Project 3: *"Today's episode of Sesame Street has been brought to you by the letter ..."*

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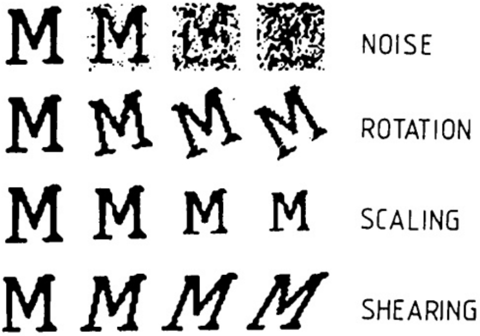
Contents

Question of Interest

Classify an image of a letter to one of the 26 capital letters in the English alphabet.



http://imagebank.osa.org/getImage.xqy?img=dTcqLmxhcmdlLGFvLTIzLTEwLTE1MDktZzAxMA

Data Set Information

- All 26 uppercase English letters
- 20 fonts for each letter
- Randomly distorted
    - File of 20,000 unique observations
- Each observation converted into 16 primitive numerical attributes

16 Variables Used:

1. **lettr**: True capital letter (26 values from A to Z)
2. **x-box**: Horizontal position of box (integer)
3. **y-box**: Vertical position of box (integer)
4. **width**: Width of box (integer)
5. **high**: Height of box (integer)
6. **onpix**: Total number on pixels (integer)
7. **x-bar**: Mean x of on pixels in box (integer)
8. **y-bar**: Mean y of on pixels in box (integer)
9. **x2bar**: Mean x variance (integer)
10. **y2bar**: Mean y variance (integer)
11. **xybar**: Mean xy correlation (integer)
12. **x2ybr**: Mean of xxy (integer)
13. **xy2br**: Mean of xyy (integer)
14. **x-ege**: Mean edge count left to right (integer)
15. **xegvy**: Correlation of x-ege with y (integer)
16. **y-ege**: Mean edge count bottom to top (integer)
17. **yegvx**: Correlation of y-ege with x (integer)

Description of Methods

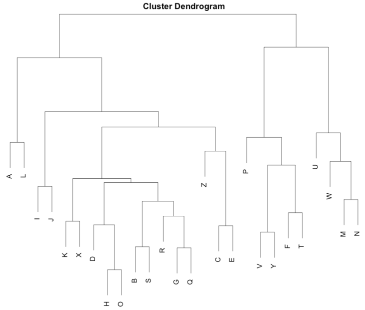
Algorithms for:

1. Logistic Regression Binary Search Tree (BST)
2. Decision Trees for Classification
    1. CART Method
    2. Bag Method

Logistic Regression BST Algorithm

Preparing Binary Tree (Using Learning Set):

1. Summarize by unique letter (average over observations from a given letter for each of the metrics)
2. Find distance between letters (uses Euclidean distance)
3. Use hclust() with "complete" method to create dendrogram



Logistic Regression BST Algorithm

Traversing Binary Tree with Logistic Regression Models:

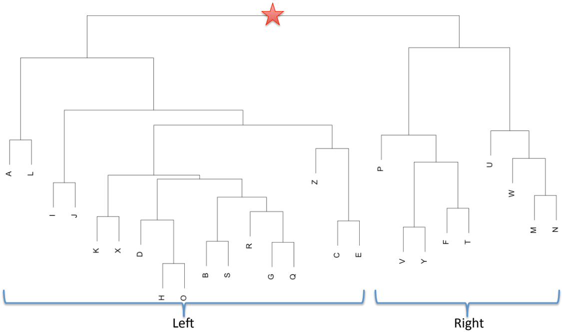
1. Subset letters are to the left and right of current intersection location. Right letters $= 1$, Left letters $= 0$
2. Create logistic regression model for probability of right (uses all 15 explanatory variables)
3. Evaluate logistic regression model with new covariates from observation in validation set.

$$\begin{cases} \text{move right} & : \text{if } \hat{\pi} \geq 0.5 \\ \text{move left} & : \text{if } \hat{\pi} < 0.5 \end{cases}$$

