Introduction to Propositional Logic

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01.

1 Definition of Propositional Logic

Overview

What is Propositional Logic?

Propositional logic is a branch of logic that deals with propositions (statements that can either be true or false)

Importance in AI

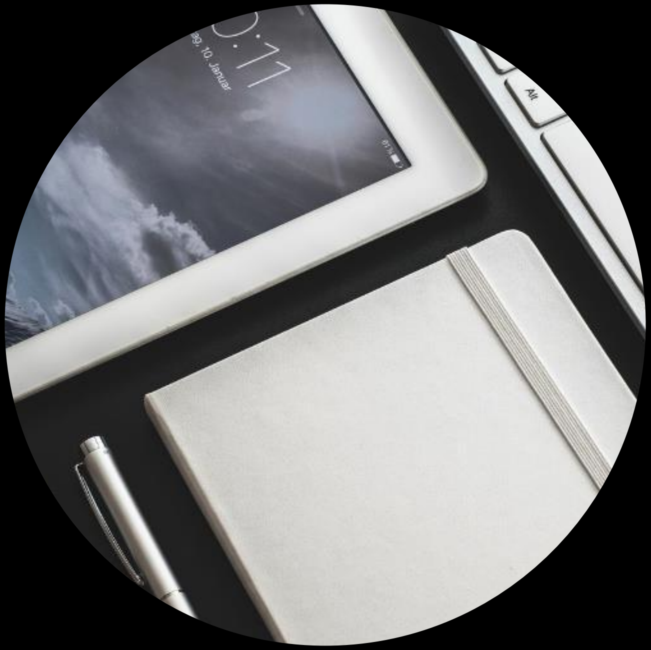
Propositional logic provides a framework for knowledge representation, allowing AI to store information in a machine-processable format

Differences from First-Order Logic

Propositional logic deals with complete statements. First-order logic deals with objects, properties, and relations

Propositional logic cannot represent statements like “All humans are mortal” while first-order logic can represent statements about collections and objects

Propositional logic has no variables or quantifiers, first-order logic does.



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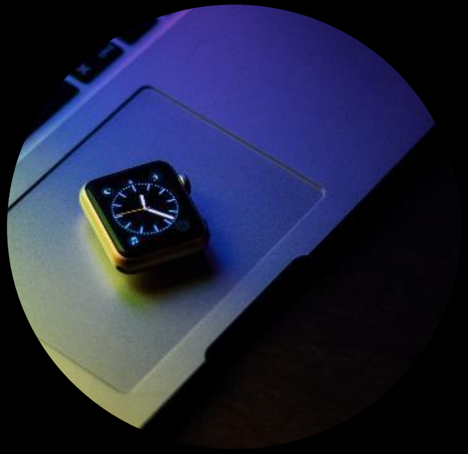
Key Components of Propositional Logic

Propositions



Definition of Propositions

Propositions are statements with truth values, i.e. can be true or false.

Examples of Propositions

"The sky is blue" (True) and "The Earth is flat" (False),

Logical Connectives

AND, OR, NOT Operators

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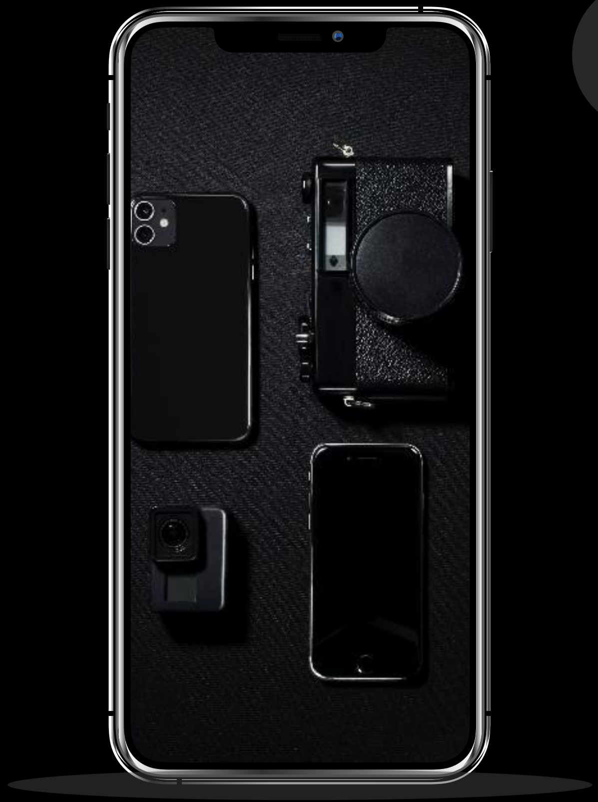
Logical operators such as AND (∧), OR (∨), and NOT (¬) are used to combine propositions. Each connective modifies the truth values based on the logical relationship, enabling more complex expressions.



