HQ19
MT23013

- Prove, using logical arguments (i.e., without calculating all possibilities) that the minimum maxload must be 6. I am looking for 3-4 sentence proof.

Ans : Suppose that we have n number of tasks and D number of days to do that task. If t is the largest work to be done, then the optimal minimum max load must t. This is because inorder to maximise the split of workloads in days, we have do the bigger tasks first and smaller tasks at the last. If the minimum max load is less than t(6), then we have to do the work having t(6) in two days, which isn't allowed in the given scenario.

- Define the decision problem corresponding to the above optimization problem (name the decision problem HWPLAN).

Ans : The problem is defined as follows:
Suppose there are n tasks and there is an array C=[….] with n elements and has D days to complete the tasks and has the minimum max load as L. HWPLAN returns True iff it is possible to complete the n tasks in D days with minimum max load/ minimum number of hours as L.

- Show that HWPLAN is NP hard. Clearly state the NP-hard problem that should be used for reduction (use a problem that has been covered in the lectures or homeworks), explain the reduction algorithm, and briefly discuss the proofs of the correctness claims.

Ans : We could reduce an already known NP hard problem, ie the Partition problem into HWPLAN to show that it is an NP hard problem.
We can input Partition problem instance and get an output as HWPLAN problem.
Def function(set S={….}):

given D= number of days to complete
C= array containing workloads
for j in size(S):
C[j]=S[j]

set_v=floor($\sum_i S = 1$)

return (D,N,C,T)

Claim: For all instances of Partition problem, we can reduce that to an instance of HWPLAN
Partition problem states that there exists a way to partition a set of integers into two subsets such that the sum of these subsets equals set_v. In the above instance, if we can find a partition where we could divide the tasks in D=2 days and set_v=floor($\sum_i S = 1$). If we don't have a solution that could divide the set into two partitions in partition problem ,

then in the HWPLAN also we won’t be having a partition where we could do the N tasks in 2 days in such a way that we can’t have minimum max load is set_v, which means we can’t divide set into two partitions such that sum is set_v, which are contradicting statements.

Claim 2: For Reduce(q) is a yes instance of HWPLAN, then q is a yes instance of PARTITION

Suppose Reduce(q) is a yes instance of HWPLAN, where we can do N tasks with minimum max load equal to set_val with in 2 days with various workloads in C. Then we can be sure that there would be a partition as we can see that for D=2, we will be doing all the workloads that has been assigned to do in day 1 could be considered as the first partition set and the remaining work loads as the second partition and sum of the workloads in both sets is set_val, we can be sure it can be reduced correctly.