

QUANTUM GRAVITY, THE PLANCK LATTICE AND THE STANDARD MODEL

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ABSTRACT

A possible ground state of Quantum Gravity is Wheeler's "space-time foam", which can be modeled as a "Planck-lattice", a space-time cubic lattice of lattice constant $a_P \simeq 10^{33}$ cm, the Planck length. I analyse the structure of the Standard Model defined on the Planck Lattice, in the light of the "no-go" theorem of Nielsen and Ninomiya, which requires an extension of the continuum model through Nambu-Jona Lasinio terms, quadrilinear in the Fermi-fields. As a result, a theory of masses (of both fermions and gauge bosons) is seen to emerge that, without Higgs excitations, agrees well with observations.

1. Introduction

The Standard Model (SM) of fundamental particle interactions represents the achievement of a long scientific path that through three decades of theory and experiment finally unifies the three different types of interactions (strong, electromagnetic and weak) in a single and well defined theoretical structure, that of Gauge Theories (GT).

It is remarkable that throughout the intellectual adventure that leads to the firm establishment of the SM, Quantum Gravity (QG) is seen to play essentially no rôle: the gravitational interactions, after all, at the typical space-time distances of particle interactions ($\sim 10^{13}$ cm) are so incredibly tiny that the phenomenology of the SM can well do without them. Even though, and this is a tantalizing thought, the structure of QG too is that of a GT, whose gauge group, as Einstein and his friend Marcel Grossmann showed at the beginning of our Century, is just the Poincaré group.

It is only recently, after the completion of the SM synthesis, achieved at the beginning of the Eighties with the "announced" discovery of the weak gauge bosons (W^\pm, Z^0), that the widespread theoretical urge to go "beyond" the SM has put the limelight on QG as one of the "new" interactions that would be unified with those of the SM within the plethora of supersymmetric models, which have been proposed since the mid-Seventies. Curiously enough, the various supersymmetric extensions of the SM were so prodigal in new particles and interactions, that even the now discredited "Fifth Force" could easily and "naturally" be accommodated in their wide and flexible

framework. However, such renewed interest in QG as a limited, particular sector of an all-embracing, unified Theory of Everything (TOE), the main aim of Superstring Theories, is quite different in character from that line of thought which, at the periphery of particle physics, has been pursued by J.A. Wheeler and his school, that has been called Geometrodynamics.¹

The main focus of this latter research program, in fact, is to obtain a deeper understanding of the structure of space-time, the arena of fundamental particle interactions, that according to Einstein's General Relativity (GR) is fully determined by the dynamics of the gravitational field. But, as Wheeler correctly remarks, a quantized gravitational field does acquire an independent dynamical rôle, i.e. it is no more uniquely determined by the distribution of matter (the energy-momentum tensor). Thus it may well happen that space-time itself, i.e. the web of relationships among different physical events, owing to the fundamental quantum-field fluctuations acquires some very peculiar structure, for instance that of a “foam”, whose discontinuities have the size of the Planck length $a_P \simeq 10^{-33}$ cm. In Wheeler's view, differently from what is being contemplated in Supersymmetric and String Theories, QG is not a sector of an extension of the SM, but rather the dynamical theory of space-time upon which the matter and fields of the SM stand and evolve.

It is just this far reaching view that I will follow in this talk with the goal to understand if and how the SM can be formulated upon a hypothesized peculiar “foam” structure, that can be modeled by a simple discrete space-time Planck Lattice (PL), whose lattice constant is a_P . This line of research has been pursued in the last three years in collaboration with the bright chinese theorist She-Sheng Xue.

