Homework 1

Aiden Kenny
STAT GR5204: Statistical Inference
Columbia University

November 10, 2020

Question 1

When rolling two dice, there are six possible ways for their total to sum up to seven: $(1,6)$, $(2,5)$, $(3,4)$, $(4,3)$, $(5,2)$, and $(6,1)$, so the probability of the sum being seven is $6/36 = 1/6$. If X is the number of trials where the total of both rolls is seven, then we can think of $X \sim \text{Bin}(120, 1/6)$, and so $\mathbb{E}X = 20$ and $\text{Var}X = 50/3$. Using the Central Limit Theorem, we then have

$$\Pr\big(|X-20| \le k\big) = \Pr\left(\left|\frac{X-20}{\sqrt{50/3}}\right| \le k\sqrt{\frac{3}{50}}\right) = 2\Phi\left(k\sqrt{\frac{3}{50}}\right) - 1 \overset{\text{set}}{=} 0.95 \implies \Phi\left(k\sqrt{\frac{3}{50}}\right) = 0.975.$$

Using a table of values for $\Phi(z)$, we can see that $k\sqrt{3/50} = 1.96$, and so $k = 1.96\sqrt{50/3} \approx 8$.

Question 2

Let $X \sim \text{Pois}(10)$, and so $\mathbb{E}X = \text{Var}X = 10$. Using the CLT without any continuity correction, we have $(X-10)/\sqrt{10} \approx \text{N}(0,1)$, and so

$$\Pr(8 \le X \le 12) = \Pr\left(\frac{8-10}{\sqrt{10}} \le Z \le \frac{12-10}{\sqrt{10}}\right) = \Pr\big(|Z| \le \sqrt{2/5}\big) \approx 2\Phi\big(\sqrt{2/5}\big) - 1 = 0.4714.$$

If we do use continuity correction, then we have

$$\begin{aligned}\Pr(8 \le X \le 12) &\approx \Pr(7.5 \le X \le 12.5) \\ &= \Pr\left(\frac{7.5-10}{\sqrt{10}} \le Z \le \frac{12.5-10}{\sqrt{10}}\right) = \Pr\big(|Z| \le 2.5/\sqrt{10}\big) \approx 2\Phi\big(2.5/\sqrt{10}\big) - 1 = 0.5704.\end{aligned}$$