ECO862 - International Trade
Lecture 4: Dynamic Trade Models - Sunk cost model

Model outline

1. CES + monopolistic competition math
2. Firm decision problem in partial equilibrium
3. Success and challenges

CES + monopolistic competition: Consumers

- m is mass of varieties available
- Total expenditure E exogenous but time-varying

$$\max_{c(\nu)>0} \left(\int_0^m \omega_t(\nu) c(\nu)^{\frac{\theta-1}{\theta}} d\nu \right)^{\frac{\theta}{\theta-1}}$$
$$\text{s.t.} \int_0^m p(\nu) c(\nu) d\nu = E_t$$

- Demand function

$$d(\nu, p) = \omega_t(\nu)^\theta \left(\frac{p(\nu)}{P_t} \right)^{-\theta} \frac{E_t}{P_t}$$

  - Price of ν relative to aggregate price
  - Price elasticity of demand θ
  - Other demand curves get used, linear, translog, etc. often feature ”chokepoint”.

CES + monopolistic competition: Variety producers (ν)

- Monopolistic competitors: monopolists in their variety.
- Choose prices taking residual demand as given (atomistic)
- Linear (generally CRS) production
- Variable trade costs: $\tau_t \geq 1$ = tariffs; ξ_t = common trade cost; $\hat{\xi}_t(\nu)$ = idio trade cost

$$\max_{p,l} \; p(\nu)d(\nu, \tau p) - wl(\nu)$$
$$\text{s.t. } \xi_t \hat{\xi}_t(\nu) d(\nu, \tau p) = z_t(\nu) l(\nu)$$

- The ex-tariff pricing decision

$$p(\nu) = \frac{\theta}{\theta - 1} \frac{w}{z_t(\nu)} \xi_t \hat{\xi}_t(\nu)$$

- Markup decreasing in $\theta > 1$ (Why?)
- More productive (z, ξ) firms charge lower prices. Higher demand firms don't.

CES + monopolistic competition: Variety producers

- Substitute price and labor demand functions into the objective

$$p(\nu)c(\nu) = E_t P_t^{\theta-1} \omega_t(\nu)^\theta \left(\frac{\theta}{\theta-1} \frac{w}{z_t} \tau_t \xi_t \hat{\xi}_t(\nu) \right)^{-(\theta-1)}$$

- and profits…

$$\pi(\nu) = \frac{1}{\theta} p(\nu)c(\nu)$$

- These are very special properties of CES + monop. competition

- Notice that p, c, π do not depend on ν. They depend on $z, \hat{\xi}$, and ω.
- Index goods by $(z, \hat{\xi}, \omega)$ and use the measure over them to aggregate.

Static "entry" model intuition

- Assume there is a fixed cost to export, f
- Then decisions depends on

$$\pi(\nu) = \frac{1}{\theta} E_t P_t^{\theta-1} \omega^{\theta} \left(\frac{\theta}{\theta - 1} \frac{w}{z_t} \tau_t \xi_t \hat{\xi} \right)^{-(\theta-1)} \geq f(\nu)$$

Sunk-cost model: decision problem

- Now we introduce the sunk-cost model, sometimes with a more general notation
- Three key features in firm-level models of trade
  1. An investment in “market access” technology
  2. An uncertain future return to that investment
  3. A depreciation process of that investment

Sunk-cost model: decision problem

- Consider a firm i making a decision to export: $x_{it} = \{0, 1\}$

$$V_t = \max E_t \sum_{s=t}^{\infty} \frac{1}{1+r_s} x_{is} \left(\pi_{is}\left(\cdot\right) - f_{is}(\cdot)\right)$$

- Fixed export costs: $f_{it}\left(\epsilon_{it,} x_{it-1}, x_{it-2}, ..., x_{it-k}\right)$ depend on random variable and experience
- Flow profits: $\pi\left(x_{it}, z_{it}, d_{it}\right)$
  - z_{it} = variables related to productive efficiency
  - d_{it} = variables related to foreign demand for firm i's
  - Assumes constant returns to scale, otherwise $z_{it}\left(s_{it}, d_{it}\right)$ where s_{it} is sales at home
- Future discounted by interest rate, r_s. Role of interest rate depends on how interest rate affects rest of the economy.

