# Graduate Trade (II): ECON 8433

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University of Colorado Boulder
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# Organizational details

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|---|---|
| LECTURE: | Mon. & Wed., 1:55pm – 3:10pm |
| EMAIL: | sergey.nigai@colorado.edu |
| WEB-SITE: | Canvas |
| OFFICE HOURS: | Mon., 9:30am-11:30pm, and by appointment (same ZOOM link) |

# Why trade?

# Why trade?

International trade is a field that studies movement of goods and factors of production across borders. What makes international trade special?

1. Large variety of topics to study.

The field is on the cross-over of different fields:

- Trade and IO
- Trade and Macro
- Trade and Labor
- Trade and Development

# Why trade?

2. Data sources

You can choose to examine trade issues at the world-, regional-, country-, state-, industry-, firm-, and worker-level

- Aggregate trade data exist virtually for all countries and sectors
- Many countries also have firm-level data sets
- There are also transaction-level data
- Household scanner data

# Why trade?

3. Unique convergence of theory and empirics

There is a unique synergy between theory and empirics in trade. Empirical findings and stylized facts (e.g. heterogeneity of firm-level balance sheet data) lead to development of trade models (e.g. Melitz, 2003). On the other hand, theory often informs estimation.

# Main objectives and expectations

This course is designed to:

- Get you started on your research early
  - By the end of the semester you will have a good basis for one of the chapters in your dissertation
  - You should <u>aim</u> for original research publishable in a decent field journal (I will inform you in the end of the semester if I think you should polish and submit your work somewhere)
- Introduce to you econometric and computational techniques used to answer questions in international economics in a structural quantitative way

# Main objectives and expectations

I expect you to:

- Read the relevant materials, complete assignments and participate in discussions
- Make three presentations
  - Introduction
  - Intermediate
  - Final
- Write a report (close to an actual paper)

# Main objectives and expectations

You will also make *many* presentations throughout this course on your take home assignments.

- Typically, we will have one assignment per week
- Your assignment presentations will take place on Mondays
- You will work in groups (or alone).
- If time permits, each group will have a chance to present. If we are pressed for time, I will randomly choose a group.

All further details are in the syllabus.

# Software and Hardware

This is a quantitative course and you will learn a lot about calibration and estimation. You should have access to:

- STATA
- MATLAB

In principle, you can use any other software but I won't be able to help you should you have difficulties with coding. I also expect you to make presentations and the final report in LATEX. Sample LATEX codes are on Canvas.

# Plan

| Week | Topic |
|---|---|
| Week 1 | Introduction to Structural Gravity Equation |
| Week 2 | Calibration and Estimation |
| Week 3 | Mapping Models to the Data |
| Week 4 | Designing Counterfactual Experiments in General Equilibrium |
| Week 5 | Presentations (I) and Catch-up |
| Week 6 | Heterogeneous Firms (I) |
| Week 7 | Heterogeneous Firms (II) |
| Week 8 | Ricardian Models |
| Week 9 | Presentations (II) and Catch-up |
| Week 11 | Extensions: Demand Side |
| Week 12 | Extensions: Supply Side |
| Week 13 | Extensions: Migration and Geography |
| Week 14 | Presentations (III) and Catch-up |

# Structural Gravity Equation

Many trade models (and all models that we will cover in this class) lead to a log-additive gravity equation. This means that the bilateral trade between countries $i$ (exporter) and $j$ (importer) can be written as:

$$Trade_{ij} = EX_i \times PAIR_{ij} \times IM_j$$

In it's simplest form, it is equivalent to:

$$Trade_{ij} = CONSTANT \times \frac{GDP_i \times GDP_j}{DISTANCE_{ij}}$$

Jan Tinbergen was the first one to propose this equation for trade in 1962. The first formal models were Armington (1969) and Anderson (1979).

# Armington model

# Armington model: Set-up

- There are $S$ countries in the world
- Each country $i \in S$ has $L_i$ workers. The number of workers is exogenous and constant.
- Workers supply their labor inelastically. (What does that imply?)
- Workers/consumers are identical within each country. There is a representative consumer
- There are exogenous productivity differences across countries captured by the t.f.p. parameter, $A_i$

# Trade costs

In this course, we will talk a lot about trade costs. Trade costs are specified as in Samuelson (1954). Here is what he wrote:

*"I now propose to come directly to grips with transport costs. The simplest assumption is the following: To carry each good across the ocean you must pay some of the good itself. Rather than set up elaborate models of a merchant marine, invisible items, etc., we can achieve our purpose by assuming that just as only a fraction of ice exported reaches its destination as unmelted ice, so will $a_x$ and $a_y$ be the fractions of exports $X$ and $Y$ that respectively reach the other country as imports. Of course, $a_x < 1$ and $a_y < 1$, except in the costless model, where they were each unity."*

# Trade costs

Let us use $\tau_{ij}$ to denote trade costs between $i$ and $j$.

