ECO862 - International Trade
Lecture 5: Dynamic Trade Models - Solving PE sunk-cost models and an estimate of the costs

Solving discrete models

- Goal: Solve and estimate a sunk-cost model
  1. Solve model
  2. Estimate model
- Focus on solution for now

Algorithm

1. Initial set up
   - Set parameter values
   - Construct grids; Discretize continuous stochastic processes
   - Initialize policy and value functions

2. Solve decision problem
   - Value/policy function iteration to convergence
   - Key output: Policy functions

3. Create simulated panel of data
   - Set initial firm states; Draw sequences of shocks
   - Use policy functions to model firm behavior, record panel
   - Use panel to compute moments in simulated data

4. Compare model-moments to data-moments
   - If moments match, finished
   - If moments do not match, update parameters, return to step 2.

1. Initial setup

- Parameters
  - θ = elasticity of substitution in demand
  - τ = tariff (constant, could be stochastic)
  - β = discount factor
  - δ = survival probability (constant, could depend on char.)
  - f_0, f_1 = export entry, continuation costs
  - A process for z ($\bar{z}$, ρ, σ_ϵ)

$$\log(z') = (1-\rho)\log(\bar{z}) + \rho\log(z) + \epsilon$$

$$\epsilon \sim \text{iid } \; N(0, \sigma_\epsilon)$$

  - $\xi_H > \xi_L$ export variable costs (constant, could be stochastic)
  - $g = g(z)$ is the probability mass function of new producers

1. Initial setup

- Construct a grid for z
  - Equally spaced points
  - Importance-weighted: Use CDF of ergodic distribution
- Use Tauchen-like method to convert AR(1) to discrete Markov chain f_{ij}
- Initialize value and policy functions
  - $V^1(x, z, \xi)$ value function for exporter ($N_z \times N_\xi \times 2$)
  - $V^0(x, z, \xi)$ value function for non-exporter
  - $V(x, z, \xi)$ value of the firm (need two of these, old and new)
  - $X(x, z, \xi)$ export decision
- Initialize V to something like $\pi(x, z, \xi)/(1 - \beta\delta\rho)$
- Precompute and store $\pi(x, z, \xi)$ (assumes delays, otw $\pi_{x'}(x, z, \xi)$)
- Ancillary functions: $I(x, z, \xi)$, $ex(z, \xi)$. Compute after convergence.

2. Solve decision problem

- Value function iteration. Loop over z_i

$$V^1(x, z_i, \xi) = \pi(x, z, \xi) - xf_1 - (1-x)f_0 + \beta \sum_{z_j} V_{\text{old}}(1, z_j, \xi)\text{prob}(z_j|z)$$

$$V^0(x, z_i, \xi) = \pi(x, z, \xi) + \beta \sum_{z_j} V_{\text{old}}(0, z_j, \xi)\text{prob}(z_j|z)$$

$$V_{\text{new}}(x, z_i, \xi) = \max\left\{V^1(x, z_i, \xi), V^0(x, z_i, \xi)\right\}$$

- Check: $\|V_{\text{new}}(x, z_i, \xi) - V_{\text{old}}(x, z_i, \xi)\|$
- If not converged, set $V_{\text{old}}(x, z_i, \xi) = V_{\text{new}}(x, z_i, \xi)$, repeat
- Once converged, compute $X(x, z_i, \xi)$, $I(x, z_i, \xi)$, $ex(z_i, \xi)$

2. Decision rules - Interpolation

- With a discrete choice, there is a cutoff for entry z_0, z_1 that is generally between nodes.
- Thus small changes in parameters can lead to discrete change in the mass of firms making the choice.
- This can lead to some instability in convergence or parameter estimation, especially with sparse grid.
- Solution I: interpolate and randomize.
  - Find the cutoffs using the value functions.
  - Assume firms are distributed uniformly between the nodes and then let the decision rule be be based on the share of firms that meet the threshold.
  - Alternatively could introduce small idiosyncratic fixed costs shock
- Solution II: Add nodes in area of cutoffs.

