Graduate Trade (II): ECON 8433

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Plan

Week	Topic
Week 1	Introduction to Structural Gravity Equation
Week 2	Calibration and Estimation
Week 3	Mapping Models to the Data
Week 4	Designing Counterfactual Experiments in General Equilibrium
Week 5	Presentations (I) and Catch-up
Week 6	Heterogeneous Firms (I)
Week 7	Heterogeneous Firms (II)
Week 8	Ricardian Models
Week 9	Multi-Sector Models
Week 10	Global Value Chains
Week 11	Presentations (II) and Catch-up
Week 12	Extensions: Demand Side
Week 13	Extensions: Supply Side
Week 14	Extensions: Migration and Geography
Week 15	Presentations (III) and Catch-up

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Structural Estimation: Fixed Effects

Log-linearize the structural gravity equation:

$$X_{ij} = \frac{Y_i}{\Phi_i} \times K_{ij} \times \frac{Y_j}{\Pi_j}$$

Parameterize K_{ij} and replace i-specific and j-specific variables with the appropriate fixed effects:

$$\ln X_{ij} = exp_i + \alpha \ln d_{ij} + imp_j + \epsilon_{ij}$$

Then, the following must hold:

$$exp(\widehat{exp_i}) = \frac{Y_i}{\Phi_i}; \;\; exp(\widehat{imp_i}) = \frac{Y_i}{\Pi_i}; \;\; exp(\widehat{d_{ij}^{\alpha}}) = K_{ij}$$

Structural Estimation: Fixed Effects

Plug in the expressions for Φ_i, P_i and K_{ij} to get the following system of equations:

$$
\begin{aligned}
exp(\widehat{exp_i}) &= \frac{Y_i}{\sum_k \left(a_{ik}\tau_{ik}^{1-\sigma}\right) \times \left(Y_k P_k^{\sigma-1}\right)} \\
exp(\widehat{imp_i}) &= Y_i P_i^{\sigma-1} \\
exp(\widehat{d_{ij}^{\alpha}}) &= a_{ij}\tau_{ij}^{1-\sigma}
\end{aligned}
$$

Recall that we need A_i, L_i, σ, τ_{ij}, and a_{ij}. So what has the structural estimation bought us?

Structural Estimation: Fixed Effects

Equilibrium is a sequence of w_i such that the following relationships hold:

$$
\begin{aligned}
Y_j &= w_j L_j \\
P_j &= \left(\sum_{i \in S} a_{ij} A_i^{\sigma-1} \left(\tau_{ij} w_i\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
X_{ij} &= \left(A_i^{\sigma-1} w_i^{1-\sigma}\right) \times \left(a_{ij} \tau_{ij}^{1-\sigma}\right) \times \left(Y_j P_j^{\sigma-1}\right) \\
w_i &= \sum_{j \in S} X_{ij} / L_i
\end{aligned}
$$

Now replace things that we can with the fitted values from the gravity equation.

Structural Estimation: Fixed Effects

We can use the structural estimates in the following system of equations:

$$
\begin{aligned}
w_j L_j &= \sum_i \widehat{X}_{ij} \\
P_j &= \left(\sum_{i \in S} \widehat{K}_{ij} A_i^{\sigma-1} w_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
\widehat{X}_{ij} &= \left(A_i^{\sigma-1} w_i^{1-\sigma} \right) \times \widehat{K}_{ij} \times \left(exp(\widehat{imp_i}) \right) \\
w_i &= \sum_{j \in S} \widehat{X}_{ij} / L_i
\end{aligned}
$$

We can calculate $A_i^{\sigma-1} w_i^{1-\sigma}$. Is this enough to inform the model?

Structural Estimation: Fixed Effects

We still need to do the following:

- Separate A_i and w_i
- Calculate L_i
- Calculate σ

Any ideas?

Structural Estimation: Fixed Effects

One approach is to measure one of the variables that we need. For example, we can use data on w_i to separate A_i from w_i as follows:

$$\left(A_i^{\sigma-1} w_i^{1-\sigma}\right) = \frac{\widehat{X}_{ij}}{\widehat{K}_{ij}\left(exp(\widehat{imp_i})\right)},$$

and calculate L_i from:

$$L_i = \sum_{j \in S} \widehat{X}_{ij}/w_i$$

For now ignore σ. We will see different ways to estimate it.

