Mobile Robot Navigation Amidst Humans with Intents and Uncertainties:A Time Scaled Collision cone Approach

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Outline

Motivation

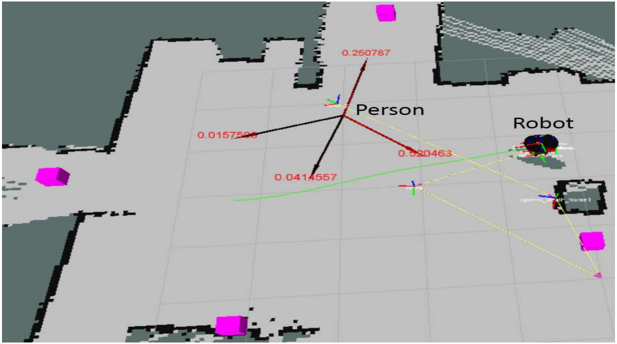
- Robots and humans are beginning to occupy the same work spaces
- Account for human intent in robot's navigation and avoidance Maneuver
- Uncertain and Haphazard local movements of human

Outline

Human Intention prediction

- Characterize intents as the final destinations a person might reach
- Let $D = \{\mathbf{d^1}, \mathbf{d^2}, ..., \mathbf{d^m}\}$ be the set of final destinations a person can go to in a given environment
- compute the probability of each of these intents Using Hidden Markov Model.
- Characterize local Haphazard movements as a gaussian $\mathcal{N}(\mu_i(\mathbf{x}^t), \sigma_t)$

Human Intention prediction

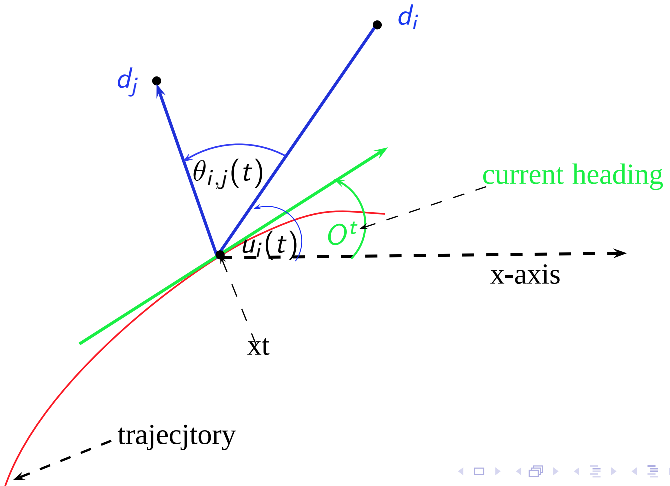


HMM for Intention prediction

- Let $S^t \in D$ represent the intent of a person to reach destination S^t at time t.
- D represents set of states in HMM.
- Human trajectories are represented as $X(T) = \{\mathbf{x^1}, \mathbf{x^2}, ..., \mathbf{x^T}\}$

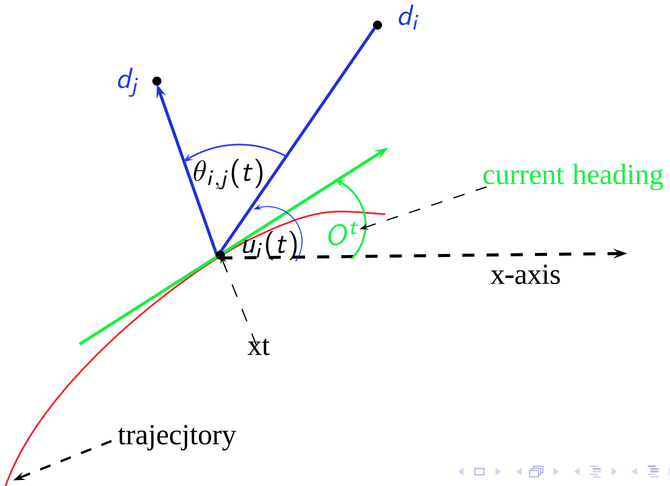
HMM for Intention prediction

- O^t is the angle defined by the first derivative of the trajectory at point $\mathbf{x^t}$
- Given the current position and orientation we compute the probability of reaching each of the destination $d^i \in D$



HMM for Intention prediction

- $\mu_i(t)$ is the measure relative to the destination $\mathbf{d^i}$
- O^t is the global measure of the target orientation
- $\theta_{ij}(t)$ is the measure between final destinations $\mathbf{d^i}$ and $\mathbf{d^j}$ relative to the current position $\mathbf{x^t}$



HMM for Intention prediction

- $b_i(O^t)$ is the probability of observing heading O^t given that the person is following the intent $\mathbf{d^i}$ at time t.