2. SM: what if space-time were a Planck lattice?

Let me first remind you very briefly what is the SM on a continuous, Minkowskian space-time (CSM). Its Lagrangian is

$$\mathcal{L}_{CSM}(x) = \mathcal{L}_G(x) + \mathcal{L}_{Higgs}(x), \quad (1)$$

where the gauge lagrangian is

$$\mathcal{L}_G(x) = -\frac{1}{4} \left(\sum_{a=1}^8 C_{\mu\nu}^a C^{a\mu\nu} + \sum_{i=1}^3 A_{\mu\nu}^i A^{i\mu\nu} + B_{\mu\nu} B^{\mu\nu} \right) + \sum_q \bar{\Psi}_q i \not{D} \Psi_q + \sum_l \bar{\Psi}_l i \not{D} \Psi_l, \quad (2)$$

and describes the fundamental matter fields Ψ_q (quarks) and Ψ_l (leptons) coupled to the gauge fields of the colour $SU(3)$ -group ($C_{\mu\nu}^a$), the $SU(2)_L$ -group ($A_{\mu\nu}^i$) and the $U(1)_Y$ -group ($B_{\mu\nu}$):

$$\not{D} = \partial + ig_1 \not{B} \left\{ \frac{1}{2} (1 - \gamma_5) \frac{Y_L}{2} + \frac{1}{2} (1 + \gamma_5) \frac{Y_R}{2} \right\} + ig_2 \not{A}^i \frac{\tau^i}{2} (1 - \gamma_5) + ig_3 \not{C}^a \frac{\lambda_a}{2}, \quad (3)$$

with

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (4)$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig_2 [A_\mu, A_\nu], \quad (5)$$

$$C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu + ig_3 [C_\mu, C_\nu]. \quad (6)$$

All this is of course very beautiful and elegant, giving blood and flesh to the simple and very general idea that **all fundamental symmetries must be defined locally**, thus tying intimately symmetries to space-time.

As for the Higgs Lagrangian, its simplest form is

$$\mathcal{L}_{Higgs}(x) = -(D^\mu \phi)^\dagger (D_\mu \phi) - V_H(\phi^\dagger \phi), \quad (7)$$

with

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

a weak isospin doublet,

$$D_\mu = \partial_\mu + ig_1 B_\mu \frac{Y_H}{2} + ig_2 A_\mu^k \frac{\tau_k}{2} \quad (8)$$

and

$$V_H(\phi^\dagger \phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad (9)$$

with the “ad hoc” negative mass squared term to ensure “spontaneous electroweak symmetry breaking”.

And this is of course very ugly for:

- (a) the elegance of the gauge principle, as applied to the fundamental building blocks of matter leads to a **massless world** and in CSM no dynamical mechanism has been found that, respecting the Ockam’s razor, would produce the observed **massive world** by using the two fundamental building blocks only;
- (b) to generate mass no better way has been found than **graft** upon the beautifully simple gauge lagrangian \mathcal{L}_G (2) the ugly Higgs-mechanism, induced by \mathcal{L}_{Higgs} (7), that
 - (i) is motivated by an unpretentious pedagogical model, developed with the aim to impressionistically describe the main features of the Meissner effect of superconductivity;
 - (ii) extends in a totally arbitrary fashion the fundamental building blocks of nature, spin-1/2 fermions and vector bosons, to include a very odd scalar field ϕ ;
 - (iii) introduces the instability that leads to spontaneous chiral symmetry breaking by means of a totally arbitrary “negative mass squared” in V_H (9);

- (iv) introduces scalar local fields that in QFT are very peculiar objects, there being no way, besides “tuning”, to keep its mass from diverging like the ultraviolet cut-off Λ_{UV} . This is one of the main reasons why Supersymmetry, that could cure such disease, survived that remarkable “physics fasting” that goes on since more than twenty years.
- (c) the regularization/renormalization program of the QFT defined by \mathcal{L}_{CSM} is in bad shape, for
 - (i) the lattice regularization is blocked by the Nielsen-Ninomiya “no-go” theorem (see below);
 - (ii) dimensional regularization is still incapable of metabolizing no less than γ_5 !

In spite of all these difficulties the low energy phenomenology of electroweak interactions is fundamentally decoupled from all such flaws, that are all of a conceptual, theoretical nature, and stands as a beautiful confirmation of the simple and powerful $SU(2)_L \otimes U(1)_Y$ gauge principle. And this is the strong message of 20 years of electroweak interactions.

In view of all this, the question of the title of this Section appears much less philosophical and may well lead us to a SM without all the defects that have been exposed above, as I shall try to argue in the rest of this Lecture. As mentioned in the Introduction, I side completely with John Wheeler who sees the violent quantum fluctuations at the Planck scale to tear continuous space-time apart and create a foamlike structure, full of voids and discontinuities*.

