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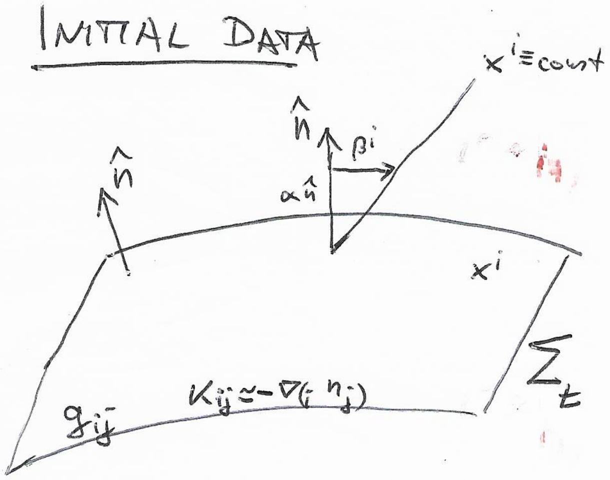
MPI for Grav. Physics

INITIAL DATA



gr-qc/~~0510016~~ 0412002

Greg Cook's thesis

Lichnerowicz (1944)

York, O'Murchadha (1970s)

Brandt + Brügmann (1999)

York, Cook, HP (2000's)

Lovelace, Varma, (~2010)

"T^{00}" many students 2010+

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

"00" $\Rightarrow$ $R[g_{ij}] + K^2 - K_{ij}K^{ij} = 16\pi\rho$ (H)

"0i" $\Rightarrow$ $\nabla_j (K^{ij} - g^{ij} K) = 8\pi j^i$ (M)

3D wrt. g_{ij} "T^{0j}"

Goal: find (g_{ij}, K^{ij})

1) Math: make H, M well-posed
2) Physics: find those solns that we want (BBH)
3) Numerics: actually solve $\rightarrow$ Nils

~~Step~~ Divide & Conquer

rewrite in other variables, s.t. some are determined, others "free"

$$g_{ij} = \psi^4 \tilde{g}_{ij} \qquad (1)$$

$$\Rightarrow \quad \underset{g_{ij}}{R} = \psi^{-4} \underset{\tilde{g}_{ij}}{\tilde{R}} - 8\psi^{-5} \underset{\tilde{g}_{ij}}{\tilde{\nabla}^2} \psi \qquad (2)$$

(2) in (H) $\Rightarrow$ <u>Elliptic eqn for ψ</u>: $\tilde{\nabla}^2 \psi = \dots$ ($\tilde{g}_{ij}$ free) (H')

E.g. vacuum, $K_{ij} \equiv 0$, $\tilde{g}_{ij} = \delta_{ij}$ = flat metric

(M) ✓

$$(H) \Rightarrow \tilde{\nabla}^2 \psi \doteq 0 \Rightarrow \psi = \frac{A}{r} + 1 \Rightarrow g_{ij} = \left(\frac{A}{r} + 1\right)^4 \delta_{ij}$$

Schwarzschild in isotropic coords

What about (M)?

$$(1) \Rightarrow \nabla_j \left(\psi^{-10} \tilde{S}^{ij}\right) = \psi^{-10} \tilde{\nabla}_j \tilde{S}^{ij} \qquad \text{~~for trace-free, symmetric $\tilde{S}^{ij}$~~} \qquad (3)$$

for $\tilde{S}^{ij}$ symmetric + trace-free

Prepare to use (3):

$$K^{ij} \equiv A^{ij} + \frac{1}{3} g^{ij} K$$

$$A^{ij} = \psi^{-10} \tilde{A}^{ij} \qquad \text{trace-free ex. curv.}$$

$$(M) \Rightarrow \underset{\text{conformal divergence}}{\tilde{\nabla}_j} \tilde{A}^{ij} - \frac{2}{3} \psi^6 \tilde{\nabla}^i K = 8\pi \tilde{j}^i$$

E.g. vacuum, $\tilde{g}_{ij} = $ flat, $K=0$

$$\not{\tilde{\nabla}}_j \tilde{A}^{ij} = 0$$

Bowen-York solution $W^i_{BY} = -\frac{1}{4r}\left(7P^i + \tilde{s}^i \tilde{s}_j P^j\right) + \frac{1}{r^2}\varepsilon^{ijk}\tilde{s}_j S_k$

$\tilde{s}^i = \frac{x^i}{r}$ radial unitvector

$$\tilde{A}^{ij}_{BY} = (\tilde{\mathbb{L}}W)^{ij}_{BY}$$

$$(\tilde{\mathbb{L}}V)^{ij} = \tilde{\nabla}^i V^j + \tilde{\nabla}^j V^i - \frac{2}{3}\tilde{g}^{ij}\tilde{\nabla}_k V^k$$

$$(H) \Rightarrow \not{\tilde{\nabla}}^2 \psi + \frac{1}{8}\psi^{-7}\tilde{A}^{ij}_{BY}\tilde{A}^{BY}_{ij} = 0$$

Enforce BH by singularity in ψ

$$\psi = \frac{2\, m_{bare}}{r} + 1 + u$$

$$\Rightarrow \not{\tilde{\nabla}}^2 u = \ldots, \quad x \in \mathbb{R}^3$$

BH with linear & angular momenta P^i, S^i

<u>puncture data</u>

Note: $\neq$ Kerr !

