Content Caching in MCPs

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Contribution

- We studied about Matern Cluster Process and content caching through various references given in Wang and Zhu, 2018; Wang et al., 2017; Azimi-Abarghouyi, Nasiri-Kenari, and Debbah, 2019; Chen, Pappas, and Kountouris, 2016
- We derived Expression for Cache Aided Throughput (CAT) for closest selection in Clustered D2D network's modeled as a Matern cluster process.
- In the above step we only considered D2D devices and ignored any interaction with BS.
- Then we derived the same results including the BS's and assuming that both the BS and D2D transmitter share the same band.
- Our above analysis can be used to compare Cache Aided Throughput for 2 types of D2D communication – *In-band* and *Out-of-Band* D2D communications.

Outline

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- Analysis of Case 2
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Background

- Poisson Cluster Processes(PCP) are widely used to model wireless networks because of their ability to model the network coupling/hotspots.
- MCP a special case of PCP.
- Union of point processes.
- At every point of a uniform PPP, there is another uniform PPP over a circular disc.
- The former is called the parent PP.
- Conditioned on the number of points in a given cluster, the points become uniformly distributed.

System Model

- We have a parent point point process Φ_p and intensity λ_p, which points denotes the cluster center.
- The daughter point process is taken to be uniform Poisson Point Process λ_u on a disk of radius R_d i.e. the user devices are distributed as a Matern Cluster Process.
- Each device acts as a possible transmitter or receiver, independently with probability ρ and $1 - \rho$ respectively. The transmitting devices are transmitting with power P_d.
- Each cluster has a finite content library $\mathcal{F}_x = [f_1^x, f_2^x, \cdots, f_{N_x}^x]$ for each cluster centered at Φ_x and $x \in \Phi_p$, with possibly different files. The N_x files are sorted in decreasing order of their popularity, which is modelled using Zipf(γ_x) distribution, where γ_x denotes the skewness of the Zipf distribution. Each receiver requests a file f_i^x with probability q_i^x, which is given by

$$q_i^x = \frac{i^{-\gamma_x}}{\sum_{j=1}^{N_X} j^{-\gamma_x}}$$

System Model

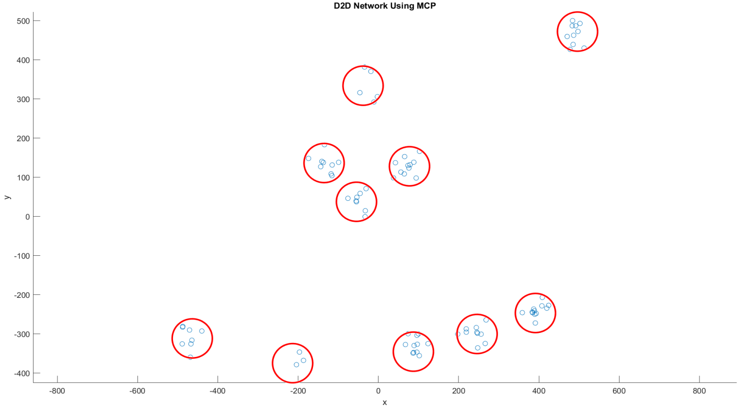

Figure 1: Illustration denoting BSs and devices (dots)

System Model

- The transmitting devices have a local cache memory of size $M < N_{\mathbf{x}}$. Each unit of memory carries a single file. Each file has a size L.
- Each device independently caches the file(s) $f_i^{\mathbf{x}}$ with probabilities $p_i^{\mathbf{x}}$, where the index i corresponds to the popularity index of the file.
- For a representative cluster centered at $\mathbf{x}_0$, $P_{i,\mathbf{x}_0}^{hit}$ denotes the conditional hit probability conditioned on the user requesting the i^{th} most popular file($f_i^{\mathbf{x}_0}$).

$$P_{i,x_0}^{hit} = \mathbb{P}(\mathrm{SINR}_{i,\mathbf{x}_0} > \beta)$$

where β is the minimum threshold for a successful transmission and

$$\mathrm{SINR}_{i,\mathbf{x}_0} = \frac{P_d g_z ||\mathbf{z}||^{-\alpha}}{\sigma^2 + I_{inter} + I_{intra}}$$

where $\mathbf{z}$ is the location of service device, based on closest association, g_z is the random Rayleigh fading coefficient (exponentially distributed), I_{inter} and I_{intra} denote the inter cluster and intra cluster interference respectively.

