Final Project

Abhinav Khemka

January 2020

1 Simple KS Algorithm

1.1 Proof for Proposition 3

Guess and verify method

- Since all households are identical we can make a guess for assets at t+2.
- Using the above guess we derive the law of motion of capital.
- Then we can derive the equation for consumption today and tomorrow by substituting the guess for assets.
- The next step involves substituting the budget constraint for consumption tomorrow.
- Now we can use the Euler's equation and substitute the consumption we found in the previous steps. This finally will give us an equation for savings that will be function of phi and beta. Where, phi is an expectation in terms of eta,alpha,tau and lambda.

Formal proof for proposition 3 is as follows:

- Using the guess and verify method we get the law of motion of assets tomorrow as:
$$a2_{t+1} = s(1-\tau)w$$
- Now we can substitute the wage ewuation and γ to get capital tomorrow:
$$k_{t+1} = \frac{s(\tau)(1-\tau)(1-\alpha)\zeta_t k_t^\alpha}{(1+g)(1+\lambda)}$$
- Using the step 1 guess we can get the consumption today and tomorrow by substituting the BC,rate of interest and social security formula.
- Now we can finally substitute the consumption from last step into the Euler's equation which is given as
$$1 = \beta E \frac{c_t(1+r_{t+1})}{c_{t+1}}$$

- if $\lambda > 0$, the argument of $\Phi >= 0$ which ensures $\Phi <= 1$, and this implies equation in the paper which gives savings as follows:

$$s = \frac{\beta\Phi}{1 + \beta\Phi}$$

1.2 Law of motion of capital

To simulate the law of motion of capital the following steps were applied:

- Setup initial parameters and shock distribution according to specification provided.
- Derive phi using the formula provided in the paper. This gives us the savings function.
- Calculate capital today as a function of savings,tau and alpha. Since now we have capital today and we can then derive the law of motion of capital according the equation 1.

1.3 KS Algorithm

a. To find the theoretical values for ψ_i we can use equation 1 and 2 where :

$$\psi_0 = log(saving) + log(1 - \tau) + log(zeta_t)$$

$$\psi_1 = \alpha$$

b. The following steps were applied to solve the KS algorithm:

- Setup initial parameters and shock distribution according to specification provided.
- Setup a grid for capital given the specification and policy functions for savings and capital tomorrow accordingly.
- Given the guess of psi in part a and kt we can derive the law of motion of capital.
- Now we can derive the Euler's equation using the constraints in equation 1-3 in the paper where the only unknown parameter is assets. Therefore, by using a unit root solver we can find our policy function for savings.
- Now that we have are policy function we simulate capital, consumption today and tomorrow for a given t.
- In the next step now we can find psi by running regressions for capital in both states for a given t today and tomorrow burning 500 iterations.

- Finally we can iterate using the psi we find and our psi policy function until we get convergence in order to evaluate our consumption and utility functions in each state.

c.Comparison of theoretical and empirical solutions:

$$\psi_0 = \begin{bmatrix} -1.024 & 0.3 \\ -8.139 & 0.3 \end{bmatrix}$$

when $\tau = 0$:
when $\tau = 0.1$:

$$\psi = \begin{bmatrix} -0.667429 & 0.303729 \\ -0.551233 & 0.304805 \end{bmatrix}$$

$$Savingsgrid = \begin{bmatrix} 0.300384 & 0.302413 & 0.303913 & 0.305109 & 0.306108 \\ 0.33984 & 0.340523 & 0.341038 & 0.341456 & 0.34181 \end{bmatrix}$$

$$\psi = \begin{bmatrix} -7.915 & 3.051 \\ -7.595 & 3.051 \end{bmatrix}$$

$$Savingsgrid = \begin{bmatrix} 0.33414 & 0.334766 & 0.335242 & 0.33563 & 0.33596 \\ 0.345434 & 0.345844 & 0.346157 & 0.346415 & 0.346634 \end{bmatrix}$$

First the values for psi are consistent.We can observe for both cases the magnitude is correct although psi0 is more negative in the case where tau is equal to one. Savings increase with pension this is unexpected since we would expect lower savings.

d Welfare analysis

$$g = \frac{V1 - V0}{\beta} - 1$$

V0 corresponds to the case tau=0.0, while V1 to tau=0.1. My value for g=-0.78 approximately indicating the welfare loss given the pension has a higher effect than insurance.This could be affected by the risk aversion of the household.

2 Complex KS Algorithm

I tried implementing the algorithm in Project 3 but after multiple attempts it still gives me errors. I have stated below the approach I would follow for this problem: In the complex KS algorithm the main difference is that now the policy function are 5 dimensional and depends on age,assets,eta,z and capital. To implement this we can apply the following algorithm:

- After setting up the framework as before we can take capital steady state as before. This will give us the law of motion of capital.
- Now we can modify the household problem to solve for policy function taking into account now we have a new function.

- the next step would be to solve the household problem by backward iteration and get our policy function and then we can use the aggregation and check for convergence as in the previous exercise.