Math 110A, Winter 2018
Problem Set 7: Abstract Algebra Homework
Due 2018-03-06

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1 Some problem

Let $f(x)$ be a monic polynomial in $\mathbb{R}[x]$ of degree 3 that has $1+2i$ and 2 as roots. Then $\overline{1+2i} = 1-2i$ is a root as well as $f(x)$ has real coefficients. $f(x)$ has degree 3, so these three roots are all the roots of $f(x)$.
$f(x) = 1(x-2)(x-(1+2i))(x-(1-2i)) = x^3 - 4x^2 + 9x - 10$ then, which has constant term $10 = (1-2i)(1+2i)(2)$.

2 Monic polynomials of degree 2 in $\mathbb{Z}_{11}[x]$

Monic polynomials of degree 2 in $\mathbb{Z}_{11}[x]$ are of the form

$$f(x) = 1x^2 + bx + c : \; b, c \in \mathbb{Z}_{11}$$

There are 11 choices each for b and c, so there are 121 such polynomials in all.

3 Monic degree 2 irreducible polynomials in $\mathbb{Z}_3[x]$

There's only 9 such unique polynomials and 3 possible roots in $\mathbb{Z}_3$, so I can just make a table to find out how many irreducible ones there are.
(Here I use the fact that irreducible $\equiv$ no roots).

$f(x)$	$f(0)$	$f(1)$	$f(2)$
x^2	0	1	1
x^2+1	1	1	2
x^2+2	2	2	0
x^2+x	0	2	0
x^2+x+1	1	0	1
x^2+x+2	2	1	2
x^2+2x	0	0	2
x^2+2x+1	1	1	0
x^2+2x+2	2	2	1

So there's 3 irreducible polynomials: $x^2 + 1$, $x^2 + x + 2$, and $x^2 + 2x + 2$.

4 $\mathbb{Z}_3[x]/(x^2 + 1)$ is an integral domain

This is because the modulus $x^2 + 1$ is irreducible in $\mathbb{Z}_3[x]$ as shown earlier.

5 Polynomial roots of Unity

I will find $\omega \in \mathbb{Z}_2[x]/(x^2 + x + 1)$ such that $\omega \neq 1$ and $\omega^3 = 1$. $\omega = [x + 1]$.

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1 = x(x^2 + x + 1) + 1 \ (1 \equiv 3 \text{ in } \mathbb{Z}_2)$$
$$\text{so}$$
$$\omega^3 \equiv (x+1)^3 \equiv 1 \bmod (x^2 + x + 1)$$