Overview of the package BuyseTest

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This vignette describes the main functionalities of the **BuyseTest** package. This package implements the Generalized Pairwise Comparisons (GPC) as defined in Buyse (2010) for complete observations, and extended in Péron et al. (2018) to deal with right-censoring. When considering a single endpoint, the GPC procedure can be summarized as follow. Denote the endpoint by Y in the treatment group and by X in the control group. Given a threshold of clinical relevance τ , the aim of GPC is to estimate the proportion in favor of treatment $\mathbb{P}[Y \geq X + \tau]$ and the proportion in favor of control $\mathbb{P}[X \geq Y + \tau]$. Other statistics such as the net benefit $\mathbb{P}[Y \geq X + \tau] - \mathbb{P}[X \geq Y + \tau]$ or the win ratio $\mathbb{P}[X \geq Y + \tau]$ can then be deduced. It is assumed that the reader is familiar with the GPC terminology $\mathbb{P}[X \geq X + \tau]$ can then be deduced. It is assumed that the reader is familiar with the GPC terminology $\mathbb{P}[X \geq X + \tau]$ or the software aspect of the **BuyseTest** package (not on the underlying statistical model).

The **BuyseTest** package contains three main functions:

- the function BuyseTest is the main function of the package. It performs the GPC, estimates the net benefit/win ratio, and output a BuyseRes object. The user can interact with BuyseRes objects using:
 - summary to obtain a nice display of the results
 - coef to extract the estimates.
 - confint to extract estimates, confidence intervals, and p.values.
 - getIid to extract the iid decomposition of the estimator.
 - getPairScore to extract the contribution of each pair to the net benefit/win ratio.
 - getSurvival to extract the estimates of the survival (only relevant for right-censored endpoints).
- the powerBuyseTest function performs simulation studies, e.g. to estimate the statistical power or assess the bias / type 1 error rate of a test for a specific design.
- the BuyseTest.options function enables the user to display the default values used in the BuyseTest package (essentially used by the BuyseTest function). function. The function can also change the default values to better match the user needs.

¹if not, Buyse (2010) is a good place to start.

Before going further we need to load the **BuyseTest** package in the R session:

```
library(BuyseTest)
library(data.table)
```

To illustrate the functionalities of the package, we will used the **veteran** dataset from the **survival** package:

```
data(veteran,package="survival")
head(veteran)
```

```
trt celltype time status karno diagtime age prior
    1 squamous
                                            69
                 72
                         1
                               60
                                         7
1
                                                   0
2
    1 squamous 411
                         1
                               70
                                         5
                                            64
                                                   10
    1 squamous 228
3
                         1
                               60
                                         3
                                           38
                                                   0
    1 squamous 126
4
                         1
                                         9
                                                  10
                               60
                                            63
    1 squamous 118
5
                         1
                               70
                                        11
                                            65
                                                   10
    1 squamous
                                            49
                                                   0
6
               10
                         1
                               20
                                         5
```

See ?veteran for a presentation of the database.

<u>Note:</u> the **BuyseTest** package is under active development. Newer package versions may include additional functionalities and fix previous bugs. The version of the package that is being is:

```
utils::packageVersion("BuyseTest")
```

[1] '2.2.2'

For completness, the details of the R session used to generate this document are:

```
sessionInfo()
```

```
R version 3.6.3 (2020-02-29)
Platform: x86_64-pc-linux-gnu (64-bit)
Running under: Ubuntu 16.04.7 LTS

Matrix products: default
BLAS: /usr/lib/libblas/libblas.so.3.6.0
LAPACK: /usr/lib/lapack/liblapack.so.3.6.0
```

locale:

[10] LC_TELEPHONE=C LC_MEASUREMENT=da_DK.UTF-8 LC_IDENTIFICATION=C

attached base packages:

[1] stats graphics grDevices utils datasets methods base

other attached packages:

[1] data.table_1.13.7 BuyseTest_2.2.2 Rcpp_1.0.5

loaded via a namespace (and not attached):

[1] compiler_3.6.3 Matrix_1.2-18 tools_3.6.3 parallel_3.6.3 [5] survival_3.1-12 prodlim_2019.11.13 splines_3.6.3 grid_3.6.3

[9] lava_1.6.8.1 stats4_3.6.3 lattice_0.20-41

1 Performing generalized pairwise comparisons (GPC) using the BuyseTest function

To perform generalized pairwise comparisons, the BuyseTest function needs:

• where the data are stored - argument data

• the name of the endpoints - argument endpoint

• the type of each endpoint - argument type

• the variable defining the two treatment groups - argument treatment

The BuyseTest function has many optional arguments to specify for example:

- the threshold of clinical relevance associated to each endpoint argument threshold
- the censoring associated to each endpoint (for time to event endpoints) argument status

There are two equivalent ways to define the GPC:

• using a separate argument for each element:

Generalized Pairwise Comparisons

```
Settings
```

Point estimation and calculation of the iid decomposition

```
Estimation of the estimator's distribution - method: moments of the U-statistic
```

Gather the results in a S4BuyseTest object

- or via a formula interface. In the formula interface endpoint are wrapped by parentheses. The parentheses must be preceded by their type:
 - binary (b, bin, or binary)
 - continuous (c, cont, or continuous)
 - time to event (t, tte, or timetoevent)

```
BT.f <- BuyseTest(trt ~ tte(time, threshold = 20, status = "status"),
data = veteran, trace = 0)
```

Here we set in addition the argument trace to 0 to force the function to be silent (i.e. no display in the terminal). We can check that the two approaches are equivalent:

```
testthat::expect_equal(BT.f,BT)
```

