Project title: Content Caching in MCPs

Akhilesh Chauhan 170070

Japneet Singh 17807313

May 10, 2021

Abstract

In this work, we derive expressions for Cache Aided Throughput in Matern Clustered Process (MCPs) for different situations. First, we start with a simpler model where only Device to Device (D2D) users are present and cache the files. Then we extend the derived expressions to the cooperative case where Base Station's (BS) are also present along with the users and both of them cache the files. Here we consider two different mode of D2D communication namely: In-Band and Out-band D2D. In the Out-band case, the user devices participating in D2D communication and Base Station's (BS) do not share the same band and hence do not interfere with each other. While, in Inband case, both the D2D devices and BS's share same band.

1 Introduction

Over past few years, the global mobile data traffic has increased explosively, and increased Video traffic has been the major contributor in this explosive growth. Caching at network edge such as in mobile devices and in small cells has emerged as a promising approach. In principle, Caching can be done at the BS's and also at User's Equipment's (UE's). However, to get significant increase in cache aided transmissions, caching should be done at both BS's and UE's. In this work, we derive expressions for cache aided throughput for closest association in the co-operative case where caching is done at both BS's and users and both of them form a Matern Cluster Process.

Many works [1, 2] model the location of nodes as a Poisson Point Process (PPP). However, Poisson clustered Processes have been found to be more suited to model dense urban areas where users tend to form clusters around the hotspots. [1] uses PPP to model the location of users and calculates an approximate expression of cache-aided throughput, which is the density of successfully served requests by local caches. [2] also uses PPP and solves for the optimal caching strategies by maximizing the Density of Successful Receptions.

Previous works [3,4] which try to analyze caching in clustered networks, either derive expression for a simpler models and considers only the users or the BS's. [4] studies the performance of clustered D2D using MCP. Their model is quite simple and only includes the user's containing the requested file and ignores any BS's or other user not containing the file which may also be interfering. [3]

calculates expression for cache hit probability in Cloud caching networks and does not assume any D2D communication in the system model. Very less studies have focused on the Co-operative case where both Users and BS's participate in caching. To best of our knowledge [5, 6] are the only works which focus on Cooperative Caching Placement. Both [5,6] focus on calculating the probability that the contents are successfully transmitted within a given threshold time in PPP's.

The question of how to share the spectrum resources between cellular and D2D communications has remained quite unclear. Depending on the type of spectrum sharing, D2D communications can be subsumed into two categories: in-band and out-of-band D2D. In In-band D2D users and BS's share the same cellular spectrum, while in out-of-band D2D users and BS's uses bands (for e.g. unlicensed spectrum) other the cellular band. In-band D2D can further subsumed into two categories: Overlay and Underlay. In Overlay mode BS's and D2D transmitters use orthogonal time/frequency resource blocks, while in underlay mode D2D transmitters access the time/frequency resource blocks occupied by cellular users. Practically, Overlay scheme is easier to implement but underlay scheme leads to more efficient spectrum usage. We only consider underlay case in in-band D2D. While in out-band D2D there is no interference between D2D and cellular users although interference may occur from other devices (like Bluetooth and WiFi) operating in that band. We ignore this type of interference while analyzing the out-band case.

2 System Model for Out-band D2D

In this section we describe the model for the D2D communication network. We will first start with a simple model for D2D communication and consider the non-cooperative case where the communication happens only between the users. Then we will generalize our expression to the cooperative case where BS's and users both participate in caching.

2.1 System model

We have a parent Poisson point process Φ_p with intensity λ_p, whose points denotes the cluster center. The daughter point process indicating the location of users is taken to be uniform Poisson Point Process λ_u on a disk of radius R_d i.e. the user devices are distributed as a Matern Cluster Process. Each user acts as a possible transmitter or receiver, independently with probability ρ and $1 - \rho$ respectively. The D2D transmitters transmit with power P_{DT}.

Different clusters may be interested in different contents. We assume a finite content library $\mathcal{F}_x = [f_1^{\mathbf{x}}, f_2^{\mathbf{x}}, \cdots, f_{N_{\mathbf{x}}}^{\mathbf{x}}]$ for each cluster centered at $\Phi_{\mathbf{x}}$ and $\mathbf{x} \in \Phi_p$, with possibly different files. The $N_{\mathbf{x}}$ files are sorted in decreasing order of their popularity, which is modelled using Zipf($\gamma_{\mathbf{x}}$) distribution, where $\gamma_{\mathbf{x}}$ denotes the skewness of the Zipf distribution. Each receiver requests a file $f_i^{\mathbf{x}}$ with probability $q_i^{\mathbf{x}}$, which is given by

$$q_i^{\mathbf{x}} = \frac{i^{-\gamma_{\mathbf{x}}}}{\sum_{j=1}^{N_{\mathbf{x}}} j^{-\gamma_{\mathbf{x}}}}$$

The transmitting devices have a local cache memory of size $M < N_{\mathbf{x}}$. Each unit of memory carries a single file. Each device independently caches the file(s) $f_i^{\mathbf{x}}$ with probabilities $p_i^{\mathbf{x}}$, where the index i corresponds to the popularity index of the file. For a representative cluster centered at $\mathbf{x}_0$, $P_{i,\mathbf{x}_0}^{hit}$