4. Keep track of path traversed
5. Repeat steps 1-4 until you arrived at an end node, which is the predicted letter

Logistic Regression BST Algorithm Example

New observation: (T, 2, 6, 3, 4, 2, 7, 12, 2, 7, 7, 11, 8, 1,11, 1, 8)

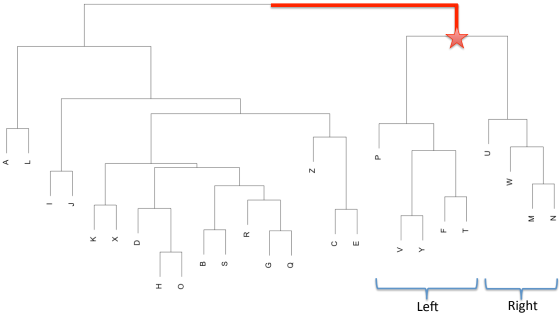


$$log(\frac{\pi_i}{1-\pi_i}) = -.5 + .32x_1 - .08x_2 + .07x_3 - .1x_4 + .11x_5 - .05x_6 + .41x_7 - .09x_8 - .3x_9 - .05x_{10} + .54x_{11} - .68x_{12} + .56x_{13} + .23x_{14} - .58x_{15} - .24x_{16} \rightarrow \hat{\pi} = 0.929$$

Move right!

Logistic Regression BST Algorithm Example

New observation: (T, 2, 6, 3, 4, 2, 7, 12, 2, 7, 7, 11, 8, 1,11, 1, 8)

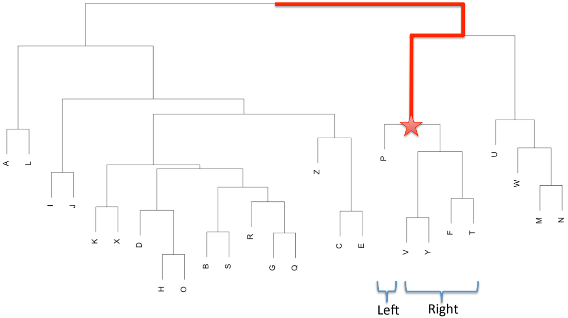


$$log(\frac{\pi_i}{1-\pi_i}) = 4.12 - .37x_1 + .15x_2 + .83x_3 - 1.07x_4 + .3x_5 - .64x_6 + .23x_7 + 1.17x_8 + .58x_9 - .39x_{10} - .83x_{11} + .88x_{12} + 1.87x_{13} - .51x_{14} - 2x_{15} - .57x_{16} \rightarrow \hat{\pi} = 0.0007$$

Move left!

Logistic Regression BST Algorithm Example

New observation: (T, 2, 6, 3, 4, 2, 7, 12, 2, 7, 7, 11, 8, 1,11, 1, 8)

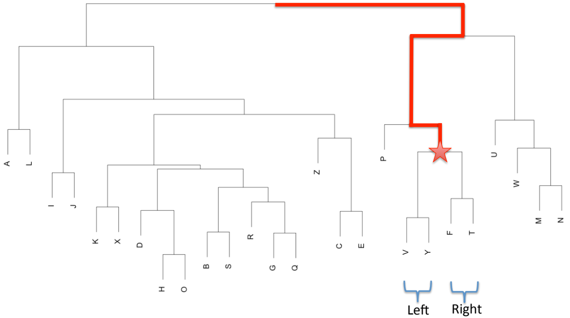


$$log(\frac{\pi_i}{1-\pi_i}) = -23.41 + .16x_1 + .17x_2 + .04x_3 - .25x_4 - .49x_5 + .38x_6 + .67x_7 - .65x_8 + .69x_9 + .23x_{10} + .91x_{11} + 1.79x_{12} + .36x_{13} - .1x_{14} + .07x_{15} - .29x_{16} \rightarrow \hat{\pi} = 0.999$$

Move right!

