Project 2

Ex1

1. The BC can be defined as follows:

$$X' = (X - C)(1+r) + Y'$$

Multiplying by $\frac{P_t}{P_{t+1}}$ & detrending:

$$\Rightarrow \quad x' = (x - c)(1+r)\frac{P_t}{P_{t+1}} + \varepsilon'$$

$$\Rightarrow \quad x' = (x - c)\frac{(1+r)}{(1+g)} + \varepsilon'$$

2. $$V(X) = \max_C \{ u(C) + \beta E_t V(X') \}$$

$$\Rightarrow V\left(\frac{X}{P}\right) = \max_c \left\{ u\left(\frac{C}{P}\right) + \beta E_t V(X') \frac{P_{t+1}}{P_{t+1}} \right\}$$

$$\Rightarrow V(x) = \max_c \{ u(c) + \beta E_t V_{t+1}(x_{t+1}) \cdot P_{t+1}^{1-\theta} \}$$

$$\Rightarrow V(x) = P_t^{1-\theta}\left[\frac{1}{1-\theta} c_t^{1-\theta} + \beta E_t V_{t+1}(x_{t+1}) \frac{P_t^{1-\theta}}{P_t^{1-\theta}}\right]$$

$$\Rightarrow V(x) = \left[\frac{1}{1-\theta} c_t^{1-\theta} + \beta (1+g)^{1-\theta} E\, V_{t+1}(x_{t+1})\right]$$

$$\text{or } V(x) = \max_c \{ u(c) + \beta (1+g)^{1-\theta} E\, V'(x') \}$$

3. → These transformation are useful since now we can remove a state variable & solve the problem using only cash in hand making it relatively easier.

→ If we solve for FOC we get:

$$c_t^{-\theta} = \beta \frac{R}{(1+g)^{\theta}} E_t [c_{t+1}^{-\theta}]$$

Therefore, for convergence we require that:

$$\frac{\beta R}{(1+g)^{\rho}} < 1.$$

- We require $s = x - c \geq 0$ since now we have a condition for precautionary savings.
Therefore at some point in the life cycle the household would like to borrow more than his savings. This is not possible according to the budget constraint.
Therefore, the household will respond by decreasing their consumption.

Ex 2

1. Pseudo code:

a) Set original parameters of the model.

b) Set an evenly spaced grid for cash in hand, such that $x \geq -\frac{\phi}{1+r} + c$. The code instead of using a equidistant grid space uses triple exponential grid.

c) Set $c = x$ & $a' = 0$ for period T. Now we can Solve for the eulers equation & iterate making sure we do not violate the BC.

d) This iteration involves interpolating over x and since c appears on both sides we have to use a nonlinear Solver.

e) Finally we can check for convergence & update the function to solve the problem.

- The simulation allows us to observe the dynamics over time. In particular we can check the evolution of our consumption policy & the eulers error. which permits us to make better approximation.
The eulers error is the error generated at each simulation; where

$$\varepsilon_t = 1 - \beta R E_t \left[\left(\frac{c_{t+1}}{c_t} u_{t+1} \right)^{-\theta} \right]$$

Therefore, by better estimating the error term we can get more precise approximations.

2. As we increase theta, the household becomes more risk averse, hence the precautionary savings increases leading to a fall in consumption today, changing the consumption policy over time.
Increasing the income shock, leads to an increase in the variance. Therefore, consumption is different over time and has a higher volatility. This happens since now the uncertainty is higher in the economy inducing higher consumption and a similar effect in x.

3. If we change the grid size the eulers equation error falls considerably. This is expected since, increasing the grid points we are approximating over more points making are estimation more precise. On average the errors fall from 0.0266 to 0.114. The time taken by using 50 grid points is approximately 4 seconds more. Lastly increasing theta, increases the average

error, since as agents are more risk averse the eulers equation would show a greater difference in Ct & Ct+1 Therefore increasing the error.

Ex 3

1 a) Howard's Iteration converges much faster than ~~Fct~~ Value function iteration. This happens since now we use the policy function to iterate and this decision rule applies for all periods, while in value function iteration we assume that the decision rule is only for 2 periods. This is the reason for quicker convergence. The main difference between these methods is the way we iterate to find convergence. In VFI we make initial guess of our value function at period T and ~~solve~~ iterate backwards. In Howards Improvement we identify our policy function and identify a value function given the decision rule. While in the eulers method ~~requires~~ requires that FOC is sufficient to solve the problem. The three methods are related since they all require the satisfaction of the ~~FOC~~. the maximization problem with the BC. They all use the Bellman equation to find the solution to the problem.

$\boxed{\text{Problem d}}$

$T = \infty$ Interpolate V_x.

