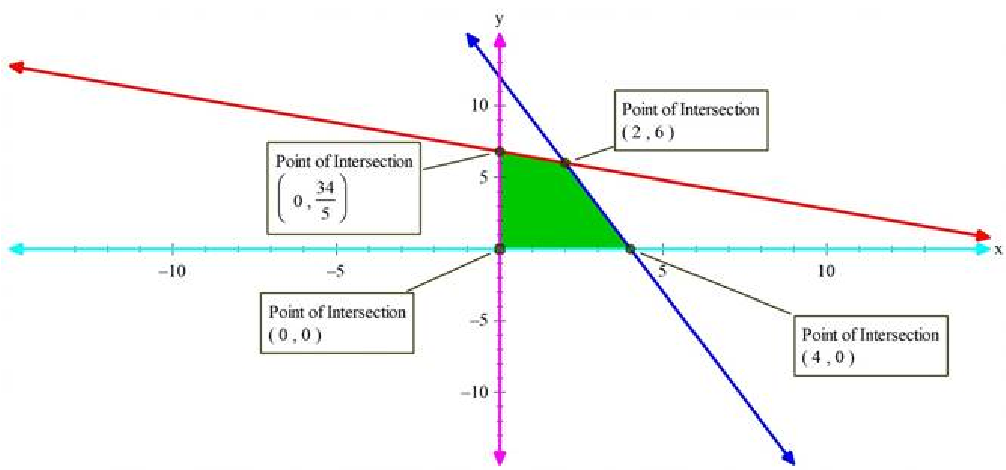
3x + y ≤ 12 → 3x + y = 12 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 12 (0,12) - - - - first coordinate.

Put, y = 0 ⇒ x = 12/3 = 4 (4,0) - - - - second coordinate

x = 0 is the y-axis and y = 0 is the x-axis



Hence, we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving 3x + y = 12 and 2x + 5y = 34 gives (2, 6)

Similarly solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

(0, 0), (4, 0), (0, 34/5), (2, 6)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ $Z = 10x + 6y$

∴ Z at (2, 6) = $10 \times (2) + 6 \times (6) = 56$

Z at (0,34/5) = $10 \times (0) + 6 \times (34/5) = 204/5$

Z at (4,0) = $10 \times (4) + 10 \times 0 = 40$

Z at (0,0) = 0

We can see that Z is maximum at (2,6) and max. value is 56

∴ Z is maximum at x = 2 and y = 16 ; and max value is 56

8. Question

Solve each of the following linear programming problems by graphical method.

Maximize $Z = 3x + 4y$

Subject to :

$2x + 2y \le 80$

$2x + 4y \le 120$

Answer

Given,

Objective function is: $Z = 3x + 4y$

Constraints are:

$2x + 2y \le 80$

$2x + 4y \le 120$

First convert the given inequations into corresponding equations and plot them:

$2x + 2y \le 80 \rightarrow 2x + 2y = 80$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 40$ (0,40) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 40$ (40,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in a plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$2x + 4y \le 120 \rightarrow 2x + 4y = 120$ (corresponding equation)

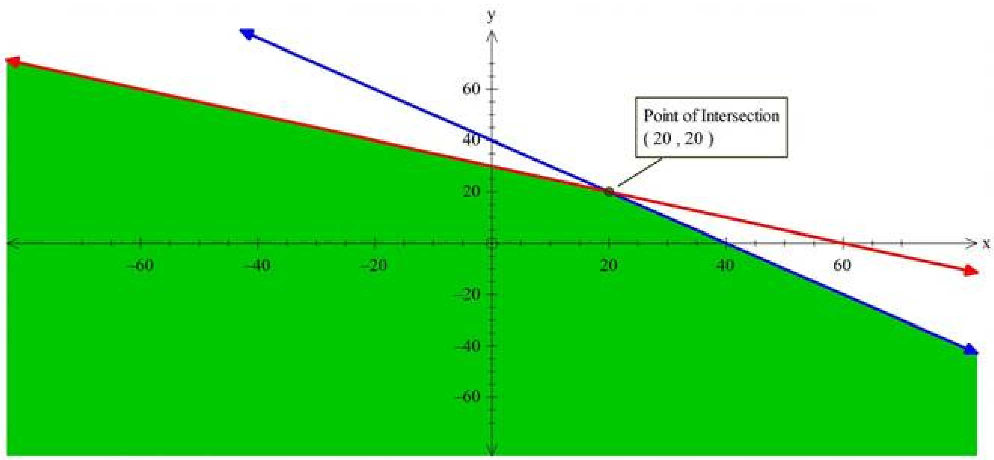
Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 30$ (0,30) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 60$ (60,0) - - - - second coordinate

x = 0 is the y-axis and y = 0 is the x-axis

Hence, we obtain a plot as shown in figure:



The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $2x + 4y = 120$ and $2x + 2y = 80$ gives (20, 20)

There are no other corners in the region obtained.

So maxima will occur at (20,20)

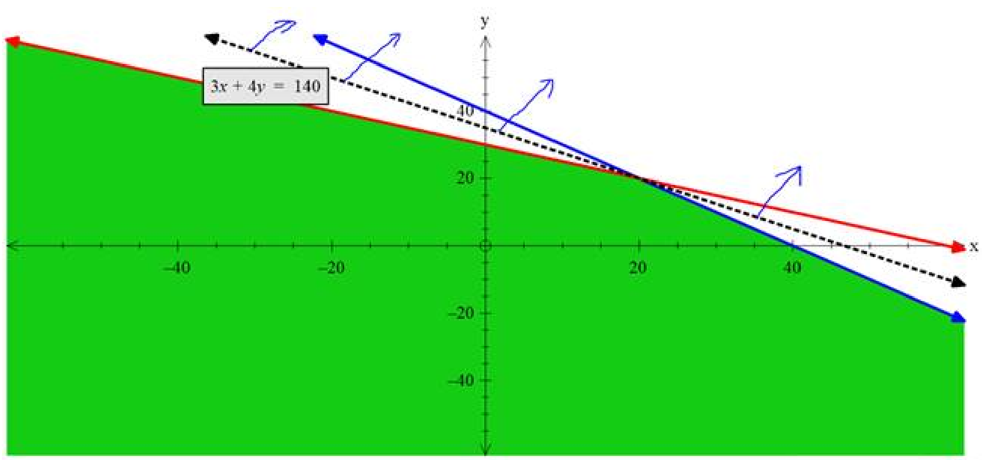
Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 3x + 4y$

$\therefore$ Z at (20,20) $= 3\times(20) + 4\times(20) = 140$

Note: As the region is unbounded, so we need to check whether maxima occurs or not.

For this we define inequation using the optimal function if the solution region of the inequation does not coincide with the feasible region, it means it has a maximum.



$\therefore$ inequation is : $3x + 4y > 140$

Clearly, from the graph we observe that $3x + 4y>140$ does not overlap with the feasible region

$\therefore$ Z is maximum at (20,20) and max. value is 140

$\therefore$ Z is maximum at $x = 20$ and $y = 20$; and max value is 140

9. Question

Solve each of the following linear programming problems by graphical method.

Maximize $Z = 7x + 10y$

Subject to :

$x + y \leq 30000$

$y \leq 12000$

$x \geq 6000$

$x, y \geq 0$

$x \geq y$

Answer

Given,

Objective function is: $Z = 7x + 10y$

Constraints are:

$x + y \leq 30000$

$y \leq 12000$

$x \geq 6000$

$x, y \geq 0$

$x \geq y$

First convert the given inequations into corresponding equations and plot them:

$x + y \leq 30000 \rightarrow x + y = 30000$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 30000$ (0,30000) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 30000$ (30000,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

For a line passing through origin, we put any other coordinate to check the region.

$x \geq y \rightarrow x - y = 0$ (corresponding equation)

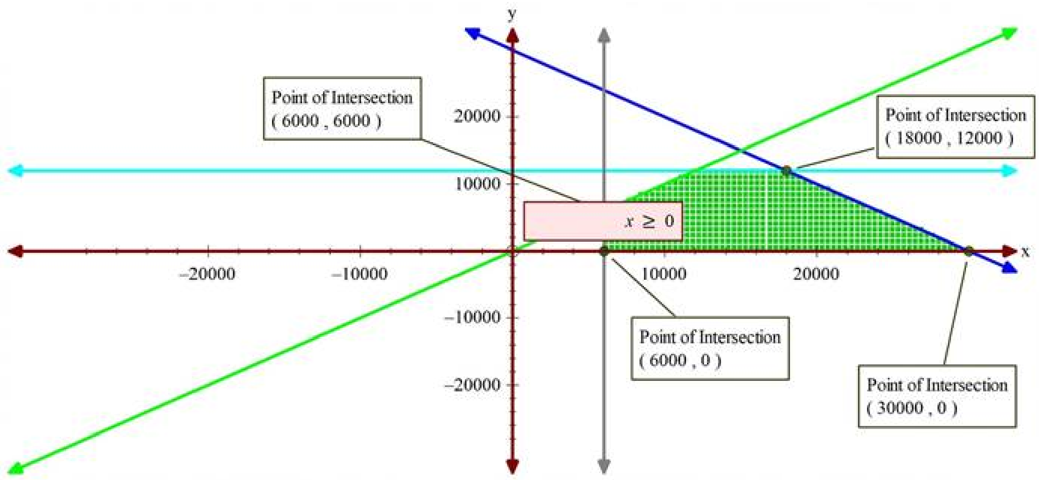
Line passing through origin (0,0) and (1,1)

$x = 0$ is the y - axis and $y = 0$ is the x - axis

$y = 12000$ (line parallel to x - axis passing through (0,12000))

$x = 6000$ (line parallel to x – axis passing through (6000,0))

Hence, we obtain a plot as shown in figure:



The shaded region in the above figure represents the region of feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 30000 and y = 12000 gives (18000, 12000)

Similarly, solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

(18000,12000) , (12000,12000) , (6000,0) , (6000,6000) and (30000,0)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because$ Z = 7x + 10y

$\therefore$ Z at $(18000,20000) = 7 \times (18000) + 10 \times (20000) = 246000$

Z at (12000,12000) = 7 × (12000) + 10 × (12000) = 204000

Z at (6000,0) = 7 × (6000) + 10 × 0 = 42000

Z at (6000,6000) = 7×6000 + 10×6000 = 67000

Z at (30000,0) = 7×30000 + 10×0 = 210000

Z at (0,0) = 0

We can see that Z is maximum at (18000, 20000) and max. value is 246000

$\therefore$ Z is maximum at x = 18000 and y = 20000 ; and max value is 246000

10. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 2x + 4y

Subject to :

x + y ≥ 8

x + 4y ≥ 12

x ≥ 3

y ≥ 2

Answer

Given,

Objective function is: Z = 2x + 4y

Constraints are:

x + y ≥ 8

x + 4y ≥ 12

x ≥ 3

y ≥ 2

First convert the given inequations into corresponding equations and plot them:

x + y ≥ 8 → x + y = 8 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 8 (0,8) - - - - first coordinate.

Put, y = 0 ⇒ x = 8 (8,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

As, x + 4y ≥ 12 → x + 4y = 12

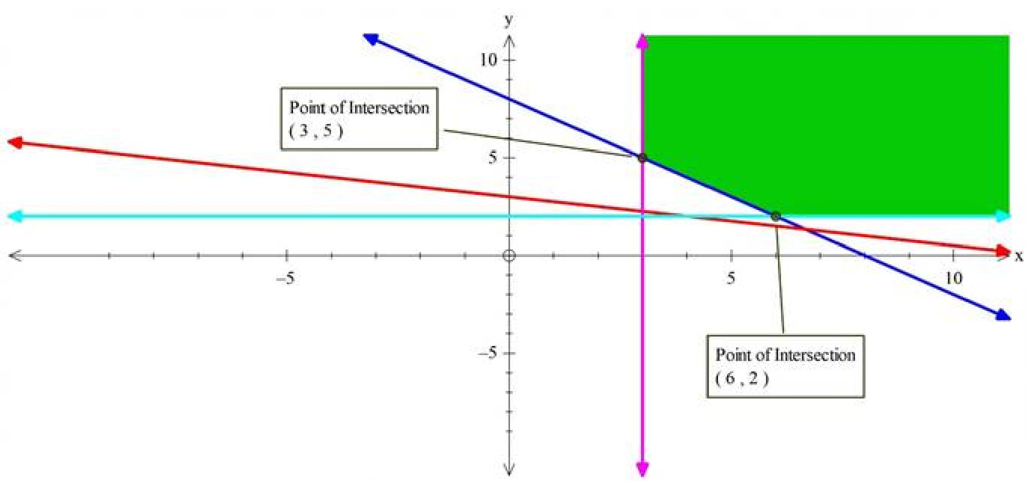
Put x = 0 ⇒ y = 3 coordinate - - - - - (0,3)

Put y = 0 ⇒ x = 12 coordinate - - - - - - (12,0)

y = 2 (line parallel to x - axis passing through (0,2))

x = 3 (line parallel to x – axis passing through (3,0))

Hence, we obtain a plot as shown in figure:



The shaded region in the above figure represents the region of feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 8 and x = 3 gives (3, 5)

Similarly, solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(3,5) and (6,2)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 2x + 4y

∴ Z at (3,5) = 2×(3) + 4×(5) = 26

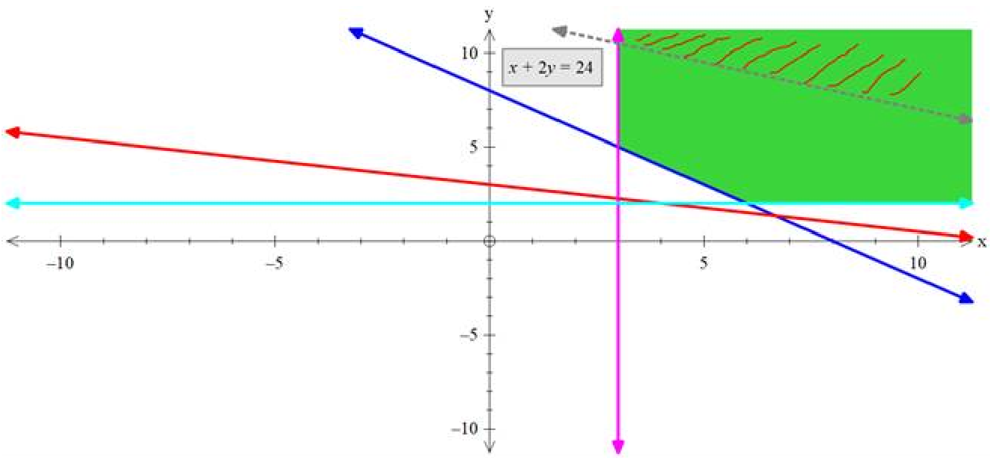
Z at (6,2) = 2 × (6) + 4 × (8) = 20

Note: As the region is unbounded as we can't say blindly that Z = 24 is maximum because there might be other points in the feasible region that may Make Z even greater.

So we need to check whether Z is maximum or not or Z greater than 24 or not.

For this we define inequation using the optimal function if the solution region of the inequation does not coincide with the feasible region, it means it has a maxima

Inequation : 2x + 4y > 24 or x + 2y > 12



Now we again plot the graph with the constraints and the above inequation

Clearly, x + 2y > 24 has solutions in feasible region.

This proves that the values of Z greater than 24 are possible.

∴ Optimal value of Z is not possible.

11. Question

Solve each of the following linear programming problems by graphical method.

Minimize Z = 5x + 3y

Subject to :

2x + y ≥ 10

x + 3y ≥ 15

x ≤ 10

y ≤ 8

x, y ≥ 0

Answer

Given,

Objective function is: Z = 5x + 3y

Constraints are:

2x + y ≥ 10

x + 3y ≥ 15

x ≤ 10

y ≤ 8

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

2x + y ≥ 10 → 2x + y = 10 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 10 (0,10) - - - - first coordinate.

Put, y = 0 ⇒ x = 5 (5,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

For a line passing through origin, we put any other coordinate to check the region.

x + 3y ≥ 15 → x + 3y = 15 (corresponding equation)

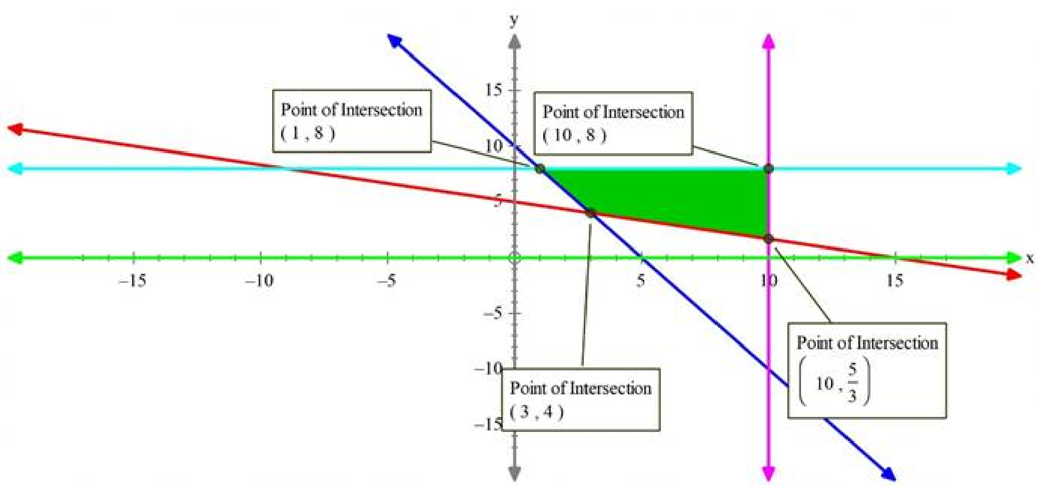
Put x = 0 ⇒ y = 5 coordinate - - - - (0,5)

Put y = 0 ⇒ x = 15 coordinate - - - - (15,0)

x = 0 is the y - axis and y = 0 is the x - axis

y = 8 (line parallel to x - axis passing through (0,8))

x = 10 (line parallel to x – axis passing through (10,0))



Hence, we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + 3y = 15 and 2x + y = 10 gives (3,4)

Similarly solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

(3,4) , (10,8), (1,8) and (10, 5/3)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because$ Z = 5x + 3y

$\therefore$ Z at (3,4) = 5×(3) + 3×(4) = 27

Z at (10,8) = 5 × (10) + 3 × (8) = 74

Z at (1,8) = 5 × (1) + 3 × 8 = 29

Z at (10,5/3) = 5 × 10 + 3 × 5/3 = 55

We can see that Z is minimum at (3,4) and min. value is 27

$\therefore$ Z is minimum at x = 3 and y = 4 ; and min value is 27

12. Question

Solve each of the following linear programming problems by graphical method.

Minimize Z = 30x + 20y

Subject to :

x + y ≤ 8

x + 4y ≥ 12

5x + 8y = 20

x, y ≥ 0

Answer

Given,

Objective function is: Z = 30x + 20y

Constraints are:

x + y ≤ 8

x + 4y ≥ 12

5x + 8y = 20

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

x + y ≤ 8 → x + y = 8 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 8 (0,8) - - - - first coordinate.

Put, y = 0 ⇒ x = 8 (8,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

For a line passing through origin, we put any other coordinate to check the region.

x + 4y ≥ 12 → x + 4y = 12 (corresponding equation)

Put x = 0 ⇒ y = 3 coordinate - - - - (0,3)

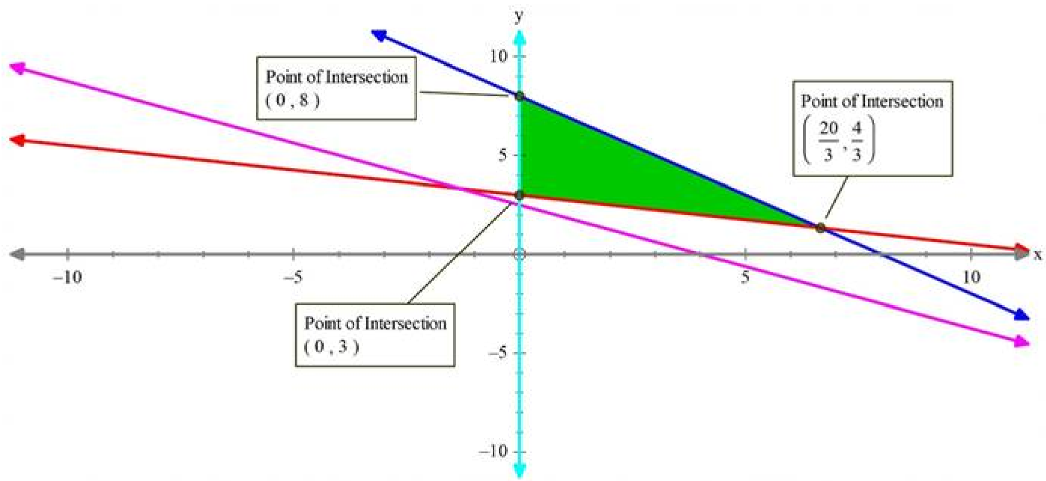
Put y = 0 ⇒ x = 12 coordinate - - - - (12,0)

x = 0 is the y - axis and y = 0 is the x - axis

5x + 8y = 20

On putting x = 0 , y = 20/8 (0,2.5)

Putting y = 0 , x = 4 (4,0)



Hence, we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 8 and x + 4y = 12 gives (20/3,4/3)

Similarly, solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

(0,3) , (0,8) and (20/3 , 4/3)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 30x + 20y

∴ Z at (0,8) = 30×(0) + 20×(8) = 160

Z at (0,3) = 30 × (0) + 20 × (3) = 60

Z at (20/3,4/3) = 30 × (20/3) + 20 × 4/3 = 226.666We can see that Z is minimum at(0,3) and min. value is 60

∴ Z is minimum at x = 0 and y = 3 ; and min value is 60

Maximum value is 226.66 and it occurs at x = 20/3 and y = 4/3

13. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 4x + 3y

Subject to :

8x + 6y ≤ 48

3x + 4y ≤ 24

x ≤ 5 , y ≤ 6

x, y ≥ 0

Answer

Ideas required to solve the problem:

• Fundamentals of plotting a linear equation. 2 coordinates are sufficient to plot a straight line.

• A linear inequation represents a region of XY plane when plotted.

• For linear programming we define various linear constraints and combining them we get a region in the XY plane which represents a region of feasible operation subject to various constraints.

• But our objective in the linear programming problem is to optimize (maximize or minimize) our objective function and it will be optimal only at one of the corner points of the feasible region. So we check the value of the objective function at every corner points and hence find maximum or minimum

Given,

Objective function is: Z = 4x + 3y

Constraints are:

8x + 6y ≤ 48

3x + 4y ≤ 24

x ≤ 5

y ≤ 6

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

8x + 6y ≤ 48 → 8x + 6y = 48 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 8 (0,8) - - - - first coordinate.

Put, y = 0 ⇒ x = 6 (6,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

3x + 4y ≤ 24 → 3x + 4y = 24 (corresponding equation)

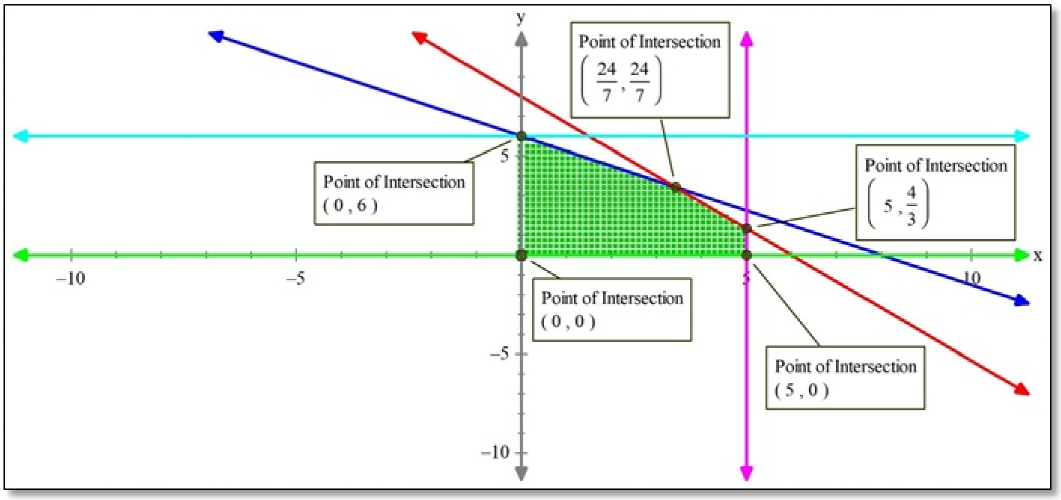
Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 6 (0,6) - - - - first coordinate.

Put, y = 0 ⇒ x = 8 (8,0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x - axis

x = 5 and y = 6 are lines parallel to y - axis and x - axis respectively.



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving 3x + 4y = 24 and 8x + 6y = 48 gives $\left(\frac{24}{7}, \frac{24}{7}\right)$

Similarly, solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(0,0) (5,0) (0,6), $\left(\frac{24}{7}, \frac{24}{7}\right)$, $\left(5, \frac{4}{3}\right)$

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 4x + 3y

∴ Z at $\left(\frac{24}{7}, \frac{24}{7}\right) = 4 \times \left(\frac{24}{7}\right) + 3 \times \left(\frac{24}{7}\right) = 24$

Z at (0,6) = 4 × 0 + 3 × 6 = 18

Z at (5,0) = 4 × (5) + 3 × 0 = 20

Z at (0,0) = 0

Z at $\left(5, \frac{4}{3}\right) = 4 \times 5 + 3 \times \left(\frac{4}{3}\right) = 24$

We can see that Z is maximum at $\left(\frac{24}{7}, \frac{24}{7}\right)$ and max. value is 24

∴ Z is maximum at x = 24/7 and y = 24/7 ; and max value is 24

Also it has a maximum at x = 5 and y = 4/3 with max value = 24

14. Question

Solve each of the following linear programming problems by graphical method.

Minimize Z = x - 5y + 20

Subject to :

x - y ≥ 0

- x + 2y ≥ 2

$x \geq 3$

$y \leq 4$

$x, y \geq 0$

Answer

Given,

Objective function is: $Z = x - 5y + 20$

Constraints are:

$x - y \geq 0$

$- x + 2y \geq 2$

$x \geq 3$

$y \leq 4$

$x, y \geq 0$

First convert the given inequations into corresponding equations and plot them:

$x - y \geq 0 \rightarrow x = y$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 0$ (0,0) - - - - first coordinate.

Put, $y = 1 \Rightarrow x = 1$ (1,1) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$- x + 2y \geq 2 \rightarrow - x + 2y = 2$ (corresponding equation)

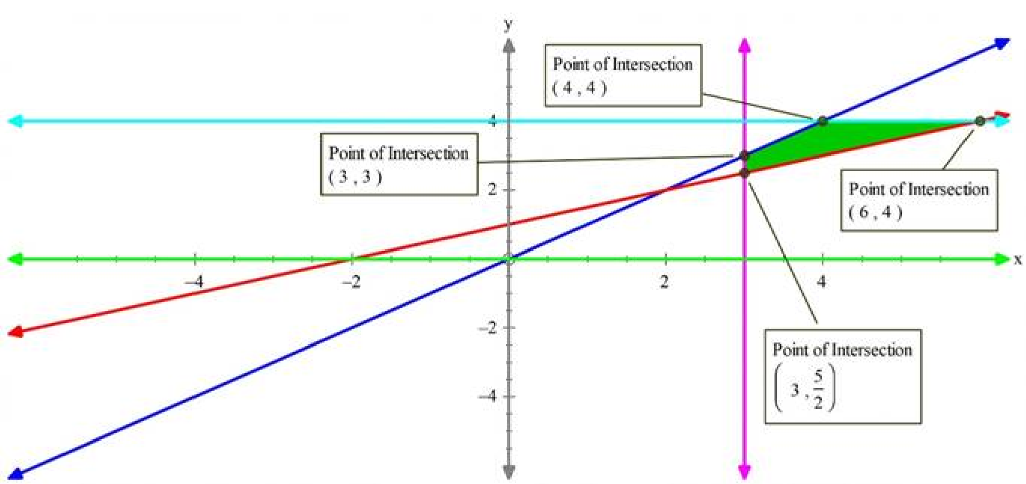
Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 1$ (0,1) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = - 2$ (- 2,0) - - - - second coordinate

$x = 0$ is the y - axis and $y = 0$ is the x - axis

$x = 3$ and $y = 4$ are lines parallel to y - axis and x - axis respectively.



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x - y = 0 and x = 3 gives (3,3)

Similarly, solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(3,3) (4,4) (6,4), $\left(3,\frac{5}{2}\right)$

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = x - 5y + 20

∴ Z at $\left(3,\frac{5}{2}\right) = (3) - 5 \times \left(\frac{5}{2}\right) + 20 = 10.5$

Z at (3,3) = 3 – 5×3 + 20 = 8

Z at (4,4) = 4 - 5×4 + 20 = 4

Z at (6,4) = 6 – 5×4 + 20 = 6

We can see that Z is minimum at (4,4) and min. value is 4

∴ Z is minimum at x = 4 and y = 4 ; and min value is 4

15. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 3x + 5y

Subject to :

x + 2y ≤ 20

x + y ≤ 15

y ≤ 5

X, y ≥ 0

Answer

Given,

Objective function is: Z = 3x + 5y

Constraints are:

x + 2y ≤ 20

x + y ≤ 15

y ≤ 5

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

x + 2y ≤ 20 → x + 2y = 20 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 10 (0,10) - - - - first coordinate.

Put, y = 0 ⇒ x = 20 (20,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$x + y \leq 15 \rightarrow x + y = 15$ (corresponding equation)

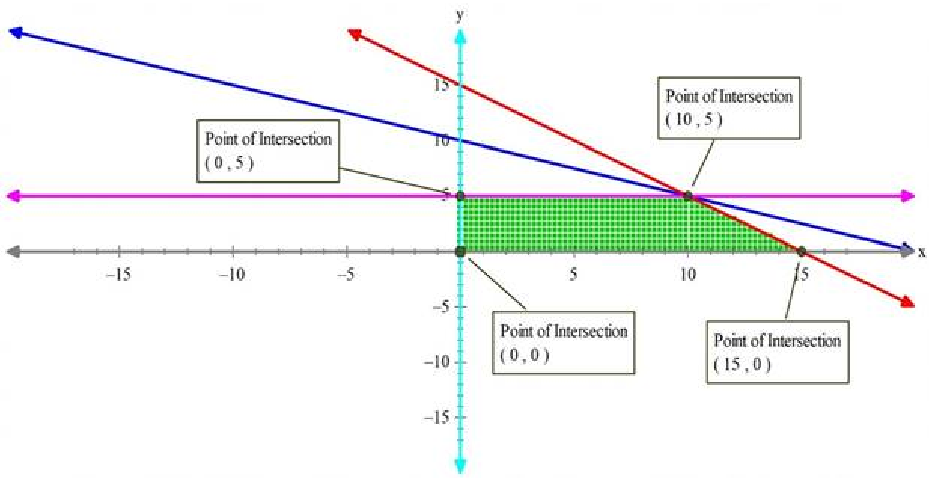
Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 15 (0,15) - - - - first coordinate.

Put, y = 0 ⇒ x = 15 (15,0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x - axis

y = 5 is line parallel to x - axis passing through (0,5).



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 15 and x + 2y = 20 gives (10,5)

Similarly solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

(0,0) (10,5) (15,0), (0,5)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ $Z = 3x + 5y$

∴ Z at (0,0) = 0

Z at (10,5) = $3\times 10 + 5\times 5 = 55$

Z at (15,0) = $3\times 15 + 5\times 0 = 45$

Z at (0,5) = 3×0 + 5×5 = 25

We can see that Z is maximum at (10,5) and max. value is 55

∴ Z is maximum at x = 10 and y = 5 ; and max value is 55

16. Question

Solve each of the following linear programming problems by graphical method.

Minimize $Z = 3x_1 + 5x_2$

Subject to :

$x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

Answer

Given,

Objective function is: $Z = 3x_1 + 5x_2$

Constraints are:

$x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$

$x_1, x_2 \geq 0$

First convert the given inequations into corresponding equations and plot them:

$x_1 + 3x_2 \geq 3 \rightarrow x_1 + 3x_2 = 3$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x_1 = 0 \Rightarrow x_2 = 1$ (0,1) - - - - first coordinate.

Put, $x_2 = 0 \Rightarrow x_1 = 3$ (3,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

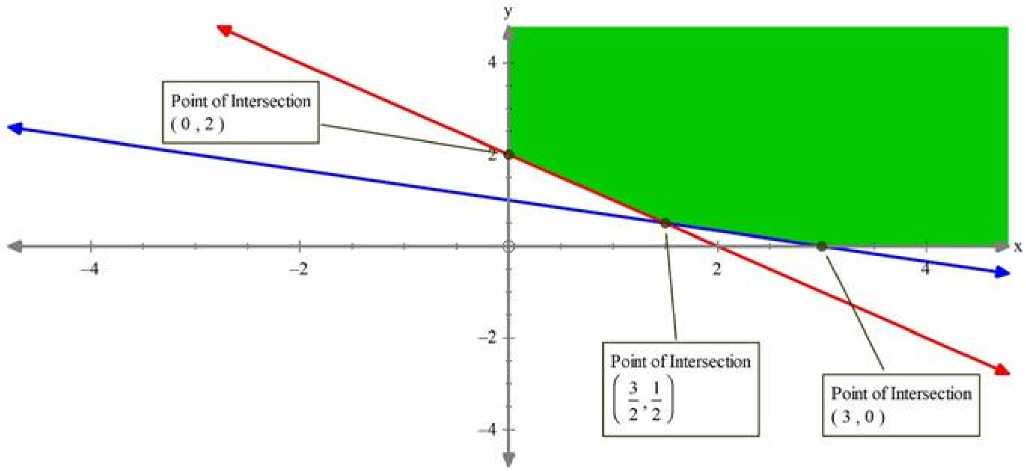
$x_1 + x_2 \geq 2 \rightarrow x_1 + x_2 = 2$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x_1 = 0 \Rightarrow x_2 = 2$ (0,2) - - - - first coordinate.

Put, $x_2 = 0 \Rightarrow x_1 = 2$ (2,0) - - - - second coordinate

$x_1 = 0$ is the y-axis and $x_2 = 0$ is the x-axis



Hence we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $x_1 + x_2 = 2$ and $x_1 + 3x_2 = 3$ gives (3/2,1/2)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(3/2,1/2) ,(3,0) and (0,2)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 3x_1 + 5x_2$

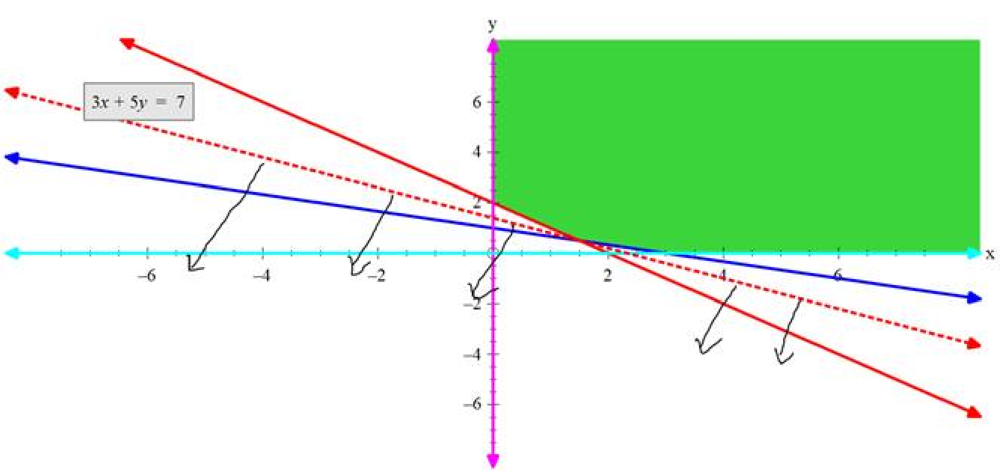
$\therefore$ Z at (3,0) = $3\times 3 + 5\times 0 = 9$

Z at (0,2) = $3\times 0 + 5\times 2 = 10$

Z at (3/2,1/2) = $3\times(3/2) + 5\times(1/2) = 7$

Note: As the region is unbounded, so we need to check whether any value less than 7 is possible for Z or not. If it is unique, we will say that under given constraints we found the minimum Z

For this we define inequation as: $3x + 2y < 7$



As (0,0) satisfies the inequation, so this is the region specified by above inequation. As our feasible region does not coincide with the region specified by $3x + 2y < 7$.

∴ Z can be minimized

We can see that Z is minimum at (3/2,1/2) and min. value is 7

∴ Z is minimum at x = 3/2 and y = 1/2 ; and min value is 7

17. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 2x + 3y

Subject to :

$x + y \geq 1$

$10x + y \geq 5$

$x + 10y \geq 1$

$x, y \geq 0$

Answer

Given,

Objective function is: Z = 2x + 3y

Constraints are:

$x + y \geq 1$

$10x + y \geq 5$

$x + 10y \geq 1$

$x, y \geq 0$

First convert the given inequations into corresponding equations and plot them:

$x + y \geq 1 \rightarrow x + y = 1$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 1 (0,1) - - - - first coordinate.

