Graduate Trade (II): ECON 8433

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Plan

Week	Topic
Week 1	Introduction to Structural Gravity Equation
Week 2	Calibration and Estimation
Week 3	Mapping Models to the Data
Week 4	Designing Counterfactual Experiments in General Equilibrium
Week 5	Presentations (I) and Catch-up
Week 6	Heterogeneous Firms (I)
Week 7	Heterogeneous Firms (II)
Week 8	Ricardian Models
Week 9	Multi-Sector Models
Week 10	Global Value Chains
Week 11	Presentations (II) and Catch-up
Week 12	Extensions: Demand Side
Week 13	Extensions: Supply Side
Week 14	Extensions: Migration and Geography
Week 15	Presentations (III) and Catch-up

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Heterogeneous Firms and International Trade

So far, we have talked about how countries trade:

- In the real world, firms (not countries) decide how much to produce and how much to export
- Armington model of international trade, while useful, fails to capture some of the firm-level evidence that economists have uncovered in the last 20 years
- Melitz (2003) is a very important model of international trade that captures many firm-level stylized facts about exporters

Heterogeneous Firms and International Trade

Bernard, Jensen, Redding and Schott (2007) use census data on the US firms and uncover several stylized facts

- Firm exporting is relatively rare
- Exporters are different
- Trade is concentrated and scarce

These facts have been confirmed using data on other countries.

Firm exporting is rare

Exporting By U.S. Manufacturing Firms, 2002

NAICS industry	*Percent of firms*	*Percent of firms that export*	*Mean exports as a percent of total shipments*
311 Food Manufacturing	6.8	12	15
312 Beverage and Tobacco Product	0.7	23	7
313 Textile Mills	1.0	25	13
314 Textile Product Mills	1.9	12	12
315 Apparel Manufacturing	3.2	8	14
316 Leather and Allied Product	0.4	24	13
321 Wood Product Manufacturing	5.5	8	19
322 Paper Manufacturing	1.4	24	9
323 Printing and Related Support	11.9	5	14
324 Petroleum and Coal Products	0.4	18	12
325 Chemical Manufacturing	3.1	36	14
326 Plastics and Rubber Products	4.4	28	10
327 Nonmetallic Mineral Product	4.0	9	12
331 Primary Metal Manufacturing	1.5	30	10
332 Fabricated Metal Product	19.9	14	12
333 Machinery Manufacturing	9.0	33	16
334 Computer and Electronic Product	4.5	38	21
335 Electrical Equipment, Appliance	1.7	38	13
336 Transportation Equipment	3.4	28	13
337 Furniture and Related Product	6.4	7	10
339 Miscellaneous Manufacturing	9.1	2	15
Aggregate manufacturing	**100**	**18**	**14**

Sources: Data are from the 2002 U.S. Census of Manufactures.

Source: Bernard, Jensen, Redding and Schott (2007)

Exporters are different

Exporter Premia in U.S. Manufacturing, 2002

	Exporter premia		
	(1)	*(2)*	*(3)*
Log employment	1.19	0.97	
Log shipments	1.48	1.08	0.08
Log value-added per worker	0.26	0.11	0.10
Log TFP	0.02	0.03	0.05
Log wage	0.17	0.06	0.06
Log capital per worker	0.32	0.12	0.04
Log skill per worker	0.19	0.11	0.19
Additional covariates	None	Industry fixed effects	Industry fixed effects, log employment

Sources: Data are for 2002 and are from the U.S. Census of Manufactures.
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm's export status. Regressions in column 2 include industry fixed effects. Regressions in column 3 include industry fixed effects and log firm employment as controls. Total factor productivity (TFP) is computed as in Caves, Christensen, and Diewert (1982). "Capital per worker" refers to capital stock per worker. "Skill per worker" is nonproduction workers per total employment. All results are significant at the 1 percent level.

