Quant Macroeconomics Homework 5 Name: Giada Bozzelli, Abhinav Khemka, Richard Sparkes

Question 1: Factor Input Misallocation

1. In this part of the exercise we plot the joint density in logs and in levels: assume that $\ln z_i$ and $\ln k_i$ follow a joint normal distribution. Assume that the correlation between $\ln z_i$ and $\ln k_i$ is zero, the variance of $\ln z_i$ is equal to 1.0, the variance of $\ln k_i$ is equal to 1.0, and that average s and k is equal to one.

Figure 1: Joint Density in logs

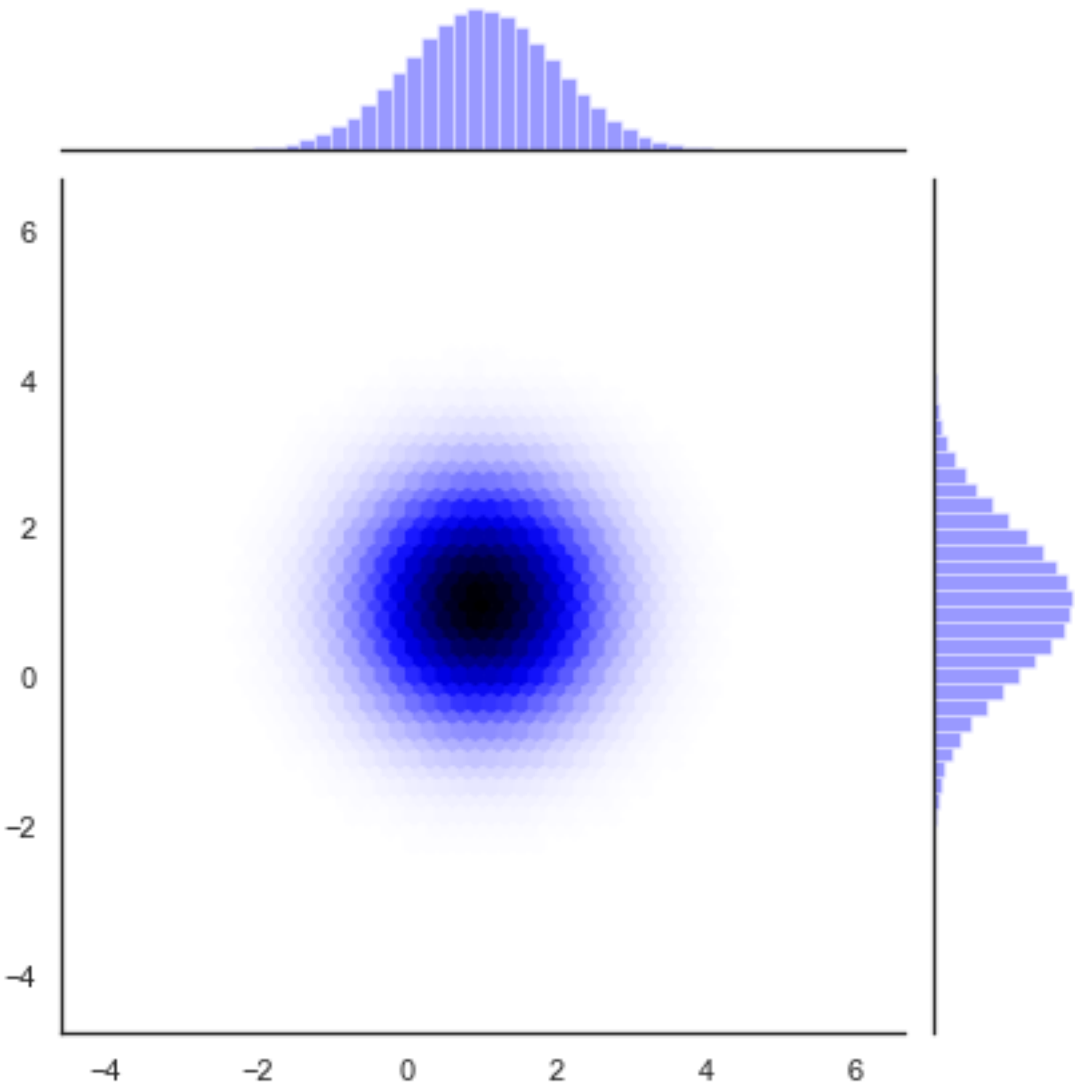
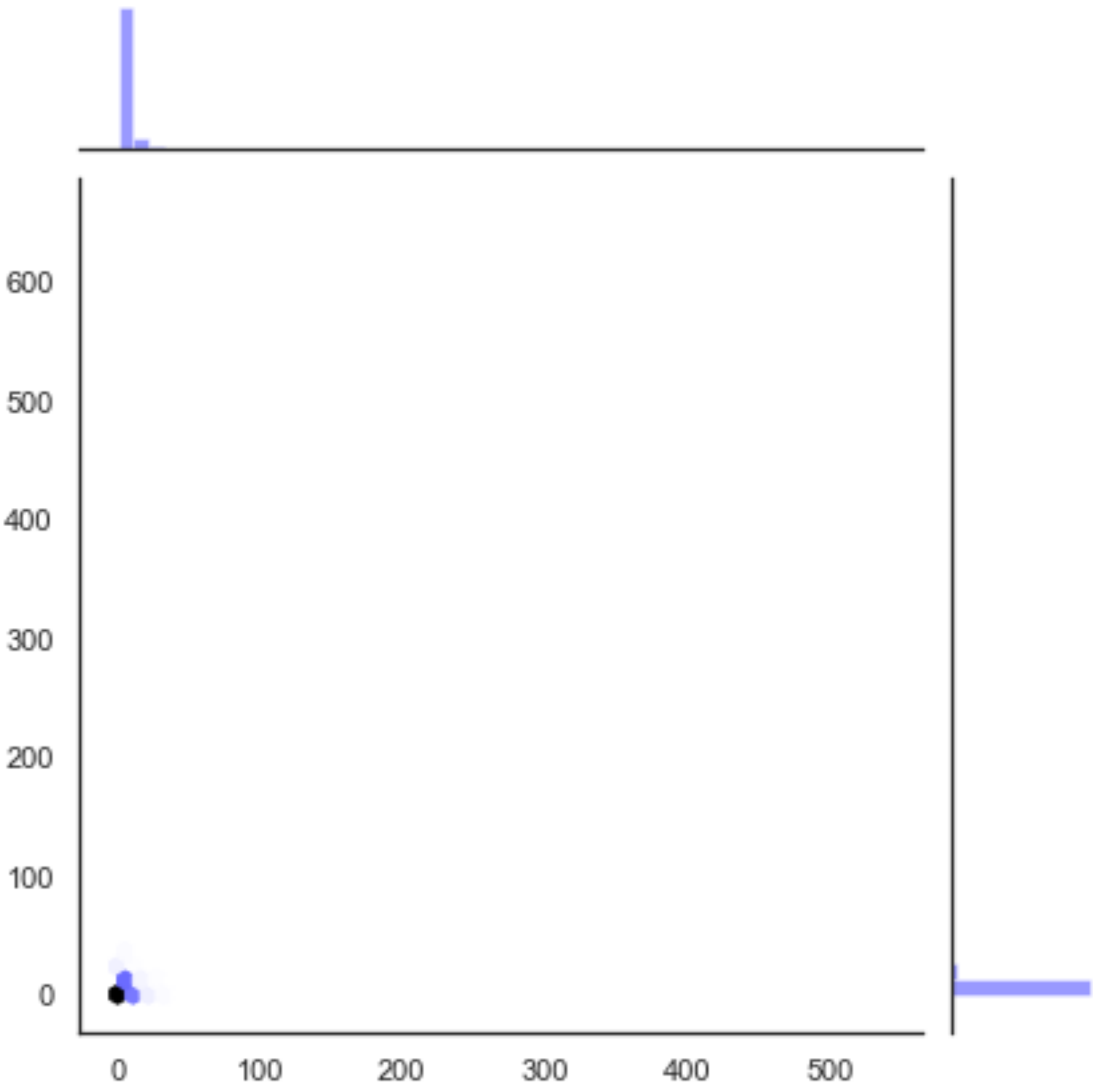


