Static Trade Models

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1 The Armington Model

1.1 Introduction

One of the most robust results in economics is the so-called gravity equation of international trade, which states that the bilateral trade of two countries is explained by their size and distance – the economic analog of Newton's law of gravitation. Precisely, the statistical model that illustrates this relation is:

$$X_{ij} = \alpha \frac{Y_i Y_j}{D_{ij}}$$

where X_{ij} are exports from source country i to destination country j, α is a constant, Y_i and Y_j are source and destination GDP's, and D_{ij} is a measure of bilateral trade barriers, including geography, culture, language, policy related trade costs such as tariffs, and non-policy related trade costs such as shipping costs. The generalized expression of the gravity equation is:

$$X_{ij} = \kappa_{ij} \gamma_i \delta_j \tag{1}$$

where κ_{ij} are all bilateral trade barriers, and γ_i and δ_j are source- and destination-country fixed effects. While this result had for a long time been shown empirically using statistical models, it lacked a theoretical foundation. The reason was that classic theories of trade, namely Ricardian trade and the Heckscher-Ohlin model, while useful to understand patterns of specialization and comparative advantage, were difficult to generalize to settings with more than two countries and trade costs. The breakthrough occurred thanks to the insights of Armington (1969) and Anderson (1979). The key insight of the former was that each country produces one good or variety, which is consumed by all other countries in the World; the key contribution of the latter was to incorporate a CES demand function to this setting.

1.2 Setup: Supply

In the Armington World there is a continuum of countries and each country $i \in N$ produces a variety $\omega \in \Omega$, which you can interpret as an aggregate good. Exports from i to j are denoted by X_{ij}. Production is linear in labor and productivity is denoted by A_i. Workers supply labor, L_i, inelastically and are compensated with the wage, W_i, determined in equilibrium. To export their good to destination j, export country i faces an iceberg cost, denoted by κ_{ij}. The iceberg cost is so-called because it is resource that melts away in the export from i to j, as exporting 1 unit requires producing $\kappa \geq 1$ units. It is generally assumed that $\kappa_{ii} = 1$. Note κ is commonly viewed as a capturing all sorts of bilateral trade barriers and is sometimes estimated as the residual from regressing trade flows on observables (Anderson and van Wincoop, 2003).

The Armington model assumes perfect competition and firms take prices as given. Given the linear production function $Q_i(\omega) = \sum_{j\in N} q_{ij}(\omega) = A_i L_i$, we can focus on a representative firm (see proof for similar argument in the case of the representative consumer). The profit maximization problem is as follows:

$$\max_{\{q_{ij}(\omega)\}} = \sum_{N} p_{ij} q_{ij}(\omega) - \kappa_{ij} W_i L_i \text{ subject to } \sum_{j\in N} q_{ij}(\omega)/A_i \leq L_i \tag{2}$$

After setting up the Lagrangian, we can simply substitute the constraint into the maximization problem and solve for the first order condition to obtain an expression for the price:

$$\frac{\partial \mathcal{L}}{\partial q_{ij}(\omega)} = 0 \Longleftrightarrow p_{ij}(\omega) = \frac{\kappa_{ij} W_i}{A_i} \tag{3}$$

Note the differences of prices of ω across destinations only varies with the iceberg cost, since the marginal production cost times is the same across destinations (W_i/A_i).

1.3 Setup: Demand

In each country there is a representative consumer who obtains utility according to the following CES utility function:

$$U_j = C_j = \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \tag{4}$$

where $\nu(\omega)$ is a demand shifter, $q(\omega)$ is the quantity of ω consumed, and σ is the elasticity of substitution. Note the representative consumer assumption is innocuous since we can write it as the sum of all (identical) workers' per-capita utility. To see this consider:

$$\begin{aligned} U_j^{pc} &= \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} (q(\omega)/L_j)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \\ &= (1`/L_j)\Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \\ \Longleftrightarrow U_j \equiv L_j U_j^{pc} &= \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where the second line uses homotheticity property of the CES utility function (or homogeneous of degree). Thus, under the representative consumer assumption, utility can be viewed as the total welfare in the country.