1.1 Displaying the results

The results of the GPC can be displayed using the summary method:

```
summary(BT)
```

Generalized pairwise comparisons with 1 endpoint

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- confidence level: 0.95
                 : H-projection of order 1

    inference

- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                               Delta
                                   37.78
                                                  46.54
                                                              15.68
                                                                           0 - 0.0877
CI [2.5%; 97.5%] p.value
 [-0.2735;0.1045] 0.37162
```

To display the number of pairs instead of the percentage of pairs that are favorable/unfavorable/neutral/uniformative, set the argument percentage to FALSE:

```
summary(BT, percentage = FALSE)
```

Generalized pairwise comparisons with 1 endpoint

```
- statistic : net benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference : H-projection of order 1
- treatment groups: 1 (control) vs. 2 (treatment)
```

By default summary displays results relative to the net benefit. To get results for the win ratio set the argument statistic to "winRatio":

```
summary(BT, statistic = "winRatio")
```

Generalized pairwise comparisons with 1 endpoint

```
- statistic
                  : win ratio (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 1
- confidence level: 0.95
                  : H-projection of order 1
- inference
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta
                        100
                                   37.78
                                                  46.54
                                                              15.68
                                                                           0 0.8117
                20
CI [2.5%; 97.5%] p.value
  [0.5134;1.2833] 0.37195
```

See help(BuyseRes-summary) for more detailed explanations about the summary method and its output. Note that a more concise output, in a data.frame format, can be obtained via the confint method:

```
confint(BT, statistic = "winRatio")
```

```
estimate se lower.ci upper.ci p.value
time_20 0.8116692 0.1896937 0.5133887 1.283252 0.3719466
attr(,"n.resampling")
time_20
NA
```

1.2 Stratified GPC

GPC can be performed for subgroups of a categorical variable

- argument strata

For instance, the celltype may have huge influence on the survival time and the investigator would like to only compare patients that have the same celltype. In the formula interface this is achieved by adding a single variable in the right hand side of the formula:

```
ff2strata <- update(ff2, . \sim . + celltype)
BT2 <- BuyseTest(ff2strata, data = veteran, trace = 0)
```

The fact the it is not wrapped by bin, cont or tte indicates differentiate it from endpoint variables.

When doing a stratified analysis, the summary method displays the global results as well as the results within each strata²:

Generalized pairwise comparisons with 2 prioritized endpoints and 4 strata

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
 confidence level: 0.95
                  : H-projection of order 1
- inference
- treatment groups: 1 (control) vs. 2 (treatment)
 censored pairs : probabilistic score based on the survival curves
                  : re-analyzed using lower priority endpoints
- uninformative pairs: no contribution at the current endpoint, analyzed at later endpoints
- results
                      strata total(%) favorable(%) unfavorable(%)
endpoint threshold
                                                                      delta
                                                                               Delta
                                                              45.77 -0.0971 -0.0971
    time
                20
                      global
                                100.00
                                              36.06
                    squamous
                                 25.38
                                              14.33
                                                               8.77 0.2193
                   smallcell
                                 45.69
                                              12.69
                                                              20.88 -0.1792
                                                               6.15 -0.1034
                       adeno
                                 13.71
                                               4.74
                       large
                                 15.23
                                               4.30
                                                               9.97 - 0.3722
   karno
             1e-12
                                 18.17
                                                6.72
                                                               8.07 -0.0135 -0.1106
                      global
                                  2.28
                                               0.76
                                                               0.94 -0.0071
                    squamous
                   smallcell
                                 12.12
                                                4.33
                                                               5.75 -0.0311
                                  2.81
                                                1.46
                                                               0.85 0.0448
                       adeno
                                  0.96
                                                               0.54 -0.0241
                                               0.17
                       large
```

Note that here the numbers in the total/favorable/unfavorable/ columns are relative to the overall sample while the delta is only relative to the strata. The global delta is a sum of the strata specific delta weighted by the empirical proportion of pairs for each strata.

²the strata-specific results can be removed by setting the argument strata to "global" when calling summary.

1.3 Using multiple endpoints

More than one endpoint can be considered by indicating a vector of endpoints, types, and thresholds. In the formula interface, the different endpoints must be separated with a "+" on the right hand side of the formula:

Generalized pairwise comparisons with 2 prioritized endpoints

```
: net benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : H-projection of order 1
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
                  : re-analyzed using lower priority endpoints
- neutral pairs
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                               delta
                                                                                       Delta
                     100.00
                                   37.78
             2e+01
                                                   46.54
                                                              15.68
                                                                           0 -0.0877 -0.0877
    time
             1e-12
                      15.68
                                    5.78
                                                    7.11
                                                               2.78
                                                                           0 -0.0133 -0.1009
  karno
CI [2.5%; 97.5%] p.value
 [-0.2735;0.1045] 0.37162
 [-0.2901;0.0959] 0.31478
```

The hierarchy of the endpoint is defined from left (most important endpoint, here time) to right (least important endpoint, here karno). It is also possible to perform the comparisons on all endpoints setting the argument hierarchical to FALSE:

```
BT.nH <- BuyseTest(ff2, hierarchical = FALSE, data = veteran, trace = 0)
summary(BT.nH)</pre>
```