Thus our question can be reformulated as: what if Wheeler’s idea were right and space-time at distances smaller than a_P would dissolve into the nothingness of wormholes?. A positive answer would lead us to make the following hypothesis: space-time is **not** a 4-dimensional continuum but rather a (random) Planck lattice.

As a consequence the SM and all QFT’s, that so far have been defined on a continuous manifold, must be reformulated as Lattice QFT’s. It is amusing to note that our hypothesis, if right, would turn the various “theories of everything” into theories of nothing, for at the scales where superstrings become relevant space-time would dissolve. But if we are to formulate the SM as a chiral Lattice Gauge Theory (LGT), our research program clashes immediately with the “no-go” theorem of Nielsen and Ninomiya,³ which informs us that we cannot simply transcribe the usual SM on a lattice, for when a LGT is chiral the low energy spectrum (the massless fermions) gets doubled in such a way that the fundamental chiral symmetry is violated. The physical origin of such unpleasant result is the peculiarity of the dispersion relations in a discrete

*The analogy with the non-trivial vacuum structure that has been demonstrated² to emerge in another non-abelian GT, QCD, is to my mind particularly relevant. I hope to be able to make it more precise in the near future

space-time: for a Weyl fermion $[\Psi_L = \frac{1}{2}(1 - \gamma_5)\Psi]$ the free-field dispersion relation is

$$\omega(\vec{p}) = \frac{1}{a} \sum_i \sin(\vec{p}_i a), \quad (10)$$

yielding low-energy ($\ll 1/a$) solutions not only for $\sum_i \vec{p}_i a \simeq 0$, but also for $\sum_i \vec{p}_i a \simeq \pi$. And the existence of these “doublers” is enough, not only to create problems with the observed spectrum, but also to destroy the very notion of left-handedness that permeates the electroweak phenomenology. Indeed it is easy to show that “doublers” do not have the same chirality of the Weyl field. Thus the “no-go” theorem of Nielsen and Ninomiya would appear to bar as velleitarian and impossible the idea of a Planck Lattice Standard Model (PLSM), whose foundation is the object of this talk. However, a careful analysis of its proof turns out to suggest very clearly the necessary ingredients towards a viable formulation of PLSM. Indeed, one of the crucial hypotheses of the “no-go” theorem is that the SM action be **bilinear** in the matter fermionic fields. As well known, this is precisely the structure of the usual Wilson action⁴ ($P_{L,R} = 1/2(1 \pm \gamma_5)$, F is a fermion index)

$$S_D^F = \frac{i}{2} \sum_{x,\mu,F} \left\{ \bar{\Psi}^F(x) \gamma_\mu U_\mu(x) [L_\mu^F(x) P_L + R_\mu^F(x) P_R] \Psi^F(x + u_\mu) - h.c. \right\}, \quad (11)$$

where $U_\mu(x) \in SU(3)$ and $L_\mu(x) \in SU(2)_L \otimes U(1)$ and $R_\mu \in U(1)_Y$.

Thus if one violates this hypothesis by introducing the simplest generalization of the SM action, i.e. adding interaction terms that are **quadrilinear** in the Fermi fields, one may have a chance to overcome this unpleasant obstacle. Furthermore, this addition would introduce into the SM a term of the Fermi type, that was proposed more than 30 years ago by Y. Nambu and G. Jona-Lasinio,⁵ in the context of chiral symmetry breaking in QFT! Also note that quadrilinear terms of the type we will consider are expected to arise as “effective” interactions among matter fields due to the gravitational interactions at the Planck scale, where gravitation ceases to be so utterly negligible.