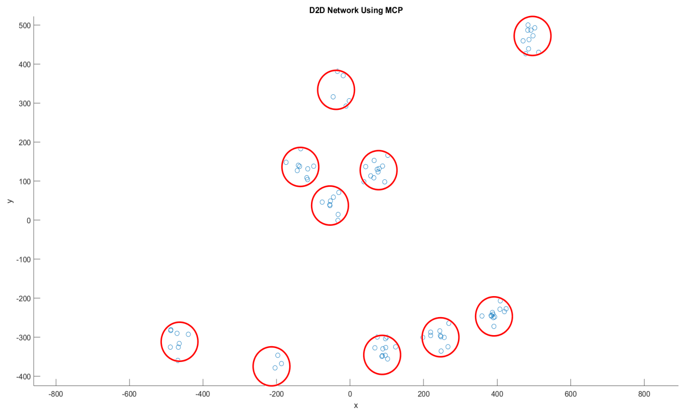

Figure 1: Illustration denoting BSs and devices (dots) in MCPs

denotes the conditional hit probability conditioned on the user requesting the i^{th} most popular file($f_i^{\mathbf{x}_0}$).

$$P_{i,\mathbf{x}_0}^{hit} = \mathbb{P}(\text{SINR}_{i,\mathbf{x}_0} > \beta)$$

where β is the minimum threshold for a successful transmission and

$$\text{SINR}_{i,\mathbf{x}_0} = \frac{P_{DT} g_{\mathbf{z}} r^{-\alpha}}{\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}_{\text{intra}}}$$

where $\mathbf{z}$ is the location of serving device, based on the closest association, $g_{\mathbf{z}}$ is the random Rayleigh fading coefficient (exponentially distributed), $\mathcal{I}_{\text{inter}}$ and $\mathcal{I}_{\text{intra}}$ denote the inter cluster and intra cluster interference respectively.

$$\mathcal{I}_{\text{inter}} = \sum_{\mathbf{x} \in \Phi_p \setminus \mathbf{x}_0} \sum_{\mathbf{y} \in \Phi_{\mathbf{x}}} P_{DT} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha} \qquad \text{and} \qquad \mathcal{I}_{\text{intra}} = \sum_{\mathbf{y} \in \Phi_{\mathbf{x}_0} \setminus \mathbf{y}_0} P_{DT} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha} \tag{1}$$

The total hit probability $P_{\mathbf{x}_0}^{hit}$ is given as:

$$P_{\mathbf{x}_0}^{hit} = \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{hit}$$

The hit probability can be maximized by choosing the optimum values of caching probabilities of each file. The cache aided throughput [1] $\mathcal{T}$ (for the representative cluster) is the average number of requests that can be successfully and simultaneously handled by the local caches per unit area.

$$\mathcal{T} = \rho \lambda_u \left[\sum_{i=1}^{N_{\mathbf{x}_0}} p_i^{\mathbf{x}_0} q_i^{\mathbf{x}_0} \cdot 1 + \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} (1 - p_i^{\mathbf{x}_0}) P_{i,\mathbf{x}_0}^{hit} \right]$$

We will derive expressions for cache aided throughput for the above model as explained in next section.

3 Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

There are 2 possible ways a cache hit can happen:

- **Self request**: The file requested by the user could have been cached by the user itself. Self request refers to the user requesting the file from the cache of the user itself. The self request success probability can be given by:

$$P_{\mathbf{x}_0}^{\text{self}} = \sum_{i=1}^{N_{\mathbf{x}_0}} p_i^{\mathbf{x}_0} q_i^{\mathbf{x}_0}$$

- **D2D Cache hit**: Let $P_{i,\mathbf{x}_0}^{\text{hit}}$ be the conditional hit probability that the user requests $i-$th most popular file.

$$P_{i,\mathbf{x}_0}^{\text{hit}} = \mathbb{P}(\text{SINR}_{i,\mathbf{x}_0} > \beta)$$

  and define

$$P_{\mathbf{x}_0}^{\text{hit}} = \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{\text{hit}} (1 - p_i^{\mathbf{x}_0})$$

- Total cache hit probability = $P_{\mathbf{x}_0}^{\text{self}} + P_{\mathbf{x}_0}^{\text{hit}}$ and Cache aided throughput is

$$\mathcal{T} = \rho\lambda_u \left[\sum_{i=1}^{N_{\mathbf{x}_0}} p_i^{\mathbf{x}_0} q_i^{\mathbf{x}_0} + \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{\text{hit}} (1 - p_i^{\mathbf{x}_0}) \right]$$

- The distance distribution of possible transmitters in $\Phi_{\mathbf{x}}$ $\forall \mathbf{x} \in \Phi_P$ to the origin as a function of $\|\mathbf{x}\|$ as already derived in [3] is given as
  Case 1 : $||\mathbf{x}|| < R_d$

$$f^{||\mathbf{x}||}(y) = \begin{cases} \frac{2y}{R_d^2} & 0 \le y < R_d - ||\mathbf{x}|| \\ \frac{1}{\pi R_d^2} \frac{\partial \boldsymbol{B}_{||\mathbf{x}||}(y)}{\partial y} & R_d - ||\mathbf{x}|| \le y < R_d + ||\mathbf{x}|| \\ 0 & y \ge R_d + ||\mathbf{x}|| \end{cases}$$

