ISLR Chapter3

Linear Regression: Labs

ACO

2025-02-13

Contents

Linear Regression

3.6.1 Libraries

```
library(MASS)
library(ISLR2)
```

3.6.2 Simple Linear Regression

Predict `medv` using 12 predictors such as `rm` (average number of rooms per house), `age` (proportion of owner-occupied units built prior to 1940) and `lstat` (percent of households with low socioeconomic status).

```
str(Boston, 2)
```

```
## 'data.frame':    506 obs. of  13 variables:
##  $ crim   : num  0.00632 0.02731 0.02729 0.03237 0.06905 ...
##  $ zn     : num  18 0 0 0 0 0 12.5 12.5 12.5 12.5 ...
##  $ indus  : num  2.31 7.07 7.07 2.18 2.18 2.18 7.87 7.87 7.87 7.87 ...
##  $ chas   : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ nox    : num  0.538 0.469 0.469 0.458 0.458 0.458 0.524 0.524 0.524 0.524 ...
##  $ rm     : num  6.58 6.42 7.18 7 7.15 ...
##  $ age    : num  65.2 78.9 61.1 45.8 54.2 58.7 66.6 96.1 100 85.9 ...
##  $ dis    : num  4.09 4.97 4.97 6.06 6.06 ...
##  $ rad    : int  1 2 2 3 3 3 5 5 5 5 ...
##  $ tax    : num  296 242 242 222 222 222 311 311 311 311 ...
##  $ ptratio: num  15.3 17.8 17.8 18.7 18.7 18.7 15.2 15.2 15.2 15.2 ...
##  $ lstat  : num  4.98 9.14 4.03 2.94 5.33 ...
##  $ medv   : num  24 21.6 34.7 33.4 36.2 28.7 22.9 27.1 16.5 18.9 ...
```

```
lm_fit <- lm(medv ~ lstat , data = Boston)
summary(lm_fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.168  -3.990  -1.318   2.034  24.500
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34.55384    0.56263   61.41   <2e-16 ***
## lstat       -0.95005    0.03873  -24.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.216 on 504 degrees of freedom
## Multiple R-squared:  0.5441, Adjusted R-squared:  0.5432
## F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

confidence interval for coeficient estimates

```
confint(lm_fit)
```

```
##                 2.5 %     97.5 %
## (Intercept) 33.448457 35.6592247
## lstat       -1.026148 -0.8739505
```

confidence interval for predictions

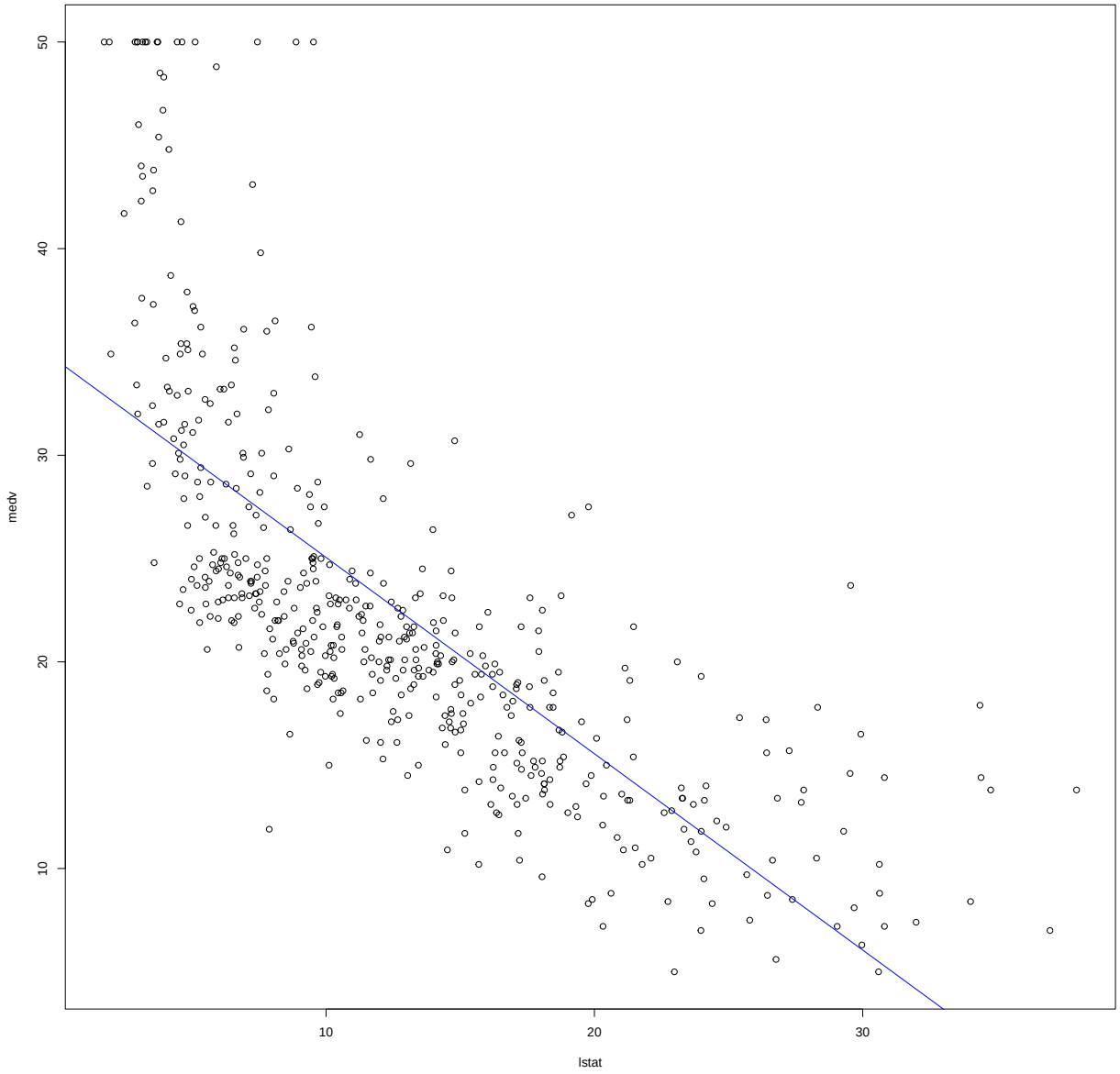
```
predict(
  lm_fit , data.frame(lstat = (c(5, 10, 15))), interval = "confidence")
```

