Mathematics Tutor Interview Questions

Quantitative & Science Center

Franklin & Marshall College

1. Compute the following indefinite integral:

$$\int x\sqrt{x^2-1}\ \mathrm{d}x.$$

Solution. By letting $u = x^2 - 1$, we have $\mathrm{d}u/2 = x\mathrm{d}x$, so the integral becomes

$$\int x\sqrt{x^2-1}\ \mathrm{d}x = \frac{1}{2}\int \sqrt{u}\ \mathrm{d}u = \frac{1}{2}\cdot\frac{2u^{3/2}}{3} + C = \frac{(x^2-1)^{3/2}}{3} + C.$$

The purpose of this question was to see if the tutor was able to correct the student's algebra mistake while computing this integral.

2. Given the function

$$F(x) = \int_0^{x^2} \sin^2(t)\mathrm{d}t,$$

determine $F'(x)$.

Solution. By using the Fundamental Theorem of Calculus, we have

$$F'(x) = \sin^2(x^2)\cdot 2x = 2x\sin^2(x^2).$$

The purpose of this question was to see if the tutor was able to correctly explain *why* this calculation was taken this way: they should know that this is a composite function, and so the chain rule must be used.

3. Determine if the series

$$\sum_{n=0}^{\infty}\frac{1+\sin(n)}{10^n}$$

converges or not.

Solution. The series can be re-written as

$$\sum_{n=0}^{\infty}\frac{1+\sin(n)}{10^n} = \sum_{n=0}^{\infty}\left(\frac{1}{10^n}+\frac{\sin(n)}{10^n}\right) = \sum_{n=0}^{\infty}\frac{1}{10^n}+\sum_{n=0}^{\infty}\frac{\sin(n)}{10^n}.$$

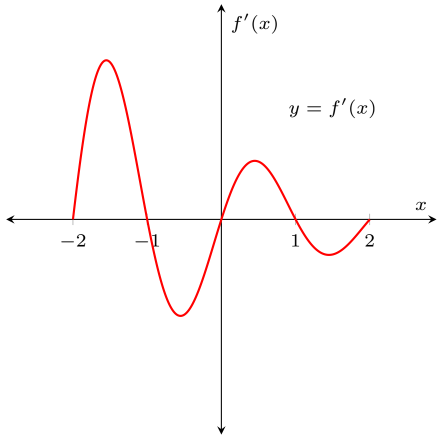
The left series is a geometric series with common ration $r = 1/10$, so it converges. For the right series, observe that

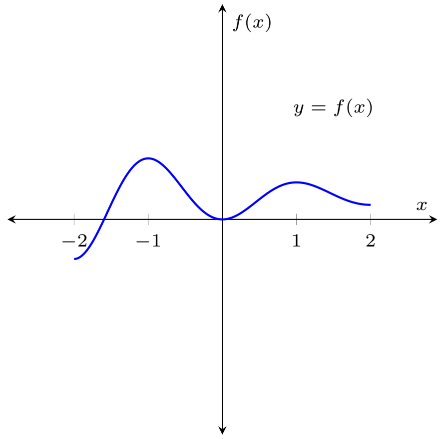
$$\frac{\sin(n)}{10^n} \le \frac{1}{10^n},$$

since $-1 \le \sin(n) \le 1$, and so by the Limit comparison test, the series converges. Therefore, the initial series converges.

The purpose of this question was content review; to see if the tutor was able to answer the question correctly.

4. For a function $f(x)$, the graph of its derivative, $f'(x)$, is given below:





Determine the following:

(a) On what intervals if $f(x)$ increasing? Decreasing?

(b) At what value of x does $f(x)$ have a local maximum? Local minimum?

(c) If it is known that $f(0) = 0$, sketch a possible graph of $f(x)$.

Solution. For our reference, the functions are given by

$$f'(x) = e^{-\frac{x}{2}} \sin(\pi x)$$

and

$$f(x) = -\frac{4e^{-\frac{x}{2}}}{4\pi^2 + 1} \left(\frac{\sin(\pi x)}{2} + \pi \cos(\pi x) \right) + \frac{4\pi}{4\pi^2 + 1},$$

given that $f(0) = 0$.

(a) Increasing on $(-2, -1) \cup (0, 1)$, decreasing on $(-1, 0) \cup (1, 2)$.

(b) Local maxima at $x = -1$ and $x = 1$, local minimum at $x = 0$.

(c) See the plot of $y = f(x)$ in blue above.

The purpose of this question was content review; to see if the tutor was able to answer the question correctly.