LESSON-I → VECTOR SPACE

NOTES CUM ASSIGNMEN

1. INTERNAL COMPOSITION $(A,+)$ $\alpha,\beta \in A \Rightarrow \alpha+\beta \in A$
2. EXTERNAL COMPOSITION $(A,+)$ and $(B,+,\cdot)$
   $\alpha,\beta \in A$ $b \in B$
   $\alpha \cdot b \in A$ $\beta \cdot B \in B$

VECTOR SPACE

Vectors
↓
$V(F)$
↓
over F (scalars)
↳ Field

↓

$V \neq \phi$
elts of V are vectors
elts of F are scalars
then $V(F)$ is

1. V is an abelian gp w.r.t. '+' $[\alpha \in V, \beta \in V \Rightarrow \alpha+\beta \in V]$

↓

This is ____ Compos.

What do we mean? → V satisfies

(i) $\alpha+\beta \in V \ \forall \ \alpha,\beta \in V$
(ii) $\alpha+\beta = \beta+\alpha$
(iii) $\alpha+(\beta+\gamma)=(\alpha+\beta)+\gamma$
(iv) $\exists \ 0 \in V$ s.t $0+\alpha=\alpha$
(v) $\exists \ -\alpha \in V$ s.t $\alpha+(-\alpha)=0$

2. $\exists$ scalar Multiplication in V. $\alpha \in V$ and $a \in F$
   $\Rightarrow a\alpha \in V$ and This is ____ Composition

→ Non empty set
→ Mult. and Add defined
→ Additive identity $(e) \in F$
   $a+e=a \ \forall \ a \in F$
→ Multiplic inverse $\frac{1}{a} \in F$
   $a \cdot \frac{1}{a} = 1(e)$ $\forall a \neq 0 \in F$
→ Additive inverse
   $-a \in F$ s.t $\forall \ a \in F$
   $a+(-a)=0$ [e]
→ Multi Identity $e' \in F$
   s.t. $a \cdot e' = a$
→ COMMUTATIVE [+ and ·]
   $a+b=b+a$ $a \cdot b = b \cdot a$
→ ASSOCIATIVE [+ and ·]
   $(a+b)+c = a+(b+c)$
   $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
   $\forall \ a,b,c \in F$
→ DISTRIBUTIVE
   $a \cdot (b+c) = a \cdot b + a \cdot c$
→ Ex. → IR and C and Q are fields

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3. V satisfies

(i) $a(\alpha+\beta) = a\alpha + b\beta \quad \forall \ a \in F, \ \alpha, \beta \in V$

(ii) $(a+b)\alpha = a\alpha + b\alpha \quad \forall \ a, b \in F \quad \alpha \in V$

(iii) $(ab)\alpha = a(b\alpha) \quad \forall \ a, b \in F$ and $\alpha \in V$

(iv) $1 \cdot \alpha = \alpha \quad \forall \ \alpha \in V$ and $1 \in F$ [UNITY]

Note → V CONTAIN ELEMENTS CALLED VECTORS (NOT TO BE CONFUSED WITH VECTOR QUANTITY]

→ USE $\alpha, \beta, \gamma, --$ for vectors and a, b, c for scalars.

Ex. I The set of all polynomials over F is a vector space [POLY. VECTOR SPACE]

Ex. II $V = \{ f \mid f : S \to F \}$

$(f+g)(x) = f(x) + g(x) \quad \forall \ x \in S$

$(cf)(x) = cf(x) \quad \forall \ x \in S.$

S is any NON EMPTY set and F is any field. [THIS IS CALLED F^n SPACE]

Ex. III PROVE THAT

$(x, y) + (x_1, y_1) = (x + x_1, y + y_1)$

$c(x, y) = (c^2 x, c^2 y)$

is a vector space or Not over the field of real Numbers

Ex. 4. $\{f : R \to R \mid \frac{df}{dx} \text{ exists}\} = V$

V is a VS over F or NOT?

Ex. 5. $V = \{\text{solution set of } \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\}$

CHECK V is a vector space or Not

Ex. 6 $P = \{\begin{pmatrix} a \\ b \end{pmatrix} \mid a, b \geq 0\}$

P is a VS or Not over $F = R$.

Ex. 7. $V = \{f : R \to R \mid f(x) \neq 0 \text{ for any } x \in R\}$

CHECK V is a vector space or NOT?

Ex. 8 The set of all polynomials over field of rationals? YES/NO

Ex. 9. The set of all Nonzero Polynomials over ~~real~~ reals? YES/NO

Ex. 10 The set of all Polynomials of deg ≤ 3 over Real Field? YES/NO

Ex.11. Complex Field C over real field R?

YES/NO

Ans.

Ex.12. $(x,y)+(x_1,y_1)=(3y+y_1,\ -x-x_1)$

$c(x,y)=(3cy,\ -cx)$

$(x,y)\in \mathbb{R}^2$

$F=\mathbb{R}$.

YES/NO

MUST

Ex.13 HOW MANY ELEMENTS ARE THERE IN THE VECTOR SPACE OF POLYNOMIALS OF DEG AT MOST n in which coefficient are elements of Field $I(P)$, P being a prime No. Means $I(P)=\{0,1,2,--,P-1\}$.

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