```
##        fit      lwr      upr
## 1 29.80359 29.00741 30.59978
## 2 25.05335 24.47413 25.63256
## 3 20.30310 19.73159 20.87461
```

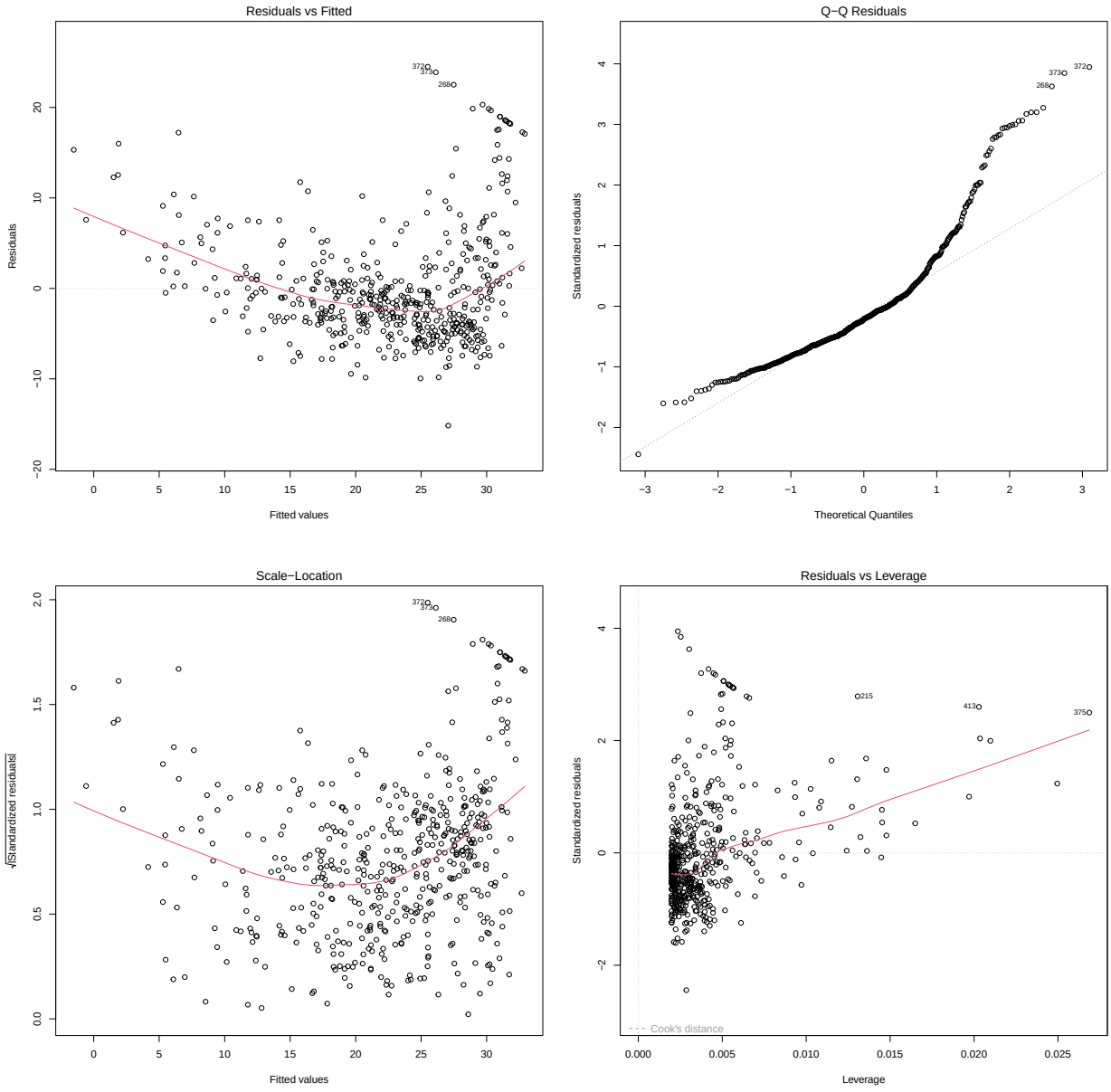
```
predict(
  lm_fit , data.frame(lstat = (c(5, 10, 15))), interval = "prediction")
```

```
##        fit       lwr      upr
## 1 29.80359 17.565675 42.04151
## 2 25.05335 12.827626 37.27907
## 3 20.30310  8.077742 32.52846
```

```
plot(medv ~ lstat , data = Boston)
abline(lm(medv ~ lstat , data = Boston), col = 'blue')
```

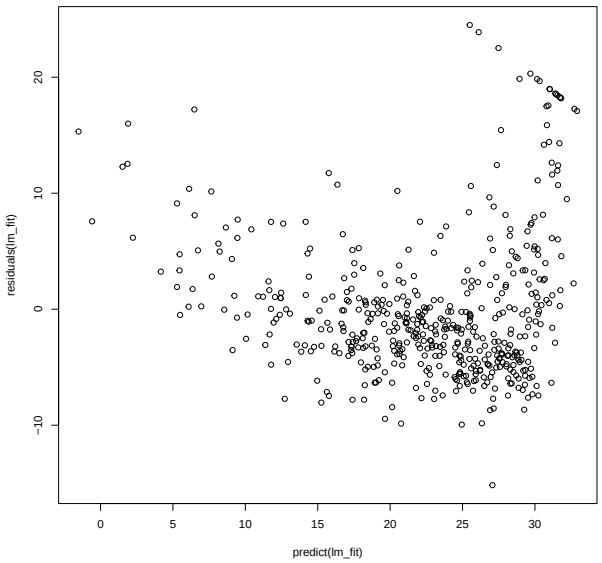


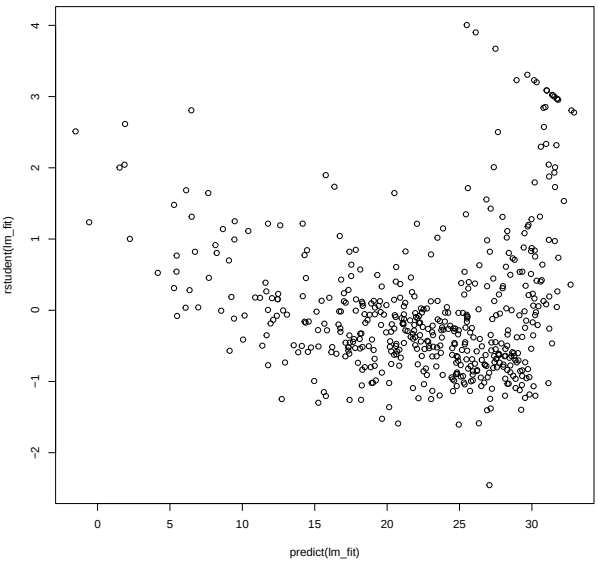