Generalized pairwise comparisons with 2 endpoints

```
- statistic
                  : net benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : H-projection of order 1
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
                  : re-analyzed using lower priority endpoints
- neutral pairs
endpoint threshold weight total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                                      delta
    time
             2e+01
                        1
                               100
                                          37.78
                                                          46.54
                                                                     15.68
                                                                                  0 - 0.0877
             1e-12
                        1
                                          41.82
                                                          44.95
                                                                     13.24
                                                                                  0 -0.0313
 Delta CI [2.5%; 97.5%] p.value
-0.0877 [-0.2735;0.1045] 0.37162
-0.1190 [-0.4346;0.2226] 0.49821
```

In that case the score of a pair is the weighted sum of the score relative to each endpoint. By default the weights are all set to 1 but this behavior can be changed by setting the argument weight when calling BuyseTest, e.g.:

Generalized pairwise comparisons with 2 endpoints

```
- statistic
                  : net benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : H-projection of order 1
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
                  : re-analyzed using lower priority endpoints
- neutral pairs
- results
endpoint threshold weight total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                                      delta
                                          37.78
   time
             2e+01
                      0.8
                               100
                                                         46.54
                                                                     15.68
                                                                                  0 - 0.0877
             1e-12
                      0.2
                                          41.82
                                                         44.95
                                                                                  0 -0.0313
  karno
                               100
                                                                     13.24
 Delta CI [2.5%; 97.5%] p.value
-0.0701 [-0.2204;0.0834] 0.37073
-0.0764 [-0.2504;0.1024] 0.40269
```

This has been referred as the O'Brien test in the litterature (Verbeeck et al. (2019), section 3.2).

1.4 What if smaller is better?

By default BuyseTest will always assume that higher values of an endpoint are favorable. This behavior can be changed by specifying operator = "<0" for an endpoint:

```
ffop <- trt ~ tte(time, status = "status", threshold = 20, operator = "<0")
BTinv <- BuyseTest(ffop, data = veteran, trace = 0)
summary(BTinv)</pre>
```

Generalized pairwise comparisons with 1 endpoint

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- confidence level: 0.95

    inference

            : H-projection of order 1
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta
                                                 37.78
               20
                       100
                                  46.54
                                                            15.68
                                                                      0 0.0877
CI [2.5%; 97.5%] p.value
 [-0.1045;0.2735] 0.37162
```

Internally BuyseTest will compute the favorable and unfavorable score as usual and then switch them around if operator if "<0".

1.5 Stopping comparison for neutral pairs

In presence of neutral pairs, BuyseTest will, by default, continue the comparison on the endpoints with lower priority. For instance let consider a dataset with one observation in each treatment arm:

```
Id treatment tumor size
1: 1 Yes Yes 15
2: 2 No Yes 20
```

If we use the GPC with tumor as the first endpoint and size as the second endpoint:

Generalized pairwise comparisons with 2 prioritized endpoints

```
: net benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- treatment groups: No (control) vs. Yes (treatment)
- neutral pairs : re-analyzed using lower priority endpoints
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) delta Delta
   tumor
             5e-01
                        100
                                        0
                                                       0
                                                                 100
                                                                            0
                                                                                  0
                                                                                        0
    size
             1e-12
                        100
                                      100
                                                       0
                                                                  0
                                                                            0
                                                                                  1
                                                                                        1
```

the outcome of the comparison is neutral for the first priority, but favorable for the second. Setting the argument neutral.as.uninf to FALSE will stop the comparison when a pair is classified as neutral:

```
BT.pair2 <- BuyseTest(treatment ~ bin(tumor) + cont(size, operator = "<0"), data = dt.sim, trace = 0, method.inference = "none", neutral.as.uninf = FALSE) summary(BT.pair2)
```

Generalized pairwise comparisons with 2 prioritized endpoints

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- treatment groups: No (control) vs. Yes (treatment)
- neutral pairs : ignored at lower priority endpoints
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) delta Delta
             5e-01
                        100
                                        0
                                                       0
                                                                100
  tumor
                                                                            0
                                                                                  0
                                                                                        0
                                        0
                                                                  0
    size
             1e-12
                          0
                                                                                        0
```

So in this case no pair is analyzed at second priority.

1.6 What about p-value and confidence intervals?

Several methods are available in BuyseTest to perform statistical inference:

• permutation test setting the argument method.inference to "permutation". Assuming exchangeability under the null hypothesis, this approach gives valid p-values (regardless to the sample size) for testing the absence of a difference between the groups.

Generalized pairwise comparisons with 1 endpoint

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : permutation test with 1000 samples
                    p-value computed using the permutation distribution
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta p.value
                20
                        100
                                   37.78
                                                  46.54
                                                             15.68
                                                                          0 -0.0877
                                                                                      0.366
   time
```

• bootstrap resampling setting the argument method.inference to "bootstrap". In large enough samples, this approach gives valid p-values and confidence intervals.

Generalized pairwise comparisons with 1 endpoint

```
- statistic
                  : net benefit (delta: endpoint specific, Delta: global)
- null hypothesis : Delta == 0
- confidence level: 0.95
- inference
                  : bootstrap resampling with 1000 samples
                    CI computed using the percentile method; p-value by test inversion
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                   37.78
                                                  46.54
                                                             15.68
                20
                        100
                                                                          0 -0.0877
CI [2.5%; 97.5%] p.value
 [-0.2797;0.1108] 0.363
```

• asymptotic distribution setting the argument method.inference to "u-statistic". In large enough samples, this approach gives valid p-values and confidence intervals.