Thus the PLSM action is:

$$S_{PL} = S_G + \sum_F (S_D^F + S_{NJL1}^F + S_{NJL2}^F), \quad (12)$$

where ($G_{1,2}$ are $O(a^2)$, F denotes the fermions – leptons and quarks – i, j denote the $SU(2) \times U(1)$ indices) S_G is the usual Wilson gauge lagrangian,⁴ S_D the bilinear Dirac lagrangian, and

$$S_{NJL1}^F = -G_1 \sum_x \bar{\Psi}_L^F(x)^i \Psi_R^F(x)_j \bar{\Psi}_R^F(x)^j \Psi_L^F(x)_i, \quad (13)$$

$$S_{NJL2}^F = -\frac{G_2}{2} \sum_{x\mu} \bar{\Psi}_L^F(x)^i L_\mu^F(x)_i^{i'} U_\mu^F(x) \Psi_R^F(x + a_\mu)_{j'} \bar{\Psi}_R^F(x)^j R_\mu^F(x)_j^{j'} U_\mu^F(x) \Psi_L^F(x + a_\mu)_{i'}. \quad (14)$$

Complicated though they appear, $S_{N\text{JL}1,2}^F$ simply enforce the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge-principle and evade the “no-go” theorem, at least in principle, **without the introduction of new matter or gauge fields**. As noted above, I wish to emphasize once more that terms like these are indeed expected to arise as effective matter interactions, induced by the gravitational interactions at the Planck scale. Their size too [$O(a_P^2)$] is just what one expects from interactions of gravitational origin.

The question now is: does the action S_{PL} (Eqs. (12)-(14)) give rise to a SM consistent with the multiform and sophisticated “low-energy” phenomenology? The next Section is devoted to giving a convincing, positive answer to this most important question.

3. A Consistent SM on the Planck lattice

As I have just stressed, our S_{PL} action evades in principle the “no-go” theorem, will it also evade it in practice? This is the question on which She-Sheng and myself are devoting quite a lot of hard thinking since three years, and this is where we have got so far⁶

(a) The origin of masses

We start our analysis by computing the energy of the ground state (the effective potential) as a function of (we work from now on with a euclidean lattice; V_4 is the 4-dimensional euclidean volume)

$$M^F = -\frac{G_1}{2V_4} \sum_x \left\langle \bar{\Psi}^F(x) \Psi^F(x), \right\rangle \quad (15)$$

and

$$r^F = -\frac{G_2}{8V_4} \sum_{x,\mu} \left\langle \bar{\Psi}_L^F(x) U_\mu(x) \Psi_R^F(x + a^\mu) + \text{h.c.} \right\rangle. \quad (16)$$

Note that both M^F and r^F , when different from zero, violate the original chiral gauge symmetry, causing some or all the fermions to acquire a mass and **all** the doublers to “go to Heaven”, i.e. to get a mass of the order of the Planck mass $m_P \simeq 10^{19}$ GeV.

In other words, what we are trying to ascertain is whether, by generating non-zero values of M^F and r^F , the ground state energy gets lowered with respect to the symmetric situation where $M^F = r^F = 0$. Note that the symmetric situation corresponds to a totally unacceptable, because unrealistic, scenario where all the fermions and gauge bosons are massless, the low-energy spectrum is doubled and no evasion from the “no-go” theorem is possible.

The problem we must now solve is formally identical to the one that occurs in the theory of Superconductivity, where a key rôle is played by the possible non-trivial solution(s) of the gap-equation(s).

To have a clearer idea of what kind of physics is involved, as a first step we switch off the gauge-interactions and keep the Nambu-Jona Lasinio (NJL) terms only. In the mean-field approximation (the same that is effectively employed in the theory of

Superconductivity) it is a completely straightforward business to derive the appropriate “gap equations” and to solve them with the following results:

- (i) r^F is non-vanishing for **all** fermions and for all choices of the adimensional coupling constants $g_2 = \frac{G_2}{a_P^2}$ (Fig. 1). This result fulfills the necessary and sufficient condition for the removal of doublers;

Fig. 1. The function $r^F(g_2)$ for quarks (q) and leptons(l) ($ma \simeq 0$).

- (ii) M^F – the fermion mass matrix – also turns out to be in general different from zero, depending however on the values of g_1 and g_2 (see Fig. 2). Many solutions, however, are possible with the following features:
 - (α) all leptons are massless;
 - (β) either one, two or three quark families become massive (the other two, one or zero remaining massless).

The real minimum of the ground state energy (the effective potential) can be ascertained, and the true solution can be found, only when the problem of composite particles (the Goldstone modes) is solved and their contribution to the ground-state

energy is computed. This will be discussed in a moment, for the time being, however, we remark that:

- (i) mass is generated in the SM;
- (ii) doublers are removed;

and all this without arbitrarily extending the basic building blocks (fermionic matter and gauge-fields) of the SM.

