Assignment 4

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Part 1 (a)

Figure 1.1 Time series plot of growth of GNP

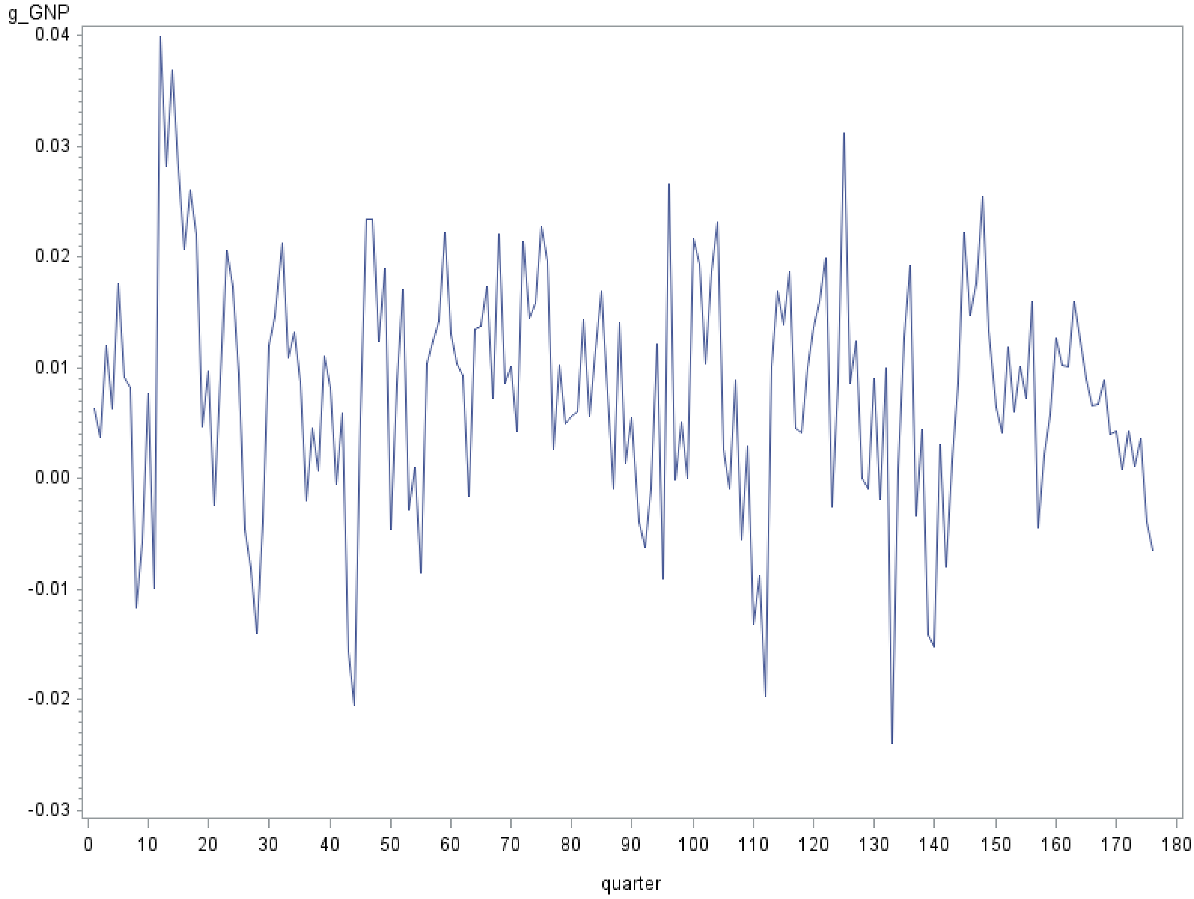

Table 1.2. Output of ARIMA

(a). Summary output

Name of Variable = g_GNP	
Mean of Working Series	0.007741
Standard Deviation	0.010697
Number of Observations	176

(b). Autocorrelation

Lag	Covariance	Correlation	-1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1	Std Error
0	0.00011443	1.00000	| |********************|	0
1	0.00004312	0.37687	| . |******** |	0.075378
2	0.00002905	0.25391	| . |***** |	0.085416

