Static Trade Models*

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1 The Armington Model

1.1 Introduction

One of the most robust results in economics is the so-called gravity equation of international trade, which states that the bilateral trade of two countries is explained by their size and distance – the economic analog of Newton's law of gravitation. Precisely, the statistical model that illustrates this relation is:

$$X_{ij} = \alpha \frac{Y_i Y_j}{D_{ij}}$$

where X_{ij} are exports from source country i to destination country j, α is a constant, Y_i and Y_j are source and destination GDP's, and D_{ij} is a measure of bilateral trade barriers, including geography, culture, language, policy related trade costs such as tariffs, and non-policy related trade costs such as shipping costs. The generalized expression of the gravity equation is:

$$X_{ij} = \kappa_{ij} \gamma_i \delta_j \tag{1}$$

where κ_{ij} are all bilateral trade barriers, and γ_i and δ_j are source- and destination-country fixed effects. While this result had for a long time been shown empirically using statistical models, it lacked a theoretical foundation. The reason was that classic theories of trade, namely Ricardian trade and the Heckscher-Ohlin model, while useful to understand patterns of specialization and comparative advantage, were difficult to generalize to settings with more than two countries and trade costs. The breakthrough occurred thanks to the insights of Armington (1969) and Anderson (1979). The key insight of the former was that each country produces one good or variety, which is consumed by all other countries in the World; the key contribution of the latter was to incorporate a CES demand function to this setting.

1.2 Setup supply

In the Armington World there is a continuum of countries and each country $i \in N$ produces a variety $\omega \in \Omega$, which you can interpret as an aggregate good. Exports from i to j are denoted by X_{ij}. Production is linear in labor and productivity is denoted by A_i. Workers supply labor, L_i, inelastically and are compensated with the wage, W_i, determined in equilibrium. To export their good to destination j, export country i faces an iceberg cost, denoted by κ_{ij}. The iceberg cost is so-called because it is resource that melts away in the export from i to j, as exporting 1 unit requires producing $\kappa \geq 1$ units. It is generally assumed that $\kappa_{ii} = 1$. Note κ is commonly viewed as a capturing all sorts of bilateral trade barriers and is sometimes estimated as the residual from regressing trade flows on observables (Anderson and van Wincoop, 2003).

*These notes are mostly based on the online notes by Allen and Arkolakis and Chris Edmond.

The Armington model assumes perfect competition and firms take prices as given. Given the linear production function $Q_i(\omega) = \sum_{j\in N} q_{ij}(\omega) = A_i L_i$, we can focus on a representative firm (see proof for similar argument in the case of the representative consumer). The profit maximization problem is as follows:

$$\max_{\{q_{ij}(\omega)\}} = \sum_{N} p_{ij} q_{ij}(\omega) - \kappa_{ij} W_i L_i \text{ subject to } \sum_{j\in N} q_{ij}(\omega)/A_i \leq L_i \tag{2}$$

After setting up the Lagrangian, we can simply substitute the constraint into the maximization problem and solve for the first order condition to obtain an expression for the price:

$$\frac{\partial \mathcal{L}}{\partial q_{ij}(\omega)} = 0 \Longleftrightarrow p_{ij}(\omega) = \frac{\kappa_{ij} W_i}{A_i} \tag{3}$$

Note the differences of prices of ω across destinations only varies with the iceberg cost, since the marginal production cost times is the same across destinations (W_i/A_i).

1.3 Setup demand

In each country there is a representative consumer who obtains utility according to the following CES utility function:

$$U_j = C_j = \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where $\nu(\omega)$ is a demand shifter, $q(\omega)$ is the quantity of ω consumed, and σ is the elasticity of substitution. Note the representative consumer assumption is innocuous since we can write it as the sum of all (identical) workers' per-capita utility. To see this consider:

$$\begin{aligned} U_j^{pc} &= \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} (q(\omega)/L_j)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \\ &= (1`/L_j)\Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \\ \Longleftrightarrow U_j \equiv L_j U_j^{pc} &= \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where the second line uses homotheticity property of the CES utility function (or homogeneous of degree). Thus, under the representative consumer assumption, utility can be viewed as the total welfare in the country.

We now derive the optimal demand for each variety $\omega \in \Omega$ by setting up the following constrained utility maxmization problem:

$$\max_{\{q(\omega)\}} \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} (q(\omega)/L_j)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \text{ subject to } \int_{\omega\in\Omega} p(\omega) q(\omega) d\omega \leq Y_j$$

which can be as the following Lagrangian:

$$\mathcal{L} = \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} (q(\omega)/L_j)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} - \lambda\Big(\int_{\omega\in\Omega} p(\omega) q(\omega) d\omega - Y_j\Big)$$

The first order condition for $q(\omega)$ is:

$$\frac{\partial \mathcal{L}}{\partial q(\omega)} = 0 \Longleftrightarrow \frac{\sigma}{\sigma-1}\Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}-1} = \lambda p(\omega)$$
$$\Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{1}{\sigma-1}} \nu(\omega)^{1/\sigma} q(\omega)^{-1/\sigma} = \lambda p(\omega)\ ,\ \forall \omega \in \Omega \tag{5}$$

Now consider the ratio of the first order conditions of ω, ω' from (5):

$$\frac{p(\omega)}{p(\omega')} = \frac{\nu(\omega)^{1/\sigma} q(\omega)^{-1/\sigma}}{\nu(\omega')^{1/\sigma} q(\omega')^{-1/\sigma}} \Longleftrightarrow \frac{q(\omega)}{q(\omega')} = \frac{\nu(\omega)^{p}(\omega)^{-\sigma}}{\nu(\omega')p(\omega')^{-\sigma}}$$
$$\Longleftrightarrow q(\omega) = \frac{\nu(\omega)p(\omega)^{-\sigma}}{\nu(\omega')p(\omega')^{-\sigma}} q(\omega')$$

Now multiply by $p(\omega)$ and integrate both sides:

$$\int_{\omega\in\Omega} p(\omega)q(\omega)d\omega = \int_{\omega\in\Omega} \frac{\nu(\omega)p(\omega)^{1-\sigma}}{\nu(\omega')p(\omega')^{-\sigma}} q(\omega')d\omega$$
$$= \frac{q(\omega')}{\nu(\omega')p(\omega')^{-\sigma}} \int_{\omega\in\Omega} \nu(\omega)p(\omega)^{-\sigma} d\omega$$

Finally we use $Y_j \equiv \int_\Omega p(\omega)q(\omega)d\omega$ and the aggregate price index $P_j = (\int_\Omega \nu(\omega)p(\omega)^{1-\sigma}d\omega)^{\frac{1}{1-\sigma}}$ so that:

$$Y_j = \frac{q(\omega')}{\nu(\omega')p(\omega')^{-\sigma}} P_j^{1-\sigma}$$
$$\Longleftrightarrow q(\omega) = \nu(\omega)p(\omega)^{-\sigma} Y_j P_j^{1-\sigma} = \nu(\omega)p(\omega/P_j)^{-\sigma}(Y_j/P_j) \tag{6}$$

Note, because demand for all varieties is the same, we can set $\omega' = \omega$ in the second line. Thus, for $\sigma > 1$, demand is increasing in the demand shock ν, aggregate expenditures Y_j, the aggregate price index, P_j, and decreasing in the price $p(\omega)$. Note real income Y_j/P_j is sometimes denoted as X_j or C_j. Before we continue two small side notes. First, using (5) we can show that indeed the elasticity of substitution is constant:

$$-\frac{\partial \log(q(\omega)/q(\omega'))}{\partial \log(p(\omega)/p(\omega'))} = -\frac{\partial \log\left(\frac{\nu(\omega)}{\nu(\omega')}(\frac{p(\omega)}{p(\omega')})^{-\sigma}\right)}{\partial \log(p(\omega)/p(\omega'))} = \sigma$$