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Implication and Biconditional

Implication (⇒) signifies a conditional statement where the truth of one proposition asserts the truth of another. Biconditional (⇔) indicates that two propositions are interchangeable concerning their truth values.

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Syntax and Semantics of Propositional Logic

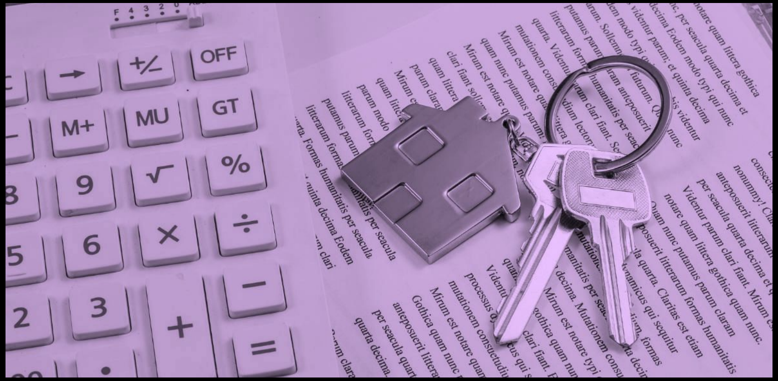
Syntax: Well-formed Formulas

Atomic and Compound Propositions

Atomic propositions are individual statements that cannot be further decomposed, while **compound propositions** are created by combining atomic propositions through logical connectives, allowing greater complexity in expression.

Formation Using Connectives

The formation of well-structured logical statements requires systematic use of connectives to ensure clarity and adherence to syntactic rules, resulting in coherent logical formulas.



Semantics of Propositional Logic

Semantics defines the meaning of formulas — determines their truth value.

Each formula is **True (1)** or **False (0)** under **interpretation**

Truth Tables

Shows truth values for all possible combinations of atomic propositions.

Truth Table

P	Q	Conjunction AND P ∧ Q	Disjunction OR P v Q	Negation NOT P ~P
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

Logical Equivalence

Two formulas are logically equivalent if they have the same truth values for all interpretations

Truth Table for (p→q)∨(q→p)				
p	q	p→q	q→p	(p→q)∨(q→p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Tautologies, Contradictions, and Contingencies

- Tautologies: Always True
- Contradictions: Always false
- Contingency: Sometimes true other times false

p	q	~p	~q	p∨q	~(p)∧(~q)	(p∨q) ∧ [(~p)∧(~q)]
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Semantics: Truth Tables

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Definition of Truth Tables

Truth tables are comprehensive tools that enumerate all possible truth values of a set of propositions, providing a visual representation of how compound propositions' truth can be derived from their atomic components.

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

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Example of a Truth Table

An example truth table showcases the relationships among propositions p and q, detailing their combined truth values under different logical operators, crucial for understanding logical operations.

