SECTION A

Q1.

i) Define the term Operations Research

Operations Research (OR) is a scientific approach used for decision-making and problem-solving in complex systems. It utilizes mathematical models, statistical analysis, and optimization techniques to improve decision-making in areas such as logistics, production, and resource allocation.

ii) Origin of Operations Research

Operations Research emerged during World War II in the late 1930s. It was first applied in the UK, particularly in military operations to optimize the use of limited resources like aircraft, radar, and weapons. This interdisciplinary field involved mathematicians, engineers, and scientists collaborating to solve logistical problems in warfare. After the war, the methods and principles of OR were adopted in industrial, business, and governmental decision-making processes.

iii) Methodology of Operations Research

The Operations Research methodology generally follows these steps:

1. **Problem Definition**: Identify and clearly define the problem, collecting data to understand the variables involved.
2. **Model Formulation**: Construct a mathematical model to represent the system and its constraints, often in the form of equations or inequalities.
3. **Solution Derivation**: Solve the mathematical model using optimization or simulation methods to obtain a solution.
4. **Model Validation**: Test the solution and model against real-world scenarios to ensure its accuracy.
5. **Implementation**: Apply the solution in practice, monitor the results, and make adjustments if necessary.

iv) Definitions of Terms in Operations Research

1. **Model**: A simplified representation of a system or process used to analyze problems and test solutions.
2. **Objective Function**: The function that needs to be maximized or minimized in a problem, such as profit maximization or cost minimization.
3. **Constraints**: The limitations or restrictions on decision variables, such as resource availability.

4. **Model Formulation**: The process of developing a mathematical representation of the problem, incorporating the objective function and constraints.
5. **Feasible Solution**: A solution that satisfies all the constraints of the problem without violating any restrictions.
6. **Transportation Problem**: A type of problem focused on determining the optimal way to transport goods from sources to destinations at minimum cost.
7. **Allocation Problems**: These involve distributing limited resources among competing activities in a way that optimizes a desired objective.
8. **Non-Negative Conditions**: A condition where decision variables must be greater than or equal to zero, ensuring the problem's variables make sense in a real-world context.

v) Operations Research Techniques

The most common techniques used in Operations Research include:

1. **Linear Programming (LP)**: A method used to achieve the best outcome (such as maximizing profit or minimizing cost) in a mathematical model whose constraints and objective function are linear.
2. **Game Theory**: Used to make decisions in situations where multiple players with competing interests are involved.
3. **Queuing Theory**: Helps in optimizing the performance of systems where there is a need to manage waiting lines, such as customer service or manufacturing systems.
4. **Inventory Control Models**: Used to minimize the cost of holding and ordering inventory while meeting demand.
5. **Network Analysis (PERT/CPM)**: Project management techniques used to optimize scheduling by analyzing the critical path and project completion time.
6. **Simulation**: A method of testing different decision scenarios by creating a digital or physical representation of a system.

Q2:

i) Discuss the significance of Operations Research

Operations Research (OR) plays a vital role in optimizing decision-making by:

1. **Improving Decision Quality**: It provides a structured and quantitative basis for making decisions, reducing the reliance on intuition or guesswork.

2. **Resource Optimization:** OR techniques help in efficient utilization of limited resources such as labor, materials, time, and capital, thereby maximizing productivity or minimizing costs.
3. **Enhanced Productivity**: By optimizing production schedules, inventory control, and resource allocation, OR increases productivity in manufacturing, logistics, and services.
4. **Risk Management**: It enables better forecasting and planning, which helps in mitigating risks in uncertain environments like finance, logistics, and operations.
5. **Solving Complex Problems**: OR is especially useful for handling complex systems with multiple variables and constraints, offering insights that might not be immediately apparent.

ii) Identify the limitations of Operations Research

While OR is valuable, it has certain limitations:

1. **Complexity**: Developing mathematical models can be complex and time-consuming, requiring specialized knowledge in mathematics, statistics, and programming.
2. **Data Dependency:** OR models rely heavily on accurate and comprehensive data. If data is incomplete or incorrect, the results of the model will be unreliable.
3. **Quantitative Bias**: OR focuses on quantifiable factors and may ignore qualitative aspects like human emotions, motivation, or ethics, which are difficult to model mathematically.
4. **High Costs**: The implementation of OR solutions, especially those that involve sophisticated software or hardware, can be expensive.
5. **Resistance to Change**: Human factors, such as resistance to change by employees or managers, can hinder the successful implementation of OR models.

iii) Outline and briefly explain the five principal phases of Operations Research

The five key phases of Operations Research are:

1. **Problem Definition**: Clearly defining the problem, objectives, and constraints. This involves identifying key factors that affect the decision-making process.
2. **Data Collection**: Gathering relevant data about the problem, including both quantitative (numbers, statistics) and qualitative (judgment, expert opinion) data.
3. **Model Formulation**: Constructing a mathematical model that represents the problem and incorporates the objective function and constraints.

4. **Solution Derivation**: Solving the model using techniques like linear programming, simulation, or optimization to find the best solution.
5. **Implementation and Monitoring**: Applying the solution in real-life scenarios and monitoring its effectiveness. Adjustments are made if necessary based on performance.

iv) Define the term linear programming and outline the five steps followed when formulating a linear programming model mathematically

- **Linear Programming (LP)** is a mathematical technique used to optimize a linear objective function, subject to a set of linear constraints. LP is commonly used to maximize profit or minimize cost in resource allocation problems.

Steps in formulating an LP model:

  - **Define Decision Variables**: Identify the variables that will represent the decisions to be made.
  - **Formulate the Objective Function**: Develop an equation that reflects the goal of the problem (e.g., maximizing profit or minimizing cost).
  - **Formulate the Constraints**: List the restrictions or limitations on the decision variables (e.g., resource availability, time).
  - **Non-Negativity Conditions**: Ensure that the decision variables are non-negative, meaning they cannot take negative values.
  - **Solve the Model**: Use graphical methods or optimization algorithms (such as the Simplex method) to solve the LP problem.

v) List the basic properties of linear programming models

The basic properties of linear programming models include:

1. **Linearity**: Both the objective function and constraints must be linear (i.e., no variables are raised to a power or multiplied by each other).
2. **Decision Variables**: The model must include variables that represent the quantities to be determined.
3. **Objective Function**: The goal of the model is either to maximize or minimize a linear objective function.
4. **Constraints:** The model includes a set of linear equations or inequalities that represent the restrictions or limitations on the decision variables.
5. **Non-Negativity**: The values of the decision variables must be non-negative (i.e., they cannot be less than zero).

Q3 i) Let X and Y represent Wheat and rye respectively

Maximize $Z = 500x + 300y$

Subjected to: Constraints:

	X	Y
Cost	\$200	\$100
Time	1	2

$x + y \geq 7$ ---(i)

$200(x) + 100(y) \leq 1200$

$\frac{200x}{100} + \frac{100y}{100} \leq \frac{1200}{100}$

$2x + y \leq 12$ ---(ii)

$X \geq 0, y \geq 0$

$x + 2y \leq 12$ -----(iii)

<u>solution</u>

$x + y = 7$ $\quad x + 2y = 12$

$2x + y = 12$

$X \geq 0, y \geq 0$

Y-Intercept

When $X = 0$, $y = ?$

$x + y = 7$

$0 + y = 7$

$y = 7$

When $y = 0$, $X = ?$

$x + y = 7$

$x + 0 = 7$

$x = 7$

X	0	7
Y	7	0

$2x + y = 12$

When $X = 0$, $y = ?$

$2(0) + y = 12$

$y = 12$

When $y = 0$, $X = ?$

$2x + y = 12$

$2x + 0 = 12$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$

X	0	6
Y	12	0

y-intercept

When $x=0$, $y=?$

$x+2y=12$

$0+2y=12$

$\frac{2y}{2}=\frac{12}{2}$

$y=6$

X	0	12
Y	6	0

x-intercept

When $y=0$, $x=?$

$x+2y=12$

$x+2(0)=12$

$x+0=12$

$x=12$

Maximize : $Z=500x+300y$.

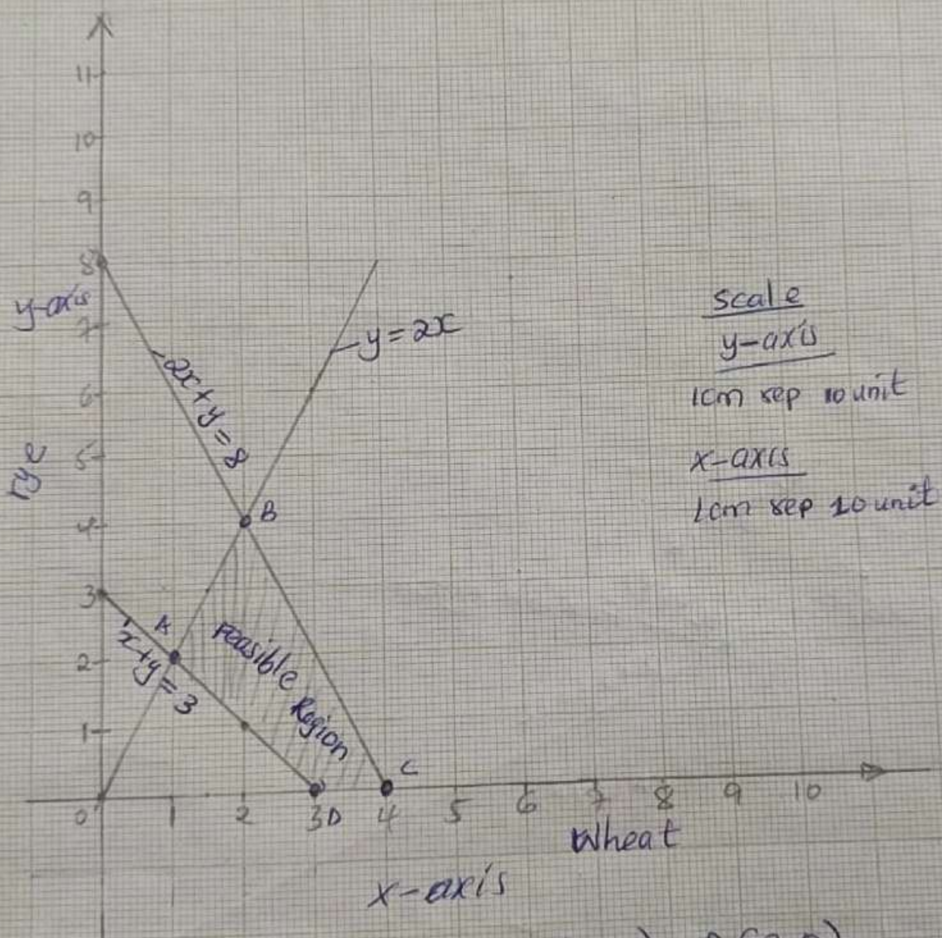
A: $(5\times500)+(2\times300) = 2500+600 \Rightarrow 3100$

