



# Mining Trees

---

CS 145  
Fall 2015

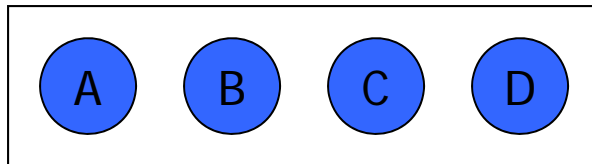
# Mining Complex Patterns

---

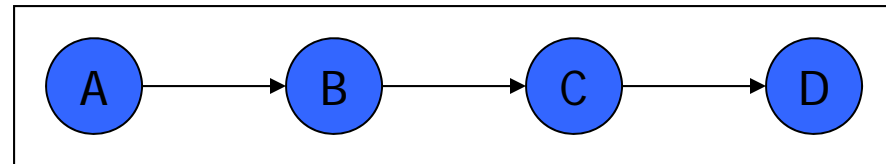
- Common Pattern Mining Tasks:
  - **Itemsets** (transactional, unordered data)
  - **Sequences** (temporal/positional: text, bioseqs)
  - **Tree patterns** (semi-structured/XML data, web mining)
  - **Graph patterns** (protein structure, web data, social network)

# Example Pattern Types

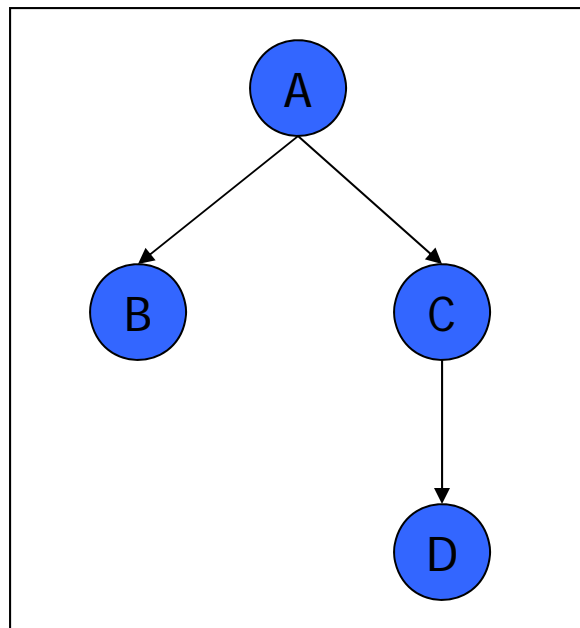
Itemset



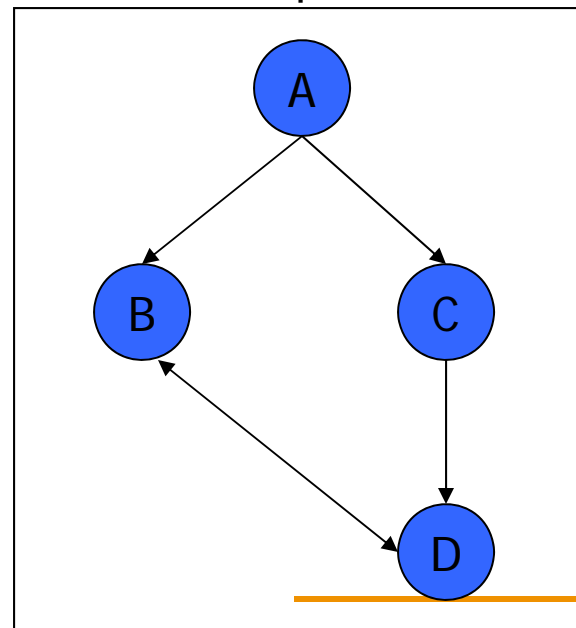
Sequence



Tree



Graph



- Can add attributes
  - To nodes
  - To edges
- **Attributes**
  - Labels
  - Type (directed or undirected)
  - Set-valued