Fig. 2. The critical line for $m = 0$ in the $g_1 - g_2$ plane.

(b) The composite particles' spectrum

When one analyzes the correlation functions

$$= -\frac{G_1}{\sqrt{2}} \left\langle \bar{\Psi}^F(x) \gamma_5 \Psi^F(x) \bar{\Psi}^F(0) \gamma_5 \Psi^F(0) \right\rangle, \quad (17)$$

one finds, as predicted by the Goldstone theorem, massless poles – the Goldstone bosons – whose number is equal to $(2N_F)^2$, where N_F is the number of quark families that acquire a non-zero mass.

Note that the solution $N_F \neq 1$ is phenomenologically disastrous, for only four Goldstone bosons can be “incorporated” in the longitudinal dynamics of the gauge fields, which become thus massive, as experimentally observed. All Goldstone bosons in excess of four should therefore remain in the observable spectrum, thus clashing with the experimental fact that no such particles have ever been detected, even though they could be copiously produced.

In addition one can analyze the scalar correlation function

$$\frac{G_1}{\sqrt{2}} \left\langle \bar{\Psi}^F(x) \Psi^F(x) \bar{\Psi}^F(0) \Psi^F(0) \right\rangle = \quad (18)$$

and look for possible poles in this channel too. This exercise is far from academic, for in the continuum SM supplemented by NJL-interactions, studied in refs.⁷, it was found that the ugly and unnatural Higgs particle, chased away from the theory and replaced by the NJL-term, slyly reappears in the particle spectrum as an “almost pointlike” composite scalar particle with mass

$$m_S \simeq 2m_F = 2m_{top}, \quad (19)$$

a very high mass, which according to current knowledge should be around 400 GeV: a very natural goal of the future (if any) supercolliders. The remarkable and surprising consequence of the novel structure of the PLSM at the Planck scale is that instead of Eq. (19), we obtain

$$m_S^2 = (2m_F)^2 + 0.8m_F \frac{r_F}{a_P} + 0.9 \frac{(2r_F)^2}{a_P^2}, \quad (20)$$

where $r_F \simeq 0.25$ is the coefficient of the Wilson-term, that is responsible for the lifting of the doublers’ mass to the Planck scale. Thus, according to (20), the finite mass (19) of the CSM receives in the PLSM corrections of the order of the Planck mass itself $1/a_P = m_P$. The physical origin of the extra terms that lift the Higgs mass to m_P is quite easy to track: it stems from the doublers that, due to $r^F \neq 0$, dominate the spectrum at the Planck scale.

4. The origin of masses

Having found a consistent way to build a PLSM, by evading the constraints of the “no-go theorem” through a minimal set of NJL quadrilinear terms, we shall

now investigate how such PLSM takes account of the most fundamental and puzzling aspect of the electroweak phenomenology, that of fermion (quarks and leptons) and boson masses.

In the preceding Section we have seen that in the simple mean-field approximation to the ground state energy density, the gap equations for the fermion masses allow a plurality of solutions, where one, two or three quark generations acquire a non-zero mass, while the rest of fermions, and in particular **all** leptons, remain massless. There exist also solutions where the situation between quarks and leptons gets interchanged, depending on the (very) fine-tuning of the coupling constants $g_{1,2}$. It is interesting that such asymmetry between quarks and leptons finds its origin in the colour degree of freedom, which the leptons lack.

For obvious phenomenological reasons we must choose the former (exclusive) class of solutions, and in order to see whether the degeneracy noted above may be lifted we must go beyond the mean-field approximation to the vacuum energy, to include the contributions from the composite fields, that we have just seen to emerge from the solutions of the gap equations. By adding such contributions to the effective potential we obtain the energy density $\Delta E(r)$ ($\Delta E = 0$ for the symmetric configuration) as a function of r (the Wilson parameter) for $m^F a_p \simeq 0$ depicted in Fig. 3.

