

11. (i) Annular region between the circles of radii 2 and 4 with centre $(-3, 0)$ including boundary of inner circle; (ii) Region of complex plane above the line $y = 2$; (iii) Infinite region bounded by the lines $\theta = \pi/3$ and $\theta = \pi/2$; (iv) Real axis and region above it between $x = \pm 2$
13. (i) Ellipse with foci at $z = \pm 1$ and major for axis = 3
(ii) (a) Right bisector of the line joining $z = 3$ and $z = -1$;
(b) Circle through the points $z = 3$ and $z = -1$;
14. (i) Right bisector of the points 0 and 2; (ii) Circle through the points $\frac{1}{3}$ and 3
15. (i) A straight line; (ii) Circle with centre $(1, 1/2)$ and radius $\sqrt{5}/2$.

Problems 19.2, page 650

5. $4m\pi/n(n+1)$

7. $2^{n+1} \sin^n \frac{\alpha-\beta}{2} \cos n\left(\frac{\pi+\alpha+\beta}{2}\right).$

Problems 19.3, page 653

1. (i) $(2)^{1/8} [0.98 \pm i(0.195)]6$; $(2)^{1/2} [-0.195 \pm i(0.98)]$
(ii) $(2)^{1/5} \cos \frac{4n+3}{10}\pi$, where $n = 0, 1, 2, 3, 4$; (iii) $\pm 2\sqrt{2}$
(iv) $2^3\sqrt{2} \cos r\pi/9$, where $r = 1, 7$ or 13
3. $\pm i, \frac{1}{2}(\sqrt{3} \pm i), \frac{1}{2}(-\sqrt{3} \pm i)$
5. (i) $1, -1, \cos(\pm \pi/5), \cos(\pm 3\pi/5)$
(ii) $-1, \frac{1}{2}(1 \pm i\sqrt{3}), \pm(1+i)/\sqrt{2}, \pm(-1+i)/\sqrt{2}$
(iii) $\frac{1 \pm i}{\sqrt{2}}, \frac{-1 \pm i}{\sqrt{2}}; 1, \cos\left(\pm \frac{2\pi}{5}\right), \cos\left(\pm \frac{4\pi}{5}\right)$.
(iv) $\cos(2n+1)\pi/6$, where $n = 0, 1, 2, 3, 4, 5$; $2(2m+1)\pi/3$, where $m = 0, 1, 2$
6. $(-1+i)/\sqrt{2}, (1-i)/\sqrt{2}$
7. $\pm 1, \pm i, \pm \left(\cos \frac{\pi}{6} \pm i \sin \frac{\pi}{6}\right), \pm \left(\cos \frac{\pi}{3} \pm i \sin \frac{\pi}{3}\right)$; Last four values
9. $x^3 + x^2 - 2x - 1 = 0$.

Problems 19.4, page 655

1. $32 \cos^5 \theta - 24 \cos^3 \theta + 6 \cos \theta$
13. $-(2)^{-11} (\sin 120 - 2 \sin 100 - 4 \sin 80 + 10 \sin 60 + 5 \sin 40 - 20 \sin 20)$
14. $\sin^5 \theta = A \sin \theta - B \sin 3\theta + C \sin 5\theta$.

Problems 19.5, page 661

1. (i) $e^{x^2-y^2} \cos 2xy, e^{x^2-y^2} \sin 2xy$ (ii) ie^5 . (iii) $e^{16} \cos 30, e^{16} \sin 30$
5. $[(pq' - p'q)(qr' - q'r)]^2 = [(pq' - p'q)^2 + (qr' - q'r)^2](pr' - p'r)^2$
10. $\frac{1}{64} (\cosh 7\theta + 7 \cosh 5\theta + 21 \cosh 3\theta + 35 \cosh \theta)$
17. $-\log^3$ 18. $-13/12$.

Problems 19.6, page 664

10. $\pm \frac{\pi}{4} + \frac{i}{4} \log \frac{1+\sin\theta}{1-\sin\theta}$ according as $\cos\theta$ is + ve or - ve

11. $\sin^{-1}(\sqrt{\sin\theta}) + i \log [\sqrt{(1+\sin\theta)} - \sqrt{\sin\theta}]$.

Problems 19.7, page 667

1. (i) $\log_e 10 + i [\tan^{-1}(4/3) \pm 2n\pi]$; (ii) $\log_e 1 + i (\pi + 2n\pi)$
4. (i) $\sqrt{2}e^{-(2n-\frac{1}{4})\pi}, (2n-\frac{1}{4})\pi + \log \sqrt{2}$; (ii) $e^{-\pi^2/8}, (\pi/4) \log_e 2$
9. $\sqrt{[\frac{1}{2}(\cos 2x + \cosh 2y)]} - i \tan^{-1}(\tan x \tanh y)$
10. (i) $2n\pi \pm i \log(2 + \sqrt{3})$; (ii) $-\frac{1}{2} \log 3 + (n + \frac{1}{2})i\pi$.

Problems 19.8, page 669

1. $e^{\sin\theta \cos\theta} \cos(\theta + \sin^2\theta)$
2. $\sin\alpha \cos(\cos\beta) \cosh(\sin\beta) - \cos\alpha \sin(\cos\beta) \sinh(\sin\beta)$
3. $\tan^{-1} \frac{x \sin\alpha}{1+x \cos\alpha}$, except when $x = 1$ and $\alpha = (2n+1)\pi$
4. $\log(2 \cos\theta/2)$
5. $-\frac{1}{2} \tan^{-1}(\cos\beta \operatorname{cosech}\alpha)$
6. $\frac{1}{2} \tan^{-1} \frac{2c \sin\alpha}{1-c^2}$
7. $(2 \cos\theta)^{-1/2} \cos\theta/2$
8. $\sin \frac{n(\pi-\alpha)}{2} / (2 \sin\alpha/2)^n$
9. $\sin\left(\alpha + \frac{n-1}{2}\beta\right) \sin \frac{n\beta}{2} \operatorname{cosec} \frac{\beta}{2}$
10. $\frac{\cos(\alpha + \frac{1}{2}(n-1)\beta) \sin n \frac{\beta}{2}}{\sin \frac{1}{2}\beta}$
11. $\frac{\sin\alpha(\cos\alpha - \sin\alpha)}{1 - \sin 2\alpha + \sin^2\alpha}$
12. $\frac{1 - x \cos\theta - x^n \cos n\theta + x^{n+1} \cos(n-1)\theta}{1 - 2x \cos\theta + x^2}$.

Problems 19.9, page 670

2. 0.053 radians
3. $1^\circ 59'$
4. 39.7.

Problems 19.10, page 670

- | | | | |
|--|-----------------------|----------------------------|--|
| 1. (b) | 2. (c) | 3. (b) | 4. (c) |
| 5. (b) | 6. (d) | 7. odd | 8. $32 \cos^6\theta - 48 \cos^4\theta + 18 \cos^2\theta - 1$ |
| 9. $6(1-2i)$ | 10. $2i \sin n\theta$ | 11. $\frac{1}{25}(-6+17i)$ | 12. $\cosh x \cos y$ |
| 13. $\frac{1}{19}$ radians | 14. 1 | 15. $-\cos x \sinh y$ | 16. $e^{-\pi/4\sqrt{2}}$ |
| 18. $2i n\pi$ | 19. real | 20. $\sinh \phi$ | 21. $\sinh 2\phi / (\cosh 2\theta + \cosh 2\phi)$ |
| 22. $16 \cos^5\alpha - 20 \cos^3\alpha + 5 \cos\alpha$ | 26. a circle | 23. an equilateral | 24. $\pi/2$ |
| 25. $x = \pm 1, y = -4$. | 29. True | 27. True | 28. True |
| 29. True | 30. True | 31. False | 32. True |
| 33. False. | | | |

Problems 20.1, page 682

4. $a = 1, b = -6, c = 1, d = 2, e = 4$

6. (i) and (ii) Not analytic. (iii) Analytic

7. $p = -1$

11 & 12. $f(z)$ is not analytic at origin although C-R equations are satisfied there

14. (i) $z^3 + 3z^2 + 1 + iz$; (ii) $\cos z$; (iii) $\log z$; (iv) iz ; (v) $e^z + i(c-z)$; (vi) $ze^{2z} + ic$; (vii) $z \sin z$; (viii) $x^2e^z + ic$

15. (i) $(1+i)/z + c$; (ii) $\cos z + c$; (iii) e^z ; (iv) $\bar{z}e^{-\bar{z}} + c$; (v) $1 + ize^{-z}$; (vi) $(2 \cos x \cosh y)/(\cos 2x + \cosh 2y)$

16. (i) $f(z) = c - iz^3$; (ii) $f(z) = \left(\frac{1}{2} - \cot \frac{z}{2}\right)$; (iii) $\frac{\cot z}{1+c} + c\left(\frac{1+3i}{5}\right)e^z + c$

17. $\psi = 3xy^2 - x^3 + c$

18. $2 \tan^{-1}(y/x); 2 \log z + c$ 20. $v = C - e^{-2xy} \cos(x^2 - y^2); f(z) = C - ie^{iz^2}$

22. (i) $x^4 - 6x^2y^2 + y^4 = c$ (ii) $x^2 - y^2 + 2e^x \sin y = c$ (iii) $r^2 \sin 2\theta = c'$

23. (i) $x/(x^2 + y^2)$; (ii) $\frac{1}{2} \log(x^2 + y^2)$

24. (i) $a(1 + \cos \theta + i \sin \theta \log r)$; (ii) $(r + 1/r) \cos \theta + (r - 1/r) \sin \theta + c$

27. $-2 \tan^{-1}[(y-2)/(x-1)], 2i \log(z-1-2i)$.

Problems 20.2, page 687

1. $z = \pm i$

2. $w = -1/z$

3. $w = (2i - 6z)/(iz - 3)$; fixed points $z = i, 2i$; no critical points

4. $w = \frac{(20 + 18i) - (32 + 12i)z}{(29 + 17i) - (11 - 3i)z}$

5. (i) $w = i(1-z)/(1+z)$; (ii) $w = \frac{(4i-2)+(5-3i)z}{2i+(1+i)z}$ (iii) $w = (1-z)/(1+z)$.

Problems 20.3, page 692

1. (i) $I(w) > 0$; (ii) Region bounded by the parabolas $v^2 = 4(1 \pm u)$; and $u^2 = 1 - 2v$; (iii) Region bounded by parabolas $v^2 = \frac{1}{4} \pm u$, $v^2 = 4(1 \pm u)$; (iv) Region boundary $\rho = 2 \sqrt{\rho} \cos \frac{\phi}{2} + 3$.

2. $w = z^6$

3. Lines parallel to x and y axes map into two families of rectangular hyperbolas in the w -plane which cut each other at right angles. Lines parallel to u and v axes map into two families of parabolas in the z plane which cut each other orthogonally. It is conformal at $z = 0$

4. (a) Line $4v + 1 = 0$

5. (b) Every circle through the origin ($z = 0$) transforms into a st. line not passing through the origin ($w = 0$). If a line passes through $z = 0$, its image is a line through $w = 0$. (b) Circle with centre $(1/2, 1/2)$ and radius $(1/\sqrt{2})$ (c) Lemniscate $p^2 = \cos 2\phi$

10. $z = \pm a$

11. See § 20.10 (3)

14. See § 20.10(4).

Problems 20.4, page 694

1. $z = \cosh w$

3. $w = \sin z$

4. $w = \log z$.

Problems 20.5, page 696

1. (a) $(5-i)/6$ (b) $(5+i)/6$

2. (i) $4 + (25/3)i$; (ii) $4 + 8i$

3. $\frac{1}{6}(64i - 103)$

6. (i) i ; (ii) $2i$; (iii) 0

7. (a) $\frac{1}{3}(i-1)$, (b) $\frac{1}{6}(5i-3)$

9. (i) $\frac{2}{3}$; (ii) $-\frac{2}{3}$.

Problems 20.6, page 702

1. (i) 0 ; (ii) $2\pi i$
 4. (i) $5\pi i$; (ii) $\pi i/2$ (iii) $4\pi i$
 7. (i) $8\pi i$; (ii) 0
2. 0
 5. (a) $-10\pi i$ (b) $2\pi ie$
 8. (i) 0 ; (ii) $2\pi(6 + 13i)$; (iii) $12\pi i$
3. (a) zero ; (b) zero
 6. (i) $4\pi i$; (ii) $2\pi ie^{-1}$; (iii) $-\pi i$
 9. zero.

Problems 20.7, page 709

1. $\sum_{n=1}^{\infty} (-1)^{n+1} n(z-1)^n$; Convgt. in $|z-1| < 1$
2. $f(z) = \frac{1}{3} + \frac{u}{9}(z+i) - \frac{7}{27}(z+1)^2 + \dots$. Region of convergence is $|z+i| < 1$
3. (i) $\frac{1}{2}(z-1) - \frac{1}{2^2}(z-1)^2 + \frac{1}{2^3}(z-1)^3 - \dots$ (ii) $-(z-\pi/2) + (z-\pi/2)^3/3! - (z-\pi/2)^5/5! + \dots$
 (iii) (a) $f(z) = -\frac{1}{5} \sum_{n=0}^{\infty} (-1)^n (z+1)^n - \frac{1}{20} \sum_{n=0}^{\infty} \left(\frac{z+1}{4}\right)^n$ in the region $|z+1| < 1$. Also, $(z+1) < 4$
 (b) $f(z) = -\frac{1}{5} \sum_{n=0}^{\infty} \left(\frac{z-1}{5}\right)^n \frac{1}{10} \sum_{n=0}^{\infty} \left(\frac{z-1}{2}\right)^n$ in the region $|z-1| < 2$. Also $|z-1| < 3$
4. (i) $\frac{1}{z+1} + \frac{1}{(z+1)^2} + \frac{1}{(z+1)^3} + \dots + \frac{1}{2} \left[1 + \frac{z+1}{2} + \frac{(z+1)^2}{2^2} + \frac{(z+1)^3}{2^3} \right]$
 (ii) $\frac{1}{4z} \left(1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right) - \frac{1}{12} \left(1 - \frac{z}{3} + \frac{z^2}{9} - \dots \right)$. (iii) $-\frac{1}{2(z-1)} - 3 \sum_{n=1}^{\infty} \frac{(z-1)^{n-1}}{2^{n+1}}$
5. (i) $e \left[(z-1)^{-2} + (z-1)^{-1} + \frac{1}{2!} + \frac{1}{3!}(z-1) + \frac{1}{4!}(z-1)^2 + \dots \right]$
 (ii) $e^2(z-1)^{-3} + 2e^2(z-1)^{-2} + 2e^2(z-1)^{-1} + \frac{4e^2}{3} + \frac{2e^2}{3}(z-1) + \dots$
6. (i) $\sum_{n=1}^{\infty} (-1)^{n-1} n \cdot (z-1)^{-n}$ for $|z-1| > 1$. (ii) $-\sum_{n=2}^{\infty} \frac{z^{2n-5}}{2(n-1)!}$
7. (i) $1 + \frac{3}{z} \left(1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + \dots \right) - \frac{8}{3} \left(1 - \frac{z}{3} + \frac{z^2}{3^2} - \frac{z^3}{3^3} + \dots \right)$
 (ii) $\frac{2}{z+2} + \frac{3}{(z+2)^2} + \frac{3^2}{(z+2)^3} + \dots + \frac{1}{5} \left(1 + \frac{z+2}{5} + \frac{(z+2)^2}{5^2} + \frac{(z+2)^3}{5^3} + \dots \right)$
 (iii) (a) $\frac{7}{z} - \frac{9}{z^2} - \frac{45}{z^3} - \frac{81}{z^4} + \dots$ (b) $\frac{5}{2(z-3)} + \frac{7}{12} - \frac{z-3}{24} - \frac{5(z-3)^2}{432} + \frac{7(z-3)^2}{864} + \dots$
8. (a) $\frac{z}{4} - \frac{5z^2}{16} + \frac{21}{64} z^5 - \dots$; (b) $\frac{1}{3} \left(\frac{1}{z^5} - \frac{1}{z^3} - \frac{z}{4} + \frac{z^3}{16} - \frac{z^5}{64} + \dots \right)$; (c) $\frac{1}{z^3} - \frac{3}{z^5} + \dots$
9. $z=0, z=2$ are the isolated singularities
 10. $z=0$ is an isolated essential singularity
11. $z=0$, is a non-isolated essential singularity
12. $z=1$ is a pole of order 2
 13. $z=1$ is a pole of order 4
14. $z=a$ is a double pole and $z=0, \pm 1, \pm 2, \dots$ are simple poles.