Source: Bernard, Jensen, Redding and Schott (2007)

Trade is concentrated and scarce

Distribution of Exporters and Export Value by Number of Products and Export Destinations, 2000

A: Share of Exporting Firms

Number of products	Number of countries: *1*	*2*	*3*	*4*	*5+*	*All*
1	40.4	1.2	0.3	0.1	0.2	**42.2**
2	10.4	4.7	0.8	0.3	0.4	**16.4**
3	4.7	2.3	1.3	0.4	0.5	**9.3**
4	2.5	1.3	1.0	0.6	0.7	**6.2**
5+	6.0	3.0	2.7	2.3	11.9	**25.9**
All	**64.0**	**12.6**	**6.1**	**3.6**	**13.7**	**100**

Source: Bernard, Jensen, Redding and Schott (2007)

Heterogeneous Firms and International Trade

In sum, we need a model that would capture the fact that:

- Selection into exporting
- Exporters are larger
- Exporters are more productive
- Exporters pay higher wages
- Exporters use capital and skilled-labor more intensively

Melitz (2003) was able to capture the first three facts. Numerous extensions of the model captured the other stylized facts. Why do we want to capture stylized facts?

Heterogeneous Firms and International Trade

The model:

- S countries
- Exogenous labor endowment
- CES preferences
- Monopolistic competition
- Heterogeneous productivity levels
- Fixed costs of entry and exporting

Heterogeneous Firms and International Trade

As before, consumers have CES preference over different varieties specified as:

$$U_j = \left(\sum_{i \in S} \int_{\Phi_{ij}} (q_{ij}(\phi))^{\frac{\sigma}{\sigma-1}} d\phi \right)^{\frac{\sigma-1}{\sigma}}$$

Consumers maximize U_j subject to the usual budget contraint such that the optimal demand is:

$$q_{ij}(\phi) = p_{ij}(\phi)^{-\sigma} Y_j P_j^{\sigma-1},$$

and the usual CES price index:

$$P_j = \left(\sum_{i \in S} \int_{\Phi_{ij}} p_{ij}(\phi)^{1-\sigma} d\phi \right)^{\frac{1}{1-\sigma}}$$

Heterogeneous Firms and International Trade

- There is a measure N_i in each country $i \in S$
- Firms produce according to: $q(\phi) = \phi \ell(\phi)$
- Before starting production, each firm has to pay entry cost fe_i and draw its productivity level ϕ from a known distribution
- If a firm in i wants to serve market j, it must pay fixed cost of exporting f_{ij}

Heterogeneous Firms and International Trade

Firm ϕ solves the following optimization problem:

$$\max \sum_{j \in S} \left(p_{ij}(\phi) q_{ij}(\phi) - \frac{w_i}{\phi} \tau_{ij} q_{ij}(\phi) - w_j f_{ij} \right)$$

Let us plug the expression for q_{ij} from the optimal demand:

$$q_{ij}(\phi) = p_{ij}(\phi)^{-\sigma} Y_j P_j^{\sigma - 1},$$

to get the following:

$$\max_{p_{ij}} \sum_{j \in S} \left(p_{ij}(\phi)^{1-\sigma} Y_j P_j^{\sigma-1} - \frac{w_i}{\phi} \tau_{ij} p_{ij}(\phi)^{-\sigma} Y_j P_j^{\sigma-1} - w_j f_{ij} \right)$$

Heterogeneous Firms and International Trade

Take the first order conditions to get:

$$(1-\sigma)p_{ij}(\phi)^{-\sigma}Y_jP_j^{\sigma-1}+\sigma\frac{w_i}{\phi}\tau_{ij}p_{ij}(\phi)^{-\sigma-1}Y_jP_j^{\sigma-1}=0$$

This can be reformulated as follows:

$$p_{ij}=\frac{\sigma}{\sigma-1}\frac{w_i}{\phi}\tau_{ij}$$

So there is a constant markup that is common across all firms! Plug this expression back into the profit function to get:

$$\pi_{ij}(\phi)=\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\phi}\tau_{ij}\right)^{1-\sigma}\frac{1}{\sigma}Y_jP_j^{\sigma-1}-w_jf_{ij}$$

When will firm ϕ export to market j ?