The Log of Gravity

The Log of Gravity

We can log-linearize and estimate the gravity equation:

$$\ln X_{ij} = exp_i + \alpha \ln d_{ij} + imp_j + \epsilon_{ij}$$

If ϵ_{ij} are statistically independent of the regressors.

- The expected value of a log variable is a function of many moments of the underlying distribution
- If the variance of ϵ_{ij} depends on one of the regressors, the expected value will depend on it too.
- Why is it a problem?

The Log of Gravity

We have also seen that the Armington model (almost never) predicts zero trade. What if we see zeros in the real data?

$$\ln X_{ij} = exp_i + \alpha \ln d_{ij} + imp_j + \epsilon_{ij}$$

Can we just exclude zero observations? Why?

The Log of Gravity

Santos Silva and Tenreyro (2006) argue that the following must hold on average:

$$X_{ij} = e^{exp_i + \alpha \ln d_{ij} + imp_j}$$

Then, the stochastic error term is as follows:

$$\epsilon_{ij} = X_{ij} - E(X_{ij} | exp_i, \alpha \ln d_{ij}, imp_j)$$

Then, the estimating equation is:

$$X_{ij} = e^{exp_i + \alpha \ln d_{ij} + imp_j} + \epsilon_{ij}$$

Recall, Jensen's inequality and you will realize that OLS is inconsistent!

The Log of Gravity

One could estimate:

$$X_{ij} = e^{exp_i + \alpha \ln d_{ij} + imp_j} + \epsilon_{ij}$$

using NLLS. Then, the objective function (in a standard notation):

$$\sum_i^n (y_i - exp(x_i b))^2 ,$$

the corresponding first-order conditions:

$$\sum_i^n \left(y_i - exp(x_i \widehat{b})\right) exp(x_i \widehat{b}) x_i$$

FOC's show that more weight is given to "large" observations. These are also noisy observations. Hence, NLLS is inefficient.

The Log of Gravity

It has become standard in the literature to estimate the gravity equation using Poisson Pseudo Maximum Likelihood (PPML) estimator with the following first-order conditions:

$$\sum_{i}^{n} \left(y_i - exp(x_i \widetilde{b}) \right) x_i$$

Which gives the same weight to all observations. This is important if we are unsure about the exact form of heteroskedasticity.

Structural Estimation: Problems

Structural Estimation: Problems

In practice, we may encounter several problems when estimating the gravity equation:

- Misspecified bilateral frictions
- Measurement error about certain variables
- Dependency on σ, we still do not know how to estimate it

Misspecified bilateral frictions

Recall that we have assumed the following relationship:

$$\ln K_{ij} = \alpha \ln(distance_{ij}) + \beta \ln(border_{ij})$$

The explanatory variables are generally symmetric. What if the trade costs/preferences are asymmetric?

To capture this asymmetry, we could introduce either exporter-specific or importer-specific fixed effect such that:

$$\ln K_{ij} = z_i + \alpha \ln(distance_{ij}) + \beta \ln(border_{ij}),$$

where z_i is the exporter fixed effect.

Misspecified bilateral frictions

The problem with including z_i is that it will be collinear with exp_i:

$$X_{ij} = e^{exp_i + \gamma \ln(distance_{ij}) + \beta \ln(border_{ij}) + z_i + imp_j} + \epsilon_{ij}$$

How can we estimate exp_i and z_i if they are perfectly collinear?

Since we need to estimate $S \times 1$ additional fixed effects, we must add at least $S \times 1$ constraints!

Tetrad Method

To introduce additional constraints let us first specify trade share as:

$$\pi_{ij} = \frac{X_{ij}}{Y_j} = \left(A_i^{\sigma-1} w_i^{1-\sigma}\right) \times \left(a_{ij}\tau_{ij}^{1-\sigma}\right) \times P_j^{\sigma-1}$$

Assuming that intra-trade frictions are unity,

$$\pi_{jj} = \left(A_j^{\sigma-1} w_j^{1-\sigma}\right) \times P_j^{\sigma-1}$$

We can then define the following tetrad:

$$\frac{\pi_{ij}}{\pi_{jj}} = \frac{\left(A_i^{\sigma-1} w_i^{1-\sigma}\right) \times \left(a_{ij}\tau_{ij}^{1-\sigma}\right)}{\left(A_j^{\sigma-1} w_j^{1-\sigma}\right)}$$

Tetrad Method

Now we can estimate the modified equation:

$$\frac{\pi_{ij}}{\pi_{jj}} = e^{exp_i + \gamma \ln(distance_{ij}) + \beta \ln(border_{ij}) + z_i - imp_j} + \epsilon_{ij}$$

Note that we now have additional $S \times 1$ constraints:

$$exp_i = imp_j \quad \text{for} \quad \{i, j\}$$

This allows us to estimate exp_i and z_i!

