Electrical Engineering

Electronics and Communication Engineering

NETWORK THEORY

Lecture No. 10

Transient Analysis

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1. Question Discussion
2.
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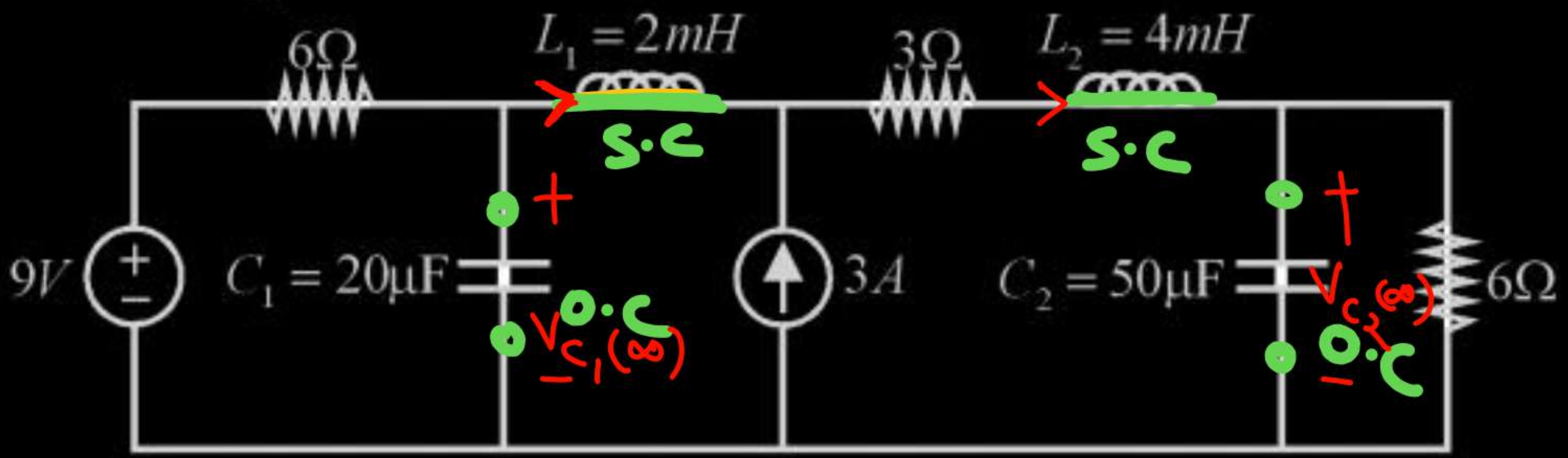
6.

Question

Initial or final (S.S).

Find the energy stored in circuit shown below.

By default take → 'S.S'

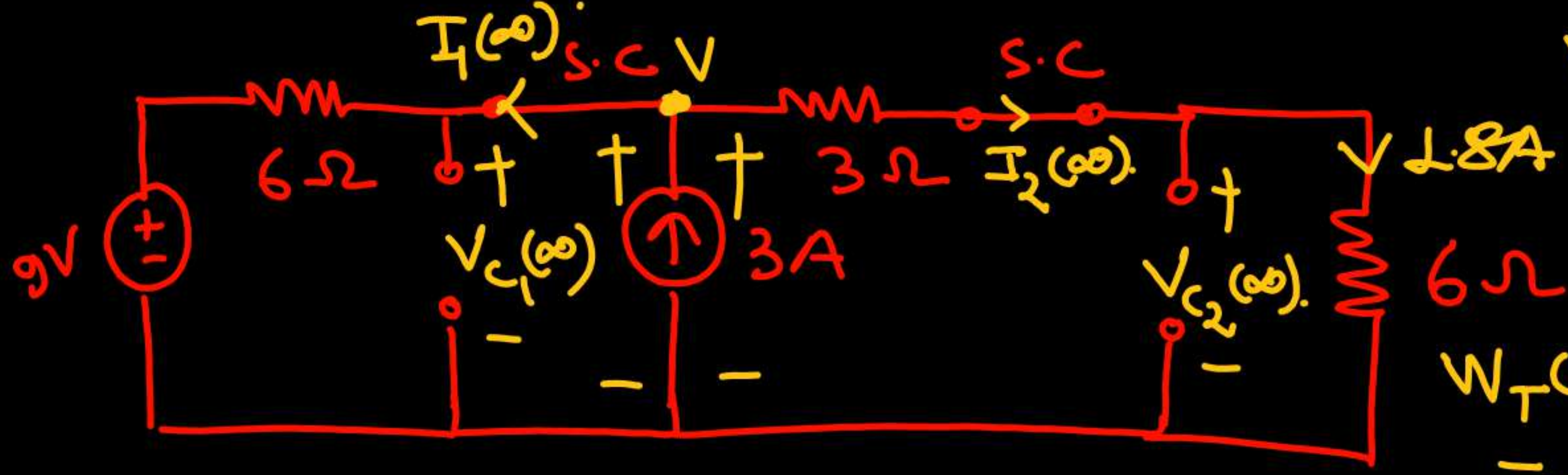


(A) 13.46mJ (B) 14.90mJ
(C) 12.28mJ (D) 16.17mJ

At Steady state.

L → S.C

C → O.C



$$V_{c_2}(\infty) = (6\times 1.8) = 10.8\,V$$

$$W_T(\infty) = (13.46\,mJ)$$

$$= \frac{1}{2}\times 2\times 10^{-3}\times(1.2)^2 + \frac{1}{2}\times 4\times 10^{-3}\times(1.8)^2 + \frac{1}{2}\times 20\times 10^{-6}\times(16.24)^2 + \frac{1}{2}\times 50\times 10^{-6}\times(10.8)^2$$

Nodal → $V\left(\frac{1}{6}+\frac{1}{9}\right) = 3 + \frac{9}{6}$

$$V = 16.24\,V = V_{c_1}(\infty)$$

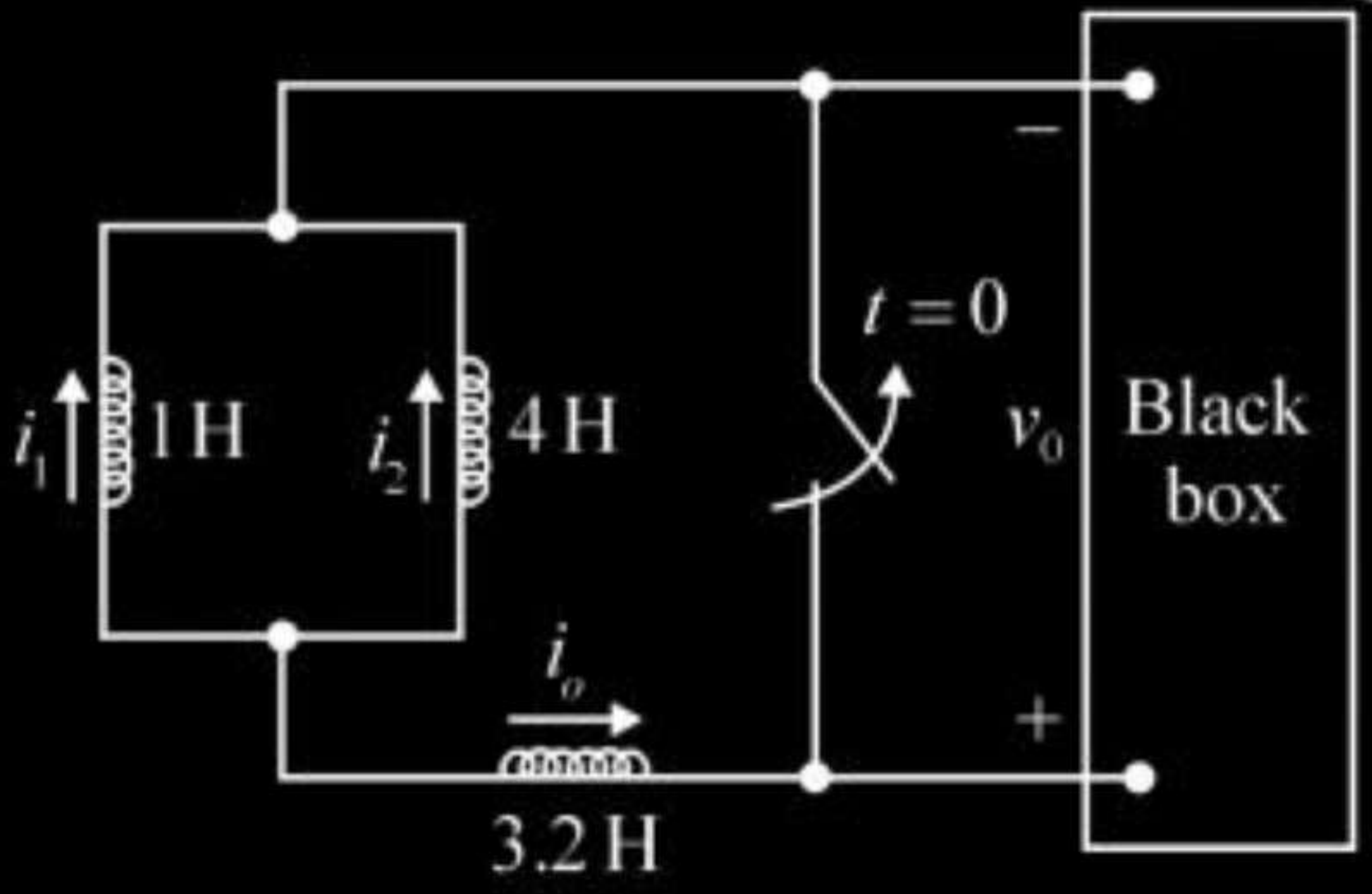
$$I_2(\infty) = \frac{V}{9} = \left(\frac{16.24}{9}\right) = (1.8\,A)$$

$$I_1(\infty) = 3 - I_2(\infty) = 3 - 1.8 = (1.2\,A)$$

Question H.W telegram

The three inductors in the circuit in figure are connected across the terminals of a black box at $t = 0$. The resulting voltage for $t > 0$ is known to be

$v_o = 2000e^{-100t}$ V.