Logistic Regression BST Algorithm Example

New observation: (T, 2, 6, 3, 4, 2, 7, 12, 2, 7, 7, 11, 8, 1,11, 1, 8)

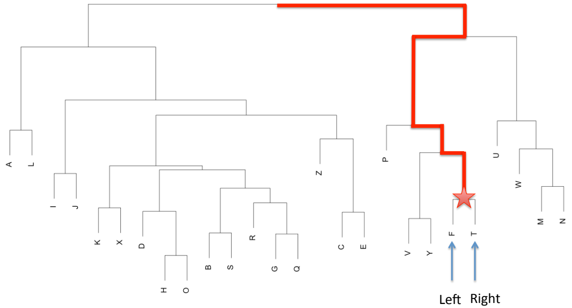


$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = -13.86 - .61x_1 + .5x_2 - .96x_3 - .49x_4 + 1.57x_5 + .57x_6 + 1.64x_7 + .69x_8 + 1.56x_9 + .85x_{10} - 1.71x_{11} + .32x_{12} - .65x_{13} - .96x_{14} - .55x_{15} + .58x_{16} \rightarrow \hat{\pi} = 0.991$$

Move right!

Logistic Regression BST Algorithm Example

New observation: (T, 2, 6, 3, 4, 2, 7, 12, 2, 7, 7, 11, 8, 1,11, 1, 8)

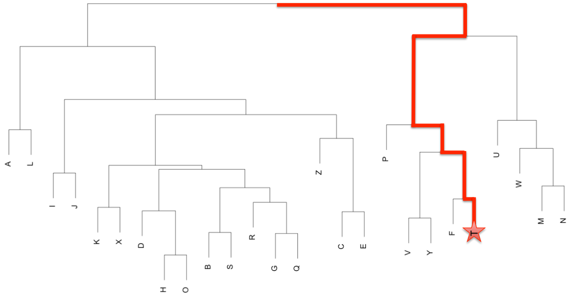


$$log(\frac{\pi_i}{1-\pi_i}) = -33.85 + .99x_1 + .77x_2 - .59x_3 - 1.36x_4 - .04x_5 + 1.5x_6 + 2.41x_7 + 1.22x_8 + 3.35x_9 - 1.96x_{10} - .87x_{11} + 1.61x_{12} + .33x_{13} + .66x_{14} - 1.25x_{15} - 1.32x_{16} \rightarrow \hat{\pi} = 0.999$$

Move right! and STOP

Logistic Regression BST Algorithm Example

New observation: (T, 2, 6, 3, 4, 2, 7, 12, 2, 7, 7, 11, 8, 1,11, 1, 8)



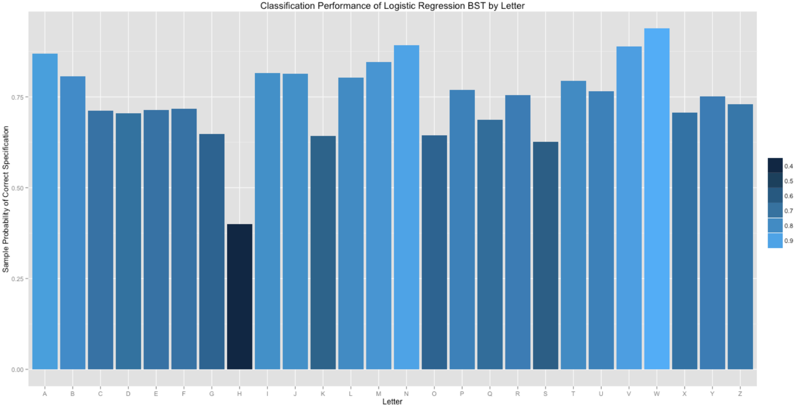
Prediction: T
Conclusion: Correctly classified! Yay!