$$b_i(O^t) = p(O^t | S^t = \mathbf{d^i}) = \mathcal{N}(O^t | \mu_i(t), \sigma_o)$$

- $a_{ij}(t)$ is the probability that the human changes his intent from $\mathbf{d^i}$ to $\mathbf{d^j}$ at any discrete instant t

$$a_{ij}(t) = p(S^{t+1} = \mathbf{d^j} | S^t = \mathbf{d^i}) = \eta \mathcal{N}(\theta_{ij}(t) | 0, \sigma_a)$$

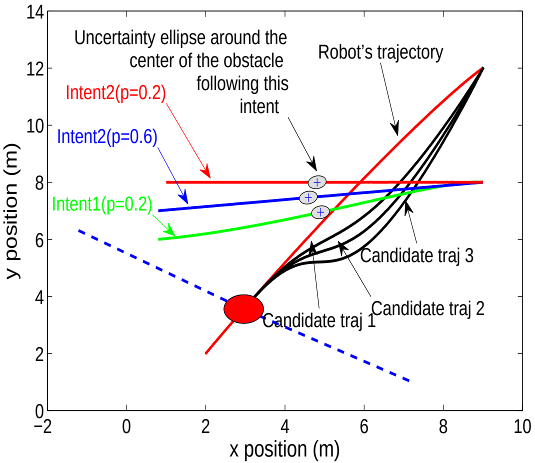
HMM for Intention prediction

- Let $O^{1:T} = \{O^1, O^1, ..., O^T\}$ is the set of measurements obtained till time T.
- Our task is to calculate $p(S^t = \mathbf{d^i} | O^{1:T}, \lambda)$
- In HMM this term is usually referred to as $\gamma_t(i)$ To find this we use standard forward and backward algorithms.

Proactive collision avoidance in intent space

- To propose an optimization framework, That achieves an elegant balance between minimizing risk and ease of collision avoidance maneuver.
- Ease of Collision avoidance maneuver directly relates to factors like deviation from current path and acceleration and deceleration capabilities of robot.
- Minimizing risk boils down to biasing the maneuver towards avoiding the most likely intent with higher confidence.

Proactive collision avoidance in intent space



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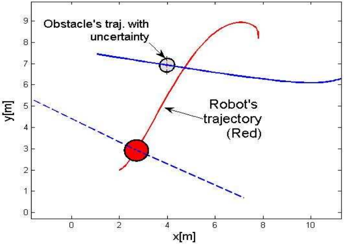
Formulation steps

- Formulation for finding a relation between a particular collision avoidance maneuver and its confidence of safety, for a particular obstacle/intent.



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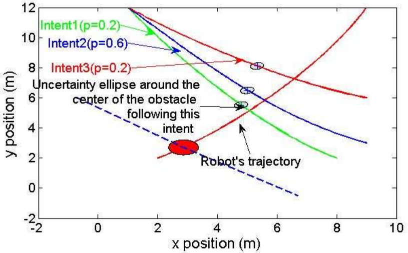
Formulation steps

- Formulation extending it to multiple intent space



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Explanation of Formulation one

- Finding a relation between a particular collision avoidance maneuver and its confidence of safety, for a particular obstacle/intent [1]

[1]: Bharath Gopalakrishnan*, Arun Kumar Singh*, K.Madhava Krishna, Closed form characterization of Collison free velocities and confidence boinds for Non- holonomic robots in uncertain dynamic environments- To appear in IEEE Proc of IROS 2015

Proactive collision avoidance in intent space

Recap of time scaled collision cone:

- Time scaled collision cone constraint takes the following from

$$f_i^s \geq 0$$

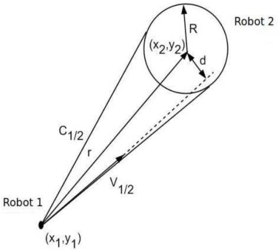
- where f_i^s is given by

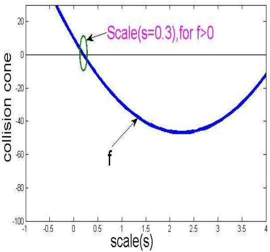
$$\begin{aligned} f_i = (x^{t_c} - x_i^{t_c})^2 + (y^{t_c} - y_i^{t_c})^2 - R^2 \\ - \frac{(s\dot{x}^{t_c} - \dot{x}_i^{t_c})(x^{t_c} - x_i^{t_c}) + (s\dot{y}^{t_c} - \dot{y}_i^{t_c})(y^{t_c} - y_i^{t_c})^2}{(s\dot{x}^{t_c} - \dot{x}_i^{t_c})^2 + (s\dot{y}^{t_c} - \dot{y}_i^{t_c})^2} \\ , \forall i = 1, 2...n \end{aligned} \tag{1}$$