Second, utility is equal to the real income. To see this, consider the demand function and divide both sides of the line above (6) by P_j to obtain:

$$Y_j/P_j = \frac{q(\omega)}{\nu(\omega)} p(\omega)^\sigma P_j^{-\sigma} = \frac{q(\omega)}{\nu(\omega)} p(\omega)^\sigma \Big(\int_\Omega \nu(\omega)p(\omega)^{1-\sigma}d\omega\Big)^{\frac{-\sigma}{1-\sigma}}$$
$$= \Big(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} \nu(\omega)^{1+\frac{1-\sigma}{\sigma}} p(\omega)^{\sigma-1} p(\omega)^{1-\sigma} d\omega\Big)^{\frac{\sigma}{\sigma-1}}$$
$$= \Big(\int_\Omega \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \equiv U_j$$

1.4 The Gravity Equation

Let's go back to the CES demand function we derived. Note that because each country only produces and sells one variety, in the Armington setup the demand function is equal to the import demand of j

from i, which in values can be expressed as:

$$X_{ij}(\omega) = p_{ij}(\omega)q_{ij}(\omega) = \nu(\omega)p(\omega/P_j)^{1-\sigma}Y_j \tag{7}$$

To fully express the gravity equation in terms of parameters, we can substitute the price under perfect competition (2) into (7):

$$X_{ij}(\omega) = \nu(\omega)\Big(\frac{W_i}{A_i}\Big)^{1-\sigma}\kappa_{ij}^{1-\sigma}P_j^{\sigma-1}Y_j \tag{8}$$

Thus, for $\sigma > 1$, bilateral trade is increasing in the exporter's productivity, the demand shifter, the exporter and importer's size (GDP), and the importer's aggregate price index; while it is decreasing in the bilateral iceberg cost and the exporter's wage. We can further derive the generalized form of the gravity equation by considering first considering

$$Y_i = \sum_{j\in N} X_{ij} = \sum_{j\in N} \nu_{ij}\kappa_{ij}(W_i/A_i)^{1-\sigma}Y_jP_j^{1-\sigma}$$
$$\iff (W_i/A_i)^{1-\sigma} = Y_i/(\sum_{j\in N}\nu_{ij}\kappa_{ij}Y_jP_j^{1-\sigma})$$

And then substituting this expression into (8):

$$X_{ij}(\omega) = \nu(\omega)\Big(\frac{Y_i}{\sum\limits_{j\in N}\nu_{ij}\kappa_{ij}Y_jP_j^{1-\sigma}}\Big)^{1-\sigma}\kappa_{ij}^{1-\sigma}P_j^{\sigma-1}Y_j = \nu_{ij}\kappa_{ij}^{1-\sigma}\frac{Y_i}{\pi_i^{1-\sigma}}\frac{Y_j}{P_j^{1-\sigma}}$$

where $\pi_{ij} = \sum\limits_{j\in N}\nu_{ij}\kappa_{ij}Y_jP_j^{1-\sigma}$. This formulation of the gravity equation in the Armington maps directly into the generalized form in (1). An important insight from the gravity equation, is that bilateral trade between i and j not only depends on the size of i and j and their trade barriers, but also on (1) the trade barriers other exporters $k \neq i \in N$ face in j, and (2) on the trade barriers i faces when exporting to $k \neq j \in N$. These forces are often referred to as the multilateral resistance of the gravity equation and can be seen in the terms P_j and π_i (Anderson and van Wincoop, 2003). Intuitively, (1) the greater i's cost of exporting to all destinations, the smaller is $\pi^{1-\sigma}$ and the greater is X_{ij}; (2) the greater j's costs of importing are, the smaller is $P_j^{1-\sigma}$ and the greater is X_{ij}.

1.5 Welfare

We can now characterize how welfare is linked to trade openness. In effect, a convenient feature of the Armington model of other static models that we will see later in class, is that welfare is a function of trade openness and a set of parameters, oif which the most important is the trade elasticity. To illustrate this let's begin by defining the expenditure share λ of source i in destination j:

$$\lambda_{ij} \equiv \frac{X_{ij}}{\sum_k X_{kj}} = \frac{\nu_{ij}\kappa_{ij}^{1-\sigma}(W_i/A_i)^{1-\sigma}}{\sum\limits_k \nu_{kj}\kappa_{kj}^{1-\sigma}(W_k/A_k)^{1-\sigma}} = \nu_{ij}\kappa_{ij}^{1-\sigma}A_i^{1-\sigma}(W_i/P_j)^{1-\sigma}$$

where the third equality uses the definition of the aggregate price index.
Now we can use the fact that $U_j = Y_j/P_j$ and that the budget constraint implies $W_j = P_j$ (we normalize $L_j = 1$), so that $U_j = W_j/P_j$. By substituting this expression in the equation above and setting $i = j$

we derive:

$$\lambda_{jj} = \nu_{ij}\kappa_{ij}^{1-\sigma}A_i^{1-\sigma}U_j^{1-\sigma}$$
$$\iff U_j = \nu_{jj}^{\frac{1}{\sigma-1}}A_j\lambda_{jj}^{\frac{1}{1-\sigma}} \tag{9}$$

This expression indicates that welfare depends positively on the home bias (ν_{jj}), domestic productivity, and trade openness. Note welfare increases with trade-to-GDP (trade openness) because increases in imports are due to reductions in iceberg costs (note: less resources are wasted) and these result in lower import prices and thus, a lower price index. Moreover, the elasticity with which welfare decreases with the domestic expenditure share is the inverse of the trade elasticity, which is $1-\sigma$ (why?).

2 The Krugman (1980) Model

2.1 Introduction

The trade patterns resulting from the classic Ricardian and Heckscher-Ohlin trade theories imply that only one country exports a given good. Thus, they are only able to explain inter-industry trade and/or trade between countries characterized by very different comparative advantages. Nevertheless, with the advent of new data collection, by the 1960s and 70s it became apparent that most trade actually occurred intra-industry and between countries – developed countries – that were relatively similar. The seminal 1980 paper by Krugman not only allowed for the justification of intra-industry trade, but also revealed a new and important source of gains from trade: Trade increases market size and thus allows for larger scale economies. Krugman (1980) derived the first contribution by extending the Armington(1969)-Anderson(1979) assumption that all countries consume all countries varieties' to firms. Precisely, it develops an industry model in which firms compete monopolistically and face a CES demand by consumers. The second contribution results from the assumption that firms need to pay a fixed cost to produce, so that the average production cost declines with market size. Larger markets allow for more firms to operate, which, in turn, results in more varieties available to consumers and thus gains from trade due to "love of variety" in the form of lower aggregate prices indexes.

2.2 Setup demand

The demand-side of Krugman (1980) is very similar to the Armington(1969)-Anderson(1979) setup, with one main difference: Instead of countries, varieties are produced by firms and each country has continuum of firms that can potentially produce this product. Thus, we shall think of the model as that of an industry with each country producing a set of varieties that are imperfect substitutes. Again, we assume there is a representative consumer who supplies labor, L_i, inelastically. The utility function takes the following form:

$$U_j = C_j = \Big(\sum_{i\in N}\int_{\omega\in\Omega_i} q(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\Big)^{\frac{\sigma}{\sigma-1}}$$

Note the set of varieties Ω_i refers to the exporter country, that is, all varieties produced in i. The demand function that results from the UMP is the familiar:

$$q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma}P_j^{\sigma-1}Y_j \tag{10}$$

where $P_j = \left(\sum_{i \in N} \int_{\omega \in \Omega_i} p_{ij}(\omega)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$. Thus, expenditures on each variety are characterized by:

$$x_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma} P_j^{\sigma-1} Y_j.$$

Importantly, these are only firm-level exports. To characterize aggregate (industry) exports from i to j, we need to sum over all firms exporting from i to j:

$$X_{ij} \equiv \int_{\omega \in \Omega_i} x_{ij}(\omega) = P_j^{\sigma-1} Y_j \int_{\omega \in \Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega.$$

