

Answers to Problems

Problems 1.1, page 5

1. $x^4 - 6x^3 + 3x^2 + 42x - 70 = 0$ 2. (i) $-2, 1 + 3i, 1 - 3i$ (ii) $2 \pm \sqrt{3}, 3, -5$
 5. Two roots between $(1, 2)$ and $(-3, -4)$
 6. $2, 2, -\frac{1}{3}$ 7. 3 8. $a = 2, b = 1$
 9. $6, 4, -1$ 10. $-4, 2, 6$ 11. $1, 1, 2, 2$
 12. $\frac{1}{2}(3 \pm \sqrt{5}; \frac{1}{2}(5 \pm \sqrt{5})$ 13. $1, 4, 7$ 14. $1, \frac{1}{2}, \frac{1}{4}$
 16. (i) $-5, -2, 1, 4$ (ii) $1, -2, 4, -8$ 17. (i) $m^2 - 2ln$, (ii) $lm - n$
 18. 36 19. (i) $4/3$, (ii) $16/9$.

Problems 1.2, page 8

1. $x^3 + 6x^2 - 36x + 27 = 0$ 2. $6x^5 - 7x^4 - 13x^3 + 4x^2 - 2 = 0$ 3. $10x^4 + 9x^3 + 8x^2 - 7x + 1 = 0$
 4. $-\frac{1}{2}, \frac{1}{3}, 2$ 5. (i) $x^3 - 9x^2 + 26x - 24 = 0$; (ii) $x^4 + 13x^3 + 60x^2 + 116x + 80 = 0$;
 (iii) $x^5 + 7 = 0$
 6. $x^3 + 15x^2 + 52x - 36 = 0$ 7. $y^3 + (p^3 + 3q)y^2 + 3q^2y + q^3 = 0$ 8. $3x^3 - 11x^2 + 9x - 2 = 0$
 9. $y^3 - qy^2 + py - r^2 = 0$
 10. (a) $y^3 + 4my - 8n = 0$; (b) $nx^3 + m^2x^2 - 2mnx + n^2 = 0$; (c) $x(nx + m)^2 = n$
 11. $y^3 - 30y^2 + 225y - 68 = 0$
 12. (i) $\frac{-5 \pm \sqrt{21}}{2}, \frac{5 \pm \sqrt{91}i}{12}$ (ii) $2, 2, 1/2, 1/2$ (iii) $1, -2, 4, -1/2, 1/4$;
 (iv) $-1, -2, 3, -1/2, 1/3$; (v) $\pm 1, -3, -1/3, \frac{3 \pm \sqrt{5}}{2}$
 13. $\frac{1}{2}(5 \pm \sqrt{21})$; $\frac{1}{2}(-3 \pm \sqrt{5})$ 14. $-1, -2, -6, -7$.

Problems 1.3, page 11

1. $-6, 3, 3$ 2. $5, \frac{1}{2}(-5 \pm i\sqrt{3})$ 3. $6, -3 \pm 2\sqrt{(-3)}$
 4. $-1, -2, \frac{1}{2}$ 5. $\frac{1}{2}, -\frac{1}{6}(3 \pm i\sqrt{3})$ 6. $5, \frac{1}{2}(1 + i\sqrt{3})$
 7. $2 \cos \frac{2\pi}{9}, 2 \cos \frac{8\pi}{9}, 2 \cos \frac{14\pi}{9}$ 8. $\frac{1}{2}, \frac{-7 \pm 9i\sqrt{3}}{6}$

Problems 1.4, page 12

1. 1, 2, 3, 4
 4. $-1, 3, 3 \pm \sqrt{30}$
 7. $2 \pm \sqrt{3}, -2 \pm i\sqrt{3}$

2. $-3, 1, \pm 2$
 5. $1 \pm \sqrt{7}, 2 \pm \sqrt{3}$
 8. $2, 4, 2 \pm 2i\sqrt{2}$

3. $4, -2, -1 \pm i$
 6. $1 \pm 2i, -1 \pm \sqrt{2}$

Problems 1.5, page 15

1. 1.32
 4. (i) 0.71 rad

2. 2.29
 5. 1.81 rad

3. 0.45
 6. 0.26

Problems 1.6, page 15

- | | | |
|---------------------------------|-------------------------------------|--|
| 1. (d) | 2. (c) | 3. (c) |
| 4. (c) | 5. (c) | 6. (a) |
| 7. (d) | 8. (b) | 9. (c) |
| 10. (a) | 11. (c) | 12. minus |
| 13. § 15.1 (v) | 14. p/q | 15. 21 |
| 16. -3 and -2 | 17. Conjugate pairs | 18. $f(x)$ is continuous in (a, b) |
| 19. $x^3 - 9x^2 + 29x - 24 = 0$ | 20. $3, 6, -2$ | 21. $x^3 - 200x - 7000 = 0$ |
| 22. p/r | 23. $x^4 + 2x^3 - x^2 - 6x - 6 = 0$ | 24. 6 |
| 25. minus | 26. $pq = r$ | 27. $1, \frac{1}{2}(-1 \pm \sqrt{3}i)$ |
| 28. (iii) | 29. $1, 1, -2$ | 30. $x^3 - 7x^2 + 12x - 10 = 0$ |
| 31. Zero and 2 | 32. 21 | 33. True |
| 34. True. | | |

Problems 2.1 page 25

- | | |
|--------------------------------------|--|
| 5. (i) 1 (ii) 0 | 13. $(a-b)(b-c)(c-a)$ |
| 14. $(a-b)(b-c)(c-a)(ab+bc+ca)$ | 15. $(b-c)(c-a)(a-b)(a-1)(b-1)(c-1)$ |
| 16. $(a-b)(a-c)(a-d)(b-c)(b-d)(c-d)$ | 17. $x = 0, \pm \sqrt{(a^2 + b^2 + c^2 - ab - bc - ca)}$ |
| 18. $0, -\frac{1}{2}$. | |

Problems 2.2 page 31

1. $x = 0, 3$

2. $x = -3, y = -2, z = -4, a = 3$

3. $x = 3, y = 8$

4. $-2 \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$

5. $AB = \begin{bmatrix} 4 & 4 & 7 & 10 \\ 10 & 7 & 11 & 21 \\ 4 & 3 & 3 & 9 \end{bmatrix}$

7. (i) $[ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy]$

(ii) $\begin{bmatrix} 8 & 7 \\ 122 & 104 \\ -365 & -131 \end{bmatrix}$ (iii) $\begin{bmatrix} 9 & 6 \\ -18 & -12 \\ 27 & 18 \end{bmatrix}$

10. $3I$

11. $\begin{bmatrix} -6 & 1 & 2 \\ 5 & 4 & 4 \\ 2 & 8 & -3 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 0 & 0 \\ 7/5 & 1 & 0 \\ 3/5 & 41/19 & 1 \end{bmatrix} \begin{bmatrix} 5 & -2 & 1 \\ 0 & 19/5 & -32/5 \\ 0 & 0 & 327/19 \end{bmatrix}$

Problems 2.3, page 35

$$\begin{array}{l}
 2. (i) \begin{bmatrix} 3 & 0 & 5.5 \\ 0 & 7 & 1.5 \\ 5.5 & 1.5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -2 & 5 \\ 2 & 0 & -2.5 \\ -0.5 & 2.5 & 0 \end{bmatrix} \quad (ii) \begin{vmatrix} a & \frac{1}{2}(a+c) & \frac{1}{2}(b+c) \\ \frac{1}{2}(a+c) & b & \frac{1}{2}(a+b) \\ \frac{1}{2}(b+c) & \frac{1}{2}(a+b) & c \end{vmatrix} + \begin{vmatrix} 0 & \frac{1}{2}(a-c) & \frac{1}{2}(b-c) \\ \frac{1}{2}(c-a) & 0 & \frac{1}{2}(b-a) \\ \frac{1}{2}(c-b) & \frac{1}{2}(a-b) & 0 \end{vmatrix} \\
 4. (i) \begin{bmatrix} 2 & 4/5 & 9/5 \\ 3 & -4/5 & -14/5 \\ -1 & 1/5 & 6/5 \end{bmatrix} \quad (ii) \begin{bmatrix} 1/33 & -4/33 & 2/11 \\ -4/33 & 14/33 & 13/33 \\ 2/11 & 13/33 & -1/33 \end{bmatrix} \\
 5. B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 6. (i) \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}.
 \end{array}$$

Problems 2.4, page 40

$$1. 3 \quad 2. 2 \quad 3. 3 \quad 4. 2 \quad 5. 3$$

6. No value of p is possible.

$$\begin{array}{ll}
 7. (i) \begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix} & (ii) \frac{1}{21} \begin{bmatrix} 1 & 10 & -7 \\ 1 & -11 & 14 \\ -3 & 12 & 0 \end{bmatrix} \\
 (iii) \begin{bmatrix} -1/9 & 2/9 & 2/9 \\ 2/9 & -1/9 & 2/9 \\ 2/9 & 2/9 & -1/9 \end{bmatrix} & (iv) \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -1/2 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} \\
 8. P = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & 2/3 & -7/24 & 5/6 \\ 0 & -1/3 & 0 & 1/3 \\ 0 & 0 & -5/24 & 1/2 \\ 0 & 0 & -1/12 & 0 \end{bmatrix}, \text{Rank}(A) = 3
 \end{array}$$

$$9. \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad 10. (i) P = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}, Q = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$(ii) P = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}, Q = \begin{bmatrix} 1 & \frac{1}{3} & -\frac{4}{3} & -\frac{1}{3} \\ 0 & -\frac{1}{6} & -\frac{5}{6} & \frac{7}{6} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$11. (i) 3 \quad (ii) 3 \quad (iii) 2 \quad (iv) 3.$$

Problems 2.5, Page 43

$$\begin{array}{ll}
 1. \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} & 2. \frac{1}{8} \begin{bmatrix} 2 & 2 & -2 \\ -9 & 11 & 5 \\ 5 & -7 & -1 \end{bmatrix} \\
 3. \begin{bmatrix} 1 & -2 & 1 & 0 \\ 1 & -2 & 2 & -3 \\ 0 & 1 & -1 & 1 \\ -2 & 3 & -2 & 3 \end{bmatrix} & 4. \begin{bmatrix} 7 & -3 & 0 & -5 \\ 8 & 1 & -2 & -11 \\ -5 & 0 & 1 & 6 \\ 19 & 5 & -6 & -28 \end{bmatrix}
 \end{array}$$

Problems 2.6, page 45

$$\begin{array}{ll}
 1. x = 1, y = 2, z = 1 & 2. x = 2, y = -1, z = 1/2 \\
 3. x = 1.2, y = 2.2, z = 3.2 & 4. x = y = z = e^2
 \end{array}$$

5. $u = 1, v = 1/2, w = 1/3$
 7. $x = 2, y = 1, z = 0$
 9. $x = y = z = 2$
 11. $x = 1, y = -1, z = 1$
 13. $i_1 = 369/175, i_2 = 24/25, i_3 = 72/175$
6. $x_1 = 1, x_2 = -5, x_3 = 5$
 8. $x = 1/7, y = 10/7, z = 1/7$
 10. $x_1 = 2, x_2 = 1, x_3 = -1, x_4 = 3$
 12. $i_1 = 1.5, i_3 = 2.5$
 14. $x_1 = 2, x_2 = 1/5, x_3 = 0, x_4 = 4/5.$

Problems 2.7, page 50

1. Consistent; $x = 1, y = 3k - 2, z = k$ for all k
2. $k = 1, x = -3z, y = 2z + 1$; $k = 2, x = 1 - 3z, y = 2z$
3. (i) $\lambda = 3, \mu \neq 10$; (ii) $\lambda \neq 3$; (iii) $\lambda = 3, \mu = 10$
4. (i) Equations are inconsistent; (ii) consistent; $x = -1, y = 1, z = 2$; (iii) Equations are inconsistent; (iv) Consistent: $x = 2, y = 1, z = -4$
5. If $a = -1, b = 6$, equations will be consistent and have infinite number of solutions
 If $a = -1, b \neq 6$, equations will be inconsistent;
 If $a \neq -1, b$ has any value, equations will be consistent and have a unique solution
6. $\lambda \neq -5, x = 4/7, y = -9/7, z = 0$; $\lambda = -5, x = \frac{1}{7}(4 - 5k), y = \frac{1}{7}(13k - 9), z = k$ for all k
9. $\lambda = -1, 1, 12$; $x = -1/11, y = -15/11, z = -5, y = 1$; $x = \frac{1}{2}, y = 1$
11. $k = 3$ is the only real value for which $x = y = z$
12. $\lambda = 1, x_1 = 2t - s, x_2 = t, x_3 = s$; $\lambda = -3, x_1 = -t, x_2 = -2t, x_3 = t$
13. $\lambda = 1, -9$. For $\lambda = 1$, sol. is $x = k, y = -k, z = 2k$
 For $\lambda = -9$, sol. is $x = 3k, y = 9k, z = -2k$
14. (i) Have infinite number of non-trivial solutions; $x = \lambda - 5\mu/3, y = \lambda - 4\mu/3, z = \lambda, w = \mu$ for all values of λ and μ . (ii) $x = 11k_2 + 6k_1, y = -8k_2 - 3k_1, z = k_2, w = k_1$ where k_1, k_2 are arbitrary constants.

Problems 2.8, page 54

1. $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 11 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$
2. $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}, A^{-1} = A' = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$
3. $Z = (BA)X$, where $BA = \begin{bmatrix} 1 & 4 & -1 \\ -1 & 9 & -1 \\ -3 & 14 & -1 \end{bmatrix}$
4. $x_1 = 19y_1 - 9y_2 + 2y_3$; $x_2 = -4y_1 + 2y_2 - y_3$; $x_3 = -2y_1 + y_2$
6. $a = \pm \frac{1}{\sqrt{2}}, b = \pm \frac{1}{\sqrt{6}}, c = \pm \frac{1}{\sqrt{3}}$
9. (i) No. (ii) No. (iii) Yes, $9x_1 - 12x_2 + 5x_3 - 5x_4 = 0$.

Problems 2.9, page 60

1. 10; 30
2. (a) 10, 3; (1, 2), (3, -1); (b) -1, 6; (1, 1), (2, -5)
3. (a) 0, 3, 5; (1, 2, 2), (2, 1, -2), (2, -2, 1) (b) 1, 2, 3; (1, 0, -1), (0, 1, 0), (1, 0, 1)
 (c) 5, -3, -3; (1, 2, -1), (2, -1, 0), (3, 0, 1)
 (d) 8, 2, 2; (2, -1, 1), (1, 0, -2), (1, 2, 0) (e) 2, 3, -1; (3, 1, 1), (-4, 1, -3), (0, 5, 5)
5. (i) 8, 12, 6 (ii) 49, 121, 25
6. 1, 1, 1/5
9. (i) $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$; (ii) $\frac{1}{9} \begin{bmatrix} 0 & 3 & 3 \\ 3 & 2 & -7 \\ 3 & -1 & -1 \end{bmatrix}$; (iii) $\frac{1}{4} \begin{bmatrix} 12 & 4 & 6 \\ -5 & -1 & -3 \\ -1 & -1 & -1 \end{bmatrix}$; (iv) $\begin{bmatrix} 1 & 1/2 & -2/3 \\ 0 & -1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$
10. $\lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0, \frac{1}{35} \begin{bmatrix} -4 & 11 & -5 \\ -1 & -6 & 25 \\ 6 & 1 & -10 \end{bmatrix}$

11. (i) $\frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ (ii) $\frac{1}{3} \begin{bmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{bmatrix}$ (iii) $\frac{1}{27} \begin{bmatrix} 1 & 10 & -8 \\ -8 & 1 & 10 \\ 10 & -8 & 1 \end{bmatrix}$

12. $625I$

13. $\lambda^3 - 6\lambda^2 + 8\lambda - 3 = 0$, $\begin{bmatrix} 124 & -123 & 162 \\ -95 & 96 & -123 \\ 95 & -95 & 124 \end{bmatrix}$

14. $\begin{bmatrix} 1/5 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$

$$A^{-3} = \frac{1}{64} \begin{bmatrix} 1 & 78 & 78 \\ -21 & 90 & 26 \\ 21 & -154 & -90 \end{bmatrix}$$

15. $A^{-1} = \frac{1}{4} \begin{bmatrix} 1 & 6 & 6 \\ -1 & 6 & 2 \\ 1 & -10 & -6 \end{bmatrix}$, $A^{-2} = \frac{1}{16} \begin{bmatrix} 1 & -18 & -18 \\ -5 & 10 & -6 \\ 5 & 6 & 22 \end{bmatrix}$,

16. $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$.

Problems 2.10, page 67

3. $\begin{bmatrix} -5 & 0 \\ 0 & 2 \end{bmatrix}$

4. $A^n = \begin{bmatrix} 2^n + 3 \cdot 6^n & -3 \cdot 2^n + 3 \cdot 6^n \\ -2^n + 6^n & 3 \cdot 2^n + 6^n \end{bmatrix}$; $A^4 = \begin{bmatrix} 976 & 960 \\ 320 & 336 \end{bmatrix}$

5. $\begin{bmatrix} 251 & -405 & 235 \\ -405 & 891 & -405 \\ 235 & -405 & 251 \end{bmatrix}$

7. $(1, 1, -1), (1, 1, -1), (2, -1, 1); 4x^2 + y^2 + z^2$

8. $x^2 + y^2 - 2z^2$
 10. (a) $1, 2, 4; (1, 0, 0), (0, 1, 1), (0, 1, -1)$; $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$
 (b) $x_1^2 + 2x_2^2 + 4x_3^2$

10. (i) $x_1^2 + 4x_2^2 + 4x_3^2$, $\begin{bmatrix} -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$, positive definite;

(ii) $3y^2 + 15z^2$, $\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$, positive semidefinite

11. 2, 1

12. Indefinite.

Problems 2.11, Page 71

8. $\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} + i \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ -1 & 3 & 0 \end{bmatrix}$

Problems 2.12, page 72

- | | | | | | |
|---------|---------|---------|---------|---------|----------|
| 1. (b) | 2. (a) | 3. (c) | 4. (c) | 5. (c) | 6. (a) |
| 7. (a) | 8. (b) | 9. (d) | 10. (a) | 11. (d) | 12. (e) |
| 13. (b) | 14. (d) | 15. (c) | 16. 2 | 17. sum | 18. 0, 8 |

19. 2 20. $\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$

21. 0

22. All the eigen values are ≥ 0 and at least one eigen value is zero.

23. (a) $n = p$, (b) $m = p, n = q$

24. 8

25. (b)

26. $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 6 & 2 \\ 6 & 9 & 3 \end{bmatrix}$ 27. $x^2 + 4xy - 4y^2$ 28. $x = y = z = 0$ 29. 2, 2, 8
 30. $A^2 = A$ 31. 2 32. 1, 4, 9 33. (iv)
 34. 4 35. zero 36. Indefinite 37. 1–1
 38. The elements of its leading diagonal 39. 2 40. $\lambda_i, i = 1, 2, \dots, n$
 41. (c) 42. A or A^T 43. 1, 1/2, 1/3 44. $\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$
 45. 1, 1/3 46. $x = 3 - t$ 47. Symmetric ; skew-symmetric 48. 7 ; 5
 49. $\begin{bmatrix} \cos 30 & \sin 30 \\ -\sin 30 & \cos 30 \end{bmatrix}$ 50. its determinant 51. $\lambda^2 - 6\lambda + 3 = 0$ 52. Augmented matrix
 53. $\begin{bmatrix} 4 & -1 & 3 \\ -1 & -2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$ 54. $\lambda_1^3, \lambda_2^3, \lambda_3^3$ 55. $1/\lambda$ 56. $\begin{bmatrix} 1 & -0.5 & -0.5 \\ -0.5 & 1 & -0.5 \\ -0.5 & -0.5 & 1 \end{bmatrix}$
 57. 38 58. 2 59. Index = 2, Signature = 1
 60. False 61. False 62. False 63. True
 64. False 65. True 66. False 67. True
 68. False 69. True 70. True 71. True.