Put, y = 0 ⇒ x = 1 (1,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$10x + y \geq 5 \rightarrow 10x + y = 5$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 5 (0,5) - - - - first coordinate.

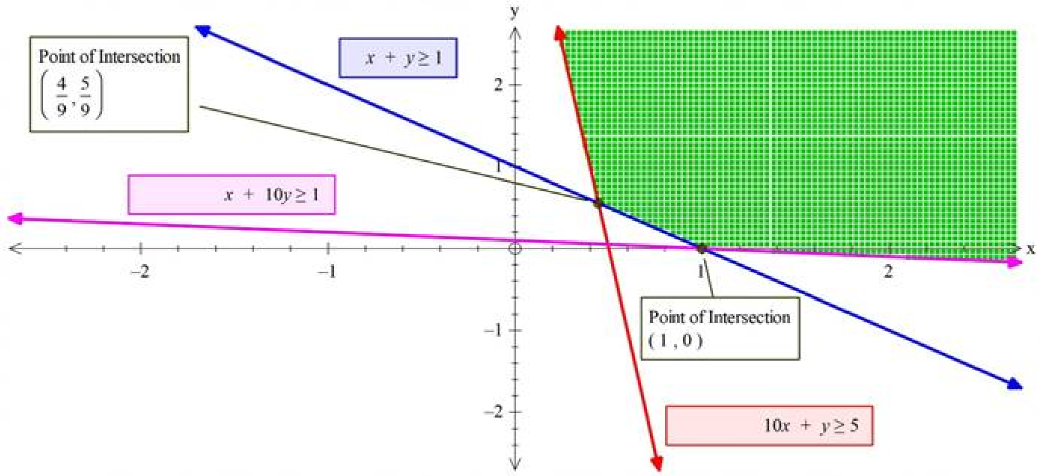
Put, y = 0 ⇒ x = 1/2 (1/2,0) - - - - second coordinate

$x + 10y \geq 1 \rightarrow x + 10y = 1$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 1/10 (0,0.1) - - - - first coordinate.

Put, y = 0 ⇒ x = 1 (1,0) - - - - second coordinate x = 0 is the y - axis and y = 0 is the x - axis



Hence, we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 1 and 10x + y = 5 gives (4/9,5/9)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(4/9,5/9) (1,0) and (0,5)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 2x + 3y

∴ Z at (1,0) = 2

Z at (0,5) = 2× 0 + 3×5 = 15

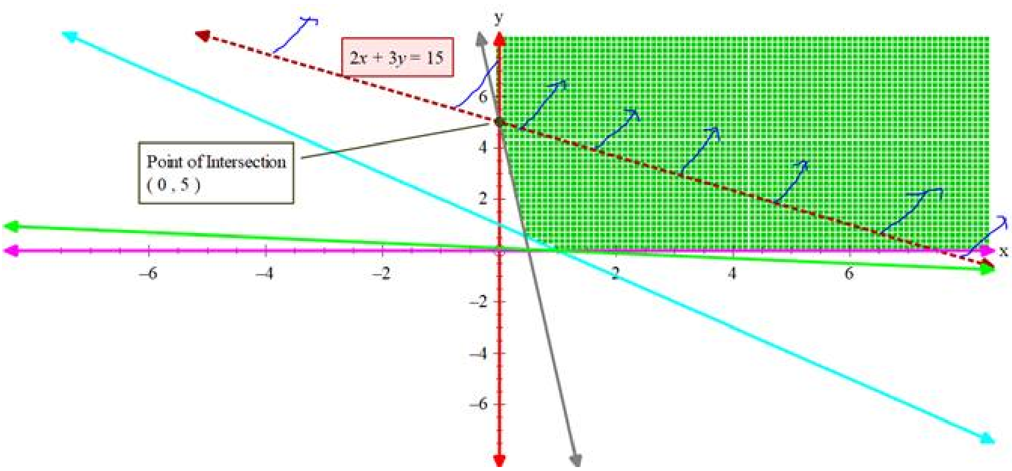
Z at (4/9,5/9) = 2×(4/9) + 3× (5/9) = 23/9

Note: As the region is unbounded as we can’t say blindly that Z = 15 is maximum because there might be other points in the feasible region that may Make Z even greater.

So we need to check whether Z is maximum or not or Z greater than 15 or not.

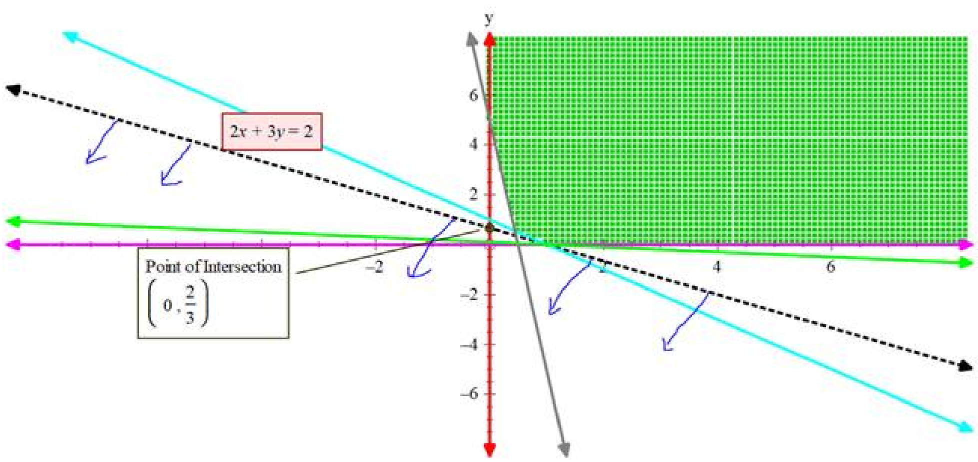
For this we define inequation using the optimal function if the solution region of the inequation does not coincide with the feasible region, it means it has a maxima

Inequation : 2x + 3y > 15



We can see that the inequation 2x + 3y > 15 overlaps with a feasible region. So there are no maxima possible

If we want to check regarding a minimum of Z, from the corner we have a min of Z = 2



∴ required inequation is : 2x + 3y<2

Clearly no overlap with the feasible region is there for 2x + 3y < 2

∴ Z has minimum at x = 1 and y = 0 and min value of Z is 2

18. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = - x_1 + $2x_2$

Subject to :

- x_1 + $3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

Answer

Given,

Objective function is: Z = - x_1 + $2x_2$

Constraints are:

- x_1 + $3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

First convert the given inequations into corresponding equations and plot them:

$-x_1 + 3x_2 \leq 10 \rightarrow -x_1 + 3x_2 = 10$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x_1 = 0 \Rightarrow x_2 = 10/3$ (0,10/3) - - - - first coordinate.

Put, $x_2 = 0 \Rightarrow x_1 = -10$ (- 10,0) - - - - second coordinate

$x_1 + x_2 \leq 6 \rightarrow x_1 + x_2 = 6$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x_1 = 0 \Rightarrow x_2 = 6$ (0,6) - - - - first coordinate.

Put, $x_2 = 0 \Rightarrow x_1 = 6$ (6,0) - - - - second coordinate

$x_1 - x_2 \leq 2 \rightarrow x_1 - x_2 = 2$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x_1 = 0 \Rightarrow x_2 = -2$ (0, - 2) - - - - first coordinate.

Put, $x_2 = 0 \Rightarrow x_1 = 2$ (2,0) - - - - second coordinate

$x_1 \geq 0$ and $x_2 \geq 0$

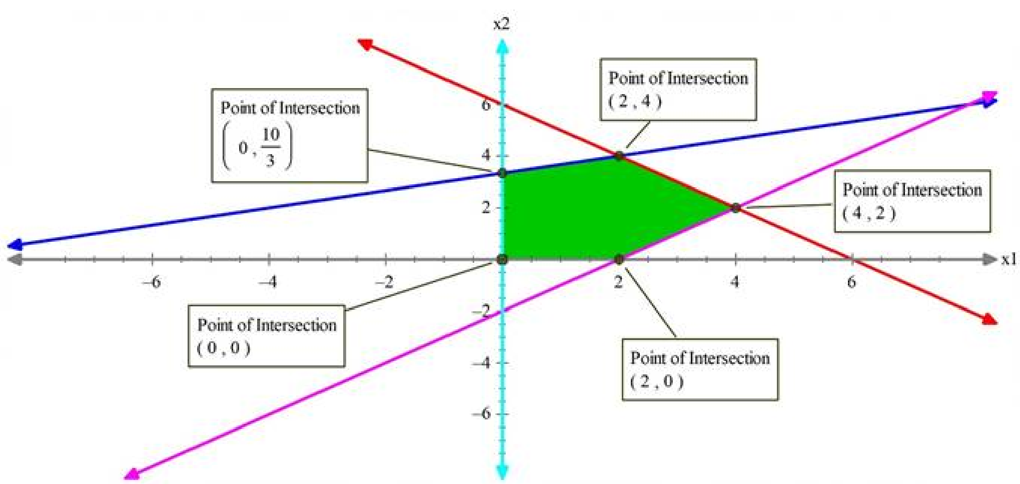
Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

Hence, we obtain the following plot:



The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $x_1 - x_2 = 2$ and $x_1 + x_2 = 6$ gives (6,6)

Similarly solve other combinations by observing graph to get other coordinates.

From figure we have obtained coordinates of corners as:

(0,0),(2,0),(4,2),(2,4) and (0,10/3)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = -x_1 + 2x_2$

$\therefore$ Z at (0,0) = 0

Z at (2,0) = - 2 + 2×0 = - 2

Z at (4,2) = - 4 + 2× 2 = 0

Z at (2,4) = - 2 + 2×4 = 6

Z at (0,10/3) = - 0 + 2× (10/3) = 20/3

We can see that Z is maximum at (0,10/3) and max. value is 20/3

$\therefore$ Z is maximum at x = 0 and y = 10/3 ; and max value is 20/3

19. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = x + y

Subject to :

$-2x + y \le 1$

$x \le 2$

$x + y \le 3$

$x, y \ge 0$

Answer

Given,

Objective function is: Z = x + y

Constraints are:

$-2x + y \le 1$

$x \le 2$

$x + y \le 3$

$x, y \ge 0$

First convert the given inequations into corresponding equations and plot them:

$-2x + y \le 1 \rightarrow -2x + y = 1$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 1 (0,1) - - - - first coordinate.

Put, y = 0 ⇒ x = - 1/2 (- 1/2,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$x + y \leq 3 \rightarrow x + y = 3$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

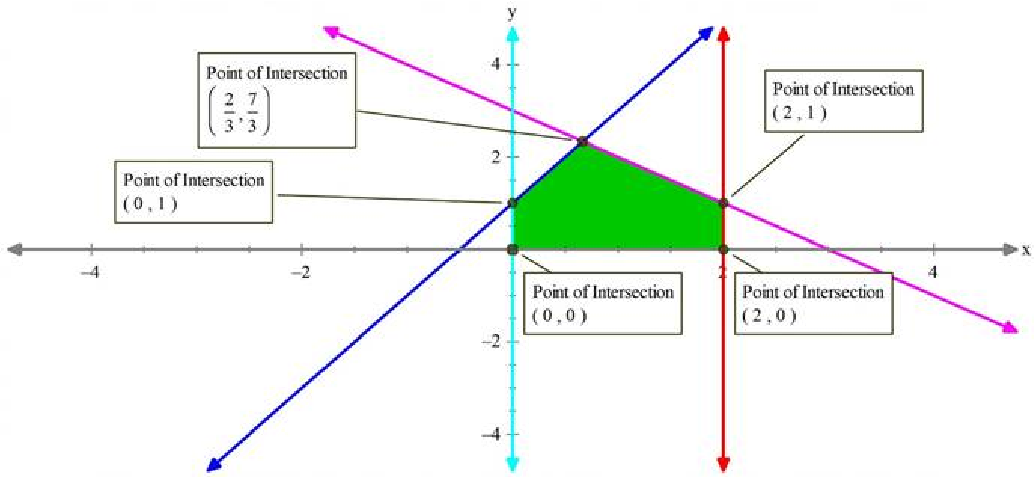
Put, $x = 0 \Rightarrow y = 3$ (0,3) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 3$ (3,0) - - - - second coordinate

$x = 0$ is the y - axis and $y = 0$ is the x - axis

$x = 2$ is line parallel to y - axis passing through (2,0).

Hence we obtain a plot as shown in figure:



The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $-2x + y = 1$ and $x + y = 3$ gives (2/3,7/3)

Similarly, solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(2/3,7/3), (0,1), (0,0), (2,0) and (2,1)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = x + y$

$\therefore$ Z at (0,0) = 0

Z at (2,0) = 2 + 0 = 2

Z at (2,1) = 2 + 1 = 3

Z at (0,1) = 0 + 1 = 1

Z at (2/3,7/3) = 2/3 + 7/3 = 3

Clearly, we can see that Z is maximum at (2,1) and max. value is 3

$\therefore$ Z is maximum at x = 2 and y = 1 also at x = 2/3 and y = 7/3; and max value is 3

20. Question

Solve each of the following linear programming problems by graphical method.

Maximize $Z = -x_1 + 2x_2$

Subject to :

$-x_1 + 3x_2 \leq 10$

$x_1 + x_2 \leq 6$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

Answer

Given,

Objective function is: $Z = 3x_1 + 4x_2$

Constraints are:

$x_1 - x_2 \leq -1$

$-x_1 + x_2 \leq 0$

$x_1, x_2 \geq 0$

First convert the given inequations into corresponding equations and plot them:

$x_1 - x_2 \leq -1 \rightarrow x_1 - x_2 = -1$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x_1 = 0 \Rightarrow x_2 = 1$ (0,1) - - - - first coordinate.

Put, $x_2 = 0 \Rightarrow x_1 = -1$ (- 1,0) - - - - second coordinate

$-x_1 + x_2 \leq 0 \rightarrow -x_1 + x_2 = 0$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x_1 = 0 \Rightarrow x_2 = 0$ (0,0) - - - - first coordinate.

Put, $x_2 = 1 \Rightarrow x_1 = -1$ (- 1,1) - - - - second coordinate

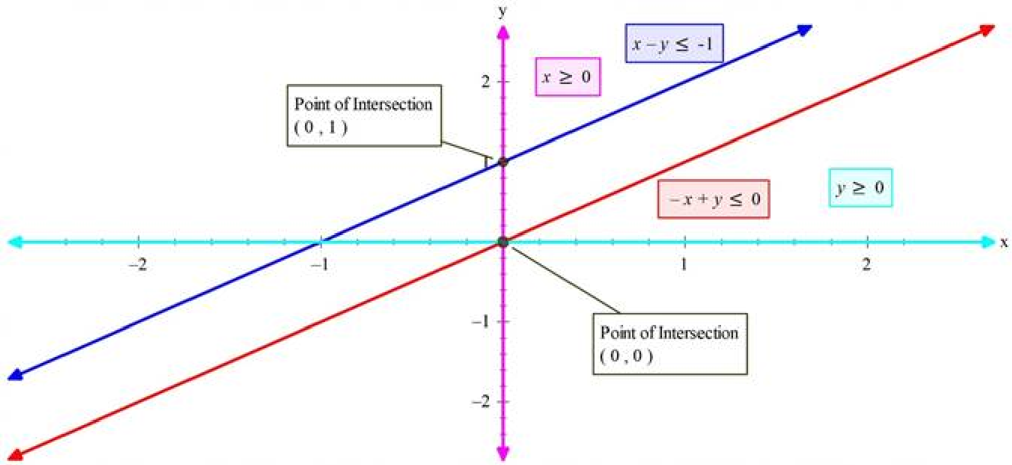
$x_1 \geq 0$ and $x_2 \geq 0$

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.



Hence, we obtain the following plot:

The given constraints don't enclose any feasible region. No common shaded region so no maxima exists.

21. Question

Solve each of the following linear programming problems by graphical method.

Maximize Z = 3x + 4y

Subject to :

x – y ≤ 1

x + y ≥ 3

x, y ≥ 0

Answer

Given,

Objective function is: Z = 3x + 4y

Constraints are:

x – y ≤ 1

x + y ≥ 3

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

x – y ≤ 1 → x – y = 1 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = - 1 (0, - 1) - - - - first coordinate.

Put, y = 0 ⇒ x = 1 (1,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

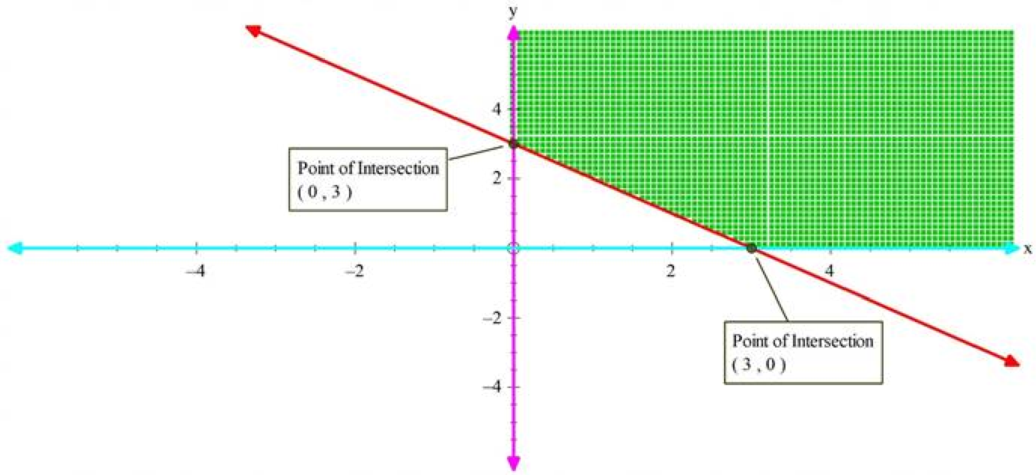
x + y ≤ 3 → x + y = 3 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 3 (0,3) - - - - first coordinate.

Put, y = 0 ⇒ x = 3 (3,0) - - - - second coordinate

x = 0 is the y-axis and y = 0 is the x-axis



Hence, we obtain a plot as shown in figure:

The shaded region in the above figure represents the region of a feasible solution.

Now to maximize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 3 and x = 0 gives (0,3)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(0,3) and (3,0)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 3x + 4y

∴ Z at (0,3) = 4×3 = 12

Z at (3,0) = 3×3 + 4×0 = 9

As the region is unbounded as we can't say blindly that Z = 12 is maximum because there might be other points in the feasible region that may Make Z even greater.

So we need to check whether Z is maximum or not or Z greater than 12 or not.

For this we define inequation using the optimal function if the solution region of the inequation does not coincide with the feasible region, it means it has a maxima

Inequation: 3x + 4y > 12

It will be away from the origin side so that it will overlap with the feasible region.

∴ No maxima are possible.

22. Question

Solve each of the following linear programming problems by graphical method.

Show the solution zone of the following inequalities on a graph paper:

5x + y ≥ 10

$x + y \geq 6$

$x + 4y \geq 12$

$x \geq 0, y \geq 0$

Find x and y for which $3x + 2y$ is minimum subject to these inequalities. Use a graphical method.

Answer

Given,

$Z = 3x + 2y$

Constraints:

$5x + y \geq 10$

$x + y \geq 6$

$x + 4y \geq 12$

$x \geq 0, y \geq 0$

First convert the given inequations into corresponding equations and plot them:

$5x + y \geq 10 \rightarrow 5x + y = 10$(corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 10$ (0,10) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 2$ (2,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$x + y \geq 6 \rightarrow x + y = 6$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 6$ (0,6) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 6$ (6,0) - - - - second coordinate

$x + 4y \geq 12 \rightarrow x + 4y = 12$ (corresponding equation)

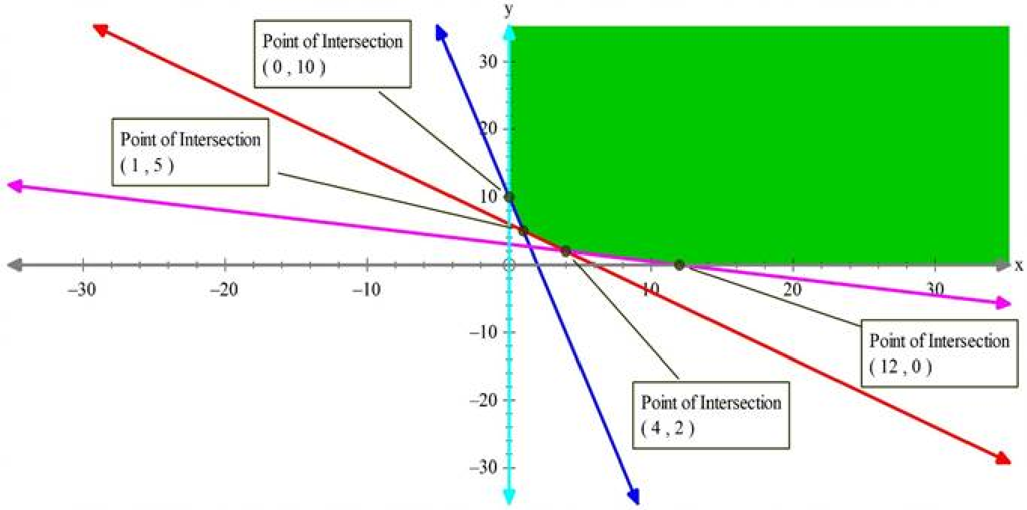
Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 3$ (0,3) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 12$ (12,0) - - - - second coordinate

$x = 0$ is the y - axis and $y = 0$ is the x - axis.

Hence, we have the following plot:



The shaded region in the above figure represents the region of a feasible solution.

Now to minimize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $x + 4y = 12$ and $x + y = 6$ gives (4,2)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(4,2),(12,0),(1,5) and (0,10)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 3x + 2y$

$\therefore$ Z at (4,2) = $4\times3 + 2\times2 = 16$

Z at (12,0) = $3\times12 + 2\times0 = 36$

Z at (1,5) = $3\times 1 + 2\times 5 = 13$

Z at (0,10) = $3\times 0 + 2\times10 = 20$

As the region is unbounded as we can't say blindly that Z = 13 is minimum because there might be other points in feasible region that may Make Z even lesser.

So we need to check whether Z is minimum or not or Z lesser than 13 or not.

For this we define inequation using the optimal function if the solution region of the inequation does not coincide with the feasible region, it means it has a maxima

Inequation: $3x + 2y < 13$

It will be towards origin side, so it will not overlap with the feasible region.

$\therefore$ Minima are possible.

Minimum is possible and minimum occurs at $x = 1$ and $y = 5$ and value is Z = 13.

23. Question

Solve each of the following linear programming problems by graphical method.

Find the maximum and minimum value of $2x + y$ subject to the constraints :

$x + 3y \geq 6,$

$x – 3y \leq 3$,

$3x + 4y \leq 24$,

$- 3x + 2y \leq 6$,

$5x + y \geq 5$,

$x, y \geq 0$.

Answer

Given,

$Z = 2x + y$

Constraints:

$x + 3y \geq 6$

$x – 3y \leq 3$

$3x + 4y \leq 24$

$- 3x + 2y \leq 6$,

$5x + y \geq 5$,

$x, y \geq 0$.

First convert the given inequations into corresponding equations and plot them:

$x + 3y \geq 6 \rightarrow x + 3y = 6$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 2$ (0,2) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 6$ (6,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$x – 3y \leq 3 \rightarrow x – 3y = 3$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = - 1$ (0, - 1) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 3$ (3,0) - - - - second coordinate

$3x + 4y \leq 24 \rightarrow 3x + 4y = 24$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 6$ (0,6) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 8$ (8,0) - - - - second coordinate

$- 3x + 2y \leq 6 \rightarrow - 3x + 2y = 6$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 3$ (0,3) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = - 2$ (- 2,0) - - - - second coordinate

$5x + y \geq 5 \rightarrow 5x + y = 5$, (corresponding equation)

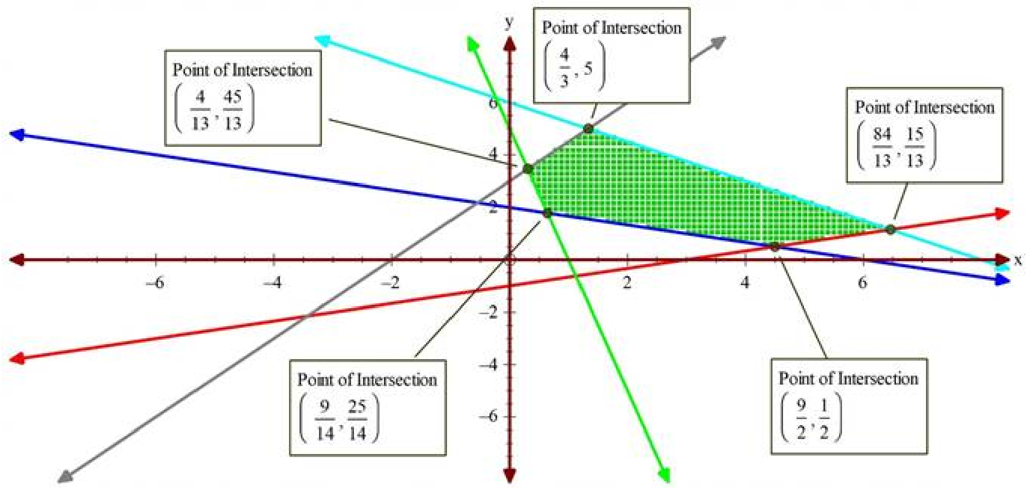
Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 5$ (0,5) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 1$ (1,0) - - - - second coordinate

$x = 0$ is the y - axis and $y = 0$ is the x - axis.

Hence, we have the following plot:



The shaded region in the above figure represents the region of a feasible solution.

Now to minimize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $5x + y = 5$ and $- 3x + 2y = 6$ gives (4/13,45/13)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(4/13, 45/13) ,(4.5, 0.5), (9/14, 25/14), (4/3, 5), (84/13, 15/13)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 2x + y$

$\therefore$ Z at $\left(\frac{4}{13}, \frac{45}{13}\right) = 2\times(4/13) + 45/13 = 57/13 = 4.38$

Z at $\left(\frac{9}{2}, \frac{1}{2}\right) = 2\times(9/2) + 1/2 = 19/2 = 9.5$

Z at $(9/14,25/14) = 2\times (9/14) + (25/14) = 43/14 = 3.07$

Z at $\left(\frac{4}{3}, 5\right) = 2\times(4/3) + 5 = 23/3 = 7.67$

Z at $\left(\frac{84}{13}, \frac{15}{13}\right) = \left(2 \times \frac{84}{13} + \frac{15}{13}\right) = 183/13 = 14.07$

Clearly, Z is maximum at $x = 84/13$ and $y = 15/13$ and maximum value is 14.07

Z is minimum at $x = 9/14$ and $y = 25/14$ and minimum value is 3.07

24. Question

Solve each of the following linear programming problems by graphical method.

Find the minimum value of $3x + 5y$ subject to the constraints

$- 2x + y \leq 4$

$x + y \geq 3$

$x – 2y \leq 2$

$x, y \leq 0$

Answer

Given,

$Z = 3x + 5y$

Constraints:

$- 2x + y \leq 4$

$x + y \geq 3$

$x – 2y \leq 2$

$x, y \leq 0$

First convert the given inequations into corresponding equations and plot them:

$- 2x + y \leq 4 \rightarrow - 2x + y = 4$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 4$ (0,4) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = - 2$ (- 2,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$x + y \geq 3 \rightarrow x + y = 3$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 3$ (0,3) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 3$ (3,0) - - - - second coordinate

$x – 2y \leq 2 \rightarrow x – 2y = 2$ (corresponding equation)

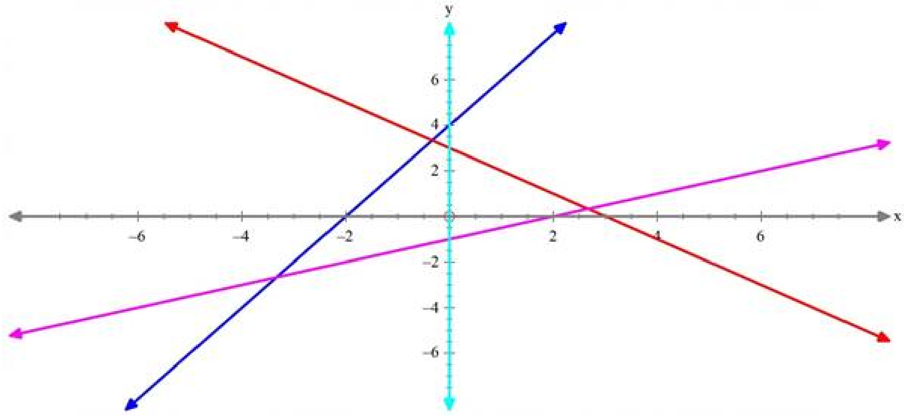
Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = - 1$ (0, - 1) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 2$ (2,0) - - - - second coordinate

$x = 0$ is the y - axis and $y = 0$ is the x - axis.

Hence, we have the following plot:



There is no shaded region in the above figure represents that there is no region of a feasible solution.

x + y ≥ 3 will not be bounded by x, y ≤ 0. Thus, no feasible region is there.

∴ There is no possible minimum value Z.

25. Question

Solve each of the following linear programming problems by graphical method.

Solved the following linear programming problem graphically :

Maximize Z = 60x + 15y

Subject to constraints

x + y ≤ 50

3x + y ≤ 90

x, y ≥ 0

Answer

Given,

Z = 60x + 15y

Constraints:

x + y ≤ 50

3x + y ≤ 90

x, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

x + y ≤ 50 → x + y = 50 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 50 (0,50) - - - - first coordinate.

Put, y = 0 ⇒ x = 50 (50,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

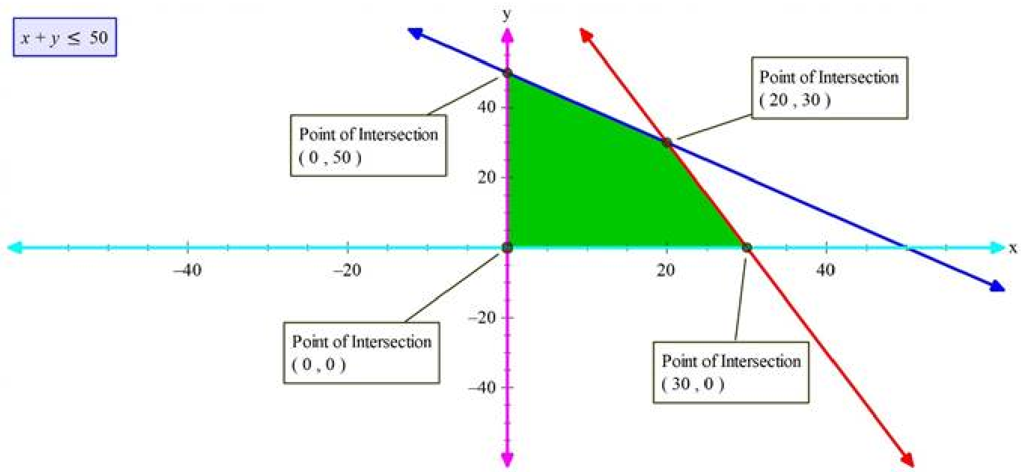
3x + y ≤ 90 → 3x + y = 90 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 90 (0,90) - - - - first coordinate.

Put, y = 0 ⇒ x = 30 (30,0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x - axis.



Hence, we have the following plot:

The shaded region in the above figure represents the region of a feasible solution.

Now to minimize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 50 and 3x + y = 90 gives (20,30)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(0,0),(30,0),(0,50) and (20,30)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

∵ Z = 60x + 15y

∴ Z at (0,0) = 0

Z at (30,0) = 60×30 + 15×0 = 1800

Z at (0,50) = 60×0 + 15×50 = 750

Z at (20,30) = 60×20 + 15×30 = 1650

Clearly Z is maximum at x = 30 and y = 0 and maximum value is 1800

26. Question

Solve each of the following linear programming problems by graphical method.

Find graphically, the maximum value of Z = 2x + 5y, subject to constraints given below :

2x + 4y ≤ 8

3x + y ≤ 6

x + y ≤ 4

x ≥ 0, y ≥ 0

Answer

Given,

Z = 2x + 5y

Constraints:

2x + 4y ≤ 8

3x + y ≤ 6

x + y ≤ 4

x ≥ 0, y ≥ 0

First convert the given inequations into corresponding equations and plot them:

2x + 4y ≤ 8 → 2x + 4y = 8 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 2 (0,2) - - - - first coordinate.

Put, y = 0 ⇒ x = 4 (4,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

3x + y ≤ 6 → 3x + y = 6 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 6 (0,6) - - - - first coordinate.

Put, y = 0 ⇒ x = 2 (2,0) - - - - second coordinate

x + y ≤ 4 → x + y = 4 (corresponding equation)

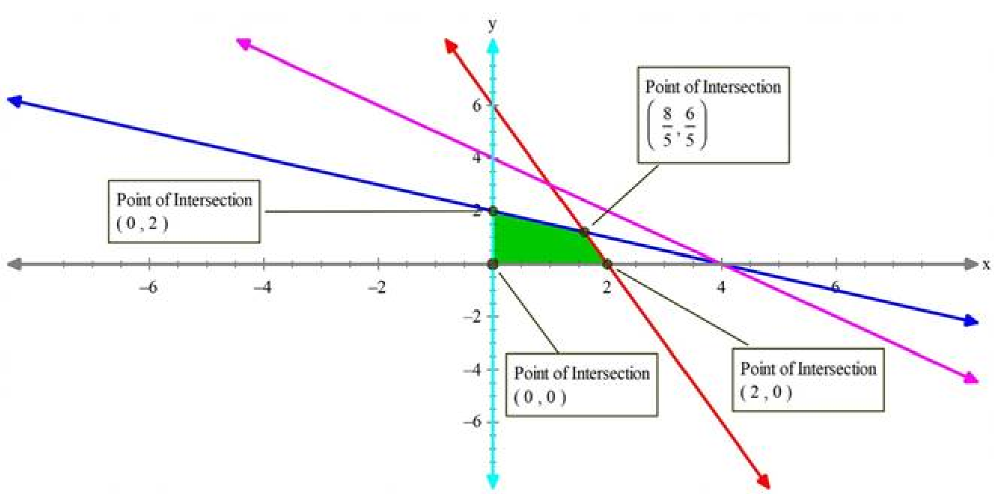
Two coordinates required to plot the equation are obtained as:

Put, x = 0 ⇒ y = 4 (0,4) - - - - first coordinate.

Put, y = 0 ⇒ x = 4 (4,0) - - - - second coordinate

x = 0 is the y - axis and y = 0 is the x - axis.

Hence, we have the following plot:



The shaded region in the above figure represents the region of a feasible solution.

Now to minimize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $2x + 4y = 8$ and $3x + y = 6$ gives $(8/5,6/5)$

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

$(0,0),(2,0),(0,2)$ and $(8/5,6/5)$

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 2x + 5y$

$\therefore Z$ at $(0,0) = 0$

Z at $(2,0) = 2\times2 + 5\times0 = 4$

Z at $(0,2) = 2\times0 + 5\times2 = 10$

Z at $(8/5,6/5) = 2\times(8/5) + 5\times(6/5) = 46/5 = 9.2$

Clearly Z is maximum at $x = 0$ and $y = 2$ and maximum value is 10

27. Question

Solve each of the following linear programming problems by graphical method.

Solve the following LPP graphically :

Maximize $Z = 20x + 10y$

Subject to the following constraints

$x + 2y \leq 28$

$3x + y \leq 24$

$x \geq 2$

Answer

Given,

$Z = 20x + 10y$

Constraints:

$x + 2y \leq 28$

$3x + y \leq 24$

$x \geq 2$

First convert the given inequations into corresponding equations and plot them:

$x + 2y \leq 28 \rightarrow x + 2y = 28$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 14$ $(0,14)$ - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 28$ $(28,0)$ - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$3x + y \le 24 \rightarrow 3x + y = 24$ (corresponding equation)

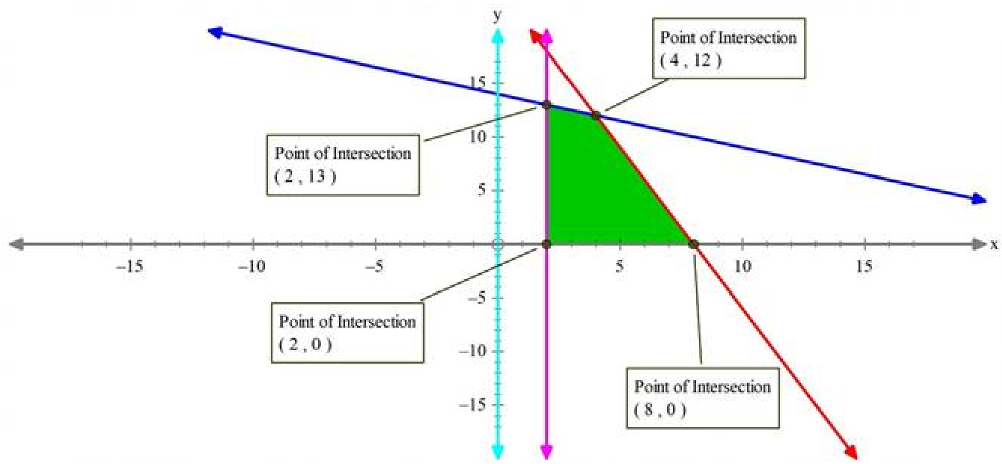
Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 24$ (0,24) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 8$ (8,0) - - - - second coordinate

$x = 2$ is the line parallel to y-axis passing through (2,0)

Hence, we have the following plot:



The shaded region in the above figure represents the region of a feasible solution.