2.3 Setup supply: closed economy

Although each firm produces a differentiated variety they are homogeneous in everything else and produce with productivity a_i and $y_i(\omega) = a_i l_i(\omega)$, so that producing one unit requires $1/a_i$ units of labor. In this subsection, for simplicity I will drop the ω from the notation. Firms only start producing after paying a fixed entry cost, f. Thus, total labor demand by the firm is $l_i = f + q_i/a_i$ and average cost is $f/q_i + 1/a_i$, while the marginal cost is only $1/a_i$. Thus, the average cost drops as y increases, creating increasing returns to scale. Note the fixed cost is not a fixed cost of exporting, thus once a firm produces it will sell in any market. To model some firms as exporters and others not, or with different intensities, one requires either heterogeneity in the fixed cost of exporting or heterogeneity in productivity. We will see the latter in the next model.
Firms compete monopolistically – i.e. they internalize the demand for their variety. The profit maximization problem is as follows:

$$\max_q pq - Wl = pq - W(q/A + f) = (p - W/A)q - Wf \text{ subject to } y = q = p^{-\sigma} P^{\sigma-1} Y.$$

We can simply substitute the constraint into the objective function and maximize with respect to the price:

$$\max_p (p - W/A) p^{-\sigma} P^{\sigma-1} Y - Wf = p^{1-\sigma} P^{\sigma-1} Y - W/A p^{-\sigma} P^{\sigma-1} Y.$$

The first order condition with respect to p is the following:

$$\begin{aligned} (1-\sigma) p^{-\sigma} P^{\sigma-1} Y &= -\sigma W/A p^{-\sigma-1} P^{\sigma-1} Y \\ \iff p &= \frac{\sigma}{\sigma-1} W/A \end{aligned} \tag{11}$$

Equation 11 indicates that the price is a constant markup over the marginal cost, where the markup is decreasing in the elasticity of substitution, that is, lower elasticities are associated with more market power. Note given the same σ across firms (countries), all firms (countries) charge the same markup.

2.4 Equilibrium in the closed economy

To characterize the equilibrium in the closed economy we use two conditions:

1. Free entry: Firms enter the industry until profits are zero.

2. Labor market clearing: Labor supply equals labor demand.

Before we write out the free entry condition, it helps to define firm-level operating profits, $\tilde{\pi}$, using the price:

$$\tilde{\pi} = (p - W/A)y = (\frac{\sigma}{\sigma - 1} - 1)(W/A)y = \frac{1}{\sigma - 1}(W/A)y$$

Note that operating profits are scalable in the output level. This is an important feature, since trade will expand demand and thereby output and profits. We can now characterize the free entry condition as follows:

$$\begin{aligned} \pi = 0 &\iff \tilde{\pi} = Wf \\ &\iff \frac{1}{\sigma - 1}(W/A)y = Wf \\ &\iff y = (\sigma - 1)Af \end{aligned} \tag{12}$$

Thus, the free entry condition allows us to pin down the output level of all firms. Note, given productivity and fixed cost homogeneity, all firms produce the same output.
In turn, the labor market clearing will allow us to pin down the number of firms operating in equilibrium. To see this consider:

$$\begin{aligned} L = \int_{\omega \in \Omega_i} l(\omega)\ d\omega = \int_{\omega \in \Omega_i} f + y(\omega)/A\ d\omega = f + y/A \int_{\omega \in \Omega_i} d\omega = n(f + y/A) \\ \iff n = L/(\sigma f) \end{aligned} \tag{13}$$

where the third equality follows from the fact that all firms produce the same, the fourth equality defines the number of firms operating as $n \equiv \int_{\omega \in \Omega_i} d\omega$, and the second line uses (12). Equation 13 states that the equilibrium number of firms is increasing in L – which is a good proxy of the economy's size (in the closed economy W can be normalized to 1, so that the budget constraint implies $Y = W$); and is decreasing in the elasticity of substitution and the fixed cost. Higher values of σ increase firm size as can be seen in (12) but because they also shrink markups, in equilibrium fewer firms are able to operate. Lower values of f allow more firms to enter, while leading to smaller firm size.

2.5 Welfare

To characterize welfare in the closed economy it is helpful to first characterize the aggregate price index as a function of parameters only:

$$P = \Big(\int_{\omega \in \Omega_i} p(\omega)^{1-\sigma} d\omega\Big)^{\frac{1}{1-\sigma}} = pn^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma - 1} W/A \Big(\frac{L}{\sigma f}\Big)^{\frac{1}{1-\sigma}}$$

Given $\sigma > 1$, the second equality indicates the price index is decreasing in the number of varieties/firms available. Similarly, we can show that utility increases with the number of varieties/firms:

$$U = \Big(\int_{\omega \in \Omega_i} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} = yn^{\frac{\sigma}{\sigma-1}} = (\sigma - 1)Afn^{\frac{\sigma}{\sigma-1}}\ (= Y)$$

If we continue to substitute n for the equation (12) derived above we obtain an expression of welfare as a function of parameters only:

$$\begin{aligned}U =&(\sigma-1)Af(L/(\sigma f))^{\frac{\sigma}{\sigma-1}} = (\sigma-1)AfL^{\frac{\sigma}{\sigma-1}}(\sigma f)^{\frac{\sigma}{1-\sigma}} = (\sigma-1)AL^{\frac{\sigma}{\sigma-1}}\sigma^{\frac{\sigma}{1-\sigma}}f^{\frac{1}{1-\sigma}}\\ =&\frac{\sigma-1}{\sigma}AL^{\frac{\sigma}{\sigma-1}}(\sigma f)^{\frac{1}{1-\sigma}}\end{aligned}$$

The first term of the second line is the inverse markup – lower markups in the economy increase welfare; the second term is productivity; the third indicates that welfare is increasing in market size, which again is due to the existence of more varieties in larger markets (and "love of variety" in the utility); the final term indicates that welfare is decreasing in both the elasticity of substitution and the fixed cost. While the fixed cost is intuitive as it implies resources that are not used for production, the reason welfare is decreasing in σ is that higher values of σ not only reduce markups (positive welfare effect) but also lower the number of firms (negative welfare effect).
Finally, note indeed, real income is equal to welfare, i.e. $U = (WL)/P$:

$$WL/P = \frac{WL}{\frac{\sigma}{\sigma-1}W/A\left(\frac{L}{\sigma f}\right)^{\frac{1}{1-\sigma}}} = \frac{\sigma-1}{\sigma}AL^{\frac{\sigma}{\sigma-1}}(\sigma f)^{\frac{1}{1-\sigma}}$$

2.6 Equilibrium in Two Country Open Economy

Now we consider an open economy version of the model with two countries, Home and Foreign. We denote all variables of Foreign with an asterisk. Both countries are identical in size, i.e. $L = L^*$. And to export from one to the other countries, firms in each countries face an iceberg cost $\kappa > 1$ that is symmetric. The iceberg cost leads to two different prices for the Home good sold at Home, $p_H = \frac{\sigma}{\sigma-1}W/A$, and at Foreign, $p_F = \frac{\sigma}{\sigma-1}\kappa W/A$. Note $p_F^* = p_H$ and $p_H^* = p_F$.
The aggregate price index in each country are as follows:

$$\begin{aligned}P &= \left(np_H^{1-\sigma} + n^*(\kappa p_H^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}\\ P^* &= \left(n(\kappa p_F)^{1-\sigma} + n^*p_F^{*1-\sigma}\right)^{\frac{1}{1-\sigma}}\end{aligned}$$

