Exercise 1.1:

i)

Exercise 1.2:

i) If $\mathcal{F}_i,\ i \in I$ are σ-fields then $\cap_{i\in I}\mathcal{F}_i$ is.

> Suppose $\mathcal{F}_i$ are σ-fields.
>
> - Consider $A \in \cap_{i\in I}\mathcal{F}_i$. Then $\exists i$ such that $A \in \mathcal{F}_i$. So $A^c \in \mathcal{F}_i$ because $\mathcal{F}_i$ is a σ-field σ-field. Then $A^c \in \cap_{i\in I}\mathcal{F}_i$.
> - Consider $A_j \in \cap_{i\in I}\mathcal{F}_i$ a countable sequence of sets. Then $\forall j, \forall i,\ A_j \in \mathcal{F}_i$. Then $\cup A_j \in F_i,\ \forall i$ because $\mathcal{F}_i$ is a σ-field. Then $\cup A_j \in \cap_{i\in I}\mathcal{F}_i$

ii) Use the result in (i) to show if we are given a set Ω and a collection $\mathcal{A}$ of subsets of Ω then there is a smallest σ-field containing $\mathcal{A}$.

> Let $\mathcal{A}$ be a collection of subsets of Ω. let $\mathcal{F}_{\mathcal{A}}$ be the set of sigma fields that contain $\mathcal{A}$. Define $\mathcal{F} = \cap_{\mathcal{A}}\mathcal{F}_{\mathcal{A}}$. From (i) we know that $\mathcal{F}$ is a sigma field. By definition $\mathcal{F}$ is the smallest sigma field containing $\mathcal{A}$ since for sigma field $\mathcal{C}$ such that $\mathcal{A} \subset \mathcal{C}$, we have $\mathcal{F} = \cap_{\mathcal{A}}\mathcal{F}_{\mathcal{A}} \subset \mathcal{C}$

With $(\mathbb{R}, \mathcal{F}, P)$ and $\mathcal{B}$ the borel sets, define a random variable as a real valued function such that X is $\mathcal{F}$ measurable for every borel set

$$X^{-1}(B) \in \mathcal{F}, \quad B \in \mathcal{B}$$

Then X induces a probability measure on $\mathbb{R}$ called its distribution

$$\mu(A) = P(X \in A) = P(X^{-1}(A)), \quad A \in \mathcal{B}$$

The distribution function is defined as

$$F(x) = P(X \leq x)$$