

Math 1110 Midterm Exam Solutions

Solutions

Problem 1: Find the indicated derivatives

1.a)

$$y'(x) = 5x^4 + 2.5$$

1.b)

$$\frac{d}{d\theta} \sin(\cos(\theta)) = -\cos(\cos(\theta)) \cdot \sin(\theta)$$

1.c) Given $f(x) = 2\sin(2x) + e^{-x} + x$, the derivatives are:

$$f^{(1)}(x) = 4\cos(2x) - e^{-x} + 1$$

$$f^{(2)}(x) = -8\sin(2x) + e^{-x}$$

$$f^{(3)}(x) = -16\cos(2x) - e^{-x}$$

$$f^{(4)}(x) = 32\sin(2x) + e^{-x}$$

$$f^{(5)}(x) = 64\cos(2x) - e^{-x}$$

Problem 2: Continuity and differentiability

2.a) For f to be continuous at every point, we require:

$$a = 2 \quad \text{and} \quad b = \frac{1}{2}$$

2.b) For f to be differentiable, the only value of x at which f may not be differentiable is $x = 1$. Upon verification, f is differentiable at $x = 1$ for $a = 2$ and $b = \frac{1}{2}$. Therefore, f is differentiable on $(-\infty, \infty)$.

2.c) For f differentiable on $(-\infty, \infty)$, we have:

$$f'(x) = \begin{cases} \cos(x) & \text{if } x < 0 \\ 2 & \text{if } 0 \leq x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$$