B: $(4\times500)+(4\times300) = 2000+1200 \Rightarrow 3200$

C: $(2\times500)+(5\times300) = 1000+1500 \Rightarrow 2500$

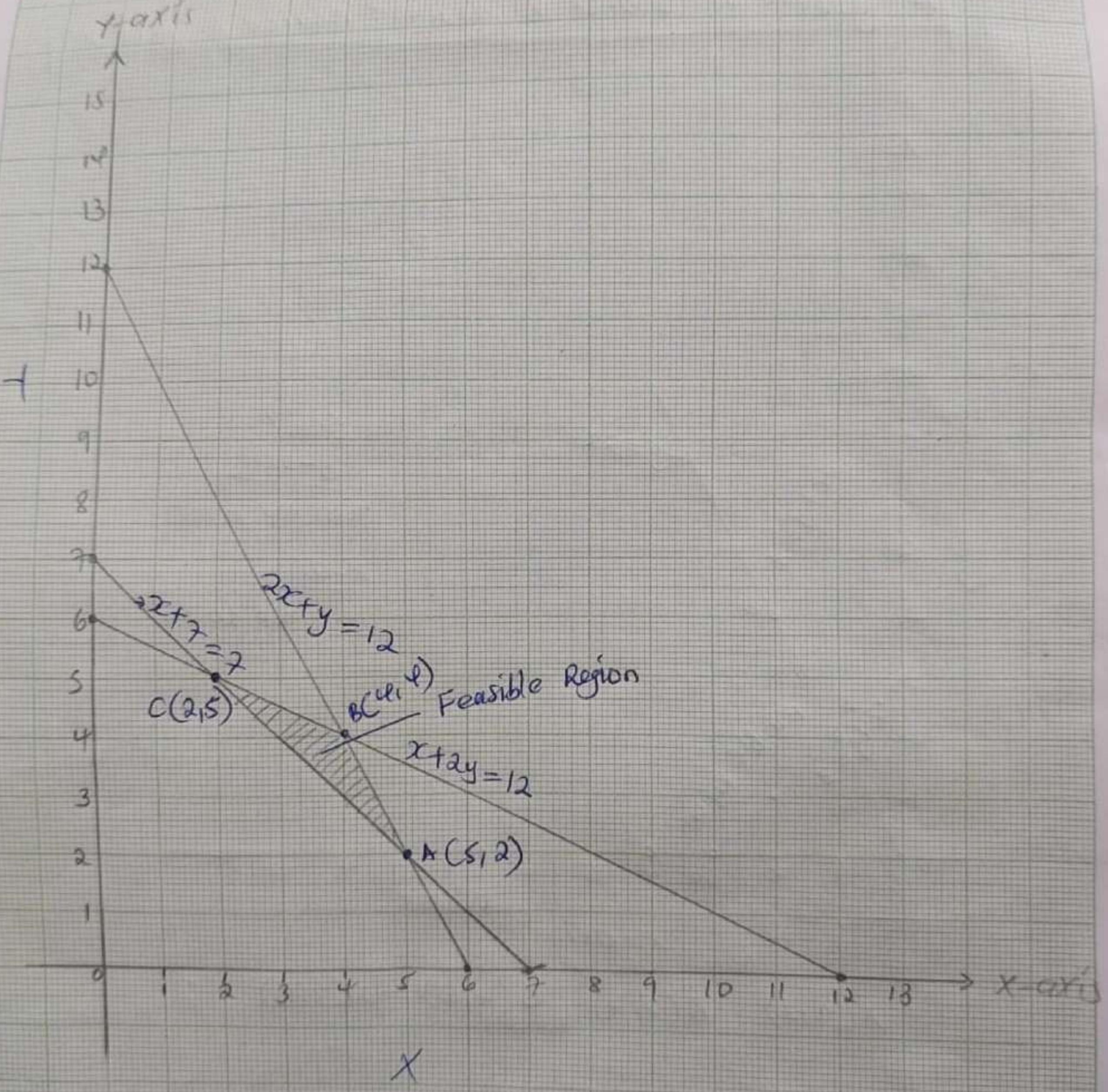
To maximize profit, wheat should be 4 acres and rye should be 4 acres.

A graph of source B against source A gold A



A(1,2), B(2,4), C(4,0), D(3,0)

A Graph of rye against Wheat



(ii) Let X and Y represent source A and Source B of gold ore respectively.

Maximize $Z = 2X + 3Y$
subjected to
Constraints
$X + y \geq 3$ --- (i)
$20(x) + 10(y) \leq 80$
$\frac{20x}{10} + \frac{10y}{10} \leq \frac{80}{10}$
$2x + y \leq 8$ --- (ii)
$Y \leq 2X$ --- (iii)

soln.
X-intercept
When $y = 0$, $x = ?$
$x + y = 3$
$x + 0 = 3$
$X = 3$

$x + y = 3$
$2x + y = 8$
$Y = 2X$

X	3	0
Y	0	3

~~When~~ Y-intercepts
When $x = 0$, $y = ??$
$x + y = 3$
$0 + y = 3$
$y = 3$

(ii) $2x + y = 8$
X-intercept
When $y = 0$, $x = ?$
$2x + y = 8$
$2x + 0 = 8$
$\frac{2x}{2} = \frac{8}{2}$
$x = 4$

Y-intercept,
When $x = 0$, $y = ??$
$2x + y = 8$
$2(0) + y = 8$
$0 + y = 8$
$y = 8$

X	4	0
Y	0	8

When $x = 1$, $y = 2$,

X	0	1	2	3	4
Y	0	2	4	6	8

Cost Maximization

A : $(1\times2) + (3\times2) = 4+6 = 10$

B : $(2\times2) + (4\times3) \Rightarrow 4+12 = 16$

C : $(4\times2) + (0\times3) = 8+0 = 8$

D : $(3\times2) + (0\times3) = 6+0 = 6$

To maximize the amount of gold, 2 tons from Source A and 4 tons from Source B should be extracted.

(iii)

Let:

- x_1 be the number of copier shipped from Novato to San Francisco
- x_2 be the number of copier shipped from Novato to Sacramento
- x_3 be the number of copier shipped from Lodi to San Francisco
- x_4 be the number of copier shipped from Lodi to Sacramento

Supply Constraints

- From Novato: $x_1 + x_2 \leq 700$
- From Lodi: $x_3 + x_4 \leq 800$

Demand Constraints

- For San Francisco: $x_1 + x_3 = 600$
- For Sacramento: $x_2 + x_4 = 400$

Objective function $C = 5x_1 + 10x_2 + 15x_3 + 4x_4$

Using the NWCR method

	San Francisco	Sacramento		Supply
Novato	5 $\boxed{600}$	10 $\boxed{100}$	0	~~700~~ ~~100~~ 0
Lodi	15	4 $\boxed{300}$	0 $\boxed{500}$	~~800~~ ~~500~~ 0
Demand	~~600~~ 0	~~400~~ ~~300~~ 0	~~500~~ 0	1500

Total cost: $(5 \times 600) + (10 \times 100) + (4 \times 300) + (0 \times 500)$

$= 3000 + 1000 + 1200 + 0 = \$5,200$.

Novato: 600 copies should be supplied to San Francisco
100 copies should be supplied to Sacramento

Lodi: 300 copies should be supplied to Sacramento.

(iv)

$1hr = 60min$
$40hr \; x?$

$$\frac{40hr \times 60min}{1hr} = 2400mins$$

$1hr = 60 \, min$
$35hr \; x?$

$$\frac{35hr \times 60min}{1hr} = 2100 \, min$$

Constraint:

$13x + 19y \leq 2400$
$20x + 29y \leq 2100$
$x \geq 10, \; y \geq 0$

Maximize: $(20x + 30y) - \left(\frac{1}{6}(13x + 19y) + \frac{1}{30}(20x + 29y)\right)$

Solving:

$13x + 19y = 2400$
$20x + 29y = 2100$
$x = 10, \; y = 0$

When $x = 0, y = ??$
$13x + 19y = 2400$
$0 + 19y = 2400$
$\frac{19y}{19} = \frac{2400}{19}$
$y = \frac{2400}{19} = 126$

When $x = 0, x = ??$
$13x + 19y = 2400$
$13x + 19(0) = 2400$
$13x = 2400$
$x = \frac{2400}{13} = 185$

X	0	185
Y	126	0

When $y = 0$, $x = ??$
$20x + 29y = 2100$
$20x + 29(0) = 2100$
$\frac{20x}{20} = \frac{2100}{20}$
$x = 105$

When $x = 0$, $y = ??$
$20x + 29y = 2100$
$20(0) + 29y = 2100$
$29y = 2100$
$y = \frac{2100}{29}$
$= 74$

X	0	105
Y	72	0

Maximize : $(20x + 30y) - \left(\frac{1}{6}(13x + 19y) + \frac{1}{30}(20x + 29y)\right)$

A: $(20 \times 20) + (30 \times 0) - \left[\frac{1}{6}((13 \times 10) + (19 \times 0))\right] + \frac{1}{30}((20 \times 10) + (29 \times 0))$
$= 400 - \left(\frac{130}{6} + \frac{200}{30}\right)$
$= 371.67$

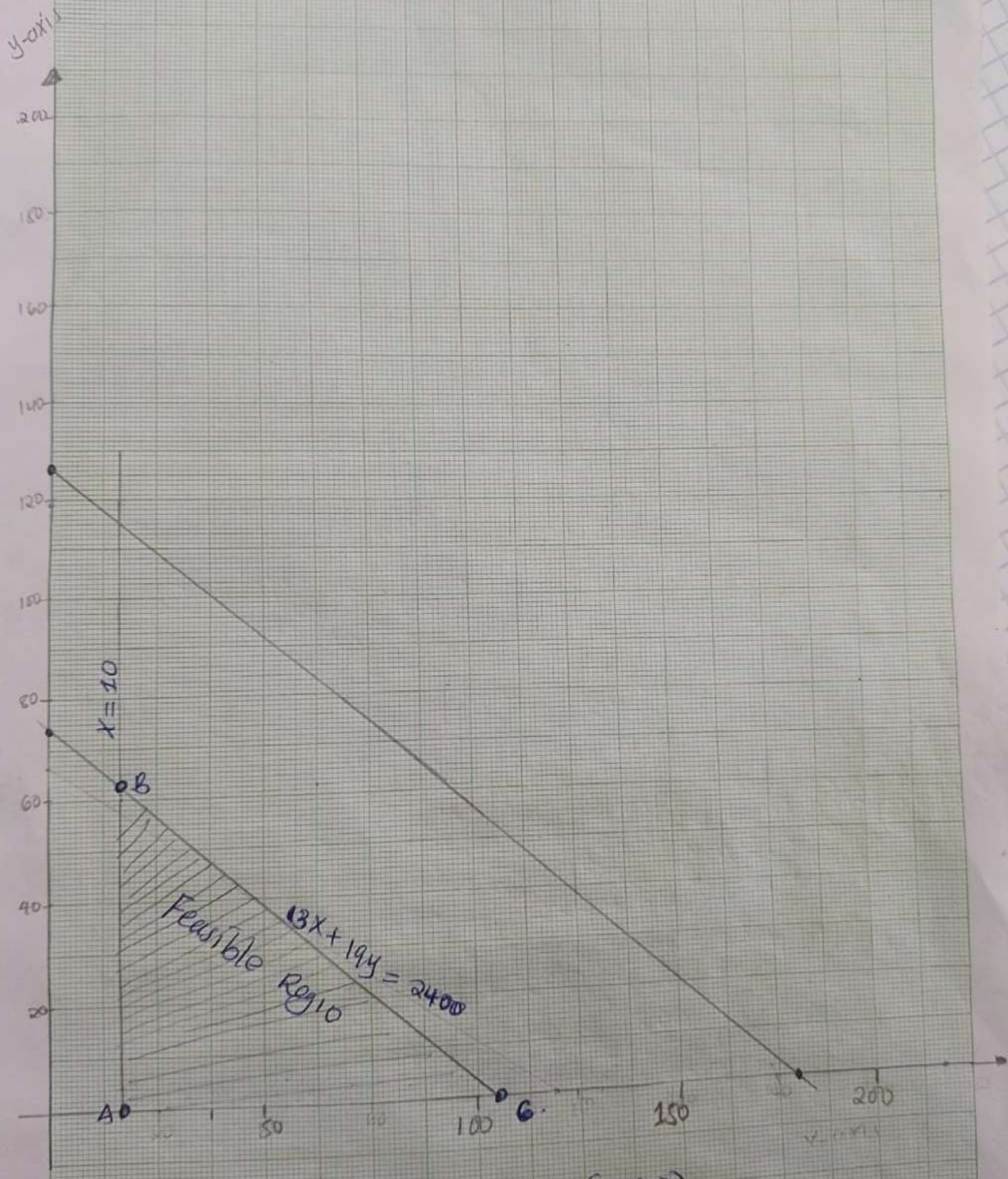
B: $\cancel{(20 \times 10) + (30 \times 63) - \frac{1}{6}(13x + 10}$

$(20 \times 10) + (30 \times 63) - \frac{1}{6}(((13 \times 10) + (19 \times 63)) + \frac{1}{30}((20 \times 10) + 29 \times 63)$
$\Rightarrow 200 + 1890 - (222 + 68)$
$\Rightarrow 2090 - 290$
$\Rightarrow \$1800$

C: $(20 \times 105) + 0 - \frac{1}{6}(13 \times 105 + 0) + \frac{1}{30}(20 \times 105) + 0)$
$= 2100 - \frac{1}{6}[1365] + [2210]\frac{1}{30}]$
$\Rightarrow 2100 - 301 = 1799$

To maximize profit, Machine time must be **10** and Craftsman time should be **63** Item

A GRAPH OF MACHINE TIME AGAINST CRAFTSMAN TIME



A(10,0) , B(10,63) C(105,0)

~~North West co.~~

SECTION B

Q1.)

a) North-West corner Rule

Destination

	D1	D2	D3	Dummy	Supply
S1	3	2	1	0	350
S2	1	1	2	0	450
S3	2	1	1	0	520
S4	1	2	3	0	340
Demand	460	500	480	220	1660

- The Demand and supply were not equal so we add a dummy column to balance.