Problems 3.1, page 80

1. (i) $\sqrt{159}; 6/\sqrt{159}, 1/\sqrt{159}, 11/\sqrt{159}$; (ii) $\sqrt{131}; 9/\sqrt{131}, -7/\sqrt{131}, 1/\sqrt{131}$
 3. 90° 5. $x = 1, y = -1$ 11. 2 : 1.

Problems 3.2, page 88

1. 5 2. (ii) A, B, C form a Δ , rt. \angle ed at C 4. $\cos^{-1}(.62)$
 6. 2.11 7. 13; 12/13, 4/13, 3/13 13. 60°
 14. $\cos^{-1}\left(-\frac{2}{3\sqrt{21}}\right)$.

Problems 3.3, page 89

1. $\mathbf{I} - 10\mathbf{J} - 18\mathbf{K}, \frac{1}{5\sqrt{17}}\mathbf{I} - \frac{2}{\sqrt{17}}\mathbf{J} - \frac{18}{5\sqrt{17}}\mathbf{K}, \sin^{-1}\left(\frac{5\sqrt{17}}{21}\right)$ 3. $(2\mathbf{J} + \mathbf{K})/\sqrt{5}$
 5. $\frac{1}{2}\sqrt{(94)}$ 6. (b) $10\sqrt{3}$ 7. $-2/\sqrt{26}$.

Problems 3.4, page 92

1. 40 2. 17; $-24\mathbf{I} + 13\mathbf{J} + 4\mathbf{K}$ 3. 3.33
 4. 70.5 5. $2\mathbf{I} - 7\mathbf{J} - 2\mathbf{K}; \sqrt{(57)}$ 6. (1, 2, 2)
 7. 6 8. 8.25 9. $\frac{5}{6}(-3\mathbf{I} + 2\mathbf{J} + 10\mathbf{K}), \frac{5}{6}\sqrt{113}$.

Problems 3.5, page 96

1. 7 2. -4 3. (ii) Yes 4. Not linearly dependent
 5. 5/6 6. (i) 15; (ii) $1\frac{1}{3}$ 11. (i) $-7\mathbf{I} - 11\mathbf{J} + 5\mathbf{K}$; (ii) $-30\mathbf{I} - 15\mathbf{J} + 15\mathbf{K}$
 15. (b) $\frac{1}{6}abc \begin{vmatrix} 1 & \cos \psi & \cos \phi \\ \cos \psi & 1 & \cos \theta \\ \cos \phi & \cos \theta & 1 \end{vmatrix}$.

Problems 3.6, page 101

1. $2x - y + 3z = 9$
2. $\mathbf{R} \cdot (2\mathbf{I} + 2\mathbf{J} + \mathbf{K}) = 5$
3. $\frac{1}{3}(2\mathbf{I} + 2\mathbf{J} + \mathbf{K})$
4. 3
5. $4x - 3y + 2z = 3$
7. $x - 5y - 2z + 7 = 0$
8. $2; 2x + 2y - 3z = 6$
9. $\sum(x_1 - x_2) \left\{ x - \frac{1}{2}(x_1 + x_2) \right\} = 0$
10. $y = 2$
11. $3x + 4y - 5z = 9$
12. $k = 10.2; 5x - 15y - 21z = 34$
13. $1/6 \cdot 2x - 3y + 6z + 5 = 0$
14. $\cos^{-1}(\sqrt{2}/3)$
15. (i) $\frac{5}{\sqrt{83}}, \frac{-7}{\sqrt{83}}, \frac{-3}{\sqrt{83}}$ (ii) 83.7° (iii) $5x - 7y - 3z + 7 = 0$
17. $6x + 3y - 2z = 18; 2x - 3y - 6z = 6$
20. $x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$
21. $xyz = 6k^3$
22. (i) $25x + 17y + 62z - 78 = 0$; (ii) $x + 35y - 10z - 156 = 0$; (iii) bisects the acute angle.

Problems 3.7, page 105

1. $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z-4}{-2}$
2. $43^\circ 3'$
3. 90°
4. $x+2 = \frac{y-3}{2} = \frac{z-4}{2}$
5. 3
6. $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z+1}{1}$
7. $\frac{x-1}{1} = \frac{y+2}{-19} = \frac{z-3}{3.5}; \frac{x-1}{11} = \frac{y+1}{13} = \frac{z-3}{-21.5}$
8. (3, 4, 5)
9. 4.1
10. 8.57
11. (3, 4, 5); (ii) $(26/7, -15/7, 17/7)$
12. $40^\circ 15'$
13. $29x - 27y - 22z = 85$
14. $2-x = y+1 = (z+1)/3$.

Problems 3.8, page 107

1. $7x - 2y - 3z = 0$
2. $2x + 3y + 6z = 38$
3. $11x + 12y - 8z = 5$
4. $3y - z = 2$
5. $\frac{x-4}{7} = \frac{y-6}{-13} = \frac{z+2/3}{9}$
6. $x+y+2z=1, x+y+(2/5)z=1$
7. $\frac{x+4/15}{-11} = \frac{y-2/5}{9} = \frac{z}{15}$
8. $\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z-1}{1}$.

Problems 3.9, page 110

1. $x - 2y + z = 0$
2. (5, -7, 6)
3. $\frac{a\alpha + b\beta + c\gamma + d}{al + bm + cn} = \frac{a'\alpha + b'\beta + c'\gamma + d'}{a'l + b'm + c'n}$
4. (0, 1, 2); $4x + y - 2z + 3 = 0$
7. $-\frac{1}{6}(x-5) = y-3 = \frac{1}{2}(z-13)$
8. $\frac{x-2}{7} = \frac{y-3}{4} = \frac{z-1}{-5}$
9. (2, 8, -3); (0, 1, 2); 8.83.

Problems 3.10, page 113

1. $1/\sqrt{6}; 11x + 2y - 7z + 0 = 0; 7x + y - 5z + 7 = 0$
2. $10.77; \frac{1}{2}(x-3) = \frac{1}{3}(y-5) = \frac{1}{4}(z-7); (3, 5, 7); (-1, -1, -1)$
3. $\frac{1}{\sqrt{5}}; 3x - 10y + 6z - 1 = 0 = x + 2z$.

Problems 3.11, page 115

4. First and second planes cut along $x - 36 = -\frac{1}{2}(y + 22) = z$.

Problems 3.12, page 118

1. $x^2 + y^2 + z^2 - 4x + 6y - 2z + 5 = 0 ; (2, -3, -1) ; 3$
2. $x^2 + y^2 + z^2 - 2x + 2y - 4z = 0 ; (1, -1, 2) ; \sqrt{6}$
3. (a) $x^2 + y^2 + z^2 - 4x - 4y - 4z + 3 = 0$ (b) $3(x^2 + y^2 + z^2) - 2(x + y + z) - 1 = 0$
7. (i) $x^2 + y^2 + z^2 - ax - by - cz = 0$,

$$(ii) x^2 + y^2 + z^2 - ax - by - cz = 0 \text{ and } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 ; \left[\frac{a(b^{-2} + c^{-2})}{2\sum a^{-2}}, \frac{b(c^{-2} + a^{-2})}{2\sum a^{-2}}, \frac{c(a^{-2} + b^{-2})}{2\sum a^{-2}} \right]$$

8. $(1, 3, 4) ; \sqrt{7}$
10. $x^2 + y^2 + z^2 + 2(x + y + z + 1) = 0$
11. $13(x^2 + y^2 + z^2) - 35x - 21y + 43z + 176 = 0$
12. $3(x^2 + y^2 + z^2) - 7x - 8y + z + 10 = 0$
13. $x^2 + y^2 + z^2 + 7y - 8z + 24 = 0$.

Problems 3.13, page 120

1. (i) $x + 3 = 0, x - 7 = 0$ (ii) $x + 2y + 2z = 9, 2x + y - 2z = 9$
2. $x^2 + y^2 + z^2 + 2x + 4y + 6z - 11 = 0$ and $5(x^2 + y^2 + z^2) - 4x - 8y - 12z - 13 = 0$
3. (i) $x^2 + y^2 + z^2 - 10y - 10z - 31 = 0$ 4. (ii) $x^2 + y^2 + z^2 - 14(x + y + z) + 98 = 0$
6. $\frac{x - 0.6}{-2} = \frac{y - 2.4}{7} = \frac{z}{5}$ 8. $3\sqrt{6}, \sqrt{6}$
9. $3x + y + z + 6 = 0$ 10. $(12/5, 4, 9/5)$.

Problems 3.14, page 124

1. $(\beta z - \gamma y)^2 = 4a(\alpha z - \gamma x)(z - \gamma)$
2. $528x^2 + 363y^2 + 76z^2 - 528xy - 264yz + 353zx + 704x + 1352z - 4436 = 0$
3. $5x^2 + 3y^2 + z^2 - 2xy - 6yz - 4zx + 6x + 8y + 10z - 26 = 0$
4. $x^2 + y^2 - 3z^2 - 2x - 2y + 6z - 1 = 0$
5. $x^2 + y^2 = z^2 \tan^2 \alpha$
6. $x^2 + 7y^2 + z^2 + 8xy + 8yz - 16zx = 0$
7. $4x^2 + 40y^2 + 19z^2 - 48xy - 72yz + 36zx = 0$
8. $x^2 - y^2 + z^2 + 4y - 4z = 0$
9. $yz \pm zx \pm xy = 0, \cos^{-1}(1/\sqrt{3}) ; x = y/\pm 1 = z/\pm 1$
10. $\cos^{-1} 4/\sqrt{41} ; 25x^2 - 16y^2 - 16z^2 = 0$
11. $4x^2 + 4y^2 - z^2 + 20z - 100 = 0$
12. $x = y/2 = z/-1 ; x/-2 = y = z$
14. $-2x^2 + y^2 - 2z^2 + 4xy - 8xz + 4yz + 8x - 10y + 8z - 3 = 0$.

Problems 3.15, page 126

1. $5x^2 + 8y^2 + 5z^2 + 4yz + 8xz - 4xy - 144 = 0$
2. $3x^2 + 6y^2 + 3z^2 + 8yz - 2zx + 6x - 24y - 18z + 24 = 0$
3. $45x^2 + 40y^2 + 13z^2 + 12xy + 36yz - 24zx - 42x - 112y - 126z - 392 = 0$
4. $x^2 + y^2 + z^2 - yz - zx - xy = a^2$
5. $9x^2 + 5y^2 + 9z^2 + 12xy + 6yz - 36x - 30y - 18z + 36 = 0 ; \pi \text{ units}$
6. $x^2 + y^2 - 2x - 4y - 11 = 0$
7. $a(nx - lz)^2 + 2h(nx - lz)(ny - mz) + b(ny - mz)^2 + 2gn(nx - lz) + 2fn(ny - mz) + cn^2 = 0$.

Problems 3.16, page 131

1. Ellipsoid, 33.51
2. Hyperboloid of revolution of one sheet; Hyperbola $5x^2 - y^2 = 6$. No area

3. Right circular cylinder with axis along z -axis
 4. Hyperbolic paraboloid
 5. Hyperboloid of two sheets
 6. Parabolic cylinder
 7. Right circular cylinder
 8. Cone with vertex at the origin
 9. Hyperbolic paraboloid
 10. Hyperboloid of two sheets.

Problems 3.17, page 131

1. (b) 2. (a) 3. (c) 4. (b) 5. (b) 6. (a)
 7. (c) 8. (d) 9. (c) 10. (c) 11. (d) 12. (c)
 13. (b) 14. (c) 15. (c) 16. (c) 17. (c) 18. (b)
 19. (c) 20. (c) 21. (b) 22. (b) 23. (c) 24. (c)
 25. (b) 26. (a) 27. $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$ 28. 0, 0, 1 29. $x = 0, y = 0$
 30. $(-3, 2, -1)$ 31. 8 or -10 32. $\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} = 0$
 33. $y^2 + z^2 = (bx/a)^2$ 34. $12x + 31y - 20z = 66$ 35. 523.6
 36. $\cos^{-1}(6/\sqrt{42})$ 37. $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = (x + y + z)^2$ 38. $-(x - 1) = y - 2 = \frac{1}{6}(z - 3)$
 39. $(3, 5, 7), (5, 8, 11)$ 40. $x^2 + y^2 + z^2 + x - 6y - 7z + 9 = 0$
 41. $al + bm + cn = 0, ax_1 + by_1 + cz_1 + d = 0$ 42. $2/\sqrt{26}$
 43. $(3/2, -2, 2), 3\sqrt{5}/2$ 44. $\sqrt{44}/3$ 45. $\frac{x+1/3}{1} = \frac{y+2/3}{-2} = \frac{z}{1}$
 46. Parabolic cylinder 47. Hyperboloid of two sheets
 48. $\cos^{-1}\left(\frac{7}{\sqrt{84}}\right)$ 49. 6, -4, 12 50. 6
 51. True 52. True 53. True
 54. Elliptic cylinder 55. $4(x^2 + y^2 + z^2) + 9(xy + yz + zx) = 0$
 56. $(\mathbf{I} - 2\mathbf{J} - 8\mathbf{K})/\sqrt{69}$

Problems 4.1, page 135

6. $8t^3/(1-t^2)^3$ 7. $-\frac{3}{2}$ 8. $\sin t/a \cos^4 t$.

Problems 4.2, page 138

1. $(-1)^{n-1} (n-1)! 2^n [(2x+1)^{-n} + (2x-1)^{-n}]$ 2. $(-1)^n \left\{ \frac{n!}{(x+1)^{n+1}} - \frac{(n-1)!}{(x+2)^n} + \frac{(n-1)!}{(x+1)^n} \right\}$
 3. $\frac{1}{16} [2 \sin(x+n\pi/2) + 3^n \sin(3x+n\pi/2) - 5^n \sin(5x+n\pi/2)]$
 4. $\frac{1}{256} [9^n \cos(9\theta + n\pi/2) + 9.7^n \cos(7\theta + n\pi/2) + 36.5^n \cos(5\theta + n\pi/2) + 84.3^n \cos(3\theta + n\pi/2) + 126 \cos\theta]$
 5. $\frac{(20)^{n/2}}{2} [e^{2x} \sin(2x + n \tan^{-1} 2) - e^{-2x} \sin(4x - n \tan^{-1} 2)]$
 6. $\frac{1}{2} e^{5x} S(41)^{n/2} \cos[4x + n \tan^{-1}(0.8)] + (29)^{n/2} \cos[2x + n \tan^{-1}(0.4)]$

7. $\frac{(-1)^n n!}{3} \left\{ \frac{4}{(x-1)^{n+1}} \frac{1}{(x+2)^{n+1}} \right\}$

8. $(-1)^n n! \left\{ \frac{9(2)^{n-1}}{(2x+3)^{n+1}} - \frac{4}{(x+2)^{n+1}} \right\}$

9. $\frac{(-1)^n n!}{3} \left\{ \frac{4}{(x+1)^{n+1}} + \frac{i-1}{4} \frac{1}{(x+i)^{n+1}} - \frac{i+1}{4} \frac{1}{(x-i)^{n+1}} \right\}$

10. $\frac{(-1)^n n! \cos(n+1)\theta}{(x^2+a^2)^{(n+1)/2}}$ where $\theta = \tan^{-1}(a/x)$

11. $2(-1)^{n-1} (n-1)! \sin n\alpha \sin^n \alpha$ where $\alpha = \cot^{-1} x$.

Problems 4.3, page 141

1. (i) $\frac{(-1)^{n-3} (n-3)!}{x^n} [(n-1)(n-2) + n(3-n)x^2];$

(ii) $\frac{1}{256} \{(\log 2)^n 2^x (\cos 9\theta + 9 \cos 7\theta + 3 \cos 5\theta + 84 \cos 3\theta + 126 \cos \theta) + {}^n C_1 (\log 2)^{n-1} 2^x \{\cos(9\theta + \pi/2) + 9 \cos(7\theta + n\pi/2) + 36 \cos(5\theta + \pi/2) + 84 \cos(3\theta + \pi/2) + 126 \cos(\theta + \pi/2) + \dots + 2^x [\cos(9\theta + n\pi/2) + 9 \cos(7\theta + n\pi/2) + 36 \cos(5\theta + n\pi/2) + 84 \cos(3\theta + n\pi/2) + 126 \cos(\theta + n\pi/2)]\}$

5. $y_{2m}(0) = 0, y_{2m+1}(0) = (-1)^m \cdot (2m)!$

7. $(y_n)_o = 0$, if n is even
 $= m(1^2 - m^2)(3^2 - m^2) \dots [(2n-1)^2 - m^2]$, if n is odd

8. $(y_{2n})_0 = e^{m\pi/2} m^2(2^2 + m^2)(4^2 + m^2) \dots [(2n-2)^2 + m^2]$

$(y_{2n+1})_o = -e^{m\pi/2} m (1^2 + m^2)(3^2 + m^2) \dots [(2n-1)^2 + m^2]$

17. $\{m^2 - (n-2)^2\} \{m^2 - (n-4)^2\} \dots (m^2 - 2^2) m^2, n$ even

$\{m^2 - (n-2)^2\} \{m^2 - (n-4)^2\} \dots (m^2 - 1^2) m, n$ odd.

Problems 4.4, page 146

2. $x = (2m-1)a/(2m+2n-1)$

3. (i) $c = 3.154, 0.846$; (ii) $c = \pi/2$; (iii) $c = e-1$. (iv) $c = 0.5413$

6. 0.36

12. $\theta = 0.25$.

Problems 4.5, page 150

1. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \infty$

2. $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \infty$

3. $1+x - \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$

4. $x + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{x^7}{7} + \dots \infty$

5. $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \infty$

6. $\frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \dots$

24. $\frac{m \sin \theta}{1!} - \frac{m(m^2-1^2)}{3!} \sin^3 \theta + \frac{m(m^2-1^2)(m^2-3^2)}{5!} \sin^5 \theta - \dots$

25. $4 + 21(x-1) + 13(x-1)^2 + 2(x-1)^3$

26. (i) $e \left\{ 1 + \frac{(x-1)}{1!} + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!} + \dots \right\}$

(ii) $\frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3$

27. $1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!} - \dots ; .9998$

29. $\log(0.5) = \sqrt{3}(x - \pi/3) - 2(x - \pi/3)^2 - \frac{4\sqrt{3}}{3}(x - \pi/3)^3 + \dots$

30. 0.8482

31. (i) 2.6121. (ii) 1.12.

Problems 4.6, page 154

1. $\log_e(a/b)$	2. $-1/3$	3. $1/3$	4. $a \log a$	5. 1	6. $1/18$
7. $1/2$	8. $1/12$	9. $3/2$	10. 0	11. $1/30$	12. 1
13. $1/3$	14. 2	15. 1	16. 1	17. 2	18. $11e/24$
19. $a = 2; 1$	20. $a = 5, b = -5$	21. $a = 1, b = 2, c = 1$.			

