Graduate Trade (II): ECON 8433

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Plan

Week	Topic
Week 1	Introduction to Structural Gravity Equation
Week 2	Calibration and Estimation
Week 3	Mapping Models to the Data
Week 4	Designing Counterfactual Experiments in General Equilibrium
Week 5	Presentations (I) and Catch-up
Week 6	Heterogeneous Firms (I)
Week 7	Heterogeneous Firms (II)
Week 8	Ricardian Models
Week 9	Multi-Sector Models
Week 10	Global Value Chains
Week 11	Presentations (II) and Catch-up
Week 12	Extensions: Demand Side
Week 13	Extensions: Supply Side
Week 14	Extensions: Migration and Geography
Week 15	Presentations (III) and Catch-up

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Heterogeneous Firms and International Trade

Recall the system: Given primitives $\{S, L_i, \sigma, f(\phi), F(\phi), \tau_{ij}, f_{ij}, fe_i\}$, we need to solve:

$$\phi_{ij}^* = \left(w_j f_{ij} \left(\left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{1-\sigma} \frac{1}{\sigma} Y_j P_j^{\sigma-1} \right)^{-1} \right)^{\frac{1}{\sigma-1}} \quad (1)$$

$$Y_j = L_j w_j \quad (2)$$

$$P_j^{1-\sigma} = \sum_{i \in S} N_i \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w_i^{1-\sigma} \tau_{ij}^{1-\sigma} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi \quad (3)$$

$$w_i fe_i = \sum_j (\phi_{ij}^*)^{1-\sigma} w_j f_{ij} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi - \sum_j \int_{\phi_{ij}^*} w_j f_{ij} f(\phi) d\phi \quad (4)$$

$$L_i = \sum_{j \in S} N_i (\sigma - 1) \frac{w_j}{w_i} (\phi_{ij}^*)^{1-\sigma} f_{ij} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi + \sum_{j \in S} N_j \int_{\phi_{ji}^*} f_{ji} f(\phi) d\phi + \frac{N_i}{1 - F(\min_j \{\phi_{ij}^*\})} fe_i \quad (5)$$

The main puzzle to solve here is to determine how to use (4) and (5)! Are they both useful?

Heterogeneous Firms and International Trade

Let me reformulate equation (4) by substituting $\int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi$ and $\int_{\phi_{ij}^*} f(\phi) d\phi$:

$$w_i fe_i = \sum_j (\phi_{ij}^*)^{1-\sigma} w_j f_{ij} \frac{\theta b_i^\theta}{\theta + 1 - \sigma} (\phi_{ij}^*)^{(\sigma-1)-\theta} - \sum_j w_j f_{ij} b_i^\theta (\phi_{ij}^*)^{-\theta}$$

Simplify this to get:

$$w_i fe_i = \sum_j (\phi_{ij}^*)^{-\theta} w_j f_{ij} b_i^\theta \left(\frac{\theta}{\theta + 1 - \sigma} - 1 \right) = \sum_j (\phi_{ij}^*)^{-\theta} w_j f_{ij} \left(\frac{b_i^\theta (\sigma - 1)}{\theta + 1 - \sigma} \right)$$

This is equivalent to:

$$\theta fe_i = \sum_j (\sigma - 1) \frac{w_j}{w_i} f_{ij} (\phi_{ij}^*)^{-\theta} \left(\frac{\theta b_i^\theta}{\theta + 1 - \sigma} \right)$$

Keep this relationship in mind!

Heterogeneous Firms and International Trade

Let me reformulate equation (5) in the same way:

$$\begin{aligned} L_i &= \sum_{j \in S} N_i(\sigma - 1)\frac{w_j}{w_i}(\phi_{ij}^*)^{1-\sigma} f_{ij} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi \\ &+ \sum_{j \in S} N_j \int_{\phi_{ji}^*} f_{ji} f(\phi) d\phi + \frac{N_i}{1 - F(\min_j\{\phi_{ij}^*\})} fe_i \end{aligned}$$

which is equivalent to:

$$\begin{aligned} L_i &= N_i \sum_{j \in S} (\sigma - 1)\frac{w_j}{w_i} f_{ij} (\phi_{ij}^*)^{-\theta} \left(\frac{\theta b_i^\theta}{\theta + 1 - \sigma} \right) \\ &+ \sum_{j \in S} N_j b_j^\theta (\phi_{ji}^*)^{-\theta} f_{ji} + \frac{N_i}{1 - F(\min_j\{\phi_{ij}^*\})} fe_i \end{aligned}$$

Does anything look familiar?

