Chapter : 9. QUADRILATERALS AND PARALLELOGRAMS

Exercise : 9A

Question: 1

Three angle

Solution:

Let the measure of the fourth angle be xo.Since the sum of the angles of a quadrilateral is 360°, we have:∴ 56° + 115° + 84° + x° = 360° ∴ 255° + x° = 360° ∴ x° = 105°Hence, the measure of the fourth angle is 105°.

Question: 2

The angles of a q

Solution:

Our given ratio of angles is 2:4:5:7. Let common multiplying factor be x°.

Hence, ∠A = 2x°, ∠B = 4x°, ∠C = 5x° and ∠D = 7x°Since the sum of the angles of a quadrilateral is 360°, we have:∴ 2x + 4x + 5x + 7x = 360°

∴ 18 x = 360° ∴ x = 20°∴ ∠A = 40°; ∠B = 80°; ∠C = 100°; ∠D = 140°

Hence, the measure of the angles are 40°, 80°, 100° and 140°

Question: 3

In the adjoining

Solution:

Here given that ABCD is trapezium where AB || DC.

We observe that ∠ A and ∠ D are the interior angles on the same side of transversal line AD, whereas ∠ B and ∠ C are the interior angles on the same side of transversal line BC.

As ∠A and ∠D are interior angles, we have,

∠A + ∠D = 180°∴ ∠D = 180° − ∠A ∴ ∠ D = 180° − 55° = 125°Similarly for ∠ B and ∠C,

∠ B + ∠C = 180°∴ ∠C = 180°− ∠B ∴ ∠ C = 180° − 70° = 110°Hence, measure of ∠ D and ∠ C are 125°and 110° respectively.

Question: 4

In the adjo

Solution:

(i) Here it is given that in ABCD is a square and ΔEDC is an equilateral triangle.

Hence, we say that AB = BC = CD = DA and ED = EC = DC

Now in ΔADE and ΔBCE, we have,AD = BC … given

DE = EC … given∠ADE = ∠BCE … as both angles are sum of 60° and 90°

∴ $\Delta ADE \cong \Delta BCE$

Now by cpct,

AE = BE …(1)

(ii) Here ∠ADE = 90°+ 60° = 150°

DA = DC … givenDC = DE … given

∴ DA = DE

This means that sides of square and triangles are equal.

∴ ΔADE and ΔBCE are isosceles triangles.

Hence, ∠DAE = ∠DEA = $\frac{1}{2}$(180° − 150°) = 30°/2 =15°

Question: 5

In the adjo

Solution:

Given: In ABCD, in which BM ⊥ AC and DN ⊥ AC and BM = DN.

To prove: AC bisects BD ie. DO = BO

Proof:

Now, in ΔOND and ΔOMB, we have,∠OND = ∠OMB …90° each∠DON = ∠ BOM …Vertically opposite anglesAlso, DN = BM …GivenHence, by AAS congruence rule,

ΔOND ≅ ΔOMB.∴ OD = OB …CPCTHence, AC bisects BD.

Question: 6

In the give

Solution:

Given: In ABCD, $AB = AD$ and $BC = DC$.

To prove: (i) AC bisects $\angle A$ and $\angle C$,

(ii) $BE = DE$,

(iii) $\angle ABC = \angle ADC$.

Proof:

(i) In ΔABC and ΔADC, we have, AB = AD …given

BC = DC …givenAC = AC … common sideHence, by SSS congruence rule,

ΔABC ≅ ΔADC

∴ ∠BAC = ∠DAC and ∠BCA = ∠DCA …By cpctThus, AC bisects ∠A and ∠ C.(ii) Now, in ΔABE and ΔADE, we have,

AB = AD …given∠BAE = ∠DAE …from iAE = AE …common sideHence, by SAS congruence rule,

ΔABE ≅ ΔADE.∴ BE = DE …by cpct(iii) ΔABC ≅ ΔADC from ii

∴ ∠ABC = ∠ADC …by cpct

Question: 7

In the give

Solution:

Given: ABCD is where $\angle PQR = 90^\circ$. and $PB = QC = DR$,

To prove: (i) $QB = RC$, (ii) $PQ = QR$,

(iii) $\angle QPR = 45^\circ$.

Proof:

(i) Here,

BC = CD …Sides of square

CQ = DR …Given

BC = BQ + CQ

∴ CQ = BC − BQ

∴ DR = BC – BQ …(1)

Also,

CD = RC+ DR

∴ DR = CD − RC = BC − RC ...(2)

From (1) and (2), we have,

BC − BQ = BC − RC

∴ BQ = RC

(ii) Now in ΔRCQ and ΔQBP, we have,

PB = QC ...Given

BQ = RC ...from (i)

∠RCQ = ∠QBP ...90° each

Hence by SAS congruence rule,

ΔRCQ ≅ ΔQBP

∴ QR = PQ ...by cpct

(iii) ΔRCQ ≅ ΔQBP and QR = PQ ... from (ii)

∴ In ΔRPQ,

∠QPR = ∠QRP = $\frac{1}{2}$ (180° − 90°) = $\frac{90°}{2}$ = 45°

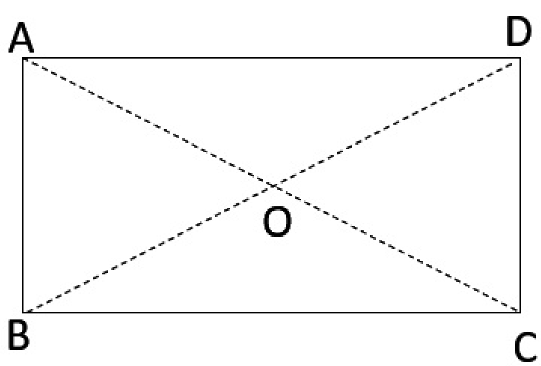
∴ ∠QPR = 45°

Question: 8

If is a poi

Solution:

Given: In *ABCD*, *O* is any point within the quadrilateral. To prove: $OA + OB + OC + OD > AC + BD$. Proof:



We know that the sum of any two sides of a triangle is greater than the third side.So, in Δ*AOC*,

OA + *OC* > *AC* ...(1)

Also, in Δ *BOD*,

OB + *OD* > *BD* ...(2)

Adding 1 and 2, we get,

(*OA* + *OC*) + (*OB* + *OD*) > (*AC* + *BD*)∴ *OA* + *OB* + *OC* + *OD* > *AC* + *BD*

Hence proved.

Question: 9

In the adjo

Solution:

Given: In ABCD, AC is one of diagonals.

To prove:

(i) $AB + BC + CD + DA > 2AC$

(ii) $AB + BC + CD > DA$

(iii) $AB + BC + CD + DA > AC + BD$

Proof:

(i) We know that the sum of any two sides of a triangle is greater than the third side.In ΔABC,

$AB + BC > AC$ ...(1)

In ΔACD,

$CD + DA > AC$ ...(2)

Adding (1) and (2), we get,

$AB + BC + CD + DA > 2AC$

(ii) In ΔABC, we have,$AB + BC > AC$ ...(1)We also know that the length of each side of a triangle is greater than the positive difference of the length of the other two sides.In ΔACD, we have:$AC > DA - CD$ ...(2)From (1) and (2), we have,$AB + BC > DA - CD$$\therefore AB + BC + CD > DA$

(ii) In ΔABC,

$AB + BC > AC$ ...(1)

In ΔACD,

$CD + DA > AC$ ...(2)

In Δ BCD,

$BC + CD > BD$ ...(3)

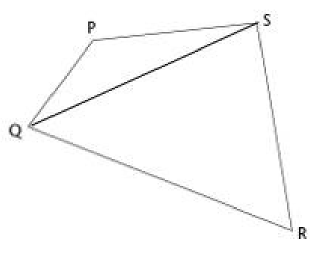
In Δ ABD,

$DA + AB > BD$ ...(4)Adding 1, 2, 3 and 4, we get,$2(AB + BC + CD + DA) > 2(AC + BD)$$\therefore AB + BC + CD + DA > AC + BD$

Question: 10

Prove that

Solution:



Given: Consider a PQRS where QS is diagonal.

To prove: ∠P + ∠Q + ∠R + ∠S = 360°

Proof:

For ΔPQS, we have,

∠P + ∠PQS + ∠PSQ = 180° ... (1) ...Using Angle sum property of Triangle

Similarly, in ΔQRS, we have,

∴ ∠SQR + ∠R + ∠QSR = 180° ... (2) ...Using Angle sum property of Triangle

On adding (1) and (2), we get

$\angle P + \angle PQS + \angle PSQ + \angle SQR + \angle R + \angle QSR = 180° + 180°$

$\therefore \angle P + \angle PQS + \angle SQR + \angle R + \angle QSR + \angle PSQ = 360°$

$\therefore \angle P + \angle Q + \angle R + \angle S = 360°$

$\therefore$ The sum of all the angles of a quadrilateral is 360°.