Problems 4.7, page 156

1. $-1/3$	2. $1/2$	3. -2	4. $-1/3$	5. $2/3$	6. $1/e$
7. ae	8. 1	9. e	10. $1/\sqrt{e}$	11. $1/e$	12. 0
13. 0	14. 1	15. $e^{-1/6}$	16. e	17. $e^{2/\pi}$	18. $e^{1/12}$
19. $-\frac{1}{2}$	20. $(6)^{1/3}$.				

Problems 4.8, page 160

1. $x - 20y = 7; 20x + y = 140$ 2. (a, b) 10. $\pi/4$
 14. $T = 2a \sin t/2; N = 2a \tan t/2 \cdot \sin t/2; S.T. = a \sin t; S.N. = 2a \sin^2 t/2 \cdot \tan t/2$
 15. $a \sin^3 \theta \tan \theta$.

Problems 4.9, page 162

7. (i) $\pi/2$; (ii) $\pi/2$.

Problems 4.10, page 166

4. $r^3 = 2ap^2$ 5. $r^3 = a^2p$ 6. $pa^m = r^{m+1}$ 7. $r^{m+1} = \sqrt{2} a^m p$
 8. $(1+m^2)p^2 = r^2$ 9. (i) $\sqrt{1+9x/4a}$; (ii) $\cosh x/c$ 10. $a\theta$
 11. (i) $2a \sin \theta/2$; (ii) $a\sqrt{\sec 2\theta}$; (iii) $r\sqrt{(8r-3)}$.

Problems 4.11, page 172

1. (i) $2a(1+t^2)^{3/2}$; (ii) y^2/c (iii) $(1+a^3)^{3b}/6a^2$
 5. (i) $(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^{3/2}/ab$; (ii) $4a \sin \theta/2$; (iii) at
 11. (i) $3/2$; (ii) 1; (iii) $\sqrt{2a}$
 12. (i) $\frac{4a}{3} \sin \frac{\theta}{2}$; (ii) $a^n/(n+1)r^{n-1}$ 14. $2\sqrt{(r^3/a)}$

Problems 4.12, page 176

1. $a(2+3t)t^2, -4\sqrt{2}at^{3/2}$ 4. (i) $x = a(t - \sin t), y - 2a = a(1 + \cos t)$, (ii) $x = a \cos \theta, y = a \sin \theta$
 5. $(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$
 7. (i) $(x-3a/4)^2 + (y+3a/4)^2 = a^2/2$ (ii) $x^2 + y^2 - \frac{21}{8}(x+y) + \frac{432}{128} = 0$
 11. $y^2 = 4ax$ 12. $(x/a)^2 + (y/b)^2 = 1$ 13. $27ay^2 = 4(x-2a)^3$.

14. $(x/a)^2 + (y/b)^2 = 1$

15. $y = \frac{u^2}{2g} - \frac{gx^2}{2u^2}$

16. (i) $\sqrt{x} + \sqrt{y} = \sqrt{c}$; (ii) $4xy = c^2$; (iii) $x^{2/3} + y^{2/3} = c^{2/3}$

17. $x^{2/3} + y^{2/3} = c^{2/3}$

Problems 4.13, page 181

2. $a = 1, b = 1/4$, Point of minima 4. $x = 0.42l$

8. $\theta = \frac{\pi}{4} - \frac{\alpha}{2}; (1 - \sin \alpha)/(1 + \sin \alpha)$

13. $8 + 2\sqrt{7}, 2 + 2\sqrt{7}, 5 - \sqrt{7}$ 14. Depth is half the width

16. $3\sqrt{3}a/4$ 25. 2.5 km/hr.

5. $v = (aw^2/3b)^{1/4}$

10. Sq. with side $\sqrt{2}a$

15. $(a^{2/3} + b^{2/3})^{3/2}$

Problems 4.14, page 185

1. $x + y + a = 0$

2. $x = \pm a, y = \pm b$

3. $x = \pm a, y = \pm b$

4. $y = 0; x + 1 = 0; x + y = 0$

5. $y = x, y + 2x = 0, y + 2x + 1 = 0$

6. $x + a = 0; x - a = 0; x - y + \sqrt{2}a = 0; x - y - \sqrt{2}a = 0$

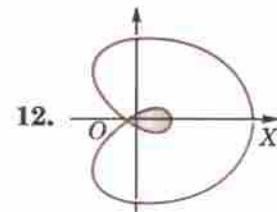
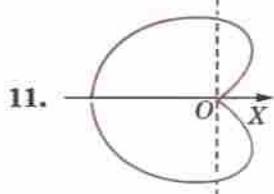
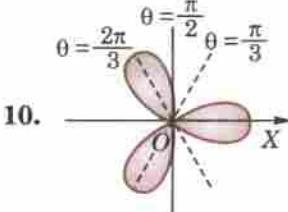
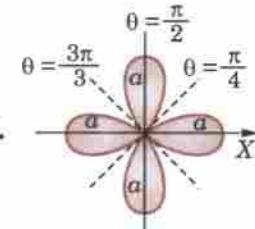
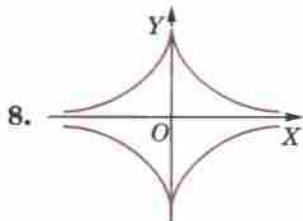
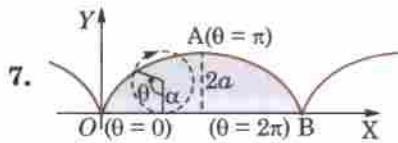
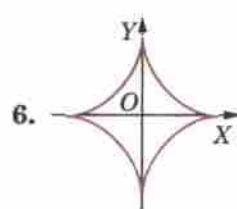
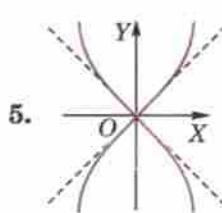
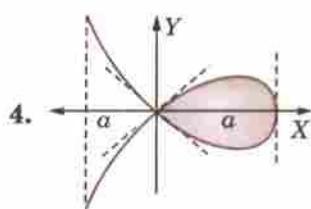
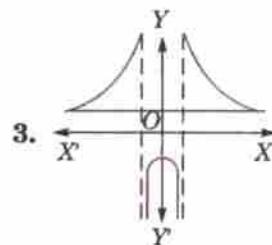
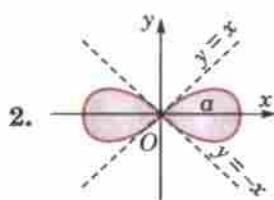
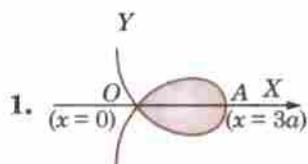
7. $x + 2y + 2 = 0, x + y = \pm 2\sqrt{2}$ 10. $r \cos \theta = a; r \cos \theta = -a$

11. $r \cos \theta = 0; r \cos \theta = 2a$

12. $r \sin \theta = 2$

13. $r \sin(\theta - m\pi/n) = a/n \cos m\pi$.

Problems 4.15, page 194



Problems 4.16, page 194

1. c 2. $x^2 + 4ay = 0$ 3. $1/5$ 4. (c) 5. (a) 6. (b)
 7. (c) 8. (b) 9. (b) 10. (c) 11. (b) 12. (c)
 13. (c) 14. (b) 15. $x^2 = 4y$ 16. of constant length
 17. $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ 18. $\frac{1}{4} \left[2^n \cos\left(2x + n \frac{\pi}{2}\right) + 4^n \cos\left(4x + n \frac{\pi}{2}\right) + 6^n \cos\left(6x + n \frac{\pi}{2}\right) \right]$
 19. $-32/3a$ 20. True 21. $-a$ 22. $2a(1+t^2)^{3/2}$
 23. $(x-a)^2 + (y-b)^2 = k^{-2}$ 24. envelope 25. $xy = c^2$
 26. α 27. $2a$ 28. $e^x(x^3 - 12x^2 - 36x - 24)$
 29. (iii) 30. $(x/a)^2 + (y/b)^2 = 1$ 31. (B)
 32. $c = 2.5$ 33. $x = y$ 34. node
 35. Four loops of $r = a \sin 2\theta$ and three loops of $r = a \cos 3\theta$.
 36. $y = \pm x$ 37. $x = 4$ 38. 4b
 39. (A) 40. $r > a$ 41. (D)
 42. (D) 43. (C).

Problems 5.1, page 198

1. $2/3$ 2. Does not exist 3. Zero 4. Does not exist
 7. Discontinuous.

Problems 5.2, page 202

1. (i) $xy(2 - \cos xy) - \sin xy ; x^2(1 - \cos xy) ;$
 (ii) $2x/(x^2 + y^2), 2y/(x^2 + y^2) ;$
 (iii) $(x^2 + 2xy - y^2)/[(x^2 + y^2)^2 + (x + y)^2] ; (y^2 + 2xy - x^2)/[(x^2 + y^2)^2 + (x + y)^2] ;$
 (iv) $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = \frac{z}{1-z}$
11. $n = 2, -3$ 18. $e^{xyz}(x^2y^2z^2 + 3xyz + 1).$

Problems 5.4, page 208

13. $2u.$

Problems 5.5, page 211

2. $4a^2t(t^2 + 2)$ 2. $-2/(e^{2t} + e^{-2t})$ 3. zero
 4. 6.5 sq. ft./sec 6. $8e^{4t}.$

Problems 5.6, page 214

9. $0 ; 0.$

Problems 5.7, page 218

6. zero 7. $x(yv + 1 - w) + z - 2uv$ 10. $0 ; u = \tan v$
 11. $u^2 - v^2 = 8w$

Problems 5.8, page 220

1. $4X + Y + Z = 6$; $\frac{X - 2}{4} = Y - 1 = Z + 3$ 2. $3Y + 2Z - X - 3 = 0$, $1 - X = \frac{Y - 2}{3} = \frac{Z + 1}{2}$
3. $\frac{X}{x_1} + \frac{Y}{y_1} + \frac{Z}{z_1} = 3$; $x_1(X - x_1) = y_1(Y - y_1) = z_1(Z - z_1)$
4. $7X - 3Y + 8Z = 26$; $\frac{X - 1}{7} = \frac{Y + 1}{-3} = \frac{Z - 2}{8}$
5. $(-1, 2, 2/3)$ 7. $\frac{X - x}{x} = \frac{Y - y}{y} = \frac{Z - z}{z}$.

Problems 5.9, page 226

1. (i) $x - \frac{1}{6}(x^3 + 3xy^2)$
(ii) $\frac{1}{2\sqrt{2}} \left[1 + \{(x+1) + (y-\pi/4)\} + \frac{1}{2} \{(x+1)^2 - 2(x+1)(y-\pi/4) + (y-\pi/4)^2\} \right. \\ \left. + \frac{1}{6} \{(x+1)^3 + 3(x+1)^2(y-\pi/4) - 3(x+1)(y-\pi/4)^2 - (y-\pi/4)^3\} + \dots \right]$
(iii) $1 + x + \frac{1}{2!}(x^2 - y^2) + \frac{1}{6}(y^3 - 3xy^2) + \dots$
2. $1 + (x-1) + (x-1)(y-1) + \frac{1}{2}(x-1)^2(y-1) + \dots$
3. -0.8232 4. -4500 units 5. 2% 7. 2%
11. $\frac{\alpha}{2} \cot \alpha + 2$ 12. Rs. 43.20 13. $(p - 3q - 4r)\%$ 14. $-1\frac{1}{3}\%$ 15. $5r$.

Problems 5.10, page 233

1. (i) (a, a) gives maximum if $a < 0$ and minimum if $a > 0$
(ii) Min. at (a, a) (iii) Max. at $(4, 0)$, Min. at $(6, 0)$
(iv) Max. at $(\pm 1, 0)$; Min. at $(0, \pm 1)$ (v) Max. at $(\pi/3, \pi/3)$; Min. at $(2\pi/3, 2\pi/3)$
2. $4, 2, 1$ 3. (i) $3a^2$; (ii) $p^2/(a^2 + b^2 + c^2)$; (iii) $3a^2$ 4. $12 \times 12 \times 6$ cm
6. $(0, 0, \pm 1)$ 8. 4, 1 9. 50
10. $4, 8, 12$ 11. Two stationary values of u are given by $\frac{l^2}{au-1} + \frac{m^2}{bu-1} + \frac{n^2}{cu-1} = 0$.

Problems 5.11, page 236

1. $\frac{1}{2a^3} \tan^{-1} \frac{x}{a} + \frac{x}{2a^2(x^2 + a^2)}$ 2. $\frac{(-1)^n n!}{(m+1)^{n+1}}$
3. $\pi \log \left[\frac{1}{2} + \frac{1}{2} \sqrt{(1-a^2)} \right]$ 4. $-\pi/(a^2 - 1)^{3/2}$.

Problems 5.12, page 236

- | | | | | | |
|---------|---------|---------|---------|----------|---------------|
| 1. zero | 2. (a) | 3. 1 | 4. (b) | 5. (b) | 6. (b) |
| 7. (c) | 8. (c) | 9. (d) | 10. (d) | 11. (b) | 12. (d) |
| 13. (a) | 14. (d) | 15. (c) | 16. (b) | 17. zero | 18. $2/(x+y)$ |

19. $rt - s^2 < 0$ 20. (d) 21. $4u$ 22. $\partial(u, v)/\partial(x, y)$
 23. $f_x(a, b) = 0, f_y(a, b) = 0$ 24. (c) 25. $\frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt}$
 26. (c) 27. -1 28. (c) 29. equal
 30. False.

Problems 6.1, page 244

1. (i) $128/315$; (ii) $8/45$ 2. (i) $128/315$; (ii) $11\pi/192$
 3. (i) $\frac{(2n-3)(2n-5)}{(2n-2)(2n-4)} \dots \frac{3.1}{4.2} \frac{\pi}{2}$ (ii) $\frac{1}{8} \left(\frac{\pi}{8} + \frac{1}{6} \right)$ 4. $35\pi/10240$
 5. (i) $3\pi/512$ (ii) $1/144$ 6. (i) $5\pi/256$; (ii) $1/15$ 7. (i) $35\pi a^4/8$; (ii) $5\pi a^3/2$
 8. (i) $5\pi/8$; (ii) 28π .

Problems 6.2, page 247

1. (i) $\frac{1}{5} \tan^5 x - \frac{1}{3} \tan^3 x + \tan x - x$ (ii) $-\frac{1}{4} \cot^4 x + \frac{1}{2} \cot x + \log \sin x$
 3. $\frac{1}{2} \log 2 - \frac{1}{4}$ 4. $\frac{\pi}{4} - \frac{2}{3}$ 5. $I_n = \frac{(2)^{(n-2)/2}}{n-1} + \frac{n-2}{n-1} + I_{n-2}$
 6. (i) $\frac{1}{5} \sec^4 x \cdot \tan x + \frac{4}{15} (\sec^2 x + 2) \tan x$ (ii) $\frac{11\sqrt{3}}{4} + \frac{3}{8} \log(2 + \sqrt{3})$
 (iii) $-\frac{1}{4} \cot x \operatorname{cosec} x - \frac{3}{8} \cot x \operatorname{cosec} x + \frac{3}{8} \log(\operatorname{cosec} x - \cot x)$
 7. $\left\{ \frac{67\sqrt{2}}{48} + \frac{5}{16} \log(1 + \sqrt{2}) \right\} a^6$ 8. $\frac{t^5}{5} - \frac{t^3}{3} + t - \tan^{-1} t$.

Problems 6.3, page 250

1. $e^x (1 - x + x^2 - x^3 + x^4)$
 3. $\int x^m (\log x)^n dx = \frac{x^{m+1}}{m+1} (\log x)^n - \frac{n}{m+1} \int x^m (\log x)^{n-1} dx$; $\int_0^1 x^5 (\log x)^3 dx = -1/216$
 5. $149/225$ 6. $3\pi^2/64 - 1/4$ 7. $\frac{5}{16}\pi^4 - 15\pi^2 + 120$
 11. $I_n = \frac{e^{ax} \cos^{n-1} x (a \cos x + n \sin x)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2} I_{n-2}$; $\int_0^{\pi/2} e^{2x} \cos^3 x dx = \frac{2}{65} (3e^\pi - 11)$
 12. $24/85$.

Problems 6.4, page 254

7. (i) $3\pi/8$; (ii) $5\pi/8$; (iii) $3\pi/256$; (iv) $15\pi/640$ 8. (i) $16\pi/35$; (ii) $8\pi/315$.

Problems 6.5, page 256

1. $\log 2$ 2. $\frac{1}{3} \log 2$ 3. $1/3$ 4. $\pi/2$ 5. $\frac{1}{4} \log 2$ 6. $2e^{(\pi-4)/2}$.

Problems 6.6, page 260

1. (i) πab ; (ii) $8a^2/3$ 2. $21\frac{1}{12}$ 3. $2a^2/5$

Problems 6.7, page 262

- $$1. (i) 3\pi a^2/2; \quad (ii) a^2 \qquad 2. (i) \pi a^2/8; \quad (ii) \pi a^2/12 \qquad 5. (1 - \pi/4)a^2 \qquad 6. \pi a^2/2.$$

Problems 6.8, page 265

- 1.** $12\frac{11}{27}a$ **2.** (i) $\log(2 + \sqrt{3})$, (ii) $\log_e(e + 1/e)$

3. (i) $a[\sqrt{2} + \log(1 + \sqrt{2})]$; (ii) $(15/16 + \log 2)a$ **4.** (i) $4a/\sqrt{3}$; (ii) $4\sqrt{3}$

5. 37.85 **7.** (i) $8a$ **8.** $6a$

9. $4\sqrt{3}$ **11.** $2 + \frac{1}{2}\log 3$ **12.** $8a$

13. $\sqrt{2}\pi a \left\{ 1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 \cdot \left(\frac{1}{2}\right)^2 + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 \cdot \left(\frac{1}{2}\right)^2 + \dots \right\}$ **14.** $2a[\sqrt{2} + \log(\sqrt{2} + 1)]$.

Problems 6.9, page 269

1. $\pi e^3(1 + \sinh 1 \cosh 1)$
 2. $\pi h^2(a - h/3)$
 3. $2\pi a^3$
 4. $\pi a^3/12$
 5. (i) $\frac{4}{3} \pi ab^2$; (ii) $\frac{4}{3} \pi a^2 b$
 6. $\frac{\pi h}{3}(r^2 + rR + R^2)$
 7. $48 \pi a^3$
 8. (i) $2\pi a^3(\log 2 - 2/3)$; (ii) $\pi a^3/24$; (iii) $\pi/48$
 9. (i) $5\pi^2 a^3$; (ii) $5\pi^2 a^3$
 10. $32 \pi a^3/105$
 11. $4\pi^2 a^3$
 13. (i) $\frac{4}{3} \pi a^3$; (ii) $\frac{8}{3} \pi a^3$
 14. $\frac{4}{3} \pi a(a^2 + b^2)$
 15. $\frac{\pi a^3}{4} \left\{ \frac{1}{\sqrt{2}} \log (\sqrt{2} + 1) - \frac{1}{3} \right\}$.

