# ECO862 - International Trade
# Lecture 4: Dynamic Trade Models - Sunk cost model

# Model outline

**1.** CES + monopolistic competition math

**2.** Firm decision problem in partial equilibrium

**3.** Success and challenges

# CES + monopolistic competition: Consumers

- $m$ is mass of varieties available
- Total expenditure $E$ exogenous but time-varying

$$\max_{c(\omega)>0} \left( \int_0^m \nu_t(\omega) c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}$$
$$\text{s.t. } \int_0^m p(\omega) c(\omega) d\omega = E_t$$

- Demand function

$$d(\omega, p) = \nu_t(\omega)^\sigma \left( \frac{p(\omega)}{P_t} \right)^{-\sigma} \frac{E_t}{P_t}$$

  - Price of $\omega$ relative to aggregate price
  - Price elasticity of demand $\sigma$
  - Other demand curves get used, linear, translog, etc. often feature ”chokepoint”.

# CES + monopolistic competition: Variety producers ($\omega$)

- Monopolistic competitors: monopolists in their variety.
- Choose prices taking residual demand as given (atomistic)
- Linear (generally CRS) production
- Variable trade costs: $\tau_t \geq 1$ = tariffs; $\xi_t$ = common trade cost; $\hat{\xi}_t(\omega)$ = idio trade cost

$$\max_{p,l} \; p(\omega)d(\omega,\tau p) - Wl(\omega)$$
$$\text{s.t.} \;\; \xi_t\hat{\xi}_t(\omega)d(\omega,\tau p) = z_t(\omega)l(\omega)$$

- The ex-tariff pricing decision

$$p(\omega) = \frac{\sigma}{\sigma-1}\frac{w}{z_t(\omega)}\xi_t\hat{\xi}_t(\omega)$$

- Markup decreasing in $\sigma > 1$ (Why?)
- More productive $(z, \xi)$ firms charge lower prices. Higher demand firms don't.

# CES + monopolistic competition: Variety producers

- Substitute price and labor demand functions into the objective

$$p(\omega)c(\omega) = E_t P_t^{\sigma-1} \nu_t(\omega)^{\sigma} \left( \frac{\sigma}{\sigma-1} \frac{w}{z_t} \tau_t \xi_t \hat{\xi}_t(\omega) \right)^{-(\sigma-1)}$$

- and profits...

$$\pi(\omega) = \frac{1}{\sigma} p(\omega)c(\omega)$$

- These are very special properties of CES + monop. competition

- Notice that $p, c, \pi$ do not depend on $\omega$. They depend on $z, \hat{\xi}$, and $\omega$.
- Index goods by $(z, \hat{\xi}, \omega)$ and use the measure over them to aggregate.

# Static "entry" model intuition

- Assume there is a fixed cost to export, $f$
- Then decisions depends on

$$\pi(\omega) = \frac{1}{\sigma} E_t P_t^{\sigma-1} \nu^{\sigma} \left( \frac{\sigma}{\sigma-1} \frac{W}{z_t} \tau_t \xi_t \hat{\xi} \right)^{-(\sigma-1)} \geq f(\omega)$$

## Sunk-cost model: decision problem

- Now we introduce the sunk-cost model, sometimes with a more general notation
- Three key features in firm-level models of trade
  1. An investment in “market access” technology
  2. An uncertain future return to that investment
  3. A depreciation process of that investment

# Sunk-cost model: decision problem

- Consider a firm $i$ making a decision to export: $x_{it} = \{0, 1\}$

$$V_t = \max E_t \sum_{s=t}^{\infty} \frac{1}{1+r_s} x_{is} \left(\pi_{is}(\cdot) - f_{is}(\cdot)\right)$$

- Fixed export costs: $f_{it}\left(\epsilon_{it,} x_{it-1}, x_{it-2}, ..., x_{it-k}\right)$ depend on random variable ($\epsilon$) and experience (past realizations of $x_i$)
- Flow profits: $\pi\left(x_{it}, z_{it}, d_{it}\right)$
  - $z_{it}$ = variables related to productive efficiency
  - $d_{it}$ = variables related to foreign demand for firm i's
  - Assumes constant returns to scale, otherwise $z_{it}\left(s_{it}, d_{it}\right)$ where $s_{it}$ is sales at home
- Future discounted by interest rate, $r_s$. Role of interest rate depends on how interest rate affects rest of the economy.

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- Future discounted by interest rate, $r_s$. Role of interest rate depends on how interest rate affects rest of the economy.
  - Fast growth or high distortions.

