Problem Set 6
1 December, 2019 Name: Giada Bozzelli, Abhinav Khemka, Richard Sparkes

Quantitative Economics

Part A

1. 1. Solve the code with low values for the income shock sig2e=sqrt(eps) and the taste shock param.sigma=sqrt(eps) and look at the plot of the comparison between the explicit and the numerical solution for the three different solution methods determined by parameter opt.meth. Throughout, set opt.comp=1 so that the solution in opt.meth will always be compared to the global solution. With that setting, you only have to run opt.meth=2 and opt.meth=3.

   Solution:

   - Time elapsed method 2: 1.5921. Maximum distance from closed form solution: 0.15118
   - time elapsed for method 3 is: 14.9107. Maximum distance from closed form solution: 0.30769

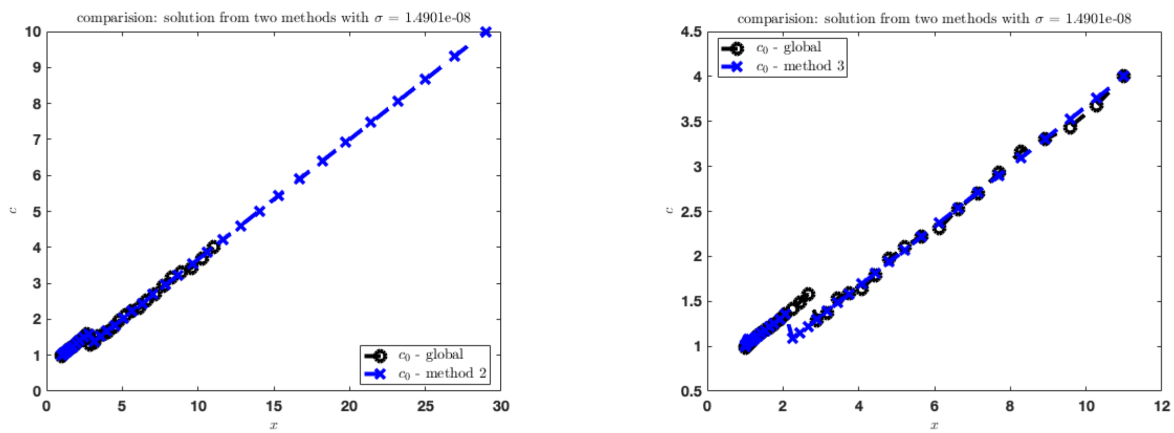

Figure 1: Comparison from two methods Global method 2 and Global vs method 3

   Interpretation:

2. Describe in words those three solution methods.

   Global Solution

   - Create a constant grid for xt
   - Create a grid for ct (initial guess)
   - To get value of consumption on each state:
     - Define utility function;
     - Define Value function as utility and define Budget Constraint;
     - In last period we have a discrete choice, so we can solve implicitly and get xT and then solve for the value function by interpolating on xT.
     - Therefore, if there is no taste shocks we can get the value function otherwise we
     - Now we can derive cash in hand in next period and interpolating on that we can get our next value function and so on.

- The we can update our consumption policy guess to get our consumption values for each state and find the maximum over the grid points.
- The last step is to use the golden search to find whether the maximum values satisfy the optimal conditions.

DC-EGM Solution

- Create an endogenous grid on savings.
- Derive policy functions for consumption at terminal period.
- Now we can use backward iteration using EGM.
- Loop over value of exogenous grid on savings, calculate next period assets and choice probabilities.
- Calculate optimal choice-specific consumption in next period and value function in period t+1.
- If no taste shock we maximize, otherwise we take the log sum.
- Compute current period optimal consumption from Euler equation and value function for current period.
- Compute endogenous point.
- After that check monotonicity of endogenous coh grid, and if necessary apply correction according to Upper Envelope Theorem.

EXOGM Solution

- Define constant grid for cash on hand.
- Derive FOC conditons.
- Derive policy functions and value function at terminal period.
- For every next period income, define next period cash at hand and for every possible shock interpolate over final consumption, final value function, cash at hand in next period to obtain value function in period t+1.
- If there is no taste shock we can maximize over the specific value function, if there is taste shock we take the log sum.
- We must check if the borrowing constrain is binding.
- Then for given consumption and cash at hand we can compute value function.
- Interpolate on tomorrow's choice specific value function.