System Model

- The total hit probability $P_{x_0}^{hit}$ is given as:

$$P_{\mathbf{x}_0}^{hit} = \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{hit}$$

  The hit probability can be maximized by choosing the optimum values of catching probabilities of each file.

- The cache aided throughput Chen, Pappas, and Kountouris, 2016 $\mathcal{T}$ (for the representative cluster) is the average number of requests that can be successfully and simultaneously handled by the local caches per unit area.

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

- **Self request:**

$$P^{\text{self}}_{\mathbf{x}_0} = \sum_{i=1}^{N_{\mathbf{x}_0}} p_i^{\mathbf{x}_0} q_i^{\mathbf{x}_0} \cdot 1$$

- **D2D Cache hit**: Let $P^{\text{hit}}_{i,\mathbf{x}_0}$ be the conditional hit probability that the user requests $i-$th most popular file.

$$P^{\text{hit}}_{i,\mathbf{x}_0} = \mathbb{P}(\text{SINR}_{i,\mathbf{x}_0} > \beta)$$

and define

$$P^{\text{hit}}_{\mathbf{x}_0} = \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P^{\text{hit}}_{i,\mathbf{x}_0} (1 - p_i^{\mathbf{x}_0})$$

- Total cache hit probability $= P^{\text{self}}_{\mathbf{x}_0} + P^{\text{hit}}_{\mathbf{x}_0}$ and Cache aided throughput is

$$\rho \lambda_u \left[\sum_{i=1}^{N_{\mathbf{x}_0}} p_i^{\mathbf{x}_0} q_i^{\mathbf{x}_0} + \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P^{\text{hit}}_{i,\mathbf{x}_0} (1 - p_i^{\mathbf{x}_0}) \right]$$

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

- For closest selection

$$
\begin{aligned}
P_{i,\mathbf{x}_0}^{\text{hit, CS}} &= \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta] \\
&= \sum_{k=1}^{\infty} \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta | n(\Phi_{\mathbf{x}_0}^i) = k] \mathbb{P}[n(\Phi_{\mathbf{x}_0}^i) = k]
\end{aligned}
$$

$$
\mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta | n(\Phi_{\mathbf{x}_0}^i) = k] = \int_0^{R_d + \|\mathbf{x}_0\|} \mathbb{P}\left[\frac{P_d g_{\mathbf{z}} r^{-\alpha_i}}{\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}_{\text{intra}}} > \beta\right] f_{\|z_{CS}\|}(r|k) dr
$$

- Here, $\|z_{CS}\|$ denotes the serving distance, i.e. the distance between the user of interest and its serving DT.
- $f_{\|z_{CS}\|}(r|k)$ corresponds to the distribution of the serving distance conditioned on the number of active DTs in the cluster and it can be obtained by the distribution of $f^{||\mathbf{x}||}(y)$, which denotes the distance distribution of possible transmitters in $\Phi_{\mathbf{x}}$ $\forall \mathbf{x} \in \Phi_P$ to the origin as a function of $\|\mathbf{x}\|$ and order statistics.