Now to minimize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $x + 2y = 28$ and $3x + y = 24$ gives (4,12)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(2,0),(8,0),(2,13) and (4,12)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 20x + 10y$

$\therefore Z$ at $(2,0) = 20\times 2 + 10\times 0 = 40$

Z at $(8,0) = 20\times8 + 10\times0 = 160$

Z at $(2,13) = 20\times2 + 10\times13 = 170$

Z at $(4,12) = 20\times(4) + 10\times(12) = 200$

Clearly Z is maximum at $x = 4$ and $y = 12$ and maximum value is 200

28. Question

Solve each of the following linear programming problems by graphical method.

Solve the following linear programming problem graphically :

Minimize $z = 6x + 3y$

Subject to the constraint

$4x + y \geq 80$

$x + 5y \geq 115$

$3x + 2y \leq 150$

$x \geq 0, y \geq 0$

Answer

Given,

$z = 6x + 3y$

Constraints:

$4x + y \geq 80$

$x + 5y \geq 115$

$3x + 2y \leq 150$

$x \geq 0, y \geq 0$

First convert the given inequations into corresponding equations and plot them:

$4x + y \geq 80 \rightarrow 4x + y = 80$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 80$ (0,80) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 20$ (20,0) - - - - second coordinate

Join them to get the line.

As we know, Linear inequation will be a region in the plane, and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation,

If the given line does not pass through origin then just put (0,0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is the required region else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

$x + 5y \geq 115 \rightarrow x + 5y = 115$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 23$ (0,23) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 115$ (115,0) - - - - second coordinate

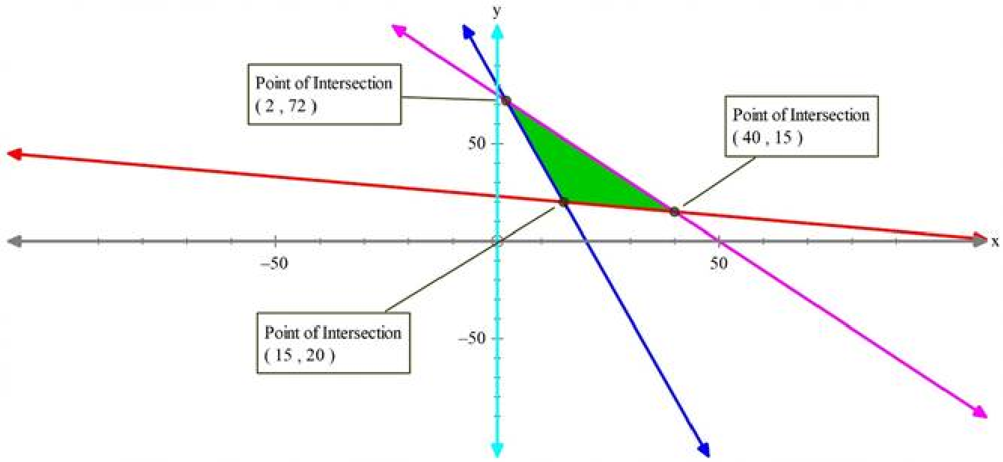
$3x + 2y \leq 150$ $3x + 2y = 150$ (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, $x = 0 \Rightarrow y = 75$ (0,75) - - - - first coordinate.

Put, $y = 0 \Rightarrow x = 50$ (50,0) - - - - second coordinate

$x = 0$ is y - axis and $y = 0$ is the x – axis.



Hence, we have the following plot:

The shaded region in the above figure represents the region of a feasible solution.

Now to minimize our objective function, we need to find the coordinates of the corner points of the shaded region.

We can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving $x + 5y = 115$ and $3x + 2y = 150$ gives (40,15)

Similarly solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corners as:

(15,20),(40,15) and (2,72)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because Z = 6x + 3y$

$\therefore$ Z at (15,20) = $6\times 15 + 3\times 20 = 150$

Z at (40,15) = $6\times 40 + 3\times 15 = 285$

Z at (2,72) = $6\times 2 + 10\times 72 = 732$

Z is minimum at x = 15, and y = 20 and the minimum value is 150

Exercise 30.3

1. Question

A diet of two foods F1 and F2 contains nutrients thiamine, phosphorous and iron. The amount of each nutrient in each of the food (in milligrams per 25 gms) is given in the following table :

Food / Nutrients	F1	F2
Thiamine	0.25	0.10
Phosphorous	0.75	1.50
Iron	1.60	0.80

The minimum requirement of the nutrients in the diet is 1.00 mg of thiamine, 7.50 mg of phosphorous and 10.00 mg of iron. The cost of F1 is 20 paise per 25 gms while the cost of F2 is 15 paise per 25 gms. Find the minimum cost of diet.

Answer

The information above can be expressed in the following table:

Food(↓)/Nutrients(⟹)	Thiamine	Phosphorous	Iron	Price/25g(₹)
F1	0.25	0.75	1.60	0.20
F2	0.10	1.50	0.80	0.15
Minimum Requirement	1.00	7.50	10.00	

Let the amount of food F1 and F2 required to be ‘x’ and ‘y’ units.

Cost of F1 = 0.20x

Cost of F2 = 0.15y

So, Cost of diet = 0.20x + 0.15y

Now,

⟹ 0.25x + 0.10y ≥ 1.00

i.e. the minimum requirement of Thiamine should be 1.00mg, from both the foods F1 and F2, each of which have 0.25mg and 0.10mg of Thiamine respectively. So, this is the first constraint.

⟹ 0.75x +1.50y ≥ 7.50

i.e. the minimum requirement of Phosphorous should be 7.50mg, from both the foods F1 and F2, each of which have 0.75mg and 1.50mg of Phosphorous respectively. This is the second constraint.

⟹ 1.60x + 0.80y ≥ 10.00

i.e. the minimum requirement of Iron should be 10.00mg, from both the foods F1 and F2, each of which have 1.60mg and 0.8mg of Iron respectively.

Hence, mathematical formulation of LPP is as follows:

Find ‘x’ and ‘y’ which

Minimise Z = 0.20x + 0.15y

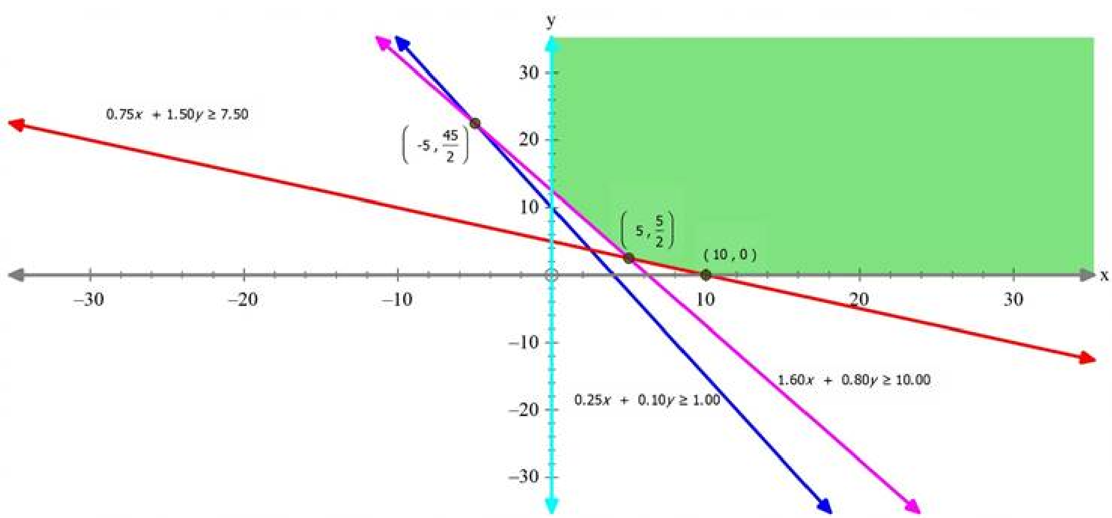
Subject to the following constraints:

(i) 0.25x + 0.10y ≥ 1.00

(ii) 0.75x +1.50y ≥ 7.50

(iii) 1.60x + 0.80y ≥ 10.00

(iv) x,y ≥0 (∵ quantity cant be negative)



The feasible region is unbounded

The end points of the feasible region are as follows:

Point	Value of Z
A(10,0)	2
B(5,2.5)	**1.375**
C(0,12.5)	1.875

So, Z is smallest at B(5,2.5)

Let us consider the inequation $0.20x + 0.15y \leq 1.375$

As this has no intersection with the feasible region, the smallest value is the minimum value.

So, the minimum cost of diet is ₹1.375

2. Question

A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 of calories. Two foods A and B, are available at the cost of ₹ 4 and ₹ 3 per unit respectively. If one unit of A contains 200 units of vitamin, 1 unit of mineral and 40 calories and one unit of food B contains 100 units of vitamin, 2 units of minerals and 40 calories, find what combination of foods should be used to have the least cost?

Answer

The above information can be expressed in the form of the following table:

Food(↓)/Nutrients(⟹)	Vitamins	Minerals	Calories	Price/unit
Food A	200	1	40	₹4
Food B	100	2	40	₹3
Minimum Requirement	4000	50	1400	

Let the quantity of the foods be ‘x’ and ‘y’ respectively.

Cost of food A = 4x

Cost of food B = 3y

Total cost of the combination = 4x + 3y

Now,

$\Longrightarrow 200x + 100y \geq 4000$

i.e. the minimum requirement of vitamins from the two foods should be 4000.

$\Longrightarrow x + 2y \geq 50$

i.e. the minimum requirement of minerals from the two foods should be 50.

$\Longrightarrow 40x + 40y \geq 1400$

i.e. the minimum requirement of calories from the two foods should be 1400

Hence, mathematical formulation of LPP is as follows:

Find ‘x’ and ‘y’ which

Minimize $Z = 4x + 3y$

Subject to the following constraints:

(i) $200x + 100y \geq 4000$

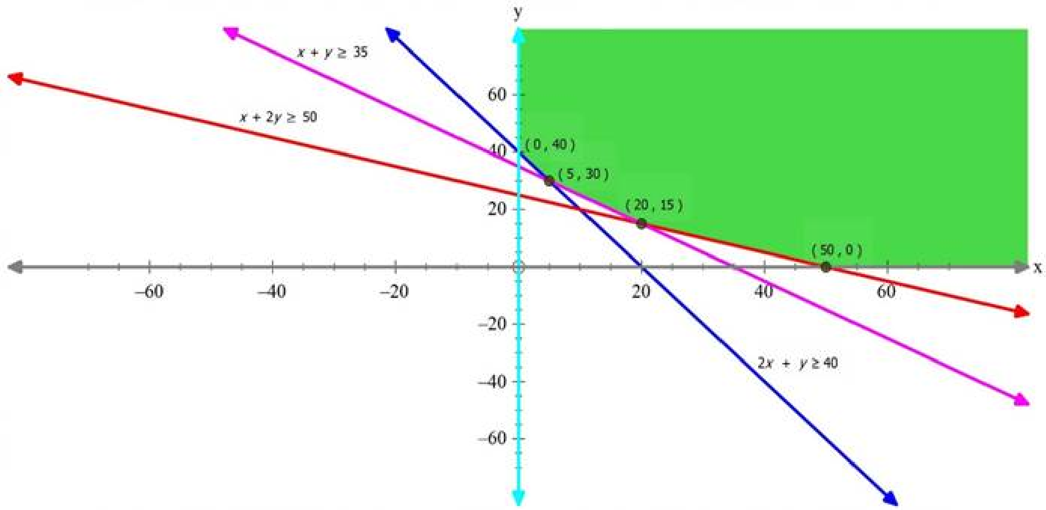
i.e. $2x + y \geq 40$

(ii) $x + 2y \geq 50$

(iii) $40x + 40y \geq 1400$

i.e. $x + y \geq 35$

(iv) $x, y \geq 0$ (∵ quantity cant be negative)



The feasible region is unbounded.

The corner points of the feasible region is as follows:

Point	Value of Z = 4x + 3y
A(0,40)	120
B(5,30)	**110**
C(20,15)	125
D(50,0)	200

Z is smallest at B(5,30)

Let us consider 4x + 3y ≤ 110

As it has no intersection with the feasible region, the smallest value is the minimum value.

The minimum cost of foods is ₹110

3. Question

To maintain one's health, a person must fulfill certain minimum daily requirement for the following three nutrients : calcium, protein and calories. The diet consists of only items I and II whose prices and nutrient contents are shown below :

	Food I	Food II	Minimum daily requirement
Calcium	10	4	20
Protein	5	6	20
Calories	2	6	12
Price	₹0.60 per unit	₹1.00 per unit	

Answer

Let the quantity of foods chosen be 'x' and 'y'

Cost of food X = 0.6x

Cost of food Y = y

Cost of diet = 0.6x + y

Now,

⟹ 10x + 4y ≥ 20

i.e. the minimum daily requirement of calcium in the diet is 20 units.

⟹ 5x + 6y ≥ 20

i.e. the minimum daily requirement of protein in the diet is 20 units.

$\Longrightarrow 2x + 6y \geq 12$

i.e. the minimum daily requirement of calories in the diet is 12 units.

Hence, mathematical formulation of the LPP is as follows:

Find ‘x’ and y’ such that

Minimises Z = 0.6x + y

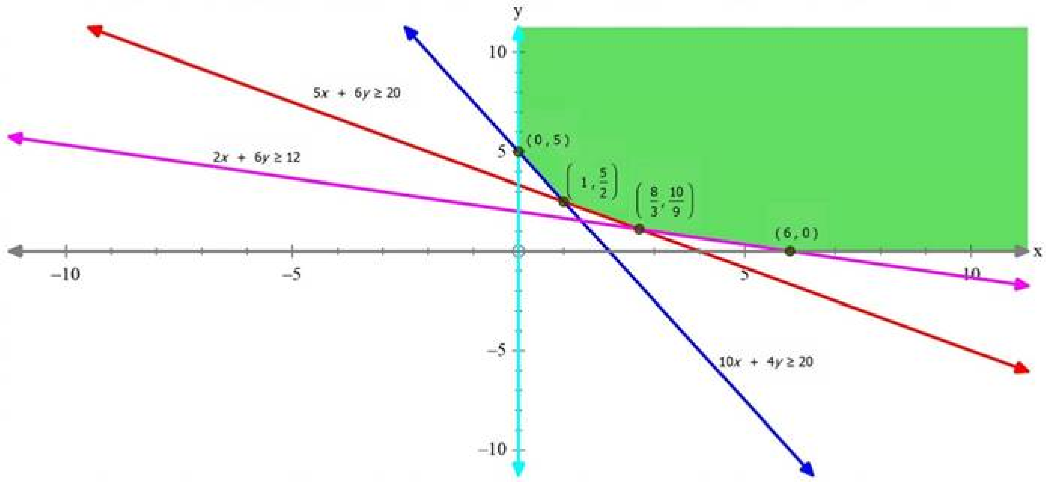
Subject to the following constraints:

(i) $10x + 4y \geq 20$

(ii) $5x + 6y \geq 20$

(iii) $2x + 6y \geq 12$

(iv) $x,y \geq 0$ ($\because$ quantity cant be negative)



The feasible region is unbounded

The corner points of the feasible region is as follows:

Point	Value of Z = 0.6x + y
A(0,5)	5
B(1,2.5)	3.1
$C\left(\frac{8}{3},\frac{10}{9}\right)$	**2.712**
D(6,0)	3.6

Z is smallest at $C\left(\frac{8}{3},\frac{10}{9}\right)$

Let us consider $0.6x + y \leq 2.712$.

As it has no intersection with the feasible region, the smallest value is the minimum value.

The minimum value of Z is ₹2.712

4. Question

A hospital dietician wishes to find the cheapest combination of two foods, A and B, that contains at least 0.5 milligram of thiamine and at least 600 calories. Each unit of A contains 0.12 milligram of thiamine and 100 calories, while each unit of B contains 0.10 milligram of thiamine and 150 calories. If each food costs 10 paise per unit, how many units of each should be combined at a minimum cost?

Answer

The above information can be expressed using the following table:

	Food A	Food B	Minimum daily requirement
Thiamine Calories	0.12 100	0.10 150	0.5 600
Price	₹0.10 per unit	₹0.10 per unit	

Let the quantity of the foods A and B be ‘x’ and ‘y’ respectively.

Cost of food A = 0.10x

Cost of food B = 0.10y

Cost of diet = 0.10x + 0.10y

Now,

$\Longrightarrow$ 0.12x + 0.10y ≥ 0.5

i.e. the minimum requirement of thiamine in the foods is 0.5mg

$\Longrightarrow$ 100x + 150y ≥ 600

i.e. the minimum requirement of calories in the foods is 600.

Hence, mathematical formulation of the LPP is as follows:

Find ‘x’ and ‘y’ that:

Minimises Z = 0.10x + 0.10y

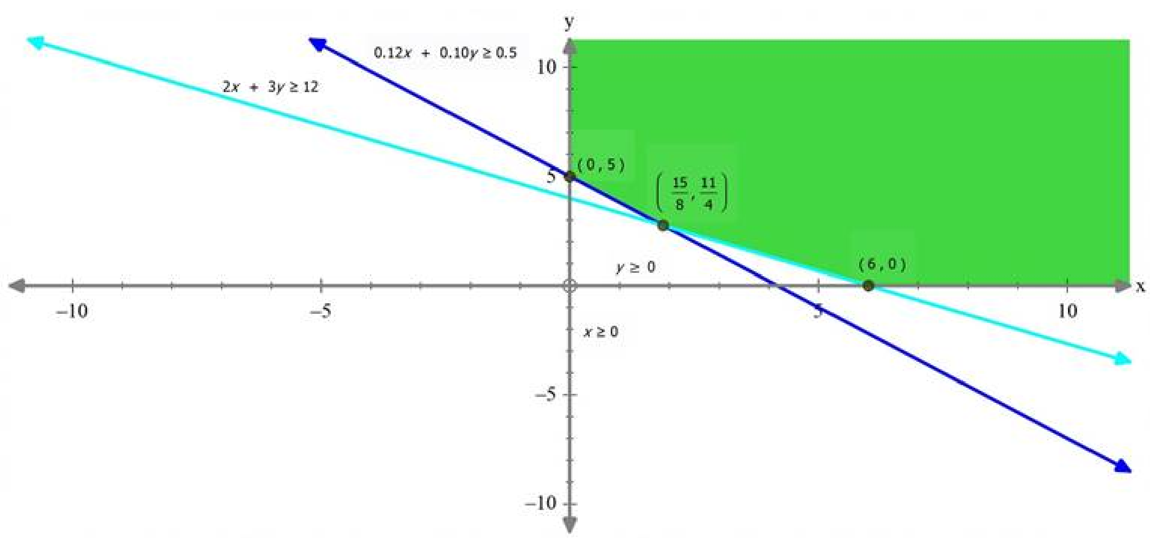
Subject to the following constraints:

(i) 0.12x + 0.10y ≥ 0.5

(ii) 100x + 150y ≥ 600

i.e. 2x + 3y ≥ 12

(iii) x,y ≥ 0 (∵ quantity cant be negative)



The feasible region is unbounded.

The corner points of the feasible region is as follows:

Point	Value of Z = 0.1x+0.1y
A(1.875,2.75)	**0.4625**
B(6,0)	0.6
C(0,5)	0.5

Z is smallest at A(1.875,2.75)

Let us consider $0.1x+0.1y \leq 0.4625$

As it has no intersection with the feasible region, the smallest value is the minimum value.

The minimum cost of the foods is ₹0.4625

5. Question

A dietician mixes together two kinds of food in such a way that the mixture contains at least 6 units of vitamin A, 7 units of vitamin B, 11 units of vitamin C and 9 units of vitamin D. The vitamin contents of 1 kg of food X and 1 kg of food Y are given below :

	Vitamin A	Vitamin B	Vitamin C	Vitamin D
Food X	1	1	1	2
Food Y	2	1	3	1

One kg of food X costs ₹ 5, whereas one kg of food Y costs ₹ 8. Find the least cost of the mixture which will produce the desired diet.

Answer

The above information can be expressed with the help of the following table:

Food(↓)/Nutrients(⇒)	Vitamin A	Vitamin B	Vitamin C	Vitamin D	Price/kg
Food X	1	1	1	2	₹5
Food Y	2	1	3	1	₹8
Minimum Requirement	6	7	11	9	

Let the quantity of foods X and Y be ‘x’ and ‘y’.

Cost of food X = 5x

Cost of food Y = 8y

Cost of the meal 5x+8y

Now,

$\Longrightarrow x + 2y \geq 6$

i.e. the minimum requirement of Vitamin A in the foods X and Y is 6units, each of which has 1unit and 2 unit of Vitamin A.

$\Longrightarrow x + y \geq 7$

i.e. the minimum requirement of Vitamin B in the two foods is 7units, each of which has 1 unit of Vitamin B.

$\Longrightarrow x + 3y \geq 11$

i.e. the minimum requirement of vitamin C in the two foods is 11units, each of which has 1 unit and 3 units of vitamin C.

$\Longrightarrow 2x + y \geq 9$

i.e. the minimum requirement of Vitamin D in the foods is 9units, each of which has 2 units and 1 unit of Vitamin D.

Hence, mathematical formulation of the LPP is as follows:

Find ‘x’ and ‘y’ that

Minimises Z = 5x + 8y

Subject to the following constraints:

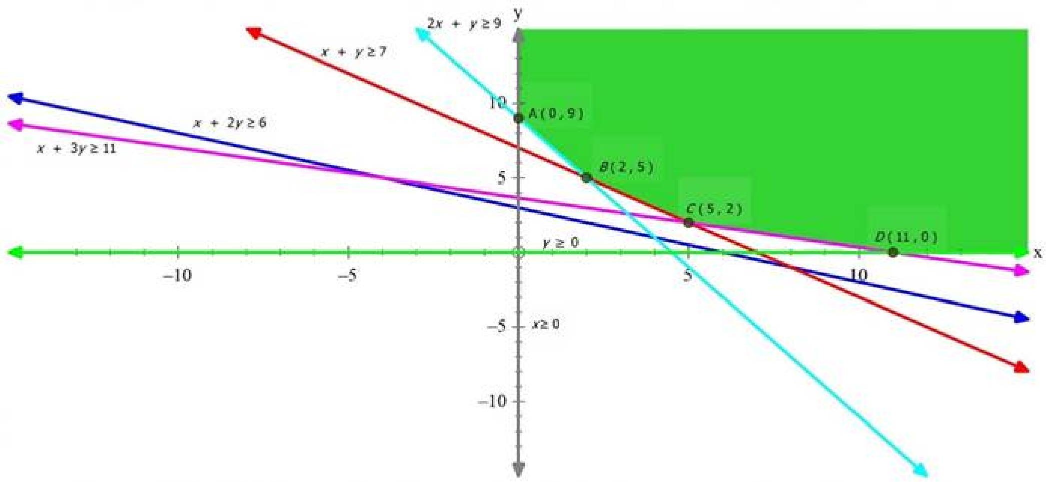
(i) x + 2y ≥ 6

(ii) x + y ≥ 7

(iii) x + 3y ≥ 11

(iv) 2x + y ≥ 9

(v) x,y ≥ 0 (∵ quantity cant be negative)



The feasible region is unbounded.

The corner points of the feasible region are as follows:

Point	Value of Z = 5x + 8y
A(0,9)	72
B(2,5)	50
C(5,2)	**41**
D(11,0)	55

Z is smallest at C(5,2)

Let us consider 5x + 8y ≤ 41.

As it has no intersection with the feasible region, the smallest value is the minimum value.

The minimum cost of the diet is ₹41

6. Question

A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F1 and F2 are available. Food F2 costs ₹ 4 per unit F2 costs ₹ 6 per unit one unit of food F1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these foods and also meets the mineral nutritional requirements.

Answer

The above information can be expressed in the following table:

	F1	F2	Minimum daily requirement
Vitamins Minerals	3 4	6 3	80 100
Price	₹4 per unit	₹6 per unit	

Let the quantity of the foods F1 and F2 be ‘x’ and ‘y’ respectively.

Cost of food F1 = 4x

Cost of food F2 = 6y

Cost of Diet = 4x + 6y

Now,

$\Longrightarrow$ 3x + 6y ≥ 80

i.e. the minimum requirement of Vitamins from the two foods is 80units, each of which contains 3units and 6units of Vitamins.

$\Longrightarrow$ 4x + 3y ≥ 100

i.e. the minimum requirement of minerals firm the two foods is 100units, each of which contains 4unit and 3 units of vitamins.

Hence, mathematical formulation of the LPP is as follows:

Find ‘x’ and ‘y’ that:

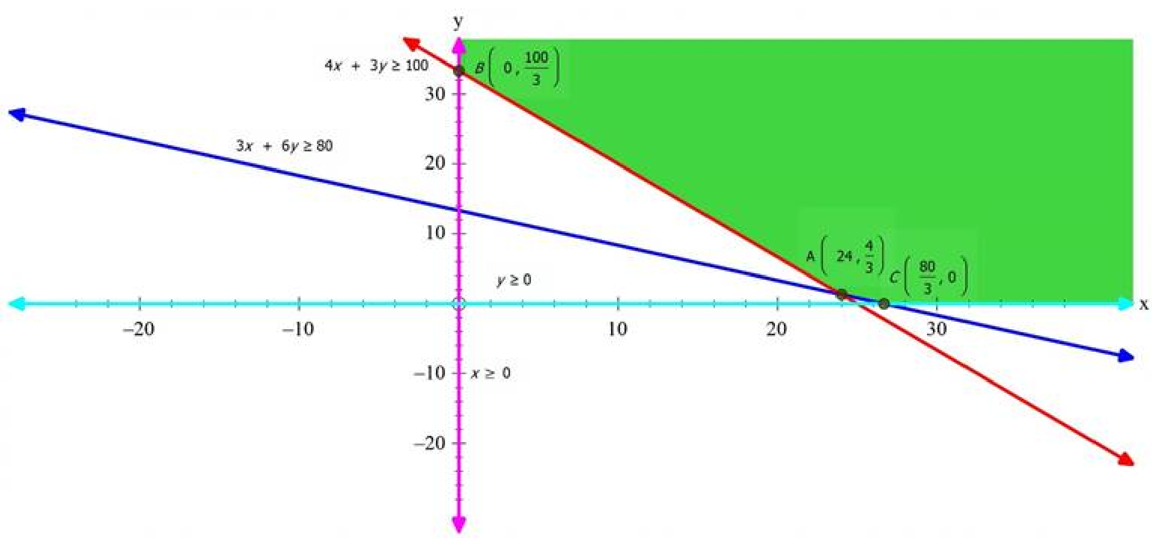
Minimise Z = 4x + 6y

Subject to the following constraints:

(i) 3x + 6y ≥ 80

(ii) 4x + 3y ≥ 100

(iii) x,y ≥ 0 (∵ quantity cant be negative)



The feasible region is unbounded.

The corner points of the feasible region is as follows:

Point	Value of Z = 4x + 6y
$A\left(24,\frac{4}{3}\right)$	**104**
$B\left(0,\frac{100}{3}\right)$	200
$C\left(\frac{80}{3},0\right)$	106.667

Z is smallest at $A\left(24,\frac{4}{3}\right)$

Let us consider 4x + 6y ≤ 104

As it has no intersection with the feasible region, the smallest value is the minimum value.

The minimum cost of diet is ₹104.

7. Question

Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams of protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs ₹ 5 per kg and rice costs ₹ 4 per kg.

Answer

The above information can be expressed using the following table:

	Bran	Rice	Minimum requirement
Proteins(g) Iron (mg)	80 40	100 30	88 36
Price	₹5 /kg	₹4 /kg	

Let the amount of Bran and Rice required be ‘x’ and ‘y’ kgs respectively.

Cost of Bran = $5x$

Cost of Rice = $4y$

Cost of the cereal = $5x + 4y$

Now,

$\Longrightarrow 80x + 100y \geq 88$

i.e. the minimum requirement of protein in the cereal, from Bran and Rice combined, is 88g, each of which have 80g and 100g of proteins respectively.

$\Longrightarrow 40x + 30y \geq 36$

i.e. the minimum requirement of iron in the cereal, from Bran and Rice combined, is 36mg, each of which have 40mg and 30mg of iron.

Hence, mathematical formulation of LPP is as follows:

Find ‘x’ and ‘y’ that:

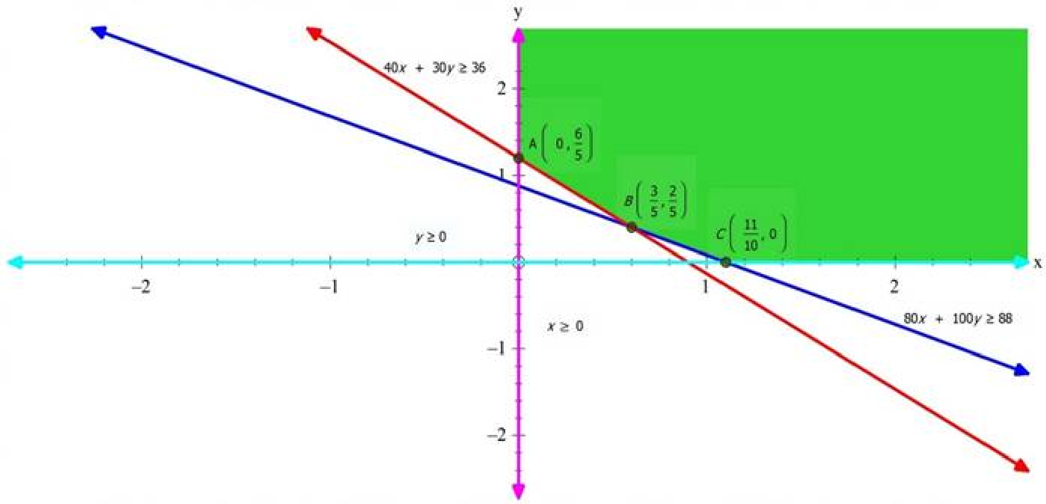
Minimises $Z = 5x + 4y$

Subject to the following constraints:

(i) $80x + 100y \geq 88$

(ii) $40x + 30y \geq 36$

(iii) $x,y \geq 0$ ($\because$ quantity cant be negative)



The feasible region is unbounded.

The corner points of the feasible region is as follows:

Point	Value of Z = 5x + 4y
A(0,1.2)	4.8
B(0.6,0.4)	**4.6**
C(1.1,0)	5.5

Z is smallest at B(0.6,0.4)

Let us consider 5x + 4y ≤ 4.6

As it has no intersection with the feasible region, the smallest value is the minimum value.

The minimum cost of the cereal is ₹4.6

8. Question

A wholesale dealer deals in two kinds, A and B (say) of mixture of nuts. Each kg of mixture A contains 60 grams of almonds, 30 grams of cashew nuts and 30 grams of hazel nuts. Each kg of mixture B contains 30 grams of almonds, 60 grams of cashew nuts and 180 grams of hazel nuts. The remainder of both mixtures is per nuts. The dealer is contemplating to use of cashew nuts and 540 grams of hazel nuts. Mixture A costs ₹ 8 per kg. and mixture B costs ₹ 12 per kg. Assuming that mixtures A and B are uniform, use graphical method to determine the number of kg. of each mixture he should use to minimize the cost of the bag.

Answer

The above information can be expressed in the form of the following table:

	Bag A	Bag B	Minimum Requirement (g)
Almonds(g) Cashew Nuts(g) Hazel Nuts(g)	60 30 30	30 60 180	240 300 540
Price	₹8 /kg	₹12 /kg	

Let the number of bags chosen of A and B be ‘x’ and ‘y’ respectively.

Cost of Bag A = 8x

Cost of Bag B = 12y

Total Cost of Bags = 8x + 12 y

Now,

$\Longrightarrow 60x + 30y \geq 240$

i.e. the minimum requirement of almonds from both the bags is 240g, each of which contains 60g and 30g of almonds respectively.

$\Longrightarrow 30x + 60y \geq 300$

i.e. the minimum requirement of Cashew Nuts from both the bags is 300g, each of which contains 30g and 60g of cashew nuts respectively.

$\Longrightarrow 30x + 180y \geq 540$

i.e. the minimum requirement of Hazel Nuts from both the bags is 540g, each of which contains 30g and 180g of hazelnut respectively.

Hence, mathematical formulation of the LPP is as follows:

Find ‘x’ and ‘y’ that

Minimises $Z = 8x + 12y$

Subject to the following constraints:

(i) $60x + 30y \geq 240$

i.e. $2x + y \geq 8$

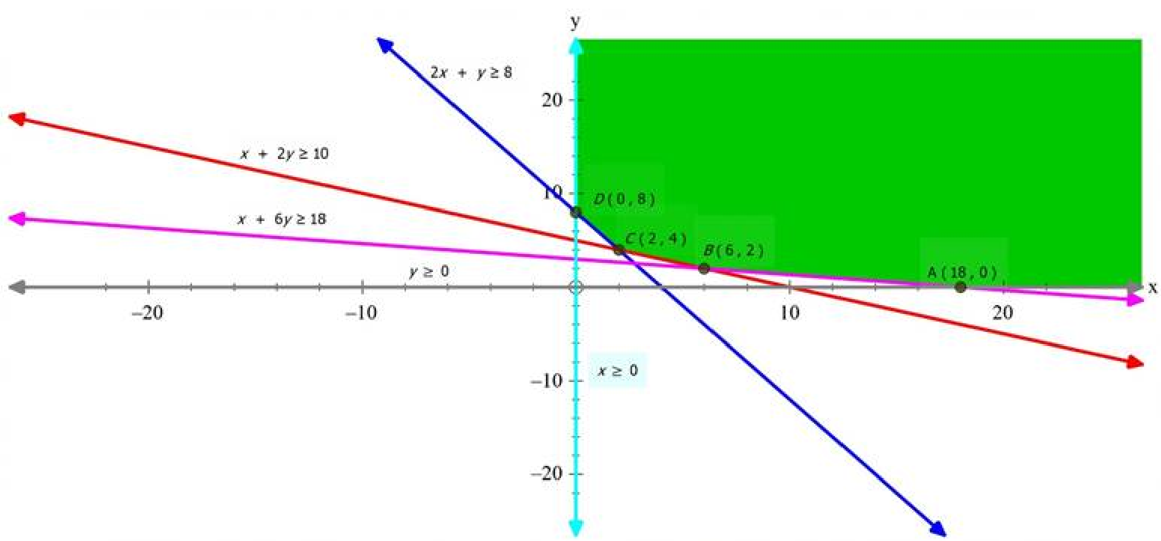
(ii) $30x + 60y \geq 300$

i.e. $x + 2y \geq 10$

(iii) $30x + 180y \geq 540$

i.e. $x + 6y \geq 18$

(iv) $x,y \geq 0$ ($\because$ quantity cant be negative)



The feasible region is unbounded.

The corner points of the feasible region are as follows:

Point	Value of Z = 8x + 12y
A(18,0)	144
B(6,2)	72
C(2,4)	**64**
D(0,8)	96

Z is smallest at C(2,4)

Let us consider $8x + 12y \leq 64$

As this has no intersection with the feasible region, the smallest value is the minimum value.

The minimum cost of the bags is ₹64

9. Question

One kind of cake requires 300 gm of flour and 15 gm of fat, another kind of cake requires 150 gm of flour and 30 gm of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 gm of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an LPP and solve it graphically.

Answer

The above information can be expressed in the form of the following table

	Cake 1	Cake 2	Maximum Availability (g)
Flour(g) Fat(g)	300 15	150 30	7500 600
Price	₹5 /kg	₹4 /kg	

Let ‘x’ and ‘y’ units of cake 1 and cake 2 be made.

Number of cakes made = x + y

Now,

$\Longrightarrow 300x + 150y \leq 7500$

i.e. the maximum availability of flour is 7500g for both cakes, each of which requires 300g and 150g of flour respectively

$\Longrightarrow 15x + 30y \geq 600$

i.e. the maximum availability of fat is 600g for both the cakes, each of which requires 15g and 30g of fat.