Destinations

	D1	D2	D3	Dummy	Supply
S1	350 3	2	1	0	~~350~~
S2	1	1	2	0	450
S3	2	1	1	0	520
S4	1	2	3	0	340
Demand	~~460~~ 110	500	480	220	1660

Destination

	D1	D2	D3	Dummy	Supply
S2	110 1	1	2	0	~~450~~ 340
S3	2	1	1	0	520
S4	1	2	3	0	340
Demand	~~110~~	500	480	220	1310

Destination

	D2	D3	Dummy	Supply
S2	340 1	2	0	~~340~~
S3	1	1	0	520
S4	2	3	0	340
Demand	~~500~~ 160	480	220	1200

Destination

	D2	D3	Dummy	Supply
S3	160 1	1	0	~~520~~ 360
S4	2	3	0	340
Demand	160	480	220	860

Destinations

	D3	Dummy	Supply
S3	360 1	0	~~360~~
S4	3	0	340
Demand	~~480~~ 120	220	700

Destinations

	D3	Dummy	Supply
S4	120 3	0	~~340~~ 220
Demand	~~120~~	220	340

Destination

	Dummy	Supply
S4	220 0	220
Demand	220	220

Initial feasible cost.

$(350 \times 3) + (110 \times 1) + (340 \times 1) + (160 \times 1) + (360 \times 1)$
$+ (120 \times 3)$

$= \$2,380$

b) Least Cost method

	Destinations D1	D2	D3	Dummy	Supply
S1	3	2	130 / 1	220 / 0	~~350~~ ~~130~~
S2	450 / 1	1	2	0	~~450~~
S3	2	500 / 1	20 / 1	0	~~525~~ ~~20~~
S4	10 / 1	2	330 / 3	0	~~340~~ ~~330~~
Demand	~~460~~ ~~10~~ 1	~~500~~	~~480~~ ~~350~~ ~~330~~	~~220~~	1660

initial feasible cost:
$(220\times0)+(130\times1)+(450\times1)+(500\times1)+(20\times1)+(10\times1)+(330\times3)$
$= \$2100$

c) Vogel's Approximation Method

	Destination D1	D2	D3	Dummy	Supply
S1	3	2	130 / 1	220 / 0	~~350~~ ~~130~~
S2	450 / 1	1	2	0	~~450~~
S3	2	170 / 1	350 / 1	0	~~520~~ ~~170~~
S4	10 / 1	330 / 2	3	0	~~340~~ ~~10~~
	~~460~~ ~~10~~	~~500~~ ~~330~~	~~480~~ ~~350~~	~~220~~	1660

R.D

S1	1	1	–	–	–	–	–
S2	1	1	1	–	–	–	–
S3	1	1	1	1	1	–	–
S4	1	1	1	1	1	1	1

C.D

D1	D2	D3	Dummy
1	1	1	0
1	1	1	–
1	1	1	–
1	1	2	–
1	1	–	–
1	2	–	–
1	–	–	–

initial feasible cost:
$(220\times0)+(130\times1)+(450\times1)+(170\times1)+(350\times1)+(10\times1)+(330\times2)$
$= \$1{,}770$

Getting optimal solution using VAM

	$v_1 = 0$	$v_2 = 1$	$v_3 = 1$	$v_4 = 0$
$u_1 = 0$	3	2	130 1	220 0
$u_2 = 1$	450 1	1	2	0
$u_3 = 0$	2	170 1	350 1	0
$u_4 = 1$	P 1	330 2	3	0

$m+n-1 = 7$

$A+n-1 = 7$

$m+n-1 = 4+3-1 = 6$

$u_i + v_j = C_{ij}$

$P_{ij} = U_i + V_j - C_{ij}$

$C_{11} = 0+0-3 = -3$

$C_{12} = 0+1-2 = -2$

$C_{22} = 1+1-1 = 0$

$C_{23} = 1+1-2 = -2$

$C_{24} = 1+0-0 = \underline{1}$

$C_{31} = 0+0-2 = -2$

$C_{34} = 0+0-0 = 0$

$C_{43} = 1+1-3 = -1$

$C_{44} = 1+0-0 = \underline{1}$ can't form loop

Q2.)

i.) A Transportation problem is a type of linear programming problem that seeks to minimize cost of distributing a product from multiple sources to multiple destinations, while meeting supply constraints at the sources and demand requirements at the destinations.

ii)

North West Corner Rule

Origin	Destination 1	2	3	Supply
1	(50) 2	7	4	~~50~~
2	(20) 3	(60) 3	1	~~80~~ ~~60~~
3	5	(30) 4	(40) 7	~~70~~ ~~40~~
4	1	6	(140) 2	140
Demand	~~70~~ ~~20~~	~~90~~ ~~30~~	~~180~~ ~~140~~	340

Total Initial feasible cost:

$(50\times2)+(20\times3)+(60\times3)+(30\times4)$
$+(40\times7)+(140\times2)$
$= \$1{,}020$

Applying MODI for optimal solution

$v_1=2 \quad v_2=2 \quad v_3=5$

	$v_1=2$	$v_2=2$	$v_3=5$
$u_1=0$	(50) 2	7	4
$u_2=1$	(20) 3	(60(-)) 3	(+) 1
$u_3=2$	5	(30(+)) 4	(40(-)) 7
$u_4=-3$	1	6	(140) 2

$u_i+v_j=C_{ij}$

Penalties

$P_{ij}=U_i+V_j-C_{ij}$

$C_{12}=0+2-7=-5$
$C_{13}=0+5-4=1$
$C_{23}=1+5-1=5$
$C_{31}=2+2-5=-1$
$C_{41}=-3+2-1=-2$
$C_{42}=-3+2-6=-7$

$v_1=2 \quad v_2=2 \quad v_3=0$

	$v_1=2$	$v_2=2$	$v_3=0$
$u_1=0$	(50) 2	7	4
$u_2=1$	(20) 3	(20) 3	(40) 1
$u_3=2$	5	(70) 4	7
$u_4=2$	1	6	(140) 2

$C_{12}=0+2-7=-5$
$C_{13}=0+2-4=-2$
$C_{31}=2+2-5=-1$
$C_{32}=2+0-7=-5$
$C_{41}=2+2-1=\underline{3}$ *Can't form loop
$C_{42}=2+2-6=-4$

Least Cost Method

Origin	Destination 1	2	3	Supply
		20	30	
1	2	7	4	~~50~~ 20
			80	
2	3	3	1	~~80~~
		70	20	
3	5	4	7	~~70~~
	70		70	
4	1	6	2	~~140~~ 70
Demand	~~70~~	~~90~~	~~180~~	340
		20	100	
			30 20	

Total cost:
$(20\times7)+(30\times4)+(80\times1)+(70\times4)+(70\times1)$
$+(70\times2)$
$= \$830$

Vogel's Approximation Method

Origin	Destination 1	2	3	Supply	R.D					
1	50 2	7	4	~~50~~	2	–	–	–	–	–
2	3	3	80 1	~~80~~	2	2	–	–	–	–
3	5	70 4	7	~~70~~ ~~140~~	1	1	1	1	4	4
4	20 1	20 6	100 2	~~140~~ 40 20	1	1	1	5	6	–
Demand	~~70~~ ~~20~~	~~90~~ 70	~~180~~ ~~100~~	340						

C.D		
1	1	1
2	1	1
4	2	5
4	2	–
–	2	–
–	–	–

Total cost:
$(50\times2)+(80\times1)(70\times4)+(20\times1)+(20\times6)$
$+(100\times2)$
$= \$800$

SECTION C

Q1) Mean interval time : 8 minutes
Mean service time : 4 minutes.

i) Mean service rate and Mean arrival rate.
→ Arrival rate $(\lambda) = \frac{1}{8}$ customers per minute.
→ Service rate $(\mu) = \frac{1}{4}$ customers per minute.

ii) Traffic intensity (ρ)

$$\rho = \frac{\lambda}{\mu} = \frac{1/8}{1/4} = \frac{1}{2} = 0.5$$

$$\rho = 0.5$$

iii) Mean time a customer spends in the Queue and in the system.
- Mean time in the system (W)

$$W = \frac{1}{\mu - \lambda} = \frac{1}{\frac{1}{4} - \frac{1}{8}} = \frac{1}{\frac{1}{8}} = 8 \text{ minutes.}$$

- Mean time in the queue (W_q)

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{1/8}{\frac{1}{4}\left(\frac{1}{4} - \frac{1}{8}\right)} = \frac{1/8}{1/32} = \frac{1}{8} \times 32 = 4$$

$= 4$ minutes

iv) Expected Number of Customers in the Queue and in the System
→ Expected number of customers in the system (L)

$$L = \frac{\lambda}{\mu - \lambda} = \frac{1/8}{\frac{1}{4} - \frac{1}{8}} = \frac{1}{8} \times 8 = 1 \text{ customer}$$

→ Expected number of customers in the queue (L_q)

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\left(\frac{1}{8}\right)^2}{\frac{1}{4}\left(\frac{1}{4}-\frac{1}{8}\right)} = \frac{1/64}{1/32} = \frac{1}{2} = 0.5$$

$= 0.5$ customers

v) Probability of having at most 4 customers in the system.

$$P(n) = (1-\rho)\rho^n$$

$$P(\text{at most 4 customers}) = \sum_{n=0}^{4} (1-\rho)\rho^n$$

$$= (1-0.5)(0.5^0 + 0.5^1 + 0.5^2 + 0.5^3 + 0.5^4)$$

$$= 0.5 \times 1.9375$$

$$= 0.96875$$

$$= 96.88\%$$

Q2) Mean inter-arrival time = 10 minutes

Arrival rate $\lambda = \frac{1}{10}$ customers per minute

Mean service time = 3 minutes

Service rate $\mu = \frac{1}{3}$ customers per minute

i) $\rho = \frac{\lambda}{\mu} = \frac{1/10}{1/3} = \frac{1}{10} \times \frac{3}{1} = 0.3$

$= 30\%$

ii) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\left(\frac{1}{10}\right)^2}{\frac{1}{3}\left(\frac{1}{3}-\frac{1}{10}\right)} = \frac{\frac{1}{100}}{\frac{7}{90}} = \frac{9}{70}$

$= 0.129$ customers

Queueing Model

ii.) $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$W_q = 3$

$3 = \dfrac{\lambda}{\frac{1}{3}(\frac{1}{3}-\lambda)}$ $\quad 3 = 3 \times \dfrac{\lambda}{\frac{1}{3}-\lambda}$

$1 = \dfrac{\lambda}{\frac{1}{3}-\lambda}$ $\quad \frac{1}{3}-\lambda = \lambda$

$\dfrac{\frac{1}{3}}{2} = \dfrac{2\lambda}{2}$ $\quad \lambda = \frac{1}{6}$

$= \frac{1}{6} - \frac{1}{10} = \frac{1}{15}$ customers per minutes

Arrival rate must increase by $\frac{1}{15}$ customers per minute

Q3) Mean arrival rate $(\lambda) = 12$ trucks per day
Mean service rate $(\mu) = 18$ trucks per day

i) $P_0 = 1-\rho$

$\rho = \dfrac{\lambda}{\mu} = \dfrac{12}{18} = 0.6667$

$P_0 = 1-0.6667 = 0.3333$

$= 33.33\%$

ii) $L_q = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

$= \dfrac{12^2}{18(18-12)} = \dfrac{144}{108} = 1.333$ trucks

iii) $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$= \dfrac{12}{18(18-12)} = \dfrac{12}{108} = 0.1111$ days

iv) $P(\text{waiting}) = \rho = \dfrac{\lambda}{\mu} = \dfrac{12}{18} = 0.6667$

$= 66.67\%$

SECTION E

Q1 i) expected ordering Quantity / fixed order:

Economic Order Quantity, EOQ $= \sqrt{\frac{2DS}{H}}$

where:

$H \leftarrow$ holding cost/unit $= \$5$

$D \leftarrow$ demand/year $= 20\,000$ units.