Heterogeneous Firms and International Trade

Between each *ij* pair there is a cutoff level of productivity which determines the marginal exporter:

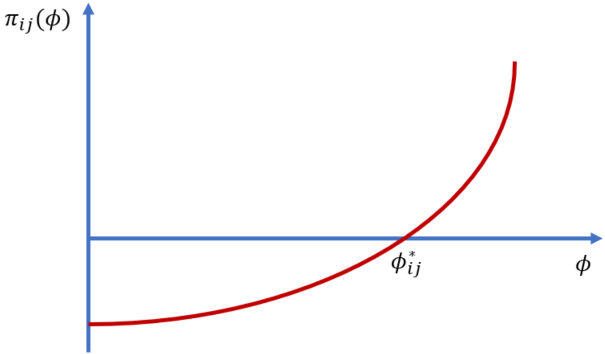
$$\pi_{ij}(\phi^*) = 0$$

This translates into:

$$(\phi_{ij}^*)^{\sigma-1} = \frac{w_j f_{ij}}{\left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}\right)^{1-\sigma} \frac{1}{\sigma} Y_j P_j^{\sigma-1}}$$

Heterogeneous Firms and International Trade

Graphically, the zero profit cutoff condition is as follows:



How to trade barriers f_{ij} and τ_{ij} affect this graph? What do these relationships imply?

Heterogeneous Firms and International Trade

Conditional on the export status, we can derive more intuitive expressions for revenue and operating profit (without fixed cost):

$$
\begin{aligned}
r_{ij} &= p_{ij}q_{ij} = \left(\frac{\sigma}{\sigma-1}\frac{w_i}{\phi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1} \\
o_{ij} &= \left(p_{ij} - \frac{w_i}{\phi}\tau_{ij}\right) q_{ij} = \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\phi}\tau_{ij}\right)^{1-\sigma} Y_j P_j^{\sigma-1}
\end{aligned}
$$

Note that the following holds:

$$o_{ij} = \frac{1}{\sigma} r_{ij}$$

Both r_{ij} and o_{ij} are increasing in ϕ!

Aggregation

We now know r_{ij} for each firm ϕ, which should allow us to calculate aggregate trade flows from i to j:

- N_i is a measure of firms operating in i (for now take it as given)
- N_{ij} is a measure of firms that export from i to j

What is the probability that a firm that draws ϕ from $Pareto(b_i, \theta)$ in i exports to j?

$$N_{ij} = (1 - F(\phi_{ij}^*))N_i$$

Now we need to find the average revenue of firms exporting from i to j!

Heterogeneous Firms and International Trade

To find average r_{ij} across ϕ we must condition on the entry to market j. The conditional p.d.f. is:

$$f(\phi|\phi > \phi^*_{ij}) = \frac{f(\phi)}{1 - F(\phi^*_{ij})}$$

The relevant support of ϕ also changes from (b_i, ∞) to (ϕ^*_{ij}, ∞). In this case, the average revenues can be derived as:

$$\int_{\phi^*_{ij}} r_{ij}(\phi) \frac{f(\phi)}{1 - F(\phi^*_{ij})} d\phi$$

Heterogeneous Firms and International Trade

How do we find total exports from i to j?

$$X_{ij} = N_{ij} \int_{\phi_{ij}^*} r_{ij}(\phi) \frac{f(\phi)}{1 - F(\phi_{ij}^*)} d\phi$$

Plug in the expressions for N_{ij} and r_{ij} to get:

$$X_{ij} = N_i \int_{\phi_{ij}^*} \left(\frac{\sigma}{\sigma - 1} \frac{w_i}{\phi} \tau_{ij} \right)^{1-\sigma} Y_j P_j^{\sigma-1} f(\phi) d\phi$$

We can reorganize it as:

$$X_{ij} = \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w_i^{1-\sigma} \tau_{ij}^{1-\sigma} Y_j P_j^{\sigma-1} N_i \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi$$

How is this gravity equation different from the Armington model?

Heterogeneous Firms and International Trade

We can derive CES price index in the same way:

$$P_j^{1-\sigma} = \sum_{i \in S} N_i \int_{\phi_{ij}^*} p_{ij}(\phi)^{1-\sigma} d\phi$$

Note that P_j is decreasing in N_i and increasing in ϕ_{ij}^*. How can we interpret that?