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

- As already given in Azimi-Abarghouyi, Nasiri-Kenari, and Debbah, 2019,

$$f_{\|z_{CS}\|}(r|k) = \begin{cases} \frac{2kr}{R_d^2}\left(1 - \frac{r^2}{R_d^2}\right)^{k-1} & 0 \leq r < R_d - \|\mathbf{x}_0\| \\ \frac{k}{\pi R_d^2}\frac{\partial \boldsymbol{B}_{\|\mathbf{x}_0\|}(r)}{\partial r}\left(1 - \frac{\boldsymbol{B}_{\|\mathbf{x}_0\|}(r)}{\pi R_d^2}\right)^{k-1} & R_d - \|\mathbf{x}_0\| \leq r < R_d + \|\mathbf{x}_0\| \\ 0 & r \geq R_d + ||\mathbf{x}|| \end{cases}$$

$$\boldsymbol{B}_{\|\mathbf{x}\|}(y) = R_d^2 \cos^{-1}\left(\frac{R_d^2 + ||\mathbf{x}||^2 - y^2}{2||\mathbf{x}||R_d}\right) + y^2 \cos^{-1}\left(\frac{y^2 + ||\mathbf{x}||^2 - R_d^2}{2||\mathbf{x}||y}\right)$$
$$-\frac{1}{2}\sqrt{[(y + ||\mathbf{x}||)^2 - R_d^2]\,[R_d^2 - (y - ||\mathbf{x}||^2)]}$$

Just by doing calculations it is easy to verify that

$$\frac{\partial \boldsymbol{B}_{\|\mathbf{x}\|}(r)}{\partial r} = \frac{2y}{\pi R_d^2} \cos^{-1}\left(\frac{y^2 + \|\mathbf{x}\|^2 - R_d^2}{2y\|\mathbf{x}\|}\right)$$

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

- The general expression for $f^{\|\mathbf{x}\|}(y)$

$$f^{\|\mathbf{x}\|}(y) = \begin{cases} \frac{2y}{R_d^2} & \text{for } \|\mathbf{x}\| \le R_d, 0 \le y \le R_d - \|\mathbf{x}\| \\ \frac{2y}{\pi R_d^2} \cos^{-1}\left(\frac{y^2 + \|\mathbf{x}\|^2 - R_d^2}{2y\|\mathbf{x}\|}\right) & \text{for } \|\mathbf{x}\| \le R_d, R_d - \|\mathbf{x}\| \le y \le R_d + \|\mathbf{x}\| \\ \frac{2y}{\pi R_d^2} \cos^{-1}\left(\frac{y^2 + \|\mathbf{x}\|^2 - R_d^2}{2y\|\mathbf{x}\|}\right) & \text{for } \|\mathbf{x}\| > R_d, \|\mathbf{x}\| - R_d \le y \le \|\mathbf{x}\| + R_d \end{cases}$$

- Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables from a distribution $f_X(x)$. The order statistics is given by $X_{(1)}, X_{(2)}, ..., X_{(n)}$, where $X_{(1)} = \min\{X_1, ..., X_n\}$ and so on. The distribution of $X_{(1)}$ is given by,

$$f_{X_{(1)}}(x) = n\left[1 - F_X(x)\right]^{n-1} f_X(x)$$

where $F_X(x)$ is the CDF of X.

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

$$\mathbb{P}\left[\frac{P_d g_{\mathbf{z}} r^{-\alpha_i}}{\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}_{\text{intra}}} > \beta\right] = \mathbb{P}\left[g_{\mathbf{z}} > \frac{\beta(\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}_{\text{intra}})}{P_d r^{-\alpha_i}}\right]$$
$$= \mathbb{E}\left[\exp\left(-\frac{\beta(\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}_{\text{intra}})}{P_d r^{-\alpha_i}}\right)\right]$$
$$= e^{-\frac{\beta\sigma^2}{P_d r^{-\alpha_i}}} \mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(-\frac{\beta}{P_d r^{-\alpha_i}}\right) \mathcal{L}_{\mathcal{I}_{\text{intra}}}\left(-\frac{\beta}{P_d r^{-\alpha_i}}\right)$$