$S \leftarrow$ ordering/setup cost per order $= \$100$

$\therefore EOQ = \sqrt{\frac{2 \times 20000 \times 100}{5}} = 894.427191 \simeq 894$

This is the expected ordering quantity/fixed order.

ii) Computing Retailer's Economic Order Quantity -

- We're working with an interest rate of 10% p.a, and £100 total cost/unit

$\therefore$ with storage cost/year $\leftarrow$ £20

Holding cost H or C $\leftarrow$ storage cost + cost from interest.

$\therefore H = £20 + (0.1)(100) = £30,$

Retailer Demand/p.a $\leftarrow D = 200$ units

ordering cost/setup, $S = £35$

Economic Order Quantity, EOQ $= \sqrt{\frac{2DS}{H}}$

$EOQ = \sqrt{\frac{2 \times 200 \times 35}{30}} = \sqrt{\frac{1400\cancel{0}}{3\cancel{0}}} = \sqrt{\frac{1400}{3}} = 21.60246$

Economic Order Quantity is roughly 22 units

iii) Computing new EOQ from known:

Economic order Quantity, EOQ $= \sqrt{\frac{2DS}{H}}$

where demand is not given so let's use D

$S \leftarrow$ ordering/setup cost _ we use S

Let C be unit cost:

$\therefore$ for EOQ 120 - holding cost $H_1 = 0.2C$

for new EOQ - holding cost $H_2 = 0.15C$

comparing ratios:

$\frac{EOQ_1}{EOQ_2} = \sqrt{\frac{H_1}{H_2}}$; $EOQ_2 = EOQ_1 \left(\sqrt{\frac{H_1}{H_2}}\right)$

$\sqrt{0.2}$ $= 138.564064$

Q2 (i) Three Main Reasons: For: Holding Inventory

① Demand uncertainty - holding inventory allows you to deal with fluctuations in demand effectively. This can help you mitigate loss of sales.

② Helps in Tracking Lead Time: Inventory management allows you to manage delays between placing an order and the receipt of it. The lead time can vary but if tracked well, can ensure continuous supply of products.

③ Economies of Scale: businesses/organizations can leverage on purchasing goods in large quantities through inventories. This helps minimize unit costs and setup.

Reasons for: Why Only Minimal Inventories are Held:

① Minimizing Spoilage/Obsolescence: products with short life span and perishable are better handled using minimal inventories.

② Minimizing Risk of Damage/Overstocking: damage that comes from collisions in overstocked goods can lead to damage hence incurring loss on large scale.

③ Reducing Holding Costs: holding cost can come in form of storage costs and can be reduced by use of minimal inventory.

④ Capital Tie-ups: inventory represents a significant investment. Holding excess inventory ties up capital that could have been used for other things. Holding minimal inventory alleviates this problem.

Q2 (ii) Economic Order Quanty (EOQ) and FNSD
The Economic Order Quantity refers to the optimal quantity that minimises the total cost for an inventory - consisting of ordering/setup and holding cost.
EOQ is based on demand, ordering and holding costs.

FNSD — Fixed Number of Supplier Deliveries -
This is a technique used to determine the optimal number of deliveries per year. and comprises transport expenses, ordering cost etc.
FNFD analysis enables the concerned party to find a balanced between minimizing costs and reliability in inventory supply.
FNSD analysis categorises inventory into
a) Fast moving to meet high demand.
b) Normal moving for regular demand.
c) slow moving for low demand
d) Dead stock for no demand at all.

Q2 (iii) Steps in Calculation of EOQ for:
discounted quantities, ABC Analysis in inventory control
- This technique typically involves the steps listed below.
① Compute the EOQ Ignoring The Discount.
You determine the optimal order quantity but not considering the discounts
② Evaluate Discount Breakpoints: identify points where the discount makes it worthwhile to order a larger quantity.
③ Compute the Total Costs: for each discount breakpoint determine the holding costs and ordering costs and cost of units /purchase.
④ Select the optimal quantity - this is the quantity that minimises the total costs.

ABC analysis ← next page.

ABC Analysis is a way of categorizing inventory items based on importance and uses alphabets in this case to denote each category.

① A Items: these are high value items that account for a significant portion of the inventory costs. This require close monitoring

② B Items: moderately important items with medium value.

③ C items: these are low value items and are managed using simpler systems compared to categories in A and B items.

Q3 Super market TOYS:

Delivery in one batch is over 2000 so we use price of 35/= from table:

Order quantity	Price per unit
1 to 1000	40/=
1000 to 2000	38/=
over 2000	35/=

- For this problem we consider only holding and ordering cost.

(a) $EOQ = \sqrt{\frac{2DS}{H}}$

where $D \leftarrow$ delivery $= 6000$

$S \leftarrow$ ordering cost (can be C) $= 80$

$H \leftarrow$ holding cost $(0.15(\text{unit price}) = 0.15 \times 35 = 5.25/=$

$\therefore EOQ = Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 6000 \times 80}{5.25}} = 427.6179871 \simeq 428$

Economic Order quantity is 428 (units)

(b) Optimum Total cost; we still consider only the last row since we are doing instantaneous delivery

Total cost, TC = ordering cost + holding cost $= \frac{D}{Q^*}S + \frac{Q^*}{2}H$

[next page]

from $TC = \frac{D}{Q^*}S + \frac{Q^*}{2}H$

where $Q^* = EOQ = 428$, $D = 6000$, $S = 80$, $H = 5.25$

$$\text{Total cost} = \frac{6000}{428}(80) + \frac{428}{2}(5.25)$$

$$= 1121.495327 + 1123.5$$

$$= 2244.995327 \simeq 2244.995$$

$$\simeq 2245 /=$$

Optimum total cost is 2,245.00 /=

(c) Number of orders per year;

$$\text{No. of orders} = \frac{\text{Demand}}{EOQ} = \frac{D}{Q^*} = \frac{6000}{428}$$

$$= 14.01869159$$

$$\simeq 15 \text{ orders}$$

We round up for orders that have fractions parts.

(c) Time between orders in days:

* no working days given so we take 365

$$\text{Time between orders} = \frac{\text{working days}}{\text{No. of orders}}$$

$$= \frac{365}{15}$$

$$= 24.3333 \text{ (days)}$$

Members:

SCT211-0848/2018	Jany Muong
SCT211-0079/2022	Joram Kireki
SCT211-0504/2021	Gatmach Yuol
SCT211-0003/2022	Josphat Waweru
SCT211-0535/2022	Akech Atem
SCT211-0070/2022	Vincent Ohanga

SECTION D

Q1 Find the dual program of the following linear programming problem

Maximize $Z = 5x_1 - 2x_2$

sub to

$$3x_1 + 2x_2 \geq 16$$
$$x_1 - x_2 \leq 4$$
$$x_1 \geq 5$$

$x_1 \geq 0$ x_2 is unconstrained.

Soln.

let $x_2 = x_3 - x_4$ Max $\rightarrow \leq$

Rewrite The problem.

Max $Z = 5x_1 - 2(x_3 - x_4)$

sub to $-3x_1 - 2(x_3 - x_4) \leq -16$

$$x_1 - (x_3 - x_4) \leq 4$$
$$-x_1 \leq -5$$

$$x_1, x_3, x_4 \geq 0.$$

Max $Z = 5x_1 - 2x_3 + 2x_4$

sub to

$$-3x_1 - 2x_3 + 2x_4 \leq -16 \quad a$$
$$x_1 - x_3 + x_4 \leq 4 \quad b$$
$$-x_1 \leq -5 \quad c$$

$$x_1, x_3, x_4 \geq 0.$$

Dual is :

let a, b, and c be dual variables of the dual problem.

$\therefore$ Min $V = -16a + 4b - 5c$

$$-3a + b - c \geq 5$$
$$-2a - b + 0c \geq -2$$
$$2a + b + 0c \geq 2$$
$$a, b, c \geq 0$$

$$\text{Min } V = -16a + 4b - 5c$$

sub to

$$-3a + b - c \geq 5$$
$$\left.\begin{array}{l} -2a - b \geq -2 \\ -2a - b \leq -2 \end{array}\right\}.$$
$$a, b, c \geq 0.$$

Replace them equal sign.

$$\text{Min } V = -16a + 4b - 5c$$

Sub to

$$-3a + b - c \geq 5$$
$$-2a - b = -2$$
$$a \geq 0,\ b \geq 0,\ c \geq 0.$$

Q2 Find the dual program of the following linear programming problem.

Minimize $Z = 30x_1 - 50x_2 + 10x_3$

Subject to

$$3x_1 + 2x_2 - x_3 \geq 44$$
$$x_1 - x_2 + x_3 = 7$$

x_1 is unconstrained, $x_2 \geq 0$, $x_3 \geq 0$.

Soln.

let $x_1 = (x_4 - x_5)$

Rewrite the problem.

$$\text{Min } Z = 30(x_4 - x_5) - 50x_2 + 10x_3$$

Sub to

$$3(x_4 - x_5) + 2x_2 - x_3 \geq 44$$
$$x_4 - x_5 - x_2 + x_3 \geq 7$$
$$x_4 - x_5 - x_2 + x_3 \leq 7.$$
$$x_2, x_3, x_4, x_5 \geq 0.$$

$$\text{Min } Z = 30x_4 - 30x_5 - 50x_2 + 10x_3$$

sub to

$$3x_4 - 3x_5 + 2x_2 - x_3 \geq 44 \qquad a$$
$$x_4 - x_5 - x_2 + x_3 \geq 7 \qquad b$$
$$x_4 - x_5 - x_2 + x_3 \leq 7 \qquad c$$

$$x_2, x_3, x_4, x_5 \geq 0.$$

~~Dual.~~
~~let a, b, and c be the dual variable~~

~~Max $w = 44a + 7b$~~

$$\text{Min } Z = 30x_4 - 30x_5 - 50x_2 + 10x_3$$

sub to

$$3x_4 - 3x_5 + 2x_2 - x_3 \geq 44$$
$$x_4 - x_5 - x_2 + x_3 \geq 7$$
$$-x_4 + x_5 + x_2 - x_3 \geq -7$$

$$x_2, x_3, x_4, x_5 \geq 0.$$

Dual.
let a, b, c be dual variable

$$\text{Max } V = 44a + 7b - 7c$$

sub to

$$3a + b - c \leq 30$$
$$-3a - b + c \leq -30$$
$$2a - b + c \leq -50$$
$$-a + b - c \leq 10$$

let $d = b - c$

$$\text{Max } v = 44a + 7d$$

$$\left.\begin{array}{l} 3a + b - c \leq 30 \\ 3a + b - c \geq 30 \end{array}\right\}$$
$$2a - b + c \leq -50$$
$$-a + b - c \leq 10.$$

$$\text{Max } V = 44a + d$$
$$2a + d = 30$$
$$2a - d \leq -50$$
$$-a + d \leq 10.$$

$a \geq 0$, d is unrestricted / unconstrained

Q3. Find the dual program of the following linear programming problem.

Min $p = 16x - 2y - 5z$

subject to

$$x + 4y - z \geq 120$$
$$x + y + 3z \leq 130$$

$x \geq 0$, $y \geq 0$, z is unconstrained

soln.

Min $p = 16x - 2y - 5z$

sub to

$$x + 4y - z \geq 120$$
$$-x - y - 3z \geq -130$$

let $z = (a - b)$

$\therefore$

Min $P = 16x - 2y - 5(a-b)$

sub to

$$x + 4y - (a-b) \geq 120$$
$$-x - y - 3(a-b) \geq -130$$

$$x, y, a, b \geq 0$$

Min $P = 16x - 2y - 5a + 5b$

sub to

$$x + 4y - a + b \geq 120$$
$$-x - y - 3a + 3b \geq -130$$

Dual is·

let c and d be dual variables·

$$
\begin{aligned}
\text{Max } w &= 120c - 130d \\
\text{sub to } \quad & c - d \le 16 \\
& 4c - d \le -2 \\
& -c - 3d \le -5 \\
& c + 3d \le 5
\end{aligned}
$$

$$
\begin{aligned}
\text{Max } w &= 120c - 130d \\
\text{sub to } \quad & c - d \le 16 \\
& 4c - d \le -2 \\
& \left.\begin{aligned} -c - 3d &\le -5 \\ -c - 3d &\ge -5 \end{aligned}\right\} =
\end{aligned}
$$

$$
\begin{aligned}
\text{Max } w &= 120c - 130d \\
\text{sub to } \quad & c - d \le 16 \\
& 4c - d \le -2 \\
& -c - 3d = -5 \\
& c, d \ge 0.
\end{aligned}
$$

4. **Solution Derivation**: Solving the model using techniques like linear programming, simulation, or optimization to find the best solution.
5. **Implementation and Monitoring**: Applying the solution in real-life scenarios and monitoring its effectiveness. Adjustments are made if necessary based on performance.

iv) Define the term linear programming and outline the five steps followed when formulating a linear programming model mathematically

- **Linear Programming (LP)** is a mathematical technique used to optimize a linear objective function, subject to a set of linear constraints. LP is commonly used to maximize profit or minimize cost in resource allocation problems.

