

We are asked to determine the equation for the function with the following three properties:

1. There is a single  $x$ -intercept at the origin (this is what we did not consider 😊)
2. There are two asymptotes at  $x = -2$  and  $x = -5$
3. The function passes through the point  $(6, -\frac{3}{4})$

Let's try to construct this function from scratch. We'll start by just having our function equal some generic constant  $c$  such that

$$f(x) = c.$$

Now, since we have an  $x$ -intercept at  $x = 0$ , we need this function to equal 0 when  $x = 0$ ; in other words, we want  $f(0) = 0$ . We can get this by multiplying our function by  $x$ , giving us

$$f(x) = cx,$$

and we can see that if  $x = 0$ , then  $f(x) = 0$ .

Now, we have two asymptotes at  $x = -2$  and  $x = -5$ , meaning this function is not defined at these two values. Remember, an asymptote occurs when we have a 0 in the denominator, so we want to modify our function to get

$$f(x) = \frac{cx}{(x+2)(x-5)}.$$

We can see that if we plug in either  $x = -2$  or  $x = 5$ ,  $f(x)$  will have a 0 in the denominator.

Finally, we have to determine the constant  $c$ . Since we want our function to pass through the point  $(6, -\frac{3}{4})$ , we need a  $c$  such that  $f(6) = -\frac{3}{4}$ . We just have to plug 6 into  $f(x)$ , set the function equal to  $-\frac{3}{4}$ , and solve for  $c$ :

$$\begin{aligned} -\frac{3}{4} &= \frac{6c}{(6+2)(6-5)} \\ -\frac{3}{4} &= \frac{3}{4}c \\ c &= -1 \end{aligned}$$

We now have our final equation, given by

$$f(x) = -\frac{x}{(x+2)(x-5)}.$$

Graphing this function, which can be seen in Figure 1, gives us the desired result.

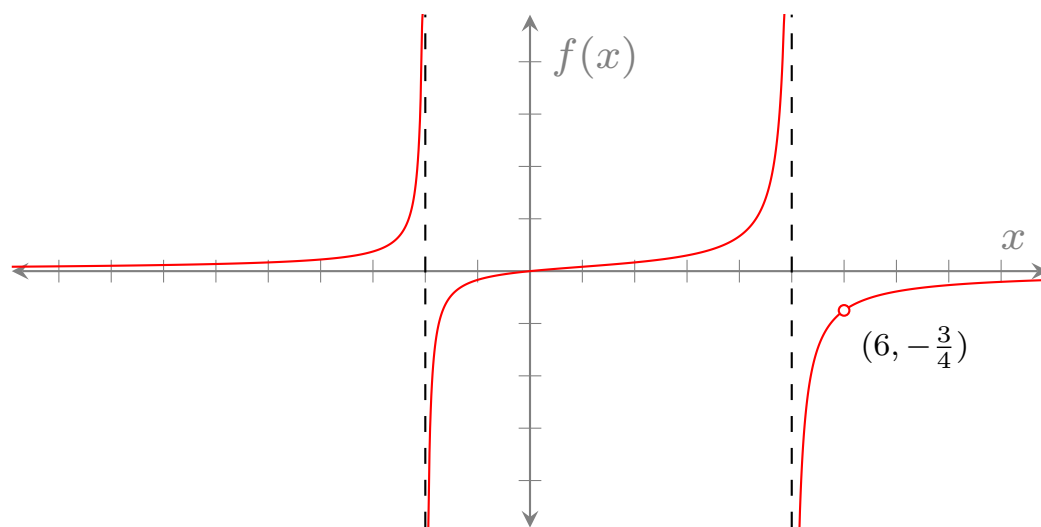


Figure 1: Plotting  $f(x) = -\frac{x}{(x+2)(x-5)}$ .