  Case 2 : $||\mathbf{x}|| > R_d$

$$f^{||\mathbf{x}||}(y) = \begin{cases} 0 & 0 \le y < ||\mathbf{x}|| - R_d \\ \frac{1}{\pi R_d^2} \frac{\partial \boldsymbol{B}_{||\mathbf{x}||}(y)}{\partial y} & ||\mathbf{x}|| - R_d \le y < ||\mathbf{x}|| + R_d \\ 0 & y \ge R_d + ||\mathbf{x}|| \end{cases}$$

  where $\boldsymbol{B}_{||\mathbf{x}||}(y)$ is defined on $|\|\mathbf{x}\| - D| \le y \le D + \|\mathbf{x}\|$ and is given by

$$\boldsymbol{B}_{||\mathbf{x}||}(y) = R_d^2 \cos^{-1}\left(\frac{R_d^2 + ||\mathbf{x}||^2 - y^2}{2||\mathbf{x}||R_d} \right) + y^2 \cos^{-1}\left(\frac{y^2 + ||\mathbf{x}||^2 - R_d^2}{2||\mathbf{x}||y} \right)$$

$$-\frac{1}{2}\sqrt{\left[(y+||\mathbf{x}||)^2-R_d^2\right]\left[R_d^2-(y-||\mathbf{x}||^2)\right]}$$

Just by doing calculations it is easy to verify that

$$\frac{\partial \boldsymbol{B}_{\|\mathbf{x}_0\|}(r)}{\partial r}=\frac{2y}{\pi R_d^2}\cos^{-1}\left(\frac{y^2+\|\mathbf{x}\|^2-R_d^2}{2y\|\mathbf{x}\|}\right)$$

- For closest selection

$$\begin{aligned} P_{i,\mathbf{x}_0}^{\text{hit, CS}} &= \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0}>\beta] \\ &= \sum_{k=1}^{\infty}\mathbb{P}[\text{SINR}_{i,\mathbf{x}_0}>\beta|n(\Phi_{\mathbf{x}_0}^i)=k]\mathbb{P}[n(\Phi_{\mathbf{x}_0}^i)=k] \end{aligned}$$

$\mathbb{P}[n(\Phi_{\mathbf{x}_0}^i)=k]=\frac{(p_i^{\mathbf{x}_0}(1-\rho)\lambda_u\pi R_d^2)^k}{k!}e^{-p_i^{\mathbf{x}_0}(1-\rho)\lambda_u\pi R_d^2}$ is the probability that there are k users in the cluster containing file i.

$$\mathbb{P}[\text{SINR}_{i,\mathbf{x}_0}>\beta|n(\Phi_{\mathbf{x}_0}^i)=k]=\int_0^{R_d+\|\mathbf{x}_0\|}\mathbb{P}\left[\frac{P_{DT}g_{i,\mathbf{x}_0}r^{-\alpha_i}}{\sigma^2+\mathcal{I}_{\text{inter}}+\mathcal{I}_{\text{intra}}}>\beta\;|n(\Phi_{\mathbf{x}_0}^i)=k\right]f_{\|z_{CS}\|}(r|k)dr$$

Here, $\|z_{CS}\|$ denotes the serving distance, i.e. the distance between the user of interest and its serving DT. $f_{\|z_{CS}\|}(r|k)$ corresponds to the distribution of the serving distance conditioned on the number of active DTs in the cluster and it can be obtained by the distribution of $f^{||\mathbf{x}||}(y)$ and order statistics. Let $X_1\leq X_2......\leq X_n$, denote the distances of the DTs in the cluster. Now due to the closest selection, the serving DT would be X_1. Conditioned on k, its distribution is given as

$$f_R(r|k)=f_{X_1}(r|k)=n\left[1-F_X(r|k)\right]^{k-1}f_X(r|k)$$

$$f_{\|z_{CS}\|}(r|k)=kf^{\|\mathbf{x}_0\|}(r)(1-F^{\|\mathbf{x}_0\|}(r))^{k-1}$$

$$f_{\|z_{CS}\|}(r|k)=\begin{cases}\frac{2kr}{R_d^2}\left(1-\frac{r^2}{R_d^2}\right)^{k-1} & 0\leq r<R_d-\|\mathbf{x}_0\| \\ \frac{k}{\pi R_d^2}\frac{\partial \boldsymbol{B}_{\|\mathbf{x}_0\|}(r)}{\partial r}\left(1-\frac{\boldsymbol{B}_{\|\mathbf{x}_0\|}(r)}{\pi R_d^2}\right)^{k-1} & R_d-\|\mathbf{x}_0\|\leq r<R_d+\|\mathbf{x}_0\| \\ 0 & r\geq R_d+||\mathbf{x}|| \end{cases} \tag{2}$$

$$\begin{aligned} \mathbb{P}\left[\frac{P_{DT}g_{i,\mathbf{x}_0}r^{-\alpha_i}}{\sigma^2+\mathcal{I}_{\text{inter}}+\mathcal{I}_{\text{intra}}}>\beta\right] &= \mathbb{P}\left[g_{i,\mathbf{x}_0}>\frac{\beta(\sigma^2+\mathcal{I}_{\text{inter}}+\mathcal{I}_{\text{intra}})}{P_{DT}r^{-\alpha_i}}\right] && (3)\\ &= \mathbb{E}\left[\exp\left(-\frac{\beta(\sigma^2+\mathcal{I}_{\text{inter}}+\mathcal{I}_{\text{intra}})}{P_{DT}r^{-\alpha_i}}\right)\right] && (4)\\ &= \exp\left(-\frac{\beta\sigma^2}{P_{DT}r^{-\alpha_i}}\right)\mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(-\frac{\beta}{P_{DT}r^{-\alpha_i}}\right)\mathcal{L}_{\mathcal{I}_{\text{intra}}}\left(-\frac{\beta}{P_{DT}r^{-\alpha_i}}\right) && (5)\end{aligned}$$