3. Simulate a panel

- We have the decision rules...
- Want to create a panel data set of firms in the stationary distribution
  1. $t = 0$: Create N_f firms, assign each a ξ and a z_0; all nonexporters
  2. $t = 1, \dots, t = T$; $f = 0, \dots, N_f$
     - Draw a z_t for firm f (use ergodic dist and uniform random)
     - Compute export decision, production, exports, etc.
  3. To avoid initial conditions problem, throw out first several hundred observations. Check that moments do not change (much) over the panel.
- Now we have a panel of data...
- If we structured out panel correctly we can **literally** use the same code we used on the data on the model panel.

3. Distribution dynamics

- Let $\mu_t(x, z_i, \xi)$ denote the mass of each type of firms.
- Vector μ_t evolves according to difference equation

$$\mu_{t+1} = \Psi_t \mu_t + m_t \, g_t, \qquad t = 0, 1, ...$$

  where $n \times n$ coefficient matrix Ψ_t has elements governing the exogenous and endogenous transitions

- Mass of firms at node (x, z_i, ξ) at $t+1$ depends on transition probabilities and export entry and exit decisions of incumbents at t plus flow of new entrants

Speeding up the last step

- With linear law of motion for μ, the stationary distribution is linearly homogeneous in m
- In terms of the discretized system above

$$\boldsymbol{\mu} = \boldsymbol{\Psi}\boldsymbol{\mu} + m\mathbf{g} \qquad \Rightarrow \qquad \boldsymbol{\mu} = m(\mathbf{I} - \boldsymbol{\Psi})^{-1}\mathbf{g}$$

  where $\mathbf{I}$ is an identity matrix

- Two implications
  - don't need simulations to find stationary distribution $\boldsymbol{\mu}$, just set up coefficient matrix $\boldsymbol{\Psi}$ (implied by $x^*(x, z_i, \xi)$) and calculate directly
  - only invert $(\mathbf{I} - \boldsymbol{\Psi})$ once, then just rescale by m
- We wrote down one $\mu_t(x, z_i, \xi)$ but could write this down as several distributions $\mu_t^j(z_i, \xi)$ which allows for easier inversion.

Computing moments: two approaches

1. Simulate panel.
   - Sampling errors can be large, may require long burn ins.
   - Can be very slow.
   - But data is from finite samples.
2. Using decisions rules and ergodic distributions
   - Yields exact moments using integration and and some iterations of transition matrices.
   - Can be very fast.
   - Exact - no errors

Aggregate shocks - exchange rate

- No aggregate uncertainty here
- Make E_t an AR(1) process that affects all firms identically
- Need to discretize and add to the firm's state variables

$$V(x, z, \xi, E) = \pi(z, \xi, E) + \cdots + \sum_{z', E'}$$

- Easy in PE;
  - Typically overstates the effect of a foreign price/demand shock — price dynamics will attenuate
  - Misses out on rich interactions between other macro variables (income, wages, interest rates, ...)

Aggregate shocks - tariffs

- Likewise it is straightforward to introduce shocks to tariffs.
- The process for tariffs is quite complicated
  - Anticipation
  - Temporary
  - Risk
  - Specific

Free entry

- Let $g_i = g(z_i)$ denote probability mass function of initial distribution over nodes z_i

- Given $V(w)$ that solves incumbent's problem, free entry condition is

$$V^e := \beta \sum_{i=1}^{n} V_i g_i = f_e$$

  whenever there is positive entry, $m > 0$ (for some parameter values, may have $m = 0$ in which case $V^e < f_e$, see below)

- Easy to show that $V^e(0) > 0$ and $V^e(w)$ monotone decreasing in w, so interior solutions (with $m > 0$) can be found by *bisection*

- Intuitively, if $V^e(w) > f_e$ then increase wage to discourage entry but if $V^e(w) < f_e$ then increase wage to encourage entry

- For now can choose f_e to satisfy this condition.

