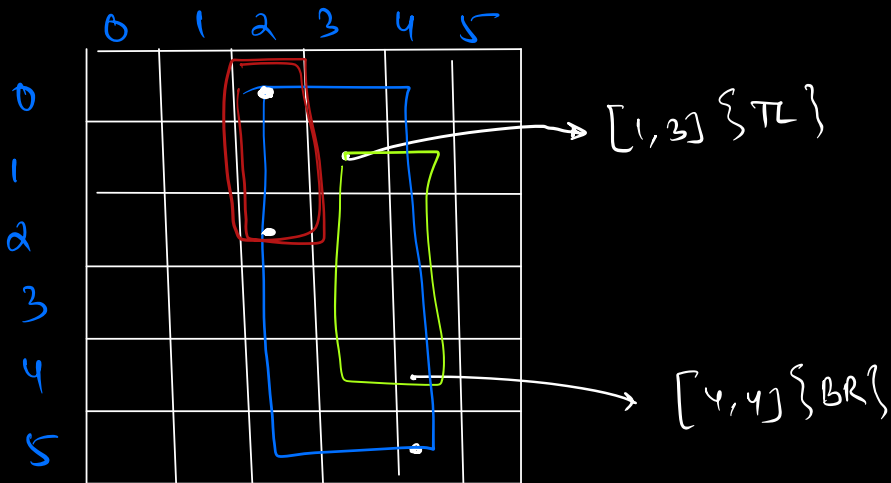


Q1. Given a matrix $N \times M$ and Q queries. Find submatrix sum for every query $[TL, BR]$



$[0, 2, 5, 4]$
 $\downarrow \quad \downarrow$
 $TL \quad BR$

* Brute force

* Iterate through each query
and find sum

$$\begin{aligned}
 TC &\approx O(Q \times N \times M) \\
 SC &\approx O(1)
 \end{aligned}$$

* optimized

pfsum of 1D array

1D array \rightarrow $pfsum[i] \Rightarrow$ sum of elements
[0 to i] ind

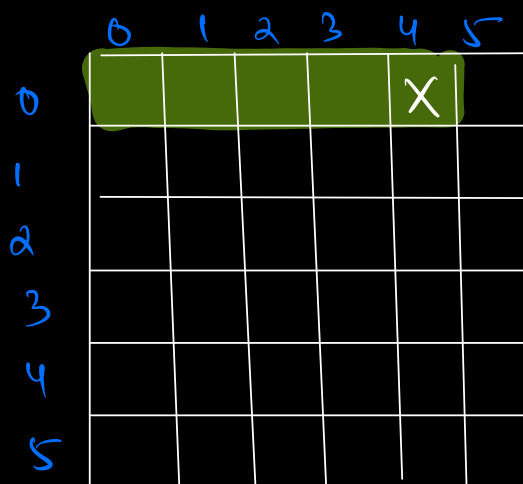
2D array \rightarrow $pfsum[i][j] \Rightarrow$ sum of all elements
from [0,0] to i^{th} row
& j^{th} column [i,j]