Plug in the expression for p_{ij} and simplify to get the following:

$$P_j^{1-\sigma} = \sum_{i \in S} N_i \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w_i^{1-\sigma} \tau_{ij}^{1-\sigma} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi$$

We are now ready to close the model!

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We are now ready to close the model!

Heterogeneous Firms and International Trade

The first equilibrium condition is free entry. Before entering production each firm draws ϕ. The draw costs $w_i fe_i$. Free entry ensures that ex ante expected profits are zero:

$$E_\phi\left(\sum_j \pi_{ij}(\phi)\right) = w_i fe_i$$

Taking expectation with respect to ϕ gives the following:

$$\sum_j \frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} w_i^{1-\sigma}\tau_{ij}^{1-\sigma} Y_j P_j^{\sigma-1}\int_{\phi_{ij}^*}\phi^{\sigma-1}f(\phi)d\phi - \sum_j \int_{\phi_{ij}^*} w_j f_{ij} f(\phi)d\phi = w_i fe_i$$

or using the definition of ϕ_{ij}^*:

$$\sum_j (\phi_{ij}^*)^{1-\sigma} w_j f_{ij}\int_{\phi_{ij}^*}\phi^{\sigma-1}f(\phi)d\phi - \sum_j \int_{\phi_{ij}^*} w_j f_{ij} f(\phi)d\phi = w_i fe_i$$

Heterogeneous Firms and International Trade

The second equilibrium condition is labor market clearing (or equivalently trade balance). Each country is endowed with labor L_i. The labor market clearing condition makes sure that labor demand and supply are equal. Labor is used for three purposes in the model:

- For production: $l_{ij}(\phi) = \frac{q_{ij}\tau_{ij}}{\phi}$
- To pay fixed cost: f_{ji} is paid in terms of L_i
- To pay for the cost of entry: fe_i is also paid in terms of L_i

To find total labor used in production:

$$
\begin{aligned}
l_{ij}(\phi) &= \frac{\tau_{ij}}{\phi} q_{ij}(\phi) = \frac{\tau_{ij}}{\phi} p_{ij}(\phi)^{-\sigma} Y_j P_j^{\sigma-1} = \frac{\tau_{ij}}{\phi} \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\phi} \tau_{ij} \right)^{-\sigma} Y_j P_j^{\sigma-1} \\
&= \frac{\sigma-1}{\sigma} \frac{1}{w_i} \left(\frac{\sigma}{\sigma-1} \frac{w_i}{\phi} \tau_{ij} \right)^{1-\sigma} Y_j P_j^{\sigma-1} = (\sigma-1) \frac{w_j}{w_i} (\phi_{ij}^*)^{1-\sigma} f_{ij} \phi^{\sigma-1}
\end{aligned}
$$

Heterogeneous Firms and International Trade

We are now ready to close the model by specifying the labor market clearing condition:

$$L_i = \sum_{j \in S} N_i \int_{\phi^*_{ij}} l_{ij}(\phi) f(\phi) d\phi + \sum_{j \in S} N_j \int_{\phi^*_{ji}} f_{ji} f(\phi) d\phi + \frac{N_i}{1 - F(\min_j\{\phi^*_{ij}\})} fe_i,$$

Substituting expression for $l_{ij}(\phi)$ gives:

$$L_i = \sum_{j \in S} N_i (\sigma - 1) \frac{w_j}{w_i} (\phi^*_{ij})^{1-\sigma} f_{ij} \int_{\phi^*_{ij}} \phi^{\sigma-1} f(\phi) d\phi + \sum_{j \in S} N_j \int_{\phi^*_{ji}} f_{ji} f(\phi) d\phi + \frac{N_i}{1 - F(\min_j\{\phi^*_{ij}\})} fe_i,$$

Why do we divide N_i by $(1 - F(\min_j\{\phi^*_{ij}\})$?