Market clearing

- Several ways to clear the market
- Export demand curve, exogenous $D(p(w))$
- Industry supply curve, endogenous

$$EX(m, w) = \sum_{i=1}^{n} ex_i(x, z, \xi; w)\mu_i(x, z, \xi; m, w)$$

- We have solved for w^* from free entry condition (supposing $m > 0$). So now want to find measure of entrants m^* that solves

$$EX(m, w) = D(p(w^*))$$

- **Trick:** $\mu(m, w)$ is linear in m, so write $\mu(m, w) = m \times \mu(1, w)$ and solve for m^* as

$$m^* = \frac{D(w^*)}{EX(1, w^*)} = \frac{D(p^*)}{\sum_{i=1}^{n} ex_i(w^*)\mu(1, w^*)}$$

Solution Methods

- Ben Moll Continuous Time approach
- Winberry: A Toolbox for Solving and Estimating Heterogeneous Agent Macro Models
- Steven Terry
- QuantEcon
- Mongey: http://www.simonmongey.com/teaching–notes.html
- VFI Toolkit

Putting the model to work

- Sunk-cost models models have been used
  - To recover trade costs across time, industries, and countries
  - To study the response of exports, entry, etc. to changes in policy, shocks,...
- Today: Das et al. (2007)
  - A "structural IO" approach
  - Lots of heterogeneity
  - High-powered econometrics
  - First estimate of costs of export investment rather than presence from persistence of exporting.

Key assumptions

1. Domestic and foreign product markets are monopolistically competitive and segmented
2. Marginal costs do not respond to output shocks.
3. Producers are heterogeneous in marginal production costs and foreign demand schedules
4. Future realizations on exchange rate, marginal cost and foreign demand are stochastic, following Markov process.

Export profits

- Firm i, potential export profit

$$\log(\pi_{it}^{*}) = \psi_0 z_i + \psi_1 e_t + \nu_{it}$$

- z_i = firm-specific, time-invariant characteristics (vector)
- e_t = log real exchange rate, follow AR(1) process
- ν_{it} = serially correlated shocks to factor prices, productivity, demand,...
  - Model this as sum of m AR(1) processes
  - Let x be the components

- No data on export profit (no export production costs, just total)
- Cannot directly estimate the parameters

Some profit function math

- Assume monopolistic competition, $\eta_1 > 1$ is the elasticity

$$\pi^*_{it} = \eta_i^{-1} \times R^{f*}_{it}$$

- Take logs, substitute into LHS of profit function,

$$\log(R^{f*}_{it}) = \log(\eta_i) + \psi_0 z_i + \psi_1 e_t + \nu_{it}$$

- Notice the extra heterogeneity in η_i
- This also introduces a "incidental parameters problem"
  - Assume that $\eta_i = (1 + v)\eta_i^d$, a bit more math...

$$1 - \frac{C_{it}}{R_{it}} = \eta_i^{-1} \left(1 + v \frac{R^d_{it}}{R_{it}} \right) + \xi_{it}$$

  - Adds data on domestic revenue and costs, introduces *n* more equations and three (v, σ_ξ, λ_ξ) new parameters

Fixed costs

- The "continuation cost:" $\gamma_F - \epsilon_{1it}$
  - γ_F common across firms, ϵ_{1it} idiosyncratic
- The start-up cost: $\gamma_s z_i + \epsilon_{1it} - \epsilon_{2it}$
  - γ_s vector of coefficients, z_i same characteristics in π
- Fixed cost shocks (ϵ) are
  - Normal, serially uncorrelated
  - Orthogonal to e and ν
- Let $y \in 0, 1$ be the export indicator

$$
\begin{aligned}
u() &= \pi^*(e_t, x_{it}, z_i) - \gamma_F + \epsilon_{1it} && \text{if } y_{it} = 1 \text{ and } y_{it-1} = 1 \\
u() &= \pi^*(e_t, x_{it}, z_i) - \gamma_F - \gamma_s z_i + \epsilon_{2it} && \text{if } y_{it} = 1 \text{ and } y_{it-1} = 0 \\
u() &= 0 && \text{if } y_{it} = 0
\end{aligned}
$$

