Statics Group Project

Akhil Ganesan, Anahat Sahay, Daryn Anthony Roberts, John Spencer Lawing, Peter Franklin Grafstein, & Samuel Roby

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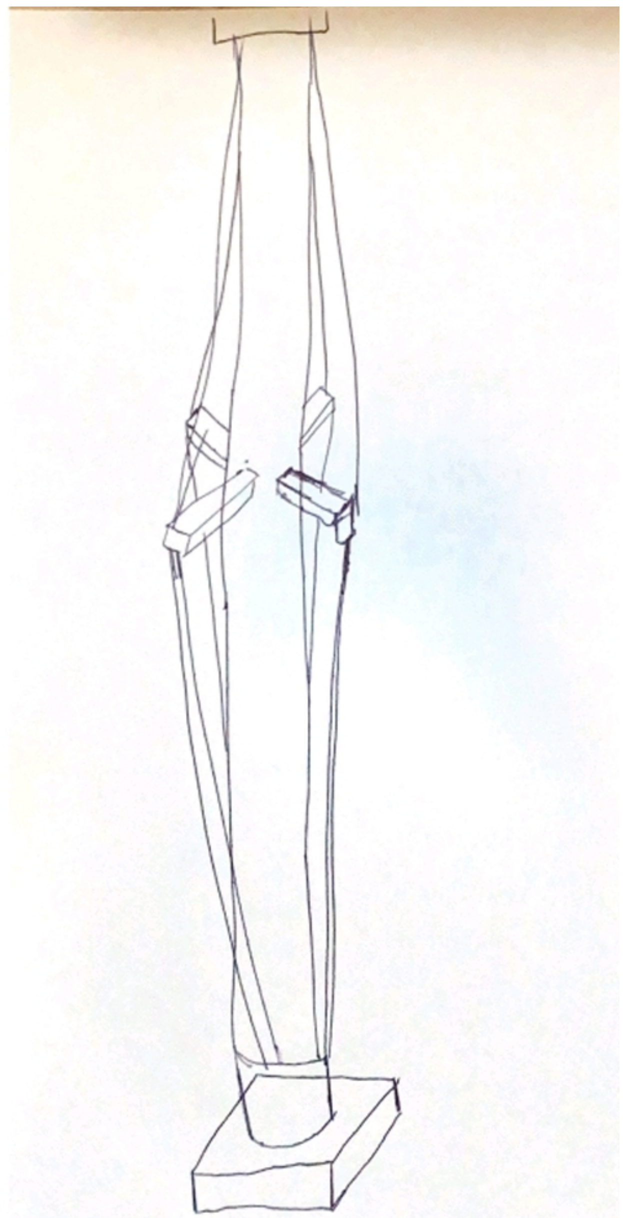
Submission 1: System

We decided to perform the statics analysis on one of the outside stilts of the Kendeda Building. We chose this 3D structure because there are many forces being applied on a single support beam at one time, including tension in the cables, rotational forces on the side supports, and the forces from supporting the weight of the roof. We also chose this specific structure because it is part of the building that catches our eyes when we first see the Kendeda building. These forces combine together to form a complex structure that not only is aesthetically pleasing to the Kendeda building but also provides an integral part of structural support to the building. Our plan is to have 2 force body diagrams (one for the stilt, the other for the roof), including tension, rotational force, and force caused by the weight. We also plan to link the force body diagrams using the estimated weight of the roof it is supporting, as well as the weights of cables, interior beams, and various other metals for a singular support beam. Once we have found all the masses and other forces, we will complete the FBDs and all other calculations needed.