Exercise : 9B

Question: 1

In the adjo

Solution:

In ABCD, $\angle A = 72°$

We know that opposite angles of a parallelogram are equal.Hence, $\angle A = \angle C$ and $\angle B = \angle D$ $\therefore \angle C = 72°$$\angle A$ and $\angle B$ are adjacent angles.$\therefore \angle A + \angle B = 180°$$\angle B = 180°° - \angle A$ $\angle B = 180° - 72° = 108°$$\therefore \angle B = \angle D = 108°$Hence, $\angle B = \angle D = 108°$ and $\angle C = 72°$

Question: 2

In the adjo

Solution:

It is given that ABCD is parallelogram and $\angle DAB = 80°$ and $\angle DBC = 60°$We need to find measure of $\angle CDB$ and $\angle ADB$In $ABCD, AD \parallel BC, BD$ *as transversal,*$\angle DBC = \angle ADB = 60°$ …Alternate interior angles ...(i)As $\angle DAB$ and $\angle ADC$ are adjacent angles,

$\angle DAB + \angle ADC = 180°$$\therefore \angle ADC = 180°° - \angle DAB$$\angle ADC = 180° - 80° = 100°$Also,

$\angle ADC = \angle ADB + \angle CDB$$\therefore \angle ADC = 100°$ $\angle ADB + \angle CDB = 100°$ ...(ii)From (i) and (ii), we get:$60° + \angle CDB = 100° \Rightarrow \angle CDB = 100° - 60° = 40°$Hence, $\angle CDB = 40°$ and $\angle ADB = 60°$

Question: 3

In the adjo

Solution:

Given: ABCD is a parallelogram. The bisectors of $\angle A$ and $\angle B$ meet DC at P, .To prove: (i) $\angle APB = 90°$, (ii) $AD = DP$ and $PB = PC = BC$, (iii) $DC = 2AD$.

Proof:

$\therefore \angle A = \angle C$ and $\angle B = \angle D$ ... Opposite anglesAnd $\angle A + \angle B = 180°$ … Adjacent angles$\therefore \angle B = 180° - \angle A$ $180° - 60° = 120°$ … as $\angle A = 60°$$\therefore \angle A = \angle C = 60°$ and $\angle B = \angle D = 120°$(i) In Δ *APB,*

$\angle PAB = \frac{60°}{2} = 30°$and $\angle PBA = \frac{120°}{2} = 60°$$\therefore \angle APB = 180° - (30° + 60°) = 90°$(ii) In Δ *ADP,* $\angle PAD =$

$30°$ and $\angle ADP = 120°$$\therefore \angle APB = 180° - (30° + 120°) = 30°$

Hence,

$\angle PAD = \angle APB = 30°$Hence, ΔADP is an isosceles triangle and $AD = DP$.In Δ *PBC,*

$\angle$ PBC = 60°

$\angle BPC = 180° - (90° + 30°) = 60°and\angle$ BCP = 60° …Opposite angle of $\angle A$.$\therefore \angle PBC = \angle BPC = \angle$ BCPHence, ΔPBC is an equilateral triangle and, therefore, $PB = PC = BC$.(iii) $DC = DP + PC$From (ii), we have

$DC = AD + BC$ …$AD = BC$ $DC = AD + AD$

$DC = 2\ AD$

Question: 4

In the adjo

Solution:

In ABCD, $\angle BAO = 35^\circ$, $\angle DAO = 40^\circ$ and $\angle COD = 105^\circ$.

(i) In ΔAOB,

$\angle BAO = 35°$

$\angle AOB = \angle COD = 105°$ …Vertically opposite angels $\therefore$ $\angle ABO = 180° - (35° + 105°) = 40°$ … Using Angle sum property of Triangle(ii) $\angle ODC$ and $\angle ABO$ are alternate angles for transversal BD.∴ $\angle ODC = \angle ABO = 40°$(iii) $\angle ACB = \angle$CAD = 40°° …Alternate angles for transversal AC(iv) $\angle CBD = \angle ABC - \angle ABD$ …(1)

$\angle ABC = 180° - \angle BAD$ …Adjacent angles are supplementary

$\angle ABC = 180° - 75° = 105°\angle CBD = 105° - \angle ABD$ … *as* $\angle ABD = \angle ABO \angle CBD = 105° - 40° = 65°$

Question: 5

In a||gm

~~**Solution:**~~

~~It is given that in ABCD, $\angle A = (2x + 25)^\circ$ and $\angle B = (3x - 5)^\circ$, We know that opposite angles of parallelogram are equal.~~

~~$\therefore \angle A = \angle C$ and $\angle B = \angle$DAlso,~~

~~$\angle A + \angle$B = 180° …Adjacent angles of parallelogram are supplementary.∴ $(2x + 25)° + (3x - 5)° = 180° 5x° + 20° = 180° 5x° = 160° x° = 32° \therefore \angle A = 2 \times 32 + 25 = 89°$~~

~~$\therefore \angle B = 3 \times 32 - 5 = 91°$Hence, $x = 32°$, $\angle A = \angle C = 89°$ and $\angle B = \angle$D = 91°~~

~~**Question: 6**~~

~~If an angle~~

~~**Solution:**~~

~~Let ABCD be the parallelogram.~~

~~We know that opposite angles of parallelogram are equal.~~

~~$\therefore \angle A = \angle C$ and $\angle B = \angle$DBy given conditions,~~

~~Let $\angle A = x°$ *and* $\angle$B = $\frac{4x^\circ}{5}$~~

~~Also, adjacent angles of parallelogram are supplementary,~~

~~$\therefore x° + \frac{4x^\circ}{5} = 180°$~~

~~$\frac{9x^\circ}{5} = 180°$~~

~~∴ x = 100°~~

~~Hence, $\angle A$ = 100° and $\angle$B = $\frac{4 \times 100^\circ}{5}$ = 80°~~

~~Hence, $\angle A = \angle C = 100°$; $\angle B = \angle D = 80°$~~

~~**Question: 7**~~

~~Find the me~~

~~**Solution:**~~

~~Let ABCD be the parallelogram.~~

~~We know that opposite angles of parallelogram are equal.~~

~~$\therefore \angle A = \angle C$ and $\angle B = \angle$DLet $\angle A$ be the smallest angle whose measure is $x°$.∴ $\angle B = (2x - 30)°$We~~

know that adjacent angles of parallelogram are supplementary,

$\angle A + \angle B = 180°$ $x + 2x - 30° = 180°$ $3x = 210°$ $x = 70°$ $\therefore \angle B = 2 \times 70° - 30° = 110°$ Hence, $\angle A = \angle C = 70°$ and $\angle B = \angle D = 110°$

Question: 8

Solution:

Here ABCD is parallelogram.

We know that the opposite sides of a parallelogram are parallel and equal.

Hence, $AB = DC = 9.5$ cm

Also let $BC = AD = x\ cm$

Now,

Perimeter of $ABCD = 30$ cm ...(given)

$\therefore AB + BC + CD + DA = 30$ cm

$\therefore 9.5 + x + 9.5 + x = 30$

$\therefore 19 + 2x = 30 \therefore 2x = 11 \therefore x = 5.5$ cm

Hence, length of each side is AB = DC = 9.5 cm and $BC = DA = 5.5$ cm

Question: 9

In each of

Solution:

(i) ABCD is a rhombus.

We know that rhombus is type of parallelogram whose all sides are equal.

In ΔABC, $\angle BAC = \angle BCA = \frac{1}{2}(180° - 110°) = 35°$

Hence x = 35°

But AB || DC ...opposite sides of rhombus are parallel

$\angle BAC = \angle$DCA ...for transversal AC

$\therefore \angle BAC = \angle$DCA = 35°

Hence, x = y = 35°

(ii) ABCD is a rhombus.

We know that the diagonals of a rhombus are perpendicular bisectors of each other.

$\therefore$ in ΔAOB,

$\angle OAB = 40°$, $\angle AOB = 90°$

$\therefore \angle ABO = 180° - (40° + 90°) = 50°$

Hence x = 50°

Now in ΔDAB,

AB = AD ... as rhombus has all sides equal.

ie. Δ*AOB is isosceles triangle.*

Also base angles of *isosceles triangle are equal.*

Hence, $x = y = 50°$

(iii) ABCD is a rhombus.

We know that rhombus is type of parallelogram whose all sides are equal.

So in ΔDCB,

DC = BC

$\therefore \angle CDB = \angle CBD = y°$ base angles of *isosceles triangle are equal.*

Now, x = $\angle CAB$ ...alternate angles with transversal AC

$\therefore x = \frac{1}{2} \angle BAD$

$\therefore x = \frac{1}{2} \times 62°$

$\therefore x = 31°$

In ΔDOC,

We know sum of angles of triangle is 180°

$\angle CDO + \angle DOC + \angle OCD = 180°$

$\therefore \angle CDO + 90° + 31° = 180°$

$\therefore \angle CDO = 59°$

$\therefore y = 59°$

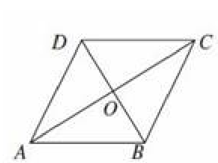
Hence, $x = 31°$ *and* $y = 59°$

Question: 10

The lengths

Solution:

Let ABCD be rhombus.



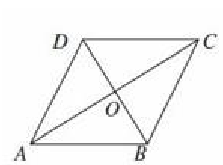
Here, *AC* and *BD* are the diagonals of *ABCD,* where *AC* = 24 cm and *BD* = 18 cm.Let the diagonals intersect each other at O.We know that the diagonals of a rhombus are perpendicular bisectors of each other.$\therefore$ Δ*AOB* is a right angle triangle in which $OA = \frac{24}{2} = 12$ cm and $OB = \frac{18}{2} = 9$ cm.Now, $AB^2 = OA^2 + OB^2$ ...Pythagoras theorem.$\therefore AB^2 = (12)^2 + (9)^2 \therefore AB^2 = 144 + 81 = 225 \therefore AB = 15$ cmHence, the side of the rhombus is 15 cm

Question: 11

Each side o

Solution:

Let ABCD be rhombus.



