Sample Solutions

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Notebook 11, Sample solutions

airport-rank (Score: 18.0 / 18.0)

1. Test cell (Score: 3.0 / 3.0)
2. Test cell (Score: 3.0 / 3.0)
3. Test cell (Score: 2.0 / 2.0)
4. Test cell (Score: 2.0 / 2.0)
5. Test cell (Score: 1.0 / 1.0)
6. Test cell (Score: 3.0 / 3.0)
7. Test cell (Score: 1.0 / 1.0)
8. Test cell (Score: 1.0 / 1.0)
9. Test cell (Score: 2.0 / 2.0)

Important note! Before you turn in this lab notebook, make sure everything runs as expected:

- First, **restart the kernel** -- in the menubar, select Kernel→Restart.
- Then **run all cells** -- in the menubar, select Cell→Run All.

Make sure you fill in any place that says `YOUR CODE HERE` or "YOUR ANSWER HERE."

Markov chain analysis of the US airport network

One way to view the airline transportation infrastructure is in the form of a directed *network* or *graph*, in which vertices are airports and edges are the direct-flight segments that connect them. For instance, if there is a direct flight from Atlanta's Hartsfield-Jackson International Airport ("ATL") to Los Angeles International Airport ("LAX"), then the airport network would have a directed edge from ATL to LAX.

Given the airport network, one question we might ask is, which airports are most critical to disruption of the overall network? That is, if an airport is shut down, thereby leading to all inbound and outbound flights being cancelled, will that catastrophic event have a big impact or a small impact on the overall network?

You would expect "importance" to be related to whether an airport has lots of inbound or outgoing connections. In graph lingo, that's also called the *degree* of a vertex or node. But if there are multiple routes that can work around a highly connected hub (i.e., a vertex with a high indegree or outdegree), that might not be the case. So let's try to use a PageRank-like scheme to see what we get and compare that to looking at degree.

As it happens, the US Bureau of Transportation Statistics collects data on all flights originating or arriving in the United States. In this notebook, you'll use this data to build an airport network and then use Markov chain analysis to rank the networks by some measure of "criticality."

> Sources: This notebook is adapted from the following: https://www.mongodb.com/blog/post/pagerank-on-flights-dataset (https://www.mongodb.com/blog/post/pagerank-on-flights-dataset). The dataset is a small subset of what is available here:

https://www.transtats.bts.gov/DL_SelectFields.asp?Table_ID=236 (https://www.transtats.bts.gov/DL_SelectFields.asp?Table_ID=236)

The formal analysis problem

Let's model the analysis problem as follows.

Consider a "random flyer" to be a person who arrives at an airport i, and then randomly selects any direct flight that departs from i and arrives at j. We refer to the direct flight from i to j as the *flight segment* $i \rightarrow j$. Upon arriving at j, the flyer repeats the process of randomly selecting a new segment, $j \rightarrow k$. He or she repeats this process forever.

Let $x_i(t)$ be the probability that the flyer is at airport i at time t. Take t to be an integer count corresponding to the number of flight segments that the flyer has taken so far, starting at $t = 0$. Let p_{ij} be the probability of taking segment $i \rightarrow j$, where $p_{ij} = 0$ means the segment $i \rightarrow j$ is unavailable or does not exist. If there are n airports in all, numbered from 0 to $n - 1$, then the probability that the flyer will be at airport i at time $t + 1$, given all the probabilities at time t, is

$$x_i(t+1) = \sum_{j=0}^{n-1} p_{ji} \cdot x_j(t).$$

Let $P \equiv [p_{ij}]$ be the matrix of transition probabilities and $x(t) = [x_i(t)]$ the column vector of prior probabilities. Then we can write the above more succinctly for all airports as the matrix-vector product,

$$x(t+1) = P^T x(t).$$

Since P is a probability transition matrix then there exists a steady-state distribution, x^* , which is the limit of $x(t)$ as t goes to infinity:

$$\lim_{t \rightarrow \infty} x(t) = x^* \equiv [x_i^*].$$

The larger x_i^* , the more likely it is that the random flyer is to be at airport i in the steady state. Therefore, we can take the "importance" or "criticality" of airport i in the flight network to be its steady-state probability, x_i^* .

Thus, our data pre-processing goal is to construct P and our analysis goal is to compute the steady-state probability distribution, x^* , for a first-order Markov chain system.

Modules you'll need

For this notebook, let's use Pandas for preprocessing the raw data and SciPy's sparse matrix libraries to implement the analysis.

One of the cells below defines a function, `spy()`, that can be used to visualize the non-zero structure of a sparse matrix.

In [1]:
```
import numpy as np
import scipy as sp
import scipy.sparse

import pandas as pd
```

In [2]:
```
import matplotlib.pyplot as plt
%matplotlib inline

def spy(A, figsize=(6, 6), markersize=0.5):
    """Visualizes a sparse matrix."""
    fig = plt.figure(figsize=figsize)
    plt.spy(A, markersize=markersize)
    plt.show()
```

In [3]:
```
from IPython.display import display, Markdown # For pretty-printing tibbles
```

In [4]:
```
def canonicalize_tibble(X):
    var_names = sorted(X.columns)
    Y = X[var_names].copy()
```

```
    Y.sort_values(by=var_names, inplace=True)
    Y.reset_index(drop=True, inplace=True)
    return Y

def tibbles_are_equivalent (A, B):
    A_canonical = canonicalize_tibble(A)
    B_canonical = canonicalize_tibble(B)
    cmp = A_canonical.eq(B_canonical)
    return cmp.all().all()
```

Part 0: Downloading, unpacking, and exploring the data

You'll need some data for this assignment. The following code cell will check for it in your local environment, and if it doesn't exist, attempt to download it.