Hence , mathematical formulation of the LPP is as follows :

Find ‘x’ and ‘y’ that,

Maximises $Z = x + y$

Subject to the following constraints:

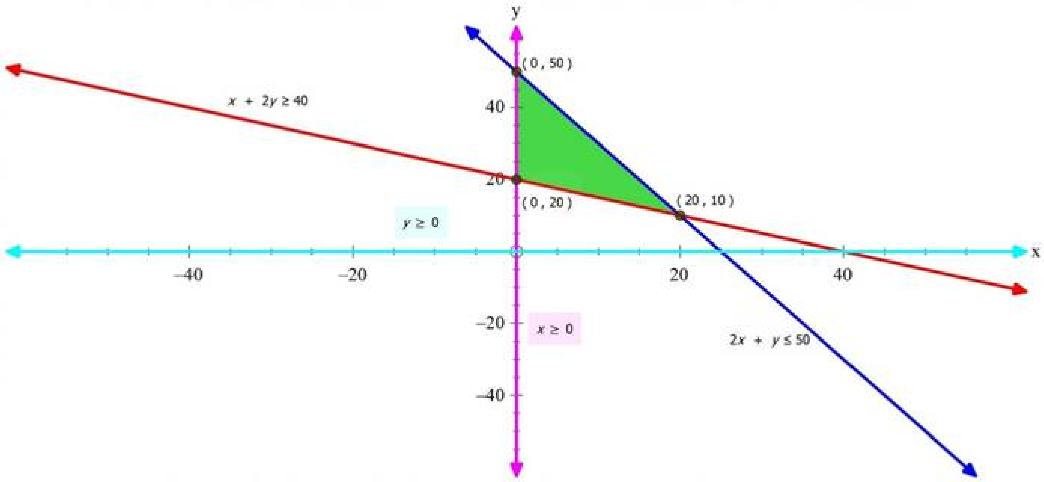
(i) $300x + 150y \leq 7500$

i.e. $2x + y \leq 50$

(ii) $15x + 30y \geq 600$

i.e. $x + 2y \geq 40$

(iii) $x,y \geq 0$ ($\because$ quantity cant be negative)



The feasible region is bounded (ABO)

The corner points of the feasible region is as follows:

Point	Value of Z = x + y
A(25,0)	25
B(20,10)	**30**
C(0,20)	20
O(0,0)	0

Z is maximised at B(20,10)

The maximum number of cakes that can be made are 20 and 10 of each kind i.e. 30 in total.

10. Question

Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs ₹ 60 kg and Food Q costs ₹ 80 kg. Food P contains 3 units / kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units / kg of Vitamin A and 2 units / kg of vitamin B. Determine the minimum cost of the mixture.

Answer

The above information can be expressed in the form of the following table:

	P	Q	Minimum requirement
Vitamin A Vitamin B	3 5	4 2	8 11
Price	₹60 per kg	₹80 per kg	

Let the mixture contain ‘x’ kgs and ‘y’ kgs of food P and Q respectively.

Cost of food P = 60x

Cost of food Q = 80y

Cost of mixture = 60x + 80y

Now,

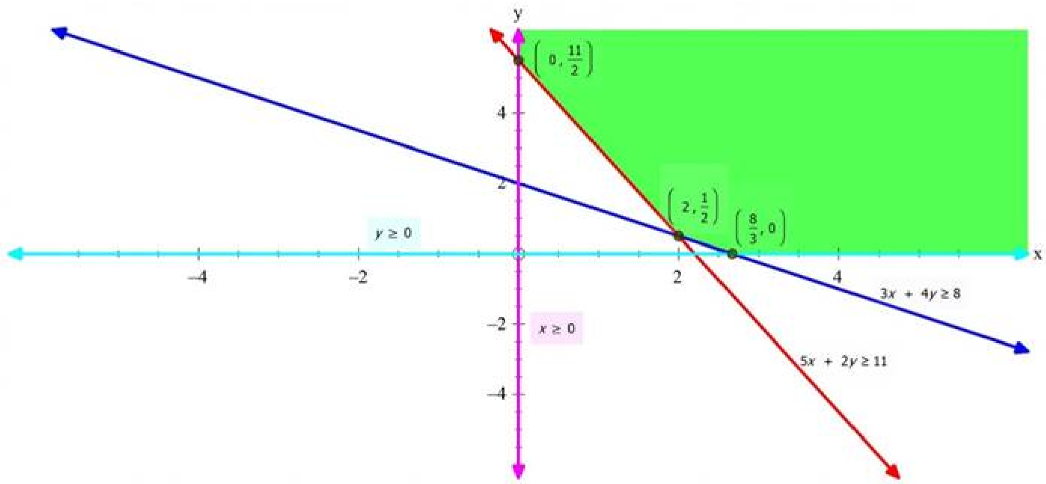
$\Longrightarrow 3x + 4y \geq 8$

i.e. the minimum requirement of vitamin A from the mixture of P and Q is 8units, each of which contains 3units and 4units respectively.

$\Longrightarrow 5x + 2y \geq 11$

i.e. the minimum requirement of vitamin B from the mixture of P and Q is 11 units, each of which contains 5units and 2units respectively.

Hence, mathematical formulation of the LPP is as follows:



The feasible region is Unbounded.

The corner points of the feasible region are as follows:

Point	Value of Z = 60x + 80y
A(0,5.5)	440
B(2,0.5)	**160**
$C\left(\frac{8}{3},0\right)$	**160**

Z is minimised on the line joining points B(2,0.5) and $C\left(\frac{8}{3},0\right)$.

The minimum cost of mixture is ₹160

11. Question

One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

Answer

The above information can be expressed in the form of the following table:

	Cake 1	Cake 2	Maximum Availability
Flour(g)	200	100	5000
Fat(g)	25	50	1000

Let the number of Cake 1 and Cake 2 be made be ‘x’ and ‘y’

Number of cakes made = x + y

Now,

$\Longrightarrow$ 200x + 100y ≤ 5000

i.e. the maximum flour available for both the cakes combined is 5000g, each of which requires 200g and 100g of flour respectively.

$\Longrightarrow$ 25x + 50y ≤ 1000

i.e. the maximum fat available for the two cakes combined is 1000g, each of which requires 25g and 50g of fat respectively.

Hence, the mathematical formulation of the LPP is as follows:

Find 'x' and 'y' that:

Maximises Z = x + y

Subject to the following constraints:

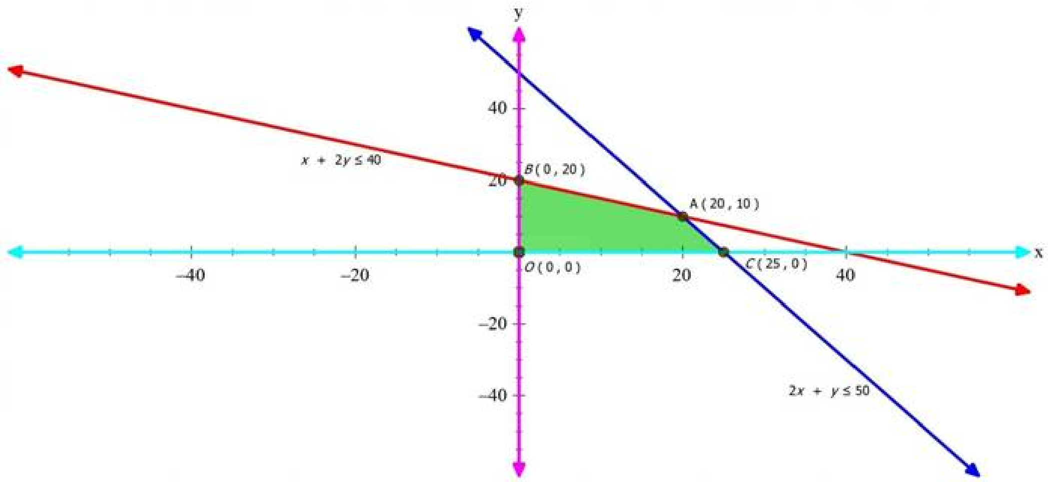
(i) 200x + 100y ≤ 5000

i.e. 2x + y ≤ 50

(ii) 25x + 50y ≤ 1000

i.e. x + 2y ≤ 40

(iii) x,y ≥ 0 (∵ quantity cant be negative)



The feasible region is bounded (OBAC)

The corner points of the feasible region is as follows:

Point	Value of Z = x + y
A(20,10)	**30**
B(0,20)	20
C(25,0)	25
O(0,0)	0

Z is maximised at A(20,10)

The maximum number of cakes that can be made are 30.

12. Question

A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

Answer

The above information can be expressed with the help of the following table

	P	Q	Requirement
Calcium	12	3	At least 240
Iron	4	20	At least 460
Cholesterol	6	4	At most 300
Vitamin A	6	3	

Let the number of packets bought, of P and Q, be ‘x’ and ‘y’

Vitamin A from P = 6x

Vitamin A from Q = 3y

Vitamin A in the diet = 6x + 3y

Now,

$\Longrightarrow 12x + 3y \geq 240$

i.e. the minimum requirement of Calcium in the diet, form both the foods combined, is 240units, each of which has 12units and 3units of calcium respectively.

$\Longrightarrow 4x + 20y \geq 460$

i.e. the minimum requirement of Iron from P and Q combined is 460 units, each of which has 4units and 20units of iron respectively.

$\Longrightarrow 6x + 4y \leq 300$

i.e. the maximum requirement of Cholesterol from P and Q combined is 300 units, each of which contains 6units and 4units of cholesterol respectively.

Hence, the mathematical formulation of the LPP is as follows:

Find ‘x’ and ‘y’ that:

Minimises Z = 6x + 3y

Subject to the following constraints:

(i) $12x + 3y \geq 240$

i.e. $4x + y \geq 80$

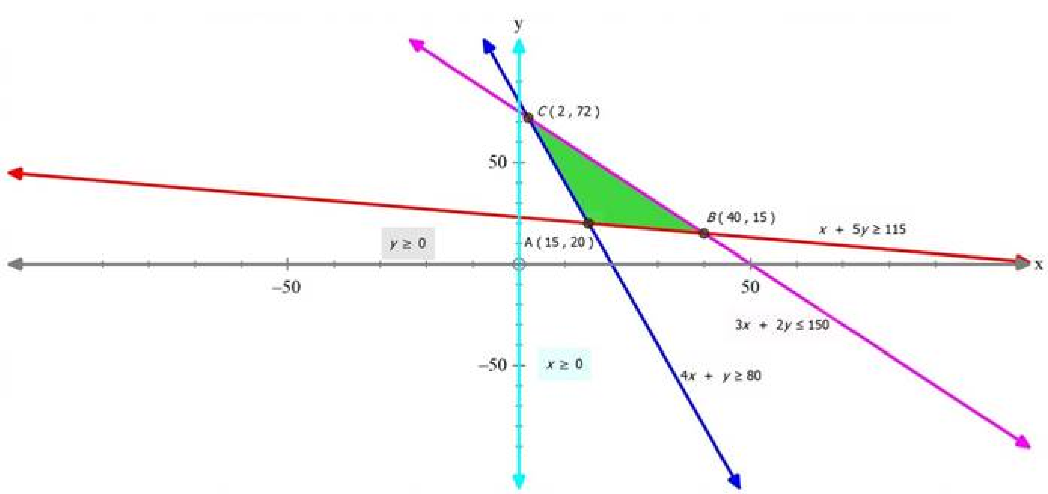
(ii) $4x + 20y \geq 460$

i.e. $x + 5y \geq 115$

(iii) $6x + 4y \leq 300$

i.e. $3x + 2y \leq 150$

(iv) $x, y \geq 0$ (∵ quantity cant be negative)



The feasible region is bounded (ABC)

The corner points of the feasible region are as follows:

Point	Value of Z = 6x + 3y
A(15,20)	**150**
B(40,15)	285
C(2,72)	228

Z is minimised at A(15,20) i.e. 15 packets of P and 20 packets of Q should be used to minimise the amount of vitamin A.

The minimum amount of vitamin A is 150 units.

13. Question

A farmer mixes two brands P and Q of cattle feed. Brand P, costing ₹ 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing ₹ 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

Answer

The above information can be expressed with the help of the following table:

	P	Q	Minimum Requirement
Element A	3	1.5	18
Element B	2.5	11.25	45
Element C	2	3	24
Cost per bag	₹250	₹200	

Let ‘x’ bags of P and ‘y’ bags of Q be bought.

Cost of P = 250x

Cost of Q = 200y

Cost of mixture = 250x + 200y

Now,

$\Longrightarrow 3x + 1.5y \geq 18$

i.e. the minimum requirement of element A from both P and Q combined is 18units, each of which has 3units and 1.5units of element A.

$\Longrightarrow 2.5x + 11.25y \geq 45$

i.e. the minimum value of element B from both P and Q combined is 45units, each of which contains 2.5units and 11,25units of element B.

$\Longrightarrow 2x + 3y \geq 24$

i.e. the minimum value of element C from both P and Q combined is 24units, each of which contains 2units and 3 units of element C.

Hence, mathematical formulation of the above LPP is as follows :

Find ‘x’ and ‘y’ that:

Minimises Z = 250x + 200y

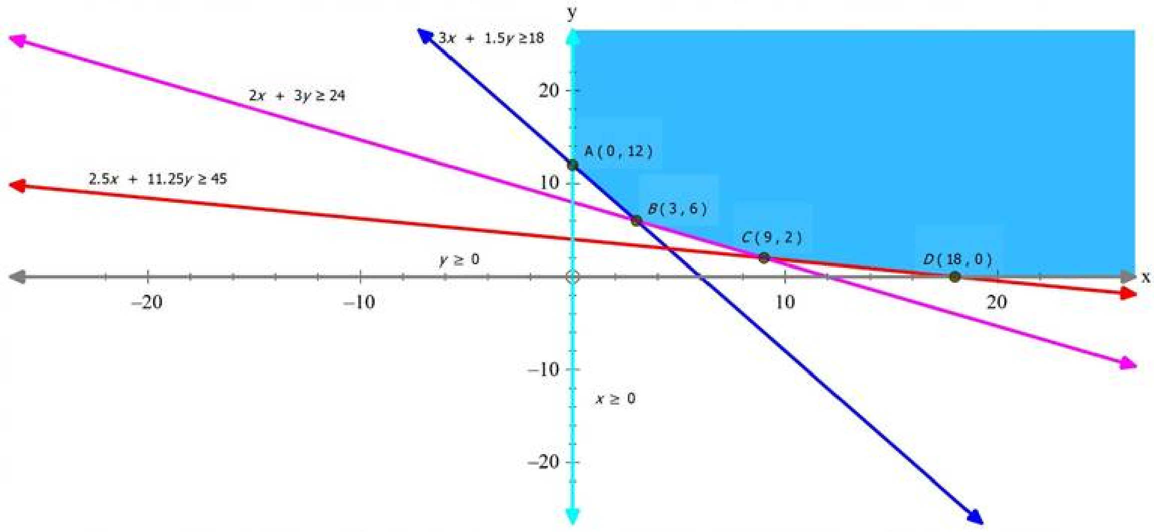
Subject to the following constraints:

(i) $3x + 1.5y \geq 18$

(ii) $2.5x + 11.25y \geq 45$

(iii) $2x + 3y \geq 24$

(iv) $x,y \geq 0$ (∵ quantity cant be negative)



The feasible region is unbounded

The corner points of the feasible region are as follows:

Point	Value of Z = 250x + 200y
A(0,12)	2400
B(3,6)	**1950**
C(9,2)	2650
D(18,0)	4500

Z is minimised at B(3,6) i.e. 3 bags of P and 6 bags of Q should be purchased to achieve the minimum cost of the mixture per bag.

The minimum cost of the mixture is ₹1950.

14. Question

A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below :

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs ₹16 and one kg of food Y costs ₹ 20. Find the least cost of the mixture which will produce the required diet?

Answer

The above information can be expressed in the form of the following table:

	X	Y	Minimum Requirement
Vitamin A	1	2	10
Vitamin B	2	2	12
Vitamin C	3	1	8
Cost per kg	₹16	₹20	

Let the quantity of X and Y purchased be ‘x’ and ‘y’ kgs

Cost of X = 16x

Cost of Y = 20y

Cost of the mixture = 16x + 20y

Now,

$\Longrightarrow x + 2y \geq 10$

i.e. the minimum requirement of Vitamin A from the mixture of X and Y is 10 units, each of which contains 1 unit and 2 units of Vitamin A respectively.

$\Longrightarrow 2x + 2y \geq 12$

i.e. the minimum requirement of Vitamin B from the mixture of X and Y is 12 units, each of which contains 2units of vitamin B each.

$\Longrightarrow 3x + y \geq 8$

i.e. the minimum requirement of vitamin C from the mixture of X and Y is 8 units, each of which contains 3 units and 1 unit of vitamin C respectively.

Hence, the mathematical formulation of the LPP is as follows :

Find 'x' and 'y' that:

Minimises $Z = 16x + 20y$

Subject to the following constraints:

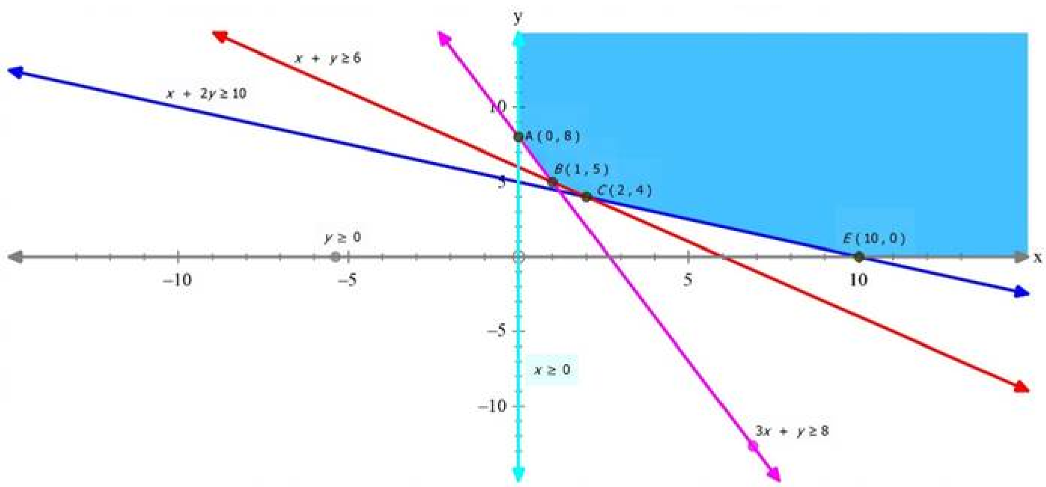
(i) $x + 2y \geq 10$

(ii) $2x + 2y \geq 12$

i.e. $x + y \geq 6$

(iii) $3x + y \geq 8$

(iv) $x,y \geq 0$ ($\because$ quantity cant be negative)



The feasible region is unbounded

The corner points of the feasible region is as follows:

Point	Value of Z = 16x + 20y
A(0,8)	160
B(1,5)	116
C(2,4)	**112**
D(6,2)	136
E(10,0)	160

Z is smallest at C(2,4)

Let us consider $16x + 20y \leq 112$

As it has no intersection with the feasible region, the smallest value is the minimum value.

The minimum cost of the mixture is ₹112.

15. Question

A fruit grower can use two types of fertilizer in his garden, brand P and Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine.

Kg per bag		
	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

Answer

The above information can be expressed with the help of the following table:

	P	Q	Requirement
Phosphoric Acid	1	2	At least 240
Potash	3	1.5	At least 270
Chlorine	1.5	2	At most 310
Nitrogen	3	3.5	

Let the number of bags of P and Q chosen be ‘x’ and ‘y’ units.

Nitrogen from P = 3x

Nitrogen from Q = 3.5y

Nitrogen form the mixture = 3x + 3.5y

Now,

$\Longrightarrow x + 2y \geq 240$

i.e. the minimum requirement of phosphoric acid in the mixture of P and Q is 240 kgs, each of which contains 1kg and 2kgs of phosphoric acid respectively

$\Longrightarrow 3x + 1.5y \geq 270$

i.e. the minimum requirement of Potash in the mixture of P and Q is 270 kgs, each of which contains 3kgs and 1.5kgs of Potash respectively.

$\Longrightarrow 1.5x + 2y \leq 310$

i.e. the maximum requirement of Chlorine in the mixture of P and Q is 310 kgs, each of which contains 1.5kgs and 2kgs of Chlorine respectively.

Hence, mathematical formulation of the LPP is as follows:

Find ‘x’ and ‘y’ that

Minimises Z = 3x + 3.5y

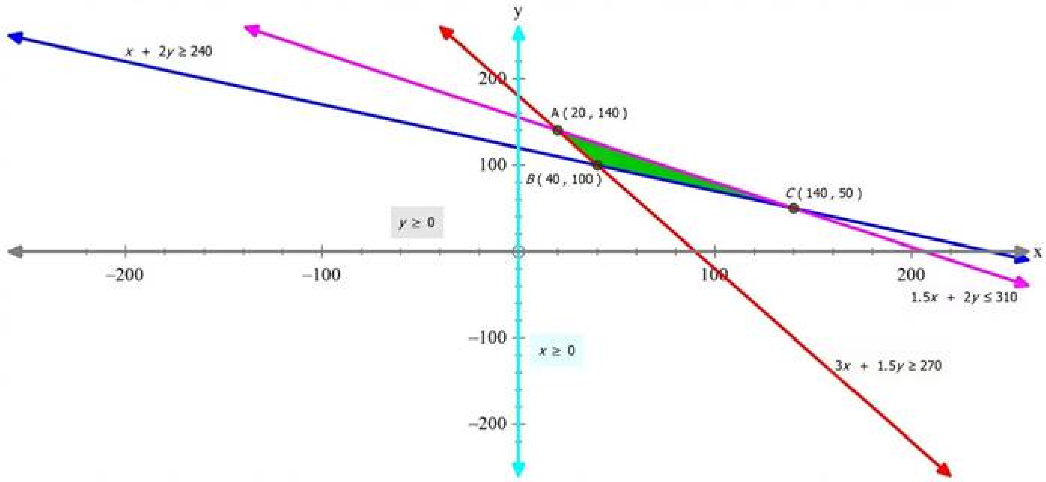
Subject to the following constraints:

(i) $x + 2y \geq 240$

(ii) $3x + 1.5y \geq 270$

(iii) $1.5x + 2y \leq 310$

(iv) $x,y \geq 0$ ($\because$ quantity cant be negative)



The feasible region is bounded (ABC)

The corner points of the feasible region is as follows:

Point	Value of Z = 3x + 3.5y
A(20,140)	550
B(40,100)	**470**
C(140,50)	595

Z is minimised at B(40,100)

The minimum amount of Nitrogen in the mixture is 470kgs

Exercise 30.4

1. Question

If a young man drives his scooter at a speed of 25 km/hr, he has to spend Rs2 per km on petrol. If he drives the scooter at a speed of 40 km/hour, it produces air pollution and increases his expenditure on petrol to Rs 5 per km. He has a maximum of Rs100 to spend on petrol and travel a maximum distance in one hour time with less pollution. Express this problem as an LPP and solve it graphically. What value do you find here?

Answer

Let young man drives x km at a speed of 25 km/hr and y km at a speed of 40 km/hr. Clearly,

x, y $\geq$ 0

It is given that, he spends Rs 2 per km if he drives at a speed of 25 km/hr and Rs 5 per km if he drives at a speed of 40 km/hr. Therefore, money spent by him when he travelled x km and y km are Rs 2x and Rs 5y respectively.

It is given that he has a maximum of Rs 100 to spend.

Thus, 2x + 5y $\leq$ 100

Time spent by him when travelling with a speed of 25 km/hr = $\frac{x}{25}$ hr

Time spent by him when travelling with a speed of 40km/hr = $\frac{y}{40}$ hr

Also, the available time is 1 hour.

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

Or, 40x + 25y$\leq$1000

The distance covered is Z = x + y which is to be maximized.

Thus, the mathematical formulation of the given linear programming problem is Max Z = x + y subject to

2x + 5y $\leq$ 100

40x + 25y $\leq$ 1000

x, y $\geq$ 0

First we will convert inequations as follows:

2x + 5y = 100

40x + 25y = 1000

x = 0 and y = 0.

The region represented by 2x + 5y $\leq$ 100

The line 2x + 5y = 100 meets the coordinate axes at A(50,0) and B(0,20) respectively. By joining these points, we obtain the line 2x + 5y = 100. Clearly (0, 0) satisfies the 2x + 5y = 100. So, the region which contains the origin represents the solution set of the inequation 2x + 5y $\leq$ 100

The region represented by 40x + 25y $\leq$ 1000

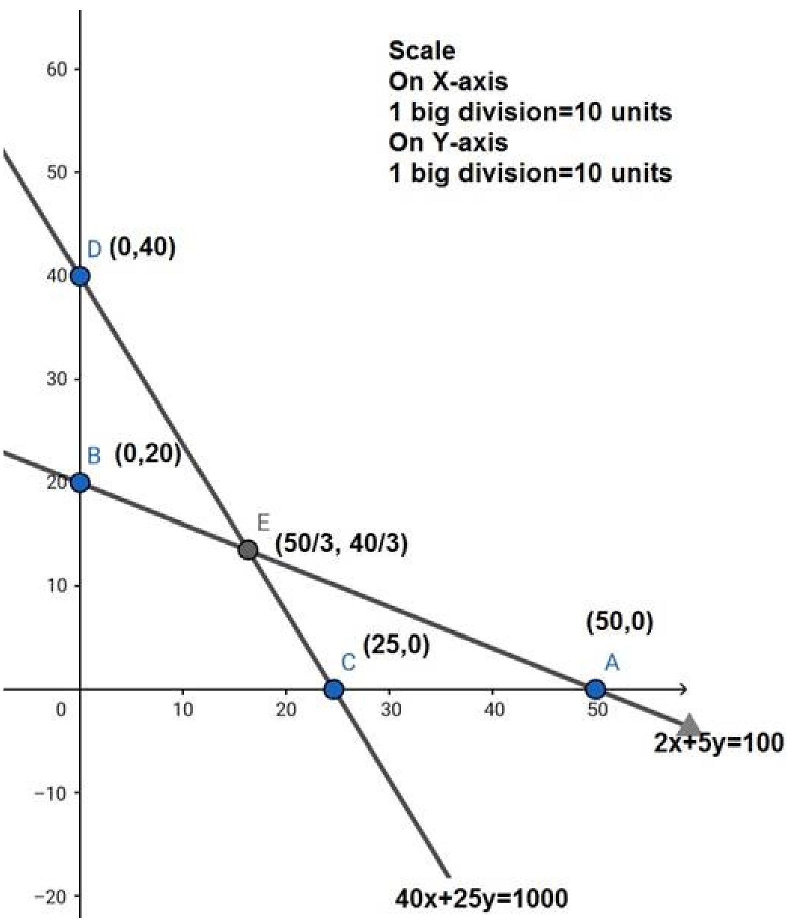
The line 40x + 25y = 1000 meets the coordinate axes at C(25,0) and D(0,40) respectively. By joining these points, we obtain the line 2x + y = 12. Clearly (0, 0) satisfies the 40x + 25y = 1000. So, the region which contains the origin represents the solution set of the inequation 40x + 25y $\leq$ 1000

The region represented by x $\geq$ 0, y $\geq$ 0 :

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0.

The feasible region determined by the system of constraints

2x + 5y $\leq$ 100, 40x + 25y $\leq$ 1000, x $\geq$ 0 and y $\geq$ 0 are as follows



The corner points are O(0,0), B(0,20), E$\left(\frac{50}{3}, \frac{40}{3}\right)$, and C(25,0). The value of Z at these corner points are as follows:

Corner Points	Z = x + y
(0,0)	0
(0,20)	20
$\left(\frac{50}{3}, \frac{40}{3}\right)$	30
(25,0)	25

The maximum value of Z is 30 which is attained at E.

Thus, the maximum distance travelled by the young man is 30 kms, if he drives $\frac{50}{3}$ km at a speed of 25 km/hr and $\frac{40}{3}$ km at a speed of 40 km/hr.

2. Question

A manufacturer has three machines installed in his factory. Machines I and II are capable of being operated for at most 12 hours whereas Machine III must operate at least for 5 hours a day. He produces only two items, each requiring the use of three machines. The number of hours required for producing one unit each of the items on the three machines is given in the following table :

Item	Number of hours required by the machine		
	I	II	III
A B	1 2	2 1	1 5/4

He makes a profit of Rs 6.00 on item A and Rs 4.00 on item B. Assuming that he can sell all that he produces, how many of each item should he produce to maximize his profit? Determine his maximum profit. Formulate this LPP mathematically and then solve it.

Answer

Let x units of item A and y units of item B be manufactured. Therefore, x, y $\geq$ 0.

As we are given,

Item	Number of hours required by the machine		
	I	II	III
A B	1 2	2 1	1 $\frac{5}{4}$

Machines I and II are capable of being operated for at most 12 hours whereas Machine III must operate at least for 5 hours a day.

According to the question, the constraints are

$x + 2y \leq 12$

$2x + y \leq 12$

$x + \frac{5}{4}y \geq 5$

He makes a profit of Rs 6.00 on item A and Rs. 4.00 on item B. Profit made by him in producing x items of A and y items of B is 6x + 4y.

Total profit Z = 6x + 4y which is to be maximized

Thus, the mathematical formulation of the given linear programming problem is

Max Z = 6x + 4y, subject to

$x + 2y \leq 12$

$2x + y \leq 12$

$x + \frac{5}{4}y \geq 5$

$x, y \geq 0$

First, we will convert the inequations into equations as follows:

$x + 2y = 12$, $2x + y = 12$, $x + \frac{5}{4}y = 5$, $x = 0$ and $y = 0$.

The region represented by $x + 2y \leq 12$

The line $x + 2y = 12$ meets the coordinate axes at A(12,0) and B(0,6) respectively. By joining these points, we obtain the line $x + y = 12$. Clearly (0, 0) satisfies the $x + 2y = 12$. So, the region which contains the origin represents the solution set of the inequation $x + 2y \leq 12$

The region represented by $2x + y \leq 12$

The line $2x + y = 12$ meets the coordinate axes at C(6,0) and D(0,12) respectively. By joining these points, we obtain the line $2x + y = 12$. Clearly (0, 0) satisfies the $2x + y = 12$. So, the region which contains the origin represents the solution set of the inequation $2x + y \leq 12$

The region represented by $x + \frac{5}{4}y \geq 5$

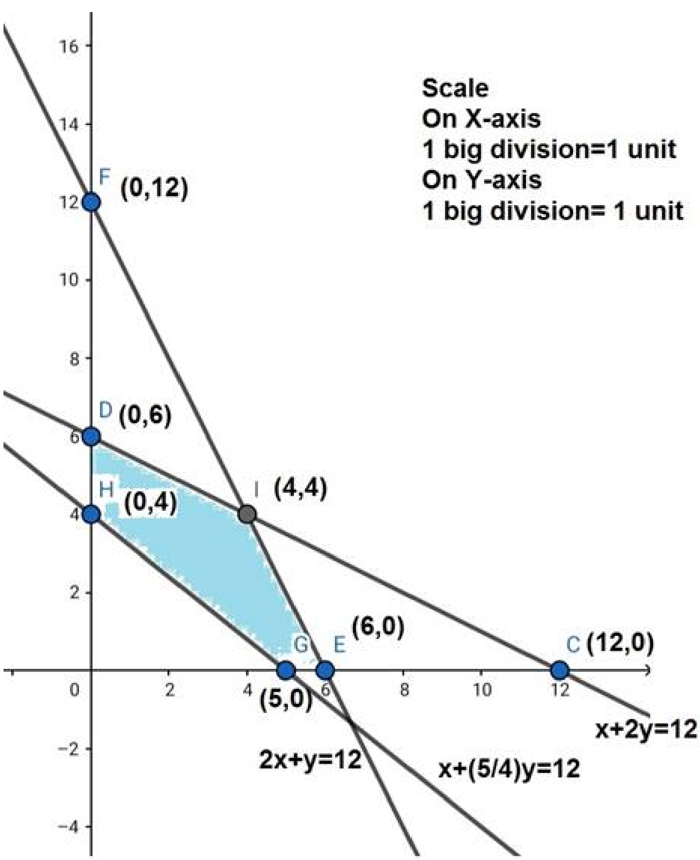
The line $x + \frac{5}{4}y \geq 5$ meets the coordinate axes at E(5,0) and F(0,4) respectively. By joining these points, we obtain the line $x + \frac{5}{4}y = 5$. Clearly (0, 0) satisfies the $x + \frac{5}{4}y \geq 5$. So, the region which does not contain the origin represents the solution set of the inequation $x + \frac{5}{4}y \geq 5$

The region represented by $x \geq 0$, $y \geq 0$:

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$ and $y \geq 0$.

The feasible region determined by the system of constraints

$x + 2y \leq 12$, $2x + y \leq 12$, $x + \frac{5}{4}y \geq 5$, $x, y \geq 0$ are as follows.



Thus the maximum profit is of Rs 40 obtained when 4 units each of item A and B are manufactured.

The corner points are D(0,6), I(4,4), C(6,0), G(5,0), and H(0,4). The values of Z at these corner points are as follows:

Corner Points	$Z = 6x + 4y$
D	24
I	40
C	36
G	30
H	16

The maximum value of Z is 40 which is attained at I(4, 4).

3. Question

Two tailors, A and B earn ₹ 15 and ₹ 20 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce (at least) 60 shirts and 32 pants at a minimum labour cost?

Answer

Let tailor A work for x days and tailor B work for y days.

In one day, A can stitch 6 shirts and 4 pants whereas B can stitch 10 shirts and 4 pants.

Thus in x days, A can stitch 6x shirts and 4x pants whereas in y days B can stitch 10y shirts and 4y pants.

It is given that the minimum requirement of the shirt and pants are respectively 60 and 32.

Thus,

$6x + 10y \geq 60$

$4x + 4y \geq 32$

Further it is given that A and B earn Rs 15 and Rs 20 per day respectively. Thus, A earn Rs 15x and B earns Rs 20y.

Let Z denotes the total cost

$Z = 15x + 20y$

Days cannot be negative.

$x,y \geq 0$.

MIN $Z = 15x + 20y$

Subject to

$6x + 10y \geq 60$

$4x + 4y \geq 32$

$x,y \geq 0$

First we will convert inequations into equations as follows:

$6x + 10y = 60$, $4x + 4y = 32$, $x = 0$, $y = 0$

Region represented by $6x + 10y \geq 60$

The line $6x + 10y = 60$ meets the coordinate axes at A(10,0) and B(0,6) respectively. By joining these points we obtain the line $6x + 10y = 60$. Clearly (0, 0) satisfies the $6x + 10y \geq 60$. So, the region which does not contains the origin represents the solution set of the inequation $6x + 10y \geq 60$

Region represented by $4x + 4y \geq 32$

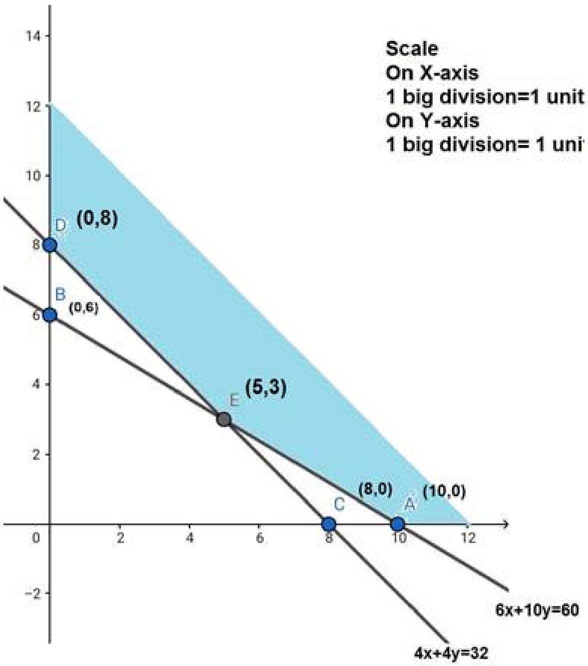
The line 4x + 4y = 32 meets the coordinate axes at C(8,0) and D(0,8) respectively. By joining these points we obtain the line 4x + 4y = 32. Clearly (0, 0) satisfies the $4x + 4y \geq 32$. So, the region which does not contains the origin represents the solution set of the inequation $4x + 4y \geq 32$.

Region represented by $x \geq 0$, $y \geq 0$:

Since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$ and $y \geq 0$.

The feasible region determined by the system of constraints $6x + 10y \geq 60$, $4x + 4y \geq 32$

$x,y \geq 0$ are as follows.



The corner points are D(0,8), E(5,3), A(10,00. The values of Z at these corner points are as follows:

Corner points	Z = 15x + 20y
D	160
E	135
A	150

The minimum value of Z is 135 which is attained at E(5,3).

Thus, for minimum labour cost, A should work for 5 days and B should work for 3 days.

4. Question

A factory manufactures two types of screws, A and B, each type requiring the use of two machines - an automatic and a hand - operated. It takes 4 minute on the automatic and 6 minutes on the hand - operated machines to manufacture a package of screws 'A', while it takes 6 minutes on the automatic and 3 minutes on the hand - operated machine to manufacture a package of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a package of screws 'A' at a profit of 70 P and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he can manufacture, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

Answer

Let the factory manufacture x screws of type A and y screws of type B on each day,

Therefore, $x \geq 0$ and $y \geq 0$.