We know that rhombus is type of parallelogram whose all sides are equal.

$\therefore AB = BC = CD = DA = 10$ cm

Let the diagonals AC and BD intersect each other at O, where AC = 16 cm and let BD = xWe know that the diagonals of a rhombus are perpendicular bisectors of each other.∴ ΔAOB is a right angle triangle, in which $OB = BD \div 2 = x \div 2$ and $OA = AC \div 2 = 16 \div 2 = 8$ cm.Now, $AB = OA^2 + OB^2$...by pythagoras theorem∴ $10^2 = (\frac{x}{2})^2 + 8^2$

ie. $100 - 64 = \frac{x^2}{4}$

$36 \times 4 = x^2 \therefore x^2 = 144 \therefore x = 12$ cm

Hence, the length of the other diagonal is 12 cm

We know that area of rhombus is,

Area of rhombus = $\frac{1}{2} \times$ (Diagonal1) × (Diagonal2)

Hence,

Area of ABCD = $\frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 16 \times 12$

$= 96$ cm^2

Hence, the area of rhombus is 96 cm^2

Question: 12

In each of

Solution:

(i) Here, ABCD is rectangle.

We know that the diagonals of a rectangle are congruent and bisect each other.

∴ In Δ AOB, we have $OA = OB$

This means that Δ AOB is isosceles triangle.

We know that base angles of isosceles triangle are equal.

$\therefore \angle OAB = \angle OBA = 35°$

$\therefore \because x = 90° - 35° = 55°$

Also, $\angle AOB = 180° - (35° + 35°) = 110°$

$\therefore y = \angle AOB = 110°$ ...Vertically opposite angles

Hence, $x = 55°$ and $y = 110°$

(ii) Here, ABCD is rectangle.

We know that the diagonals of a rectangle are congruent and bisect each other.

∴ In Δ AOB, we have $OA = OB$

This means that Δ AOB is isosceles triangle.

We know that base angles of isosceles triangle are equal.

$\therefore \angle OAB = \angle OBA = \frac{1}{2} \times (180° - 110°) = 35°$

$\therefore y = \angle BAC = 35°$ ... alternate angles with transversal ACAlso, $x = 90° - y$ ... $\because \angle C = 90° = x + y$ $\therefore x = 90° - 35° = 55°$ Hence, $x = 55°$ and $y = 35°$

Question: 13

In the adjo

Solution:

Here, ABCD is square.

Here AC and BD are diagonals.

We know that the angles of a square are bisected by the diagonals.

$\therefore\ \angle OBX = 45°$ $\because \angle ABC = 90°$ and BD bisects $\angle ABC$And $\angle BOX = \angle COD = 80°$ … Vertically opposite angles.$\because$ In ΔBOX, we have:$\angle AXO = \angle OBX + \angle BOX$ … Exterior angle theorem$\Rightarrow \angle AXO = 45° + 80° = 125°$$\therefore\ x = 125°$

Question: 14

In the adjo

Solution:

Here, ABCD is parallelogram.

Hence, AD || BC and AD = BC

(i) In ΔALD and ΔCMB, we have,AD = BC

$\angle ALD = \angle$CMB (90° each)

$\angle ADL = \angle$CBM (Alternate interior angle).$\therefore\ \Delta ALD \cong \Delta CMB$

(ii) As $\Delta ALD \cong \Delta CMB$ …from 1$\therefore AL = CM$ …by cpct

Question: 15

In the adjo

Solution:

ABCD is parallelogram.

We know that the sum of the adjacent angles in parallelogram is 180°

$\therefore \angle A + \angle B = 180°$

$\therefore \frac{\angle A}{2} + \frac{\angle B}{2} = \frac{180°}{2} = 90°$

In ΔAPB, we have:$\angle PAB = \angle A/2$$\angle PBA = \angle B/2$$\therefore\ \angle$APB = 180 − ($\angle PAB + \angle PBA$) …Angle sum property of triangle$\therefore\ \angle APB = 180 - (\frac{\angle A}{2} + \frac{\angle B}{2})$$\therefore\ \angle APB = 180 - 90 = 90°$Hence, proved.

Question: 16

In the adjo

Solution:

ABCD is parallelogram

We know that opposite sides and angles of parallelogram *are equal.*

$\therefore \angle B = \angle D$ *and* $AD = BC$ and $AB = DC$

Also, $AD \parallel BC$ and $AB \parallel DC$

It is given that $AP = \frac{1}{3} AD$ and $CQ = \frac{1}{3} BC$,

Hence, $AP = CQ$ … $\because AD = BC$

In ΔDPC and ΔBQA, we have,

$AB = CD$

$\angle B = \angle D$

~~$DP = QB$ ... as $AP = \frac{1}{3}AD$ and $CQ = \frac{1}{3}BC$,~~

~~*Hence, by SAS test for congruency,*~~

~~$\Delta DPC \cong \Delta BQA$~~

~~$\therefore PC = QA$ ... *by cpct*~~

~~Hence, from above, in AQCP, we have,~~

~~AP = CQ and PC = QA~~

~~$\therefore$ AQCP is a parallelogram.~~

~~**Question: 17**~~

~~In the adjo~~

~~**Solution:**~~

~~ABCD is parallelogram.~~

~~$\therefore$ in ΔODF and ΔOBE, we have:~~

~~$OD = OB$ ... Diagonals bisects each other $\angle DOF = \angle BOE$ ... Vertically opposite angles $\angle FDO = \angle OBE$ ... Alternate interior angles~~

~~Hence, by SAA *test for congruency,* $\Delta ODF \cong \Delta OBE \therefore OF = OE$ ...by cpctHence, proved.~~

~~**Question: 18**~~

~~In the adjo~~

~~**Solution:**~~

~~ABCD is parallelogram.~~

~~In ΔODC and ΔOEB, we have, $DC = BE$ ...as $DC = AB$ $\angle COD = \angle BOE$ ... Vertically opposite angles are equal $\angle OCD = \angle OBE$ ... Alternate angles with transversal BCHence, by SAA *test for congruency, we get,* $\Delta ODC \cong \Delta OEB$~~

~~$\therefore OC = OB$ ...*by cpct*We know that $BC = OC + OB$.$\therefore$ ED bisects BC.~~

~~**Question: 19**~~

~~In the adjo~~

~~**Solution:**~~

~~ABCD is parallelogram.~~

~~Also given that BE = CE~~

~~In ABCD, AB || DC~~

~~∠DCE = ∠EBF ... Alternate angles with transversal DF~~

~~In ΔDCE and ΔBFE, we have,∠DCE = ∠EBF ...from above~~

~~∠DEC = ∠BEF ... Vertically opposite anglesAlso, BE = CE ... givenHence, by ASA congruence rule,~~

~~ΔDCE ≅ ΔBFE ∴ DC = BF ... by cpct~~

~~But DC = AB, as ABCD is a parallelogram.∴ DC = AB = BFNow, AF = AB + BFFrom above, we get,AF = AB + AB = 2ABHence, proved.~~

~~**Question: 20**~~

~~A~~

~~**Solution:**~~

~~Here given that *BC* || *QA* and *CA* || *QB which means that BCQA* is a parallelogram.~~

~~∴ BC = QA ...(1)~~

Similarly, $BC \parallel AR$ and $AB \parallel CR$, *which means* $BCRA$ is a parallelogram.

$\therefore BC = AR \ldots(2)$

But $QR = QA + AR$From (1) and (2), we get,$QR = BC + BC$

$\therefore QR = 2BC$

Hence, BC = $\frac{1}{2}$ QR

Question: 21

In the adjo

Solution:

Here, Perimeter of $\Delta ABC = AB + BC + CA$

And Perimeter of $\Delta PQR = PQ + QR + PR$

Given that $BC \parallel QA$ and $CA \parallel QB$ *which means* $BCQA$ is a parallelogram.

$\therefore$ BC = QA ...(1)

Similarly, $BC \parallel AR$ and $AB \parallel CR$, which means $BCRA$ is a parallelogram.$\therefore BC = AR \ldots(2)$

But, $QR = QA + AR$

From 1 and 2,

$QR = BC + BC$

$\therefore QR = 2BC$

$\therefore$ BC = $\frac{1}{2}$ QR

Similarly, $CA = \frac{1}{2}$ PQ and AB = $\frac{1}{2}$ PR

Now,

Perimeter of $\Delta ABC = AB + BC + CA$

$= \frac{1}{2}\text{QR} + \frac{1}{2}\text{PQ} + \frac{1}{2}\text{PR}$

$= \frac{1}{2}(\text{PR} + \text{QR} + \text{PQ})$

This states that,

Perimeter of $\Delta ABC = \frac{1}{2}$ (Perimeter of ΔPQR)

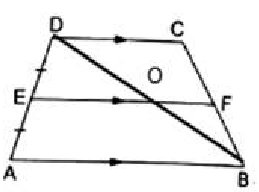
$\therefore$ Perimeter of ΔPQR = 2 × Perimeter of ΔABC

Exercise : 9C

Question: 1

In the adjo

Solution:



Here, ABCD is trapezium.

