Graduate Trade (II): ECON 8433

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Plan

Week	Topic
Week 1	Introduction to Structural Gravity Equation
Week 2	Calibration and Estimation
Week 3	Mapping Models to the Data
Week 4	Designing Counterfactual Experiments in General Equilibrium
Week 5	Presentations (I) and Catch-up
Week 6	Heterogeneous Firms (I)
Week 7	Heterogeneous Firms (II)
Week 8	Ricardian Models
Week 9	Multi-Sector Models
Week 10	Global Value Chains
Week 11	Presentations (II) and Catch-up
Week 12	Extensions: Demand Side
Week 13	Extensions: Supply Side
Week 14	Extensions: Migration and Geography
Week 15	Presentations (III) and Catch-up

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Ricardian Model of Trade

An interesting result is that as long as $\theta > \sigma - 1$ holds, the price index does not depend on σ! This is different from the Armington and Melitz models:

$$P_j = \Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \Phi_j^{-\frac{1}{\theta}},$$

Notice, that now θ which measures the dispersion of the productivity distribution plays the role of σ! What is the intuition for this result?

Ricardian Model of Trade

Next, we will derive the trade shares. Let π_{ij} be the fraction of goods (in j's total consumption) that it buys from i relative to all other origin countries:

$$
\begin{aligned}
\pi_{ij} &= Prob\left(p_{ij}(\omega) \leq \min_{k \neq i} \{p_{kj}\}\right) \\
&= \int_0^\infty \prod_{k \neq i} (1 - G_{kj}(p))\, dG_{ij}(p)
\end{aligned}
$$

Substitute the values of G_{kj} and G_{ij} and integrate to get:

$$\pi_{ij} = \frac{T_i (c_i \tau_{ij})^{-\theta}}{\Phi_j}$$

Ricardian Model of Trade

We still haven't talked about c_i. Eaton and Kortum (2002) assume the following structure:

$$c_i = w_i^{\beta} p_i^{1-\beta},$$

where w_i is the wage rate and p_i is the price of intermediate input (CES price index).

Closing the model

To close the model we need to specify the trade balance condition. First, let's calculate total expenditure in country j:

$$Expenditure = \text{Final Demand} + \text{Intermediate Demand}$$

Workers spend all income such that the final demand is :

$$Y_i = L_i w_i$$

Intermediate demand is $(1-\beta)$ share of total output:

$$\frac{1}{\beta} L_i w_i (1-\beta)$$

Then the expenditure is:

$$E_i = L_i w_i + \frac{1-\beta}{\beta} L_i w_i = \frac{1}{\beta} L_i w_i$$

Closing the model

What is the share of total output that accrues to workers?

Closing the model

What is the share of total output that accrues to workers?

The trade balance condition is thus:

$$L_i w_i = \beta \sum_k \pi_{ik} E_k.$$

Substitute the expression for E_k to get the following:

$$L_i w_i = \sum_k \pi_{ik} L_k w_k.$$

Hence, the trade balance condition when labor is not the only factor of production is the same as before. We will see that this will change if we add more industries.

Eaton and Kortum (2002)

With the assumption of Frechet, Eaton and Kortum (2002) is specified by the following equations:

$$
\begin{aligned}
c_i &= w_i^{\beta} p_i^{1-\beta} && (1) \\
\Phi_j &= \sum_{i \in S} T_i \left(c_i \tau_{ij}\right)^{-\theta} && (2) \\
p_j &= \Gamma \left(\frac{\theta + 1 - \sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \Phi_j^{-\frac{1}{\theta}} && (3) \\
\pi_{ij} &= \frac{T_i (c_i \tau_{ij})^{-\theta}}{\Phi_j} && (4) \\
L_i w_i &= \sum_k \pi_{ik} L_k w_k && (5)
\end{aligned}
$$

Is this isomorphic to the Armington model and Melitz model?

