GG5204: STATISTICAL INFERENCE
Autumn 2020
Homework 2

Homework is due Thursday, November 26 at 11pm. Read Sections 8.1-8.7 and 9.1-9.8 of DeGroot and Schervish. You may nd solutions to some problems in various sources, including solution manual for the textbook. I encourage you to solve the problems yourself rather than obtain solutions, homework is your most valuable tool in studying and preparing for the exams. As I mentioned previously, you may work in groups but do write up on your own and mention the people you collaborated with (no points will be taken o for collaboration as long as there is no evidence of copying).

1. Let $Y \sim \text{Bin}(100, p)$. To test $H_0 : p = 0.08$ against $H_A : p < 0.08$, we reject H_0 and accept H_A if and only if $Y = 6$.
   (a) Determine the significance level α of the test.
   (b) Find the probability of type II error if in fact $p = .04$.
2. Let $Y \sim \text{Bin}(300, p)$. If the observed value of Y is $y = 75$, find an approximate 90 percent confidence interval for p.
3. Let $\bar{X}$ be the sample mean of a random sample of size 25 from a gamma distribution with density:
   $$f(x) = \frac{1}{6\beta^\alpha} x^3 e^{-x/\beta}, \quad x > 0.$$
   Use the central limit theorem to find an approximate .954 confidence interval for $\mu = 4\beta$, the mean of this distribution.
4. Let $X \sim \text{Bin}(10, p), \quad p \in (1/4, 1/2)$. The simple hypothesis $H_0 : p = 1/2$ is rejected and alternative simple hypothesis $H_A : p = 1/4$ is accepted, if the observed value of X_1, the random sample of size 1, is less than of equal to 3. Find the power function of the test.
5. If X_1, X_2 is a random sample of size 2 from a distribution having a pdf
   $$f(x; \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 < x < \infty, \quad 0 < \theta < \infty.$$
   Find the joint pdf of the sufficient statistic $Y_1 = X_1 + X_2$ for θ and $Y_2 = X_2$. Show that Y_2 is an unbiased estimator of θ with variance 2. Find $\text{E}(Y_2|y_1) = \phi(y_1)$ and the variance of $\phi(Y_1)$.

6. Let X have a Poisson distribution with mean θ. Consider the simple null hypothesis $H_0 : \theta = 1/2$ and the alternative hypothesis $H_A : \theta < 1/2$. Thus $\Omega = \theta : 0 < \theta \leq 1/2$. Let $X_1, X_2, \ldots, X_{12}$ denote a random sample of size 12 from this distribution. We reject H_0 if and only if the observed value of $Y = X_1 + X_2 + \ldots + X_{12} \leq 2$. If $\beta(\theta)$ is the power function of the test, nd the powers $\beta(1/2), \beta(1/3), \beta(1/4), \beta(1/6)$, and $\beta(1/12)$. Sketch the graph of $\beta(\theta)$. What is the signicance level of the test?

7. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size $n = 4$ from a distribution with p.d.f. $f(x; \theta) = 1/\theta, \quad 0 < x < \theta, \quad \theta > 0$. The hypothesis $H_0 : \theta = 1$ is rejected and $H_A : \theta > 1$ is accepted if $Y_4 \geq c$.

   (a) Find the constant c so that the signicance level is $\alpha = .05$.

   (b) Determine the power function of the test.

8. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with unknown mean. In testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_A : \sigma^2 > \sigma_0^2$, use the critical region defined by $nS^2/\sigma_0^2 \geq c$. If $n = 13$ and the signicance level $\alpha = .025$, find c.

9. DeGroot and Schervish, #10 on page 529.

10. DeGroot and Schervish, #13 on page 529.

11. DeGroot and Schervish, #22 a,b,d on page 529.

12. DeGroot and Schervish, #9 on page 622.

13. DeGroot and Schervish, #11 on page 622.

14. DeGroot and Schervish, #12 on page 622.

15. DeGroot and Schervish, #13 on page 622.