Join BD to cut EF at O.

It is given that, in ΔDAB, E is the mid point of AD and $EO \parallel AB$.

$\therefore O$ is the midpoint of BD ...By converse of mid point theorem

Now in ΔBDC, O is the mid point of BD and O$F \parallel DC$.$\therefore F$ is the midpoint of BC ... By converse of mid point theorem

Question: 2

In the adjo

Solution:

Here, ABCD is parallelogram.

By the properties of parallelogram,

$AD \parallel BC$ and $AB \parallel DC$

$AD = BC$ and $AB = DC$

Also,

$AB = AE + BE$ and $DC = DF + FC$

This means that,

$AE = BE = DF = FC$

Now, $DF = AE$ and DF || AE, that is $AEFD$ is a parallelogram.

Hence, $AD \parallel EF$

Similarly, $BEFC$ is also a parallelogram.

Hence, $EF \parallel BC \therefore AD \parallel EF \parallel BC$

Thus, AD, EF and BC are three parallel lines cut by the transversal line DC at D, F and C, respectively such that DF = FC.

Also, the lines AD, EF and BC are also cut by the transversal AB at A, E and B, respectively such that $AE = BE$. Similarly, they are also cut by GH.

Hence by intercept theorem,$\therefore$ GP = PH

Hence proved.

Question: 3

In the adjo

Solution:

Here, ABCD is trapezium.

Hence, $AB \parallel DC$

Also given that $AP = PD$ and $BQ = CQ$

(i) In ΔQCD and ΔQBE, we have,$\angle DQC = \angle BQE$ ...Vertically opposite angles

$\angle DCQ = \angle EBQ$ ...Alternate angles with transversal BC$BQ = CQ$ ... P is the midpoint

Hence, by AAS test of congruency,$\Delta QCD \cong \Delta QBE$Hence, $DQ = QE$ ...by cpct

(ii) Also, in ΔADE, P and Q are the midpoints of AD and DE respectively

$\therefore PQ \parallel AE$

Hence, $PQ \parallel AB \parallel DC$

ie. $AB \parallel PR \parallel DC$

(iii) PQ, AB and DC are cut by transversal AD at P such that $AP = PD$.Also they are cut by transversal BC at Q such that $BQ = QC$. Similarly, lines PQ, AB and DC are also cut by AC at R.

Hence, by intercept theorem,$\therefore AR = RC$

Question: 4

In the adjo

Solution:

In ΔABC, *AD is median.*

$\therefore BD = DC$

We know that the line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.

So, in ΔABC, D is the mid point of BC and $DE \parallel BA$.

Hence, DE bisects *AC.*

$\therefore AE = EC$

This means that E is the midpoint of AC.

$\therefore$ *BE is median of* ΔABC.

Question: 5

In the adjo

Solution:

Here in ΔABC AD and BE are medians.

Hence, in ΔABC, we have: $AC = AE + EC$

But $AE = EC$ *… as E is midpoint of AC*

$\therefore AC = 2EC$ …(1)

Now in ΔBEC,

$DF \parallel BE$

Also, $EF = CF$ *… by* midpoint theorem, as D is the midpoint of BC

But,

$EC = EF + CF$

$\therefore EC = 2\ CF$ …(2)

From 1 and 2, we get,

$AC = 4\ CF$

$\therefore CF = \frac{1}{4} AC.$

Question: 6

In the adjoining

Solution:

ABCD is parallelogram.

(i) In $\Delta\ DCG$, we have:

$DG \parallel EB$ $DE = EC$ … E is the midpoint of DC)Also, $GB = BC$ … by midpoint theorem $\because B$ is the midpoint of GC.Also, $GC = GB + BC$ $GC = 2BC$

$GC = 2\ AD$ …as $AD = BC$

$\therefore AD = \frac{1}{2} GC$

(ii) Now, in $\Delta\ DCG$, $DG \parallel EB$ and E is the midpoint of DC and B is the midpoint of GC.

$\therefore$ EB = $\frac{1}{2}$ DG … by midpoint theorem

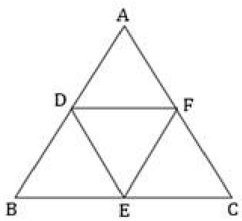
$\therefore$ DG = 2 EB

Question: 7

Prove that

Solution:

Let triangle be ΔABC. D, E and F are the midpoints of sides AB, BC and CA, respectively.



By midpoint theorem, for D and E as midpoints of sides AB and BC,

$DE \parallel AC$

Similarly, $DF \parallel BC$ and $EF \parallel AB$.

$\therefore$ *ADEF*, *BDFE* and *DFCE* are all parallelograms.

But, DE is the diagonal of the *BDFE*.

$\therefore \Delta BDE \cong \Delta FED$ ...(1)

Similarly, DF is the diagonal of the parallelogram ADEF.

$\therefore \Delta DAF \cong \Delta FED$ ...(2)And, EF is the diagonal of the parallelogram DFCE.

$\therefore \Delta EFC \cong \Delta FED$ ...(3)

Hence, all the four triangles are congruent.

Question: 8

In the adjo

Solution:

Here, in $\Delta ABC.$, D, E, F are the midpoints of the sides BC, CA and AB respectively.

By mid point theorem, as F and E are the mid points of sides AB and AC,

$FE \parallel BC$

Similarly, $DE \parallel FB$ and $FD \parallel AC$.

Therefore, *AFDE*, *BDEF* and *DCEF* are all parallelograms.

We know that opposite angles in parallelogram are equal.

$\therefore$ In AFDE, we have,

$\angle A = \angle EDF$

In *BDEF*, we have,

$\angle B = \angle DEF$

In DCEF, we have,

$\angle C = \angle DFE$

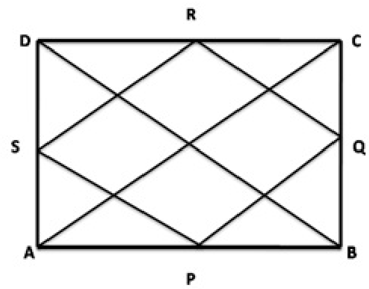
Hence proved.

Question: 9

Show that t

Solution:

Let *ABCD* be the rectangle and *P*, *Q*, *R* and *S* be the midpoints of *AB*, *BC*, *CD* and *DA*, respectively.



~~Join diagonals of the rectangle.~~

~~In Δ ABC, we have, by midpoint theorem,$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$~~

~~*Similarly,* $SR \parallel AC$ and $SR = \frac{1}{2} AC$.~~

~~*As,* $PQ \parallel AC$ and $SR \parallel AC$, *then also* $PQ \parallel SR$~~

~~*Also,* $PQ = SR$, *each equal to* $\frac{1}{2} AC$ *...(1)*~~

~~*So, PQRS is a parallelogram*~~

~~*Now, in* ΔSAP and ΔQBP, we have,~~

~~$AS = BQ$ $\angle A = \angle B = 90°$$AP = BP$~~

~~$\therefore$ *By SAS test of congruency,*~~

~~$\Delta SAP \cong \Delta QBP$~~

~~*Hence,* $PS = PQ$ *...by cpct ...(2)*~~

~~Similarly, $\Delta SDR \cong \Delta QCR$~~

~~$\therefore$ $SR = RQ$ *... by cpct ...(3)*~~

~~*Hence, from 1, 2 and 3 we have,*~~

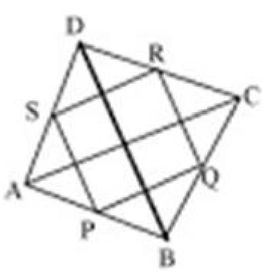
~~$PQ = PQ = SR = RQ$Hence, *PQRS* is a rhombus.~~

~~Hence, the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rectangle is a rhombus.~~

~~**Question: 10**~~

~~Show that t~~

~~**Solution:**~~



~~In ΔABC, P and Q are mid points of AB and BC respectively.$\therefore$ PQ|| AC and PQ = 1/2AC ... (1) ... Mid point theoremSimilarly in ΔACD, R and S are mid points of sides CD and AD respectively.$\therefore$ SR||AC and SR = 1/2AC ...(2) ...Mid point theoremFrom (1) and (2), we getPQ||SR and PQ = SRHence, PQRS is parallelogram (pair of opposite sides is parallel and equal)~~

~~Now, RS || AC and QR || BD.~~

~~Also, AC $\perp$ BD ... as diagonals of rhombus are perpendicular bisectors of each other.~~

~~$\therefore$RS $\perp$ QR.~~

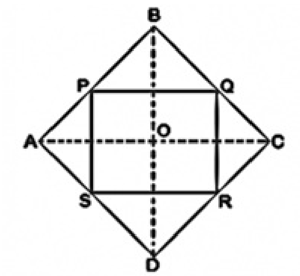
~~Thus, PQRS is a rectangle.~~

Hence, the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rhombus is a rectangle.

Question: 11

Show that t

Solution:



Let $ABCD$ be the square and P, Q, R and S be the midpoints of AB, BC, CD and DA, respectively.

Join diagonals of the square.

