Homework 1

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Question 1

When rolling two dice, there are six possible ways for their total to sum up to seven: $(1,6)$, $(2,5)$, $(3,4)$, $(4,3)$, $(5,2)$, and $(6,1)$, so the probability of the sum being seven is $6/36 = 1/6$. If X is the number of trials where the total of both rolls is seven, then we can think of $X \sim \text{Bin}(120, 1/6)$, and so $\mathbb{E}X = 20$ and $\text{Var}X = 50/3$. Using the Central Limit Theorem, we then have

$$\Pr\big(|X-20| \le k\big) = \Pr\left(\left|\frac{X-20}{\sqrt{50/3}}\right| \le k\sqrt{\frac{3}{50}}\right) = 2\Phi\left(k\sqrt{\frac{3}{50}}\right) - 1 \overset{\text{set}}{=} 0.95 \implies \Phi\left(k\sqrt{\frac{3}{50}}\right) = 0.975.$$

Using a table of values for $\Phi(z)$, we can see that $k\sqrt{3/50} = 1.96$, and so $k = 1.96\sqrt{50/3} \approx 8$.