3. The code only compares the closed form solution in period 0 to the numerical solution. Do so also for the closed form solution in period 1. Explain your findings.

   Solution We know the closed form solution for consumption in period 0 and the closed form solution for assets in period. Using this we can derive the closed form solution for consumption in period 1. Where C1= 2*CO +1 if there is a taste shock. We also have the solution for C1 if there is no taste shock. Adding this to the code we can get the consumption policy when t=1. We find that now the analytical solution further diverges from the numerical solution. Why ??

4. Measure the running times of the three alternative methods. Report and interpret your findings.

   Solution

- Global solution time: 26.5351
- EGM time:1.7803
- Root finder (EXOGM) time: 17.9554

Interpretation The global solution method takes the longest since we first make an initial guess for consumption and then interpolate on cash in hand and update our value functions to get our consumption policy. While, in the case of EXOGM we use the grid of cash on hand to get our consumption policy using the Euler's equation and a root finding tool. This saves us time since we reduce our computation by one dimension. The EGM method is significantly faster then the other two methods since now we interpolate on savings and save time on costly non linear root finding processes.

5. Why does opt.meth=3 have problems in characterizing the solution?

   Solution The main problem of using the endogenous grid method is that it cannot determine the region where, the budget constraint is unbinding. This could be solved by adding grid points at the region where, we get this issue but the code does not account for that. Due to this lack of determination of the borrowing constraint at the kink, we can observe the lack in accuracy in EXOGM method.

6. What happens if you increase the variance of the income shocks to sig2e=0.01 under the three methods? Interpret your findings.

   Solution In general, with income shocks the discontinuity of the value function reduces, but the problem still remains. For example we report the comparison of Method 3 with global in the two cases.

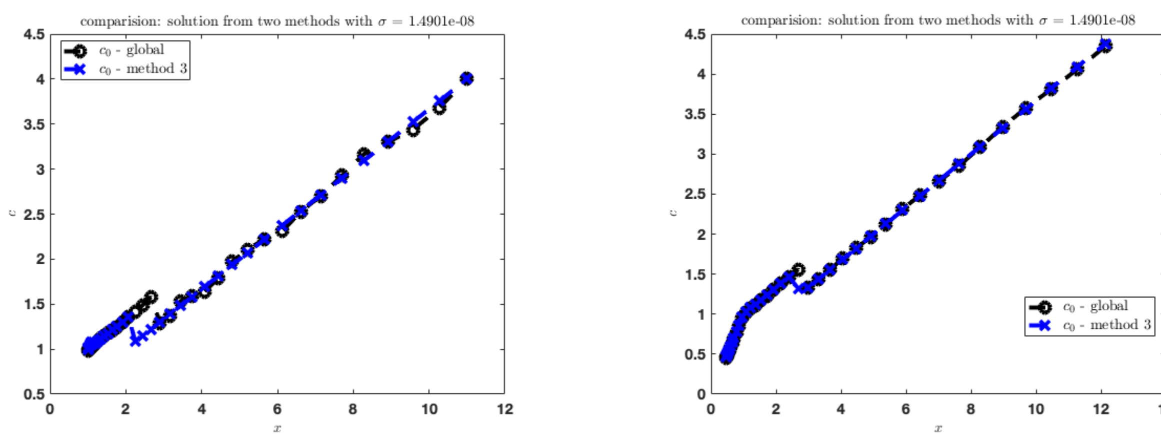

Figure 2: Comparison from two methods with higher income variance shock

With higher variance consumption and saving policies functions are going to be binding by the Budget constrain. For example, we report the consumption and saving policies for method 1.

7. Now increase the variance of the taste shocks to sigma=1.0, and set the variance of the income shocks back to sigma.square=square-root(epsilon). Interpret your findings.

   Solution Increasing the variance of taste shocks reduce discontinuity even further, as example we can again compare Method 3 with global method in the two cases.