Figure 2: Joint Density in levels



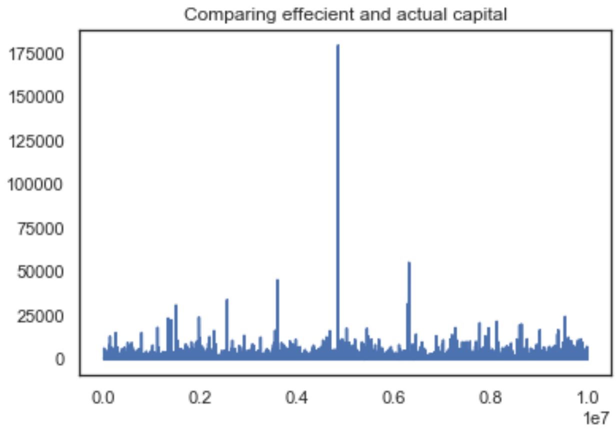
2. We then follow Problem Set's instructions and we compute firm output y_i for each of your observations, and we solved the maximization problem. We took FOC from lecture slides:

$$k_1 = \frac{z_1}{\sum_{i=1}^{I} z_i} \sum_{i=1}^{I} k_i = \frac{z_1}{Z} K$$

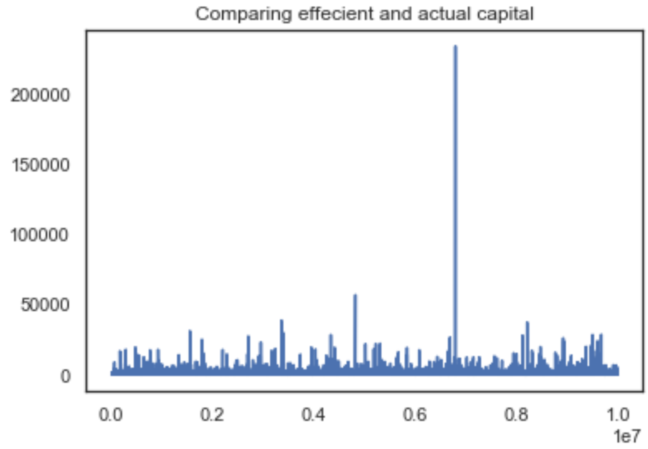
3. Compare the optimal allocations k^e against the data: the optimal allocation of capital is different from the data as expected. Figures 3 and 4 represent the difference between optimal capital allocations and the data with gamma 0,6 and 0,8 respectively.

Figure 3: Gamma = 0.6

Correlation 0



Correlation 0.50



Correlation -050

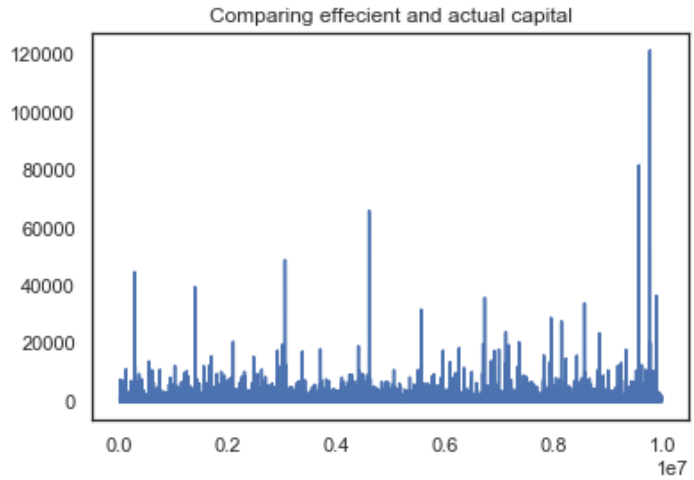
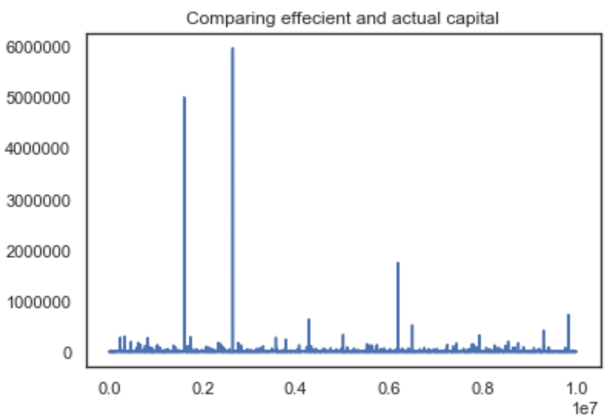
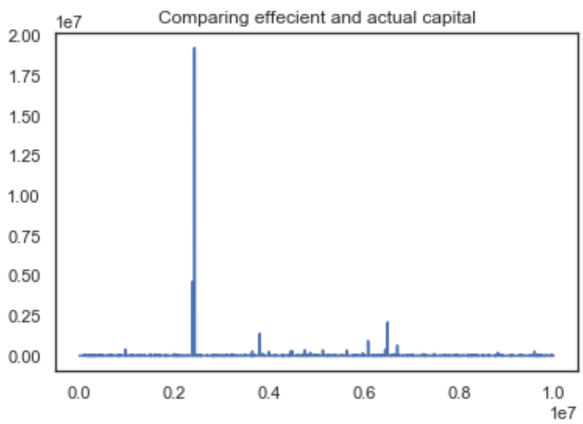