Steps in formulating an LP model:

- **Define Decision Variables**: Identify the variables that will represent the decisions to be made.
- **Formulate the Objective Function**: Develop an equation that reflects the goal of the problem (e.g., maximizing profit or minimizing cost).
- **Formulate the Constraints**: List the restrictions or limitations on the decision variables (e.g., resource availability, time).
- **Non-Negativity Conditions**: Ensure that the decision variables are non-negative, meaning they cannot take negative values.
- **Solve the Model**: Use graphical methods or optimization algorithms (such as the Simplex method) to solve the LP problem.

v) List the basic properties of linear programming models

The basic properties of linear programming models include:

1. **Linearity**: Both the objective function and constraints must be linear (i.e., no variables are raised to a power or multiplied by each other).
2. **Decision Variables**: The model must include variables that represent the quantities to be determined.
3. **Objective Function**: The goal of the model is either to maximize or minimize a linear objective function.
4. **Constraints:** The model includes a set of linear equations or inequalities that represent the restrictions or limitations on the decision variables.
5. **Non-Negativity**: The values of the decision variables must be non-negative (i.e., they cannot be less than zero).

2. **Resource Optimization:** OR techniques help in efficient utilization of limited resources such as labor, materials, time, and capital, thereby maximizing productivity or minimizing costs.
3. **Enhanced Productivity**: By optimizing production schedules, inventory control, and resource allocation, OR increases productivity in manufacturing, logistics, and services.
4. **Risk Management**: It enables better forecasting and planning, which helps in mitigating risks in uncertain environments like finance, logistics, and operations.
5. **Solving Complex Problems**: OR is especially useful for handling complex systems with multiple variables and constraints, offering insights that might not be immediately apparent.

ii) Identify the limitations of Operations Research

While OR is valuable, it has certain limitations:

1. **Complexity**: Developing mathematical models can be complex and time-consuming, requiring specialized knowledge in mathematics, statistics, and programming.
2. **Data Dependency:** OR models rely heavily on accurate and comprehensive data. If data is incomplete or incorrect, the results of the model will be unreliable.
3. **Quantitative Bias**: OR focuses on quantifiable factors and may ignore qualitative aspects like human emotions, motivation, or ethics, which are difficult to model mathematically.
4. **High Costs**: The implementation of OR solutions, especially those that involve sophisticated software or hardware, can be expensive.
5. **Resistance to Change**: Human factors, such as resistance to change by employees or managers, can hinder the successful implementation of OR models.

iii) Outline and briefly explain the five principal phases of Operations Research

The five key phases of Operations Research are:

1. **Problem Definition**: Clearly defining the problem, objectives, and constraints. This involves identifying key factors that affect the decision-making process.
2. **Data Collection**: Gathering relevant data about the problem, including both quantitative (numbers, statistics) and qualitative (judgment, expert opinion) data.
3. **Model Formulation**: Constructing a mathematical model that represents the problem and incorporates the objective function and constraints.

4. **Model Formulation**: The process of developing a mathematical representation of the problem, incorporating the objective function and constraints.
5. **Feasible Solution**: A solution that satisfies all the constraints of the problem without violating any restrictions.
6. **Transportation Problem**: A type of problem focused on determining the optimal way to transport goods from sources to destinations at minimum cost.
7. **Allocation Problems**: These involve distributing limited resources among competing activities in a way that optimizes a desired objective.
8. **Non-Negative Conditions**: A condition where decision variables must be greater than or equal to zero, ensuring the problem's variables make sense in a real-world context.

v) Operations Research Techniques

The most common techniques used in Operations Research include:

1. **Linear Programming (LP)**: A method used to achieve the best outcome (such as maximizing profit or minimizing cost) in a mathematical model whose constraints and objective function are linear.
2. **Game Theory**: Used to make decisions in situations where multiple players with competing interests are involved.
3. **Queuing Theory**: Helps in optimizing the performance of systems where there is a need to manage waiting lines, such as customer service or manufacturing systems.
4. **Inventory Control Models**: Used to minimize the cost of holding and ordering inventory while meeting demand.
5. **Network Analysis (PERT/CPM)**: Project management techniques used to optimize scheduling by analyzing the critical path and project completion time.
6. **Simulation**: A method of testing different decision scenarios by creating a digital or physical representation of a system.

Q2:

i) Discuss the significance of Operations Research

Operations Research (OR) plays a vital role in optimizing decision-making by:

1. **Improving Decision Quality**: It provides a structured and quantitative basis for making decisions, reducing the reliance on intuition or guesswork.

SECTION A

Q1.

i) Define the term Operations Research

Operations Research (OR) is a scientific approach used for decision-making and problem-solving in complex systems. It utilizes mathematical models, statistical analysis, and optimization techniques to improve decision-making in areas such as logistics, production, and resource allocation.

ii) Origin of Operations Research

Operations Research emerged during World War II in the late 1930s. It was first applied in the UK, particularly in military operations to optimize the use of limited resources like aircraft, radar, and weapons. This interdisciplinary field involved mathematicians, engineers, and scientists collaborating to solve logistical problems in warfare. After the war, the methods and principles of OR were adopted in industrial, business, and governmental decision-making processes.

iii) Methodology of Operations Research

The Operations Research methodology generally follows these steps:

1. **Problem Definition**: Identify and clearly define the problem, collecting data to understand the variables involved.
2. **Model Formulation**: Construct a mathematical model to represent the system and its constraints, often in the form of equations or inequalities.
3. **Solution Derivation**: Solve the mathematical model using optimization or simulation methods to obtain a solution.
4. **Model Validation**: Test the solution and model against real-world scenarios to ensure its accuracy.
5. **Implementation**: Apply the solution in practice, monitor the results, and make adjustments if necessary.

iv) Definitions of Terms in Operations Research

1. **Model**: A simplified representation of a system or process used to analyze problems and test solutions.
2. **Objective Function**: The function that needs to be maximized or minimized in a problem, such as profit maximization or cost minimization.
3. **Constraints**: The limitations or restrictions on decision variables, such as resource availability.

Q3 i) Let X and Y represent Wheat and rye respectively

Maximize $Z = 500x + 300y$

Subjected to: Constraints:

	X	Y
Cost	\$200	\$100
Time	1	2

$x + y \geq 7$ ---(i)

$200(x) + 100(y) \leq 1200$

$\frac{200x}{100} + \frac{100y}{100} \leq \frac{1200}{100}$

$2x + y \leq 12$ ---(ii)

$x \geq 0, y \geq 0$

$x + 2y \leq 12$ -----(iii)

solution

$x + y = 7$ $\quad x + 2y = 12$

$2x + y = 12$

$x \geq 0, y \geq 0$

Y-Intercept

When $x = 0$, $y = ?$

$x + y = 7$

$0 + y = 7$

$y = 7$

X	0	7
Y	7	0

When $y = 0$, $x = ?$

$x + y = 7$

$x + 0 = 7$

$x = 7$

$2x + y = 12$

When $x = 0$, $y = ?$

$2(0) + y = 12$

$y = 12$

X	0	6
Y	12	0

When $y = 0$, $x = ?$

$2x + y = 12$

$2x + 0 = 12$

$\frac{2x}{2} = \frac{12}{2}$

$x = 6$

y-intercept

When $x = 0$, $y = ?$

$x + 2y = 12$

$0 + 2y = 12$

$\frac{2y}{2} = \frac{12}{2}$

$y = 6$

x	0	12
y	6	0

x-intercept

When $y = 0$, $x = ?$

$x + 2y = 12$

$x + 2(0) = 12$

$x + 0 = 12$

$x = 12$

Maximize : $Z = 500x + 300y$.

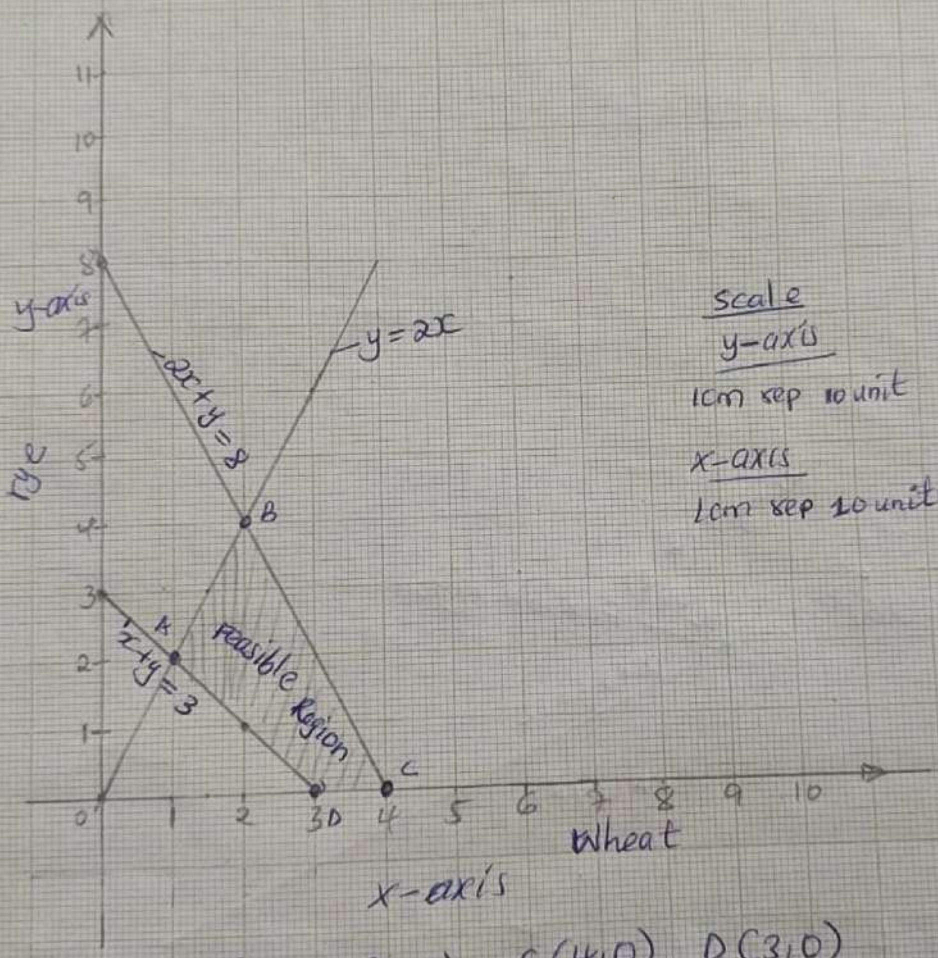
A: $(5 \times 500) + (2 \times 300) = 2500 + 600 \Rightarrow 3100$

