End Semester Examination : MTH673A

Maximum Points = 40

1. Generate $(y_1, x_1), \ldots, (y_{1000}, x_{1000})$ from the model

$$y_i = e^{-x_i} + \epsilon_i,$$

where ϵ_i $(i = 1, \ldots, 1000)$ are generated from standard normal distribution, and x_i $(i = 1, \ldots, 1000)$ are generated from uniform distribution on $(-3, 3)$. Propose a test to check whether X and ϵ are independent or not and estimate the power of the test when

(i) X and ϵ jointly follow bivariate normal distribution with $E(X) = 0$, $E(\epsilon) = 0$, $Var(X) = 1$, $Var(\epsilon) = 1$ and $E(X\epsilon) = \frac{2}{3}$. points = 6

(ii) X and ϵ jointly follow standard Cauchy distribution. points = 6

2. Let the data $\mathcal{X} = (x_1, \ldots, x_n)$ be generated from an unknown distribution F, and the α-trimmed mean based on the data $\mathcal{X}$ is defined as $\bar{x}_\alpha = \frac{1}{n-2[n\alpha]} \sum\limits_{i=[n\alpha]+1}^{n-[n\alpha]} x_{(i)}$, where $\alpha \in (0, \frac{1}{2})$, [.] denotes the greatest integer inside [.], and $x_{(i)}$ denotes the i-th order statistic. The population version of the α-trimmed mean is defined as $\theta_\alpha = \frac{1}{1-2\alpha} \int\limits_{F^{-1}(\alpha)}^{F^{-1}(1-\alpha)} x dF(x)$. Does $\sqrt{n}(\bar{x}_\alpha - \theta_\alpha)$ follow asymptotically normal distribution? If F is standard normal distribution, is the asymptotic variance of $\sqrt{n}(\bar{x}_\alpha - \theta_\alpha)$ an increasing function of α? Justify your answers. points = 8 + 8

3. Let the data $\mathcal{X} = (x_1, \ldots, x_n)$ be generated from an unknown distribution F. Suppose that $\hat{\theta}_{LTS}$, $\hat{\theta}_{LMS}$, $\hat{\theta}_{median}$, $\hat{\theta}_{mode}$ and $\hat{\theta}_{mean}$ denote the LTS, the LMS, the median, the mode and the mean, respectively based on the data $\mathcal{X}$. Compute the asymptotic efficiencies of the $\hat{\theta}_{LTS}$, $\hat{\theta}_{LMS}$, $\hat{\theta}_{median}$ and $\hat{\theta}_{mode}$ with respect to $\hat{\theta}_{mean}$ when F is a standard normal distribution.

points = 3 + 3 + 3 + 3