Measurement error about trade costs

If we truly believed the model, we would expect it to have zero deviations from the data! Such deviations then would be considered measurement error about trade costs such that the following must always hold exactly:

$$\ln X_{ij} = exp_i + k_{ij} + imp_j$$

This highlights the difference between estimation and calibration!

Measurement error about trade costs

To calibrate the model we first measure A_i, w_i, σ and then solve for $a_{ij}\tau_{ij}$ from the following system:

$$
\begin{aligned}
Y_j &= w_j L_j \\
P_j &= \left(\sum_{i \in S} a_{ij} A_i^{\sigma-1} \left(\tau_{ij} w_i\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\
X_{ij} &= \left(A_i^{\sigma-1} w_i^{1-\sigma}\right) \times \left(a_{ij}\tau_{ij}^{1-\sigma}\right) \times \left(Y_j P_j^{\sigma-1}\right) \\
w_i &= \sum_{j \in S} X_{ij} / L_i
\end{aligned}
$$

Which should allow us to solve for $S \times (S-1)$ terms of $a_{ij}\tau_{ij}^{1-\sigma}$ and match the data perfectly!

Estimating Trade Elasticity

Trade Elasticity

- The trade elasticity parameter $1-\sigma$ determines how trade flows (and welfare) are affected by the reduction in trade barriers.
- However, it is very difficult to estimate σ from the real data:
- We do not directly observe τ_{ij}. Otherwise, we could estimate $1-\sigma$ as a coefficient on log trade costs.

Trade Elasticity

Approach 1: Use tariff data

Tariffs are (arguably) the only directly observable component of trade costs. We thus can use them to estimate σ from the following:

$$X_{ij} = e^{exp_i + \gamma \ln(distance_{ij}) + \beta \ln(border_{ij}) + (1-\sigma) \ln tariff_{ij} + imp_j} + \epsilon_{ij}$$

In practice, $tariff_{ij}$ may be subject to endogeneity problem and it is unclear how well we can estimate σ without IV's.

Trade Elasticity

Approach 2: Use unit cost data

Recall that the structural gravity gives us the following relationship:

$$\frac{\widehat{X}_{ij}}{\widehat{K}_{ij}\left(exp(\widehat{imp_i})\right)} = \left(\frac{w_i}{A_i}\right)^{1-\sigma},$$

where $\frac{w_i}{A_i}$ is interpreted as unit cost. If we had data on unit cost we could estimate the following:

$$\ln\left[\frac{\widehat{X}_{ij}}{\widehat{K}_{ij}\left(exp(\widehat{imp_i})\right)}\right] = (1-\sigma)\ln(unitcost_i) + e_i,$$

Beware of the attenuation bias!

Trade Elasticity

APPROACH 3: USE PRICE DATA

Let's transform the gravity equation as follows:

$$\frac{\pi_{ij}}{\pi_{ii}} = e^{(\sigma-1)(\ln P_j - \ln P_i) + \gamma \ln(distance_{ij}) + \beta \ln(border_{ij})} + \epsilon_{ij}$$

This approach is also subject to the endogeneity concerns as well as attenuation bias!

