Al Assignment-1

Resources used: Textbook, resource1

Logic (5 marks): The universe either will simply exist as it is or end in a heat death. If there was no big bang then the universe simply existed. If and only if the universe is expanding then there was a big bang. If the universe is expanding and accelerated then it will end in a heat death.

- (a) Write the sentences using logical connectives.(1)
- (b) Write the contra-positive of the sentence using logical connectives. (1)
- (c) What can be inferred and not inferred from the statement. (1)
- (d) Draw the And-OR graph (2)

Answers:

U: Universe will simply exist as it is

D: Universe end in heat death

B: There was big bang

X: Universe is expanding

A : Universe is accelerated

I.
$$(U \lor D)$$

II.
$$\neg B \implies U$$

III.
$$B \iff X$$

IV.
$$(X \wedge A) \implies D$$

(b)

I.
$$(U \lor D)$$
 is same as $(\neg U \implies D)$

$$(\neg D \implies U)$$

II.
$$\neg U \Longrightarrow B$$

III.
$$\neg X \iff \neg B$$

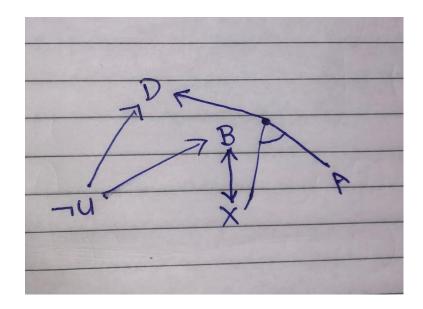
IV.
$$\neg D \implies (\neg X \lor \neg A)$$

(c)
$$(U \lor D) \land (B \lor U) \land (\neg B \lor X) \land (\neg X \lor B) \land ((\neg X \lor \neg A) \lor D)$$

Cancel out B and ¬B, X and ¬X and we will be left with

$$(U \vee D) \wedge (U \vee B) \wedge ((\neg X \vee \neg A) \vee D)$$

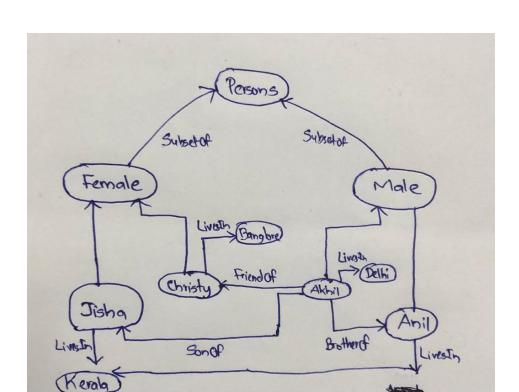
(d) (a) (i) could be converted into $\neg U \implies B$



2. Semantic Network (5 marks): Construct a Semantic network based on the relation between three people and yourself.(2) Include the information of their: Location and Gender

Explain inheritance and multiple inheritance using examples from your Network.

Ans:



<u>Inheritance</u>: It is a concept in which a particular entity or class (in terms of OOP) acquires characteristics of the parent class to the off spring class. Whenever we are creating a new class and are inheriting from a parent class, we would be having all the characteristics of the parent class in the child class.

In the above example, consider the class Persons as parent class. From Persons class Male and Female classes are inherited. The type of inheritance where a class inherits the properties of two parent classes is called multiple inheritance.

3. Show that the proof by resolution approach for propositional logic is sound and complete.

Ans:

<u>Soundness and completeness</u> are two very important concept in propositional logic. Soundness refers to the property of an inference algorithm that derives only entailed sentences. Completeness is another property that states that an algorithm can derive all the sentences that is entailed.

Proof of resolution is a technique to obtain information that is being derived from a set of premises. It starts with connecting all the premises by using Conjuctive Normal Form, ie, we would taking all the premises and add the conjuctive(\land) symbol. This would effectively cancel out contradicting symbols from the expression.

For **soundness**, consider the following premises $(\neg y \lor x)$, $(\neg x \lor z)$. Lets us assume that applying resolution does not give us an entailment. So if we apply resolution, it might entail wrong statements in β , if we have α entails β . But on applying conjuctive normal form and checking, we get that whenever we put values for the variables in α that would make it true, ie on the premises $(\neg y \lor x)$, $(\neg x \lor z)$, we see that those values would be giving true on the β as well α , ie on $(\neg y \lor z)$ or simply $(y \Longrightarrow z)$. Hence our assumption was wrong and proof by resolution is indeed sound.

For **Completeness** also, we can prove in a similar way. For the premises that we had $(\neg y \lor x)$, $(\neg x \lor z)$, which had two premises, we proved only entailed sentences are derived. When we have n number of premises, we can use the same logic to prove that for all the sentences β that is derived from the set of n premises, we get that $\alpha \models \beta$.