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- Future discounted by interest rate, r_s. Role of interest rate depends on how interest rate affects rest of the economy.
  - Fast growth or high distortions.

Model: foreign demand

- Assume a firm charging price p_{it} sells

$$d_{it}\left(p_{it}\right) = \omega_{it}\left(p_{it}\frac{\tau_t \xi_t \tilde{\xi}_{it}}{P_t}\right)^{-\theta} D_t$$

- Common factors: market size (D_t), real exchange rate (P_t), ad-valorem tariff (τ_t), iceberg trade costs (ξ_t)
- Idiosyncratic factors: demand shifter (ω_{it}) and $\left(\tilde{\xi}_{it}\right)$ e.g., shipping/distribution technology
  - Two idiosyncratic factors redundant, combine into ξ_{it}
  - No congestion effects on distribution
- CES framework is common

Fixed costs

- Following Baldwin and Krugman (1989); Roberts and Tybout (1997)
- $f(\epsilon_{it}, x_{it-1})$: only $t-1$ export status matters (full depreciation of market-access investment)
- $f(\epsilon_{it}, 1) < f(\epsilon_{it}, 0)$: cost of entering exceeds continuation cost (upfront investment in market access)
- fixed cost lowers iceberg cost from $\xi = \infty$ to $\xi < \infty$ (return on investment)
- When fixed trade cost only depends on last period's export status the fixed cost and history variable are redundant.
- A richer model in which fixed costs depend on experience requires tracking longer history

Uncertainty

- Microeconomic $(z, \xi, f(\epsilon_{it}, x_{it-1}))$
  - Let z, ξ follow AR1 process $\left(\rho_z, \sigma_z^2, \rho_\xi, \sigma_\xi^2\right)$
  - Fixed cost component follow $\epsilon_{it} \sim \log \text{Normal}\left(0, \sigma_\epsilon^2\right)$
  - Often assume aspect of ξ is learned upon entry (Learning)
- Macroeconomic
  - Processes for exchange rate (P_t) & demand (D_t) depend on equilibrium concept
  - In partial equilibrium (P, D) are exogenous AR processes
  - In general equilibrium, (P, D) depend on shocks and transmission (can be highly non-linear)
  - For tariffs no standard (indeed lots of processes).

Bellman Equation

- The firm solves a standard discrete-choice problem

$$V_t\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) = \max\left\{V_t^0\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right), V_t^1\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right)\right\}$$

- To solve this problem we will need to know
  - A firm's survival probability (δ_{it})
  - The interest rate (r_t)
- The *t*s capture non-stationary functions from aggregate shocks
  - Most partial equilibrium models assume stationarity

Bellman Equation

- Value of not exporting

$$
\begin{aligned}
V_t^0\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) &= \pi_t\left(0, z_{it}, \xi_{it}\right) \\
&\quad + \delta_{it} \underset{z,\xi,f}{\mathrm{E}} \frac{1}{1+r_{t+1}} V_{t+1}\left(0, z_{it+1}, \xi_{it+1}, f_{it+1}\right)
\end{aligned}
$$

- Value of exporting

$$
\begin{aligned}
V_t^1\left(x_{it-1}, z_{it}, \xi_{it}, f_{it}\right) &= \pi_t\left(1, z_{it}, \xi_{it}\right) - f(\epsilon_t, x_{i,t-1}) \\
&\quad + \delta_{it} \underset{z,\xi,f}{\mathrm{E}} \frac{1}{1+r_{t+1}} V_{t+1}\left(1, z_{it+1}, \xi_{it+1}, f_{it+1}\right)
\end{aligned}
$$

- Focus on a stationary environment for now (drop ts)

