Linear programming

simplex method

This presentation will help you to solve linear programming problems using the

Simplex tableau

This is a typical linear programming problem

Keep left clicking the mouse to reveal the next part

Maximise $P = 2x + 3y + 4z$

The objective function

Subject to

$3x + 2y + z < 10$

$2x + 5y + 3z < 15$

$x, y > 0$

The constraints

The first step is to rewrite objective formula so it is equal to zero

We then need to rewrite $P = 2x + 3y + 4z$ as:

$$P - 2x - 3y - 4z = 0$$

This is called the objective equation

Next we need to remove the inequalities from the constraints

This is done by adding *slack variables*

Adding *slack variables s* and *t* to the inequalities, we can write them as equalities:

$$3x + 2y + z + s = 10$$

$$2x + 5y + 3z + t = 15$$

These are called the constraint equations

Next, these equations are placed in a tableau

P	x	y	z	s	t	**values**

First the Objective equation

$$P \quad - 2x \;\; - 3y \;\; - 4z \qquad\qquad = 0$$

P	***x***	***y***	***z***	***s***	***t***	**values**
1	-2	-3	-4	0	0	0

Next, the constraint equations

$$3x + 2y + z + s = 10$$

$$2x + 5y + 3z + t = 15$$

P	*x*	*y*	*z*	*s*	*t*	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2	5	3	0	1	15

We now have to identify the **pivot**

First, we find the **pivot column**

The pivot column is the column with the greatest negative value in the objective equation

Mark this column with an arrow

P	x	y	z	s	t	values
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2	5	3	0	1	15

The next step in identifying the **pivot** is to find the **pivot row**

This is done by dividing the values of the constraints by the value in the pivot column

P	*x*	*y*	*z*	*s*	*t*	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2	5	3	0	1	15

10 divided by 1 = **10**

15 divided by 3 = **5**

The lowest value gives the **pivot row**

This is marked with an arrow

P	*x*	*y*	*z*	*s*	*t*	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2	5	3	0	1	15

The **pivot** is where the pivot column and pivot row intersect

This is normally marked with a circle

P	*x*	*y*	*z*	*s*	*t*	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2	5	3	0	1	15

We now need to make the pivot equal to zero

To do this divide the pivot row by the pivot value

P	*x*	*y*	*z*	*s*	*t*	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2	5	3	0	1	15

P	*x*	*y*	*z*	*s*	*t*	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2/3	5/3	3/3	0	1/3	15/3

It is best to keep the values as fractions

P	***x***	***y***	***z***	***s***	***t***	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2/3	5/3	1	0	1/3	5

Next we need to make the pivot column values into zeros

To do this we add or subtract multiples of the pivot column

P	*x*	*y*	*z*	*s*	*t*	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2/3	5/3	1	0	1/3	5

The pivot column in the objective function will go to zero if we add four times the pivot row

The pivot column value in the other constraint will go to zero if we subtract the pivot row

P	***x***	***y***	***z***	***s***	***t***	**values**
1	-2	-3	-4	0	0	0
0	3	2	1	1	0	10
0	2/3	5/3	1	0	1/3	5

Subtracting 4 X the pivot row from the objective function gives:

P	*x*	*y*	*z*	*s*	*t*	**values**
1+ 4(0)	-2+ 4(2/3)	-3+ 4(5/3)	-4+ 4(1)	0+ 4(0)	0+ 4(1/3)	0+ 4(5)
0	3	2	1	1	0	10
0	2/3	5/3	1	0	1/3	5

Subtracting 4 X the pivot row from the objective function gives:

P	*x*	*y*	*z*	*s*	*t*	values
1	2/3	11/3	0	0	4/3	20
0	3	2	1	1	0	10
0	2/3	5/3	1	0	1/3	5

Subtracting 1 X the pivot row from the other constraint gives:

P	*x*	*y*	*z*	*s*	*t*	values
1	2/3	11/3	0	0	4/3	20
0	3- 2/3	2- 5/3	1- 1	1- 0	0- 1/3	10- 5
0	2/3	5/3	1	0	1/3	5

Subtracting 1 times the pivot row from the other constraint gives:

P	*x*	*y*	*z*	*s*	*t*	values
1	2/3	11/3	0	0	4/3	20
0	7/3	1/3	0	1	-1/3	5
0	2/3	5/3	1	0	1/3	5

This then is the **first iteration** of the Simplex algorithm

P	*x*	*y*	*z*	*s*	*t*	values
1	2/3	11/3	0	0	4/3	20
0	7/3	1/3	0	1	-1/3	5
0	2/3	5/3	1	0	1/3	5

If any values in the objective equation are negative we have to repeat the whole process

Here, there are no negative values. This is then the **optimal solution**.

P	x	y	z	s	t	**values**
1	2/3	11/3	0	0	4/3	20
0	7/3	1/3	0	1	-1/3	5
0	2/3	5/3	1	0	1/3	5

We now have to determine the values of the variables that give the optimum solution

P	*x*	*y*	*z*	*s*	*t*	values
1	2/3	11/3	0	0	4/3	20
0	7/3	1/3	0	1	-1/3	5
0	2/3	5/3	1	0	1/3	5

Reading the objective function:

$$1P + \frac{2}{3}x + \frac{11}{3}y + \frac{4}{3}t = 20$$

P	*x*	*y*	*z*	*s*	*t*	values
1	2/3	11/3	0	0	4/3	20
0	7/3	1/3	1	1	-1/3	5
0	2/3	5/3	1	0	1/3	5

Reading the objective function:

$$1P + \frac{2}{3}x + \frac{11}{3}y + \frac{4}{3}t = 20$$

The maximum value for P is then 20

For this to be the case *x*, y and *t* must be zero

In general any variable that has a value in the objective function is zero.

From the objective function x, y and t are zero – substituting these values in to the constraint rows gives:

$s = 5$ and $z = 5$

P	x	y	z	s	t	**values**
1	2/3	11/3	0	0	4/3	20
0	7/3	1/3	0	1	-1/3	5
0	2/3	5/3	1	0	1/3	5

The solution to the linear programming problem:

maximise: $P = 2x + 3y + 4z$

subject to:

$3x + 2y + z < 10$

$2x + 5y + 3z < 15$

$x, y > 0$

is:

$$P = 5$$

$$x = 0,\ y = 0 \text{ and } z = 5$$

There are 9 steps in the Simplex algorithm

1. Rewrite objective formula so it is equal to zero
2. Add slack variables to remove inequalities
3. Place in a tableau
4. Look at the negative values to determine pivot column
5. Divide end column by the corresponding pivot value in that column
6. The least value is the pivot row
7. Divide pivotal row by pivot value – so pivot becomes 1
8. Add/subtract multiples of pivot row to other rows to zero
9. When top element are ≥ 0 then optimised solution

Go to this web site to solve any linear programme problem using the simplex method:

For on-line exercises go here: