Past pps- Complex Analysis 1 10-Mar-2021 13-18-42

Actuarial science (Jomo Kenyatta University of Agriculture and Technology)

SMA 2305: COMPLEX ANALYSIS CAT II BFE 3.1

(a) Show that the function $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate function v.

(b) Calculate the residues of the following function at all singularities

$$f(z) = \frac{z}{(z-1)(z^2+2)}$$

(c) Find the Laurent series expansion of the following function about $z = 3$

$$f(z) = \frac{1}{z^2(z-3)^2}$$

(d) Evaluate the following integral

cauchy integral formula

$$\int_{1+i}^{2+4i} z^2 dz$$

Along:

(i) The parabola $x = t, y = t^2, 1 \le t \le 2$

(ii) Straight lines from $1 + i$ to $2 + i$ and then to $2 + 4i$

(a^2+b)

$z \quad 4+a$

(a^2+1)

$(a+1)^2$

$a(a-1)-1(a-1)$

$a^2-a-a+1$

$(a+1)(a-1)$

$a(a-1)+1(a-1)$

$a^2-a+a-1$

$z^2=-2$

$(2+2)^2 = 4+2$

$1+x+nx+\frac{n(n-1)x^2}{2!}$

$+\frac{n(n-1)(n-2)x^3}{3!}$

$(z+\sqrt{2})(z-\sqrt{2})$

$z(z-\sqrt{2})+\sqrt{2}(z-\sqrt{2})$

$z^2 - z\sqrt{2}+\sqrt{2}$

5: COMPLEX ANALYSIS CAT I

Evaluate:

i. $(\sqrt{3} - i) + i(1 - i\sqrt{3})$

ii. $(2 - 3i)/(-2 + i)$

(b) Solve $z^3 - 1 = i\sqrt{3}$

$z^3 = 1 + i\sqrt{3}$ $\quad z = (1 + i\sqrt{3})^{1/3}$

(c) Use the Cauchy-Riemann equations to show that $f(z) = x^3 - 4iy$ is not analytic

(d) Show that $\lim_{z \to 0} \left(z/\bar{z} \right)$ does not exist

(e) If $z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$, evaluate $(\bar{z})^5$

(f) If $z = x + iy$, find the real and imaginary parts of $w = 1/\bar{z}$

(g) Express $-1 + i\sqrt{3}$ in polar form

$(\sqrt{3} - i) + i + \sqrt{3}$

$2\sqrt{3}$

$z^3 = i\sqrt{3} + 1$

$z = \sqrt[3]{i\sqrt{3} + 1}$

11.14

$z = (i\sqrt{3} + 1)^{1/3}$

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY

University Examinations 2018/2019

THIRD YEAR SUPPLEMENTARY/SPECIAL EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE FINANCIAL ENGINEERING

SMA 2305 COMPLEX ANALYSIS I

DATE: JUNE 2019 **TIME 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a. Construct the analytic function $f(z)$ for which the real part is $e^x \cos y$ (5 marks)

b. Solve the equation $z^4 = 1 + i\sqrt{3}$ (4 marks)

c. Determine and sketch the set represented by $\arg\left\{\frac{z-1}{z+2}\right\} = \frac{\pi}{2}$ (4 marks)

d. Find the singularities and zeros of the function

$$\frac{z-1}{z^4 - z^2(1+i) + i}$$ (5 marks)

e. Using Cauchy's integral theorem, evaluate the contour integral

$\oint_C \frac{3z^2 + z + 1}{(z^2-1)(z+3)}dz$ where C is the circle $|z| = 2$ (12 marks)

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY

OF

AGRICULTURE AND TECHNOLOGY

University Examinations 2019/2020

THIRD YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN BAS,BOR,BST,BFE, BBS AND MCS.