Summary of Findings

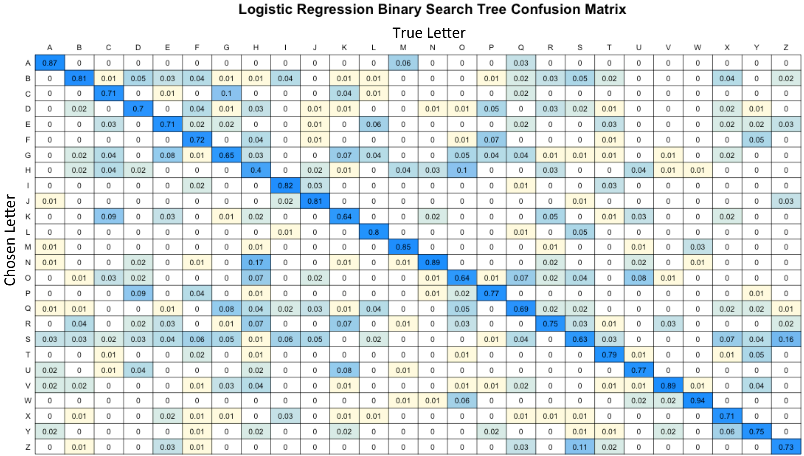
Findings for:

1. Logistic Regression BST Confusion Matrix
2. Decision Trees for Classification
    1. CART Method Confusion Matrix
    2. Bag Method Confusion Matrix

Logistic Regression BST Distribution of Specification



Logistic Regression BST Confusion Matrix



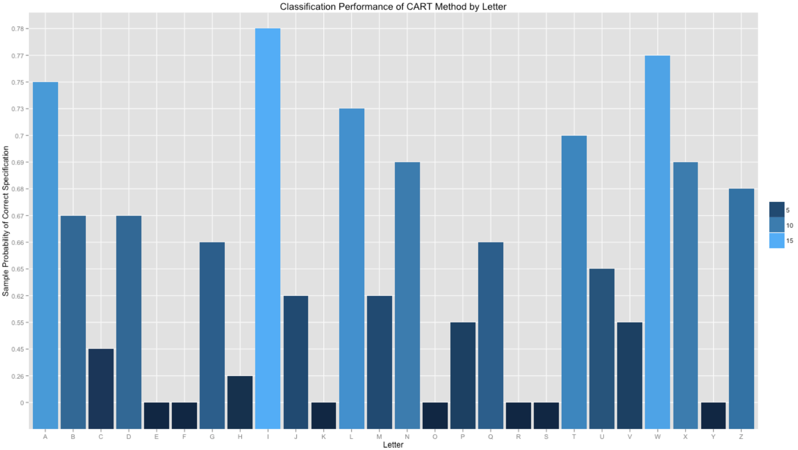
Logistic Regression Binary Search Tree Confusion Matrix