Problems 6.10, page 271

1. $\frac{\pi a^2}{2} (2 + \sinh 2)$
 2. $\frac{8\pi a^2}{3} (2\sqrt{2} - 1)$
 3. $2\pi ab \left\{ \frac{b}{a} + \frac{a}{\sqrt{(a^2 - b^2)}} \sin^{-1} [\sqrt{(a^2 - b^2)/a}] \right\}$
 4. $\frac{1}{2}\pi r^2 h ; \pi r \sqrt{(r^2 + h^2)}$, where r is the base-radius and h the height of the cone
 5. $4\pi a^2$
 8. $\frac{64}{3}\pi a^2$
 9. $\frac{64}{3}\pi a^2$
 12. $4\pi a^2$
 13. $\frac{32}{5}\pi a^2$
 14. $4\pi a^2(1 - 1/\sqrt{2})$
 15. $\pi a^2 [3\sqrt{2} - \log (\sqrt{2} + 1)]$.

Problems 6.11, page 271

- 1.** (b) **2.** (c) **3.** (b) **4.** (c) **5.** (b) **6.** (c)
7. (c) **8.** (d) **9.** (b) **10.** (d) **11.** (c) **12.** (c)
13. (a) **14.** $\frac{3\pi a^2}{2}$ **15.** (iii) **16.** $\pi a^2/12$ **17.** 1 **18.** $7\pi/8$

19. (iii)

20. (iii)

21. $\pi h(r_1^2 + r_1 r_2 + r_2^2)$

22. (c)

23. (a)

24. (b) or (c)

25. (a).

Problems 7.1, page 280

1. 13

2. 3/35

3. $\frac{1}{2}(e - 1)$

4. $\frac{1}{4}\pi \log(1 + \sqrt{2})$

5. $a^4/8$

6. $\frac{\pi}{4}ab(a^2 + b^2)$

7. 3/56

8. $\pi a/4$

9. 241/60

10. $1 - 1/\sqrt{2}$

11. $\frac{\pi a^2}{4}(\log e - \frac{1}{2})$

12. 1/24

13. $\frac{2}{3}a^4$

14. $\pi a^2/b$

15. 1

16. (i) $\int_0^{2a} \int_{y\sqrt{2a}}^{2a} f(x, y) dx dy - \int_0^a \int_{a-\sqrt{(a^2-y^2)}}^{a+\sqrt{(a^2-y^2)}} f(x, y) dx dy,$ (ii) $\int_0^{\pi/2} \int_a^{ae^{\theta/2}} f(r, \theta) r dr d\theta$

18. $4a^2/3$

19. $45\pi/2.$

Problems 7.2, page 283

1. 4.5

2. 7/6

3. πa^2

4. $\frac{3}{2} \log_e 3 - \frac{2}{3}$

5. a^2

6. $a^2(1 - \pi/4)$

7. 4/3

8. $a^2/4.$

Problems 7.3, page 284

1. $\frac{abc}{3}(a^2 + b^2 + c^2)$

2. $\frac{8}{3}abc(a^2 + b^2 + c^2)$

3. 4/35

4. $\frac{1}{8}e^{4a} - \frac{3}{4}e^{2a} + e^a - \frac{3}{8}$

5. $\frac{8}{3} \log 2 - \frac{19}{9}$

6. $\frac{1}{4}(13 - 8e + e^2)$

7. $5\pi a^3/64.$

Problems 7.4, page 291

1. $\pi/8$

2. $\pi/2$

3. $8\left(\frac{\pi}{2} - \frac{5}{3}\right)a^2$

4. $\frac{2^{n+3}}{n+4}$

5. $\frac{\pi}{4} - \frac{1}{2}$

6. 0

7. $\frac{15\pi a^4}{64}$

9. π

10. $\pi^2/8$

11. $4\pi a$

12. $\frac{1}{2}\left(\log 2 - \frac{5}{8}\right)$

14. $\pi a^8/12$

15. 3π

16. 4π

17. $\pi a^3(2 - \sqrt{2})/3$

18. $16a^3/3$

19. $\pi a^3/8$

20. $128a^3/15$

21. $3\pi a^3$

22. $4\sqrt{3}\pi$

23. $8a^4/3$

25. $\frac{1}{4}$

26. $\frac{1}{6}abc.$

Problems 7.5, page 293

2. 64

3. $2(\pi - 2)a^2$

4. $2\pi a^2$

5. $\frac{3\pi a^2}{4}.$

Problems 7.6, page 297

1. $182 \frac{7}{24} \lambda$

2. $21 \pi \mu a^4 / 32$

3. 30.375

4. $\left(\frac{3a}{20}, \frac{3a}{16} \right)$

5. $\left[\frac{a(4a+3b)}{6(a+b)}, \frac{b(3a+b)}{6(a+b)} \right]$

6. $\left(\frac{\pi a \sqrt{2}}{8}, 0 \right)$

8. $\bar{x} = 3a/5$, $\bar{y} = 9a/40$ where $a = OA$ 9. $(1/5, 1/5, 2/5)$.

10. $\bar{x} = 3/4$

11. $\left(\frac{16a}{35}, \frac{16b}{35}, \frac{16c}{35} \right)$

12. $\frac{27}{26}$ metres

13. $\left(\frac{3a}{8}, \frac{3\pi a}{16} \right)$

14. Divides the diagonal in the ratio 7 : 5

15. $\left(\frac{a}{2}, \frac{2}{3} h \right)$ where a is the base, h the depth

16. C.P. lies on the radius \perp to the bounding diameter at a depth $32a/(15\pi)$ from the centre.

Problems 7.7, page 301

1. $ab^3/12$

2. $5Ma^2/4$

3. $2M/9$

4. $\frac{1}{3}M(a^2 + b^2)$

5. $(21/32)\pi\rho a^4$

6. $\frac{2}{5}MR^2$

7. $\frac{1}{2}Mr^2 ; \frac{1}{12}M(3r^2 + 4h^2)$

8. (i) $\frac{3Mr^2}{10}$; (ii) $\frac{3M}{20}(r^2 + 4h^2)$; (iii) $\frac{M}{20}(3r^2 + 2h^2)$

9. 104803770ρ

10. $\frac{1}{30}$

11. $\frac{\pi\rho abc(a^2 + b^2)}{30}$ 12. $\frac{\rho a^2 b^2}{8}$.

Problems 7.8, page 309

1. (i) 3.323, (ii) 11.629; (iii) $\pi\sqrt{2}$; (iv) 0.1964; (v) 0.1227

2. (i) $\sqrt{\pi}/2$; (ii) $\Gamma(5/4)$; (iii) $\sqrt{\pi}/3$; (iv) $2^{p+q-1}\beta(p, q)$

4. $\pi/4\sqrt{2}$

7. $-3/8$

9. $\frac{\Gamma\left(\frac{m+1}{n}\right)\Gamma(p+1)}{n\Gamma\left(\frac{m}{n}+p+1+\frac{1}{n}\right)}$, (i) $\frac{1}{396}$ (ii) $\frac{\sqrt{\pi}}{n} \cdot \frac{\Gamma\left(\frac{1}{n}\right)}{\Gamma\left(\frac{1}{n}+\frac{1}{2}\right)}$

10. $16/35$

15. $\frac{ka^2b^2c^2}{48}$.

Problems 7.9, page 312

2. $\frac{1}{2}K(\sqrt{3}/2)$

3. $\frac{2}{\sqrt{3}} \left\{ F\left(\sqrt{\frac{2}{3}}, \frac{1}{2}\pi\right) - F\left(\sqrt{\frac{2}{3}}, \frac{1}{4}\pi\right) \right\}$

4. $2\sqrt{2}E(1/\sqrt{2}) - \sqrt{2}K(1/\sqrt{2})$

5. $erf(x) = \frac{2}{\pi} \left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \dots \right)$; $erf(0) = 0$ 6. (i) 0.3248; (ii) 0.5204.

Problems 7.10, page 313

1. 4

2. Area of the triangle having vertices (0, 0), (0, 1), (1, 0)

3. $\sqrt{\pi}/2$

4. 3.1416

5. $15\sqrt{\pi}/8$

6. 92π

6. 26

8. $-1/3$

9. $1/2\beta(4, 3/4)$

10. 1

11. $3/4$ 12. $\frac{\pi a^3}{6}$ 13. (d) 14. $27/4$
 15. $\pi a^2/12$ 16. $-1/3$ 17. ∞ 18. $44/105$
 19. $r^2 \sin \theta dr d\theta d\phi$ 20. $e^2 - 1$ 21. $\frac{1}{4} \pi ab (a^2 + b^2)$. 22. $\frac{1}{4} \pi \log(1 + \sqrt{2})$
 23. $3/256$ 24. (c) 25. $\frac{6}{25} + \frac{1}{2} \sin \frac{3}{5}$ 26. $48/5$
 27. $1/6$ 28. $16/3$ 29. $\int_0^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} f(x, y) dy dx$
 30. $\sqrt{2} \pi$ 31. $3\pi/256$ 32. $1/2$ 33. $\int_0^{\pi/2} \int_0^{2a \cos \theta} r d\theta dr$
 34. $\frac{7h}{12}$ 35. $\left(\frac{3}{20}, \frac{3}{16}\right)$ 36. 16 37. $3Mr^2/10$
 38. 1 39. $\log 2$ 40. $\frac{1}{2} \sqrt{\pi}$ 41. (c) 42. (b).

Problems 8.1, page 318

3. (i) $t^3 \sin t + 7t^2 \cos t + 20t \sin t - 10t$; (ii) $(20t^3 + t \sin t - \cos t)\mathbf{I} - (2t \cos t + 2 \sin t + 75t^2)\mathbf{J} - t(t \sin t + 2t^2 \cos t + 10 \cos t)\mathbf{K}$
 5. $-4(\mathbf{I} + 2\mathbf{J})$ 6. (i) $(ua^2 \sec \alpha)$. (ii) $a^3 \tan \alpha$; $(\cos t \mathbf{J} - \sin t \mathbf{I}) \cos \alpha + \sin \alpha \mathbf{K}$
 7. $|t\mathbf{I} + 2\mathbf{J} + (2t - 3)\mathbf{K}|/\sqrt{(5t^2 - 12t + 13)}$; $\frac{1}{3}(2\mathbf{I} + 3\mathbf{J} + \mathbf{K})$
 8. $(x - a/\sqrt{2}) = y - a/\sqrt{2} = \left(z - \frac{a\pi}{4} \tan \alpha\right)/\sqrt{2} \tan \alpha$
 9. (i) $ab/(a^2 \sin^2 t + b^2 \cos^2 t)^{3/2}$; (ii) $1/4\sqrt{2}$
 10. (i) $\mathbf{R} = (p+q)\mathbf{I} + q\mathbf{J} + 2q\mathbf{K}$; $\frac{2\mathbf{J} - \mathbf{K}}{\sqrt{5}}$
 (ii) $\mathbf{R} = p\mathbf{I} + (p+2q)\mathbf{J} + (p+q)\mathbf{J} + (p+q)\mathbf{K}$; $(2\mathbf{K} - \mathbf{I} - \mathbf{J})/\sqrt{6}$.

Problems 8.2 page 321

1. $v(\text{at } t=0) = \sqrt{37}$, $a(\text{at } t=0) = \sqrt{325}$ 2. $\alpha = \pm 1/\sqrt{6}$
 3. $8\sqrt{14/7}; -\sqrt{14/7}$ 6. (a) d^2s/dt^2 ; v^2/p ; (b) 0; 3
 7. $\sqrt{17}$ m.p.h. in the direction $\tan^{-1}(0.25)$ North of East
 8. 21.29 knots/hr. in the direction $74^\circ 47'$ South of East.

Problems 8.3, page 325

1. (a) $2(x\mathbf{I} + y\mathbf{J} + z\mathbf{K})/(x^2 + y^2 + z^2)$. (b) $\frac{6}{3} + c$ 2. $(-\mathbf{I} + 3\mathbf{J} + 2\mathbf{K})/\sqrt{14}$
 3. $12\frac{1}{3}$ 4. $15/\sqrt{17}$ 5. $a = 6, b = 24, c = -8$
 6. $-260/(69); \sqrt{1056}$ 7. $a = \pm \frac{20}{9}, b = \pm \frac{55}{9}, c = \pm \frac{55}{9}$ 8. $96(\mathbf{I} + 3\mathbf{J} - 3\mathbf{K}); 96\sqrt{19}$
 9. 9 10. $\frac{1}{3}(2\mathbf{I} + 2\mathbf{J} - \mathbf{K})$ 11. $\cos^{-1}(1/\sqrt{22})$
 12. $\cos^{-1}(-1/\sqrt{30})$ 13. $a = -6, b = -10$.

Problems 8.4, page 333

1. (i) $12; 5\mathbf{I} - 16\mathbf{J} + 9\mathbf{K}$; (ii) $278; 5(27I - 54J + 8K)$; (iii) $-32; 0$
 4. $a = -2; 4x(z - xy)\mathbf{I} + (y - 2yz + 4xy^2)\mathbf{J} + (2x^2 + y^2 - z^2 - z)\mathbf{K}$
 13. (i) 0 ; (ii) $2(x+z)\mathbf{J} + 2y\mathbf{K}$ 14. (a) $2n(2n-1)x^2 + y^2 + z^2)^{n+1}; n = 1/2$
 16. (i) $2(y^3 + 3x^2y - 6xy^2)z\mathbf{I} + 2(3xy^2 + x^3 - 6x^2y)z\mathbf{J} + 2(xy^2 + x^3 - 3x^2y)y\mathbf{K}$; (ii) Zero
 17. $1724/\sqrt{21}$.

Problems 8.5, page 335

1. $75\frac{1}{3}\mathbf{I} + 360\mathbf{J} - 42\mathbf{K}$ 2. $(t^3 - t + 2)\mathbf{I} + (1 - t^4)\mathbf{J} + (4 - 4 \cos t - 3t)\mathbf{K}$
 3. $\mathbf{V} = 6 \sin 2t\mathbf{I} + 4(\cos 2t - 1)\mathbf{J} + 8t^2\mathbf{K}; \mathbf{R} = 3(1 - \cos 2t)\mathbf{I} + 2 \sin 2t\mathbf{J} + \frac{8t^3}{3}\mathbf{K}$.

Problems 8.6, page 336

1. 0 2. 35 3. $-2/3$ 4. 5 5. $\frac{\pi^3\sqrt{2}}{3}$ 6. zero
 7. 303 8. $8\frac{8}{35}$ 9. 9π 10. $\left(2 - \frac{\pi}{4}\right)\mathbf{I} - \left(\pi - \frac{1}{2}\right)\mathbf{J}$.

Problems 8.7, page 339

2. $3\frac{1}{3}$ 3. 8.

Problems 8.8, page 341

3. πab 4. πa^2 5. Zero 6. $128/5$ 7. $35\pi a^4/16$.

Problems 8.9, page 345

3. $-2ab^2$ 5. $\frac{19}{2}\pi$ 6. Zero 10. 2 11. 0 12. π .

Problems 8.10, page 350

4. 108π 7. (i) $\frac{12}{5}\pi a^5$ (ii) $12(e - e^{-1})$ 8. $4\pi a^3$
 9. $\frac{\pi a^6}{12}$ 10. $\frac{5}{4}\pi a^4 b$ 11. -4π 12. $8/3$.

Problems 8.11, page 354

3. $14\frac{2}{3}$ 4. $x^3y - y^2z^2 + z^3$
 5. (i) $\frac{1}{3}(x^3 + y^3 + z^3 - 3xyz)$; (ii) $x^2y + y^2z + z$;
 (iii) $xz^3 - yz + 3x^2y$. (iv) $x^2y^2 + y^2z^2 + xyz = 0$
 6. (i) Yes, $\frac{a}{2}(x^2 + y^2 - 2z^2)$; (ii) Yes
 7. $xy \sin z + \cos x + y^2 z + c$ 8. $x^2y + xz^3; 202$
 9. $a = 4, b = 2, c = -1$ 10. $a = 4; 2x^2y - xz^3; 47$.

Problems 8.12, page 362

- (i) $(\rho \sin 2\phi - z \sin \phi) \mathbf{T}_\rho - (2\rho \sin^2 \phi + z \cos \phi) \mathbf{T}_\phi + 3\rho \cos \phi \mathbf{T}_z$
(ii) $(2\rho \cos^2 \phi - 3\rho^2 \sin^3 \phi) \mathbf{T}_\rho - (\rho \sin 2\phi + 3\rho^2 \sin^2 \phi \cos \phi) \mathbf{T}_\phi + \rho z \cos \phi \mathbf{T}_z$
- (i) $r \sin \theta [(\sin \theta (1 + \sin^2 \phi) + r \cos^2 \theta \sin \phi) \mathbf{T}_r + (\cos \theta (1 + \sin^2 \phi) - r \sin \theta \cos \theta \sin \phi) \mathbf{T}_\theta + \sin \phi \cos \phi \mathbf{T}_\phi]$
(ii) $r^2 \sin \theta [(\sin^2 \theta \cos^2 \phi \sin \phi + \sin \theta \cos \theta \sin^2 \phi + \cos^2 \theta \cos \phi) \mathbf{T}_r + (\sin \theta \cos \theta \cos^2 \phi \sin \phi + \cos^2 \theta \sin^2 \phi - \sin \theta \cos \theta \cos \phi) \mathbf{T}_\theta + (\cos \theta \sin \phi \cos \phi - \sin \theta \sin^2 \phi \cos \phi) \mathbf{T}_\phi]$
- $\rho z \sin 2\phi \mathbf{T}_\rho + \rho z \cos 2\phi \mathbf{T}_\phi + \frac{1}{2} \rho^2 \sin 2\phi \mathbf{T}_z$.