If $i_1(0) = -6$ A and $i_2(0) = 1$ A, find

(a) $i_o(0)$;

(b) $i_o(t), t \geq 0$;

(c) $i_1(t), t \geq 0$;

(d) $i_2(t), t \geq 0$;

(e) The initial energy stored in the three inductors;

(f) The total energy delivered to the black box; and

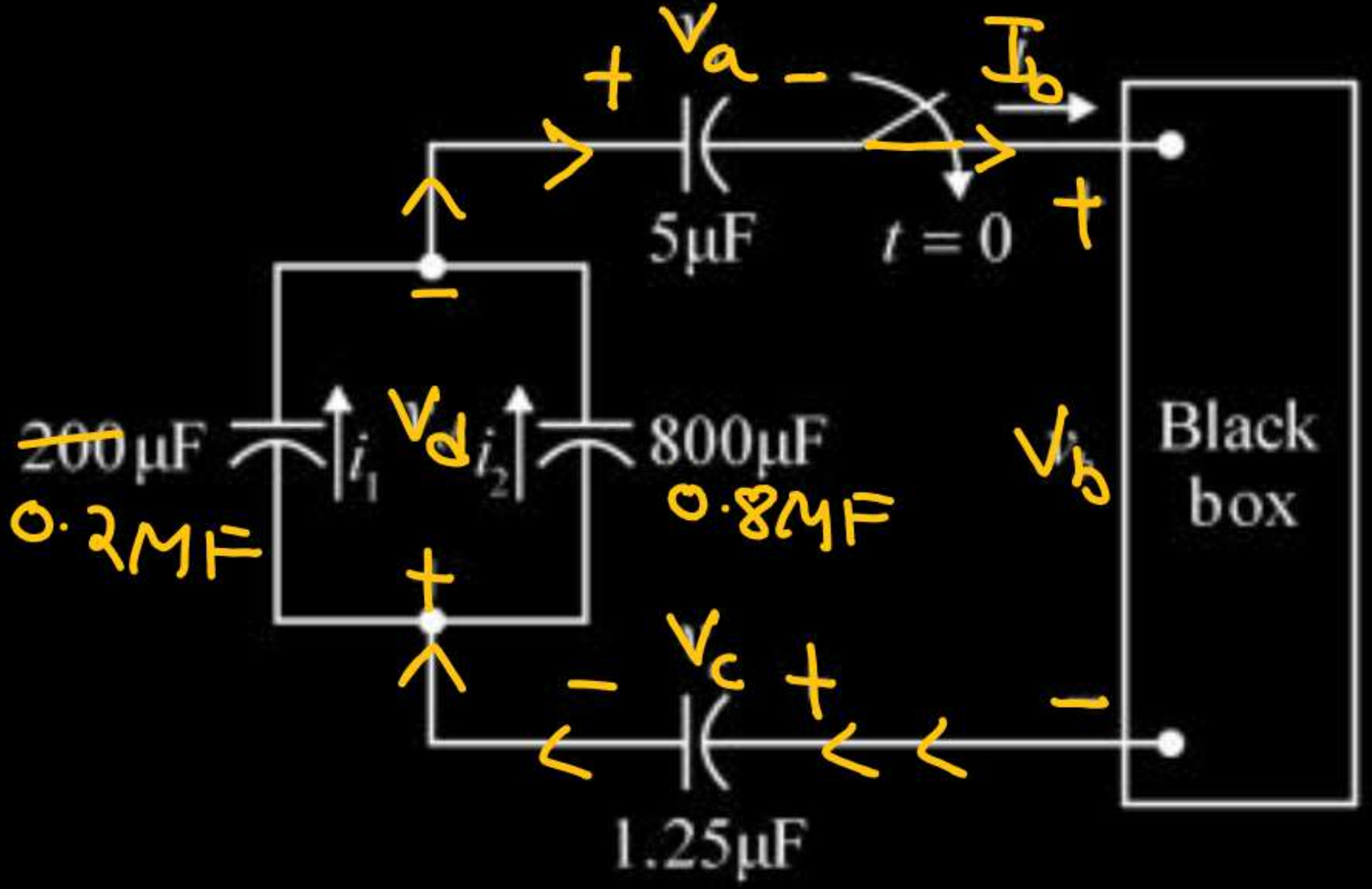
(g) The energy trapped in the ideal inductors.

Question

The four capacitors in the circuit in figure are connected across the terminals of a black box at $t = 0$. The resulting current i_b for $t > 0$ is known to be