True Letter

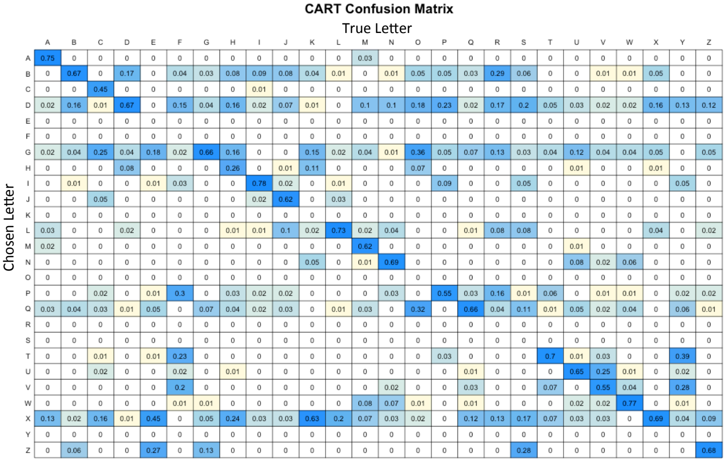
Chosen Letter

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	0.87	0	0	0	0	0	0	0	0	0	0	0	0.06	0	0	0	0.03	0	0	0	0	0	0	0	0	0
B	0	0.81	0.01	0.05	0.03	0.04	0.01	0.01	0.04	0	0.01	0.01	0	0	0	0.01	0.02	0.03	0.05	0.02	0	0	0	0.04	0	0.02
C	0	0	0.71	0	0.01	0	0.1	0	0	0	0.04	0.01	0	0	0	0	0.02	0	0	0	0	0	0	0	0	0
D	0	0.02	0	0.7	0	0.04	0.01	0.03	0	0.01	0.01	0	0	0.01	0.01	0.05	0	0.03	0.02	0.01	0	0	0	0.02	0.01	0
E	0	0	0.03	0	0.71	0.02	0.02	0	0	0.01	0	0.06	0	0	0	0	0.02	0	0	0.03	0	0	0	0.02	0.02	0.03
F	0	0	0	0	0	0.72	0	0.04	0	0.01	0	0	0	0	0.01	0.07	0	0	0	0.01	0	0	0	0	0.05	0
G	0	0.02	0.04	0	0.08	0.01	0.65	0.03	0	0	0.07	0.04	0	0	0.05	0.04	0.04	0.01	0.01	0.01	0	0.01	0	0.02	0	0
H	0	0.02	0.04	0.02	0	0	0	0.4	0	0.02	0.01	0	0.04	0.03	0.1	0	0	0.03	0	0	0.04	0.01	0.01	0	0	0
I	0	0	0	0	0	0.02	0	0	0.82	0.03	0	0	0	0	0	0	0.01	0	0	0.03	0	0	0	0	0	0
J	0.01	0	0	0	0	0	0	0	0.02	0.81	0	0	0	0	0	0	0	0	0.01	0	0	0	0	0	0	0.03
K	0	0	0.09	0	0.03	0	0.01	0.02	0	0	0.64	0	0	0.02	0	0	0	0.05	0	0.01	0.03	0	0	0.02	0	0
L	0	0	0	0	0	0	0	0	0.01	0	0	0.8	0	0	0	0	0.01	0	0.05	0	0	0	0	0	0	0
M	0.01	0	0	0	0	0	0	0.01	0	0	0	0	0.85	0	0	0	0	0.01	0	0	0.01	0	0.03	0	0	0