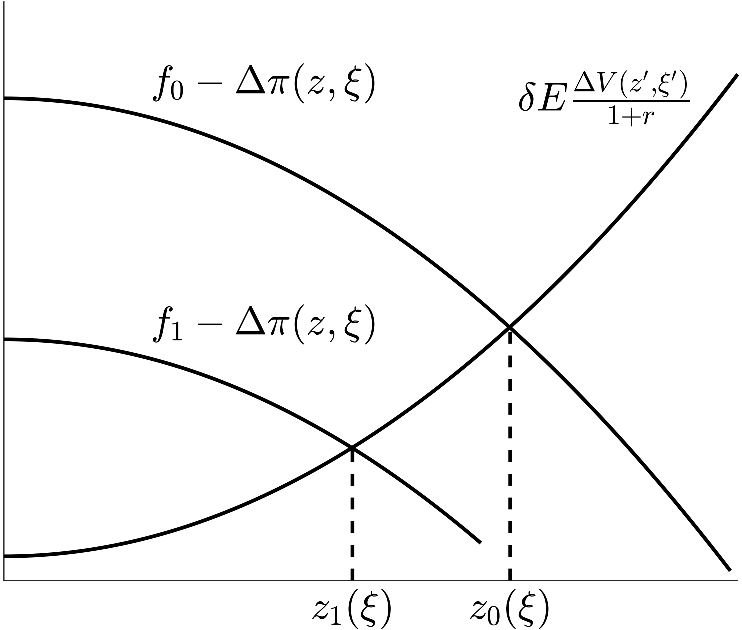
Decision Rules

- Assume 1) f is deterministic (i.e. $\sigma_\epsilon = 0$) and 2) export and domestic profit increasing in z
- Optimal policy is a cutoff rule $z_m(\xi)$ s.t. $x_{it} = 1$ iff $z \geq z_m(\xi)$

$$
\begin{aligned}
f_m - [\pi(1, z_m(\xi), \xi) - \pi(0, z_m(\xi), \xi)] &= \frac{\delta}{1+r} E\left[\begin{array}{c} V^1(z', \xi', f_1) \\ -V^0(z', \xi', f_0) \end{array}\right] \\
f_m - \Delta\pi(z_m(\xi), \xi) &= \frac{\delta}{1+r} E[\Delta V(z', \xi', f_1, f_0)]
\end{aligned}
$$

- The LHS is the current cost of exporting net of increased profits
- The RHS is the future benefit (increase in market value of the firm)

Breakevens



$f_0 - \Delta\pi(z, \xi)$
$\delta E \frac{\Delta V(z', \xi')}{1+r}$
$f_1 - \Delta\pi(z, \xi)$
$z_1(\xi)$
$z_0(\xi)$

The gain in firm value from exporting

- The RHS of the break-even condition
- The upward sloping line in the figure
- Depends on fixed costs and persistence of shock
- The slope is increasing in the persistence of shocks
  - It determines both how long and how much you earn exporting
- The intercept is mostly determined by the gap between $f_0 - f_1$
  - If $f_0 = f_1$ then $\Delta V = 0$
  - Holding f_1 constant, $\frac{\partial \Delta V}{\partial f_0} > 0$

The current cost of exporting

- The LHS of the break-even condition
- The downward sloping lines in the figure

- Holding fixed ξ cost decreases in z
  - Exporting more profitable to more productive firms

Distributions

- The cutoff thresholds and the process for (z, ξ) determine the measure of firm types $\mu(z, \xi, f)$
- $\mu(z, \xi, f_0)$ [$\mu(z, \xi, f_1)$] denotes the beginning of period non-exporters [exporters]
- The measures of current nonexporters and exporters

$$N_N = \int_{\xi} \int_{0}^{z_0(\xi)} \mu(z, \xi, f_0) + \int_{\xi} \int_{0}^{z_1(\xi)} \mu(z, \xi, f_1)$$

$$N_X = \int_{\xi} \int_{z_0(\xi)}^{\infty} \mu(z, \xi, f_0) + \int_{\xi} \int_{z_1(\xi)}^{\infty} \mu(z, \xi, f_1)$$

- The export participation share is $N_X / (N_N + N_X)$

Laws of motion

$$N'_X = \delta_{X,X} \Pr(\text{continue}) N_X + \delta_{N,X} \Pr(\text{start}) N_N$$