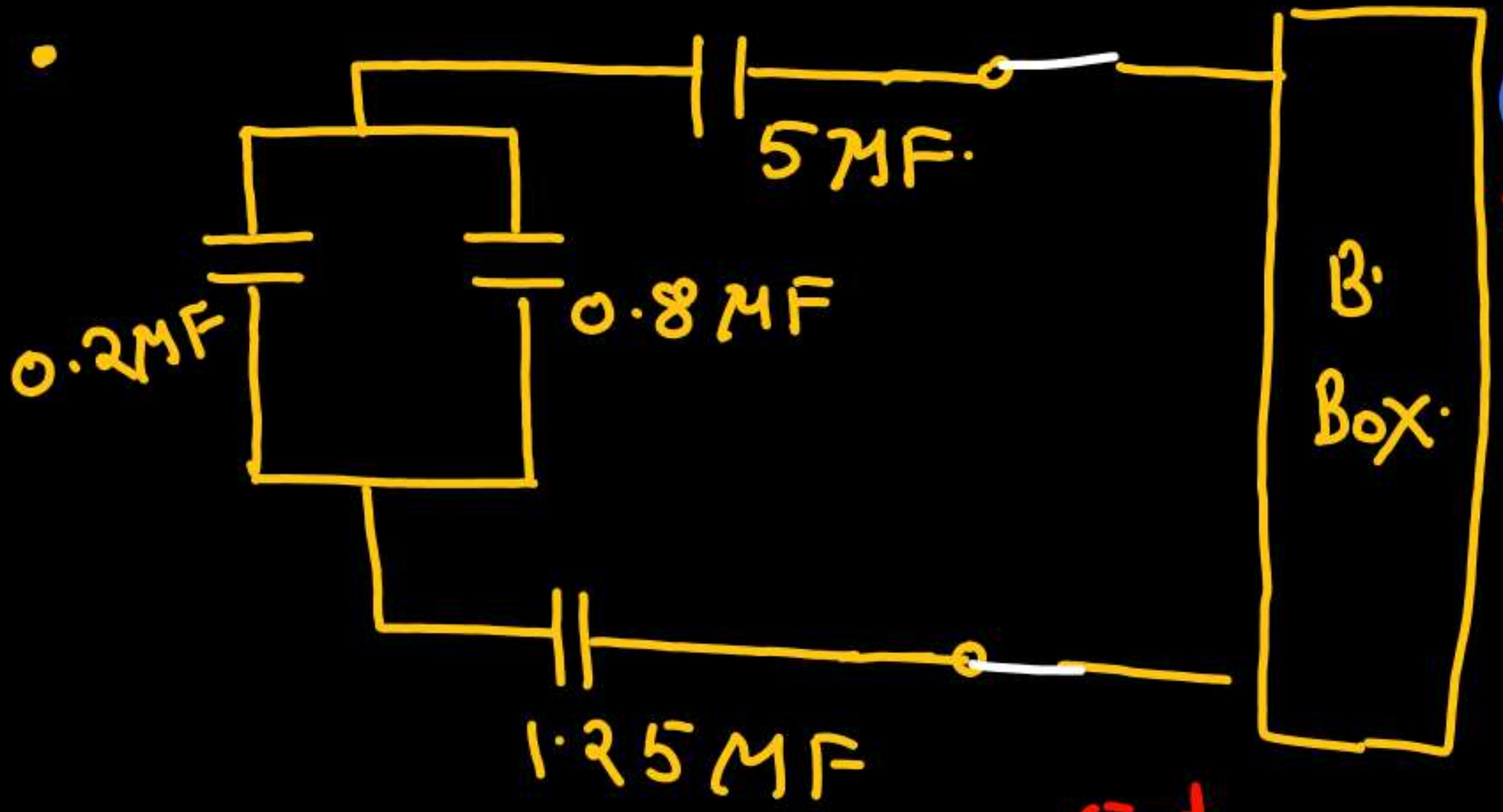
$$i_b = -5e^{-50t}\,\text{mA}$$

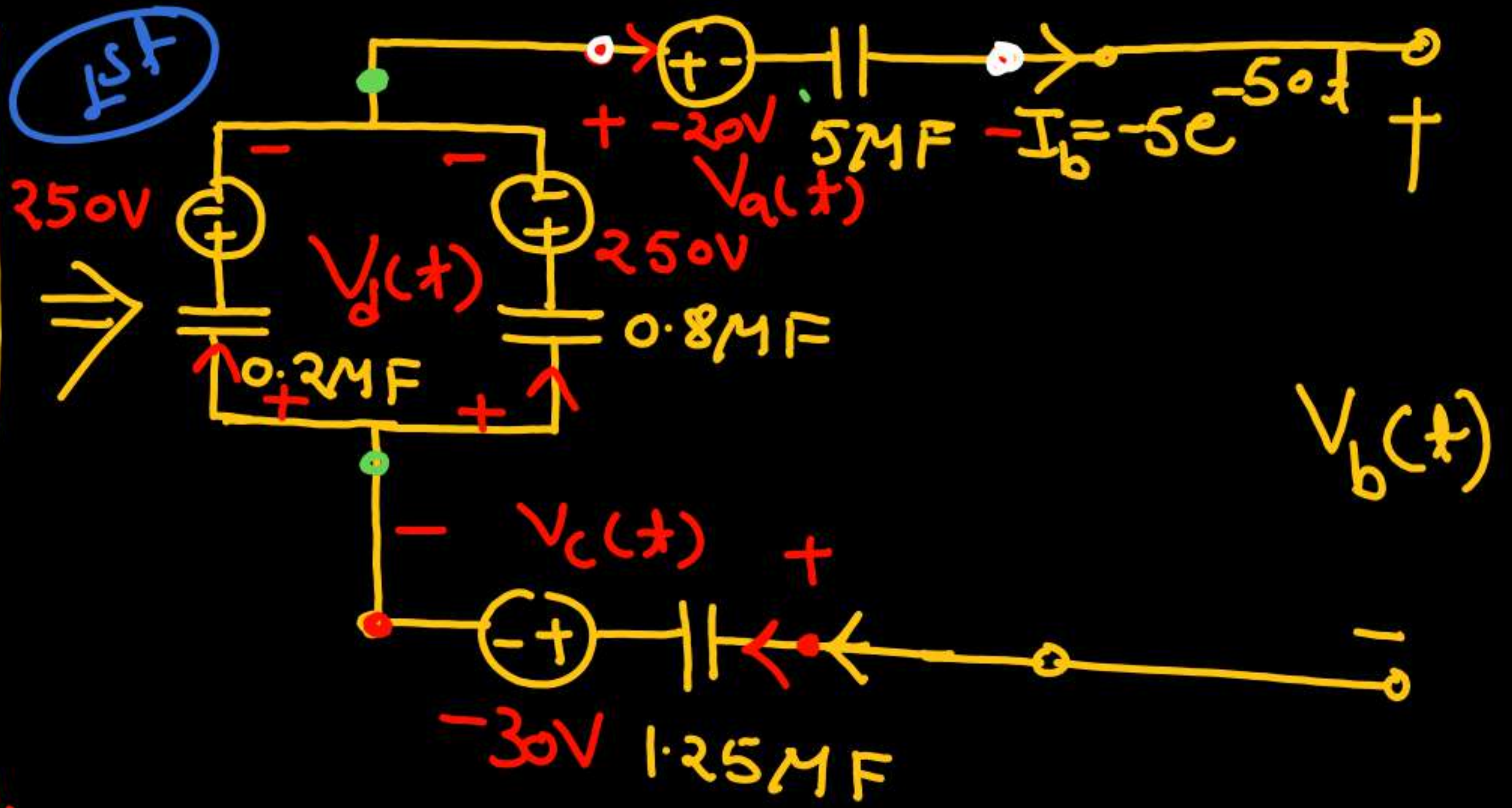
If $v_a(0) = -20\,\text{V}$, $v_c(0) = -30\,\text{V}$, and $v_d(0) = 250\,\text{V}$, find the following for $t \geq 0$:
(a) $v_b(t)$, (b) $v_a(t)$, (c) $v_c(t)$, (d) $v_d(t)$, (e) $i_1(t)$ and (f) $i_2(t)$.

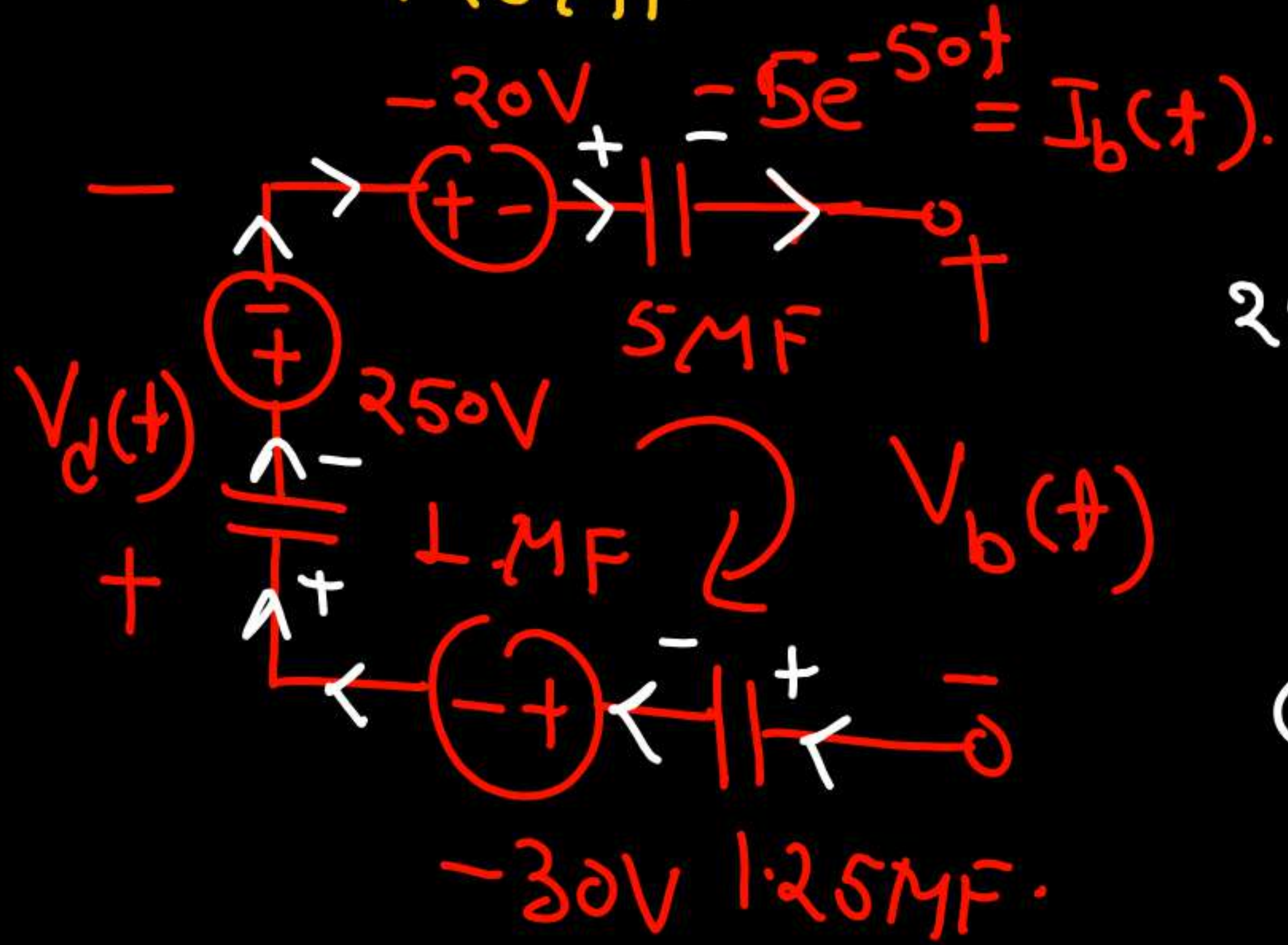


For the circuit in figure calculate

(a) The initial energy stored in the capacitors;
(b) The final energy stored in the capacitors;
(c) The total energy delivered to the black box;
(d) The percentage of the initial energy stored that is delivered to the black box; and
(e) The time in milliseconds, it takes to deliver 7.5 mJ to the black box.







$$250-20-30+\frac{1}{C_{eq}}\int I_b(t)\,dt = -V_b(t)$$

$$\frac{1}{C_{eq}} = \frac{1}{1.25}+\frac{1}{1}+\frac{1}{5} = 2$$

$$C_{eq} = \frac{1}{2} = 0.5\,MF$$

$$250 - 20 - 30 + \frac{1}{0.5\times10^{-6}} \times \int_0^t \left(-5e^{-50t}\times10^{-3}\right)\cdot dt = v_b(t).$$