- f_i^s denotes the collision cone constraint for the i^{th} obstacle as a function of scale s. which depends on the state of the robot and obstacle at time $t = t^c$ which gets reduced to

$$a_i s^2 + b_i s + c_i \geq 0$$

Proactive collision avoidance in intent space





Proactive collision avoidance in intent space

Probabilistic version of time scaled collision cone

- if at time $t = t_c$ the obstacles state are given by

$$x_i^{t_c} = \mathcal{N}(\mu_i^x, \sigma_i^x), \dot{x}_i^{t_c} = \mathcal{N}(\mu_i^{\dot{x}}, \sigma_i^{\dot{x}})$$

$$y_i^{t_c} = \mathcal{N}(\mu_i^y, \sigma_i^y), \dot{y}_i^{t_c} = \mathcal{N}(\mu_i^{\dot{y}}, \sigma_i^{\dot{y}})$$

- Then the objective would be to find the scale that maximizes

$$P(f_i^s \geq 0)$$

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Objective

$$\underset{s}{\operatorname{argmax}}\{P(f_i^s \geq 0)\}$$

Challenge

- f_i^s *is* a random variable with unknown analytical expression for its probability distribution.

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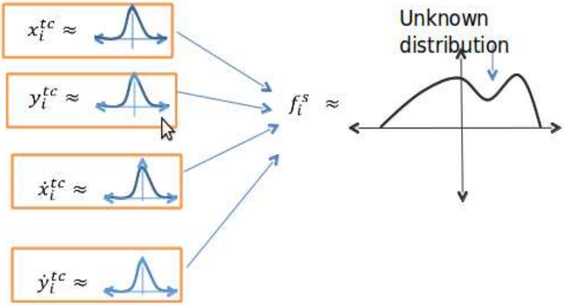

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Solution

- Though the pdf of $f_i^s is$ does not have an analytical expression we can get its mean and standard deviation in closed form as a function of s
- By the law of unconscious statistician

$$E[f_i^s] = \mu_{f_i^s} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_i^s(.)P_i(.)dx_i^{t_c}dy_it_c d\dot{x}_i{}^{t_c}d\dot{y}_i{}^{t_c}$$

- Which evaluates as

$$\mu_{f_i^2} = A_is^2 + B_is + C_i$$

Where A_i, B_i and C_i are the function of robot states and obstacle distribution parameters , $\mu_i^1, \mu_i^2, \sigma_i^1, \sigma_i^2$

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Solution

- Similarly

$$\sigma_{f_i^s} = \sqrt{E[(f_i^s - E[f_i^s])^2} = \sqrt{D_i s^4 + E_i^3 + F_i^2 + G_i s + H}$$

Where D_i, E_i, F_i, G_i, and H_i are the function of robot states and obstacle distribution parameters , $\mu_i^1, \mu_i^2, \sigma_i^1, \sigma_i^2$

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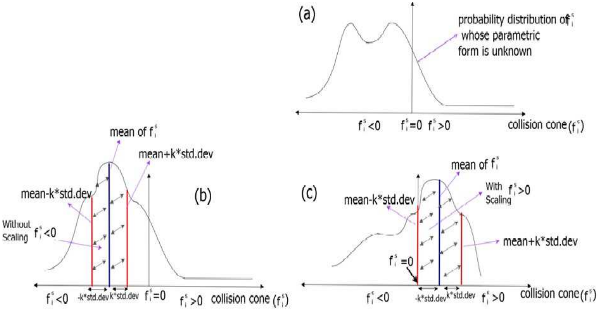
Solution

$$\underset{s}{\text{argmax}}\{P(f_i^s \geq 0)\} \implies \mu_{f_i^s} \pm k * \sigma_{f_i^s}$$

This can be suitably achieved by suitably changing the value of k



Proactive collision avoidance in intent space

Lower bound on $P(f_i^s) \geq 0)$

- In the previous section we found out on how to obtain scale s for various values of k that would end up maximizing $P(f_i^s) \geq 0$.
- Since the pdf of $P(f_i^s) \geq 0$ does not have an analytical form ,it is not possible to get the probability of f_i^s for a particular value of s.
- Hence we can only bound $P(f_i^s) \geq 0$ by a lower bound and this can be done by Cantelli's inequality.