We now derive the optimal demand for each variety $\omega \in \Omega$ by setting up the following constrained utility maxmization problem:

$$\max_{\{q(\omega)\}} \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} (q(\omega)/L_j)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \text{ subject to } \int_{\omega\in\Omega} p(\omega) q(\omega) d\omega \leq Y_j$$

which can be as the following Lagrangian:

$$\mathcal{L} = \Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} (q(\omega)/L_j)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} - \lambda\Big(\int_{\omega\in\Omega} p(\omega) q(\omega) d\omega - Y_j\Big)$$

The first order condition for $q(\omega)$ is:

$$\frac{\partial \mathcal{L}}{\partial q(\omega)} = 0 \Longleftrightarrow \frac{\sigma}{\sigma - 1}\Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}-1} \frac{\sigma-1}{\sigma}\nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}-1} = \lambda p(\omega)$$

$$\Big(\int_{\omega\in\Omega} \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{1}{\sigma-1}} \nu(\omega)^{1/\sigma} q(\omega)^{-1/\sigma} = \lambda p(\omega)\ ,\ \forall \omega \in \Omega \tag{5}$$

Now consider the ratio of the first order conditions of ω, ω' from (5):

$$\frac{p(\omega)}{p(\omega')} = \frac{\nu(\omega)^{1/\sigma} q(\omega)^{-1/\sigma}}{\nu(\omega')^{1/\sigma} q(\omega')^{-1/\sigma}} \Longleftrightarrow \frac{q(\omega)}{q(\omega')} = \frac{\nu(\omega)^{p}(\omega)^{-\sigma}}{\nu(\omega')p(\omega')^{-\sigma}}$$

$$\Longleftrightarrow q(\omega) = \frac{\nu(\omega)p(\omega)^{-\sigma}}{\nu(\omega')p(\omega')^{-\sigma}} q(\omega')$$

Now multiply by $p(\omega)$ and integrate both sides:

$$\int_{\omega\in\Omega} p(\omega)q(\omega)d\omega = \int_{\omega\in\Omega} \frac{\nu(\omega)p(\omega)^{1-\sigma}}{\nu(\omega')p(\omega')^{-\sigma}} q(\omega')d\omega$$

$$= \frac{q(\omega')}{\nu(\omega')p(\omega')^{-\sigma}} \int_{\omega\in\Omega} \nu(\omega)p(\omega)^{-\sigma}d\omega$$

Finally we can use the facts that $Y_j \equiv \int_\Omega p(\omega)q(\omega)d\omega$ and the aggregate price index $P_j = (\int_\Omega \nu(\omega)p(\omega)^{1-\sigma}d\omega)^{\frac{1}{1-\sigma}}$ so that:

$$Y_j = \frac{q(\omega')}{\nu(\omega')p(\omega')^{-\sigma}} P_j^{1-\sigma}$$

$$\Longleftrightarrow q(\omega) = \nu(\omega)p(\omega)^{-\sigma} Y_j P_j^{1-\sigma} = \nu(\omega)p(\omega/P_j)^{-\sigma}(Y_j/P_j) \tag{6}$$

Note, because demand for all varieties is the same, we can set $\omega' = \omega$ in the second line. Thus, for $\sigma > 1$, demand is increasing in the demand shock ν, aggregate expenditures Y_j, the aggregate price index, P_j, and decreasing in the price $p(\omega)$. Note real income Y_j/P_j is sometimes denoted as X_j or C_j. Before we continue two small side notes. First, using (5) we can show that indeed the elasticity of substitution is constant:

$$-\frac{\partial \log(q(\omega)/q(\omega'))}{\partial \log(p(\omega)/p(\omega'))} = -\frac{\partial \log\left(\frac{\nu(\omega)}{\nu(\omega')}(\frac{p(\omega)}{p(\omega')})^{-\sigma}\right)}{\partial \log(p(\omega)/p(\omega'))} = \sigma$$