B: $(4 \times 500) + (4 \times 300) = 2000 + 1200 \Rightarrow 3200$

C: $(2 \times 500) + (5 \times 300) = 1000 + 1500 \Rightarrow 2500$

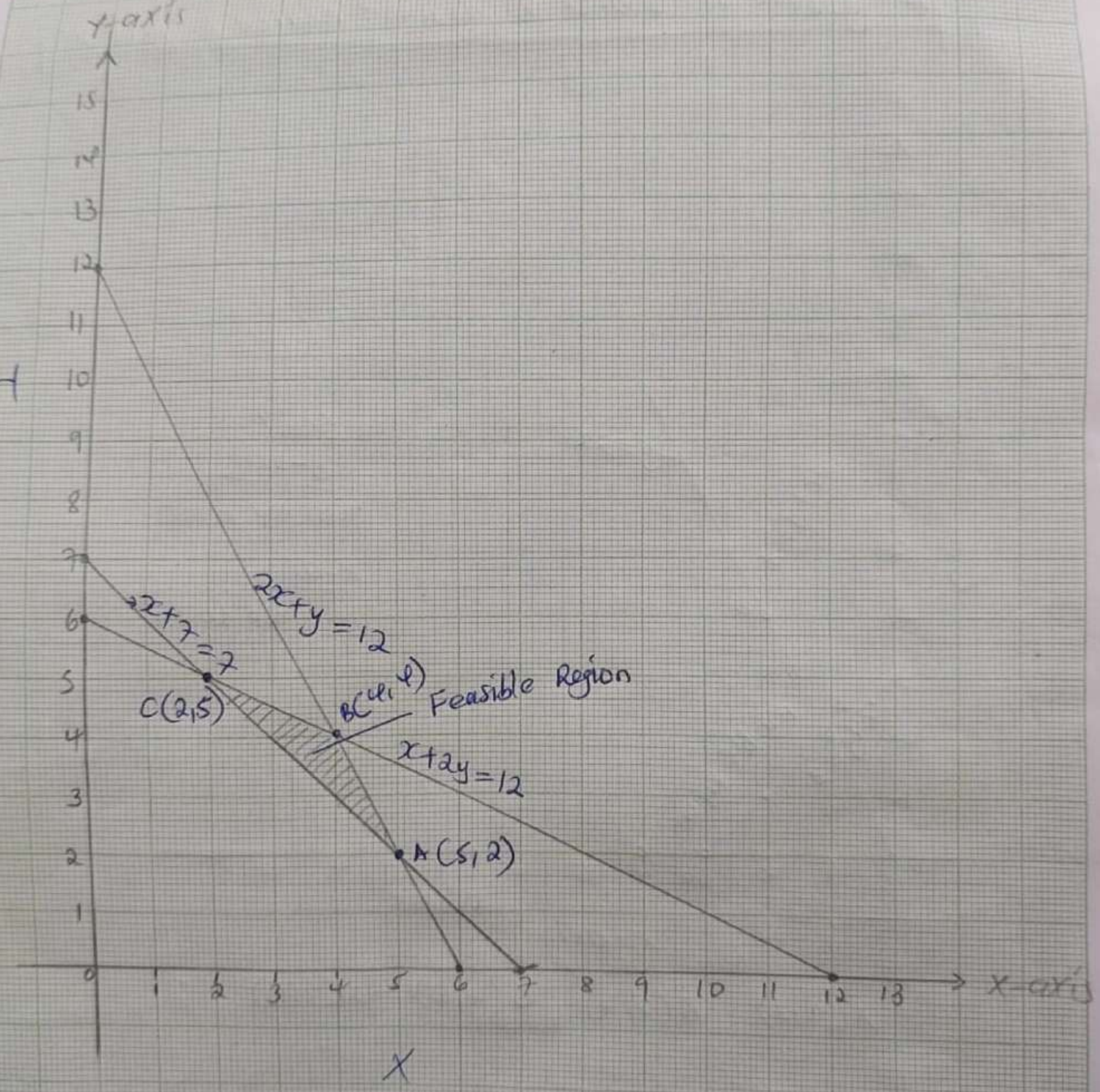
To maximize profit, wheat should be 4 acres and rye should be 4 acres.

A graph of source B against source A gold A



A(1,2) , B(2,4) , C(4,0) , D(3,0)

A Graph of rye against Wheat



(ii) Let X and Y represent source A and source B of gold ore respectively.

Maximize $Z = 2X + 3Y$

subjected to

Constraints

$X + y \geq 3$ --- (i)

$20(x) + 10(y) \leq 80$

$\frac{20x}{10} + \frac{10y}{10} \leq \frac{80}{10}$

$2x + y \leq 8$ --- (ii)

$Y \leq 2X$ --- (iii)

soln.

$x + y = 3$

$2x + y = 8$

$Y = 2X$

X-interapt

When $\cancel{Y} = 0$, $X = \cancel{8}$?

$x + y = 3$

$x + 0 = 3$

$X = 3$

X	3	0
Y	0	3

~~When~~ Y-intercepts

When $X = 0$, $y = ??$

$x + y = 3$

$0 + y = 3$

$y = 3$

(ii) $2x + y = 8$

X-interapt

When $\cancel{Y} = 0$, $X = ?$

$2x + y = 8$

$2x + 0 = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

Y-interapt,

When $X = 0$, $y = ??$

$2x + y = 8$

$2(0) + y = 8$

$0 + y = 8$

$y = 8$

X	4	0
Y	0	8

When $X = 1$, $y = 2$,

X	0	1	2	3	4
Y	0	2	4	6	8

Cost Maximization

A : $(1\times2) + (3\times2) = 4+6 = 10$

B : $(2\times2) + (4\times3) \Rightarrow 4+12 = 16$

C : $(4\times2) + (0\times3) = 8+0 = 8$

D : $(3\times2) + (0\times3) = 6+0 = 6$

~~ke~~

To maximize the amount of gold, 2 tons from Source A and 4 tons from Source B should be extracted.

(iii)

Let:

- x_1 be the number of copies shipped from Novato to San Francisco
- x_2 be the number of copies shipped from Novato to Sacramento
- x_3 be the number of copies shipped from Lodi to San Francisco
- x_4 be the number of copies shipped from Lodi to Sacramento

Supply Constraints

- From Novato: $x_1 + x_2 \leq 700$
- From Lodi: $x_3 + x_4 \leq 800$

Demand Constraints

- For San Francisco: $x_1 + x_3 = 600$
- For Sacramento: $x_2 + x_4 = 400$

Objective function $C = 5x_1 + 10x_2 + 15x_3 + 4x_4$

Using the NWCR Method

	San Francisco	Sacramento		Supply
Novato	5 $\boxed{600}$	10 $\boxed{100}$	0	~~700~~ ~~100~~ 0
Lodi	15	4 $\boxed{300}$	0 $\boxed{500}$	~~800~~ ~~500~~ 0
Demand	~~600~~ 0	~~400~~ ~~300~~ 0	~~500~~ 0	1500

Total cost: $(5 \times 600) + (10 \times 100) + (4 \times 300) + (0 \times 500)$
$= 3000 + 1000 + 1200 + 0 = \$5,200$.

Novato: 600 copies should be supplied to San Francisco
100 copies should be supplied to Sacramento

Lodi: 300 copies should be supplied to Sacramento.

(iv)

1hr = 60min

40hr x??

$\frac{40hr \times 60min}{1hr} = 2400mins$

1hr = 60 min

35hr x??

$\frac{35hr \times 60min}{1hr} = 2100 min$

Constraint:

$13x + 19y \leq 2400$

$20x + 29y \leq 2100$

$x \geq 10$, $y \geq 0$

Maximize: $(20x + 30y) - (\frac{1}{6}(13x + 19y) + \frac{1}{30}(20x + 29y))$

Solving:

$13x + 19y = 2400$

$20x + 29y = 2100$

$x = 10$, $y = 0$

When $x = 0$, $y = ??$

$13x + 19y = 2400$

$0 + 19y = 2400$

$\frac{19y}{19} = \frac{2400}{19}$

$y = \frac{2400}{19} = 126$

When $x = 0$, $x = ??$

$13x + 19y = 2400$

$13x + 19(0) = 2400$

$13x = 2400$

$x = \frac{2400}{13} = 185$

X	0	185
Y	126	0

When $y=0$, $x=??$

$20x + 29y = 2100$

$20x + 29(0) = 2100$

$\frac{20x}{20} = \frac{2100}{20}$

$x = 105$

When $x=0$, $y=??$

$20x + 29y = 2100$

$20(0) + 29y = 2100$

$29y = 2100$

$y = \frac{2100}{29}$

$= 74$

X	0	105
Y	72	0

Maximize : $(20x + 30y) - \left(\frac{1}{6}(13x + 19y) + \frac{1}{30}(20x + 29y)\right)$

A: $(20\times20) + (30\times0) - \left[\frac{1}{6}((13\times20) + (19\times0))\right] + \frac{1}{30}((20\times20) + (29\times0))$

$= 400 - \left(\frac{130}{6} + \frac{200}{30}\right)$

$= 371.67$

B: ~~$(20\times10) + (30\times63) - \frac{1}{6}(13x + 10$~~

$(20\times10) + (30\times63) - \frac{1}{6}((13\times10) + (19\times63) + \frac{1}{30}(20\times10) + 29\times63)$

$\Rightarrow 200 + 1890 - (222 + 68)$

$\Rightarrow 2090 - 290$

$\Rightarrow \$1800$

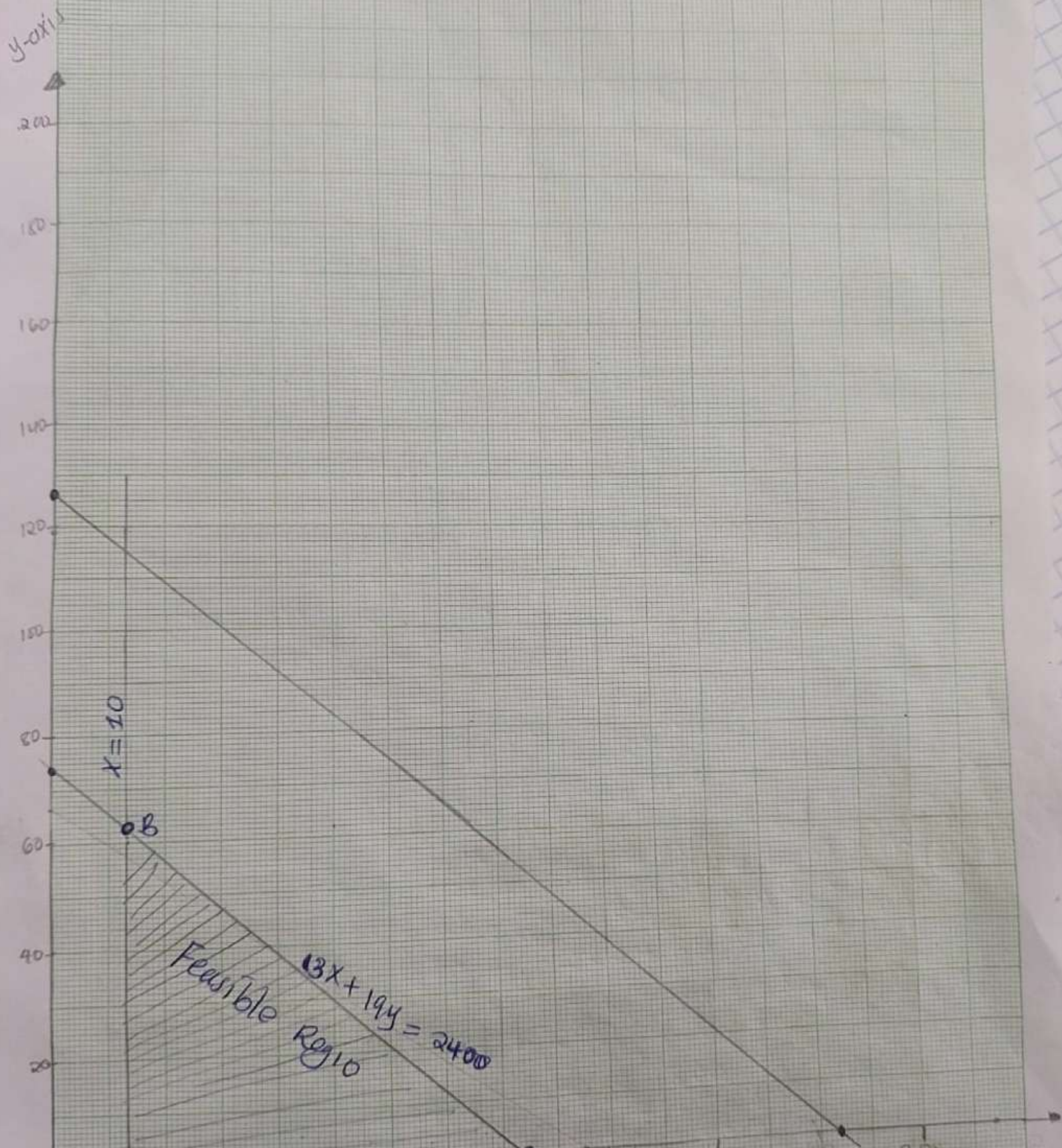
C: $(20\times105) + 0 - \frac{1}{6}(13\times105 + 0) + \frac{1}{30}(20\times105) + 0)$

$= 2100 - \frac{1}{6}[1365] + [2210]\frac{1}{30}]$

$\Rightarrow 2100 - 301 = 1799$

To maximize profit, Machine time must be **10** and Craftsman time should be **63** Item

A GRAPH OF MACHINE TIME AGAINST CRAFTSMAN TIME



A(10, 0) , B (10, 63) C (105, 0)

~~North West Co.~~
SECTION B

Q1.)

a) North-West corner Rule

Destination

	D1	D2	D3	Dummy	Supply
S1	3	2	1	0	350
S2	1	1	2	0	450
S3	2	1	1	0	520
S4	1	2	3	0	340
Demand	460	500	480	220	1660

- The Demand and Supply were not equal so we add a dummy column to balance

Destinations

	D1	D2	D3	Dummy	Supply
S1	350 3	2	1	0	~~350~~
S2	1	1	2	0	450
S3	2	1	1	0	520
S4	1	2	3	0	340
Demand	~~460~~ 110	500	480	220	1660

Destination

	D1	D2	D3	Dummy	Supply
S2	110 1	1	2	0	~~450~~ 340
S3	2	1	1	0	520
S4	1	2	3	0	340
Demand	110	500	480	220	1310

Destination

	D2	D3	Dummy	Supply
S2	340 1	2	0	~~340~~
S3	1	1	0	520
S4	2	3	0	340
Demand	~~500~~ 160	480	220	1200

Destination

	D2	D3	Dummy	Supply
S3	160 1	1	0	~~520~~ 360
S4	2	3	0	340
Demand	160	480	220	860

Destinations

	D3	Dummy	Supply
S3	360 1	0	~~360~~
S4	3	0	340
Demand	~~480~~ 120	220	700

Destinations

	D3	Dummy	Supply
S4	120 3	0	~~340~~ 220
Demand	~~120~~	220	340

Destination

	Dummy	Supply
S4	220 0	220
Demand	220	220

Initial feasible cost.