```
par(mfrow = c(2,2))
plot(lm(medv ~ lstat , data = Boston))
```

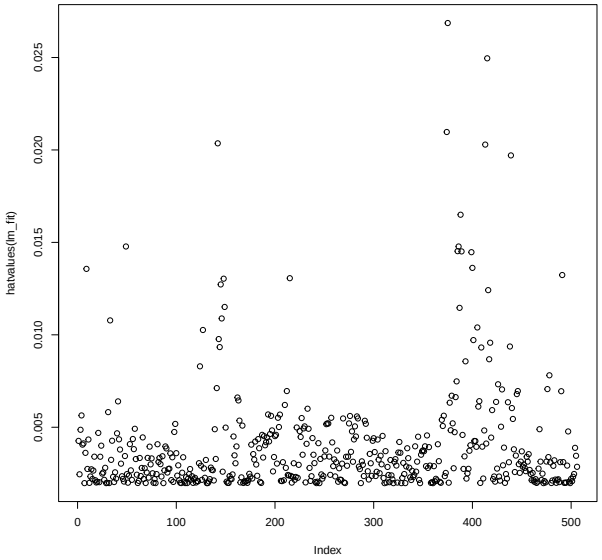



```r
par(mfrow = c(1,1))
```

```r
par(mfrow = c(2,2))
plot(predict(lm_fit), residuals(lm_fit))
plot(predict(lm_fit), rstudent(lm_fit))
plot(hatvalues(lm_fit))
par(mfrow = c(1,1))
```








```
which.max(hatvalues(lm_fit))
```

```
## 375
## 375
```

3.6.3 Multiple Linear Regression

```
lm_fit <- lm(medv ~ lstat + age , data = Boston)
summary(lm_fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat + age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.981  -3.978  -1.283   1.968  23.158
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.22276    0.73085  45.458  < 2e-16 ***
## lstat       -1.03207    0.04819 -21.416  < 2e-16 ***
## age          0.03454    0.01223   2.826  0.00491 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.173 on 503 degrees of freedom
## Multiple R-squared:  0.5513, Adjusted R-squared:  0.5495
## F-statistic:   309 on 2 and 503 DF,  p-value: < 2.2e-16
```

```
lm_fit <- lm(medv ~ ., data = Boston)
summary(lm_fit)
```

```
##
## Call:
## lm(formula = medv ~ ., data = Boston)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -15.1304  -2.7673  -0.5814   1.9414  26.2526
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  41.617270   4.936039   8.431 3.79e-16 ***
## crim         -0.121389   0.033000  -3.678 0.000261 ***
## zn            0.046963   0.013879   3.384 0.000772 ***
## indus         0.013468   0.062145   0.217 0.828520
## chas          2.839993   0.870007   3.264 0.001173 **
## nox         -18.758022   3.851355  -4.870 1.50e-06 ***
## rm            3.658119   0.420246   8.705  < 2e-16 ***
## age           0.003611   0.013329   0.271 0.786595
## dis          -1.490754   0.201623  -7.394 6.17e-13 ***
## rad           0.289405   0.066908   4.325 1.84e-05 ***
## tax          -0.012682   0.003801  -3.337 0.000912 ***
## ptratio      -0.937533   0.132206  -7.091 4.63e-12 ***
## lstat        -0.552019   0.050659 -10.897  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.798 on 493 degrees of freedom
## Multiple R-squared:  0.7343, Adjusted R-squared:  0.7278
## F-statistic: 113.5 on 12 and 493 DF,  p-value: < 2.2e-16
```

Hence R-Squared = rsummary(lm.fit)$r.sq$`and` RSE = `$summary(lm.fit)$sigma`

```
library(car)
```

```
vif(lm_fit)
```

```
##     crim       zn    indus     chas      nox       rm      age      dis
## 1.767486 2.298459 3.987181 1.071168 4.369093 1.912532 3.088232 3.954037
##      rad      tax  ptratio    lstat
## 7.445301 9.002158 1.797060 2.870777
```

3.6.4 Interaction Terms

```
# interaction, use : or *
lm_fit <- lm(medv ~ lstat:age , data = Boston)
summary(lm_fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat:age, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -13.347  -4.372  -1.534   1.914  27.193
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 30.1588631  0.4828240   62.46   <2e-16 ***
## lstat:age   -0.0077146  0.0003799  -20.31   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.827 on 504 degrees of freedom
## Multiple R-squared:  0.4501, Adjusted R-squared:  0.449
## F-statistic: 412.4 on 1 and 504 DF,  p-value: < 2.2e-16
```

3.6.5 Non-linear Transformations of the Predictors

```
lm_fit <- lm(medv ~ lstat , data = Boston)
summary(lm_fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.168  -3.990  -1.318   2.034  24.500
```

```
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34.55384    0.56263   61.41   <2e-16 ***
## lstat       -0.95005    0.03873  -24.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.216 on 504 degrees of freedom
## Multiple R-squared:  0.5441, Adjusted R-squared:  0.5432
## F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
lm_fit2 <- lm(medv ~ lstat + I(lstat^2) , data = Boston)
summary(lm_fit)
```