- Laplace transform of Inter cluster interference [Wang and Zhu, 2018]

$$\mathcal{L}_{\mathcal{I}_{\text{inter}}}(s) = \exp\left[-2\pi\lambda_p \int_0^\infty \left[1 - \exp\left(-\bar{m}\int_0^\infty \frac{f^\eta(y)}{1+sy^{-\alpha}}dy\right)\right]\eta d\eta\right]$$

where $\eta = \|\mathbf{x}\|$

- Laplace transform of Intra-cluster interference [Wang and Zhu, 2018]

$$\mathcal{L}_{\mathcal{I}_{\text{intra}}}(s) = \frac{e^{-\bar{m}}\left[e^{\left(\bar{m}_{-i}\int_0^\infty \frac{1}{1+sr^{-\alpha}} f^{-i}(r|a)dr\right) + \left(\bar{m}_i \int_r^\infty \frac{1}{1+sr^{-\alpha}} f^{i}(r|a)dr\right) - 2}\right]}{\int_r^\infty \frac{f^i(r|a)}{1+sr^{-\alpha}}dr}$$

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

This Intra cluster interference can be given as :

$$\mathcal{L}_{\mathcal{I}_{\text{intra}}}(s) = \mathbb{E}\left[\exp\left(-s(\sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,i}\setminus\mathbf{y}_0} P_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,-i}} P_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha})\right)\right]$$

$$= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,i}\setminus\mathbf{y}_0} \mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right) \prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,-i}} \mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right)\right]$$

$$= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,i}\setminus\mathbf{y}_0} \frac{1}{1+sP_d\|\mathbf{y}\|^{-\alpha}} \prod_{\mathbf{y}\in\Phi_{\mathbf{x}_0}^{u,-i}} \frac{1}{1+sP_d\|\mathbf{y}\|^{-\alpha}}\mathbf{I}[\|y\|>r]\right]$$

$$= \sum_{n=1}^{\infty}\frac{\bar{m}_{-i}^{n}e^{-\bar{m}_{-i}}}{n!}\left[\int_0^{\infty}\frac{f^{-i}(r|a)}{1+sP_d r^{-\alpha}}dr\right]^n \sum_{n=1}^{\infty}\frac{\bar{m}_{i}^{n}e^{-\bar{m}_{i}}}{n!}\left[\int_r^{\infty}\frac{f^{i}(r|a)}{1+sP_b r^{-\alpha}}dr\right]^{n-1}$$

This can be simply rewritten as the product of 2 exponential series and hence the result given above follows.

System Model for Case 2

- In this case, we can treat the user and the BS processes to be two independent MCPs.
- In this case, we consider probabilistic caching in the BSs as well, with probabilities denoted by $p_{i,BS}^{\mathbf{x}_0}$.
- **Self Request**

$$P_{\mathbf{x}_0}^{\text{self}} = \sum_{i=1}^{N_{\mathbf{x}_0}} p_i^{\mathbf{x}_0} q_i^{\mathbf{x}_0}$$

- **D2D Cache hit**: Let $P_{i,\mathbf{x}_0}^{\text{hit}}$ be the conditional hit probability that the $i-$th most popular file is present in cluster.

$$P_{\mathbf{x}_0}^{\text{hit}} = \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{\text{hit}} (1 - p_i^{\mathbf{x}_0})$$

- Total cache hit probability $= P_{\mathbf{x}_0}^{\text{self}} + P_{\mathbf{x}_0}^{\text{hit}}$ and Cache aided throughput is

$$\rho \lambda_u \left[\sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} p_i^{\mathbf{x}_0} + \sum_{i=1}^{N_{\mathbf{x}_0}} q_i^{\mathbf{x}_0} P_{i,\mathbf{x}_0}^{\text{hit}} (1 - p_i^{\mathbf{x}_0}) \right]$$

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

$$P^{\text{hit}}_{i,\mathbf{x}_0} = \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta] =$$

