Assignment 6

Presented by: Qi (Daniel) ZHENG G44426724, Sizhe WU G26890251, Ruibo WANG G24256043, Yingyu LIN G48092483, Xinyue ZHAO G27620695

Part 1

The REG Procedure
Model: MODEL1
Dependent Variable: NIKKEIDIFF

Number of Observations Read	132
Number of Observations Used	131
Number of Observations with Missing Values	1

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	9078283	4539142	3.52	0.0324
Error	128	164923103	1288462		
Corrected Total	130	174001387			

Root MSE	1135.10429	**R-Square**	0.0522
Dependent Mean	-97.33221	**Adj R-Sq**	0.0374
Coeff Var	-1166.21645		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	**1**	2409.73805	951.70013	2.53	0.0125
TIME	**1**	-7.57578	3.66169	-2.07	0.0406
NIKKEILAG	**1**	-0.11003	0.04176	-2.63	0.0095

Dickey-Fuller Unit Root Tests

Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	F	Pr > F
Zero Mean	**0**	-0.8890	0.4911	-1.24	0.1958		
Single Mean	**0**	-6.5157	0.3013	-1.64	0.4584	1.82	0.6075
Trend	**0**	-14.4145	0.1893	-2.63	0.2657	3.52	0.4734

Dickey-Fuller τ-test for stationarity

H_0: δ = 0, the series has unit root.

H_a: δ < 0, the series does not have unit root

Firstly, it is crucial to mention that we cannot use ANOVA t-test (P = 0.0095 < 0.05) to test for unit root, only because $\hat{\rho}$ no longer follows t-distribution. Instead, by observing the Dickey-Fuller table, we obtain that the P-value of the τ-statistics is 0.2657, which is greater than 0.05; therefore, we retain the null hypothesis at 95% confidence level, and conclude that the original series has unit root on a statistical basis. Consequently, the Nikkei series is not stationary.

Dickey-Fuller F-test for trend stationarity

H0: δ =0; β = 0, the series is not trend stationary.

H1: δ < 0 or β ≠ 0, the series is trend stationary.

According to the Dickey-Fuller unit root test table, we obtain the P-value of the F is 0.4734, which is greater than 0.05; therefore, we retain the null hypothesis at 95% confidence level. This means the original series is neither stationary nor trend stationary, instead, the series is a random walk. In this case, Nikkei would be difference stationary and not trend stationary; the necessary order of differencing to make original Nikkei series stationary would be one.

Part 2

(1)

(a)

Original model: $\mathrm{Y_t} = 10 + \mathrm{\varepsilon_t} - 0.65 \cdot \varepsilon_{t-1} - 0.24 \cdot \varepsilon_{t-2}$ (1.1)

To calculate $\widehat{\mathrm{Y}}_{\mathrm{n}}(1)$, substitute $\mathrm{t = n + 1}$, thus getting:

$\widehat{\mathrm{Y}}_{\mathrm{n}}(1) = \mathrm{E(T_{n+1}|H_n)} = 10 + \hat{\varepsilon}_{\mathrm{n+1}} - 0.65 \cdot \varepsilon_{\mathrm{n}} - 0.24 \cdot \varepsilon_{\mathrm{n-1}}$ (1.2) where the estimated future error $\hat{\varepsilon}_{n+1} = 0$, and historical errors are obtained by calculating their deviation from their most recent estimates, i.e.,

$$\begin{aligned}\hat{Y}_n(1) &= 10 - 0.65 \cdot \varepsilon_n - 0.24 \cdot \varepsilon_{n-1} \\ &= 10 - 0.65\left(Y_n - \hat{Y}_{n-1}(1)\right) - 0.24\left(Y_{n-1} - \hat{Y}_{n-2}(1)\right)\end{aligned} \tag{1.3}$$

Similarly,

$$\begin{aligned}\hat{Y}_n(2) &= 10 - 0.24 \cdot \varepsilon_n \\ &= 10 - 0.24\left(Y_n - \hat{Y}_{n-1}(1)\right)\end{aligned} \tag{1.4}$$

$$\begin{aligned}\hat{Y}_n(3) &= 10 + \hat{\varepsilon}_{n+3} - 0.65 \cdot \hat{\varepsilon}_{n+2} - 0.24 \cdot \hat{\varepsilon}_{n+1} \\ &= 10\end{aligned} \tag{1.5}$$

(b) .

Substitute n = 100 into (1.3), thus getting

$$\begin{aligned}\hat{Y}_{100}(1) &= 10 - 0.65\left(Y_{100} - \hat{Y}_{99}(1)\right) - 0.24\left(Y_{99} - \hat{Y}_{98}(1)\right) \\ &= 10 - 0.65(11 - 10) - 0.24(13 - 11) \\ &= 8.87\end{aligned}$$

Substitute n = 100 into (1.4), thus getting

$$\begin{aligned}\hat{Y}_{100}(2) &= 10 - 0.24\left(Y_{100} - \hat{Y}_{99}(1)\right) \\ &= 10 - 0.24(11 - 10) \\ &= 9.76\end{aligned}$$

Substitute n = 100 into (1.5), thus getting

$$\hat{Y}_{100}(3) = 10$$

Substitute n = 101 into (1.3), thus getting

$$\begin{aligned}\hat{Y}_{101}(1) &= 10 - 0.65\left(Y_{101} - \hat{Y}_{100}(1)\right) - 0.24\left(Y_{100} - \hat{Y}_{99}(1)\right) \\ &= 10 - 0.65 \times (12 - 8.87) - 0.24 \times (11 - 10) \\ &= 7.7225\end{aligned}$$

(2)

Original model:

$$\begin{cases} Y_t = 750 - 100X_t + \varepsilon_t \\ \varepsilon_t = 0.575\varepsilon_{t-1} + a_t \end{cases}$$

Data input gives

$$Y_{49} = 250, X_{49} = 0.55, X_{50} = 0.45, X_{51} = 0.3$$

Historical error is the deviation between actual data and its estimate, therefore,

$$\begin{aligned} \varepsilon_{49} &= Y_{49} - 750 + 100\,X_{49} \\ &= 250 - 750 + 100 \times 0.55 \\ &= -445 \end{aligned}$$

Forecast $\hat{Y}_{50}$:

$$\begin{aligned} \hat{Y}_{50} &= 750 - 100\,X_{50} + 0.575\varepsilon_{49} + \hat{a}_{50} \\ &= 750 - 100 \ \times 0.45 + 0.575 \times (-445) + 0 \\ &= 449.125 \end{aligned}$$

where

$$\begin{aligned} \hat{\varepsilon}_{50} &= 0.575\varepsilon_{49} + \hat{a}_{50} \\ &= 0.575 \times (-445) + 0 \\ &= -255.875 \end{aligned}$$

Therefore, forecast $\hat{Y}_{51}$:

$$\begin{aligned} \hat{Y}_{51} &= 750 - 100\,X_{51} + 0.575\hat{\varepsilon}_{50} + \hat{a}_{51} \\ &= 750 - 100 \ \times 0.3 + 0.575 \times (-255.875) + 0 \\ &= 572.872 \end{aligned}$$