```
BT.ustat <- BuyseTest(trt ~ tte(time, threshold = 20, status = "status"),
data = veteran, trace = 0, method.inference = "u-statistic")
summary(BT.ustat)
```

Generalized pairwise comparisons with 1 endpoint

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- confidence level: 0.95
                  : H-projection of order 1
- inference
- treatment groups: 1 (control) vs. 2 (treatment)
- censored pairs : probabilistic score based on the survival curves
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                20
                        100
                                   37.78
                                                  46.54
                                                              15.68
                                                                           0 -0.0877
CI [2.5%; 97.5%] p.value
 [-0.2735;0.1045] 0.37162
```

The first two approaches require simulating a large number of samples and applying the GPC to each of these samples. The number of samples is set using the arugment n.resampling and it should large enough to limit the Monte Carlo error when estimating the p-value. Typically should be at least 10000 to get, roughtly, 2-digit precision, as examplified below:

```
set.seed(10)
sapply(1:10, function(i){mean(rbinom(1e4, size = 1, prob = 0.05))})
```

[1] 0.0511 0.0491 0.0489 0.0454 0.0516 0.0522 0.0468 0.0483 0.0491 0.0508

Indeed, here we get a reasonnable approximation of 0.05 (if we round and only keep 2 digits). Note that to get 3 digits precision we would need more samples. The last method does not rely on resampling but on the computation of the influence function of the estimator. Fortunately, when using the Gehan's scoring rule, this does not really involve any extra-calculations and this is therefore very fast to perform. When using the Peron's scoring rule, more serious extra-calculations are involved so the computation time is expected to increase by a factor 5 to 10 compared to the point estimate alone (i.e. method.inference equal to "none").

2 Getting additional inside: looking at the pair level

So far we have looked at the overall score and probabilities. But it is also possible to extract the score relative to each pair, as well as to "manually" compute this score. This can give further inside on what the software is actually doing and what is the contribution of each individual on the evaluation of the treatment.

2.1 Extracting the contribution of each pair to the statistic

The net benefit or the win ratio statistics can be expressed as a sum of a score over all pairs of patients. The argument keep.pairScore enables to export the score relative to each pair in the output of BuyseTest:

The method getPairScore can then be used to extract the contribution of each pair. For instance the following code extracts the contribution for the first endpoint:

```
getPairScore(BT.keep, endpoint = 1)
```

| | index.1 | index.2 | favorable | ${\tt unfavorable}$ | neutral | uninf | weight |
|-------|---------|---------|-----------|---------------------|---------|-------|--------|
| 1: | 1 | 70 | 1 | 0 | 0 | 0 | 1 |
| 2: | 2 | 70 | 1 | 0 | 0 | 0 | 1 |
| 3: | 3 | 70 | 1 | 0 | 0 | 0 | 1 |
| 4: | 4 | 70 | 1 | 0 | 0 | 0 | 1 |
| 5: | 5 | 70 | 1 | 0 | 0 | 0 | 1 |
| | | | | | | | |
| 4688: | 65 | 137 | 0 | 1 | 0 | 0 | 1 |
| 4689: | 66 | 137 | 0 | 1 | 0 | 0 | 1 |
| 4690: | 67 | 137 | 0 | 1 | 0 | 0 | 1 |
| 4691: | 68 | 137 | 0 | 1 | 0 | 0 | 1 |
| 4692: | 69 | 137 | 0 | 1 | 0 | 0 | 1 |

Each line corresponds to different comparison between a pair from the control arm and the treatment arm. The column strata store to which strata the pair belongs (first, second, ...). The columns favorable, unfavorable, neutral, uninformative contains the result of the comparison, e.g. the first pair was classified as favorable while the last was classified as favorable with a weight of 1. The second and third columns indicates the rows in the original dataset corresponding to the pair:

```
veteran[c(70,1),]
```

```
trt celltype time status karno diagtime age prior
70
     2 squamous
                  999
                            1
                                 90
                                           12
                                               54
                                                      10
1
     1 squamous
                   72
                            1
                                 60
                                            7
                                               69
                                                       0
```

For the first pair, the event was observed for both observations and since 999 > 72 + 20 the pair is rated favorable. Substracting the average probability of the pair being favorable minus the average probability of the pair being unfavorable:

```
getPairScore(BT.keep, endpoint = 1)[, mean(favorable) - mean(unfavorable)]
```

[1] -0.08765836

gives the net benefit in favor of the treatment for the first endpoint:

```
BT.keep
```

```
endpoint threshold delta Delta
time 2e+01 -0.0877 -0.0877
karno 1e-12 -0.0133 -0.1009
```

More examples and explanation can be found in the documentation of the method getPairScore.

2.2 Extracting the survival probabilities

When using scoring.rule equals "Peron", survival probabilities at event time, and event times +/threshold in the control and treatment arms are used to score the pair. Setting keep.survival to TRUE
and precompute to FALSE in BuyseTest.options enables to export the survival probabilities in the output
of BuyseTest:

The method getSurvival can then be used to extract these survival probabilities. For instance the following code extracts the survival for the first endpoint:

```
outSurv <- getSurvival(BT.keep2, endpoint = 1, strata = 1)
str(outSurv)</pre>
```

```
List of 5
 $ survTimeC: num [1:69, 1:13] 72 411 228 126 118 10 82 110 314 100 ...
  ..- attr(*, "dimnames")=List of 2
  .. ..$ : NULL
  ....$ : chr [1:13] "time" "survivalC-threshold" "survivalC_0" "survivalC+threshold" ...
 $ survTimeT: num [1:68, 1:13] 999 112 87 231 242 991 111 1 587 389 ...
  ..- attr(*, "dimnames")=List of 2
  ....$ : NULL
  ....$ : chr [1:13] "time" "survivalC-threshold" "survivalC_0" "survivalC+threshold" ...
 $ survJumpC: num [1:57, 1:6] 3 4 7 8 10 11 12 13 16 18 ...
  ..- attr(*, "dimnames")=List of 2
  .. ..$ : NULL
  ....$ : chr [1:6] "time" "survival" "dSurvival" "index.survival" ...
 $ survJumpT: num [1:51, 1:6] 1 2 7 8 13 15 18 19 20 21 ...
  ..- attr(*, "dimnames")=List of 2
  .. ..$ : NULL
  ....$ : chr [1:6] "time" "survival" "dSurvival" "index.survival" ...
 $ lastSurv : num [1:2] 0 0
```