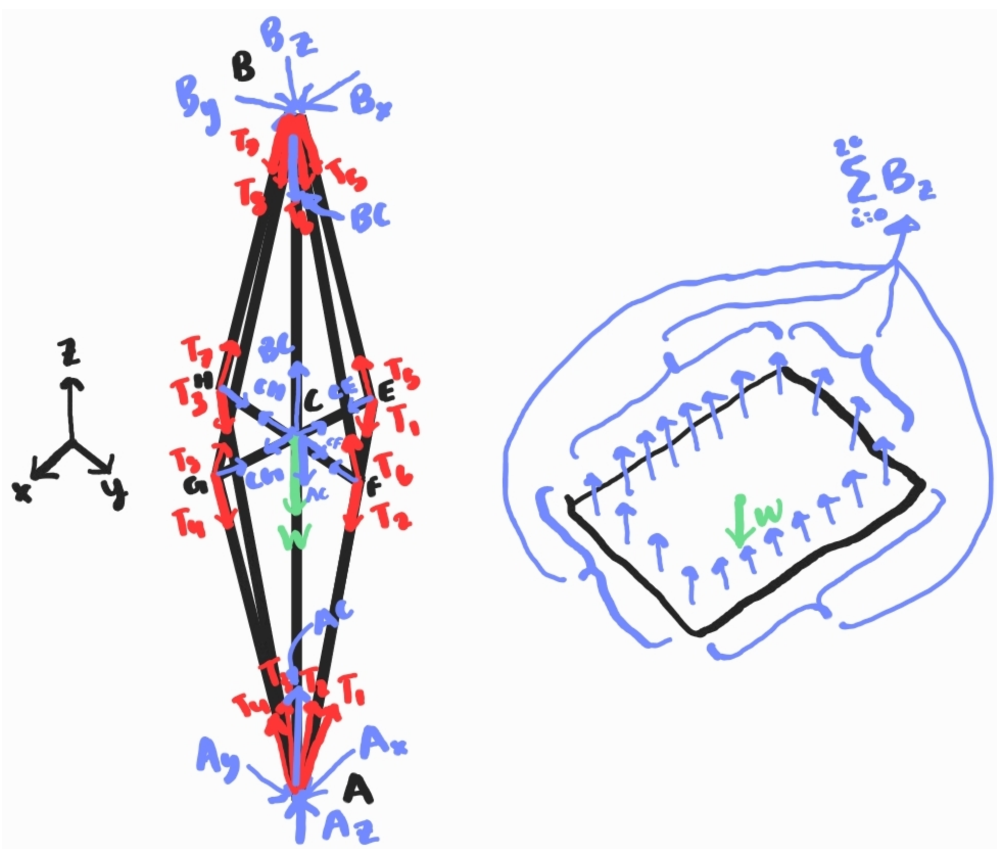
Submission 2: Force List/Preliminary FBD

We developed 2 free-body diagrams: one analyzing the stilt using method of joints & the other analyzing the building's roof, which we assume to be held up entirely by 20 stilts. The following forces were chosen to perform the statics analysis for the stilt (note the FBD doesn't satisfy solution existence ($M + 6 = J * 3$) unless we make the assumption EG & FH are perpendicular, essentially adding a virtual bar between them):

- **Weight** (W): The weight of the stilt was significant in the analysis and was placed at the geometric center of the stilt (assuming the stilt has a uniform mass distribution)
- **Pin Connection Normal Forces** ($A_x, A_y, A_z, B_x, B_y, B_z$): There are 2 pin connections at the bottom (A) & top (B) points of the stilt, transmitting normal forces
- **Beam Internal Forces** (Between points $A, B, C, D, E, F, G, \& H$): Each beam has either a compressive or tension force associated with it
- **Angled Beam Internal Forces** (T_n, n ranges from 1 to 8): There are 8 forces transmitted by 4 beams (using symmetry, we predict these forces will have the same magnitude)

The following forces were chosen to perform the statics analysis for the stilt:

- **Weight** (W): The weight of the roof was significant in the analysis and was placed at the geometric center of the roof (assuming the roof has a uniform mass distribution)
- **Pin Connection Normal Forces** (ΣB_z): There are 20 pin connections from each stilt, transmitting normal forces upwards equal to upper pin connection on the stilt (using symmetry to equate all moments to zero, we predict these normal forces will have all have the same magnitude)



Submission 3: Statics Analysis

We calculated the forces on a quarter of the stilt and symmetrically translated this to the entire structure. One change here from the previous plan is the structure was analyzed with each member having weight. The FBD (Figure 1) and calculations for this are as follows. Note W_n & l_n are the weight/length of members AB ($n = 1$), CD ($n = 2$), BD ($n = 3$), & AD ($n = 4$).

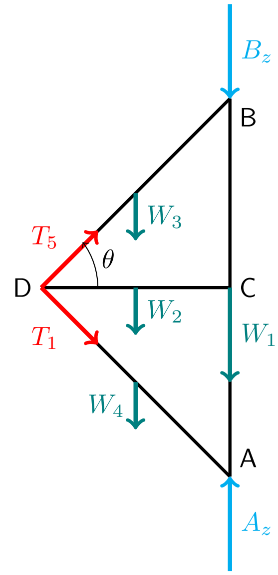


Figure 1: Section Free Body Diagram

$$\Sigma F_y = A_z - B_z - W_1 - W_2 - W_3 - W_4 = 0$$

$$A_z = B_z + W_1 + W_2 + W_3 + W_4$$

Then, we analyzed beam CD with the free body diagram in Figure 2.

