Graduate Trade (II): ECON 8433

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Fall Semester 2020

Plan

Week	Topic
Week 1	Introduction to Structural Gravity Equation
Week 2	Calibration and Estimation
Week 3	Mapping Models to the Data
Week 4	Designing Counterfactual Experiments in General Equilibrium
Week 5	Presentations (I) and Catch-up
Week 6	Heterogeneous Firms (I)
Week 7	Heterogeneous Firms (II)
Week 8	Ricardian Models
Week 9	Multi-Sector Models
Week 10	Global Value Chains
Week 11	Presentations (II) and Catch-up
Week 12	Extensions: Demand Side
Week 13	Extensions: Supply Side
Week 14	Extensions: Migration and Geography
Week 15	Presentations (III) and Catch-up

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Timing

FCQ's:

- `colorado.campuslabs.com/courseeval`

Outline

1. Overview of current approaches
2. Two-piece distribution
3. Data and estimation
4. Workhorse trade model with heterogeneous firms
5. Counterfactual experiments
6. Sensitivity analysis and extensions
7. Follow-up research
8. Conclusion

Estimation

The data come from ORBIS and cover almost 1 mil. French entities in 2012

1. Keep firms in production sectors
2. Calculate domestic sales as total sales net of export revenues
3. Calculate productivity distribution
4. Generate 100,000 quantiles (for numerical purposes)
   - This grid covers c.d.f. on the support $[0.00001; 0.99999]$

QQ-estimator

The estimator solves the following:

$$\min_{\Theta_\ell} \left\{ \sum_q \left(\ln\left[Q_e(q)\right] - \ln\left[Q_\ell(q|\Theta_\ell)\right] \right)^2 \right\}$$

- q is the grid of the c.d.f. 100,000 data points in $[0.00001; 0.99999]$
- $Q_e(q)$ the empirical quantile function evaluated at q
- $Q_\ell(q)$ is the parametric quantile function of type ℓ evaluated at q
- Θ_ℓ is the vector of parameters of the parametric function of type ℓ to estimate

Example of QQ-estimator

To estimate parameters of the Pareto distribution, recall the c.d.f. of Pareto:

$$F_P(x) = 1 - \left(\frac{x_m}{x}\right)^\alpha \text{ for } x \geq x_m$$

Invert the c.d.f. to get the quantile function and take logs:

$$\ln(Q_P(q)) = \ln x_m - \frac{1}{\alpha} \ln(1 - F_q) + \epsilon_q$$

Since F_q is observed, QQ-estimator minimizes the sum of squared residuals and boils down to OLS.

Results I: fit on different intervals of the support

	Parameters			Root Mean Squared Error				
	(I)	(II)	(III)	All	Bot. 1%	Bot. 5%	Top 5%	Top 1%
Two-piece	3.033	1.185	0.938	0.058	0.465	0.221	0.026	0.033
	(0.006)	(0.005)	(0.001)					
Log-normal	0.569	-0.701		0.069	0.415	0.194	0.156	0.304
	(0.001)	(0.001)						
Pareto	1.914	0.294		0.236	1.405	0.840	0.344	0.648
	(0.005)	(0.001)						
B. Pareto	0.372	0.214	1.438	0.183	1.105	0.582	0.406	0.799
	(0.026)	(0.001)	(0.015)					

Table notes: In the case of the Two-piece distribution, parameter (I) refers to the shape parameter, α, (II) and (III) refer to the scale parameters, θ and ρ, respectively; in the case of the Log-normal distribution, (I) and (II) refer to the scale and location parameters; in the case of the Pareto, (i) and (II) refer to the shape and scale parameters; in the case of the Bounded Pareto distribution, (I) refers to the shape parameter and (II) and (III) to two location parameters. All parameters are estimated using 100,000 quantile data points.