SMA 2321 NUMERICAL ANALYSIS I

DATE: DECEMBER, 2019 **TIME 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

Question One: 30 Marks

a) Evaluate the following and discuss the error involved.

$$\frac{15.36+27.1-1.672}{2.36\times1.043}.$$ (6 Marks)

b) Evaluate $3x^3+5x^2-2x+9$ at $x=\frac{1}{2}-\sqrt{2}\,i$ correct to 4 dp. (6 Marks)

c) Using the Lagrange's interpolation formula, find the value of y corresponding to x =10 from the table:

x	5	6	9	11
y	12	13	14	16

(6 Marks)

$\sum \phi_i(x) = \frac{x-x_0}{x_0-x_1}$

$x_{10} = x$

QUESTION TWO (20 MARKS)

a. Using complex variables, evaluate the real integral $\int_0^{2\pi} \frac{\cos\theta}{3+\sin\theta}\,d\theta$ (10 marks)

b. Determine the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at each of its poles in the finite z-plane (10 marks)

QUESTION THREE (20 MARKS)

a. Evaluate by using contour integration, the real integral

$$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(x^2+4)}dx$$

b. Determine the Laurent Series expansion $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $|z| > 3$

QUESTION FOUR (20 MARKS)

a. Find the image in the w-plane of the circle $|z| = 2$ in the z-plane under the bilinear mapping $z = \frac{z-i}{z+i}$. Sketch the curves in both z-plane and w-plane and shade the region in the w-plane corresponding to the region inside the circle in the z-plane

b.

i. Show that if the function $f(z) = u + iv$ is analytic, then it is harmonic

ii. If $f(z) = u + iv$, where $u = 2x - x^3 + 3xy^2$ show that u is harmonic. Hence or otherwise find v that will make the function $f(z)$ harmonic

d) Estimate $\int_{1.5}^{3.0} \frac{1}{x-1}dx$, to 4 decimal places using Simpson's rule with n = 5. Obtain a bound for the truncation error. (6 Marks)

e) Write the equation solvable by the iterative method $x_{n+1} = \sqrt{\frac{5x_n + 1}{2}}$. Show that the equatio has a root between 2 and 3. With $x_0 = 2$, approximate the root correct to 2 decimal place

(6 Marks)

$(x-1)^{-1}$ $(x-1)^{-2}$

Question Two: 20 Marks

a) Show that the equation $3x^3 - 5x^2 - 4x + 4 = 0$ has a root in the interval $[0,1]$. Use the method of Bisection to find an interval of length $\frac{1}{8}$ containing the root. (8 Marks)

b) Determine the quadratic factor of $x^3 - 3.1x^2 + 3.7x - 1.6$ which is close to $x^2 - 2x + 1.5$. Perform only two trials. Determine the zeros of the function. (12 Marks)

$-(x-1)^{-2}$ $2(x-1)^{-3}$ $2(x-1)^{-3}$ $-6(x-1)^{-4}$

Question Three: 20 Marks

a) Using the Newton –Raphson method with synthetic division, approximate correct to 3dp the root near 0 ,of the equation $x^4 - 4x^3 + 7x^2 - 5x - 2 = 0$.

(10 Marks)

b) Find the number of integration points needed to evaluate $\frac{1}{\sqrt{2\pi}}\int_0^2 e^{-x^2/2}dx$, with a truncation error less than 10^{-7} using the trapezoidal rule. (10 Marks)

Question Four: 20 Marks

a) Find a real root of the equation $xe^x = 2$ using the rule of false position correct to 4 decimal places. (8 Marks)

b) Show that the equation $e^x - x - 5 = 0$ has a root between -5 and -4 and another between 1 and 2. Show that $x_{n+1} = \ln(x_n + 5)$ is a convergent iterative formula for getting the root near 2. Use the iterative formula and Aitken's $\delta^2 - process$ to obtain this root correct to 6 decimal places.