The export decision

- The recursive problem is

$$V_{it} = \max_{y_{it}} u(e_t, x_{it}, z_i, \epsilon_{it}, y_{it}, y_{i,t-1|\theta}) + \delta E_t V_{it+1}$$

- Where θ is the parameter vector, δ is the discount rate, and

$$E_t V_{it+1} = \int_{e'} \int_{x'} \int_{\epsilon'} V_{it+1} \times f_e(e'|e_t, \theta) f_x(x'|x_{it}, \theta) f_\epsilon(\epsilon'|\epsilon_t, \theta) d\epsilon' dx' de'$$

- Which generates the usual discrete-choice exporting rule

$$y_{it} = 1 \text{ if } u(e_t, x_{it}, z_i, \epsilon_{it}, 1, y_{i,t-1|\theta}) + \delta \Delta E_t V_{it+1} > 0$$

where

$$\Delta E_t V_{it+1} = E_t[V_{it+1}|y_{it} = 1] - E_t[V_{it+1}|y_{it} = 0]$$

Data

- Colombian firm-level data 1981–1991
  - $R^f_{it}, R^d_{it}, C_{it}, e_t, z_i$
- Focus on three industries: leather, knitted fabrics, basic chemicals
- Colombian export boom over this period (RER depreciation)
  - RER depreciates 33 percent (many interventions)
  - Exports grow: 26, 16, 19 percent per year
  - Export participation: 12→18, 50→58, 42→50

Identification

- η_i, v: variation in R_{it}/C_{it}, R^f_{it}/R_{it}
- ψ and x process params: variation in export revenue across plants and time
- Sunk cost params: export behavior of firms with similar potential export profits, but different export history
- fixed cost params: exit behavior of firms (given a sunk cost value)

- Estimation: Bayesian MCMC
- Number of x = 2; $\delta = 0.9$,
- z_i = big and small by 1981 domestic revenues
- Exchange rate estimated directly from data

TABLE I

POSTERIOR PARAMETER DISTRIBUTIONS (MEANS AND STANDARD DEVIATIONS)