In [5]:

```
import requests
import os
import hashlib
import io

def on_vocareum():
    return os.path.exists('.voc')

def download(file, local_dir="", url_base=None, checksum=None):
    local_file = "{}{}".format(local_dir, file)
    if not os.path.exists(local_file):
        if url_base is None:
            url_base = "https://cse6040.gatech.edu/datasets/"
        url = "{}{}".format(url_base, file)
        print("Downloading: {} ...".format(url))
        r = requests.get(url)
        with open(local_file, 'wb') as f:
            f.write(r.content)

    if checksum is not None:
        with io.open(local_file, 'rb') as f:
            body = f.read()
            body_checksum = hashlib.md5(body).hexdigest()
            assert body_checksum == checksum, \
                "Downloaded file '{}' has incorrect checksum: '{}' instead of '{}'".forma
t(local_file,

body_checksum,

checksum)
    print("'{}' is ready!".format(file))

if on_vocareum():
    URL_BASE = "https://cse6040.gatech.edu/datasets/us-flights/"
    DATA_PATH = "../resource/lib/publicdata/"
else:
    URL_BASE = "https://github.com/cse6040/labs-fa17/raw/master/lab11-markov_chains/"
    DATA_PATH = ""

datasets = {'L_AIRPORT_ID.csv': 'e9f250e3c93d625cce92d08648c4bbf0',
            'L_CITY_MARKET_ID.csv': 'f430a16a5fe4b9a849accb5d332b2bb8',
            'L_UNIQUE_CARRIERS.csv': 'bebe919e85e2cf72e7041dbf1ae5794e',
            'us-flights--2017-08.csv': 'eeb259c0cdd00ff1027261ca0a7c0332',
            'flights_atl_to_lax_soln.csv': '4591f6501411de90af72693cdbcc08bb',
            'origins_top10_soln.csv': 'de85c321c45c7bf65612754be4567086',
            'dests_soln.csv': '370f4c632623616b3bf26b6f79993fe4',
            'dests_top10_soln.csv': '4c7dd7edf48c4d62466964d6b8c14184',
            'segments_soln.csv': '516a78d2d9d768d78bfb012b77671f38',
            'segments_outdegree_soln.csv': 'b29d60151c617ebafd3a1c58541477c8'
           }

for filename, checksum in datasets.items():
    download(filename, local_dir=DATA_PATH, url_base=URL_BASE, checksum=checksum)

print("\n(All data appears to be ready.)")
```

```
'L_UNIQUE_CARRIERS.csv' is ready!
```

```
'us-flights--2017-08.csv' is ready!
'L_CITY_MARKET_ID.csv' is ready!
'segments_outdegree_soln.csv' is ready!
'segments_soln.csv' is ready!
'L_AIRPORT_ID.csv' is ready!
'dests_top10_soln.csv' is ready!
'origins_top10_soln.csv' is ready!
'dests_soln.csv' is ready!
'flights_atl_to_lax_soln.csv' is ready!

(All data appears to be ready.)
```

Airport codes. Let's start with the airport codes.

In [6]:
```
airport_codes = pd.read_csv("{}{}".format(DATA_PATH, 'L_AIRPORT_ID.csv'))
airport_codes.head()
```

Out[6]:

	Code	Description
0	10001	Afognak Lake, AK: Afognak Lake Airport
1	10003	Granite Mountain, AK: Bear Creek Mining Strip
2	10004	Lik, AK: Lik Mining Camp
3	10005	Little Squaw, AK: Little Squaw Airport
4	10006	Kizhuyak, AK: Kizhuyak Bay

Flight segments. Next, let's load a file that contains all of US flights that were scheduled for August 2017.

In [7]:
```
flights = pd.read_csv('{}{}'.format(DATA_PATH, 'us-flights--2017-08.csv'))
print("Number of flight segments: {} [{:.1f} million]".format (len(flights), len(flights)
*1e-6))
del flights['Unnamed: 7'] # Cleanup extraneous column
flights.head()
```

```
Number of flight segments: 510451 [0.5 million]
```

Out[7]:

	FL_DATE	UNIQUE_CARRIER	FL_NUM	ORIGIN_AIRPORT_ID	ORIGIN_CITY_MARKET_ID	DEST_AIRPORT_ID
0	2017-08-01	DL	2	12478	31703	14679
1	2017-08-01	DL	4	12889	32211	12478
2	2017-08-01	DL	6	12892	32575	14869
3	2017-08-01	DL	7	14869	34614	12892
4	2017-08-01	DL	10	11292	30325	13487

Each row of this tibble is a *(direct) flight segment*, that is, a flight that left some origin and arrived at a destination on a certain date. As noted earlier, these segments cover a one-month period (August 2017).

Exercise 0 (3 points). As a warmup to familiarize yourself with this dataset, determine all direct flight segments that originated at Atlanta's Hartsfield-Jackson International Airport and traveled to Los Angeles International. Store the result in a dataframe named `flights_atl_to_lax`, which should be the corresponding subset of rows from `flights`.