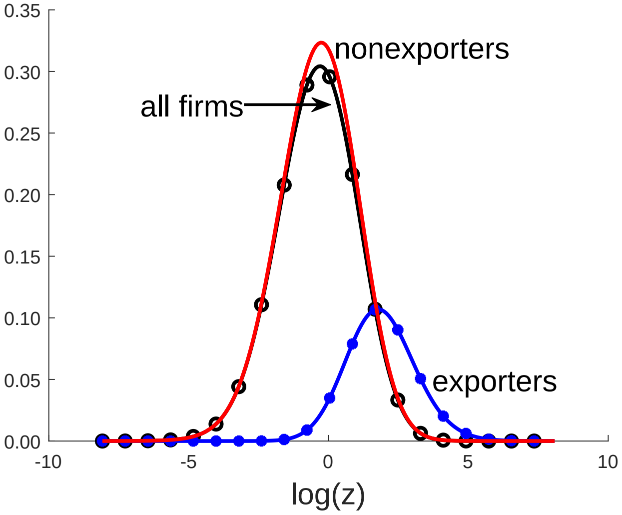
$$N'_N = \delta_{X,N} [1 - \Pr(\text{continue})] N_X + \delta_{NN} [1 - \Pr(\text{start})] N_N + N_E$$

- A more careful exposition would focus fully on

$$\mu'(z, \xi, f) = T(\mu(z, \xi, f))$$

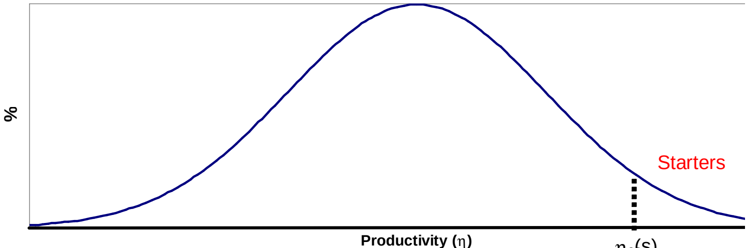
- See the appendix to Alessandria et al. (2021a) for details

Distributions

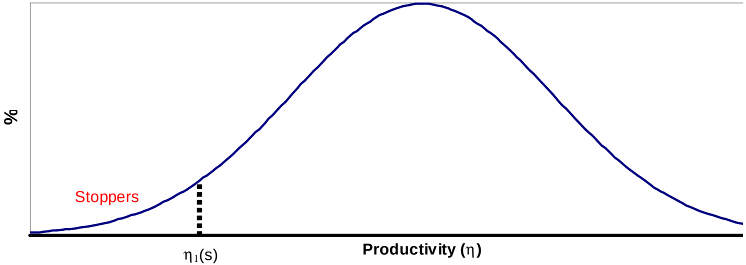


... with $f_0 > f_1$. What happens when $f_0 = f_1$?

Last Period's Non-exporters



Last Period's Exporters



$$N(s') = Pr(\eta > \eta_1(s)) * N(s) + (1 - N(s)) Pr(\eta > \eta_0(s))$$

Properties

- Crucial outcome of dynamic decision: $z_1(\xi) < z_0(\xi)$
  - Harder to break into exporting than to stay
- This generates
  - **Exporter hysteresis:** Firms continue exporting after conditions deteriorate
  - **Low exit rate:** Exporters will delay exiting to avoid paying the entry cost again
  - **Export Premium:** Exporters are larger than nonexporters
    - Increasing in the average fixed cost
    - Falling in the difference in fixed costs

Properties

- Consider Δ in current and future primitives abstracting from GE interactions
- Let's look at
  1. Trade barriers
  2. Uncertainty

Trade costs and Tariffs

- Consider three possible reductions in variable trade costs, either (ξ, τ)
  1. Current trade costs temporary
  2. Future trade costs permanent
  3. Current and future trade costs

Temporary current

- Experiment: $\tau_t \downarrow$, $\tau_s = \tau_{t-1}, \quad s = t+1, t+2, \ldots$
- Lowering today's tariff will shift down the $\text{LHS}_m(z)$
- Increasing entry and decreasing exit
- Through law of motion, trade will remain persistently high, only gradually mean-reverting

Permanent future

- Lowering tariff in the future will shift up the $\text{RHS}_m(z)$
- Increasing entry and decreasing exit today
- Trade grows in advance of liberalization
- Through law of motion trade will increase gradually

Permanent current

- Lowering tariff in the current will shift up the $\text{RHS}_m(z)$ and $\text{LHS}_m(z)$
- Combination of previous two shocks
- Increasing entry and decreasing exit today
- Trade grows by more on impact
- Through law of motion trade will increase gradually.