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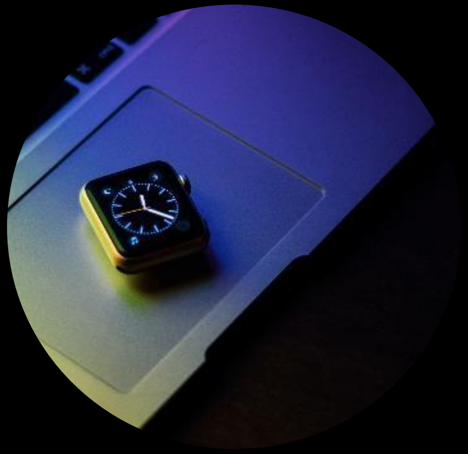
Logical Equivalence and Inference

Logical Equivalence



Definition and Examples

Logical equivalence occurs when two statements consistently have identical truth values across all scenarios, ensuring their interchangeable use in logical deductions and proofs.

De Morgan’s Law

De Morgan’s Laws provide essential relationships between conjunctions and disjunctions, illustrating how negation interacts with these operators, thereby enabling simplification of logical expressions.

Tautologies and Contradictions

Definition of Tautologies

A tautology refers to a proposition that is invariably true, regardless of the truth values of its components, exemplifying logical certainty in reasoning.

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Definition of Contradictions

Conversely, a contradiction is a proposition that is always false, highlighting logical impossibility, thereby reinforcing constraints in propositional logic.

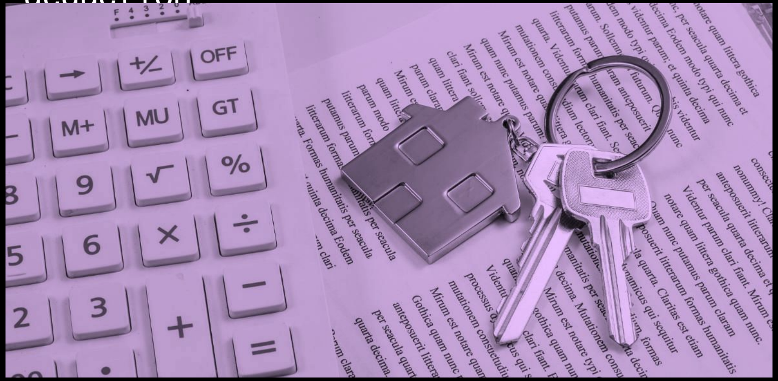
Logical Inference

Definition of Logical Inference

Logical inference encompasses the methodology of deriving conclusions from established premises. This process utilizes a series of logical rules to ascertain valid statements and conclusions.

Rules of Inference

Rules of inference, such as Modus Ponens and Modus Tollens, are systematic methods that facilitate the drawing of conclusions from premises, central to logical reasoning and deduction.



