Homework 2

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STAT GR5205: Linear Regression Models
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Question 1 Supposed for $\mathbf{x}, \mathbf{y}, \boldsymbol{\epsilon}, \mathbf{1} \in \mathbb{R}^n$, where $\mathbf{x}, \mathbf{1}$ are *fixed* vectors and $\mathbf{y}, \boldsymbol{\epsilon}$ are *random* vectors, the simple linear regression model

$$\mathbf{y} = \beta_0 \mathbf{1} + \beta_1 \mathbf{x} + \boldsymbol{\epsilon}$$

holds, with $\mathbb{E}[\boldsymbol{\epsilon}] = \mathbf{0}$, $\text{Var}[\boldsymbol{\epsilon}] = \sigma^2 \mathbf{I}$, and $\mathbb{E}[\mathbf{y}] = \mathbb{E}[\beta_0 \mathbf{1} + \beta_1 \mathbf{x} + \boldsymbol{\epsilon}] = \beta_0 \mathbf{1} + \beta_1 \mathbf{x}$. The least-squares estimators are given by

$$\hat{\beta}_1 = \frac{(\mathbf{x} - \bar{x}\mathbf{1})^T (\mathbf{y} - \bar{y}\mathbf{1})}{\|\mathbf{x} - \bar{x}\mathbf{1}\|^2}, \quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}, \quad \text{and} \quad \hat{\sigma}^2 = \frac{1}{n-2} \left\| \mathbf{y} - \hat{\beta}_0 \mathbf{1} - \hat{\beta}_1 \mathbf{x} \right\|^2 .$$

(a) We first show that