Heterogeneous Firms and International Trade

It is important for you to understand which equations are relevant for the solution of the model!

- Decide how you are going to approach solution
- As before our job is to solve for $S \times 1$ vector of wages w_i
- Relative to the Armington model, what other primitives do we have in the Melitz model?

Heterogeneous Firms and International Trade

Given primitives $\{S, L_i, \sigma, f(\phi), F(\phi), \tau_{ij}, f_{ij}, fe_i\}$, we need to solve:

$$\phi_{ij}^* = \left(w_j f_{ij} \left(\left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{1-\sigma} \frac{1}{\sigma} Y_j P_j^{\sigma-1} \right)^{-1} \right)^{\frac{1}{\sigma-1}} \quad (1)$$

$$Y_j = L_j w_j \quad (2)$$

$$P_j^{1-\sigma} = \sum_{i \in S} N_i \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w_i^{1-\sigma} \tau_{ij}^{1-\sigma} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi \quad (3)$$

$$w_i fe_i = \sum_j (\phi_{ij}^*)^{1-\sigma} w_j f_{ij} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi - \sum_j \int_{\phi_{ij}^*} w_j f_{ij} f(\phi) d\phi \quad (4)$$

$$L_i = \sum_{j \in S} N_i (\sigma - 1) \frac{w_j}{w_i} (\phi_{ij}^*)^{1-\sigma} f_{ij} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi + \sum_{j \in S} N_j \int_{\phi_{ji}^*} f_{ji} f(\phi) d\phi + \frac{N_i}{1 - F(\min_j \{\phi_{ij}^*\})} fe_i \quad (5)$$

Heterogeneous Firms and International Trade

- So far, we have not taken a stance on the distribution of ϕ
- However, in order to solve the model we have to know the productivity distribution
- It is customary to assume that ϕ follows Pareto

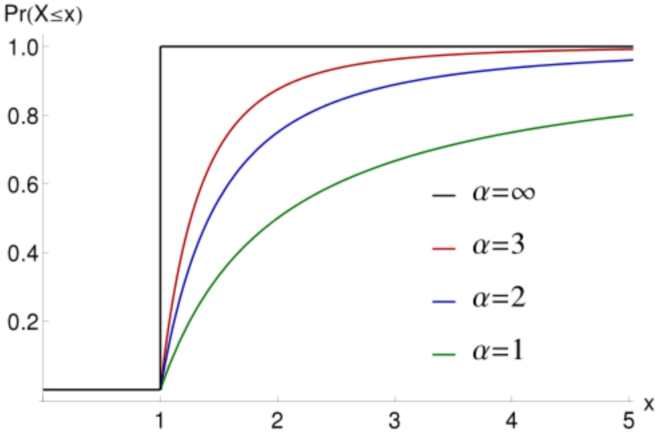
The c.d.f. or Pareto is:

$$F(\phi) = 1 - b_i^{\theta}\phi^{-\theta}$$

The p.d.f. of Pareto is:

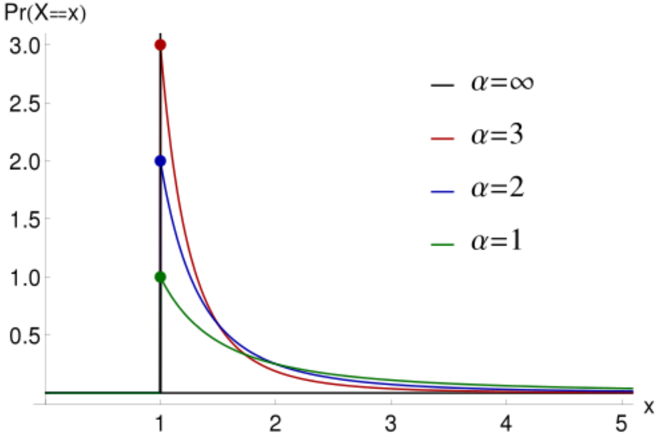
$$f(\phi) = \theta b_i^{\theta}\phi^{-\theta-1}$$

CDF of Pareto



Source: Wikipedia

PDF of Pareto



Source: Wikipedia