2

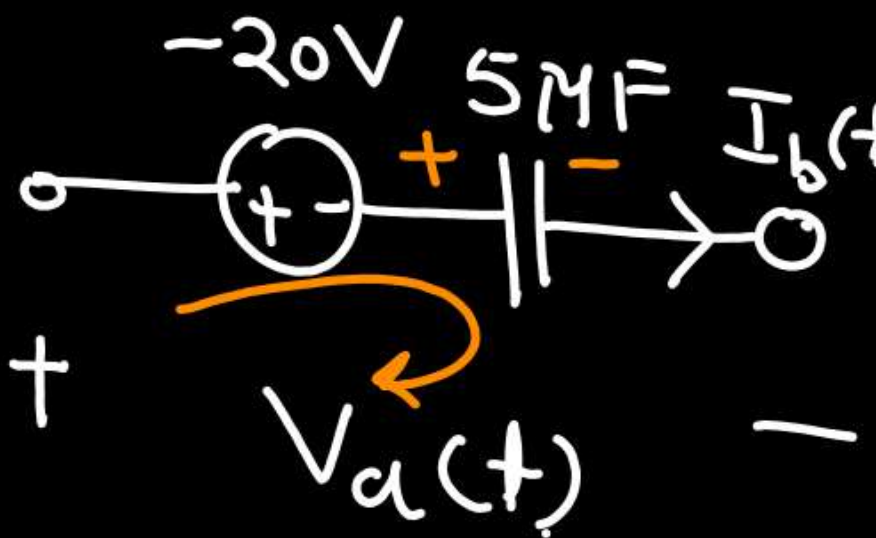
$$\boxed{V_b(t) = -200e^{-50t}\ V.}$$

- $V_a(t)$, $v_c(t)$, $v_d(t)$.

3

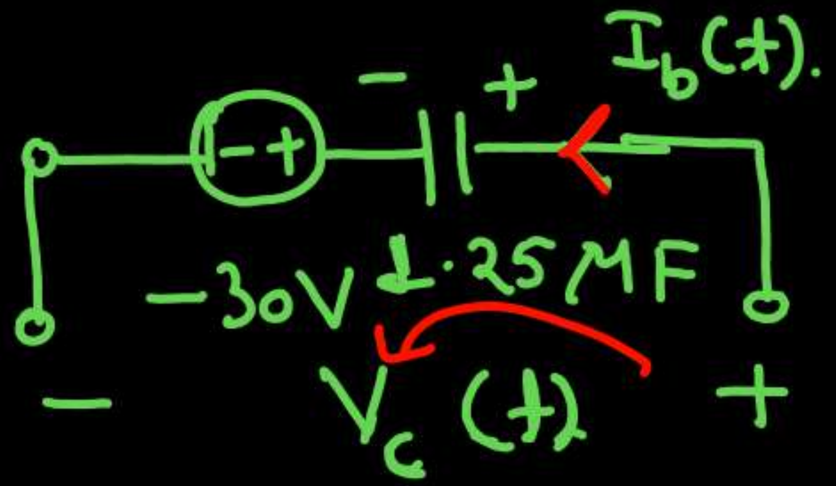
$$V_a(t) = -20 + \frac{1}{5\times10^{-6}}\int_0^t \left(-5e^{-50t}\times10^{-3}\right)dt = \left(20e^{-50t} - 40\right)$$

$I_b(t) = -5e^{-50t}$ mA.



$V_a(\infty) = -40V$

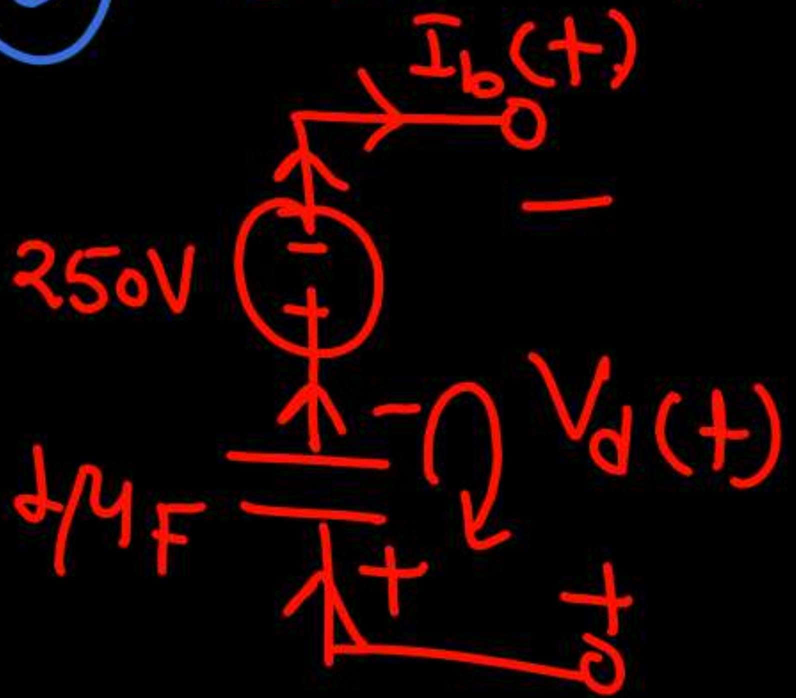
(4) $V_c(t) = -30 + \frac{1}{1.25\times10^{-6}}\int_0^t (-5\times10^{-3})e^{-50t}\,dt$



$$\boxed{V_c(t) = 80e^{-50t} - 110\text{ V}}$$

$(V_c(\infty) = -110V)$

(5) $V_d(t) = ?$



$$V_d(t) = 250 + \frac{1}{1\times10^{-6}}\int_0^t \left(-5\times10^{-3}e^{-50t}\right)dt$$

$$\boxed{V_d(t) = \left(100e^{-50t} + 150\right)\text{ volt}}$$

$$V_d(\infty) = 150V$$

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$$W_T(0^+) = \frac{1}{2}\times 5\times 10^{-6}\times(-20)^2 + \frac{1}{2}\times 1.25\times 10^{-6}\times(-30)^2 + \boxed{\frac{1}{2}\times 0.2\times 10^{-6}\times(250)^2 + \frac{1}{2}\times 0.8\times 10^{-6}\times(250)^2}$$

$$= (32812.5\ \mu\text{Joule})$$

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$$W_T(\infty) = W_{T(\text{trapped})} = \frac{1}{2}\times 5\times 10^{-6}\times(-40)^2 + \frac{1}{2}\times 1.25\times 10^{-6}(-110)^2 + \frac{1}{2}\times 1\times 10^{-6}\times(150)^2$$

$$= 22812.5\ \mu\text{Joule}$$

8

$$W_{\text{Delivered to the B. Black box}} = W_{B.B}(\text{Absorbing}) = W_T(0) - W_T(\infty) = (10{,}000\ \mu J)$$

10

- What Percentage of initial energy delivered to the B.B. during reaching to S.S.

$$\frac{W_{B.B}}{W_T(0)} = \frac{10,000}{32812.5} \times 100 = (30.47\,\%)$$

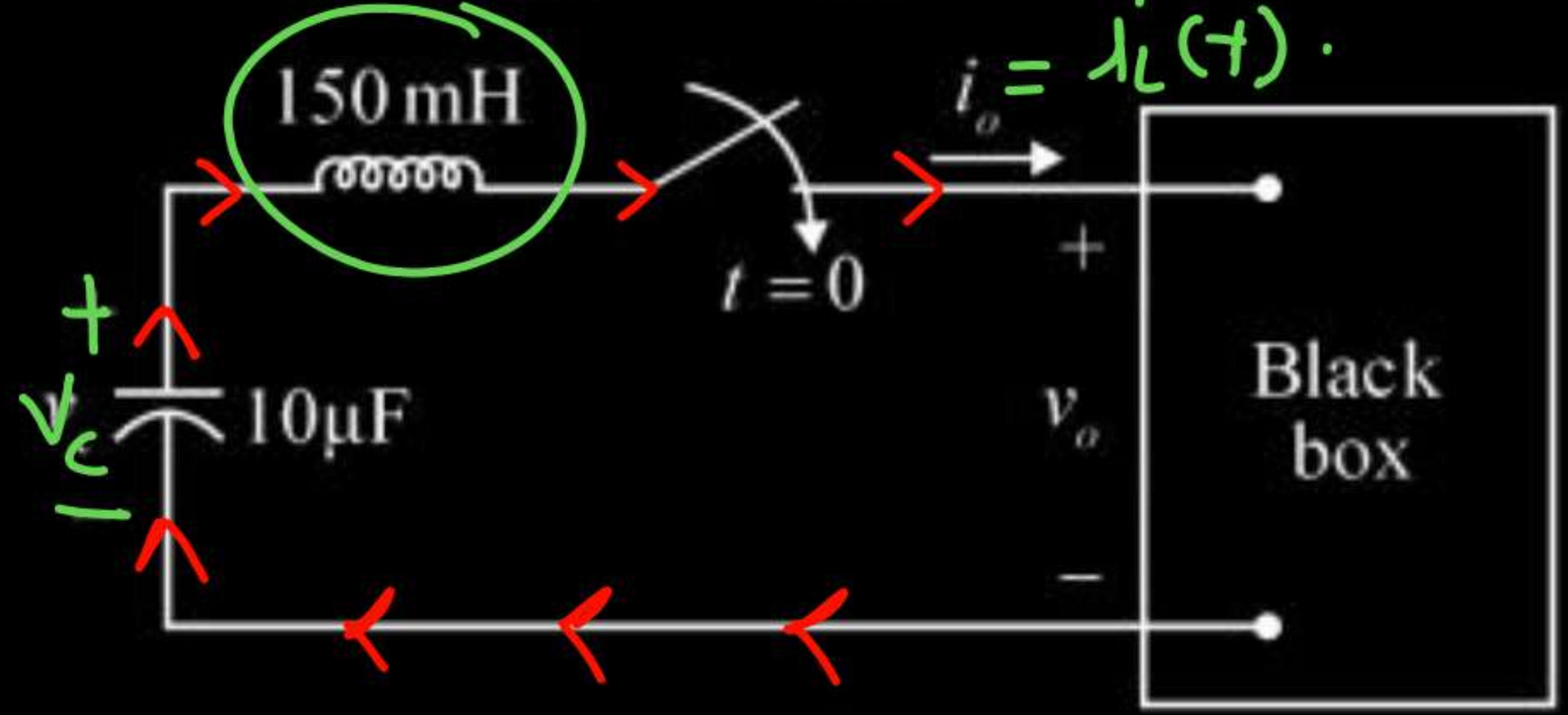
Question

At $t = 0$, a series-connected capacitor and inductor are placed across the terminals of a black box, as shown in figure. For $t > 0$, it is known that