The given information can be compiled in a table as follows

	Score A	Score B	Score C
Automatic Machine (min)	4	6	$4 \times 60 = 240$
Hand Operated Machine (min)	6	3	$4 \times 60 = 240$

$4x + 6y \leq 240$

$6x + 3y \leq 240$

The manufacturer can sell a package of screws 'A' at a profit of Rs 0.7 and screws 'B' at a profit of Re 1.

Total profit, Z = 0.7x + 1y

The mathematical formulation of the given problem is

Maximize Z = 0.7x + 1y

subject to the constraints,

$4x + 6y \leq 240$

$6x + 3y \leq 240$

$x, y \geq 0$

First we will convert the inequations into equations as follows:

4x + 6y = 240, 6x + 3y = 240, x = 0, y = 0.

Region represented by $4x + 6y \geq 240$

The line 4x + 6y = 240 meets the coordinate axes at A(60,0) and B(0,40) respectively. By joining these points we obtain the line 4x + 6y = 240. Clearly (0, 0) satisfies the $4x + 6y \geq 240$. So, the region which contains the origin represents the solution set of the inequation $4x + 6y \geq 240$.

Region represented by $6x + 3y \geq 240$

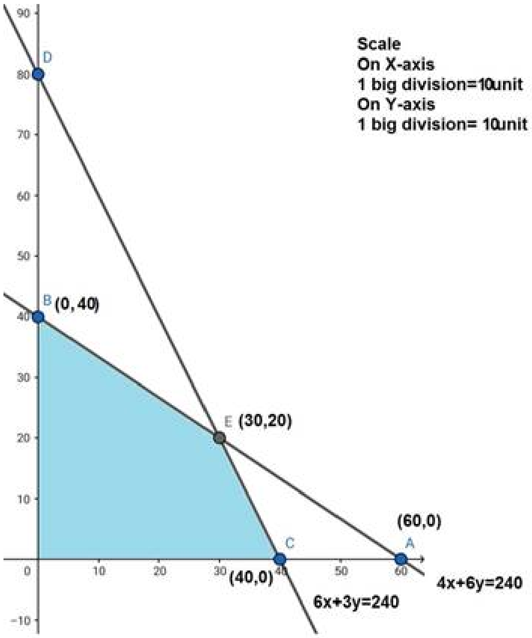
The line 6x + 3y = 240 meets the coordinate axes at C(40,0) and d(0,80) respectively. By joining these points we obtain the line 6x + 3y = 240. Clearly (0, 0) satisfies the $6x + 3y \geq 240$. So, the region which contains the origin represents the solution set of the inequation $6x + 3y \geq 240$.

Region represented by $x \geq 0$, $y \geq 0$:

Since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$ and $y \geq 0$.

The feasible region determined by the system of constraints $4x + 6y \leq 240$, $6x + 3y \leq 240$, $x \geq 0$,

$y \geq 0$ are as follows.



The corner points are C(40,0), E(30,20), B(0,40). The values of Z at these corner points are as follows

Corner Point	Z = 7x + 10y
C(40,0)	280
E(30,20)	410
B(0,40)	400

The maximum value of Z is 410 at (30, 20).

Thus, the factory should produce 30 packages of screws A and 20 packages of screws b to get the maximum profit of Rs 410.

5. Question

A company produces two types of leather belts, say type A and B. Belt A is a superior quality and belt B is of a lower quality. Profits on each type of belt are 2 and 1.50 per belt, respectively. Each belt of type A requires twice as much time as required by a belt of type B. If all belts were of type B, the company could produce 1000 belts per day. But the supply of leather is sufficient only for 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 fancy buckles are available for this per day. For belt of type B, only 700 buckles are available per day.

How should the company manufacture the two types of belts in order to have a maximum overall profit?

Answer

Let the company produces x belts of types A and y belts of type B. Number of belts cannot be negative. Therefore, x,y $\geq$ 0.

It is given that leather is sufficient only for 800 belts per day (both A and B combined).

Therefore,

x + y $\leq$ 800

It is given that the rate of production of belts of type B is 1000 per day. Hence the time taken to produce y belts of type B is $\frac{y}{1000}$.

And, since each belt of type A requires twice as much time as a belt of type B, the rate of production of belts of type A is 500 per day and therefore, total time taken to produce x belts of type A is $\frac{x}{500}$

Thus, we have,

$$\frac{x}{500} + \frac{y}{1000} \leq 1$$

Or, $2x + y \leq 1000$

Belt A requires fancy buckle and only 400 fancy buckles are available for this per day.

$x \leq 400$

For Belt of type B only 700 buckles are available per day.

$y \leq 700$

profits on each type of belt are Rs 2 and Rs 1.50 per belt, respectively. Therefore, profit gained on x belts of type A and y belts of type B is Rs 2x and Rs 1.50y respectively. Hence, the total profit would be Rs(2x + 1.50y). Let Z denote the total profit

$Z = 2x + 1.50y$

Thus, the mathematical formulation of the given linear programming problem is;

Max $Z = 2x + 1.50y$ subject to

$x + y \leq 800$

$2x + y \leq 1000$

$x \leq 400$

$y \leq 700$

First we will convert these inequations into equations as follows:

$x + y = 800$

$2x + y = 1000$

$x = 400$

$y = 700$

Region represented by $x + y = 800$

The line $x + y = 800$ meets the coordinate axes at A(800,0) and B(0,800) respectively. By joining these points we obtain the line $x + y = 800$. Clearly (0, 0) satisfies the $x + y \geq 800$. So, the region which contains the origin represents the solution set of the inequation $x + y \geq 800$.

Region represented by $2x + y \geq 1000$

The line $2x + y = 1000$ meets the coordinate axes at C(500,0) and D(0,1000) respectively. By joining these points we obtain the line $2x + y = 1000$. Clearly (0, 0) satisfies the $2x + y \geq 1000$. So, the region which contains the origin represents the solution set of the inequation $2x + y \geq 1000$.

Region represented by $x \leq 400$

The line $x = 400$ will pass through (400,0). The region to the left of the line $x = 400$ will satisfy the inequation $x \leq 400$

Region represented by $y \leq 700$

The line $y = 700$ will pass through (0,700). The region to the left of the line $y = 700$

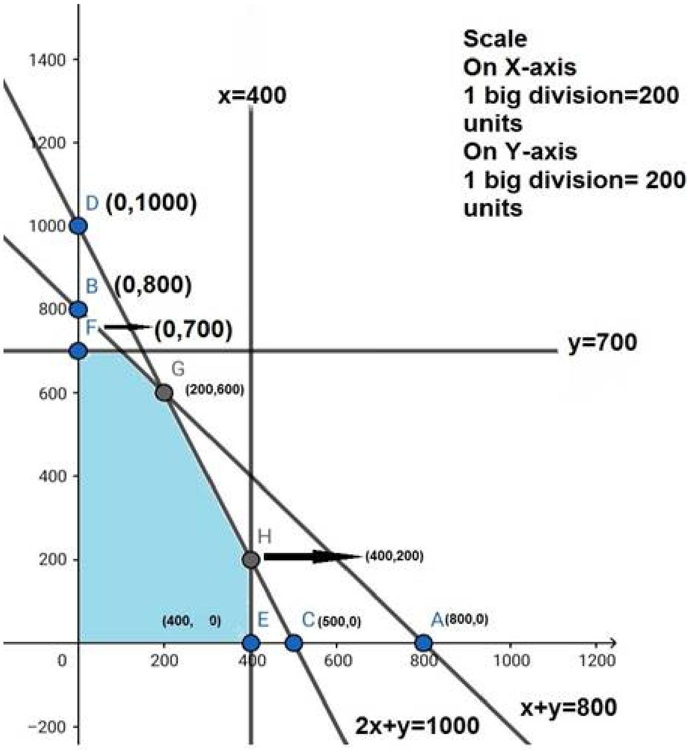
will satisfy the inequation $y \leq 700$.

Region represented by $x \geq 0$, $y \geq 0$:

Since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x \geq 0$ and $y \geq 0$.

The feasible region determined by the system of constraints $x + y \leq 800$, $2x + y \leq 1000$, $x \leq 400$,

$y \leq 700$



The corner points are F(0,700), G(200,600), H(400,200), E(400,0). The values of Z at these corner points are as follows

Corner Point	Z = 2x + 1.5y
F(0,700)	1050
G(200,600)	1300
H(400,200)	1100
E(400,0)	800

The maximum value of Z is 1300 which is attained at G(200,600).

Thus, the maximum profit obtained is Rs 1300 when 200 belts of type A and 600 belts of type B are produced.

6. Question

A small manufacturer has employed 5 skilled men and 10 semi - skilled men and makes an article in two qualities deluxe model and an ordinary model. The making of a deluxe model requires 2 hrs. work by a skilled man and 2 hrs. work by a semi - skilled man. The ordinary model requires 1 hr by a skilled man and 3 hrs. by a semi - skilled man By union rules no man may work more than 8 hrs per day. The manufacturers clear profit on deluxe model is Rs 15 and on an ordinary model is Rs 10. How many of each type should be made in order to maximize his total daily profit.

Answer

Let x articles of deluxe model and y articles of an ordinary model be made.

Numbers cannot be negative.

Therefore,

x, y $\geq$ 0

According to the question, the profit on each model of deluxe and ordinary type model are Rs 15 and Rs 10 respectively.

So, profits on x deluxe model and y ordinary models are 15x and 10y.

Let Z be total profit, then,

Z = 15x + 10y

Since, the making of a deluxe and ordinary model requires 2 hrs. and 1 hr work by skilled men, so, x deluxe and y ordinary models require 2x and y hours of skilled men but time available by skilled men is 5$\times$8 = 40 hours.

So,

2x + y $\leq$ 40 { First Constraint}

Since, the making of a deluxe and ordinary model requires 2 hrs. and 3 hrs work by semi skilled men, so, x deluxe and y ordinary models require 2x and 3y hours of skilled men but time available by skilled men is 10 $\times$8 = 80 hours.

So,

2x + 3y $\leq$ 80 {Second constraint}

Hence the mathematical formulation of LPP is,

Max Z = 15x + 10y

subject to constraints,

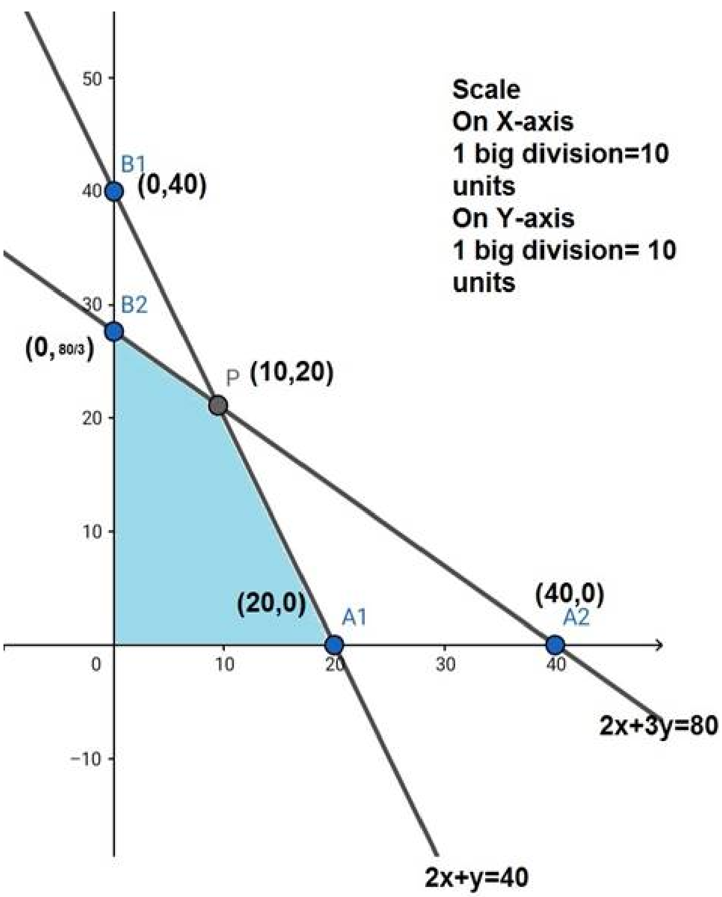
2x + y $\leq$ 40

2x + 3y $\leq$ 80

x, y $\geq$ 0

Region 2x + y $\leq$ 40: line 2x + 4y = 40 meets axes at A_1(20,0), B_1 (0,40) respectively. Region containing origin represents 2x + 3y$\leq$ 40 as (0,0) satisfies 2x + y $\leq$ 40

Region 2x + 3y $\leq$ 80: line 2x + 3y = 80 meets axes at A_2(40,0), B_2(0,$\frac{80}{3}$) respectively. Region containing origin represents 2x + 3y $\leq$ 80.



The corner points are A_1(20,0), P(10,20), B_2(0,$\frac{80}{3}$).

The value of Z = 15x + 10y at these corner points are

Corner Points	Z = 15x + 10y
A_1	$\frac{800}{3}$
P	350
B_2	300

The maximum value of Z is 300 which is attained at P(10,20).

Thus, maximum profit is obtained when 10 units of deluxe model and 20 units of ordinary model is produced.

7. Question

A manufacturer makes two types A and B of tea - cups. Three machines are needed for the manufacture and the time in minutes required for each cup on the machines is given below :

	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each cup A is 75 paise and that on each cup B is 50 paise, show that 15 tea - cups of type A and 30 of type B should be manufactured in a day to get the maximum profit.

Answer

Let the required number of tea cups of Type A and B are x and y respectively.

Since, the profit on each cup A is 75 paise and that on each cup B is 50 paise. So, the profit on x tea cup of type A and y tea cup of type B are 75x and 50y respectively.

Let total profit on tea cups be Z, so

Z = 75x + 50y

Since, each tea cup of type A and B require to work machine I for 12 and 6 minutes respectively so, x tea cups of Type A and y tea cups of Type B require to work on machine I for 12x and 6y minutes respectively.

Total time available on machine I is 6×60 = 360 minutes. So,

12x + 6y $\leq$360 {First Constraint}

Since, each tea cup of type A and B require to work machine II for 18 and 0 minutes respectively so, x tea cups of Type A and y tea cups of Type B require to work on machine IIII for 18x and 0y minutes respectively.

Total time available on machine I is 6×60 = 360 minutes. So,

18x + 0y $\geq$ 360

x $\leq$ 20 {Second Constraint}

Since, each tea cup of type A and B require to work machine III for 6 and 9 minutes respectively so, x tea cups of Type A and y tea cups of Type B require to work on machine I for 6x and 9y minutes respectively.

Total time available on machine I is 6×60 = 360 minutes. So,

6x + 9y $\leq$360 {Third Constraint}

Hence mathematical formulation of LPP is,

Max Z = 75x + 50y

subject to constraints,

12x + 6y $\leq$ 360

x $\leq$ 20

$6x + 9y \leq 360$

$x, y \geq 0$ [Since production of tea cups can not be less than zero]

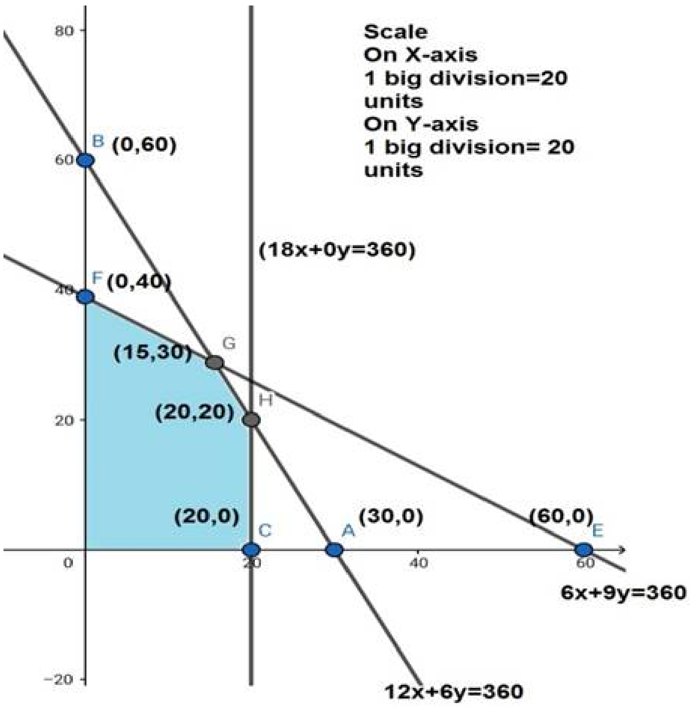
Region $12x + 6y \leq 360$: line $12x + 6y = 360$ meets axes at A(30,0), B(0,60) respectively. Region containing origin represents $12x + 6y \leq 360$ as (0,0) satisfies $12x + 6y \leq 360$

Region $x \leq 20$: line $x = 20$ is parallel to y axis and meets x - axes at C(20,0). Region containing origin represents $x \leq 20$

as (0,0) satisfies $x \leq 20$.

Region $6x + 9y \leq 360$: line $6x + 9y = 360$ meets axes at E(60,0), F(0,40) respectively. Region containing origin represents $6x + 9y \leq 360$ as (0,0) satisfies $6x + 9y \leq 360$.

Region $x, y \geq 0$: it represents the first quadrant.



The shaded region is the feasible region determined by the constraints,

$12x + 6y \leq 360$

$x \leq 20$

$6x + 9y \leq 360$

$x, y \geq 0$

The corner points are F(0,40), G(15,30), H(20,20), C(20,0).

The values of Z at these corner points are as follows

Corner Points	Z = 75x + 50y
F	2000
G	2624
H	2500
C	1500

Here Z is maximum at G(15,30).

Therefore, 15 teacups of Type A and 30 tea cups of Type B are needed to maximize the profit.

8. Question

A factory owner purchases two types of machines, A and B, for his factory. The requirements and limitations for the machines are as follows :

	Area occupied by the machine	Labour force for each machine	Daily output in units
Machine A Machine B	1000 sq. m 1200 sq. m	12 men 8 men	60 40

He has an area of 7600 sq.m available and 72 skilled men who can operate the machines. How many machines of each type should he buy to maximize the daily output?

Answer

Let required number of machine A and B are x and y respectively.

Since, products of each machine A and B are 60 and 40 units daily respectively. So, production by by x number of machine A and y number of machine B are 60x and 40y respectively.

Let Z denotes total output daily, so,

Z = 60x + 40y

Since, each machine of type A and B requires 1000sq. m and 1200 s. m area so, x machine of type A and y machine of type B require 1000x and 1200y sq. m area but,

Total available area for machine is 7600 sq. m. So,

$1000x + 1200y \leq 7600$

or, $5x + 6y \leq 38$. {First Constraint}

Since each machine of type A and B requires 12 men and 8 men to work respectively. So, x machine of type A and y machine of type B require 12x and 8y men to work respectively.

But total men available for work are 72.

So,

$12x + 8y \leq 72$

$3x + 2y \leq 18$ {Second Constraint}

Hence mathematical formulation of the given LPP is,

Max Z = 50x + 40y

Subject to constraints,

$5x + 6y \leq 38$

$3x + 2y \leq 18$

$x,y \geq 0$ [Since number of machines can not be less than zero]

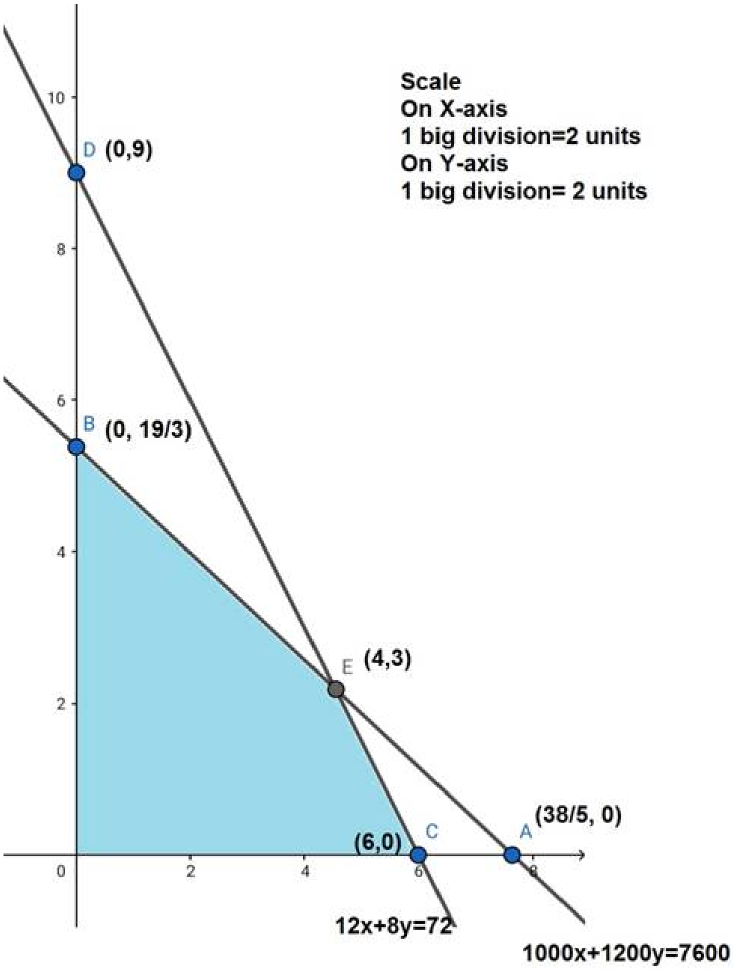
Region $5x + 6y \leq 38$: line 5x + 6y = 38 meets the axes at $A(\frac{38}{5},0)$, $B(0,\frac{19}{3})$ respectively.

Region containing the origin represents $5x + 6y \leq 38$ as origin satisfies $5x + 6y \leq 38$

Region $3x + 2y \leq 18$: line 3x + 2y = 18 meets the axes at C(6,0), D(0,9) respectively.

Region containing the origin represents $3x + 2y \leq 18$ as origin satisfies $3x + 2y \leq 18$.

Region $x,y \geq 0$: it represents the first quadrant.



Shaded region represents the feasible region.

The corner points are O(0,0), B(0,$\frac{19}{3}$), E(4,3), C(6,0).

Thus the values of Z at these corner points are as follows:

Corner Points	Z = 60x + 40y
O	0
B	253.3
E	360
C	360

The maximum value of Z is 360 which is attained at E(4,3), C(6,0).

Thus,the maximum output is Rs 360 obtained when 4 units of type A and 3 units of type B or 6 units of type A and 0 units of type B are manufactured.

9. Question

A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 gm of silver and 1 gm of gold while that of type B requires 1 gm of silver and 2 gm of gold. The company can produce 9 gm of silver and 8 gm of gold. If each unit of type A brings a profit of ₹ 40 and that of type B ₹ 50, find the number of units of each type that the company should produce to maximize the profit. What is the maximum profit?

Answer

Let required number of goods A and B are x and y respectively.

Since, profits of each A and B are Rs. 40 and Rs. 50 respectively. So, profits on x number of type A and y number of type B are 40x and 50y respectively.

Let Z denotes total output daily, so,

Z = 40x + 50y

Since, each A and B requires 3 grams and 1 gram of silver respectively. So, x of type A and y of type B require 3x and y of silver respectively. But,

Total silver available is 9 grams. So,

$3x + y \leq 9$ {First Constraint}

Since each A and B requires 1 gram and 2 grams of gold respectively. So, x of type A and y of type B require x and 2y respectively.

But total gold available is 8 grams.

So,

$x + 2y \leq 8$ {Second Constraint}

Hence mathematical formulation of the given LPP is,

Max $Z = 40x + 50y$

Subject to constraints,

$3x + y \leq 9$

$x + 2y \leq 8$

$x, y \geq 0$ [Since production of A and B can not be less than zero]

Region $3x + y \leq 9$: line $3x + y = 9$ meets the axes at A(3,0), B(0,9) respectively.

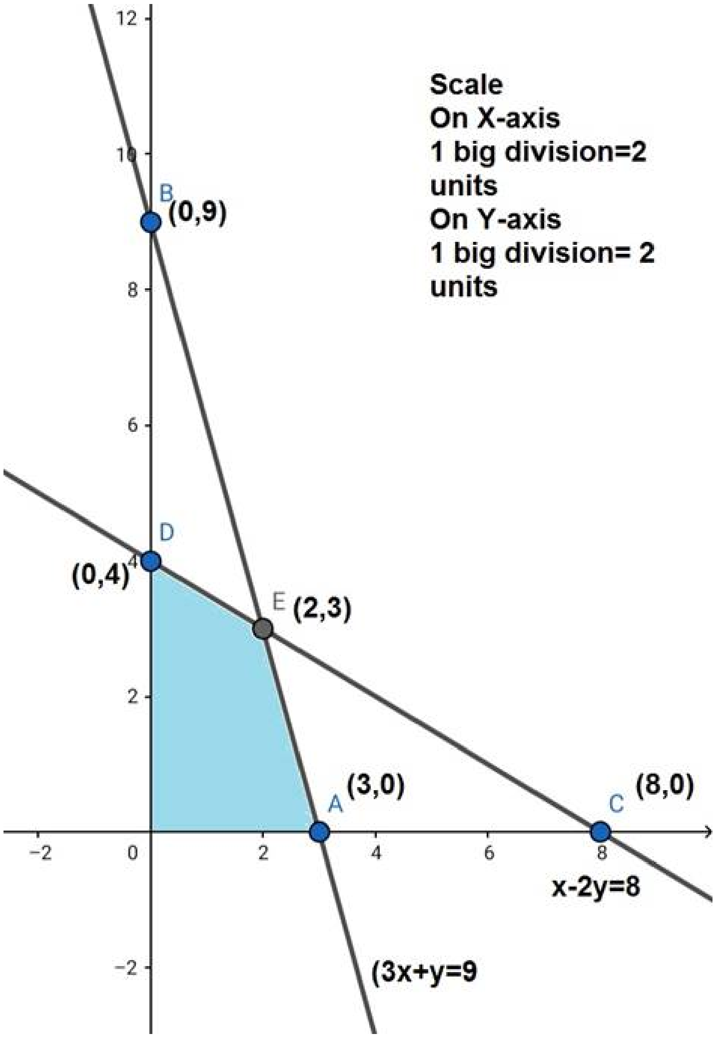
Region containing the origin represents $3x + y \leq 9$

as origin satisfies $3x + y \leq 9$.

Region $x + 2y \leq 8$: line $x + 2y = 8$ meets the axes at C(8,0), D(0,4) respectively.

Region containing the origin represents $x + 2y \leq 8$ as origin satisfies $x + 2y \leq 8$.

Region $x, y \geq 0$: it represents the first quadrant.



The corner points are O(0,0), D(0,4), E(2,3), A(3,0)

The values of Z at these corner points are as follows

Corner Points	Z = 40x + 50y
O	0
D	200
E	230
A	120

The maximum value of Z is 230 which is attained at E(2,3).

Thus the maximum profit is of Rs 230 when 2 units of Type A 3 uniits of Type B are produced.

10. Question

A manufacturer of Furniture makes two products : chairs and tables. Processing of these products is done on two machines A and B. A chair requires 2 hrs on machine A and 6 hrs on machine B. A table requires 4 hrs on machine A and 2 hrs on machine B . There are 16 hrs of time per day available on machine A and 30 hrs on machine B. profit gained by the manufacturer from a chair and a table is ₹ 3 and ₹ 5 respectively. Find with the help of graph what should be the daily production of each of the two products so as to maximize his profit.

Answer

Let daily production of chairs and tables be x and y respectively.

Since, profits of each chair and table is Rs. 3 and Rs. 5 respectively. So, profits on x number of type A and y number of type B are 3x and 5y respectively.

Let Z denotes total output daily, so,

$Z = 3x + 5y$

Since, each chair and table requires 2 hrs and 3 hrs on machine A respectively. So, x number of chair and y number of table require 2x and 4y hrs on machine A respectively. But,

Total time available on Machine A is 16 hours. So,

$2x + 3y \leq 16$

$x + 2y \leq 8$ {First Constraint}

Since, each chair and table requires 6 hrs and 2 hrs on machine B respectively. So, x number of chair and y number of table require 6x and 2y hrs on machine B respectively. But,

Total time available on Machine B is 30 hours. So,

$6x + 2y \leq 30$

$3x + y \leq 15$ {Second Constraint}

Hence mathematical formulation of the given LPP is,

Max $Z = 3x + 5y$

Subject to constraints,

$x + 2y \leq 8$

$3x + y \leq 15$

$x,y \geq 0$ [Since production of chairs and tables can not be less than zero]

Region $x + 2y \leq 8$: line $x + 2y = 8$ meets the axes at A(8,0), B(0,4) respectively.

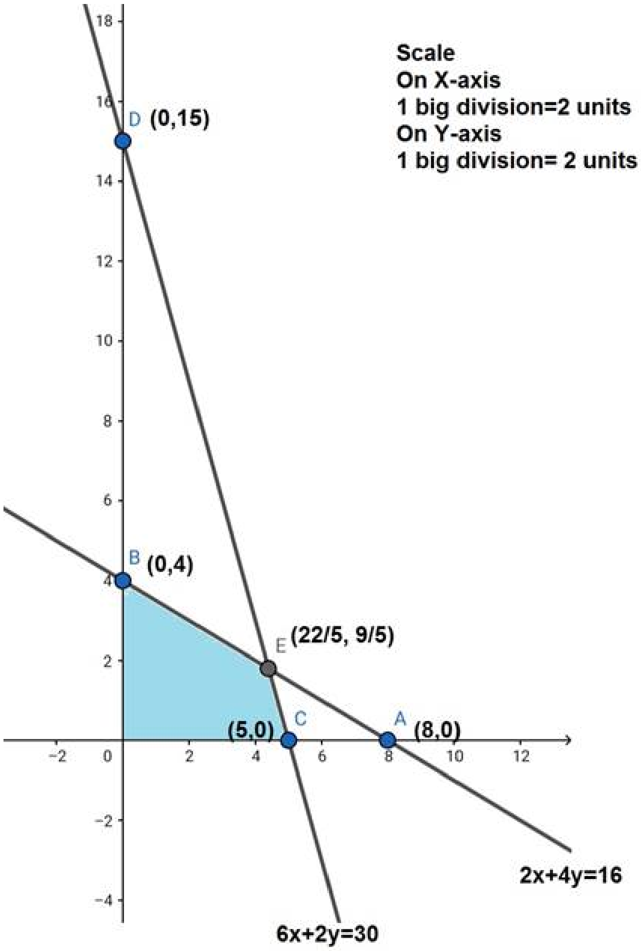
Region containing the origin represents $x + 2y \leq 8$

as origin satisfies $x + 2y \leq 8$.

Region $3x + y \leq 15$: line $3x + y = 15$ meets the axes at C(5,0), D(0,15) respectively.

Region containing the origin represents $3x + y \leq 15$ as origin satisfies $3x + y \leq 15$

Region $x,y \geq 0$: it represents the first quadrant.



The corner points are O(0,0), B(0,4), E($\frac{22}{5}$,$\frac{9}{5}$), and C(5,0).

The values of Z at these corner points are as follows,

Corner Points	Z = 3x + 5y
O	0
B	20
E	22.2
C	15

The maximum value of Z is 22.2 which is attained at E($\frac{22}{5}$,$\frac{9}{5}$).

Thus the maximum profit of Rs 22.2 when $\frac{22}{5}$ units of chair and $\frac{9}{5}$ units of table are produced.

11. Question

A furniture manufacturing company plans to make two products : chairs and tables. From its available resources which consists of 400 square feet of teak wood and 450 man hours. It is known that to make a chair requires 5 square feet of wood and 10 man - hours and yields a profit of ₹ 45, while each table uses 20 square feet of wood and 25 man - hours and yields a profit of ₹ 80. How many items of each product should be produced by the company so that the profit is maximum?

Answer

Let required production of chairs and tables be x and y respectively.

Since, profits of each chair and table is Rs. 45 and Rs. 80 respectively. So, profits on x number of type A and y number of type B are 45x and 80y respectively.

Let Z denotes total output daily, so,

Z = 45x + 80y

Since, each chair and table requires 5 sq. ft and 80 sq. ft of wood respectively. So, x number of chair and y number of table require 5x and 80y sq. ft of wood respectively. But,

But 400 sq. ft of wood is available. So,

$5x + 80y \leq 400$

$x + 4y \leq 80$ {First Constraint}

Since, each chair and table requires 10 and 25 men - hours respectively. So, x number of chair and y number of table require 10x and 25y men - hours respectively. But, only 450 hours are available . So,

$10x + 25y \leq 450$

$2x + 5y \leq 90$ {Second Constraint}

Hence mathematical formulation of the given LPP is,

Max $Z = 45x + 80y$

Subject to constraints,

$x + 4y \leq 80$

$2x + 5y \leq 90$

$x,y \geq 0$ [Since production of chairs and tables can not be less than zero]

Region $x + 4y \leq 80$: line $x + 4y = 80$ meets the axes at A(80,0), B(0,20) respectively.

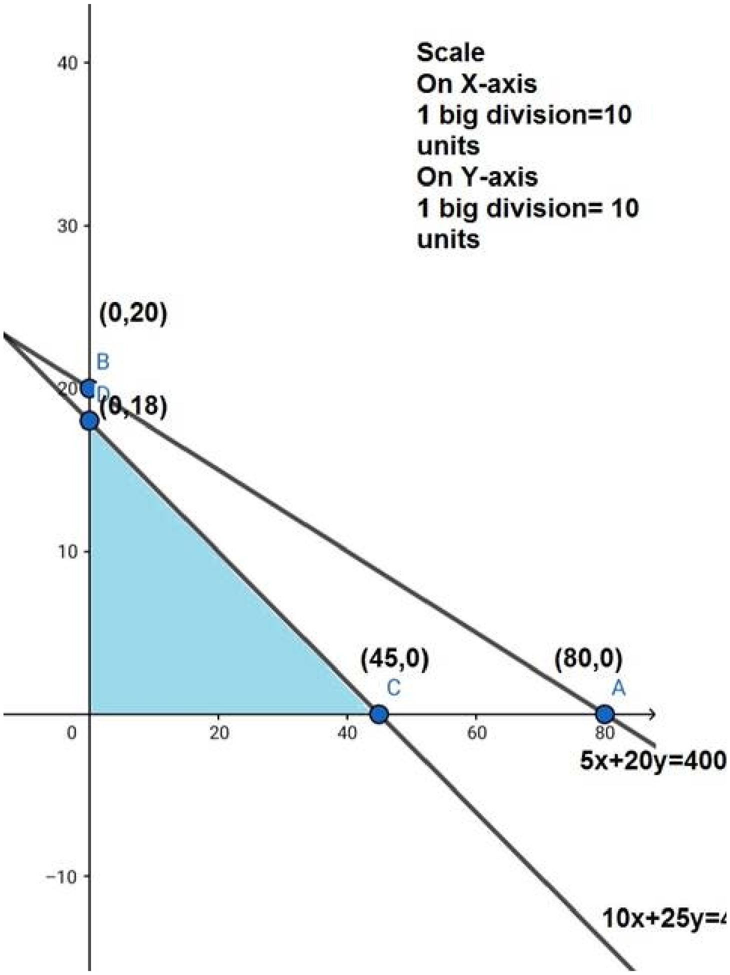
Region containing the origin represents $x + 4y \leq 80$ as origin satisfies $x + 4y \leq 80$

Region $2x + 5y \leq 90$: line $2x + 5y = 90$ meets the axes at C(45,0), D(0,20) respectively.

Region containing the origin represents $2x + 5y \leq 90$

as origin satisfies $2x + 5y \leq 90$

Region $x,y \geq 0$: it represents the first quadrant.



The corner points are O(0,0), D(0,18), C(45,0).

The values of Z at these corner points are as follows:

Corner Points	Z = 45x + 80y
O	0
D	1440
C	2025

The maximum value of Z is 2025 which is attained at C(45,0).

Thus maximum profit of Rs 2025 is obtained when 45 units of chairs and no units of tables are produced.

12. Question

A firm manufactures two products A and B. Each product is processed on two machines M_1 and M_2. Product A requires 4 minutes of processing time on M_1 and 8 min. on M_2; product B requires 4 minutes on M_1 and 4 min. on M_2. The machine M_1 is available for not more than 8 hrs 20 min. while machine M_2 is available for 10 hrs. during any working day. The products A and B are sold at a profit of ₹ 3 and ₹ 4 respectively.Formulate the problem as a linear programming problem and find how many products of each type should be produced by the firm each day in order to get maximum profit.

Answer

Let required production of product A and B be x and y respectively.

Since profit on each product A and B are Rs. 3 and Rs. 4 respectively. So, profits on x number of type A and y number of type B are 3x and 4y respectively.

Let Z denotes total output daily, so,

Z = 3x + 4y

Since, each A and B requires 4 minutes each on machine M_1. So, x of type A and y of type B require 4x and 4y minutes respectively. But, Total time available on machine M_1 is 8 hours 20 minutes = 500 minutes.

So,

4x + 4y $\leq$ 500

x + y $\leq$ 125 {First Constraint}

Since, each A and B requires 8 minutes and 4 minutes on machine M_2 respectively. So, x of type A and y of type B require 8x and 4y minutes respectively. But,

Total time available on machine M_1 is 10 hours = 600 minutes.