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{l\in\{\text{BS, DR}\}} \mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta | n(\Phi^{i,BS}_{\mathbf{x}_0}) = k, n(\Phi^{i,u}_{\mathbf{x}_0}) = m, \mathbb{1}_{\text{CS}} = l] \times$$

$$\mathbb{P}[n(\Phi^{i,u}_{\mathbf{x}_0}) = k, n(\Phi^{i,BS}_{\mathbf{x}_0}) = m, \mathbb{1}_{\text{CS}} = l]$$

$$\mathbb{P}[n(\Phi^{i,u}_{\mathbf{x}_0}) = k, n(\Phi^{i,BS}_{\mathbf{x}_0}) = m, \mathbb{1}_{\text{CS}} = l]$$
$$= \mathbb{P}[\mathbb{1}_{\text{CS}} = l | n(\Phi^{i,u}_{\mathbf{x}_0}) = k, n(\Phi^{i,BS}_{\mathbf{x}_0}) = m]\mathbb{P}[n(\Phi^{i,u}_{\mathbf{x}_0}) = k]\mathbb{P}[n(\Phi^{i,BS}_{\mathbf{x}_0}) = m]$$

$$\mathbb{P}[\text{SINR}_{i,\mathbf{x}_0} > \beta | n(\Phi^{i,u}_{\mathbf{x}_0}) = k, n(\Phi^{i,BS}_{\mathbf{x}_0}) = m, \mathbb{1}_{\text{CS}} = l]$$
$$= \int_0^{R_d + \|\mathbf{x}_0\|} \mathbb{P}\left[\frac{P_l g_{\mathbf{z}} r^{-\alpha_i}}{\sigma^2 + \mathcal{I}_{\text{inter}} + \mathcal{I}^l{}_{\text{intra}}} > \beta\right] f_{\|\mathbf{z}_{CS}\|}(r|k,m,l)dr$$

Analysis

- We now calculate $\mathbb{P}[\mathbb{1}_{\text{CS}} = \text{BS} | n(\Phi_{\mathbf{x}_0}^{i,u}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m]$.
- Let $\{\mathbf{y}_1^u, \mathbf{y}_2^u, \cdots, \mathbf{y}_k^u\} \in \Phi_{\mathbf{x}_0}^{i,u}$ and $\{\mathbf{y}_1^{BS}, \mathbf{y}_2^{BS}, \cdots, \mathbf{y}_m^{BS}\} \in \Phi_{\mathbf{x}_0}^{i,BS}$.
- We need to find $\mathbb{P}[\|\mathbf{y}_{(1)}^{BS}\| < \|\mathbf{y}_{(1)}^u\|]$ where $\mathbf{y}_i^{BS} \sim f^{\|\mathbf{x}\|}(y)$ and $\mathbf{y}_i^u \sim f^{\|\mathbf{x}\|}(y)$ all of them are independent of each other.
- To compute $\mathbb{P}[\mathbb{1}_{\text{CS}} = \text{BS} | n(\Phi_{\mathbf{x}_0}^{i,u}) = k, n(\Phi_{\mathbf{x}_0}^{i,BS}) = m]$ we use the fact that for independent RV's X, Y

$$\mathbb{P}[X < Y] = \int_0^\infty \int_0^y f_X(x) g_Y(y) dx dy$$

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

$$
\begin{aligned}
\mathbb{P}\left[\frac{P_l g_{\mathbf{z}} r^{-\alpha}}{\sigma^2+\mathcal{I}_{\text{inter}}+\mathcal{I}^l{}_{\text{intra}}} > \beta\right] &= \mathbb{P}\left[g_{\mathbf{z}} > \frac{\beta(\sigma^2+\mathcal{I}_{\text{inter}}+\mathcal{I}^l{}_{\text{intra}})}{P_l r^{-\alpha}}\right] \\
&= \mathbb{E}\left[\exp\left(-\frac{\beta(\sigma^2+\mathcal{I}_{\text{inter}}+\mathcal{I}^l{}_{\text{intra}})}{P_l r^{-\alpha}}\right)\right] \\
&= e^{-\frac{\beta\sigma^2}{P_l r^{-\alpha}}}\mathcal{L}_{\mathcal{I}_{\text{inter}}}\left(-\frac{\beta}{P_l r^{-\alpha}}\right)\mathcal{L}_{\mathcal{I}^l{}_{\text{intra}}}\left(-\frac{\beta}{P_l r^{-\alpha}}\right)
\end{aligned}
$$