(12 Marks)

-5 -4 -3 -2 -1 0 1 2 0.3690 $-6(x-1)^{-4}$ $\frac{1}{x+5}$

-9.993 -8.98 -7.95 -6.5.632 -6.665 -4 -3.282 $24(x-1)^{-5}$ $\frac{1}{6.5}$

BFE & MCS DEC 2017

MA 2305 COMPLEX ANALYSIS CAT II

answer in

a. Show that the function $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate v.

b. If $f(z) = \dfrac{z}{(z-1)(z+2)^2}$, identify the singularities and calculate the corresponding residues.

c. Find the Laurent series for the function $f(z) = \dfrac{1}{z^2(z-3)^2}$ about the singular point $z = 3$

d. Evaluate the integral

$$\int_{i+1}^{2+4i} z^2 dz$$

i. Along the parabola $x = t, y = t^2, 1 \le t \le 2$

ii. Along straight lines from $1 + i$ to $2 + i$ and then to $2 + 4i$.

i. $z = {}^{i}/(1+i)$

ii. $z = \sqrt{3} + i$

25/4

232.5

142.5

-37.5

52.5

277.5

[4 marks]

MA 2305: COMPLEX ANALYSIS I, CAT I, OCTOBER 2018

(a) Given that $z_1 = 6 + 7i, z_2 = 4 - 2i, z_3 = 2 - i\sqrt{3}$. Evaluate the following giving your answer in the form $x + iy$.

i. $|z_1 z_2|$

ii. z_1/z_3

(b) If $z = \frac{-1}{2} + \frac{\sqrt{3}}{2} i$ evaluate $(\bar{z})^4$ $r^n \cos n\theta + i \sin n\theta$

(c) Find the real and imaginary parts of w given that

$$w = \frac{5}{(1-i)(2-i)(3-i)}$$

(d) If $z = -2\sqrt{3} - 2i$ find $z^{1/4}$ $r^{1/n}$ $\cos\frac{\theta + 2\pi k}{n} + i \sin\frac{\theta + 2\pi k}{n}$

$\tan^{-1}\left(\frac{-\cancel{2}}{-\cancel{2}\sqrt{3}}\right)$

(e) Find the principal argument and exponential form of

i. $z = {}^{i}/_{(1+i)}$ $e^{i\theta}$ Arg.

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

ii. $z = \sqrt{3} + i$

25/4

232.5

277.5 −37.5 52.5 142.5

SMA 2305: COMPLEX ANALYSIS CAT I BFE & MCS DEC 2017

SMA 2305: COMPLEX ANALYSIS I, OCTOBER 2016
BFE/ MCS YEAR 3

1. If $z_1 = -2+3i$ and $z_2 = 1-4i$, find

(a) $z_1 + 2z_2$ (b) $z_2 - iz_1$ (c) $z_1 z_2$ (d) $\frac{z_1}{z_2}$

2. Let $z_1 = 1+\sqrt{3}i$ and $z_2 = 3-\sqrt{3}i$.

(a) By expressing z_1 and z_2 in polar form, find $z_1 z_2$ and $\frac{z_1}{z_2}$.

(b) Find the modulus and the principal value of the argument of $\left(\frac{iz_2}{z_1}\right)^2$ To be revised

3. Let $z = \frac{1-i}{2+i}$. Find $\mathrm{Re}(z)$.

4. Solve the equation $z^5 = 1+i$

5. Show that $\lim_{z\to 0} \bar{z}$ does not exist.

b) Solve $z^3 = -8i$ [4 marks]

c) Evaluate

i. $|(1+i)(2+i)|$ [2 marks]

SMA 2305: COMPLEX ANALYSIS CAT I BFE & MCS DEC 2017

(a) Express each of the following functions in the form $u(x, y) + iv(x, y)$

(i) z^3

(ii) $1/(1 - z)$

(b) Solve the equation $z^4 + i = 0$

(c) Show that the function $f(z) = x - iy$ is not differentiable anywhere in the complex plane

(d) Show that the function $f(z) = \frac{1}{z}$ is analytic everywhere except at $z = 0$

(e) Given that $z = x + iy$. Show that $x = \frac{z+\bar{z}}{2}$ and $y = \frac{z-\bar{z}}{2i}$

