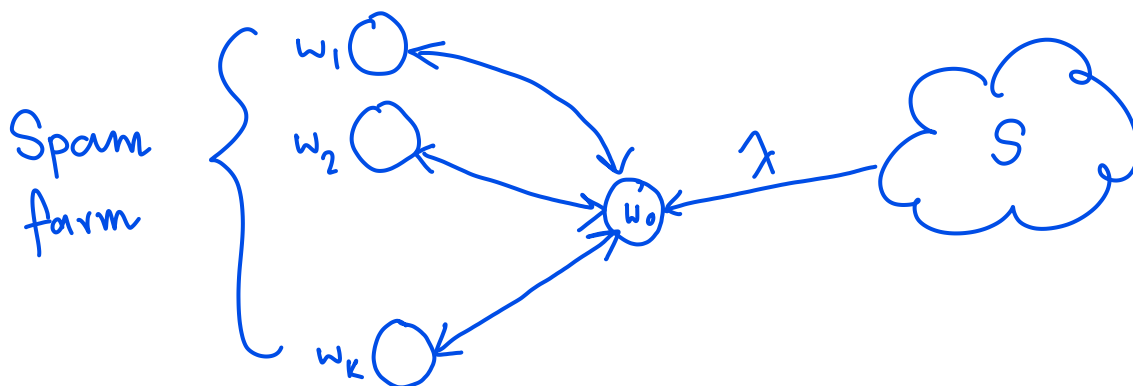


## CS328: Homework 2

The usual agreement with respect to collaboration being okay (mention names of collaborators) but not copying. Other aspects of honor code are also to be followed. All code needs to be written by self. Code to be submitted using colab as usual. Also give names of collaborators.

1. Suppose that you've already computed the personalized PageRank vectors of a set of users (denote the computed vectors  $V$ ). What is the set of all personalized PageRank vectors that you can compute from  $V$  without accessing the web graph?
2. Consider the design of a spam farm as shown below. The farm has pages  $\{w_1, \dots, w_k\}$  and the target page is  $w_0$ . The rest of the web is  $S$ .  $\lambda$  denotes the amount of pagerank that flows in through edges from  $S$  to  $p_0$ , i.e.  $\lambda = \sum_{j \in S} \frac{p_j}{d_j}$ , where  $p_j$  is the pagerank of page  $j$  in  $S$  and  $d_j$  is its outdegree. Let  $N = |S| + k + 1$ , i.e. the number of pages in entire web. Write down pagerank  $p_0$  of the target page in terms of  $\lambda, k, N$  as well as  $\alpha$  (the teleportation parameter). Notice that  $\lambda$  does not include the teleportation, but  $p_0$  should.



3. Suppose you have a turnstile stream of  $n$  distinct items. Suppose frequency distributions follow a power law with exponent 3, i.e. number of items with frequency  $k$  is  $C/k^3$ . What should roughly  $C$  be (order notation)? If you fix  $w$  and  $d$ , does CM or CS give a better guarantee for this distribution.
4. Implement CM sketch, CS sketch and Misra-Gries sketch, all should use the same size. For CM and CS, fix  $d = 5$ . Choose a  $k$  for MG and set  $w = k/d$  for CM and CS. Try all  $k \in \{100, 200, 500, 1000, 2000\}$ .

Compare their performance as follows. The input data should be the entire train.data obtained from <http://qwone.com/~jason/20Newsgroups/>. Use the Matlab file format (actually just a text-file), throwing away the docIdx. That is, the stream is just tuples (wordIdx, count).

For preparing the query data, find out the exact frequencies. Then choose 100 words at random out of the top 1000 most frequent words. These are your query set. Find out the relative error averaged over the 100 queries i.e.

$$\frac{1}{100} \sum_{x \leq 100} |f_x - \hat{f}_x| / f_x,$$

where  $f_x$  is true frequency and  $\hat{f}_x$  is estimated by the sketch. There should be 3 curves, one for each of the sketches. Report the numbers in a table and in a plot.

For each of the sketches, also report the minimum  $w$  that you need for the average error over the 100 queries is less than 1%.

5. Consider the <https://grouplens.org/datasets/movielens/> We will use a low-rank approximation of to estimate the entries that are queried for in . Follow the procedure below.

Train = 80% of set of labeled triplets. Denoted as  $(i, j, T(i, j))$   
 Test = 20% of set of labeled triplets. Denoted as  $(i, j, S(i, j))$

```

Represent Train as a matrix and find a rank-k approximation to Train.
Let this matrix be named Pred.
For every triplet (i, j, S(i,j)) in Test:
    err = err + (S(i, j) - Pred(i, j))^2
Return err

```

Plot  $k$  versus the error for  $k$  in a range (0, 100). Compare this with the following baseline algorithm: every test entry  $(i,j)$  is predicted as  $\alpha * \mu_i + \beta * \eta_j$  where  $\mu_i$  is the average rating of user  $i$ , and  $\eta_j$  that of movie  $j$  over all ratings in ,  $\alpha$  and  $\beta$  are fitted using training data.