$(350\times3)+(110\times1)+(340\times1)+(160\times1)+(360\times1)$
$+(120\times3)$

$=\$2,380$

b) Least Cost method

Destinations

	D1	D2	D3	Dummy	Supply
S1	3	2	130 1	220 0	~~350~~ ~~130~~
S2	450 1	1	2	0	~~450~~
S3	2	500 1	20 1	0	~~525~~ ~~20~~
S4	10 1	2	330 3	0	~~340~~ ~~330~~
Demand	~~460~~ ~~10~~ 1	~~500~~	~~480~~ ~~350~~ ~~330~~	~~220~~	1660

initial feasible cost :

$(220\times 0)+(130\times 1)+(450\times 1)+(500\times 1)$
$+(20\times 1)+(10\times 1)+(330\times 3)$
$=\underline{\$2100}$

c) Vogel's Approximation Method

Destination

	D1	D2	D3	Dummy	Supply
S1	3	2	130 1	220 0	~~350~~ ~~130~~
S2	450 1	1	2	0	~~450~~
S3	2	170 1	350 1	0	~~520~~ ~~170~~
S4	10 1	330 2	3	0	~~340~~ ~~10~~
	~~460~~ ~~10~~	~~500~~ ~~330~~	~~480~~ ~~350~~	~~220~~	1660

R.D

1	1	—	—	—	—	—
1	1	1	—	—	—	—
1	1	1	1	1	—	—
1	1	1	1	1	1	1

C.D

1	1	1	0
1	1	1	—
1	1	1	—
1	1	2	—
1	1	—	—
1	2	—	—
1	—	—	—

initial feasible cost:

$(220\times 0)+(130\times 1)+(450\times 1)+(170\times 1)+(350\times 1)+(10\times 1)+(330\times 2)$
$=\underline{\$1770}$

Getting optimal solution using VAM

	$v_1 = 0$	$v_2 = 1$	$v_3 = 1$	$v_4 = 0$
$u_1 = 0$	3	2	130 1	220 0
$u_2 = 1$	450 1	1	2	0
$u_3 = 0$	2	170 1	350 1	0
$u_4 = 1$	P 1	330 2	3	0

$m+n-1 = 7$

$A + n - 1 = 7$

$$m+n-1 = 4+3-1 = 6$$

$$u_i + v_j = C_{ij}$$

$$P_{ij} = U_i + V_j - C_{ij}$$

$C_{11} = 0+0-3 = -3$

$C_{12} = 0+1-2 = -2$

$C_{22} = 1+1-1 = 0$

$C_{23} = 1+1-2 = -2$

$C_{24} = 1+0-0 = \underline{1}$

$C_{31} = 0+0-2 = -2$

$C_{34} = 0+0-0 = 0$

$C_{43} = 1+1-3 = -1$

$C_{44} = 1+0-0 = \underline{1}$ can't form loop

Q2.)

i.) A Transportation problem is a type of linear programming problem that seeks to minimize cost of distributing a product from multiple sources to multiple destinations, while meeting supply constraints at the sources and demand requirements at the destinations.

ii)

North West Corner Rule

Destination

origin	1	2	3	Supply
1	50 / 2	7	4	~~50~~
2	20 / 3	60 / 3	1	~~80~~ ~~60~~
3	5	30 / 4	40 / 7	~~70~~ ~~40~~
4	1	6	140 / 2	140
Demand	~~70~~ ~~20~~	~~90~~ ~~30~~	~~180~~ ~~140~~	340

Total Initial feasible cost:

$(50\times2)+(20\times3)+(60\times3)+(30\times4)$
$+(40\times7)+(140\times2)$
$= \$1,020$

Applying MODI for optimal solution

$v_1=2 \quad v_2=2 \quad v_3=5$

$u_1=0$	50 / 2	7	4
$u_2=1$	20 / 3	60(-) / 3	+ / 1
$u_3=2$	5	30(+) / 4	40(-) / 7
$u_4=-3$	1	6	140 / 2

$u_i+v_j=C_{ij}$

Penalties

$P_{ij}=U_i+V_j-C_{ij}$

$C_{12} = 0+2-7=-5$
$C_{13} = 0+5-4=1$
$C_{23} = 1+5-1=5$
$C_{31} = 2+2-5=-1$
$C_{41} = -3+2-1=-2$
$C_{42} = -3+2-6=-7$

$v_1=2 \quad v_2=2 \quad v_3=0$

$u_1=0$	50 / 2	7	4
$u_2=1$	20 / 3	20 / 3	40 / 1
$u_3=2$	5	70 / 4	7
$u_4=2$	1	6	140 / 2

$C_{12} = 0+2-7=-5$
$C_{13} = 0+2-4=-2$
$C_{31} = 2+2-5=-1$
$C_{32} = 2+0-7=-5$
$C_{41} = 2+2-1=\underline{3}$ *Can't form loop
$C_{42} = 2+2-6=-4$

Least Cost Method

	Destination			
Origin	1	2	3	Supply
1	2	20 7	30 4	~~50~~ 20
2	3	3	80 1	~~80~~
3	5	70 4	20 7	~~70~~
4	70 1	6	70 2	~~140~~ 70
Demand	~~70~~	~~90~~ 20	~~180~~ 100 30	340

Total cost:
$(20\times7)+(30\times4)+(80\times1)+(70\times4)+(70\times1)$
$+(70\times2)$
$=\underline{\$830}$

Vogel's Approximation Method

	Destination				R.D					
Origin	1	2	3	Supply						
1	50 2	7	4	~~50~~	2	-	-	-	-	-
2	3	3	80 1	~~80~~	2	2	-	-	-	-
3	5	70 4	7	~~70~~ ~~140~~	1	1	1	1	4	4
4	20 1	20 6	100 2	~~140~~ 40 20	1	1	1	5	6	-
Demand	~~70~~ ~~20~~	~~90~~ 70	~~180~~ ~~100~~	340						

C.D

	1	2	3
	1	1	1
	2	1	1
	4	2	5
	4	2	-
	-	2	-
	-	-	-

Total cost:
$(50\times2)+(80\times1)(70\times4)+(20\times1)+(20\times6)$
$+(100\times2)$
$=\underline{\$800}$

SECTION C

Q1) Mean interval time : 8 minutes
Mean service time : 4 minutes

i) Mean service rate and Mean arrival rate.
→ Arrival rate (λ) = $\frac{1}{8}$ customers per minute.
→ Service rate (μ) = $\frac{1}{4}$ customers per minute.

ii) Traffic intensity (ρ)

$$\rho = \frac{\lambda}{\mu} = \frac{1/8}{1/4} = \frac{1}{2} = 0.5$$

$$\rho = 0.5$$

iii) Mean time a customer spends in the Queue and in the system.
- Mean time in the system (W)

$$W = \frac{1}{\mu - \lambda} = \frac{1}{\frac{1}{4} - \frac{1}{8}} = \frac{1}{\frac{1}{8}} = 8 \text{ minutes.}$$

- Mean time in the queue (W_q)

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{1/8}{\frac{1}{4}(\frac{1}{4} - \frac{1}{8})} = \frac{1/8}{1/32} = \frac{1}{8} \times 32 = 4$$

$= 4$ minutes

iv) Expected Number of Customers in the Queue and in the System
→ Expected number of customers in the system (L)

$$L = \frac{\lambda}{\mu - \lambda} = \frac{1/8}{\frac{1}{4} - \frac{1}{8}} = \frac{1}{8} \times 8 = 1 \text{ customer}$$

→ Expected number of customers in the queue (L_q)

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\left(\frac{1}{8}\right)^2}{\frac{1}{4}\left(\frac{1}{4}-\frac{1}{8}\right)} = \frac{\frac{1}{64}}{\frac{1}{32}} = \frac{1}{2} = 0.5$$

$= 0.5$ customers

v) Probability of having at most 4 customers in the system.

$$P(n) = (1-\rho)\rho^n$$

$$P(\text{at most 4 customers}) = \sum_{n=0}^{4} (1-\rho)\rho^n$$

$$= (1-0.5)(0.5^0 + 0.5^1 + 0.5^2 + 0.5^3 + 0.5^4)$$

$$= 0.5 \times 1.9375$$

$$= 0.96875$$

$$= 96.88\%$$

Q2) Mean inter-arrival time = 10 minutes

Arrival rate $\lambda = \frac{1}{10}$ customers per minute

Mean service time = 3 minutes

Service rate $\mu = \frac{1}{3}$ customers per minute

i) $\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{10}}{\frac{1}{3}} = \frac{1}{10} \times \frac{3}{1} = 0.3$

$= 30\%$

ii) $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\left(\frac{1}{10}\right)^2}{\frac{1}{3}\left(\frac{1}{3}-\frac{1}{10}\right)} = \frac{\frac{1}{100}}{\frac{7}{90}} = \frac{9}{70}$

$= 0.129$ customers

Queuing Model

ii) $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

$W_q = 3$

$3 = \frac{\lambda}{\frac{1}{3}(\frac{1}{3}-\lambda)}$ $\quad 3 = 3 \times \frac{\lambda}{\frac{1}{3}-\lambda}$

$1 = \frac{\lambda}{\frac{1}{3}-\lambda}$ $\quad \frac{1}{3} - \lambda = \lambda$

$\frac{\frac{1}{3}}{2} = \frac{2\lambda}{2}$ $\quad \lambda = \frac{1}{6}$

$= \frac{1}{6} - \frac{1}{10} = \frac{1}{15}$ customers per minutes

Arrival rate must increase by $\frac{1}{15}$ customers per minute

Q3) Mean arrival rate $(\lambda) = 12$ trucks per day
Mean service rate $(\mu) = 18$ trucks per day

i) $P_0 = 1 - \rho$

$\rho = \frac{\lambda}{\mu} = \frac{12}{18} = 0.6667$

$P_0 = 1 - 0.6667 = 0.3333$

$= 33.33\%$

ii) $L_q = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

$= \dfrac{12^2}{18(18-12)} = \dfrac{144}{108} = 1.333$ trucks

iii) $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$= \dfrac{12}{18(18-12)} = \dfrac{12}{108} = 0.1111$ days

iv) $P(\text{waiting}) = \rho = \dfrac{\lambda}{\mu} = \dfrac{12}{18} = 0.6667$

$= 66.67\%$

SECTION D

Q1 Find the dual program of the following linear programming problem

Maximize $Z = 5x_1 - 2x_2$

sub to

$$3x_1 + 2x_2 \geq 16$$
$$x_1 - x_2 \leq 4$$
$$x_1 \geq 5$$

$x_1 \geq 0$ x_2 is unconstrained.