	Leather Products	Basic Chemicals	Knitted Fabrics	Priors
		Profit Function Parameters		
ψ_{01} (intercept)	−13.645 (4.505)	1.143 (3.642)	−12.965 (3.058)	$\psi_{01} \sim N(0, 500)$
ψ_{02} (domestic size dummy)	1.544 (0.789)	1.862 (0.813)	1.362 (0.449)	$\psi_{02} \sim N(0, 500)$
ψ_1 (exchange rate coefficient)	4.323 (0.957)	0.975 (0.745)	4.047 (0.640)	$\psi_1 \sim N(0, 500)$
λ_x^1 (root, first AR process)	0.787 (0.180)	−0.383 (0.186)	0.458 (0.258)	$\lambda_x^1 \sim U(-1, 1)$
λ_x^2 (root, second AR process)	0.952 (0.018)	0.951 (0.022)	0.709 (0.103)	$\lambda_x^2 \sim U(-1, 1)$
$\sigma_{\omega 1}^2$ (variance, first AR process)	0.282 (0.144)	0.320 (0.109)	0.469 (0.250)	$\ln(\sigma_{\omega 1}^2) \sim N(0, 20)$
$\sigma_{\omega 2}^2$ (variance, second AR process)	0.422 (0.146)	0.491 (0.137)	0.809 (0.264)	$\ln(\sigma_{\omega 2}^2) \sim N(0, 20)$
v (foreign elasticity premium)	−0.016 (0.022)	0.849 (0.126)	0.950 (0.047)	$v \sim U(-1, 1)$
λ_ξ (root, measurement error)	0.336 (0.070)	0.962 (0.011)	0.935 (0.013)	$\lambda_\xi \sim U(-1, 1)$
σ_ξ (std. error, ξ innovations)	0.011 (0.001)	1.277 (0.389)	1.312 (0.264)	$\ln(\sigma_\xi) \sim N(0, 20)$
		Foreign Demand Elasticities (quintiles only)		
η_{Q1} (demand elasticity, quintile 1)	8.020 (2.907)	12.098 (13.881)	10.289 (12.032)	$\ln(\eta - 1) \sim N(2, 1)$
η_{Q2} (demand elasticity, quintile 2)	12.282 (13.351)	12.974 (18.682)	12.314 (8.330)	$\ln(\eta - 1) \sim N(2, 1)$
η_{Q3} (demand elasticity, quintile 3)	17.866 (11.089)	14.139 (13.363)	13.780 (16.725)	$\ln(\eta - 1) \sim N(2, 1)$
η_{Q4} (demand elasticity, quintile 4)	37.189 (25.331)	24.604 (27.253)	36.279 (32.844)	$\ln(\eta - 1) \sim N(2, 1)$
		Dynamic Discrete Choice Parameters		
γ_{S_1} (sunk cost, size class 1)	63.690 (1.934)	62.223 (3.345)	61.064 (2.628)	$\gamma_{S_1} \sim N(0, 500)$
γ_{S_2} (sunk cost, size class 2)	52.615 (4.398)	50.561 (5.043)	59.484 (2.361)	$\gamma_{S_2} \sim N(0, 500)$
γ_F (fixed cost)	−0.610 (1.042)	1.635 (0.983)	1.372 (1.340)	$\gamma_F \sim N(0, 500)$
$\sigma_{\varepsilon 1}$ (std. error, ε_1)	12.854 (6.171)	7.517 (4.109)	32.240 (8.382)	$\ln(\sigma_{\varepsilon 1}) \sim N(0, 20)$
$\sigma_{\varepsilon 2}$ (std. error, ε_2)	30.627 (7.831)	32.432 (3.196)	17.630 (4.737)	$\ln(\sigma_{\varepsilon 2}) \sim N(0, 20)$
		Initial Conditions Parameters		
α_0 (intercept)	−3.559 (6.523)	−13.693 (7.069)	−40.811 (21.379)	$\alpha_0 \sim N(0, 500)$
α_1 (domestic size dummy)	16.484 (9.965)	25.868 (11.959)	23.397 (14.762)	$\alpha_1 \sim N(0, 500)$
$\alpha_2(x_1)$	29.388 (11.675)	−18.028 (11.658)	31.603 (18.165)	$\alpha_2 \sim N(0, 500)$
$\alpha_3(x_2)$	3.451 (4.861)	8.908 (5.710)	16.561 (15.519)	$\alpha_3 \sim N(0, 500)$

Profit function parameter estimates

- Big firms earn bigger export profits
- Leather and knitted have big RER elasticity
- Strong serial correlation in plant shocks (λ_x)
- Average elasticities: 14.2, 13.0, 12.7
- $\eta = \eta^d(1 + v)$; for chem and knits, foreign markets are tougher

Fixed cost estimates

- Entry costs
  - Small producers \$412k–430k
  - Large producers \$344k–402k
  - These are big!
  - Actual entry costs paid are lower: Enter when ϵ draws are favorable
  - Modest variation across industries, (usually better to discuss relative to profits)
- Fixed costs
  - Close to zero
  - Large standard deviation—fixed cost matter sometimes

Takeaways

1. Depreciation shifts up value of exporting
2. Luck is key in drawing most plants in except for leather products.
3. History is almost everything.
4. Value is much larger than flow profits
5. Exchange rate shocks have heterogeneous effects (not crazy about how it is modelled) depending on initial export participation and distribution
    - Would rather model changing monetary policy rule or shocks