In [8]: Student's answer (Top)

```
def lookup_airport(index):
    return airport_codes.loc[index, 'Code'], airport_codes.loc[index, 'Description']

ATL_ID, ATL_DESC = lookup_airport(373)
```

```
LAX_ID, LAX_DESC = lookup_airport(2765)

print("{}: ATL -- {}".format(ATL_ID, ATL_DESC))
print("{}: LAX -- {}".format(LAX_ID, LAX_DESC))
print()

is_atl_origin = (flights['ORIGIN_AIRPORT_ID'] == ATL_ID)
is_lax_dest = (flights['DEST_AIRPORT_ID'] == LAX_ID)
is_atl_to_lax = is_atl_origin & is_lax_dest
flights_atl_to_lax = flights[is_atl_to_lax]

# Displays a few of your results
print("Your code found {} flight segments.".format(len(flights_atl_to_lax)))
display(flights_atl_to_lax.head())
```

```
10397: ATL -- Atlanta, GA: Hartsfield-Jackson Atlanta International
12892: LAX -- Los Angeles, CA: Los Angeles International

Your code found 586 flight segments.
```

	FL_DATE	UNIQUE_CARRIER	FL_NUM	ORIGIN_AIRPORT_ID	ORIGIN_CITY_MARKET_ID	DEST_AIRPORT_
64	2017-08-01	DL	110	10397	30397	12892
165	2017-08-01	DL	370	10397	30397	12892
797	2017-08-01	DL	1125	10397	30397	12892
806	2017-08-01	DL	1133	10397	30397	12892
858	2017-08-01	DL	1172	10397	30397	12892

In [9]: Grade cell: `flights_atl_to_lax_test` Score: 3.0 / 3.0 (Top)

```
# Test cell: `flights_atl_to_lax_test`

if False:
    flights_atl_to_lax.to_csv('flights_atl_to_lax_soln.csv', index=False)
flights_atl_to_lax_soln = pd.read_csv('{}{}'.format(DATA_PATH, 'flights_atl_to_lax_soln
.csv'))

assert tibbles_are_equivalent(flights_atl_to_lax, flights_atl_to_lax_soln), \
       "Sorry, your solution does not match the instructor's solution."

print("\n(Passed!)")
```

```
(Passed!)
```

Aggregation. Observe that an (origin, destination) pair may appear many times. That's because the dataset includes a row for *every* direct flight that occurred historically and there may have been multiple such flights on a given day.

However, for the purpose of this analysis, let's simplify the problem by collapsing *all* historical segments $i \rightarrow j$ into a single segment. Let's also do so in a way that preserves the number of times the segment occurred (i.e., the number of rows containing the segment).

To accomplish this task, the following code cell uses the `groupby()` (http://pandas.pydata.org/pandas-docs/stable/generated/pandas.DataFrame.groupby.html) function available for Pandas tables and the `count()` (http://pandas.pydata.org/pandas-docs/stable/groupby.html) aggregator in three steps:

1. It considers just the flight date, origin, and destination columns.
2. It *logically* groups the rows having the same origin and destination, using `groupby()`.
3. It then aggregates the rows, counting the number of rows in each (origin, destination) group.

In [10]:
```
flights_cols_subset = flights[['FL_DATE', 'ORIGIN_AIRPORT_ID', 'DEST_AIRPORT_ID']]
```

```
In [10]: flights_cols_subset = flights[['FL_DATE', 'ORIGIN_AIRPORT_ID', 'DEST_AIRPORT_ID']]
segment_groups = flights_cols_subset.groupby(['ORIGIN_AIRPORT_ID', 'DEST_AIRPORT_ID'], as
_index=False)
segments = segment_groups.count()
segments.rename(columns={'FL_DATE': 'FL_COUNT'}, inplace=True)
segments.head()
```

Out[10]:

	ORIGIN_AIRPORT_ID	DEST_AIRPORT_ID	FL_COUNT
0	10135	10397	77
1	10135	11433	85
2	10135	13930	18
3	10140	10397	93
4	10140	10423	4

As a last sanity check, let's verify that the counts are all at least 1.

```
In [11]: assert (segments['FL_COUNT'] > 0).all()
```

Actual (as opposed to "all possible") origins and destinations. Although there are many possible airport codes stored in the `airport_codes` dataframe (over six thousand), only a subset appear as actual origins in the data. The following code cell determines the actual origins and prints their number.

```
In [12]: origins = segments[['ORIGIN_AIRPORT_ID', 'FL_COUNT']].groupby('ORIGIN_AIRPORT_ID', as_ind
ex=False).sum()
origins.rename(columns={'FL_COUNT': 'ORIGIN_COUNT'}, inplace=True)
print("Number of actual origins:", len(origins))
origins.head()
```

```
Number of actual origins: 300
```

Out[12]:

	ORIGIN_AIRPORT_ID	ORIGIN_COUNT
0	10135	180
1	10140	1761
2	10141	62
3	10146	41
4	10154	176

To get an idea of what airports are likely to be the most important in our Markov chain analysis, let's rank airports by the total number of *outgoing* segments, i.e., flight segments that originate at the airport.

Exercise 1 (3 points). Construct a dataframe, `origins_top10`, containing the top 10 airports in descending order of outgoing segments. This dataframe should have three columns:

- `ID`: The ID of the airport
- `Count`: Number of outgoing segments.
- `Description`: The plaintext descriptor for the airport that comes from the `airport_codes` dataframe.

> *Hint*: Look up and read about `numpy.argsort()`, which you can also apply to any Pandas Series object.

In [13]: Student's answer (Top)

```
# Rank each origin
origin_ranks = np.argsort(-origins['ORIGIN_COUNT'])
origins_top10 = origins.iloc[origin_ranks[:10]]
origins_top10 = origins_top10.rename(columns={'ORIGIN_AIRPORT_ID': 'ID', 'ORIGIN_COUNT'
: 'Count'})
origins_top10 = origins_top10.merge(airport_codes, how='left', left_on='ID', right_on='
Code')
```

```
Code')
del origins_top10['Code']

# Prints the top 10, according to your calculation:
origins_top10
```

Out[13]:

	ID	Count	Description
0	10397	31899	Atlanta, GA: Hartsfield-Jackson Atlanta Intern...
1	13930	25757	Chicago, IL: Chicago O'Hare International
2	11292	20891	Denver, CO: Denver International
3	12892	19399	Los Angeles, CA: Los Angeles International
4	14771	16641	San Francisco, CA: San Francisco International
5	11298	15977	Dallas/Fort Worth, TX: Dallas/Fort Worth Inter...
6	14747	13578	Seattle, WA: Seattle/Tacoma International
7	12889	13367	Las Vegas, NV: McCarran International
8	14107	13040	Phoenix, AZ: Phoenix Sky Harbor International
9	13487	12808	Minneapolis, MN: Minneapolis-St Paul Internati...