The Laplace transform has 2 components, $\mathcal{L}_{\mathcal{I}_{\text{intra}}}$ and $\mathcal{L}_{\mathcal{I}_{\text{inter}}}$, which are formed by the intra cluster and the inter cluster interference respectively. The Laplace transform of the inter cluster interference, as already derived in [4] is given by

$$\mathcal{L}_{\mathcal{I}_{\text{inter}}}(s) = \mathbb{E}\left[\exp\left(-s\sum_{\mathbf{x}\in\Phi_p\backslash\mathbf{x}_0}\sum_{\mathbf{y}\in\Phi_\mathbf{x}} g_\mathbf{y}\|\mathbf{y}\|^{-\alpha}\right)\right] = \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p}\mathbb{E}_{\Phi_\mathbf{x}}\left[\prod_{\mathbf{y}\in\Phi_\mathbf{x}}\mathbb{E}_{g_\mathbf{y}}\left[\exp\left(-sg_\mathbf{y}\|\mathbf{y}\|^{-\alpha}\right)\right]\right]\right] \tag{6}$$

$$= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p}\mathbb{E}_{\Phi_\mathbf{x}}\left[\prod_{\mathbf{y}\in\Phi_\mathbf{x}}\frac{1}{1+s\|\mathbf{y}\|^{-\alpha}}\right]\right] = \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p}\sum_{n=0}^{\infty}\frac{e^{-\bar{m}}\bar{m}^n}{n!}\left(\int_0^\infty\frac{f^{\|\mathbf{x}\|}(y)}{1+sy^{-\alpha}}dy\right)^n\right] \tag{7}$$

$$= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p}\exp\left(-\bar{m}\left(1-\int_0^\infty\frac{f^{\|\mathbf{x}\|}(y)}{1+sy^{-\alpha}}dy\right)\right)\right] \tag{8}$$

$$= \exp\left[-2\pi\lambda_p\int_0^\infty\left[1-\exp\left(-\bar{m}\int_0^\infty\frac{f^r(y)}{1+sy^{-\alpha}}dy\right)\right]rdr\right] \quad \text{where r} = \|\mathbf{x}\| \tag{9}$$

where $g_\mathbf{y}$ denotes the channel fading gain between the typical device receiver(DR) and the inter cluster interferer at $\mathbf{y}$, and $\Phi_\mathbf{x}$ denotes the cluster centered at $\mathbf{x}$. Here, $f^{\|\mathbf{x}\|}(y)$ denotes the PDF of the inter cluster interfering distance conditioned on the distance between $\mathbf{x}$ and a typical DR. $\bar{m}$ denotes the expected number of DTs in the cluster of the typical DR. Therefore, $\bar{m} = \rho\lambda_u\pi R_d^2$

$$\begin{aligned}
\mathcal{L}_{\mathcal{I}_{\text{intra}}}(s) &= \mathbb{E}\left[\exp\left(-s\sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}\backslash\mathbf{y}_0} g_\mathbf{y}\|\mathbf{y}\|^{-\alpha}\right)\right] \\
&= \mathbb{E}\left[\exp\left(-s\left(\sum_{\mathbf{y}\in\Phi^i_{\mathbf{x}_0}\backslash\mathbf{y}_0} g_\mathbf{y}\|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi^{-i}_{\mathbf{x}_0}} g_\mathbf{y}\|\mathbf{y}\|^{-\alpha}\right)\right)\right] \\
&= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\prod_{\mathbf{y}\in\Phi^i_{\mathbf{x}_0}\backslash\mathbf{y}_0}\mathbb{E}_{g_\mathbf{y}}\left(\exp\left(-sg_\mathbf{y}\|\mathbf{y}\|^{-\alpha}\right)\right)\prod_{\mathbf{y}\in\Phi^{-i}_{\mathbf{x}_0}}\mathbb{E}_{g_\mathbf{y}}\left(\exp\left(-sg_\mathbf{y}\|\mathbf{y}\|^{-\alpha}\right)\right)\right] \\
&= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\prod_{\mathbf{y}\in\Phi^i_{\mathbf{x}_0}\backslash\mathbf{y}_0}\frac{1}{1+s\|\mathbf{y}\|^{-\alpha}}\mathbb{1}(\|y\|>r)\prod_{\mathbf{y}\in\Phi^{-i}_{\mathbf{x}_0}}\frac{1}{1+s\|\mathbf{y}\|^{-\alpha}}\right]
\end{aligned}$$

$$= \left[\bar{m}_i^0 e^{-\bar{m}_i}.0 + \sum_{n=1}^{\infty}\frac{\bar{m}_i^n e^{-\bar{m}_i}}{n!}\left[\int_r^\infty\frac{f(r|a)}{1+sr^{-\alpha}}dr\right]^{n-1}\right] + \left[\sum_{n=0}^{\infty}\frac{\bar{m}_{-i}^n e^{-\bar{m}_{-i}}}{n!}\left[\int_0^\infty\frac{f(r|a)}{1+sr^{-\alpha}}dr\right]^n\right]$$

$$\mathcal{L}_{\mathcal{I}_{\text{intra}}}(s) = \frac{e^{-\bar{m}}\left[e^{\left(\bar{m}_{-i}\int_0^\infty\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr\right)+\left(\bar{m}_i\int_r^\infty\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr\right)-1}\right]}{\int_r^\infty\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr}$$