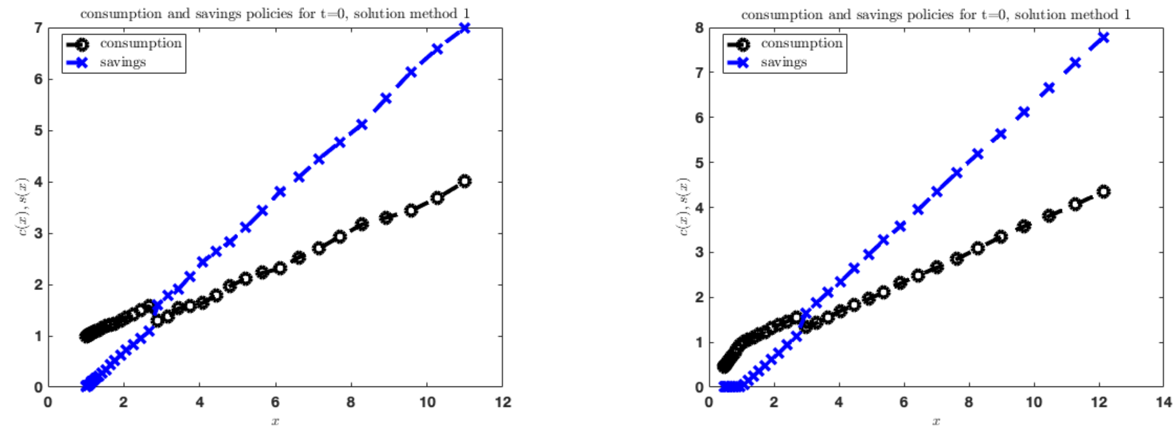

Figure 3: Comparison policies functions with higher income variance shock

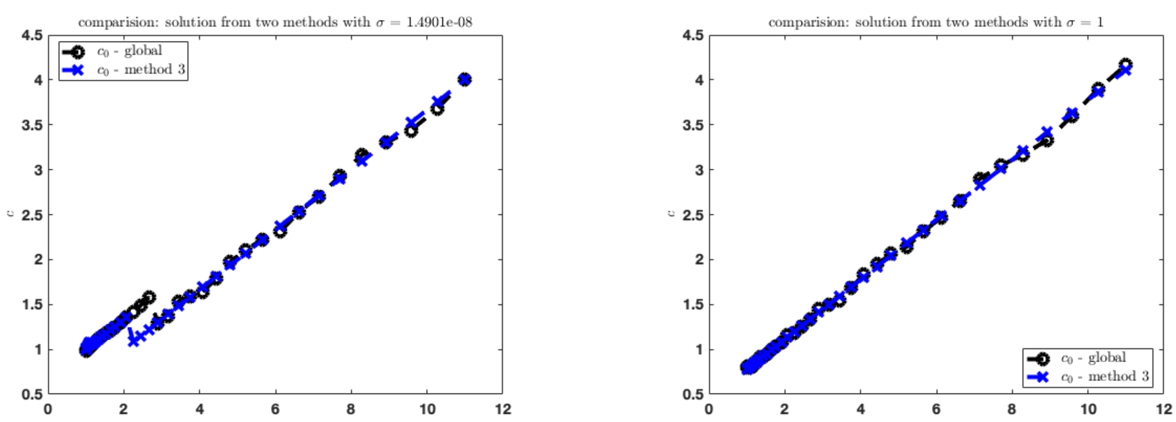

Figure 4: Comparison from two methods with higher taste variance shock

Part B 1

8. Explain all sections of the code.

   Household Problem

   - Define a grid for savings which is independent on type such as age.
   - Define a grid for cash on hand that is dependent on type. Define minimum and maximum cash on hand
   - Derive policy functions for terminal period for consumption and assets. Also we can define the value function and its derivative. Now using the previous step we can iterative backwards where,jc=nj-1:
   - The first step for the iteration is to derive consumption for all states. In order to do this first we derive the Euler's equation analytically.. Now we can interpolate using savings for all cash on hand not belonging to grid by using the inverted value function to achieve linearity.
   - Now that we have derived our consumption for all states we can derive a grid for cash in hand and update our functions for consumption and assets for all states.
   - Using the above step we can update the value function and its derivative for all points on the grid by interpolating. Aggregation:
   - For a given prices, rate of interest, pension system we can derive an aggregate for assets.
   - The first step is to create a distribution for assets conditional on age and income shock. Defined as phi in the code
   - Now we can define a asset holding and income for newborns. Using this we can find the income and asset holdings for each period and age using a transfer distribution function.
   - The next step we check if the distribution for each country sums to 1 and create a big enough grid to store the aggregations.
   - Now that we have the income, consumption and assets holding for each age we can finally do the aggregation.
   - For this we calculate the total living population and using the above steps now find the aggregate income, assets holding and consumption for the economy.