Structural Gravity Equation

Today:

- Monte Carlo Design
- Tetrad Method
- Calibration

Monte Carlo: Armington model

1. Open your code for Contraction Mapping for the Armington model
2. For simplicity assume that $a_{ij} = 1$ for all i, j
3. Solve model such that X_{ij} is fully consistent with the GE framework
4. Add various stochastic error terms and see how well we can estimate the variables of interest

Monte Carlo: Armington model

After modifying and solving the model you have to record the:

- Exporter-fixed effect $= \ln\left(A_i^{\sigma-1} w_i^{1-\sigma}\right)$, `exp_true`
- Importer-fixed effect $= \ln\left(Y_i P_i^{\sigma-1}\right)$, `imp_true`
- Bilateral trade costs $= \ln\left(\tau_{ij}^{1-\sigma}\right)$, `k_true`
- True trade flows $= X_{ij}$, `X`

Monte Carlo: Armington model

Now let's produce *observable* trade values:

$$Trade = X_{ij} + \epsilon_{ij},$$

where ϵ_{ij} is drawn from some distribution with zero mean. For example:

```
sigma_trade = 2;
Trade = X + normrnd(0,sigma_trade,S*S,1);
Trade = max(0,Trade);
```

Also produce *observable* distance related to trade costs:

```
sigma_tau = 1;
distance = reshape(k_true./(1-sigma),S*S,1) +
normrnd(0,sigma_tau,S*S,1);
```

Monte Carlo: Armington model

We need to generate fixed effects. Recall that:

$$X = \begin{pmatrix} X_{11} & X_{12} & X_{13} & ... & X_{1S} \\ X_{21} & X_{22} & X_{23} & ... & X_{2S} \\ X_{31} & X_{32} & X_{33} & ... & X_{3S} \\ ... & ... & ... & ... & ... \\ X_{S1} & X_{S2} & X_{S3} & ... & X_{SS} \end{pmatrix}$$

How should we code exporter- and importer-fixed effects?

Monte Carlo: Armington model

Exporter-fixed effects:

$$\begin{pmatrix} 1 & 0 & 0 & \ldots \\ 0 & 1 & 0 & \ldots \\ 0 & 0 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ 1 & 0 & 0 & \ldots \\ 0 & 1 & 0 & \ldots \\ 0 & 0 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ 1 & 0 & 0 & \ldots \\ 0 & 1 & 0 & \ldots \\ 0 & 0 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

Monte Carlo: Armington model

Importer-fixed effects:

$$\begin{pmatrix} 1 & 0 & 0 & ... \\ 1 & 0 & 0 & ... \\ 1 & 0 & 0 & ... \\ ... & ... & ... & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ 0 & 1 & 0 & ... \\ ... & ... & ... & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ 0 & 0 & 1 & ... \\ ... & ... & ... & ... \end{pmatrix}$$

Monte Carlo: Armington model

One way to code the fixed-effects:

```
exp_fe = repmat(eye(S,S),S,1);
exp_fe(:,1) = [];
imp_fe = kron(eye(S,S),ones(S,1));
```

Why do we exclude the first column in `exp_fe` ?

Monte Carlo: Armington model

First, run OLS using `fitlm` command in MATLAB:

```
XX = [exp_fe(Trade~=0,:)  imp_fe(Trade~=0,:)
distance(Trade~=0,:)]
YY = log(Trade(Trade~=0))
ols_results = fitlm( XX, YY,'linear','Intercept',false);
OLS = ols_results.Coefficients(:,1:2);
OLS = table2array(OLS);
```

Monte Carlo: Armington model

Second, run PPML using `glmfit` command in MATLAB:

```
[coef_PPML, dev, stats] = glmfit([exp_fe imp_fe
distance],Trade,'Poisson', 'constant','off');
PPML = [coef_PPML stats.se];
```

Monte Carlo: Armington model

Now we are ready to check for bias:

```
norm_exp = exp_true - exp_true(1);
plot(norm_exp, norm_exp,
norm_exp(2:S),PPML(1:S-1,1),'b*',norm_exp(2:S),OLS(1:S-1,1),'ko');
```

We can a similar thing for the importer fixed effects and for the coefficient on trade costs. What does the latter measure?

How does the bias change with the variance of the error terms?

Discussion and in-class exercise

By now you have all instruments necessary to estimate asymmetric trade costs when you observe only the symmetric component such that $\ln \tau_{ij} = ac_i + \ln sc_{ij}$

- Download the data set called `tetrad_data.mat`. The data set has the following variables:
  - Matrix of trade flows, X
  - Observable symmetric component of trade costs, SC
  - Observable wages, w
- Structurally estimate/calibrate all parameters necessary for the Armington model

Choose and show using graphs/tables:

- How the asymmetries in trade costs matter for trade and welfare!
- How the asymmetries in trade costs matter for world income inequality!