Here, i and $-i$ correspond to the DTs which contain the requested file i and the remaining DTs respectively. This means that $\bar{m}_i$ denote the expected number of DTs which have the i^{th} file cached with them. $\bar{m}_{-i}$ denotes the expected number of DTs which do not have the i^{th} file cached. Therefore, $\bar{m}_i = p_i^{\mathbf{x}_0}\rho\lambda_u\pi R_d^2$ and $\bar{m}_{-i} = (1-p_i^{\mathbf{x}_0})\rho\lambda_u\pi R_d^2$ and $\bar{m} = \bar{m}_i + \bar{m}_{-i}$.

$$P_{i,\mathbf{x}_0}^{\text{hit, CS}} = \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta] = \sum_{k=1}^{\infty} \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta | n(\Phi_{\mathbf{x}_0}^i) = k]\mathbb{P}[n(\Phi_{\mathbf{x}_0}^i) = k]$$
$$= \sum_{k=1}^{\infty} \mathbb{P}[n(\Phi_{\mathbf{x}_0}^i) = k] \int_0^{R_d+\|x_0\|} e^{\frac{-\beta\sigma^2}{P_{DT}r^{-\alpha_i}}} \mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(\frac{-\beta}{P_{DT}r^{-\alpha_i}}\right) \mathcal{L}_{\mathcal{I}_{\text{intra}}}\left(\frac{-\beta}{P_{DT}r^{-\alpha_i}}\right) f_{\|z_{CS}\|}(r|k)dr$$
$$= \sum_{k=1}^{\infty} \mathbb{P}[n(\Phi_{\mathbf{x}_0}^i) = k] \int_0^{R_d+\|x_0\|} e^{\frac{-\beta\sigma^2}{P_{DT}r^{-\alpha_i}}} \mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(\frac{-\beta}{P_{DT}r^{-\alpha_i}}\right) \mathcal{L}_{\mathcal{I}_{\text{intra}}}\left(\frac{-\beta}{P_{DT}r^{-\alpha_i}}\right) kf^{\|\mathbf{x}_0\|}(r)(1-F^{\|\mathbf{x}_0\|}(r))^{k-1}dr$$

$$P_{i,\mathbf{x}_0}^{\text{hit, CS}} = \sum_{k=1}^{\infty}[\int_0^{R_d-\|x_0\|} e^{\frac{-\beta\sigma^2}{P_{DT}r^{-\alpha_i}}} \mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(-\frac{\beta}{P_{DT}r^{-\alpha_i}}\right) \mathcal{L}_{\mathcal{I}_{\text{intra}}}\left(-\frac{\beta}{P_{DT}r^{-\alpha_i}}\right) \frac{2kr}{D^2}\left(1-\frac{r^2}{D^2}\right)^{k-1} dr$$

$$+ \int_{R_d-\|x_0\|}^{R_d+\|x_0\|} e^{\frac{-\beta\sigma^2}{P_{DT}r^{-\alpha_i}}} \mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(-\frac{\beta}{P_{DT}r^{-\alpha_i}}\right) \mathcal{L}_{\mathcal{I}_{\text{intra}}}\left(-\frac{\beta}{P_{DT}r^{-\alpha_i}}\right) \frac{k}{\pi D^2}\frac{\partial \boldsymbol{B}_{\|\mathbf{x}_0\|}(r)}{\partial r}\left(1-\frac{\boldsymbol{B}_{\|\mathbf{x}_0\|}(r)}{\pi D^2}\right)^{k-1} dr]$$
$$\times \mathbb{P}[n(\Phi_{\mathbf{x}_0}^i) = k]$$

4 Co-operative communication

Now we will slightly modify our model and consider the cooperative case where multiple BS's are also present in the same cluster and both BSs as well as devices are caching the data. In the modified system model, we will also have base stations in addition to the previous model. The BS's are distributed as MCP having the same parent process as that of the user's. The base stations are distributed around the cluster center as a finite homogeneous (of size R_d) PPP of intensity λ_b independent of user point process given the parent points. The transmitting powers of the base stations is P_{BS}. The size of the base station cache is given as $K < N_x$. Now We obtain the expressions for the coverage probability, hit probability, DSR, and the cache aided throughput for cooperative caching.

4.1 Inband Communication using modified system model

In this case, we can treat the user and the BS processes to as two independent MCPs which share the same band and interfere with each other.