The two expressions illustrate how trade lowers the price index by making available more varieties.
Firms now produce using the same technology by allocate some of the production to domestic sales and some to exports. A convenient feature of a CRS production function (homogeneous of degree 1) is that is allows to separate production for both destinations. Total firm-level production of a Home firm is as follows:

$$y = y_H + \kappa y_H^*$$

And the profit function is the following:

$$\begin{aligned}\pi &= p_H y_H + p_H^*\kappa y_H^* - \frac{W}{A}(y_H + \kappa y_H^*) - Wf\\ &= \Big(\frac{\sigma}{\sigma-1}\frac{W}{A} - \frac{W}{A}\Big)y_H + \Big(\frac{\sigma}{\sigma-1}\frac{W}{A} - \frac{W}{A}\Big)\kappa y_H^* - Wf\\ &= \frac{1}{\sigma-1}\frac{W}{A}(y_H + \kappa y_H^*) - Wf = \frac{1}{\sigma-1}\frac{W}{A}y - Wf\end{aligned}$$

Thus, just as in the closed economy, the free entry condition pins down firm-level (total) output and thus profits:

$$\pi = 0 \iff \frac{1}{\sigma - 1}\frac{W}{A}y = Wf \iff y = (\sigma - 1)Af$$

And similarly, $y^* = (\sigma - 1)A^* f^*$. Note firm-level production is the same in the open and closed economy, indicating that trade does not affect firm's output. Therefore, it also does not affect the labor market clearing condition ($L = \int_{\omega \in \Omega_i}(y/A + f)$) and $n = L/(\sigma f)$ and $n^* = L^*/(\sigma f)$. Thus, the scale of production is unaffected by trade: The same number of firms operate producing the same output level. The reason is that with trade, domestic consumers spend less on domestic varieties and domestic producers export their variety, and these two forces exactly balance each other out.

An important point with respect to welfare is that as soon as iceberg costs are finite, the CES demand structure implies that foreign varieties are consumed; that is, when going from autarky to trade, the number of varieties increases from n to $n + N^*$. With trade, further reductions in iceberg costs lead consumers to spend more on imports and less on domestic goods, and the gains accrue only due to lower price levels (and not new varieties).

To solve the open economy model we also need to pin down the relative wage, W/W^*. We do this by assuming balanced trade, i.e. $X = X^*$. Let's write down the expression for aggregate exports of Home to Foreign:

$$X = n\kappa p_H^* y_H^* = n\kappa p_H^* \Big(\frac{\kappa p_H^*}{P^*}\Big)^{-\sigma} Y^* = n\Big(\frac{\kappa p_H^*}{P^*}\Big)^{1-\sigma}\frac{W^* L^*}{P^*}$$

Similarly, exports from Foreign to Home are $X^* = n^*\Big(\frac{\kappa p_F}{P}\Big)^{1-\sigma}\frac{WL}{P}$. When use the free entry expressions for n, n^* and the optimal prices we can write aggregate exports as:

$$X = \varphi L L^* \Big(\frac{\kappa W}{P^*}\Big)^{1-\sigma} W^*$$
$$X = \varphi L L^* \Big(\frac{\kappa W^*}{P}\Big)^{1-\sigma} W,$$

where $\varphi \equiv \frac{1}{\sigma f}\Big(\frac{\sigma}{(\sigma-1)A}\Big)^{1-\sigma}$. If we further use:

$$P = \Big(\varphi(LW^{1-\sigma} + L^*(\kappa W^*)^{1-\sigma}\Big)^{\frac{1}{1-\sigma}}$$
$$P^* = \dots,$$

to substitute into the aggregate export expressions and then impose the balanced trade condition ($X = X^*$) we obtain:

$$\frac{(\kappa W)^{1-\sigma}}{L(\kappa W)^{1-\sigma} + L^* W^{*1-\sigma}} W^* = \frac{(\kappa W^*)^{1-\sigma}}{LW^{1-\sigma} + L^*(\kappa W^*)^{1-\sigma}} W,$$

which implicitly determines the relative wage W/W^*. The following conditions will hold:

1. If $\kappa > 1$ and $L = L^*$, $\Rightarrow\ W = W^*$
2. If $\kappa > 1$ and $L > L^*$, $\Rightarrow\ W > W^*$. The intuition is as follows: A larger market leads to a larger number of firms operating in equilibrium and a lower price index. Thus consumers are less inclined to consume foreign varieties and in order to balance trade W has to rise relative to W^*.

3. If $\kappa = 1, \Rightarrow \; W = W^* \;\perp\; L, L^*$ and $X = \frac{LL^*}{L+L^*}$

2.7 Gravity and Welfare

Given CES demand, the gravity equation in the Krugman model is going to be very similar to (8) in the Armington model, with two differences: (1) monopolistic competition implies a markup; and (2) the entry margin of the Krugman model implies that the number of operating firms enters the gravity equation. To see this, let's consider the CES demand function without demand shifter specified in (10) multiplied by $p(\omega)$. Note here we consider N countries as we did in the Armington model:

$$\begin{aligned}
X_{ij} &\equiv \int_{\Omega_i} x_{ij}(\omega) = \int_{\Omega_i} q_{ij}(\omega) q_{ij}(\omega) = X_j P_j^{\sigma-1} \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega \\
&= X_j P_j^{\sigma-1} \int_{\Omega_i} \Big(\frac{\sigma}{\sigma-1}\frac{W_i}{A_i}\kappa_{ij}\Big)^{1-\sigma} d\omega = \Big(\frac{\sigma}{\sigma-1}\Big)^{1-\sigma} \kappa_{ij}^{1-\sigma} \Big(\frac{W_i}{A_i}\Big)^{1-\sigma} N_i X_j P_j^{\sigma-1}
\end{aligned}$$

where $N_i = \int_{\Omega_i}$ is the number of firms producing in i. The new terms in the gravity are the constant due to the markup, which decreases bilateral trade, and N_i, which increases bilateral trade.

Not surprisingly, these two terms are also going to be the only difference in the welfare expression (9). To derive, let's write down the price index in the N country model version:

$$P_j^{1-\sigma} \equiv \sum_{i \in N} \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma} d\omega = \sum_{i \in N} \int_{\Omega_i} \Big(\frac{\sigma}{\sigma-1}\frac{W_i}{A_i}\kappa_{ij}\Big)^{1-\sigma} d\omega = \Big(\frac{\sigma}{\sigma-1}\Big)^{1-\sigma} \sum_{k \in N} \kappa_{kj}^{1-\sigma} \Big(\frac{W_k}{A_k}\Big)^{1-\sigma} N_k$$

As before, define $\lambda_{ij} = X_{ij} / \sum_k X_{kj}$. Then using the above expression for the price index, we can define λ as:

$$\lambda_{ij} = \Big(\frac{\sigma}{\sigma-1}\Big)^{1-\sigma} \frac{\kappa_{ij}^{1-\sigma}(W_i/A_i)^{1-\sigma} N_i}{P_j^{1-\sigma}}$$

which can be rewritten as:

$$P_j = \Big(\frac{\sigma}{\sigma-1}\Big)\kappa_{ij}(W_i/A_i) N_i^{\frac{1}{1-\sigma}} \lambda_{ij}^{\frac{1}{\sigma-1}}$$

By considering $i = j$ and $\kappa_{jj} = 1$, we can rewrite this as:

$$\begin{aligned}
P_j/W_j &= \Big(\frac{\sigma}{\sigma-1}\Big)(1/A_j) N_j^{\frac{1}{1-\sigma}} \lambda_{jj}^{\frac{1}{\sigma-1}} \\
\Longleftrightarrow U_j = W_j/P_j &= \Big(\frac{\sigma-1}{\sigma}\Big) A_j N_j^{\frac{1}{\sigma-1}} \lambda_{jj}^{\frac{1}{1-\sigma}} \qquad (14)
\end{aligned}$$

Thus, as with the Armington-Anderson model, the real wage is a function of trade openness, domestic productivity and the trade elasticity; but, in the Krugman model also increases with the number of domestic firms and decreases with the markup. Note, the real wage equals welfare thanks to the free entry condition: If firms make positive or negative profits, the real wage no longer captures the welfare of a location, as the households' income includes such profits.