Figure 4: Gamma= 0.8

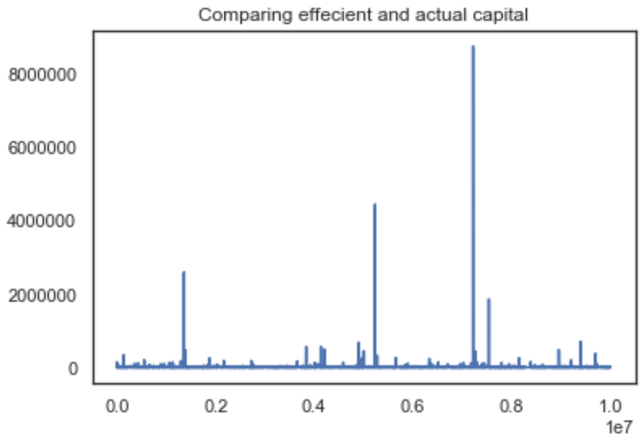
Correlation 0



Correlation 0.50



Correlation -0.50



From the graphs we can see that a higher value of gamma reduce the difference between efficient and actual capital, and this is positive since we are getting closer to the efficient capital at equilibrium. Moreover, correlation of 0 and -0.50 work better than a positive correlation.

4. Table 1 describe the result in terms of output gains with different correlations and different span of control, i.e. when gamma = 0.8 and when gamma = 0.6.

Table 1:

OUTPUT GAINS			
	Correlation 0	Correlation 0.50	Correlation -0.50
Gamma = 0.6	137.69	77.4	221.6
Gamma = 0.8	620.0	455.7	1040.2

The output gain increases for all cases when compared to the actual data. The gain is drastically higher with a rise in gamma. As the correlation falls we can also observe that the output gain increases.

Question 3. From Complete Distributions to Random Samples

1. Randomize the sample with 10,000 observations: Some initial results are presented in Table 2.1 which report the variance of capital and productivity for different random sample size. Moreover, Table 2.1 reports the correlation and the covariance matrix of the sample.

Table 2:

VARIANCE Sample				
	Pop. 10000	Pop. 1000	Pop. 100	Pop. 100000
Capital K	1.01	1.003	0.875	0.998
Productivity Z	1.01	0.966	0.806	1.00

CORRELATION Sample				
	Pop. 10000	Pop. 1000	Pop. 100	Pop. 100000

Capital K and Productivity Z	1.01 Covariance sample [[1.019 ,0.013] [0.013,1.018]]	0.04 Covariance sample [1.004,0.042] [0.042,0.967]]	-0.036 Covariance sample [[0.884, -0.036] [-0.036, 0.814]]	Very small negative number Covariance sample [[9.98119139e-01 4.23070733e-05] [4.23070733e-05 1.00029581e+00]]

We can observe that the sample data follows a similar distribution to the actual data but as sample size fall considerably to 100 observations the distribution changes.

2- Compare your results for misallocation using your random sample to the results obtained using the complete distribution.