Problems 20.8, page 715

1. $-\frac{1}{t} - 2i + 3t + 4i t^2 + \dots$ where $t = z - i ; -1$

2. (i) $3e/2$; (ii) $\text{Res } f(z = -ai/2)$
3. (i) $\text{Res } f(0) = -1/2$, $\text{Res } f(2) = 2\frac{1}{2}$;
(ii) $\text{Res } f(-1) = 0$, $\text{Res } f(i) = \frac{1+2i}{2(1-i)}$, $\text{Res } f(-i) = \frac{2i-1}{2(i-1)}$
(iii) $\text{Res } f(-1) = 1$, $\text{Res } f(i) = \frac{2+i}{-1+i}$, $\text{Res } f(-i) = \frac{-2+i}{1+i}$
4. (i) $\text{Res } f(0) = -4/3$; (ii) $\text{Res } f(i) = \frac{1}{2}e^{-1}$; $\text{Res } f(-i) = \frac{1}{2}e$; (iii) $-i\pi/4$, $\text{Res } f(n\pi) = 1$, n an integer.
5. (i) 0; (ii) $16\pi i/(2+3i)$
6. (i) $\frac{\pi ie^{-4}}{5}$ (ii) 0; (iii) $i\pi/4$ 7. (i) πi ; (ii) $\pi/2(3+2i)$, (iii) zero.
8. (i) $-2\pi i$; (ii) πi ; (iii) $\pi/16$
9. (i) $-2\pi i$; (ii) $8\pi i/3e^2$; (iii) $\frac{\pi e^{2i}}{2}$; (iv) $\frac{8\pi}{3}ie^{-2}$
10. (i) $\frac{21\pi i}{16}$; (ii) $2\pi i \sec 1(1 + \tan 1)$; (iii) $-2\pi i$
11. $2\pi i$ 12. $\frac{1}{z^3} - \frac{1}{6z} + \frac{7z}{360} - \frac{31z^6}{15120} + \dots; -\frac{1}{3}\pi i$.

Problems 20.10, page 723

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|--|-----------|--------------------------------------|----------|---|---------|
| 1. (i) | 2. (iii) | 3. (a) | 4. (iii) | 5. (ii) | 6. (ii) |
| 7. (ii) | 8. (ii) | 9. (iii) | 10. (ii) | 11. (ii) | 12. (i) |
| 13. (ii) | 14. (iii) | 15. $v(x, y) = x^2 - y^2 + 2y + c$ | | | |
| 16. $3x^2y - y^3 + c$ | | 17. $u + iv$ is an analytic function | | 18. $2i/3$ | |
| 19. 1 | | 20. $z = \frac{1}{2}(a+b)$ | | 21. $2u + 1 = 0$ | |
| 22. $z = 1, \frac{1}{2}(1 \pm \sqrt{3})$ | | 23. $z = 0$ | | 24. -1 | |
| 25. $z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \dots$ | | 26. zero | | 27. (iii) | |
| 28. $\frac{1}{2} \left[\frac{d^2}{dz^2} \{(z-a)^3 f(z)\} \right] z = a$ | | 29. zero | | 30. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ | |
| 31. zero | | 32. $e^x \sin y$ | | 33. zero | |
| 34. $z = 2$ | | 35. no point in the z -plane | | 36. $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$ | |
| 37. zero | | | | | |
| 38. A simply connected region is one in which any closed curve, lying entirely within it, can be shrunk to a point without going out of the region | | | | | |
| 39. which is analytic or regular | | 40. $z = 1, 2$ | | | |
| 41. $(z_1 - z_2)(z_3 - z_4)/(z_1 - z_4)(z_3 - z_2)$ | | 42. i | | | |
| 43. $\frac{1}{2} + \frac{3}{4}z + \frac{7}{8}z^2 + \frac{15}{16}z^3 + \dots$ | | 44. $\pm i$ | | | |
| 45. 1 | | 46. Magnification and rotation | | | |
| 47. The coefficient of $(z-a)^{-1}$ in the expansion of $f(z)$ around an isolated singularity ($z=a$) is called the residue of $f(z)$ at that point. | | | | | |
| 48. $2\pi i$ | | 49. zero | | 50. $z = 1, 2$ | |
| 51. $-\frac{1}{2} \left\{ 1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots \right\}$ | | 52. $i/2$ | | 53. $z = 0, 2$ | |
| 54. § 20.14 | | 55. $\pm i$ | | 56. zero | |
| 57. Zeroes are at $z = \pm 1$, singularity is at $z = 0$ | | | | 58. -1 | |

59. essential singularity

60. zero

$$61. \sin z = \frac{1}{\sqrt{2}} \left\{ 1 + (z - \pi/4) - \frac{(z - \pi/4)^2}{2!} - \frac{(z - \pi/4)^3}{3!} + \frac{(z - \pi/4)^4}{4!} + \dots \right\}$$

62. a circle with centre (3, 2) and radius 2 in w -plane.63. $n\pi$, n an integer64. $\phi(a)/\psi'(a)$

65. circles

66. True

67. False

68. True

69. True

70. False

71. True

72. True

73. True

74. True

75. True

76. True

77. False

78. False

79. False

80. True

81. True

82. True.

Problems 21.1, page 732

1. $\frac{1}{s-2} + \frac{24}{s^4} + \frac{3(s-2)}{s^4+9}$.

2. $\frac{1}{s} + \frac{\sqrt{\pi}}{s^{3/2}} + \sqrt[3]{\left(\frac{\pi}{s}\right)}$

3. $\frac{3s-20}{s^2-25}$

4. $\frac{s \cos b - a \sin b}{s^2 + a^2}$

5. $\frac{s^2 - 2s + 4}{s(s^2 + 4)}$

6. $\frac{2(s^2 - 5)}{(s^2 + 1)(s^2 + 25)}$

7. $\frac{\sqrt{\pi} - e^{1(1/4s)}}{2s^3/2}$

8. $\frac{5}{4} \left\{ \frac{1}{s^2 + 1} - \frac{3/2}{s^2 + 9} + \frac{1/2}{s^2 + 25} \right\}$

9. $\frac{s(s^2 + 28)}{(s^2 + 4)(s^2 + 36)}$

10. $\frac{b}{(s+a)^2 - b^2}$

11. $\frac{60}{s-2} - \frac{s-2}{s^2 - 4s + 20}$

12. $\frac{30(s+3)}{(s^2 + 6s + 13)(s^2 + 6s + 73)}$

13. $\frac{2}{(s+1)(s^2 + 2s + 5)}$

14. $\frac{1}{8} \left\{ \frac{3}{s-2} - \frac{4(s-2)}{s^2 - 4s + 8} + \frac{s-4}{s^2 - 8s + 32} \right\}$

15. $\frac{a(s^2 + 2a^2)}{s^4 + 4a^4}$

16. $\frac{3}{2} \left[\frac{1}{s^2 - 9} + \frac{s^2 - 13}{s^4 - 10s^2 + 169} \right]$

17. $\frac{2}{(s+2)^3}$

18. $\frac{1}{n} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{6}{(s+3)^4}$

19. $4 \frac{(4s^2 + 4s - 1)}{(4s^2 + 1)^2}$

20. $\frac{4}{s} - \frac{e^{-s}}{s}$

21. $\frac{1 + e^{-\pi s}}{s^2 + 1}$

22. $\frac{e^{-\pi s/3}}{s^2 + 1}$

23. $e^{-2\pi s/3} \frac{s}{s^2 + 1}$

24. $\frac{2}{s^3} - \frac{e^{-2s}}{s^3} (2 + 3s + 3s^2) + \frac{e^{-3s}}{s^2} (5s - 1)$

25. $\frac{4}{(s-1)(s^2 - 2s + 5)}$.

Problems 21.2, page 734

1. $(1/s^2 T) - e^{-sT}/s(1 - e^{sT})$

2. $[Ew/(s^2 + w^2)] \coth(\pi s/2w)$

3. $(a/s) \tanh(as/2)$

4. $(1/s^2) \tanh \frac{1}{2} as$

5. $\frac{1}{\sqrt{(s^2 + a^2)}}$

6. $(s^2 - 2as + a^2 + b^2)^{-1/2}$

7. $\frac{2}{(s-2)\sqrt{(s+1)}}$.

Problems 21.3, page 740

1. $\frac{s+1}{s(s^2 + 2s + 2)}$

4. $\frac{2(3s^2 + 4)}{s^2(s^2 + 4)^2}$

5. $\frac{16}{(s^2 + 4)^2}$

6. $\frac{2s^3 - 6s^2 s}{(s^2 + a^2)^3}$

7. $\frac{2as}{(s^2 - a^2)^2}$

8. $\frac{6(s-2)}{(s^2 - 4s + 13)^2}$

9. $\frac{8(s+2)}{s^2 + 4s + 20}$

10. $\frac{2(s^3 + 6s^2 + 9s + 2)}{(s^2 + 4s + 5)^3}$

11. $\log \{(s+b)/(s+a)\}$

12. $\cot^{-1}(s)$

13. $\frac{1}{2} \log \{(s^2 + 36)/(s^2 + 16)\}$

14. $\frac{1}{2} \log \left(\frac{s^2 + b^2}{(s-a)^2} \right)$

15. $\cot^{-1}(s+1)$

16. $\frac{1}{2} \log \left(\frac{s^2 + 9}{s^2} \right)$

17. $\cot^{-1}s - \frac{1}{2}s \log(1+s^{-2})$

18. $\frac{1}{s-\log 2} + \frac{2s}{(s^2+1)^2} + \frac{1}{2} \log \left(\frac{s^2+9}{s^2+4} \right)$

19. (i) $\log 2/3$; (ii) $\pi/8$; (iii) $12/169$; (iv) $\frac{8(s+1)}{s(s^2+2s+17)}$

21. (i) $\frac{1}{s} \cot^{-1}(s)$; (ii) $\frac{1}{s} \cdot \frac{s+1}{s^2+2s+2}$; (iii) $\frac{\cot^{-1}(s-1)}{s}$.

Problems 21.4, page 743

1. $\frac{1}{2} \left(\cos \frac{5t}{2} - \sin \frac{5t}{2} \right) - 4 \cosh 3t + 6 \sinh 3t$

2. $e^{3t} - e^{2t}$

3. $3e^{t/2} + 2e^{t/3}$

4. $e^{2t} + 2e^{-4t}$

5. $\frac{1}{3} (8e^{2t} - e^{-t})$

6. $\cosh t$

7. $e^t + e^{-2t} - 2e^{3t}$

8. $2e^{3t} - \frac{3}{5}e^{2t} - \frac{2}{5}e^{7t}$

9. $\frac{1}{2}e^t - e^{2t} + \frac{5}{2}e^{3t}$

10. $\frac{1}{2}t \sinh t$

11. $\frac{1}{3}t(e^t - e^{-2t})$

12. $\frac{1}{13} (3e^{3t} - 3 \cos 2t + 2 \sin 2t)$

13. $\frac{1}{2}(\sin t - te^{-t})$

14. $\frac{1}{2} [\cos at + \cosh at]$

15. $(\frac{1}{3}a^2) [e^{at} - e^{-at/2} (\cos(\sqrt{3}at/2) + \sqrt{3}\sin(\sqrt{3}at/2))]$

16. $\frac{1}{3} (5 \sin t - \sin 2t)$

17. $\frac{1}{3}e^{-2t} (6 \cos 3t - 7 \sin 3t)$

18. $\frac{1}{5} (1 + e^{-t}) \sin t + \frac{3}{5} (1 - e^{-t}) \cos t$

19. $\frac{1}{3}e^{-t} (\sin t + \sin 2t)$

20. $(2/\sqrt{3}) \sinh(\frac{1}{2}t) \sin(\frac{1}{2}\sqrt{3}t)$

21. $\cos at \sinh at$.

Problems 21.5, page 750

1. $\frac{1}{25} (e^{-5t} + 5t - 1)$

2. $\frac{1}{8} - \frac{1}{4} \left(t^2 + t + \frac{1}{2} \right) e^{-2t}$

3. $\frac{1}{a^2} \cos \left(\frac{bt}{a} \right)$

4. $\frac{1}{a^2} \left(t - \frac{1}{a} \sin at \right)$

5. $\frac{1}{2} t^2 + \cos t + 1$

6. $\frac{1}{2} t e^{-2t} \sin 2t$

7. $t \sin at$

8. $\frac{1}{2} (a^2 t^2 - 4at + 2) e^{-at}$

9. $\frac{1-e^t}{t}$

10. $\frac{1}{t} (e^{-bt} - e^{-at})$

11. $e^{-t} - e^{-2t} - e^{-3t}$

12. $\frac{1}{t} (\cos at - \cos bt)$

13. $\frac{2}{t} (1 - \cosh at)$

14. $\frac{2}{t} (e^t - \cos t)$

15. $\frac{\sin 2t}{t}$

16. $\frac{\sin t}{t}$

17. $\frac{2(\sinh t - t \cosh t)}{t^2}$

18. $\frac{e^{-bt} - e^{-at}}{a - b}$

19. $\frac{1}{2a^3} (\sin at - at \cos at)$

20. $\frac{1}{a^3} (at - \sin at)$

21. $t(e^{-t} + 1) + 2(e^{-t} - 1)$

22. $\frac{1}{16} (e^{2t} - e^{-2t} - 4te^{-2t})$

23. $\frac{1}{13} (3 \sin 3t + 2 \cos 2t - 2e^{-2t})$

24. $\frac{t^2}{2} + \cos t - 1$

25. $\frac{e^{-2t}}{54} (\sin 3t - 3t \cos 3t)$.

Problems 21.6, page 754

1. $y = \frac{7}{4} e^{-t} - \frac{3}{4} e^{-3t} - \frac{1}{2} te^{-t}$

2. $x = \frac{at}{2} \sinh t$

3. $y = t - 3 \sin t + \cos t$

4. $y = 2t + 3 + \frac{1}{2}(e^{3t} - e^t) - 2e^{2t}$

5. $y = 4e^{2t}(1+t) - 7e^t$

6. $y = 2 \cos 5t + t \sin 5t$

7. $y = \frac{1}{2\omega} \sin \omega t$

8. $y = \frac{1}{8} e^t - \frac{1}{40} e^{-3t} - \frac{1}{10} (2 \sin t + \cos t)$

9. $y = \frac{1}{2} (\cos kt + \cosh kt)$

10. $y = \frac{1}{8} [(3-t^2) \sin t - 3t \cos t]$

11. $y = \frac{11}{3} e^{-t} (\sin t + \sin 2t)$

12. $y = \frac{-12}{5} + \frac{12}{5} e^{-t} \cos 2t + \frac{7}{10} e^{-t} \sin 2t$

13. $y = e^{2t}(x^2 - 6x + 12) - e^t(15x^2 + 7x + 11)$

14. $x = \frac{4}{9} \sin 2t - \frac{5}{9} \sin t - \frac{1}{3} t \cos 2t$

15. $y = \frac{1}{2} \left(\frac{3 \sin t}{t} - \cos t \right)$

16. $y = e^{2t}$

17. $y = t$

18. $y = 3J_0(2t)$

21. $(n \sin at - a \sin nt) F_0 / mn(n^2 - a^2)$, where $n^2 = k/m$.

Problems 21.7, page 756

1. $x = \frac{1}{2} (e^t + \cos t + 2 \sin t - t \cos t)$, $y = \frac{1}{2} (t \sin t - e^t + \cos t - \sin t)$

2. $x = e^t + e^{-t}$, $y = e^{-t} - e^t + \sin t$

3. $x = 2 + t^2/2$, $y = -1 - t^2/2$

4. $x = \frac{1}{10} (5 - 2e^{-t} - 3e^{-6t/11})$, $y = \frac{1}{5} (e^{-t} - e^{-6t/11})$

5. $x = e^6 (1 + 2t) + 2e^{3t}$, $y = \sinh t + \cosh t - e^{-3t} - te^t$

6. $i_1 = \frac{a}{p+\omega} (\sin \omega t + \sin pt)$; $i_2 = \frac{a}{p-\omega} (\cos \omega t - \cos pt)$.

Problems 21.8, page 762

1. $\frac{2}{s^2 + 4} (e^{-2\pi s} - e^{-4\pi s})$

2. (i) $(1-2t) u(t-\pi) + 2tu(t)$, $\frac{2}{s^2} + \left(\frac{1-2\pi}{s} - \frac{2}{s^2} \right) e^{-as}$

(ii) $t^2 [u(t) - u(t-2)]$, $\frac{2(1-e^{-2s})}{s^3} - \frac{4e^{-2s}(1+s)}{s^2}$

(iii) $\{u(t) - u(t-T)\} \cos(\omega t + \phi)$; $[(s \cos \phi - \omega \sin \phi) - e^{-sT} \times \{s \cos(\phi + \omega T) - \omega \sin(\phi + \omega T)\}] / (s^2 + \omega^2)$

3. (i) $\frac{s}{s^2+1} + \left(\frac{1}{s} + \frac{s}{s^2+1} \right) e^{-\pi s} - \left(\frac{1}{s} - \frac{1}{s^2+1} \right) e^{-2\pi s}$

(ii) $\frac{1}{s^2+1} + e^{-\pi s} \left(\frac{s}{s^2+4} - \frac{s}{s^2+1} \right) + e^{-2\pi s} \left(\frac{s}{s^2+9} - \frac{s}{s^2+4} \right)$

(iii) $\frac{2}{s^3} \{1 + e^{-2s} (2s^2 - 1) - 2e^{-4s} (1 + 4s)\}$

4. (i) $e^{-s}/(s-1)$; (ii) $2e^{-s}/s^3$; (iii) $e^{-2s} \left\{ \frac{24}{s^4} + \frac{42}{s^3} + \frac{28}{s^2} + \frac{25}{s} \right\}$; (iv) $e^{-s}(2+2s+s^2+s^3)/s^3$

5. $20e^{-2}$

6. (i) $-\sin t \cdot u(t-\pi)$; (ii) $\frac{1}{3} e^{-4(t-2)} \sin 3(t-2) \cdot u(t-2)$;

(iii) $\frac{1}{2} e^{-(t-1)} (t-1)^2 u(t-1)$ (iv) $3 - 4(t-1)u(t-1) + 4(t-3)u(t-3)$

7. $y = \frac{1}{2} \sin 2t + \frac{1}{4} (1 - \cos 2t) - \left\{ \frac{1}{4} [1 - \cos(t-1)] u(t-1) \right\}$

8. $x = 3 - 2 \cos t + 2[t-4 - \sin(t-4)] u(t-4)$.

