University of Illinois at Urbana-Champaign
Department of Mechanical and Industrial Engineering

IE411 Optimization of Large-Scale Linear Systems

Fall 2021

Coding project: A Revised Simplex Method

The input to your code is the following linear program:

$$\begin{aligned} \min \quad & cx \\ s.t. \quad & Ax = b \\ & x \geq 0, \end{aligned}$$

where A is a $m \times n$ matrix, b is a column vector in $\Re^m$, c is a row vector in $\Re^n$.

Step One

The first step in the project is to implement a function called simplex_step for executing a single step of the revised simplex method. We will keep track of the current basic feasible vector with three variables: iB, iN, and xB. The vector iB will hold the indices of the current set of basic variables, iN will hold the indices of the current set of nonbasic variables, and xB will hold the values of the basic variables.

The function simplex_step should be placed in a file simplex_step.m and it should have the calling sequence:

```
function [istatus,iB,iN,xB, Binv] = simplex_step(A,b,c,iB,iN,xB,Binv,irule)
%
% Take a single simplex method step for the linear program
%
%      min  cx
%      s.t.   Ax=b
%               x>=0,
%
% where A is an (m,n) matrix.
%
% That is, given a basic feasible vector described by the variables iB,iN,xB return the values
% of iB,iN, and xB corresponding to the adjacent basic feasible vector arrived at via a
% simplex method step.
%


%      Input Parameters:
```

```
%
% A - (m,n) constraint matrix

% b - (m,1) POSITIVE vector appearing in the constraint equation above

% c - (1,n) vector giving the coefficients of the objective function
%
% iB - (1,m) integer vector specifying the indices of the basic
%       variables at the beginning of the simplex step
% iN - (1,n-m) integer vector specifying the indices of the nonbasic
%        variables at the beginning of the simplex step
% xB - (m,1) vector specifying the values of the basic
%        variables at the beginning of the simplex step
% Binv – (m,m) inverse matrix of the basis B
%
% irule - integer parameter speciying which pivot rule to use:
%      irule = 0 indicates that the smallest coefficient rule should be
%              used
%      irule = 1 indicates that Bland’s rule should be used
%
%            Output Parameters:
%
% istatus - integer parameter reporting on the progress or lake thereof
%              made by this function
%        istatus = 0 indicates normal nondegenerate simplex method step
%              completed
%        istatus = 16 indicates the program is unbounded
%        istatus = -1 indicates an optimal feasible vector has been
%              found
%
% iB - integer vector specifying the m indices of the basic variables
%       after the simplex step
% iN - integer vector specifying the n-m indices of the nonbasic
%       variables after the simplex step
% xB - vector of length m specifying the values of the basic
%       variables after the simplex step
%
```

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Step Two

The second step in the project is to implement a function for performing initialization as described in the two-phase method. The function should be called simplex_init and it should be placed in the file simplex_init.m. The calling sequence for this function is:

```
function [istatus,iB,iN,xB] = simplex_init(A,b,c)
%
% Attempt to find a basic feasible vector for the linear program
%
%     min    cx
%     s.t.     Ax=b
%              x>=0,
%
% where A is a (m,n) matrix.
%
%         Input Parameters:
%
% A - (m,n) constraint matrix
% b - (m,1) vector appearing in the constraint equation above
% c - (1,n) vector giving the coefficients of the objective function
%
%        Output Parameters:
%
% istatus - integer parameter reporting the result of the initialization procedure
%       istatus = 0 indicates a basic feasible vector was found
%       istatus = 4 indicates that the initialization procedure failed
%       istatus = 16 indicates that the problem is infeasible
%
% iB - integer vector of length m specifying the indices of the basic variables
% iN - integer vector of length n-m specying the indices of the nonbasic variables

% xB - vector of length m specifying the values of the basic variables
%
```

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Step Three

The final step in the project is the implementation of a function simplex_method which used the preceding two functions in order to compute a solution to a linear program.

The calling sequence for the function simplex_method, which should reside in the file simplex_method.m, is as follows:

```
function [istatus,X , eta ,iB,iN,xB] = simplex_method(A,b,c,irule)
%
% Find a basic optimal solution for the linear program
%
%    min   cx
%    s.t.    Ax=b
%              x>=0,
%
% where A is an (m,n) matrix.
%
%        Input Parameters:
%
% A - (m,n) constraint matrix
% b - (m,1) a vector appearing in the constraint equation above
% c - (1,n) vector giving the coefficients of the objective function
%
% irule - integer parameter speciying which pivot rule to use:
%      irule = 0 indicates that the smallest coefficient rule should be used
%      irule = 1 indicates that Bland's rule should be used
%
%       Output Parameters:
%

% istatus - integer parameter reporting the results obtained by this function
%       istatus = 0 indicates normal completion (i.e., a solution has been found and reported)
%       istatus = 4 indicates the program is infeasible
%       istatus = 16 indicates the program is feasible but our initialization procedure has failed
%       istatus = 32 indicates that the program is unbounded
%
```

```
% X - vector of length n specifying the solution
% eta - the minimum value of the objective function
% iB - integer vector specifying the m indices of the basic variables after the simplex step
% iN - integer vector specifying the n-m indices of the nonbasic variables after the simplex step
% xB - vector of length m specifying the values of the basic variables after the simplex step
%
```