Graduate Trade (II): ECON 8433

Sergey Nigai
University of Colorado Boulder
Fall Semester 2018

Plan

Week	Topic
Week 1	Introduction to Structural Gravity Equation
Week 2	Calibration and Estimation
Week 3	Mapping Models to the Data
Week 4	Designing Counterfactual Experiments in General Equilibrium
Week 5	Presentations (I) and Catch-up
Week 6	Heterogeneous Firms (I)
Week 7	Heterogeneous Firms (II)
Week 8	Ricardian Models
Week 9	Multi-Sector Models
Week 10	Global Value Chains
Week 11	Presentations (II) and Catch-up
Week 12	Extensions: Demand Side
Week 13	Extensions: Supply Side
Week 14	Extensions: Migration and Geography
Week 15	Presentations (III) and Catch-up

Plan

WEEK	TOPIC
Week 1	Introduction to Structural Gravity Equation
Week 2	Calibration and Estimation
Week 3	Mapping Models to the Data
Week 4	Designing Counterfactual Experiments in General Equilibrium
Week 5	Presentations (I) and Catch-up
Week 6	Heterogeneous Firms (I)
Week 7	Heterogeneous Firms (II)
Week 8	Ricardian Models
Week 9	Multi-Sector Models
Week 10	Global Value Chains
Week 11	Presentations (II) and Catch-up
Week 12	Extensions: Demand Side
Week 13	Extensions: Supply Side
Week 14	Extensions: Migration and Geography
Week 15	Presentations (III) and Catch-up

Ricardian Model of Trade

In terms of quantitative work, there are three canonical multi-country models of international trade:

- Anderson and van Wincoop (2003)
- Melitz (2003)
- Eaton and Kortum (2002)

We have already covered Anderson and van Wincoop (2003) in our discussion of the Armington-type models. We have also seen Melitz (2003). Today, we will look at Eaton and Kortum (2002).

Ricardian Model of Trade

- The classical Ricardian model features two countries and two goods
- Dornbusch, Fischer, and Samuelson (1977) extended the framework to a continuum of goods but for only two countries
- Eaton and Kortum (2002) formulated a model with many countries, multiple industries and arbitrary trade costs.

Ricardian Model of Trade

The original paper was titled "Technology, Geography and Trade". The authors argued that trade models need to come to grips with the following four facts:

- Trade diminishes dramatically with distance
- Prices vary across locations, with greater difference between places farther apart
- Factor rewards are far from equal across countries
- Countries' relative productivities vary substantially across industries

Eaton and Kortum (2002) argue that the first two facts have to do with geography, whereas the latter two – with different technologies. Their model is able to capture all four facts.

Ricardian Model of Trade

The general set-up is similar to what we've already seen. You, however, will see that Eaton and Kortum (2002) has conceptual differences relative to the Armington and Melitz models.

- There are S countries in the world
- Continuum of goods Ω
- Every country is able to produce every good subject to their technology and factor prices
- The productivity levels differ across countries
- The model features perfect competition

Ricardian Model of Trade

Consumers have CES preferences and maximize the following utility function:

$$U_j = \left(\int_\Omega q_j(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

subject to the usual budget constraint. Note that unlike the Armington model, country j does not consume <u>all</u> goods produced in the world.

The CES price index is:

$$P_j = \left(\int_\Omega p_j(\omega)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}}$$

Ricardian Model of Trade

Due to perfect competition, the concept of a firm is problematic in Eaton and Kortum (2002).

- Each country has a representative producer for each variety: ω
- The cost of input bundle is the same for all producers in i: c_i
- The productivity level of producing good ω in country i, $z_i(\omega)$, is drawn from a country-specific distribution

Given the cost of input bundle and productivity level, the cost of production is:

$$\frac{c_i}{z_i(\omega)}$$

The are iceberg trade costs τ_{ij} between countries i and j.

Ricardian Model of Trade

Given production and trade costs, country i can supply good ω to country j at the following price:

$$p_{ij}(\omega) = \frac{c_i \tau_{ij}}{z_i(\omega)}$$

If consumers can buy ω from any $i \in S$. How do they choose?

$$p_j(\omega) = \min_{i \in S} \{p_{ij}(\omega)\} = \min_{i \in S} \left\{ \frac{c_i \tau_{ij}}{z_i(\omega)} \right\}$$

The probability that i will be the supplier to country j:

- Decreasing in c_i and τ_{ij}
- Increasing in $z_i(\omega)$

Ricardian Model of Trade

What is the distribution of $z_i(\omega)$?

Eaton and Kortum (2002) assume that $z_i(\omega)$ is drawn from Frechet with the following p.d.f.:

$$F_i(z) = e^{-T_i z^{-\theta}},$$

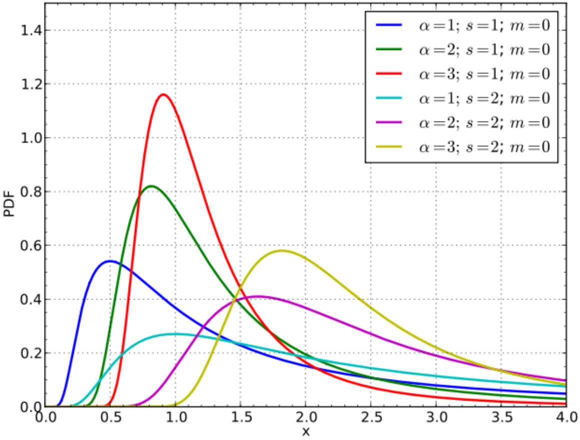
where $T_i > 0$ and $\theta > 1$. In terms of the model, we need to make a stricter assumption $\theta > \sigma - 1$.

