Homework 2

Aiden Kenny
STAT GR5203: Propbability
Columbia University
October 12, 2020

Question 1 Let X have a pdf of $f(x) = cx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ elsewhere.

(a) For this to be a valid pdf, it must integrate to 1 over the support. So

$$\int_0^1 cx^2 \, \mathrm{d}x = \left. \frac{cx^3}{3} \right|_0^1 = \frac{c}{3} \overset{\text{set}}{=} 1,$$

which leads to $c = 3$. So $f(x) = 3x^2$ for $0 \leq x \leq 1$.

(b) The cdf is given by

$$F(x) = \int_{-\infty}^{x} f(t) \, \mathrm{d}t = \int_0^x 3t^2 \, \mathrm{d}t = t^3 \big|_0^x = x^3$$

for $0 \leq x \leq 1$. We also have $F(x) = 0$ when $x < 0$ and $F(x) = 1$ when $x > 1$.

(c) We have

$$\Pr\left(\frac{1}{10} \leq X \leq \frac{1}{2} \right) = F\left(\frac{1}{2} \right) - F\left(\frac{1}{10} \right) = \frac{1}{2^3} - \frac{1}{10^3} = \frac{31}{250}.$$