# Model: foreign demand

- Assume a firm charging price $p_{it}$ sells

$$d_{it}\left(p_{it}\right) = \nu_{it}\left(p_{it}\frac{\tau_t \xi_t \tilde{\xi}_{it}}{P_t}\right)^{-\sigma} D_t$$

- Common factors: market size ($D_t$), real exchange rate ($P_t$), ad-valorem tariff ($\tau_t$), iceberg trade costs ($\xi_t$)
- Idiosyncratic factors: demand shifter ($\nu_{it}$) and $\left(\tilde{\xi}_{it}\right)$ e.g., shipping/distribution technology
  - Two idiosyncratic factors redundant, combine into $\xi_{it}$
  - No congestion effects on distribution
- CES framework is common

# Fixed costs

- Following Baldwin and Krugman (1989); Roberts and Tybout (1997)
- $f(\epsilon_{it}, x_{it-1})$: only $t-1$ export status matters (full depreciation of market-access investment)
- $f(\epsilon_{it}, 1) < f(\epsilon_{it}, 0)$ : cost of entering exceeds continuation cost (upfront investment in market access)
- fixed cost lowers iceberg cost from $\xi = \infty$ to $\xi < \infty$ (return on investment)
- When fixed trade cost only depends on last period's export status the fixed cost and history variable are redundant.
- A richer model in which fixed costs depend on experience requires tracking longer history

# Uncertainty

- Microeconomic $(z, \xi, f(\epsilon_{it}, x_{it-1}))$
  - Let $z, \xi$ follow AR1 process $\left(\rho_z, \sigma_z^2, \rho_\xi, \sigma_\xi^2\right)$
  - Fixed cost component follow $\epsilon_{it} \sim \log \text{Normal}\left(0, \sigma_\epsilon^2\right)$
  - Often assume aspect of $\xi$ is learned upon entry (Learning)
- Macroeconomic
  - Processes for exchange rate $(P_t)$ & demand $(D_t)$ depend on equilibrium concept
  - In partial equilibrium $(P, D)$ are exogenous AR processes
  - In general equilibrium, $(P, D)$ depend on shocks and transmission (can be highly non-linear)
  - For tariffs no standard (indeed lots of processes).

# Bellman Equation

- The firm solves a standard discrete-choice problem

$$V_t\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) = \max\left\{V_t^0\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right), V_t^1\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right)\right\}$$

- To solve this problem we will need to know
  - A firm's survival probability ($\delta_{it}$)
  - The interest rate ($r_t$)
- The *t*s capture non-stationary functions from aggregate shocks
  - Most partial equilibrium models assume stationarity

# Bellman Equation

- Value of not exporting

$$
\begin{aligned}
V_t^0\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) &= \pi_t\left(0, z_{it}, \xi_{it}\right) \\
&\quad + \delta_{it} \underset{z,\xi,f}{\mathrm{E}} \frac{1}{1+r_{t+1}} V_{t+1}\left(0, z_{it+1}, \xi_{it+1}, f_{it+1}\right)
\end{aligned}
$$

- Value of exporting

$$
\begin{aligned}
V_t^1\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) &= \pi_t\left(1, z_{it}, \xi_{it}\right) - f(\epsilon_t, x_{i,t-1}) \\
&\quad + \delta_{it} \underset{z,\xi,f}{\mathrm{E}} \frac{1}{1+r_{t+1}} V_{t+1}\left(1, z_{it+1}, \xi_{it+1}, f_{it+1}\right)
\end{aligned}
$$

- Focus on a stationary environment for now (drop $t$s)

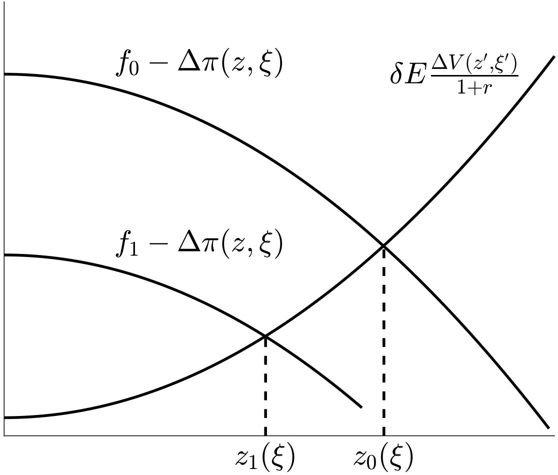
# Decision Rules

- Assume 1) $f$ is deterministic (i.e. $\sigma_\epsilon = 0$) and 2) export and domestic profit increasing in $z$ (monopolistic competition + CES demand).
- Optimal policy is a cutoff rule $z_m(\xi)$ s.t. $x_{it} = 1$ iff $z \geq z_m(\xi)$

$$
\begin{aligned}
f_m - \left[\pi\left(1, z_m(\xi), \xi\right) - \pi\left(0, z_m(\xi), \xi\right)\right] &= \frac{\delta}{1+r} E \left[ \begin{array}{c} V^1\left(z', \xi', f_1\right) \\ -V^0\left(z', \xi', f_0\right) \end{array} \right] \\
f_m - \Delta\pi\left(z_m(\xi), \xi\right) &= \frac{\delta}{1+r} E\left[\Delta V\left(z', \xi', f_1, f_0\right)\right]
\end{aligned}
$$

- The LHS is the current cost of exporting net of increased profits
- The RHS is the future benefit (increase in market value of the firm)

# Breakevens




- Holding fixed the firm-size distribution, when the thresholds are farther apart, exporters become and smaller, non-exporters larger, and the exporter premium decreases.
- The exporter premium is a key empirical moment to discipline the model's parameters.