- We will use the inverse of Samuelson's $a_x$ such that $\tau_{ij}$ reflects how much of a good you have to ship from $i$ to receive one unit of that good in $j$
- In case of costless trade, $\tau_{ij} = 1$. In all other cases, $\tau_{ij} > 1$
- Usually, we also impose the triangularity condition: $\tau_{ij}\tau_{jk} > \tau_{ik}$. Why?

# Demand Side

In more than 90% of all trade papers, preferences are CES (constant elasticity of substitution). You must know the properties and implications of CES functions:

- Homothetic
- Encompasses other special demand systems such as Cobb-Douglas and Leontieff
- Tractable

CES preferences are extremely convenient. However, as we'll see they are very special and generally do not come to grips with the data.

# Demand Side

In each country, there is a representative consumer with the following CES preferences:

$$U_j = \left( \sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}},$$

where $\sigma > 0$ is the elasticity of substitution, $a_{ij}$ is an exogenous demand shifter, and $q_{ij}$ is the quantity of a good from $i$ consumed in $j$. The utility is maximized subject to the budget constraint:

$$\sum_{i \in S} q_{ij} p_{ij} \leq Y_j,$$

where $Y_j$ is total income of country $j$.

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# Demand Side

The Lagrangian can be formulated as follows:

$$\mathcal{L} : \left( \sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \lambda \left( \sum_{i \in S} q_{ij} p_{ij} - Y_j \right)$$

Then, the FOC's are:

$$\frac{\partial \mathcal{L}}{\partial q_{ij}} : \qquad \left( \sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{1}{\sigma-1}} a_{ij}^{\frac{1}{\sigma}} q_{ij} = \lambda p_{ij}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : \qquad \sum_{i \in S} q_{ij} p_{ij} = Y_j$$

# Demand Side

For any two countries $i$ and $i'$, the following must hold according to the FOC's:

$$\frac{a_{ij}^{\frac{1}{\sigma}} q_{ij}^{-\frac{1}{\sigma}}}{a_{i'j}^{\frac{1}{\sigma}} q_{i'j}^{-\frac{1}{\sigma}}} = \frac{p_{ij}}{p'_{ij}} \Rightarrow p_{i'j} q_{i'j} = \frac{1}{a_{ij}} q_{ij} p_{ij}^{\sigma} a_{i'j} p_{i'j}^{1-\sigma}$$

Summing up across $i'$ and realizing that $\sum_{i' \in S} p_{i'j} q_{i'j} = Y_j$ gives the following:

$$Y_j = \frac{1}{a_{ij}} q_{ij} p_{ij}^{\sigma} \left( \sum_{i' \in S} a_{i'j} p_{i'j}^{1-\sigma} \right)$$

Note that $\left( \sum_{i' \in S} a_{i'j} p_{i'j}^{1-\sigma} \right)$ is $j$ specific. We will establish the following relationship:

$$P_j^{1-\sigma} = \left( \sum_{i' \in S} a_{i'j} p_{i'j}^{1-\sigma} \right)$$

This is the CES price index (often called Spence-Dixit-Stiglitz or Dixit-Stiglitz).

# Demand Side

CES preferences lead to the following demand equation:

$$q_{ij} = a_{ij} \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} Y_j,$$

where consumers take $p_{ij}$ as given. We can also derive total expenditure of $j$ on the good from $i$ as:

$$p_{ij} q_{ij} \equiv X_{ij} = a_{ij} \frac{p_{ij}^{1-\sigma}}{P_j^{1-\sigma}} Y_j$$

How do we get $p_{ij}$?

# Supply Side

The optimal supply depends on the market structure. We can consider various set-ups:

- Perfect competition
- Monopolistic competition
- Endowment economy
- Others

We will focus on the first two structures.