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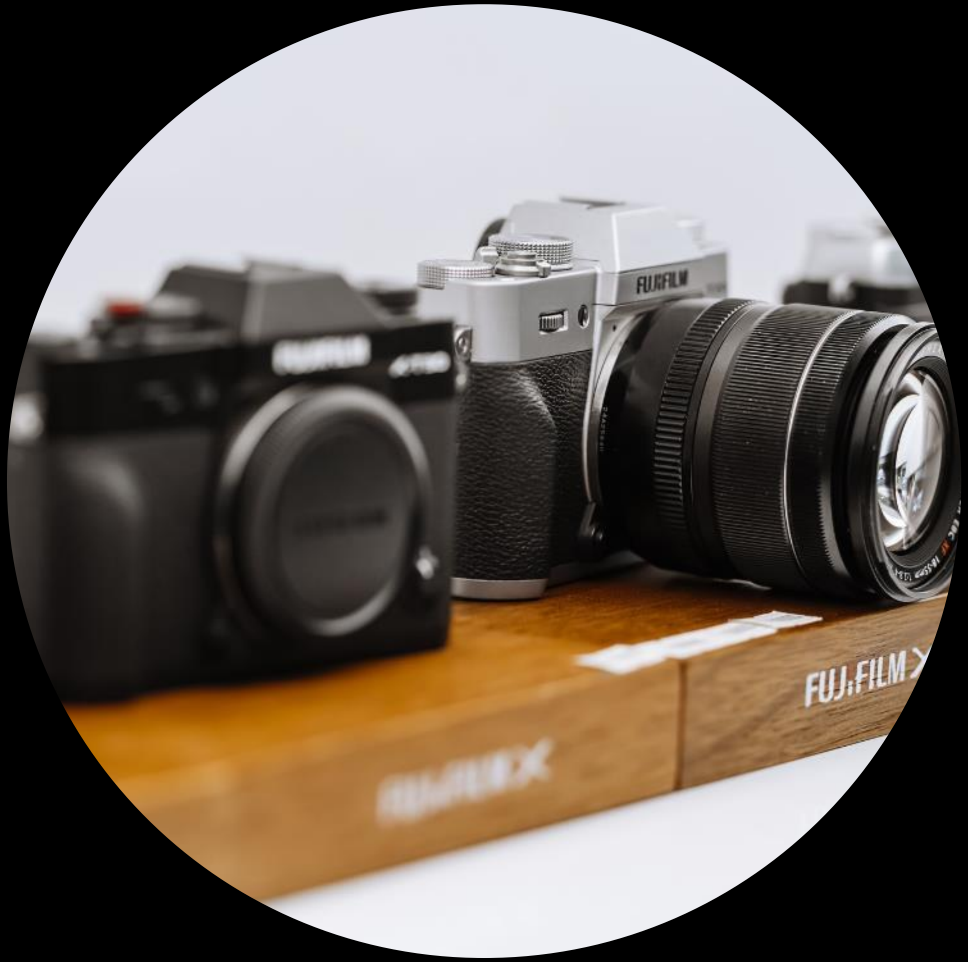
Representation and Reasoning

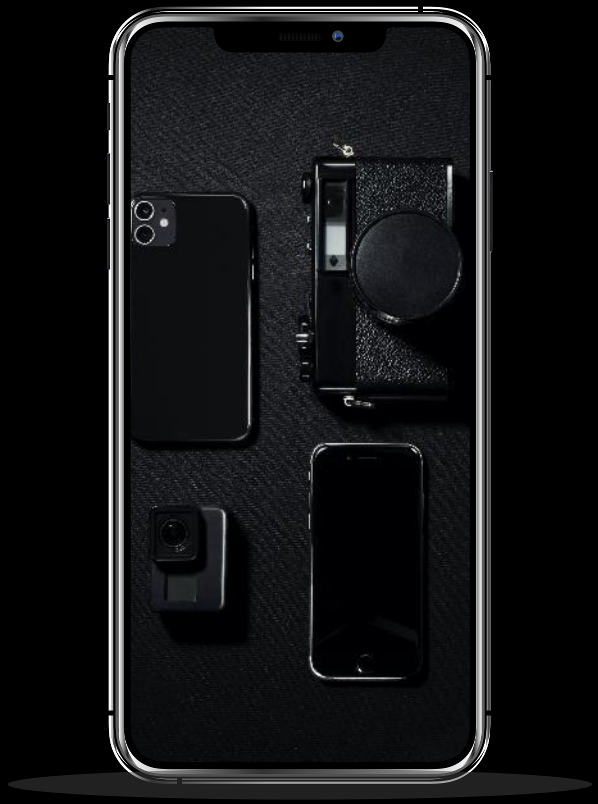
Knowledge Representation



Formal Rules in Logic

Knowledge representation in propositional logic often employs formal rules that are explicitly defined, allowing for automated processing and reasoning across various applications.

CNF and DNF



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Disjunctive Normal Form

Disjunctive Normal Form (DNF) organizes logical arguments as a disjunction of conjunctions, enabling comprehensive evaluations of propositions' truth conditions.