In $\Delta\ ABC$, we have, by midpoint theorem,$\therefore\ PQ \parallel AC$ and $PQ = \frac{1}{2}AC$

Similarly, $SR \parallel AC$ and $SR = \frac{1}{2}AC$.

As, $PQ \parallel AC$ and $SR \parallel AC$, *then also* $PQ \parallel SR$

Also, $PQ = SR$, *each equal to* $\frac{1}{2}AC$ *...(1)*

So, PQRS is a parallelogram

Now, in ΔSAP and ΔQBP, we have,

$AS = BQ\ \angle A = \angle B = 90° AP = BP$

$\therefore$ *By SAS test of congruency,*

$\Delta SAP \cong \Delta QBP$.

Hence, $PS = PQ$ *...by cpct ...(2)*

Similarly, $\Delta SDR \cong \Delta QCR$

$\therefore\ SR = RQ$ *... by cpct ...(3)*

Hence, from 1, 2 and 3 we have,

$PQ = PQ = SR = RQ$

We know that the diagonals of a square bisect each other at right angles.$\therefore\ \angle EOF = 90^{\circ}$Now, $RQ \parallel DB \Rightarrow RE \parallel FO$Also, $SR \parallel AC \Rightarrow FR \parallel OE$.$\therefore\ OERF$ is a parallelogram. So, $\angle FRE = \angle EOF = 90^{\circ}$. (Opposite angles are equal)Thus, $PQRS$ is a parallelogram with $\angle R = 90^{\circ}$ and $PQ = PS = SR = RQ$.

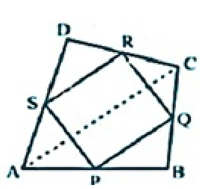
This means that PQRS is square.

Hence, the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a square is a square.

Question: 12

Prove that

Solution:



In ΔADC, S and R are the midpoints of AD and DC respectively.

By midpoint theorem,Hence SR || AC and SR = $\frac{1}{2}$ AC ... (1)Similarly, in ΔABC, P and Q are midpoints of AB and BC respectively.PQ || AC and PQ = $\frac{1}{2}$ AC ...(2) ...By midpoint theoremFrom equations (1) and (2), we getPQ || SR and PQ = SR ...(3)Here, one pair of opposite sides of quadrilateral PQRS is equal and parallel.Hence PQRS is a parallelogramHence the diagonals of parallelogram PQRS bisect each other.Thus PR and QS bisect each other.

Hence, the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other.

Question: 13

In the give

Solution:

Here, in ABCD, diagonals intersect at 90°

Also, in *ABCD*, *P*, *Q*, *R* and *S* be the midpoints of *AB*, *BC*, *CD* and *DA*, respectively.

In Δ *ABC*, we have,∴ *PQ* || *AC* and *PQ* = $\frac{1}{2}$ *AC* ...*by midpoint theorem*

Similarly, in ΔD*AC*,

SR || *AC* and *SR* = $\frac{1}{2}$ *AC* ...*by midpoint theorem*

Now, *PQ* || *AC* and *SR* || *AC*

∴ *PQ* || *SR*

Also, *PQ* = *SR* = $\frac{1}{2}$ *AC*

Hence, PQRS is parallelogram.

We know that the diagonals of the given quadrilateral bisect each other at right angles.∴ ∠*EOF* = 90°Also, *RQ* || *DB* ∴ *RE* || *FO*Also, *SR* || *AC*∴ *FR* || *OE*∴ *OERF* is a parallelogram.

So, ∠*FRE* = ∠*EOF* = 90° ...Opposite angles of parallelogram are equalThus, *PQRS* is a parallelogram with ∠*R* = 90°.∴ *PQRS* is a rectangle.

Exercise : CCE QUESTIONS

Question: 1

Three angles of a

Solution:

Let the fourth angle be x

$80^\circ + 95^\circ + 112^\circ + x^\circ = 360^\circ$ (Sum of angles of quadrilateral)

$287^\circ + x^\circ = 360^\circ$

$x = 360^\circ - 287^\circ$

$= 73^\circ$

Hence, option (B) is correct

Question: 2

Three angles of a

Solution:

Let the angles be 3x, 4x, 5x and 6x

$3x + 4x + 5x + 6x = 360^\circ$ (Sum of angles of a quadrilateral)

$18x = 360^\circ$

~~$x = \frac{360}{18}$~~

~~$x = 20^\circ$~~

~~∴ Angles of the quadrilateral are:~~

~~$3x = 3 \times 20^\circ = 60^\circ$~~

~~$4x = 4 \times 20^\circ = 80^\circ$~~

~~$5x = 5 \times 20^\circ = 100^\circ$~~

~~$6x = 6 \times 20^\circ = 120^\circ$~~

~~Hence, the smallest angle is 60°~~

~~∴ Option (B) is correct~~

~~**Question: 3**~~

~~In the given figu~~

~~**Solution:**~~

~~It is given in the question that,~~

~~In parallelogram ABCD: $\angle BAD = 75^\circ$, $\angle CBD = 60^\circ$~~

~~Now, $\angle DAB = \angle DCB = 75^\circ$ (Opposite angles)~~

~~Also, in triangle DBC we know that sum of angles of a triangle is 180°~~

~~$\angle DBC + \angle BDC + \angle DCB = 180^\circ$~~

~~$60^\circ + \angle BDC + 75^\circ = 180^\circ$~~

~~$135^\circ + \angle BDC = 180^\circ$~~

~~$\angle BDC = 180^\circ - 135^\circ$~~

~~$\angle BDC = 45^\circ$~~

~~Hence, option (C) is correct~~

~~**Question: 4**~~

~~In which of the f~~

~~**Solution:**~~

~~As we know that from all the quadrilaterals given below, diagonals of a rectangle are equal~~

~~Hence, option (D) is correct~~

~~**Question: 5**~~

~~If the diagonals~~

~~**Solution:**~~

~~As we know that from all the quadrilaterals given below the diagonals of rhombus bisect each other at right angles~~

~~Hence, option (D) is correct~~

~~**Question: 6**~~

~~The lengths of th~~

~~**Solution:**~~

~~Let us assume a rhombus ABCD where,~~

~~AB = BC = CD = DA~~

~~Now, in triangle OBC by using Pythagoras theorem we get:~~

~~$BC^2 = OB^2 + OC^2$~~

~~$BC^2 = 6^2 + 8^2$~~

~~$BC^2 = 36 + 64$~~

~~$BC^2 = 100$~~

~~$BC = \sqrt{100}$~~

~~BC = 10 cm~~

~~∴ AB = BC = CD = DA = 10 cm~~

~~Hence, option (A) is correct~~

~~Question: 7~~

~~The length of eac~~

~~Solution:~~

~~It is given in the question that,~~

~~ABCD is rhombus where, AB = BC = CD = DA~~

~~Now, by using Pythagoras theorem in triangle BOC we have:~~

~~$BC^2 = OB^2 + OC^2$~~

~~$(10)^2 = OB^2 + (8)^2$~~

~~$100 = OB^2 + 64$~~

~~$OB^2 = 100 - 64$~~

~~$OB^2 = 36$~~

~~OB = 6 cm~~

~~∴ Length of diagonal, BC = OB + OD~~

~~BC = 6 + 6~~

~~BC = 12 cm~~

~~Hence, option (B) is correct~~

~~Question: 8~~

~~If ABCD is a para~~

~~Solution:~~

~~It is given in the question that,~~

~~ABCD is a parallelogram where two adjacent angles ∠ A = ∠ B~~

~~We know that, sum of adjacent angles is 180°~~

~~$\therefore \angle A + \angle B = 180^\circ$~~

~~$2\angle A = 180^\circ$~~

~~$\angle A = 180/2$~~

~~$\angle A = 90^\circ$~~

~~As, $\angle A = \angle B = \angle C = \angle D = 90^\circ$~~

~~∴ ABCD is a rectangle as all the angles are equal to 90°~~

~~Hence, option (C) is correct~~

~~**Question: 9**~~

~~In a quadrilatera~~

~~**Solution:**~~

~~It is given in the question that, ABCD is a quadrilateral where AO and BO are the bisectors of $\angle A$ and $\angle B$~~

~~We know that, sum of all angles of a quadrilateral is equal to 360°~~

~~$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ$~~

~~$\angle A + \angle B + 70^\circ + 30^\circ = 360^\circ$~~

~~$\angle A + \angle B = 360^\circ - 100^\circ$~~

~~$\angle A + \angle B = 260^\circ$~~

~~$1/2\ (\angle A + \angle B) = 1/2 \times 260^\circ$~~

~~$1/2\ (\angle A + \angle B = 130^\circ$~~

~~Now, in triangle AOB~~

~~$1/2\ (\angle A + \angle B) + \angle AOB = 180^\circ$~~

~~$130^\circ + \angle AOB = 180^\circ$~~

~~$\angle AOB = 180^\circ - 130^\circ$~~

~~$\angle AOB = 50^\circ$~~

~~Hence, option (B) is correct~~

~~**Question: 10**~~

~~The bisectors of~~

~~**Solution:**~~

~~We know that,~~

~~Sum of two adjacent angles = 180°~~

~~Also, sum of bisector of adjacent angles = $180/2 = 90^\circ$~~

~~As sum of angles of a triangle = 180°~~

~~$\therefore$ Sum of 2 adjacent angles + Intersection angle = 180°~~

~~90° + Intersection angle = 180°~~

~~$\therefore$ Intersection angle = $180^\circ - 90^\circ$~~

~~$= 90^\circ$~~

~~Hence, option (D) is correct~~

~~**Question: 11**~~

~~The bisectors of~~

~~**Solution:**~~

~~From all the given quadrilateral we know that the bisectors of the angles of a parallelogram enclose a rectangle~~

