

Figure 1: Plotting the curve when r = 1.

The function for "The Witch of Agnesi," for some constant r, is given by

$$f(x) = \frac{8r^3}{4r^2 + x^2}. (1)$$

Using the quotient rule, we can show that the derivative of the function is given by

$$f'(x) = -\frac{16r^3x}{(4r^2 + x^2)^2}. (2)$$

Again, using the quotient rule, we can take the derivative once more to show that

$$f''(x) = -\frac{(16r^3)(4r^2 + x^2)^2 - (4r^2 + x^2)(64r^3x^2)}{(4r^2 + x^2)^4}.$$
 (3)

Looking at Figure 1, all of the characteristics that we talked about have been met. Try taking the derivatives yourself to see if you can get the resulting formulas.