# Supply Side: Perfect Competition

There is one representative firm (or equivalently many identical small firms) with the following production function:

$$q_i = A_i L_i,$$

where $A_i$ is t.f.p. and $L_i$ is labor. This implies that the factory gate price is:

$$p_i = \frac{w_i}{A_i}.$$

However, we have to remember that there are trade costs such that the price of consuming the same one unit of good in country $j$ is:

$$p_{ij} = \tau_{ij} \frac{w_i}{A_i}$$

# Equilibrium: Perfect Competition

If we substitute optimal $p_{ij}$ in $X_{ij}$ we get the structural gravity equation:

$$X_{ij} = a_{ij} \frac{\left(\tau_{ij} \frac{w_i}{A_i}\right)^{1-\sigma}}{P_j^{1-\sigma}} Y_j,$$

which can be rearranged as:

$$X_{ij} = \left(A_i^{\sigma-1} w_i^{1-\sigma}\right) \times \left(a_{ij} \tau_{ij}^{1-\sigma}\right) \times \left(Y_j P_j^{\sigma-1}\right),$$

which is equivalent to:

$$Trade_{ij} = EX_i \times PAIR_{ij} \times IM_j$$

# Equilibrium: Perfect Competition

$$X_{ij} = \left(A_i^{\sigma-1} w_i^{1-\sigma}\right) \times \left(a_{ij} \tau_{ij}^{1-\sigma}\right) \times \left(Y_j P_j^{\sigma-1}\right),$$

The structural gravity equation captures several important features:

- Trade is increasing in the productivity of the exporter, market size of the importer, and preferences of $j$ for the good from $i$
- Trade is decreasing in the wage of the exporter, price level of the importer and the level of trade costs

# Closing the model: Perfect Competition

Total spending in $j$ is $Y_j = L_j w_j$. Then the gravity equation is:

$$X_{ij} = \left(A_i^{\sigma-1} w_i^{1-\sigma}\right) \times \left(a_{ij} \tau_{ij}^{1-\sigma}\right) \times \left(w_j L_j P_j^{\sigma-1}\right)$$

What is the only variable that we *do not* know at this point? To close the model we need to solve for $w_i$!

In equilibrium, markets must clear such that total income must equal total expenditure! This is equivalent to the trade balance condition:

$$\sum_{j \in S} X_{ij} = \sum_{j \in S} X_{ji}$$

# Closing the model: Perfect Competition

The trade balance condition:

$$\sum_{j \in S} X_{ij} = \sum_{j \in S} X_{ji}$$

can be rewritten as:

$$L_i w_i = \sum_{j \in S} X_{ij}.$$

Then, given a numeraire we can solve for unique $S \times 1$ wages using $S$ conditions as:

$$w_i = \frac{\sum_{j \in S} X_{ij}}{L_i}.$$

# Solving the model

Given primitives and parameters of the model:

- Number of countries: $S$
- Preferences parameters: $a_{ij}$, $\sigma$
- Productivity parameters and labor endowment: $A_i$ and $L_i$
- Trade costs: $\tau_{ij}$

Equilibrium is a sequence of $w_i$ such that the following relationships hold:

$$\begin{aligned}
Y_j &= w_j L_j \\
P_j &= \left( \sum_{i \in S} a_{ij} A_i^{\sigma-1} \left(\tau_{ij} w_i\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
X_{ij} &= \left( A_i^{\sigma-1} w_i^{1-\sigma} \right) \times \left( a_{ij} \tau_{ij}^{1-\sigma} \right) \times \left( Y_j P_j^{\sigma-1} \right) \\
w_i &= \sum_{j \in S} X_{ij} / L_i
\end{aligned}$$

# Discussion

- Why do countries trade in the Armington model?
- If you fully believed the Armington model, would you expect to see zero flows between certain countries?
- Are $a_{ij}$ and $\tau_{ij}$ distinguishable in the real world? Why/How ?
- When is it OK to use the Armington model and when is it not?

# References

Anderson James E. 1979. "A Theoretical Foundation for the Gravity Equation", American Economic Review Vol. 69, No. 1, pp. 106-116

Armington Paul S. 1969. "A Theory of Demand for Products Distinguished by Place of Production," Staff Papers (International Monetary Fund) Vol. 16, No. 1, pp. 159-178

Samuelson, P. A. (1954). "The transfer problem and transport costs, ii: Analysis of effects of trade impediments," The Economic Journal, 64(254):264-289

Tinbergen, Jan. 1962. "An Analysis of World Trade Flows," in Shaping the World Economy, edited by Jan Tinbergen. New York, NY: Twentieth Century Fund.