STA601: Lab 8

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Lognormal distribution

$f_X(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{(2\pi)}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, x > 0$

mean: $e^{\mu+\sigma^2/2}$
variance: $\big(e^{\sigma^2}-1\big)e^{2\mu+\sigma^2}$

1. Likelihood:

$L(x|\mu,\sigma^2) = \prod_{i=1}^n \frac{1}{x_i\sigma\sqrt{2\pi}}e^{-\frac{(\ln x_i-\mu)^2}{2\sigma^2}}, x_i > 0$

Priors:

$\mu \sim N(\mu_0,\tau_0) = N(0,15)$
$\sigma^{-2} \sim Gamma(a,b) = Gamma(0.5,0.5)$

Full conditionals:

$\pi(\sigma^{-2}|\mu,x) \propto (\sigma^{-2})^{\frac{n}{2}}e^{-0.5\sigma^{-2}\sum_{i=1}^n(lnx_i-\mu)^2} \propto (\sigma^{-2})^{a+\frac{n}{2}-1}e^{-\sigma^{-2}\big(b+0.5\sum_{i=1}^n(\ln x_i-\mu)^2\big)}$
$\propto Gamma\big(a+\frac{n}{2}, b+0.5\sum_{i=1}^n(\ln x_i-\mu)^2\big)$

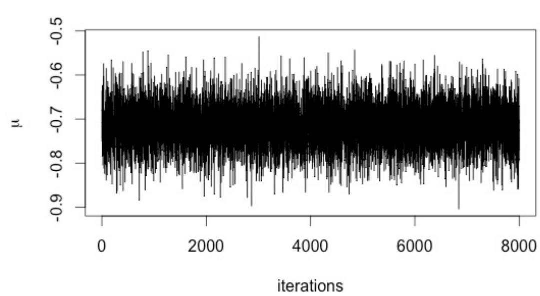
$\pi(\mu|\sigma^2,x) \propto \prod_{i=1}^n e^{-\frac{(\ln x_i-\mu)^2}{2\sigma^2}}e^{-\frac{(\mu-\mu_0)^2}{2\tau_0}} \propto \exp\Bigg(\frac{1}{2\sigma^2}\big(n\mu^2-2\mu\sum_{i=1}^n\ln x_i\big)-\frac{1}{2\tau_0}\big(\mu^2-2\mu\mu_0\big)\Bigg) \propto$
$\exp\Bigg(-\frac{1}{2}\big[(\frac{n}{\sigma^2}+\frac{1}{\tau_0})\mu^2-2\mu(\frac{\sum_{i=1}^n\ln x_i}{\sigma^2}+\frac{\mu_0}{\tau_0})\big]\Bigg) \propto N(M,V)$, where
$M = V\big(\frac{\sum_{i=1}^n\ln x_i}{\sigma^2}+\frac{\mu_0}{\tau_0}\big)$
$V = \big(\frac{n}{\sigma^2}+\frac{1}{\tau_0}\big)^{-1}$

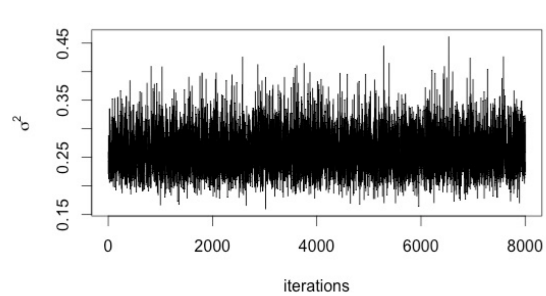
2. Run Gibbs sampling in R:

```
setwd('/Users/akbota/Documents/STA601/Labs/lab8')
data <- read.table("data.txt", stringsAsFactors=F)
```

```
x=as.numeric(data[2:101,1])

a=0.5
b=0.5
mu0=0
tau0=15
N=10000
n=100
sig_sq<-c(2)
mu<-c(1)


for(i in 2:N){
  V=(n/sig_sq[i-1]+1/tau0)^(-1)
  M=V*(sum(log(x))/sig_sq[i-1]+mu0/tau0)
  mu[i]<-rnorm(1,M,sqrt(V))

  sumforf=0
  for(j in 1:n){
    sumforf=(log(x[j])-mu[i])^2+sumforf
  }
  f<-rgamma(1,shape=a+n/2, rate=b+0.5*sumforf)
  sig_sq[i]=1/f
}

plot(mu[2001:10000], type='l', ylab=expression(mu), xlab="iterations")
plot(sig_sq[2001:10000], type='l', ylab=expression(sigma^2), xlab="iterations")
```

Both post burn-in plots illustrate that sampler has converged:





3. First calculate samples of mean and variance using samples of μ and σ^2 generated using Gibbs sampler, just by simply pluging in to the formulas of mean and variance given above.

Find 95% CI for mean and variance using quantile function in R.

<u>CI for mean</u>: (0.503, 0.602)
<u>CI for variance</u>: (0.061, 0.128)

```
mean<-exp(mu[2001:10000]+sig_sq[2001:10000]/2)
var<-(exp(sig_sq[2001:10000])-1)*exp(2*mu[2001:10000]+sig_sq[2001:10000])

quantile(mean, probs=c(0.025, 0.925))
quantile(var, probs=c(0.025, 0.925))
```