G5203: PROBABILITY
Fall 2020
Final

1. Please **print** your name and student ID number in the upper right corner of this page.
2. This is a closed book, closed-notes examination. You can refer to 2 two-sided pages of notes.
3. Please write the answers in the space provided. If you do not have enough space, use the back of a nearby page or ask for additional blank paper. Make sure you sign any loose pages.
4. In order to receive full credit for a problem, you should show all of your work and explain your reasoning. Good work can receive substantial partial credit even if the final answer is incorrect.

Question	Total Points	Credit
1	20	
2	20	
3	20	
4	20	
5	20	
total	100	

1. Suppose X_1 and X_2 are independent and distributed according to the following pdf

$$f(x) = \frac{1}{2}e^{-x/2}, \quad x > 0.$$

(a) Let $Y = X_1 + X_2$. Find the distribution of Y.

(b) Let $Y_1, \ldots, Y_{50}$ be a random sample with marginal distribution the same as that of Y. Find the mean of $Y_{(1)}$.

2. Suppose (X, Y) is a point chosen on the unit square with probability governed by the joint pdf

$$f(x, y) = x + y, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1.$$

What is the pdf of Z, which denotes the area of the rectangle formed by $(0, 0)$, $(x, 0)$, $(0, y)$, and (x, y)?

3. Let X and Y be jointly distributed random variables with joint pdf given by

$$f(x, y) = 2(x + y), \quad 0 < x < y < 1$$

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(a) Find the marginal pdf of Y.

(b) Find $E(X|Y)$.

4. (a) Compute the moment generating function for $X \sim N(0,1)$. Recall the pdf of a standard normal is

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad -\infty < x < \infty.$$

(b) Using (a) compute the moment generating function for $X \sim N(\mu, \sigma^2)$.

5. It is known that blank DVDs produced by a certain company will be defective with probability .05, independently of each other. The company sells the blank DVDs in packages of 10 and offers a money-back guarantee that at most 1 of the blank DVDs in the package will be defective. You may assume that the number of defectives in packages are independent of each other.

   (a) What is the probability that a package is returnable under the terms of the money-back guarantee?

   (b) The cost, C, to the manufacturer is given by $C = Y^2 + 5Y + 1$, where Y is the number of returnable (under terms of the money-back guarantee) packages shipped. Find the expected cost associated with a shipment that contains 100 packages of CDs.