- **Self Request**: The user in this case as well, can have the requested file cached with himself.

$$P_{\mathbf{x}_0}^{\text{self}} = \sum_{i=1}^{N_{\mathbf{x}_0}} p_i^{\mathbf{x}_0} q_i^{\mathbf{x}_0}$$

- **D2D Cache hit**: In this case, we consider uniform caching in the BSs as well with probabilities $q_{i,BS}^{\mathbf{x}_0}$. Let $P_{i,\mathbf{x}_0}^{\text{hit}}$ be the conditional hit probability that the user is able to receive $i-$th most popular file. and define

$$P_{\mathbf{x}_0}^{\text{hit}} = \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{\text{hit}} (1 - p_i^{\mathbf{x}_0})$$

- Total cache hit probability = $P_{\mathbf{x}_0}^{\text{self}} + P_{\mathbf{x}_0}^{\text{hit}}$ and Cache aided throughput is

$$\mathcal{T} = \rho \lambda_u \left[\sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} p_i^{\mathbf{x}_0} + \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{\text{hit}} (1 - p_i^{\mathbf{x}_0}) \right]$$

- The conditional hit probability in this case will depend on whether the receiver is being served by the DT or the BS. Here we consider an indicator variable $\mathbb{1}_{CS}$ which takes values $\{$BS, DT$\}$ and denotes whether the closest selection based serving device is a BS or a DT respectively. Now the conditional hit probability, $P_{i,\mathbf{x}_0}^{\text{hit}}$ can be calculated as:

$$\begin{aligned} &P_{i,\mathbf{x}_0}^{\text{hit}} = \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0}^{l} > \beta] = \\ &\sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \sum_{l \in \{\text{BS, DT}\}} \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0}^{l} > \beta | n(\Phi_{\mathbf{x}_0}^{i,BS}) = k, n(\Phi_{\mathbf{x}_0}^{i,u}) = m, \mathbb{1}_{\text{CS}} = l] \times \\ &\qquad\qquad\qquad\qquad\qquad\qquad \mathbb{P}[n(\Phi_{\mathbf{x}_0}^{i,u}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m, \mathbb{1}_{\text{CS}} = l] \end{aligned}$$

$$\begin{aligned} &\mathbb{P}[n(\Phi_{\mathbf{x}_0}^{i,u}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m, \mathbb{1}_{\text{CS}} = l] \\ &\qquad = \mathbb{P}[\mathbb{1}_{\text{CS}} = l | n(\Phi_{\mathbf{x}_0}^{i,u}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m] \mathbb{P}[n(\Phi_{\mathbf{x}_0}^{i,u}) = k] \mathbb{P}[n(\Phi_{\mathbf{x}_0}^{i,BS}) = m] \end{aligned}$$

- The above expression can be simplified as:

$$\begin{aligned} &\mathbb{P}[\text{SINR}_{i,\mathbf{x}_0}^{l} > \beta | n(\Phi_{\mathbf{x}_0}^{i,u}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m, \mathbb{1}_{\text{CS}} = l] \\ &\qquad = \int_0^{R_d + \|\mathbf{x}_0\|} \mathbb{P}\left[\frac{P_l g_{\mathbf{z}} r^{-\alpha_i}}{\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}^l{}_{\text{intra}}} > \beta \right] f_{\|\mathbf{z}_{CS}\|}(r | k, m, l) dr \end{aligned}$$

- Now, we have

$$\begin{aligned} \mathbb{P}\left[\frac{P_l g_{\mathbf{z}} r^{-\alpha}}{\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}^l{}_{\text{intra}}} > \beta \right] &= \mathbb{P}\left[g_{\mathbf{z}} > \frac{\beta(\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}^l{}_{\text{intra}})}{P_l r^{-\alpha}} \right] \\ &= \mathbb{E}\left[\exp\left(-\frac{\beta(\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}^l{}_{\text{intra}})}{P_l r^{-\alpha}} \right) \right] \\ &= e^{-\frac{\beta \sigma^2}{P_l r^{-\alpha}}} \mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(-\frac{\beta}{P_l r^{-\alpha}} \right) \mathcal{L}_{\mathcal{I}^l{}_{\text{intra}}}\left(-\frac{\beta}{P_l r^{-\alpha}} \right) \end{aligned}$$

Note here that the intra-cluster laplace transform depends on the serving device.

- We now compute $\mathbb{P}[\mathbb{1}_{\text{CS}} = l | n(\Phi_{\mathbf{x}_0}^{i,DT}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m]$. If l =BS (or DT) it is the probability of getting served by a BS(or DT). Hence, we consider 2 independentt random variables $X = \min\{\mathbf{y}_1^{DT}, \mathbf{y}_2^{DT}, \cdots, \mathbf{y}_k^{DT}\}, \mathbf{y}_j^{DT} \in \Phi_{\mathbf{x}_0}^{i,DT}$ and $Y = \min\{\mathbf{y}_1^{BS}, \mathbf{y}_2^{BS}, \cdots, \mathbf{y}_m^{BS}\}, \mathbf{y}_j^{BS} \in \Phi_{\mathbf{x}_0}^{i,BS}$ and each of $\mathbf{y}_j^{DT}$ and $\mathbf{y}_j^{BS}$ are independent samples from $f^{\|\mathbf{x}\|}$ and hence using the following property we can calculate $\mathbb{P}[X < Y]$ the

$$\mathbb{P}[\mathbb{1}_{\text{CS}} = DT | n(\Phi_{\mathbf{x}_0}^{i,DT}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m] = \mathbb{P}[X < Y] = \int_0^\infty \int_0^y f_X(x) g_Y(y) dx dy$$

  as the expression of $f_X(x)$ and $f_Y(y)$ can be computed in Eq. 2.