2.2.1 Computation of the score with only one censored event

Let's look at pair 91:

```
getPairScore(BT.keep2, endpoint = 1, rm.withinStrata = FALSE)[91]
```

In the dataset this corresponds to:

```
veteran[c(22,71),]
```

```
celltype time status karno diagtime age prior
22
     1 smallcell
                    97
                             0
                                   60
                                              5
                                                 67
                                   80
                                              6
                                                 60
                                                         0
71
     2
        squamous
                   112
                             1
```

The observation from the control group is censored at 97 while the observation from the treatment group has an event at 112. Since the threshold is 20, and (112-20)<97, we know that the pair is not in favor of the treatment. The formula for probability in favor of the control is $\frac{S_c(97)}{S_c(112+20)}$. The survival at the event time in the censoring group is stored in survTimeC. Since observation 22 is the 22th observation in the control group:

```
iSurv <- outSurv$survTimeC[22,]
iSurv</pre>
```

```
survivalC-threshold
                                                                    survivalC 0
                     time
               97.000000
                                           0.5615232
                                                                      0.5171924
                                                                    survivalT_0
      survivalC+threshold
                                 survivalT-threshold
                0.4235463
                                           0.4558824
                                                                      0.3643277
      survivalT+threshold index.survivalC-threshold
                                                              index.survivalC_0
                0.2827500
                                          25.0000000
                                                                     28.0000000
index.survivalC+threshold index.survivalT-threshold
                                                              index.survivalT_0
               33.0000000
                                          27.0000000
                                                                     32.0000000
index.survivalT+threshold
               35.0000000
```

Since we are interested in the survival in the control arm exactly at the event time:

```
Sc97 <- iSurv["survivalC_0"]
Sc97
```

```
survivalC_0
0.5171924
```

The survival at the event time in the treatment group is stored in survTimeC. Since observation 71 is the 2nd observation in the treatment group:

```
iSurv <- outSurv$survTimeT[2,] ## survival at time 112+20
iSurv</pre>
```

```
time
                                 survivalC-threshold
                                                                    survivalC_0
              112.0000000
                                           0.5319693
                                                                      0.4549201
      survivalC+threshold
                                 survivalT-threshold
                                                                    survivalT_0
                0.3594915
                                           0.3801681
                                                                      0.2827500
      survivalT+threshold index.survivalC-threshold
                                                              index.survivalC 0
                0.2827500
                                          27.0000000
                                                                     32.0000000
index.survivalC+threshold index.survivalT-threshold
                                                              index.survivalT_0
               37.0000000
                                          31.0000000
                                                                     35.0000000
index.survivalT+threshold
               35.0000000
```

Since we are interested in the survival in the control arm at the event time plus threshold:

```
Sc132 <- iSurv["survivalC+threshold"]
Sc132
```

```
survivalC+threshold 0.3594915
```

The probability in favor of the control is then:

```
Sc132/Sc97
```

```
survivalC+threshold 0.6950827
```

2.2.2 Computation of the score with two censored events

When both observations are censored, the formula for computing the probability in favor of treatment or control involves an integral. This integral can be computed using the function calcIntegralSurv_cpp that takes as argument a matrix containing the survival and the jumps in survival, e.g.:

```
head(outSurv$survJumpT)
```

| | time | survival | dSurvival | <pre>index.survival</pre> | index.dsurvival1 | index.dsurvival2 |
|------|------|-----------|-------------|---------------------------|------------------|------------------|
| [1,] | 1 | 0.7681159 | -0.02941176 | 12 | 0 | 1 |
| [2,] | 2 | 0.7536232 | -0.01470588 | 13 | 1 | 2 |
| [3,] | 7 | 0.7388463 | -0.02941176 | 14 | 2 | 3 |
| [4,] | 8 | 0.7388463 | -0.02941176 | 14 | 3 | 4 |
| [5,] | 13 | 0.7092924 | -0.01470588 | 16 | 4 | 5 |
| [6,] | 15 | 0.6945155 | -0.02941176 | 17 | 5 | 6 |

and the starting time of the integration time. For instance, let's look at pair 148:

```
getPairScore(BT.keep2, endpoint = 1, rm.withinStrata = FALSE)[148]
```

which corresponds to the observations:

```
veteran[c(10,72),]
```

```
trt celltype time status karno diagtime age prior 10 1 squamous 100 0 70 6 70 0 72 2 squamous 87 0 80 3 48 0
```

The probability in favor of the treatment (p_F) and control (p_{UF}) can be computed as:

$$p_F = -\frac{1}{S_T(x)S_C(y)} \int_{t>y} S_T(t+\tau) dS_C(t)$$

$$p_{UF} = -\frac{1}{S_T(x)S_C(y)} \int_{t>x} S_C(t+\tau) dS_T(t)$$

where x = 87 and y = 100. To ease the call of calcIntegralScore_cpp we create a warper:

and then call it to compute the probabilities:

```
favorable unfavorable lowerBound 0.5058685 0.3770426 upperBound 0.5058685 0.3770426
```

