Supplementary Material

RTI beyond relighting: Image analysis for Oseberg textiles

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1 Statistical Analysis of RTI data

1.1 Mean Map

The mean function calculates the average luminance of each point on a surface across all the images taken from different illumination directions. For a stack of RTI images represented as a 3D array $I_{m,n,k}$ where (m, n) represents spatial coordinates of pixels $k \in 1, ..., K$ indicates the image index in the stack. The mean image $\mu_{m,n}$ is calculated as [1]:

$$\mu_{m,n} = \frac{1}{K} \sum \left(I^1_{(m,n)}, I^2_{(m,n)}, \dots, I^K_{(m,n)} \right) \tag{1}$$

This operation provides a baseline representation of the surface's overall reflectance properties, effectively averaging out the variations caused by different lighting directions. The resulting mean map reveals the general albedo distribution across the surface, helping to identify regions of consistently high or low reflectance regardless of illumination conditions.

1.2 Median Map

The median map calculates the central luminance value of each point on a surface across all the images taken under different illumination directions. The median image $M_{m,n}$ is calculated as:

$$M_{m,n} = \text{median}\left(I^1_{(m,n)}, I^2_{(m,n)}, \dots, I^K_{(m,n)} \right) \tag{2}$$

We call the median image a median map. It is more robust to outliers compared to the mean map, providing a representation that is less sensitive to extreme specular highlights or dark grazing angles. It gives the middle value of the luminance distribution at each pixel location, effectively capturing the typical appearance of the surface while minimizing the impact of outlier illumination directions.

1.3 Standard Deviation Map

The standard deviation map measures the variation in luminance of each point on a surface across all the images taken under different illumination directions. The standard deviation image $\sigma_{m,n}$ is calculated as [1]:

$$\sigma_{m,n} = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \left(I^k_{(m,n)} - \mu_{m,n} \right)^2} \tag{3}$$

where $\mu_{m,n}$ is the mean value at position (m, n). We call the standard deviation image a standard deviation map. It quantifies how much the luminance values at each pixel vary from their mean across different illumination conditions. High values in the standard deviation map indicate areas where the surface appearance changes significantly with different lighting directions, which often corresponds to regions with strong geometric features or varying material properties.

1.4 Max Map

The maximum map captures the highest luminance value of each point on a surface across all the images taken under different illumination directions. The maximum image $Max_{m,n}$ is calculated as:

$$Max_{m,n} = \max \left(I^1_{(m,n)}, I^2_{(m,n)}, \dots, I^K_{(m,n)} \right) \tag{4}$$

We call the maximum image a maximum map. It represents the brightest appearance of each surface point in any of the illumination directions. High values in the maximum map often correspond to regions with specular reflections or areas that are highly exposed to certain lighting directions, providing insights into the surface's reflective properties and geometric features.

1.5 Min Map

The minimum map captures the lowest luminance value of each point on a surface across all the images taken under different illumination directions. The minimum image $Min_{m,n}$ is calculated as:

$$Min_{m,n} = \min\left(I^1_{(m,n)}, I^2_{(m,n)}, \ldots, I^K_{(m,n)}\right) \tag{5}$$

We call the minimum image a minimum map. It represents the darkest appearance of each surface point under any of the illumination directions. Low values in the minimum map often correspond to regions that remain in shadow under certain lighting conditions or areas with deep surface features, providing information about the surface's geometric characteristics and self-shadowing behavior.

1.6 Skewness Map

The skewness map measures the asymmetry of the luminance distribution of each point on a surface across all images taken under different illumination directions. The skewness image $S_{m,n}$ is calculated as [1]:

$$S_{m,n} = \frac{\frac{1}{K}\sum_{k=1}^{K}\left(I^k_{(m,n)} - \mu_{m,n}\right)^3}{\sigma^3_{m,n}} \tag{6}$$

where $\mu_{m,n}$ is the mean value and $\sigma_{m,n}$ is the standard deviation at position (m, n). We call the skewness image a skewness map. It quantifies the degree of asymmetry in the luminance distribution at each pixel. A positive skewness indicates a distribution with a longer tail towards higher values (suggesting specular highlights or higher reflectance response), while a negative skewness indicates a longer tail towards lower values (suggesting predominant shadows or towards matt surface). A skewness of zero indicates a symmetric distribution around the mean.