Second, utility is equal to the real income. To see this, consider the demand function and divide both sides of the line above (6) by P_j to obtain:

$$Y_j/P_j = \frac{q(\omega)}{\nu(\omega)} p(\omega)^\sigma P_j^{-\sigma} = \frac{q(\omega)}{\nu(\omega)} p(\omega)^\sigma \Big(\int_\Omega \nu(\omega)p(\omega)^{1-\sigma}d\omega\Big)^{\frac{-\sigma}{1-\sigma}}$$

$$= \Big(\int_\Omega q(\omega)^{\frac{\sigma-1}{\sigma}} \nu(\omega)^{1+\frac{1-\sigma}{\sigma}} p(\omega)^{\sigma-1} p(\omega)^{1-\sigma} d\omega\Big)^{\frac{\sigma}{\sigma-1}}$$

$$= \Big(\int_\Omega \nu(\omega)^{1/\sigma} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} \equiv U_j$$

1.4 The Gravity Equation

Let's go back to the CES demand function we derived. Note that because each country only produces and sells one variety, in the Armington setup the demand function is equal to the import demand of j

from i, which in values can be expressed as:

$$X_{ij}(\omega) = p_{ij}(\omega)q_{ij}(\omega) = \nu(\omega)p(\omega/P_j)^{1-\sigma}Y_j \tag{7}$$

To fully express the gravity equation in terms of parameters, we can substitute the price under perfect competition (2) into (7):

$$X_{ij}(\omega) = \nu(\omega)\Big(\frac{W_i}{A_i}\Big)^{1-\sigma}\kappa_{ij}^{1-\sigma}P_j^{\sigma-1}Y_j \tag{8}$$

Thus, for $\sigma > 1$, bilateral trade is increasing in the exporter's productivity, the demand shifter, the exporter and importer's size (GDP), and the importer's aggregate price index; while it is decreasing in the bilateral iceberg cost and the exporter's wage. We can further derive the generalized form of the gravity equation by considering first considering

$$\begin{aligned} Y_i = \sum_{j\in N} X_{ij} = \sum_{j\in N} \nu_{ij}\kappa_{ij}(W_i/A_i)^{1-\sigma}Y_jP_j^{1-\sigma} \\ \iff (W_i/A_i)^{1-\sigma} = Y_i/(\sum_{j\in N}\nu_{ij}\kappa_{ij}Y_jP_j^{1-\sigma}) \end{aligned}$$

And then substituting this expression into (8):

$$X_{ij}(\omega) = \nu(\omega)\Big(\frac{Y_i}{\sum\limits_{j\in N}\nu_{ij}\kappa_{ij}Y_jP_j^{1-\sigma}}\Big)^{1-\sigma}\kappa_{ij}^{1-\sigma}P_j^{\sigma-1}Y_j = \nu_{ij}\kappa_{ij}^{1-\sigma}\frac{Y_i}{\pi_i^{1-\sigma}}\frac{Y_j}{P_j^{1-\sigma}}$$

where $\pi_{ij} = \sum\limits_{j\in N}\nu_{ij}\kappa_{ij}Y_jP_j^{1-\sigma}$. This formulation of the gravity equation in the Armington maps directly into the generalized form in (1). An important insight from the gravity equation, is that bilateral trade between i and j not only depends on the size of i and j and their trade barriers, but also on (1) the trade barriers other exporters $k \neq i \in N$ face in j, and (2) on the trade barriers i faces when exporting to $k \neq j \in N$. These forces are often referred to as the multilateral resistance of the gravity equation and can be seen in the terms P_j and π_i (Anderson and van Wincoop, 2003). Intuitively, (1) the greater i's cost of exporting to all destinations, the smaller is $\pi^{1-\sigma}$ and the greater is X_{ij}; (2) the greater j's costs of importing are, the smaller is $P_j^{1-\sigma}$ and the greater is X_{ij}.