9. $y(x) = \begin{cases} \frac{2Wx^2(3l-5x)}{81EI}, & 0 < x < l/3 \\ \frac{2Wx^2(3l-5x)}{81EI} + \frac{W}{6EI} \left(x - \frac{l}{3} \right)^3, & \frac{l}{3} < x < l \end{cases}$

10. $y(x) = \frac{wl^2}{16EI} x^2 - \frac{wl}{12EI} x^3 + \frac{w}{24EI} x^4 - \frac{w}{24EI} (x-l/2)^4 u(x-l/2)$

11. $x = \frac{I}{mn} e^{-\mu t/2m} \sin nt$; $\frac{dx}{dt} = \frac{I}{m} e^{-\mu t/2m} \left(\cos nt - \frac{\mu}{2mn} \sin nt \right)$, where $n^2 = \frac{k}{m} - \frac{\mu^2}{4m^2}$.

Problems 21.9, page 764

1. $\frac{2s}{(s^2+1)^2}$

2. (d)

3. (b)

4. $\frac{1}{s^2 - 4s + 5}$

5. te^{-2t}

6. $\frac{1}{3} e^{-2t} \sin 3t$

7. $s \bar{f}(s) - f(0)$

8. $\frac{1}{8} (2-3t) e^{-3t/2}$

9. $\frac{1}{2} \left[\frac{1}{s} + \frac{s}{s^2 - 16} \right]$

10. $\frac{1}{s - \log 2}$

11. $\frac{k!}{(s+1)^{k+1}}$

12. $\frac{2}{13}$

13. $e^{-at/s}$

14. $\Gamma(3/2)/s^{3/2}$

15. $s^2 \bar{f}(s) - sf(0) - f'(0)$

16. $\frac{1}{4} \left[\frac{3s}{s^2 + 16} + \frac{s}{s^2 + 144} \right]$

17. $\cot^{-1}(s/a)$

18. $\frac{e^{-3t} t^4}{24}$

19. $\frac{s \cos 3 - 2 \sin 3}{s^2 + 4}$

20. (c)

21. $f(t-a) u(t-a)$

22. e^{-as}

23. $1 - 3t + 2t^2$

24. $\int_0^t f(t) dt$

25. $\int_0^T \frac{e^{-st}}{(1-e^{-st})} f(t) dt$

26. $\frac{1}{(s+1)^2(s+2)}$

27. $\frac{2(s-3)}{s^2-6s+34} + \frac{12}{s^2-6s+25}$

28. $1/(s - \log 4)$

29. $\frac{e^{-3t}}{\sqrt{\pi t}}$

30. (d)

31. $\frac{t}{2} \sin \frac{t}{2}$

32. (c)

33. (c)

34. (ii)

35. (iii)

36. (ii)

37. (iv)

38. (i)

39. $\int_a^b f(x)dx$ 40. (ii) 41. (iii) 42. (d)
 43. True 44. False 45. False.

Problems 22.1, page 776

1. $\frac{2}{\pi} \int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} d\lambda ; \frac{\pi}{2}$ for $|x| < 1, \frac{\pi}{4}$ for $|x| = 1, 0$ for $|x| > 1$
2. (i) $\frac{4}{\pi} \int_0^\infty \frac{\sin \omega - \omega \cos \omega}{\omega^3} \cos \omega x d\omega$ (ii) $\frac{2}{\pi} \int_0^\infty \cos \omega x \frac{a}{a + \omega^2} d\omega$
4. (i) $\frac{2\sin as}{s}, \pi$; (ii) $2[(a^2 s^2 - 2)\sin as + 2as \cos as]/s^3$
5. $\frac{4}{s^3} (\sin sa - sa \cos sa)$ 6. (i) $\sqrt{(3\pi)} e^{-3s^2/4}$ (ii) $\frac{\sqrt{\pi}}{2} e^{(3is - s^2/4)}$
7. $\frac{1 - \cos 2s}{s}, \frac{\sin 2s}{s}$ 8. $\sqrt{(\pi/2)} e^{-as}$ 9. $\frac{a}{a^2 + s^2}; \frac{\pi}{2a} e^{-a\lambda}$
10. 1
13. (i) $\frac{\pi}{2a^2} (1 - e^{-as})$ (ii) $\tan^{-1}(s/a)$
11. $\frac{1}{a\sqrt{2}} e^{-s^2/4a^2}; \frac{1}{2a^3\sqrt{2}} e^{-s^2/4a^2}$
14. (i) $\frac{1}{2} \left\{ \frac{\sin [a(1-s)]}{1-s} - \frac{\sin [a(1+s)]}{1+s} \right\}$ (ii) $(2\cos s - \cos 4s - 1)/s^2 - (2\sin s)/s$
15. $2/(\pi s^2)$ 16. $F_s(p) = -32(-1)^p/p\pi; F_c(p) = 32\frac{(-1)^p - 1}{p^2\pi^2}$
17. $(\pi/s) \cos s/c$ 19. $f(x) = (2 + 2\cos x - 4\cos 2x)/\pi x$ 20. $2/\pi(1+x^2)$.

Problems 22.2, page 780

2. $\frac{1}{4} \int_{-\infty}^{\infty} e^{-[|x-t|+|t|]} dt$
5. $2\left(\frac{1 - \cos s}{s^2}\right).$

Problems 22.3, page 783

1. $\frac{1}{9} [e^{-t} + e^{2t} (3t - 1)]$
2. $\frac{1}{5} (e^{-2t} - 2 \sin t - \cos t)$
3. $(\sinh at - at)/a^2$
4. $\frac{1}{2} [e^x (x - 1) + \cos x]$
5. $\frac{1}{2} (\sin t - t \cos t).$

Problems 22.4, page 791

1. $y = 30 e^{-75t} \cos 5x$
2. $\sum_{n=1}^{\infty} \frac{V_o}{n\pi} (1 - \cos n\pi) e^{-n^2 t \sin nx_0}$
3. $u(x, t) = \frac{2}{\pi} \int_0^{\infty} e^{-s^2 t} \left\{ \frac{\sin (1+x)s + \sin (1-x)s}{s} \right\} ds$
6. $\theta(x, t) = \theta_0 \operatorname{erf} \left(\frac{x}{2\sqrt{(kt)}} \right) + \theta_0 \sum_{n=1}^{\infty} (-1)^n \left\{ \operatorname{erf} \frac{nl - x}{2\sqrt{(kt)}} - \operatorname{erf} \frac{nl + x}{2\sqrt{(kt)}} \right\}.$

Problems 22.5, page 792

1. $F_c(s) = \int_0^\infty f(t) \cos st dt$
2. $s^2/2$
3. The Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transforms.
4. $f(x) = \frac{1}{2\pi} \int_{-\infty}^\infty F(s) e^{-isx} ds$
5. $\int_{-\infty}^\infty t^n f(t) e^{ist} dt$
6. $e^{isa} F(s)$
7. $\frac{2}{\pi} \int_0^\infty \sin \lambda x \int_0^\infty f(t) \sin \lambda t dt d\lambda$
8. $\frac{1}{a}$
9. $-s^2 [F(u)]$
10. $\frac{2}{\pi} \int_0^\infty \sin(\lambda x) d\lambda \int_0^\pi \sin(\lambda t) dt$
11. $-\frac{n\pi}{l} \int_0^1 f(x) \cdot \cos \frac{n\pi x}{l} dx$
12. $\frac{1}{a} F\left(\frac{\lambda}{a}\right)$
13. $f(x) = \frac{2}{\pi^3} \sum_{p=1}^\infty \left(\frac{1 - \cos p\pi}{p^2} \right) \sin px$
14. $\frac{1}{2} F(s/2)$
15. $1/(s^2 + 1)$
16. True
17. False
18. True
19. True
20. False
21. True.

Problems 23.1, page 800

1. (i) e^{az} ; (ii) $z(e^{-z} - 1)$; (iii) $\frac{z}{z - e^{i\theta}}$
2. (i) $\frac{2z}{(z-1)^2} + \frac{z/\sqrt{2}}{z^2 - \sqrt{2}z + 1} + \frac{z}{z-1}$; (ii) $\frac{z^3 - 3z^2 + 4z}{(z-1)^3}$; (iii) $\frac{z(z+1)}{(z-1)^3} + \frac{3z}{(z-1)^2} + \frac{2z}{z-1}$
6. (i) $\frac{z^2 \sin \theta}{z^2 - 2z \cos \theta + 1}$ (ii) $\cos \alpha \frac{z(z - \cos \pi/8)}{z^2 - 2z \cos \pi/8 + 1} - \sin \alpha \frac{z \sin \pi/8}{z^2 - 2z \cos \pi/8 + 1}$
7. $\frac{z(z - \cos \theta)}{z^2 - 2z \cos \theta + 1}, |z| > 1; \frac{z(z^2 \cos \theta - 2z + \cos \theta)}{(z^2 - 2z \cos \theta + 1)^2}$
8. $\frac{z^2}{z^2 + 1}, |z| > 1; \frac{z^2}{z^2 + a^2}$
9. (i) $\frac{z}{z - e^{-a}}$; (ii) $\frac{ze^{-a}}{(z - e^{-a})^2}$; (iii) $\frac{(z + e^{-a})e^{-a}}{(z - e^{-a})^3}$
12. $\frac{z^2(1 + 3z^2)}{(1 - z)(1 + z^2)}$
13. $u_2 = 2, u_3 = 11.$

Problems 23.2 page 804

1. $\frac{z}{z-4}; |z| > 4$
2. $\frac{z}{2-z}; |z| < 2$
3. $\frac{3z}{(4-z)(z-3)}; 3 < |z| < 4.$
4. $\frac{5z}{(4-5z)^2}; |z| > 5$
5. $-\log(1 - 3/z); |z| > 3$
6. $e^{3/z}$, ROC is z -plane
7. $(1 - e^a/z)^{-1}; |z| > |e^a|.$

Problems 23.3, page 807

1. $\frac{1}{2}(3^{n+1} - 1)$
2. $(n+1)a^n$
3. $\frac{1}{2}n(n-1)$
5. $4a^n$
6. $(1/3)^n - 2^n$
7. n
8. $\frac{3}{4} \left\{ \frac{1}{(2)^{n-1}} + \frac{1}{(-4)^n} \right\}$
9. $(n^2 + 7n + 4)(4)^{n-1}$

13. $u_n = (2)^{n-1} + (3)^{n-1} + (4)^{n-1}$ ($n > 0$)
12. $(-1)^{n+1} - 2n + \cos n\pi/2$
13. $1 - e^{-at}$
14. (i) $\left(-\frac{1}{3} - \frac{z}{3^2} - \frac{z^2}{3^3} - \frac{z^3}{3^4} \dots\right) + \left(\frac{1}{2} + \frac{z}{2^2} + \frac{z^2}{2^3} + \frac{z^3}{2^4} + \dots\right)$; (ii) $(-2^{n-1})z^{-n}$, $n > 0$
 (iii) $(3^{n-1} - 2^{n-1})z^{-n}$, $n \geq 1, 0, n \leq 0$
15. $\frac{1}{2}(n-1)(n-2)5^{n-3}$, $n \geq 3$ and $= 0, n < 3$
16. $2(-i)^{n-1} - (-2)^{n-1}$
17. $\frac{1}{3} + \frac{2}{3}\left(-\frac{1}{2}\right)^n$
18. $\frac{1}{3} - \frac{1}{3}\left(-\frac{1}{2}\right)^{n-1}$
19. $u_n = 1 + \frac{1}{2}[(i)^{n-2} + (-i)^{n-2}]$
20. $2n \sin(n\pi/2)$, $n = 0, 1, 2, \dots$

Problems 23.4, page 811

1. $y_k = \frac{8}{5}\left(\frac{1}{2}\right)^k - \frac{3}{5}\left(\frac{-1}{3}\right)^k$
2. $y(n) = (n-1)(-1)^{n-2}y(n-2) - 2^n$
3. $y_n = 2^{n-1} + (-2)^{n-1}$
4. $f(n) = 2 + (-4)^n$
5. $y_n = \frac{4}{3}[2(-1)^n + (2)^n]$
6. $36\left[\frac{1}{2} - (2)^n + \left(\frac{1}{2}\right)^n\right]$
7. $y_n = (c_1 + c_2 n)3^n + \frac{1}{2}n(n-1)3^{n-2}$
8. $y_n = c_1 + c_2 \cdot 3^n + 5^n/8$
9. $y_n = 2\left[\left(\frac{1}{4}\right)^n - \left(-\frac{1}{4}\right)^n\right]$
10. $5 \cdot 2^n$
11. $y_n = \frac{1}{3}(-1)^n - \frac{2}{5}(-3)^n + \frac{1}{15}(2)^n$
12. $y_k = 1 - 2k + 2^k$
13. $y_n = c_1 4^n + \left(c_2 - \frac{n}{4}\right)2^n + 2n - \frac{8}{3}$
14. $y_k = \frac{1}{2}(k+2)\frac{1}{5^k} \cos \frac{k\pi}{2}$
15. $y_n = \left[\frac{1}{4} - \frac{9}{4}(-3)^n\right] \mu(n)$
16. $y_n = (-2)^{n-1}$, ($n \geq 1$).

Problems 23.5, page 811

1. $z/(z-1)$
2. $\sum_{n=0}^{\infty} u_n z^{-n}$
3. $z/(z-1)^2$
4. $az/(z-a)^2$
5. $\frac{z \sin \theta}{z^2 - 2z \cos \theta + 1}$
6. $e^{1/z}$
7. $(z^2 + z)/(z-1)^3$
8. $Z(au_n + bv_n + cw_n) = aZ(u_n) + bZ(v_n) + cZ(w_n)$
9. 2^{n-1}
10. $(-1)^{n-1}n$
11. $u_0 = \lim_{z \rightarrow \infty} \{Z(u_n)\}$
12. False
13. False
14. True
15. True
16. False
17. False
18. False.

Problems 24.1, page 815

1. $a = 2.28, b = 6.1879, p = 30.46$
2. $a = 1120, b = 55.1$
3. $a = 0.2, b = 0.0044$
4. $a = 0.5012, n = 0.5$
5. $a = 0.115, b = 11.8$
6. $a = 4.1, b = 0.43$
7. $a = 0.0498, b = -0.02$

Problems 24.2, page 819

1. $y = 13.6x$
 2. 15.2 thousand tons
 3. $Y = 0.004P + 0.048$
 4. $R = 70.052 + 0.292t$
 5. $a = 0.545, b = 0.636$
 6. (a) $y = 4.193 + 1.117x$
 (b) $y = 8 - 0.5x$
 7. $y = 1.243 - 0.004x + 0.22x^2$
 8. $y = 0.34 - 0.78x + 0.99x^2$
 9. $y = 18.866 + 66.158x - 4.333x^2$
 10. $R = 3.48 - 0.002V + 0.0029V^2$
 11. $V = 2.593 - 0.326T + 0.023T^2$.

Problems 24.3, page 823

1. $6.32, b = 0.0095$
 2. $a = 1.52, b = 0.49$
 3. $a = 3, b = 2$
 4. $y = 7.187 - 5.16 \frac{1}{x}; 4.894$
 5. $a = 0.988, b = 3.275$
 6. $y = 2.978 x^{0.5143}; 5.8769$
 7. $a = 0.1839, b = 0.0221$
 8. $f(t) = 0.678 e^{-3t} + 0.312 e^{-2t}$
 9. $a = 146.3, k = -0.412$
 10. $a = 99.86, b = 1.2$.

Problems 24.4, page 826

1. $a = 11.1, b = 0.71$
 2. $y = 46.05 + 6.1x$
 3. $a = 0.0028, b = 0.01, c = 4.18$
 4. $a = 15.8, b = 2.1, c = -0.5$
 5. $a = 1.459, b = 0.062$.

Problems 24.5, page 828

1. $y = 0.12 + 0.47x$
 2. $y = 1.184 + .523x$
 3. $y = 1.53 + 0.063x + 0.074x^2$
 4. $y = 0.485 + 0.397x + 0.124x^2$.