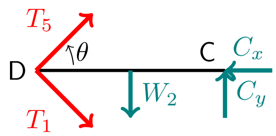


Figure 2: CD Free Body Diagram

This started with first defining the tension vectors as shown below. This requires defining the angle between the truss θ.

$$T_{n,z} = T_n \sin\theta$$

Then, we took a couple moment and force equilibrium equations.

$$\Sigma M_C = (T_{1,z})l_2 + (W_2)\frac{l_2}{2} - (T_{5,z})l_2 = 0 \Rightarrow T_{5,z} = T_{1,z} + \frac{W_2}{2}$$

$$T_5 = T_1 + \frac{W_2}{2\sin\theta} \Rightarrow T_1 = T_5 - \frac{W_2}{2\sin\theta}$$

$$\Sigma F_x = T_5\cos\theta + T_1\cos\theta - C_x = 0 \Rightarrow C_x = \cos\theta(T_1 + T_5)$$

We then analyzed joint B as shown below. This required adding a normal force in the x-direction to the free body diagram (Figure 3) which would maintain x-axis equilibrium & cancel out by symmetry with the other trusses.

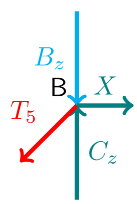


Figure 3: Joint B Free Body Diagram

$$\Sigma F_z = C_z - B_z - T_{5,z} = 0 \Rightarrow T_5 = \frac{C_z - B_z}{\sin\theta}$$

After, we analyzed member BD (the FBD is Figure 4; the forces at D are simplified because M_D is solved for).

$$\Sigma M_D = C_z l_2 - B_z l_2 - \frac{W_3 l_2}{2} = 0 \Rightarrow C_z = B_z + W_3/2$$

Overall, these equations reduce to the following for the beams (note $BD = T_5$, $AD = T_1$, $BD = T_5$, $AB = C_z$, & $CD = C_x$)

1. $AB = B_z + W_3/2$
2. $CD = (BD + AD)\cos\theta$
3. $BD = (AB - B_z)/\sin\theta$
4. $AD = BD - W_2/2\sin\theta$
5. $\theta = \arccos(\frac{l_2}{l_1})$

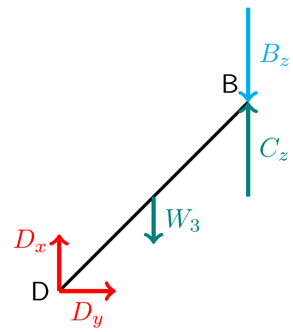


Figure 4: BD Free Body Diagram

After this, we calculated the weight of the roof through estimating the roof's density (researched as around 2 lb/sq ft as a lower bound[1]) and multiplying that Kendeda's area[2] (37000 sq ft), giving ΣB_z, which was divided by 20 for B_z on each stilt.

$$B_z = \frac{\Sigma B_z}{20} = \frac{2\,\mathrm{lb/ft}^2 \times 37000\,\mathrm{ft}^2}{20} = 3700\,\mathrm{lb}$$

We also retrieved the lengths & radii of the stilts, and used the weight density of steel[3] (the assumed material, $e_{steel} = 490\,\mathrm{lb/ft}^3$) to calculate the weight of the members through the following formula: $W_n = e\pi r_n^2 l_n$. The following table gives the researched measured values for the beam radii & length & calculated weight. Note the weight on AB is $3920.6/4 = 980.2$ because only a quarter of the beam is represented in the single truss.

Measurement	1	2	3	4
l_n (ft)	40.75	2.50	20.25	20.25
r_n (ft)	0.25	0.10	0.04	0.04
W_n (lb)	980.2	38.50	49.875	49.875

We also calculated $\theta = \arccos(\frac{2.5}{40.75}) = 86.5°$. Through this, we resolved for the internal forces between the 4 beams, ultimately getting the following (note AB is $950 \times 4 = 3800\,\mathrm{lb}$ to account for the 4 truss symmetry):

1. $AB = 3800\,\mathrm{lb}$ Compression
2. $CD = 1.89\,\mathrm{lb}$ Compression
3. $BD = 25\,\mathrm{lb}$ Tension
4. $AD = 5.57\,\mathrm{lb}$ Tension

[1] https://www.whatnextnow.com/home/solar/is-my-roof-suitable-for-solar-panels-and-what-is-the-weight-of-a-solar-panel
[2] https://livingbuilding.gatech.edu/key-living-building-details
[3] https://www.kloecknermetals.com/blog/what-is-the-density-of-stainless-steel/