# Induced vs Embedded Sub-trees

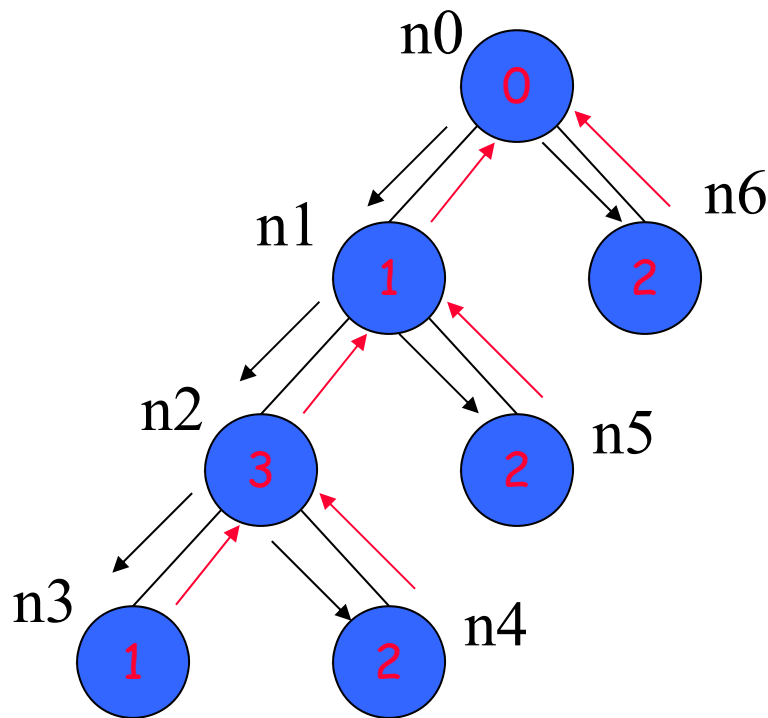
- ▶ **Induced Sub-trees:**  $S = (V_s, E_s)$  is a sub-tree of  $T = (V, E)$  if and only if
  - ▶  $V_s \subseteq V$
  - ▶  $e = (n_x, n_y) \in E_s$  iff  $(n_x, n_y) \in E$  ( $n_x$  directly connected to  $n_y$ )
- ▶ **Embedded Sub-trees:**  $S = (V_s, E_s)$  is a sub-tree of  $T = (V, E)$  if and only if
  - ▶  $V_s \subseteq V$
  - ▶  $e = (n_x, n_y) \in E_s$  iff  $n_x \leq_l n_y$  in  $T$  ( $n_x$  connected to  $n_y$ )
- ▶ An induced sub-tree is a special case of embedded sub-tree.
- ▶ We say  $S$  *occurs* in  $T$  and  $T$  *contains*  $S$  if  $S$  is an embedded sub-tree of  $T$
- ▶ If  $S$  has  $k$  nodes, we call it a  $k$ -sub-tree

# Mining Frequent Trees

---

- ▶ Support: the *support* of a subtree in a database of trees, is the number of trees containing the subtree.
- ▶ A subtree is frequent if its support is at least the minimum support.
- ▶ TreeMiner: Given a database of trees (a forest) and a minimum support, find all frequent subtrees.