N	0.01	0	0	0.02	0	0.01	0	0.17	0	0	0.01	0	0.01	0.89	0	0	0	0.02	0	0	0.02	0	0.01	0	0	0
O	0	0.01	0.03	0.02	0	0	0	0.07	0	0.02	0	0	0	0.01	0.64	0.01	0.07	0.02	0.04	0	0.08	0.01	0	0	0	0
P	0	0	0	0.09	0	0.04	0	0.01	0	0	0	0	0	0.01	0.02	0.77	0	0	0	0	0	0	0	0	0.01	0
Q	0.01	0.01	0	0	0.01	0	0.08	0.04	0.02	0.03	0.01	0.04	0	0	0.05	0	0.69	0.02	0.02	0	0	0	0	0.02	0.02	0.01
R	0	0.04	0	0.02	0.03	0	0.01	0.07	0	0	0.07	0	0.01	0	0.03	0	0	0.75	0.03	0.01	0	0.03	0	0	0	0.02
S	0.03	0.03	0.02	0.03	0.04	0.06	0.05	0.01	0.06	0.05	0	0.02	0	0	0	0.01	0.04	0	0.63	0.03	0	0	0	0.07	0.04	0.16
T	0	0	0.01	0	0	0.02	0	0.01	0	0	0	0	0	0	0.01	0	0	0	0	0.79	0.01	0	0	0.01	0.05	0
U	0.02	0	0.01	0.04	0	0	0	0.02	0	0	0.08	0	0.01	0	0	0	0	0	0	0	0.77	0	0	0	0	0
V	0.02	0.02	0	0	0	0.01	0.03	0.04	0	0	0.01	0	0	0	0.01	0.01	0.02	0	0	0.01	0.01	0.89	0.01	0	0.04	0
W	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0.01	0.06	0	0	0	0	0	0.02	0.02	0.94	0	0	0
X	0	0.01	0	0	0.02	0.01	0.01	0	0.03	0	0.01	0.01	0	0	0	0	0.01	0.01	0.01	0	0	0	0	0.71	0	0
Y	0.02	0	0	0	0	0.01	0	0.02	0	0	0.02	0	0	0	0	0.02	0	0	0.01	0.01	0	0.02	0	0.06	0.75	0
Z	0	0.01	0	0	0.03	0.01	0	0	0	0	0	0	0	0	0	0	0.03	0	0.11	0.02	0	0	0	0	0	0.73