9. Compare the solution procedure in funchouse to a standard implementation of the exogenous grid method described in the lecture notes. Where are the differences? What is the advantage of working with a savings grid? What is the disadvantage?

   Solution The main difference is that with EXGM we use cash on hand grid. Since consumption appears on both side of Euler equation, the method is quite costly once we have to solve a system of non-linear equation.

   The advantage of working with a saving grid is that we can remove consumption on right side of Euler equation, so the problem is no longer non-linear. In theory, ENDGM is 8 times faster than EXGM. The speed advantage of ENDGM relative to EXGM is achieved because the mapping from saving to (assets next period, consumption) has a closed form solution.

   The disadvantage and the problem comes with higher dimensions. When we have to compute today assets holdings we can observe that, for some regular grids on the saving grid, the grid of assets today is generally irregular. In subsequent iterations, we need to interpolate on such irregular grid. This does not cause problem with one dimension, but the problem arises with more dimensions.

(Reference: Endogenous Grids in Higher Dimensions: Delaunay Interpolation and Hybrid Methods,Alexander Ludwig Matthias Schon (2014))

10. Implement the standard method of exogenous grid points by working with an exogenous grid for cash-on-hand x. Compare the running time to the endogenous grid method.

    Solution Exogenous grid time elapsed: 38.0655 When we compare EXOGM with EGM the time increases drastically. This is expected since now we have a non-linear solution and need a root finding method.

11. Have a look at funcaggr. What is the role of the objects TT and Phi? Compare this with the corresponding definitions in the lecture notes. Both these objects are defined and used in lines 402-434 in the code. Opening the loops for xc,yc and ycc twice is actually inefficient. Defining separately the object TT can be avoided by merging the steps of constructing TT and then looping forward on Phi. Modify the code by combining the steps and measure the gain in running time. Also, you can exit the for loop if the current measure $Phi(jc-1, xc, yc) = 0$. Implement this by using the command break. Measure the gains in speed from these modifications.

    Solution Phi is the sequence of cross sectional distribution which is depended on age and income shocks. TT is the transition function that is induced by the Markov process of the income shocks and assets. It summarizes how individuals move within the distribution over ages, income and assets from one period to the next.Using this we can simulate the distribution required to find the assets holdings for the economy. Modifying code by removing extra loop:

    - Original Time elapsed:11.1661
    - **Modification 1** Time elapsed: 10.2254

    As we can observe there is a significant improvement in time by removing the extra loop. Now if add the modification Phi(jc-1,xc,yc)=0:

    - **Modification 2** time elapsed: 9.9537

    We can see there is a further improvement in time if we add this extra modification.

12. Solve the model under the following three alternative settings with the options optdet and optnosr:

    Solution

    - Model 1: optdet=true and optnosr=true :

    This is the case when we have the deterministic model with no survival risk. In this model since now there is no discrete choice and survival risk the household can smooth their consumption better. So we can observe that the agent accumulates assets when he is working and then uses this after retirement.Therefore, now we can see that the household can consume the same amount over his lifetime. To calibrate the model we can derive the Euler equations using the budget constraint. Then since we know our policy functions at the the last period we can iterate backwards to find the solution for each period to find our consumption at each state interpolating on savings. Then we can get an updated value function and policy functions and solve the problem. When r =rho we see no change in the savings profile as we change tetta: When r is significantly different from rho: We can observe that the household starts saving at