- We need to find $\mathbb{P}[\|\mathbf{y}_{(1)}^{BS}\| < \|\mathbf{y}_{(1)}^{u}\|]$ where $\mathbf{y}_i^{BS} \sim f^{\|\mathbf{x}\|}(y)$ and $\mathbf{y}_i^{u} \sim f^{\|\mathbf{x}\|}(y)$ all of them are independent of each other. Using this property we can compute the expression of $\mathbb{P}[\mathbb{1}_{\text{CS}} = \text{BS} | n(\Phi_{\mathbf{x}_0}^{i,u}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m]$

- Laplace transform of Inter-cluster interference:

$$\mathcal{L}_{\mathcal{I}_{\text{inter}}}(s) = \mathbb{E}\left[\exp\left(-s \sum_{\mathbf{x}\in\Phi_p\setminus\mathbf{x}_0} (\sum_{\mathbf{y}\in\Phi_{\mathbf{x}}^u} P_{DT} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi_{\mathbf{x}}^{BS}} P_{BS} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha})\right)\right] \tag{10}$$

$$\begin{aligned}
&= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p} \mathbb{E}_{\Phi_{\mathbf{x}}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^u} \mathbb{E}_{g_{\mathbf{y}}}\left[\exp\left(-s P_{DT} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha}\right)\right] \prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^{BS}} \mathbb{E}_{g_{\mathbf{y}}}\left[\exp\left(-s P_{BS} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha}\right)\right]\right]\right] \\
&= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p} \mathbb{E}_{\Phi_{\mathbf{x}}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^u} \frac{1}{1 + s P_{DT} \|\mathbf{y}\|^{-\alpha}} \prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^{BS}} \frac{1}{1 + s P_{BS} \|\mathbf{y}\|^{-\alpha}}\right]\right] \\
&= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p} \sum_{n=0}^{\infty} \frac{e^{-\bar{m}_1}\bar{m}_1^n}{n!} \left(\int_0^\infty \frac{f^{\|\mathbf{x}\|}(y)}{1 + s P_{DT} y^{-\alpha}} dy\right)^n \sum_{n=0}^{\infty} \frac{e^{-\bar{m}_2}\bar{m}_2^n}{n!} \left(\int_0^\infty \frac{f^{\|\mathbf{x}\|}(y)}{1 + s P_{BS} y^{-\alpha}} dy\right)^n\right] \\
&= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p} \exp\left(-\bar{m}_1\left(1 - \int_0^\infty \frac{f^{\|\mathbf{x}\|}(y)}{1 + s P_{DT} y^{-\alpha}} dy\right) - \bar{m}_2\left(1 - \int_0^\infty \frac{f^{\|\mathbf{x}\|}(y)}{1 + s P_{BS} y^{-\alpha}} dy\right)\right)\right] \\
&= \exp\left[-2\pi\lambda_p \int_0^\infty \left[2 - \exp(-(\int_0^\infty \bar{m}_1 \frac{f^r(y)}{1 + s P_{DT} y^{-\alpha}} + \bar{m}_2 \frac{f^r(y)}{1 + s P_{BS} y^{-\alpha}} dy))\right] r dr\right]
\end{aligned} \tag{11}$$

  where r = $\|\mathbf{x}\|$, $\bar{m}_1$ and $\bar{m}_2$ are the expected number of DTs and BSs in the representative cloud $\mathbf{x}_0$. Therefore,

$$\bar{m}_1 = \rho\lambda_u \pi R_d^2$$
$$\bar{m}_2 = \lambda_b \pi R_d^2$$

- Laplace transform of Intra-cluster interference(given the serving device is DT)

$$\mathcal{L}_{\mathcal{I}_{\text{intra}}^{DT}}(s) = \mathbb{E}\left[\exp\left(-s(\textstyle\sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^u\setminus\mathbf{y}_0} P_{DT} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{BS}} P_{BS} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha})\right)\right]$$

$$\mathcal{L}_{\mathcal{I}_{\text{intra}}^{DT}}(s) = \mathbb{E}\left[\exp\left(-s(\textstyle\sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,i}\setminus\mathbf{y}_0} P_{DT} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,-i}} P_{DT} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{BS,i}} P_{BS} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{BS,-i}} P_{BS} g_{\mathbf{y}} \|\mathbf{y}\|^{-\alpha})\right)\right]$$

$$= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,i}\setminus\mathbf{y}_0}\mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_{DT}g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right)\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,-i}}\mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_{DT}g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right)\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{BS,i}}\mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_{BS}g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right)\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{BS,-i}}\mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_{BS}g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right)\right]$$

$$= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,i}\setminus\mathbf{y}_0}\frac{1}{1+sP_{DT}\|\mathbf{y}\|^{-\alpha}}\mathbb{1}(\|y\|>r)\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,-i}}\frac{1}{1+sP_{DT}\|\mathbf{y}\|^{-\alpha}}\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{BS,i}}\frac{1}{1+sP_{BS}\|\mathbf{y}\|^{-\alpha}}\mathbb{1}(\|y\|>r)\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{BS,-i}}\frac{1}{1+sP_{BS}\|\mathbf{y}\|^{-\alpha}}\right]$$

