Question 10.1.4

Find a parameterization, using $\cos(t)$ and $\sin(t)$, of the intersection of the plane $y = 6$ with the sphere $x^2 + y^2 + z^2 = 117$.

Solution. We want to first determine where the plane and the sphere intersect. This happens whenever $y = 6$ on the sphere, so we just have to plug $y = 6$ into the equation of the sphere. By doing this, we get

$$x^2 + 36 + z^2 = 117,$$

and by subtracting 36 from both sides, we have

$$x^2 + z^2 = 81.$$

That is, the intersection is a circle in the x-z plane centered at the origin with a radius of 9. Parameterizing a circle is not hard to do; for us we have $x = 9\cos(t)$ and $z = 9\sin(t)$. And since $y = 6$ regardless of the value of t, a parameterization of the intersection is given by

$$\mathbf{r}(t) = \langle 9\cos(t), 6, 9\sin(t)\rangle.$$

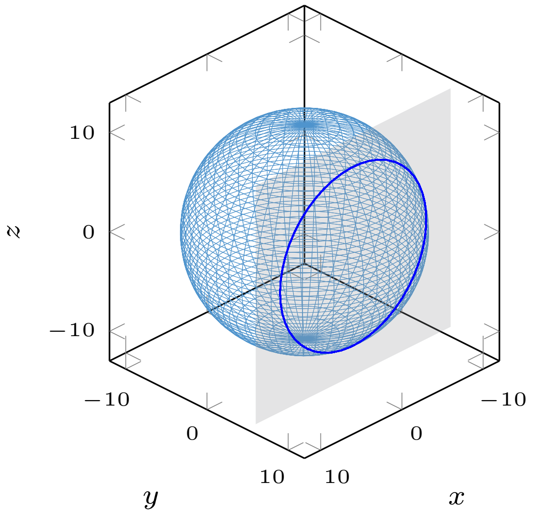

FIGURE 1: *The intersection between the plane and the sphere.*

Question 10.2.4

Find the parametric equation for the tangent line to the curve $x = t^3 - 1$, $y = t^3 + 1$, and $z = t$ at the point $(0, 2, 1)$. Use the variable t as your parameter.

Solution. Since we are given that

$$\mathbf{r}(t) = \langle t^3 - 1, t^3 + 1, t\rangle,$$

we can determine $\mathbf{r}'(t)$ by differentiating each of the components of $\mathbf{r}(t)$; doing this gives us

$$\mathbf{r}'(t) = \langle 3t^2, 3t^2, 1\rangle.$$

What value of t corresponds to the point $(0, 2, 1)$? Since $z = 1$ at that point, its easy enough to see that $t = 1$; plugging $t = 1$ into the parameterizations of x and y give you 0 and 2, which confirms that this is our point. Then the tangent vector at that point is given by

$$\mathbf{r}'(1) = \langle 3, 3, 1\rangle.$$

Then the parametric equation for the tangent line, which we will denote as $\mathbf{r}_t(t)$, is given by

$$\mathbf{r}_t(t) = \mathbf{r}(1) + t\mathbf{r}'(1) = \langle 0, 2, 1\rangle + t\langle 3, 3, 1\rangle = \langle 3t, 3t + 2, t + 1\rangle,$$

or in other words,

$$x = 3t \qquad y = 3t + 2 \qquad z = t + 1.$$