1.7 Kurtosis Map

The kurtosis map measures the "tailedness" of the luminance distribution of each point on a surface across all the images taken under different illumination directions. The kurtosis image (excess kurtosis)$\kappa_{m,n}$ is calculated as:

$$\kappa_{m,n} = \frac{\frac{1}{K}\sum_{k=1}^{K}\left(I^k_{(m,n)} - \mu_{m,n}\right)^4}{\sigma^4_{m,n}} - 3 \tag{7}$$

where $\mu_{m,n}$ is the mean value and $\sigma_{m,n}$ is the standard deviation at position (m, n). We call the kurtosis image a kurtosis map. It describes the shape of the luminance distribution at each pixel. A high kurtosis value indicates that the luminance distribution has heavier tails (more outliers) compared to a normal distribution, which often corresponds to regions with specular reflections or deep shadows. The subtraction of 3 makes this the excess kurtosis, normalizing it so that a normal distribution has a kurtosis of 0.

2 Geometric Analysis of RTI Data

2.1 Normal Map

The normal map calculates the orientation of the surface at each point by estimating the surface normals from the observed reflectance variations under different illumination directions. The figure 1 demonstrates the normal for the Lambertian reflectance model.

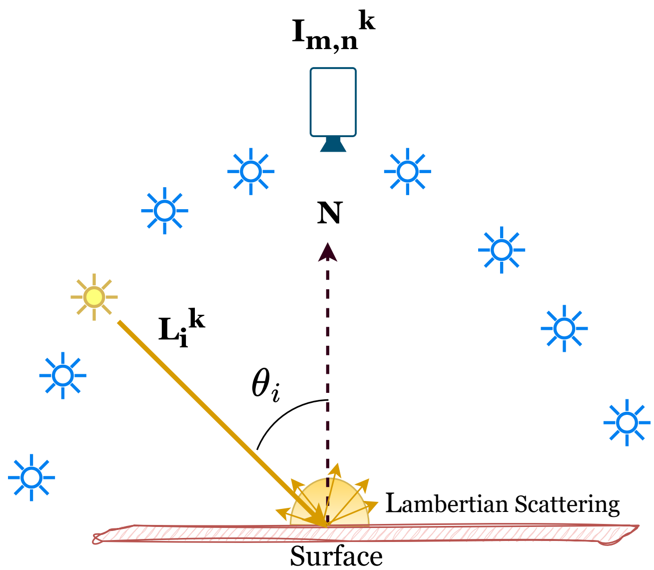

Figure 1: RTI acquisition setup and Lambertian scattering model. The camera captures intensity $I^k_{m,n}$ at pixel position (m,n) under k^{th} light direction. The incident angle θ_i is angle between incident light source L^k_i and normal N to the surface. The yellow scattering arrows illustrate the diffuse reflection characteristic of a lambertian surface where incident light hits the surface and scatters in all directions uniformly.

Following the Lambertian reflectance model, the surface normal $N_{m,n}$ at each pixel can be calculated as derived from [2]:

$$I^k_{m,n} = \rho_{m,n} N_{m,n} \cdot L^k_i \tag{8}$$

where $\rho_{m,n} = 1$ for lambertian surface, $I^k_{m,n}$ is a vector with camera obersvations of pixel (m, n) for the k^{th} light direction, and L^k_i represents the light direction for the k^{th} light direction. For multiple number of light directions L^k_i is a matrix where each row represents a light direction vector.

$$L = \begin{bmatrix} L^1_x & L^1_y & L^1_z \\ L^2_x & L^2_y & L^2_z \\ \vdots & \vdots & \vdots \\ L^K_x & L^K_y & L^K_z \end{bmatrix}, \qquad I^k_{m,n} = \begin{bmatrix} I^1_{m,n} \\ I^2_{m,n} \\ \vdots \\ I^K_{m,n} \end{bmatrix} \tag{9}$$

Since we typically have more equations than unknowns ($k > 3$), this is an overdetermined system. The least squares solution minimizes the sum of squared errors. The normal vectors are estimated by solving using a least squares solution.

$$N_{m,n} = (L_i^{k^T} L_i^k)^{-1} L_i^{k^T} I_{m,n}^k \tag{10}$$

The final normal vectors are normalized to unit length:

$$N_{m,n} = \frac{N_{m,n}}{||N_{m,n}||} \tag{11}$$

We call this three-channel image a normal map. It encodes the surface orientation at each point, where the RGB channels correspond to the X, Y, and Z components of the normal vector. This representation provides detailed information about the local surface geometry and is fundamental for surface characterization and visualization.