In [14]: Grade cell: `origin_ranks_test` Score: 3.0 / 3.0 (Top)

```
# Test cell: `origin_ranks_test`

if False:
    origins_top10.to_csv('origins_top10_soln.csv', index=False)
origins_top10_soln = pd.read_csv('{}{}'.format(DATA_PATH, 'origins_top10_soln.csv'))

print("=== Instructor's solution ===")
display(origins_top10_soln)

assert tibbles_are_equivalent(origins_top10, origins_top10_soln), \
       "Your table does not have the same entries as the solution."

counts_0_9 = origins_top10['Count'].iloc[:9].values
counts_1_10 = origins_top10['Count'].iloc[1:].values
assert (counts_0_9 >= counts_1_10).all(), \
       "Are your rows sorted in descending order?"

print("\n(Passed!)")
```

```
=== Instructor's solution ===
```

	ID	Count	Description
0	10397	31899	Atlanta, GA: Hartsfield-Jackson Atlanta Intern...
1	13930	25757	Chicago, IL: Chicago O'Hare International
2	11292	20891	Denver, CO: Denver International
3	12892	19399	Los Angeles, CA: Los Angeles International
4	14771	16641	San Francisco, CA: San Francisco International
5	11298	15977	Dallas/Fort Worth, TX: Dallas/Fort Worth Inter...
6	14747	13578	Seattle, WA: Seattle/Tacoma International
7	12889	13367	Las Vegas, NV: McCarran International
8	14107	13040	Phoenix, AZ: Phoenix Sky Harbor International
9	13487	12808	Minneapolis, MN: Minneapolis-St Paul Internati...

```
(Passed!)
```

Exercise 2 (2 points). The preceding code computed a tibble, `origins`, containing all the unique origins and their number of outgoing flights. Write some code to compute a new tibble, `dests`, which contains all unique destinations and their number of *incoming* flights. Its columns should be named `DEST_AIRPORT_ID` (airport code) and `DEST_COUNT` (number of direct inbound segments).

The test cell that follows prints the number of unique destinations and the first few rows of your result, as well as some automatic checks.

In [15]: Student's answer (Top)

```
dests = segments[['DEST_AIRPORT_ID', 'FL_COUNT']].groupby('DEST_AIRPORT_ID', as_index=F
alse).sum()
dests.rename(columns={'FL_COUNT': 'DEST_COUNT'}, inplace=True)

print("Number of unique destinations:", len(dests))
dests.head()
```

```
Number of unique destinations: 300
```

Out[15]:

	DEST_AIRPORT_ID	DEST_COUNT
0	10135	179
1	10140	1763
2	10141	62
3	10146	40
4	10154	176

In [16]: Grade cell: `dests_test` Score: 2.0 / 2.0 (Top)

```
# Test cell: `dests_test`

if False:
    dests.to_csv('dests_soln.csv', index=False)
dests_soln = pd.read_csv('{}{}'.format(DATA_PATH, 'dests_soln.csv'))

assert tibbles_are_equivalent(dests, dests_soln), "Your solution does not match the ins
tructors'."

print("\n(Passed!)")
```

```
(Passed!)
```

Exercise 3 (2 points). Compute a tibble, `dests_top10`, containing the top 10 destinations (i.e., rows of `dests`) by inbound flight count. The column names should be the same as `origins_top10` and the rows should be sorted in decreasing order by count.

In [17]: Student's answer (Top)

```
dest_ranks = np.argsort(-dests['DEST_COUNT'])
dests_top10 = dests.iloc[dest_ranks[:10]]
dests_top10 = dests_top10.rename(columns={'DEST_AIRPORT_ID': 'ID', 'DEST_COUNT': 'Count
'})
dests_top10 = dests_top10.merge(airport_codes, how='left', left_on='ID', right_on='Code
')
del dests_top10['Code']

print("Your computed top 10 destinations:")
dests_top10
```

```
Your computed top 10 destinations:
```

Out[17]:

	ID	Count	Description
0	10397	31901	Atlanta, GA: Hartsfield-Jackson Atlanta Intern...
1	13930	25778	Chicago, IL: Chicago O'Hare International
2	11292	20897	Denver, CO: Denver International

3	12892	19387	Los Angeles, CA: Los Angeles International
4	14771	16651	San Francisco, CA: San Francisco International
5	11298	15978	Dallas/Fort Worth, TX: Dallas/Fort Worth Inter...
6	14747	13582	Seattle, WA: Seattle/Tacoma International
7	12889	13374	Las Vegas, NV: McCarran International
8	14107	13039	Phoenix, AZ: Phoenix Sky Harbor International
9	13487	12800	Minneapolis, MN: Minneapolis-St Paul Internati...