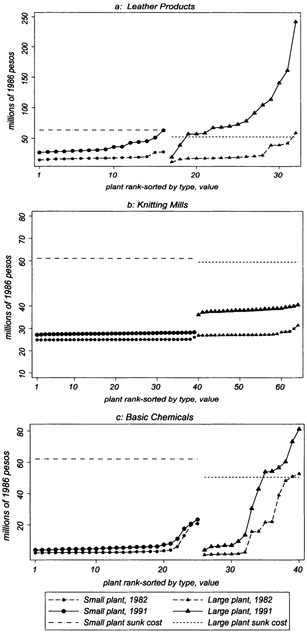


FIGURE 1.—Plant export value and sunk entry costs.

- Compares gross expected value with expected sunk entry costs $\gamma_S z_i$, plant by plant
- Both the first (1982) and the last (1991) sample year illustrate sensitivity to the exchange rate changes.
- The two horizontal lines are the posterior mean sunk costs for each plant type.
- Responsiveness on the entry–exit margin will vary considerably across industries: similar across small plants, insufficient to cover the entry costs
- Among large producers, more within-industry heterogeneity in leather products and basic chemicals: the entry margin will probably be active, but relatively few non-exporters are likely to respond to devaluation.
- Large knitting mills are much more homogeneous. Non-exporters are likely to respond similarly to devaluation, so a sufficiently large change in the exchange rate may well induce a wave of entry
- Note: does not predict which plants actually participate in the export market because it omits realizations x_{it} and ϵ_{it}, and does it distinguish which producers were already in the export market

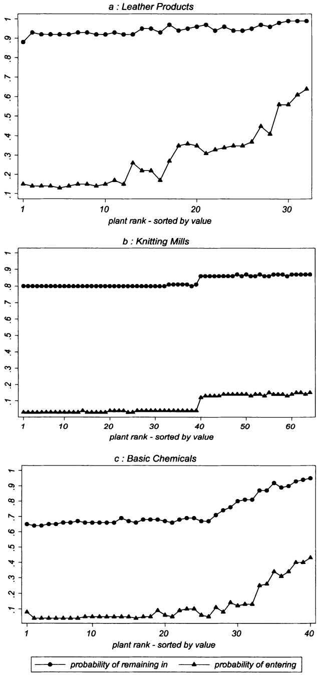


FIGURE 2.—Probability of exporting conditional on plant history.

- The probability of remaining an exporter, once in, is above 0.8 for knitted fabric producers, above 0.9 for leather producers, and varies from 0.6 to 0.9 for basic chemical producers.
- In contrast, the probability of entering the market is generally below 0.2 in knitting and basic chemicals, although entry is more likely for a subset of leather products producers.

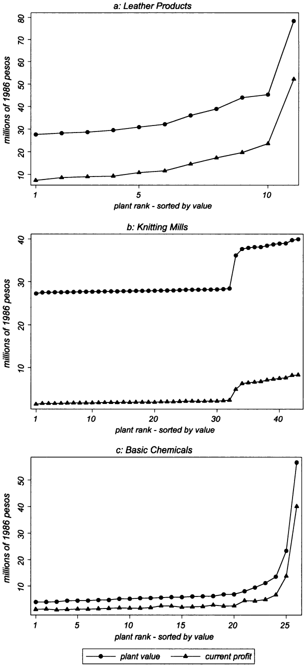
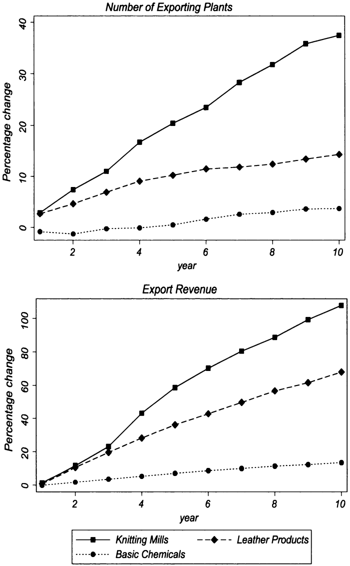


FIGURE 3.—Value of exporting and current profit for nonexporting plants.