Table: Estimation results

Results II: QQ-plot

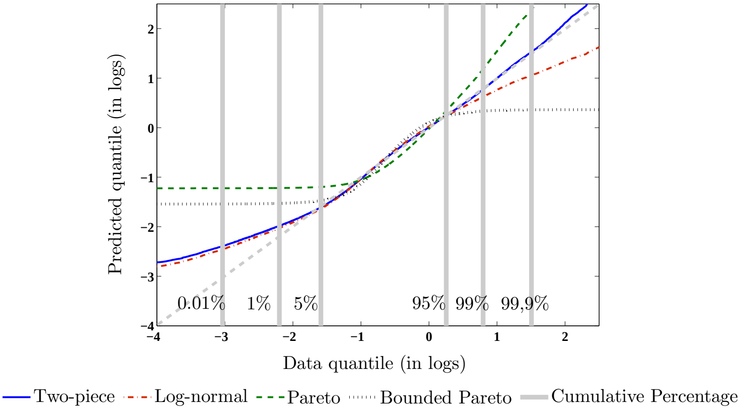

Figure: QQ plot of two-piece, log-normal and pareto vs. data

Results III: p.d.f.

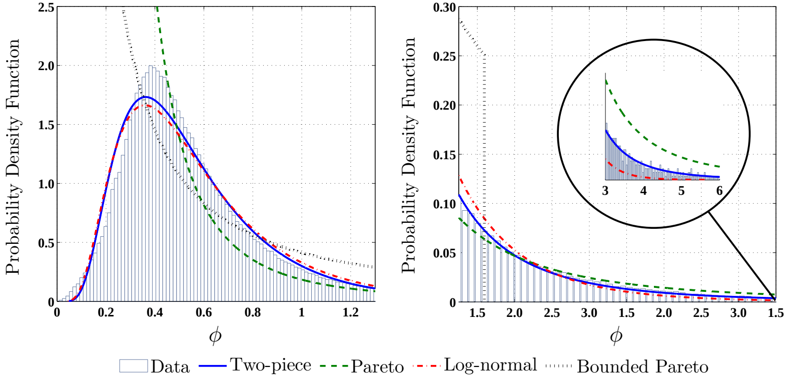

Figure: Density of two-piece, log-normal and pareto vs. data

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Workhorse trade model with heterogeneous firms

The model is standard and follows Arkolakis, Demidova, Klenow and Rodríguez-Clare (2008) and only slightly deviates from Arkolakis, Costinot and Rodríguez-Clare (2012) and Melitz and Redding (2014):

- Heterogeneous firms: ϕ
- CES preferences: $q(\phi)$
- Labor is the only factor of production: L_i
- Variable and fixed cost of exporting: τ_{ij} and f_{ij}
- Fixed cost of entry: f_i^e
- Productivity is randomly drawn upon paying entry cost: f_i^e
- Free entry, markets clear

Workhorse trade model with heterogeneous firms

Consumers maximize utility according to the CES function:

$$U_j = \left(\sum_{i \in J} \int_{\Omega_{ij}} q_{ij}(\phi)^{\frac{\sigma-1}{\sigma}} d\phi \right)^{\frac{\sigma}{\sigma-1}}$$

where Ω_{ij} is the set of goods from i available in j, and σ is the elasticity of the substitution parameter. Consumer optimization leads to:

$$\begin{aligned} x_{ij}(\phi) &= \frac{1}{p_{ij}(\phi)} \left(\frac{p_{ij}(\phi)}{P_j} \right)^{1-\sigma} L_j w_j \\ P_j &= \left(\sum_{i \in J} \int_0^{\bar{\phi}} p_{ij}(\phi)^{1-\sigma} d\phi \right)^{\frac{1}{1-\sigma}} \end{aligned}$$

Workhorse trade model with heterogeneous firms

Firms from i maximize their profits in market j according to the following function:

$$\pi_{ij}(\phi) = \left(\frac{p_{ij}(\phi)}{P_j}\right)^{1-\sigma} L_j w_j - \frac{w_i}{\phi} p_{ij}(\phi)^{-\sigma} \tau_{ij} P_j^{\sigma-1} L_j w_j - w_j f_{ij}$$

where f_{ij} is the fixed cost of exporting from i to j. The solution is:

$$p_{ij}(\phi) = \frac{\sigma}{\sigma-1}\frac{w_i}{\phi}\tau_{ij}$$

Not all firms in i choose to export to j but only those that have a productivity higher than the cut-off, defined as:

$$\phi_{ij}^* = \left(\frac{\sigma}{L_j}\right)^{\frac{1}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right) P_j^{-1} f_{ij}^{\frac{1}{\sigma-1}}$$