	0	1	2	3	4	5
0						
1						
2						
3						
4						X
5						

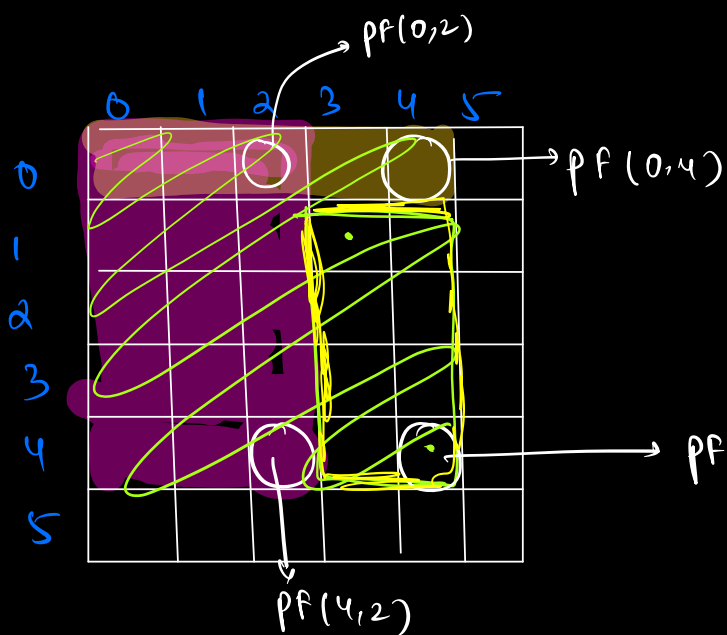
$pf[4,5]$

	0	1	2	3	4	5
0						
1						
2				X		
3						
4						
5						

$pfsum[2,3]$

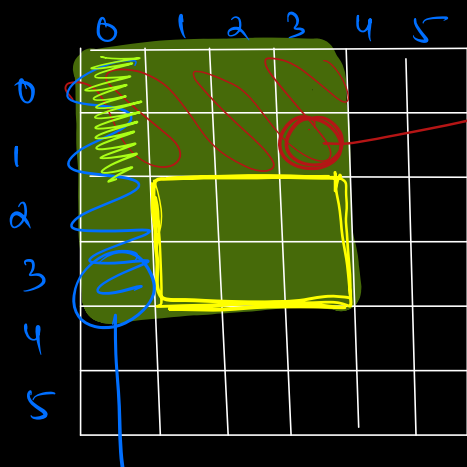


$pfsum[0,4]$



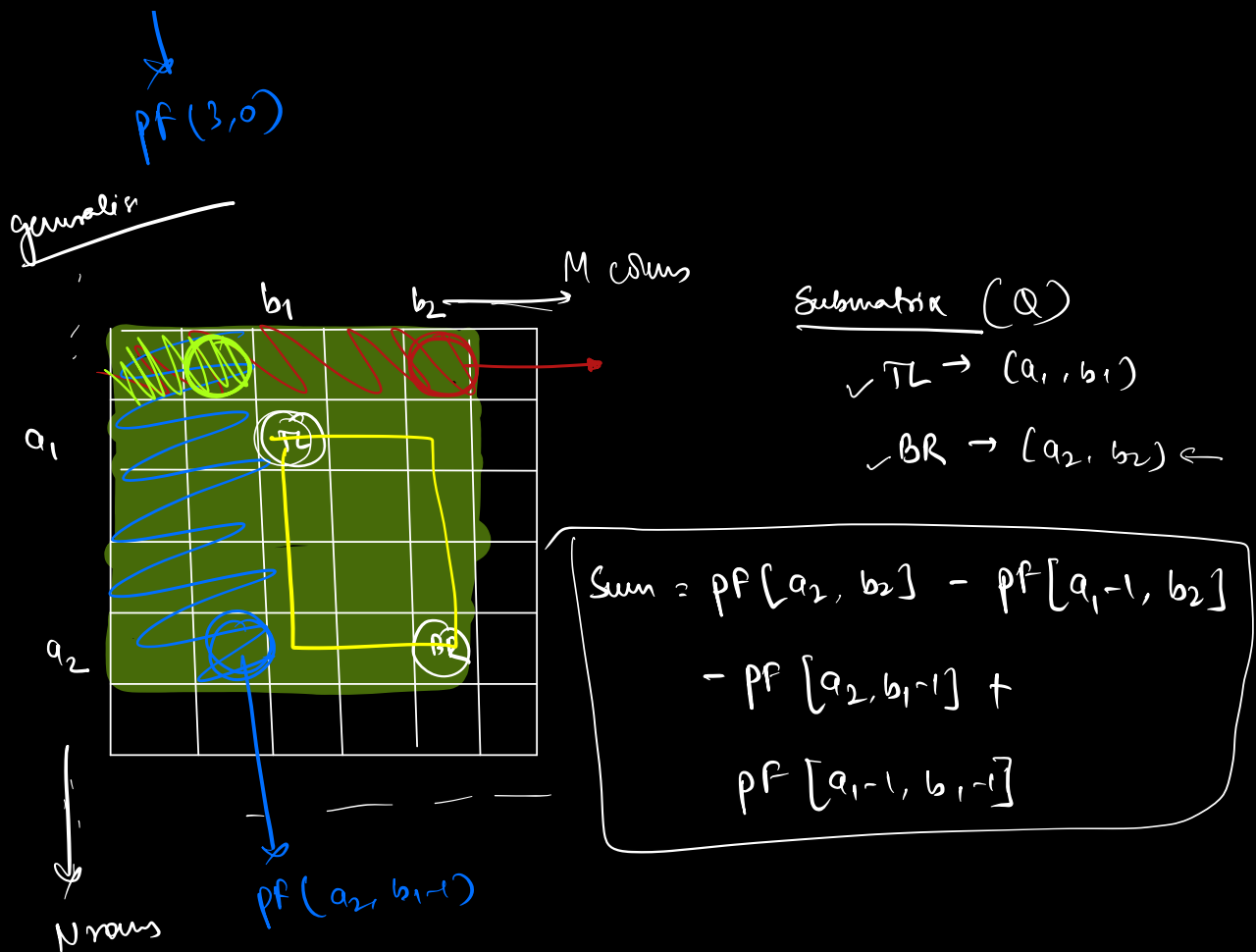
Submatrix
 $TL = (1,3)$
 $BR = (4,4)$

$$sum = pf(4,4) - pf(0,4) - pf(4,2) + pf(0,2)$$



$pf(1,3)$ $BR \rightarrow 3,3$

$$pf(3,3) - pf(1,3) - pf(3,0) + pf(1,0)$$



[sum of submatrix ([a₁, b₁], [a₂, b₂])]

