G5203: PROBABILITY
Fall 2020
Midterm

1. Please **print** your name and student ID number in the upper right corner of this page.
2. This is a closed book, closed-notes examination. You can refer to 2 two-sided pages of notes.
3. Please write the answers in the space provided. If you do not have enough space, use the back of a nearby page or ask for additional blank paper. Make sure you sign any loose pages.
4. In order to receive full credit for a problem, you should show all of your work and explain your reasoning. Good work can receive substantial partial credit even if the final answer is incorrect.

Question	Total Points	Credit
1	20	
2	20	
3	10	
4	10	
5	20	
6	20	
total	100	

1. The joint density of X and Y is given by

$$f(x, y) = k(x + y), \qquad 0 \leq y \leq 1 \quad \text{and} \quad 1 \leq x \leq 1.$$

(a) Are X and Y independent? Find k.

(b) What is the marginal density of X and the marginal density of Y?

2. Five separate awards are to be presented to selected students from a class of 30. How many outcomes are possible if

   (a) a student can receive any number of awards;

   (b) each student can receive at most one award?

3. If A and B alternate rolling a pair of dice, stopping either when A rolls the sum of 9 or when B rolls the sum of 6. Assuming that A rolls first, what's the probability that the final roll is made by A?

4. If 4 married couples are arranged in a row, find the probability that no husband sits next to his wife.

5. You arrive at a bus stop at 10am, knowing that the bus will arrive at some times uniformly distributed between 10 and 10:30.

   (a) What is the probability that you will have to wait longer than 10 minutes?

   (b) If at 10:15 the bus has not arrived yet, what's the probability that you will have to wait at least an additional 10 minutes?

6. Suppose that the cumulative distribution function of X is given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{4}, & 0 \le x < 1 \\ \frac{1}{2} + \frac{x-1}{4}, & 1 \le x < 2 \\ \frac{11}{12}, & 2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

(a) Find $P(X = i)$ for $i = 1, 2, 3$.

(b) Find $P(\frac{1}{2} < X < \frac{3}{2})$.