1.5 Welfare

We can now characterize how welfare is linked to trade openness. In effect, a convenient feature of the Armington model of other static models that we will see later in class, is that welfare is a function of trade openness and a set of parameters, oif which the most important is the trade elasticity. To illustrate this let's begin by defining the expenditure share λ of source i in destination j:

$$\lambda_{ij} \equiv \frac{X_{ij}}{\sum_k X_{kj}} = \frac{\nu_{ij}\kappa_{ij}^{1-\sigma}(W_i/A_i)^{1-\sigma}}{\sum\limits_k \nu_{kj}\kappa_{kj}^{1-\sigma}(W_k/A_k)^{1-\sigma}} = \nu_{ij}\kappa_{ij}^{1-\sigma}A_i^{1-\sigma}(W_i/P_j)^{1-\sigma}$$

where the third equality uses the definition of the aggregate price index.
Now we can use the fact that $U_j = Y_j/P_j$ and that the budget constraint implies $W_j = P_j$ (we normalize $L_j = 1$), so that $U_j = W_j/P_j$. By substituting this expression in the equation above and setting $i = j$

we derive:

$$\lambda_{jj} = \nu_{ij}\kappa_{ij}^{1-\sigma}A_i^{1-\sigma}U_j^{1-\sigma}$$
$$\iff U_j = \nu_{jj}^{\frac{1}{\sigma-1}}A_j\lambda_{jj}^{\frac{1}{1-\sigma}}$$

This expression indicates that welfare depends positively on the home bias (ν_{jj}), domestic productivity, and trade openness. Note welfare increases with trade-to-GDP (trade openness) because increases in imports are due to reductions in iceberg costs (note: less resources are wasted) and these result in lower import prices and thus, a lower price index. Moreover, the elasticity with which welfare decreases with the domestic expenditure share is the inverse of the trade elasticity, which is $1-\sigma$ (why?).

2 The Krugman (1980) Model

2.1 Introduction

The trade patterns resulting from the classic Ricardian and Heckscher-Ohlin trade theories imply that only one country exports a given good. Thus, they are only able to explain inter-industry trade and/or trade between countries characterized by very different comparative advantages. Nevertheless, with the advent of new data collection, by the 1960s and 70s it became apparent that most trade actually occurred intra-industry and between countries – developed countries – that were relatively similar. The seminal 1980 paper by Krugman not only allowed for the justification of intra-industry trade, but also revealed a new and important source of gains from trade: Trade increases market size and thus allows for larger scale economies. Krugman (1980) derived the first contribution by extending the Armington(1969)-Anderson(1979) assumption that all countries consume all countries varieties' to firms. Precisely, it develops an industry model in which firms compete monopolistically and face a CES demand by consumers. The second contribution results from the assumption that firms need to pay a fixed cost to produce, so that the average production cost declines with market size. Larger markets allow for more firms to operate, which, in turn, results in more varieties available to consumers and thus gains from trade due to "love of variety" in the form of lower aggregate prices indexes.

2.2 Setup: Demand

The demand-side of Krugman (1980) is very similar to the Armington(1969)-Anderson(1979) setup, with one main difference: Instead of countries, varieties are produced by firms and each country has continuum of firms that can potentially produce this product. Thus, we shall think of the model as that of an industry with each country producing a set of varieties that are imperfect substitutes. Again, we assume there is a representative consumer who supplies labor, L_i, inelastically. The utility function takes the following form:

$$U_j = C_j = \Big(\sum_{i\in N}\int_{\omega\in\Omega_i} q(\omega)^{\frac{\sigma-1}{\sigma}}d\omega\Big)^{\frac{\sigma}{\sigma-1}}$$

Note the set of varieties Ω_i refers to the exporter country, that is, all varieties produced in i. The demand function that results from the UMP is the familiar:

$$q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma}P_j^{\sigma-1}Y_j$$

where $P_j = \Big(\sum_{i\in N}\int_{\omega\in\Omega_i} p_{ij}(\omega)^{1-\sigma}\Big)^{\frac{1}{1-\sigma}}$. Thus, expenditures on each variety are characterized by:

$$x_{ij}(\omega) = p_{ij}(\omega)^{1-\sigma}P_j^{\sigma-1}Y_j.$$

Importantly, these are only firm-level exports. To characterize aggregate (industry) exports from i to j, we need to sum over all firms exporting from i to j:

$$X_{ij} \equiv \int_{\omega\in\Omega_i} x_{ij}(\omega) = P_j^{\sigma-1}Y_j\int_{\omega\in\Omega_i} p_{ij}(\omega)^{1-\sigma}d\omega.$$