Problems 24.6, page 829

1. $Y = aX + b$ where $X = x, Y = y/x$
 2. $Y = A + BX$, where $X = \log_{10} p, Y = \log_{10} v, A = \frac{1}{r} \log b, B = -1/r$
 3. § 24.4
 4. $y = aX + c$, where $X = x^b$
 5. (ii)
 6. $\Sigma y = nA + B\Sigma x, \Sigma xy = A\Sigma x + B\Sigma x^2$ where $y = \log_{10} y, A = \log_{10} a, B = \log_{10} b$
 7. § 24.12
 8. Zero
 9. $y = aX + b$ where $X = x^2/\log_{10} z, Y = y/\log_{10} x$
 10. $a = 0.0167, b = 1.05$.
 11. The moments of the observed values of y are respectively equal to the moments of the calculated values of y .
 12. $a = 1.7, b = 1.26$
 13. $y = a + bx$ where $x = 1/x, y = 1/y$
 14. (r)
 15. (b)

Problems 25.1, page 837

1. 336.79
 2. 64% get more than 50 marks ; median 54.7, $Q_1 = 46, Q_3 = 61.5$
 3. Mean = 27.9 ; Median = 25.66 ; Mode = 21.85
 4. Mean = 32.58 ; Median = 32.6 ; Mode = 35.1
 5. 3.1%
 6. 1.3%
 7. 192 km/hr
 8. 60 km/hr
 9. 38.6 ; 36.2
 10. Median = 12.2 days ; Mode = 11.4 days

Problems 25.2, page 842

1. 4.45, 0.39
 3. 4, 7
 4. 10.04, 10.13, 11.69, 5.54, 2.35

5. 32, 32.6, 12.4 6. Q.D. = 10.9, S.D. = 15.26 7. 1.845 ; 1.8175
 8. 0.55, 1.24 ; first, yes 9. Height 10. A
 11. B is a better player and more consistent
 12. A is more efficient, B is more consistent 13. 161.3, 5.68.

Problems 25.3, page 845

1. $\mu_1 = \mu_3 = 0, \mu_2 = 2, \mu_4 = 11; \beta_1 = 0, \beta_2 = 2.75$ 2. 8.85 ; 5.25 ; 0.32 ; 1.09
 3. -0.2064 4. 0.22 ; 1.157 6. 0 ; 2.9
 7. $\beta_1 = 0.493; \beta_2 = 0.655$; platykurtic.

Problems 25.4, page 854

1. $r = 0.81; x = 0.5y + 0.5, y = 1.3x + 1.1$ 2. $r = 0.96$
 3. $r = 0.92$ 4. $r = -0.055$ 5. 0.7291
 6. $r = 0.4517$ 7. $r = 0.632; y = 0.467 + 0.8x, x = 0.167 + 0.5y$
 9. $m = (\beta - b)/(a - \alpha)$ 10. $\bar{x} = 4, \bar{y} = 7, r = -0.5$ 11. $\bar{x} = 9.06, \bar{y} = 5.52, r = 0.46$
 12. $r = 0.7395; \bar{x} = -0.1034; \bar{y} = 0.5172$ 13. 134.5
 14. 1.28 inch 15. 0.8545 16. 0.932.

Problems 25.5, page 855

1. (d) 2. (d) 3. (a) 4. (a)
 5. (b) 6. (a) 7. (b) 8. Coefficient of correlation
 9. No 10. 13.83 11. $\bar{x} = 2, \bar{y} = 3, r = \sqrt{3}$
 12. Zero 13. $\frac{1}{2}(Q_3 - Q_1)$ 14. -1
 15. § 25.9 16. $\frac{\Sigma XY}{n \sigma_x \sigma_y}$ 17. (\bar{x}, \bar{y})
 18. Reliability or consistency 19. $\sqrt{\beta_1}$ 20. degree of peakedness
 21. $y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$ 22. 100 σ/\bar{x} 23. $\tan^{-1} \left\{ \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \right\}$
 24. 3 25. Two regression coefficients 26. perpendicular
 27. greater 28. ± 1 29. $1 - \frac{6 \sum d_i^2}{n^3 - n}$
 30. Coefficient of standard deviation 31. zero
 32. -1 and 1 33. -0.6 34. False
 35. True 36. False

Problems 26.1, page 858

1. 4096 2. 360 ; 120 3. 120 ; 115 4. (a) 676000 (b) 468000

Problems 26.2, page 868

1. (i) 7/12 ; (ii) $P(A/B) = 3/4, P(A \cup B) = 7/12, P(A' B') = 3/8$
 2. (a) 1/36 ; (b) 1/6, Yes 3. 36 : 30 : 25
 4. 1/7 5. (i) 1/4, (ii) 7/8, (iii) 11/16 6. 15/1024

7. 3/28 8. 20/81 9. (i) 2816/4165 ; (ii) 2197/2025,
 10. 0.11 11. (a) 6.739 ; (b) 0.024 12. (a) 1/114 ; (b) 685/1140
 13. 2/801 14. 10/21 15. $1 - (1 - p_1)(1 - p_2) \dots (1 - p_n)$; 0.518
 16. 15/17 17. 1/2 18. 5/12 19. (i) 83/110 (ii) 25/83
 20. $1 - 2/(n - 1)$ 21. 7/20 22. (a) 1/6 ; (b) 3/4 23. 61/90
 24. 0.72 25. 0.2223 26. 0.88

Problems 26.3, page 871

1. 3/11 2. 25/69, 28/69, 16/69 3. 0.3175, 0.254 4. 15/59.

Problems 26.4, page 878

1. $k = 1$; $\mu = .8$, $\sigma^2 = 2.232$ 2. $2\sqrt{5}$
 3. $F(x) = 0$, $-\infty < x < 0$ 5. ₹ 32
 $= 1/8$, $0 \leq x \leq 1$
 $= 1/2$, $1 \leq x \leq 2$
 $= 7/8$, $2 \leq x \leq 3$
 $= 1$, $3 \leq x \leq \infty$
 6. 2 7. $f(x)$ is a p.d.f. $\bar{x} = \frac{1}{2}$, $\sigma^2 = \frac{1}{20}$ 8. (i) 9/16, 7/16 ; (ii) $k = 0.45$
 9. (i) 0.37, (ii) 0.63 10. 4/9
 12. $y_0 = 3/4$; Mean = 1; Variance = 1/5. 13. 0.2
 14. 1/3, 2/9 15. $F(x) = 0$, $x < x_1$; $(x - x_1)/(x_2 - x_1)$, $x_1 \leq x < x_2$; 1, $x \geq x_2$.

Problems 26.5, page 881

1. $n = 4$, $p = q = \frac{1}{2}$; $\frac{15}{16}$ 2. ${}^4C_r(1/6)^{4-r}(5/6)^r$; $r = 0, 1, 2, 3, 4$
 3. (a) 0.02579 ; (b) 0.04571 ; (c) 1.024×10^{-7} 4. 0.3456
 5. 45927/50000 6. (i) 0.246 ; (ii) 0.345
 7. (i) ${}^{20}C_1(1/20)(19/20)^{19}$; (ii) $\sum_{r=0}^5 {}^{20}C_r(1/20)^r(19/20)^{20-r}$ (iii) 19
 8. (a) 0.08 ; (b) 0.26 ; (c) 0.92 9. (a) 250 ; (b) 25 ; (c) 500
 10. (i) 0.59049 ; (ii) 0.32805 ; (iii) 0.08146 11. 11
 12. 99.83 13. 0.3585, 0.3773, 0.1887 ; 0.0596
 14. 600 15. 100 $(.432 + .568)^5$
 16. 200 $(0.554 + 0.446)^6$.

Problems 26.6, page 884

1. (i) 2 ; (ii) $\frac{2}{3} e^{-2}$ 2. $P(0) = 0.2636$, $P(3) = 0.1041$, $P(> 3) = 0.1506$
 4. $(10)^{15} e^{-10}/15! = 0.035$ 5. 0.08
 6. (i) 0.2231 (ii) 0.1913 7. 0.0008
 8. $m = 0.51 = \sigma^2$; Poisson frequencies of 0, 1, 2, 3, 4 accidents are 180.1, 91.9, 23.4, 4, 0.6
 9. 0.6 11. Theoretical frequencies are 44, 43, 21, 7, 1
 12. Theoretical frequencies are 109, 142, 92, 40, 13, 3, 1, 0, 0, 0, 0.

Problems 26.7, page 890

2. (i) 0.1644 ; (ii) 0.7686 3. (i) 0.095 ; (ii) -0.995 4. 36.4
 5. (i) 16, (ii) 2 6. 294 7. 543
 8. (i) 79 ; (ii) 35% ; (iii) 11 10. 52 11. 67
 12. ₹ 866 13. $y = \frac{100}{\sqrt{(3.4\pi)}} e^{-\frac{(x-2)^2}{3.4}}$ 14. $\mu = 13.64, \sigma = 3.98.$

Problems 26.8, page 892

1. (a) 0.0287, (b) 0.9672, (c) 0.5111 2. (a) 0.7854, (b) 0.1815, (c) 0.1815
 3. (a) 0.97815, (b) 0.00595, (c) 0.01209.

Problems 26.9, page 893

3. $\frac{1}{2}(n+1), \frac{1}{12}(n^2 - 1)$ 5. Mean = $a + b$, variance = b^2
 6. $[(1 - e^t)/t]^2$ 8. $(1 - t)^{3/4}.$

Problems 26.10, page 894

- | | | | | | |
|---|--|---|--|---|--------|
| 1. (a) | 2. (b) | 3. (d) | 4. (b) | 5. 1/7 | 6. 1/2 |
| 7. (b) | 8. (b) | 9. (a) | 10. (c) | 11. 0.1288 | 12. 2 |
| 13. 0.21 | 14. 0.24 | | | | |
| 15. $X: 0 \quad 1 \quad 2$
$p(x): 1/4 \quad 2/4 \quad 1/4$ | | 16. § 25.5 | | 17. 0.7837 | |
| 18. zero | | 19. equal | | 20. $P(A) + P(B)$ | |
| 21. $\beta_1 = 0, \beta_2 = 3$ | | 22. 120 | | 23. 0.2646 | |
| 24. 1/9 | | 25. 0.2222 | | 26. 1/6 | |
| 27. e^{-3} | | 28. 5/36 | | 29. 2 | |
| 30. symmetrical | | 31. $1 - e^{-m}$ | | 32. six | |
| 33. 0.6915 | | 34. $(q + pe^t)^n$ | | 35. 4 : 5 | |
| 36. 1/6 | | 37. 3.5 | | 38. $\sqrt{2}$ | |
| 39. unity | | 40. $n \rightarrow \infty, p \rightarrow 0$ such that np is fixed | | | |
| 41. $P(A) + P(B)$ | | 42. $yC_x/y + zC_x$ | | 43. np | |
| 44. $P(A \cup B) = 0.72, P(A \cap B') = 0.1653$ | | | | 45. $\left(\frac{1}{3} + \frac{2}{3}\right)^{18}$ | |
| 46. 1 | 47. $\mu_r' = \left[\frac{d^r}{dt^r} (\sum p_i e^{tx_i}) \right]_{t=0}$ | | 48. 1/6 | | |
| 49. 3/4 | 50. 1/2 | 51. 0.2 | 52. 2/7 | | |
| 53. $(q + p)^n$ | 54. 2/3 | 55. Mean and S.D. | 56. 1/13 | | |
| 57. 2 | 58. § 26.6 | 59. $e^{-4/3}$ | 60. 1/3 | | |
| 61. True | 62. False | 63. True | 64. False | | |
| 65. False | 66. True | 67. False | 68. False | | |
| 69. $k = 2$ | 70. 8 | 71. $f(x) = \lambda e^{-\lambda x}$ for $x > 0, \lambda$ is a parameter | | | |
| 72. 25/12 | 73. 2 | 74. l | 75. 4 | | |
| 76. 8 | 77. n, m degrees of freedom | | 78. $\int_{-\infty}^{\infty} e^{t(x-a)} f(x) dx$ | | |

79. $-5/7$

80. $\frac{e^{-x} x^{l-1}}{\Gamma(l)}, 0 < x < \infty$

81. $\frac{1}{2}(b+a)$

82. 50%

83. 3/8

84. 1

85. 3/4

86. (iii)

87. $F(x) = \int_{-\infty}^x f(x) dx$ 88. $\frac{\lambda^{2r} e^{-\lambda}}{(2r)!}$

89. $-\infty < t < \infty$

90. 6

91. 1/9

92. False

93. 2

94. $4(1-x)^3$

95. 0.264

Problems 27.1, page 901

1. Die is biased

2. No

3. Yes

4. 8.91% and 15.07%

5. Consistent

6. $p = 65/500$, S.E. = 0.015

7. 37.5%; 30.3 and 44.7 respectively

8. No

9. Difference is not significant

10. $Z \sim 6.56$ so that the difference is significant

11. No.

Problems 27.2, page 904

1. No

2. Mean weight lies between 64.6 and 69.4 lbs.

3. 0.0774

5. 62 6. 2.696

7. No

8. No

9. (i) Yes, (ii) No 10. Yes.

Problems 27.3, page 910

1. 0.25

2. $t = 0.62$, Yes

3. 11.887 and 12.113 cm

4. Refute the claim

5. Process is not under control

6. No

7. Sample mean = 575.2 kg., S.E. = 2.75 kg

8. Accept null hypothesis

9. Yes with 75% confidence.

10. No

11. No

12. No

Problems 27.4, page 914

1. 0.41

2. Hypothesis is correct

3. Significant at 5% level

4. Yes

5. f_e : 33.82 161.78 315.98 308.48 150.54 29.4 $\chi^2 = 7.97$. Binomial distribution gives a good fit at 5% level6. f_o : 305 365 210 80 18 12 f_e : 301.2 361.4 216.8 86.7 26 7.9 ; $\chi^2 = 3.097$

Poisson distribution gives a good fit at 5% level

7. f_o : 314 355 204 85 29 12 f_e : 301 362.2 217.3 86.9 26.1 6.5

Poisson distribution can be fitted to the data

8. $\chi^2 = 1.2$. The fit is quite good at 5% level.**Problems 27.5, page 917**

1. First variance cannot be regarded as significantly greater than the second

2. Not significant as $F = 2.1$ and $F_{0.05} = 4.15$ 4. Not significant as $F = 2.4$ and $F_{0.01} = 3.2$

5. Product of firm B cannot be said to be of better quality than those of firm A.

6. Not significant at 1% level and just significant at 5% level as $F = 2$, $F_{0.01} = 2.62$ and $F_{0.05} = 1.98$
 7. $F = 1.49$, Not-significant 8. $F = 1.025$; Yes

Problems 27.6, page 917

- | | | |
|--|-------------|----------------------------|
| 1. § 27.3 (3) | 2. § 27.15 | 3. § 27.3 (2) |
| 4. We are testing the hypothesis that one process is better than another | | |
| 5. § 27.11 | 6. 1 | 7. 50 |
| 9. II | 10. 8; 16 | 11. $r = n - 1$ |
| 13. Less than 30 | 14. § 27.17 | 15. $-\infty < t < \infty$ |
| 17. True | | 16. (ii) |

Problems 28.1, page 926

- | | |
|---|---|
| 1. (i) 2.687, (ii) 1.46, (iii) 2.375, (iv) 2.875 | 2. (i) 0.519, (ii) 2.875, (iii) 1.146, (iv) 0.367 |
| 3. (i) -0.686, (ii) 2.7065, (iii) 0.686, (iv) 1.4036 | |
| 4. (i) 0.853, (ii) 0.607, (iii) 2.798, (iv) 3.789, (v) -0.134 | |
| 5. 1.861 | 6. (i) 1.532, (ii) 2.095, (iii) 1.834, (iv) 1.226 |
| 7. (i) 1.855 (ii) 2.198 (iii) 1.662 | 8. -16.56 |
| 9. (i) 0.853, (ii) -1.9338, (iii) 2.7985, (iv) 4.545 | |
| 10. (i) 0.518, (ii) 0.052, (iii) 0.695, (iv) 2.911 | 11. $x_{n+1} = \frac{1}{2}(x_n + N/x_n)$; (i) 3.605 (ii) 3.162 |
| 12. 3.4482 | 13. 2.3784 |
| 14. (i) 0.055 (ii) 0.258 (iii) 0.4347 | |

Problems 28.2, page 929

- | | | |
|--|-----------|----------|
| 1. (i) 1.532, (ii) 0.684, (iii) 3.18, (iv) 1.168 | 2. 1.674 | |
| 3. 2.231 | 4. -1.328 | 5. 2.924 |

Problems 28.3, page 936

- | | | |
|--|---|---|
| 1. $x = 7, y = -9, z = 5$ | 2. $x = -51/4, y = 115/8, z = 35/4$ | 3. $x = 1, y = 2, z = 3$ |
| 4. $x_1 = 2, x_2 = -1, x_3 = 3$ | 5. $x_1 = 1, x_2 = 2, x_3 = -1, x_4 = -2$ | 6. $x = 1, y = 3, z = 5$ |
| 7. $x = 8.7, y = 5.7, z = -1.3$ | 8. $x = 1, y = 2, z = 3$ | 9. $x = 7, y = -9, z = 5$ |
| 10. $x_1 = 2, x_2 = 1/5, x_3 = 0, x_4 = 4/5$ | 11. $x = y = z = 1$ | 12. $x = 1, y = 2, z = 3$ |
| 13. $x = 35/18, y = 29/18, z = 5/18$ | 14. $x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2$ | 15. $\begin{bmatrix} 1.2 & -0.4 & 0.2 \\ -0.2 & -0.1 & 0.3 \\ -0.4 & 0.3 & 0.1 \end{bmatrix}$ |

Problems 28.4, page 942

- | | |
|--|--|
| 1. $x = 2.556, y = 1.722, z = -1.055$ | 4. $x = 0.998, y = 1.723, z = 2.024$ |
| 2. (a) $x = 2.426, y = 3.573, z = 1.926$ (b) $x = 2.426, y = 3.573, z = 1.926$ | 8. $x = -13.223, y = 16.766, z = -2.306$ |
| 3. $x = 1, y = 1, z = 1$ | 10. $x = 1.36, y = 2.103, z = 2.845$ |
| 6. $x = 1.052, y = 1.369, z = 1.962$ | 12. $x = 52.5, y = 44.5, z = 59.7$ |
| 9. $x = 1, y = 2, z = 3, u = 4$ | |
| 11. $x = y = z = 1$ | |
| 13. $x = 1.93, y = 3.57, z = 2.43$ | |

Problems 28.5, page 943

- 1.** $x = 2, y = 1$
2. $x = -1.853, y = -1.927$

- 3.** $x = 0.7974, y = 0.4006$
5. $x = -3.131, y = 2.362$

- 4.** $x = 3.162, y = 6.45$

Problems 28.6, page 945

- 1.** (a) $5.38, \begin{bmatrix} 0.46 \\ 1 \end{bmatrix}$; (b) $4.418, \begin{bmatrix} 1 \\ 0.618 \end{bmatrix}$
2. $3.41; [0.74, -1, 0.67]'$
3. (a) $6, [1, 1, -1]'$ (b) $8, [1, -0.5, 0.5]'$
4. (a) $7; [2.099/7, 0.467/7, 1]$ (b) $25.182, [1, 0.045, 0.068]'$ (d) $11.66 [0.025, 0.422, 1.000]$.