3 The Melitz (2003) - Chaney (2008) Model

3.1 Empirical motivation: firm heterogeneity in productivity & exporting

The Krugman (1980) trade model was the first to go from firms to aggregate trade characterized by a gravity equation. However, it makes the simplifying assumption that all firms are equal. By the early 2000s, data from many countries clearly falsified this assumption: (1) Firms within the same industry differed widely along several dimensions; and (2) firms involved in international trade were very different to firms that only operated domestically. While the first fact is documented in Bernard et al. (2003) (see Table 1), Bernard et al. (2007) provides a review of some of dimensions of the second fact:

- Only 4% of all firms export; and only 18% of manufacturing firms export (2002 U.S. Manufacturing Census). See Table 2
- Among those 4% of exporting firms, 96% of the value of exports came from only 10% of exporters, while the top 1% of exporters accounts for 80% of all exports, but only 24 and 14% of employment, respectively.
- Within the manufacturing sector, there is wide heterogeneity in export participation across more disaggregate 3-digit NAICS industries.
- Exports represent on average 14% of sales among manufacturing firms that export.
- In U.S. manufacturing, exporting firms have 119% more employment, 148% higher sales, 26% higher value-added per worker. See Table 3.

The Melitz-Chaney model will capture some of these facts: (1) heterogeneity in productivity; (2) selection into exporting; and (3) the productivity, sales, and profit premium of exporters. To do so it introduces two critical elements, productivity differences across firms and a fixed cost of exporting. By doing so it provides a new source of gains from trade: As trade barriers drop, aggregate productivity increases as the least competitive firms are replaced by import competition. Moreover, the model includes a new margin when it comes to the trade elasticity: As trade barriers drop, new firms enter the export market, resulting in a trade elasticity that is larger than the intensive margin response of the Armington-Anderson and Krugman models. The Melitz(-Chaney) model has become a workhorse model in trade.

3.2 Setup Demand

The demand setup is the same as in the previous two models. Recall that expenditures on any variety are characterized by:

$$x(\omega) = p(\omega)^{1-\sigma} P^{1-\sigma} X.$$

where X are aggregate expenditures (previously denoted as Y).

3.3 Setup Supply: Closed economy

Firms differ in their productivity $a \in [0, \infty)$, which follows an exogenous distribution $G(a)$. However, the equilibrium distribution of productivity will be endogenous due to selection into operating (and later into exporting). Firms compete monopolistically and thus price their variety at

$$p(\omega) = \frac{\sigma}{\sigma-1}\frac{W}{a} = \frac{\sigma}{\sigma-1}\frac{1}{a} \tag{15}$$

where the second equality is due to the normalization of the wage to 1. Note the producer of variety ω charges the same price as producers of variety ω' with the same productivity a; therefore, we will from now on identify firms with their productivity and denote the price in (15) as $p(a)$.
In the closed economy there are two fixed costs: (1) a (sunk) cost of entering denoted f_e which is paid before observing productivity; and (2) a fixed cost of operating f. Neglecting the entry cost, the firm's profit function is as follows:

$$\begin{aligned}\pi(a) &= (p(a) - \frac{1}{a})c - f = (\frac{\sigma}{\sigma-1} - 1)\frac{1}{a}\Big(\frac{\sigma}{\sigma-1}\frac{1}{a}\Big)^{-\sigma} P^{1-\sigma}X - f = \frac{\sigma}{\sigma}\frac{1}{\sigma-1}\frac{1}{a}\Big(\frac{\sigma}{\sigma-1}\frac{1}{a}\Big)^{-\sigma} P^{1-\sigma}X - f \\ &= \frac{1}{\sigma}\Big(\frac{\sigma}{\sigma-1}\frac{1}{a}\Big)^{1-\sigma} P^{1-\sigma}X - f = \frac{x(a)}{\sigma} - f \end{aligned} \tag{16}$$

Thus, firm-level profits are scalable in revenues, and both, revenues and profits increasing in a.

3.4 Aggregation

As in the Krugman model, the number of firms that enter the market is endogenous. However, here, differences in productivity across firms makes aggregate or average productivity an endogenous object, which we denote as $\mu(a)$. To define aggregate variables it helps to define the following aggregate productivity index:

$$A = \Big(\int_0^\infty a^{\sigma-1}\mu(a)da\Big)^{\frac{1}{\sigma-1}} \tag{17}$$

Consider for example the aggregate price index:

$$\begin{aligned}P &= \Big(\int_\Omega p(\omega)^{1-\sigma}d\omega\Big)^{\frac{1}{1-\sigma}} = \Big(\int_\Omega (\frac{\sigma}{\sigma-1}\frac{1}{a})^{1-\sigma}d\omega\Big)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1}\Big(\int_\Omega d\omega \int_0^\infty (a^{\sigma-1}\mu(a)da\Big)^{\frac{1}{1-\sigma}} \\ &= \frac{\sigma}{\sigma-1}n^{\frac{1}{1-\sigma}}\Big(\int_0^\infty (a^{\sigma-1}\mu(a)da\Big)^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1}n^{\frac{1}{1-\sigma}}\int_0^\infty (a^{\sigma-1}\mu(a)da\Big)^{\frac{-1}{\sigma-1}} = n^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}\frac{1}{A} \\ &= n^{\frac{1}{1-\sigma}}p(A)\end{aligned}$$

Similarly we can define $Y = ny(A)$, $X = nx(A)$, and $\Pi = n\pi(A)$. In other words, all aggregate (endogenous) variables are determined by A.

3.5 Entry and exit

The timing of firm entry and exit is as follows: To draw a productivity, firms first need to pay a fixed cost of entering f_e; second they draw productivity $a \sim G(a)$, $a \in [0,\infty]$; after observing their productivity, firms decide to enter or not; if so, they pay the fixed cost of operating f; finally, each period a constant fraction δ exits. Note empirically exit is decreasing in firm size, productivity and other variables. Here we abstract from this to maintain tractability. Later we will see quantitative models in which exit is endogenous.

The value of a firm making the decision to enter or not is defined as follows:

$$v(a) = \max\{0, \sum_{t=0}^{\infty}\{(1-\delta)^t \pi(a)\} = \max\{0, \pi(a)/\delta\}$$

Given that profits increase monotonically in a and $f > 0$, there will be a cutoff a^* at which firms enter if $a > a^*$ and exit if $a < a^*$, i.e.

$$v(a^*) = 0 \iff \pi(a^*) = 0$$

Thus, the equilibrium productivity distribution is $\mu(a) = \frac{g(a)}{1-G(a^*)}$ for $a \geq a^*$ and 0 otherwise, where $1 - G(a^*)$ is the ex-ante probability of entering. We can express aggregate productivity as:

$$A(a^*) = \Big(\int_{a^*}^{\infty} a^{\sigma-1} \frac{g(a)}{1-G(a^*)} da\Big)^{\frac{1}{\sigma-1}}$$

3.6 Equilibrium in closed economy

To determine the equilibrium prices and quantities, we need to first determine a^*. To do so, we use the free entry condition (FEC):

$$\begin{aligned}
\mathbb{E}v(a) = \int_0^{\infty} v(a)g(a)da &\leq f_e/\delta \\
\iff \int_0^{\infty} \max\{0, \pi(a)/\delta\}g(a)da &\leq f_e/\delta \\
\iff \int_{a^*}^{\infty} \pi(a)g(a)da &\leq f_e.
\end{aligned}$$