⇒ sum = PF(a₂, b₂)

if (a₁ > 0)

sum = sum - PF(a₁-1, b₂)

if (b₁ > 0)

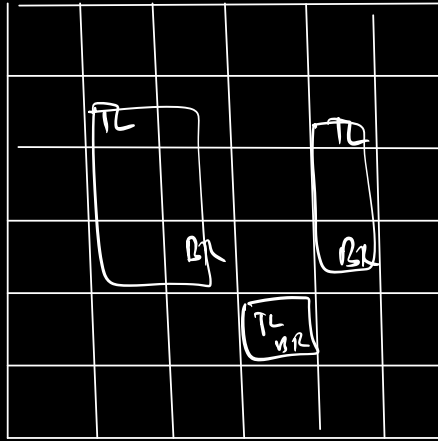
sum = sum - PF(a₂, b₁-1)

if (a₁ > 0 && b₁ > 0)

sum = sum + PF(a₁-1, b₁-1)

OX OCU

OCU



// construction of Pfaffian matrix

a_0	b_0	c_0
a_1	b_1	c_1
a_2	b_2	c_2

→
row wise
pfaffian

a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
a_1	$a_1 + b_1$	$a_1 + b_1 + c_1$
a_2	$a_2 + b_2$	$a_2 + b_2 + c_2$

↓
column wise pfaffian

a_0	$a_0 + b_0$	$a_0 + b_0 + c_0$
$a_0 + a_1$	$a_0 + b_0 + a_1 + b_1$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1$
$a_0 + a_1 + a_2$	$a_0 + b_0 + a_1 + b_1 + a_2 + b_2$	$a_0 + b_0 + c_0 + a_1 + b_1 + c_1 + a_2 + b_2 + c_2$

$$TC \Rightarrow O(d + NM)$$

$$SC \Rightarrow O(N \times M)$$

Q2. Given a matrix $N \times M$. Find sum of all submatrices sum

Bruteforce

Sum through all possible matrices

submatrix \Rightarrow TL, BR



all possible
combs

\Rightarrow no. of
submatrices

		TL			

TL \Rightarrow all the cells can
act as top left—

BR \geq TL

\hookrightarrow TL $\Rightarrow (a_1, b_1)$

BR $\Rightarrow (a_2, b_2) \Rightarrow a_2 \geq a_1 \quad b_2 \geq b_1$

$N \times M$

$$\left[\begin{array}{l} \text{for}(a_1 = 0; a_1 < N; a_1++) \{ \\ \quad \text{for}(b_1 = 0; b_1 < M; b_1++) \{ \\ \quad \quad \parallel [a_1, b_1] \rightarrow RL \end{array} \right.$$

$N \times M$

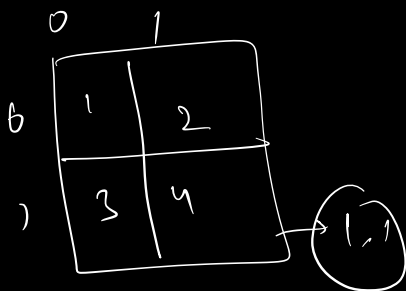
$$\left[\begin{array}{l} \text{for}(a_2 = a_1; a_2 < N; a_2++) \{ \\ \quad \text{for}(b_2 = b_1; b_2 < M; b_2++) \{ \\ \quad \quad \parallel [a_2, b_2] \rightarrow BR \end{array} \right.$$