3 Dealing with missing values or/and right censoring

In presence of censoring or missing values, it is often not be possible to classify all pairs without a model for the censoring mechanism. The unclassified pairs, called uninformative, have a score of 0 which will typically bias the estimate of the net net benefit towards 0^{-3} . Consider the following dataset:

treatment eventtimeUncensored eventtime status toxicity eta_toxicity status1

```
yes -0.30786498
1:
                        1.3499793 0.4546612
                                                   0
2:
           C
                        1.3022440 0.8234702
                                                                 0.75808558
                                                                                   1
                                                   0
                                                           no
           C
                        0.9800451 0.3656312
                                                                                   1
3:
                                                   0
                                                          yes -0.57386341
           C
4:
                        0.1809881 0.1809881
                                                   1
                                                                -0.93874446
                                                                                   1
                                                          yes
5:
           C
                        0.2747980 0.2747980
                                                   1
                                                          yes
                                                               -0.02769932
                                                                                   1
           C
                        0.1351895 0.1351895
                                                          yes -1.06624865
                                                                                   1
6:
                                                   1
```

where we have the uncensored event times (eventtimeUncensored) as well as the censored event times (eventtime). The percentage of censored observations is:

```
100*dt[,mean(status==0)]
```

[1] 46

We would like to be able to recover the net benefit estimated with the uncensored event times:

```
BuyseTest(treatment ~ tte(eventtimeUncensored, status1, threshold = 0.5),
data = dt,
scoring.rule = "Gehan", method.inference = "none", trace = 0)
```

```
eventtimeUncensored 0.5 -0.2314
```

using the censored survival times.

³While the power is typically reduced, the type 1 error will still be controlled if censoring is at random

The BuyseTest function handles missing values via two arguments:

- scoring.rule indicates how pairs involving missing data are compared.
 - the Gehan's scoring rule compares the observed values. If it is not possible to decide whether one observation has a better endpoint than the other (e.g. because both are right-censoring) then the paired is scored uninformative.
 - the Peron's scoring rule compares the probability of one observation having a better endpoint
 than the other given the observed values. This require a model for the censoring distribution.
 If the full survival curve can be identified then all pairs can be fully classified otherwise some
 of the pair will be partially uninformative.
- correction.uninf indicates what to do with the uninformative scores. Setting this argument to TRUE will re-distribute this score to favorable/unfavorable/neutral scores.

When the survival curve can be fully identified, the default (and recommanded) approach is to use the Peron's scoring rule where the censoring model rely on Kaplan Meier curve is fitted in each treatment group.

When the last observation are censored, then part of the survival curve is unknown and there is no perfect solution. One can:

- only use the Peron's scoring rule, which will lead to a non-0 uninformative score and therefore a "conservative" estimate of the net benefit.
- use the Peron's scoring rule in conjonction with the correction which will led to an unbiased estimator if certain assumption are met.
- only use the Peron's scoring rule with a parametric model which if appropriate will lead to an unbiased (and rather efficient) estimator.

3.1 Gehan's scoring rule

In the example, Gehan's scoring rule:

```
e.G <- BuyseTest(treatment ~ tte(eventtime, status, threshold = 0.5),
    data = dt, scoring.rule = "Gehan", trace = 0)
summary(e.G, print=FALSE)$table.print</pre>
```

```
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta
1 eventtime    0.5    100     4.27     13.09    20.42    62.22 -0.0881
CI [2.5%; 97.5%]    p.value significance
1 [-0.1146; -0.0616] 8.7323e-11    ***
```

leads to many uninformative pairs (about 60%) and an estimate much closer to 0 than the truth.

3.2 Peron's scoring rule

In the example, Peron's scoring rule:

```
e.P <- BuyseTest(treatment ~ tte(eventtime, status, threshold = 0.5),
    data = dt, scoring.rule = "Peron", trace = 0)
summary(e.P, print=FALSE)$table.print</pre>
```

leads to no uninformative pairs. Indeed the last observation in each group is an (uncensored) event:

```
dt[,.SD[which.max(eventtime)],by="treatment"]
```

so the full survival curve could be identified. As a result the estimate is very close to the truth.

<u>Note 1:</u> the censoring model can be specified by first fitting a Kaplan Meier model for the survival time:

```
library(prodlim)
e.prodlim <- prodlim(Hist(eventtime, status) ~ treatment, data = dt)</pre>
```

Then passing the model to the BuyseTest via the model.tte argument:

```
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta

1 eventtime 0.5 100 14.61 40.43 44.96 0 -0.2583

CI [2.5%; 97.5%] p.value significance

1 [-0.3029; -0.2124] < 2.22e-16 ***
```

Note that the CI/p-value have changed since, unless stated otherwise, BuyseTest assumes no uncertainty about the survival model when using model.tte. One can force it to account for the uncertainty adding an attribute:

```
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta

1 eventtime 0.5 100 14.61 40.43 44.96 0 -0.2583

CI [2.5%; 97.5%] p.value significance

1 [-0.3292; -0.1844] 2.6247e-11 ***
```

Note 2: it is possible to use a parametric model via the survreg function:

Then passing the model to the BuyseTest via the model.tte argument:

2500 obs. of 3 variables:

\$ index.jump: logi NA NA NA NA NA NA ...

summary(e.P4, print=FALSE)\$table.print

'data.frame':