# The gain in firm value from exporting

- The RHS of the break-even condition
- The upward sloping line in the figure depends on fixed costs and persistence of shock
- The slope is increasing in the persistence of shocks
  - It determines both how long and how much you earn exporting
  - Higher persistence lowers both thresholds, and the gap
- The intercept is mostly determined by the gap between $f_0 - f_1$
  - If $f_0 = f_1$ then $\Delta V = 0$
  - Holding $f_1$ constant, $\frac{\partial \Delta V}{\partial f_0} > 0$

# The current cost of exporting

- The LHS of the break-even condition
- The downward sloping lines in the figure : Holding fixed $\xi$ profits increase $z$, so $f_m - \pi_m$ decrease in $z$
- Exporting more profitable to more productive firms

# Distributions

- The cutoff thresholds and the process for $(z, \xi)$ determine the measure of firm types $\mu(z, \xi, f)$
- $\mu(z, \xi, f_0)$ [$\mu(z, \xi, f_1)$] denotes the beginning of period non-exporters [exporters]
- The measures of current non-exporters and exporters

$$N_N = \int_{\xi} \int_{0}^{z_0(\xi)} \mu(z, \xi, f_0) + \int_{\xi} \int_{0}^{z_1(\xi)} \mu(z, \xi, f_1)$$

$$N_X = \int_{\xi} \int_{z_0(\xi)}^{\infty} \mu(z, \xi, f_0) + \int_{\xi} \int_{z_1(\xi)}^{\infty} \mu(z, \xi, f_1)$$

- The export participation share is $N_X / (N_N + N_X)$

# Laws of motion

$$N'_X = \delta_{X,X} \Pr(\text{continue}) N_X + \delta_{N,X} \Pr(\text{start}) N_N$$

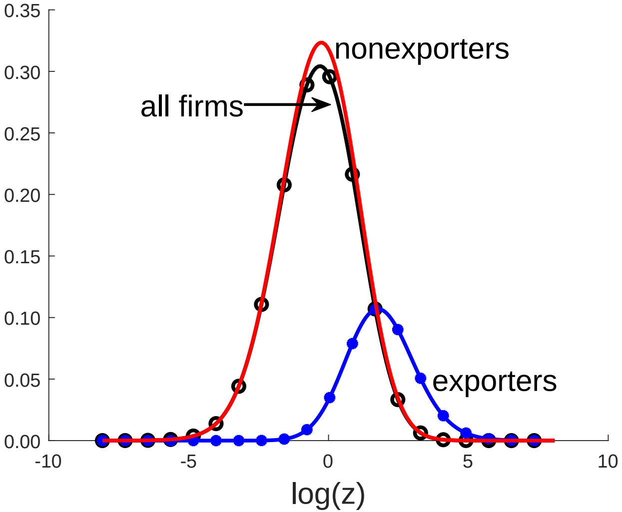
$$N'_N = \delta_{X,N} [1 - \Pr(\text{continue})] N_X + \delta_{NN} [1 - \Pr(\text{start})] N_N + N_E$$

- A more careful exposition would focus fully on

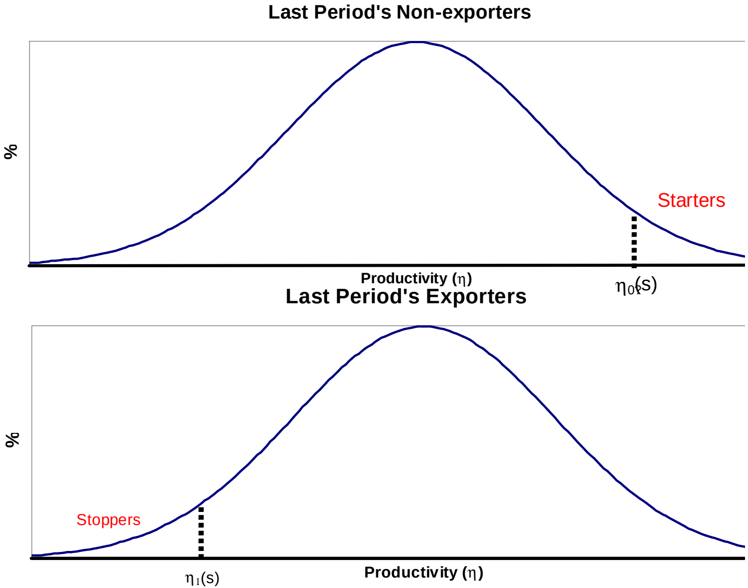
$$\mu'(z, \xi, f) = T(\mu(z, \xi, f))$$

- See the appendix to Alessandria et al. (2021a) for details

# Distributions




... with $f_0 > f_1$. What happens when $f_0 = f_1$?