In [18]: Grade cell: `dests_top10_test` Score: 2.0 / 2.0 (Top)

```
# Test cell: `dests_top10_test`

if False:
    dests_top10.to_csv('dests_top10_soln.csv', index=False)
dests_top10_soln = pd.read_csv('{}{}'.format(DATA_PATH, 'dests_top10_soln.csv'))

print("=== Instructor's solution ===")
display(dests_top10_soln)

assert tibbles_are_equivalent(dests_top10, dests_top10_soln), \
       "Your table does not have the same entries as the solution."

counts_0_9 = dests_top10['Count'].iloc[:9].values
counts_1_10 = dests_top10['Count'].iloc[1:].values
assert (counts_0_9 >= counts_1_10).all(), \
       "Are your rows sorted in descending order?"

print("\n(Passed!)")
```

```
=== Instructor's solution ===
```

	ID	Count	Description
0	10397	31901	Atlanta, GA: Hartsfield-Jackson Atlanta Intern...
1	13930	25778	Chicago, IL: Chicago O'Hare International
2	11292	20897	Denver, CO: Denver International
3	12892	19387	Los Angeles, CA: Los Angeles International
4	14771	16651	San Francisco, CA: San Francisco International
5	11298	15978	Dallas/Fort Worth, TX: Dallas/Fort Worth Inter...
6	14747	13582	Seattle, WA: Seattle/Tacoma International
7	12889	13374	Las Vegas, NV: McCarran International
8	14107	13039	Phoenix, AZ: Phoenix Sky Harbor International
9	13487	12800	Minneapolis, MN: Minneapolis-St Paul Internati...

```
(Passed!)
```

The number of actual origins does equal the number of actual destinations. Let's store this value for later use.

In [19]:

```
n_actual = len(set(origins['ORIGIN_AIRPORT_ID']) | set(dests['DEST_AIRPORT_ID']))
print("Number of actual locations (whether origin or destination):", n_actual)
```

```
Number of actual locations (whether origin or destination): 300
```

Part 1: Constructing the state-transition matrix

Now that you have cleaned up the data, let's prepare it for subsequent analysis. Start by constructing the *probability state-transition matrix* for the airport network. Denote this matrix by $P \equiv [p_{ij}]$, where p_{ij} is the conditional probability that a random flyer departs from airport i and arrives at airport j given that he or she is currently at airport i.

To build P, let's use SciPy's sparse matrix facilities. To do so, you will need to carry out the following two steps:

1. *Map airport codes to matrix indices.* An `m`-by-`n` sparse matrix in SciPy uses the zero-based values 0, 1, ..., `m`-1 and 0, ..., `n`-1 to refer to row and column indices. Therefore, you will need to map the airport codes to such index values.
2. *Derive weights, $p{ij}.$Y ouwillneedtodecidehowtodetermine*p_{ij}$.

Let's walk through each of these steps next.

Step 1: Mapping airport codes to integers. Luckily, you already have a code-to-integer mapping, which is in the column `airport_codes['Code']` mapped to the dataframe's `index`.

As a first step, let's make note of the number of airports, which is nothing more than the largest index value.

In [20]:
```
n_airports = airport_codes.index.max() + 1
print("Note: There are", n_airports, "airports.")
```

```
Note: There are 6436 airports.
```

Next, let's add another column to `segments` called `ORIGIN_INDEX`, which will hold the id corresponding to the origin:

In [21]:
```
# Recall:
segments.columns
```

Out[21]:
```
Index(['ORIGIN_AIRPORT_ID', 'DEST_AIRPORT_ID', 'FL_COUNT'], dtype='object')
```

In [22]:
```
# Extract the `Code` column and index from `airport_codes`, storing them in
# a temporary tibble with new names, `ORIGIN_AIRPORT_ID` and `ORIGIN_INDEX`.
origin_indices = airport_codes[['Code']].rename(columns={'Code': 'ORIGIN_AIRPORT_ID'})
origin_indices['ORIGIN_INDEX'] = airport_codes.index

# Since you might run this code cell multiple times, the following
# check prevents `ORIGIN_ID` from appearing more than once.
if 'ORIGIN_INDEX' in segments.columns:
    del segments['ORIGIN_INDEX']

# Perform the merge as a left-join of `segments` and `origin_ids`.
segments = segments.merge(origin_indices, on='ORIGIN_AIRPORT_ID', how='left')
segments.head()
```

Out[22]:

	ORIGIN_AIRPORT_ID	DEST_AIRPORT_ID	FL_COUNT	ORIGIN_INDEX
0	10135	10397	77	119
1	10135	11433	85	119
2	10135	13930	18	119
3	10140	10397	93	124
4	10140	10423	4	124

Exercise 4 (1 point). Analogous to the preceding procedure, create a new column called `segments['DEST_INDEX']` to hold the integer index of each segment's *destination*.

In [23]:

Student's answer (Top)

```
dest_indices = airport_codes[['Code']].rename(columns={'Code': 'DEST_AIRPORT_ID'})
dest_indices['DEST_INDEX'] = airport_codes.index

# Since you might run this code cell multiple times, the following
# check prevents `ORIGIN_ID` from appearing more than once.
if 'DEST_INDEX' in segments.columns:
    del segments['DEST_INDEX']

# Perform the merge as a left-join of `segments` and `origin_ids`.
segments = segments.merge(dest_indices, on='DEST_AIRPORT_ID', how='left')
```

```
# visually inspect your result:
segments.head()
```

Out[23]:

	ORIGIN_AIRPORT_ID	DEST_AIRPORT_ID	FL_COUNT	ORIGIN_INDEX	DEST_INDEX
0	10135	10397	77	119	373
1	10135	11433	85	119	1375
2	10135	13930	18	119	3770
3	10140	10397	93	124	373
4	10140	10423	4	124	399

In [24]: Grade cell: `dest_id_test` Score: 1.0 / 1.0 (Top)

```
# Test cell: `dest_id_test`

if False:
    segments.to_csv('segments_soln.csv', index=False)
segments_soln = pd.read_csv('{}{}'.format(DATA_PATH, 'segments_soln.csv'))

assert tibbles_are_equivalent(segments, segments_soln), \
       "Your solution does not match the instructors'."

print("\n(Passed!)")
```

```
(Passed!)
```

Step 2: Computing edge weights. Armed with the preceding mapping, let's next determine each segment's transition probability, or "weight," p_{ij}.