(f) Find the principal argument and exponential form of

i. $z = {}^{i}/_{(1 + i)}$

ii. $z = \sqrt{3} + i$

$\bar{z} = z^{-1}$

$-i^2 \cdot -1 \times -1$

UNIVERSITY EXAMINATION 2017/2018

SUPPLIMENTARY/SPECIAL EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE, MATHEMATICS, MATHEMATICS AND COMPUTER SCIENCE

SMA 2305 COMPLEX ANALYSIS 1

DATE: OCTOBER 2018 **2HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE (30MARKS)

a) Compute $\lim\limits_{z\to 2+i}\left(\dfrac{z^2-(2+i)^2}{z-(2+i)}\right)$ [3 marks]

b) Solve $z^3=-8i$ [4 marks]

$\dfrac{8-2i-3i^2}{4-i^2}$

c) Evaluate

i. $|(1+i)(2+i)|$ [2 marks]

ii. $\mathrm{Re}\left(\dfrac{4-3i}{2-i}\right)$ $\dfrac{(4-3i)(2+i)}{(2-i)\cdot(2+i)} = \dfrac{8+4i-6i-3i^2}{4+2i-2i-i^2}$ [3 marks]

$4-i^2$

$\dfrac{x_1y_2+x_1y_2}{x_2^2+y_2^2}$

d) (i) Use Cauchy's integral theorem to evaluate $\int_C \dfrac{e^z}{(z^2-1)}dz$, *where* $C:|z|=1$ [3 marks]

(ii) Describe the poles of the function $f(z)=(z^2-\pi z)\sin z$ [2 marks]

e) Evaluate the integral $\int_0^\infty xe^{-xz}dx$ when $R(z)<0$ [4 marks]

f) Show that the function $f(z)=x-2iy$ is not differentiable anywhere (4 marks)

g) (i) Express $e^{\frac{i\pi}{2}}$ in the form $a+ib$ where a and b are constants [2 marks]

(ii) Find all values of z if $z=(-i)^i$ [3 marks]

QUESTION TWO (20MARKS)

a) Verify that $u=x^2-y^2-y$ is harmonic in the whole complex plane and find a harmonic conjugate function v of u [7 marks]

b) Find the principal argument and exponential form of $z = {}^{i}/{(1+i)}$ [3 marks]

c) Use Cauchy integral formula to evaluate $\oint_c \frac{e^{2z}}{(z+1)^4}\,dz$ where c is the circle $|z| = 3$ [8 marks]

d) Explain what you understand by the following terms [2 marks]

i. A singular point
ii. Removable singularity

QUESTION THREE (20MARKS)

✓ a) State Cauchy Residue theorem, hence using Cauchy Residue theorem evaluate

$$\int_0^{2\pi} \frac{d\theta}{3\cos\theta + 5}$$

[10 marks]

✓ b) Use the Cauchy-Riemann Equations to show that the function $f(z) = \frac{x}{x^2+y^2} + i\frac{y}{x^2+y^2}$ is not analytic.

[5 marks]

c) Find Mobius transformation which maps points $(-1,0,1)$ into $(1,\infty,i)$ [5 marks]

QUESTION FOUR (20MARKS)

a) Find the Laurent series of the function $f(z) = \frac{z+4}{z^2(z^2+3z+2)}$ about all singularities. [6 marks]

b) Find the poles and residues of the function $f(z) = \frac{1}{z^4+5z^2+6}$ [5 marks]

c) Determine $\log(-1+i\sqrt{3})$ [4 marks]

d) Find the poles and residues of the function $f(z) = \frac{1}{z^4+5z^2+6}$ [5 marks]

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JOMO KENYATTA UNIVERSITY
OF
AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2018/2019

THIRD YEAR EXAMINATION FOR BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING

SMA 2305: COMPLEX ANALYSIS I

DATE: DECEMBER 2018 **TIME: 2 HOURS**

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE

a) Solve for x and y if

$$\left(\frac{1+i}{1-i}\right)^2 + \frac{1}{x+iy} = 1+i$$

[4 marks]

b) If $z = -2\sqrt{3} - 2i$ find $z^{1/4}$ $= r^{1/n}\left(\cos\frac{\theta+\pi k}{n} + i\sin\frac{\theta+\pi k}{n}\right)$ [6 marks]

c) Evaluate

$$\lim_{z\to i}\frac{(3+i)z^4 - z^2 + 2z}{z+1}$$

leaving your answer in the from $a+bi, a, b \in \mathbb{R}$ [5 marks]

d) Show that the function $u = x^3 - 3xy^2 - 5y$ is harmonic and find its harmonic conjugate v [5 marks]

e) Find the Laurent series for $f(z) = \frac{z}{(z+1)(z+2)}$ about the singularity $z = -2$ [5 marks]

f) Evaluate

$$\oint_C \frac{1}{(z-1)^2(z-3)}\,dz$$

where C is the circle $|z| = 2$ $\lim_{z\to 1}$ $n!$ [5 marks]

$\frac{u}{u+1} = 1 + \frac{1}{u}$ $\frac{u}{u} + \frac{u}{1}$

$\cos = \frac{z + z^{-1}}{2}$

QUESTION TWO

a) Using complex variables, evaluate the real integral $\int_0^{2\pi} \frac{\cos\theta}{3+\sin\theta}\,d\theta$ [10 marks]

b) Determine the residues of $f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at each of its poles in the finite z-plane [10 marks]

QUESTION THREE

a) Evaluate $\lim_{z \to 1+i}\left[\frac{z^2 - z + 1 - i}{z^2 - 2z + 2}\right]^3$ [5 marks]
marks]

b) Show that the function $f(z) = \frac{x}{x^2+y^2} + i\frac{y}{x^2+y^2}$ is not analytic at any point [5 marks]

c) Show that $z = 0$ is a removable singularity of the function $f(z) = \frac{e^{2z} - 1}{z}$ [3 marks]

d) Determine the Laurent Series expansion $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $|z| > 3$ [7 Marks]

$p = 2$
$s = 2$

$\frac{1}{2}$

$z = 1\theta$

$1 + i.(1+i)$
$1 + 2i - 1$

$\frac{dz}{d\theta} = i$

QUESTION FOUR

a) Evaluate the integral $\int_{(1,3)}^{(4,5)} (2y + x^2)dx + i(3x - y)dy$ along the straight line from $P_1(1, 3)$ to $P_2(4, 5)$ [10 marks]

b) i) Show that if the function $f(z) = u + iv$ is analytic, then ~~it is~~ u's v are harmonic.
marks] [4 marks]

ii) If $f(z) = u + iv$, where $u = 2x - x^3 + 3xy^2$ show that u is harmonic. Hence or otherwise find v that will make the function $f(z)$ harmonic.

[6 marks]

$(z+i)(z-i)$ $\quad z^2 + 4$ $\quad 2$ $\quad 4$ $\quad 16i^2 - 28$

$z^2 \cdot 2z \times 1$ $\quad (z+2)(z-2)$ $\quad -16 - 8i$

$z^2 = 4$ $\quad (4i+1)(4i+1)$

z^2 $\quad z = \pm\sqrt{-4}$ $\quad 16i^2 + 4i + 4i + 1$

Complex A
Exams.

(5)

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

University Examinations 2009/2010

THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

2305: COMPLEX ANALYSIS I

AUGUST 2009 — TIME: 2 HOURS

ructions: Answer Question ONE and Any Other Two Questions

$u(x,y)$ $v(x,y)$

$z = x + iy$

QUESTION ONE (COMPULSORY 30 MARKS)

a. Find the Cartesian (x, y) equation for $|z - i| = |z + 4i|$. (5 marks)

b. If $f(z) = u + iv = w$, find u and v if $w = \dfrac{1}{1-z}$. (5 marks)

c. Find $z^{\frac{1}{4}}$ if $z = 2 + 2i$. (5 marks)

d. If $f(z) = \dfrac{1}{z^4 - 16}$ and c is the circle $|z - 3i| = 4$, find all the singularities of f(z) inside c. (5 marks)

