

## GR5203: PROBABILITY

### Autumn 2020

#### Homework 1

Homework is due Tuesday, September 29 at 11pm, EDT. You need to upload it to course-works assignments tab. Read Sections 1.1-1.10 and 2.1-2.3 of DeGroot and Schervish. You may find solutions to some problems in various sources, including solution manual for the textbook. I encourage you to solve the problems yourself rather than obtain solutions, homework is your most valuable tool in studying and preparing for the exams. As I mentioned previously, you may work in groups but do write up on your own and mention the people you collaborated with (no points will be taken off for collaboration as long as there is no evidence of copying).

1. A coin is tossed three times and the sequence of heads and tails is recorded.
  - (a) List the sample space
  - (b) List the elements that make up the following events: 1)  $A$  = at least two heads; 2)  $B$  = the first two tosses are heads; 3)  $C$  = the last toss is a tail.
  - (c) List the elements of the following events: 1)  $A^c$ ; 2)  $A \cap B$ ; 3)  $A \cup C$ .
2. A poker hand consists of 5 cards dealt from a well shuffled standard 52 card deck. Assuming that all possible hands have the same probability, calculate the probability of each of the following combinations below (exclude higher combinations where needed):
  - (a) Royal Flush: ace, king, queen, jack, then, all of the same suit
  - (b) Straight Flush: 5 consecutive cards of the same suit
  - (c) Four of a Kind: four cards of the same value
  - (d) Flush: five cards of the same suit
  - (e) Three of a Kind: three cards of the same value
  - (f) Two pairs: two pairs of cards of the same value
3. A committee of 48 members needs to choose a president and a vice president. They decide to pick two of them at random (a method that may prove to be dangerous in practice). Suppose that of the 48, 16 are women and 32 are men. Let  $E$  represent the event that “the president is a woman”,  $F$  the event that “the vice president is a man”, and  $G$  the event that “the president and the vice president are of the same sex”.
  - (a) Calculate  $P(E)$ ,  $P(F)$ , and  $P(G)$ .
  - (b) Calculate  $P(E \cap F)$ ,  $P(E \cup F)$ , and  $P(E \cap F \cap G)$ .
  - (c) Calculate  $P(G|E \cup F)$ .

4. A deck of 52 cards is shuffled thoroughly. What is the probability that the four aces are all next to each other?
5. A group of 60 second graders is to be randomly assigned to two classes of 30 each. Five of the second graders, Marcelle, Sarah, Michelle, Katy, and Camerin, are friends.
  - (a) What is the probability that they will all be in the same class?
  - (b) What is the probability that exactly four of them will be in the same class?
  - (c) What is the probability that Marcelle will be in one class and her friends in the other?
6. A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability  $p$  or it moves down 1 unit with probability  $1 - p$ . The changes on different days are assumed to be independent.
  - (a) What is the probability that after two days the stock will be at its original price?
  - (b) What is the probability that after three days the stock's price will have increased by 1 unit?
  - (c) Given that after three days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?
7. A true-false question is to be posed to a husband and wife team on a quiz show. Both the husband and the wife will, independently, give the correct answer with probability  $p$ . Which of the following is a better strategy for this couple?
  - (a) Choose one of them and let that person answer the question, or
  - (b) have them both consider the question and then either give the common answer if they agree or, if they disagree, flip a coin to determine which answer to give?
8. In the previous problem, if  $p = .6$  and the couple uses strategy in (b), what is the conditional probability that the couple gives the correct answer given that they (a) agree; (b) disagree?
9. A total of  $n$  independent tosses of a coin that lands on heads with probability  $p$  are made. How large need  $n$  be so that the probability of obtaining at least one head is at least  $\frac{1}{2}$ ?
10. There are three cabinets,  $A$ ,  $B$ , and  $C$ , each of which has two drawers. Each drawer contains one coin;  $A$  has two gold coins,  $B$  has two silver coins, and  $C$  has one gold coin and one silver coin. A cabinet is chosen at random, one drawer is opened, and a silver coin is found. What is the probability that the other drawer in that cabinet contains a silver coin?
11. Urn  $A$  has four red, three blue, and two green balls. Urn  $B$  has two red, three blue, and four green balls. A ball is drawn from  $A$  and put into urn  $B$ , and then a ball is drawn from urn  $B$ .
  - (a) What is the probability that a red ball is drawn from urn  $B$ ?

(b) If a red ball is drawn from urn  $B$ , what is the probability that a red ball was drawn from urn  $A$ ?

12. DeGroot & Schervish, #3 on page 25.
13. DeGroot & Schervish, #16 on page 41.
14. DeGroot & Schervish, #6 on page 46.
15. DeGroot & Schervish, #6 on page 50.
16. DeGroot & Schervish, #10 on page 54.