Soln.

let $x_2 = x_3 - x_4$ Max $\rightarrow \leq$

Rewrite The problem.

$$\text{Max } Z = 5x_1 - 2(x_3 - x_4)$$

$$\text{sub to } -3x_1 - 2(x_3 - x_4) \leq -16$$
$$x_1 - (x_3 - x_4) \leq 4$$
$$-x_1 \leq -5$$

$$x_1, x_3, x_4 \geq 0.$$

$$\text{Max } Z = 5x_1 - 2x_3 + 2x_4$$

sub to

$$\left.\begin{aligned} -3x_1 - 2x_3 + 2x_4 &\leq -16 \\ x_1 - x_3 + x_4 &\leq 4 \\ -x_1 &\leq -5 \end{aligned}\right\} \begin{aligned} a \\ b \\ c \end{aligned}$$

$$x_1, x_3, x_4 \geq 0.$$

Dual is :

let a, b, and c be dual variables of the dual problem.

$$\therefore \text{ Min } V = -16a + 4b - 5c$$
$$-3a + b - c \geq 5$$
$$-2a - b + 0c \geq -2$$
$$2a + b + 0c \geq 2$$
$$a, b, c \geq 0$$

$$\text{Min } V = -16a + 4b - 5c$$

sub to

$$-3a + b - c \geq 5$$
$$\left.\begin{array}{l} -2a - b \geq -2 \\ -2a - b \leq -2 \end{array}\right\}.$$
$$a, b, c \geq 0.$$

Replace Them equal sign.

$$\text{Min } V = -16a + 4b - 5c$$

Sub to

$$-3a + b - c \geq 5$$
$$-2a - b = -2$$
$$a \geq 0,\ b \geq 0,\ c \geq 0.$$

Q2 Find the dual program of the following linear programming problem.

Minimize $Z = 30x_1 - 50x_2 + 10x_3$

Subject to

$$3x_1 + 2x_2 - x_3 \geq 44$$
$$x_1 - x_2 + x_3 = 7$$

x_1 is unconstrained, $x_2 \geq 0$, $x_3 \geq 0$.

Soln.

let $x_1 = (x_4 - x_5)$

Rewrite the problem.

Min $Z = 30(x_4 - x_5) - 50x_2 + 10x_3$

Sub to

$$3(x_4 - x_5) + 2x_2 - x_3 \geq 44$$
$$x_4 - x_5 - x_2 + x_3 \geq 7$$
$$x_4 - x_5 - x_2 + x_3 \leq 7.$$
$$x_2, x_3, x_4, x_5 \geq 0.$$

$$\text{Min } Z = 30x_4 - 30x_5 - 50x_2 + 10x_3$$

sub to

$$3x_4 - 3x_5 + 2x_2 - x_3 \geq 44 \quad a$$
$$x_4 - x_5 - x_2 + x_3 \geq 7 \quad b$$
$$x_4 - x_5 - x_2 + x_3 \leq 7 \quad c$$

$$x_2, x_3, x_4, x_5 \geq 0.$$

~~Dual.~~
~~let a, b, and c be the dual variable~~

$$\cancel{\text{Max } w = 44a + 7b}$$

$$\text{Min } Z = 30x_4 - 30x_5 - 50x_2 + 10x_3$$

sub to

$$30x_4 - 30x_5 + 2x_2 - x_3 \geq 44$$
$$x_4 - x_5 - x_2 + x_3 \geq 7$$
$$-x_4 + x_5 + x_2 - x_3 \geq -7$$

$$x_2, x_3, x_4, x_5 \geq 0.$$

Dual.
let a, b, c be dual variable

$$\text{Max } V = 44a + 7b - 7c$$

sub to

$$3a + b - c \leq 30$$
$$-3a - b + c \leq -30$$
$$2a - b + c \leq -50$$
$$-a + b - c \leq 10$$

let $d = b - c$

$$\text{Max } v = 44a + 7d$$
$$\left.\begin{array}{l} 3a + b - c \leq 30 \\ 3a + b - c \geq 30 \end{array}\right\}$$
$$2a - b + c \leq -50$$
$$-a + b - c \leq 10.$$

$$\text{Max } V = 44a + d$$
$$3a + d = 30$$
$$2a - d \leq -50$$
$$-a + d \leq 10.$$

$a \geq 0$, d is unrestricted / unconstrained

Q3. Find the dual program of the following linear programming problem.

$$\text{Min } p = 16x - 2y - 5z$$
subject to
$$x + 4y - z \geq 120$$
$$x + y + 3z \leq 130$$

$x \geq 0$, $y \geq 0$, z is unconstrained

soln.

$$\text{Min } p = 16x - 2y - 5z$$
sub to
$$x + 4y - z \geq 120$$
$$-x - y - 3z \geq -130$$

let $z = (a - b)$

$\therefore$

$$\text{Min } P = 16x - 2y - 5(a-b)$$
sub to
$$x + 4y - (a-b) \geq 120$$
$$-x - y - 3(a-b) \geq -130$$

$x, y, a, b, \geq 0$

$$\text{Min } P = 16x - 2y - 5a + 5b$$
sub to
$$x + 4y - a + b \geq 120$$
$$-x - y - 3a + 3b \geq -130$$

Dual is:

let c and d be dual variables

$$\text{Max } w = 120c - 130d$$

sub to

$$c - d \leq 16$$
$$4c - d \leq -2$$
$$-c - 3d \leq -5$$
$$c + 3d \leq 5$$

$$\text{Max } w = 120c - 130d$$

Sub to

$$c - d \leq 16$$
$$4c - d \leq -2$$
$$\left.\begin{array}{l} -c - 3d \leq -5 \\ -c - 3d \geq -5 \end{array}\right\} =$$

$$\text{Max } w = 120c - 130d$$

sub to

$$c - d \leq 16$$
$$4c - d \leq -2$$
$$-c - 3d = -5$$
$$c, d \geq 0.$$

SECTION E

Q1 i) expected ordering Quantity / fixed order:

Economic Order Quantity, $EOQ = \sqrt{\frac{2DS}{H}}$

where:

$H \leftarrow$ holding cost/unit = \$ 5

$D \leftarrow$ demand/year = 20 000 units.

$S \leftarrow$ ordering/setup cost = \$ 100 per order

$\therefore EOQ = \sqrt{\frac{2 \times 20000 \times 100}{5}} = 894.427191 \approx 894$

This is the expected ordering quantity / fixed order.

ii) Computing Retailer's Economic Order Quantity -

- We're working with an interest rate of 10% p.a, and £ 100 total cost/unit

$\therefore$ with storage cost/year $\leftarrow$ £20

Holding cost H or C $\leftarrow$ storage cost + cost from interest.

$\therefore H = £20 + (0.1)(100) = £30,$

Retailer Demand/p.a $\leftarrow D = 200$ units

ordering cost/setup, $S = £35$

Economic Order Quantity, $EOQ = \sqrt{\frac{2DS}{H}}$

$EOQ = \sqrt{\frac{2 \times 200 \times 35}{30}} = \sqrt{\frac{1400\cancel{0}}{3\cancel{0}}} = \sqrt{\frac{1400}{3}} = 21.60246$

Economic Order Quantity is roughly 22 units

iii) Computing new EOQ from known:

Economic order Quantity, $EOQ = \sqrt{\frac{2DS}{H}}$

where demand is not given so let's use D

$S \leftarrow$ ordering/setup cost _ we use S

Let C be unit cost:

$\therefore$ for EOQ 120 - holding cost $H_1 = 0.2C$

for new EOQ - holding cost $H_2 = 0.15C$

Comparing ratios:

$\frac{EOQ_1}{EOQ_2} = \sqrt{\frac{H_1}{H_2}}$; $EOQ_2 = EOQ_1 \left(\sqrt{\frac{H_1}{H_2}}\right)$

$\sqrt{0.2}$ $= 138.564064$

Q2 (i) Three Main Reason: For: Holding Inventory

① Demand uncertainty - holding inventory allows you to deal with fluctuations in demand effectively. This can help you mitigate loss of sales.

② Helps in Tracking Lead Time: Inventory management allows you to manage delays between placing an order and the receipt of it. The lead time can vary but if tracked well, can ensure continuous supply of products.

③ Economies of Scale: businesses/organizations can leverage on purchasing goods in large quantities through inventories. This helps minimize unit costs and setup.

Reasons for: Why Only Minimal Inventories are Held:

① Minimizing Spoilage/Obsolescence: products with short life span and perishable are better handled using minimal inventories.

② Minimizing Risk of Damage/Overstocking: damage that comes from collisions in overstocked goods can lead to damage hence incurring loss on large scale.

③ Reducing Holding Costs: holding cost can come in form of storage costs and can be reduced by use of minimal inventory.

④ Capital Tie-ups: inventory represents a significant investment. Holding excess inventory ties up capital that could have been used for other things. Holding minimal inventory alleviates this problem.

Q2 (ii) Economic Order Quanty (EOQ) and FNSD
The Economic Order Quantity refers to the optimal quantity that minimises the total cost for an inventory - consisting of ordering / setup and holding cost.
EOQ is based on demand, ordering and holding costs.

FNSD — Fixed Number of Supplier Deliveries -
This is a technique used to determine the optimal number of deliveries per year. and comprises transport expenses, ordering cost etc.
FNFD analysis enables the concerned party to find a balance between minimizing costs and reliability in inventory supply.
FNSD analysis categorises inventory into
a) Fast moving to meet high demand.
b) Normal moving for regular demand.
c) Slow moving for low demand
d) Dead stock for no demand at all.

Q2 (iii) Steps in Calculation of EOQ for:
discounted quantities, ABC Analysis in inventory control
- This technique typically involves the steps listed below.
① Compute the EOQ Ignoring The Discount.
You determine the optimal order quantity but not considering the discounts
② Evaluate Discount Breakpoints: identify points where the discount makes it worthwhile to order a larger quantity.
③ Compute the Total Costs: for each discount breakpoint determine the holding costs and ordering costs and cost of units / purchase.
④ Select the optimal quantity - this is the quantity that minimises the total costs.

ABC analysis ← next page.

ABC Analysis is a way of categorizing inventory items based on importance and uses alphabets in this case to denote each category.

① A Items: these are high value items that account for a significant portion of the inventory costs. This require close monitoring

② B Items: moderately important items with medium value.

③ C items: these are low value items and are managed using simpler systems compared to categories in A and B items.

Q3 Super market TOYS:

Delivery in one batch is over 2000 so we use price of 35/= from table:

Order quantity	Price per unit
1 to 1000	40/=
1000 to 2000	38/=
over 2000	35/=

- For this problem we consider only holding and ordering cost.

a) $EOQ = \sqrt{\frac{2DS}{H}}$

where D ← delivery = 6000

S ← ordering cost (can be C) = 80

H ← holding cost (0.15 (unit price) = 0.15 × 35 = 5.25/=

$\therefore EOQ = Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 6000 \times 80}{5.25}} = 427.6179871 \simeq 428$

Economic Order quantity is 428 (units)

b) Optimum Total cost; we still consider only the last row since we are doing instantaneous delivery

Total cost, TC = ordering cost + holding cost $= \frac{D}{Q^*}S + \frac{Q^*}{2}H$

[next page]

from $TC = \frac{D}{Q^*}S + \frac{Q^*}{2}H$

where $Q^* = EOQ = 428$, $D = 6000$, $S = 80$, $H = 5.25$

$$\text{Total cost} = \frac{6000}{428}(80) + \frac{428}{2}(5.25)$$

$$= 1121.495327 + 1123.5$$

$$= 2244.995327 \simeq 2244.995$$

$$\simeq 2245/=$$

Optimum total cost is 2,245.00/=

(c) Number of orders per year;

$$\text{No. of orders} = \frac{\text{Demand}}{EOQ} = \frac{D}{Q^*} = \frac{6000}{428}$$

$$= 14.01869159$$

$$\simeq 15 \text{ orders}$$

We round up for orders that have fractions parts.

(c) Time between orders in days:

* No working days given so we take 365

$$\text{Time between orders} = \frac{\text{working days}}{\text{No. of orders}}$$

$$= \frac{365}{15}$$

$$= 24.3333 \text{ (days)}$$

Members:

SCT211-0848/2018	Jany Muong
SCT211-0079/2022	Joram Kireki
SCT211-0504/2021	Gatmach Yuol
SCT211-0003/2022	Josphat Waweru
SCT211-0535/2022	Akech Atem
SCT211-0070/2022	Vincent Ohanga