At this point it is helpful to define profits as follows:

$$\pi(a) = x(a)/\sigma - f = \Big(\frac{\sigma-1}{\sigma}\Big)^{\sigma-1}\frac{P^{\sigma-1}X}{\sigma}a^{\sigma-1} - f = Ba^{\sigma-1} - f,$$

where $B \equiv \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{P^{\sigma-1}X}{\sigma}$, which captures (and is increasing in) aggregate demand conditions. Now we can write the FEC as:

$$\int_{a^*}^{\infty}[Ba^{\sigma-1} - f]g(a)da \leq f_e$$

However, B is an endogenous object that also depends on $g(a)$ (since P, X do), thus we can't further simplify this expression. Instead, we can use the cutoff a^* at which $\pi(a^*) = 0$ to define $B = fa^{*1-\sigma}$, so that:

$$\begin{aligned}
\int_{a^*}^{\infty}[f(a/a^*)^{\sigma-1} - f]g(a)da &\leq f_e \\
\iff \int_{a^*}^{\infty}[(a/a^*)^{\sigma-1} - 1]g(a)da &\leq f_e/f \\
\iff J(a^*) &\leq f_e/f, \qquad (18)
\end{aligned}$$

where $J(a^*) = \int_{a^*}^{\infty}[(a/a^*)^{\sigma-1} - 1]g(a)da$ is a strictly decreasing function of a^*, and therefore the FEC implies a unique solution for a^*. Figure 4 summarizes relation between productivity, profits and the entry and exit decisions in the closed economy. There are three types of firms: (1) those that exit because $a < a^*$ and obtain $\pi(a) = 0$ and overall lose f_e; (2) those that enter because $a > a^*$ but $\pi(a) < f_e$ and thus overall still make negative profits; (3) those that enter and make positive profits. Once a^* is determined, we can obtain $B(a^*) = a^{*1-\sigma} f$ and $A(a^*) = \left(\int_{a^*}^{\infty} a^{\sigma-1} \frac{g(a)}{1-G(a^*)} da \right)^{\frac{1}{\sigma-1}}$. Once we have determined $A(a^*)$ we market clearing yields the following expression for the number of operating firms:

$$n(a^*) = X/x(A) = \frac{L}{\sigma B(a^*) A(a^*)^{\sigma-1}}$$

With $n(a^*)$ and $A(a^*)$ in hand, we can determine the aggregate price index $P(a^*)$, and real income (consumption per worker), $W/P = 1/P(a^*)$.

3.7 Open economy

We consider the following open economy version: There are $i = 1, ..., N+1$ countries of size L, an iceberg cost κ to ship goods from one country to another, and a fixed cost of exporting f_x, again in units of labor. Because of symmetry, the wage is equal in all countries, i.e. $W_i = W = 1 \; \forall i \in N+1$. Firms can now sell domestically and export. Note symmetry also implies that if a firm exports it will export to all countries. The domestic price of home producer with productivity a is the familiar $p_H(a) = \frac{\sigma}{\sigma-1}\frac{1}{a}$, while the export price is $p_H^*(a) = \kappa p_H(a)$. Firm-level domestic sales by domestic firms are $x_H(a) = \left(\frac{\sigma}{\sigma-1} aP\right)^{\sigma-1} X$ and their export sales are $x_H^*(a) = \kappa^{1-\sigma} x_H(a)$, since by symmetry $X = X^*, P = P^*$. Also note $x_F^*(a) = x_H(a)$ and $x_F(a) = x_H^*(a)$. Therefore, domestic firm profits at Home are $\pi_H(a) = x_H(a)/\sigma - f$ and their profits from exporting to each destination are $\pi_H^*(a) = x_H^*(a)/\sigma - f_x$. Thus, total firm-level profits can be expressed as:

$$\pi(a) = \pi_H(a) + N \max\{0, \pi_H^*(a)\}.$$

And again, $v(a) = \max\{0, \pi(a)\}$. The firm-level optimization problem implies two cutoffs:

1. Entry: $v(a^*) = 0 \iff \pi(a^*) = 0$
2. Exporting: $\pi(a_x^*) = 0$

We will focus on the relevant parameter space in which $a^* < a_x^*$, that is when the cutoff for exporting is above the cutoff for entry and only some firms that operate domestically also export, i.e. exporting is tougher. To determine the parameter restriction that satisfies this condition we obtain expressions for a^* and a_x^* and their zero profit conditions:

$$\pi_H(a^*) = 0 \iff \left(\frac{\sigma}{\sigma-1} a^* P\right)^{\sigma-1} X = \sigma f \iff a^{*\sigma-1} = \frac{\left(\frac{\sigma}{\sigma-1}P\right)^{1-\sigma} \sigma f}{X}$$

and (we apply symmetry $P = P^*, X = X^*$)

$$\pi_H^*(a_x^*) = 0 \iff \kappa^{1-\sigma}\left(\frac{\sigma}{\sigma-1} a_x^* P\right)^{\sigma-1} X = \sigma f_x \iff a_x^{*\sigma-1} = \kappa^{\sigma-1} \frac{\left(\frac{\sigma}{\sigma-1}P\right)^{1-\sigma} \sigma f_x}{X}$$

and then set $a^* < a_x^*$:

$$\frac{\left(\frac{\sigma}{\sigma-1}P\right)^{1-\sigma}\sigma f}{X} < \kappa^{\sigma-1}\frac{\left(\frac{\sigma}{\sigma-1}P\right)^{1-\sigma}\sigma f_x}{X} \iff f < \kappa^{\sigma-1}f_x$$

3.8 Equilibrium in the open economy

To determine the equilibrium in the open economy we follow the same steps as in the closed economy. First, we consider the zero profit conditions of the two cutoffs:

$$\pi(a^*) = \pi_H(a^*) = 0 \iff \pi_H(a^*) = Ba^{*\sigma-1} - f = 0 \iff B = fa^{*1-\sigma}$$
$$\pi(a_x^*) = \pi_H^*(a_x^*) = 0 \iff \pi_H^*(a_x^*) = B\kappa^{1-\sigma}a_x^{*\sigma-1} - f_x = 0 \iff B = \kappa^{\sigma-1}f_x a_x^{*1-\sigma}$$

which by equalizing the two expressions from above allows us to define a_x^* as a function of a^*:

$$a_x^* = \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\kappa a^* \tag{19}$$

Second, we use the free entry condition to pin down a^*. Note now expected value of entry includes profits from exporting:

$$\begin{aligned}
&\mathbb{E}v(a) = \int_0^\infty v(a)g(a)da \leq f_e/\delta \\
\iff &\int_{a^*}^\infty [\pi_H(a) + \max\{0, N\pi_H^*(a)\}]g(a)da \leq f_e \\
\iff &\int_{a^*}^\infty \pi_H(a)g(a)da + N\int_{a_x^*}^\infty \pi_H^*(a)g(a)da \leq f_e \\
\iff &\int_{a^*}^\infty (Ba^{\sigma-1} - f)g(a)da + N\int_{a_x^*}^\infty (B(a/\kappa)^{\sigma-1} - f_x)g(a)da \leq f_e
\end{aligned}$$

Here we can eliminate the market condition B term at the cutoffs a^*, a_x^* to obtain:

$$\begin{aligned}
&\int_{a^*}^\infty [(a/a^*)^{\sigma-1} - 1]g(a)da + Nf_x/f\int_{a_x^*}^\infty](a/a_x^*)^{\sigma-1} - 1]g(a)da \leq f_e/f \\
\iff &J(a^*) + f_x/fNJ(a_x^*) \leq f_x/f
\end{aligned} \tag{20}$$

Thus, equations 20 and 19 form a system of two equations and two unknowns that allow us to pin down a^* and a_x^*. Once we have this we can proceed to obtain $A(a^*)$ and the aggregate variables $X, P, = C$ and C.

Figure 5 illustrates the firm dynamics in the open economy version of Melitz (2003). Firms with productivity $a < a^*$ exit upon drawing their productivity; firms with productivity $a^* < a < a_x^*$ enter but only sell domestically; and firms with $a > a_x^*$ sell domestically and export to all destinations. Thus, export participation is limited to the most productive firms, which also are larger both in terms of revenues and profits.