Problems 28.7, page 945

- 1.** Newton-Raphson method **2.** $x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$ **3.** Chord AB
4. $x_{n+1} = \frac{1}{2}(x_n + N/x_n)$ **5.** initial approximation x_0 is chosen sufficiently close to the root
6. diagonal **7.** (c) **8.** (a)
9. $x_{n+1} = \frac{1}{3}(2x_n + N/x_n^2)$ **10.** $x_{n+1} = \frac{1}{2}(x_n + x_{n-1})$ **11.** $x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$
12. (a) **13.** $x_{n+1} = x_n(2 - Nx_n)$ **14.** (b)
15. Newton-Raphson method **16.** § 28.6 **17.** Upper triangular matrix
18. False **19.** True **20.** $x = 1, y = 1$.

Problems 29.1, page 952

- 1.** 0.4 **2.** -7459 **5.** 239 **6.** 4.68, 2.68, 55.8, 99.88
8. (i) $1 - 2 \sin(x + 1/2) \sin 1/2$; (ii) $\tan^{-1}(1/2n^2)$;
 (iii) $192[x(x+4)(x+8)(x+12)(x+16)]$ (iv) $-2/[(x+2)(x+3)(x+4)]$
9. (i) $e^{3x}[e^3 \log(1+1/x) + (e^3 - 1) \log 2x]$ (ii) $2^x(1-x)/(1+x)$
 (iii) $(a-1)^n a^x$; (iv) $(-1)^n n!/[x(x+1)(x+2)\dots(x+n)]$.
12. (i) -36; (ii) $24 \times 2^{10} \times 10!$ **14.** $u = [x]^4 - 6[x]^3 + 13[x]^2 + x + 9$
15. $4x^3 - 12x^2 + 8x + 1$; $12x(x-1)$ **16.** $\frac{1}{2}[x]^4 + 3[x]^3 + 4[x] + c$
17. $y(4) = 74, y(6) = 261$ **19.** 15.

Problems 29.2, page 957

- 1.** $\left(\frac{\Delta^2}{E}\right)u_x = u_{x+h} - 2u_x + u_{x-h}; \frac{\Delta^2 u_x}{Eu_x} = \frac{u_{x+2h} - 2u_{x+h} + u_x}{u_{x+h}}$
2. (i) $2(\cos h - 1) \sin x$; (ii) $6x$; (iii) $2(\cos h - 1)[\sin(x+h) + 1]$; (iv) 8
8. Error = 10 **9.** 31 **10.** $f(1.5) = 0.222, f(5) = 22.022$
11. $y(4) = 74, y(6) = 261$ **12.** -99 **13.** $y_4 = 1$ approx
15. (i) $n(3n^2 + 6n + 1)$; (ii) $\frac{n(n+1)(n+2)(n+3)}{4}$ **16.** $2/(1-x)^3$.

Problems 29.3, page 961

1. 5.54 2. 6.36 3. 1.1312 4. 0.788
 5. ₹ 110.52 6. 8666 7. 352 ; 219 8. 0.9623, 0.2903
 9. 24 approx 10. $f(x) = 9x - 4x^2$ 11. 1.625 12. 0.1955
 13. 4.219 14. 2530 15. $y_1 = 0.1, y_{10} = 100$ 16. $u_2 = 42, u_4 = 49$
 17. 10, 22 18. 755.

Problems 29.4, page 971

1. 19.4 2. 12.826 3. 54000 4. 3.2219
 5. 3.0375 6. 395 7. 3.347 8. 9
 9. 3250.875 11. 2.5283 by all formulae.

Problems 29.5, page 974

1. 14.63 2. 2.8168 3. 0.89 4. 100
 5. $648 + 30x - x^2$ 6. $x^3 - 3x^2 + 5x - 6$ 7. $x^5 - 9x^4 + 18x^3 - x^2 + 19x - 18$
 8. 3 9. $\frac{0.5}{x-1} - \frac{0.5}{x+1} + \frac{1}{x-2}$

Problems 29.6, page 977

1. 1 2. 3.09 3. 448, 3150 4. 133.19
 5. $f(x) = \frac{1}{24}x^3 - 25x + 24 - \frac{7}{6}x^2 + \frac{557}{60}x - 25$ 6. $f(x) = \frac{1}{20}x^3$ 7. $f(x) = x^4 - 3x^3 + 5x^2 - 6$
 8. 31.

Problems 29.7, page 978

1. 11.5 2. 6.5928 3. 37.23

Problems 29.8, page 978

1. \$ 7.3 2. (b)

x	$f(x)$	I.D.D.	II.D.D.
5	7		
		2.9	
15	36		0.87
22	160	17.7	

4. Intermediate value of the variables. 5. \$ 7.8
 6. $\frac{[x_1, x_2, x_3, x_4] - [x_0, x_1, x_2, x_3]}{x_4 - x_0}$ 7. $\frac{-1}{4}$ and $\frac{1}{4}$ 8. \$ 7.14
 9. $f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + \frac{(x - x_0)(x - x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$
 10. $\frac{13}{5}$ 11. (c) 12. 1.857
 13. Extrapolation is the process of estimating the value of a function outside the given range of values.
 14. $1/(abc)$ 15. (a) 16. $x^3 - 7x^2 - 18x - 12$
 17. (b) 18. $6h^2(x + h)$.

Problems 30.1, page 987

1. $-27.9, 117.67$ 2. $4.75, 9$ 3. $0.63, 6.6$
 4. (a) $0.493, -1.165$ (b) $0.4473, -0.1583$; (c) $0.4662, -0.2043$ 5. 2.8326
 6. $-0.06; 0.5$ 7. 0.175 8. 13.13 m/sec
 9. (i) -52.4 , (ii) -0.0191 10. 44.92 11. 0.085
 12. $3.82 \text{ rad/sec}, 6.75 \text{ rad/sec}^2$ 13. 0.2561 15. 0.0186
 16. 135 17. $y_{\max}(1) = 0.25, y_{\min}(0) = 0$ 18. $\text{Max } f(10.04) = 1340.03$

Problems 30.2, page 995

1. (i) 0.695 (ii) 0.693 (iii) 0.693 2. (i) 0.7854 (ii) 0.7854 , (iii) 0.78535 , (iv) 0.7854
 3. 1.61 4. -6.436 5. (i) 70.16 (ii) 0.635
 6. 0.6305 7. (i) 2.0009 ; (ii) 1.1873 8. (i) 1.1249 (ii) 0.911
 9. (a) $1.8276551, .0001924$; (b) $1.8278472, .0000003$; (c) $1.8278470, 0000005$; (d) $1.8278474, .00000001$
 10. 1.3028 11. 403.67 12. 7.78
 13. 710 sq.ft 14. 3.032 15. 408.8 cub. cm.
 16. $1.063 \text{ sec}; 1.064 \text{ sec}$ 17. $552 \text{ m}; 3 \text{ m/sec}^2$. 18. 30.87 m/sec.
 19. 29 min nearly.

Problems 30.3, page 996

1. (c) 2. $\frac{1}{h} \left[\Delta f(a) - \frac{1}{2} \Delta^2 f(a) + \frac{1}{3} \Delta^3 f(a) - \dots \right]$
 3. h should be small 4. 0.775 5. $2\frac{2}{3}$
 6. 30.8 7. (b)
 8. larger number of sub-intervals 9. 0.7854 10. (d)
 11. $y'_{x_n} \frac{1}{h} = \left[\nabla_{y_n} + \frac{1}{2} \nabla^2 y_n + \frac{1}{3} \nabla^3 y_n + \dots \right]$ 12. a multiple of 6
 13. (c) 14. 0.783 15. 0.69
 16. 1.36125 17. 1.36
 18. if the entire curve is itself a parabola
 19. even and multiples of 3 20. False

Problems 31.1, page 999

1. $y_{x+3} - 2y_{x+2} + 2y_{x+1} = 0$ 2. $\Delta y_n = (-1)^{n+1}/(n+1)$ 3. $u_{n+1} - 2u_n = 0$
 4. (i) $(x+2)y_{x+2} - 2(2x+1)y_{x+1} + xy_x = 0$; (ii) $(x^2+x)y_{x+2} - (2x^2+4x)y_{x+1} + (x^2+3x+2)y_x = 0$
 5. (ii) $y_{n+2} - 8y_{n+1} + 15y_n = 0$; (ii) $y_{n+2} - 6y_{n+1} + 4y_n = 0$
 6. (i) $(x-1)y_{x+2} - (3x-2)y_{x+1} + 2xy_x = 0$; (ii) $y_{x+2} - 4y_x = 0$;
 (iii) $y_{x+3} - 6y_{x+2} + 11y_{x+1} - 6y_x = 0$.

Problems 31.2, page 1002

1. $u_p = (c_1 + c_2 p) 3p$ 2. $y_n = c_1 \cos \frac{2n\pi}{3} + c_2 \sin \frac{2n\pi}{3}$ 3. $u_n = c_1 \cos n\pi/2 + c_2 \sin n\pi/2$
 4. $y_n = c_1 \cdot 2^n + c_2 \cdot 3^n$. 5. $y_n = (2)^{n-1} + (-2)^{n-1}$ 6. $u_k = c_1 (-1)^k + (c_2 + c_3 k) 2^k$.
 7. $f(x) = (c_1 + c_2 x)(-1)^x + c_3 \cdot 2^x$ 8. $u_n = 2n + (-2)^n$ 9. $y_n = 6 + (n-3) 2^n$
 10. $u_n = 2^{n/2} [c_1 \cos n\pi/4 + c_2 \sin n\pi/4]$
 11. $y_m = 2^m \left\{ c_1 \cos \frac{m\pi}{4} + c_2 + \sin \frac{m\pi}{4} + c_3 \cos \frac{3m\pi}{4} + c_4 \sin \frac{3m\pi}{4} \right\}$ 14. $y_n = c_1 (-1)^n + c_2 (10)^n$

Problems 31.3, page 1005

1. $y_n = c_1(-1)^n + c_2(6)^n - 2^n/12$

2. $y_n = \left(\frac{n}{15} - \frac{1}{25}\right)(-3)^n + \frac{2^n}{25}$

3. $y_p = c_1 + c_2p + c_3p^2 + \frac{1}{6}p(p-1)(p-2)$

4. $y_n = 2^n \left(\frac{2}{\sqrt{3}} \sin \frac{n\pi}{3} - 2 \cos \frac{n\pi}{3}\right) + 2$

5. $y = c_1 + c_2 \cdot 3^x + \frac{1}{2}x \cdot 3^{x-1}$

6. $y_x = (c_1 + c_2x)2^x + 3x(x-1)2^{x-3} + 5 \cdot 4^{x-1}$

7. $u_n = c_1 + c_2(-1)^n + \frac{1}{2} \frac{\cos\left(\frac{n}{2}-1\right) - \cos\frac{n}{2}}{1-\cos 1}$

8. $y_p = c_1 \cos \frac{p}{2} + c_2 \sin \frac{p}{2} + \frac{p \cos\left(p - \frac{1}{2}\right)}{2 \sin \frac{1}{2}}$

9. $y_x = c_1 + 2^x + c_2(-2)^x - \frac{1}{27}(9x^2 + 12x + 11)$

10. $y_n = c_1(-1)^n + c_2 \cos \frac{n\pi}{3} + c_3 + \sin \frac{n\pi}{3} + \frac{1}{2}n(n-3)$

11. $y_n = (c_1 + c_2n)(3)^n + c_3(-1)^n + \frac{1}{3}(2)^n - \frac{3n}{4}$ 12. $y_n = (c_1 + c_2n)2^{-n} + \frac{2^n}{9} + n(n-1)\left(\frac{1}{2}\right)^{n-1}$

13. $y_n = c_1(-2)^n + c_2(-3)^n + \frac{n}{12} - \frac{7}{144}$ 14. $u_x + (c_1 + c_2x)(-3)^x + \frac{2^x}{25}(5x-2) + \frac{2}{4}x^{x-2} + \frac{7}{16}$

15. $y_n = c_1(-2)^n + 2^n(c_2 \cos n\pi/3 + c_3 \sin n\pi/3) + \frac{3}{16}(2)^n + 2^{n-4}(2n+3)$

16. $u_n = \left\{c_1 + c_2n + \frac{1}{48}n(n-1)^2(n-2)\right\}2^n$. 17. $y_k = c_1 \cdot 2^k + c_2 \cdot 3^k + \frac{4^k}{2}(k^2 - 13k + 61)$

18. $y_n = 2^n \left\{(c_1 + n) \cos \frac{n\pi}{3} + c_2 + \sin \frac{n\pi}{3}\right\}$.

Problems 31.4, page 1006

1. $y_x = a + b(-1)^x + x, z_x = a + b(-1)^{x+1} - (x+1)$

2. $y_x = (a + bx)(-1)^x - \frac{1}{9} \cdot 2^{x+2}, z_x = \frac{2^x}{9} - (-1)^x [a + b(x - \frac{1}{2})]$

3. $u_n = 2.4^n - 2\frac{1}{2}n(n-1), v_n = 4^n + 2 + \frac{1}{n} + n(n+1)$

4. $u_x = -2a + b(-2)^x - c + \frac{1}{2x}(3-x), v_x = a + c + b(-2)^x + \frac{1}{2}x(x-1)$.

Problems 31.5, page 1007

1. $y_{i+1} - 2y_i + y_{i-1} = -\frac{l_m}{P}y_i$. Solve it for y_i .

Problems 31.6, page 1007

1. $y_{n+2} - 5y_{n+1} + 6y_n = 0$

2. $u_n = c_1 + c_2n + c_3n^2$

3. $u_n = c_1 + c_2(-2)^n + c_3(3)^n$

4. $y_n = c + 2^n$.

5. $y_n = c(2)^n - (n+1)$

7. $y_n = c(2)^k + 1$

6. $(x^2 + x)y_{x+2} - (2x^2 + 4x)y_{x+1} - (x^2 + 3x + 2)y_x = 0$

9. Third

8. $y_n = (2)^{n+2} + (-2)^{n-1}$

10. $(x+2)y_{n+2} - 2(n+1)y_{n+1} + ny_n = 0$

12. $y_n = (C_1 + C_2 n)2^n$

15. True.

11. Second

13. $\frac{1}{2}x(x-1)(3)^{x-2}$

14. $y_{n+2} - 6y_{n+1} + 9y_n = 0$

Problems 32.1, page 1012

1. $y = 1 - \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^6}{48}$

2. 0.0214

3. $y = \frac{1}{3}x^3 - \frac{1}{81}x^9 + \dots$

4. (a) 0.9138, (b) 0.1938

5. $y(1.1) = 0.1103, y(1.2) = 0.2428$. Exact $y(1.1) = 0.1103$ and $y(1.2) = 0.2428$

6. 1.1053425

7. 1.1272

8. 1.005.

