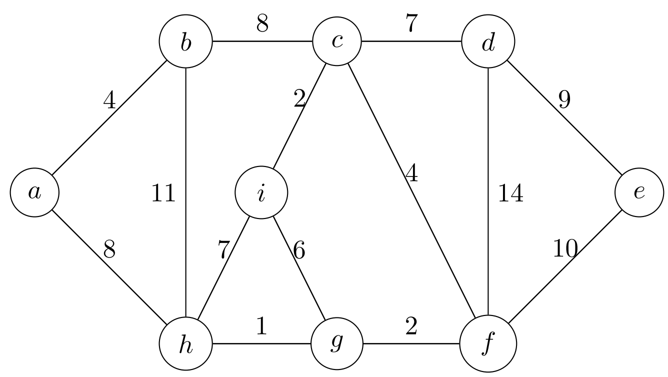
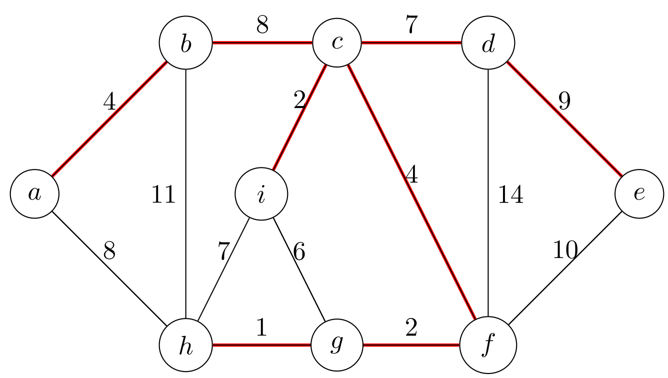
# DSC 40B - Discussion 09

**Problem 1.**

Compute the minimum spanning tree for the following graph using Kruskal's algorithm. (Also compute the MST using Prim's algorithm and compare the results.)




**Solution:**




**Problem 2.**

Suppose we are given both an undirected graph G with weighted edges and a minimum spanning tree T of G.

**a)** Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e in T is decreased.

**Solution:** The minimum spanning tree of the updated graph would be T.

**b)** Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e not in T is increased.

> **Solution:** The minimum spanning tree of the updated graph would be T.

**c)** Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e in T is increased.

> **Solution:** Let $e = (u, v)$ be the edge whose weight is increased. Remove the edge e from the minimum spanning tree T. This divides the tree T into two connected components. Let $T_u$ be the component which contains u and $T_v$ be the component which contains v. We can identify $T_u$ and $T_v$ by running a BFS or DFS with u and v as the sources. This takes time $\Theta(V + E)$. While running BFS we can also label each node as 0 if it is a part of $T_u$ and 1 if it is a part of $T_v$. We can now examine each edge e in the graph and find the minimum weight edge which connects a node labelled 0 to a node labelled 1. This takes time $\Theta(E)$. We can then add this edge to T to get the minimum spanning tree of the updated graph. The total time complexity is $\Theta(V + E)$.

**d)** Describe an efficient algorithm to update the minimum spanning tree when the weight of one edge e not in T is decreased.

> **Solution:** Let $e = (u, v)$ be the edge whose weight is decreased. Add the edge e to the minimum spanning tree T. This would result in a cycle in T. We can identify the nodes and the edges on the cycle by running BFS or DFS, with minimal modifications, with u or v as the source. This takes time $\Theta(V + E)$. Find the maximum weight edge on the cycle and remove it from T. This takes time $\Theta(E)$. The resultant tree would be a minimum spanning tree for the updated graph. The total time complexity is $\Theta(V + E)$.