3.9 Reallocation from autarky to trade

Although at the firm-level productivity is exogenous, aggregate (or industry) productivity is the outcome of the endogenous distribution over productivity that results from firms' entry decision. Thus, we may ask how productivity responds when moving from autarky to trade. Equations 18 and 18 provide

the key insight: Because $J'(x) < 0$, the entry cutoff in autarky (a^*_{aut}) has to be smaller than in the open economy (a^*), i.e. $a^*_{aut} < a^*$. Thus, in the open economy average or aggregate productivity is larger. In turn, this results in a smaller aggregate price index and higher real wages in the open economy compared to autarky, providing an additional source of gains from trade.
Figure 6 illustrates the firm dynamics when moving from autarky to trade. The reallocation effects are as follows: First, some firms exit, those are firms with $a^*_aut < a < a^*$; second, some firms with $a^* < a < a^*_x$ contract in terms of their revenues, as the slope B is smaller than under autarky than under trade due to the aggregate market conditions; finally, a third group of firms $a > a^*_x$ begin exporting, but only those for which a is larger than the intersection between $\pi(a)$ and $\pi_{aut}(a)$ grow in terms of profits.

3.10 Chaney (2008): The margins of trade

An important implication of the Melitz (2003) model is that when trade barriers drop, trade may adjust along two margins: (1) an *intensive margin* which is the increase in exports of (incumbent) firms that are already exporting; and (2) an *extensive margin* of new firms that previously were only selling domestically. It turns out, that both empirically and theoretically, the second and new adjustment margin is sizable. The importance of the extensive margin in trade adjustments was forcefully delivered by Chaney (2008). By making some simplifying assumptions and imposing more structure on the Melitz (2003) model, Chaney (2008) is able to derived some closed form solutions of the adjustment process to trade reforms. Here I lay out the basics of these assumptions and discuss the main analytical results.
Chaney (2008) considers the following version of the Melitz (2003) model:

- Many asymmetric countries, $s \in S$ sectors, asymmetric trade costs ($\kappa_{ijs} \geq 1, f_{ijs} \geq 0$) for each sector-specific bilateral pair.
- No free entry, number potential firms proportional to country size. Thus, the only cutoff is for exporting. Also note, without free entry there are positive profits in equilibrium and the ownership of firms across the World matters.
- Pareto distribution for firm productivity, i.e. $a \sim G(a) = 1 - (a/a_{min})^{-\varepsilon}$
  Note, $J(a) = \frac{\sigma-1}{\varepsilon-(\sigma-1)}\left(\frac{a}{a_{min}}\right)^{-\varepsilon}$ under the Pareto assumption, allowing one to define (18) in closed form as $a^* = \left(\frac{\sigma-1}{\varepsilon-(\sigma-1)}\frac{f}{f_e}\right)^{1/\varepsilon} a_{min}$. Importantly, The conditional probability distribution of a Pareto-distributed random variable is a Pareto distribution with the same shape parameter (ε) but with the new minimum.
- Numeraire good produced by competitive firms with country-specific productivity A_i, costlessly traded, allows one to write the wage of each country in units of the numeraire, $W_i = A_i$.

Some of the most important expressions that result from solving the model under this setup are (note the notation excludes sector-specific index s):

- The productivity cutoff to export from i to j:

  $$a^*_{ij} = K\Big(\frac{\kappa_{ij}W_i}{\theta_j}\Big)\Big(f_{ij}W_i\Big)^{\frac{1}{\sigma-1}} Y_j^{-1/\varepsilon}$$

  where K is some constant, and θ_j is the multilateral resistance term. This expression illustrates that the exporting cutoff is increasing with both trade costs. The elasticity of the cutoff to the variable cost is 1. The elasticity of the cutoff to the fixed cost is $\frac{1}{\sigma-1}$ and thus decreases with σ.

- Firm-level exports (for firms with $a > a^*$):

$$x_{ij}(a) = \overline{x}\Big(\frac{\kappa_{ij} W_i}{\theta_j}\Big)^{1-\sigma} Y_j^{\frac{\sigma-1}{\varepsilon}} a^{\sigma-1}$$

where $\overline{x}$ is some constant. Thus, at the firm-level, the elasticity of exports to the iceberg cost is the same as the trade elasticity in the Krugman and Armington model: $\sigma - 1$.

- Aggregate exports are:

$$X_{ij} = \overline{x} n_i \Big(\frac{\kappa_{ij} W_i}{\theta_j}\Big)^{1-\sigma} Y_j^{\frac{\sigma-1}{\varepsilon}} \Big(\int_{a^*}^{\infty} a^{\sigma-1} dG(a)\Big)$$

Note the elasticity of aggregate exports to the iceberg cost now also depends on the productivity cutoff a^*, which we know from the expression above varies with κ. In fact, one can re-write it as:

$$X_{ij} = \overline{X} n_i \Big(\frac{\kappa_{ij} W_i}{\theta_j}\Big)^{-\varepsilon} (f_{ij} W_i)^{-\frac{\varepsilon}{\sigma-1}-1} Y_j$$

where $\overline{X}$ is some constant. Note the elasticity of aggregate exports to κ is ε, i.e. it now longer depends on the elasticity of substitution but rather the dispersion of productivity. The elasticity of aggregate exports to the fixed cost, in turn, is $\frac{\varepsilon}{\sigma-1} - 1$ and decreases with σ. This is because when σ is low, new entrants, despite being less productive than incumbents, are relatively large. Thus, aggregate exports behave very differently than firm-level exports.

- Finally, after some tedious math, one can decompose the trade elasticity (wrt κ) into two margins:

$$-\frac{\log \partial X_{ij}}{\partial \kappa_{ij}} = \underbrace{\sigma - 1}_{\text{intensive margin}} + \underbrace{(\varepsilon - (\sigma - 1))}_{\text{new extensive margin}} = \varepsilon$$

Note the effect of σ on the intensive margin is exactly canceled out by the dampening effect on the extensive margin. Moreover, because $\varepsilon > \sigma - 1$, the trade elasticity in the Melitz (2003)-Chaney (2008) model has to be larger than in the Krugman (1980) model.

4 Tables & Figures

Table 2: Plant-Level Productivity Facts

Productivity measure (value added per worker)	Variability (standard deviation of log productivity)	Advantage of exporters (exporter less nonexporter avg. log productivity, %)
Unconditional	0.75	33
Within 4-digit industries	0.66	15
Within capital intensity bins	0.67	20
Within production labor share bins	0.73	25
Within industries (capital bins)	0.60	9
Within industries (prod. labor bins)	0.64	11

The statistics are calculated from all plants in the 1992 Census of Manufactures. The "within" measures subtract the mean value of log productivity for each category. There are 450 4-digit industries, 500 capital-intensity bins (based on total assets per worker), 500 production labor share bins (based on payments to production workers as a share of total labor cost). When appearing within industries there are 10 capital-intensity bins or 10 production labor share bins.

