G5204: STATISTICAL INFERENCE
Fall 2020
Midterm

1. Please **print** your name and student ID number in the upper right corner of this page.
2. This is a closed book, closed-notes examination. You can refer to five two-sided pages of notes.
3. Please write the answers in the space provided. If you do not have enough space, use the back of a nearby page or ask for additional blank paper. Make sure you sign any loose pages.
4. In order to receive full credit for a problem, you should show all of your work and explain your reasoning. Good work can receive substantial partial credit even if the final answer is incorrect.

Question	Total Points	Credit
1	25	
2	25	
3	25	
4	25	
total	100	

1. Consider a Pareto distribution with density function

$$f(x;\theta) = (\theta - 1)x^{-\theta}, \quad \theta > 2, \quad 1 \leq x < \infty$$

and suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. with density $f(x;\theta)$.

(a) Find the method of moments estimate of θ.

(b) What is the maximum likelihood estimator of θ?

(c) Find the distribution of $Z = (\theta - 1)\ln(X)$.

(d) Use the Central Limit Theorem to approximate the distribution of $\bar{Z}_n$ and find an approximate 95% confidence interval for θ.

2. Suppose a sequence of independent trials, each with probability of success θ, are performed until there are 3 total successes. Let X denote the total number of trials, we observe $x = 18$. Using a uniform prior on $[0, 1]$, find a Bayes estimator for θ under squared-error loss function.

3. Suppose that $X_1, X_2, \ldots, X_n$ form a random sample from the exponential distribution with mean $1/\beta$. Find the distribution of $\sum_1^n X_i$ and use it to find a confidence interval for β. Hint: exploit the connection between the gamma distribution and the χ^2 distribution.

4. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. with densitry function

$$f(x;\theta) = e^{-(x-\theta)}, \quad x \geq \theta.$$

(a) Find a sufficient statistic for θ.

(b) Is it minimally sufficent?