Analysis

Laplace transform of Inter-cluster interference:

$$\mathcal{L}_{\mathcal{I}_{\text{inter}}}(s) = \mathbb{E}\left[\exp\left(-s\sum_{\mathbf{x}\in\Phi_p\setminus\mathbf{x}_0}\left(\sum_{\mathbf{y}\in\Phi_{\mathbf{x}}^u} P_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi_{\mathbf{x}}^{BS}} P_b g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}\right)\right)\right]$$

$$= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p\setminus\mathbf{x}_0}\mathbb{E}_{\Phi_{\mathbf{x}}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^u}\mathbb{E}_{g_{\mathbf{y}}}\left[\exp\left(-sP_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}\right)\right]\prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^{BS}}\mathbb{E}_{g_{\mathbf{y}}}\left[\exp\left(-sP_b g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}\right)\right]\right]\right]$$

$$= \mathrm{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p\setminus\mathbf{x}_0}\mathbb{E}_{\Phi_{\mathbf{x}}}\left[\prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^u}\frac{1}{1+sP_d\|\mathbf{y}\|^{-\alpha}}\prod_{\mathbf{y}\in\Phi_{\mathbf{x}}^{BS}}\frac{1}{1+sP_b\|\mathbf{y}\|^{-\alpha}}\right]\right]$$

$$= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p\setminus\mathbf{x}_0}\left(\sum_{n=0}^{\infty}\frac{e^{-\bar{m}}\bar{m}^n}{n!}\left(\int_0^{\infty}\frac{f^{\|\mathbf{x}\|}(y)}{1+sP_d y^{-\alpha}}dy\right)^n\right)\left(\sum_{n=0}^{\infty}\frac{e^{-\bar{m}}\bar{m}^n}{n!}\left(\int_0^{\infty}\frac{f^{\|\mathbf{x}\|}(y)}{1+sP_b y^{-\alpha}}dy\right)^n\right)\right]$$

$$= \mathbb{E}_{\Phi_p}\left[\prod_{\mathbf{x}\in\Phi_p}\exp\left(-\bar{m}\left(1-\int_0^{\infty}\frac{f^{\|\mathbf{x}\|}(y)}{1+sP_d y^{-\alpha}}dy-\int_0^{\infty}\frac{f^{\|\mathbf{x}\|}(y)}{1+sP_b y^{-\alpha}}dy\right)\right)\right]$$

$$= \exp\left[-2\pi\lambda_p\int_0^{\infty}\left[1-\exp(-\bar{m}(\int_0^{\infty}\frac{f^r(y)}{1+sP_d y^{-\alpha}}+\frac{f^r(y)}{1+sP_b y^{-\alpha}}dy))\right]rdr\right]$$

Analysis for Representative cloud $\Phi_{\mathbf{x}_0}$

$$\mathcal{L}_{\mathcal{I}^{DT}_{\text{intra}}}(s) = \mathbb{E}\left[\exp\left(-s(\textstyle\sum_{\mathbf{y}\in\Phi^{u}_{\mathbf{x}_0}\setminus\mathbf{y}_0} P_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi^{BS}_{\mathbf{x}_0}} P_b g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha})\right)\right]$$

$$\mathcal{L}_{\mathcal{I}^{DT}_{\text{intra}}}(s) = \mathbb{E}\left[\exp\left(-s(\textstyle\sum_{\mathbf{y}\in\Phi^{u,i}_{\mathbf{x}_0}\setminus\mathbf{y}_0} P_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi^{u,-i}_{\mathbf{x}_0}} P_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi^{BS,i}_{\mathbf{x}_0}} P_b g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha} + \sum_{\mathbf{y}\in\Phi^{BS,-i}_{\mathbf{x}_0}} P_b g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha})\right)\right]$$