Heterogeneous Firms and International Trade

$$L_i \quad = \quad N_i \theta fe + \sum_{j \in S} N_j b_j^{\theta} (\phi_{ji}^*)^{-\theta} f_{ji} + \frac{N_i}{1 - F(\min_j\{\phi_{ij}^*\})} fe_i$$

This already looks much simpler but we have one more term here which can be simplified further: $\sum_{j \in S} N_j b_i^{\theta} (\phi_{ij}^*)^{-\theta} f_{ij}$. Recall the expression for trade flows:

$$X_{ji} = \left(\frac{\sigma}{\sigma - 1} w_j \tau_{ji}\right)^{1-\sigma} Y_i P_i^{\sigma - 1} N_j \int_{\phi_{ji}^*} \phi^{\sigma - 1} f(\phi) d\phi,$$

and the cut-offs:

$$\left(\frac{\sigma}{\sigma - 1} w_j \tau_{ji}\right)^{1-\sigma} Y_i P_i^{\sigma - 1} = (\phi_{ji}^*)^{1-\sigma} (\sigma w_i f_{ji})$$

This leads to:

$$X_{ji} = (\phi_{ji}^*)^{1-\sigma} (\sigma w_i f_{ji}) N_j \frac{\theta b_j^{\theta}}{\theta + 1 - \sigma} (\phi_{ji}^*)^{(\sigma - 1) - \theta} = (\sigma w_i) \frac{\theta}{\theta + 1 - \sigma} N_j b_j^{\theta} (\phi_{ji}^*)^{-\theta} f_{ji}$$

Heterogeneous Firms and International Trade

The following is true:

$$N_j b_j^{\theta} (\phi_{ji}^*)^{-\theta} f_{ji} = \frac{1}{\sigma w_i} \frac{\theta + 1 - \sigma}{\theta} X_{ji}$$

Then equation (5) becomes:

$$L_i \quad = \quad N_i \theta f e_i + \frac{1}{\sigma w_i} \frac{\theta + 1 - \sigma}{\theta} \sum_{j \in S} X_{ji} + \frac{N_i}{1 - F(\min_j\{\phi_{ij}^*\})} f e_i,$$

where $\sum_j X_{ji} = ?$

Heterogeneous Firms and International Trade

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$$L_i \quad = \quad N_i \theta f e_i + \frac{1}{\sigma w_i} \frac{\theta + 1 - \sigma}{\theta} \sum_{j \in S} X_{ji} + \frac{N_i}{1 - F(\min_j\{\phi_{ij}^*\})} f e_i,$$

where $\sum_j X_{ji} = ?$

$$L_i \left(1 - \frac{\theta + 1 - \sigma}{\sigma \theta}\right) = N_i f e_i \left(\theta + \frac{1}{1 - F(\min_j\{\phi_{ij}^*\})}\right),$$

$$N_i = \frac{L_i}{f e_i} \frac{(\theta + 1)(\sigma - 1)}{\sigma \theta} \left(\frac{1 - F(\min_j\{\phi_{ij}^*\})}{\theta(1 - F(\min_j\{\phi_{ij}^*\})) + 1}\right)$$

Heterogeneous Firms and International Trade

Important note!

- We assume that fe_i is non-recoverable! All firms pay entry cost before drawing their productivity. This is why, we have to divide $N_i fe_i$ by $1 - F(\min_j\{\phi^*_{ij}\})$!
- There is an alternative way to think about it. If there is a perfect capital market, firms borrow fe_i and pay it before drawing. If their ϕ is low enough they do not enter production and return fe_i. In this case, labor is not "wasted" and all entry costs amount to $N_i fe_i$. In this case, things simplify even further!

$$N_i = \frac{L_i}{fe_i}\frac{(\sigma - 1)}{\sigma\theta}$$

Heterogeneous Firms and International Trade

- We have used equations (4)-(5) and the identity $\sum_i X_{ij} = L_j w_j$ to pin down N_i
- How do we calculate w_i ?

Heterogeneous Firms and International Trade

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- How do we calculate w_i ?

We once again invoke the trade balance condition:

$$L_i w_i = \sum_j X_{ij}$$

Heterogeneous Firms and International Trade

We can now reformulate the Melitz model:

$$
\begin{aligned}
\phi_{ij}^* &= \left(w_j f_{ij} \left(\left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{1-\sigma} \frac{1}{\sigma} Y_j P_j^{\sigma-1} \right)^{-1} \right)^{\frac{1}{\sigma-1}} \\
Y_j &= L_j w_j \\
N_i &= \frac{L_i}{fe_i} \frac{(\sigma - 1)}{\sigma\theta} \\
P_j &= \left(\sum_{i \in S} N_i \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w_i^{1-\sigma} \tau_{ij}^{1-\sigma} \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi \right)^{\frac{1}{1-\sigma}} \\
X_{ij} &= \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w_i^{1-\sigma} \tau_{ij}^{1-\sigma} Y_j P_j^{\sigma-1} N_i \int_{\phi_{ij}^*} \phi^{\sigma-1} f(\phi) d\phi \\
w_i &= \left(\sum_j X_{ij} \right) / L_i
\end{aligned}
$$

When coding remember about the implicit relationship between ϕ_{ij}^* and b_i!

Heterogeneous Firms and International Trade

- Download *melitz data.mat* from D2L
- Solve the model (find equilbrium)
- Calculate equilibrium values:
  - Number of Firms
  - Share of Exporters
  - Real wage
- Suppose variable trade costs decrease by 20%. Calculate the effect on the numbers of firms, exporters and real wage in each country

Tips:

- There are many different ways to solve the model. Sometimes simplifying the system of equations may help!