~~Hence, option (C) is correct~~

~~**Question: 12**~~

~~The figure formed~~

~~**Solution:**~~

We know that, the figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a parallelogram

Hence, option (D) is correct

Question: 13

The figure formed

Solution:

We know that, the figure formed by joining the mid-points of the adjacent sides of a square is a square

Hence, option (B) is correct

Question: 14

The figure formed

Solution:

We know that, the figure formed by joining the mid-points of the adjacent sides of a parallelogram is parallelogram

Hence, option (D) is correct

Question: 15

The figure formed

Solution:

We know that, the figure formed by joining the mid-points of the adjacent sides of a rectangle is a rhombus

Hence, option (A) is correct

Question: 16

The figure formed

Solution:

We know that, the figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle

Hence, option (C) is correct

Question: 17

If an angle of a

Solution:

We know that,

Sum of two adjacent angles is equal to 180°

$\therefore \angle A + \angle B = 180^\circ$

According to the condition given in the question, we have

$\angle A = x^\circ$ then $\angle B = 2/3\ x^\circ$

$\therefore x^\circ + 2x/3\ ^\circ = 180^\circ$

$5x/3\ ^\circ = 180^\circ$

$\Rightarrow x = \frac{180\times 3}{5}$

$\Rightarrow x = 540^\circ/5$

$\Rightarrow x = 108^\circ$

$\therefore \angle A = 108^\circ$ and,

~~$\angle B = 2/3 \times 108°$~~

~~$\angle B = 2\times 36° = 72°$~~

~~Thus, the smallest angle = $\angle B = 72°$~~

~~Hence, option (C) is correct~~

~~**Question: 18**~~

~~If one angle of a~~

~~**Solution:**~~

~~As per the question,~~

~~Let the smallest angle be x° and the largest angle be (2x – 24)°~~

~~Since, the sum of adjacent angles of a parallelogram is 180°~~

~~$\therefore x + (2x – 24) = 180°$~~

~~$3x – 24 = 180°$~~

~~$x = 68°$~~

~~Hence, the largest angle is: $2x – 24 = 2(68) – 24 = 136 – 24 = 112$~~

~~$\therefore$Option A is correct~~

~~**Question: 19**~~

~~In the given figu~~

~~**Solution:**~~

~~As per the question,~~

~~$\angle BAD = \angle BCD = 75°$ (opposite angles of parallelogram)~~

~~Now, in ΔBCD,~~

~~$\angle BCD + \angle CBD + \angle BCD = 180°$~~

~~$45 + \angle CBD + 75 = 180°$~~

~~$\angle CBD = 60°$~~

~~$\therefore$ Option C is correct~~

~~**Question: 20**~~

~~If area of a ||gm~~

~~**Solution:**~~

~~Let the height of the parallelogram be ‘h’~~

~~Now, h < b (Since, perpendicular distance is the shortest)~~

~~$\therefore a \times h < a \times b$~~

~~$A < B$~~

~~$\therefore$Option C is correct~~

~~**Question: 21**~~

~~In the given figu~~

~~**Solution:**~~

~~According to the condition given in the question, we have~~

~~In triangle DCE and FBE~~

~~BE = EC (E is the mid-point of BC)~~

~~∠ CED = ∠ BEF (Vertically opposite angles)~~

~~∠ CDE = ∠ EFB (Alternate interior angles)~~

~~∴ΔDCE ≅ ΔFBE (By AAS congruence rule)~~

~~DC = BF (BY CPCT)~~

~~As AB is parallel to DC, then AB = DC~~

~~∴ AB = DC = BF~~

~~AF = AB + BF~~

~~AF = AB + AB~~

~~AF = 2AB~~

~~Hence, option (B) is correct~~

~~Question: 22~~

~~The parallel side~~

~~Solution:~~

~~It is given in the question that,~~

~~ABCD is a trapezium~~

~~Draw EF parallel to AB and DC, and join BD intersecting EF at point M.~~

~~Now, E is the midpoint of AD and EM ∥ AB. Hence, using midpoint theorem,~~

~~EM = 1/2 AB~~

~~⇒ EM = 1/2 b~~

~~Similarly, FM = 1/2~~

~~⇒ DC = 1/2 a~~

~~EF = EM + FM~~

~~EF = 1/2 a + 1/2 b~~

~~EF = 1/2 (a + b)~~

~~∴Option B is correct~~

~~Question: 23~~

~~In a trapezium AB~~

~~Solution:~~

~~Construction: Join CF and extent it to cut AB at point M~~

~~Firstly, in triangle MFB and triangle DFC~~

~~DF = FB (As F is the mid-point of DB)~~

~~∠DFC = ∠MFB (Vertically opposite angle)~~

~~∠DFC = ∠FBM (Alternate interior angle)~~

~~∴ By ASA congruence rule~~

~~ΔMFB ≅ DFC~~

~~Now, in triangle CAM~~

~~E and F are the mid-points of AC and CM respectively~~

~~∴ EF = 1/2 (AM)~~

~~EF = 1/2 (AB – MB)~~

EF = 1/2 (AB-CD)

Hence, option D is correct

Question: 24

In the given figu

Solution:

Since, ABCD is a parallelogram,

$\therefore \angle B = \angle D$ (opposite angle)

$1/2\ \angle B = 1/2\ \angle D$

$\angle ADB = \angle ABD$

$\therefore$ ADB is an isosceles triangle.

Since, M is the midpoint of BD

$\therefore$ AM is a median of ΔADB.

Now, $\angle AMB = 90°$ (AM is perpendicular to BD)

$\therefore$Option C is correct

Question: 25

In the given figu

Solution:

Since, we know that the diagonals of a rhombus bisect each other at 90°.

Hence, $OA = \frac{1}{2} AC$, $OB = \frac{1}{2} BD$ and $\angle AOB = 90°$

$AB^2 = OA^2 + OB^2$

$AB^2 = (\frac{1}{2} AC)^2 + (\frac{1}{2} BD)^2$

$= \frac{1}{4}(AC)^2 + \frac{1}{4}(BD)^2$

$AB^2 = \frac{1}{4}(AC^2 + BD^2)$

$4AB^2 = (AC^2 + BD^2)$

$\therefore$ Option C is correct

Question: 26

In a trapezium AB

Solution:

Draw perpendicular from D on AB meeting it on E and from C on AB meeting AB at F

$\therefore$ DEFC will be a parallelogram and thus, EF = CD

Now, In ΔABC

Since, $\angle B$ is acute

$\therefore AC^2 = BC^2 + AB^2 - 2AB \times AE$ (i)

Similarly, In ΔABD,

Since $\angle A$ is acute

$\therefore BD^2 = AD^2 + AB^2 - 2AB \times AF$ (ii)

Adding (i) and (ii),

$AC^2 + BD^2 = (BC^2 + AD^2) + (AB^2 + AB^2) - 2AB\ (AE + BF)$

$= (BC^2 + AD^2) + 2AB\ (AB - AE - BF)$ [Since, AB = AE + EF + FB and AB - AE = BE]

$= (BC^2 + AD^2) + 2AB\ (BE - BF)$

$= (BC^2 + AD^2) + 2AB.EF$

Now, we know that CD = EF

Thus, $AC^2 + BD^2 = (BC^2 + AD^2) + 2AB.CD$

∴ Option D is correct

Question: 27

Two parallelogram

Solution:

We know that,

Area of a parallelogram = base × height

Now, if both parallelograms are on the same base and between the same parallels, then their heights will be equal.

Hence, their areas will also be equal

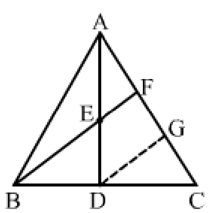
∴ Option D is correct

Question: 28

In the given figu

Solution:

Let G be the mid-point of FC and join DG



In ΔBCF,

G is the mid-point of FC and D is the mid-point of BC

Thus, DG|| BF

DG || EF

Now, In Δ ADG,

E is the mid-point of AD and EF is parallel to DG.

Thus, F is the mid-point of AG.

AF = FG = GC [G is the mid-point of FC]

Hence, AF = $\frac{1}{3}$ AC

∴ Option B is correct

Question: 29

If 3x + 7x + 6x + 4x = 360° (Sum of angles of quadrilateral)

20x = 360°

x = 18°

Hence, angles are:

$3x = 3 \times 18° = 54°$

$7x = 7 \times 18° = 126°$

$6x = 6 \times 18° = 108°$

$4x = 4 \times 18° = 72°$

Now we can observe that, $54° + 126° = 180°$ and $72° + 108° = 180°$

Thus, ABCD is a trapezium.

Hence option C is correct.

Question: 30

Which of the foll

Solution:

We know that,

In any parallelogram, opposite angles are bisected by the diagonals

$\therefore$ Option C is correct

Question: 31

If APB and CQD ar

Solution:

It is given in the question that,

APB and CQD are two parallel lines,

Thus, the bisectors of $\angle CQP$, $\angle APQ$, $\angle BPQ$ and $\angle PQD$ enclose a rectangle.