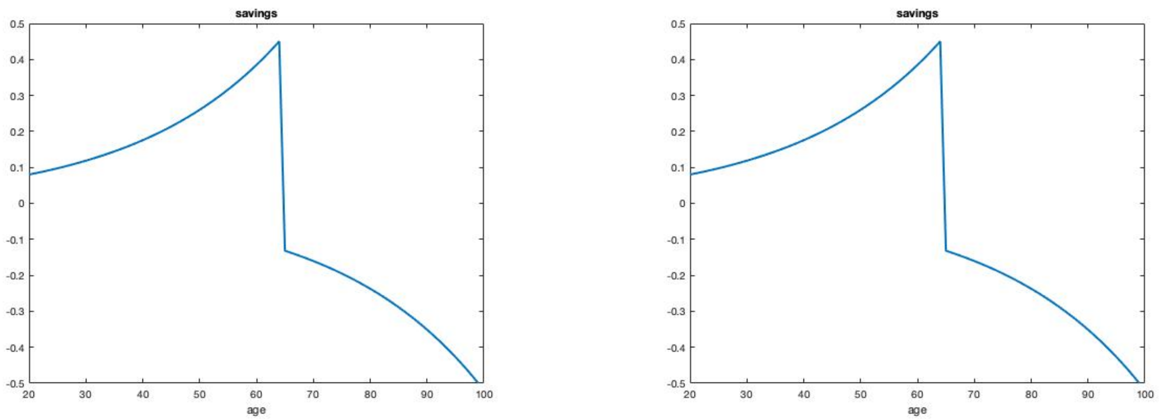

Figure 5: Comparison for different thetas

much earlier age and saves higher amount before retirement as tetta increases as we expect due to a rise in risk aversion.

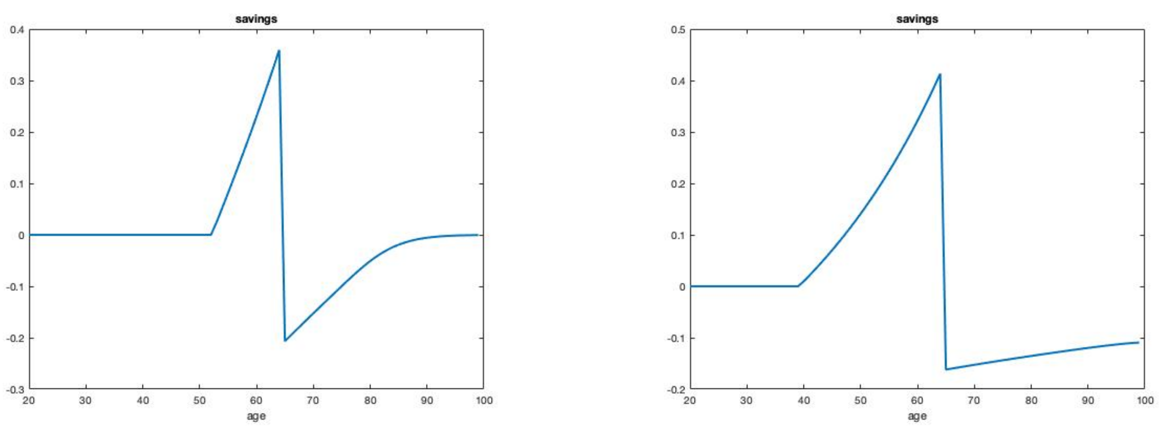

Figure 6: Comparison for different thetas and r is different from rho

- Model 2: optdet=false and optnosr=true :

This is a discrete model with no survival risk. Since in this model we add the choice of working the household saves more during his working years therefore, increasing his savings and asset holdings when the household works (at young ages) then consumes a constant amount after retirement since there is no survival risk. In comparison to discrete choice with survival risk model we can observe in that case that consumption fell after retirement since the probability of survival reduces as you grow older. Moreover, with no survival risk savings after retirement declines reaching value at age 100.

When r =rho we see now as tetta increases the household saves much more in his working age. This is because he his risk averse and now he has a risk associated to the discrete choice problem. When r is significantly different from rho: We can observe that the household starts

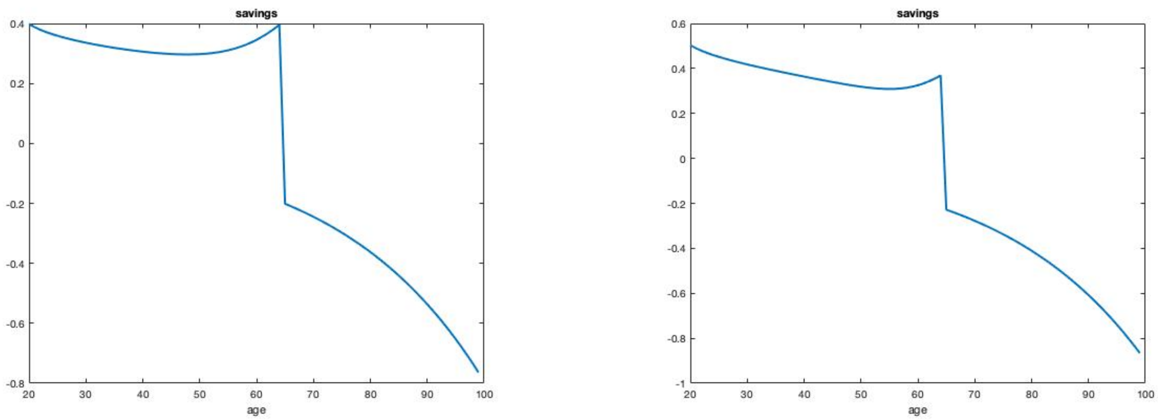

Figure 7: Comparison for different thetas

saving much more then in the last case at early ages but after a certain age has declining savings due to having no survival risk.