2.3 Setup: Supply closed economy

Although each firm produces a differentiated variety they are homogeneous in everything else and produce with productivity a_i and $y_i(\omega) = a_i l_i(\omega)$, so that producing one unit requires $1/a_i$ units of labor. In this subsection, for simplicity I will drop the ω from the notation. Firms only start producing after paying a fixed entry cost, f. Thus, total labor demand by the firm is $l_i = f + q_i/a_i$ and average cost is $f/q_i + 1/a_i$, while the marginal cost is only $1/a_i$. Thus, the average cost drops as y increases, creating increasing returns to scale.

Firms compete monopolistically – i.e. they internalize the demand for their variety. The profit maximization problem is as follows:

$$\max_q pq - Wl = pq - W(q/A + f) = (p - W/A)q - Wf \text{ subject to } y = q = p^{-\sigma}P^{\sigma-1}Y.$$

We can simply substitute the constraint into the objective function and maximize with respect to the price:

$$\max_p (p - W/A)p^{-\sigma}P^{\sigma-1}Y - Wf = p^{1-\sigma}P^{\sigma-1}Y - W/Ap^{-\sigma}P^{\sigma-1}Y.$$

The first order condition with respect to p is the following:

$$(1-\sigma)p^{-\sigma}P^{\sigma-1}Y = -\sigma W/Ap^{-\sigma-1}P^{\sigma-1}Y$$
$$\iff p = \frac{\sigma}{\sigma-1}W/A \tag{9}$$

Equation 9 indicates that the price is a constant markup over the marginal cost, where the markup is decreasing in the elasticity of substitution, that is, lower elasticities are associated with more market power. Note given the same σ across firms (countries), all firms (countries) charge the same markup.

2.4 Equilibrium in the closed economy

To characterize the equilibrium in the closed economy we use two conditions:

1. Free entry: Firms enter the industry until profits are zero.
2. Labor market clearing: Labor supply equals labor demand.

Before we write out the free entry condition, it helps to define firm-level operating profits, $\tilde{\pi}$, using the price:

$$\tilde{\pi} = (p - W/A)y = (\frac{\sigma}{\sigma-1} - 1)(W/A)y = \frac{1}{\sigma-1}(W/A)y$$

Note that operating profits are scalable in the output level. This is an important feature, since trade will expand demand and thereby output and profits. We can now characterize the free entry condition as follows:

$$\begin{aligned}\pi = 0 \iff & \tilde{\pi} = Wf \\ \iff & \frac{1}{\sigma-1}(W/A)y = Wf \\ \iff & y = (\sigma-1)Af \qquad (10)\end{aligned}$$

Thus, the free entry condition allows us to pin down the output level of all firms. Note, given productivity and fixed cost homogeneity, all firms produce the same output.
In turn, the labor market clearing will allow us to pin down the number of firms operating in equilibrium. To see this consider:

$$\begin{aligned}L = & \int_{\omega\in\Omega_i} l(\omega)\ d\omega = \int_{\omega\in\Omega_i} f + y(\omega)/A\ d\omega = f + y/A \int_{\omega\in\Omega_i} d\omega = n(f+y/A) \\ \iff & n = L/(\sigma f) \qquad (11)\end{aligned}$$

where the third equality follows from the fact that all firms produce the same, the fourth equality defines the number of firms operating as $n \equiv \int_{\omega\in\Omega_i} d\omega$, and the second line uses (10). Equation 11 states that the equilibrium number of firms is increasing in L – which is a good proxy of the economy's size (in the closed economy W can be normalized to 1, so that the budget constraint implies $Y = W$); and is decreasing in the elasticity of substitution and the fixed cost. Higher values of σ increase firm size as can be seen in (10) but because they also shrink markups, in equilibrium fewer firms are able to operate. Lower values of f allow more firms to enter, while leading to smaller firm size.