1. For each $x' \in \mathcal{G}^x$, interpolate $V_x(x')$
2. Take expectation to evaluate $E V_x(x)$

- Plot $\underline{V_x} = u_c$ compare
  $\underline{\Lambda_x} = (V_x)^{-1/\sigma} = c$

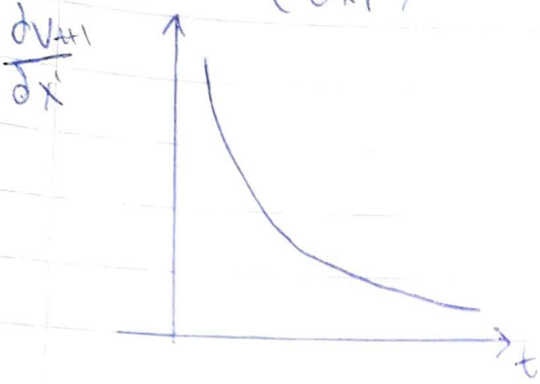
Code

```
upp1 = zeros(Ne,1)
for ec = 1:Ne
    xp1 = X - consumption (1+r)/(1+g) + ε(ec)
    upp1(ec) = func - wtp(gridx, Yofun, xp1)
```

Problem e

1. What could one possibly gain by interpolating on Λ_x instead of V_x?

$$\Lambda = \left(\frac{\partial V_{t+1}}{\partial x_1}\right)^{-\frac{1}{\theta}} = C$$



In this case, I guess Λ_x, is a more linear object than V_x; linear interpolation works better with linear objects.

The object on which im interpolating has a lot of curvature.

2. How many times do I need to evaluate the shadow value?

FOC $\quad u_c(c) = \beta R \cdot E_m \, \frac{1}{m} \left[\frac{\partial V_{t+1}}{\partial x'}(x', m') + \mu \right]$ shadow value

In each case $\mu = 0$, therefore I do not need to calculate it.

What is the lowest value of savings?

Rewriting the budget constraint as.

$$Q' \geq -\phi$$

$$Q' = (x-c)(1+r) = \sigma(1+r)$$

$$S = x - c \geq \frac{-\phi}{1+r}$$

Therefore, s must be ≥ 0, and the minimum value is $s = 0$ at period T.

What's the highest possible value of savings?

If we assume households will always consume the minimum value of ε and save the rest. We also assume that in each period they draw the best possible realisation of income shocks, therefore we can calculate the maximum amount of asset . And ultimately an alternative is to set an arbitrary x and then simulate the model. If x exceeds the simulations we must increase x.

Problem e

Exogenous grid method: Using the endogenous method consumption is in the right hand side of the marginal utility equation, therefore this method is quite costly, once we have to use non-linear systems of equations, using the exogenous method we can remove this problem, since consumption does not show on the right hand side of the equation, so the problem is no longer non-linear. Moreover, we know the lowest point in the saving grid where $S = -\frac{\phi}{1+r}$. Furthermore we know for the

of $c \in [\varepsilon, x + \frac{\phi}{1+r}]$, c is linear.

“Consumption over the Lifecycle” Gourinchas and Parker (2002).

This paper estimates a structural model of optimal life-cycle consumption expenditures the presence of realistic labor income uncertainty. Additionally, it estimates a dynamic stochastic model of the life-cycle saving behaviour of households when they face exogenous, stochastic, labor income processes. In their model, the optimal choice of consumption depends not only on lifetime resources, the real interest rate, and the discount factor, but also on the expected growth rate of income, so that consumer behaviour may vary systematically as households age. They focus on estimation of structural preference parameters and upon characterizing optimal behaviour when households face exogenous, stochastic, labor income processes.

Using weak identifying assumptions they construct average total consumption and income profiles across the working lives of households of five different educational attainments and four different occupational groupings, using high quality household-level data on consumption and income from a sample of roughly 40,000 households from the Consumer Expenditure Survey from 1980 to 1993.

They use the Consumer Expenditure Survey to construct life-cycle profiles of consumption and income. The Consumer Expenditure Survey contains information about consumption expenditures, demographics, income and assets, for a large sample of the US population. They use the household survey data alongside simulation techniques to estimate a structural model of inter-temporal consumption choice with realistic levels of income uncertainty. Each household contributes one data point to the sample. For each household they construct a measure of household income and consumption. Based on the characteristics of the household, they assign the household to an occupation group, an education group, a birth cohort, an interview year and a Census region.

The paper yields four main findings:

1. The fitted model matches the correlation between consumption and income at young ages and the general concavity of the profile that is observed in the data.
2. They find reasonable estimates of the preference parameters. The average household has a discount rate of 4%-4.5% and a coefficient of relative risk aversion varying between 0.5%-1.4%.
3. The paper contributes to the debate on the determinants of wealth accumulation. In our model, the relative shapes of the consumption and income profiles reveal the relative roles of precautionary and retirement motives for accumulating liquid assets.
4. They find strikingly different consumption behaviour for households at different ages: households behave like “buffer-stock” consumers early in the working lives and more like certainty-equivalent life-cycle hypothesis model households as retirement nears.

The model does much better in an economic sense than the certainty-equivalent life-cycle hypothesis model. In two places however, the model fit is not good. First, actual consumption exceeds simulated consumption early in life. Second, the actual consumption profile is slightly flatter and peaks slightly later.

They authors developed a new method for estimating household consumption behaviour in their mode. They model consumer behaviour in the presence of realistic levels of uninsurable income uncertainty and estimate preference parameters and household consumption behaviour using the Method of Simulated Moments. The model fits well and yields tight estimates of the discount rate and intertemporal elasticity of substitution.

Their results indicate that small holding of liquid assets by young households is an optimal response to expected income growth and the riskiness of future labor income over the life cycle. Until their early forties, household consumption behaviour, while fully optimal, appears short-sighted within the context of the certainty-equivalent life-cycle hypothesis model. Their results also imply that older households save actively for retirement purposes and behave in a manner more consistent with the certainty-equivalent life-cycle hypothesis model.