So,

8x + 4y $\leq$ 600

2x + y $\leq$ 150 {Second Constraint}

Hence mathematical formulation of the given LPP is,

Max Z = 3x + 4y

Subject to constraints,

x + y $\leq$ 125

2x + y $\leq$ 150

x,y $\geq$ 0 [Since production of A and B can not be less than zero]

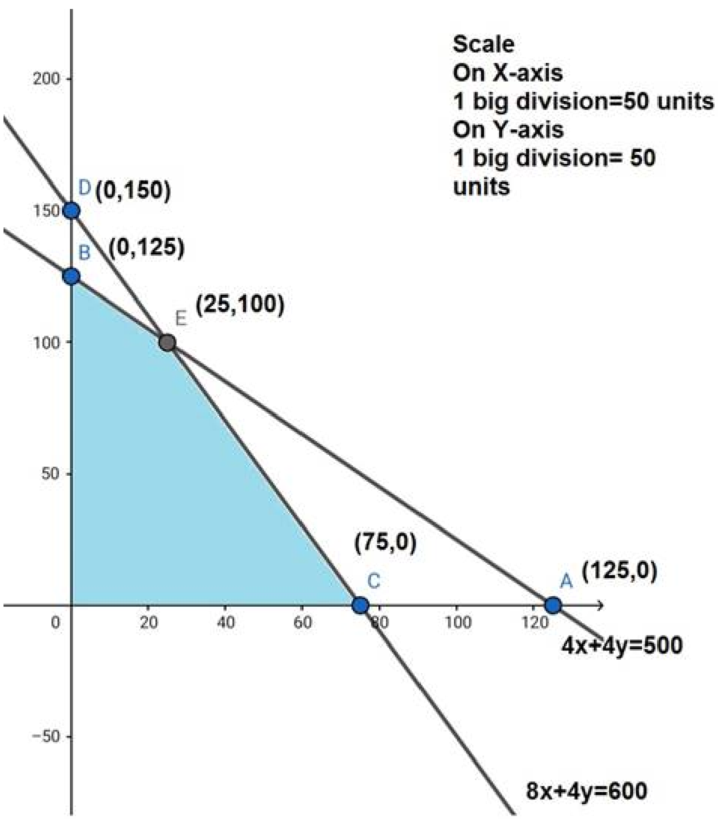
Region x + y $\leq$ 125: line x + y = 125 meets the axes at A(125,0), B(0,125) respectively.

Region containing the origin represents x + y $\leq$ 125 as origin satisfies x + y $\leq$ 125.

Region 2x + y $\leq$ 150: line 2x + y = 150 meets the axes at C(75,0), D(0,150) respectively.

Region containing the origin represents 2x + y $\leq$ 150 as origin satisfies 2x + y $\leq$ 150.

Region x,y $\geq$ 0: it represents the first quadrant.



The corner points are O(0,0), B(0,125), E(25,100), and C(75,0).

The vaues of Z at these corner points are as follows:

Corner Points	Z = 3x + 4y
O	0
B	500
E	475
C	225

The maximum value of Z is 500 which is attained at B(0,125).

Thus, the maximum profit is Rs 500 obtained when no units of product A and 125 units of product B are manufactured.

13. Question

A firm manufacturing two type of electric items, A and B, can make a profit of 20 per unit of A and ₹ 30 per unit of B. Each unit of A requires 3 motors and 4 transformers and each unit of B requires 2 motors and 4 transformers. The total supply of these per month is restricted to 210 motors and 300 transformers. Type B is an export model requiring a voltage stabilizer which has a supply restricted to 65 units per month. Formulate the linear programming problem for maximum profit and solve it graphically.

Answer

Let x units of item A and y units of item B were manufactured.

Numbers of items cannot be negative. Therefore,

$x, y \geq 0$

The given information can be tabulated as follows:

Product	Motors	Transformers
A(x)	3	4
B(y)	2	4
Availability	210	300

Further, it is given that type B is an export model, whose supply is restricted to 65 units per month.

Therefore, the constraints are

$3x + 2y \leq 210$

$4x + 4y \leq 300$

$y \leq 65$

A and B can make profit of Rs 20 and Rs 30 per unit respectively.

Therefore, profit gained from x units of item A and y units of item B is Rs 20x and 30y respectively.

Total Profit = Z = 20x + 30y which according to question is to be maximised.

Thus the mathematical formulation of the given LPP is,

Max $Z = 20x + 30y$

Subject to constraints

$3x + 2y \leq 210$

$4x + 4y \leq 300$

$y \leq 65$

$x, y \geq 0$

Region represented by $3x + 2y \leq 210$: The line $3x + 2y = 210$ meets the axes at A(70,0), B(0,105) respectively.

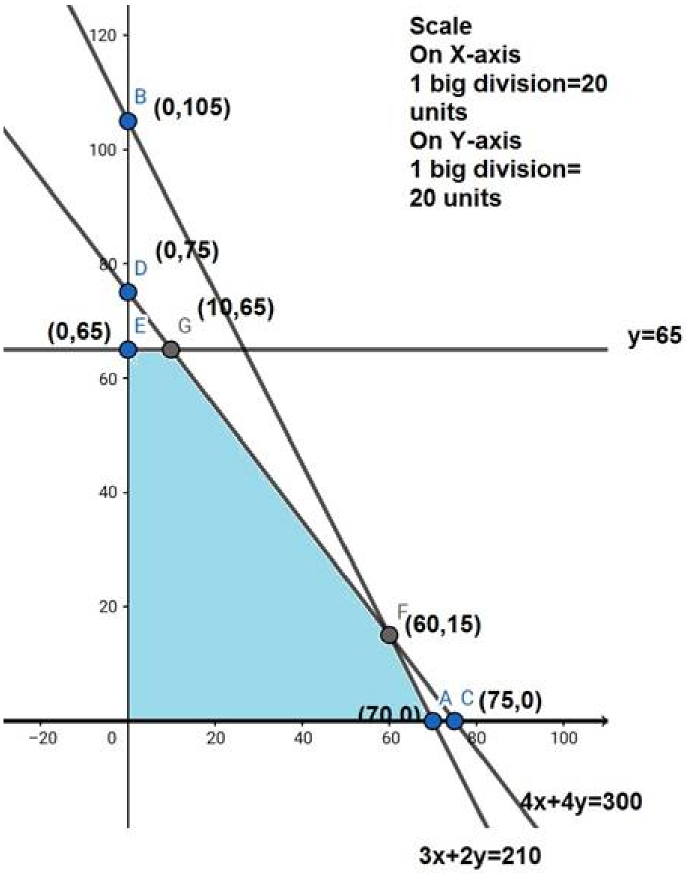
Region containing the origin represents $3x + 2y \leq 210$ as origin satisfies $3x + 2y \leq 210$.

Region represented by $4x + 4y \leq 300$: The line $4x + 4y = 300$ meets the axes at C(75,0), D(0,75) respectively.

Region containing the origin represents $4x + 4y \leq 300$ as origin satisfies $4x + 4y \leq 300$

y = 65 is the line passing through the point E(0,65) and is parallel to X - axis.

Region $x,y \geq 0$: it represents the first quadrant.



The corner points are O(0,0), E(0,65), G(10,65), F(60,15) and A(70,0).

The values of Z at these corner points are as follows:

Corner Points	Z = 20x + 30y
O	0
E	1950
G	2150
F	1650
A	1400

The maximum value of Z is 2150 which is attained at G(10,65).

Thus, the maximum profit is Rs. 2150 obtained when 10 units of item A and 65 units of item B are manufactured.

14. Question

A factory uses three different resources for the manufacture of two different products,20 units of the resources a, 12 units of B and 16 units of C being available 1 unit of the first product requires 2,2 and 4 units of the respective resources and 1 unit of the second product requires 4,2 and 0 units of respective resources. It is known that the first product gives a profit of 2 monetary units per unit and the second 3. Formulate the linear programming problem. How many units of each product should be manufactured for maximizing the profit? Solve it graphically.

Answer

Let number of product I and product II are x and y respectively.

Since, profits on each product I and II are 2 and 3 monetary unit. So, profits on x number of Product I and y number of Product II are 2x and 3y respectively.

Let Z denotes total output daily, so,

$Z = 2x + 3y$

Since, each I and II requires 2 and 4 units of resources A. So, x units of product I and y units of product II requires 2x and 4y minutes respectively. But, maximum available quantity of resources A is 20 units.

So,

$2x + 4y \leq 20$

$x + 2y \leq 10$ {First Constraint}

Since, each I and II requires 2 and 2 units of resources B. So, x units of product I and y units of product II requires 2x and 2y minutes respectively. But, maximum available quantity of resources A is 12 units.

So,

$2x + 2y \leq 12$

$x + y \leq 6$ {Second Constraint}

Since, each units of product I requires 4 units of resources C. It is not required by product II. So, x units of product I require 4x units of resource C. But, maximum available quantity of resources C is 16 units.

So,

$4x \leq 16$

$x \leq 4$ {Third Constraint}

Hence mathematical formulation of LPP is,

Max $Z = 2x + 3y$

Subject to constraints,

$x + 2y \leq 10$

$x + y \leq 6$

$x \leq 4$

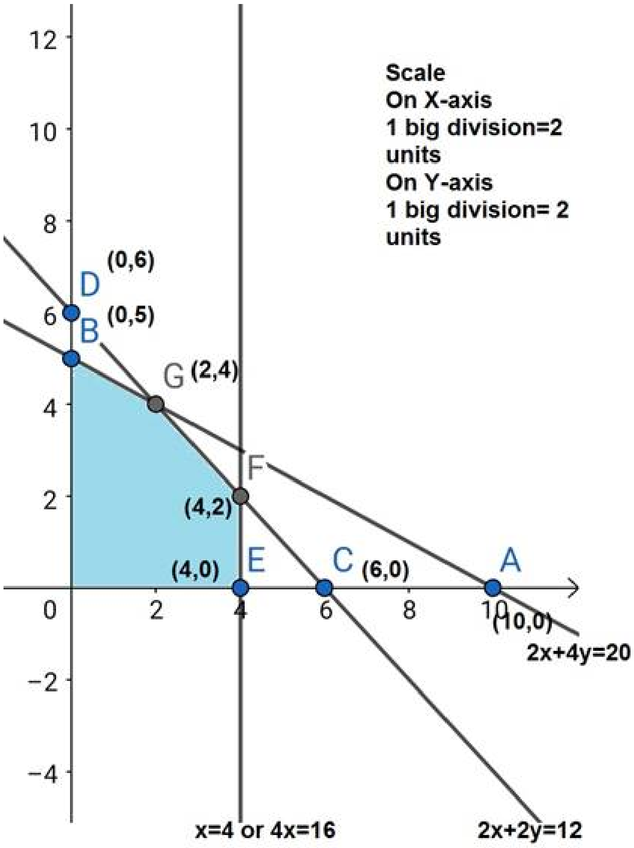
$x, y \geq 0$ [Since production for I and II can not be less than zero]

Region represented by $x + 2y \leq 10$: The line $x + 2y = 10$ meets the axes at A(10,0), B(0,5) respectively.

Region containing the origin represents $x + 2y \leq 10$ as origin satisfies $x + 2y \leq 10$.

Region represented by $x + y \leq 6$: The line $x + y = 6$ meets the axes at C(6,0), D(0,6) respectively. Region containing the origin represents $x + y \leq 6$ as origin satisfies $x + y \leq 6$

Region $x,y \geq 0$: it represents the first quadrant.



The corner points are O(0,0), B(0,5), G(2,4), F(4,2), and E(4,0).

The values of Z at these corner points are as follows:

Corner Points	Z = 2x + 3y
O	0
B	15
G	16
F	14
E	8

The maximum value of Z is 16 which is attained at G (12,4).

Thus, the maximum profit is 16 monetary units obtained when 2 units of first product and 4 units of second product were manufactured.

15. Question

A publisher sells a hard cover edition of a text book for ₹ 72.00 and a paperback edition of the same ext for ₹ 40.00. Costs to the publisher are ₹ 56.00 and ₹ 28.00 per book respectively in addition to weekly costs of ₹ 9600.00. Both types require 5 minutes of printing time, although hardcover requires 10 minutes binding time and the paperback requires only 2 minutes. Both the printing and binding operations have 4,800 minutes available each week. How many of each

type of book should be produced in order to maximize profit?

Answer

Let the sale of hand cover edition be ‘h’ and that of paperback editions be ‘t’.

SP of a hard cover edition of the textbook = Rs 72

SP of a paperback edition of the textbook = Rs 40

Cost to the publisher for hard cover edition = Rs 56

Cost to the publisher for a paperback edition = Rs 28

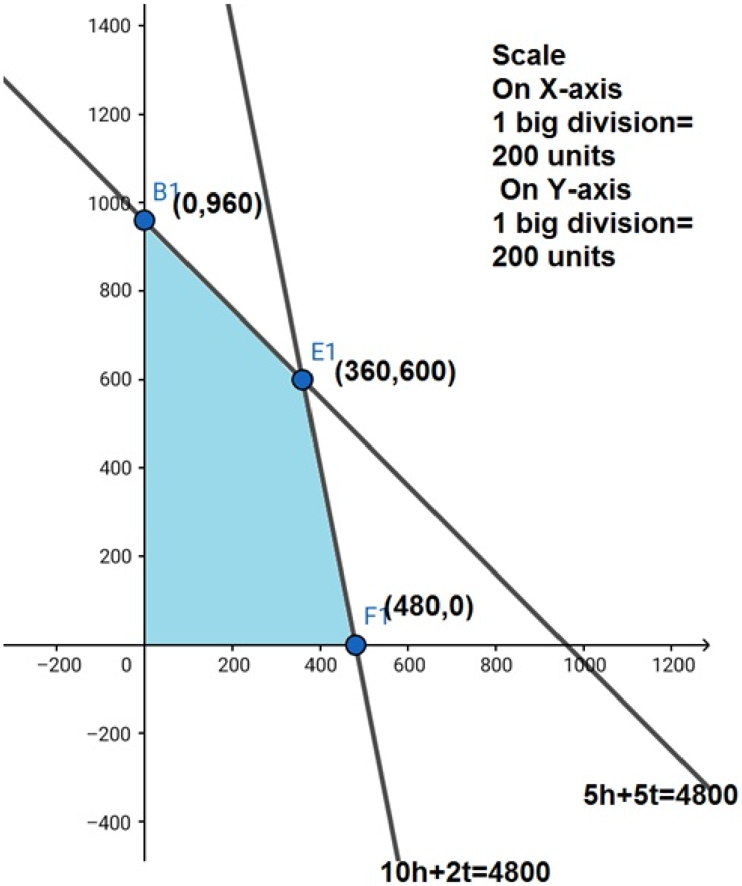
Weekly cost to the publisher = Rs 9600

Profit to be maximized, Z = (72 - 56)h + (40 - 28)t – 9600

Z = 16h + 12t - 9600

5(h + t) $\leq$ 4800

10h + 2t $\leq$ 4800.



The corner Points are O(0,0), B_1 (0,960), E_1 (360,600) and F_1 (480,00).

The values of Z at these corner points are as follows:

Corner Points	Z = 16h + 12t - 9600
O	- 9600
B	1920
E	3360
F	- 1920

The maximum value of Z is 3360 which is attained at E_1 (360,600).

The maximum profit is 3360 which is obtained by selling 360 copies of hardcover edition and 600 copies paperback edition.

16. Question

A firm manufactures headache pills in two sizes A and b. Size A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine ; size B contains 1 grain of aspirin, 8 grains of bicarbonate and 66 grains of codeine. It has been found by users that it requires at least 12 grains of aspirin, 7.4 grains of bicarbonate and 24 grains of codeine for providing immediate effects. Determine graphically the least number of pills a patient should have to get immediate relief. Determine also the quantity of codeine consumed by patient.

Answer

The above LPP can be presented in a table below,

	Pill size A	Pill size B	
	X	Y	
Aspirin	2x	1y	≥ 12
Bicarbonate	5x	8y	≥ 7.4
Codeine	1x	66y	≥ 24
Relief	X	Y	Minimize

Hence mathematical formulation of LPP is,

Max Z = x + y

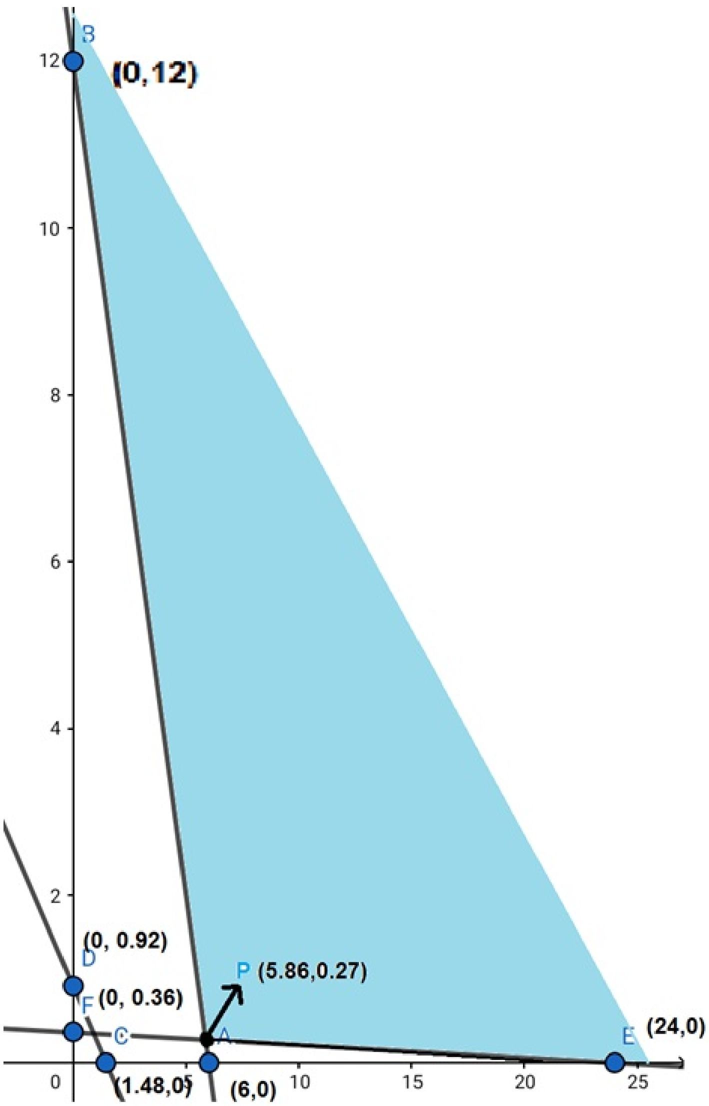
Subject to constraints,

$2x + y \geq 12$

$5x + 8y \geq 7.4$

$x + 66y \geq 24$

x, y $\geq$ 0 [Since production can not be less than zero]



The corner points are B(0,12), P(5.86, 0.27), E(24,0)

The values of Z at these corner points are as follows:

Corner Points	Z = x + y
(0,12)	12
(24,0)	24
(5.86,0.27)	6.13

The minimum value of Z is 6.13 but the region is unbounded so check whether $x + y < 6.13$

Clearly, it can be seen that it does not has any common region.

So, x = 5.86, y = 0.27

This is the least quantity of pill A and B.

Codeine quantity = $x + 66y = 5.86 + (66 \times 0.27) = 24$(approx).

17. Question

A chemical company produces two compounds, A and B. The following table gives the units of ingredients, C and D per kg of compounds A and B as well as minimum requirements of C and D and costs per kg of A and B. Find the quantities of A and B which would give a supply of C and D at a minimum cost.

	Compound		Minimum requirement
	A	B	
Ingredient C Ingredient D	1 3	2 1	80 75
Cost (in Rs) per kg	4	6	

Answer

Let required quantity of compound A and B are x and y kg.

Since, cost of one kg of compound A and B are Rs 4 and Rs 6 per kg. So,

Cost of x kg of compound A and y kg of compound B are Rs 4x and Rs 6

respectively.

Let Z be the total cost of compounds, so,

Z = 4x + 6y

Since, compound A and B contain 1 and 2 units of ingredient C per kg

respectively, So x kg of compound A and y kg of compound B contain x and 2y

units of ingredient C respectively but minimum requirement of ingredient C is 80

units, so,

$x + 2y \geq 80$ {first constraint}

Since, compound A and B contain 3 and 1 units of ingredient D per kg

respectively, So x kg of compound A and y kg of compound B contain 3x and y

units of ingredient D respectively but minimum requirement of ingredient C is 75

units, so,

$3x + y \geq 75$ {second constraint}

Hence, mathematical formulation of LPP is,

Min Z = 4x + 6y

Subject to constraints,

$x + 2y \geq 80$

$3x + y \geq 75$

$x, y \geq 0$ [Since production can not be less than zero]

Region $x + 2y \geq 80$: line $x + 2y = 80$ meets axes at A(80,0), B(0,40)

respectively. Region not containing origin represents $x + 2y \geq 80$ as (0,0) does

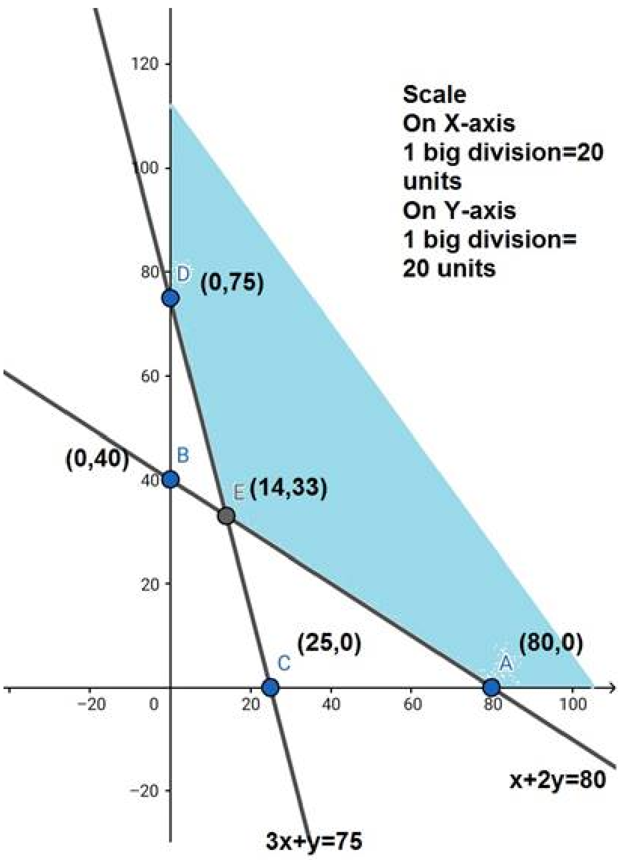
not satisfy $x + 2y \geq 80$.

Region $3x + y \geq 75$: line $3x + y = 75$ meets axes at C(25,0), D(0,75)

respectively. Region not containing origin represents $3x + y \geq 75$ as (0,0) does

not satisfy $3x + y \geq 75$.

Region $x,y \geq 0$: it represents first quadrant.



The corner points are D(0,75), E(14,33), A(80,0).

The values at Z at these corner points are as follows:

Corner Point $Z = 4x + 6y$

D 450

E 254

A 320

The minimum value of Z is 254 which is attained at E(14,33).

Thus, the minimum cost is Rs 254 obtained when 14 units of compound A

and 33 units compound B are produced.

18. Question

A company manufactures two type of novelty Souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours available for assembling. The profit is 50 paisa each for type A and 60 paisa each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

Answer

Let the company manufacture x souvenirs of Type A and y souvenirs of Type B.

Therefore, $x \geq 0$, $y \geq 0$

The given information can be compiled in a table as follows:

	Type A	Type B	Availability
Cutting (min)	5	8	$3\times60 + 20 = 200$
Assembling (min)	10	8	$4\times60 = 240$

The profit on Type A souvenirs is 50 paisa and on Type B souvenirs is 60 paisa. Therefore, profit gained on x souvenirs of Type A and y

souvenirs of Type B is Rs 0.50x and Rs 0.60y respectively.

Total Profit, $Z = 0.5x + 0.6y$

The mathematical formulation of the given problem is,

Max $Z = 0.5x + 0.6y$

Subject to constraints,

$5x + 8y \leq 200$

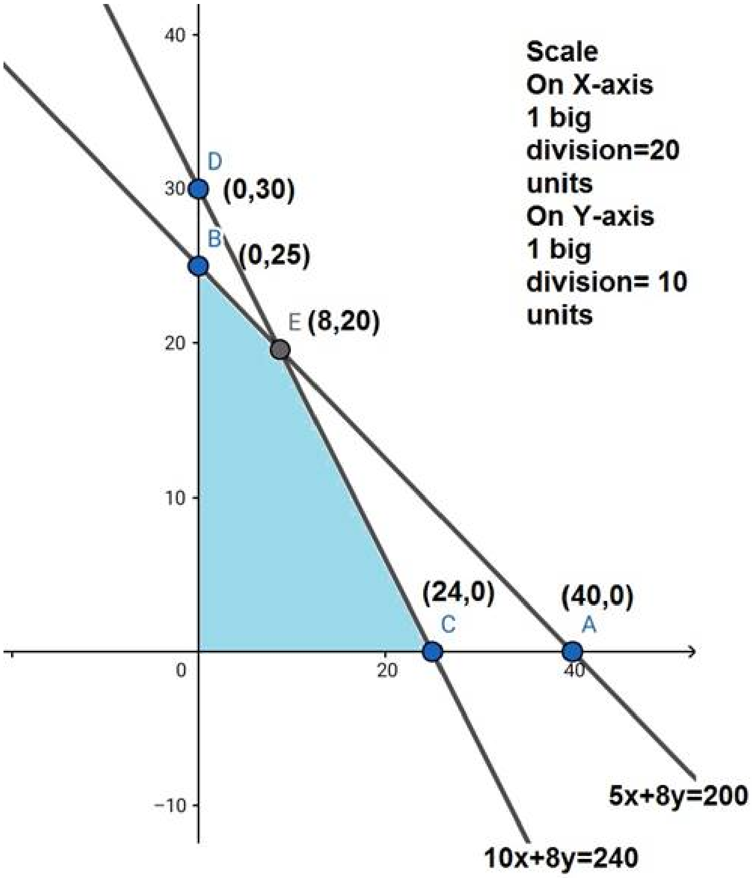
$10x + 8y \leq 240$

$x \geq 0$, $y \geq 0$

Region $5x + 8y \leq 200$: line $5x + 8y = 200$ meets axes at A(40,0), B(0,25) respectively. Region containing origin represents the solution of the inequation $5x + 8y \leq 200$ as (0,0) satisfies $5x + 8y \leq 200$.

Region $10x + 8y \leq 240$: line $10x + 8y = 240$ meets axes at C(24,0), D(0,30) respectively. Region containing origin represents the solution of the inequation $10x + 8y \leq 240$ as (0,0) satisfies $10x + 8y \leq 240$.

Region $x,y \geq 0$: it represents first quadrant.



The corner points of the feasible region are O(0,0), B(0,25), E(8,20), C(24,0).

The values of Z at these corner points are as follows:

Corner Points	Z = 0.5x + 0.6y
O	0
B	15
E	16
C	12

The maximum value of Z is attained at E(8,20).

Thus, 8 souvenirs of Type A and 20 souvenirs of Type B should be produced each day to get the maximum profit of Rs 16.

19. Question

A manufacturer makes two products A and B. Product A sells at 200 each and takes 1/2 hour to make. Product A sells at ₹ 300 each and takes 1 hours to make. There is a permanent order for 14 of product A and 16 of product B. A working week consists of 40 hours of production and weekly turnover must not be less than Rs 10000. If the profit on each of product A is ₹ 20 and on product B is Rs 30, then how many of each should be produced so that the profit is maximum. Also, find the maximum profit.

Answer

Let x units of product A and y units of product B were manufactured.

Number of units cannot be negative.

Therefore, x,y $\geq$ 0.

According to question, the given information can be tabulated as:

	Selling price (Rs)	Manufacturing time (hrs)
Product A (x)	200	0.5
Product B (y)	300	1

Also, the availability of time is 40 hours and the revenue should be atleast Rs 10000.

Further, it is given that there is a permanent order for 14 units of Product A and 16 units of product B.

Therefore, the constraints are,

$200x + 300y \geq 10000$,

$0.5x + y \leq 40$

$x \geq 14$

$y \geq 16$.

If the profit on each of product A is Rs 20 and on product B is Rs 30. Therefore, profit gained on x units of product A and y units of product B is Rs 20x and Rs 30y respectively.

Total profit = 20x + 30y which is to be maximized.

Thus, the mathematical formulation of the given LPP is,

Max Z = 20x + 30y

Subject to constraints,

$200x + 300y \geq 10000$,

$0.5x + y \leq 40$

$x \geq 14$

$y \geq 16$

$x,y \geq 0$.

Region $200x + 300y \geq 10000$: line 200x + 300y = 10000 meets the axes at A(50,0), B$(0,\frac{100}{3})$ respectively.

Region not containing origin represents $200x + 300y \geq 10000$ as (0,0) does not satisfy $200x + 300y \geq 10000$.

Region $0.5x + y \leq 40$: line 0.5x + y = 40 meets the axes at C(80,0), D(0,40) respectively.

Region containing origin represents $0.5x + y \leq 40$ as (0,0) satisfies $0.5x + y \leq 40$.

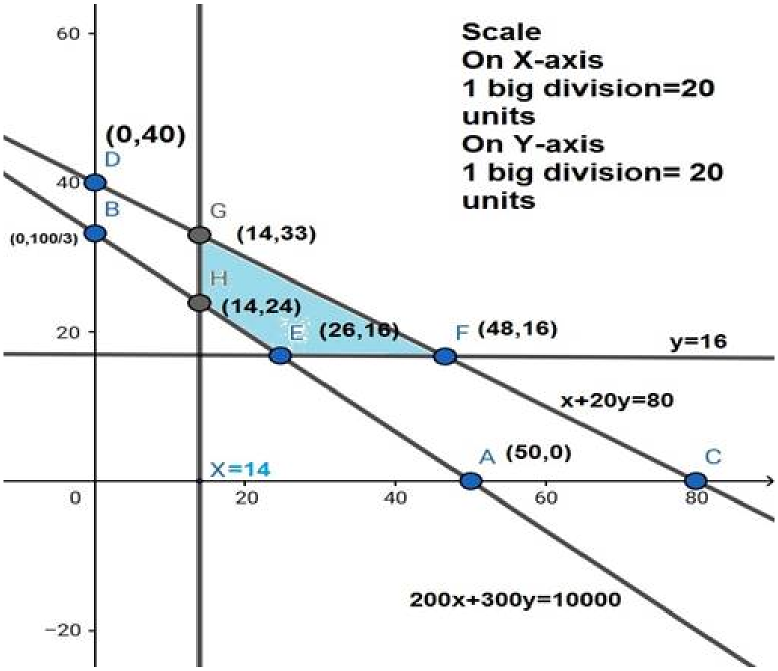
Region represented by $x \geq 14$,

x = 14 is the line passes through (14,0) and is parallel to the Y - axis. The region to the right of the line x = 14 will satisfy the inequation.

Region represented by $y \geq 16$,

y = 14 is the line passes through (16,0) and is parallel to the X - axis. The region to the right of the line y = 14 will satisfy the inequation.

Region $x,y \geq 0$: it represents first quadrant.



The corner points of the feasible region are E(26,16), F(48,16), G(14,33), H(14,24).

The values of Z at these corner points are as follow:

Corner Points	Z = 20x + 30y
E	1000
F	1440
G	1270
H	1000

The maximum value of Z is Rs 1440 which is attained at F(48,16).

Thus, the maximum profit is Rs 1440 obtained when 48 units of product A and 16 units of product B are manufactured.

20. Question

A manufacturer produces two type of steel trunks. He has two machines A and B. For completing, the first types of the trunk requires 3 hours on machine A and 3 hours on machine B, whereas the second type of the trunk requires 3 hours on machine A and 2 hours on machine B . Machines A and B can work at most for 18 hours and 15 hours per day respectively. He earns a profit of Rs 30 and Rs 25 per trunk of the first type and the second type respectively. How many trunks of each type musthe make each day to make maximum profit?

Answer

Let x trunks of first type and y trunks of second type were manufactured. Number of trunks cannot be negative.

Therefore, x, y $\geq$ 0

According to the question, the given information can be tabulated as

	Machine A (hours)	Machine B (hours)
First type (x)	3	3
Second type (y)	3	2
Availability	18	15

Therefore, the constraints are,

3x + 3y $\leq$ 18

3x + 2y $\leq$ 15.

He earns a profit of Rs 30 and Rs 25 per trunk of the first type and the second type respectively. Therefore, profit gained by him from x trunks of first type and y trunks of second type is Rs 30x and Rs 25y respectively.

Total profit Z = 30x + 25y which is to be maximized.

Thus, the mathematical formulation of the given LPP is

Max Z = 30x + 25y

Subject to

$3x + 3y \leq 18$

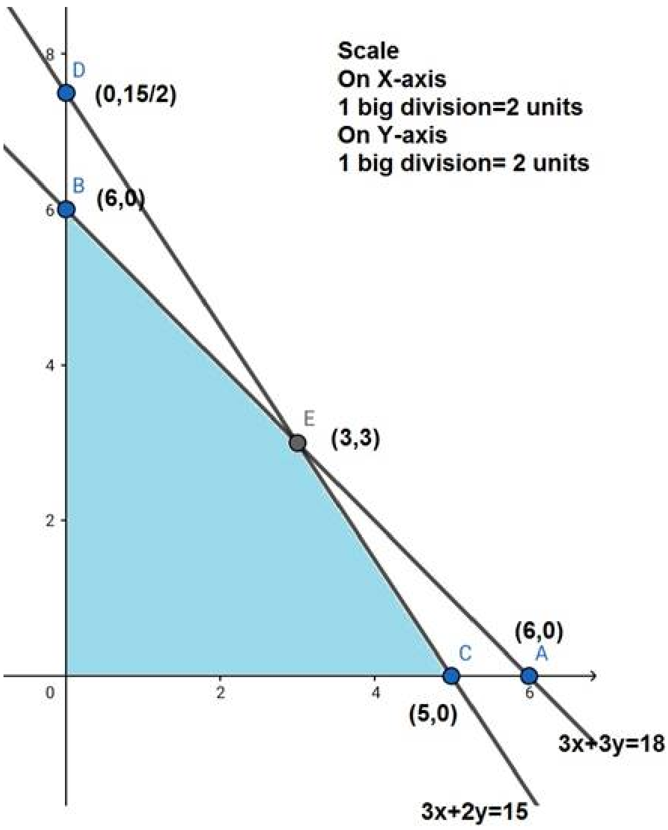
$3x + 2y \leq 15$

$x, y \geq 0$

Region $3x + 3y \leq 18$: line 3x + 3y = 18 meets axes at A(6,0), B(0,6) respectively. Region containing origin represents the solution of the inequation $3x + 3y \leq 18$ as (0,0) satisfies $3x + 3y \leq 18$.

Region $3x + 2y \leq 15$: line 3x + 2y = 15 meets axes at C(5,0), D$(0, \frac{15}{2})$ respectively. Region containing origin represents the solution of the inequation $3x + 2y \leq 15$ as (0,0) satisfies $3x + 2y \leq 15$.

Region $x,y \geq 0$: it represents first quadrant.



The corner points are O(0,0), B(0,6), E(3,3), and C(5,0).

The values of Z at these corner points are as follows:

Corner Points	Z = 30x + 25y
O	0
B	150
E	165
C	150

The maximum value of Z is 165 which is attained at E(3,3).

Thus, the maximum profit is of Rs 165 obtained when 3 units of each type of trunk is manufactured.

21. Question

A manufacturer of patent medicines is preparing a production plan on medicines, A and B. There are sufficient raw materials available to make 20000 bottles of A and 40000 bottles of B, but there are only 45000 bottles into which either of the medicines can be put. Further, it takes 3 hours to prepare enough material to fill 1000 bottles of A, it takes 1 hours to prepare enough material to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is ₹ 8 per bottle for A and ₹ 7 per bottle for B. How should the manufacturer schedule his production in order to maximize his profit?

Answer

Let production of each bottle of A and B are x and y respectively.

Since profits on each bottle of A and B are Rs 8 and Rs 7 per bottle respectively. So, profit on x bottles of A and y bottles of of B are 8x and 7y respectively. Let Z be total profit on bottles so,

Z = 8x + 7y

Since, it takes 3 hours and 1 hour to prepare enough material to fill 1000 bottles of Type A and Type B respectively, so x bottles of A and y bottles of B are preparing is $\frac{3x}{1000}$ hours and $\frac{y}{1000}$ hours respectively, bout only 66 hours are available, so,

$$\frac{3x}{1000} + \frac{y}{1000} \leq 66$$

3x + y $\leq$ 66000

Since raw materials available to make 2000 bottles of A and 4000 bottles of B but there are 45000 bottles in which either of these medicines can be put so,

x $\leq$ 20000

y $\leq$ 40000

x + y $\leq$ 45000

x,y $\geq$ 0. [Since production of bottles can not be negative]

Hence mathematical formulation of the given LPP is,

Max Z = 8x + 7y

Subject to constraints,

3x + y $\leq$ 66000

x $\leq$ 20000

y $\leq$ 40000

x + y $\leq$ 45000

x,y $\geq$ 0

Region 3x + y$\leq$ 66000: line 3x + y = 66000 meets the axes at A(22000,0), B(0,66000) respectively.