(0,3)

e. Evaluate $\int_c (5z^2 + 1)\,dz$ where c is the straight line joining the origin to the point $z = 3i$. (5 marks)

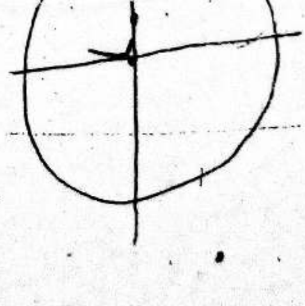
(0,0) (0,3)

f. Evaluate $\oint_c \dfrac{z+2}{(z+1)(z+3)}$ if c is the circle $|z| = 2$ (5 marks)

Fourth route of unity. Circle Centre - (0,3) r = 4

$z^4 = 16$

$z = \pm 2$



$\int \frac{z+2}{(z+1)}$ $a = 1$

$n = 0$

$f(a) = \frac{1}{2\pi i} \int \frac{f(z)}{z - a}$

QUESTION TWO (20 MARKS)

a. Find $(1+i)^i$. (4 marks)

b. i. Show that $\tan^{-1} z = \frac{i}{2}\ln\frac{1+z}{1-z}$. (6 marks)

ii. Find W such that $\tan W = 2i$. (4 marks)

c. If $W = u + iv = \frac{1+zi}{z+i}$, find u and v. (6 marks)

QUESTION THREE (20 MARKS)

a. i. If $z = \cos\theta + i\sin\theta$, show that $2\cos\theta = z + \frac{1}{z}$, $2i\sin\theta = z - \frac{1}{z}$. (2 marks)

ii. By expanding $(z+\frac{1}{z})^5(z-\frac{1}{z})^5$ into powers of z find an expression for $\sin^5\theta\cos^5\theta$ in terms of θ. (5 marks)

b. Evaluate $\lim_{z \to 1+i}\left[\frac{z^2 - z + 1 - i}{z^2 - 2z + 2}\right]^3$ (6 marks)

c. i. Prove that the function $u = x + e^{-y}\cos x$ is harmonic. (2 marks)

ii. Find a function v such that f(z) = u + iv is analytic using u in (i) above. (3 marks)

iii. Express f(z) from (ii) in terms of z. (2 marks)

QUESTION FOUR (20 MARKS)

a. Locate and name all the singularities of $f(z) = \frac{z^7 + 5z^4 + 3z^2 - 9z}{(z^2-1)(3z^2+z-2)^2}$. (4 marks)

b. Evaluate $\int_i^{2-i}(3xy + iy^2)\,dz$ along the straight line joining z = i and z = 2 − i. (6 marks)

c. Find the first five terms of the binomial expansion of $\frac{1}{(z-2)^2}$, $|z| > 2$. (4 marks)

d. Five the Laurent series expansion of $\frac{z}{(z+1)(z+2)}$ about z = -2. (6 marks)

$\frac{U-2}{U^2}\left\{\frac{1}{1-\frac{1}{U}}\right\} = \frac{U-2}{U^2}\left\{1 - \frac{1}{U} + \frac{1}{U^2}\right\}$ $\frac{z}{(z+1)(z+2)}$

$U = (z+2)$

$z = (U-2)$

$z+1 = U-1$

$\frac{U-2}{U-1[U]}$

$\frac{U-2}{U(U-1)} = \frac{U-2}{U(U)\{1-\frac{1}{U}\}}$

$\frac{U-2}{}\left\{\frac{1}{U^2} - \frac{1}{U^3} + \frac{1}{U^4}\right\}$

$2x - 2xy$

QUESTION TWO(20 MARKS)

(a) Determine the residues of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at each of its poles in the finite z-plane. [12 m

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3sin\theta}$ [8 ma

QUESTION THREE(20 MARKS)

(a) Determine the Laurent series expansion $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $|z| > 3$ [5 ma