Figure 1: **Productivity Dispersion (Bernard et al., 2003)**

Exporting By U.S. Manufacturing Firms, 2002

NAICS industry	*Percent of firms*	*Percent of firms that export*	*Mean exports as a percent of total shipments*
311 Food Manufacturing	6.8	12	15
312 Beverage and Tobacco Product	0.7	23	7
313 Textile Mills	1.0	25	13
314 Textile Product Mills	1.9	12	12
315 Apparel Manufacturing	3.2	8	14
316 Leather and Allied Product	0.4	24	13
321 Wood Product Manufacturing	5.5	8	19
322 Paper Manufacturing	1.4	24	9
323 Printing and Related Support	11.9	5	14
324 Petroleum and Coal Products	0.4	18	12
325 Chemical Manufacturing	3.1	36	14
326 Plastics and Rubber Products	4.4	28	10
327 Nonmetallic Mineral Product	4.0	9	12
331 Primary Metal Manufacturing	1.5	30	10
332 Fabricated Metal Product	19.9	14	12
333 Machinery Manufacturing	9.0	33	16
334 Computer and Electronic Product	4.5	38	21
335 Electrical Equipment, Appliance	1.7	38	13
336 Transportation Equipment	3.4	28	13
337 Furniture and Related Product	6.4	7	10
339 Miscellaneous Manufacturing	9.1	2	15
Aggregate manufacturing	**100**	**18**	**14**

Figure 2: **Exporter Participation (Bernard et al., 2007)**

Exporter Premia in U.S. Manufacturing, 2002

	Exporter premia		
	(1)	*(2)*	*(3)*
Log employment	1.19	0.97	
Log shipments	1.48	1.08	0.08
Log value-added per worker	0.26	0.11	0.10
Log TFP	0.02	0.03	0.05
Log wage	0.17	0.06	0.06
Log capital per worker	0.32	0.12	0.04
Log skill per worker	0.19	0.11	0.19
Additional covariates	None	Industry fixed effects	Industry fixed effects, log employment

Sources: Data are for 2002 and are from the U.S. Census of Manufactures.
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm's export status. Regressions in column 2 include industry fixed effects. Regressions in column 3 include industry fixed effects and log firm employment as controls. Total factor productivity (TFP) is computed as in Caves, Christensen, and Diewert (1982). "Capital per worker" refers to capital stock per worker. "Skill per worker" is nonproduction workers per total employment. All results are significant at the 1 percent level.

Figure 3: **Exporter Premium (Bernard et al., 2007)**

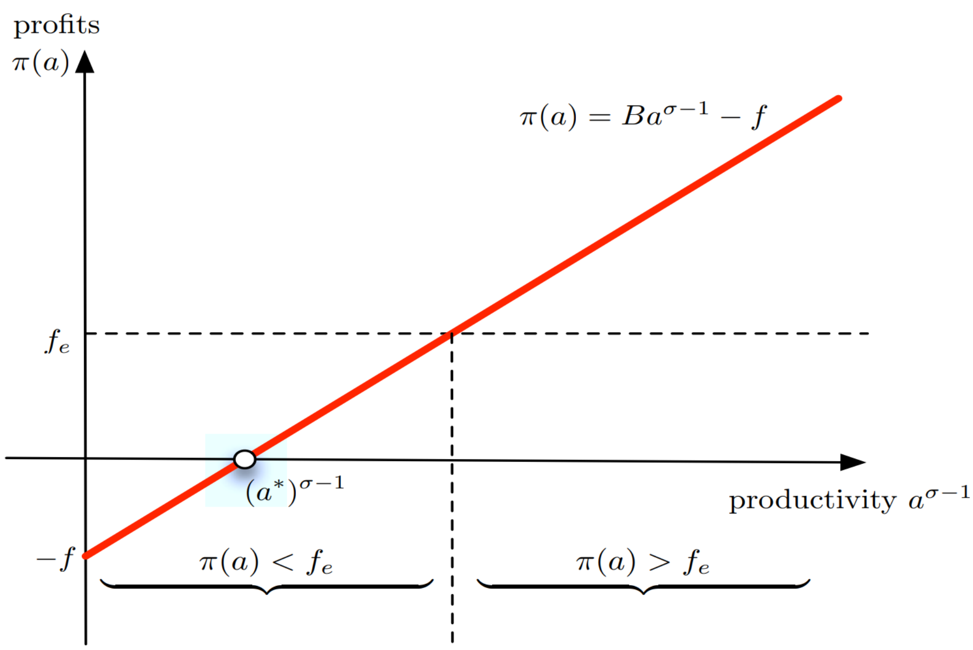

Figure 4: **Melitz (2003) - closed economy**

Note. This figure is from notes by Chris Edmond.

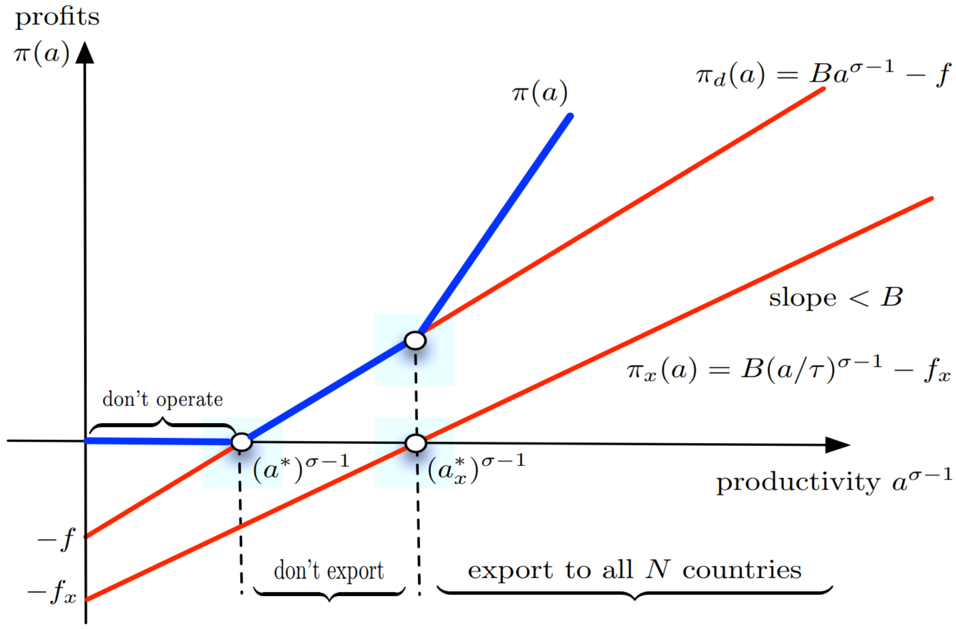

Figure 5: **Melitz (2003) - open economy**

Note. This figure is from notes by Chris Edmond.

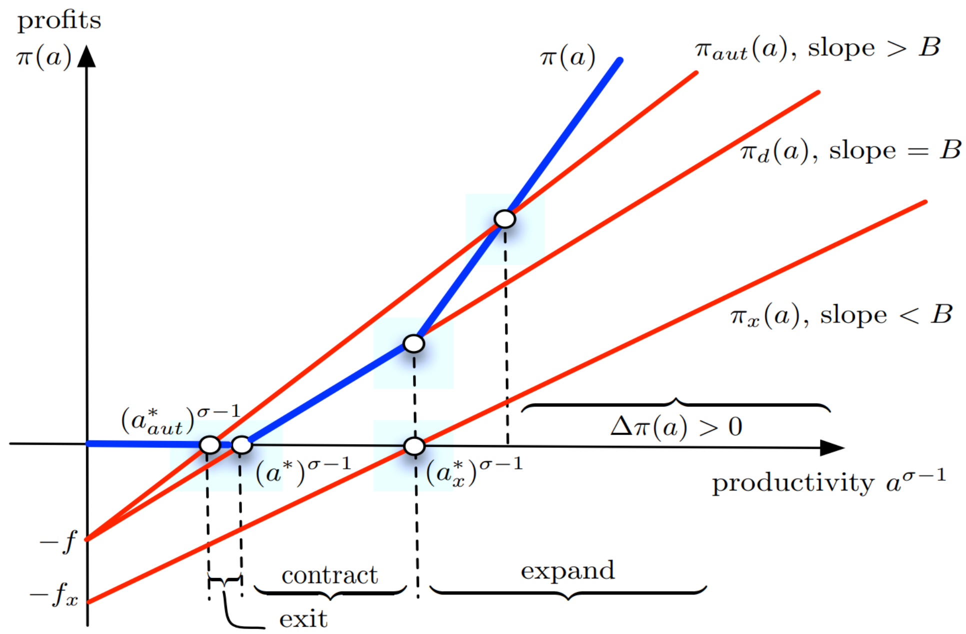

Figure 6: **Melitz (2003) - reallocation from autarky to trade**

Note. This figure is from notes by Chris Edmond.

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