Table 3:

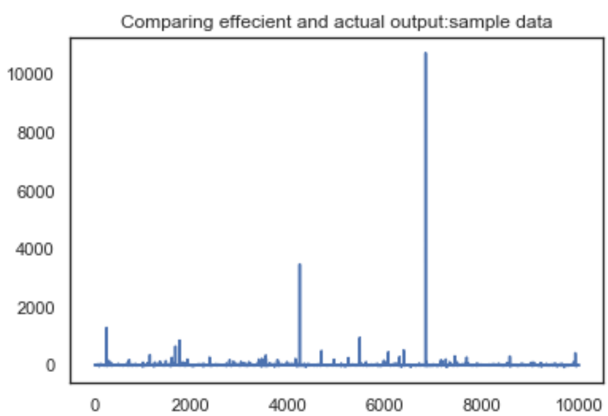
Outuput gain from reallocation				
	Pop. 10000	Pop. 1000	Pop. 100	Pop. 100000
	133.5	111.3	80.20	137.1
Comparison output gain sample and output gain entire population	-5.39	-27.5	-58.69	-1.70

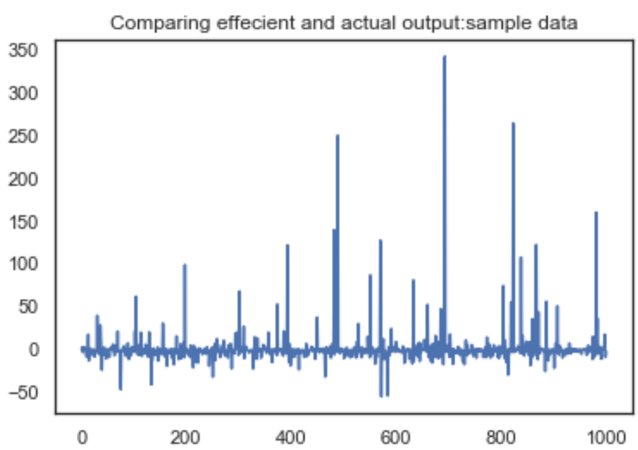
As the sample size decreases the output gain from reallocation falls. This is expected since closer we are to the actual data the more the output would be reallocated to the sample.

Figure 6:

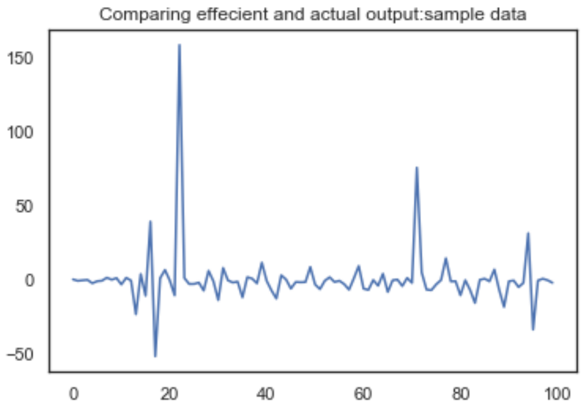
Comparison efficient and actual output: sample data

Pop. 10000 Pop. 1000

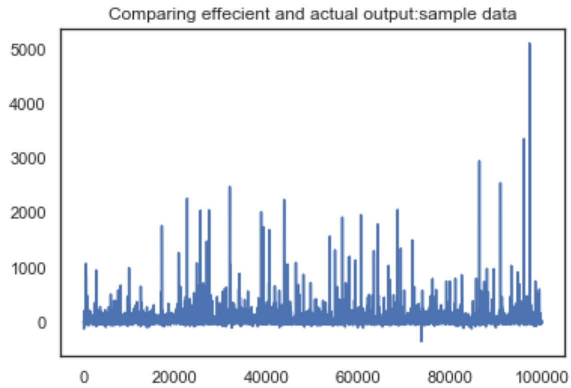




Pop. 100



Pop. 100000

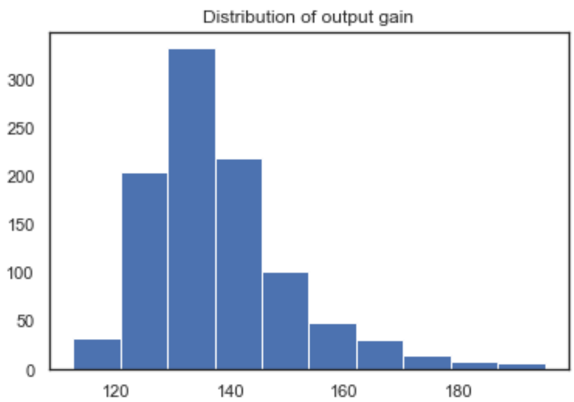


From these graphs we can observe that the difference between efficient and actual output in the sample and in the data is smaller when population size is 10000; in the contrast neither population size 100000 or 100 help to reduce the difference.