Problems 8.13, page 363

- | | | | |
|--|---|--|---|
| 1. $1/\sqrt{14}$, $2/\sqrt{14}$, $3/14$ | 2. $\frac{1}{4}(x-2) = y-1 = z+3$ | 3. $dudv = \frac{1}{h_1 h_2} dxdy$ | |
| 4. $4x - 3z + 2xz$ | 5. zero | 6. $\frac{1}{2} \int_C (xdy - ydx)$ | |
| 7. 3V | 8. 3 ; 0 | 9. Irrotational | |
| 10. 4π | 11. solenoidal | 12. $-28/\sqrt{5}$ | 13. zero. |
| 14. $-(y\mathbf{I} + z\mathbf{J} + x\mathbf{K})$. | 15. $\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$. | 16. zero | 17. $-(12\mathbf{I} + 5\mathbf{J} + 8\mathbf{K})$ |
| 18. zero | 19. zero | 20. zero | 21. zero |
| 22. \mathbf{R}/r^2 ; $nr^{n-2} \mathbf{R}$ | 23. §8.5(2) | 24. $\frac{1}{\sqrt{21}} (2\mathbf{I} + 4\mathbf{J} - \mathbf{K})$. | 25. 2, -2, 2 |
| 26. $6\frac{\sqrt{7}}{3}$ | 27. $2/r$ | 28. 7/3 | 29. zero |
| 30. (c) | 31. (c) | 32. (b) | 33. (c) |
| 34. (a) | 35. (a) | 36. $5u$ | 37. zero |
| 38. irrotational field | 39. (a) | 40. the rate at which fluid is originating at P per unit volume. | |
| 41. (a) | 42. it gives the maximum rate of change of ϕ . | 43. (iv) | |
| 44. (a) | 45. (a) | 46. (b) | 47. (a) |
| 48. (b) | 49. zero : | 50. True | 51. True. |

Problems 9.1, page 366

- | | | | |
|---------------|---------------|---------------|---------------|
| 1. Convergent | 2. Convergent | 3. Convergent | 4. Divergent |
| 5. Convergent | 6. Convergent | 7. Convergent | 8. Divergent. |

Problems 9.2, page 367

- | | | | |
|---------------|---------------|----------------|---------------|
| 1. Convergent | 2. Convergent | 3. Oscillatory | 4. Convergent |
| 5. 15 m. | | | |

Problems 9.3, page 372

- | | | | |
|---------------|--|----------------|----------------|
| 1. Convergent | 2. Convergent | 3. Divergent | 4. Divergent |
| 5. Convergent | 6. Conv. for $p > 2$; Div. for $p \leq 2$. | | 7. Divergent |
| 8. Convergent | 9. Convergent | 10. Convergent | 11. Convergent |

12. Divergent

16. Convergent

13. Divergent

17. Divergent

14. Convergent

18. Convergent.

15. Convergent

1. Conv. for $x < 1$; Div. for $x \geq 1$ 3. Conv. for $x \leq 1$; Div. for $x > 1$ 5. Convergent for all values of p

8. Convergent

11. Convergent

14. Convergent

16. Divergent

Problems 9.4, page 3762. Conv. for $x < 1$; Div. for $x \geq 1$ 4. Conv. for $x \geq 1$; Div. for $x < 1$

6. Convergent

9. Conv. for $x < 1$; Div. for $x \geq 1$

7. Convergent

10. Convergent

12. Convergent

15. Conv. for $x < 1$, Div. for $x > 1$; Conv. for $p > 1$ and Div. for $p \leq 1$ 17. Conv. if $\beta > \alpha > 0$; Div. if $\alpha \geq \beta > 0$.**Problems 9.5, page 379**1. Conv. for $x \leq 1$; Div. for $x > 1$ 3. Conv. for $x < 1$; Div. for $x \geq 1$ 5. Conv. for $x < e$; Div. for $x \geq e$ 7. Conv. for $x \leq 1$; Div. for $x > 1$ 9. Conv. for $x^2 < 4$; Div. for $x^2 \geq 4$ 11. Conv. for $x < 1/e$; Div. for $x \geq 1/e$

13. Diverges

14. Conv. for $x < 1$; Div. for $x > 1$. When $x = 1$, Conv. for $b - a > 1$, Div. for $b - a \leq 1$.2. Conv. for $x \leq 1$; Div. for $x > 1$ 4. Conv. for $x < 2$; Div. for $x \geq 2$ 6. Conv. for $x \leq 1$; Div. for $x > 1$ 8. Conv. for $x^2 \leq 1$; Div. for $x^2 > 1$

10. Convergent

12. Conv. for $x < 1$; Div. for $x \geq 1$ **Problems 9.6, page 381**

1. Convergent

4. Convergent

6. Conv. for $x < \frac{1}{2}$; Div. for $x \geq \frac{1}{2}$

2. Convergent

5. Conv. for $x < 1$; Div. for $x \geq 1$

3. Convergent

7. Convergent.

Problems 9.7, page 383

1. Oscillatory

5. Convergent

9. Convergent

2. Convergent

6. Oscillatory

10. Oscillatory.

3. Convergent

7. Convergent

4. Convergent

8. Convergent

Problems 9.8, page 387

1. (i) and (ii) conditionally convergent

3. (i) Conditionally convgt. for $0 < p \leq 1$; (ii) Conditionally convgt4. Absolutely convergent for (i) $0 < x < 1$; (ii) $-1 < x \leq 1$; (iii) $|x| \leq 1$.5. Convergent for $x \leq 1$ and not convergent for $x > 1$ 6. (i) $-1 < x \leq 1$; (ii) $-1 < x \leq 1$;7. $-e < x \leq e$

8. (i) Absolutely convergent (ii) convergent

9. Absolutely convergent.

Problems 9.9, page 3881. Conv. for $x < 1$; Div. for $x \geq 1$

4. Convergent

7. Conv. for $x < 1$; Div. for $x \geq 1$ 9. Conv. for $x < 1/4$; Div. for $x \geq 1/4$

2. Convergent

5. Divergent

8. Conv. for $x < 1$; Div. for $x \geq 1$

3. Divergent

6. Conv. for $x < 1$; Div. for $x \geq 1$ 10. Conv. for $x < 2$; Div. for $x \geq 2$

11. Convergent for all x 12. Conv. for $x < 1$; Div. for $x \geq 1$ 13. Convergent
 14. Absolutely convergent 15. Convergent
 16. Convergent for $p > 1$; divergent for $p \leq 1$.

Problems 9.10, page 391

1. Uniformly convergent for $0 \leq x \leq 1$. 2. to
 5. Uniformly convergent for all real values of x 6. Uniformly convergent for $0 \leq x \leq 1/a$
 10. (i) and (ii) Both converge uniformly for all real values of x .

Problems 9.11, page 392

- | | | | | | |
|-----------------------|-------------------|-----------------------|----------------|-----------------|-----------------|
| 1. (c) | 2. (d) | 3. (a) | 4. (b) | 5. (c) | 6. (d) |
| 7. (a) | 8. (b) | 9. (b) | 10. (d) | 11. (c) | |
| 12. (a) $(-1, 1)$ | (b) $(-1/2, 1/2)$ | 13. $-1 < x \leq 1$ | 14. $k > 1$ | 15. $a_n < k$ | 16. Oscillatory |
| 17. All values of x | 18. $k < 1$ | 19. Convergent. | 20. Divergent. | 21. $q - p > 1$ | |
| 22. Divergent | 23. Convergent. | 24. $0 < x < 4$ | 25. yes | 26. True | 27. Convergent |
| 28. Divergent | 29. $x > 1$ | 30. $0 \leq x \leq 1$ | 31. (b) | 32. (c) | 33. (d) |
| 34. (b) | 35. True. | | | | |

Problems 10.1, page 400

$$1. \frac{2 \sinh a\pi}{\pi} \left\{ \left(\frac{1}{2a} - \frac{a \cos x}{1^2 + a^2} + \frac{a \cos 2x}{2^2 + a^2} - \dots \right) + \left(\frac{\sin x}{1^2 + a^2} - \frac{2 \sin 2x}{2^2 + a^2} + \frac{3 \sin 3x}{3^2 + a^2} - \dots \right) \right\}$$

$$\frac{\pi}{\sinh \pi} = 2 \left[\frac{1}{2^2 + 1} - \frac{1}{3^2 + 1} + \frac{1}{4^2 + 1} - \dots \right].$$

Problems 10.2, page 401

1. No 2. No 3. Yes.

Problems 10.3, page 404

1. $\frac{1}{2}\pi - \frac{4}{\pi} \left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots \right)$ 2. $\frac{I_0}{\pi} + \frac{1}{2}I_0 \sin x - \frac{2I_0}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}, \frac{1}{2}$
 3. $\frac{\pi^2}{6} - 2 \left(\cos x - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \dots \right) - \frac{1}{\pi} \left\{ \left(\frac{2}{1^3} - \frac{\pi^2}{1} \right) \sin x - \left(\frac{2}{2^3} - \frac{\pi^2}{2} \right) \sin 2x + \dots \right\}$
 4. $2 \left(\pi - \frac{4}{\pi} \right) \sin x - \pi \sin 2x + \frac{2}{3} \left(\pi - \frac{4}{9\pi} \right) \sin 3x - \frac{\pi}{2} \sin 4x + \dots$
 5. $\frac{4}{\pi} \left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right).$

Problems 10.4, page 408

$$1. -\frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin n\pi x}{n}$$

2. (i) $\frac{a^2}{3} \frac{a^2}{11^2} \left\{ \frac{1}{12} \cos \frac{2\pi x}{a} + \frac{1}{2^2} \cos 4\pi \frac{x}{a} + \dots \right\}$ (ii) $\frac{a^2}{\pi} \left\{ \frac{1}{1} \sin \frac{2\pi x}{a} \frac{1}{2} \sin \frac{4\pi x}{a} + \dots \right\}$

(ii) $f(t) = \frac{2}{3} + \frac{4}{\pi^2} \left(\cos \pi t - \frac{\cos 2\pi t}{2^2} + \frac{\cos 3\pi t}{3^2} - \dots \right)$

4. $\frac{3}{2} - \frac{12}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \left\{ \cos \frac{(2n-1)\pi x}{3} \right\}$

5. $\frac{E}{\pi} + \frac{E}{2} \sin \omega t - \frac{2E}{\pi} \left[\frac{1}{1.3} \cos 2\omega t + \frac{1}{3.5} \cos 4\omega t + \frac{1}{5.7} \cos 6\omega t + \dots \right]$

6. $f(x) = \frac{\pi}{4} + 2 \sum_{n=1}^{\infty} (-1)^{n+1} \sin n\pi x$; put $x = 1/2$.

Problems 10.5, page 412

1. $\frac{a^2}{3} + \sum_{n=1}^{\infty} \frac{4a^2}{n^2 \pi^2} (-1)^n \cos \frac{n\pi x}{a}$

3. $1 - \frac{1}{2} \cos x - \frac{2}{1.3} \cos 2x + \frac{2}{2.4} \cos 3x - \frac{2}{3.5} \cos 4x - \dots$

5. $\frac{1}{2}\pi - \frac{4}{\pi} \left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots \right)$

6. (i) $\frac{2}{\pi} - \frac{4}{\pi} \left(\frac{\cos 2x}{3} + \frac{\cos 4x}{15} + \dots + \frac{\cos 2nx}{4n^2 - 1} + \dots \right)$ (ii) $\frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{(4m^2 - 1)\pi} \cos \frac{2m\pi x}{l}$

7. $\frac{\pi}{2} + 1 - \frac{4}{\pi} \left(\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right); \frac{\pi^2}{8}$

8. $\frac{4k}{\pi} \left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right).$

Problems 10.6, page 416

2. $\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cdot \cos 3x + \frac{1}{5^2} \cdot \cos 5x + \dots \right]; 2 \left[\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \dots \right]$

3. $\frac{\pi^2}{3} - 4 \left[\cos x - \frac{1}{2^2} \cos 2x + \frac{1}{3^2} \cos 3x - \frac{1}{4^2} \cos 4x + \dots \right]$

4. $\sum_{n=2}^{\infty} \frac{1}{n} \sin 2nx$

5. $\frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi x$

6. $\frac{8}{\pi^3} \left(\frac{\sin \pi t}{1^3} + \frac{\sin 3\pi t}{3^3} + \frac{\sin 5\pi t}{5^3} + \dots \right)$

7. $\frac{1}{n\pi} [1 - (-1)^n]$

8. $-\frac{1}{2} \sin x + \sum_{n=2}^{\infty} \frac{(-1)^n \cdot 2n}{n^2 - 1} \sin nx$

9. $\sum_{n=1}^{\infty} \frac{2n\pi}{1 + n^2 \pi^2} (1 - e \cos n\pi) \sin n\pi x$

10. $\sum_{n=1}^{\infty} \frac{4}{\pi n^3} [1 - (-1)^n] \sin nx$

12. $\frac{l}{4} + \sum_{n=1}^{\infty} \frac{2l}{(\pi n)^2} \left\{ 2 \cos \frac{n\pi r}{2} - 1 - (-1)^n \right\} \cos \frac{n\pi x}{l}$

13. $\frac{8}{\pi} \cos \frac{\pi}{4} \left[\frac{\sin 2x}{1 \cdot 3} - \frac{\sin 6x}{5 \cdot 7} + \frac{\sin 10x}{9 \cdot 11} + \dots \right]$

14. $\frac{2l^2 h}{a(l-a)\pi^2} \left[\sin \frac{\pi a}{l} \sin \frac{\pi x}{l} + \frac{1}{2^2} \sin \frac{2\pi a}{l} \sin \frac{2\pi x}{l} + \frac{1}{3^2} \sin \frac{3\pi a}{l} \sin \frac{3\pi x}{l} + \dots \right].$

Problems 10.7, page 419

3. $\pi^4/96$.

Problems 10.8, page 420

1.
$$\sum_{n=-\infty}^{\infty} \frac{(\sinh al \cos n\pi l - i \cosh al \sin n\pi l)(a + in\pi)}{(a^2 + n^2\pi^2)} e^{inx/l}$$

2.
$$\frac{2}{\pi} \left\{ 1 - \frac{e^{2it} + e^{-2it}}{1 \cdot 3} - \frac{e^{4it} + e^{-4it}}{3 \cdot 5} - \frac{e^{6it} + e^{-6it}}{5 \cdot 7} - \dots \right\}$$

3.
$$\frac{a}{\pi} \sin a\pi \sum_{n=-\infty}^{\infty} \frac{(-1)^n e^{inx}}{a^2 - n^2}$$

4.
$$\sin h 9 \sum_{n=-\infty}^{\infty} \frac{(-1)^n (9 + n\pi i)}{81 + (n\pi)^2} e^{n\pi ix/3}$$

5.
$$\frac{a}{2} - \frac{a}{\pi} \left[(e^u - e^{-u}) + \frac{1}{3}(e^{3u} - e^{-3u}) + \frac{1}{5}(e^{5u} - e^{-5u}) + \dots \right] \text{ where } u = i\pi x/l.$$

Problems 10.9, page 423

1.
$$11.733 - 7.733 \cos 2x - 2.833 \cos 4x + \dots - 1.566 \sin 2x - 0.116 \sin 4x + \dots$$

2.
$$1.45 + (-0.37 \cos x + 0.17 \sin x) - (0.1 \cos 2x + 0.06 \sin 2x)$$

3.
$$a_0 = 41.66, a_1 = -8.33, b_1 = -1.15 \quad 4. -0.0731$$

5.
$$y = 2.102 + 0.558 \cos x + 1.531 \sin x + 0.354 \cos 2x + 0.145 \sin 2x$$

6.
$$7.8 \sin \theta + 1.5 \sin 2\theta - 9.2 \sin 3\theta + 11.6 \sin 4\theta - \dots$$

Problems 10.10, page 424

1. $2\pi/3$

2. $\frac{1}{2} [f(c-0) + f(c+0)]$

3. $(-1, 1)$ such that $f(x) = -f(-x)$

4. $f(x) = A$ when $0 < x < \pi$ and $f(x) = -A$ when $\pi < x < 2\pi$

5. Sine

6. § 10.11 (3)

7. Zero

8. not defined

9. odd

10. Cosine

11. even

12. $x = k/n$

13. Zero

14. Cosine

15. Zero

16. $\int_0^2 x^2 \cos \frac{n\pi x}{2} dx$

17. $\frac{1}{T} \int_a^{\alpha+2T} f(x) \sin \frac{n\pi x}{T} dx$

18. § 10.3

19. Zero

20. $a_0 = \frac{2}{l} \int_0^l f(x) dx, a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l} dx$

21. $\frac{4}{\pi} \left\{ \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right\}$

22. π

23. Zero

24. $2l$

25. $\sum_{n=-\infty}^{\infty} (-1)^n \frac{(1-in\pi)}{1+n^2\pi^2} \sinh 1 e^{inx}$

26. False

27. $-\pi/2$

28. odd

29. Zero

30. 3.5355

31. zero

32. $-1/2$

33. $\frac{1}{2} a_n$

34. $\frac{\pi^2}{8}$

35. $\frac{\pi}{2} - \frac{4}{\pi} \left\{ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right\}$

36. $f(x) = \frac{3}{8} + \frac{1}{2} \cos 2x + \frac{1}{4} \cos 4x$

37. $x^2 - x$

38. $x(l+x)$

39. True

40. False

41. False.

Problems 11.1, page 429

1. $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 0$

2. $\frac{d^2y}{dx^2} + 4y = 0$

3. $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy = 0$

4. $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$

5. $\frac{d^3y}{dx^3} - 7 \frac{dy}{dx} + 6y = 0$

6. $2xy \frac{dy}{dx} + x^2 - y^2 = 0$

7. $(x^2 - 25) \left(\frac{dy}{dx} \right)^2 + x^2 = 0$

8. $y \frac{dy}{dx} = 2a$

10. $y''' - 3y'' + 3y' - y = 0$.

Problems 11.2, page 431

1. $\sqrt{1-x^2} + \sqrt{1-y^2} = c$

2. $\log \frac{x}{y} - \frac{1}{x} - \frac{1}{y} = c$

3. $\tan x \tan y = c$

4. $(1+x^2)^{3/2} - 3\sqrt{1+y^2} = c$

5. $\tan y = c(1-e^x)$

6. $2e^{-y} = e^{-x^2} + 1$

7. $x = 2 \cos y$

8. $(x^2+1)(y^2+1) = c$

9. $3e^{2x} - 2e^{3y} + 8x^3 = c$

10. $(1-ay)(a+x) = cy$

11. $(x+1)(2-e^y) = c$

12. $a \log \left(\frac{x-y-a}{x-y+a} \right) = 2y + c$

13. $y = \tan^{-1}(x+y+1) + c$

14. $\tan(x+y) = \sec(x+y) + x + c$

15. $x = \operatorname{cosec}(x+y+1) - \cot(x+y+1) + c$

16. $\log \sin(y-x) = \frac{1}{2}x^2 + cz$

17. $\cos xy + \frac{1}{2x^2} = c$.

Problems 11.3, page 432

1. $x(x^2 - 3y^2) = c$

2. $cy^3 = x^2 e^{-x/y}$

3. $(x/y)^3 = 3 \log cy$

4. $y + \sqrt{x^2 + y^2} = c$

5. $y^2 = 2x[y + x \log(cx)]$

6. $x(c+y) = ay^2$

7. $y = 2x \tan^{-1}(cx)$

8. $e^{x/y} = y + c$

9. $\log y - \frac{x^2}{4y^2} \left(z \log \frac{y}{x} + 1 \right) = c$

10. $\log x = \frac{1}{2} \left[\frac{y}{x} - \frac{1}{2} \sin \left(\frac{2y}{x} \right) \right] + c$

11. $xy \cos(y/x) = c$.

Problems 11.4, page 434

1. $(X^2 + 2Y^2)^2 = c \left(\frac{\sqrt{2}Y - X}{\sqrt{2}Y + X} \right)$ where $X = x + 1$, $Y = y - 1$

2. $(y-x)^3 = c(y+x-2)$

3. $(x+y)^7 = c(x-y-2/3)^3$

4. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

5. $3(2y-x) + \log(3x+3y+4) = c$

6. $x-y + \frac{3}{4} \log(8x-12y-5) = c$

7. $\log(x+y+\frac{1}{3}) + \frac{3}{2}(y-x) = c$.

Problems 11.5, page 437

1. $y = ce^{-\tan x} + \tan x - 1$

2. $y = \log x + c/\log x$

3. $y \sec^2 x = \sec x - 2$

4. $y \cosh x = c + \frac{2}{3} \cosh^3 x$

5. $y \sqrt{1-x^2} = \sin^{-1} x + c$

6. $y = c(1-x)^2 + (1-x^2)$

7. $y(1+\sin x) = c - x^2/2$

8. $2r \sin^2 \theta + \sin^4 \theta = c$

9. $ye^{x^2} = 2x + c$

10. $x = y^3 + cy$

11. $x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}$

12. $xy^{-2} = c - e^{-y}$

13. $xe^{\tan^{-1} y} = \tan^{-1} y + c$

14. $xe^y = c + \tan y$.