Remarkably, only the solution with $N_F = 1$ realizes an energy gain with respect to the symmetric ground state, attaining its minimum at $r \simeq 0.25$. The other two solutions of the gap equations with $N_F = 2$ and 3 are always more energetic than the symmetric, massless solutions. This results is extremely welcome, for it implies that in a first stage of approximation the fermionic spectrum contains only two massive quarks (the t- and b-quarks), while all other fermions are massless. Furthermore, as we shall see in a moment, the four Goldstone bosons that, in the absence of gauge-interactions, result from the "spontaneous" breaking of the original chiral symmetry, are in the right number and structure to get incorporated in the longitudinal dynamics of the weak gauge bosons, thus giving rise to the observed masses of such particles.

When we switch the gauge interactions on, and analyse the Dyson's equations of the PLSM for the fermion self energies we obtain for the self-mass operator of the massive doublet (top and bottom) the equation depicted in Fig. 4.

What is so interesting about such equation, when evaluated on the Planck lattice, is that, due to the doublers, at the Planck scale the charged gauge bosons (W^\pm) contribute a term which couples Σ_t to Σ_b and renders the above equation equation non diagonal. Furthermore the different e.m. charges of the top- (2/3) and bottom- (-1/3) quark introduce an asymmetry capable in principle to lift the degeneracy between the two quarks, that in a pure NJL-scenario are indeed degenerate. But, can it lift it in the dramatic way suggested by observation? A completely straightforward calculation⁶ gives the very enticing answer:

$$m_t \simeq 30m_b, \quad (21)$$

which setting $m_b \simeq 5\text{GeV}$, predicts the top mass in the ball park where it has

Fig. 3. The vacuum energy $\Delta E(r_q)$ for different N_F ($m^q a_P \simeq 0$).

been very recently announced. Another very pleasing aspect of the PLSM analysis, as compared with the one carried out within the CSM,⁷ is that the tuning of the coupling constant $g_{1,2}$, necessary to yield finite quark masses, is not so incredibly fine, i.e. of $O(m_q/m_P)$, but only of order $O(\alpha \log(m_q/m_P))$. The same exercise can be carried out for other quarks and leptons, where now the composite fields' contribution is missing, and for leptons where the NJL-term is 3 times smaller than for the quarks, due to their being colourless. When it is done (but it has not yet been completed) a complete fermion spectrum should emerge tying its apparently capricious pattern to the mixing (Kobayashi-Maskawa) matrices of the quarks and leptons. The potentiality of this approach looks really remarkable: I hope to be able to report its final results soon.

Turning now to the masses of the gauge bosons, in order to calculate them we must analyse the gauge-bosons self energy function.

$$\Pi_{\mu\nu}(k) = (-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2})\Pi(k^2) \quad (22)$$

given by the diagrams (Fig. 5).

Doing the tedious job of computing the diagrams,⁶ and in particular keeping track

Fig. 4. The Dyson equations for top and bottom quark masses.

of the composite Goldstone modes, we obtain a relation between the top-quark mass m_t and the Z -mass:

$$m_t = 1.633 M_Z = 149 \text{GeV}. \quad (23)$$

Furthermore we obtain:

$$\frac{M_W^2}{M_Z^2} = \cos^2 \Theta_W (1 + \rho), \quad (24)$$

where

$$\rho = \left(\frac{m_b}{m_t}\right)^2 \frac{\log\left(\frac{M_Z^2}{M_W^2}\right)}{\log\left(\frac{\Lambda_P^2}{m_t^2}\right)} \simeq 10^{-6}, \quad (25)$$

in agreement with well established phenomenology.

5. Conclusions

Let me now conclude this talk by trying to summarize and put in perspective its theme. As I have already emphasized in the Introduction, in its momentous development since the end of World War II, that led to the admirable synthesis of the SM, Particle Physics rarely felt any need or interest for Gravitation, the particle aspect of which, all the way down to the Planck scale, looking so incredibly remote. And if recently the attitude towards Gravitation has changed, it has only been in the framework of a far fetched search for the Theory of Everything, a definite act of hybris which may well ruin theoretical particle physics.