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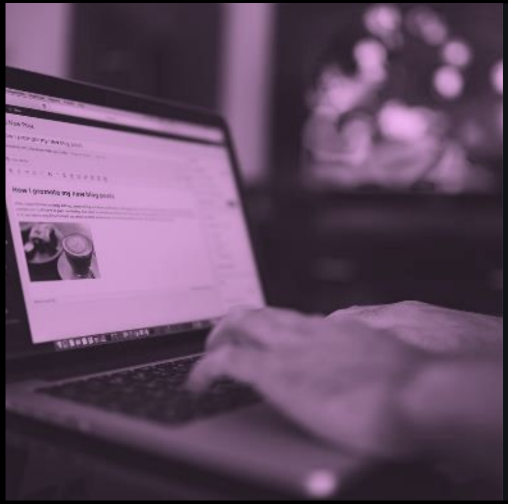
Conjunctive Normal Form

Conjunctive Normal Form (CNF) structures propositions as a conjunction of disjunctions, providing a standardized format that simplifies the analysis and resolution of logical expressions.

Applications in AI



Expert Systems

Expert systems leverage propositional logic for rule-based reasoning, facilitating decision-making processes in domains such as medical diagnosis and troubleshooting.

Automated Theorem Proving

Propositional logic is foundational in automated theorem proving which involves logic solvers that validate theorems through formal proof techniques, enhancing efficiency in mathematical fields.

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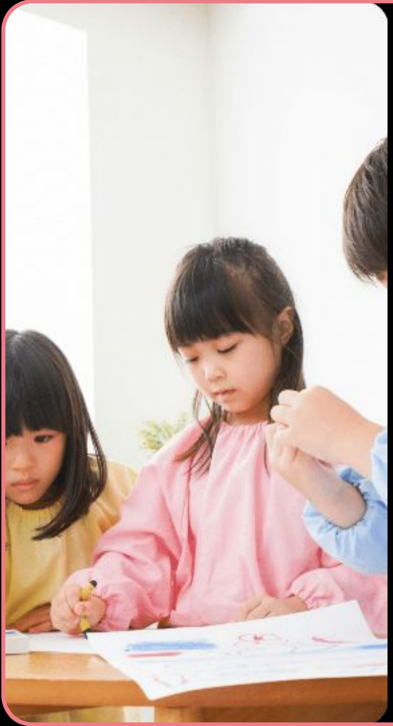
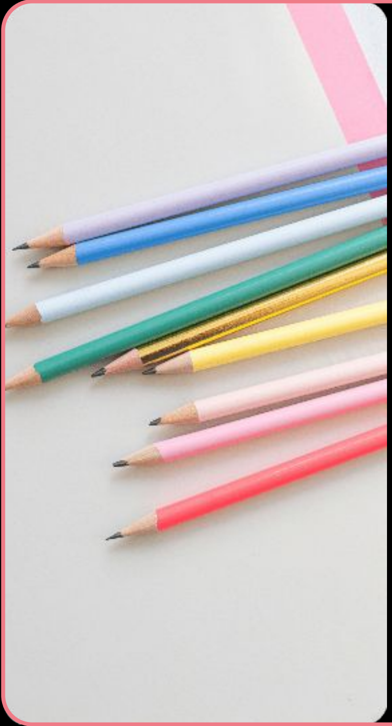
Limitations and Applications in AI



Limited Expressiveness:

Cannot represent objects, relationships, or quantifiers.
Example: Cannot express "All humans are mortal."



Scalability Issues:

Large knowledge bases require too many individual propositions.
Hard to manage and compute efficiently in AI systems.

Propositional Logic vs. First-Order Logic (FOL)

Why First-Order Logic is More Powerful?

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FOL Can Represent More Complex Relationships
Example: *All humans are mortal*
→ ∀x (Human(x) → Mortal(x))
Propositional logic would require separate statements for each human.