```
##
## Call:
## lm(formula = medv ~ lstat, data = Boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -15.168  -3.990  -1.318   2.034  24.500
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 34.55384    0.56263   61.41   <2e-16 ***
## lstat       -0.95005    0.03873  -24.53   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.216 on 504 degrees of freedom
## Multiple R-squared:  0.5441, Adjusted R-squared:  0.5432
## F-statistic: 601.6 on 1 and 504 DF,  p-value: < 2.2e-16
```

```
anova(lm_fit, lm_fit2)
```

```
## Analysis of Variance Table
##
## Model 1: medv ~ lstat
## Model 2: medv ~ lstat + I(lstat^2)
##   Res.Df   RSS Df Sum of Sq     F    Pr(>F)
## 1    504 19472
## 2    503 15347  1    4125.1 135.2 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
lm_fit <- lm(medv ~ lstat + poly(lstat, 2, raw = TRUE) , data = Boston)
summary(lm_fit)
```

3.6.6 Qualitative Predictors

```
str(Carseats)
```

```
## 'data.frame':    400 obs. of  11 variables:
##  $ Sales      : num  9.5 11.22 10.06 7.4 4.15 ...
##  $ CompPrice  : num  138 111 113 117 141 124 115 136 132 132 ...
##  $ Income     : num  73 48 35 100 64 113 105 81 110 113 ...
##  $ Advertising: num  11 16 10 4 3 13 0 15 0 0 ...
##  $ Population : num  276 260 269 466 340 501 45 425 108 131 ...
##  $ Price      : num  120 83 80 97 128 72 108 120 124 124 ...
##  $ ShelveLoc  : Factor w/ 3 levels "Bad","Good","Medium": 1 2 3 3 1 1 3 2 3 3 ...
##  $ Age        : num  42 65 59 55 38 78 71 67 76 76 ...
##  $ Education  : num  17 10 12 14 13 16 15 10 10 17 ...
##  $ Urban      : Factor w/ 2 levels "No","Yes": 2 2 2 2 2 1 2 2 1 1 ...
##  $ US         : Factor w/ 2 levels "No","Yes": 2 2 2 2 1 2 1 2 1 2 ...
```

```
lm_fit <- lm(
  Sales ~ . + Income:Advertising + Price:Age, data = Carseats)
summary(lm_fit)
```

```
##
## Call:
## lm(formula = Sales ~ . + Income:Advertising + Price:Age, data = Carseats)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.9208 -0.7503  0.0177  0.6754  3.3413
##
## Coefficients:
##                       Estimate Std. Error t value Pr(>|t|)
## (Intercept)         6.5755654  1.0087470   6.519 2.22e-10 ***
## CompPrice           0.0929371  0.0041183  22.567  < 2e-16 ***
## Income              0.0108940  0.0026044   4.183 3.57e-05 ***
## Advertising         0.0702462  0.0226091   3.107 0.002030 **
## Population          0.0001592  0.0003679   0.433 0.665330
## Price              -0.1008064  0.0074399 -13.549  < 2e-16 ***
## ShelveLocGood       4.8486762  0.1528378  31.724  < 2e-16 ***
## ShelveLocMedium     1.9532620  0.1257682  15.531  < 2e-16 ***
## Age                -0.0579466  0.0159506  -3.633 0.000318 ***
## Education          -0.0208525  0.0196131  -1.063 0.288361
## UrbanYes            0.1401597  0.1124019   1.247 0.213171
## USYes              -0.1575571  0.1489234  -1.058 0.290729
## Income:Advertising  0.0007510  0.0002784   2.698 0.007290 **
## Price:Age           0.0001068  0.0001333   0.801 0.423812
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.011 on 386 degrees of freedom
## Multiple R-squared:  0.8761, Adjusted R-squared:  0.8719
## F-statistic:   210 on 13 and 386 DF,  p-value: < 2.2e-16
```

```r
contrasts(Carseats$ShelveLoc)
```

```
##        Good Medium
## Bad       0      0
## Good      1      0
## Medium    0      1
```

Applied

Q8

```r
q8_lm = lm(mpg~horsepower, data = Auto)
summary(q8_lm)
```

```
##
## Call:
## lm(formula = mpg ~ horsepower, data = Auto)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -13.5710  -3.2592  -0.3435   2.7630  16.9240
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 39.935861   0.717499   55.66   <2e-16 ***
## horsepower  -0.157845   0.006446  -24.49   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.906 on 390 degrees of freedom
## Multiple R-squared:  0.6059, Adjusted R-squared:  0.6049
## F-statistic: 599.7 on 1 and 390 DF,  p-value: < 2.2e-16
```

i. Yes
ii. -0.7784268
iii. R squared 60% iv