Problems 32.2, page 1017

1. 1.1831808

2. 1.1448

3. 4.5559

4. $y(0.1) = 0.095, y(0.2) = 0.181, y(0.3) = 0.259$

5. $y(0.2) = 1.2046, y(0.4) = 1.4644$

6. 2.2352

7. 1.0928

8. 5.051.

Problems 32.3, page 1021

1. 1.7278

2. 1.1749

3. 1.0207, 1.0438

4. 2.5005

5. $y(0.1) = 0.9052, y(0.2) = 0.8213$

6. $y(0.1) = 2.9919, y(0.2) = 2.9627$

7. 0.3487

8. 0.8489

9. 1.1678

10. 1.0911, 1.1677, 1.2352, 1.2902, 1.338.

Problems 32.4, page 1026

1. 3.795

2. 1.2797

3. $y(1.4) = 3.0794$

4. $y(4.5) = 1.023$

5. $y(0.4) = 2.162$

6. $y(0.4) = 0.441$.

Problems 32.5, page 1030

1. 0.2416

2. 1.0408

3. 0.6897

4. $y(4.4) = 1.019$

5. 2.5751

6. $y(1.4) = 0.949$.

Problems 32.6, page 1034

1. $y_3 = 1 + \frac{x}{2} + \frac{3}{40}x^5 + \frac{1}{40}x^6 + \frac{1}{192}x^9, z_3 = \frac{1}{2} + \frac{3}{8}x^4 + \frac{1}{10}x^5 + \frac{3}{34}x^8 + \frac{7}{340}x^9 + \frac{1}{256}x^{12}$

2. $y(0.1) = 0.105, z(0.1) = 0.999; y(0.2) = 0.22, Z(0.2) = 0.997$

3. $y(0.1) = 2.0845, z(0.1) = 0.5867$

4. $y_2 = 1 + \frac{1}{2}x + \frac{3}{40}x^5$

5. $y(0.1) = 0.5075$

6. $y(0.2) = 0.9802, y(0.2) = -0.196$

7. -0.5159

8. $\theta(0.2) = 0.8367, (\frac{d\theta}{dt})_{0.2} = 3.6545$.

Problems 32.7, page 1038

1. 0.14031

2. $y(.25) = y(.75) = 2.4, y(.5) = 3.2$

3. $y(1.25) = 1.3513, y(1.5) = 1.635, y(1.75) = 1.8505$

4. $y(.25) = -0.3473, y(.5) = -0.9508, y(.75) = -1.7257$

5. $n = 2, y(0.5) = 0.1389$, true value = 0.1505 ; $n = 4, y(0.5) = 0.147$

6. $y(0.25) = 0.062, y(0.5) = 0.25, y(0.75) = 0.562$

7. $y(1) = 1.0171, y(2) = 1.094$

8. $y(1/3) = 1.1539, y(2/3) = 3.9231; y(1) = 7.4615.$

Problems 32.8, page 1038

1. (b)

4. § 31.4

7. $y_4 = y_2 + \frac{h}{3} (f_2 + 4f_3 + f_4)$

9. $y_1 = y_0 + \frac{h}{24} (55f_0 - 59f_{-1} + 37f_{-2} - 9f_{-3})$

10. Milne's method and Adam-Basforth method

11. four

12. $y = 1 + \frac{x^2}{2} + \frac{x^4}{8}$

3. $1 + x + x^2 + x^3/6$

6. (b)

14. $y_4 = y_0 + \frac{4h}{3} (2f_1 - f_2 + 2f_3)$

15. $y_1 = y_0 + \frac{h}{24} (9f_1 + 19f_0 - 5f_{-1} + f_{-2})$

16. 1.1818

17. $\frac{dy}{dx} = z, \frac{dz}{dx} + y(1 + yz) = 0$

18. § 31.7

19. starting values

20. Picards and Runge-Kutta methods

21. It agrees with Taylor's series solution upto the term in h^4

22. (d)

23. $y_{i+1} + 2(h^2 - 1)y_i + y_{i-1} = 0$

24. (a)

25. True

26. False

27. False.

Problems 33.1, page 1041

1. Parabolic

2. Hyperbolic

3. (i) Parabolic (ii) Elliptic (iii) Elliptic

4. Outside the ellipse $(x/0.5)^2 + (y/0.25)^2 = 1$.**Problems 33.2, page 1050**

1. $u_1 = 1.999, u_2 = 2.999, u_3 = 3.999$

2. 2.37, 5.6, 9.87, 2.89, 6.14, 9.89, 3.02, 6.17, 9.51

3. $u_1 = 10.188, u_2 = 0.5, u_3 = 1.188, u_4 = 0.25, u_5 = 0.625, u_6 = 1.25$

4. $u_1 = 26.66, u_2 = 33.33, u_3 = 43.33, u_4 = 46.66$

5. $u_1 = 0.99, u_2 = 1.49, u_3 = 0.49$

6. $u_1 = 1.57, u_2 = 3.71, u_3 = 6.57, u_4 = 2.06, u_5 = 4.69, u_6 = 8.06, u_7 = 2, u_8 = 4.92, u_9 = 9$

7. $u_1 = -3, u_2 = -2, u_3 = -2$.

Problems 33.3, page 1054

1.

<i>j</i>	<i>i</i>	0	1	2	3	4
0	0	0	3	4	3	0
1	0	0	2	3	2	0
2	0	0	1.5	2	1.5	0
3	0	0	1	1.5	1	0
4	0	0	0.75	1	0.75	0
5	0	0	0.5	0.75	0.5	0

2.

$j \backslash i$	0	1	2	3	4	5	6	7	8	9	10
j	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
0	0	0.08	0.15	0.20	0.23	0.24	0.23	0.20	0.15	0.08	0
1	0	0.075	0.14	0.19	0.22	0.23	0.22	0.19	0.14	0.075	0
2	0	0.07	0.133	0.18	0.21	0.22	0.21	0.18	0.133	0.07	0
3	0	0.07	0.133	0.18	0.21	0.22	0.21	0.18	0.133	0.07	0

3.

$j \backslash i$	0	1	2	3	4	5
j	0	24	84	144	144	0
0	0	42	84	114	72	0
1	0	42	78	78	57	0
2	0	39	60	67.5	39	0
3	0	30	53.25	49.5	33.75	0
4	0	26.6	39.75	43.5	24.75	0
5	0	19.88	35.06	32.25	21.75	0
6	0	0	0	0	0	0

4.

$j \backslash i$	0	1	2	3	4
j	0	0.5	1	0.5	0
0	0	0.5	0.5	0.5	0
1	0	0.25	0.5	0.25	0
2	0	0.25	0.25	0.25	0
3	0	0.25	0.25	0.25	0

Problems 33.4, page 1060

1.

$t = 0.3; x =$	0.1	0.2	0.3	0.4	0.5
Numerical sol. $u =$	0.02	0.04	0.06	0.075	0.08
Exact sol. $u =$	0.02	0.04	0.06	0.075	0.08

2.

$j \backslash i$	0	1	2	3	4	5
j	0	20	15	10	5	0
0	0	7.5	15	10	5	0
1	0	-5	2.5	10	5	0
2	0	-5	-10	10	5	0
3	0	-5	-10	-2.5	5	0
4	0	-5	-10	-15	-7.5	0
5	0	-5	-10	-15	-20	0

3.

<i>j</i>	<i>i</i>	0	1	2	3	4	5	6	7	8	9	10
0	0	10.19	10.16	10.21	10.24	10.25	10.24	10.21	10.16	10.09	0	
1	0	5.08	10.15	10.20	10.23	10.24	10.23	10.20	10.15	5.08	0	
2	0	0.06	5.12	10.17	10.20	10.21	10.20	10.17	10.12	0.06	0	
3	0	0.04	0.08	5.12	10.15	10.16	10.15	10.12	10.08	0.04	0	
4	0	0.02	0.04	0.06	5.08	10.09	10.08	10.06	10.04	0.02	0	
5	0	0	0	0	0	0	0	0	0	-0.02	0	

4.

<i>j</i>	<i>i</i>	0	0.1	0.2	0.3	0.4	0.5
0.1	0	0.037	0.07	0.096	0.113	0.119	
0.2	0	0.031	0.059	0.082	0.096	0.101	
0.3	0	0.023	0.043	0.059	0.07	0.074	
0.4	0	0.012	0.023	0.031	0.037	0.039	
0.5	0	0	0	0	0	0	

Problems 33.5, page 1060

1. (b)

2. False

3. a hyperbolic equation

4. Poisson's equation

5. $u_{i,j+1} = u_{i-1,j} + u_{i+1,j} - u_{i,j-1}$ 6. $(u_{i-1,j} - 2u_{i,j} + u_{i+1,j})/h^2$

7. § 32.8 (2)

8. $\frac{1}{4}(u_{i-1,j+1} + u_{i+1,j-1} + u_{i+1,j+1} + u_{i-1,j-1})$

9. hyperbolic

10. Bendre-Schmidt

11. $u_{i,j+1} = 2(1 - 4\alpha^2)u_{i,j} + 4\alpha^2(u_{i-1,j} + u_{i+1,j} - u_{i,j-1})$ where $\alpha = k/h$.12. $u_{i,j+1} = \frac{1}{2}(u_{i-1,j} + u_{i+1,j})$

13. \$33.5(2)

14. $\lambda < \frac{1}{2}$

15. Elliptic

16. $u(0, t) = 0 = u(1, t)$, ($t > 0$) ; $u(x, 0) = f(x)$, $0 < x < l$; $\delta u/\delta t(x, 0) = 0$, $0 < x < l$ 17. $u_{i,j+1} = u_{i+1,j} - u_{i-1,j} - u_{i,j-1}$ 18. $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} = f(x, y)$

19. 100

20. $\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = 0$ 21. $y < 0$ 22. $k = 1/4$.**Problems 34.1, page 1063**1. Max. $Z = 1.2x_1 + 1.4x_2$; subject to $40x_1 + 25x_2 \leq 1000$, $35x_1 + 28x_2 \leq 980$, $25x_1 + 35x_2 \leq 875$ and $x_1, x_2 \geq 0$ 2. Max. $Z = 3x_1 + 2x_2 + 4x_3$; subject to $4x_1 + 3x_2 + 5x_3 \leq 2000$, $2x_1 + 2x_2 + 4x_3 \leq 2500$, $100 \leq x_1 \leq 150$, $200 \leq x_2$ and $50 \leq x_3$ 3. Max. $Z = 3x_1 + 2x_2 + x_3$; subject to $3x_1 + 4x_2 + 3x_3 \leq 42$, $5x_1 + 3x_3 \leq 45$, $3x_1 + 6x_2 + 2x_3 \leq 41$ and $x_1, x_2, x_3 \geq 0$.4. Max. $Z = 400x + 300y$; subject to $x + y \leq 200$, $x \geq 20$, $y \geq 4x$, $x \geq 0$, $y \geq 0$ 5. Min. $Z = x_1 + x_2$; subject to $2x_1 + x_2 \geq 12$, $5x_1 + 8x_2 \geq 74$, $x_1 + 6x_2 \geq 24$ and $x_1, x_2, x_3 \geq 0$.

6. Max. $Z = 0.15x_1 + 0.25x_2$; subject to

$$\begin{aligned} 2x_1 + 5x_2 &\leq 480,000, \quad 5x_1 + 4x_2 \leq 720,000, \\ 8x_1 + 16x_2 &\leq 300,000, \quad 0 \leq x_1 \leq 25,000 \text{ and } 0 \leq x_2 \leq 7,000 \end{aligned}$$

7. Min. $Z = 41x_1 + 35x_2 + 96x_3$; subject to

$$\begin{aligned} 2x_1 + 3x_2 + 7x_3 &\geq 1250, \quad x_1 + x_2 \geq 250, \quad 5x_1 + 3x_2 \geq 900, \\ 6x_1 + 25x_2 + x_3 &\geq 232.5 \text{ and } x_1, x_2, x_3 \geq 0 \end{aligned}$$

8. Min. $Z = 100x_1 + 250x_2 + 160x_3$; subject to

$$\begin{aligned} 0.94x_1 + x_2 + 1.04x_3 &\leq 0.98, \quad 10x_1 + 15x_2 + 17x_3 \geq 14, \\ 470x_1 + 500x_2 + 520x_3 &\geq 495, \quad x_1 + x_2 + x_3 = 1 \text{ and } x_1, x_2, x_3 \geq 0. \end{aligned}$$

Problems 34.2, page 1069

1. $x_1 = 100, x_2 = 50$; max. $Z = 550$

2. $x_1 = 8/15, x_2 = 12/5$, max. $Z = 24.8$

3. $x_1 = 15, x_2 = 0$; max. $Z = 300$

4. $x_1 = 1000, x_2 = 500$; max. $Z = 5500$

5. 450 units of product B only; max. profit = ₹ 1800

6. $X = 2, Y = 4.5$; max. profit = ₹ 37

7. $A = 1.18$ units, $B = 0.53$ units; max. profit = ₹ 14.50 approx.

8. 2000/11 units of product A and 1000/11 units of B; max. profit = ₹ 10,000

9. $x_1 = 4, x_2 = 0$; max. $Z = 8$

10. Unbounded solution

11. $x_1 = 2, x_2 = 4$; min. $Z = 64$

12. Production cost will be min. if G and J run for 12 and 4 days respectively.

Problems 34.3, page 1074

1. Max. $Z = 3x_1 + 5x_2 + 8x_3$; subject to $2x_1 - 5x_2 + s_1 = 6$,

$$\begin{aligned} 3x_1 + 2x_2 + x_3 - s_2 &= 5, \quad 3x_1 + 4x_3 + s_3 = 3; \\ x_1, x_2, x_3, s_1, s_2, s_3 &\geq 0 \end{aligned}$$

2. Min. $Z = 3x_1 + 2x_2 + 5x_3$, subject to $-5x_1 + 2x_2 + s_1 = 5$,

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 - s_2 &= 7, \quad 2x_1 + 5x_3 + s_3 \geq 3 \\ x_1, x_2, x_3, s_1, s_2, s_3 &\geq 0. \end{aligned}$$

3. Max. $Z = 3x_1 - 2x_2 + 4x_4 - 4x_5$; subject to

$$\begin{aligned} x_1 + 2x_2 + x_4 - x_5 + s_1 &= 8, \quad 2x_1 - x_2 + x_4 - x_5 - s_2 = 2, \\ -4x_1 + 2x_2 + 3x_4 - 3x_5 &= 6; \quad x_1, x_2, x_4, x_5, s_1, s_2 \geq 0 \end{aligned}$$

4. (i) $x_1 = 2, x_3 = 1$ (Basic); $x_2 = 0$ (Non-basic). (ii) $x_1 = 5$,

$x_3 = -1$ (Basic); $x_2 = 0$ (Non-basic); (iii) $x_2 = 5/3, x_3 = 2/3$ (Basic); $x_1 = 0$ (Non-basic). All the three basic solutions are non-degenerate

6. Basic solutions are (i) $x_1 = 2, x_2 = 1$ (Basic) and $x_3 = 0$;

(ii) $x_1 = x_3 = 1$ (Basic) and $x_2 = 0$; (iii) $x_2 = -1, x_3 = 2$ (Basic) and $x_1 = 0$

(a) First two solutions are non-degenerate basic feasible solutions

(b) First solution is optimal and $Z_{\max} = 5$.

Problems 34.4, page 1081

1. $x_1 = 2, x_2 = 4$, max. $Z = 14$

2. $x_1 = 0, x_2 = 20$; max. $Z = 200$

3. $x_1 = 7/3, x_2 = 4/3$; max. $Z = 16$

4. $x_1 = 5, x_2 = x_3 = 0$; max. $Z = 50$

5. $x_1 = 0, x_2 = 100, x_3 = 230$; max. $Z = 1350$

6. $x_1 = 89/41, x_2 = 50/41, x_3 = 62/41$; max. $Z = ₹ 765/41$

7. $x_1 = 4, x_2 = 5, x_3 = 0$; min. $Z = -11$

8. $x_1 = 280/13, x_2 = 0, x_3 = 20/13, x_4 = 180/13$; max. $Z = 2280/13$

9. $x_1 = 0, x_2 = 400$ units; max. profit = ₹ 1200

10. $x_1 = 125, x_2 = 250$ units; max. profit = ₹ 2250

11. $x_1 = 400$ gms, $x_2 = 0$; min. cost = ₹ 2

12. $x_1 = 0, x_2 = x_3 = 50$; max. profit = ₹ 700

13. $x_1 = 0.5, x_2 = x_3 = 0.04$ units ; min. cost = ₹ 5.80
 14. Averages for corn, wheat, soyabean are 250, 625, zero respectively to achieve a max. profit of ₹ 32,000.