CART Method Distribution of Specification



CART Method Confusion Matrix



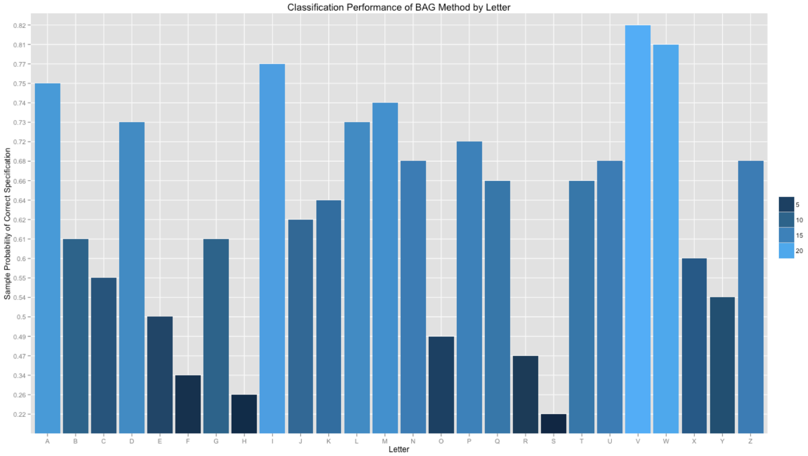
CART Confusion Matrix

True Letter

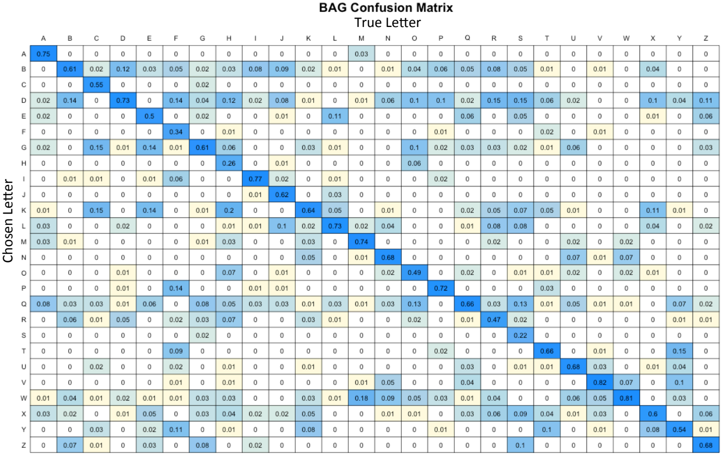
Chosen Letter

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	0.75	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0
B	0	0.67	0	0.17	0	0.04	0.03	0.08	0.09	0.08	0.04	0.01	0	0.01	0.05	0.05	0.03	0.29	0.06	0	0	0.01	0.01	0.05	0	0
C	0	0	0.45	0	0	0	0	0	0.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D	0.02	0.16	0.01	0.67	0	0.15	0.04	0.16	0.02	0.07	0.01	0	0.1	0.1	0.18	0.23	0.02	0.17	0.2	0.05	0.03	0.02	0.02	0.16	0.13	0.12
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
G	0.02	0.04	0.25	0.04	0.18	0.02	0.66	0.16	0	0	0.15	0.02	0.04	0.01	0.36	0.05	0.07	0.13	0.03	0.04	0.12	0.04	0.04	0.05	0	0.05
H	0	0	0	0.08	0	0	0	0.26	0	0.01	0.11	0	0	0	0.07	0	0	0	0	0	0.01	0	0	0.01	0	0
I	0	0.01	0	0	0.01	0.03	0	0	0.78	0.02	0	0.01	0	0	0	0.09	0	0	0.05	0	0	0	0	0	0.05	0
J	0	0	0.05	0	0	0	0	0	0.02	0.62	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
K	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L	0.03	0	0	0.02	0	0	0	0.01	0.01	0.1	0.02	0.73	0.02	0.04	0	0	0.01	0.08	0.08	0	0	0	0	0.04	0	0.02
M	0.02	0	0	0	0	0	0	0	0	0	0	0	0.62	0	0	0	0	0	0	0	0.01	0	0	0	0	0
N	0	0	0	0	0	0	0	0	0	0	0.05	0	0.01	0.69	0	0	0	0	0	0	0.08	0.02	0.06	0	0	0
O	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
P	0	0	0.02	0	0.01	0.3	0	0.03	0.02	0.02	0	0	0	0.03	0	0.55	0.03	0.16	0.01	0.06	0	0.01	0.01	0	0.02	0.02
Q	0.03	0.04	0.03	0.01	0.05	0	0.07	0.04	0.02	0.03	0	0.01	0.03	0	0.32	0	0.66	0.04	0.11	0.01	0.05	0.02	0.04	0	0.06	0.01
R	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T	0	0	0.01	0	0.01	0.23	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0.7	0.01	0.03	0	0	0.39	0
U	0	0	0.02	0	0	0.02	0	0.01	0	0	0	0	0	0	0	0	0.01	0	0	0	0.65	0.25	0.01	0	0.02	0
V	0	0	0	0	0	0.2	0	0	0	0	0	0	0	0.02	0	0	0.03	0	0	0.07	0	0.55	0.04	0	0.28	0
W	0	0	0	0	0	0.01	0.01	0	0	0	0	0	0.08	0.07	0.01	0	0.01	0	0	0	0.02	0.02	0.77	0	0.01	0
X	0.13	0.02	0.16	0.01	0.45	0	0.05	0.24	0.03	0.03	0.63	0.2	0.07	0.03	0.02	0	0.12	0.13	0.17	0.07	0.03	0.03	0	0.69	0.04	0.09
Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Z	0	0.06	0	0	0.27	0	0.13	0	0	0	0	0	0	0	0	0	0	0	0.28	0	0	0	0	0	0	0.68