```r
predict(q8_lm, newdata = data.frame(horsepower = 98))
```

```
##        1
## 24.46708
```

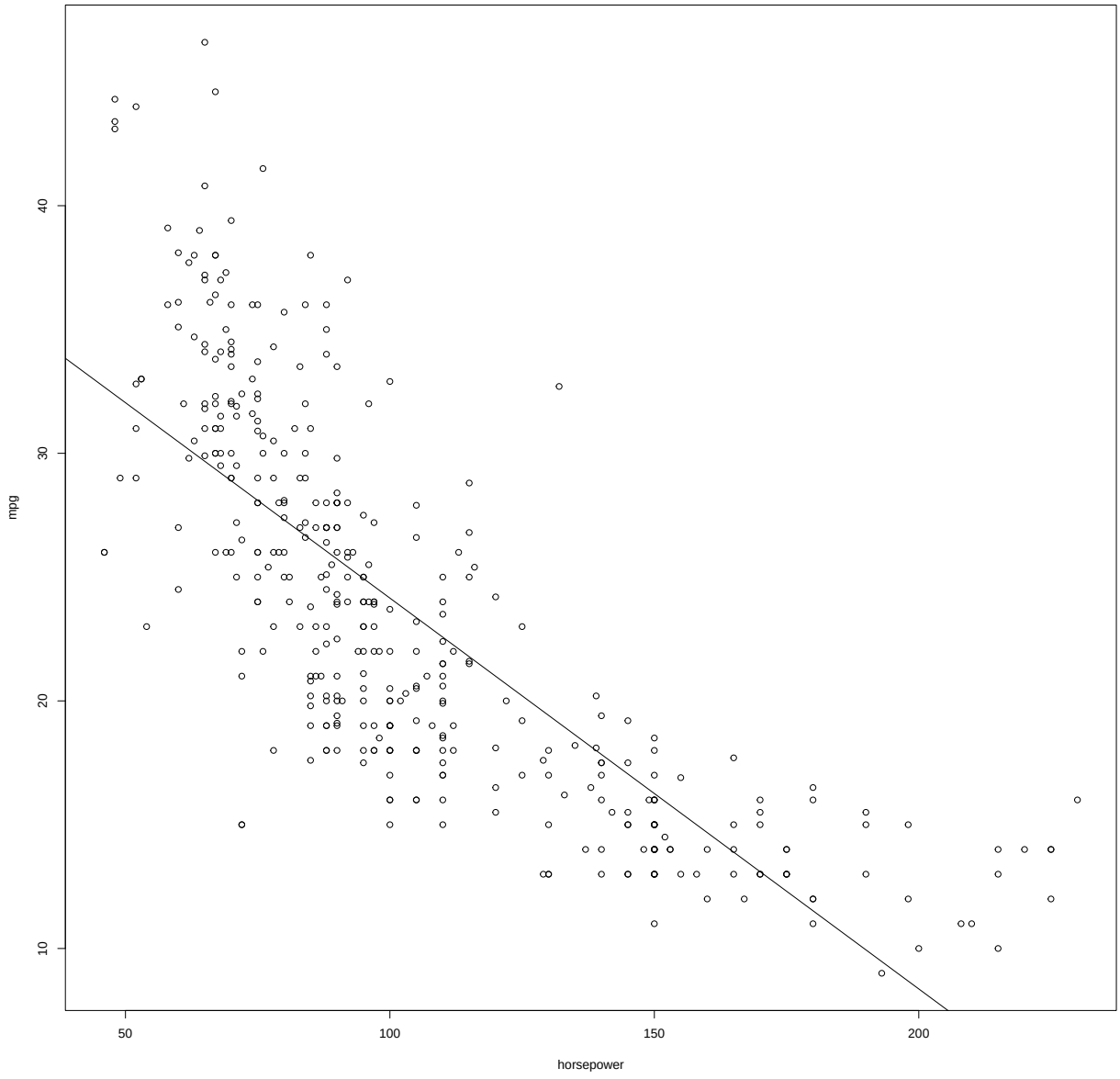
```r
predict(q8_lm, newdata = data.frame(horsepower = 98), interval = "confidence")
```

```
##        fit      lwr      upr
## 1 24.46708 23.97308 24.96108
```

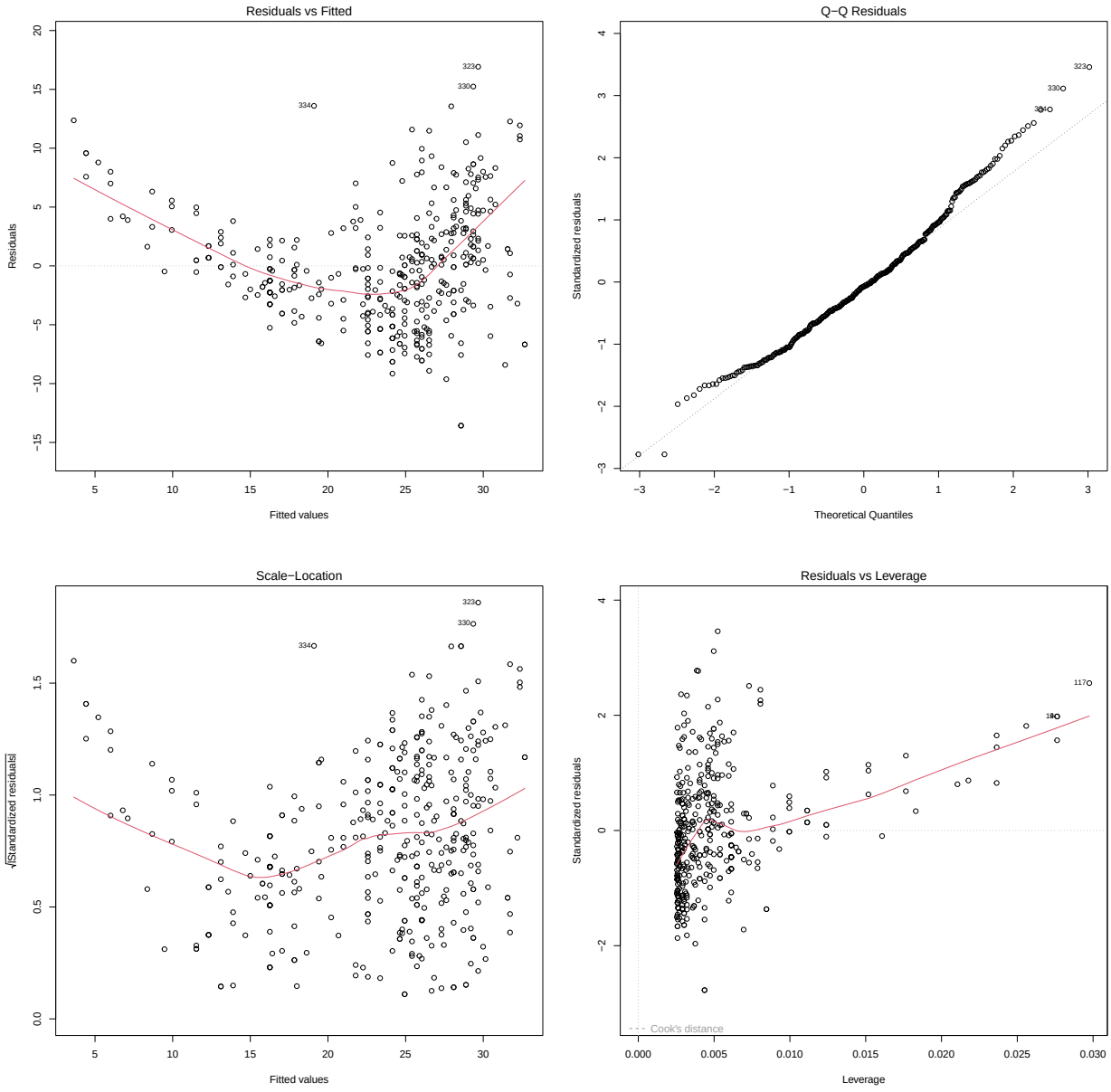
```r
predict(q8_lm, newdata = data.frame(horsepower = 98), interval = "prediction")
```

