Al Assignment-3 MT23013 Akhil P Dominic

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- (a) About 82.5 % people have travelled and have caught either corona or other diseases.
- (b) Of the people who had travelled 15 % have mild and 22 percentage have severe

cases of corona, respectively.

- (c) Given that a person travelled the chance they caught a disease other than corona is 0.485 rounded to 3 decimal places.
- (d) About 24 % of people died of diseases other than corona after travelling.
- (e) There is 0.025 probability that a person has not travelled and has severe case of corona.
- (f) Given a person has not travelled the probability that the person is severely sick is about 0.457 rounded to 3 decimal places.
- (g) The probability that a person has died and did have corona is 0.059.
- (h) About 70 % people had mild or severe cases of any disease.
- (i) There is 80 % chance that a person has travelled given that he is severely sick.
- (j) There is 50 % chance a person had corona whether they travelled or not.

Ans:

- T Person has travelled or not
- C Person has corona or not
- S Disease is severe or mild
- D Person has other disease than corona or not
- X Person died or not

(a)

(a):
$$P(T \land (C \lor D)) = 0.825$$

(b):
$$P(T \land (\neg S \land C)) = 0.15$$

 $P(T \land (S \land C)) = 0.22$

(c):
$$P(D \mid T) = 0.485$$

(d):
$$P(X \land D \mid T) = 0.24$$

(e):
$$P(\neg T \land (S \land C)) = 0.025$$

(f):
$$P(S \mid \neg T) = 0.457$$

(g):
$$P(X \wedge C) = 0.059$$

(h):
$$P(((C \lor D) \land \neg S) \lor ((C \lor D) \land S)) = 0.70$$

(i):
$$P(T | S) = 0.8$$

(j):
$$P((C \mid T)P(T) \lor (C \mid \neg T)P(\neg T)) = 0.50$$

(b) Axioms are:

- >All probabilities has to be non negative
- >All probabilities must be less than or equal to 1
- >The probability of the union of two mutually exclusive events is the sum of their individual probabilities.

(c)
Joint distribution table:

| T | D | С | Χ | S | P(T,D,C,X) | P(T,C,S,X,D,S) |
|---|---|---|---|---|------------|----------------|
| 1 | 1 | 1 | 1 | 1 | 0.048 | 0.01 |
| 1 | 1 | 1 | 0 | 1 | 0.77 | 0.6 |
| 1 | 1 | 0 | 1 | 1 | 0.77 | 0.164 |
| 1 | 1 | 0 | 0 | 1 | 0.77 | 0.164 |
| 1 | 0 | 1 | 1 | 1 | 0.048 | 0.01 |
| 1 | 0 | 1 | 0 | 1 | 0.77 | 0.16 |
| 1 | 0 | 0 | 1 | 1 | 0.16 | 0.1248 |
| 1 | 0 | 0 | 0 | 1 | 0.16 | 0.1248 |
| 0 | 1 | 1 | 1 | 1 | 0.01 | 0.0078 |
| 0 | 1 | 1 | 0 | 1 | 0.16 | 0.1248 |
| 0 | 1 | 0 | 1 | 1 | 0.16 | 0.1248 |
| 0 | 1 | 0 | 0 | 1 | 0.16 | 0.1248 |
| 0 | 0 | 1 | 1 | 1 | 0.01 | 0.0078 |
| 0 | 0 | 1 | 0 | 1 | 0.16 | 0.1248 |
| 0 | 0 | 0 | 1 | 1 | 0.16 | 0.1248 |
| 0 | 0 | 0 | 0 | 1 | 0.16 | 0.1248 |
| 1 | 1 | 1 | 1 | 0 | 0.048 | 0.03744 |
| 1 | 1 | 1 | 0 | 0 | 0.77 | 0.6006 |
| 1 | 1 | 0 | 1 | 0 | 0.77 | 0.6006 |
| 1 | 1 | 0 | 0 | 0 | 0.77 | 0.6006 |

| 1 | 0 | 1 | 1 | 0 | 0.048 | 0.03744 |
|---|---|---|---|---|-------|---------|
| 1 | 0 | 1 | 0 | 0 | 0.77 | 0.6006 |
| 1 | 0 | 0 | 1 | 0 | 0.16 | 0.1248 |
| 1 | 0 | 0 | 0 | 0 | 0.16 | 0.1248 |
| 0 | 1 | 1 | 1 | 0 | 0.01 | 0.0078 |
| 0 | 1 | 1 | 0 | 0 | 0.16 | 0.1248 |
| 0 | 1 | 0 | 1 | 0 | 0.16 | 0.1248 |
| 0 | 1 | 0 | 0 | 0 | 0.16 | 0.1248 |
| 0 | 0 | 1 | 1 | 0 | 0.01 | 0.0078 |
| 0 | 0 | 1 | 0 | 0 | 0.16 | 0.1248 |
| 0 | 0 | 0 | 1 | 0 | 0.16 | 0.1248 |
| 0 | 0 | 0 | 0 | 0 | 0.16 | 0.1248 |

(d)

T and D given C $P(T \wedge D \mid C) = P(T \mid C)P(D \mid C)$ T and D conditionally independent

C and X given S $P(C \land X \mid S) = P(C \mid S)P(X \mid S)$ C and X conditionally independent

T and C given S $P(T \land C \mid S) = P(T \mid S)P(T \mid S)$ T and C are conditionally independent

T and S given D $P(T \wedge S \,|\, D) = P(T \,|\, D)P(S \,|\, D)$ T and S are conditionally independent

2

(a)(1 mark) Should you switch your choice to the other unopened door to maximize your chance of winning the key?

Before opening the door, the probability was P(A)=P(B)=P(C)=1/3. Now after knowing that C has been opened and shown no key and lost life, we check for other probabilities

 $P(A \lor B \mid \neg C) = P((A \lor B) \land \neg C) / P(\neg C)$ $P(\neg C) = 1 - 1/3 = 2/3$ $P((A \lor B) \land \neg C) = 1/2$

So $P(A \lor BI \neg C) = 1 * 3/2 * 2 = 3/4$

Since the probability of finding the key is higher after the door is opened, it is better to switch the choice to the next unopened door.

(b)However, if he occasionally makes a mistake, he reveals the loss of life with a probability of 1/3 and correctly reveals the key with a probability of 2/3. In this scenario, should you switch your choice to maximize your chances of winning the key?

If the host revealed the correct door with the key, then you can trust him. But if he reveals a door with a loss of life(no key), then either he made a mistake and revealed the wrong door, or else he correctly revealed the door which didn't have the key.

In the latter case, it is better to stick with switching as it increases the chances of winning(has a bigger probability of winning). The the overall conditional expectation would be 2/3-1/3=1/3

In the former case, sticking with the former choice would be worse. The conditional probabilities would be 1/3 as well.

The overall expectation would be 2/3*1/3+1/3*1/3=5/9
As you can see, its still better to switch because of the higher conditional probabilities it is giving.

(c)If you choose to switch, what is the conditional probability that you win the key if the man has mistakenly revealed the door that shows life lost?

The probability for that will be 2/3.

(d)Additionally, what is the conditional expectation of your prize (key/life lost) based on your choice to switch or stick, considering both possible scenarios? Would you choose to switch or stick based on the conditional expectation?

Since in 2, we found out that its better to switch because of the higher conditional probability, we would be doing that only. We would switch and won't stick on.

Computational part accuracies

| A | В | С | |
|-------|-------|-------|--|
| 0.971 | 0.971 | 0.988 | |