2.5 Welfare

To characterize welfare in the closed economy it is helpful to first characterize the aggregate price index as a function of parameters only:

$$P = \Big(\int_{\omega\in\Omega_i} p(\omega)^{1-\sigma} d\omega\Big)^{\frac{1}{1-\sigma}} = pn^{\frac{1}{1-\sigma}} = \frac{\sigma}{\sigma-1} W/A \Big(\frac{L}{\sigma f}\Big)^{\frac{1}{1-\sigma}}$$

Given $\sigma > 1$, the second equality indicates the price index is decreasing in the number of varieties/firms available. Similarly, we can show that utility increases with the number of varieties/firms:

$$U = \Big(\int_{\omega\in\Omega_i} q(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\Big)^{\frac{\sigma}{\sigma-1}} = yn^{\frac{\sigma}{\sigma-1}} = (\sigma-1)Afn^{\frac{\sigma}{\sigma-1}}\ (=Y)$$

If we continue to substitute n for the equation (10) derived above we obtain an expression of welfare as a function of parameters only:

$$\begin{aligned}U =& (\sigma-1)Af(L/(\sigma f))^{\frac{\sigma}{\sigma-1}} = (\sigma-1)AfL^{\frac{\sigma}{\sigma-1}}(\sigma f)^{\frac{\sigma}{1-\sigma}} = (\sigma-1)AL^{\frac{\sigma}{\sigma-1}}\sigma^{\frac{\sigma}{1-\sigma}} f^{\frac{1}{1-\sigma}} \\ =& \frac{\sigma-1}{\sigma} AL^{\frac{\sigma}{\sigma-1}}(\sigma f)^{\frac{1}{1-\sigma}}\end{aligned}$$

The first term of the second line is the inverse markup – lower markups in the economy increase welfare; the second term is productivity; the third indicates that welfare is increasing in market size, which again

is due to the existence of more varieties in larger markets (and "love of variety" in the utility); the final term indicates that welfare is decreasing in both the elasticity of substitution and the fixed cost. While the fixed cost is intuitive as it implies resources that are not used for production, the reason welfare is decreasing in σ is that higher values of σ not only reduce markups (positive welfare effect) but also lower the number of firms (negative welfare effect).
Finally, note indeed, real income is equal to welfare, i.e. $U = WL/P$:

$$WL/P = \frac{WL}{\frac{\sigma}{\sigma-1}W/A\left(\frac{L}{\sigma f}\right)^{\frac{1}{1-\sigma}}} = \frac{\sigma-1}{\sigma}AL^{\frac{\sigma}{\sigma-1}}(\sigma f)^{\frac{1}{1-\sigma}}$$

2.6 Open Economy

Now we consider an open economy version of the model with two countries, Home and Foreign. We denote all variables of Foreign with an asterisk. Both countries are identical in size, i.e. $L = L^*$. And to export from one to the other countries, firms in each countries face an iceberg cost $\kappa > 1$ that is symmetric. The iceberg cost leads to two different prices for the Home good sold at Home, $p_H = \frac{\sigma}{\sigma-1}W/A$, and at Foreign, $p_F = \frac{\sigma}{\sigma-1}\kappa W/A$. Note $p_F^* = p_H$ and $p_H^* = p_F$.
The aggregate price index in each country are as follows:

$$P = \left(np_H^{1-\sigma} + n^*(\kappa p_H^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$P^* = \left(n(\kappa p_F)^{1-\sigma} + n^* p_F^{*1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

The two expressions illustrate how trade lowers the price index by making available more varieties.
Firms now produce using the same technology by allocate some of the production to domestic sales and some to exports. A convenient feature of a CRS production function (homogeneous of degree 1) is that is allows to separate production for both destinations. Total firm-level production of a Home firm is as follows:

$$y = y_H + \kappa y_H^*$$

And the profit function is the following:

$$\begin{aligned}\pi &= p_H y_H + p_H^* \kappa y_H^* - \frac{W}{A}(y_H + \kappa y_H^*) - Wf \\ &= \left(\frac{\sigma}{\sigma-1}\frac{W}{A} - \frac{W}{A}\right)y_H + \left(\frac{\sigma}{\sigma-1}\frac{W}{A} - \frac{W}{A}\right)\kappa y_H^* - Wf \\ &= \frac{1}{\sigma-1}\frac{W}{A}(y_H + \kappa y_H^*) - Wf = \frac{1}{\sigma-1}\frac{W}{A}y - Wf\end{aligned}$$