Problems 34.5, page 1088

1. $x_1 = 0, x_2 = 2, x_3 = 0$; max. $Z = 4$
2. $x_1 = 3, x_2 = 2, x_3 = 0$; max. $Z = 8$
3. $x_1 = x_2 = -6/15$; max. $Z = -48/5$
4. $x_1 = 23/3, x_2 = 5, x_3 = 0$; max. $Z = 85/3$
5. $x_1 = x_2 = x_3 = 5/2, x_4 = 0$; max. $Z = 15$
6. $x_1 = 21/13, x_2 = 10/13$; max. $Z = 31/13$
7. Infeasible
8. $x_1 = 23/3, x_2 = 5, x_3 = 0$; max. $Z = 85/3$
9. $x_1 = 55/7, x_2 = 30/7, x_3 = 0$; max. $Z = 155/7$
10. $x_1 = 2, x_2 = 0$; max. $Z = 18$
11. Degenerate solution : $x_1 = 0$ (non-basic) ; $x_2 = 1, x_3 = 0$ (basic) ; max. $Z = 3$.

Problems 34.6, page 1091

1. Min. $W = 26y_1 + 7y_2$; subject to $6y_1 + 4y_2 \geq 10$
 $5y_1 + 2y_2 \geq 13, 3y_1 + 5y_2 \geq 19; y_1, y_2, y_3 \geq 0$
2. Max. $W = 11y_1 + 7y_2 + y_3 + 5y_4$; subject to $3y_1 + 2y_2 - y_3 + 3y_4 \leq 2,$
 $4y_1 + 3y_2 + 2y_3 + 2y_4 \leq 4, y_1 - 2y_2 + 3y_3 + 2y_4 \leq 3;$
 $y_1, y_2, y_3, y_4 \geq 0$
3. Min. $W = -3y_1 + y_2 + 4y_3$; subject to $y_1 + 3y_2 - 2y_3 \leq -3,$
 $y_1 + y_3 \geq 16, y_1 - 2y_2 + y_3 \leq -7;$
 $y_1, y_2 \geq 0, y_3$, unrestricted in sign
4. Max. $W = -5y_1 + 9y_2 + 8y_3$; subject to $-2y_1 + 4y_2 - 8y_3 \leq 3,$
 $3y_1 - 2y_2 + 4y_3 \leq -2, -y_1 + 3y_3 = 1;$
 $y_1, y_2 \geq 0, y_3$ unrestricted
5. Min. $y = 3y_1 + 4y_2 + y_3 + 6y_4$; subject to $5y_1 - 2y_2 + y_3 - 3y_4 \geq 2,$
 $6y_1 + y_2 - 5y_3 - 3y_4 \geq 5, -y_1 + 4y_2 + 3y_3 + 7y_4 \geq 6, y_1, y_2, y_3, y_4 \geq 0.$

Problems 34.7, page 1094

1. $x_1 = x_2 = 0, x_3 = 5/2$; min. $Z = 2.5$
2. $x_1 = 4, x_2 = 2$; max. $Z = 10$.
3. $x_1 = 7, x_2 = 0$, max. $Z = 21$
4. $x_1 = 0, x_2 = 100, x_3 = 230$; max. $Z = 1350$.

Problems 34.8, page 1097

1. $x_1 = 0, x_2 = 1$; max. $Z = -1$
2. $x_1 = 3/5, x_2 = 6/5$; min. $Z = 12/5$
3. $x_1 = 6, x_2 = 2, x_3 = 0$; min. $Z = 10$
4. $x_1 = 65/23, x_2 = 0, x_3 = 20/23, x_4 = 0$; min. $Z = 215/23$.

Problems 34.9, page 1104

1. $x_{11} = 200, x_{12} = 50, x_{22} = 175, x_{24} = 125, x_{33} = 275, x_{34} = 125$; min. cost = ₹ 12075
2. $x_{13} = 14, x_{21} = 6, x_{22} = 5, x_{23} = 1, x_{32} = 5$; min. cost = 143
3. $x_{11} = 50, x_{12} = 100, x_{21} = 150, x_{33} = 150, x_{42} = 100, x_{43} = 50$; min. tonnage = 3300
4. $x_{11} = 140, x_{13} = 60, x_{21} = 40, x_{22} = 120, x_{33} = 90$; min. cost = ₹ 5920
5. $x_{11} = 5, x_{14} = 2, x_{22} = 3, x_{23} = 7, x_{32} = 5, x_{34} = 13$;
 min. cost = ₹ 799 and maximum saving = ₹ 201
6. $x_{11} = 150, x_{13} = 20, x_{22} = 160, x_{24} = 40, x_{33} = 90, x_{34} = 90$; max. profit = 4920
7. $x_{13} = 2, x_{22} = 1, x_{23} = 2, x_{31} = 4, x_{33} = 1$; min. cost = 33
8. $x_{13} = 0, x_{15} = 800, x_{21} = 400, x_{24} = 100, x_{32} = 400, x_{33} = 200,$
 $x_{34} = 300, x_{43} = 300$; min. cost = 9200.

Problems 34.10, page 1109

- $x_{11} = x_{22} = x_{33} = 1$; min. cost = ₹ 18
- $A \rightarrow 2, B \rightarrow 3, C \rightarrow 4, D \rightarrow 1$; min. $Z = 38$
- $I \rightarrow B, II \rightarrow A, III \rightarrow D, IV \rightarrow C$; min. cost = ₹ 49
- $A \rightarrow$ Dyn. Prog., $B \rightarrow$ Queuing Th., $C \rightarrow$ Reg. Analysis, $D \rightarrow$ L.P.; min. time = 28 hrs
- (i) $A \rightarrow I, B \rightarrow II, C \rightarrow III, D \rightarrow IV$; (ii) $A \rightarrow I, B \rightarrow III, C \rightarrow II, D \rightarrow IV$
- $1 \rightarrow IV, 2 \rightarrow II, 3 \rightarrow VI, 4 \rightarrow I, 5 \rightarrow III, 6 \rightarrow V$; max. profit = ₹ 270

Problems 34.11, page 1110

- | | | |
|---|-----------------------------------|---------------------------------|
| 1. § 34.5 Def. 2 | 2. it provides an optimality test | 3. § 34.11 |
| 4. § 34.16 (1) | 5. § 34.13 | 6. § 34.6 (1) |
| 7. Min. $W = 7y_1 + 5y_2$, subject to $2y_1 + 3y_2 \leq 4$,
$3y_1 - 2y_2 \leq 9$, $2y_1 + 4y_2 \leq 2$, $y_1 \geq 0$, y_2 is unrestricted in sign | | |
| 8. § 34.12 (2) | 9. § 34.14 | |
| 10. Minimize $Z = (2x_{11} + 3x_{12} + 11x_{13} + 4x_{14}) + (5x_{21} + 6x_{22} + 8x_{23} + 7x_{24})$,
subject to $x_{11} + x_{12} + x_{13} + x_{14} = a_1 (= 15)$, $x_{21} + x_{22} + x_{23} + x_{24} = a_2 (= 20)$,
$x_{11} + x_{21} = b_1 (= 10)$, $x_{12} + x_{22} = b_2 (= 5)$; $x_{13} + x_{23} = b_3 (= 12)$; $x_{14} + x_{24} = b_4 (= 8)$ and $x_{ij} \geq 0$.
[$\because \Sigma a_i = \Sigma b_j = 35$] | | |
| 11. (i) $x_1 = 3, x_2 = 5, x_3 = 0$; (ii) $x_1 = 0.5, x_2 = 0, x_3 = 2.5$ | | 12. § 34.5 (Def. 4) |
| 13. § 34.15 | 14. balanced | 15. § 34.9 |
| 16. § 34.7 (3) | 17. optimal | |
| 18. Minimize $y = 5y_4 - 3y_3$, subject to $y_4 + y_3 = 5$, $2y_4 - 5y_3 \geq 6$, $y_3 \geq 0$ and y_4 unrestricted | | |
| 19. 5 | 20. Max. $Z = 5/19$ | 21. § 34.7 |
| 22. § 34.16 | 23. § 34.7 [2 (ii)] | |
| 24. Min. $W = 2y_1 + 4y_2 + 3y_3$, subject to $-y_1 + y_2 + y_3 \geq 2$, $2y_1 + y_2 \geq 1$, $y_1, y_2 \geq 0$ | | |
| 25. North west corner rule and Vogeli approximation method | | 26. Slash or surplus variables. |

Problems 35.1, page 1118

- | | |
|---|--|
| 1. (i) $y = \frac{1}{4}x^2 + c_1x + c_2$; (ii) $y = c_1x^{-1} + c_2$ | 3. $y = c_1 e^x + c_2 e^{-x} + \frac{xe^x}{2}$ |
| 4. $y = -x \cos x/2$ | 5. $y = (c+x) \sin x$ |
| 7. $y = x^2 - 1$ | 8. $(x+c)^2 + y^2 = k^2$ |
| 12. The spirals of the family $r = a \sec(\phi \sin \alpha + b)$. | 6. $y = \sinh(c_1x + c_2)$ |
| | 9. $y = 2 \sin x$ |

Problems 35.2, page 1120

- $y = \pm 2 \sin m\pi x$, where m is an integer
- $y = \lambda x^2 + ax + b$, where λ, a, b are determined from the isoperimetric and boundary conditions.
- $y(x) = \frac{1}{2}(1 - \cos x) + \frac{1}{4}(2 - \pi) \sin x$.

Problems 35.3, page 1124

- $y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x$,
 $z = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x - 2c_2 \sin x + 2c_4 \cos x$
- $y = a_n \sin nx$, $n = 1, 2, 3, \dots$
- $y = \cos x$
- $y = -\frac{\lambda}{24\mu} (x^2 - a^2)^2$

6. (i) $y = c_1 e^{2x} + c_2 e^{-2x} + c_3 \cos 2x + c_4 \sin 2x$;
(ii) $y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + c_5 x^4 + c_6 x^5 + x^7/7$!.

Problems 35.4, page 1126

1. (i) and (ii) $\bar{y} = \frac{5}{18}x(1-x)$ 2. $\bar{y} = \frac{x}{4}(5x-1)$ 4. $\bar{y} = 0.58 + 0.27x$
5. (i) and (ii) $c_1 = 0.93, c_2 = -0.05$ 6. 0.05 7. $c_1 = 3.27, c_2 = -2.69$
8. $y = \frac{1}{2}(5x^2 - 3x)$.

Problems 36.1, page 1134

1. $y(x) = 6x - 5 + \int_0^x (5 - 6x + 6t) y(t) dt$.
2. $y(x) = x - \sin x + e^x(x-1) + \int_0^x [\sin x - e^x(x-t)] y(t) dt$
3. $y(x) = \int_0^x t(t-x) y(t) dt + \frac{1}{2}x^2$
4. $y(x) + \int_0^x [1 + x - 2t + (x-t)e^{-t}] y(t) dt = \frac{x^5}{20} - \frac{5x^3}{6} + x - 3$
5. $y(x) = \cos x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{2} \int_0^x (x-t)^2 y(t) dt$
6. $y(x) = \sin x - \frac{1}{2} + \int_0^x \left\{ \frac{1}{2}x(x-t)^2 - 1 \right\} y(t) dt$
7. $y(x) - \int_0^x \{4 - 6(x-t) + 2(x-t)^2 - \frac{1}{6}(x-t)^3\} y(t) dt = \frac{1}{4} \cos 2x - \frac{19}{12} + \frac{32}{3}x - \frac{85}{6}x^2 + \frac{20}{3}x^3$
8. $y''(x) - 2xy'(x) - 3y(x) = 0$; $y(0) = 1, y'(0) = 0$
9. $y''(x) - y(x) + 3 \sin x = 0$; $y(0) = 3, y'(0) = 0$
10. $y'''(x) + 6y(x) = 0$; $y(0) = 4, y'(0) = -3, y''(0) = 2$
11. $y'''(x) - 3y''(x) + 4y'(x) + 2y(x) + e^{-x} = 0$; $y(0) = 1, y'(0) = 2, y''(0) = 3$.

Problems 36.2, page 1137

1. $y(x) = \int_0^1 G(x, t) ty(t) dt + \frac{1}{2}x(x-1)$, where $G(x, t) = x(1-t)$, $(x < t)$ and $= t(1-x)$, $x > t$
2. $y(x) = \int_0^1 G(x, t) y(t) dt + \frac{1}{6}(x^3 - 3x + 6)$, where $G(x, t) = x$, $x < t$ and $= t$, $x > t$
3. $u(x) = \int_0^1 G(x, t) e^t u(t) dt + \frac{1}{6}(x^3 + x)$, where $G(x, t) = x(1-2t)$, $x < t$ and $= t(1-2x)$, $x > t$
4. $G(x, t) = \frac{\sinh x \sinh(t-1)}{\sinh 1}$, $x < t$ and $= \frac{\sinh t \sinh(x-1)}{\sinh 1}$, $x > t$
5. $u(x) = \lambda \int_0^1 G(x, t) t \cdot u(t) dt$, where $G(x, t) = \frac{1}{2}x\left(\frac{1}{t-t}\right)$, $x < t$ and $= \frac{1}{2} \cdot t\left(\frac{1}{x-x}\right)$, $x > t$

Problems 36.3, page 1141

5. $y(x) = \frac{1}{2}(\sin x + \sinh x)$ 6. $y(x) = x^2 + \frac{1}{12}x^4$ 7. $y(x) = 1$
8. $y(x) = \pm 6J_0(4x)$ 9. $y(x) = J_1(2x)$ ($x > 0$)

10. $y(x) = \frac{1}{2} e^{-2x} (\cos x + 3 \sin x) - \frac{1}{2} e^{-x}$

12. $y(x) = 1 + x^2 + x^4/24.$

15. $y(x) = \frac{3\sqrt{3}}{4\pi} x^{1/3} (3x + 2).$

11. $y(x) = (1-x)e^{-x} + 1/2 \sin x$

14. $y(x) = 1/2$

Problems 36.4, page 1145

1. Has no eigen values and eigen functions

2. Eigen value $\lambda = 1/4$; eigen function is $y(x) = x^2 + 3x/2$

3. Eigen values $\lambda = 8/(\pi - 2)$; eigen function is $y(x) = \sin^2(x)$

4. Eigen value $\lambda = 1/\pi$, $y = \sin x$ 5. Has no eigen values or eigen functions

6. Eigen values are $\lambda = \pm 1/\pi$; eigen functions are $y(x) = \cos x + \sin x$, $y(x) = \cos x - \sin x$

7. $y(x) = x + \frac{\lambda[(6-\lambda)x-4]}{12+\lambda^2}$

8. $y(x) = x + \frac{\lambda}{12(1-2\lambda)-\lambda^2} [10 + (6+\lambda)x]$

9. $y(x) = x + \frac{\lambda}{(1-\lambda\pi)\left(1-\frac{1}{2}\lambda\pi\right)+4\lambda^2} \left\{ 2\lambda\pi + \frac{1}{2}\pi^2\left(1-\frac{1}{2}\lambda\pi\right) + \pi(1-2\lambda\pi)\sin x \right\}$

10. $y(x) = 2x - \pi + \frac{\pi^2 \sin^2 x}{\pi - 1}$

11. $y(x) = \frac{2}{2-\lambda} \sin x, \lambda \neq 2$

12. $y(x) = x + \frac{2\lambda\pi}{1+2\lambda^2\pi^2} (\lambda\pi x - 4\lambda\pi \sin x + \cos x)$

13. There is no solution to the integral equation when $\lambda = 3$

14. $\lambda_1 = 2, \lambda_2 = -2$; $y_1(x) = 1 - x, y_2(x) = 1 - 3x$

15. (i) When $F(x) = x$, solution is $y(x) = x + \lambda \left\{ \frac{2\lambda\pi^2}{\lambda^2\pi^2-1} \sin x + \frac{2\pi}{\lambda^2\pi^2-1} \cos x \right\}$

(ii) When $F(x) = 1$, solution is $y(x) = 1$.

Problems 36.5, page 1148

1. $y(x) = 1 - \frac{3\lambda x}{2(3+\lambda)} (\lambda \neq -3)$

2. $y(x) = \frac{\sin x}{1+\lambda\pi}$ only if $|\lambda| < \frac{1}{\pi}$

3. $y(x) = \frac{4+2\lambda(2-3x)}{4-\lambda^2} (\lambda \neq 2)$

4. $y = e^x$

5. $y(x) = 1$

6. $y = \sin x$

7. $y(x) = 2$.

Problems 37.1, page 1154

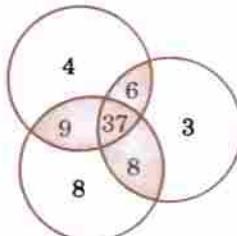
2. (i) True, since $\{a\}$ is a subset of the set $\{a, b, c\}$; (ii) and (iii) False, since the element a cannot be a subset of the set $\{a, b, c\}$; (iv) True, since the set $\{a, b\}$ is a subset of the set $\{a, b, c\}$; (v) False, since the set $\{a, b\}$ is not an element of the set $\{a, b, c\}$; (vi) True, since the null set \emptyset is a subset of every set.