(b) Use Cauchy's integral formula to evaluate $\oint_c \frac{cos\pi z+sin\pi z}{(z-1)(z+1)}$, $c : |z| = 2$ [7 ma

(c) Determine the residues of the function $\frac{z^2}{(z-2)(z^2+1)}$ at z=2 and z=i [3 ma

(d) Evaluate

$$\lim_{z\to 1+i}[\frac{z^2 - z + 1 - i}{z^2 - 2z + 2}]^3$$

[5 mark

QUESTION FOUR(20 MARKS)

(a) Evaluate $\int_{1+i}^{2+4i} z^2 dz$

(a) Along the parabola $x = t, y = t^2$ where $\{1 \leqq t \leqq 2\}$. [5 mark

(b) Along straight lines from 1+i to 2+i and then to 2+4i. [5 mark

(c) Determine the images of $x = c_1(c_1 \neq 0)$ and $y = c_2(c_2 \neq 0)$ under the transformatio $w = \frac{1}{z}$ [10 marks

THE END

$1(1+i) - i(1+i) - i^2$

$1+i-i+1$

2

$$\frac{z^2 - z + (1-i)}{z^2 - 2z + (1+i)(1-i)}$$

$-i^2$

$\frac{y\sqrt{3}}{x\sqrt{3}}$

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$n = 4$

$k = n - 1$

UNIVERSITY EXAMINATIONS 2015/2016

THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING

SMA 2305: COMPLEX ANALYSIS

DATE: DECEMBER 2015 **TIME:** 2HRS

INSTRUCTIONS: Answer question ONE and any other TWO questions

QUESTION ONE (30 MARKS COMPULSORY)

a) Evaluate the limit $\lim_{z \to 2i} \frac{z^2+4}{z-2i}$. Hence state whether the function is continuous or not. (3mks)

b) Find the fourth roots of $-2-2i\sqrt{3}$ and plot them on the complex plane. (5mks)

c) Evaluate the integral $\int_{1+i}^{2+4i} z^2 dz$ along the line segment from z = 1+i to z = 2+4i (4mks)

d) Obtain the Laurent's series expansion for f(z) = $\frac{z^2-1}{z^2-3z+2}$ about the singular point z=1. Use the series obtained to find the residue of f(z) at z=1 and hence evaluate $\int_C \frac{z^2-1}{z^2-3z+2} dz$ where C:$|z+i| = 2$ (6mks)

e) Let $u(x,y) = (x-1)^3 - 3xy^2 + 3y^2$, find v(x,y) such that f(z) = u(x,y) + iv(x,y) is harmonic. Express f(z) in terms of z. (5mks)

f) Find the fixed points of the transformation $f(z) = \frac{2z-5}{z+4}$ (2mks)

$(z-2i)\cdot(z+2i$

$z^2 - 2i$

g) Given that $f(z) = \frac{z(z-2)}{(z+1)^2(z^2+4)}$,using Cauchy's Residue Theorem evaluate $\int_c f(z)dz$ where c is the circle centre (0,0) and radius 4 units. (5mks)

QUESTION TWO (20 MARKS)

a) Solve the equation $e^{2z} + e^z + 1 = 0$ (4mks)

b) Show that if a function f(z) is analytic inside and on a simple closed contour C, then $\int_c f(z)dz = 0$ (4mks)

c) Verify that the real and imaginary parts of the function $f(z) = z^2 + 5iz + 3 - i$ satisfy the Cauchy Riemann equations. (5mks)

d) Show that $u(x,y) = \frac{1}{2}In(x^2 + y^2)$ is harmonic and hence find its harmonic conjugate v(x,y). (7mks)

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$\frac{23}{40}$

QUESTION THREE (20 MARKS)

a) Let w = f(z)=Az where A= 3 + 4i be a transformation.

i) Identify the transformation equations and find and the image of the unit square under w =Az.