<table>
<tr><th>Lag</th><th>Covariance</th><th>Correlation</th><th>-1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1</th><th>Std Error</th></tr>
<tr><td>3</td><td>1.4332E-6</td><td>0.01253</td><td>| . | . |</td><td>0.089602</td></tr>
<tr><td>4</td><td>-9.8318E-6</td><td>-.08592</td><td>| . **| . |</td><td>0.089611</td></tr>
<tr><td>5</td><td>-0.0000123</td><td>-.10706</td><td>| . **| . |</td><td>0.090078</td></tr>
<tr><td>6</td><td>-6.5788E-6</td><td>-.05749</td><td>| . *| . |</td><td>0.090798</td></tr>
<tr><td>7</td><td>-2.0849E-6</td><td>-.01822</td><td>| . | . |</td><td>0.091005</td></tr>
<tr><td>8</td><td>-8.8383E-6</td><td>-.07724</td><td>| . **| . |</td><td>0.091026</td></tr>
<tr><td>9</td><td>-8.0315E-6</td><td>-.07019</td><td>| . *| . |</td><td>0.091397</td></tr>
<tr><td>10</td><td>1.19116E-6</td><td>0.01041</td><td>| . | . |</td><td>0.091703</td></tr>
<tr><td>11</td><td>-2.6307E-6</td><td>-.02299</td><td>| . | . |</td><td>0.091710</td></tr>
<tr><td>12</td><td>-0.0000111</td><td>-.09673</td><td>| . **| . |</td><td>0.091743</td></tr>
<tr><td>13</td><td>-0.0000116</td><td>-.10106</td><td>| . **| . |</td><td>0.092320</td></tr>
<tr><td>14</td><td>-0.0000131</td><td>-.11454</td><td>| . **| . |</td><td>0.092947</td></tr>
<tr><td>15</td><td>-8.5437E-6</td><td>-.07467</td><td>| . *| . |</td><td>0.093745</td></tr>
<tr><td>16</td><td>3.3379E-6</td><td>0.02917</td><td>| . |* . |</td><td>0.094083</td></tr>
<tr><td>17</td><td>7.06801E-6</td><td>0.06177</td><td>| . |* . |</td><td>0.094134</td></tr>
<tr><td>18</td><td>0.00001049</td><td>0.09163</td><td>| . |** . |</td><td>0.094364</td></tr>
<tr><td>19</td><td>3.93535E-6</td><td>0.03439</td><td>| . |* . |</td><td>0.094868</td></tr>
<tr><td>20</td><td>2.81195E-6</td><td>0.02457</td><td>| . | . |</td><td>0.094939</td></tr>
<tr><td>21</td><td>-4.4305E-6</td><td>-.03872</td><td>| . *| . |</td><td>0.094975</td></tr>
<tr><td>22</td><td>-2.4663E-6</td><td>-.02155</td><td>| . | . |</td><td>0.095065</td></tr>
<tr><td>23</td><td>-0.0000101</td><td>-.08830</td><td>| . **| . |</td><td>0.095093</td></tr>
<tr><td>24</td><td>-8.3708E-6</td><td>-.07315</td><td>| . *| . |</td><td>0.095557</td></tr>
</table>

The time series (Figure 1.1) is behaving in a stochastic pattern, and drafting both upward and downward with no sign of exhibiting any trend, which is expected if it is a stationary time series. According to the autocorrelations table (Table 2.1(b)), only the first three autocorrelations (lag 0, lag1, and lag 2) are significant, but the value of autocorrelation from lag 1 to lag 24 decays more quickly than the benchmark where Φ_1=0.9. Therefore, we could conclude that the series looks stationary.

Part 1 (b):

Table 1.2. Output of ARIMA

(a). Autocorrelations

<table>
<tr><th>Lag</th><th>Covariance</th><th>Correlation</th><th>-1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1</th><th>Std Error</th></tr>
<tr><td>0</td><td>0.00011443</td><td>1.00000</td><td>| |********************|</td><td>0</td></tr>
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</table>

(b). Partial autocorrelation

<table>
<tr><th>Lag</th><th>Correlation</th><th>-1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1</th></tr>
<tr><td>1</td><td>0.37687</td><td>| . |******** |</td></tr>
<tr><td>2</td><td>0.13040</td><td>| . |*** |</td></tr>
<tr><td>3</td><td>-0.14209</td><td>| ***| . |</td></tr>
<tr><td>4</td><td>-0.09880</td><td>| .**| . |</td></tr>
<tr><td>5</td><td>-0.01995</td><td>| . | . |</td></tr>
<tr><td>6</td><td>0.03253</td><td>| . |* . |</td></tr>
</table>

According to the Table 2.1(b), partial autocorrelations of Lag 2 and Lag 3 seems significant; however, from Table 2.1(a), autocorrelation of Lag 2 is significant, and the autocorrelation of Lag 3 is not significant. Therefore, we were in need to conduct an autoregression process to the second order urgently to determine the significance.