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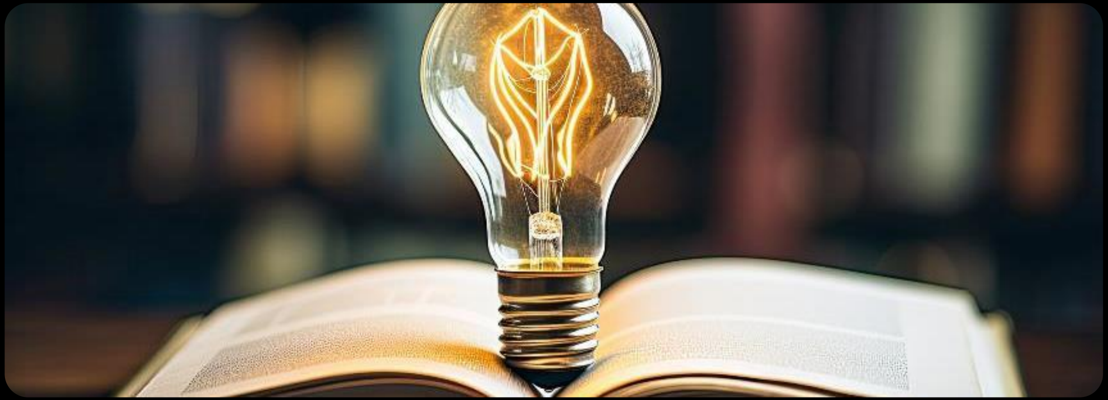
Better Suited for AI Reasoning
Enables object classification, relationships, and generalization.
Used in **knowledge graphs, semantic reasoning, and natural language processing (NLP).**

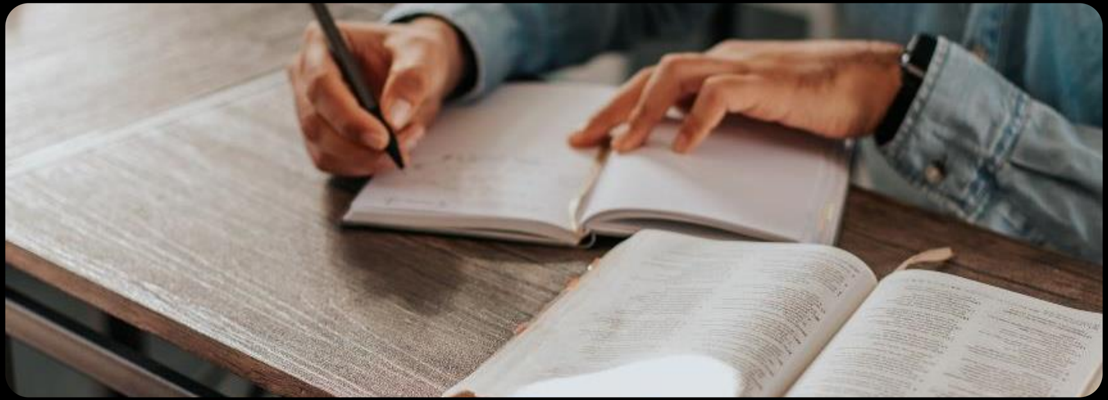
Real-World AI Applications of Propositional Logic



Chatbots

Chatbots utilize propositional logic for straightforward rule-based responses, enabling them to provide relevant information based on user inputs in a coherent manner.

e.g., IF user says "Hello" THEN reply "Hi!"



Game AI and Robotics

Propositional logic facilitates decision-making processes in game AI and robotic systems, helping to establish simple rule sets that guide behaviors and actions in dynamic environments.

Eg., IF enemy spotted THEN attack

More Applications of Propositional Logic in AI

🏥 Medical Diagnosis Systems

Use Case: AI systems in healthcare can use propositional logic to support diagnostic decision-making.
Example Rule:
IF patient has high fever AND rash, **THEN** suspect measles.

📦 Automated Customer Support

Use Case: Logic-based systems can route support tickets based on user inputs and predefined rules.
Example Rule:
IF issue = "billing" AND user = "premium", **THEN** assign to senior agent.

🚨 Intrusion Detection Systems

Use Case: Security systems use propositional rules to flag suspicious activity.
Example Rule:
IF login_attempt > 3 **AND** IP unknown, **THEN** trigger alert.

Thank you for listening.

📌 *Understanding logic improves AI decision-making and automation!*