G5203: PROBABILITY
Autumn 2018
Final

1. Please **print** your name and student ID number in the upper right corner of this page.
2. This is a closed book, closed-notes examination. You can refer to 3 two-sided pages of notes.
3. Please write the answers in the space provided. If you do not have enough space, use the back of a nearby page or ask for additional blank paper. Make sure you sign any loose pages.
4. In order to receive full credit for a problem, you should show all of your work and explain your reasoning. Good work can receive substantial partial credit even if the final answer is incorrect.

Question	Total Points	Credit
1	20	
2	20	
3	20	
4	20	
5	20	
total	100	

1. Suppose the number of hurricanes during a year has the probability mass function

$$f(x) = \frac{(3t)^x e^{-3t}}{x!}, \quad x = 0, 1, 2, \ldots,$$

where t is the time that passed since the beginning of the year, $t \in [0, 1]$, and that the occurrence of hurricanes in any given year is independent of that in any other year.

   (a) Consider a 2 year period. Find the probability that there are at least 3 hurricanes in this 2 year period.

   (b) Find the distribution of Y, the time of the first hurricane of the year.

2. Let X be a random variable with the following cumulative distribution function

$$F(x) = 1 - e^{-(x/\alpha)^\beta},\ x \geq 0,$$

where $\alpha > 0$ and $\beta > 0$ are parameters of this distribution.

(a) Find the density function of X.

(b) Find the distribution of $Y = (X/\alpha)^\beta$.

(c) How could X be generated from a uniform random number generator?

3. Two independent measurements, X and Y, are taken of a quantity μ. We know that $\mathrm{E}(X) = \mathrm{E}(Y) = \mu$, but σ_X and σ_Y are unequal. The two measurements are combined to give

$$Z = \alpha X + (1 - \alpha)Y,$$

where $0 \leq \alpha \leq 1$.

(a) Find α in terms of σ_X and σ_Y to minimize $\mathrm{Var}(Z)$.

(b) Under what circumstances is it better to use the average $(X + Y)/2$ than either X or Y alone?

4. Let X and Y be continuous r.v.s with joint density

$$f(x, y) = \frac{3x^2}{1-x}, \quad 0 < x < 1,\ x < y < 1.$$

(a) Find $E(Y|X)$, the conditional expectation of Y, given X.

(b) Find $E(Y)$.

5. Random variables $Y_1, Y_2, \ldots, Y_{10}$ are independent and have the same distribution. The moment generating function (mgf) of every Y_k, $1 \leq k \leq 10$, is given by

$$\psi(t) = \frac{e^{2t} + e^{-2t}}{2}, \;\; t \in \mathbb{R}.$$

(a) What is the distribution of $3Y_1 - 2$?

(b) Find the mgf of $S = \sum_{k=1}^{10} Y_k$.

(c) What is $\mathrm{P}(S = 0)$? Recall that $(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$.