BAG Method Distribution of Specification



BAG Method Confusion Matrix



BAG Confusion Matrix

True Letter

Chosen Letter

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	0.75	0	0	0	0	0	0	0	0	0	0	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0
B	0	0.61	0.02	0.12	0.03	0.05	0.02	0.03	0.08	0.09	0.02	0.01	0	0.01	0.04	0.06	0.05	0.08	0.05	0.01	0	0.01	0	0.04	0	0
C	0	0	0.55	0	0	0	0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
D	0.02	0.14	0	0.73	0	0.14	0.04	0.12	0.02	0.08	0.01	0	0.01	0.06	0.1	0.1	0.02	0.15	0.15	0.06	0.02	0	0	0.1	0.04	0.11
E	0.02	0	0	0	0.5	0	0.02	0	0	0.01	0	0.11	0	0	0	0	0.06	0	0.05	0	0	0	0	0.01	0	0.06
F	0	0	0	0	0	0.34	0	0.01	0	0	0	0	0	0	0	0.01	0	0	0	0.02	0	0.01	0	0	0	0
G	0.02	0	0.15	0.01	0.14	0.01	0.61	0.06	0	0	0.03	0.01	0	0	0.1	0.02	0.03	0.03	0.02	0.01	0.06	0	0	0	0	0.03
H	0	0	0	0	0	0	0	0.26	0	0.01	0	0	0	0	0.06	0	0	0	0	0	0	0	0	0	0	0
I	0	0.01	0.01	0	0.01	0.06	0	0	0.77	0.02	0	0.01	0	0	0	0.02	0	0	0	0	0	0	0	0	0	0
J	0	0	0	0	0	0	0	0	0.01	0.62	0	0.03	0	0	0	0	0	0	0	0	0	0	0	0	0	0
K	0.01	0	0.15	0	0.14	0	0.01	0.2	0	0	0.64	0.05	0	0.01	0	0	0.02	0.05	0.07	0.05	0.01	0	0	0.11	0.01	0
L	0.03	0	0	0.02	0	0	0	0.01	0.01	0.1	0.02	0.73	0.02	0.04	0	0	0.01	0.08	0.08	0	0	0	0	0.04	0	0.02
M	0.03	0.01	0	0	0	0	0.01	0.03	0	0	0.03	0	0.74	0	0	0	0	0.02	0	0	0.02	0	0.02	0	0	0
N	0	0	0	0	0	0	0	0	0	0	0.05	0	0.01	0.68	0	0	0	0	0	0	0.07	0.01	0.07	0	0	0
O	0	0	0	0.01	0	0	0	0.07	0	0.01	0	0	0	0.02	0.49	0	0.02	0	0.01	0.01	0.02	0	0.02	0.01	0	0
P	0	0	0	0.01	0	0.14	0	0	0.01	0.01	0	0	0	0	0	0.72	0	0	0	0.03	0	0	0	0	0	0
Q	0.08	0.03	0.03	0.01	0.06	0	0.08	0.05	0.03	0.03	0.01	0.03	0.01	0.03	0.13	0	0.66	0.03	0.13	0.01	0.05	0.01	0.01	0	0.07	0.02
R	0	0.06	0.01	0.05	0	0.02	0.03	0.07	0	0	0.03	0.01	0	0	0.02	0	0.01	0.47	0.02	0	0	0	0	0	0.01	0.01
S	0	0	0	0	0	0	0.02	0	0	0	0	0	0	0	0	0	0	0	0.22	0	0	0	0	0	0	0
T	0	0	0	0	0	0.09	0	0	0	0	0	0	0	0	0	0.02	0	0	0	0.66	0	0.01	0	0	0.15	0
U	0	0	0.02	0	0	0.02	0	0.01	0	0	0.01	0	0	0	0	0	0.03	0	0.01	0.01	0.68	0.03	0	0.01	0.04	0
V	0	0	0	0	0	0.01	0	0.01	0	0	0	0	0.01	0.05	0	0	0.04	0	0	0	0	0.82	0.07	0	0.1	0
W	0.01	0.04	0.01	0.02	0.01	0.01	0.03	0.03	0	0	0.03	0.01	0.18	0.09	0.05	0.03	0.01	0.04	0	0	0.06	0.05	0.81	0	0.03	0
X	0.03	0.02	0	0.01	0.05	0	0.03	0.04	0.02	0.02	0.05	0	0	0.01	0.01	0	0.03	0.06	0.09	0.04	0.01	0.03	0	0.6	0	0.06
Y	0	0	0.03	0	0.02	0.11	0	0.01	0	0	0.08	0	0	0	0	0.01	0	0	0	0.1	0	0.01	0	0.08	0.54	0.01
Z	0	0.07	0.01	0	0.03	0	0.08	0	0.02	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0.68

Overall Findings

1. Logistic Regression BST : 74.8% Correct Specification Overall
    - Highest Correct Classification: **W** with 94%
    - Lowest Correct Classification: **H** with 40%
2. CART Method: 47.1% Correct Specification Overall
    - Highest Correct Classification: **I** with 78%
    - Lowest Correct Classification: **E,F,K,O,R,S,Y** with 0%
3. Bag Method: 60.6% Correct Specification Overall
    - Highest Correct Classification: **V** with 82%
    - Lowest Correct Classification: **S** with 22%

Discussion

Usual Logistic Regression Assumptions

- The true conditional probabilities are a logistic function of the independent variables
- No important variables are omitted.
- No extraneous variables are included.
- The independent variables are measured without error.
- The observations are independent.
- The independent variables are not linear combinations of each other.

Source: IDRE UCLA (Institute for Digital Research and Education

Questions