Region containing origin represents 3x + y $\leq$ 10000 as (0,0) satisfy 3x + y $\leq$ 66000

Region x + y $\leq$ 45000: line x + y = 45000 meets the axes at C(45000,0), D(0,45000) respectively.

Region towards the origin will satisfy the inequation as (0,00 satisfies the inequation

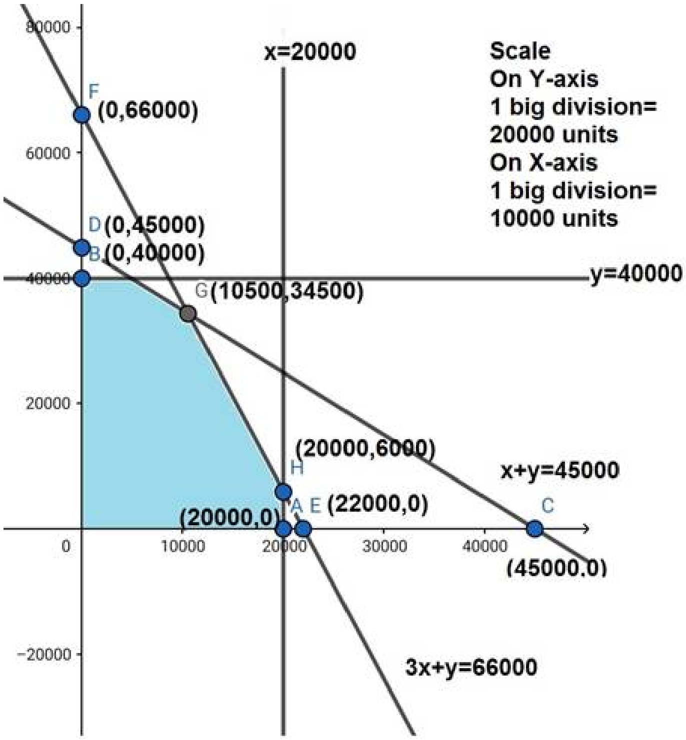
Region represented by x $\leq$ 20000,

x = 20000 is the line passes through (20000, 0) and is parallel to the Y - axis. The region towards the origin will satisfy the inequation.

Region represented by y $\leq$ 40000,

y = 40000 is the line passes through (0,40000) and is parallel to the X - axis. The region towards the origin will satisfy the inequation.

Region x,y $\geq$ 0: it represents first quadrant.



The corner points are O(0,0), B(0,40000), G(10500,34500), H(20000,6000), A(20000,0).

The values of Z at these corner points are,

Corner Points	Z = 8x + 7y
O	0
B	280000
G	325500
H	188000
A	160000

The maximum value of Z is 325500 which is attained at G(10500, 34500).

Thus the maximum profit is Rs 325500 obtained when 10500 bottles of A and 34500 bottles of B are manufactured.

22. Question

An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 400 is made on each first class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats of first class. However, at least 4 times as many passengers prefer to travel by economy class to the first class. Determine how many each type of tickets must be sold in order to maximize the profit for the airline. What is the maximum profit.

Answer

Let required number of first class and economy class tickets be x and y respectively.

Each ticket of first class and economy class make profit of Rs 400 and Rs 600 respectively.

So, x ticket of first class and y tickets of economy class make profit of Rs 400x and Rs 600y respectively.

Let total profit be Z = 400x + 600y

Given, aeroplane can carry a minimum of 200 passengers, so,

x + y$\leq$ 200

Given airline reserves at least 20 seats for first class, so,

$x \geq 20$

Also, at least 4 times as many passengers prefer to travel by economy class to the first class, so

$y \geq 4x$

Hence the mathematical formulation of the LPP is

Max Z = 400x + 600y

Subject to constraints

$x + y \leq 200$

$y \geq 4x$

$x \geq 20$

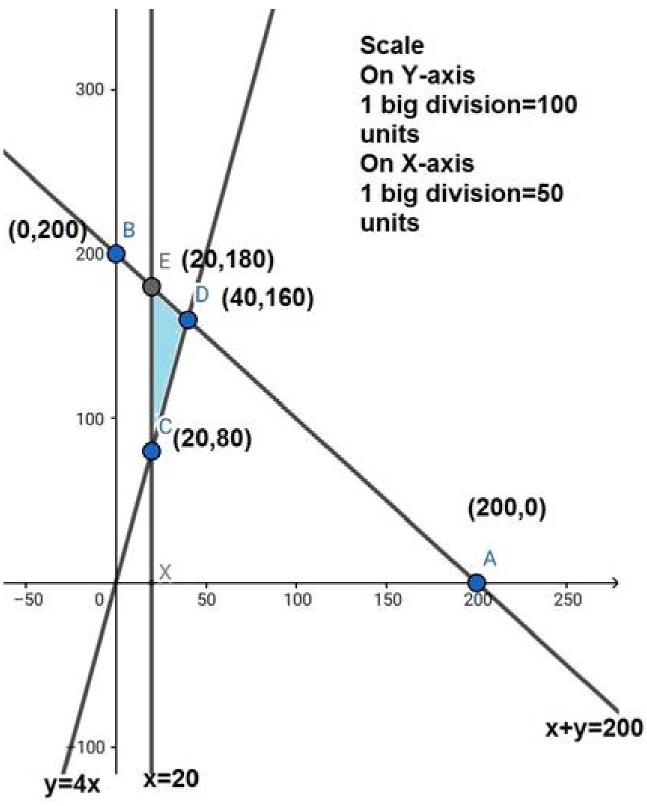
$x, y \geq 0$ [since seats in both the classes can not be zero]

Region represented by $x + y \leq 200$: the line x + y = 200 meets the axes at A(200,0), B(0,200). Region containing origin represents $x + y \leq 200$ as (0,0) satisfies $x + y \leq 200$.

Region represented by $x \geq 20$: line x = 20 passes through (20,0) and is parallel to y axis. The region to the right of the line x = 20 will satisfy the inequation $x \geq 20$

Region represented by $y \geq 4x$: line y = 4x passes through (0,0). The region above the line y = 4x will satisfy the inequation $y \geq 4x$

Region $x, y \geq 0$: it represents the first quadrant.



The corner points are C(20,80), D(40,160), E(20,180).

The values of Z at these corner points are as follows:

Corner points	Z = 400x + 600y
O	0
C	56000
D	112000
E	116000

The maximum value of Z is attained at E(20, 180).

Thus, the maximum profit is Rs 116000 obtained when 20 first class tickets and 180 economy class tickets are sold.

23. Question

A gardener has a supply of fertilizer of type I which consists of 10% nitrogen and 6% phosphoric acid and type II fertilizer which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, he finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If the type I fertilizer costs 60 paise per kg and type II fertilizer costs 40 paise per kg, determine how many kilograms of each fertilizer should be used, so that nutrient requirements are met at a minimum cost. What is the minimum cost?

Answer

Let x kg of Type I fertilizer and y kg of Type II fertilizers are supplied.

The quantity of fertilizers can not be negative.

So, $x,y \geq 0$

A gardener has a supply of fertilizer of type I which consists of 10% nitrogen and Type II consists of 5% nitrogen, and he needs at least 14 kg of nitrogen for his crop.

So,

$(10\times100) + (5\times100) \geq 14$

Or, $10x + 5y \geq 1400$

A gardener has a supply of fertilizer of type I which consists of 6% phosphoric acid and Type II consists of 10% phosphoric acid, and he needs at least 14 kg of phosphoric acid for his crop.

So,

$(6\times100) + (10\times100) \geq 14$

Or, $6x + 10y \geq 1400$

Therefore, A/Q, constraints are,

$10x + 5y \geq 1400$

$6x + 10y \geq 1400$

If the Type I fertilizer costs 60 paise per kg and Type II fertilizer costs 40 paise per kg. Therefore, the cost of x kg of Type I fertilizer and y kg of Type II fertilizer is Rs0.60x and Rs 0.40y respectively.

Total cost = Z(let) = 0.6x + 0.4y is to be minimized.

Thus the mathematical formulation of the given LPP is,

Min $Z = 0.6x + 0.4y$

Subject to the constraints,

$10x + 5y \geq 1400$

$6x + 10y \geq 1400$

$x,y \geq 0$

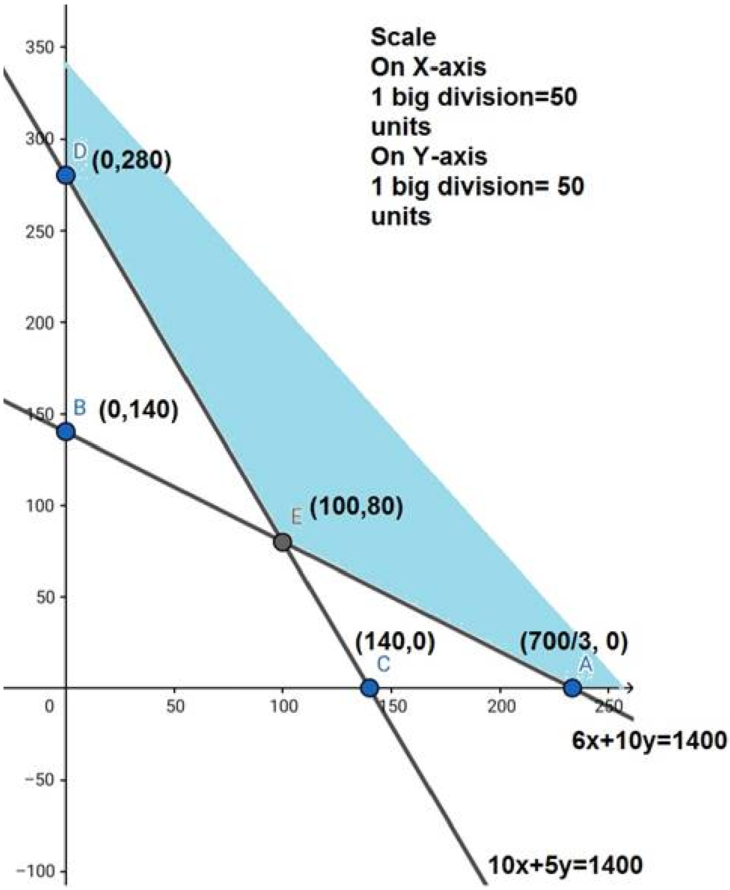
The region represented by 6x + 10y $\geq$ 1400: line 6x + 10y = 1400 passes through A($\frac{700}{3}$,0) and B(0,140). The region which doesn't contain the origin represents the solution of the inequation 6x + 10y $\geq$ 1400

As (0,0) doesn't satisfy the inequation 6x + 10y $\geq$ 1400

Region represented by 10x + 5y $\geq$ 1400: line 10x + 5y = 1400 passes through C(140,0) and D(0,280). The region which doesn't contain the origin represents the solution of the inequation 10x + 5y $\geq$ 1400

As (0,0) doesn't satisfy the inequation 10x + 5y $\geq$ 1400

The region, x,y $\geq$0: represents the first quadrant.



The corner points are D(0,280), E(100,80), A($\frac{700}{3}$,0)

The values of Z at these points are as follows:

Corner Points	Z = 0.6x + 0.4y
O	0
D	112
E	92
F	140

The minimum value of Z is Rs 92 which is attained at E(100,80)

Thus, the minimum cost is Rs92 obtained when 100 kg of Type I fertilizer and 80 kg of TypeII fertilizer is supplied.

24. Question

Anil wants to invest at most ₹ 12000 in Saving Certificates and National Saving Bonds. According to rules, he has to invest at least ₹ 2000 in Saving Certificates and at least 4000 in National Saving Bonds. If the rate of interest on saving certificate is 8% per annum and the rate of interest on National Saving Bonds is 10% per annum, how much money should he invest to earn

maximum yearly income? Find also his maximum yearly income.

Answer

Let Anil invests Rs x and Rs y in saving certificate (SC) and National saving bond (NSB) respectively.

Since, the rate of interest on SC is 8% annual and on NSB is 10% annual. So, interest on Rs x of SC is $\frac{8x}{100}$ and Rs y of NSB is $\frac{10x}{100}$ per annum.

Let Z be total interest earned so,

$Z = \frac{8x}{100} + \frac{10x}{100}$

Given he wants to invest Rs 12000 is total

$x + y \leq 12000$

According to the rules he has to invest at least Rs 2000 in SC and at least Rs 4000 in NSB.

$x \geq 2000$

$y \geq 4000$

Hence the mathematical formulation of LPP is to find x and y which

Maximizes Z

Max $Z = \frac{8x}{100} + \frac{10x}{100}$

Subject to constraints

$x \geq 2000$

$y \geq 4000$

$x + y \leq 12000$

$x, y \geq 0$

The region represented by $x \geq 2000$: line x = 2000 is parallel to the y - axis and passes through (2000,0).

The region not containing the origin represents $x \geq 2000$

As (0,0) doesn't satisfy the inequation $x \geq 2000$

The region represented by $y \geq 4000$: line y = 4000 is parallel to the x - axis and passes through (0,4000).

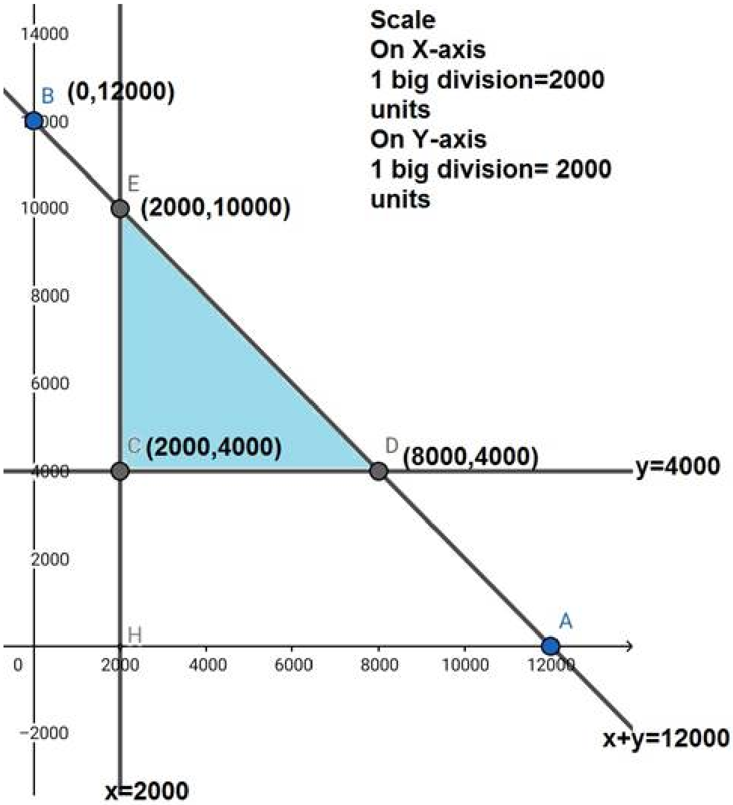
The region not containing the origin represents $y \geq 4000$

As (0,0) doesn't satisfy the inequation $y \geq 4000$

Region represented by $x + y \leq 12000$: line x + y = 12000 meets axes at A(12000,0) and B(0,12000) respectively. The region which contains the origin represents the solution set of $x + y \leq 12000$

as (0,0) satisfies the inequality $x + y \leq 12000$.

Region $x, y \geq 0$ is represented by the first quadrant.



The corner points are E(2000,10000), C(2000,4000), D(8000,4000).

The values of Z at these corner points are as follows:

Corner Points	$Z = \frac{8x}{100} + \frac{10x}{100}$
O	0
E	1160
D	1040
C	560

The maximum value of Z is Rs 1160 which is attained at E(2000,10000).

Thus the maximum earning is Rs1160 obtained when Rs 2000 were invested in SC and Rs 10000 in NSB.

25. Question

A man owns a field of area 1000 sq.m. He wants to plant fruit trees in it. He has a sum of 1400 to purchase young trees. He has the choice of two type of trees. Type A requires 10 sq.m of ground per tree and costs ₹ 20 per tree and type B requires 20 sq.m of ground per tree and costs ₹ 25 per tree. When fully grown, type A produces an average of 20 kg of fruit which can be sold at a profit of ₹ 2.00 per kg and type B produces an average of 40 kg of fruit which can be sold at a profit of ₹ 1.50 per kg. How many of each type should be planted to achieve maximum profit when the trees are fully grown? What is the maximum profit?

Answer

Let the required number of trees of Type A and B be Rs x and Rs y respectively.

Number of trees cannot be negative.

$x,y \geq 0$.

To plant tree of Type A requires 10 sq. m and Type B requires 20 sq. m of ground per tree. And it is given that a man owns a field of area 1000 sq. m. Therefore,

$10x + 20y \leq 1000$

$x + 2y \leq 100$

Type A costs Rs 20 per tree and Type B costs Rs 25 per tree. Therefore, x trees of type A and y trees of type B cost Rs 20x and Rs 25y respectively. A man has a sum of Rs 1400 to purchase young trees.

$20x + 25y \leq 1400$

$4x + 5y \leq 280$

Thus the mathematical formulation of the given LPP is

Max Z = 40x - 20x + 60y - 25y = 20x + 35y

Subject to,

$x + 2y \leq 100$

$4x + 5y \leq 280$

$x, y \geq 0$

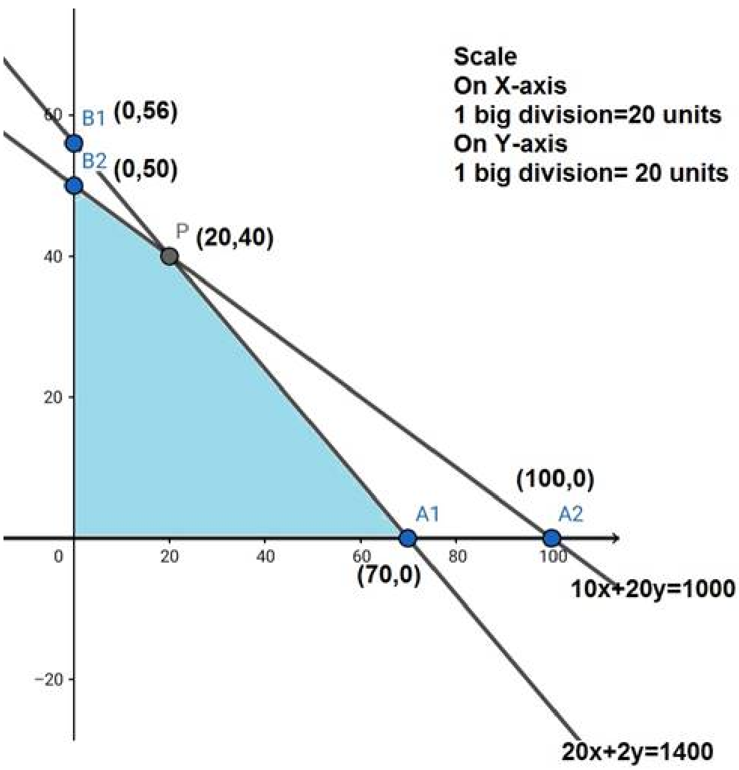
Region $4x + 5y \leq 280$: line $4x + 5y \leq 280$ meets axes at A_1(70,0), B_1(0,56) respectively.

The region containing origin represents $4x + 5y \leq 280$ as (0,0) satisfies $4x + 5y \leq 280$.

Region $x + 2y \leq 100$: line x + 2y = 100 meets axes at A_2(100,0), B_2(0,50) respectively.

Region containing origin represents $x + 2y \leq 100$ as (0,0) satisfies $x + 2y \leq 100$

Region $x, y \geq 0$: it represents the first quadrant.



The corner points are A_1 (70,0), P(20,40), B_2(0,50)

The values of Z at these corner points are as follows:

Corner Points	Z = 20x + 35y
O	0
A_1	1750
P	1800
B_2	1400

The maximum value of Z is 1800 which is attained at P(20,40).

Thus the maximum profit is Rs 1800 obtained when Rs 20 were involved in Type A and Rs 40 were involved in Type II.

Exercise 30.5

1. Question

Two godowns, A and B, have grain storage capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F, whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table :

Transportation cost per quintal (in ₹)		
From / To	A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

How should the supplies be transported in order that the transportation cost is minimum?

Answer

Let godown A supply x and y quintals of grain to shop D and E respectively. Remaining grain storage of A is 100 – x – y. So, now it is supplied to shop F by godown A.

Now, remaining requirement of shop D is 60 – x which is supplied by godown B.

Remaining requirement of shop E is 50 – y which is supplied by godown B.

Remaining requirement of shop F is 40 – (100 – x – y)

= x + y - 60 which is supplied by godown B.

where,

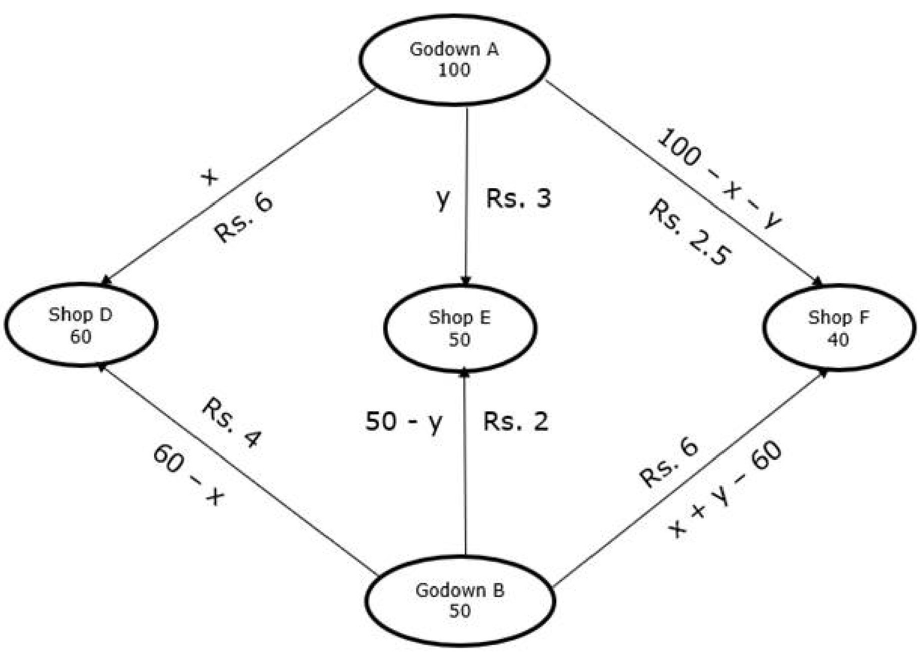
$x \geq 0$, $y \geq 0$ and $100 - x - y \geq 0$

$\Rightarrow x \geq 0$, $y \geq 0$ and $x + y \leq 100$

$60 - x \geq 0$, $50 - y \geq 0$ and $x + y - 60 \geq 0$

$\Rightarrow x \leq 60$, $y \leq 50$ and $x + y \geq 60$

This can be illustrated by this:



Transportation cost per quintal (in ₹)		
From / To	A	B
D	6.00	4.00
E	3.00	2.00
F	2.50	3.00

Cost = Number of quintals * Cost of transportation per quintal

Total transportation cost z is given by,

z = 6x + 3y + 2.5(100 – x – y) + 4(60 – x) + 2(50 – y) + 3(x + y – 60)

⇒ z = 6x + 3y + 250 – 2.5x – 2.5y + 240 – 4x + 100 – 2y + 3x + 3y – 180

⇒ z = 2.5x + 1.5y + 410

We need to minimize the cost

Hence, mathematical formulation of LPP is

Minimize z = 2.5x + 1.5y + 410

subject to the constraints,

x + y ≥ 60

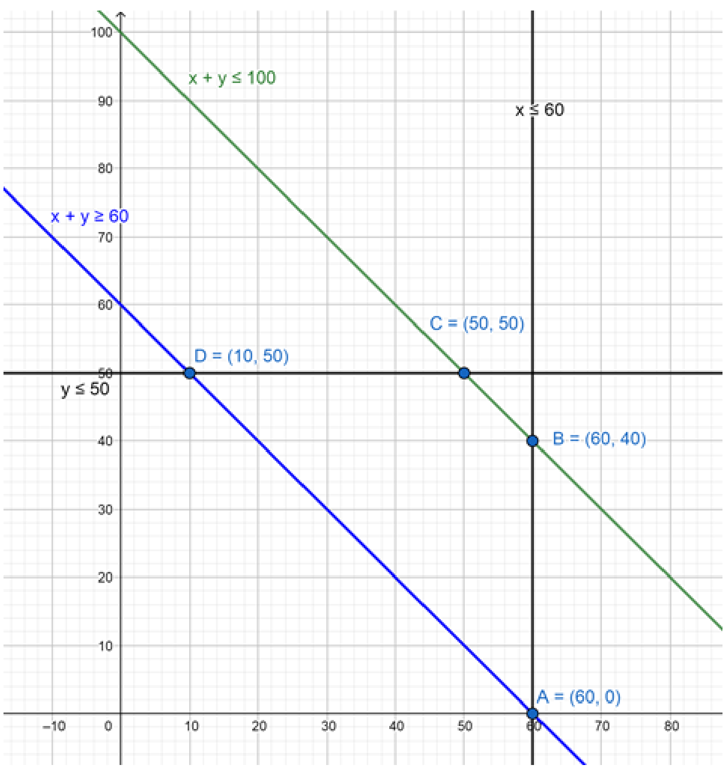
y ≤ 50

x ≤ 60

x + y ≤ 100

x, y≥0

The feasible region determined by the system of constraints is as follows:



The corner points of enclosed region are A(60, 0) , B(60, 40), C(50, 50) and D(10,50)

The value of z at these corners points is as follows:

Case 1: A(60, 0)

z = 2.5x + 1.5y + 410

⇒ z = 2.5(60) + 1.5(0) + 410

⇒ z = 150 + 0 + 410

⇒ z = 560

Case 2: B(60, 40)

z = 2.5x + 1.5y + 410

⇒ z = 2.5(60) + 1.5(40) + 410

⇒ z = 150 + 60 + 410

⇒ z = 620

Case 3: C(50, 50)

z = 2.5x + 1.5y + 410

⇒ z = 2.5(50) + 1.5(50) + 410

⇒ z = 125 + 75 + 410

⇒ z = 610

Case 4: D(10, 50)

z = 2.5x + 1.5y + 410

⇒ z = 2.5(10) + 1.5(50) + 410

⇒ z = 25 + 75 + 410

⇒ z = 510

The value of z is minimum in fourth case at point D(10, 50)

As, x = 10, y = 50

Godown A supplies to:

Shop D = x = 10 quintals

Shop E = y = 50 quintals

Shop F = 100 – x – y = 100 – 10 – 50 = 40 quintals

Godown B supplies to:

Shop D = 60 – x = 60 – 10 = 50 quintals

Shop E = 50 – y = 50 – 50 = 0 quintals

Shop F = x + y – 60 = 10 + 50 – 60 = 0 quintals

Minimum cost for transportation of these quintals to their respective shops = Rs. 510

Transportation of grain(quintals)		
From / To	A	B
D	10	50
E	50	0
F	40	0

2. Question

A medical company has factories at two places, A and B. From these places, supply is made to each of its three agencies situated at P, Q and R. The monthly requirements of the agencies are respectively, 40, 40 and 50 packets of the medicines, while the production capacity of the factories, A and B are 60 and 70 packet respectively. The transportation cost per packet from the factories to the agencies are given below :

Transportation cost per packet (in ₹)		
From / To	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each factory be transported to each agency so that the cost of transportation is minimum? Also find the minimum cost?

Answer

Let factory A supply x and y packets of medicines to agency P and Q respectively. Remaining packets of factory A are 60 – x – y. So, now they will be supplied to agency R by factory A.

Now, remaining packets requirement of agency P is 40 – x which will be supplied by factory B.

Remaining packets requirement of agency Q is 40 – y which will be supplied by factory B.

Remaining packets requirement of agency R is 50 – (60 – x – y)

= x + y – 10 which will be supplied by factory B.

where,

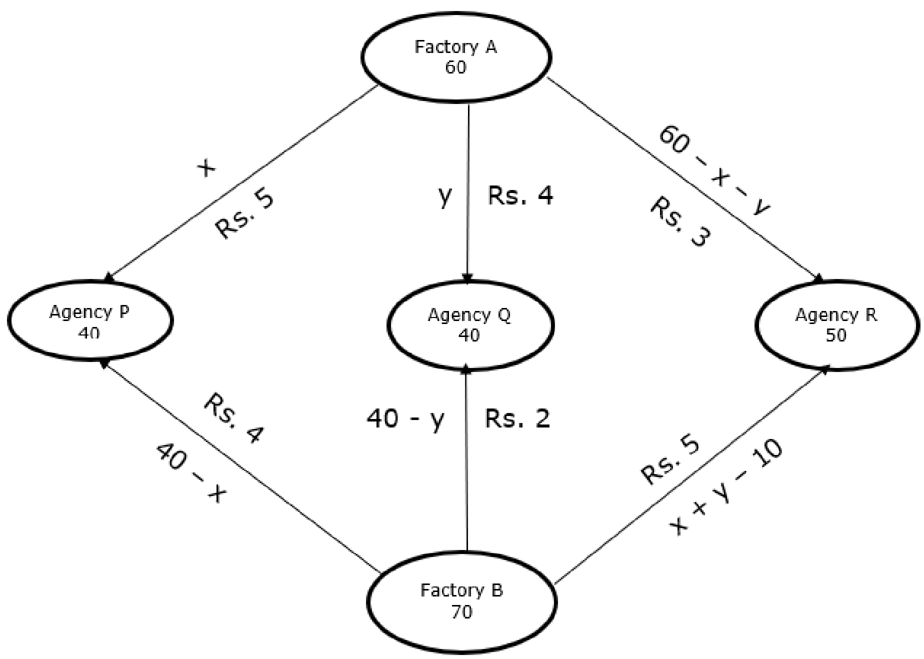
$x \geq 0$, $y \geq 0$ and $60 - x - y \geq 0$

$\Rightarrow x \geq 0$, $y \geq 0$ and $x + y \leq 60$

$40 - x \geq 0$, $40 - y \geq 0$ and $x + y - 10 \geq 0$

$\Rightarrow x \leq 40$, $y \leq 40$ and $x + y \geq 10$

This can be illustrated by this:



Transportation cost per packet (in ₹)		
From / To	A	B
P	5	4
Q	4	2
R	3	5

Cost = Number of packets * Cost of transportation per packet

Total transportation cost z is given by,

z = 5x + 4y + 3(60 – x – y) + 4(40 – x) + 2(40 – y) + 5(x + y – 10)

⇒ z = 5x + 4y + 180 – 3x – 3y + 160 – 4x + 80 – 2y + 5x + 5y – 50

⇒ z = 3x + 4y + 370

We need to minimize the cost

Hence, mathematical formulation of LPP is

Minimize z = 3x + 4y + 370

subject to the constraints,

$x+y \geq 10$

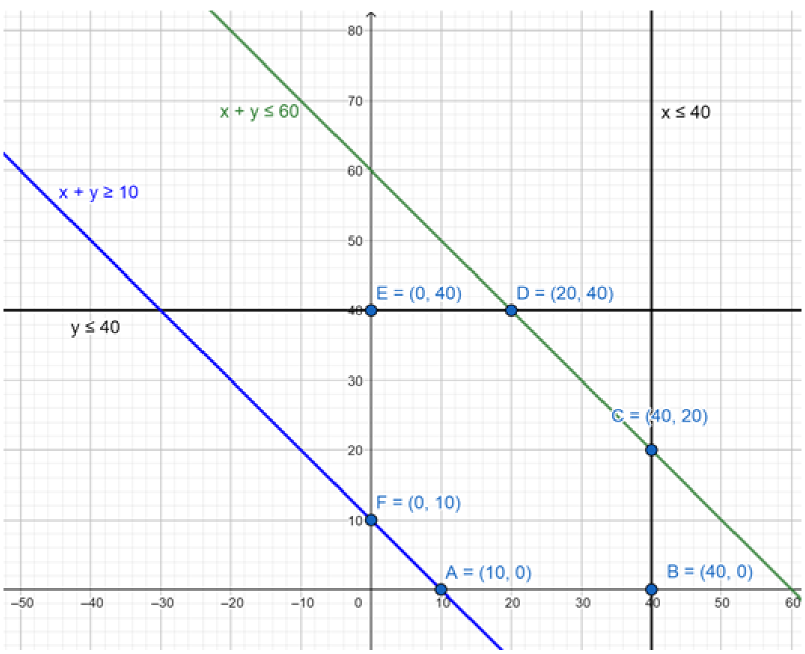
$y \leq 40$

$x \leq 40$

$x+y \leq 60$

$x, y \geq 0$

The feasible region determined by the system of constraints is as follows:



The corner points of the enclosed region are A(10, 0) , B(40, 0), C(40, 20), D(20,40), E(0, 40) and F(0, 10)

The value of z at these corners points is as follows:

Case 1: A(10, 0)

z = 3x + 4y + 370

⇒ z = 3(10) + 4(0) + 370

⇒ z = 30 + 0 + 370

⇒ z = 400

Case 2: B(40, 0)

z = 3x + 4y + 370

⇒ z = 3(40) + 4(0) + 370

$\Rightarrow z = 120 + 0 + 370$

$\Rightarrow z = 490$

Case 3: C(40, 20)

$z = 3x + 4y + 370$

$\Rightarrow z = 3(40) + 4(20) + 370$

$\Rightarrow z = 120 + 80 + 370$

$\Rightarrow z = 570$

Case 4: D(20, 40)

$z = 3x + 4y + 370$

$\Rightarrow z = 3(20) + 4(40) + 370$

$\Rightarrow z = 60 + 160 + 370$

$\Rightarrow z = 590$

Case 5: E(0, 40)

$z = 3x + 4y + 370$

$\Rightarrow z = 3(0) + 4(40) + 370$

$\Rightarrow z = 0 + 160 + 370$

$\Rightarrow z = 530$

Case 4: F(0, 10)

$z = 3x + 4y + 370$

$\Rightarrow z = 3(0) + 4(10) + 370$

$\Rightarrow z = 0 + 40 + 370$

$\Rightarrow z = 410$

The value of z is minimum in first case at point A(10, 0)

As, x = 10, y = 0

Factory A supplies to:

Agency P = x = 10 packets

Agency Q = y = 0 packets

Agency R = 60 – x – y = 60 – 10 – 0 = 50 packets

Godown B supplies to:

Agency P = 40 – x = 40 – 10 = 30 packets

Agency Q = 40 – y = 40 – 0 = 40 packets

Agency R = x + y – 10 = 10 + 0 – 10 = 0 packets

Minimum cost for transportation of these packets to their respective agencies = Rs. 400

Transportation of packets

From / To	A	B
P	10	30
Q	0	40
R	50	0

MCQ

1. Question

The solution set of the inequation 2x + y > 5 is

A. half plane that contains the origin

B. open half plane not containing the origin

C. whole xy-plane not containing the origin

D. none of these

Answer

Given inequation is 2x + y > 5.

Now, we convert the inequation into an intercept line equation form, we can clearly see the intercepts of the inequation on x-axis and y-axis.

2x + y > 5

[dividing the whole inequation by 5]

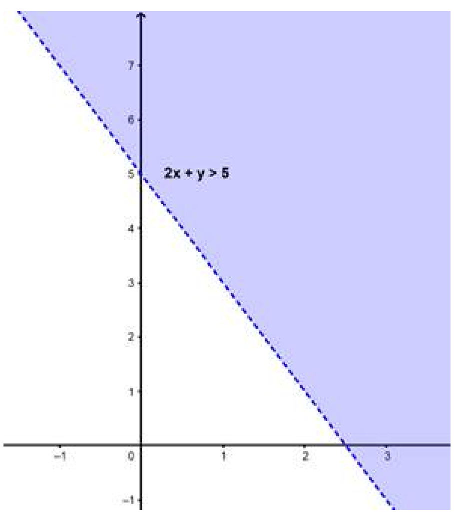
$$\frac{2x}{5}+\frac{y}{5}>\frac{5}{5}$$

$$\frac{x}{\frac{5}{2}}+\frac{y}{5}>1$$

$$\frac{x}{2.5}+\frac{y}{5}>1$$

Therefore, from the above inequation, we can say that, 2.5 and 5 are the intercepts of the x-axis and y-axis respectively.

Now by plotting these on the graph, we can clearly see the graph of the inequation.



From the graph, it is clear that, the solution set of the inequation

2x + y > 5 is the open half plane not containing origin i.e. option B.

2. Question

Objective function of a LPP is

A. a constraint

B. a function to be optimized

C. a relation between the variables

D. none of these

Answer

Given,

To define the objective of a Linear programming Problem.

As per the definition of the Linear Programming Problem,

A Linear programming problem is a linear function (also known an objective function) subjected to certain constraints for which we need to find an optimal solution (i.e. either a maximum/minimum value) depending on the requirement of the problem.