- Option value of being able to export next period without paying entry costs, for non-exporters.
- Without entry costs, the option value is zero and exporting behavior could be described with a static model.
- Here: option values are the largest component of export value for most producers (especially knitting mills).
- One implication is that changes in option values due, for example, to changing expectations about future market conditions, can induce large changes in the return to becoming an exporter, even if current profits are unaffected.



IGURE 4.—Export response to a correctly perceived 20 percent devaluation.

- The export supply response to a devaluation reflects adjustments on two margins: entry–exit and output adjustments among incumbents
- Simulate plants' reactions to a permanent change in the exchange-rate process that depreciates the steady state value of the peso by 20%.
- Relatively strong entry response among knitting mills because few mills were exporting in 1981. (Only 5/64 vs 17/32 basic chemical plants and 14/40 leather products plants.) It also reflects their relative homogeneity in terms of their expected export profit streams and proximity to the entry threshold.
- Export revenues are generally larger as incumbent exporters responded to the new regime by increasing their foreign sales volumes.
- Variation in responses across industries is due both to differences in entry patterns and to differences in the elasticity of export profits to the exchange rate.

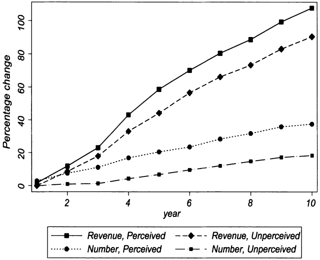


FIGURE 5.—Response to a perceived versus unperceived regime shift: 20 percent steady state devaluation, knitting mills.

- Perceptions of the policy regime can also matter.
- Once again, consider a policy reform that promotes exports by increasing the intercept of the exchange rate process. If managers persist in believing that the exchange-rate realizations they observe were generated by the pre-reform exchange-rate process, they underestimate the increase in the value of becoming an exporter.
- For a 20% depreciation, this type of misperception makes little difference for leather products producers and basic chemicals producers, because few of them are near their entry threshold, but it matters for knitting mills
- The number of exporters would have grown by 18% rather than 37% ; total export volume would have been 90% higher rather than 107%

Subsidies

- Broad set of subsidies active in developing and developed economies (some are against WTO rules).
  - Direct export subsidies
  - Entry subsidies
  - Continuation subsidies

TABLE III

EXPORT REVENUE/COST RATIOS FOR ALTERNATIVE SUBSIDY PLANS
(MEANS OVER 300 SIMULATIONS)

	Knitting Mills	Leather Products	Basic Chemicals
	Revenue Subsidies		
2 percent	19.23	11.81	11.17
5 percent	15.67	9.94	9.56
10 percent	10.09	7.74	7.52
	Entry Cost Subsidies		
25 percent	2.03	−0.54	−0.068
50 percent	1.02	−0.67	−0.49
100 percent	0.14	−0.26	−0.23
	Fixed Cost Subsidy		
2 million pesos	3.96	2.22	2.66
10 million pesos	0.99	0.68	0.69

- Total gain in export revenue divided by the direct cost of the subsidy.
- Effects do not vary greatly over time, consider 5 years after intro.
- The revenue subsidy is the most potent because it acts on the volume margin, relevant for all producers; fixed cost and entry cost subsidies affect only the export decision.
- Entry cost subsidy reduce the option value of remaining an exporter and encourages exit along with entry; turnover increases, and this effect is sufficiently strong to create a negative return to export promotion the leather products and the basic chemicals industries.

Final thoughts

- Convincing take on export entry costs.
- Points to aspects of export response related to industry heterogeneity
- Harder to implement than in a more macro-approach.