For each origin i, let d_i be the number of outgoing edges, or *outdegree*. Note that this value is *not* the same as the total number of (historical) outbound *segments*; rather, let's take d_i to be just the number of airports reachable directly from i. For instance, consider all flights departing the airport whose airport code is 10135:

In [25]:

```
display(airport_codes[airport_codes['Code'] == 10135])

abe_segments = segments[segments['ORIGIN_AIRPORT_ID'] == 10135]
display(abe_segments)

print("Total outgoing segments:", abe_segments['FL_COUNT'].sum())
```

	Code	Description
119	10135	Allentown/Bethlehem/Easton, PA: Lehigh Valley ...

	ORIGIN_AIRPORT_ID	DEST_AIRPORT_ID	FL_COUNT	ORIGIN_INDEX	DEST_INDEX
0	10135	10397	77	119	373
1	10135	11433	85	119	1375
2	10135	13930	18	119	3770

```
Total outgoing segments: 180
```

In [26]:

```
k_ABE = abe_segments['FL_COUNT'].sum()
d_ABE = len(abe_segments)
i_ABE = abe_segments['ORIGIN_AIRPORT_ID'].values[0]

display(Markdown('''
Though `ABE` has {} outgoing segments,
its outdegree or number of outgoing edges is just {}.
Thus, `ABE`, whose airport id is $i={}$, has $d_{{{}}} = {}$.
'''.format(k_ABE, d_ABE, i_ABE, i_ABE, d_ABE)))
```

Though `ABE` has 180 outgoing segments, its outdegree or number of outgoing edges is just 3. Thus, `ABE`, whose airport id is $i = 10135$, has $d_{10135} = 3$.

Exercise 5 (3 points). Add a new column named `OUTDEGREE` to the `segments` tibble that holds the outdegrees, $\{d_i\}$. That is, for each row whose airport *index* (as opposed to code) is i, its entry of `OUTDEGREE` should be d_i.

For instance, the rows of segments corresponding to airport ABE (code 10135 and matrix index 119) would look like this:

ORIGIN_AIRPORT_ID	DEST_AIRPORT_ID	FL_COUNT	ORIGIN_INDEX	DEST_INDEX	OUTDEGREE
10135	10397	77	119	373	3
10135	11433	85	119	1375	3
10135	13930	18	119	3770	3

In [27]: Student's answer (Top)

```
# This `if` removes an existing `OUTDEGREE` column
# in case you run this cell more than once.
if 'OUTDEGREE' in segments.columns:
    del segments['OUTDEGREE']

outdegrees = segments[['ORIGIN_INDEX', 'DEST_INDEX']].groupby('ORIGIN_INDEX', as_index=
False).count()
outdegrees.rename(columns={'DEST_INDEX': 'OUTDEGREE'}, inplace=True)
segments = segments.merge(outdegrees, on='ORIGIN_INDEX', how='left')

# Visually inspect the first ten rows of your result:
segments.head(10)
```

Out[27]:

	ORIGIN_AIRPORT_ID	DEST_AIRPORT_ID	FL_COUNT	ORIGIN_INDEX	DEST_INDEX	OUTDEGREE
0	10135	10397	77	119	373	3
1	10135	11433	85	119	1375	3
2	10135	13930	18	119	3770	3
3	10140	10397	93	124	373	23
4	10140	10423	4	124	399	23
5	10140	10821	64	124	792	23
6	10140	11259	143	124	1214	23
7	10140	11292	127	124	1245	23
8	10140	11298	150	124	1250	23
9	10140	12191	89	124	2106	23

In [28]: Grade cell: `weights_test` Score: 3.0 / 3.0 (Top)

```
# Test cell: `weights_test`

if False:
    segments.to_csv('segments_outdegree_soln.csv', index=False)

segments_outdegree_soln = pd.read_csv('{}{}'.format(DATA_PATH, 'segments_outdegree_soln
.csv'))

assert tibbles_are_equivalent(segments, segments_outdegree_soln), \
       "Your solution does not appear to match the instructors'."

print("\n(Passed!)")
```

```
(Passed!)
```

From outdegree to weight. Given the outdegree d_i, let $p_{ij} = \frac{1}{d_i}$. In other words, suppose that a random flyer at airport i is *equally likely* to pick any of the destinations directly reachable from i. The following code cell stores that value in a new column, `WEIGHT`.

In [29]:
```
if 'WEIGHT' in segments:
    del segments['WEIGHT']

segments['WEIGHT'] = 1.0 / segments['OUTDEGREE']
display(segments.head(10))

# These should sum to 1.0!
origin_groups = segments[['ORIGIN_INDEX', 'WEIGHT']].groupby('ORIGIN_INDEX')
assert np.allclose(origin_groups.sum(), 1.0, atol=10*n_actual*np.finfo(float).eps), "Rows
of $P$ do not sum to 1.0"
```

	ORIGIN_AIRPORT_ID	DEST_AIRPORT_ID	FL_COUNT	ORIGIN_INDEX	DEST_INDEX	OUTDEGREE	WEIGHT
0	10135	10397	77	119	373	3	0.333333
1	10135	11433	85	119	1375	3	0.333333
2	10135	13930	18	119	3770	3	0.333333
3	10140	10397	93	124	373	23	0.043478
4	10140	10423	4	124	399	23	0.043478
5	10140	10821	64	124	792	23	0.043478
6	10140	11259	143	124	1214	23	0.043478
7	10140	11292	127	124	1245	23	0.043478
8	10140	11298	150	124	1250	23	0.043478
9	10140	12191	89	124	2106	23	0.043478

