Daisy Grossman Tutoring Notes

MAT109: Calculus I
Professor Wendell Ressler
November 25, 2019

Question: *Use rectangles to find the estimate of each type for the area under the given graph of f from $x = 0$ to $x = 8$.*

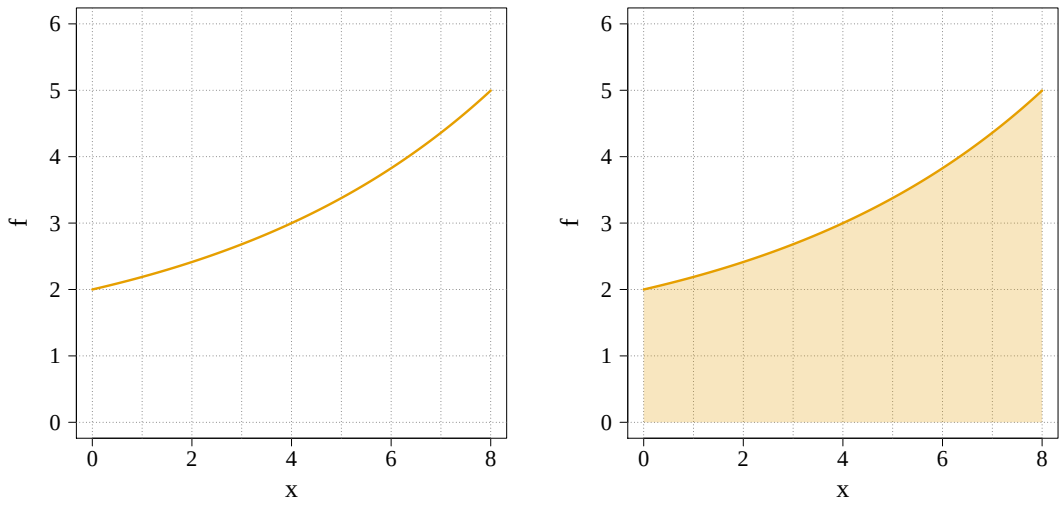

Figure 1: *The true area that we are trying to determine.*

In this question, we are using various *estimation* techniques to estimate

$$\int_0^8 f(x)\,\mathrm{d}x,$$

or in other words, the area under the function from 0 to 8. The left panel in Figure 1 plots the function, while the right panel shows the area under the curve that we are trying to estimate.

Figure 2 shows a graphical interpretation of using the left-hand rule, while Figure 3 shows a graphical interpretation of using the right-hand rule. In questions like this where we do not actually know the function, we kind of have to guess the heights of the rectangles by looking at the y-values. For example, using the left-hand rule with $n = 4$, we can see that the heights of the four rectangles are about 2, 2.5, 3, and 3.9, and since the width of each rectangle is 2, we can approximate the area under the curve by

$$\int_0^8 f(x)\,\mathrm{d}x \approx 2\big(2 + 2.5 + 3 + 3.9\big) = 22.8.$$

The same reasoning is used for the other approximations as well.

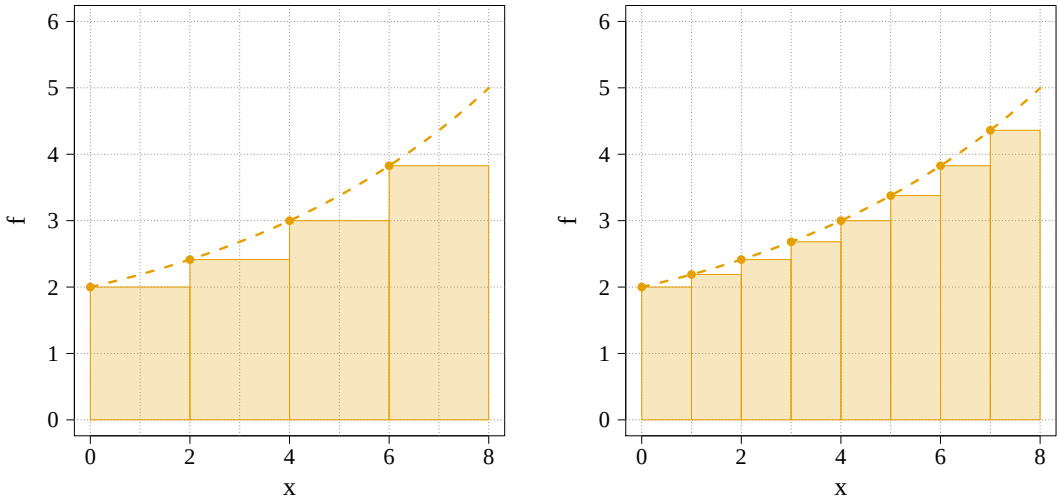

Figure 2: *Estimating the area using left rectangles, with* $n = 4$ *and* $n = 8$.

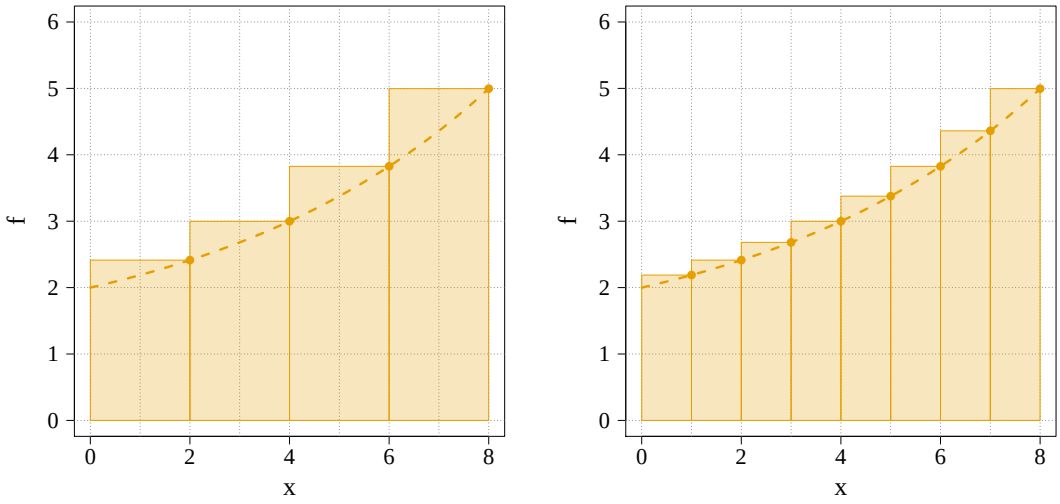

Figure 3: *Estimating the area using right rectangles, with* $n = 4$ *and* $n = 8$.