Thus, just as in the closed economy, the free entry condition pins down firm-level (total) output and thus profits:

$$\pi = 0 \Longleftrightarrow \frac{1}{\sigma-1}\frac{W}{A}y = Wf \Longleftrightarrow y = (\sigma-1)Af$$

And similarly, $y^* = (\sigma-1)A^*f^*$. Note firm-level production is the same in the open and closed economy, indicating that trade does not affect firm's output. Therefore, it also does not affect the labor market clearing condition ($L = \int_{\omega\in\Omega_i}(y/A + f)$) and $n = L/(\sigma f)$ and $n^* = L^*/(\sigma f)$. Thus, the scale of production is unaffected by trade: The same number of firms operate producing the same output level. The reason is that with trade, domestic consumers spend less on domestic varieties and domestic producers export their variety, and these two forces exactly balance each other out.

An important point with respect to welfare is that as soon as iceberg costs are finite, the CES demand structure implies that foreign varieties are consumed; that is, when going from autarky to trade, the number of varieties increases from n to $n + N^*$. With trade, further reductions in iceberg costs lead consumers to spend more on imports and less on domestic goods, and the gains accrue only due to lower price levels (and not new varieties).

To solve the open economy model we also need to pin down the relative wage, W/W^*. We do this by assuming balanced trade, i.e. $X = X^*$. Let's write down the expression for aggregate exports of Home to Foreign:

$$X = n\kappa p_H^* y_H^* = n\kappa p_H^* \left(\frac{\kappa p_H^*}{P^*}\right)^{-\sigma} Y^* = n\left(\frac{\kappa p_H^*}{P^*}\right)^{1-\sigma} \frac{W^* L^*}{P^*}$$

Similarly, exports from Foreign to Home are $X^* = n^* \left(\frac{\kappa p_F}{P}\right)^{1-\sigma} \frac{WL}{P}$. When use the free entry expressions for n, n^* and the optimal prices we can write aggregate exports as:

$$\begin{aligned} X &= \varphi L L^* \left(\frac{\kappa W}{P^*}\right)^{1-\sigma} W^* \\ X &= \varphi L L^* \left(\frac{\kappa W^*}{P}\right)^{1-\sigma} W, \end{aligned}$$

where $\varphi \equiv \frac{1}{\sigma f}\left(\frac{\sigma}{(\sigma-1)A}\right)^{1-\sigma}$. If we further use:

$$\begin{aligned} P &= \left(\varphi(LW^{1-\sigma} + L^*(\kappa W^*)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} \\ P^* &= \dots , \end{aligned}$$

to substitute into the aggregate export expressions and then impose the balanced trade condition ($X = X^*$) we obtain:

$$\frac{(\kappa W)^{1-\sigma}}{L(\kappa W)^{1-\sigma} + L^* W^{*1-\sigma}} W^* = \frac{(\kappa W^*)^{1-\sigma}}{LW^{1-\sigma} + L^*(\kappa W^*)^{1-\sigma}} W,$$

which implicitly determines the relative wage W/W^*. The following conditions will hold:

1. If $\kappa > 1$ and $L = L^*$, $\Rightarrow\ W = W^*$
2. If $\kappa > 1$ and $L > L^*$, $\Rightarrow\ W > W^*$. The intuition is as follows: A larger market leads to a larger number of firms operating in equilibrium and a lower price index. Thus consumers are less inclined to consume foreign varieties and in order to balance trade W has to rise relative to W^*.
3. If $\kappa = 1$, $\Rightarrow\ W = W^* \perp L, L^*$ and $X = \frac{LL^*}{L+L^*}$

3 The Melitz (2003) - Chaney (2008) Model

3.1 Introduction

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