Proactive collision avoidance in intent space

Lower bound on $P(f_i^s) \geq 0)$

- The lower bound are thus obtained through

$$P(f_i^s \geq 0 | \mu_{f_i^s} - k * \sigma_{f_i^s} > 0) \geq \frac{k^2}{k^2 + 1}$$

- Thus solving for larger k increases the lower bounds and thus improves the confidence measures

Proactive collision avoidance in intent space

Collision Avoidance in Multiple Intent space

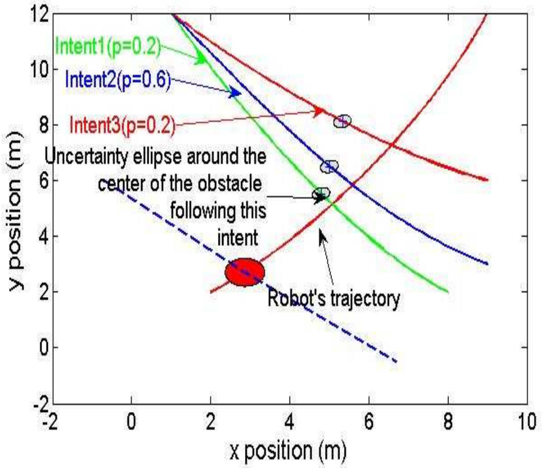
- As stated earlier, the objective is to Find maneuvers that maximize the confidence of safety for more probable intents.
- As seen in formulation one, the expression on the Lower bound of confidence is.

$$P(f_i^s \geq 0 | \mu_{f_i^s} - k * \sigma_{f_i^s} > 0) \geq \frac{k^2}{k^2 + 1}$$

- Hence confidence of safety is directly related to variable k

Proactive collision avoidance in intent space

Collision Avoidance in Multiple Intent space



Proactive collision avoidance in intent space

Cost function for Collision Avoidance in Multiple Intent space

- Optimization Formulation

$$\begin{aligned} \text{minimize} \quad & w_t \Delta t + w_r \Delta r \\ \text{subject to} \quad & \mu_{f_i^s} \pm k_i * \sigma_{f_i^s} \geq 0, \; i = 1, \ldots, n, \; . \end{aligned}$$

- Where $\Delta t = (s-1)^2$ and $\Delta r = -\sum_i^n \gamma_i * k_i$
- Here $\mu_{f_i^s}$ and $\sigma_{f_i^s}$ are the mean and the standard deviation of the collision cone respectively.
- k is the variable directly relating to the confidence of safety as described in Cantelli's inequality.
- γ_i is the probability of intent i

Proactive collision avoidance in intent space

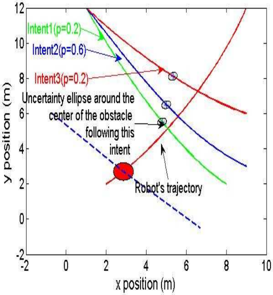
Cost function for Collision Avoidance in Multiple Intent space

- Δt ensures a collision free velocity very close to the current velocity.
- Δr relates the risk associated with the avoidance maneuver.
- This biases the solution space towards avoiding the most likely intent with higher confidence.

Proactive collision avoidance in intent space

Illustration,Scale along the current robot path

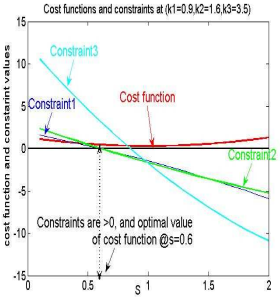
- Consider the Scenario shown in the following figure



Proactive collision avoidance in intent space

Illustration, Scale along the current robot path

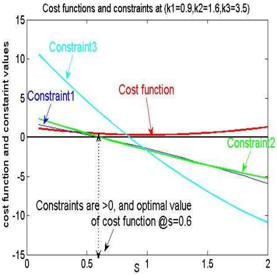
- The cost function and the constraint for the above scenario is shown in the following figure



Proactive collision avoidance in intent space

Illustration, Scale along the current robot path

- It can be noticed from the above figure, that the values of scale s, k_1, k_2,k_3, at which the cost function has an optimal value are $s = 0.6$, $k_1 = 0.9, k_2 = 1.6, k_3 = 2$

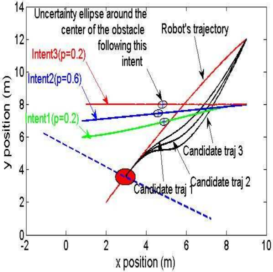


- From the values of k_1, k_2, k_3, we can say that the scale $s = 0.6$ corresponds to 75% of safety/confidence for intent two (most probable) and 30% confidence / safety for intent one

Proactive collision avoidance in intent space

Illustration, for multiple candidate trajectories

- In some cases, it may become imperative to deviate from the current path to avoid collisions.



- For example in the figure, and in the table below, it is concluded that there is no solution (scale:s) possible along the robots current path(red).

Proactive collision avoidance in intent space

Illustration, for multiple candidate trajectories

Candidate Trajectory	k_1	k_2	k_3	scale
Robot Original	NULL	NULL	NULL	NULL
Candidate 1	0.2	2	2	0.6
Candidate 2	0.67	2	2	0.916
Candidate 3	1.73	2	2	1

- From the above table it is clear that Candidate Trajectory 2 can be preferred