From the above definition, we can clearly say that, Linear programming problem’s objective is to either maximize/ minimize a given objective function, which means to optimize a function to get an optimum solution.

Hence the answer is option B.

3. Question

Which of the following sets are convex?

A. $\{(x, y) : x^2 + y^2 \geq 1\}$

B. $\{(x, y) : y^2 \geq x\}$

C. $\{(x, y) : 3x^2 + 4y^2 \geq 5\}$

D. $\{(x, y) : y \geq 2, y \leq 4\}$

Answer

Given sets are

• $\{(x,y) : x^2 + y^2 \geq 1\}$

• $\{(x, y) : y^2 \geq x\}$

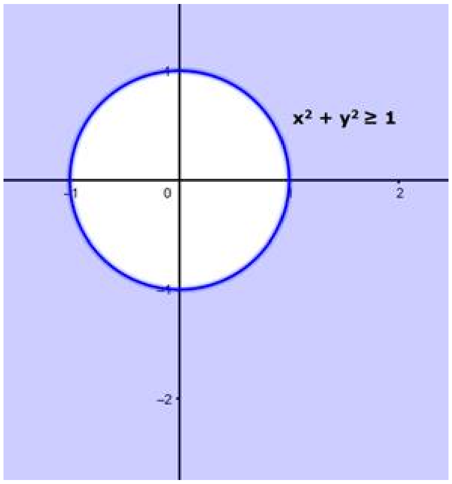
• $\{(x, y) : 3x^2 + 4y^2 \geq 5\}$

• $\{(x, y) : y \geq 2, y \leq 4\}$

A convex set, is nothing but whose solution set is in the shape of a convex polygon.

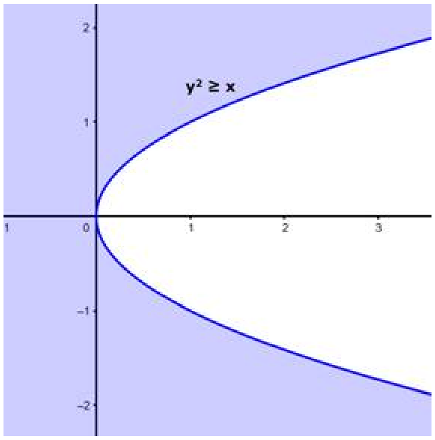
If we map these functions on a graph, we can clearly find the set with a convex solution set.

• f : $\{(x,y) : x^2 + y^2 \geq 1\}$



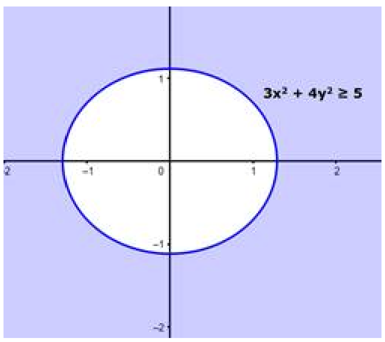
From the graph, it is evident that the solution set which is the shaded region is not convex.

• g: $\{(x, y) : y^2 \geq x\}$



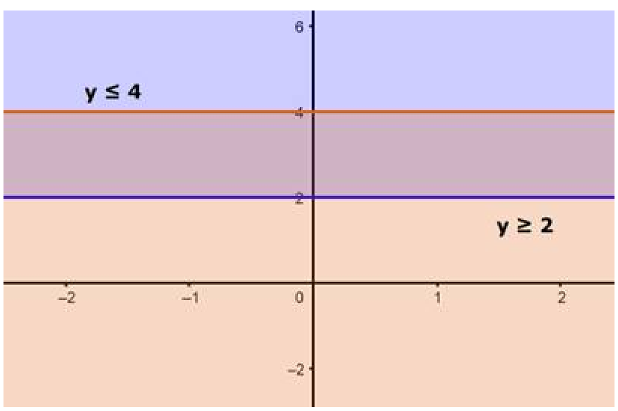
From the graph, it is evident that the solution set which is the blue shaded region is not convex.

• h:{(x, y) : $3x^2 + 4y^2 \geq 5$}



From the graph, it is evident that the solution set which is the grey shaded region is not convex.

• p:{(x, y) : y ≥ 2, y ≤ 4}



From the graph, the dark blue shaded region between the two bright lines is a convex set.

Hence, the solution is option D.

4. Question

Let X_1 and X_2 are optimal solutions of a LPP, then

A. $X = \lambda X_1 + (1 - \lambda)X_2$, $\lambda \in R$ is also an optimal solution

B. $X = \lambda X_1 + (1 - \lambda)X_2$, $0 \leq \lambda \leq 1$ gives an optimal solution

C. $X = \lambda X_1 + (1 - \lambda)X_2$, , $0 \leq \lambda \leq 1$ give an optimal solution

D. $X = \lambda X_1 + (1 + \lambda)X_2$, $\lambda \in R$ gives an optimal solution

Answer

Given, X_1 and X_2 are optimal solutions of a Linear programming problem(LPP).

This means that, $\{X_1, X_2\} \in C$ (a convex Set) as the optimal solution of a LPP is convex.

Now by using the definition of a Convex set,

A set of points C is called convex if, for all λ in the interval $0 \le \lambda \le 1$, $\lambda y + (1 - \lambda)z$ is contained in C whenever y and z are contained in C.

By using this property of Convex set,

If $\{X_1, X_2\} \in C$ (a convex set of optimal solutions), then

$X = \lambda X_1 + (1 - \lambda) X_2$ where $0 \le \lambda \le 1$, is also contained in C (the optimal solution set).

This proves that, also $X \in C$.

Hence the answer is option B.

5. Question

The maximum value of $Z = 4x + 2y$ subjected to the constraints $2x + 3y \le 18$, $x + y \ge 10$; $x, y \ge 0$ is

A. 36

B. 40

C. 20

D. none of these

Answer

Given,

$Z = 4x + 2y$

Subjected to constraints,

$2x + 3y \le 18$

$x + y \ge 10$

$x \ge 0$

$y \ge 0$

Consider, the inequalities as equalities for some time,

$2x + 3y = 18$ and $x + y = 10$,

If we convert these into intercept line format equations, we get,

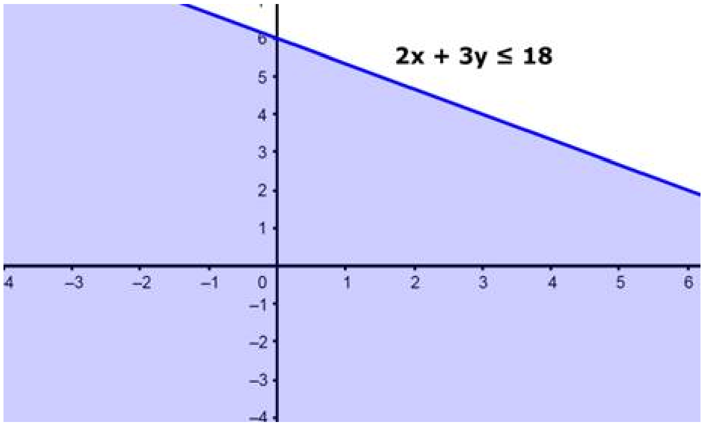
[Dividing the whole equation with the right hand side number of the equation]

$\frac{2x}{18} + \frac{3y}{18} = \frac{18}{18}$ and $\frac{x}{10} + \frac{y}{10} = \frac{10}{10}$

$\frac{x}{9} + \frac{y}{6} = 1$ and $\frac{x}{10} + \frac{y}{10} = 1$

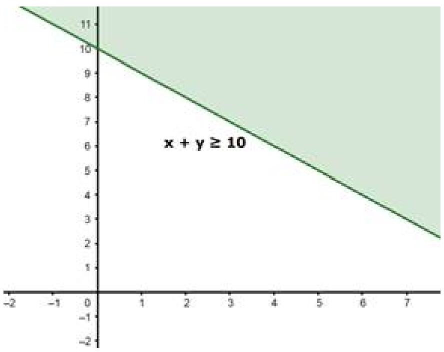
From this form of line, we can say that the line $2x + 3y = 18$ meets the x-axis at (9,0) and y-axis at (0,6).

This shows the inequality $2x + 3y \le 18$ holds good in the below blue colored region.



Similarly from the intercept line format, we can say that the line x + y = 10 meets the x-axis at (10,0) and y-axis at (0,10).

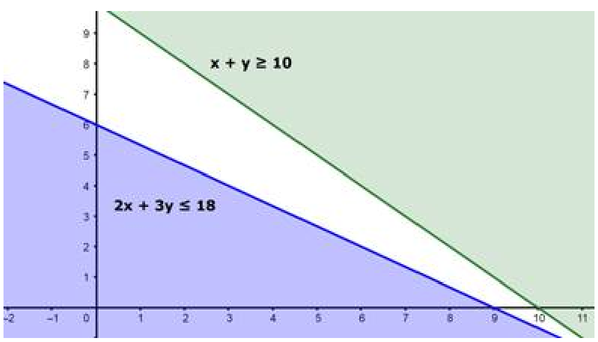
This shows the inequality x + y ≥ 10 holds above in the green colored region.



Now considering the inequalities, x ≥ 0 and y ≥ 0, this clearly shows the region where both x and y are positive. This represents the 1st quadrant of the graph.

So, the solutions of the LPP are in the first quadrant where the inequalities meet.

Now by plotting both graphs 2x + 3y ≤ 18 and x + y ≥ 10 we get the below graph.



We can clearly see that, there is no area in the 1st quadrant where the two inequalities met.

This clearly says that there is no solution for the LPP with the given constraints.

Hence the option D, is the solution to the problem.

6. Question

The optimal value of the objective function is attained at the points

A. given by intersection of inequations with the axes only

B. given by intersection of inequations with x-axis only

C. given by corner points of the feasible region

D. none of these

Answer

Given that,

• There is an objective function

• There are optimal values

From the definition of optimal value of a Linear Programming Problem(LPP):

<u>An optimal/ feasible solution is any point in the feasible region that gives a maximum or minimum value if substituted in the objective function.</u>

Here feasible region of an LPP is defined as:

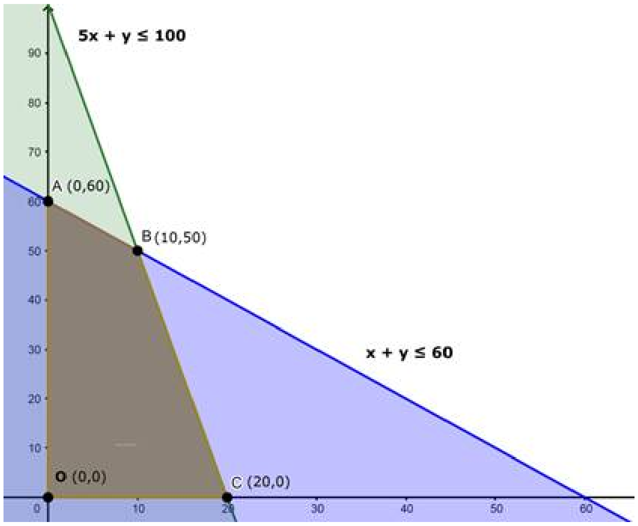
<u>A feasible region is that common region determined by all the constraints including the non-negative constraints of the LPP.</u>

So the Feasible region of a LPP is a convex polygon where, its vertices (or corner points) determine the optimal values (either maximum/minimum) of the objective function.

For Example,

5x + y ≤ 100 ; x + y ≤ 60 ; x ≥ 0 ; y ≥ 0

The feasible solution of the LPP is given by the convex polygon OADC.



Here, points O, A ,D and C will be optimal solutions of the taken LPP

Hence the answer is option C.

7. Question

The maximum value of Z = 4x + 3y subjected to the constraints 3x + 2y ≥ 160, 5x + 2y ≥ 200, x + 2y ≥ 80; x, y ≥ 0 is

A. 320

B. 300

C. 230

D. none of these

Answer

Given object function is

Z = 4x+3y

Constraints are

3x + 2y ≥ 160

5x + 2y ≥ 200

x + 2y ≥ 80

x ≥ 0

y≥ 0.

Consider, the inequalities as equalities for some time,

3x + 2y = 160 ; 5x + 2y = 200 and x + 2y = 80

If we convert these into intercept line format equations, we get,

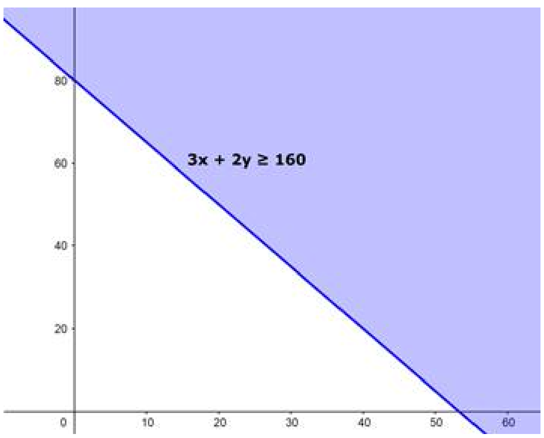
[Dividing the whole equation with the right hand side number of the equation]

$\frac{3x}{160}+\frac{2y}{160}=\frac{160}{160}$; $\frac{5x}{200}+\frac{2y}{200}=\frac{200}{200}$ and $\frac{x}{80}+\frac{2y}{80}=\frac{80}{80}$

$\frac{x}{\frac{160}{3}}+\frac{y}{80}=1$; $\frac{x}{40}+\frac{y}{100}=1$ and $\frac{x}{80}+\frac{y}{40}=1$

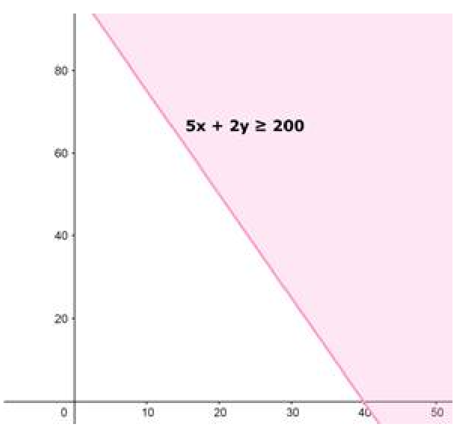
From this form of line, we can say that the line 3x + 2y = 160 meets the x-axis at ($\frac{160}{3}$,0) and y-axis at (0,80).

This shows the inequality 3x + 2y ≥ 160 holds good in the below blue colored region.



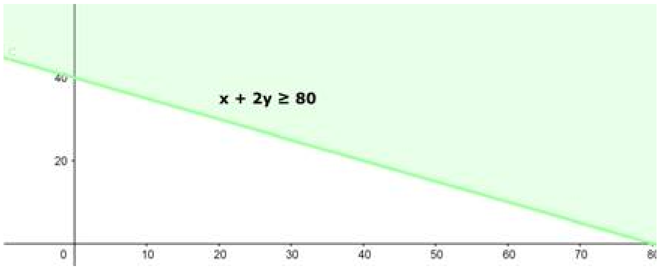
Similarly, from the intercept line format, we can say that the line 5x + 2y = 200 meets the x-axis at (40,0) and y-axis at (0,100).

This shows the inequality 5x + 2y ≥ 200 holds above in the pink colored region.



Similarly from the intercept line format, we can say that the line x + 2y = 80 meets the x-axis at (80,0) and y-axis at (0,40).

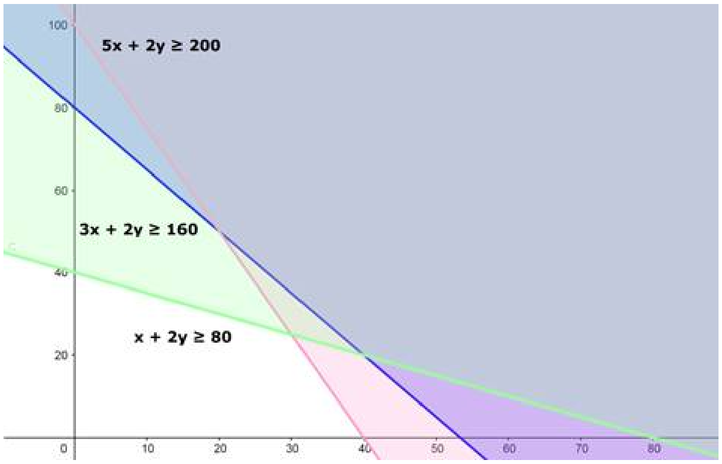
This shows the inequality x + 2y ≥ 80 holds above in the green colored region.



Now considering the inequalities, x ≥ 0 and y ≥ 0, this clearly shows the region where both x and y are positive. This represents the 1^{st} quadrant of the graph.

So, the solutions of the LPP are in the first quadrant where the inequalities meet.

Now by plotting all the graphs 3x + 2y ≥ 160 , 5x + 2y ≥ 200 and x + 2y ≥ 80 we get the below graph.



We can clearly see that, there is no area in the 1^{st} quadrant where all the three inequalities met.

This clearly says that there is no solution for the LPP with the given constraints.

Hence the option D, is the solution to the problem.

8. Question

Consider a LPP given by

Minimum Z = 6x + 10y

Subjected to x ≥ 6; y ≥ 2; 2x + y ≥ 10; x, y ≥ 0

Redundant constraints in this LPP are

A. x ≥ 0, y ≥ 0

B. x ≥ 6, 2x + y ≥ 10

C. 2x + y ≥ 10

D. none of these

Answer

Given

Objective Function is Z = 6x + 10y

Constraints are:

x ≥ 6

y ≥ 2

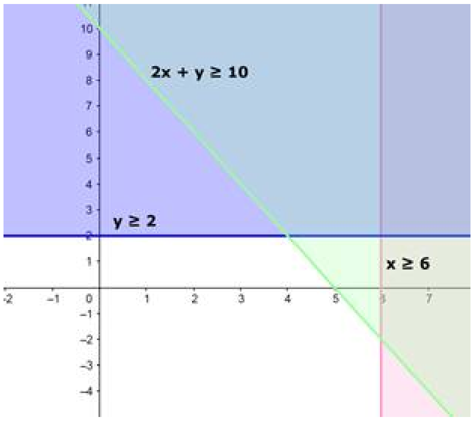
2x + y ≥ 10

x, y ≥ 0

A redundant constraint is that, which doesn't intersect with the feasible region of the out non-redundant constraints.

Here, the problem is a minimization problem and as per the constraints x≥ 0 and y ≥ 0 the feasible solution is located in the 1st quadrant.

Now, if we map all the three inequalities in a graph, we have



From the graph, it is very clear that, the graph of the inequality 2x + y ≥ 10 is not intersecting the feasible region formed by the constraints x ≥ 6; y ≥ 2; x≥ 0 and y ≥ 0.

Hence the inequality 2x + y ≥ 10 is not really making any difference to the feasible region from by x ≥ 6; y ≥ 2; x≥ 0 and y ≥ 0.

Therefore inequality 2x + y ≥ 10 remains redundant.

The answer of the question is option C.

9. Question

The objective function Z = 4x + 3y can be maximized subjected to the constraints 3x + 4y ≤ 24, 8x + 6y ≤ 48, x ≤ 5, y ≤ 6; x, y ≥ 0

A. at only one point

B. at two points only

C. at an infinite number of points

D. none of these

Answer

Given the objective function is Z = 4x + 3y

Constraints are:

3x + 4y ≤ 24

8x + 6y ≤ 48

x ≤ 5

y ≤ 6

x ≥ 0

y≥ 0

If we consider these inequalities as equalities for some time,

We will have

3x + 4y = 24

$8x + 6y = 48$

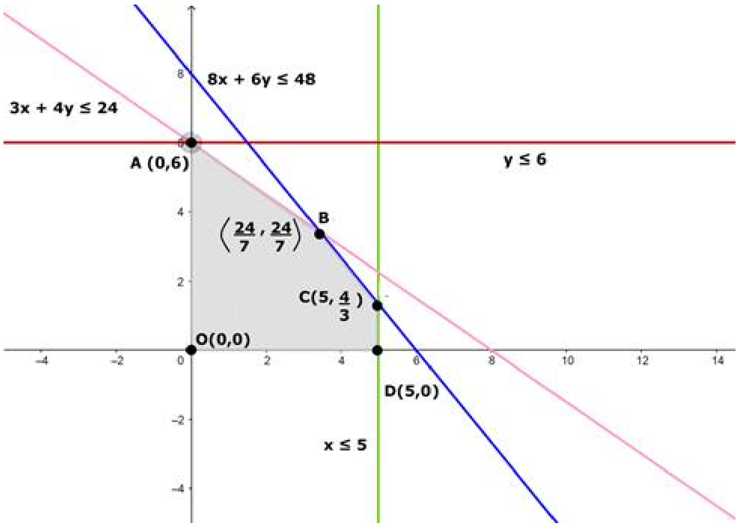
$x = 5$

$y = 6$

$x = 0$

$y= 0$

If we plot all these lines on a graph we will have optimal area formed by the vertices, OABCD.



Now, to find where the function Z has maximized, let us substitute all these points in the objective function Z.

Z at O(0,0)	$Z = 4\ (0) + 3\ (0) = 0 + 0 = 0$
Z at A (0,6)	$Z = 4\ (0) + 3\ (6) = 0 + 18 = 18$
Z at B $\left(\frac{24}{7}, \frac{24}{7}\right)$	$Z = 4\left(\frac{24}{7}\right) + 3\left(\frac{24}{7}\right) = \frac{96+72}{7} = \frac{168}{7} = 24$
Z at C $\left(5, \frac{4}{3}\right)$	$Z = 4\ (5) + 3\left(\frac{4}{3}\right) = 20 + 4 = 24$
Z at D (5,0)	$Z = 4\ (5) + 3\ (0) = 20 + 0 = 20$

Here, we can clearly see that, the function Z is maximized at two points B & C giving the value 24.

<u>There will be infinite/multiple optimal solutions for a LPP if it has more than one set of optimal solutions that can maximize/ minimize a problem.</u>

This will clear the fact that, the function Z will maximize at infinite number of points.

Hence the answer is option C.

10. Question

If the constraints in a linear programming problem are changed

A. the problem is to be re-evaluated

B. solution is not defined

C. the objective function has to be modified

D. the change in constraints is ignored

Answer

Given,

The constraints of a linear programming problem are changed.

Now, as per the definition of the Linear Programming Problem,

A Linear programming problem is a linear function (also known an objective function) subjected to certain constraints for which we need to find an optimal solution (i.e. either a maximum/minimum value) depending on the requirement of the problem.

Here, the LPP is solved using the constraints, so, if the constraints are changed, the problem is to has to be re-calculated with the new constraints provided.

Hence the answer is option A.

11. Question

Which of the following statements is correct?

A. Every LPP admits an optimal solution

B. A LPP admits unique optimal solution

C. If a LPP admits two optimal solutions it has an infinite number of optimal solutions

D. The set of all feasible solutions of a LPP is not a converse set convex set

Answer

Given,

The statements:

• Every LPP admits an optimal solution

This need not be true as all LPPs need not have optimal solutions and such LPPs are called unbound.

• A LPP admits unique optimal solution

Every LLP need not have unique optimal solutions as if there are two optimal solutions to an LLP there will be infinite number or optimal solutions to the LLP problem.

• If a LPP admits two optimal solutions it has an infinite number of optimal solutions

As mentioned in the above point, if there are two optimal solutions to an LLP there will be infinite number or optimal solutions to the LLP problem.

• The set of all feasible solutions of a LPP is not a convex set.

As per a theorem of Convex Sets,

If $\{X_1, X_2\} \in C$ (a convex set of optimal solutions), then

$X = \lambda X_1 + (1 - \lambda) X_2$ where $0 \leq \lambda \leq 1$, is also contained in C (the optimal solution set). This makes all the feasible solutions of a LPP also a convex set.

Hence, from the explanations, the answer is Option C.

12. Question

Which of the following is not a convex set?

A. {(x, y) : 2x + 5y < 7}

B. {(x, y) : $x^2 + y^2 \le 4$}

C. {x : |x| = 5}

D. {(x, y) : $3x^2 + 2y^2 \le 6$}

Answer

Given 4 sets,

i. {(x, y) : 2x + 5y < 7}

ii. {(x, y) : $x^2 + y^2 \le 4$}

iii. {x : |x| = 5}

iv. {(x, y) : $3x^2 + 2y^2 \le 6$}

By graphing them, we can clearly figure out the convex set.

A convex set, is nothing but whose solution set is in the shape of a convex polygon.

i. {(x, y) : 2x + 5y < 7}

This inequation can be converted into an equation and by applying the intercept line format, we get,

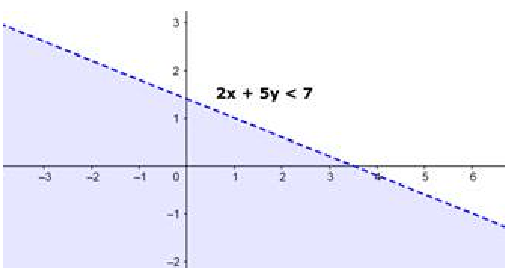
$$\frac{2x}{7} + \frac{5y}{7} = \frac{7}{7}$$

[dividing the whole by 7]

We get,

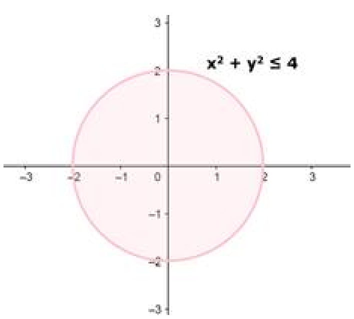
$$\frac{x}{\frac{7}{2}} + \frac{y}{\frac{7}{5}} = 1$$

So the graph is



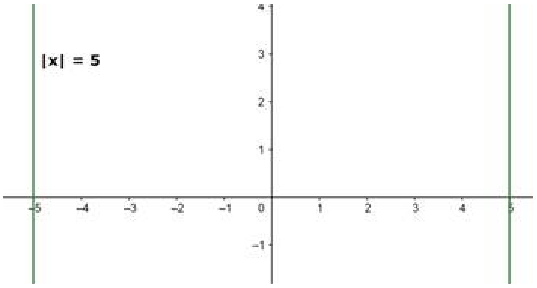
ii. {(x, y) : $x^2 + y^2 \le 4$}

So the graph for this inequality is given by



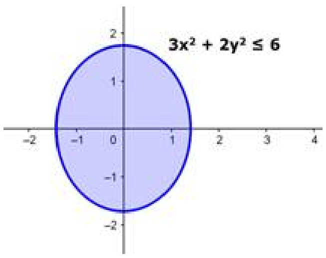
iii. {x : |x| = 5}

The graph for the set is as below:



iv. $\{(x, y) : 3x^2 + 2y^2 \leq 6\}$

The graph for the inequality is given by



From t he above graphs, we can clearly say that, only the 3rd graph is a convex set.

Hence option C is the answer.

13. Question

By graphical method, the solution of linear programming problem

Maximize $Z = 3x_1 + 5x_2$

Subject to $3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1 \geq 0, x_2 \geq 0$, is

A. $x_1 = 2, x_2 = 0, Z = 6$

B. $x_1 = 2, x_2 = 6, Z = 36$

C. $x_1 = 4, x_2 = 3, Z = 27$

D. $x_1 = 4, x_2 = 6, Z = 12$

Answer

Given objective function is $Z = 3x_1 + 5x_2$

Subject to constraints:

$3x_1 + 2x_2 \leq 18$

$x_1 \leq 4$

$x_2 \leq 6$

$x_1 \geq 0, x_2 \geq 0$

To solve this,

Consider the constraints as equations for a while, then we will have

$3x_1 + 2x_2 = 18$

$x_1 = 4$

$x_2 = 6$

$x_1 = 0, x_2 = 0$

Consider the $3x_1 + 2x_2 = 18$, for graphing this, we can find the intercepts of this equation and then plot the line.

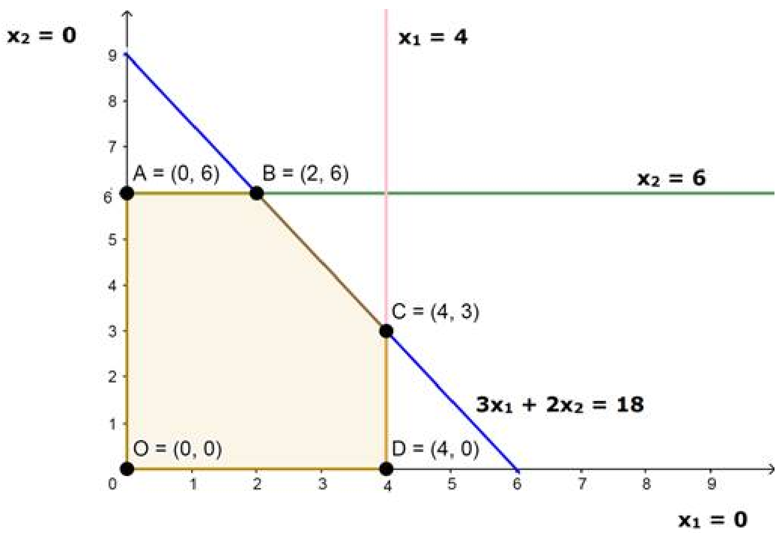
By dividing the whole equation with 18 we get,

$$\frac{3x_1}{18} + \frac{2x_3}{18} = \frac{18}{18}$$

$$\frac{x_1}{6} + \frac{x_3}{9} = 1$$

From this equation we know that, x_1 will intercept the line $x_1 = 0$ at (6,0) and x_2 will intercept the line $x_2 = 0$ at (0,9)

Using these points to plot $3x_1 + 2x_2 = 18$ in the graph, continue plotting the other equations, $x_1 = 4$, $x_2 = 6$, $x_1 = 0$, $x_2 = 0$ on he graph.



From the graph we can clearly see that the area under the polygon, OABCD is the feasible solution.

For determining the maximum optimum value from the polygon, the points at the vertices of OABCD are substituted in the Z function and see which point will maximize the value of $Z = 3x_1 + 5x_2$.

Z at O ($x_1 = 0, x_2 = 0$)	$Z = 3(0) + 5(0) = 0 + 0 = 0$
Z at A ($x_1 = 0, x_2 = 6$)	$Z = 3(0) + 5(6) = 0 + 30 = 30$
Z at B ($x_1 = 2, x_2 = 6$)	$Z = 3(2) + 5(6) = 6 + 30 = 36$
Z at C ($x_1 = 4, x_2 = 3$)	$Z = 3(4) + 5(3) = 12 + 15 = 27$
Z at D ($x_1 = 4, x_2 = 0$)	$Z = 3(4) + 5(0) = 12 + 0 = 12$

Among the above points, the Z value maximized at point B where $x_1 = 2$ and $x_2 = 6$.

Therefore the solution is, option B.

14. Question

The region represented by the inequation system x, y ≥ 0, y ≤ 6, x + y ≤ 3 is

A. unbounded in first quadrant

B. unbounded in first and second quadrants

C. bounded in first quadrant

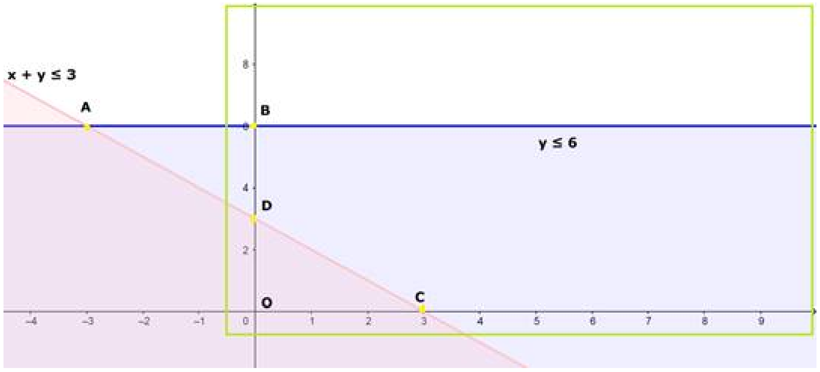
D. none of these

Answer

Given inequations are

x, y ≥ 0, y ≤ 6, x + y ≤ 3

If we graph the inequalities y ≤ 6 and x + y ≤ 3on the graph, we get the graph,



Here we can clearly see that, the region for the inequality is above the points A, B, C and D. But considering the constraints x , y ≥ 0, this clearly shows that the region can only be in the 1st quadrant.

So from the green color highlighted area, we can say that, the region of the inequalities is unbound in the first quadrant.

Hence, the answer is option A.

15. Question

The point at which the maximum value of x + y, subject to the constraints x + 2y ≤ 70, 2x + y ≤ 95, x, y ≥ 0 is obtained, is

A. (30, 25)

B. (20, 35)

C. (35, 20)

D. (40, 15)

Answer

Given objective function is Z = x + y

Constraints are:

x + 2y ≤ 70

2x + y ≤ 95

x, y ≥ 0

Let us consider these constraints as equations for a while, then we will have,

x + 2y = 70 ----- (1)

2x + y = 95 ----- (2)

Now, graph the equations, by transforming the equations to intercept form of line.

Equation (1) dividing throughout by 70

$$\frac{x}{70} + \frac{2y}{70} = \frac{70}{70}$$

$$\frac{x}{70} + \frac{y}{35} = 1$$

The line x + 2y = 70 can be plot in the graph as a line passing through the points, (70,0) and (0,35) as 70 and 35 are the intercepts of the line on the x-axis and y-axis respectively.

Similarly equation (2) can be divided 95 to get

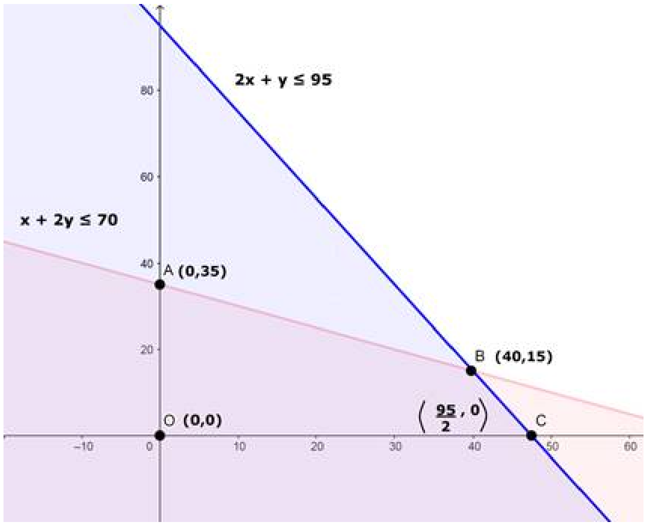
$$\frac{2x}{95} + \frac{y}{95} = \frac{95}{95}$$

$$\frac{x}{\frac{95}{2}} + \frac{y}{95} = 1$$

The line 2x + y = 95 can be plot in the graph as a line passing through the points, ($\frac{95}{2}$,0) and (0,95) as $\frac{95}{2}$ and 95 are the intercepts of the line on the x-axis and y-axis respectively.

By considering the constraints x , y ≥ 0, this clearly shows that the region can only be in the 1^{st} quadrant.

The graph of the inequations will look like,



The points OABC is the feasible region of the LPP.

Now from the points O, A ,B and C the vertices of the polygon formed by the constraints, one of the points will provide the maximum solution to the function Z = x + y

Now checking the points, O,A,B and C by substituting in Z = x + y.

Z at O (0,0)	Z = 0 + 0 = 0
Z at A (0,35)	Z = 0 + 35 = 35
Z at B (40,15)	Z = 40 + 15 = 55
Z at C ($\frac{95}{2}$, 0)	Z = $\frac{95}{2}$ + 0 = $\frac{95}{2}$ = 47.5

From the above values, it is clear that Z maximized at point B(40,15).

Hence the answer is option D.

16. Question

The value of objective function is maximum under linear constraints

A. at the centre of feasible region

B. at (0, 0)

C. at any vertex of feasible region

D. the vertex which is maximum distance from (0, 0)

Answer

Given that,

The objective function will be maximum at ______

As per the steps while solving a Linear Programming Problem, we first determine the feasible region using the constraints and then, consider the vertices of the obtained convex polygon in the objective function to see the vertex at which the objective function obtains a maximum value. Then that vertex is determines the maximum value of the objective function.

Hence, the maximum value of an objective function under linear constraints is at any vertex of the feasible region.

Therefore the answer is C.

17. Question

The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5) is

A. p = q B. p = 2q

C. p = 3q D. q = 3p

Answer

Given the vertices of the feasible region are:

O (0,0)

A (5,0)

B (3,4)

C (0,5)

Also given the objective function is Z = px+qy

Now substituting O,A,B and C in Z

Z at O(0,0)	$Z = p(0) + q(0) = 0$
Z at A(5,0)	$Z = p(5) + q(0) = 5p + 0 = 5p$
Z at B(3,4)	$Z = p(3) + q(4) = 3p + 4q$
Z at C(0,5)	$Z = p(0) + q(5) = 0 + 5q$

As per the condition on p and q so that the maximum of Z occurs at both (3, 4) and (0, 5)

Then we can equate Z values at B and C, this gives

$3p + 4q = 5q$

$3p = 5q – 4q$

$3p = q$

Therefore the answer is option D i.e. q = 3p.