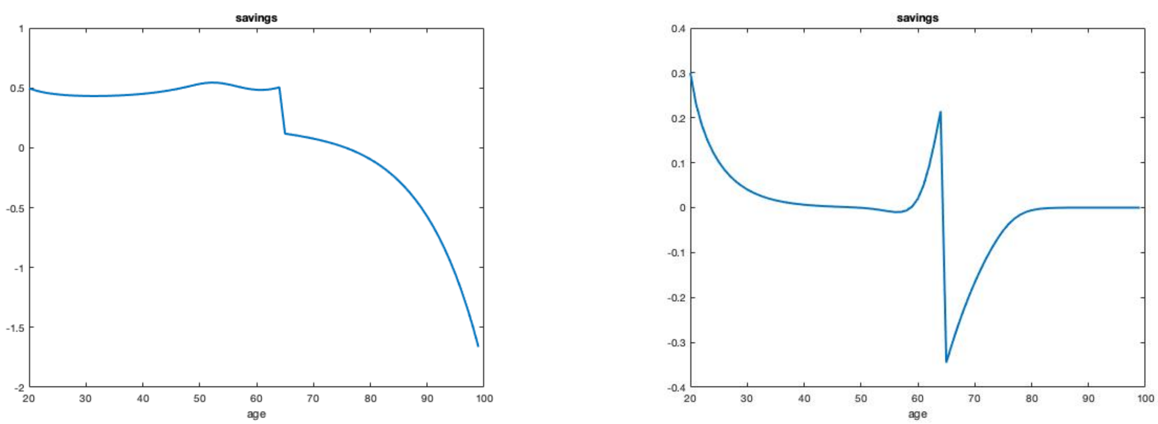

Figure 8: Comparison for different thetas and r is different from rho

- Model 3: optdet=true and optnosr=false:

In this model now we have the deterministic case with survival risk. We can observe consumption is no more hump shaped but it declines with ages and this is due to survival risk. Over the working life household decide to hold more assets in comparison to discrete case. Savings instead increase with working age, young household decide not to save, they prefer to consume.

Holding r equal to rho, increasing theta savings do not change significantly. Instead, if we make r significantly different from rho we can observe household prefer to save and smooth consumption after retirement.