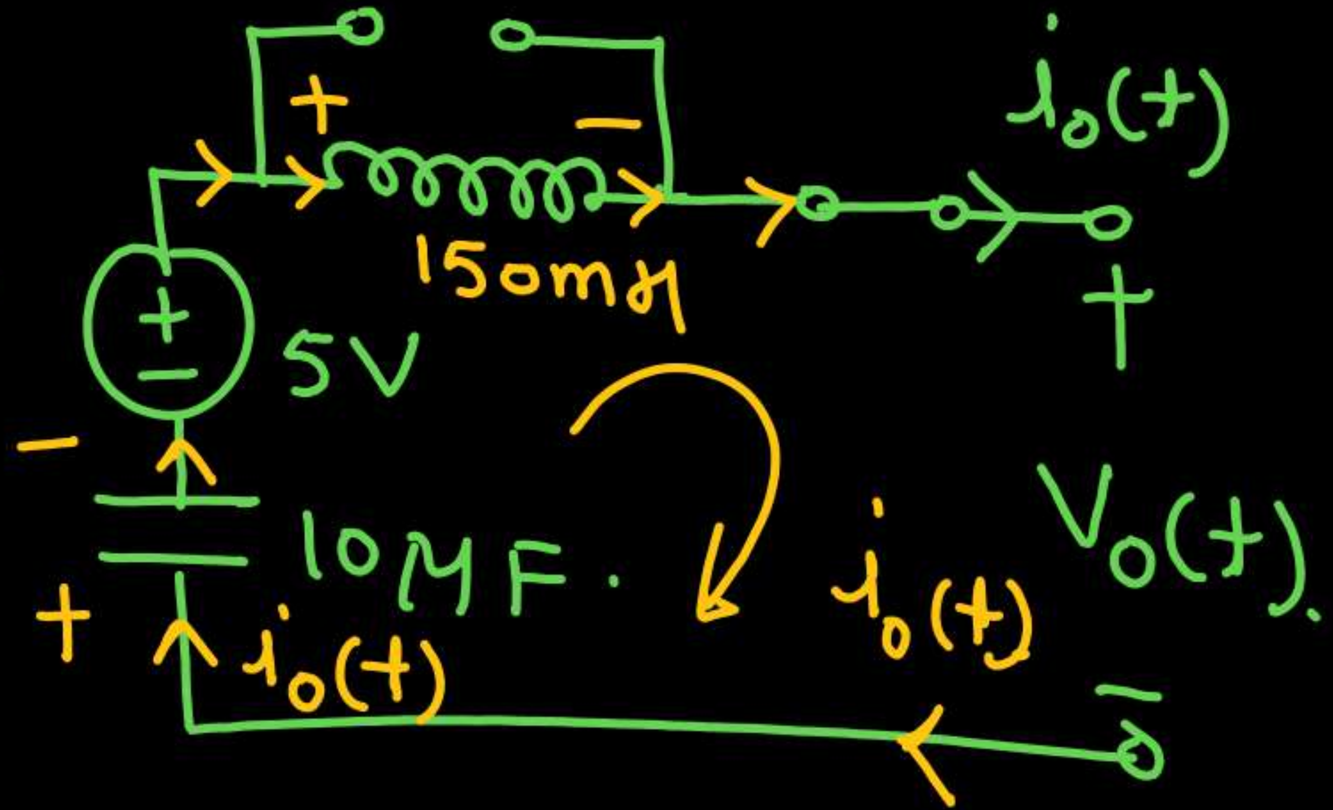
$$\boxed{i_o = 200e^{-800t} - 40e^{-200t}\,\text{mA}} \longrightarrow (t>0) \not\longrightarrow t \geq 0$$

If $v_c(0) = 5\,\text{V}$ find v_o for $t \geq 0$.

$$i_L(0^-) = i_L(0^+) = 0\,\text{A}$$



$\Rightarrow$



$$V_0(t) + \frac{1}{10\times10^{-6}}\int i_0(t)\,dt + 150\times10^{-3}\frac{di_0}{dt} = 5$$

$$V_0(t) = 5 - \frac{1}{10\times10^{-6}}\int_0^t \left(2\omega e^{-8\omega t} - 40 e^{-2\omega t}\right)dt - 150\times10^{-3}\times10^{-3} \times\left(\frac{d}{dt}\left(2\omega e^{-8\omega t} - 40 e^{-2\omega t}\right)\right)$$

1st

$$V_0(t) = 5 - \frac{1}{10\times10^{-3}}\left[\left(\frac{2\omega e^{-8\omega t}}{-8\omega}\right)_0^t + \left[\frac{40 e^{-2\omega t}}{200}\right]_0^t\right] - 150\times10^{-6} \times\left[2\omega e^{-8\omega t}\times(-8\omega) - 40 e^{-2\omega t}\times(-2\omega)\right]$$

$$\boxed{V_0(t) = \left(49 e^{-8\omega t} - 21.2 e^{-2\omega t}\right)(\text{volt})}$$

2nd

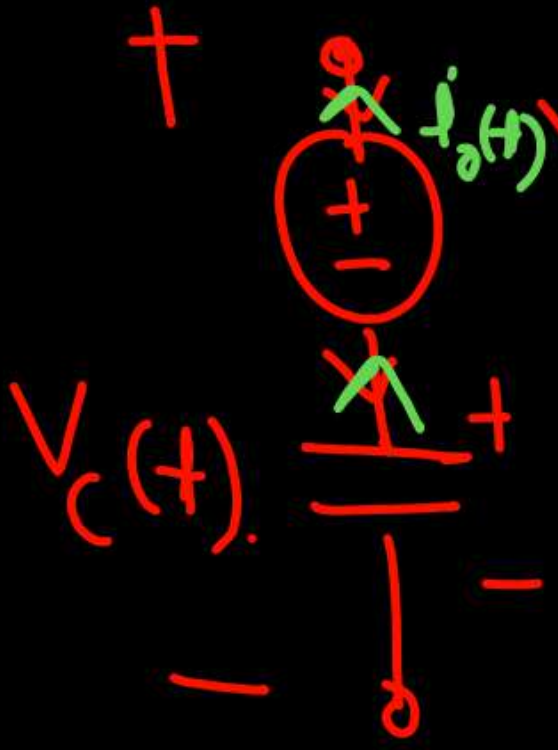
$$V_L(t) = L\,\frac{d\,i_o(t)}{dt}$$

$$= 150\times10^{-3}\,\frac{d}{dt}\left[0.200\,e^{-8\omega t} - 0.040\,e^{-2\omega t}\right]$$

$$\boxed{V_L(t) = 150\times10^{-3}\left[0.2\,e^{-8\omega t}\times(-8\omega) - 0.04\,e^{-2\omega t}\times(-2\omega)\right]}$$

3rd

$$V_c(t) = \,?$$



$$V_c(t) = 5 + \frac{1}{C}\int_0^t i_o(t)\,dt$$

H.W

(4) $W_T(0) = \underbrace{\frac{1}{2} L \times \underset{\downarrow \atop 0A}{I_L(0)}^2}_{0J} + \underbrace{\frac{1}{2} \times C \times \underset{\downarrow \atop 5V}{V_C(0)}^2}_{\left[\frac{1}{2} \times 10 \times 10^{-6} \times (25)\right]} = (125\mu \text{ joule})$

(5) $W_T(\infty) = \underbrace{\frac{1}{2} \times L \times i_L(\infty)^2}_{0J} + \underbrace{\frac{1}{2} \times C \times V_C(\infty)^2}_{\downarrow}$