Hence, option C is correct.

Question: 32

The diagonals AC

Solution:

In the given figure,

$\angle OAD = \angle OCB$ (Alternate interior angle)

$\angle OCB = 30°$

$\angle AOB + \angle BOC = 180°$ (Linear pair)

$70° + \angle BOC = 180°$

$\angle BOC = 110°$

Now, In ΔBOC,

$\angle OBC + \angle BOC + \angle OCB = 180°$

$\angle OBC + 110° + 30° = 180°$

$\angle OBC = 40°$

$\therefore \angle DBC = 40°$

Hence, Option A is correct.

Question: 33

Three statements

Solution:

~~We can clearly observe that statement I and statement II are correct. Whereas Statement III is not correct because the triangle formed by joining the midpoints of the sides of an isosceles triangle is always an isosceles triangle~~

~~Therefore, Option C is correct~~

~~Question: 34~~

~~Three statements~~

~~Solution:~~

~~We can clearly observe that statement II and statement III are correct and Statement I is wrong because the diagonals of a rectangle does not bisect ∠A and ∠C. And this is so because the adjacent sides are unequal in a rectangle.~~

~~∴ Option B is correct~~

~~Question: 35~~

~~In each of the qu~~

~~Solution:~~

~~Here, as we know that if the diagonals of a quadrilateral bisects each other, then it is a parallelogram.~~

~~But as per II, if the diagonals of a quadrilateral are equal, then it is not necessarily a parallelogram which is not true. Thus, II does not give the answer.~~

~~Therefore Option A is correct.~~

~~Question: 36~~

~~In each of the qu~~

~~Solution:~~

~~Here, we can observe that neither I not II can alone justify the answer to the given question. But if we consider both I and II together then they completely satisfies the answer.~~

~~∴ Option C is correct.~~

~~Question: 37~~

~~In each of the qu~~

~~Solution:~~

~~We know that when the diagonals of a parallelogram are equal, it might be a square or a rectangle. But if the diagonals of that parallelogram intersect at a right angle, then it is definitely a square. Thus, it can be concluded that both I and II together will give the answer.~~

~~Therefore, Option C is correct.~~

~~Question: 38~~

~~In each of the qu~~

~~Solution:~~

~~We know that a quadrilateral is a parallelogram when either I or II holds true.~~

~~Hence, the correct answer is (b)~~

~~Question: 39~~

~~Each question con~~

~~Solution:~~

~~Let the fourth angle be x,~~

~~$130^\circ + 70^\circ + 60^\circ + x^\circ = 360^\circ$ (angle sum of quadrilateral)~~

~~$x^\circ = 360^\circ - (130^\circ + 70^\circ + 60^\circ)$~~

$x° = 100°$

Thus, it can be observed that reason and assertion both are true and the reason explains the assertion.

Therefore Option A is correct.

Question: 40

Each question con

Solution:

It is given that, ABCD is a quadrilateral in which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Then, PQRS is a parallelogram

Also, the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Hence, both assertion and reason are true and reason is correct explanation of the assertion

∴ Option (a) is correct

Question: 41

Each question con

Solution:

It is given that,

In a rhombus ABCD, the diagonal AC bisects ∠A as well as ∠ C which is true

And we know that, the diagonals of a rhombus bisect each other at right angles.

Hence, both assertion and reason are true but reason is not the correct explanation of assertion

∴ Option (b) is correct

Question: 42

Each question con

Solution:

The statement given in assertion is not true as every parallelogram is not a rectangle whereas, statement given in the reason is true as the angle bisectors of a parallelogram form a rectangle

Hence, assertion is false whereas reason is true

∴ Option (d) is correct

Question: 43

Each question con

Solution:

We know that,

The diagonals of a ||gm bisect each other

Also we know that, if the diagonals of a ||gm are equal and intersect at right angles, then the parallelogram is a square

Hence, both assertion and reason are true but reason is not the correct explanation of the assertion

Hence, option (b) is correct

Question: 44

Match the followi

Solution:

The correct match for the above given table is as follows:

Column I	Column II
(a) Angle bisectors of a parallelogram form a	(q) Rectangle
(b) The quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a square is a	(r) Square
(c) The quadrilateral formed by joining the mid-points	(s) Rhombus
(d) The figure formed by joining the mid-points of the pairs of adjacent sides of a quadrilateral is a	(p) Parallelogram

~~**Question: 45**~~

~~Match the followi~~

~~**Solution:**~~

~~a) PQ =~~ $\cancel{\frac{1}{2}(AB + CD)}$

~~PQ =~~ $\cancel{\frac{1}{2}(17)}$

~~PQ = 8.5 cm~~

~~(b) OR =~~ $\cancel{\frac{1}{2}(PR)}$

~~OR =~~ $\cancel{\frac{1}{2}(13)}$

~~OR = 6.5 cm~~

~~(c) We know that,~~

~~The diagonals of a square are equal~~

~~(d) We also know that,~~

~~The diagonals of a rhombus bisect each other at right angles~~

~~∴ The correct match is as follows:~~

~~(a) - (r)~~

~~(b) -(s)~~

~~(c) - (p)~~

~~(d) - (q)~~

~~Exercise : FORMATIVE ASSESSMENT (UNIT TEST)~~

~~Question: 1~~

~~Which is false?~~

~~Solution:~~

~~from the above given four statements option A is false as we know that in any parallelogram the diagonals are not equal~~

~~Hence, option A is correct~~

~~Question: 2~~

~~If P is a point o~~

~~Solution:~~

~~In ΔABC,~~

~~Since, AD is the median~~

~~Thus, BD = DC~~

~~Let the height of ΔABC be h~~

~~ar(ΔABD) = ar(ΔABD)~~

~~1/2 × h × BD = 1/2 × h × BD~~

~~1/2 × h × BD = 1/2 × h × CD~~

~~∴ ar (ΔABD) = ar (ΔADC)~~

~~Let H be the height of ΔBPD and ΔPDC~~

~~∴ ar (ΔBPD) = ar (ΔPDC)~~

~~Now, ar(ΔABD) = ar (ΔABP) + ar (ΔBPD)~~

~~And, ar(ΔACD) = ar(ΔACP) + ar(ΔPDC)~~

~~Thus, ar(ΔABP) = ar(ΔACP)~~

~~∴ Option A is correct~~

~~Question: 3~~

~~The angles of a q~~

~~Solution:~~

~~Let the angles be x, 3x, 5x and 6x.~~

~~x + 3x + 5x + 6x = 360° (sum of angles of quadrilateral)~~

~~$15x^\circ = 360^\theta$~~

~~$x^\circ = 24^\theta$~~

~~Therefore, angles are as follows:~~

~~$x^\circ = 24^\theta$~~

~~$3x^\circ = 24^\theta \times 3 = 72^\theta$~~

~~$5x^\circ = 24^\theta \times 5 = 120^\theta$~~

~~$6x^\circ = 24^\theta \times 6 = 144^\theta$~~

~~Hence, 144° is the greatest angle.~~

~~Question: 4~~

~~In a ΔABC, D and~~

~~**Solution:**~~

~~We know that in ΔABC, D and E are the midpoints of AB and AC, respectively.~~

~~Now using mid point theorem,~~

~~DE = $\frac{1}{2}$ (BC)~~

~~BC= 2 × DE~~

~~BC= 2 × 5.6~~

~~= 11.2 cm~~

~~Thus, BC = 11.2 cm~~

~~**Question: 5**~~

~~In the given figu~~

~~**Solution:**~~

~~In ΔABC, using mid point theorem~~

~~We know that D is the mid point of BC and DE|| AB.~~

~~Thus, AE = EC and DE = $\frac{1}{2}$ (AB)~~

~~Now, E is the mid point of AC~~

~~Thus, BE is the median~~

~~**Question: 6**~~

~~In the given figu~~

~~**Solution:**~~

~~Here, we have:~~

~~l || m || n~~

~~And p and q are the transversal lines~~

~~Thus, AB : BC = 5 : 15~~

~~AB : BC = 1 : 3~~

~~∴ Using intercept theorem,~~

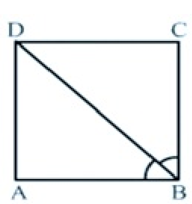
~~DE : EF = 1 : 3~~

~~**Question: 7**~~

~~ABCD is a rectang~~

~~**Solution:**~~

~~Let there be a rectangle ABCD with AB = CD and BC = AD and ∠A = ∠B = ∠C= ∠D = 90°~~



~~Since, BD bisects ∠B~~

~~∠ABD = ∠DBC (i)~~

~~And, ∠ADB = ∠DBC [Alternate interior angles]~~

~~∠ABD = ∠ADB. [From (i)]~~

~~AB = DA. (Sides opposite to equal angles)~~

~~∴ AB = CD = DA = BC~~

~~Since, all the sides are equal and all the angles are equal to 90°, thus the quadrilateral is a square.~~

~~Hence, ABCD is a square.~~

~~**Question: 8**~~

~~The diagonals of~~

~~**Solution:**~~

~~∠BOC = ∠AOD (Vertically opposite angles)~~

~~Angle AOD = 50°~~

~~In Δ AOD, Since, the diagonals are equal, thus the bisectors will also be equal)~~