Table 2.2 Output of AR(2)

<table>
<tr><th colspan="6">Conditional Least Squares Estimation AR (2)</th></tr>
<tr><td>Parameter</td><td>Estimate</td><td>Standard Error</td><td>t Value</td><td>Approx Pr > |t|</td><td>Lag</td></tr>
<tr><td>MU</td><td>0.0076333</td><td>0.0013830</td><td>5.52</td><td><.0001</td><td>0</td></tr>
<tr><td>AR1,1</td><td>0.33072</td><td>0.07556</td><td>4.38</td><td><.0001</td><td>1</td></tr>
<tr><td>AR1,2</td><td>0.13440</td><td>0.07583</td><td>1.77</td><td>0.0781</td><td>2</td></tr>
</table>

Estimated AR (2) process:

$$Y_t = 0.1344Y_{t-2} + 0.33072Y_{t-1} + 0.0076333$$

By observing the conditional least squares estimation AR(2), we obtained the P-value of the first lag term (AR1,1) to be < 0.05, and the second lag term (AR1,2) to be 0.0781 > 0.05. As a result, we concluded that the coefficient at the lag of 2 (AR 1,2) is not statistically significant, while coefficient of AR1,1 is significant. However, second-order autoregression model cannot be reduced to a first-order one, and we must conduct a first-order autoregressive process to generate the appropriate model here.

Table 2.3 Output of AR(1)

Conditional Least Squares Estimation AR(1)					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	0.0076840	0.0012065	6.37	<.0001	0
AR1,1	0.38074	0.07052	5.40	<.0001	1

Estimated AR(1) process:

$$Y_t = 0.38074Y_{t-1} + 0.0076840$$

According to Table 2.3, the P-value of the first lag term (AR1,1) is < 0.0001, and thus < 0.05, and we could conclude that the coefficient AR1,1 is statistically significant. As a result, the first-order autoregression model, $Y_t = 0.38074Y_{t-1} + 0.0076840$, turned out to be the proper model for our purpose here.

Part 1 C:

Table 3.1 Autocorrelation check of residuals

To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	8.36	5	**0.1372**	-0.051	0.170	-0.057	-0.071	-0.079	-0.019
12	11.76	11	**0.3822**	0.036	-0.063	-0.064	0.056	0.008	-0.074
18	15.79	17	**0.5389**	-0.042	-0.077	-0.062	0.044	0.027	0.080
24	18.55	23	**0.7268**	-0.005	0.040	-0.056	0.028	-0.077	-0.046

Table 3.1 is the autocorrelation table of residuals where χ^2-testing results are shown. For χ^2-testing, there are null hypothesis H_0: residual series $\{\varepsilon_t\}$ is unautocorrelated to the indicated lag, and alternative hypothesis H_a: $\{\varepsilon_t\}$ is autocorrelated to at least one lag smaller than the indicated lag. With all P-values greater than 0.05, we retained all of null hypotheses, and must conclude statistically that all autocorrelations up to a lag of 24 are 0. Since autocorrelations die out with lag increasing, all autocorrelation can then be seen as equal to 0. Because the residual term is zero-mean and homoscedastic, χ^2-testing in our case essentially tests for white noise. Therefore, the residual term is a white noise series on a statistical basis.

Part 2:

(a) Estimate the constant term C, coefficient ϕ_1 and write down the estimated AR(1) model.

Solution:

At lag=1, $\phi_1 = r_1 = 0.709$

For estimation of unconditional mean of Y_t: $\mu = \frac{C}{1-\phi_1} = 40.268$

Solve for constant: $C = \mu \times (1 - \phi_1) = 40.268 \times (1 - 0.709) = 11.718$

Model AR(1): $Y_t = 0.709Y_{t-1} + 11.718$

(b) Estimate the autocorrelations at lag 2 and 3, and the partial autocorrelation of lag 1.

Solution:

For AR(1) process: $r_k = \phi_1 r_{k-1} = \phi_1^k$

Therefore,

$r_2 = {\phi_1}^2 = 0.709^2 = 0.50268$

$r_3 = {\phi_1}^3 = 0.709^3 = 0.35640$

Also, for AR(1) process: $\text{PACF}(1) = \phi_1 = 0.709$

That is, the autocorrelations at Lag 2 and Lag 3 are 0.50268 and 0.3564 respectively. And, according to the statistically induction, we know that the partial autocorrelation at lag 1 equals 0.709.

(c) Estimate the variance of the ϵ_t's

Solution:

For estimation of unconditional variance of Y_t: $\gamma_0 = \frac{{\sigma_\varepsilon}^2}{1-{\phi_1}^2}$

$\therefore\ {\sigma_\varepsilon}^2 = \gamma_0 \times \left(1 - {\phi_1}^2\right) = 12.867 \times (1 - 0.709^2) = 6.399$