Uncertainty

- As in typical models with non-convexities, uncertainty matters (Dixit and Pindyck, 1994)

1. Current dispersion in productivity, $\sigma_z \uparrow$ [temporary]

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1. Current dispersion in productivity, $\sigma_z \uparrow$ [temporary]
   - Does not affect thresholds, but does affect distribution of ability today
   - Thicker tails $\rightarrow$ more entry and more exit
   - Volume of trade should increase since conditional mean of productivity $\uparrow$ (selection on a thicker right tail)
2. Future uncertainty/dispersion, $\sigma_z' \uparrow$ [permanent]
   - Shift up and flattening of the marginal gain curve

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2. Future uncertainty/dispersion, $\sigma'_z \uparrow$ [permanent]
   - Shift up and flattening of the marginal gain curve
   - Entry and exit fall, ambiguous effect on trade today and in the future

Success and Challenges

- Successes
  - Persistent export participation (fact #1)
  - Low export and entry rates (facts #3,4)
  - Dynamic macro adjustment (fact #7)
- Challenges
  - New exporters (too productive at entry, too likely to continue, and export intensity too high)
  - Connection in exporting across markets
  - High re-entry rates in monthly and longer frequencies
- Causes
  - Exporting technology too simple (parsimonious): f_0, f_1, ξ
  - Need to shift more investment into post-entry period and reduce depreciation

Resolutions: Starting and stopping

- Small new-exporters & low continuation rate
  - Let $f_1\left(t_e\right)$ be a decreasing function of t_e=age in market
- High re-entry data
  - Annual: Let firm that stops re-enter with $f_R \in [f_1, f_0]$
  - Monthly: set $f_0 = f_1$, hold goods in inventories at a cost abroad

Resolution: Export intensity dynamics

With CES

$$\mathit{exs}(z,\hat{\xi}) = \frac{(\tau\xi\hat{\xi})^{1-\sigma}}{1+(\tau\xi\hat{\xi})^{1-\sigma}}$$

- Modify iceberg cost structure so that they fall with experience
  - Alessandria et al. (2021b) assume firm enters at $\xi_H > \xi_L$ and then Markov transition between states
  - Reflects improvements in export distribution technology
- Alternatively could accumulate customers or build habit (Fitzgerald et al., 2023; Piveteau, 2021; Ruhl and Willis, 2017; Rodrigue and Tan, 2019)
- Both approaches have investments in improving market after entry, not just maintaining access
- Backloads profits which leads to lower estimates of entry costs.
- When growth process is uncertain, this makes it more likely to exit

References

Alessandria, George, Costas Arkolakis, and Kim J. Ruhl (2021a). “Firm Dynamics and Trade.” *Annual Review of Economics* 13 (1), pp. 253–280.

Alessandria, George, Horag Choi, and Kim J. Ruhl (2021b). “Trade adjustment dynamics and the welfare gains from trade.” *Journal of International Economics* 131, pp. 1034–58.

Baldwin, Richard E. and Paul R. Krugman (1989). “Persistnent Trade Effects of Large Exchange Rate Shocks.” *Quarterly Journal of Economics* 104 (4), pp. 635–54.

Dixit, Avinash K. and Robert S. Pindyck (1994). *Investment under Uncertainty*. Princeton University Press.

Fitzgerald, Doireann, Stefanie Haller, and Yaniv Yedid-Levi (2023). “How exporters grow.”

Piveteau, Paul (2021). “An empirical dynamic model of trade with consumer accumulation.” AEJ: Macro, forthcoming.

Roberts, Mark J. and James R. Tybout (1997). “The decision to export in Colombia: An empirical model of entry with sunk costs.” *American Economic Review* 87 (4), pp. 545–564.

Rodrigue, Joel and Yong Tan (2019). “Price, product quality, and exporter Dynamics: Evidence from China.” *International Economic Review* 60 (4).

Ruhl, Kim J. and Jonathan L. Willis (2017). “New exporter dynamics.” *International Economic Review* 58 (3), pp. 703–726.