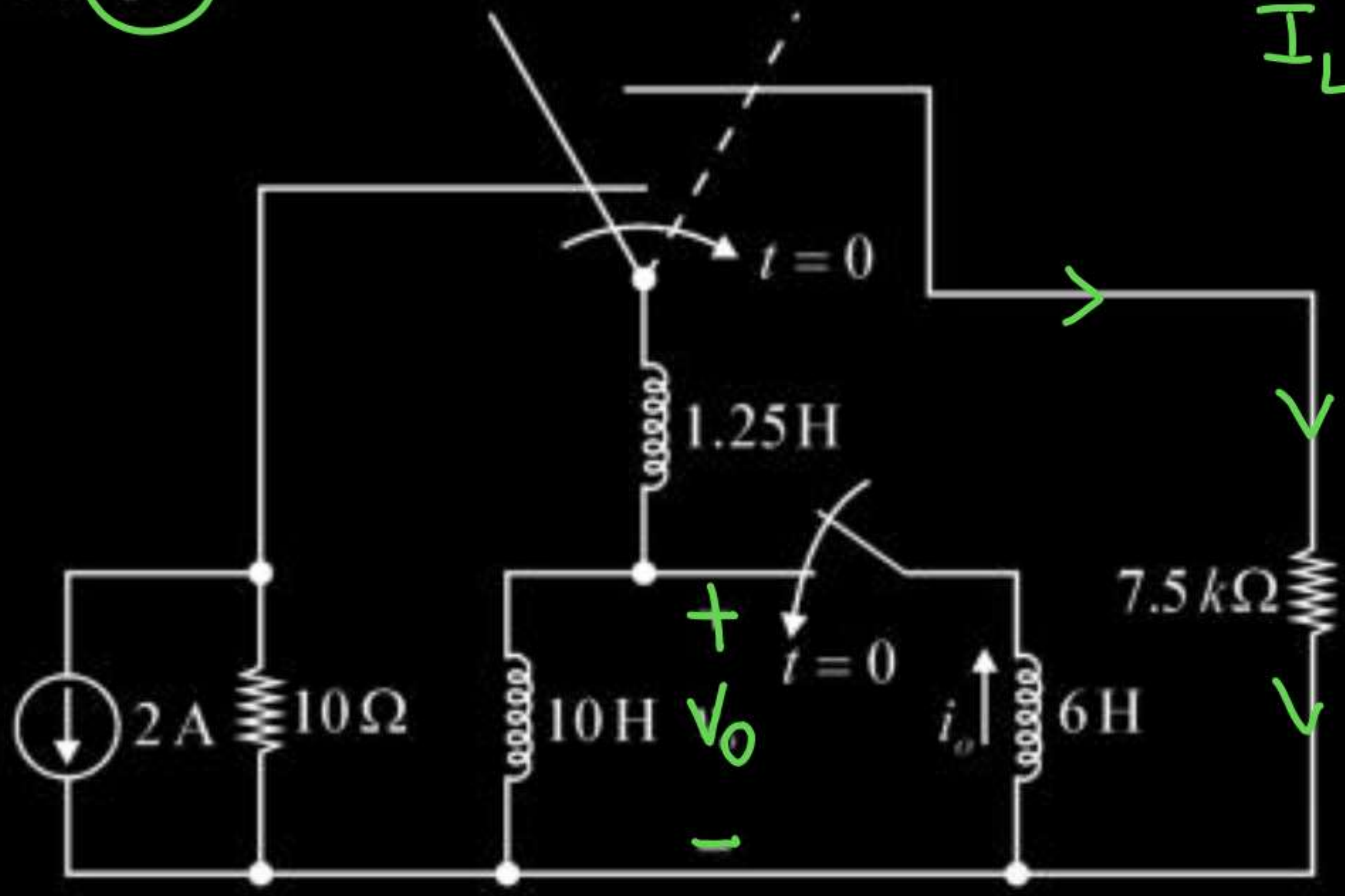
(6) Energy delivered to B.B $= W_T(0) - W_T(\infty)$

Question Beautiful: (1 minute)

The two switches shown in the circuit in Fig. operate simultaneously. Prior to $t = 0$ each switch has been in its indicated position for a long time. At $t = 0$ the two switches move instantaneously to their new positions. Find

(a) $v_o(t), t \geq 0^+$

(b) $i_o(t), t \geq 0$



• It is a first order circuit.

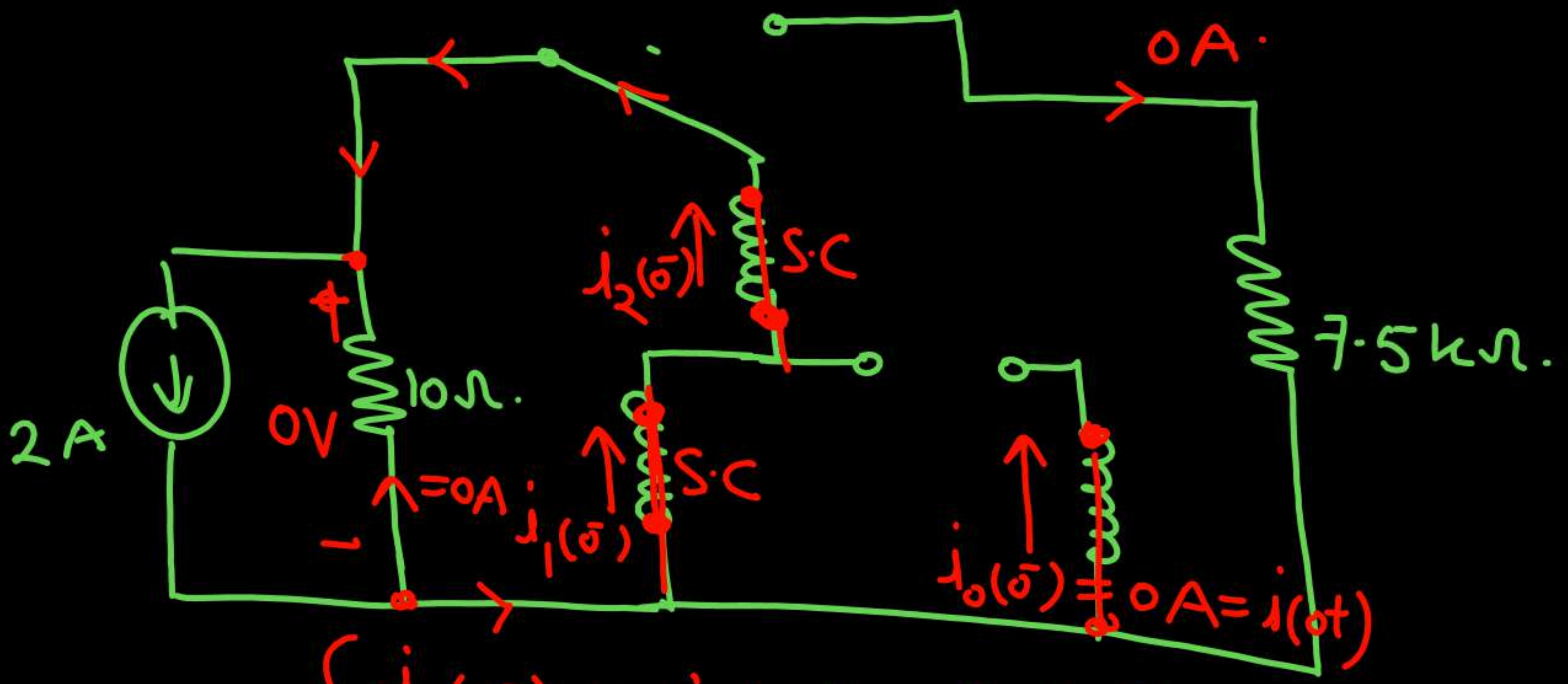
$$I_L(t) = I_L(\infty) + [I_L(0^+) - I_L(\infty)]e^{-t/\tau}$$

$I_L(\infty) \to 0\text{A}$, $I_L(\infty) \to 0\text{A}$

$$\left(I_L(t) = I_L(0^+)e^{-t/\tau}\right)$$

• switching instant → $t = 0$.

<u>case-I</u> $t < 0 \longrightarrow t = 0^-$ $L \rightarrow S \cdot C \rightarrow (S \cdot S)$.

