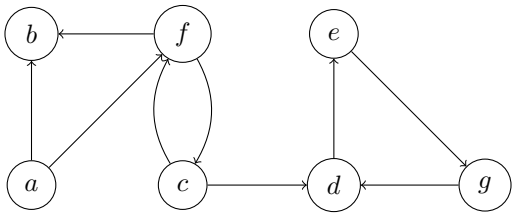
DSC 40B - Discussion 07

Problem 1.

Consider a *breadth*-first search on the graph shown in the figure, starting with node c.



a) Suppose you call bfs_shortest_paths(graph, 'c') on the graph above. This function returns dictionaries distance and predecessor. Write down the contents of these dictionaries as they are when the function exits.

```
def bfs_shortest_paths(graph, source):
    status = {node:'undiscovered' for node in graph.nodes}
    distance= {node:float('inf') for node in graph.nodes}
    predecessor = {node: None for node in graph.nodes}
    status[source] = 'pending'
    distance[source]=0
    pending = deque([source])
    # while there are still pending nodes
    while pending:
        u = pending.popleft()
        for v in graph.neighbors(u):
            # explore edge (u,v)
            if status[v] == 'undiscovered':
                status[v]='pending'
                distance[v]=distance[u]+1
                predecessor[v]=u
                # append to right
                pending.append(v)
        status[u]='visited'
    return predecessor, distance
```

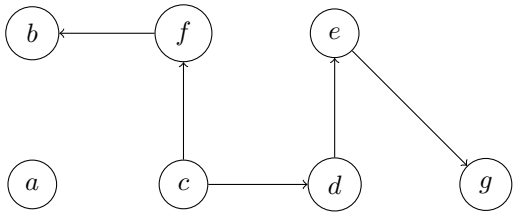
Solution:

distance: {
'a':∞,
'b':2,
'c':0,
'd':1,
'e':2,
'f':1,
'g':3
}
predecessor:{
'a':None,

'b':'f',
'c':None,
'd':'c',
'e':'d',
'f':'c',
'g':'e'
}

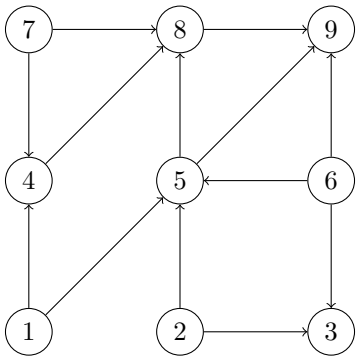
b) Mark the BFS trees produced on executing BFS on this graph.

Solution:



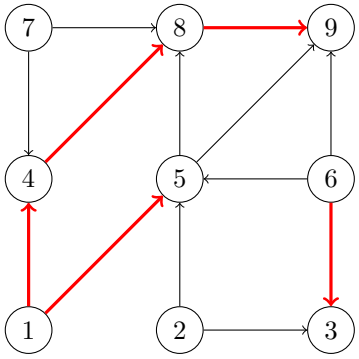
Problem 2.

Consider the following directed graph.



a) Run Full_DFS on the graph above. Make a bold arrow from node u to node v if u is the predecessor of node v in DFS. Use the convention that a node's neighbors are processed in ascending order by label.

Solution:



b) Fill in the table below so that it contains the start and finish times of each node after a Full_DFS is

performed on the above graph. Assume node 1 as the source for the first DFS call. Begin your start times with 1.

Node	Start	Finish
1	1	10
2	17	18
3	14	15
4	2	7
5	8	9
6	13	16
7	11	12
8	3	6
9	4	5

c) Topologically sort the vertices of the graph.

Solution: 2,6,3,7,1,5,4,8,9

Problem 3.

State whether the following statements are true or false.

a) Full breadth first search on a directed graph always produces same number of BFS trees irrespective of order in which vertices are given and the neighbouring nodes are visited.

Solution: False.

b) Full breadth first search on an undirected graph always produces same number of BFS trees irrespective of order in which vertices are given and the neighbouring nodes are visited.

Solution: True.

c) Both Full BFS and Full DFS require atleast $\Omega(V)$ memory.

Solution: True.

d) Consider a graph G on which BFS is run with node s as the source. Assume that BFS visits a node u in the graph before node v. Then $d(s,u) < d(s,v)$

> **Solution:** False. $d(s,u) \leq d(s,v)$

e) Every directed acyclic graph has exactly one topological ordering.

> **Solution:** False. There can be multiple topological orderings depending on the order of nodes chosen during DFS.

Problem 4.

Given an undirected graph G=(V,E), give an algorithm to find if the graph is disconnected.

> **Solution:** Run BFS or DFS on a node in G (since G is undirected, any node will do). If the size of the set of visited nodes does not equal $|V|$, then the graph is disconnected.

Figure 1: "Full" DFS

```
from data classes import dataclass
    @dataclass
    class Times:
        clock: int
        start: dict
        finish: dict

    def full_dfs_times(graph):
        status = {node:'undiscovered' for node in graph.nodes}
        predecessor = {node: None for node in graph.nodes}
        times=Times(clock=0, start={}, finish={})
        for u in graph.nodes:
            if status[u]=='undiscovered':
                dfs_times(graph, u, status, times)
        return times, predecessor

        def dfs_times(graph, u, status, predecessor, times):
            times.clock+=1
            times.start[u]=times.clock
            status[u]='pending'
            for v in graph.neighbors(u):
                # explore edge (u, v)
                if status[v]=='undiscovered':
                    predecessor[v]= u
                    dfs_times(graph, v, status, times)
            status[u]='visited'
            times.clock+=1
            times.finish[u]=times.clock
```