ii) Sketch the image on the w-plane and describe the properties of the transformation w. (6mks)

b) Consider the transformation w = f(z) = ln Z^6, show that circles with center (0,0) on the z-plane are mapped onto straight lines parallel to the v-axis. (4mks)

c) Determine the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto the points $w_1 = 1, w_2 = i, w_3 = -1$ (5mks)

d) Use Cauchy's integral formula to evaluate $\int_c \frac{z^2+5}{z^2(z+2i)}dz$, where C:|z| = 1.5 (5mks)

QUESTION FOUR (20 MARKS)

$(z)^6$

a) Prove that $sin^6\theta + cos^6\theta = \frac{1}{8}(3cos4\theta + 5)$ (6mks)

b) State without proof the Cauchy's residue theorem. Hence use it to evaluate the integral $I = \int_c \frac{2z^2+1}{(z+2)(z-3)^2}dz$ $\quad C: |z| = 5$ (7mks)

c) Expand $f(z) = \frac{3z}{(z+2)(z-4)}$ into Laurent series in the region 2 < |z| < 4 (6mks)

$(\sin\theta + \cos\theta)^6$

$\sin^2\theta + \cos^2\theta = 1$

W1-2-60-1-6

JOMO KENYATTA UNIVERSITY OF AGRICULTURE AND TECHNOLOGY

UNIVERSITY EXAMINATIONS 2017/2018

EXAMINATION FOR BACHELOR OF SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN FINANCIAL ENGINEERING YEAR III SEMESTER I

SMA 2305: COMPLEX ANALYSIS I

DATE: JANUARY 2018 **TIME: 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

(a) Evaluate:

i. $(\sqrt{2} - i) - i(1 - i\sqrt{2})$ $= \sqrt{2} - i - i + i^2\sqrt{2}$ $\cancel{\sqrt{2}} - i - i - \cancel{\sqrt{2}} = -2i$ [1 mark]

ii. $(2 - 3i)(-2 + i)$ [1 mark]

(b) Solve $z^3 + 1 = i$ [6 marks]

(c) Determine the residues of $f(z) = e^z/(1 - z^2)$ about all singularities [3 marks]

$(1-z)(1+z)$ $1+z-z-z^2$ $(1-z^2)$ $1 = z^2$ $z = 1$

(d) Show that the function $u = 2x - x^3 + 3xy^2$ is harmonic [3 marks]

(e) Show that the function $f(z) = x - 2iy$ is not differentiable anywhere [6 marks]

(f) Find all the values of z for which $e^z = 2 - 2i$ [4 marks]

(g) Write $\exp(2 + \pi i/4)$ in the form $a + ib$ [3 marks]

(h) Evaluate log 1 [3 marks]

$(1-z)$ $(z-1)(z+1)$ $z^2 - z + z - 1$

QUESTION TWO [20 MARKS]

(a) Evaluate: [8 marks]

$$\int_{2i}^{1-i} 3z^2 dz$$

(b) Use the Cauchy-Riemann Equations to show that the function $f(z) = \bar{z}$ is not analytic [7 marks]

let $z = x + iy$
$\bar{z} = x - iy$

(c) Use Cauchy's integral theorem to evaluate [5 marks]

$$\int_C \frac{e^z}{z^2 - 4} dz$$

$\frac{n!}{2\pi i} \oint_C$

QUESTION THREE [20 MARKS]

(a) Find the values of z if [10 marks]

i. $z = \log i^{i}$

ii. $z = (-i)^{i}$

(b) Find the principal argument and exponential form of

i. $z = {}^{i}/_{(1+i)}$ [3 marks]

ii. $z = \sqrt{3} + i$ [3 marks]

(c) Suppose the function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D. Show that $u(x,y)$ and $v(x,y)$ are harmonic in D [4 marks]

QUESTION FOUR [20 MARKS]

(a) Use the residue theorem to evaluate

$$\int_{0}^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta$$

within the unit circle [12 marks]

(b) State the residue theorem and hence evaluate: $\oint_{C} \frac{e^{z}}{(z+3)^{2}(z-1)}\, dz$ where C is given by $|z| = 4$ [8 marks]