3- Do the previous exercise 1000 times and show the histogram of the output gains and provide some statistics of that distribution of these output gains, in particular, the median. Figure 7 shows the histogram for the different random population size.

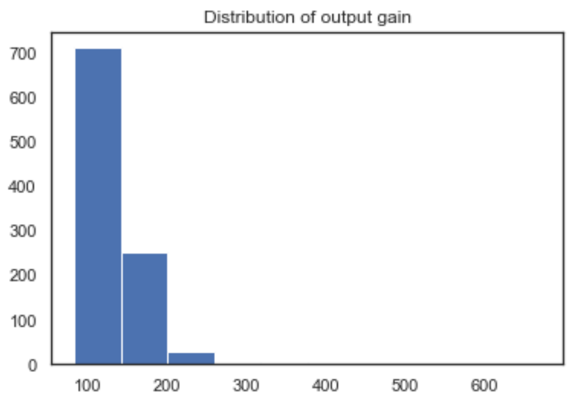
Figure 7:

Pop. 10000



Pop. 100

Pop. 1000



Pop. 100000

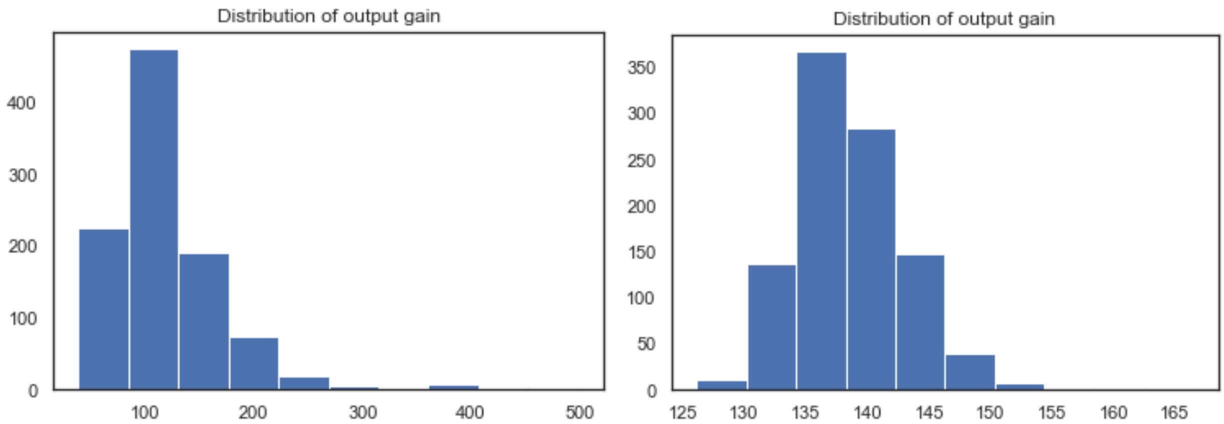


Table 4:

Statistics output gains				
	Pop. 10000	Pop. 1000	Pop. 100	Pop. 100000
Mean	138.1	135.4	120.2	138.6
Std	13.1	35.09	1.97	4.74
Min	112.5	82.9	38.2	126.1
25%	129.4	115.1	85.9	135.4
50%	135.4	129.0	107.9	138.1
75%	143.7	145.0	140.4	141.3
Max	195.0	670.1	498.7	166.4

The mean is similar for all sample sizes and as the sample size decreases it skews more towards the left. This can be also observed in the histograms provided above in Figure 7.

4- What is the probability that a random sample delivers the misallocation gains within an interval of 10% with respect to the actual misallocation gains obtained from complete data?

Table 5:

Probability of sample in 10% interval				
	Pop. 10000	Pop. 1000	Pop. 100	Pop. 100000
	78.3 %	36.19 %	14.7 %	99.2 %

We can observe as the sample size falls the probability that the sample predicts the data at the 10% probability level is lower. In other words, as the sample size increase we are closer to the true prediction of the data.