Eaton and Kortum (2002)

With the assumption of Frechet, Eaton and Kortum (2002) is specified by the following equations:

$$c_i = w_i^{\beta} p_i^{1-\beta} \tag{6}$$

$$\Phi_j = \sum_{i \in S} T_i \left(c_i \tau_{ij}\right)^{-\theta} \tag{7}$$

$$p_j = \Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right)^{\frac{1}{1-\sigma}} \Phi_j^{-\frac{1}{\theta}} \tag{8}$$

$$\pi_{ij} = \frac{T_i (c_i \tau_{ij})^{-\theta}}{\Phi_j} \tag{9}$$

$$L_i w_i = \sum_k \pi_{ik} L_k w_k \tag{10}$$

Is this isomorphic to the Armington model and Melitz model?

Structural Estimation

Eaton and Kortum (2002) structurally estimate θ by using tetrad-type method. Note that the following holds:

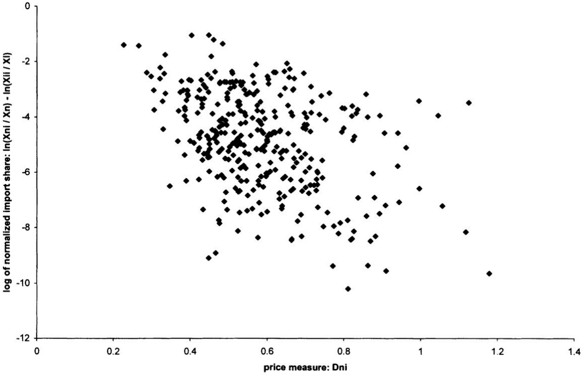
$$\frac{\pi_{ij}}{\pi_{ii}} = \left(\frac{p_i}{p_j}\tau_{ij}\right)^{-\theta}$$

They use data on 50 manufacturing products in 19 countries to approximate $\ln\left(\frac{p_i}{p_j}\tau_{ij}\right)$:

$$D_{ij} = \frac{\max 2_k\{r_{ij}(k)\}}{\sum_{k=1}^{50}[r_{ij}(k)]/50},$$

where $r_{ij}(k) = \ln p_i(k) - \ln p_j(k)$.

Structural Estimation



Source: Eaton and Kortum (2002)

Slope is -4.57.

Eaton and Kortum (2002)

- Coding (1)-(5) is straightforward to you
- We will learn a different technique this week: brute force simulation
- This method does not need analytical expressions
- It is helpful when certain assumptions that we make for analytical convenience do not hold

Eaton and Kortum (2002)

Suppose $z_i(\omega)$ does not follow Frechet. For simplicity, assume that labor is the only factor of production such that $c_i = w_i$. Then Eaton and Kortum (2002) can be specified as:

$$\begin{aligned} \pi_{ij} &= Prob\left(\frac{w_i\tau_{ij}}{z_i(\omega)} \leq \min_{k\neq i}\left\{\frac{w_k\tau_{kj}}{z_k(\omega)}\right\}\right) \\ L_i w_i &= \sum_k \pi_{ik} L_k w_k \end{aligned}$$

Technically, we do not need P_j to solve the model. However, to calculate welfare we can use the following identify:

$$\ln P_j = \frac{1}{\Omega}\sum_{\omega} \ln\left(p_i(\omega)\right),$$

where

$$p_j(\omega) = \min_{i\in S}\left\{\frac{c_i\tau_{ij}}{z_i(\omega)}\right\}$$

Random Sample Generation

There are many ways to generate random samples. There are also pre-built algorithms in MATLAB. We will use simple inverse transform sampling:

1. Draw u from a uniform distribution: $U \sim Uniform[0, 1]$
2. Find the quantile function of the desired probability distribution: F_X^{-1}
3. Generate $X = F_x^{-1}(u)$

Ricardian Model of Trade

1. Use simulations to solve the model
   - Load data on the Melitz model from the previous lecture
   - Keep the relevant primitives: b_i, θ, L_i, τ_{ij}, S
   - Solve the EK model and compare to the Melitz model with the same primitives. Do you see any differences between the two models in terms of trade outcomes? Why?
2. Solve the EK model with intermediate inputs $\beta = 0.5$ using the assumption of Frechet (same mean and variance as Pareto used in Melitz) using the contraction mapping algorithm
   - EK argue that the benefits of improvements in foreign technology depend on the level of globalization. Use the model to shed light on this issue quantitatively