```
##        fit     lwr      upr
## 1 24.46708 14.8094 34.12476
```

```r
q8_lm = lm(mpg~horsepower, data = Auto)
plot(mpg~horsepower, data = Auto)
abline(q8_lm)
```




```r
par(mfrow = c(2,2))
plot(q8_lm)
```

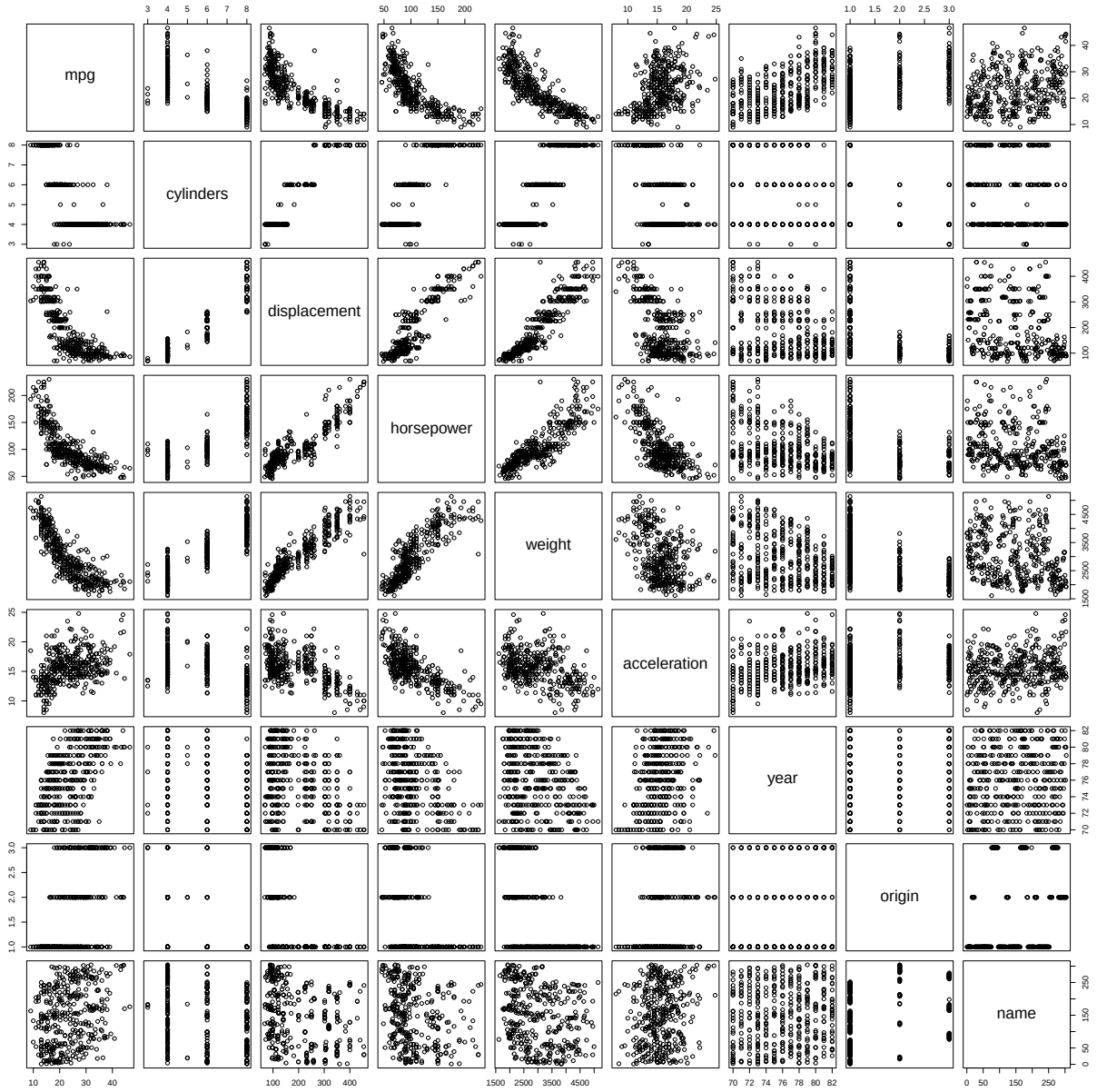



```
par(mfrow = c(1,1))
```

Q9

a

```
pairs(Auto)
```



b

```
cor(Auto[, sapply(Auto, FUN = (\(x) is.numeric(x)))])
```

```
##                     mpg  cylinders displacement horsepower     weight
## mpg           1.0000000 -0.7776175   -0.8051269 -0.7784268 -0.8322442
## cylinders    -0.7776175  1.0000000    0.9508233  0.8429834  0.8975273
## displacement -0.8051269  0.9508233    1.0000000  0.8972570  0.9329944
## horsepower   -0.7784268  0.8429834    0.8972570  1.0000000  0.8645377
## weight       -0.8322442  0.8975273    0.9329944  0.8645377  1.0000000
## acceleration  0.4233285 -0.5046834   -0.5438005 -0.6891955 -0.4168392
## year          0.5805410 -0.3456474   -0.3698552 -0.4163615 -0.3091199
```

