Ariana Keeling

16-711 HW3

3/27/2017

Part 1

The alien artifact tumbles the most when the largest angular velocity is applied to the x axis, the second largest to the y axis, and the third largest to the z axis. This causes quick rotations about the shortest sides of the artifact, while the object twists more slowly about its longer axes, giving the appearance of a complicated tumble. The artifact turns about a fixed axis if an angular velocity is applied about any one axis while there is no angular velocity about the other two axes; this is intuitive since the object is only turning in one direction.

Part 2

2a) The center of mass of the object was found using optimization. The criteria used found the point in between the markers that stayed the same distance from the markers as the object moved. One hundred points across the ten seconds were used for the optimization, and the initial location and linear velocity of the center of mass were found to be [-11, 0, 0] and [0.02, -0.05, 0.01], respectively. The orientation of the object was found using the relative rotation of a basis constructed using one of the object's corners. One marker was identified as the "origin", while the edges that passed through it were used as axes. The rotation of this artificial coordinate system was separated from the translation and compared to the initial orientation of the frame; the resulting rotation matrix was then converted to quaternions. The resultant positions and orientations over time can be found in problem_2_0.dat.

2b) The angular velocity of the object was found by converting the quaternions into XYZ Euler angles; I did it this way because Euler angles are already split into the constituent rotations and because I had the conversion program from a previous homework. The difference over time between each set of Euler angles is the angular velocity of the object. The angular velocities can be found in problem_2_1.dat. The velocities for the first time step were set to 0 to keep all matrices the same size for later calculations; these are placeholders and should be disregarded.

2c) The angular acceleration of the object was found by taking the difference for each time step of the angular velocities of the object, which were calculated in part 2b. The calculated accelerations are recorded in problem_2_2.dat. The accelerations for the first two time steps were set to 0 to keep all matrices the same size for later calculations; these are placeholders and should be disregarded.

2d) The moments of inertia of the principle axes, in ascending order, are: $I_{yy} = 0.4359$, $I_{xx} = 0.5199$, and $I_{zz} = 0.7346$. The other moments of inertia are close to 0; the matrix is

$$\begin{bmatrix} 0.5199 & 0.0050 & 0.0053 \\ 0.0050 & 0.4359 & 0.0027 \\ 0.0053 & 0.0027 & 0.7346 \end{bmatrix}$$

These moments of inertia were found by separating the components of the torque equation into two matrices, one with the angular velocity and acceleration terms and one with the moments of inertia. The matrix with the known terms was solved for the same one hundred points used in part 2a, with each resultant 3x6 matrix stacking under the previous matrices. The smallest eigenvalue of the 450x6 matrix of points was found using SVD, with the corresponding V vector representing the moments of inertia. The rotation matrix representing the original rotation between the principal axes and the assignment coordinate axes is

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

or, in quaternions, [0.6479, -0.7563, -0.0883, -0.0201]. This rotation was found using the initial orthonormal basis found in part 2a and the xyz coordinate frame.

There is something wrong with this moment of inertia calculation, but for the life of me I can't figure out where the problem is, and I don't have time to study it in depth.

2e) The future position of the center of mass of the artifact was found by shifting the previous position by the velocity multiplied by the time step for each time step from 10 – 20 seconds. The rotation was found by using Euler integration; the derivative of the rotation matrix is $\dot{R} = S(\omega)R$, with

$$S = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

The angular velocity was also found using Euler integration; the angular acceleration was found by solving the torque equation at each time step: $I\alpha + \omega \text{ x } I\omega = 0$. The resultant position and rotation data can be found in problem_2_3.dat.

Part 3

The location for the artifact in world coordinates was found using the rotation matrix found in part 2d and the original location of the lander. The resulting transformation matrix was combined with the artifact coordinates according to the lander to find the artifact coordinates in the world. The rotation was found by combining the rotation quaternion found in part 2d with the viewed quaternion. This rotation appears to be a mirror of the original rotation; the rotation matrix in 2d is actually a reflection, but the origin of the issue is unknown. In order to repair this code, the reflection should be repaired, and the desired point of the lander should be moved outside of the artifact so the two objects do not collide. In addition there should probably be a waypoint chosen to move the lander around the artifact on the way to dock. However, I have no more time to devote to this and things are not working, so I put my best attempt in “problem_3.dat”.