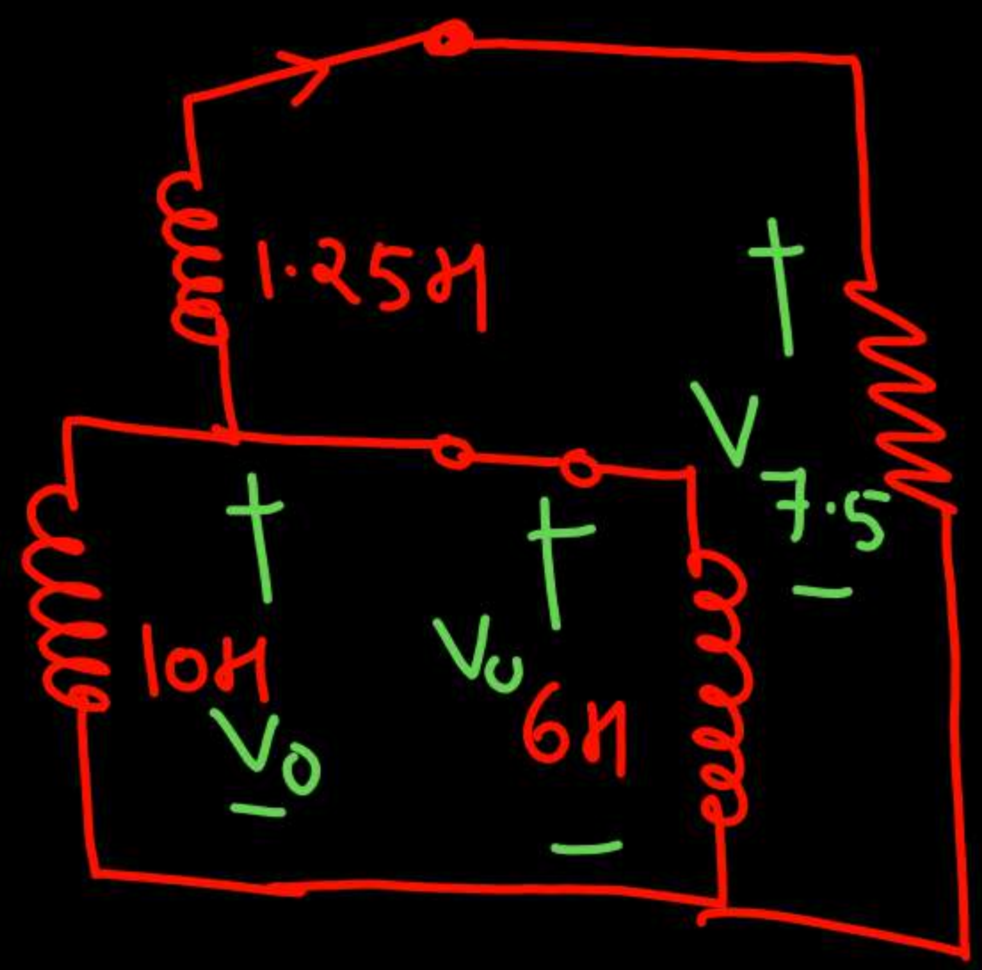


$$\left\{ \begin{array}{l} i_1(0^-) = i_2(0^-) = 2A. \\ i_1(0^+) = i_2(0^+) = 2A \end{array} \right\}$$

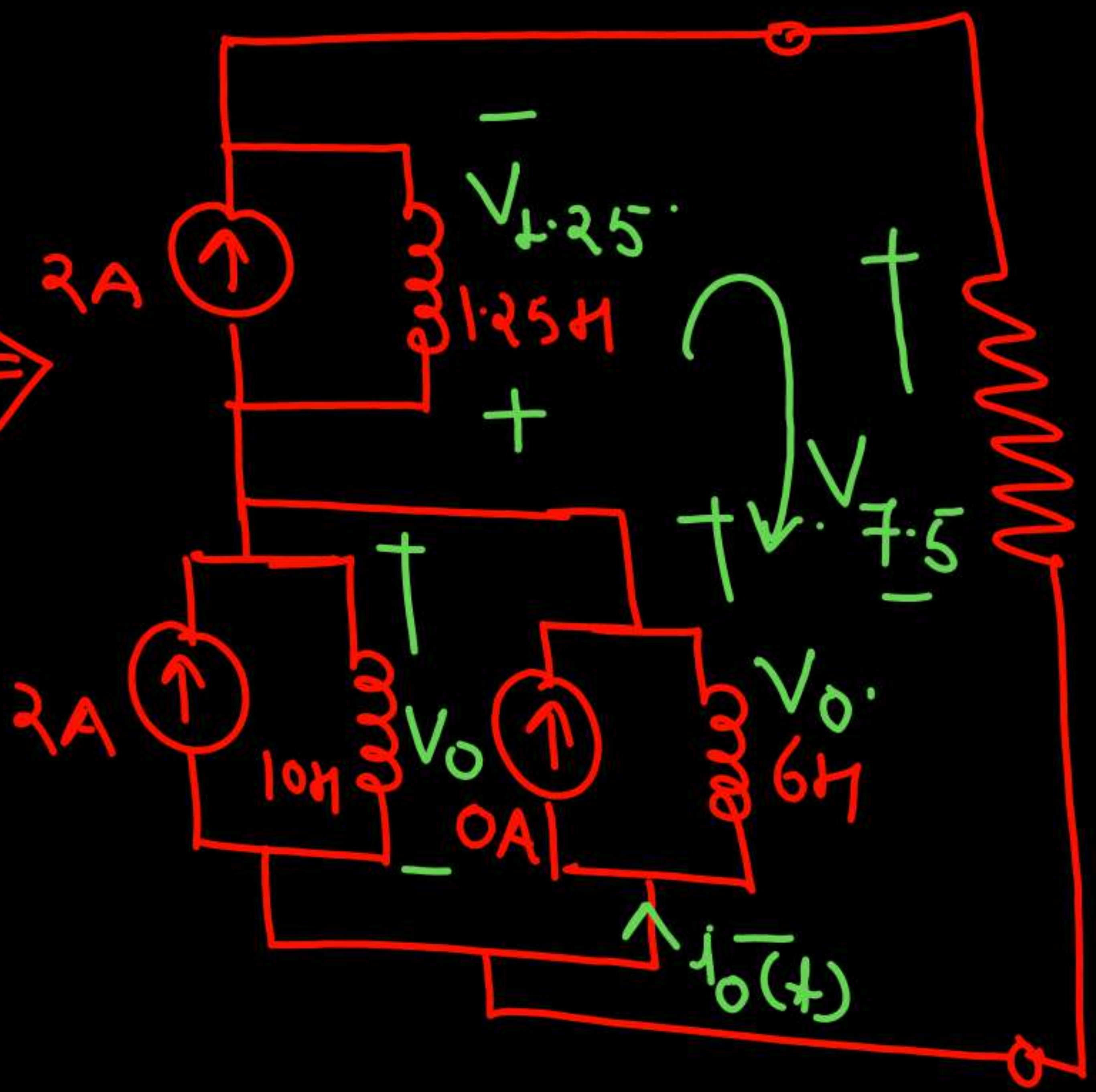
case-II ($t>0$)

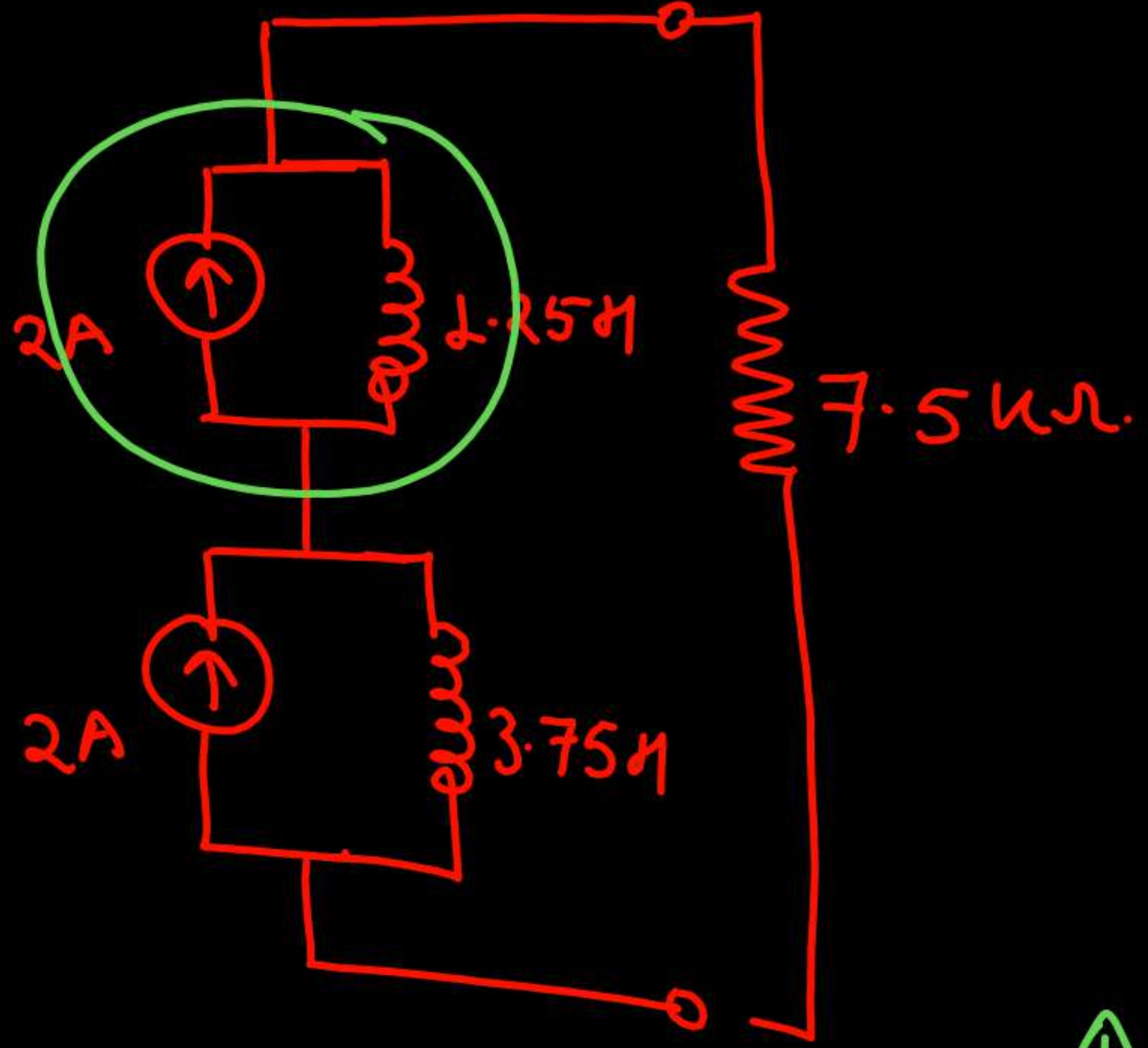
(i) $t=\infty$, $i_1(\infty)$, $i_2(\infty)$, $i_0(\infty) = 0\,A$.