Fortunately in Astrophysics and Cosmology the great tradition that from Riemann through M. Grossmann and A. Einstein enriched mankind with the classical theory of Gravitation (General Relativity), has been kept alive and keenly focused on

Fig. 5. The diagrammatic form of the Dyson equation for $\Pi_{\mu\nu}(k)$.

the fundamental problem of the structure of Space-Time, as determined by the dynamics of Gravity: Geometrodynamics, as J.A. Wheeler has aptly called it. It is this way of looking at Gravity, within the general framework of QFT, that is giving new meaning and relevance to the "neglected interaction" of Particle Physics, which now plays the fundamental role of determining the very nature of the space-time, in which the drama of the SM is played. I believe it will soon be possible to show that, as conjectured by Wheeler himself,¹ at the Planck scale Euclidean Space-Time - the ground state of classical Gravity - will cease to be the ground state of Quantum Gravity and it will be replaced by a "condensate" of "wormholes", of size a_P at a distance a_P : a foam of Planck size. The reason for believing in the soundness of Wheeler's intuition is a similar result that I have been able to derive for the (likely) ground state of another non-abelian gauge-theory: QCD.² There, the highly non-linear interactions of the quantum fluctuations – the gluons – render a finely structured network of needle-shaped chromomagnetic domains highly stable with respect to the perturbative ground state; and as a consequence on such a ground state all isolated colour charges acquire an infinite mass, thus disappearing from the physical spectrum: colour confinement is finally explained and understood.

And if space-time at the Planck scale dissolves into a foam of wormholes, it is a fascinating question to ask what will then happen to the local matter and gauge fields of the Continuum Standard Model (CSM). In such a discontinuous structure, which one can reasonably approximate with the more regular structure of a Planck lattice, She Sheng Xue and I have gone through the long and to try give an answer to this questionr. The immediate difficulty posed by the "no-go" theorem, that bans the possibility

to directly transcribe the CSM on any lattice surprisingly unveils a path – the addition of a minimal number of quadrilinear Fermi interactions (see Eqs. (2.13) and (2.14)) – that allows us to get rid of the ugly and unnatural Higgs Lagrangian (Eq. (2.7)). In fact the fundamental problem of the generation of the masses of both fermions and weak gauge bosons gets an immediate solution through a process of condensation of fermion-pairs, quite analogous to what happens in the phenomena of superconductivity. Mass does get generated in principle without adding to the beautiful and economical SM an artificial and troublesome new type of matter, the Higgs scalars. Do the generated masses conform with the seemingly chaotic observed pattern? Even though the necessary analysis is not yet completed, we can however state with confidence that in the "spontaneous" symmetry breaking mechanism of the PLSM:

- (i) a single quark family – the t,b-family – receives masses that are much larger than those of the other quark and lepton families;
- (ii) the m_t/m_b -ratio is predicted correctly [Eq. 21];
- (iii) a correct relation can be derived between the top mass and the weak boson masses [Eqs. (23), (24) and (25)]
- (iv) a theory of the mixing matrices for both quarks and leptons seems within reach.

If the promise will finally be kept and a successful PLSM will be added to the treasures of scientific rationalism, a new and intriguing type of relationship will be seen to tie Gravity to the masses of the Universe: no more and only the sources of Gravity, masses will finally have found their "raison d' être" in the violent fluctuations of the metric gravitational field that at the Planck distance tear space-time apart.

References

1. C.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation* (Freeman, San Francisco 1973).
2. See for instance G. Preparata, *Why Are Quark (and Gluons) Confined?*, MITH 93/18, Milano, June 1993, Preprint, to appear in the Proceedings of the VIII Winter School on "Hadronic Physics" – Folgaria, 1-6 February 1993.
3. H.B. Nielsen and M. Ninomya, Nucl. Phys. B185(81)20 and B193(81)173, Phys. Lett. B105 (1981) 219.
4. K. Wilson, in "New phenomena in subnuclear physics" (Erice, 1975) ed. A. Zichichi (Plenum, N.Y. 1977).
5. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122(1961)345.
6. G. Preparata and S.-S. Xue, Phys. Lett. B264(91)35; B302(93)442; B235(94)161; B329(94)87; B335(94)192; B325(94)161; "Emergence of the $t\bar{t}$ condensate and the disappearance of Higgs scalars in the Standard Model on the Planck Lattice", preprint MITH 93/5, submitted to Nucl. Phys.
7. W.A. Bardeen, C.T. Hill and M. Linder, Phys. Rev. D41(90)1647.