$$N(s')=Pr(\eta>\eta_1(s))*N(s)+(1-N(s))Pr(\eta>\eta_0(s))$$

# Properties

- Crucial outcome of dynamic decision: $z_1(\xi) < z_0(\xi)$
  - Harder to break into exporting than to stay
- This generates
  - **Exporter hysteresis:** Firms continue exporting after conditions deteriorate
  - **Low exit rate:** Exporters will delay exiting to avoid paying the entry cost again
  - **Export Premium:** Exporters are larger than non-exporters
    - Increasing in the average fixed cost
    - Falling in the difference in fixed costs

# Properties

- Consider $\Delta$ in current and future primitives abstracting from GE interactions
- Let's look at
  1. Trade barriers
  2. Uncertainty

# Trade costs and Tariffs

- Consider three possible reductions in variable trade costs, either $(\xi, \tau)$
  1. **1.** Current trade costs temporary
  2. **2.** Future trade costs permanent
  3. **3.** Current and future trade costs

# Temporary current

- Experiment: $\tau_t \downarrow$, $\tau_s = \tau_{t-1}, \quad s = t+1, t+2, \ldots$
- Lowering today's tariff will shift down the $\text{LHS}_m\,(z)$
- Increasing entry and decreasing exit
- Through law of motion, trade will remain persistently high, only gradually mean-reverting

# Permanent future

- Lowering tariff in the future will shift up the $\text{RHS}_m(z)$
- Increasing entry and decreasing exit today
- Trade grows in advance of liberalization
- Through law of motion trade will increase gradually
- Current response to a unobserved future change in trade barriers can be used to identify the expected path of future policy.

# Permanent current

- Lowering tariff in the current will shift up the $\text{RHS}_m(z)$ and $\text{LHS}_m(z)$
- Combination of previous two shocks
- Increasing entry and decreasing exit today
- Trade grows by more on impact
- Through law of motion trade will increase gradually.

# Uncertainty

- As in typical models with non-convexities, uncertainty matters (Dixit and Pindyck, 1994)

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1. Current dispersion in productivity, $\sigma_z \uparrow$ [temporary]
   - Does not affect thresholds, but does affect distribution of ability today
   - Thicker tails $\rightarrow$ more entry and more exit
   - Volume of trade should increase since conditional mean of productivity $\uparrow$ (selection on a thicker right tail)
2. Future uncertainty/dispersion, $\sigma'_z \uparrow$ [permanent]
   - Shift up and flattening of the marginal gain curve

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   - Shift up and flattening of the marginal gain curve
   - Entry and exit fall, ambiguous effect on trade today and in the future

## Success and Challenges

- Successes
  - Persistent export participation (fact #1)
  - Low export and entry rates (facts #3,4)
  - Dynamic macro adjustment (fact #7)
- Challenges
  - New exporters (too productive at entry, too likely to continue, and export intensity too high)
  - Connection in exporting across markets
  - High re-entry rates in monthly and longer frequencies
- Causes
  - Exporting technology too simple (parsimonious): $f_0, f_1, \xi$
  - Need to shift more investment into post-entry period and reduce depreciation

# Resolutions: Starting and stopping

- Small new-exporters & low continuation rate
  - Let $f_1(t_e)$ be a decreasing function of $t_e$=age in market
- High re-entry data
  - Annual: Let firm that stops re-enter with $f_R \in [f_1, f_0]$
  - Monthly: set $f_0 = f_1$ + inventory cost abroad: firms order infrequently and hold inventories. Account for fact #5 and generate a large short-run elasticity.

# Resolution: Export intensity dynamics

With CES

$$exs(z, \hat{\xi}) = \frac{(\tau\xi\hat{\xi})^{1-\sigma}}{1 + (\tau\xi\hat{\xi})^{1-\sigma}}$$

- Modify iceberg cost structure so that they fall with experience
  - Alessandria et al. (2021b) assume firm enters at $\xi_H > \xi_L$ and then Markov transition between states
  - Reflects improvements in export distribution technology
- Alternatively could accumulate customers or build habit (Fitzgerald et al., 2023; Piveteau, 2021; Ruhl and Willis, 2017; Rodrigue and Tan, 2019)
- Both approaches have investments in improving market after entry, not just maintaining access
- Backloads profits which leads to lower estimates of entry costs.
- When growth process is uncertain, this makes it more likely to exit

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