17. 20

18. 105

19. 136

20. Number of students not taking any of these courses is 71.



Problems 37.2, page 1160

1. (a) It is not true that Sam is a teacher and John is an honest boy ; (b) Sam is a teacher and John is not an honest boy ; (c) Sam is not a teacher iff John is an honest boy ; (d) If Sam is a teacher then John is not an honest boy.
2. (a) $(p \vee q) \Rightarrow r$ where $p = I$ have no car, $q = I$ do not wear good dress, $r = I$ am not, a millionaire.

3.

p	q	$\neg q$	$p \Rightarrow q$	$p \Rightarrow q \wedge \neg q$
1	1	0	1	0
1	0	1	0	0
0	1	0	1	0
0	0	1	1	1

(b)

p	q	r	$p \Leftrightarrow q$	$r \vee q$	$(p \Leftrightarrow q) \wedge (r \vee q)$
1	1	1	1	1	1
1	1	0	1	1	0
1	0	1	0	1	0
1	0	0	0	0	0
0	1	1	0	1	0
0	1	0	0	1	0
0	0	1	1	1	1
0	0	0	1	0	0

7. (i) $T_p = [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31]$

$T_q = [1, 3, 9, 27, \dots], T_r = [1, 3, 9, 7]$

(ii) $T_r \leq T_q$

10. (i)

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

15. (i) Dual of $(p \wedge q) \vee r$ is $(p \wedge q) \vee r$ (ii) Dual of $(p \wedge q) \vee t$ is $(p \vee q) \wedge t$

Problems 37.3, page 1166

1. (a) $(\forall x \in A) (x + 2 < 10)$ (b) $(\exists x \in A) (x + 2 = 10)$
2. (a) $\forall x, (x^3 \neq x)$ (b) $\neg \forall x, (x + 5 \leq x)$
- (c) None of the students are 26 or older (d) Some students do not live in the hostels.
3. $\forall x P(x)$ is false 4. $\forall (x_1, x_2) Q(x_1 + x_2) Q$
5. $\forall (a, b) R(a + x = b)$
6. (a) $\forall x [Q(x) \rightarrow R(x)]$ (b) $\neg \forall x [Q(x) \rightarrow R(x)]$, (c) $\exists x [Q(x) \wedge R(x)]$, (d) $\exists x [Q(x) \wedge \neg R(x)]$
8. $(\neg A \vee \neg A) \wedge (B \vee \neg A) \wedge (\neg A \vee C) \wedge (B \vee C)$
10. 1. $p \vee q$ (Premise), 2. $\neg p \rightarrow q$ (conditional equivalence)
 3. $q \rightarrow s$ (Premise) 4. $\neg p \rightarrow s$ (2, 3 chain rule)
 5. $p \rightarrow r$ (Premise) 6. $\neg s \rightarrow p$ (4, conditional equivalence)
 7. $\neg s \rightarrow r$ (5, 6 chain rule) 8. $s \vee r$ (7, conditional equivalence)
12. (b) $x R (x = \sqrt{Z})$ 13. (a) Conclusion is not valid (b) Conclusion is not valid

Problems 37.4, page 1170

1.

x	y	z	$x \wedge y$	z'	$y \wedge z'$	$(x \wedge y) \vee (y \wedge z')$
0	0	0	0	1	0	0
0	0	1	0	0	0	0
0	1	0	0	1	1	1
1	0	0	0	1	0	0
1	1	0	1	1	1	1
1	0	1	0	0	0	0
0	1	1	0	0	0	0
1	1	1	1	0	0	1

2. $x \vee z' \wedge y = x \wedge y$

3. (i) $x' \vee y' \vee z' \quad$ (ii) 0

14.

x_1	x_2	x_3	$x_1 \vee x_3$	x_3'	$x_2 \vee x_3'$	$x_1 \wedge (x_2 \vee x_3')$	P
0	0	1	1	0	0	0	0
0	1	0	0	1	1	0	0
0	1	0	0	1	1	0	0
1	0	0	1	1	1	1	1
1	1	0	1	1	1	1	1
0	1	1	1	0	1	0	0
1	0	1	1	0	0	0	0
1	1	1	1	0	1	1	1

Problems 37.5, page 1172

1. (i) 0 (ii) 0

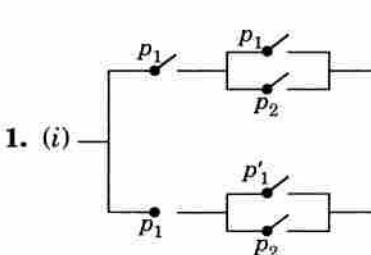
2. (i) $(x \vee y \vee z) \wedge (x \vee y \vee z') \quad$ (ii) $x \vee y \wedge (x \vee y') \wedge (x' \vee y)$

4. $(x \vee y \vee z) \wedge (x \vee y \vee z') \wedge (x \vee y' \vee z) \wedge (x \vee y' \vee z') \wedge (x \vee y' \vee z') \wedge (x' \vee y' \vee z) \wedge (x' \vee y' \vee z')$

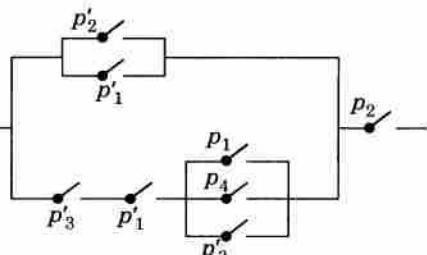
5. $(x \wedge y \wedge z) \vee (x \wedge y \wedge z') \vee (x \wedge y' \wedge z) \vee (x \wedge y' \wedge z')$

6. $F' = (x \wedge y \wedge z) \vee (x \wedge y' \wedge z) \vee (x' \wedge y \wedge z) \vee (x' \wedge y \wedge z') \vee (x' \wedge y' \wedge z)$

Problems 37.6, page 1174

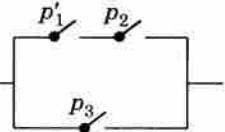


(ii)



2. (i) $p_1 \vee [p_2' \wedge (p_1 \vee p_2')]$

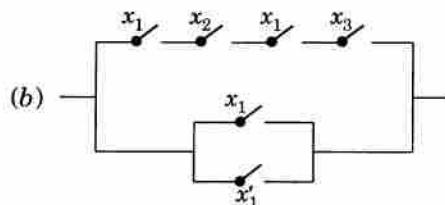
(ii) $[(p_1 \vee p_2) \vee (p_1 \vee p_3)] \wedge (p_1 \wedge p_2')$



3. $x \wedge y$

4. $p_1' \wedge p_2 \vee p_3;$

6. (a) $x_1 \wedge x_2'$



(c) $(x_1 \vee x_2' \vee x_3) \wedge (x_1 \vee x_2' \vee x_3') \wedge (x_3 \vee x_2' \vee x_1) \wedge (x_3 \vee x_2' \vee x_1')$

Problems 37.7, page 1179

- $F_2 = F_3$
- $F \cup G = [0.4 x_1, 0.7 x_2, 0.5 x_3, 0.9 x_4]$
 $F \cap G = [0.3 x_1, 0.6 x_2, 0.1 x_3, 0.8 x_4]$
- (i) Truth value of 'F is not rich' is 0.2
(ii) Truth value of 'G is not fat' is 0.4
(iii) Truth value of 'Mary is not beautiful' is 0.3
- (i) $F \neq G$
(ii) F is not a subset of G ; G is not a subset of F .
(iii) $F^c = [1, 1, 1, 1, 0.9, 0.7, 0.5, 0.1, 0, 0]$
 $F \cap G = [0, 0.1, 0.3, 0.5]$
 $F \cup G = [0.1, 0.5, 0.9, 1, 0.9, 0.9, 1, 1]$
- (i) Truth value of the conjunction of 'Latif and John are good players' is 0.6.
(ii) Truth value of the disjunction of 'Latif and John are good players' is 0.7.
- Members and its degree of membership.

Problems 38.1, page 1186

- (i) $\frac{d\phi}{dt} = \frac{d\phi}{dx^i} \cdot \frac{dx^i}{dt}$; (ii) $x^i x^i$
- (i) $a_{11}(x^1)^2 + a_{22}(x^2)^2 + a_{33}(x^3)^2 + (a_{12} + a_{21})x^1 x^2 + (a_{13} + a_{31})x^1 x^3 + (a_{23} + a_{32})x^2 x^3$
(ii) $g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2 + 2g_{12}dx^1 dx^2 + 2g_{23}dx^2 dx^3 + 2g_{31}dx^3 dx^1$.
(iii) $g_{11} = g_{12}g^{2p} + \dots + g_{ln}g^{np}$
- (i) δ_h^i ; (ii) δ_s^p .
- (i) $\bar{A}_p^{qr} = \frac{\partial \bar{x}^q}{\partial x^j} \frac{\partial \bar{x}^r}{\partial x^k} \frac{\partial x^i}{\partial \bar{x}^p} A_i^{jk}$; (ii) $\bar{C}_{pq} = \frac{\partial x^m}{\partial \bar{x}^p} \frac{\partial x^n}{\partial \bar{x}^q} C_{mn}$
- Yes, $A_{h,l}^{ij,m}$, contravariant order 3, covariant order 2, Rank 5
- (a) $2\rho \cos^2 \phi - z \cos \phi + \rho^3 \sin^2 \phi \cos^2 \phi, -\rho^2 \sin 2\phi + \rho z \sin \phi + \rho^4 \sin \phi \cos^3 \phi, \rho z \sin \phi$;
(b) $2r \sin^2 \theta \cos^2 \phi - r \sin \theta \cos \theta \cos \phi + r^3 \sin^4 \theta \sin^2 \phi \cos^2 \phi + r^2 \sin \theta \cos^2 \theta \sin \phi$;
 $2r^2 \sin \theta \cos \theta \cos^2 \phi - r^2 \cos^2 \theta \cos \phi + r^4 \sin^3 \theta \cos \theta \sin^2 \phi \cos^2 \phi - r^3 \sin^2 \theta \cos \theta \sin \phi$;
 $-2r^2 \sin^2 \theta \sin \phi \cos \phi + r^2 \sin \theta \cos \theta \sin \phi + r^4 \sin^4 \theta \sin \phi \cos^3 \phi$
- (a $\cos \phi + b \sin \phi$) $\sin \theta + c \cos \theta$; $((a \cos \phi + b \sin \phi) \cos \theta - c \sin \theta)/r$; $(b \cos \phi - a \sin \phi)/r \sin \theta$.

Problems 38.2, page 1189

6. Rank = 1

Problems 36.3, page 1193

- $g = 4, g^{11} = 2, g^{22} = 5, g^{33} = 1.5, g^{12} = 3, g^{23} = -2.5, g^{13} = -1.5,$
- $g_{11} = 1, g_{22} = \rho^2, g_{33} = 1, g_{ij} = 0 (i \neq j); g^{11} = 1, g^{22} = \rho^{-2}, g^{33} = 1, g^{ij} = 0 (i \neq j)$
- $g = r^4 \sin^2 \theta / (1 - r^2/R^2); g^{11} = 1 - r^2/R^2, g^{22} = 1/r^2, g^{33} = (r \sin \theta)^{-2}, g^{ij} = 0 (i \neq j)$.

Problems 38.4, page 1199

1. (a) $[ii, i] = \frac{1}{2} \partial g^{ij} / \partial x^i$, $[ii, k] = -\frac{1}{2} \partial g_{ik} / \partial x^k$,

$[ik, k] = [ki, k] = \frac{1}{2} \partial g_{kk} / \partial x^i$, $[ij, k] = 0$, when i, j, k are all different

(b) $\begin{Bmatrix} i \\ ii \end{Bmatrix} = \frac{1}{2} g^{ij} \frac{\partial g_{ii}}{\partial x^i}$, $\begin{Bmatrix} k \\ ii \end{Bmatrix} = -\frac{1}{2} g^{kk} \frac{\partial g_{ii}}{\partial x^k}$

$\begin{Bmatrix} k \\ ik \end{Bmatrix} = \begin{Bmatrix} k \\ ki \end{Bmatrix} = \frac{1}{2} g^{kk} \frac{\partial g_{kk}}{\partial x^i}$ (no summation over i or k)

$\begin{Bmatrix} k \\ ij \end{Bmatrix} = 0$, when i, j, k are all different

2. (a) All are zero

(b) $[21, 2] = \rho = [12, 2]$; $[22, 1] = \rho$, all others are zero

(c) $[21, 2] = r = [12, 2]$; $[31, 3] = r \sin^2 \theta = [13, 3]$; $[32, 3] = r^2 \sin \theta \cos \phi = [23, 3]$; $[22, 1] = -r$; $[33, 1] = -r \sin^2 \theta$; $[33, 2] = -r^2 \sin \theta \cos \phi$; all others are zero

3. (a) All are zero

(b) $\begin{Bmatrix} 1 \\ 22 \end{Bmatrix} = -\rho$, $\begin{Bmatrix} 2 \\ 21 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 12 \end{Bmatrix} = \frac{1}{\rho}$, all others are zero

(c) $\begin{Bmatrix} 1 \\ 22 \end{Bmatrix} = -r$, $\begin{Bmatrix} 1 \\ 33 \end{Bmatrix} = -r \sin^2 \theta$, $\begin{Bmatrix} 2 \\ 21 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 12 \end{Bmatrix} = \frac{1}{r}$,

$\begin{Bmatrix} 2 \\ 33 \end{Bmatrix} = -\sin \theta \cos \theta$, $\begin{Bmatrix} 3 \\ 31 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 13 \end{Bmatrix} = \frac{1}{r}$,

$\begin{Bmatrix} 3 \\ 32 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 23 \end{Bmatrix} = \cot \theta$, all others are zero

5. (a) $r^2 \sin \theta \cos \theta$; $r^2 \sin \theta \cos \theta$

(b) $-\sin \theta \cos \theta$; $\cot \theta$

6. (a) $-r \sin^2 \theta$; $r^2 \sin \theta \cos \theta$

(b) $-r \sin^2 \theta$; $\cot \theta$

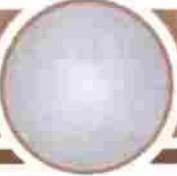
8. (a) $u^{ij}_{,k} = \frac{\partial u^{ij}}{\partial x^k} + \begin{Bmatrix} i \\ ks \end{Bmatrix} u^{sj} + \begin{Bmatrix} i \\ ks \end{Bmatrix} u^{is}$;

(b) $A^h_{ij,k} = \frac{\partial A^h_{ij}}{\partial x^k} - \begin{Bmatrix} s \\ ik \end{Bmatrix} A^h_{sj} + \begin{Bmatrix} h \\ ks \end{Bmatrix} A^s_{ij}$

10. $A^j_{k,q} B_n^{lm} + A^j_{k,l} B_{n,q}^{lm}$

11. (a) $\frac{1}{\rho} \left[\frac{\partial}{\partial p} (\rho A_p) + \frac{\partial}{\partial \phi} (A_\phi) + \frac{\partial}{\partial z} (\rho A_z) \right]$; (b) $\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial r} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$

12. (a) $\frac{\partial^2 v}{\partial \rho^2} + \frac{1}{\rho^2} \frac{\partial^2 v}{\partial \phi^2} + \frac{\partial^2 v}{\partial z^2} + \frac{1}{\rho} \frac{\partial v}{\partial \rho} = 0$ (b) $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v}{\partial \phi^2} + \frac{2}{r} \frac{\partial v}{\partial r} + \frac{\cot \theta}{r^2} \frac{\partial v}{\partial \theta} = 0$.

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Higher Engineering Mathematics

Dr. B.S. Grewal

The book provides a clear exposition of essential tools of applied mathematics from a modern point of view and meets complete requirements of engineering and computer science students. Every effort has been made to keep the presentation at once simple and lucid. It is written with the firm conviction that a good book is one that can be read with minimum guidance from the instructor. To achieve this, more than the usual number of solved examples, followed by properly graded problems have been given. Many of the examples and problems have been selected from recent papers of various university and other engineering examinations. Basic Concepts and Useful Information has been given in an Appendix. However, the subject matter has been set in eight main units:

- **Algebra & Geometry** : Solution of equations, Linear algebra: Determinants, Matrices, Vector algebra and Solid geometry.
- **Calculus** : Differential calculus, Partial differentiation, Integral calculus, Multiple integrals, Vector calculus.
- **Series** : Infinite series and Fourier series.
- **Differential Equations** : Differential equations of first order and their applications, Linear differential equations and their applications, Differential equations of different types, Series solution of differential equations and special functions, Partial differential equations and their applications.
- **Complex Analysis** : Complex numbers and functions, Calculus of complex functions.
- **Transforms** : Laplace transforms, Fourier transforms and Z-transforms.
- **Numerical Techniques** : Empirical Laws and Curve fitting, Statistical methods, Probability and Distributions, Sampling and Inference, Numerical methods, Finite differences and Interpolation, Difference equations, Numerical solution of Ordinary and Partial differential equations, Linear programming.
- **Special Topics** : Calculus of variations, Integral equations, Discrete mathematics, Tensors.

An exhaustive list of 'Objective Type of Questions' has been given at the end of each chapter. Standard Tables, Answers to Problems, and a fairly comprehensive Index is given at the end.



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