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MAT110: Calculus II
Professor Christina Weaver
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Homework question

53. *Q: A car is traveling at* 100km/h *when the driver sees an accident* 80 m *ahead and slams on the breaks. What constant deceleration is required to stop the car in time to avoid a pileup?*

 Before actually trying to go about answering this question, it is important to carefully read the entire question and extract all relevant information. Most of the time questions are written so that all of the information presented is relevant to solving the question.

 - "*A car is traveling at* 100km/h".
 - "*the driver sees an accident* 80 m *ahead*".
 - Notice the difference in units! Not taking this into account will lead you to the incorrect answer, and the first step of answering this question should be to convert everything to a single unit.
 - "*What constant deceleration is required*" means we are assuming our acceleration is constant and does not change over time.

 First, we will convert the initial velocity $v(0)$ from km/h to m/s:

$$v(0) = \left(\frac{1000 \text{ km}}{1 \text{ h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1000 \text{ m}}{1 \text{ km}}\right) = \frac{250}{9} \text{ m/s}.$$

 Next, since we are assuming that the acceleration is constant, we know that our acceleration function has the general form

$$a(t) = \alpha,$$

 where α is a constant. Because of the situation, we should have $\alpha < 0$, since our car is trying to slow down and not crash. In order to find the velocity function, we take the antiderivative of $a(t)$, which gives

$$v(t) = \alpha t + C,$$

 where C is a constant. Using our initial condition $v(0) = 250/9$, we have $v(0) = \alpha \cdot 0 + C = C = 250/9$. Similarly, to find the velocity function, we now take the antiderivative of $v(t)$, giving us

$$s(t) = \frac{\alpha}{2}t^2 + \frac{250}{9}t + C,$$

 where C is another constant. Given the situation, the car's initial position is 0 m, which implies $C = 0$.

 This question is difficult because the process for solving it is unclear and maybe not obvious, and we are not simply crunching numbers to get our answer. Recall that the question asks

"*What constant deceleration is required to stop the car in time to avoid a pileup?*" When the question says "on time," it means that we want the car's final position to be at most 80 m, otherwise it will collide with the accident. A greater deceleration will mean the car will stop before 80 m, so we will assume the worst-case scenario: the driver cuts it very close and stops the car exactly 80 m from where he started. In addition, since we want the driver to stop before he crashes, the final velocity will be 0 m/s. The plan of attack is as follows (think about what you should do to solve this before looking at what I did):

1. Determine the time at which the car stops, i.e. find t_α where $v(t_\alpha) = 0$. This answer will depend on α.
2. Plug in t_α into $s(t)$ and determine which value of α makes the final position 80 m, i.e. set $s(t_\alpha) = 80$. The idea behind this is that α determines the position where the velocity becomes 0.

For step 1, we have

$$v(t) = \alpha t + \frac{250}{9} \overset{\text{set}}{=} 0 \Rightarrow t_\alpha = -\frac{250}{9\alpha}.$$

Before moving on, it is good to ask yourself if this result makes sense. We can see that $t_\alpha > 0$ only when $\alpha < 0$, so we have a positive time (and therefore an acceptable result) when the acceleration is negative. If the acceleration is positive, then the car will not stop at all, which should be obvious. Now, the second step gives us

$$s(t_\alpha) = \frac{\alpha}{2}\left(-\frac{250}{9\alpha}\right)^2 + \frac{250}{9}\left(-\frac{250}{9\alpha}\right) = \frac{1}{\alpha}\left(-\frac{31250}{81}\right) \overset{\text{set}}{=} 80,$$

which implies

$$\alpha = \frac{1}{80}\left(-\frac{31250}{81}\right) = -\frac{3125}{648} \approx -4.82 \text{ m/s}.$$