Problems 11.6, page 439

1. $y^{-1} \sec x = \tan x + c$

2. $1/r = \sin \theta + c \cos \theta$

3. $x^2 + (4x^5 + c)y^4 = 0$

4. $1/y = x^2 - 2 + ce^{-x^2/2}$

5. $y^2 = x^2 + cx - 1$

6. $y/x = \log y + c$

7. $\sin y = (1+x)(e^x + c)$

8. $e^{x+y} = \frac{1}{2}e^{2x} + c$

9. $\tan y = x^3 - 3x^2 + 6x - 6 + ce^{-x}$

10. $\cos y = \cos x (\sin x + c)$

11. $\sqrt{x} = \sqrt{y}(\log \sqrt{y} + c)$

12. $y^{-1} = \frac{1}{2} \log x + \frac{1}{4} + cx^2$.

Problems 11.7, page 442

1. $x^3 + y^3 - 3axy = c$

2. $x^4 + 2x^2y^2 - y^4 - 2a^2x^2 - 2b^2y^2 = c$

3. $x^3 - 6x^2y - 6xy^2 + y^3 = c$

4. $\frac{x^5}{5} - x^2y^2 + xy^4 + \cos y = c$

5. $e^{xy} + y^2 = c$

6. $x^5 + x^3y^2 - x^2y^3 - y^5 = c$

7. $x^3 + 3x^2y^2 + y^4 = c$

8. $x^2 - y^2 = cy^3$

9. $3y \cos 2x + 6y + 2y^3 = c$

10. $e^x = \sec x \tan y + c$

11. $x^2y + xy - x \tan y + \tan y = c$.

Problems 11.8, page 445

1. $ax + \tan^{-1} y/x = c$

2. $x^2 + y^2 - 2a^2 \tan^{-1}(y/x) = c$

3. $y + cx + \log x + 1 = 0$

4. $3 \log x - (y/x)^3 = c$

5. $\log(y/x) + \frac{1}{2}x^2y^2 = c$

6. $xy + \log(x/y) - (1/xy) = c$

7. $(y + 2/y^2)x + y^2 = c$

8. $4x^4y + 4x^3y^2 - x^4 = c$

9. $2 \cos(xy) + x^{-2} = c$

10. $\log(x/y) = c + xy$

11. $(x/y) + e^{x^3} = c$

12. $4(xy)^{1/3} - \frac{2}{3}(x/y)^{3/2} = c$

13. $4y \log x = y^2 + c$.

Problems 11.9, page 446

1. $(x - y + c)(x^2 + y^2 + c) = 0$

2. $(2y - x^2 + c)(y + x + ce^{-x} - 1) = 0$

3. $x^2 + y^2 = cx$

4. $(y - cx)(y^2 - x^2 - c) = 0$

5. $(y - c)(y + x^2 - c)(xy + cy + 1) = 0$.

Problems 11.10, page 448

1. $x + c = \frac{a}{2} \left[\log \frac{p-1}{\sqrt{1+p^2}} - \tan^{-1} p \right]$, with the given relation

2. $xy = c^2x + c$

3. $y = 2\sqrt{(xc)} + c^2$

4. $2cy = c^2x^2 + 1$

5. $x = (\log p - p + c)(p - 1)^2$, with the given relation

6. $x = \sin p + c$, with the given relation.

Problems 11.11, page 449

1. $y = c(x - c)^2$

2. $y^2 = 2cx + c^3$

3. $(y + \alpha p)\sqrt{(p^2 - 1)} + a \cosh^{-1} p = c$, with the given relation

4. $y + (1 + p^2)^{-1} = c$, with the given relation.

Problems 11.12, page 450

1. (i) Gen. sol. : $y = cx + a/c^2$; Singular sol. : $2ax^2 = (2ac + x)^3$
(ii) Gen. sol. : $c = \log(cx - y)$; Singular sol. : $y = x(\log x - 1)$
(iii) Gen. sol. $y = cx + \sqrt{a^2c^2 + b^2}$; Singular sol. $y + \sqrt{1 - x^2} = 0$
(iv) Gen. sol. $y = cx - \sin^{-1} c$; Singular sol. $y = \sqrt{x^2 - 1} - \sin^{-1} \frac{\sqrt{x^2 - 1}}{x}$
2. $y = cx + (c - 2c^2)$ 3. $(y - cx)(c - 1) = c$
4. $(y - cx)(c + 1) + ac^2 = 0$ 5. $y^2 = cx^2 + c^2$ [Hint: Put $x^2 = u, y^2 = v$]
6. $xy = cy - c^2$ [Hint: Put $u = y, v = xy$] 7. $y^2 = cx^2 - \frac{2c}{1+c}$.

Problems 11.13, page 450

1. (i) 2. (ii) 3. (iii) 4. (i) 5. $\log y + c = x^2/2y^2$
6. $yx^2 = x^3 + c$ 7. $e^x + x^2y + cy = 0$. 8. (iii) 9. $x^2 + y^2 + 2 \tan^{-1} y/x = c$
10. $\log x + c = y^3/3x^3$ 11. (i) 12. $y^2 + 1/x + ce^{-y^2/2} = 2$
13. $y = cx + a/c^2$ 14. $c = \log(cx - y)$ 15. $xy = c$ or $x^2 - y^2 = c$
16. 2 17. $xy = c$ 18. (b) 19. (b)
20. $(1 + x^2)^{3/2} + (1 + y^2)^{3/2} = c$ 21. $y = 5e^{-x}$ 22. $x + y = u$
23. x^{-5} 24. § 11.11 (3) 25. $5x^4y^2 + 2(x^5 + y^5) = c$
26. $\sin(y/x) = cx$ 27. (a) 28. (c) 29. $x + y dy/dx = 0$
30. $e^{-x^2} + 2 \cos y = c$ 31. (c) 32. False 33. False.

Problems 12.1, page 454

1. (i) $9y + 4x^2 = 0$; (ii) $3(x + 3y) = 2(1 - e^{3x})$ 2. $y + 1 = 2e^{x^2/2}$
3. $x^2 + y^2 = cx$ 4. $y = \sqrt{a^2 - x^2} + a \log \left(\frac{\alpha - \sqrt{a^2 - x^2}}{x} \right) + c$
5. $y^2 = 4x$ 6. $y = ae^{cx}$ 7. $y = ax + b$
8. $x = 3y^2$ 9. (i) $r(\theta - \alpha) = c$; (ii) $r = a + b \cos \theta$
10. $r^2 = a^2 \sin 2\theta$ 11. $c^2x^2 = 2cy + 1$ 12. $r = ae^{\theta \cot \alpha}$.

Problems 12.2, page 457

1. $2x^2 + y^2 = c$ 2. $x^2 + 2y^2 = c^2$ 3. $3y^2 + 2x^2 = c^2$
4. $x^2 + y^2 + 2\mu y - c = 0$ 5. This system is self-orthogonal 6. $r = c(1 - \cos \theta)$
7. $r = b(\cos \theta - \sin \theta)$ 8. $r = 2b/(1 - \cos \theta)$ 9. $r^2 = c^2 \sin 2\theta$
10. $r^n n \sin \theta = b$ 13. $x^2 + y^2 + cx + 1 = 0$ 14. $y = cx$.

Problems 12.3, page 462

1. $V = \sqrt{\left(\frac{mg}{k}\right)} \tanh\left(\frac{9k}{m}t + c\right)$ 3. $\frac{1}{k} \log_e 2$
5. $2\sqrt{v_o/k}$ 6. $v^2 = 2gx - \frac{\lambda}{m} x^2$
10. $y = (\sqrt{150} - 0.001328t)^2$; $t_1 = 45$ min. 1 sec., $t_2 = 1$ hr. 16 min. 51 sec., $t_3 = 1$ hr. 38 min. 13 sec
11. 17 min. 4 sec.

Problems 12.4, page 465

1. 0.0006931 sec

2. $\frac{10}{L^2 + R^2} (R \sin t - L \cos t + L e^{-Rt/L})$

3. $i = \frac{1}{5} (1 - e^{-100})$

4. $i = k e^{-t/RC} + \frac{\omega C E_m}{\sqrt{(1+R^2 C^2 \omega^2)}} \sin(\omega t + \theta)$ where $\theta = \cot^{-1}(RC\omega)$.

Problems 12.5, page 467

1. 52.5 mts

2. 48°C

3. B drinks hotter coffee

4. 490,000 cal

5. 2.16 cm.

Problems 12.6, page 469

1. 604.9

2. $2 \log 3 / \log 2$

3. $(1 - 1/p)^{21}$ times the original amount

4. 64.5 days

5. 21.5 gm

6. $t = 300 - 5 \log 2 + 5 \log \frac{0.7-x}{0.5-x}$

7. 3 hr. 50 min. 16 sec

8. $100(2 - e^{-t/20})$; 13.9 min.

Problems 12.7, page 469

1. $6(1 - e^{-3})$

2. 54 m

3. 90.25%

4. $r(\theta - \alpha) = c$

5. $y = ae^{cx}$

6. rectangular hyperbola

7. $x^2 - y^2 = c$

8. The system is self-orthogonal

9. $2\sqrt{v_0/k}$

10. $2 \log 3 / \log 2$

11. Sunil

12. (d)

13. (c)

14. (d)

15. 2.21

16. (c)

17. (c)

18. (a)

19. False

20. True.

Problems 13.1, page 474

1. $\frac{2}{3} e^{2t} \sin 3t$

2. $y = e^x (4 \cos 3x - \sin 3x)$

3. $y = c_1 + (c_2 + c_3 x) e^{-x/2}$

4. $y = c_1 e^{-x} + e^{x/2} \left(c_2 \cos \frac{\sqrt{3}x}{2} + c_3 \sin \frac{\sqrt{3}x}{2} \right)$

5. $y = (c_1 + c_2 x + c_3 x^2) e^x$

6. $y = (c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x$

7. $y = c_1 e^{-x} + c_2 e^{2x} + e^{x/2} (c_3 \cos x/\sqrt{2} + c_4 \sin x/\sqrt{2})$

8. $y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x + c_5 e^x$.

Problems 13.2, page 486

1. $y = (c_1 + c_2 x) e^{3x} + 3x^2 e^{3x} + \frac{7}{25} e^{-2x} - \frac{1}{9} \log 2$

2. $y = \frac{3}{5} e^{-2x} (\cos x + 3 \sin x) - \frac{e^x}{10} - \frac{e^{-x}}{2}$

3. $x = c_1 \cos nt + c_2 \sin nt + \frac{kx}{2n} \sin(nt + \alpha)$

4. $x = e^{-t} (c_1 \cos \sqrt{2}t + c_2 \sin \sqrt{2}t) + \frac{1}{4} (\sin t - \cos t)$

5. $y = c_1 e^{-x} + c_2 e^{-2x} + 1 + \frac{1}{10} (3 \sin 2x - \cos 2x)$

6. $y = c_1 e^x + c_2 e^{3x} + \frac{1}{884} (10 \cos 5x - 11 \sin 5x) + \frac{1}{20} (\sin x + 2 \cos x)$

7. $y = c_1 + (c_2 + c_3 x) e^{-x} - \frac{x^2}{2} e^{-x} + \frac{3}{50} \cos 2x - \frac{2}{25} \sin 2x$

8. $y = (c_1 + c_2 x) e^{-x} + \frac{1}{2} + \frac{1}{5} (2 \sin 2x + \cos 2x)$

9. $y = (c_1 + c_2 x) e^x + c_3 e^{3x} + \frac{1}{8} (xe^{3x} - x^2 e^x)$ 10. $y = c_1 e^x + c_2 e^{-x} + \frac{e^x}{12} (2x^3 - 3x^2 + 9x)$

11. $y = c_1 + c_2 e^x + c_3 e^{-x} + xe^x - (x^2 + x) - 2 \sin x$

12. $y = e^{3x} (c_1 \cos 4x + c_2 \sin 4x) + \frac{1}{17} e^{2x} + \frac{1}{565} (23 \sin x + 6 \cos x) + \frac{x}{25} + \frac{6}{625}$

13. $y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x + x^4 - 24x^2 + 72 + \frac{1}{225} \sin 4x - \frac{1}{9} \sin 2x$

14. $y = c_1 e^{-2x} + c_2 e^{-3x} - \frac{e^{-2x}}{10} (\cos 2x + 2 \sin 2x)$

15. $y = e^{-x/2} \left\{ (c_1 + x/4) \cos(x\sqrt{3/2}) + (c_2 + x/4\sqrt{3}) \sin(x\sqrt{3/2}) \right\} + e^{x/2} \left\{ c_3 \cos \sqrt{3x/2} + c_4 \sin \sqrt{3x/2} \right\}$

16. $y = e^{-x} (c_1 \cos \sqrt{2x} + c_2 \sin \sqrt{2x}) + \frac{e^x}{41} (4 \sin x + 5 \cos x)$

17. $y = c_1 e^{-x} + c_2 e^{-3x} - \frac{e^{-x}}{5} (\sin x + 2 \cos x) + \frac{e^{3x}}{22} \left(x - \frac{5}{11} \right)$

18. $y = c_1 \cos \sqrt{2x} + c_2 \sin \sqrt{2x} + \frac{e^{3x}}{11} \left(x^2 - \frac{12}{11}x + \frac{50}{121} \right) + \frac{e^x}{17} (4 \sin 2x - \cos 2x)$

19. $y = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x - (1/5) \cos x \cosh x$

20. $y = c_1 + (c_2 + c_3 x) e^{-x} + \frac{e^{2x}}{18} \left(x^2 - \frac{7x}{8} + \frac{11}{6} \right) + \frac{1}{100} (3 \sin 2x + 4 \cos 2x)$

21. $y = c_1 \cos 4x + c_2 \sin 4x + \frac{1}{7} \left(x \sin 3x - \frac{6}{7} \cos 3x \right)$

22. $y = (c_1 + c_2 x) e^{-x} + \frac{1}{2} \cos x + \frac{1}{2} (x - 1) \sin x$

23. $y = c_1 e^x + c_2 e^{-x} - \frac{1}{2} (x \sin x + \cos x) + (xe^x/12) (2x^2 - 3x + 9)$

24. $y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{-3x} + e^{-2x} \cdot e^x$

25. $y = c_1 \cos ax + c_2 \sin ax - \frac{1}{a^2} \cos ax \log(\sec ax + \tan ax)$.

Problems 13.3, page 490

1. $y = (c_1 - x/a) \cos ax + [c_2 + (1/a^2) \log \sin ax] \sin ax$

2. $y = c_1 \cos x + c_2 \sin x + \cos x \log(\cos x) + x \sin x$

3. $y = c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x)$

4. $y = c_1 \cos x + c_2 \sin x + \frac{x}{2} \sin x - \frac{x^2}{4} \cos x$ 5. $y = (c_1 + c_2 x) e^x + x e^x \log x$

6. $y = (e^x + e^{2x}) \log(1 + e^x) + (c_1 - 1 - x) e^x + (c_2 - x) e^{2x}$

7. $y = e^x (c_1 \cos x + c_2 \sin x) - e^x \cos x \log(\sec x + \tan x)$

8. $y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x$

9. $y = c_1 \cos x + c_2 \sin x + \sin x \log(1 + \sin x) - x \cos x - 1$

10. $y = c_1 e^x + c_2 e^{2x} + \frac{1}{2} (x^2 + 3x + 3.5 - 2xe^x)$ 11. $y = c_1 \cos x + c_2 \sin x - x \sin x$

12. $y = c_1 e^{2x} + c_2 e^{3x} + x e^{3x} + \frac{1}{10} (\sin x + \cos x)$ 13. $y = c_1 e^x + c_2 e^{-2x} - \frac{1}{4} (2x + 1) - \frac{1}{10} (\cos x + 3 \sin x)$
14. $y = e^x (c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x) + \frac{1}{27} (9x^3 + 18x^2 + 6x - 8) + \frac{1}{4} (\cos x - \sin x)$
15. $y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x.$

Problems 13.4, page 495

1. $y = c_1 x^2 + c_2 x^3 - x^2 \log x$
2. $y = c_1 x^4 + c_2 x^{-1} + \frac{x^4}{5} \log x$
3. $y = (c_1 + c_2 \log x) x^2 + \frac{1}{4} + 2x + \frac{1}{2} x^2 (\log x)^2$
4. $y = c_1 x^2 + c_2 x^{-1} + \frac{1}{3} (x^2 - 1/x) \log x$
5. $u = \frac{kr}{8} (a^2 - r^2)$
6. $c_1 x^{-1} + c_2 x^{-2} + \frac{1}{2} \log x - \frac{3}{4}$
7. $y = c_1 x^{-1} + \sqrt{x} [c_2 \cos ((\sqrt{3}/2) \log x) + c_3 \sin ((\sqrt{3}/2) \log x)] + \frac{1}{2} x + \log x$
8. $y = c_1 x^{-2} + x [c_2 \cos (\sqrt{3} \log x) + c_3 \sin (\sqrt{3} \log x)] + 8 \cos (\log x) - \sin (\log x)$
9. $y = c_1 x^{-1} + [c_2 \cos (\log x) + c_3 \sin (\log x)] x + 5x + 10 \log x/x$
10. $y = \frac{1}{x} (c_1 + c_2 \log x) + \frac{1}{x} \log \frac{x}{1-x}$
11. $y = x^{-2} (c_1 + c_2 \log x) + \frac{x}{9} \left(\log x - \frac{2}{3} \right)$
12. $y = c_1 x^3 + c_2 x^{-4} + \frac{x^3}{98} \log x (7 \log x - 2)$
13. $y = c_1 (2x+3)^a + c_2 (2x+3)^b - \frac{3}{14} (2x+3) + \frac{3}{4}$ where $a, b = \frac{3 \pm \sqrt{57}}{4}$
14. $y = c_1 (x-1) + c_2 (x-1)^2 + c_3 (x-1)^{-2} + \log (x+1) + 1$
15. $y = c_1 \cos \log (1+x) + c_2 \sin \log (1+x) - \frac{1}{3} \sin [2 \log (1+x)]$
16. $y = c_1 (3x+2)^{1/3} + c_2 (3x+2)^{-1} + \frac{1}{27} \left[\frac{1}{15} (3x+2)^2 + \frac{1}{4} (3x+2) - 7 \right]$

Problems 13.6, page 499

1. $x = (c_1 + c_2 x) e^{3x}; y = [(1-2x)(c_2 - 2c_1)] e^{3x}$
2. $x = e^t + e^{-t}, y = e^{-t} - e^t + \sin t$
3. $x = c_1 e^t + c_2 e^{-5t} + \frac{6}{7} e^{2t}; y = c_2 e^{-5t} - c_1 e^t + \frac{8}{7} e^{2t}$
4. $x = e^{6t} (c_1 \cos t + c_2 \sin t), y = e^{6t} [(c_1 - c_2) \cos t + (c_1 + c_2) \sin t]$
5. $x = \frac{1}{5} e^t + \frac{2}{5} e^{-t} - c_1 \sin 2t + c_2 \cos 2t, y = \frac{2}{5} e^t + \frac{1}{5} e^{-t} + c_1 \cos 2t + c_2 \sin 2t$
6. $x = c_1 e^t + c_2 e^{-5t} + \frac{3}{7} e^{2t} - \frac{2}{5} t - \frac{13}{25}, y = c_1 e^t - c_2 e^{-5t} - \frac{4}{7} e^{2t} - \frac{3t}{5} - \frac{12}{25}$
7. $x = -t - \frac{2}{3}, y = \frac{1}{2} t^2 + \frac{4}{3} t + c$
8. $y = c_1 e^x + c_2 e^{-2x} + 2e^{-x}, z = 3c_1 e^x + 2c_2 e^{-2x} + 3e^{-x}$
9. $x = c_1 e^{-t} + c_2 e^{3t} - \frac{1}{5} (\cos t - 2 \sin t); y = 2c_1 e^{-t} - 2c_2 e^{3t} + \frac{1}{5} (\sin t + 2 \cos t)$
10. $x = \frac{1}{2} \left(t + \frac{1}{t} \right), y = \frac{1}{2} \left(-t + \frac{1}{t} \right)$
11. $x = c_1 e^{-t} + c_2 e^{3t} - \frac{1}{5} (\cos t - 2 \sin t), y = 2c_1 e^{-t} - 2c_2 e^{3t} + \frac{2}{5} \cos t + \frac{1}{5} \sin t$
12. $x = (c_1 + c_2 t) e^{-t} + (c_3 + c_4 t) e^t, y = -\frac{1}{2} [c_1 + c_2 (1+t)] e^{-t} + \frac{1}{2} [c_4 (1-t) - c_3] e^t$