2.2 Directional Derivatives

Normal maps can be used to understand surface geometry. One of the way to exploit this information is directional derivatives. Directional derivatives are the first derivative of change of normals N along some axis like x, y. D_x represents slopes estimated along the $x - axis$ and D_y represents slopes estimated along the $y - axis$ as shown in equation 12 and 13 respectively, where P_{size}^x and P_{size}^y represent the pixel size in millimeters along x and y axes respectively. The directional derivatives are calculated from the normal field as derived from [1]:

$$D_x^{m,n} = P_{size}^x \frac{N_x^{m,n}}{N_z^{m,n}} \tag{12}$$

$$D_y^{m,n} = P_{size}^y \frac{N_y^{m,n}}{N_z^{m,n}} \tag{13}$$

The total slope magnitude D is calculated as:

$$D^{m,n} = 100 \frac{\sqrt{(N_x^{m,n})^2 + (N_y^{m,n})^2}}{N_z^{m,n}} \tag{14}$$

For anisotropic surfaces, D_x and D_y maps can isolate different surface features based on their orientation Some features may appear in one directional slope feature map (e.g. D_y map) but not in other (e.g. D_x map), showing directional sensitivity.

Directional double derivatives or second order derivatives can be used to understand surface curvatures and thier changes. The second order derivative also known as curvature maps can be calculated as:

$$D_{xx}^{m,n} = \frac{\partial}{\partial x}(D_x^{m,n}) = \frac{\partial}{\partial x}\left(P_{size}^x \frac{N_x^{m,n}}{N_z^{m,n}}\right) \tag{15}$$

$$D_{yy}^{m,n} = \frac{\partial}{\partial y}(D_y^{m,n}) = \frac{\partial}{\partial y}\left(P_{size}^y \frac{N_y^{m,n}}{N_z^{m,n}}\right) \tag{16}$$

2.3 Principle Curvatures

The eigenvalues of the Hessian matrix H (also called the shape operator or second fundamental form) that describe the principal directions of curvature at each point on the surface [3, 1]. The Hessian matrix is defined in equation 17.

$$H^{m,n} = \begin{bmatrix} D_{xx}^{m,n} & D_{xy}^{m,n} \\ D_{xy}^{m,n} & D_{yy}^{m,n}] \end{bmatrix} \tag{17}$$

The principal curvatures κ_{min} and κ_{max} can be found as:

$$\begin{aligned} \kappa_{max}^{m,n} &= \max(\lambda_1, \lambda_2) \\ \kappa_{min}^{m,n} &= \min(\lambda_1, \lambda_2) \end{aligned} \tag{18}$$

The interpretation of $\kappa_{max}^{m,n}$ and $\kappa_{min}^{m,n}$ can be very significant in understanding the surface. If both principle curvatures are positive, it means surface has convex curvature and if both are negative, the surface has concave curvature. If they are zero, the surface is flat. If both principal curvatures have opposite sign, the surface has saddle point [1].

These principle curvatures can be used to construct other curvatures which can further tell us about surface. The mean curvature tells us about average bending in the surface [1]. It is defined in equation 19:

$$\kappa_{\mu}^{m,n} = \frac{\kappa_{min}^{m,n} + \kappa_{max}^{m,n}}{2} \tag{19}$$

The gaussian curvature is the product of principal curvatures [1]. It is defined in equation 20:

$$\kappa_{G}^{m,n} = \kappa_{min}^{m,n} \times \kappa_{max}^{m,n} \tag{20}$$

This curvature tells us intrinsic property about the shape of the surface as follows:

- $\kappa_G > 0$: Elliptic (dome/bowl)
- $\kappa_G = 0$: Parabolic (cylinder/plane)
- $\kappa_G < 0$: Hyperbolic (saddle)

Melhum curvature combines mean and gaussian curvatures. It is useful for detecting surface features which are independent of orientation [1]. Melhum curvature can defined in equation 21

$$\kappa_{M}^{m,n} = \sqrt{3\frac{(\kappa_{\mu}^{m,n})^2}{2} - \kappa_{G}^{m,n}} \tag{21}$$

References

[1] Nurit M 2022 *Numérisation et caractérisation de l'apparence des surfaces manufacturées pour l'inspection visuelle* Ph.D. thesis Université de Bourgogne Dijon, France

[2] Wu Y 2003 *Northwestern University*

[3] Woodham R J 1994 *JOSA A* **11** 3050–3068