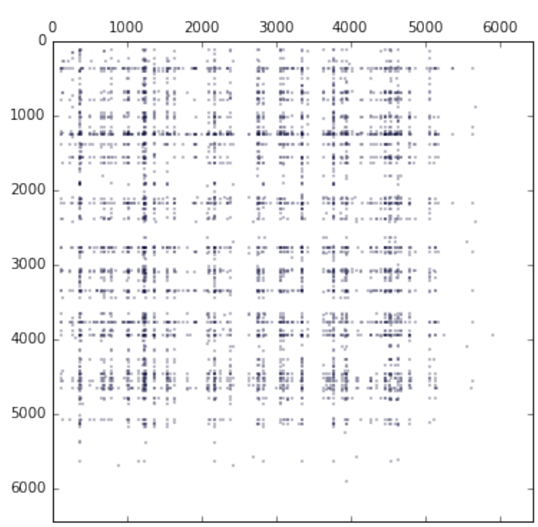
Exercise 6 (2 points). With your updated `segments` tibble, construct a sparse matrix, `P`, corresponding to the state-transition matrix P. Use SciPy's scipy.sparse.coo_matrix() (https://docs.scipy.org/doc/scipy/reference/generated/scipy.sparse.coo_matrix.html) function to construct this matrix.

The dimension of the matrix should be `n_airports` by `n_airports`. If an airport does not have any outgoing segments in the data, it should appear as a row of zeroes.

In [30]: Student's answer (Top)

```
P = sp.sparse.coo_matrix((segments['WEIGHT'],
                          (segments['ORIGIN_INDEX'], segments['DEST_INDEX'])),
                         shape=(n_airports, n_airports))

# Visually inspect your result:
spy(P)
```



In [31]: Grade cell: `P_test` Score: 1.0 / 1.0 (Top)

```
# Test cell: `P_test`

assert type(P) is sp.sparse.coo.coo_matrix, \
       "Matrix object has type {}, and is not a Numpy COO sparse matrix.".format(type(P
))
assert P.shape == (n_airports, n_airports), "Matrix has the wrong shape: it is {} x {}
instead of {} x {}".format(P.shape[0], P.shape[1], n_airports, n_airports)

# Check row sums, which must be either 0 or 1
n = P.shape[0]
u = np.ones(n)
row_sums = P.dot(u)
is_near_zero = np.isclose(row_sums, 0.0, atol=10*n*np.finfo(float).eps)
is_near_one = np.isclose(row_sums, 1.0, atol=10*n*np.finfo(float).eps)
assert (is_near_zero | is_near_one).all()
assert sum(is_near_one) == n_actual

print("\n(Passed!)")
```

```
(Passed!)
```

Note: Other formats. The preceding code asked you to use coordinate ("COO") format to store the matrix. However, you may sometimes need to convert or use other formats. For example, SciPy provides many general graph processing algorithms in its `csgraph` submodule (https://docs.scipy.org/doc/scipy/reference/sparse.csgraph.html). These routines expect the input graph as a sparse matrix, but one stored in compressed sparse row ("CSR") format rather than COO.

Part 2, analysis: Computing the steady-state distribution

Armed with the state-transition matrix P, you can now compute the steady-state distribution.

Exercise 7 (1 point). At time $t = 0$, suppose the random flyer is equally likely to be at any airport with an outbound segment, i.e., the flyer is at one of the "actual" origins. Create a NumPy vector `x0[:]` such that `x0[i]` equals this initial probability of being at airport `i`.

Note: If some airport i has *no* outbound flights, then be sure that $x_i(0) = 0$.

In [32]: Student's answer (Top)

```
# Your task: Create `x0` as directed above.

x0 = np.zeros(n_airports)
actual_airport_indices = segments['ORIGIN_INDEX'].unique()
x0[actual_airport_indices] = 1.0 / n_actual

# Visually inspect your result:
def display_vec_sparsely(x, name='x'):
    i_nz = np.argwhere(x).flatten()
    df_x_nz = pd.DataFrame({'i': i_nz, '{}[i] (non-zero only)'.format(name): x[i_nz]})
    display(df_x_nz.head())
    print("...")
    display(df_x_nz.tail())

display_vec_sparsely(x0, name='x0')
```

	i	x0[i] (non-zero only)
0	119	0.003333
1	124	0.003333
2	125	0.003333

3	130	0.003333
4	138	0.003333

...

	i	x0[i] (non-zero only)
295	5565	0.003333
296	5612	0.003333
297	5630	0.003333
298	5685	0.003333
299	5908	0.003333

In [33]: Grade cell: `x0_test` Score: 1.0 / 1.0 (Top)

```
# Test cell: `x0_test`

assert type(x0) is np.ndarray, "`x0` does not appear to be a Numpy array."
assert np.isclose(x0.sum(), 1.0, atol=10*n*np.finfo(float).eps), "`x0` does not sum to 1.0, but it should."
assert np.isclose(x0.max(), 1.0/n_actual, atol=10*n*np.finfo(float).eps), "`x0` values seem off..."

print("\n(Passed!)")
```

```
(Passed!)
```

Exercise 8 (2 points). Given the state-transition matrix `P`, an initial vector `x0`, and the number of time steps `t_max`, complete the function `eval_markov_chain(P, x0, t_max)` so that it computes and returns $x(t_{\max})$.