13. $x = c_1 e^t + c_2 e^{-t} + c_3 \cos t + c_4 \sin t - \frac{t}{4} \cos t + \frac{t}{4} \sin t$
 $y = -c_1 e^t - c_2 e^{-t} + c_3 \cos t + c_4 \sin t + \frac{1}{4}(2+t)(\sin t - \cos t)$

14. $x = \frac{8}{9}\left(1 - \cos \frac{3}{2}t\right), y = \frac{4}{3}t - \frac{8}{9} \sin \frac{3}{2}t.$

Problems 13.7, page 500

1. $y = c_1 e^{ax} + c_2 e^{-ax} + c_3 \cos ax + c_4 \sin ax$
2. $-\frac{1}{25}(3 \sin 2x + 4 \cos 2x)$
3. $1/6$
4. $e^x(x - 1)$
5. (b)
6. $y = c_1 + (c_2 + c_3 x + c_4 x^2) e^{2x}$
7. (a)
8. $y = e^x(c_1 + c_2 x) \cos 2x + (c_3 + c_4 x) \sin 2x$
9. $y = \cos x + 2 \sin x$
10. (ii)
11. $y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x$
12. $\frac{1}{10} \cosh 3x$
13. $y = a \log x + 6$
14. $y = (c_1 + c_2 x) e^{\sqrt{2}x} + (c_3 + c_4 x) e^{-\sqrt{2}x}$
15. $\frac{1}{6}x^3 e^{-x}$
16. $\frac{1}{2}e^{2x}$
17. $\sin 2x$
18. $\frac{1}{2}x^2 e^{-x}$
19. $y = (c_1 + c_2 x) e^{-x/2} + c_3$
20. (c)
21. (a)
22. $x e^{-t}$
23. $\frac{d^2y}{dt^2} + 7y = 2e^t$
24. (c)
25. (a)
26. (b)
27. $y = (c_1 + c_2 \log x)x$
28. $x^2 y_2 + xy_1 - y = 0$
29. $\frac{1}{9} \log 2$
30. e^t
31. (d)
32. $y = c_1 e^{-x} + c_2 e^{2(1+\sqrt{2})x} + c_3 e^{2(1-\sqrt{2})x}$
33. $\frac{d^3y}{dx^3} + 2$
34. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
35. False
36. False.

Problems 14.1, page 506

1. 38 sec
4. $x = \frac{ue^{-\lambda nt}}{n\sqrt{1-\lambda^2}} \sin [nt\sqrt{(1-\lambda^2)}]$
6. It must be shortened by 1/8640 of its length
7. It must be increased by 0.0074 ft./sec²
8. 4321/4319
9. $k^2 > 4\mu, \theta = c_1 e^{\frac{-k+\lambda}{2}t} + c_2 e^{\frac{-k-\lambda}{2}t}$ where $\lambda = \sqrt{(k^2 - 4\mu)}$
 $k^2 = 4\mu, \theta = (c_1 + c_2 t) e^{-kt/2}$
- $k^2 < 4\mu, \theta = c_1 e^{-kt/2} \cos \left(\frac{\sqrt{4\mu - k^2}}{2}t + c_2 \right)$
10. $x = \frac{F_0}{2n^2} (\sin nt - n \cos nt).$

Problems 14.2, page 513

2. 1 ft.; $\pi/2\sqrt{2}$ sec; $4\sqrt{2}$ ft./sec 4. $\pi/\sqrt{7}$
5. $x = e^{-5t} \left\{ \cos \sqrt{220}t + (5/\sqrt{220}) \sin \sqrt{220}t \right\}$. 6. 0.45 sec.; 1.15 sec
8. $x = \frac{10}{21}e^{-t} - \frac{17}{27}e^{-8t} - \frac{\sqrt{2}}{3} \sin \left(9t + \frac{\pi}{4} \right) \frac{\sqrt{2}}{3}, \frac{2\pi}{9}$ sec., $9/2\pi$ cycles/sec
9. 0.8 ($2 \sin 4t - \cos 4t$)
10. (i) $x = Ae^{-kt} \cos \left\{ t\sqrt{(b^2 - k^2)} + B \right\} + \{e^{-kt}/(b^2 + k^2 - n^2)\} \sin nt$
(ii) $x = Ae^{-kt} \cos \left\{ t\sqrt{(b^2 - k^2)} + B \right\} - (te^{-kt}/2n) \cos nt$

Problems 14.3, page 517

2. $i = I \sin (T/\sqrt{LC})$
3. $i = 2Eke^{-RT/2L} \sin (kt/2L)$, where $k = \sqrt{\left(\frac{4L - CR^2}{C} \right)}$
4. $R^2 > 4L/C$ for over damping; $R^2 = 4L/C$ for critical damping; $R^2 < 4L/C$ for under damping; critical resistance = $2\sqrt{L/C}$
5. $q = e^{-500t} (0.002 \cos 1323t + 0.0008 \sin 1323t)$
8. (i) $i = Ae^{-at} \cosh (\beta t + \gamma)$; (ii) $i = Ae^{-at} \cos (\beta t + \gamma) + \frac{E}{R} \cos \phi \sin (pt + \phi)$
where $\alpha = -\frac{R}{2L}$, $\beta = \pm \sqrt{\left(\frac{R}{2L} \right)^2 - \frac{1}{CL}}$ and $\phi = \tan^{-1} \{(1 - CLp^2)/CRp\}$.

Problems 14.4, page 525

4. $y = \frac{wl}{2Pn} \operatorname{cosec} \frac{nl}{2} \cos \left(nx - \frac{nl}{2} \right) - \frac{wl}{2nP} \cot \frac{nl}{2} + \frac{w}{2P} (x^2 - lx)$
6. $y = \frac{F}{P} [n \sin nx - l \cos nx + l - x]$ 7. $\pi^2 EI/4l^2$
8. $\frac{W}{2a^2} \left(\operatorname{sech} \frac{al}{2} - \operatorname{sec} al \right)$

Problems 14.5, page 528

1. $\frac{2u \sin \alpha}{g}; \frac{u^2 \sin 2\alpha}{g}$
2. (i) $\frac{2u^2 \sin(\alpha - \beta) \cos \alpha}{g \cos^2 \beta}$; (iii) $\frac{u^2}{g(1 + \sin \beta)}$
4. $4x^2 + k^2y^2 = 4$
5. $i_1 = \frac{a}{p + \omega} \sin pt, i_2 = \frac{a}{p + \omega} \cos pt$.
6. $i_1 = \frac{E}{R} \left(\frac{2}{3} - \frac{1}{2} e^{-Rt/L} - \frac{1}{6} e^{-3Rt/L} \right), i_2 = \frac{E}{R} \left(\frac{1}{3} - \frac{1}{2} e^{-Rt/L} + \frac{1}{6} e^{-3Rt/L} \right)$
7. $x = a(nt - \sin nt), y = a(1 - \cos nt)$
8. $x = \frac{E}{H\omega} (1 - \cos \omega t), y = \frac{E}{H\omega} (\omega t - \sin \omega t)$, where $\omega = eH/m$.

Problems 14.6, page 529

1. (b)

5. (b)

2. (b)

6. (b)

3. (c)

7. (b)

4. (b)

8. 60 sec

9. $30/\pi\sqrt{LC}$

10. 0.0074 sec

11. resonance

12. $EI \frac{d^2y}{dx^2} - Py = \frac{w}{2}(x^2 - lx)$

13. $y = 0$ and $\frac{dy}{dx} = 0$

Problems 15.1, page 531

1. $y = -x^2 \sin x - 4x \cos x + c_1 x + c_2$

2. $y = \frac{x^4}{24} + \frac{x^3}{6} \log x - \frac{11}{35} x^3 + c_1 x^2 + c_2 x + c_3$

Problems 15.2, page 533

1. $2(y^{1/4} - 1) = x$

2. $\sqrt{(y^2 - 8y) + 4 \cosh^{-1}(\frac{1}{4}y - 1)} = 3x$

3. $r = \frac{\sqrt{(v^2 - a^2\omega^2)}}{\omega} \sinh \left[\omega t + \sinh^{-1} \frac{a\omega}{\sqrt{(v^2 - a^2\omega^2)}} \right]$

4. $t = \frac{h^{3/2}}{\sqrt{(2g)a}} \left[\cos^{-1} \sqrt{\frac{x}{h}} - \sqrt{\left(\frac{hx - x^2}{h} \right)} \right]$

Problems 15.3 page 534

1. $y = c_1 - x^2 - c_2/x$

2. $y = c_1 x + (c_1^2 + 1) \log(x - c_1) + c_2$

3. $15c_1^2 y = 4(c_1 x + a^2)^{5/2} + c_2 x + c_3$

4. $x^2 + y^2 = a^2$

5. $\theta = \frac{m}{\mu a} \log \left(1 + \frac{\mu a \omega t}{m} \right)$

6. $v = \frac{1}{r_1 - r_2} \left[v_1 r_1 - v_2 r_2 - \frac{(v_2 - v_1)r_1 r_2}{r} \right].$

Problems 15.4, page 536

1. $y = 2x - 2 \log(1 - c_1 e^{2x}) + c_2$

2. $y^2 = x^2 + c_1 x + c_2$

3. $\log y = c_1 e^x + c_2 e^{-x}$

4. $(\log y - 1)(c_1 x + c_2) = 1$

5. $(x - a)^2 + y^2 = c^2$, circles whose centres are on the x -axis.

Problems 15.5, page 537

1. $y = e^{x^2}(c_1 x + c_2)$

2. $y = (x^2 - x + c_1 x)e^x + c_2 x$

3. $y = e^x(c_1 \log x + x + c_2)$

4. $cy = 1 + (k - x) \cot x$

5. $y = \left[c_1 - \frac{1}{2} \cos x - \frac{1}{5} c_2 e^{-2x} (\cos x + 2 \sin x) \right] e^x.$

Problems 15.6, Page 539

1. $y = c_1 \cos(\sin x) + c_2 \sin(\sin x)$

2. $y = c_1 \cos(1/x) + c_2 \sin(1/x)$

3. $y = c_1 e^t + c_2 e^{-t} - t$ where $t = \cos x$

4. $y = c_1 \cos(2 \tan^{-1} x) + c_2 \sin(2 \tan^{-1} x)$

5. $y = c_1 e^{\sqrt{2} \sin x} + c_2 e^{-\sqrt{2} \sin x} + \sin^2 x.$

Problems 15.7, page 540

$$\begin{aligned}1. \ nx - lz &= c(mz - ny) \\4. \ x^2 + y^2 - xz &= cz\end{aligned}$$

$$\begin{aligned}2. \ x^2 + y^2 + z^2 &= cx \\5. \ y(x+z) &= c(x+y+z)\end{aligned}$$

$$\begin{aligned}3. \ xy^2 &= cz^3 \\6. \ x + y + z + \log(xyz) &= c.\end{aligned}$$

Problems 15.8, page 541

$$\begin{aligned}1. \ x^3 - y^3 &= c_1, \ x^2 - z^2 = c_2 \\2. \ lx + my + nz &= c_1, \ x^2 + y^2 + z^2 = c_2 \\4. \ x^2 - y - 2xy &= c_1, \ x^2 - y^2 - z^2 = c_2 \\5. \ xyz &= c_1, \ x^2 + y^2 + z^2 = c_2 \\6. \ y = c_1z, \ x^2 + y^2 + z^2 &= c_2z.\end{aligned}$$

Problems 16.1, page 544

$$1. \ y = a_1 \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right)$$

$$\begin{aligned}2. \ y = a_0 \left(1 - \frac{x^4}{4 \cdot 3} + \frac{x^8}{8 \cdot 7 \cdot 4 \cdot 3} - \frac{x^{12}}{12 \cdot 11 \cdot 8 \cdot 7 \cdot 4 \cdot 3} + \dots \right) \\+ a_1 \left(x - \frac{x^5}{5 \cdot 4} + \frac{x^9}{9 \cdot 8 \cdot 5 \cdot 4} - \frac{x^{13}}{13 \cdot 12 \cdot 9 \cdot 8 \cdot 5 \cdot 4} + \dots \right)\end{aligned}$$

$$3. \ y = a_0 \left(1 - \frac{x^2}{2} + \frac{x^4}{2 \cdot 4} - \frac{x^6}{2 \cdot 4 \cdot 6} + \dots \right) + a_1 \left(x - \frac{x^3}{3} + \frac{x^5}{3 \cdot 5} - \frac{x^7}{3 \cdot 5 \cdot 7} + \dots \right)$$

$$4. \ y = 4 + 5x - 4x^2 - \frac{5}{3}x^3 - \frac{x^5}{3} - \frac{x^7}{7} - \dots \quad 5. \ y = a_0 \left(1 + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{16} - \frac{5x^8}{128} + \dots \right) + a_1 x$$

$$6. \ y = a_0(1 - x^2 + \frac{1}{3}x^4 - \frac{1}{5}x^6 + \dots) + a_1 x.$$

Problems 16.2, page 550

$$1. \ y = c_1 \cos \sqrt{x} + c_2 \sin \sqrt{x}$$

$$2. \ y = a_0 \left(1 - x^2 + \frac{x^4}{4} \dots \right) + a_1 \left(x - \frac{x^3}{2} + \frac{3x^5}{10} \dots \right)$$

$$3. \ y = (c_1 + c_2 \log x) \left[1 + x + \frac{1}{(2!)^2} x^2 + \frac{1}{(3!)^3} x^3 + \dots \right] - 2c_2 \left[x + \frac{1}{(2!)^2} \left(1 + \frac{1}{2} \right) x^2 + \frac{1}{(3!)^2} \left(1 + \frac{1}{2} + \frac{1}{3} \right) x^3 + \dots \right]$$

$$4. \ y = c_1(1 + x + x^2/4 + x^3/4 \cdot 7 + \dots) + c_2 x^{2/3} (1 + \frac{1}{3}x + x^2/3 \cdot 6 + x^3/3 \cdot 6 \cdot 9 + \dots)$$

$$5. \ y = a_0 \left(1 - 2x + \frac{3}{2!} x^2 - \frac{4}{3!} x^3 + \dots \right) + a_1 \left[y_1 \log x + a_0 \left(3x - \frac{13}{4} x^2 + \dots \right) \right]$$

$$6. \ y = a_0 x \left(1 + \frac{x}{5} + \frac{x^2}{70} + \dots \right) + a_1 x^{-1/2} \left(1 - x - \frac{x^2}{2} + \dots \right)$$

$$\begin{aligned}7. \ y = c_1 x^{-\frac{1}{2}} \left(1 + \frac{x}{2} + \frac{x^2}{40} + \dots \right) + c_2 x^{1/4} \left(1 + \frac{x}{14} + \frac{x^2}{616} + \dots \right) \\+ c_2 \sqrt{x} (x + x^2/2 \cdot 3 + x^4/2 \cdot 4 \cdot 3 \cdot 7 + x^6/2 \cdot 4 \cdot 6 \cdot 3 \cdot 7 \cdot 11 + \dots)\end{aligned}$$

$$8. \ y = a_0 \sqrt{x(1-x)} + a_1 \left(1 - 3x + \frac{3x^2}{1 \cdot 3} + \frac{3x^3}{3 \cdot 5} + \frac{3x^4}{5 \cdot 7} + \dots \right)$$

9. $y = a_0(1 - \frac{2}{3}x + \frac{1}{3}x^2 + \dots) + a_1x^4(1 - 2x + 3x^2 - 4x^3 + \dots)$

10. $y = c_1\left(1 + 3x^2 + \frac{3}{5}x^4 + \dots\right) + c_2x^{3/2}\left(1 + \frac{3}{8}x^2 - \frac{1.3}{8.16}x^4 + \frac{1.3.5}{8.16.24}x^6 + \dots\right).$

Problems 16.3, page 557

1. 0.224, 0.44.

Problems 16.4, page 562

1. $y = c_1J_{1/2}(x) + c_2J_{-1/2}(x)$

2. $y = c_1J_{2/5}(x) + c_2J_{-2/5}(x)$

3. $y = x^n[c_1J_n(kx) + c_2Y_n(kx)]$ where $n = \frac{1}{2}(1 - \alpha)$

4. $y = x[c_1J_1(2x) + c_2Y_1(2x)]$

5. $y = c_1\sqrt{x}J_1(2\sqrt{x}) + c_2\sqrt{x}Y_1(2\sqrt{x})$

7. $y = c_1\sqrt{x}J_n(x) + c_2\sqrt{x}J_{-n}(x)$

11. $x^2 = \sum_{n=1}^{\infty} \frac{2}{\alpha_n^2} \cdot \frac{1}{J_2^2(3\alpha_n)} (3\alpha_n J_1(3\alpha_n) - 2J_2(2\alpha_n))$

Problems 16.5, page 570

3. (i) $2P_3 + 4P_1$;

(ii) $\frac{2}{5}P_3 + \frac{4}{3}P_2 - \frac{2}{5}P_1 - \frac{7}{3}P_0$;

(iii) $\frac{8}{5}P_3 - 4P_2 + \frac{47}{5}P_1 + 4$

(iv) $\frac{8}{35}P_4 + \frac{6}{5}P_3 - \frac{2}{21}P_2 + \frac{34}{5}P_1 - \frac{224}{105}P_0$

9. (i) $f(x) = -\frac{7}{3}P_0(x) - \frac{2}{5}P_1(x) + \frac{4}{3}P_2(x) + \frac{2}{5}P_3(x)$;

(ii) $f(x) = -\frac{32}{15}P_0(x) - \frac{4}{5}P_1(x) - \frac{40}{21}P_2(x) + \frac{2}{5}P_3(x) + \frac{8}{35}P_4(x)$.

Problems 16.6, page 572

1. $x^3 = \frac{1}{4}(3T_1 + T_3)$.

Problems 16.7, page 575

1. $y_n(x) = \sin nx$, $n = 1, 2, \dots$

2. $y_n(x) = \sin [(2n+1)\pi x/2l]$, $n = 0, 1, 2, \dots$

3. $y_n(x) = \cos nx$, $n = 0, 1, 2, \dots$

4. 1, $\sin x$, $\cos x$, $\sin 2x$, $\cos 2x$, ...

5. $y_n(x) = \sin \left[(2n+1)\frac{\pi}{2} \log |x|\right]$, $n = 0, 1, 2, \dots$

6. $[xe^{-x}y']' + ne^{-x^2}y = 0$, $p(x) = e^{-x}$

7. $[e^{-x^2}y']' + 2ne^{-x^2}y = 0$, $p(x) = e^{-x^2}$.