# String Representation of Trees



0 1 3 1 -1 2 -1 -1 2 -1 -1 2 -1

With  $N$  nodes,  $M$  branches,  $F$  max fanout

Adjacency Matrix requires:  $N(F+1)$  space

Adjacency List requires:  $4N-2$  space

Tree requires (node, child, sibling):  $3N$  space

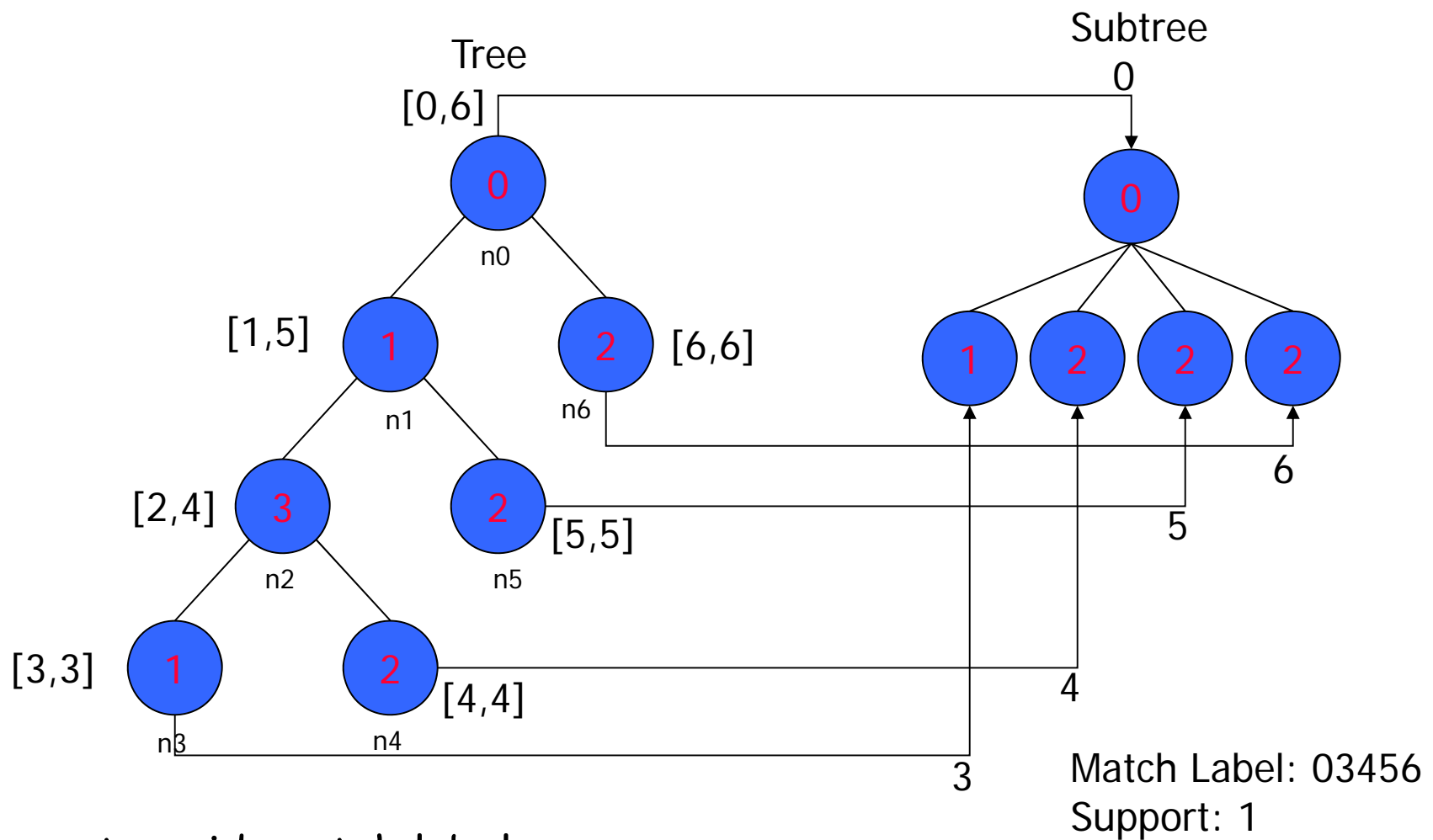
**String representation requires:  $2N-1$  space**

# Tree: String Representation

---

- ▶ Like an itemset
- ▶ -1 as the backtrack item
- ▶ Assuming only labels on nodes
- ▶ For trees labels on edges can be treated as labels on nodes:  
$$\text{edge-label} + \text{node-label} = \text{new label!}$$

# Match labels

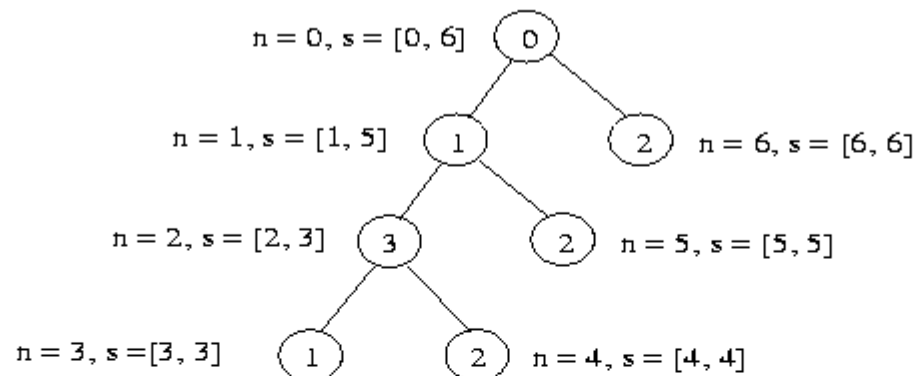


vector < id, match label, scope >



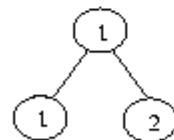
# An example

**T (a tree in D)**



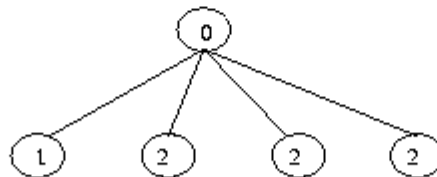
**T's String Encoding:** 0 1 3 1 -1 2 -1 -1 2 -1 -1 2 -1