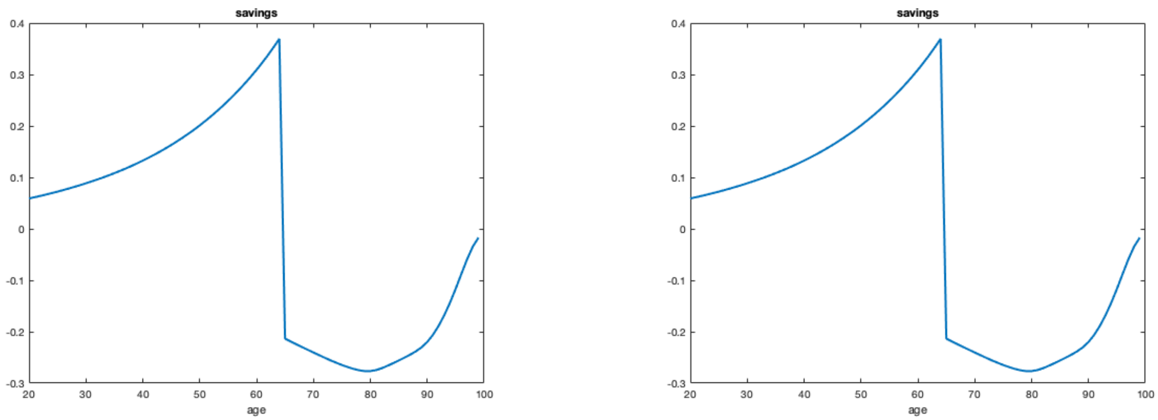

Figure 9: Comparison for different thetas

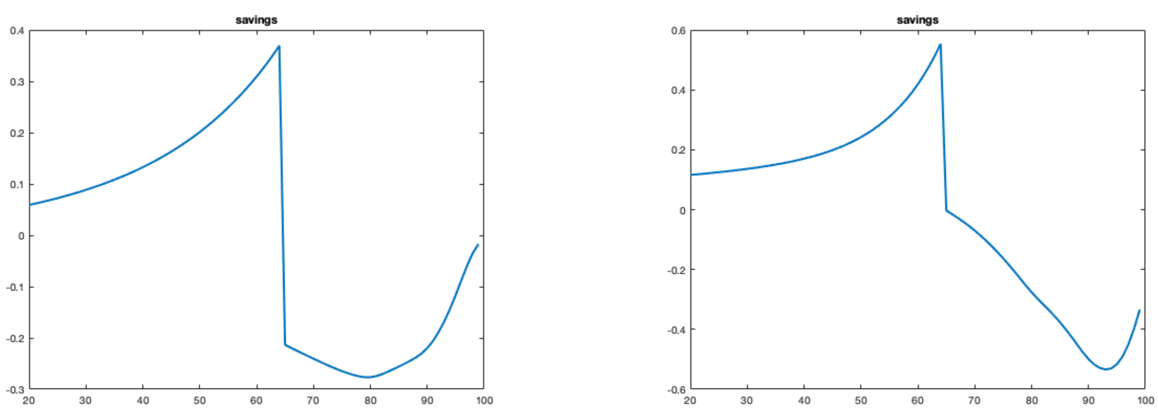

Figure 10: Comparison for different thetas and r different from rho

Part B2

13. Why can an OLG economy with idiosyncratic risk be dynamically inefficient? Make sure that the economy is dynamically efficient when there is no pension system.

    Solution Dynamically inefficiency refers to a dynamic economy which is not Pareto efficient, meaning that there could be transfers such that at least one household is better off without making other households worse off. Such a situation could happen if the economy accumulates too much capital relative to the golden rule level. In an OLG model the equilibrium capital stock maybe higher than the one in Complete Markets due to over-saving for idiosyncratic risk paired with precautionary savings. This implies also a too low rate of return. In such a situation it's possible for a government policy to improve the overall welfare with a security system. In OLG models with infinite horizons Pareto efficiency is violated.

    Reference: Lecture Notes

14. Augment the code by a welfare analysis for the life time utility of newborns by calculating the consumption equivalent variation between alternative policy scenarios (that you consider next).

    Solution

Part B3

15. Extend the code to a version that accounts for transitional dynamics. First, provide a detailed write-up (=something like a script code) that describes how you would implement the solution.

    Solution

    - Fix T;
    - Solve for Initial Steady States and Final Steady States;
    - Interpolate between initial interest rate and final interest rate to get the sequence of interest rate as the initial guess;
    - For each iteration:
    - (a) Calculate wages;
    - (b) Given wages and an interest rate, solve the Household problem;
    - (c) Since we know the Value Function in the last period, we can derive all the Value Functions, policy functions for consumption and savings by iterating backwards.
    - (d) Since we have the policy function for savings, we can derive the transition function and get the distribution for Phi by iterating forward.
    - (e) Using the previous steps, we can know derive the Aggregate Capital Supply.Setting this equal to total capital we can update the rate of interest for each iteration;
    - (f) Convergence check: if the absolute distance of the updated interest rate and the initial guess of interest rate is small enough.
    - Upon convergence: we can check whether the norm between aggregate capital supply and capital is less than epsilon. If not, we increase the time horizon.

    Reference: Kreueger and Ludwig 2007 JME

Part C

- **Krueger and Ludwig (2016)**

In this paper they compute the optimal tax and education policy transition in an economy where progressive taxes provide social insurance against idiosyncratic wage risk, but distort the education decision of households. They find that optimal education policies are always characterized by generous tuition subsidies, but the optimal degree of income tax progressivity depends crucially on whether transitional costs of policies are explicitly considered and how strongly the college premium responds to policy changes in general equilibrium.

The optimal tax deduction was d=10 percent of average income and optimal marginal taxes decrease from a status quo of 27.5 percent to 23 percent. The finding that an explicit consideration of transitional dynamics in the analysis of education finance reform in models with endogenous human capital accumulation is potentially very important for optimal tax design is the main quantitative conclusion of this paper.

