Personal statement

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Background

Having family members active in mathematics has encouraged me to engage in math since I was quite young. However, once in college I really started to experience mathematics as something I wanted to do for myself on my own terms. I originally enrolled in applied mathematics, but after my first year I was encouraged by my analysis professor to consider switching to a degree in theoretical mathematics. This was an eye-opening transition for me and I rather quickly developed a great interest for algebra and combinatorics. In my last semester I got the chance to go to UC Berkeley as an exchange student and that is when I felt the field really open up for me, with a much wider range of courses and some more advanced level activity in seminars and graduate classes. This also opened up an opportunity for an REU, under the supervision of professor Bernd Sturmfels, which I had at the Max Planck institute in Leipzig this past summer. This was a rewarding experience, and I felt good support and encouragement to continue on a path towards becoming a research mathematician.

In college, my perception was that I have a solid foundation to build on and that I gain momentum quickly in new subjects. I felt this was manifested when I received an award from the *Sigurdur Helgason award fund* last fall, first time this award is given to a female-identifying student. The award is given each year to the top student(s) in the graduating class of the mathematics department. In my current graduate courses at Berkeley I have felt comfortable and confident about my preparation and abilities.

I wrote my senior mathematics seminar paper under the supervision of professor Rögnvaldur Möller. This paper was on the Perron-Frobenius theorem, with applications to internet search engines. In my paper I presented a proof of the Perron-Frobenius theorem using linear algebra. Then I used Jordan decomposition to study convergence of the associated finite Markov chain to the stationary distribution, the Perron-Frobenius eigenvector. In the same semester I took an interesting graduate course on stochastic processes. There was a nice connection to my paper, but the approach to Markov chains was more probabilistic and covered also infinite but discrete state space and discrete and continuous time. We used coupling techniques to prove convergence to a stationary distribution for Markov chains. We also thoroughly explored the concept of irreducibility, which also arises as a matrix condition for the Frobenius theorem. The application to search engines in my paper used Markov chains, with convergence speed controlled by the spectral gap, which has some nice connections to geometry. I also considered other applications of spectral graph theory, one of combinatorial nature using interlacing of eigenvalues to show that the Petersen graph has no Hamiltonian cycles, and another of data science nature about clustering using eigenvectors (but I did not include those in my paper due to space limit). It felt quite satisfactory to explore these perspectives of the problem and see how different approaches to the same question can give it more depth.

Intellectual Merit

I gained valuable insights into abstract mathematical research this summer while at the MPI in Leipzig. As part of a project about classifying nilpotent Lie-algebras, I started with a question about algebraic geometric properties of the variety of 4-dimensional Lie algebras. I was able to use my programming background from previous summers and I explored irreducible components and their degrees using tools from computational and numerical algebraic geometry. This rather quickly gave interesting outcomes, which were different from previously published results, obtained using a different approach. This was very motivating to say the least, and I proceeded to try to understand better the theory lying behind the problem.

We considered the variety of 4-dimensional Lie-algebras over the field of complex numbers, which can be defined using structure coefficient in a given basis, where the structure coefficients satisfy algebraic relations based on anti-symmetry and Jacobi-identity for the Lie-bracket. The objective was to describe prime ideals defining the irreducible components of the variety (which turn out to all have the same dimension in this case), and then derive the degree of each of these.

The degree of a variety can be obtained by finding the intersection of the variety with a set of randomly selected hyperplanes. The number of hyperplanes is chosen to be the co-dimension of the variety, so the intersection should generically be a set of points, whose number equals the degree. The package HomotopyContinuation.jl, in simple terms, finds solution to a system of polynomial equations by tracking solutions to a simpler easily solvable system along a homotopy to the desired system of equations. This way we obtained the total degree 832 compared to previously published number of 1033. Since we actually had the numerical solutions points, each representing a Lie-algebra, we could classify these solutions based on the dimensions of their series of derived algebras, by a numerical linear algebra calculation. This indicated the degree of each irreducible component, which differed in three cases out of four from the published results.

Excited about these findings we proceeded to study ideals defining the variety and identifying their prime ideals using Gröbner bases and other techniques from computational commutative algebra. Using representation theory we expressed the variety and its components in a format that helped make the computational aspects more tractable.

This project has also encouraged me to study deeper representation theory, Gröbner bases and also intersection theory to be able to understand previous approach to the same problem, by Laurent Manivel.

This project ended up in a paper that we have now submitted to the Journal of Lie algebras. I have also been invited to give a talk about it in the Non-linear algebra seminar at Berkeley this fall. It has also led to a new collaboration with a student at the University of Leipzig where we want to study the variety of 5-dimensional nilpotent Lie algebras, leveraging what we have learned from the case of 4-dimensional algebras.

Broader impact

Looking back at my journey into research mathematics, I can very much relate to how important it is to have role models and mentors, get encouragement and feel that you belong to a community. As a girl in mathematics I have faced some challenges in being accepted as a serious mathematician in a male dominated field. I have observed this both from other students and sometimes even from teachers. On one occasion I had a professor who early in the class seemed to think I had googled my solutions from the internet, but once he got to know me better through course interaction and exams, he appeared to acknowledge my talent. Even though I broke through with that particular professor, these experiences sit with you. My desire is that I can as a professor and research mathematician serve as a role model for coming generations, not just girls, but that everyone with the aspirations I have feels they have their fair chance, whatever their background.

During my summers while in college I worked in a start-up company called PayAnalytics, a software company specializing in analyzing demographic pay gaps and providing optimal strategies to address it, based on linear programming. There I both worked on software development as well as advising clients on how to statistically interpret the results. What I have taken from this experience is seeing how I can use mathematics to help minorities in all positions get closer to equity, and I believe making tools available to organizations to assess their possible biases is very important. I also want to dive into the root of the problem since it starts much earlier than at corporate level.

As an undergraduate I took part in outreach activity to support women studying in STEM fields, in particular coding a web platform for an initiative called 'Pave the way' that presents women working in STEM fields. I have also been part of outreach to support social integration for everyone in the challenges of Covid-19, by actively contacting and running support groups for those that enrolled when classes and course activity was remote. Now as a graduate student I am excited to keep on representing women and other minorities in mathematics and I recently joined the Noetherian Ring (NRing), a graduate student organization at Berkeley that consists of mathematicians at Berkeley who identify as gender minorities. I plan to expand my contributions to this cause during my time as a PhD student.

I like to teach and I believe that the way we present and teach mathematics is important, but it has variations that are also important. I want to become as inspirational as a professor as some of my teachers have been, but interestingly they do not all fit the same mold, so there are variations in how to accomplish this. Through my graduate studies and postgraduate work I aim to develop a style of teaching that emphasizes respect for students, love of the subject and enthusiasm for the importance of access to higher education for everyone.

I have also chosen to work within the realm of what has been dubbed *non-linear algebra*, which refers to an area of mathematics that I see as having a growing impact across mathematics, applied mathematics and even beyond. I see an opportunity to make it more mainstream as a subject for a broad set of students seeking (advanced) degrees in mathematical sciences, having a role similar to linear algebra (and its numerical version) already has. This entails bringing together some tools from across e.g. algebraic geometry, commutative algebra, combinatorics, multilinear algebra, representation theory, algebraic topology and computation and make them more accessible to a broader set of students. I am excited about taking part in shaping this field, helping it gain wider acceptance and developing courses based on this concept that fit different programs.