Workhorse trade model with heterogeneous firms

In equilibrium, the expected profits must be zero such that the expected revenues exactly cover the entry cost:

$$\sum_{j \in J} \left(\int_{\phi_{ij}^*}^{\bar{\phi}} w_j f_{ij} (\phi_{ij}^*)^{1-\sigma} \phi^{\sigma-1} f(\phi) d\phi - \int_{\phi_{ij}^*}^{\bar{\phi}} w_j f_{ij} f(\phi) d\phi \right) = w_i f_i^e$$

Labor market clearing condition:

$$\frac{N_i}{1-F(\phi_{ii}^*)} \sum_{j \in J} \left(\frac{(\sigma-1) w_j}{w_i} f_{ij} (\phi_{ij}^*)^{1-\sigma} \int_{\phi_{ij}^*}^{\bar{\phi}} \phi^{\sigma-1} f(\phi) d\phi + f_i^e \right) + \sum_{j \in J} \frac{N_j}{1-F(\phi_{jj}^*)} f_{ji} \int_{\phi_{ji}^*}^{\bar{\phi}} f(\phi) d\phi = L_i$$

Solution

The solution of the system depends on two selection statistics:

- $1 - F(\phi_{ij}^*)$ which measures the probability of firms from i being active in j for all i, j
- $\int_{\phi_{ij}^*}^{\bar{\phi}} \phi^{\sigma-1} f(\phi) d\phi$, which is required to calculate total revenues of firms from i in market j for all i, j
- Third statistics $\int_{\phi_{ij}^*}^{\bar{\phi}} f(\phi) d\phi$ is redundant due to the following identity:

$$\int_{\phi_{ij}^*}^{\bar{\phi}} f(\phi) d\phi = \int_0^{\bar{\phi}} f(\phi) d\phi - \int_0^{\phi_{ij}^*} f(\phi) d\phi = 1 - F(\phi_{ij}^*)$$

International trade outcomes

Upon solving the model, one can explore different trade outcomes:

$$\text{Welfare Gains} = 100\% \times \left(\frac{w_i(\tau)}{P_i(\tau)} \frac{P_i(\tau')}{w_i(\tau')} - 1 \right)$$

International trade shares:

$$\lambda_{ij} = \frac{N_i}{1 - F(\phi^*_{ii})} \left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{1-\sigma} \left(\int_{\phi^*_{ij}}^{\bar{\phi}} \phi^{\sigma-1} f(\phi) d\phi \right) P_j^{\sigma-1}$$

Share of exporters from each origin to each destination:

$$\chi_{ij} = 1 - F(\phi^*_{ij})$$

Parameterization of the model

Without loss of generality, the model's primitives are chosen as follows:

Parameter	J	L_1	L_2	f_1^e	f_2^e	f_{11}	f_{22}	σ
	2	100	50	1	1	0.001	0.001	4

Table: Primitives of the model

Simplistic but general framework, amenable to increasing the number of countries/sectors.

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Counterfactual experiments

In the counterfactual experiments, τ_{ij} gradually goes from 3 to 1 at given levels of fixed costs of exporting:

- Experiment 1: Trade liberalization at high fixed export costs
  - Designed to look at firms in the right tail of the distribution
- Experiment 2: Trade liberalization at low fixed export costs
  - Designed to look at firms in the left tail of the distribution

Define the error in the estimates of the welfare gains from trade as:

$$Error_\ell = \text{Welfare Gains}(\tau') - \text{Welfare Gains}_\ell(\tau')$$

for model $\ell = \{\text{Two-piece, Log-normal, (un-)bounded Pareto}\}$

Benchmark

Results under all parametric distributions will be compared to those under numerical benchmark:

- $1 - F(\phi_{ij}^{*})$ is calculated by using empirical c.d.f. in a non-parametric form
- $\int_{\phi_{ij}^{*}}^{\bar{\phi}} \phi^{\sigma-1} f(\phi) d\phi$ is calculated by numerical trapezoidal integration

