Learning Objectives — Exam 01

The first exam will mostly be comprised of lab questions, primarily with only minor modifications or extensions. For your help preparing for the exam, here are the concrete learning objects for the first unit of the course:

LO 1. Define and understand the role of **training data** and **testing data** in statistical learning.

LO 2. Utilize common **loss functions**, including mean squared error, absolute error, and misclassification error. You should be able to evaluate these by hand on small datasets and in R code from larger ones.

LO 3. Apply the one-dimensional **best split** estimator by hand on small datasets.

LO 4. Understand the R code in the function `casl_utils_best_split`.

LO 5. Visualize simple linear regression from a scatter plot and understand the interpretation of the coefficients.

LO 6. Derive the simple linear regression ordinary least squares (OLS) coefficients using calculus.

LO 7. Apply the matrix format of the least squares estimator and understand the notation for y, X, and β.

LO 8. View a matrix as a linear transformation between $\mathbb{R}^n$ and $\mathbb{R}^m$ and matrix multiplication as function composition.

LO 9. Understand the matrix transpose and inverse, its notation, and rules for applying these to matrix equations.

LO 10. Apply the inner product and Euclidean-norm as matrix products between column and row vectors.

LO 11. Derive the equations for the gradient of an inner product:

$$\nabla_\beta \left(a^t \beta\right) = a$$

And the gradient of a quadratic form:

$$\nabla_\beta \left(\beta^t A \beta\right) = A^t \beta + A\beta.$$

LO 12. Understand the concept of an orthogonal matrix and how it represents rotations in n-dimensional space.

LO 13. Apply the rules for gradient functions of vector and matrix forms to derive the normal equations.

LO 14. Derive properties of the ordinary least squares estimator such as showing the residuals are perpendicular to the fitted values and that the "hat" matrix is a projection operator.

LO 15. Understand and apply the expectation and variance operators on vector equations.

LO 16. Derive formulae for the expected value and variance of β under the assumption that $y = Xb + \epsilon$ with $\mathbb{E}\epsilon = 0$ and $\mathbb{V}ar(\epsilon) = \sigma^2 I_n$.

LO 17. Construct model matrices in R from a data frame object. Fit and evaluate the ordinary least squares estimator using R's functions for matrix manipulation.