$$= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\textstyle\prod_{\mathbf{y}\in\Phi^{u,i}_{\mathbf{x}_0}\setminus\mathbf{y}_0} \mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right) \prod_{\mathbf{y}\in\Phi^{u,-i}_{\mathbf{x}_0}} \mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_d g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right) \prod_{\mathbf{y}\in\Phi^{BS,i}_{\mathbf{x}_0}} \mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_b g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right) \prod_{\mathbf{y}\in\Phi^{BS,-i}_{\mathbf{x}_0}} \mathbb{E}_{g_{\mathbf{y}}}\left(e^{-sP_b g_{\mathbf{y}}\|\mathbf{y}\|^{-\alpha}}\right)\right]$$

$$= \mathbb{E}_{\Phi_{\mathbf{x}_0}}\left[\textstyle\prod_{\mathbf{y}\in\Phi^{u,i}_{\mathbf{x}_0}\setminus\mathbf{y}_0} \frac{1}{1+sP_d\|\mathbf{y}\|^{-\alpha}}\mathbb{1}(\|y\|>r) \prod_{\mathbf{y}\in\Phi^{u,-i}_{\mathbf{x}_0}} \frac{1}{1+sP_d\|\mathbf{y}\|^{-\alpha}} \prod_{\mathbf{y}\in\Phi^{BS,i}_{\mathbf{x}_0}} \frac{1}{1+sP_b\|\mathbf{y}\|^{-\alpha}}\mathbb{1}(\|y\|>r) \prod_{\mathbf{y}\in\Phi^{BS,-i}_{\mathbf{x}_0}} \frac{1}{1+sP_b\|\mathbf{y}\|^{-\alpha}}\right]$$

$$= \textstyle\sum_{n=1}^{\infty} \frac{\bar{m}^{n}_{-i,1}e^{-\bar{m}_{-i,1}}}{n!}\left[\int_r^\infty \frac{f(r|a)}{1+sP_d r^{-\alpha}}dr\right]^n \sum_{n=1}^{\infty} \frac{\bar{m}^{n}_{i,1}e^{-\bar{m}_{i,1}}}{n!}\left[\int_0^\infty \frac{f(r|a)}{1+sP_d r^{-\alpha}}dr\right]^{n-1} \sum_{n=1}^{\infty} \frac{\bar{m}^{n}_{-i,2}e^{-\bar{m}_{-i,2}}}{n!}\left[\int_r^\infty \frac{f(r|a)}{1+sP_b r^{-\alpha}}dr\right]^n \sum_{n=1}^{\infty} \frac{\bar{m}^{n}_{i,2}e^{-\bar{m}_{i,2}}}{n!}\left[\int_0^\infty \frac{f(r|a)}{1+sP_b r^{-\alpha}}dr\right]^n$$

$$\mathcal{L}_{\mathcal{I}^{DT}_{\text{intra}}}(s) = \frac{e^{-\bar{m}_1-\bar{m}_2}\left[e^{\left(\bar{m}_{-i,1}\int_r^\infty \frac{f(r|a)}{1+sr^{-\alpha}}dr\right)+\left(\bar{m}_{i,1}\int_0^\infty \frac{f(r|a)}{1+sr^{-\alpha}}dr\right)+\left(\bar{m}_{-i,2}\int_r^\infty \frac{f(r|a)}{1+sr^{-\alpha}}dr\right)+\left(\bar{m}_{i,2}\int_0^\infty \frac{f(r|a)}{1+sr^{-\alpha}}dr\right)-4}\right]}{\int_r^\infty \frac{f(r|a)}{1+sr^{-\alpha}}dr}$$

Similarly, we can compute the laplace transform of the intra cluster interference when we are being served by a BS.

In-band and Out-band Communication

- Based on the type of spectrum sharing, D2D can be classified into two types: in-band and out-of-band.
- In-band refers to D2D using the cellular spectrum.
- out-of-band refers to D2D utilizing unlicensed bands (e.g. 2.4GHz ISM band).
- If we add the term corresponding to cache by only BS's we get the cache aided throughput for overall system.
- Expression for BS part already Derived in Azimi-Abarghouyi, Nasiri-Kenari, and Debbah, 2019

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