Ricardian Model of Trade

Here is what Eaton and Kortum (2002) write about T_i and θ:

- "The parameters T_i and θ enable us to depict very parsimoniously a world of many countries that differ in the basic Ricardian senses of absolute and comparative advantage across a continuum of goods.
- We will refer to the parameter T_i as country i's state of technology. In a trade context T_i reflects country i's absolute advantage across this continuum.
- The parameter θ regulates heterogeneity across goods in countries' relative efficiencies. In a trade context, θ governs comparative advantage within this continuum. As we show more formally below, a lower value of θ, generating more heterogeneity, means that comparative advantage exerts a stronger force for trade against the resistance imposed by the geographic barriers".

Ricardian Model of Trade



Source: Wikipedia

Ricardian Model of Trade

To get the analytical solution for the model, we will proceed in several steps:

1. Derive the probability that i supplies to j at a price lower than p:
$$G_{ij}(p) = Prob\left(p_{ij}(\omega) \leq p\right)$$
2. Derive the fraction of goods that j buys from i:
$$\pi_{ij} = Prob\left(p_{ij}(\omega) \leq \min_{k \neq i} p_{kj}(\omega)\right)$$
as a function of $G_{ij}(p)$

Ricardian Model of Trade

Technology is i.i.d. across different goods. This means that the probability that j offers the to i that is lower than p is:

$$
\begin{aligned}
G_{ij}(p) &= Prob(p_{ij}(\omega) \leq p) = Prob\left(\frac{c_i \tau_{ij}}{z_i(\omega)} \leq p\right) \\
&= Prob\left(\frac{c_i \tau_{ij}}{z_i(\omega)} \leq p\right) = Prob\left(\frac{c_i \tau_{ij}}{p} \leq z_i(\omega)\right) \\
&= 1 - F\left(\frac{c_i \tau_{ij}}{p}\right) = 1 - \exp\left\{-T_i\left(\frac{c_i \tau_{ij}}{p}\right)^{-\theta}\right\}
\end{aligned}
$$

Ricardian Model of Trade

We now can find the distribution of prices across different goods in country j as:

$$
\begin{aligned}
G_j(p) &= Prob(\min_{i \in S} p_{ij}(\omega) \leq p) = \\
&= 1 - Prob\left(\min_{i \in S} p \leq \frac{c_i \tau_{ij}}{z_i(\omega)}\right) = 1 - \prod_{i \in S} \left(1 - G_{ij}(p)\right),
\end{aligned}
$$

where we know $G_{ij}(p)$! This gives us the following identity:

$$
\begin{aligned}
G_j(p) &= 1 - \prod_{i \in S} \left(1 - G_{ij}(p)\right) = \\
&= 1 - \prod_{i \in S} \left(\exp\left\{-T_i \left(\frac{c_i \tau_{ij}}{p}\right)^{-\theta}\right\}\right),
\end{aligned}
$$

Ricardian Model of Trade

Simplify it further to get:

$$
\begin{aligned}
G_j(p) &= 1 - \exp\left(-p^{\theta} \sum_{i \in S} T_i \left(c_i \tau_{ij}\right)^{-\theta}\right) \\
&= 1 - \exp\left(-p^{\theta} \Phi_j\right),
\end{aligned}
$$

where

$$
\Phi_j = \sum_{i \in S} T_i \left(c_i \tau_{ij}\right)^{-\theta}.
$$

We now know the distribution of prices in country j ! This is useful for finding the price index.

Ricardian Model of Trade

The CES price index can be derived as follows:

$$
\begin{aligned}
P_j &= \left(\int_{\Omega} p_j(\omega)^{\sigma-1} d\omega \right)^{\frac{1}{\sigma-1}} \\
P_j^{1-\sigma} &= \int_0^{\infty} p^{1-\sigma} dG_j(p) \\
P_j^{1-\sigma} &= \int_0^{\infty} \theta \Phi_j p^{\theta-\sigma} \exp(-p^{\theta}\Phi_j) dp
\end{aligned}
$$

To find this integral, we will use the change of variables trick $x = p^{\theta}\Phi_j$:

$$
\begin{aligned}
P_j^{1-\sigma} &= \int_0^{\infty} \left(\frac{x}{\Phi_j} \right)^{\frac{1-\sigma}{\theta}} exp(-x) dx \\
P_j^{1-\sigma} &= \Gamma \left(\frac{\theta + 1 - \sigma}{\theta} \right) \Phi_j^{-\frac{1-\sigma}{\theta}},
\end{aligned}
$$

where $\Gamma(\cdot)$ is the gamma function.

Ricardian Model of Trade

An interesting result is that as long as $\theta > \sigma - 1$ holds, the price index does not depend on σ! This is different from the Armington and Melitz models:

$$P_j = \Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \Phi_j^{-\frac{1}{\theta}},$$

Notice, that now θ which measures the dispersion of the productivity distribution plays the role of σ! What is the intuition for this result?