**S1**



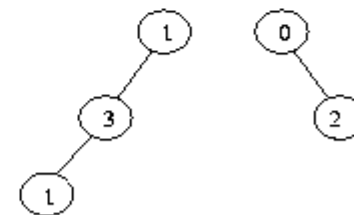
support = 1  
 weighted support = 2  
 string = 1 1 -1 2 -1

**S2**



support = 1  
 weighted support = 1  
 string = 0 1 -1 2 -1 2 -1 2 -1

**S3**



(not a subtree)

# Generic Mining Algorithms

---

- ▶ Horizontal pattern matching based
- ▶ Vertical intersection based
- ▶ BFS or DFS

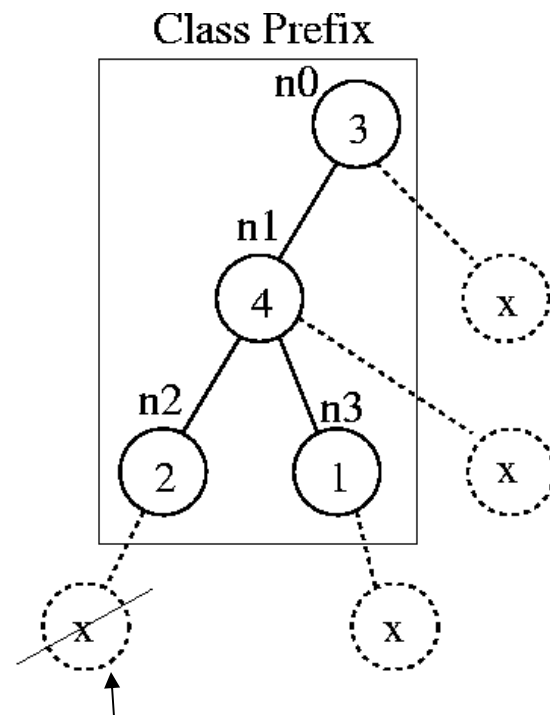
# Candidate Generation & Support Counting

---

- ▶ Candidate Generation
  - ▶ Extend by a node or an edge
  - ▶ Avoid duplicates as far as possible

# Trees: Systematic Candidate Generation

Two subtrees are in the same class iff they share a common prefix string  $P$  up to the  $(k-1)$ th node



## Equivalence Class

Prefix String: 3 4 2 -1 1

Element List: (label, attached to position)

(x, 0) // attached to n0: 3 4 2 -1 1 -1 -1 x -1

(x, 1) // attached to n1: 3 4 2 -1 1 -1 x -1 -1

(x, 3) // attached to n3: 3 4 2 -1 1 x -1 -1 -1

A valid element  $x$  attached to only the nodes lying on the path from root to **rightmost leaf** in prefix  $P$

# Candidate generation

---

- ▶ Given an equivalence class of  $k$ -subtrees, how do we generate candidate  $(k+1)$ -subtrees?
- ▶ Main idea: consider each ordered pair of elements in the class for extension, including self extension
  - ▶ Sort elements by node label and position

# Class extension

Let  $P$  be a prefix class with encoding  $P$ , and let  $(x, i)$  and  $(y, j)$  denote any two elements in the class. Let  $Px$  denote the class representing extensions of element  $(x, i)$ . Define a join operator  $\otimes$  on the two elements, denoted  $(x, i) \otimes (y, j)$ , as follows:

**case I** –  $(i = j)$ :

1. If  $P \neq \emptyset$ , add  $(y, j)$  and  $(y, \textit{ni})$  to class  $[Px]$ , where *ni* is the depth-first number for node  $(x, i)$  in tree  $Px$ .
2. If  $P = \emptyset$ , add  $(y, j + 1)$  to  $[Px]$ .

**case II** –  $(i > j)$ : add  $(y, j)$  to class  $[Px]$ .

**case III** –  $(i < j)$ : no new candidate is possible in this case.