The paper aims at characterizing the optimal progressivity of the income tax code in a life cycle economy in which the public provision of redistribution and income insurance through taxation and education policies is desirable, but where progressive taxes not only distort consumption-savings and labor-leisure choices, but also household human capital accumulation choices. They assume that financial markets are incomplete in that there is no insurance available against idiosyncratic mortality and labor productivity shocks. Households can self-insure against this risk by accumulating a risk-free one-period bond that pays a real interest rate of r. In equilibrium the total net supply of this bond equals the capital stock Kt in the economy, plus the stock of outstanding government debt B.

They find that lower-ability households go to college for a smaller range of initial assets than do high ability households. They also find that a higher non-college wage reduces the incidence of attending college. As parental transfers b increases the borrowing constraint is relaxed and even the less able households decide to go to college. Additionally, sufficiently wealthy households that expect to derive a dominant share of their lifetime income from capital income find it sub-optimal to invest in college and bear the time and resource cost in exchange for larger labor earnings after college.

The paper shows that the increase in the college subsidy and the reduction in marginal (and average) tax rates for high income earners induces a college boom on impact. The share of the youngest cohort completing college rises from 44 percent in the initial steady state to about 67 percent in the first period of the policy transition. The model used showed that although the share of the youngest cohort going to college increases immediately by close to 60 percent on policy impact, it takes approximately two generations (roughly 60 years) until the overall skill distribution has reached a level close to its new steady state value.

Due to the compelling empirical evidence for imperfect substitutability of skilled and unskilled labor that they find they conclude that the case for education subsidies and progressive labor income taxes being substitutable policies is strong.

In conclusion the authors argued that a large education subsidy and a moderately progressive labor income tax and constitute part of the optimal fiscal constitution once household college attendance decisions are endogenous and transitional dynamics are modelled explicitly. They conclude that the transitional dynamics aspect can have a crucial impact on the optimal policy.

- **Abbott, Gallipoli, Meghir, and Violante (2019)**

The paper examines the equilibrium effects of college financial aid policies building an overlapping generations life cycle model with education, labor supply, and saving decisions. The current system of federal aid improves long-run welfare by 6 percent.In this paper the authors build a life-cycle, heterogeneous-agent model with incomplete insurance and credit markets of the type popularized by Rios-Rull (1995) and Huggett (1996), featuring inter-generational links in the tradition of Laitner (1992) and set in an overlapping generations context.

Cognitive and non-cognitive skills, transmitted across generations, determine the non-pecuniary cost of education for students and productivity once entering the labor market.

The paper studies the impact of financial aid policies on college attainment, welfare, and the aggregate economy. Central to their analysis is the role of liquidity constraints and uninsurable income risk, policy-induced crowding out of private sources of funding, heterogeneity and selection, and general equilibrium feedback's.

They establish that the model fits the data along a number of crucial dimensions that are not targeted in estimation. For example, cross-sectional life-cycle profiles of the mean and dispersion of hours worked, earnings, consumption, and wealth are consistent with their empirical counterparts.

Their estimated model also implies non-trivial welfare and efficiency gains from further expansions of grant programs. An additional 1,000 dollars of grants per year for every student leads to a long-run increase in GDP of close to 1 percent. While some of this gain derives from increased college attainment, a substantial part also arises from stronger sorting into college based on ability, which is efficient in the model.

Applying the estimation approach of Goldin and Katz (2007) to data simulated from their model, the log college/high-school wage differential is estimated to be 0.58, and the HS graduate - less than HS log wage differential is 0.37. These values are close to the estimates presented in Goldin and Katz (2007) for the year 2000 which place the college premium between 0.58 and 0.61, and the high-school premium between 0.26 and 0.37.

College attainment in the model is strongly and positively correlated with reported parental income and net worth. For example, children whose parents are in the fourth quartile of the wealth distribution are nearly three times as likely to become college graduates as those whose parents are in the first quartile. More importantly, low ability children from wealthy families are much more likely to attend college than similar children from less wealthy families.

The main conclusion from the results of their model is that the current configuration of federal loans and grant programs has substantial value in terms of both output and welfare. Their results indicate that further expansions of grant programs would be welfare improving. Amongst the alternative policies they consider, the best way of expanding student aid is via ability tested grants. Part of the efficacy of this form of grants is that parental ability and education interact positively in the production of skills of the next generation.