Junk radiation in puncture data limits $\chi \leq 0.93$.

Reaching all solutions

$$\tilde{A}^{ij} = \tilde{M}^{ij} + \frac{1}{\tilde{\sigma}}(\tilde{\mathbb{L}}V)^{ij} \rightleftharpoons A^{ij} = M^{ij} + \frac{1}{\sigma}(\mathbb{L}V)^{ij}$$

$A = \psi^{-10}\tilde{A}$
$M = \psi^{-10}\tilde{M}$
$\sigma = \psi^{6}\tilde{\sigma}$ ← ensures decomp. commutes with conf. rescaling

into (M) $\Rightarrow$ $\tilde{\nabla}_j \frac{1}{\tilde{\sigma}}(\tilde{\mathbb{L}}V)^{ij} = \dots$ Elliptic eqn for V^i

choose $\tilde{g}_{ij}$, $\tilde{M}^{ij}$, K, $\tilde{\sigma}$, BC $\xrightarrow{\text{solve}}$ valid ID set.

Q: How?

Conformal thin sandwich (CTS)

Ev. eq for g_{ij}

$$\Rightarrow A^{ij} = \frac{1}{2\alpha}\left((\mathbb{L}\beta)^{ij} - u^{ij}\right)$$

$u_{ij} = (\partial_t g_{ij})^{TF}$

$\alpha = \psi^6\tilde{\alpha}$, $u^{ij} = \psi^{-10}\tilde{u}^{ij}$

$$\overset{(M)}{\Rightarrow} \quad \tilde{\nabla}_j \frac{1}{2\tilde{\alpha}}(\tilde{\mathbb{L}}\beta)^{ij} = \dots \qquad (M')$$

XCTS

consider also $\partial_t K$ as given

Ev. Eq. $\Rightarrow$ $\tilde{\nabla}^2(\alpha\psi) = \dots \partial_t K \dots \qquad (*)$

Solving (H'), (M'), $(*)$ 5 coupled PDEs satisfies constraints

and yields $(\partial_t g_{ij})^{TF} = u_{ij}$, $\partial_t K$ if α, β^i are used in evolution

Physics

- 2 BH's on controlled orbit near in equilibrium

$$\tilde{g}_{ij} \simeq g_{ij}^{KS,A} + g_{ij}^{KS,B} \qquad \text{"superposed Kerr-Schild"}$$

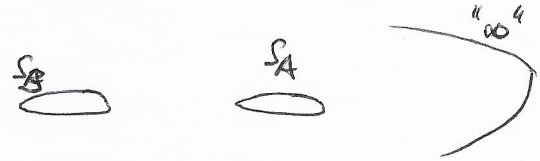
$$K \simeq K_{KS,A} + K_{KS,B}$$

- Comoving coords

$$\partial_t K = 0$$

$$\tilde{u}_{ij} = 0$$

Boundary conditions



"old"

$S_{A,B} \equiv AH \Rightarrow \partial_r \psi \hat{=} \ldots$

shear $\hat{=} 0 \Rightarrow \beta_{\parallel}^i \hat{=} \vec{\Omega}_H \times (\vec{r} - \vec{c}_A)$

AH's don't move $\Rightarrow \beta_{\perp}^i \hat{=} \ldots$

$\partial_r (\alpha\psi) = 0$

"new" [excision bdry inside AH]

$\Theta \hat{=} -\varepsilon \Rightarrow \partial_r \psi \hat{=} \ldots$

$\beta^i |_{S_A} = \beta^i_{KS,A}$

$\alpha = \alpha^i_{KS,A}$

@ infty

$\psi \to 1$, $\alpha \to 1$ asympt. flatness

$$\beta^i \to \underbrace{\vec{\Omega}_0 \times \vec{r}}_{\text{rotation}} + \underbrace{\dot{a}_0 \vec{r}}_{\text{contraction}} + \underbrace{\vec{V}_\infty}_{\text{boost}}$$

BBH - ID rootfinding

- choose Ω_0, $\dot{a}_0$, D_0 — 3 DoF
- choose $M^{A,B}_{bg}$, $\vec{\chi}^{A,B}_{bg}$ — 8

  $\vec{c}_A$, $\vec{c}_B = c_A - \vec{D}\hat{x}$ — 3

  $\vec{\Omega}_0 = \Omega_0 \hat{z}$, $\dot{a}_0$, $\vec{v}_\infty$ — 3

adjust, repeat

- Solve ID, extract

  $M^{A,B}$, $\vec{\chi}^{A,B}$, $\vec{COM}$, $\vec{P}_{ADM}$

  8 — 3 — 3

  $\stackrel{?}{=}$ target

no

yes

adjust, repeat.

- Evolve ~3 orbits, extract

  e, ℓ, evtl. T_{merger}

  $\stackrel{?}{=}$ target

no

yes

Evolve, CCE, publish