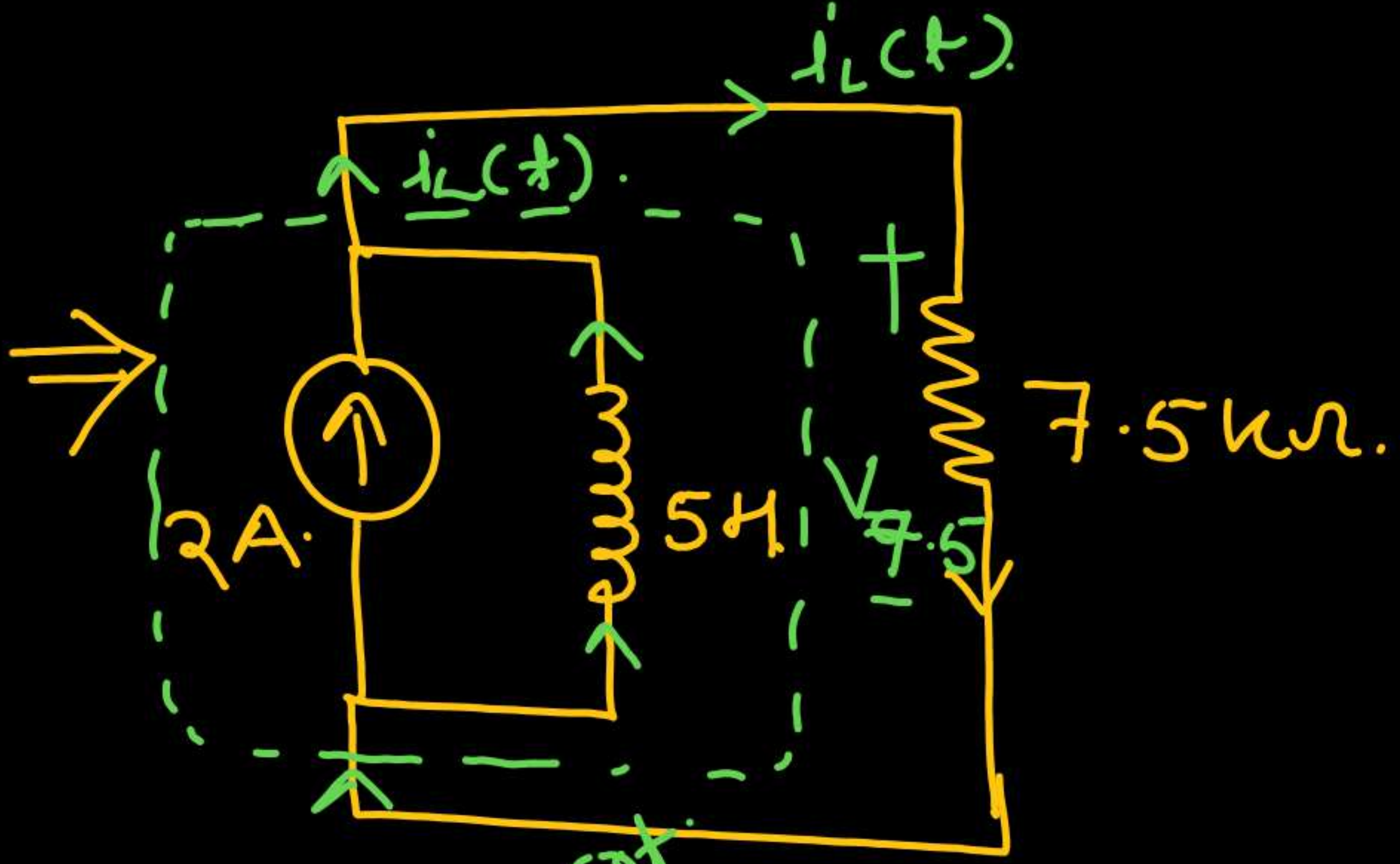
(ii) $t>0 \longrightarrow$

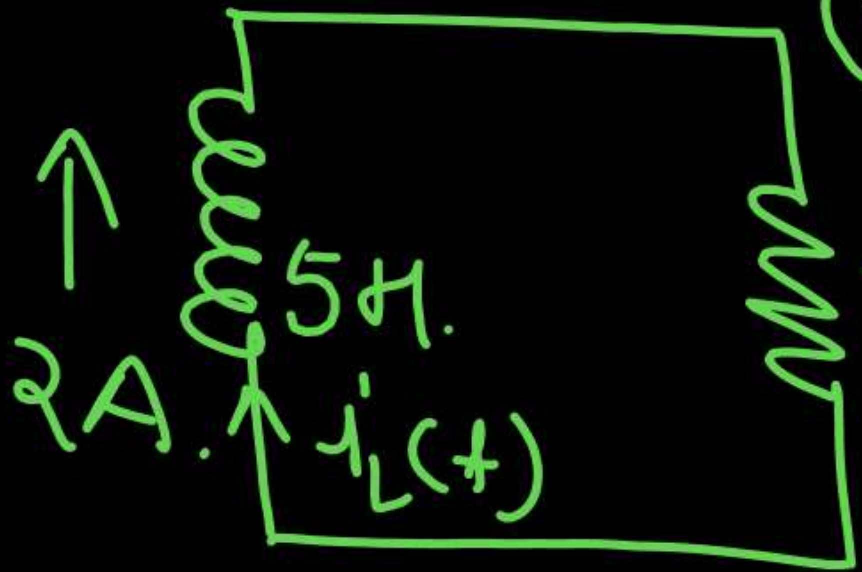


$\Rightarrow$









$$\tau = \left(\frac{5}{7.5k}\right)$$

$$(1)\quad i_L(t) = 2 \times e^{-t/\tau} = \left(2e^{-1500t}\right)$$

- Find the current in 1.25 H inductor for $t \geq 0$.

$$i_{1.25}(t) = i_L(t) = \left(2e^{-15\omega t}\right)$$

- Find the voltage across $(7.5\ k\Omega)$

$$V_{7.5}(t) = \left(2 \times e^{-15\omega t} \times 7.5 \times 10^{3}\right)$$

- Find the $V_0(t) = ?$

$$V_{1.25} = 1.25 \cdot \frac{d}{dt}\left(2e^{-15\omega t}\right) = 1.25 \times 2\, e^{-15\omega t} \times (-15\omega)$$

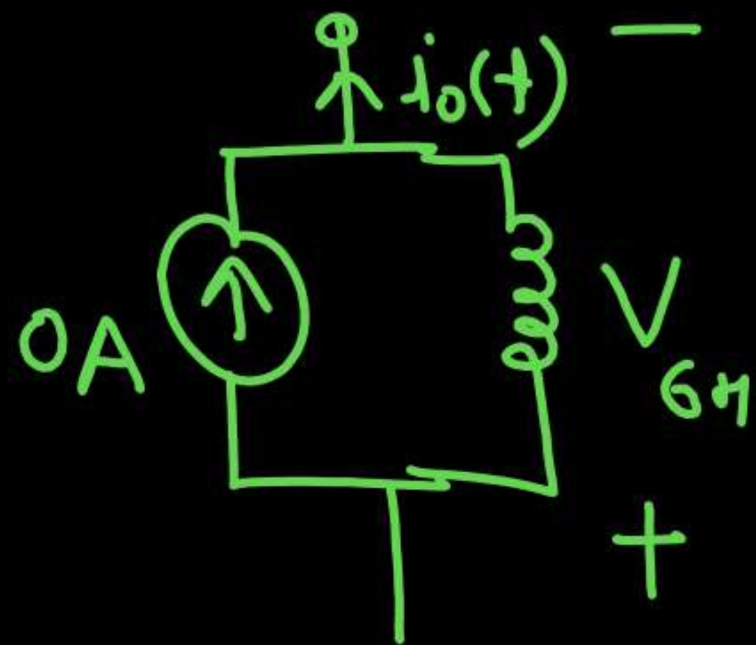
$$= -3750\, e^{-15\omega t}.$$

- $V_o(t) = V_{1.25}(t) + V_{7.5}(t)$

$$= -3750\,e^{-15\omega t} + 15000\,e^{-15\omega t}.$$

$$\boxed{V_o(t) = 11.25\,e^{-15\omega t}\ \text{kVolt}.}$$

- Find the $i_o(t)$.



$$V_{6H} = -V_o(t)$$

$$i_o(t) = \frac{1}{6}\int^{t} V_{6H}\cdot dt.$$

$$= \left[-\frac{1}{6}\int_0^t 11.25\times 10^3\, e^{-15\omega t} \right]$$

Thank you

GW Soldiers !