$$\underline{\underline{\text{sum} = O(1)}}$$

$TC \approx O(N \times M \times N \times M)$

$$\underline{\underline{= O(N^2 M^2)}}$$

Contribution technique



$[3, 4]$
 \downarrow
 7

$[1 \quad 2 \quad 3 \quad 4]$

$[1] \quad [2] \quad [3] \quad [4] \rightarrow 10$

$[1 \ 2] \quad [1 \ 3] \quad [2 \ 4]$

\downarrow
3

\downarrow
4

\downarrow
6

$\rightarrow 10$

$\frac{40}{11}$

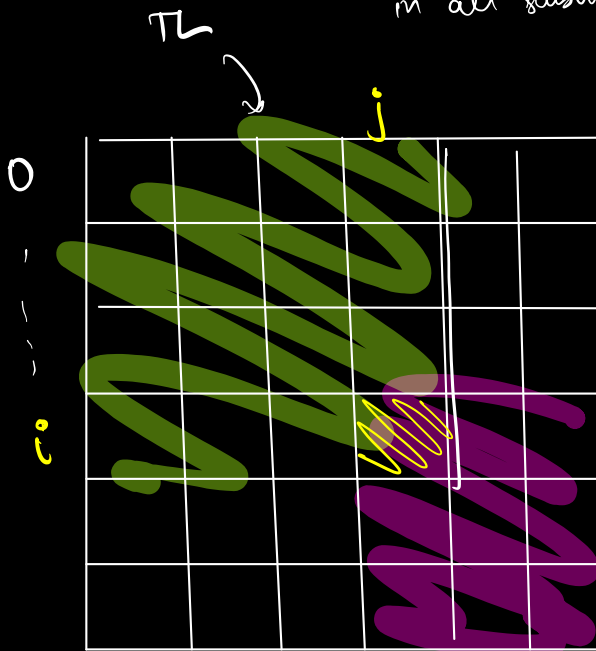
$$1 \times 4 + 2 \times 4 + 3 \times 4 + 4 \times 4$$

$$= 4 + 8 + 12 + 16 = \underline{\underline{40}}$$

Contribution
in total sum
by each cell

$$= \# \text{ count} \times \text{arr}[i][j]$$

\downarrow \downarrow
 no. of times value of
 a cell is present cell
 in all submatrices



TL BR

$$\Rightarrow \text{TL} \Rightarrow \text{rows } [i+1]$$

$$\text{column } [j+1]$$

$$\text{possible TL} \Rightarrow (i+1) \times (j+1)$$

$$\text{possible BR} \Rightarrow (N-i) \times (M-j)$$

all possible submatrices
which will include (i,j)

$$= [(i+1) \times (j+1)]$$

$$\times$$

$$[(N-i) \times (M-j)]$$

$$\text{BR} \Rightarrow \geq \text{TL}$$

\downarrow \downarrow
 rows $[N-i]$ rows $[i]$
 col $[M-j]$ column $[j]$


```

totalSum = 0
for (i = 0; i < N; i++) {
    for (j = 0; j < M; j++) {
        totalSum = totalSum + (arr[i][j] * ((i+1) * (j+1)) / ((N-i) * (M-j)))
    }
}

```

TC $\Rightarrow O(NM)$
 SC $\Rightarrow O(1)$

Q3. Given a row wise & column wise sorted matrix, Find max submatrix sum

	0	1	2	3
0	-20	-16	-4	8
1	-10	-8	12	14
2	-1	6	21	30
3	5	7	28	42

sorted \Rightarrow -6 -1 2 4 8 9 10 12
 1D array
 max element \downarrow

row wise & col wise sorted $\Rightarrow (N-1, M-1) \leftarrow$ has to be present

	0	1	2	3
0	-20	-16	-4	8
1	-10	-8	12	14
2	-1	6	21	30
3	5	7	28	42

TL BR

\rightarrow BR
 \downarrow

just need to find TL

try for all cells as TL

sum(TL, BR) \rightarrow

return max sum \Rightarrow o/p

TC $\Rightarrow O(N \times M)$
 SC $\Rightarrow O(N \times M)$

sum((i, j), (N-1, M-1))

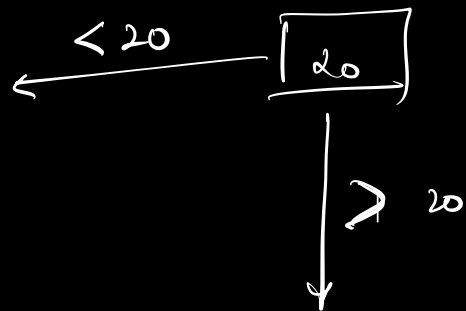
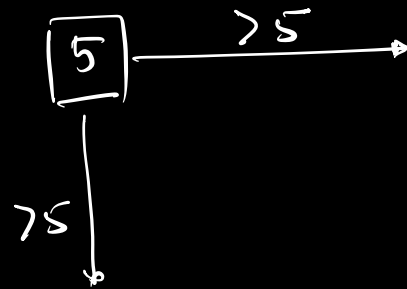
Q 4. Given a matrix sorted row wise & column wise.
Search if an element 'k' is present—

5	10	15	20
6	12	18	24
7	14	21	28
8	16	24	34

(14)

5	10	15	20
6	12	18	24
7	14	21	28
8	16	24	34

k = 14



return true

at each step, we are
rejecting a row or a
column

TC $\Rightarrow O(N+M)$

5	10	15	20
6	12	18	24
7	14	21	28
8	16	24	34

$$k = \underline{\underline{13}}$$

$$j = \underline{\underline{4}}$$

$i = \underline{\underline{3}}$
 $i = \underline{\underline{1}}$
 $\underline{\underline{3}}$
 $\underline{\underline{4}}$

	0	1	2	3	4
0	5	8	13	14	
1					
2					
3					

5	10	15	20
8	12	18	24
13	14	21	28
15	16	24	34

$$\underline{\underline{8}}$$