~~Thus, OA = OD~~

~~∴ ∠OAD = ∠ODA~~

$\cancel{= \frac{1}{2}(180° - 50°)}$

$\cancel{= \frac{1}{2}(130°)}$

$\cancel{= 65°}$

~~∴ Option C is correct~~

~~**Question: 9**~~

~~Match the followi~~

~~**Solution:**~~

~~The correct match for the above given table is as follows:~~

Column I	**Column II**
(a) Sum of all the angles of a quadrilateral is	(s) 4 right angles
(b) In a ||gm, the angle bisectors of two adjacent angles intersect at	(p) Right angles
(c) Angle bisectors of a ||gm form a	(q) Rectangle
(d) The diagonals of a square are equal and bisect each other at an angle of	(r) 90°

~~Question: 10~~

~~The diagonals of~~

~~Solution:~~

~~∠BDC = ∠ABD (Alternate interior angles)~~

~~∠ABD = 50°~~

~~Now, In ΔAOB,~~

~~∠DBA = 50° and ∠AOB = 90°~~

~~Thus, ∠OAB = 180° − (90° + 50°)~~

~~∠OAB = 180° - 140°~~

~~∠OAB = 40°~~

~~∴ Option B is correct.~~

~~Question: 11~~

~~ABCD is a trapezi~~

~~Solution:~~

~~Construction: Draw perpendicular line from D and C to AB such that it cuts AB at F and E, respectively.~~

~~Now, In ΔADF and ΔBCE,~~

~~AD = BC (Given)~~

~~∠AFD = ∠BEC (90° each)~~

~~DF = CE (Perpendicular distance between the same parallels)~~

~~∴ By SSA axiom~~

~~ΔADF ≅ ΔBCE~~

~~∠A = ∠B (by c.p.c.t.)~~

~~Therefore Option A is correct.~~

~~Question: 12~~

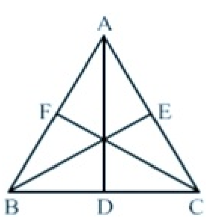
~~Look at the state~~

~~Solution:~~

~~We can clearly observe that statement I and statement III are correct.~~

~~We can prove the statement as follows:~~

~~In ΔABC, altitudes AD, BE and CF are equal~~



~~Now, In ΔABE and ΔACF,~~

~~BE = CF (Given)~~

~~∠A = ∠A (common)~~

~~∠AEB = ∠AFC (Each 90°)~~

~~Therefore, by AAS axiom,~~

~~ΔABE ≅ Δ ACF~~

~~AB = AC (by cpct)~~

~~In the same way, ΔBCF ≅ ΔBAD~~

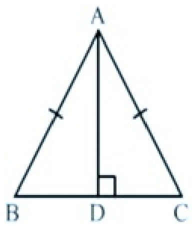
~~thus, BC = AB (by cpct)~~

~~Therefore AB = AC = BC~~

~~Thus, ΔABC is an equilateral triangle.~~

~~We can prove the IIIrd statement as follows:~~

~~Let ΔABC be an isosceles triangle with AD as an altitude~~



~~Now, In ΔABD and ΔADC,~~

~~AB = AC (Given)~~

~~∠B = ∠C (Angles opposite to equal sides)~~

~~∠BDA = ∠CDA (each 90°)~~

~~Therefore by AAS axiom,~~

~~ΔABD ≅ ΔADC~~

~~BD = DC (by congruent parts of congruent triangles)~~

~~∴ D is the mid-point of BC and hence AD bisects BC.~~

~~Question: 13~~

~~In the given figu~~

~~Solution:~~

~~Area of a triangle = 1/2 (Base × Height)~~

~~Now, draw AL perpendicular to BC and h be the height of ΔABC i.e. AL~~

~~Thus, Height of ΔABD = Height of ΔADE = Height of ΔAEC~~

~~It is given that the bases BD, DE and EC of ΔABD, ΔADE and ΔAEC respectively are equal.~~

~~Now, since base and height both are equal of all the triangles therefore,~~

~~ar(ΔABD) = ar(ΔADE) = ar(ΔAEC)~~

~~Question: 14~~

~~In the given figu~~

~~Solution:~~

~~Now, here in ΔADE and ΔBCF,~~

~~AD = BC (Opposite sides of parallelogram ABCD~~

~~DE = CF (Opposite sides of parallelogram DCEF)~~

~~AE = BF (Opposite sides of parallelogram ABFE)~~

~~∴ By SSS axiom,~~

~~ΔADE ≅ ΔBCF~~

~~And,~~

~~ar(ΔADE) = ar(ΔBCF) (By cpct)~~

~~**Question: 15**~~

~~In the given figu~~

~~**Solution:**~~

~~Here, in trapezium ABCD,~~

~~AB || DC and AC and BD are the diagonals intersecting at O.~~

~~Now, since ΔACD and ΔBCD lie on the same base and between the same parallels.~~

~~Thus, ar(ΔACD) = ar(ΔBCD)~~

~~Subtracting ar(ΔCOD) from both the sides, we get:~~

~~ar(ΔACD) − ar(ΔCOD) = ar(ΔBCD)− ar(ΔCOD)~~

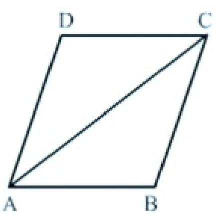
~~∴ ar(ΔAOD) = ar(ΔBOC)~~

~~**Question: 16**~~

~~Show that a diago~~

~~**Solution:**~~

~~Let there be a parallelogram ABCD and with one of its diagonal as AC.~~



~~Now, In ΔCDA and ΔABC,~~

~~DA = BC (Opposite sides of parallelogram ABCD)~~

~~AC = AC (Common)~~

~~CD = AB (Opposite sides of parallelogram ABCD)~~

~~∴ By SSS axiom~~

~~ΔCDA ≅ ΔABC~~

~~ar(ΔCDA) = ar(ΔABC) (by cpct)~~

~~Thus, we can say that the diagonal of a parallelogram divides it into two triangles of equal area.~~

~~**Question: 17**~~

~~In the given figu~~

~~**Solution:**~~

~~Here we have ABCD as a quadrilateral with one of its diagonal as AC and BL and DM are perpendicular to AC~~

~~Thus, ar(ABCD) = ar(ΔADC) + ar(ΔABC)~~

~~Since, (BL ⊥ AC) and (DM ⊥ AC)~~

~~∴ Area of ABCD = $(\frac{1}{2} \times AC \times BL) + (\frac{1}{2} \times AC \times DM)$~~

~~$= \frac{1}{2} \times AC \times (BL + DM)$~~

~~**Question: 18**~~

~~||gm ABCD and rec~~

~~**Solution:**~~

~~Here we know that parallelogram ABCD and rectangle ABEF are on the same base AB and between the same parallels such that:~~

~~AB = CD and AB = EF~~

~~So, CD = FE~~

~~Now, adding AB on both sides~~

~~AB + CD = AB + FE (i)~~

~~Since we know that hypotenuse is the longest side of a triangle~~

~~∴ AD > AF (ii)~~

~~And, BC > BE (iii)~~

~~Adding (ii) and (iii),~~

~~AD + BC > AF + BE (iv)~~

~~Now, Perimeter of ABCD = AB + BC + CD + AD~~

~~And, Perimeter of ABEF = AB + BE + FE + AF~~

~~Adding (i) and (iv),~~

~~AB + CD + AD + BC > AB + FE + AF + BE~~

~~Thus, we can say that the perimeter of parallelogram ABCD is greater than that of rectangle ABEF.~~

~~**Question: 19**~~

~~In the adjoining~~

~~**Solution:**~~

~~Here we have parallelogram ABCD with AB || DC~~

~~Thus, DC || BF~~

~~Now, in ΔDEC and ΔFEB,~~

~~∠DCF = ∠EBF (Alternate interior angle)~~

~~CE = BE (E is the mid-point of BC~~

~~∠CED = ∠BEF (Vertically opposite angle)~~

~~Therefore, by ASA axiom,~~

~~ΔDEC ≅ ΔFEB~~

~~CD = BF (by cpct)~~

~~And CD = AB (Opposite sides of a parallelogram ABCD)~~

~~So, AF = AB + BF = AB + AB = 2AB~~

~~**Question: 20**~~

~~In the adjoining~~

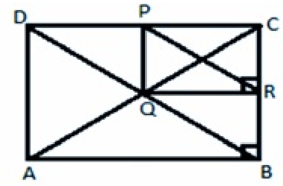
~~**Solution:**~~

~~(i) Here, we have~~

~~∠CRQ = ∠CBA = 90°~~

~~Thus, RQ || AB~~

~~Now, In ΔABC,~~



~~Q is the mid-point of AC and QR || AB.~~

~~Thus, R is the mid-point of BC.~~

~~In the same way, P is the midpoint of DC.~~

~~Hence, DP = PC~~

~~(ii) Here, let us join B to D.~~

~~Now, In ΔCDB,~~

~~P and R are the mid points of DC and BC respectively.~~

~~Since, AC = BD~~

~~Thus, PR || DB and PR =~~ $\cancel{\frac{1}{2} DB = \frac{1}{2} AC}$