

SMA 2305: COMPLEX ANALYSIS

CAT 1

1 HOUR

- a) Briefly explain the following terms
 (i) A complex number (2 Marks)
 (ii) Analytic function (2 Marks)
- b) Evaluate $\frac{1+i}{2-3i} + \frac{4-7i}{3+2i}$ (4 Marks)
- c) Solve the equation $z^2 - 4z - 53 = 0$ expressing the roots in the form $a + ib$ where $a, b \in \mathbb{R}$. Verify that the sum of the roots is 4 and the product is 53. (4 Marks)
- d) Simplify $\frac{(\cos 3\theta + i \sin 3\theta)(\cos 5\theta - i \sin 5\theta)}{(\cos 2\theta - i \sin 2\theta)(\cos 7\theta + i \sin 7\theta)}$ (4 Marks)
- e) Let $u(x, y) = e^{-x}(x \sin y - y \cos y)$
 (i) Show that $u(x, y)$ is harmonic (3 Marks)
 (ii) Find $v(x, y)$ such that $f(x, y) = u(x, y) + v(x, y)$ is analytic (3 Marks)
- f) Prove that $\sin z = \sin x \cosh y - i \cos x \sinh y$ (1 Marks)
- g) Find the poles of $\frac{2z^2 + 5}{z^4 + 16}$ (4 Marks)

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