```
## origin        0.5652088 -0.5689316   -0.6145351 -0.4551715 -0.5850054
##              acceleration       year     origin
## mpg             0.4233285  0.5805410  0.5652088
## cylinders      -0.5046834 -0.3456474 -0.5689316
## displacement   -0.5438005 -0.3698552 -0.6145351
## horsepower     -0.6891955 -0.4163615 -0.4551715
## weight         -0.4168392 -0.3091199 -0.5850054
## acceleration    1.0000000  0.2903161  0.2127458
## year            0.2903161  1.0000000  0.1815277
## origin          0.2127458  0.1815277  1.0000000
```

c

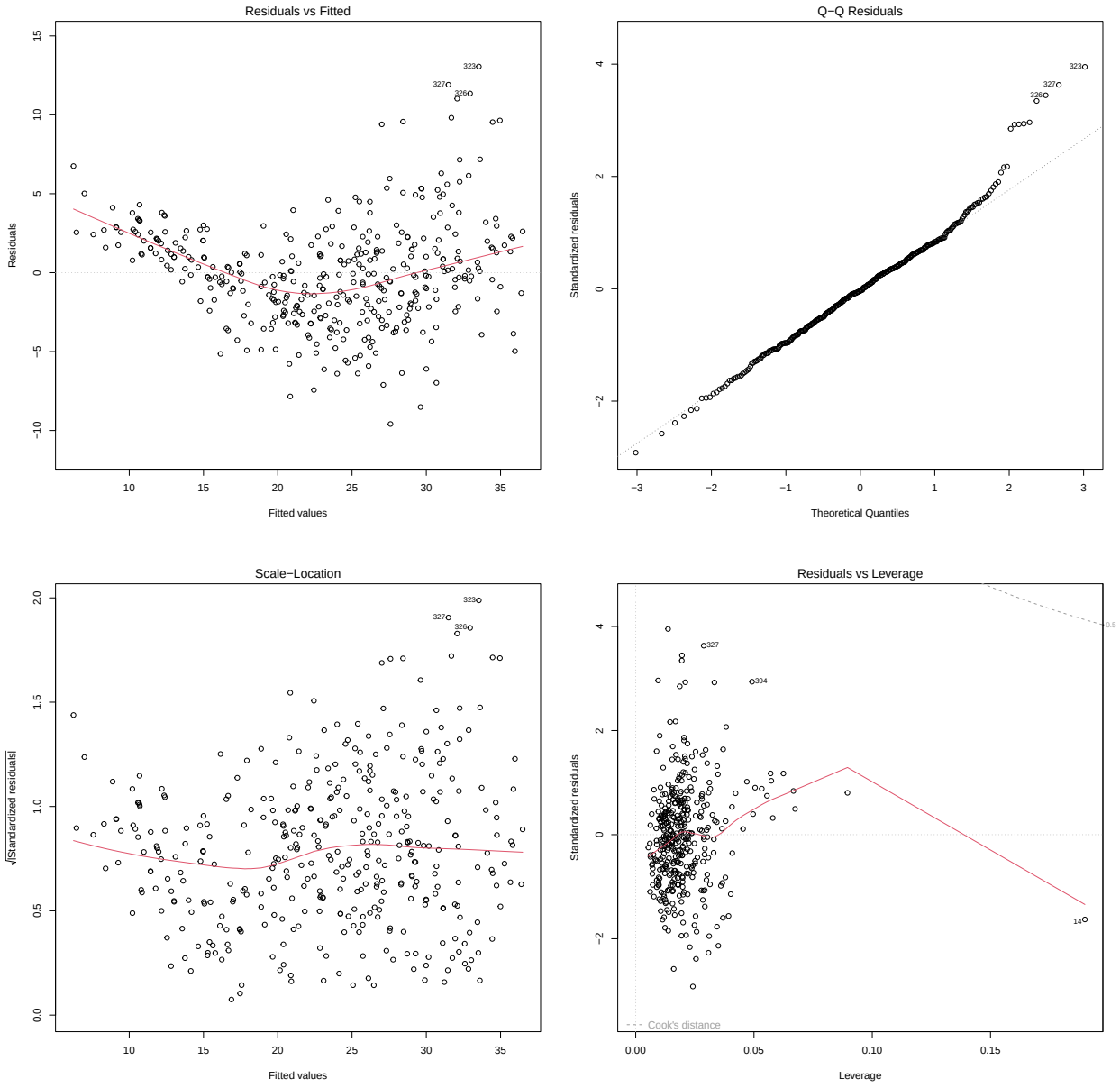
```
q9_model = lm(mpg ~. -name, data = Auto)
summary(q9_model)
```

```
##
## Call:
## lm(formula = mpg ~ . - name, data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -9.5903 -2.1565 -0.1169  1.8690 13.0604
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -17.218435   4.644294  -3.707  0.00024 ***
## cylinders     -0.493376   0.323282  -1.526  0.12780
## displacement   0.019896   0.007515   2.647  0.00844 **
## horsepower    -0.016951   0.013787  -1.230  0.21963
## weight        -0.006474   0.000652  -9.929  < 2e-16 ***
## acceleration   0.080576   0.098845   0.815  0.41548
## year           0.750773   0.050973  14.729  < 2e-16 ***
## origin         1.426141   0.278136   5.127 4.67e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.328 on 384 degrees of freedom
## Multiple R-squared:  0.8215, Adjusted R-squared:  0.8182
## F-statistic: 252.4 on 7 and 384 DF,  p-value: < 2.2e-16
```

i. Yes
ii. displacement, weight, year, origin
iii. Most recent cars have higher mpg

d

