1. Introducción

Paper:

Relative Entropy Under Mappings by Stochastic Matrices

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Relevance:

- Can provide more information on bounding the rates of convergence to equilibrium of ergodic Markov chains and Markov processes

2. Preliminares

- give the definitions of the different types of random matricies
  - positive and nonnegative probability d-vectors
  - row/column stochastic matrix
  - scrambling matrix
- And I think we need the following definitions to give context for the contraction coefficient

Definición: 2.1 → Symmetric Relative Entropy

Definición: 2.2 → relative ϕ-entropy

Definición: 2.3

Para cualquier $m \times n$ matriz A, el coeficiente de ergodicity de Dobrushin se define

$$\alpha(A) = \min_{j,k} \sum_{i=1}^{m} \min(a_{ij}, a_{ik})$$

The complement $1 - \alpha(A)$ is

$$\bar{\alpha}(A) \equiv 1 - \alpha(A) = \frac{1}{2} \max_{j,k} \sum_{i=1}^{m} |a_{ij} - a_{ik}|$$

and also satisfies

$$\bar{\alpha}(A) = \sup \left\{ \frac{\|A(x-y)\|_1}{\|x-y\|_1} : \ x \text{ and } y \text{ are positive } n\text{-vectors such that } x \neq y, \text{ y } \|x\|_1 = \|y\|_1 \right\}$$

- introduce $g(t) = \phi(1, 1+t)$ as a way to index the contraction coefficients

3. Resultados Principales

Theorem 4.1: $0 \leq \eta_\phi(A) \leq \bar{\alpha}(A) \leq 1$

Teorema: 4.1

$0 \leq \eta_\phi(A) \leq \bar{\alpha}(A) \leq 1$

Teorema: 5.4

If $g(w)$ is thrice differentiable in a neighborhood of 0 and $g''(0) > 0$, then $\eta_{w^2}(A) \leq \eta_g(A)$; in particular, $\eta_{w^2}(A) \leq \eta_{\log}(A)$

4. Elementos de las demostraciones

- Theorem 4.1
  - Introduce and explain theorem 3 and the corresponding lemas since theorem 4.1 depends on it
- Theorem 5.4
  - define homogeneous function
  - the rest seems pretty straight forward

5. Conclusión

Relación con el curso

- compare with what we've seen with DPI from KL-divergence and mutual information. Briefly, we have

$$\eta_\phi(A) = \sup\left\{\frac{H_\phi(Ax, Ay)}{H_\phi(x, y)} : x \in P_n, y \in P_n, x \neq y\right\}$$

  and also

$$\begin{aligned} A \text{ is scrambling} \iff & \eta_\phi(A) < 1 \\ \iff & H_\phi(x, y) < H_\phi(Ax, Ay) \quad \text{with equality if } A \text{ is permutation} \end{aligned}$$

  So

$$X \xrightarrow{A} Y \xrightarrow{A'} Z \implies H_\phi(X, Y) < H_\phi(X, Z)$$

  Seems like this connects easily to what we've seen if we define $P_{X|Y} \equiv A$ and $P_{Y|Z} \equiv A'$

- Compare relative g-entropy with f-divergence

- Extend symmetry to f-divergence

$$J_\phi(x,y) = H_\phi(x,y) + H_\phi(y,x) \longrightarrow J_f(x,y) = D_f(x,y) + H_f(y,x)$$