Experiment 1: falling τ_{ij} at high f_{ij}

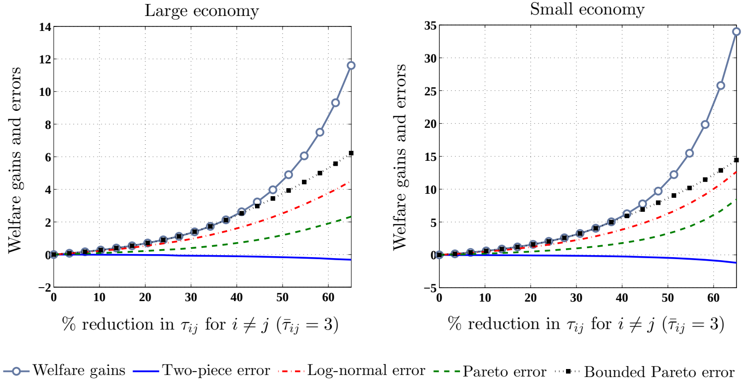

Figure: BENCHMARK WELFARE GAINS AND ERRORS: EXPERIMENT 1

Experiment 2: falling τ_{ij} at low f_{ij}

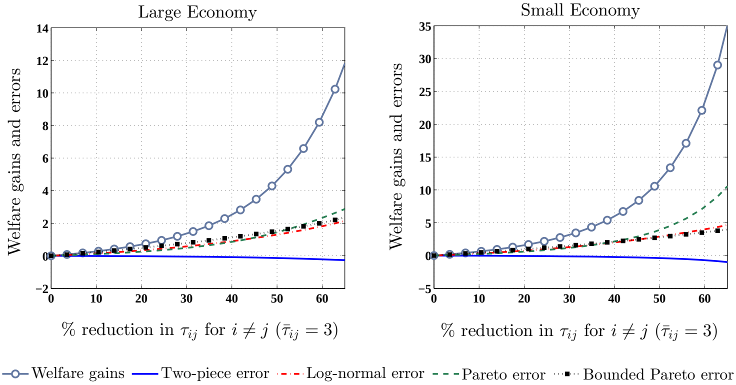

Figure: Benchmark welfare gains and errors: experiment 2

Other trade outcomes

Researchers are often interested in two additional trade statistics: intensive and extensive margins of trade. For this, define mean squared errors as follows:

$$MSE_{\ell}(\lambda) = \sqrt{\frac{1}{J}\sum_{j}\left(\lambda_{jj} - \lambda_{jj,\ell}\right)^2}; \quad MSE_{\ell}(\chi) = \sqrt{\frac{1}{J}\sum_{j}\left(\chi_{j} - \chi_{j,\ell}\right)^2}$$

where λ_{jj} is the share of intra-trade (governs intensive margin) and χ_j is the share of exporters (extensive margin).

Other trade outcomes

Repeat experiments 1 and 2 and calculate mean squared errors under the four different parametric distributions:

	Variable	Share of intratrade				Share of Exporters			
	τ_{ij} for $i \neq j$	3.0	2.4	1.8	1.2	3.0	2.4	1.8	1.2
Exp. 1	Two-piece	1.57	2.69	5.02	7.53	0.04	0.08	0.20	1.85
	Log-normal	31.38	51.16	78.91	81.65	2.29	7.80	27.77	93.02
	Bounded Pareto	12.72	22.06	40.39	63.01	0.51	1.02	2.25	7.20
	Pareto	37.30	71.05	121.29	75.83	0.75	1.51	26.44	155.79
Exp. 2	Two-piece	1.49	2.59	4.57	6.67	0.27	0.76	2.41	6.32
	Log-normal	21.61	29.11	33.42	22.13	23.66	54.58	126.29	265.54
	Bounded Pareto	14.79	25.99	48.96	80.46	1.92	3.88	9.25	23.73
	Pareto	35.13	34.51	29.28	16.23	8.67	91.78	197.69	326.43

Table notes: For expositional purposes, due to the fractional nature of the variables the results are reported in one thousandths.

Table: Mean squared errors in the share of intratrade and exporters

Outline

Sensitivity analysis and extensions

Several extensions and sensitivity checks:

1. Comparing to 3-parameter distributions 3-param. distr.
2. Removing data points at the extremes Extreme data points
3. Testing for sensitivity to the choice of the country Other countries
4. Alternative measures of productivity Other measures of ϕ
5. Truncation of the Two-piece distribution Truncation
6. External validity: city size distribution Out of sample test

Assignment

Using QQ (or other) estimators to estimate pdf:

- Download "distribution_data.mat" from CANVAS
- Choose the "best" pdf to explain the data