```
par(mfrow = c(2,2))
plot(q9_model)
```




```
par(mfrow = c(1,1))
```

e

```
summary(lm(mpg ~ cylinders*displacement, data = Auto))
```

```
##
## Call:
## lm(formula = mpg ~ cylinders * displacement, data = Auto)
##
## Residuals:
```

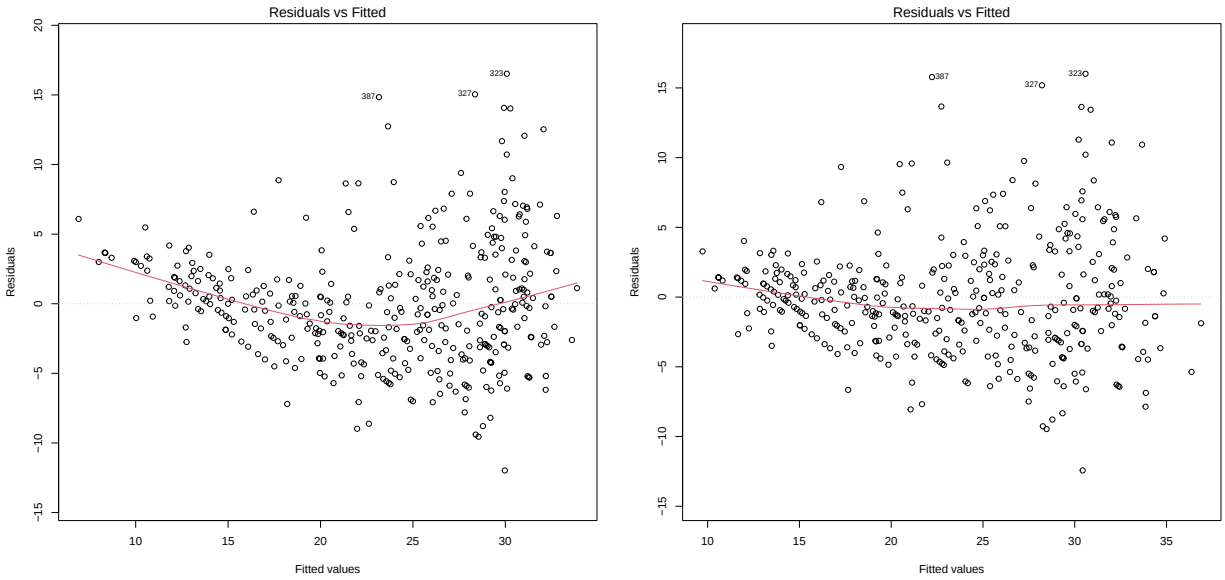
```
##      Min       1Q   Median       3Q      Max
## -16.0432  -2.4308  -0.2263   2.2048  20.9051
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)            48.22040    2.34712  20.545  < 2e-16 ***
## cylinders              -2.41838    0.53456  -4.524 8.08e-06 ***
## displacement           -0.13436    0.01615  -8.321 1.50e-15 ***
## cylinders:displacement  0.01182    0.00207   5.711 2.24e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.454 on 388 degrees of freedom
## Multiple R-squared:  0.6769, Adjusted R-squared:  0.6744
## F-statistic:   271 on 3 and 388 DF,  p-value: < 2.2e-16
```

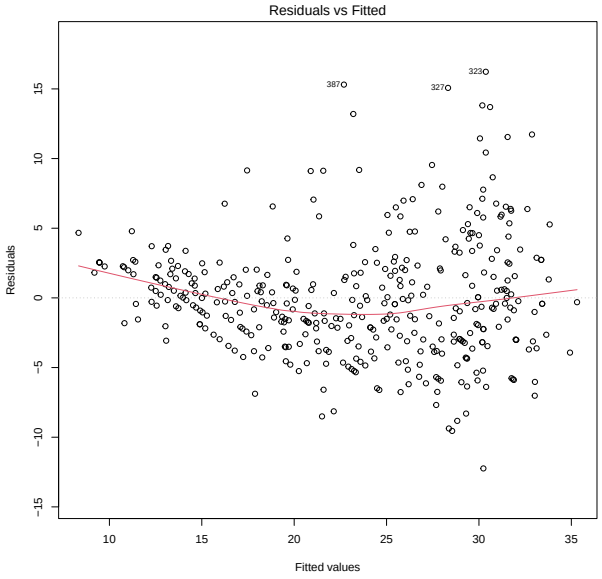
```
summary(lm(mpg ~ cylinders:displacement, data = Auto))
```

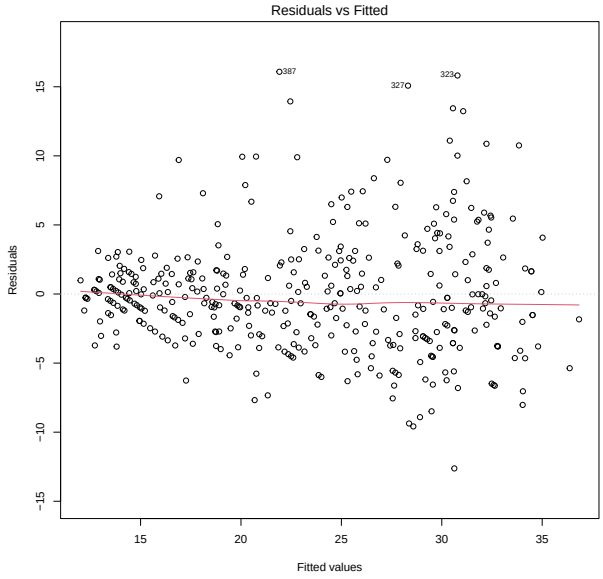
```
##
## Call:
## lm(formula = mpg ~ cylinders:displacement, data = Auto)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -11.705  -3.426  -0.450   2.704  17.715
##
## Coefficients:
##                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)            30.9896203  0.3905111   79.36   <2e-16 ***
## cylinders:displacement -0.0061177  0.0002462  -24.85   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.863 on 390 degrees of freedom
## Multiple R-squared:  0.6128, Adjusted R-squared:  0.6119
## F-statistic: 617.4 on 1 and 390 DF,  p-value: < 2.2e-16
```

f

```
par(mfrow=c(2,2))
q9_lm_f = lm(mpg~weight, data= Auto)
plot(q9_lm_f, 1)

q9_lm_f = lm(mpg~log(weight), data= Auto)
plot(q9_lm_f, 1)

q9_lm_f = lm(mpg~sqrt(weight), data= Auto)
plot(q9_lm_f, 1)

q9_lm_f = lm(mpg~poly(weight, 2, raw = T), data= Auto)
plot(q9_lm_f, 1)
```








```r
par(mfrow=c(1, 1))
```