$$= \sum_{n=1}^{\infty}\frac{\bar{m}_{-i,1}^{n}e^{-\bar{m}_{-i,1}}}{n!}\left[\int_r^{\infty}\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr\right]^n\sum_{n=1}^{\infty}\frac{\bar{m}_{i,1}^{n}e^{-\bar{m}_{i,1}}}{n!}\left[\int_0^{\infty}\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr\right]^{n-1}\sum_{n=1}^{\infty}\frac{\bar{m}_{-i,2}^{n}e^{-\bar{m}_{-i,2}}}{n!}\left[\int_r^{\infty}\frac{f(r|a)}{1+sP_{BS}r^{-\alpha}}dr\right]^n\sum_{n=1}^{\infty}\frac{\bar{m}_{i,2}^{n}e^{-\bar{m}_{i,2}}}{n!}\left[\int_0^{\infty}\frac{f(r|a)}{1+sP_{BS}r^{-\alpha}}dr\right]^n$$

$$\mathcal{L}_{\mathcal{I}_{\text{intra}}^{DT}}(s) = \frac{e^{-\bar{m}_1-\bar{m}_2}\left[e^{\left(\bar{m}_{-i,1}\int_r^{\infty}\frac{f(r|a)}{1+sr^{-\alpha}}dr\right)+\left(\bar{m}_{i,1}\int_0^{\infty}\frac{f(r|a)}{1+sr^{-\alpha}}dr\right)+\left(\bar{m}_{-i,2}\int_r^{\infty}\frac{f(r|a)}{1+sr^{-\alpha}}dr\right)+\left(\bar{m}_{i,2}\int_0^{\infty}\frac{f(r|a)}{1+sr^{-\alpha}}dr\right)-4}\right]}{\int_r^{\infty}\frac{f(r|a)}{1+sr^{-\alpha}}dr}$$

where $\bar{m}_{i,1}$ and $\bar{m}_{-i,1}$ denote the DTs which contain and donot contain the i^{th} reqested file by the user respectively. Similarly, $\bar{m}_{i,2}$ and $\bar{m}_{-i,2}$ denote the BSs which contain and donot contain the i^{th} reqested file by the user respectively.

$$\bar{m}_{i,1} = \rho\lambda_u p_i^{\mathbf{x}_0}\pi R_d^2$$

$$\bar{m}_{-i,1} = \rho\lambda_u(1-p_i^{\mathbf{x}_0})\pi R_d^2$$

$$\bar{m}_{i,2} = \lambda_b p_i^{\mathbf{x}_0}\pi R_d^2$$

$$\bar{m}_{-i,2} = \lambda_b(1-p_i^{\mathbf{x}_0})\pi R_d^2$$

4.2 Analysis for Out-Band D2D

The analysis for Out band is same as In-band D2D. The only difference is in the expression of Laplace transforms of $\mathcal{I}_{\text{inter}}$ and $\mathcal{I}_{\text{intra}}$ as they now have fewer terms. We will now highlight the differences below

- The expression for $\mathcal{L}_{\mathcal{I}_{\text{intra}}^{DT}}$ in out-band case can be obtained by substituting $P_{BS}=0$ as there is no interference from BS. Hence we get the following expression

$$\mathcal{L}_{\mathcal{I}_{\text{intra}}^{DT}}(s) = \frac{e^{-\bar{m}_1}\left[e^{\left(\bar{m}_{-i,1}\int_0^{\infty}\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr\right)+\left(\bar{m}_{i,1}\int_r^{\infty}\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr\right)-1}\right]}{\int_r^{\infty}\frac{f(r|a)}{1+sP_{DT}r^{-\alpha}}dr}$$

- The expression for $\mathcal{L}_{\mathcal{I}_{\text{inter}}^{DT}}$ can be similarly obtained by setting $P_{BS}=0$ in the expression in Eq. 10 Hence we get the following expression

$$\mathcal{L}_{\mathcal{I}_{\text{inter}}^{DT}} = \exp\left[-2\pi\lambda_p\int_0^{\infty}\left[2-\exp(-(\int_0^{\infty}\bar{m}_1\frac{f^r(y)}{1+sP_{DT}y^{-\alpha}}dy))\right]rdr\right]$$

References

[1] Z. Chen, N. Pappas, and M. Kountouris, "Probabilistic caching in wireless d2d networks: Cache hit optimal vs. throughput optimal," 2016.

[2] D. Malak, M. Shalash, and J. G. Andrews, "Optimizing content caching to maximize the density of successful receptions in device-to-device networking," p. 1–1, 2016. [Online]. Available: http://dx.doi.org/10.1109/TCOMM.2016.2600571

[3] S. M. Azimi-Abarghouyi, M. Nasiri-Kenari, and M. Debbah, "Stochastic design and analysis of wireless cloud caching networks," 2019.

[4] Y. Wang and Q. Zhu, "Performance analysis of clustered device-to-device networks using matern cluster process," p. 4849–4858, 2018.

[5] Y. Wang, X. Tao, X. Zhang, and Y. Gu, "Cooperative caching placement in cache-enabled d2d underlaid cellular network," pp. 1151–1154, 2017.

[6] S. Soleimani and X. Tao, "Cooperative crossing cache placement in cache-enabled device to device-aided cellular networks."