In [34]: Student's answer (Top)

```
def eval_markov_chain(P, x0, t_max):
    x = x0
    for t in range(t_max):
        x = P.T.dot(x)
    return x

T_MAX = 50
x = eval_markov_chain(P, x0, T_MAX)
display_vec_sparsely(x)

print("\n=== Top 10 airports ===\n")
ranks = np.argsort(-x)
top10 = pd.DataFrame({'Rank': np.arange(1, 11),
                      'Code': airport_codes.iloc[ranks[:10]]['Code'],
                      'Description': airport_codes.iloc[ranks[:10]]['Description'],
                      'x(t)': x[ranks[:10]]})
top10[['x(t)', 'Rank', 'Code', 'Description']]
```

	i	x[i] (non-zero only)
0	119	0.000721
1	124	0.005492
2	125	0.000237
3	130	0.000238
4	138	0.000715

...

	i	x[i] (non-zero only)
295	5565	0.000472

296	5612	0.000239
297	5630	0.001889
298	5685	0.000465
299	5908	0.000239

```
=== Top 10 airports ===
```

Out[34]:

	x(t)	Rank	Code	Description
373	0.037384	1	10397	Atlanta, GA: Hartsfield-Jackson Atlanta Intern...
3770	0.036042	2	13930	Chicago, IL: Chicago O'Hare International
1245	0.031214	3	11292	Denver, CO: Denver International
3347	0.026761	4	13487	Minneapolis, MN: Minneapolis-St Paul Internati...
2177	0.024809	5	12266	Houston, TX: George Bush Intercontinental/Houston
1250	0.024587	6	11298	Dallas/Fort Worth, TX: Dallas/Fort Worth Inter...
1375	0.024483	7	11433	Detroit, MI: Detroit Metro Wayne County
3941	0.021018	8	14107	Phoenix, AZ: Phoenix Sky Harbor International
4646	0.020037	9	14869	Salt Lake City, UT: Salt Lake City International
1552	0.019544	10	11618	Newark, NJ: Newark Liberty International

In [35]: Grade cell: `eval_markov_chain_test` Score: 2.0 / 2.0 (Top)

```
# Test cell: `eval_markov_chain_test`

print(x.sum())
assert np.isclose(x.sum(), 1.0, atol=T_MAX*n_actual*np.finfo(float).eps)

print("\nTop 10 airports by Markov chain analysis:\n", list(top10['Code']))
print("\nCompare that to the Top 10 by (historical) outbound segments:\n", list(origins
_top10['ID']))

A = set(top10['Code'])
B = set(origins_top10['ID'])
C = (A - B) | (B - A)
print("\nAirports that appear in one list but not the other:\n{}".format(C))
assert C == {11618, 11433, 12266, 14771, 14869, 12889, 14747, 12892}

print("\n(Passed!)")
```

```
1.0

Top 10 airports by Markov chain analysis:
 [10397, 13930, 11292, 13487, 12266, 11298, 11433, 14107, 14869, 11618]

Compare that to the Top 10 by (historical) outbound segments:
 [10397, 13930, 11292, 12892, 14771, 11298, 14747, 12889, 14107, 13487]

Airports that appear in one list but not the other:
{11618, 11433, 12266, 14771, 14869, 12889, 14747, 12892}

(Passed!)
```

Comparing the two rankings. Before ending this notebook, let's create a table that compares our two rankings, side-by-side, where the first ranking is the result of the Markov chain analysis and the second from a ranking based solely on number of segments.

In [36]:

```
top10_with_ranks = top10[['Code', 'Rank', 'Description']].copy()

origins_top10_with_ranks = origins_top10[['ID', 'Description']].copy()
origins_top10_with_ranks.rename(columns={'ID': 'Code'}, inplace=True)
origins_top10_with_ranks['Rank'] = origins_top10.index + 1
origins_top10_with_ranks = origins_top10_with_ranks[['Code', 'Rank', 'Description']]
```

```
origins_top10_with_ranks = origins_top10_with_ranks[['Code', 'Rank', 'Description']]

top10_compare = top10_with_ranks.merge(origins_top10_with_ranks, how='outer', on='Code',
                                       suffixes=['_MC', '_Seg'])
top10_compare
```

Out[36]:

	Code	Rank_MC	Description_MC	Rank_Seg	Description_Seg
0	10397	1	Atlanta, GA: Hartsfield-Jackson Atlanta Intern...	1	Atlanta, GA: Hartsfield-Jackson Atlanta Intern...
1	13930	2	Chicago, IL: Chicago O'Hare International	2	Chicago, IL: Chicago O'Hare International
2	11292	3	Denver, CO: Denver International	3	Denver, CO: Denver International
3	13487	4	Minneapolis, MN: Minneapolis-St Paul Internati...	10	Minneapolis, MN: Minneapolis-St Paul Internati...
4	12266	5	Houston, TX: George Bush Intercontinental/Houston	NaN	NaN
5	11298	6	Dallas/Fort Worth, TX: Dallas/Fort Worth Inter...	6	Dallas/Fort Worth, TX: Dallas/Fort Worth Inter...
6	11433	7	Detroit, MI: Detroit Metro Wayne County	NaN	NaN
7	14107	8	Phoenix, AZ: Phoenix Sky Harbor International	9	Phoenix, AZ: Phoenix Sky Harbor International
8	14869	9	Salt Lake City, UT: Salt Lake City International	NaN	NaN
9	11618	10	Newark, NJ: Newark Liberty International	NaN	NaN
10	12892	NaN	NaN	4	Los Angeles, CA: Los Angeles International
11	14771	NaN	NaN	5	San Francisco, CA: San Francisco International
12	14747	NaN	NaN	7	Seattle, WA: Seattle/Tacoma International
13	12889	NaN	NaN	8	Las Vegas, NV: McCarran International

Fin! That's it! You've determined the top 10 airports at which a random flyer ends up, assuming he or she randomly selects directly reachable destinations. How does it compare, qualitatively, to a ranking based instead on (historical) outbound segments? Which ranking is a better measure of importance to the overall airport network?

Be sure to submit this notebook to get credit for it.