Problems 16.8, page 575

1. $\frac{1}{3}(10 - 9P_1 + 8P_2)$

2. $\sqrt{(2/\pi x)} \cos x$

3. $\frac{2}{(2n+1)}$

4. zero

5. zero

6. $J_n(x) = \frac{x}{2n}(J_{n+1}(x) + J_{n-1}(x))$

7. $\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = 0$

8. $x \frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$

9. $\sqrt{(2/\pi x)} \sin x$

10. $x^n J_{n-1}(x)$

11. $\frac{1}{2}(3x^2 - 1)$

12. zero

13. True

14. $P_n(x) = \frac{1}{n! 2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$

15. $\alpha \neq \beta$

16. $2P_3 + 4P_1$

17. $(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} t^n P_n(x)$

18. $-J_1(x)$

19. False

20. True

21. True

22. True

23. True

24. False

(b)

26. (c)

(iv)

(iii)

(iii)

30. (iv)

(iii)

32. (iii)

33. (iii)

34. (iii)

(iv)

36. 0, 1.

Problems 17.1, page 579

1. $z = px + qy + p^2 + q^2$

2. $z^2(p^2 + q^2 + 1) = c^2$

3. $p^2 + q^2 = \tan^2 \alpha$

4. $p + q = px + qy$

5. $z^2(p^2 + q^2 + 1) = 9$

6. $py - qx = 0$

7. $py + qx = 0$

8. $qx - py = x + y$

9. $xyz = px + py - z$

10. $xyr = 2(px + qy - 2z)$

11. $\frac{\partial^2 z}{\partial y^2} = \frac{\partial z}{\partial y}$

12. $x(y - z)p + y(z - x)q = z(x - y)$

13. $z \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y}$

14. $p + q = mz$

15. $px^2 + qy = 2y^2$

16. $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = 0$

17. $\frac{\partial^2 v}{\partial t^2} = \frac{a^2}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right)$

18. $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial t} + \frac{\partial^2 z}{\partial t^2} = 0$

19. $p(x - 2z) + q(2z - y) = y - x$

20. $(y - z)p + (z - x)q = x - y$.

Problems 17.2, page 581

1. $z = \frac{x^2}{2} \log x + axy + \phi(x) + \psi(y)$

2. $z = \frac{1}{6} x^3 y + xf(y) + \phi(y)$

3. $u = -e^{-t} \sin x + \phi(x) + \psi(t)$

4. $z = f(x) + x\phi(y) + \psi(y) - \frac{1}{12} \sin(2x + 3y)$

5. $z = e^x \cosh y + e^{-x} \sinh x$

6. $z = \sin x + e^y \cos x$.

Problems 17.3, page 584

1. $x = z^3 f(x/y)$

2. $\sqrt{x} - \sqrt{y} = f(\sqrt{x} - \sqrt{z})$

3. $x^2 + y^2 + z^2 = f(x + y + z)$

4. $[\cos(x+y) + \sin(x+y)]e^{y-x} = \phi \left[z^{\sqrt{2}} \tan \left(\frac{x+y}{2} + \frac{\pi}{8} \right) \right]$

5. $x^2 - y^2 = f(y^2 - z^2)$

6. $\phi \left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z} \right) = 0$

7. $x \log(x+y) - z = f(x+y)$

8. $x^2 + y^2 + 2z = [\log(xy)]$

9. $x^2 + y^2 - z^2 = f(x+y+z)$

10. $x+y+z = f(xyz)$

11. $\phi(x^2 + y^2 + z^2, xyz) = 0$

12. $x^2 + y^2 = f(y^2 - yz)$

13. $\phi(y/z, x^2 + y^2 + z^2) = 0$

14. $x^2 + y^2 + z^2 = f(y^2 - 2yz - z^2)$

15. $f \left(\frac{y}{z}, \frac{z}{x} - \frac{y}{x} + x^2 \right) = 0$.

Problems 17.4, page 587

1. $z = ax - ay/(1+a) + b$
2. $z = ax + \sqrt{(1-a^2)y + c}$
3. $4z(1+a^2) = (x+ay+b)^2$
4. $(1-a+az) = (x+ay+b)^2$
5. $2z = ay^2 - [a/(a+1)]x^2 + b$
6. $z = a(x-y) - (\cos x + \cos y) + b$
7. $\frac{8}{9}z = (x+a)^{3/2} + (y+a)^{3/2} + b$
8. $3z = (x+a)^3 + (y-a)^3 + b$
9. $z = \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + \frac{x_1 \sqrt{(x^2+a^2)}}{2} + \frac{y \sqrt{(y^2-a^2)}}{2} - \frac{a^2}{2} \cosh^{-1} \frac{y}{a} + b$
10. $z = ax + by + \sin(a+b)$
11. $z = \frac{1}{6}(zx+a)^3 + a^2y + b$
12. $z = ax + by - 2\sqrt{(ab)}$
13. $z = axy + a^2(x+y) + b.$

Problems 17.5, page 590

1. $z = \{\sqrt{(ax)} + \sqrt{(b+y)}\}^2 / (1+a)$
2. $z = ax^b y^{1/b}$
3. $\frac{z^2}{2} \pm \left\{ \frac{z}{2} \sqrt{z^2 - 4a^2} - 2a^2 \log \left(z + \sqrt{z^2 - 4a^2} \right) \right\} = 2ax + 2y + b$
4. $\log(z-ax) = y - a \log(a+y) + b$
5. $2\sqrt{(z-a-b)} = \sqrt{ax} + \frac{1}{\sqrt{a}}y + c$
6. $z = axe^{-y} - \frac{1}{2}a^2e^{-2y} + b.$

Problems 17.6, page 595

1. $z = f_1(y) + f_2(y+2x) + xf_3(y+2x)$
2. $z = f_1(y-x) + f_2(y+2x) + xf_3(y+2x) + \frac{e^{x+2y}}{27}$
3. $z = f_1(x+y) + xf_2(x+y) + \frac{x^2}{2} \times e^{x+y}$
4. $z = f_1(y+x) + zf_2(y+x) + f_3(y+2x) - e^{2x+y}$
5. $z = f_1(y+x) + xf_2(y+x) - \sin x$
6. $y = f_1(x-at) + f_2(x+at) - \frac{E}{p^2} \sin pt$
7. $z = f_1(y) + f_2(y+2x) + xf_3(y+2x) + 3x \cos(3x+2y)$
8. $f_1(y) + f_2(y-2x) + f_3(y+3x) + \frac{1}{75} \sin(x+2y) + \frac{2}{3}x^3.$
9. $z = f_1(y+x) + f_2(y+2x) + \frac{1}{12}e^{2x-y} - xe^{x+y} - \frac{1}{3} \cos(x+2y)$
10. $z = f_1(y) + f_2(y+x) + \frac{1}{3}(\sin x \cos 2y + 2 \cos x \sin 2y)$
11. $z = f_1(y) + f_2(y+x) + \frac{1}{2}[\sin(x+2y) + \cos(x+2y)] - \frac{1}{6}[\sin(x-2y) + \cos(x-2y)]$
12. $z = f_1(y+x) + f_2(y-x) + \frac{3}{28}e^{x-y}[\sin(x+2y) - 2 \cos(x+2y)]$
13. $z = f_1(y-x) + f_2(y-2x) + 4x^3y - 3x^4$
14. $z = f_1(y-x) + xf_2(y-x) + \frac{1}{4}(x^4 - 2x^3y + 2x^2y^2)$
15. $z = f_1(y-x) + f_2(y+2x) + ye^x$
16. $z = f_1(y-x) + xf_2(y-x) + f_3(y+x) + \frac{e^x}{25}(\cos 2y + 2 \sin 2y)$
17. $z = f_1(y-x) + xf_2(y-x) + x \sin y.$

Problems 17.7, page 597

1. $z = e^{-x} \phi_1(y) + e^x \phi_2(y - x) - \frac{xe^{-x}}{2}$
2. $z = e^x \phi_1(y - x) + e^{3x} \phi_2(y - 2x) + x + 2y + 6$
3. $z = e^x \phi_1(y) + e^{-x} \phi_2(x + y) + \frac{1}{2} \cos(x + 2y)$
4. $z = f_1(y) + e^{-x} f_2(y + x) + \frac{1}{3} x^3 - x^2 + xy^2 + 6x$
5. $z = f_1(x) + e^{3y} f_2(2y - x) + \frac{3}{50} [4 \cos(3x - 2y) + 3 \sin(3x - 2y)]$

Problems 17.8, page 598

1. $z = \phi_1(x) + \phi_2(x + y + z)$
2. $z = \phi_1(y + \sin x) + \phi_2(y - \sin x)$
3. $z = \phi_1(xy^2) + \phi_2(x^2y)$
4. $z = \phi_1(y/x) + \phi_2(x^2 + y^2) + xy$
5. $y = \phi_1(z) + e^x \phi_2(z)$
6. $y = \phi_1(x + y + z) + x \phi_2(x + y + z).$

Problems 17.9, page 598

1. order two & degree two
2. $z = f_1(y + 2x) + xf_2(y + 2x)$
3. $z = -x^2 \sin xy + yf(x) + \phi(x)$
4. $x^2 + y^2 + z^2 = f(x + y + z)$
5. $-\frac{1}{2} \sin(x + y)$
6. $xp + yq = z$
7. $z = ax + (1 - \sqrt{a})^2 y + c$
8. $\sqrt{x} - \sqrt{y} = f(6/x - \sqrt{z})$
9. $x \log(x + y) = z + f(x + y)$
10. First
11. $z = 2x + y \log x + f(xy)$
12. $\partial z / \partial x = \partial z / \partial y$
13. $z = f_1(y) + f_2(y + x) + f_3(y + 2x)$
14. $4y^2 p = q^2$
15. $u = \int f(y) dy + \phi(x)$
16. $c = 1$
17. $u = \frac{1}{6} x^3 y + xf(y) + \phi(y)$
18. $f_1(y + x) + f_2(y + 6x)$
19. (iv)
20. (iii)
21. (iii)
22. (iii)
23. (ii)
24. (iv)
25. (iv)
26. (i)
27. False
28. False
29. True
30. True
31. False.

Problems 18.1, page 601

1. $z = ce^{4ax^3} \cdot e^{-3ay^4}$
2. $u = ce^{k(1/y - 1/x)}$
3. $u = 8e^{-12x-3y}$
4. $u = 3e^{x-y} - e^{2x-5y}$
5. $u = 3e^{-5x-3y} + 2e^{-3x-2y}$
6. $u = \frac{1}{\sqrt{2}} \sinh \sqrt{2x} + e^{-3y} \sin x.$

Problems 18.2, page 610

1. $y(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \cos \frac{n\pi ct}{l}$, when $b_n = \frac{2}{l} \int_0^l f(x) \cdot \sin \frac{n\pi x}{l} dx$
2. $y(x, t) = \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi ct}{l} + b_n \sin \frac{n\pi ct}{l} \right) \sin \frac{n\pi x}{l}$ where
 $a_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} dx, b_n = \frac{2}{n\pi c} \int_0^l g(x) \sin \frac{n\pi x}{l} dx$
3. $y = \frac{8k}{\pi^2} \left(\sin \frac{\pi x}{l} \cos \frac{\pi ct}{l} - \frac{1}{3^2} \sin \frac{3\pi x}{l} \cos \frac{3\pi ct}{l} + \dots \right)$
4. $y = \frac{8h}{\pi^2} \left(\sin \frac{\pi x}{l} \cos \frac{\pi ct}{l} - \frac{1}{3^2} \sin \frac{3\pi x}{l} \cos \frac{3\pi ct}{l} + \dots \right)$

6. $y(x, t) = \frac{4l^2 c}{a\pi^3} \left\{ \sin \frac{\pi x}{l} \sin \frac{\pi at}{l} - \frac{1}{33} \sin \frac{3\pi x}{l} \sin \frac{3\pi at}{l} \dots \right\}$

7. (i) $y(x, t) = a(x - x^2 - c^2 t^2)$; (ii) $y(x, t) = \frac{a}{2}(1 - \cos 2\pi x \cos 2\pi ct)$.

Problems 18.3, page 617

1. $u(x, t) = \frac{400}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} e^{-(2n+1)c\pi/100^2 t} \sin \frac{(2n+1)\pi x}{100}$

2. $\sum_{n=odd} \frac{8a}{n^3 \pi^3} \sin \frac{n\pi x}{l} e^{(n\pi c/l)^2 t}$ 3. $u(x, t) = \frac{40}{\pi} \sum_{n=1}^{\infty} \frac{[1-4(-1)^n]}{n} \sin \left(\frac{n\pi x}{30} \right) e^{-\frac{(an\pi)^2 t}{900}}$

4. $u(x, t) = -3x + 90 - \frac{80}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{5} e^{-c^2 n^2 \pi^2 t / 25}$

5. $u(x, t) = \frac{5}{2} - \frac{4l}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos \frac{(2n-1)\pi x}{l} e^{-(2n-1)^2 c^2 \pi^2 t / 25}$

6. $u(x, t) = 50 - \frac{400}{\pi^2} \sum_{n=1}^{\infty} \frac{2}{(2n-1)^2} \cos \frac{(2n-1)\pi x}{50} e^{-c^2 \pi^2 (2n-1)^2 t / 2500}$

7. $\theta = \frac{4\theta_0}{\pi} \left[e^{-(\pi/2l)^2 kt} \cos \frac{\pi x}{2l} - \frac{1}{3} e^{-(3\pi/2l)^2 kt} \cos \frac{3\pi}{2l} x + \frac{1}{5} e^{-(5\pi/2l)^2 kt} \cos \frac{5\pi}{2l} x - \dots \right]$

8. $V = V_0 e^{-\sqrt{(n/2k)x}} \sin [nt - \sqrt{(n/2k)} x]$.

Problems 18.4, page 623

1. $u = -\frac{8}{\pi} \sum_{n=1, 3, 5, \dots}^{\infty} \frac{\sin nx \sinh n(\pi - y)}{n(n^2 - 4) \sinh nx}$ 5. $u(x, y) = \frac{3200}{\pi^3} \sum_{n=1}^{\infty} \frac{\sin \frac{(2n-1)\pi x}{20} \sinh \frac{(n-1)\pi y}{20}}{(2n-1)^2 \sinh (2n-1)\pi}$

8. $u(x, y) = u_0 \cosh \frac{\pi x}{a} \cosh \frac{\pi}{a} (b-y) \operatorname{sech} \frac{\pi b}{a}$.

Problems 18.5, page 626

1. $u(r, \theta) = \frac{8k}{\pi} \sum_{n=1}^{\infty} \left(\frac{r}{a} \right)^{2n-1} \frac{\sin (2n-1)\theta}{(2n-1)^3}$ 2. $u(r, \theta) = \frac{3200}{\pi^2} \sum_{n=1}^{\infty} \left(\frac{r}{10} \right)^{2n-1} \frac{\sin (2n-1)\theta}{(2n-1)^3}$

3. $u(r, \pi) = \frac{2}{\pi} \sum_{n=1, 3, 5, \dots}^{\infty} \left(\frac{a}{r} \right)^{2n} \frac{r^{4n} - b^{4n}}{a^{4n} - b^{4n}} \cdot \frac{\sin 2n\theta}{n^3}$ 4. $u(r, \theta) = \sum \frac{2k}{n\pi} \left(\frac{r}{a} \right)^{4n} (1 - \cos n\pi) \sin 4n\theta$

5. $u(r, \theta) = 50 - \frac{200}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a} \right)^{2n-1} \sin (2n-1)\theta$

6. $u(r, \theta) = \cos \theta \left(\frac{200}{r} - \frac{r}{2} \right) + \sin \theta \left(2r - \frac{200}{r} \right)$. 7. $u(r, \theta) = 4 \cos \theta (r - 1/r) + 4 \sin \theta (r + 1/r)$.

Problems 18.6, page 630

1. $u = \sum_{m=1}^{\infty} \frac{J_2(\alpha_m)}{\alpha_m^2 J_1^2(\alpha_m)} \cos \alpha_m t J(\alpha_m r)$.

Problems 18.7, page 634

1. $e = e_0 \sin \frac{\pi x}{l} \cos \frac{\pi t}{l\sqrt{LC}}$; $i = i_0 - e_0 \sqrt{\left(\frac{C}{L}\right)} \cos \frac{\pi x}{l} \sin \frac{\pi t}{l\sqrt{CL}}$

3. $v = \frac{20(l-x)}{l} + \frac{24}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{n\pi x}{l} \exp(-n^2 \pi^2 t / RCl^2)$

$$i = \frac{20}{lR} + \frac{24}{lR} \sum_{n=1}^{\infty} (-1)^n \frac{n\pi x}{l} \exp(-n^2 \pi^2 t / RCl^2)$$

4. $v = V_0 \cos(pt - px\sqrt{LC})$.

Problems 18.9, page 638

1. $\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2}, \frac{\partial^2 i}{\partial t^2} = LC \frac{\partial^2 i}{\partial x^2}$

2. $\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$

3. If $u(x, t)$ is the temperature, then temperature gradient at a point is $\partial u / \partial x$ for all t .

4. elliptic

5. $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$

6. $u = \frac{10}{l}x + 30$

7. parabolic partial differential equation

8. $r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$

9. $y = \frac{2h}{l}x, 0 < x < \frac{l}{2}; y = \frac{l}{2h}(2h - y), \frac{l}{2} < x < l$

10. $y(0, t) = 0, y(l, t) = 0, \left(\frac{\partial y}{\partial t} \right)_{t=0} = 0$

11. zero

12. $u(0, y) = 0, v(a, y) = 0, 0 < y < a; u_x(x, 0) = 0$ for all t and $u(x, a) = u$ for $0 < x < a$

13. $\frac{\partial u(0, t)}{\partial x} = 0, \frac{\partial u(l, t)}{\partial x} = 0$ for all t

14. $y(x, t) = f(x + ct) + f(x - ct)$

15. $\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$ where c^2 is the diffusivity

16. $u(x) = x^2 + 20$

17. § 18.8-(6), (7), (8)

18. $u = 8e^{-12x-3y}$

19. $z = 4e^{3x+t}$

20. $\alpha^2 (= k/sp)$ is called the diffusivity of the substance (cm^2/sec)

21. $\frac{\partial^2 v}{\partial x^2} = RC \frac{\partial v}{\partial t}, \frac{\partial^2 i}{\partial x^2} = RC \frac{\partial c}{\partial t}$

22. $u(x, t) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} e^{-c^2 n^2 \pi^2 t / l^2}$

23. § 18.7 - (3), (4), (5)

25. False.

Problems 19.1, page 646

1. (i) $\sqrt{2} \left(\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} \right) \cos \left(\frac{\pi}{4} - \frac{\alpha}{2} \right)$. (ii) $-8i/25$

2. $\frac{-y}{x^2 + y^2 - 2x + 1}$

3. $x = \pm 1.5, y = \pm 2$

5. A circle: centre $(-1, 1)$ radius $\sqrt{2}$

8. $-1 + i\sqrt{3}, -1 - i\sqrt{3}, 1 - i\sqrt{3}, 4\sqrt{3}$

9. $-2 + 0i, 1 - i\sqrt{3}$

10. $-1 - i, \sqrt{2}(\mp \sin 15^\circ \pm i \cos 15^\circ), \sqrt{2}(\mp \cos 15^\circ \pm i \sin 15^\circ)$