Internally the survival curve is discretized using 1000 points starting from survival = 1 to survival = 0.001 (this is why there is a non-0 but small percentage of uninformative pairs). This is performed internally by applying the BuyseTTEM method. Another discretisation can be obtained by calling BuyseTTEM with another value for the n.grid argument:

```
e.TTEM <- BuyseTTEM(e.survreg, treatment = "treatment", iid = TRUE, n.grid = 2500)
attr(e.TTEM, "iidNuisance") <- TRUE
str(e.TTEM$peron$jumpSurvHaz[[1]][[1]])
```

```
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%) Delta
1 eventtime 0.5 100 11.4 32.84 55.76 0 -0.2144
CI [2.5%; 97.5%] p.value significance
1 [-0.2704; -0.157] 7.0011e-13 ***
```

It is therefore possible to extend the approach to other model by defining an appropriate BuyseTTEM method. Looking at the code use for defining BuyseTTEM.survreg can be helpful.

3.3 Correction via inverse probability-of-censoring weights (IPCW)

With IPCW, the weights of the non-informative pairs is redistributed to the informative pairs. This is only a good strategy when there are no neutral pairs or there are no lower priority endpoints. This gives an estimate much closer to the true net benefit:

```
BT <- BuyseTest(treatment ~ tte(eventtime, status, threshold = 0.5),
data = dt, keep.pairScore = TRUE, trace = 0,
scoring.rule = "Gehan", method.inference = "none", correction.uninf = 2)
summary(BT)
```

Generalized pairwise comparisons with 1 endpoint

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- treatment groups: C (control) vs. T (treatment)
 censored pairs : deterministic score or uninformative
 uninformative pairs: no contribution, their weight is passed to the informative pairs using IPCW
- results
endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                                Delta
                                    11.31
                0.5
                         100
                                                    34.64
                                                               54.05
                                                                            0 - 0.2333
eventtime
```

We can also see that no pair is finally classified as non informative. To get some inside about the correction we can look at the scores of the pairs:

```
iScore <- getPairScore(BT, endpoint = 1)
```

To get a synthetic view, we only look at the unique favorable/unfavorable/neutral/uniformative results:

```
iScore[,.SD[1],
    .SDcols = c("favorableC", "unfavorableC", "neutralC", "uninfC"),
    by = c("favorable", "unfavorable", "neutral", "uninf")]
```

```
favorable unfavorable neutral uninf favorableC unfavorableC neutralC uninfC
                                                         0.000000 0.000000
1:
           0
                        0
                                 0
                                       1
                                           0.000000
2:
           0
                        1
                                 0
                                       0
                                           0.000000
                                                         2.647043 0.000000
                                                                                  0
           0
                        0
                                 1
                                       0
                                           0.000000
                                                         0.000000 2.647043
                                                                                  0
3:
           1
                                 0
                                       0
                                           2.647043
                                                         0.000000 0.000000
                                                                                  0
4:
```

We can see that the favorable/unfavorable/neutral pairs have seen their contribution multiplied by:

```
iScore[,1/mean(favorable + unfavorable + neutral)]
```

[1] 2.647043

i.e. the inverse probability of being informative.

3.4 Correction at the pair level

Another possible correction is to distribute the non-informative weight of a pair to the average favorable/unfavorable/neutral probability observed on the sample:

```
BT <- BuyseTest(treatment ~ tte(eventtime, status, threshold = 0.5),
data = dt, keep.pairScore = TRUE, trace = 0,
scoring.rule = "Gehan", method.inference = "none", correction.uninf = TRUE)
summary(BT)
```

Generalized pairwise comparisons with 1 endpoint

```
: net benefit (delta: endpoint specific, Delta: global)
- statistic
- null hypothesis : Delta == 0
- treatment groups: C (control) vs. T (treatment)
- censored pairs : deterministic score or uninformative
- uninformative pairs: score equals the averaged score of all informative pairs
- results
 endpoint threshold total(%) favorable(%) unfavorable(%) neutral(%) uninf(%)
                                                                                Delta
eventtime
                0.5
                         100
                                    11.31
                                                   34.64
                                                               54.05
                                                                            0 - 0.2333
```

Looking at the scores of the pairs:

```
favorable unfavorable neutral uninf favorableC unfavorableC neutralC uninfC
                      0
                             0
                                   1 0.1131029
                                                  0.3464344 0.5404627
1:
                                   0 0.0000000
2:
          0
                      1
                             0
                                                  1.0000000 0.0000000
                                                                          0
                                   0 0.0000000 0.0000000 1.0000000
3:
          0
                      0
                             1
                                                                          0
4:
                      0
                             0 p 0 1.0000000 0.0000000 0.0000000
                                                                           0
```

we can see that the corrected probability have not changed for the informative pairs, but for the non-informative they have been set to:

```
iScore[, .(favorable = weighted.mean(favorable, w = 1-uninf),
     unfavorable = weighted.mean(unfavorable, w = 1-uninf),
     neutral = weighted.mean(neutral, w = 1-uninf))]
```

```
favorable unfavorable neutral
1: 0.1131029  0.3464344  0.5404627
```

4 Simulating data using simBuyseTest

You can simulate data with the simBuyseTest function. For instance the following code simulates data for 5 individuals in the treatment arm and 5 individuals in the control arm:

```
set.seed(10)
simBuyseTest(n.T = 5, n.C = 5)
```

```
eventtime status toxicity
    treatment
                                                   score
 1:
             C 0.60539304
                                0
                                        yes -1.85374045
 2:
             C 0.31328027
                                1
                                        yes -0.07794607
             C 0.03946623
                                             0.96856634
 3:
                                0
                                        yes
             C 0.32147489
 4:
                                1
                                             0.18492596
                                        yes
             C 1.57044952
                                0
                                        yes -1.37994358
 5:
 6:
             T 0.29069131
                                0
                                             1.10177950
                                         no
             T 0.19522131
                                             0.75578151
 7:
                                0
                                        yes
             T 0.04640668
                                0
                                        yes -0.23823356
 8:
 9:
             T 0.05277335
                                1
                                             0.98744470
                                        yes
10:
             T 0.43062009
                                             0.74139013
                                1
                                        yes
```

By default a categorical, continuous and time to event outcome are generated independently. You can modify their distribution via the arguments argsBin, argsCont, argsTTE. For instance the following code simulates two continuous variables with mean 5 in the treatment arm and 10 in the control arm all with variance 1:

```
treatment eventtime status toxicity tumorSize
                                                          score
             C 0.1805891
                               0
 1:
                                       yes 11.086551
                                                       8.564486
             C 0.1702538
                               1
                                            9.237455 10.362087
 2:
                                      yes
             C 0.2621793
                               1
                                            9.171337
                                                       8.240913
 3:
             C 0.2959301
                               0
                                                       9.675456
 4:
                                        no 10.834474
             C 0.4816549
                                            9.032348
                                                       9.348437
 5:
                               1
                                      yes
 6:
            T 0.6446131
                               1
                                            5.089347
                                                       6.101780
                                       no
 7:
            T 0.7372264
                                            4.045056
                                                       5.755782
                               1
                                       yes
 8:
            T 0.7213402
                                            4.804850
                                                       4.761766
                                      yes
 9:
            T 0.1580651
                               1
                                       yes
                                            5.925521
                                                       5.987445
10:
            T 0.2212117
                               0
                                       yes
                                            5.482979
                                                       5.741390
```

This functionality is based on the sim function of the lava package (https://github.com/kkholst/lava)

5 Power calculation using powerBuyseTest

The function powerBuyseTest can be used to perform power calculation, i.e., estimate the probability of rejecting a null hypothesis under a specific generative mechanism. The user therefore need to specify:

- the generative mechanism via a function argument sim
- the null hypothesis argument null
- the sample size(s) for the which the power should be computed argument sample.size

Consider the following generative mechanism where the outcome follows a Student's t-distribution in the treatment and control group, with same variance and degrees of freedom but different mean:

```
simFCT <- function(n.C, n.T){
   out <- rbind(cbind(Y=stats::rt(n.C, df = 5), group=0),
        cbind(Y=stats::rt(n.T, df = 5) + 1/2, group=1))
   return(data.table::as.data.table(out))
}
simFCT(101,101)</pre>
```

```
Y group
  1: -0.5080164
  2:
      1.3917774
                     0
      1.2909425
                     0
  3:
      1.1812472
                     0
      0.6935526
                     0
  5:
198: -0.0193772
                     1
199: -1.0573662
                     1
200: -0.7772939
                     1
201: 0.1583587
                     1
202: 4.7379910
                     1
```

We then define the null hypothesis:

```
null <- c("netBenefit" = 0)</pre>
```

Naming the value is important since that will indicate which statistic should be used (here the net benefit). We can assess the power of the Wilcoxon's test using the following syntax:

summary(powerW)

Simulation study with Generalized pairwise comparison with 1000 samples

- statistic : net benefit (null hypothesis Delta=0) endpoint threshold n.T n.C mean.estimate sd.estimate mean.se rejection.rate 1e-12 5 0.2403 Y 5 0.3845 0.3329 0.092 10 10 0.2486 0.2717 0.2444 0.157 0.2447 0.1874 0.1751 0.264 20 20 0.1518 0.1432 0.38 30 30 0.2484 50 50 0.2457 0.1159 0.1112 0.55 100 100 0.2457 0.0819 0.0787 0.847

n.T : number of observations in the treatment groupn.C : number of observations in the control group

mean.estimate: average estimate over simulations

sd.estimate : standard deviation of the estimate over simulations

mean.se : average estimated standard error of the estimate over simulations rejection : frequency of the rejection of the null hypothesis over simulations

(standard error: H-projection of order 1| p-value: after transformation)

6 Modifying default options

The BuyseTest.options method enable to get and set the default options of the BuyseTest function. For instance, the default option for trace is:

```
BuyseTest.options("trace")
```

\$trace

[1] 2

To change the default option to 0 (i.e. no output) use:

```
BuyseTest.options(trace = 0)
```

To change what the results output by the summary function use:

```
BuyseTest.options(summary.display = list(c("endpoint","threshold","delta","Delta","
    information(%)")))
summary(BT)
```

Generalized pairwise comparisons with 1 endpoint

To restore the original default options do:

```
BuyseTest.options(reinitialise = TRUE)
```

References

- Buyse, M. (2010). Generalized pairwise comparisons of prioritized outcomes in the two-sample problem. *Statistics in medicine*, 29(30):3245–3257.
- Péron, J., Buyse, M., Ozenne, B., Roche, L., and Roy, P. (2018). An extension of generalized pairwise comparisons for prioritized outcomes in the presence of censoring. *Statistical methods in medical research*, 27(4):1230–1239.
- Verbeeck, J., Spitzer, E., de Vries, T., van Es, G., Anderson, W., Van Mieghem, N., Leon, M., Molenberghs, G., and Tijssen, J. (2019). Generalized pairwise comparison methods to analyze (non) prioritized composite endpoints. *Statistics in medicine*.