Mobile Robot Navigation Amidst Humans with Intents and Uncertainties:A Time Scaled Collision cone Approach

Akhil Nagariya[1] Bharath Gopalakrishna[1] Arun Singh[2]
Krishnam Gupta [1] K Madhava Krishna [1]

[1]RRC
IIIT Hyderabad

[2]Ben-Gurion University, Isreal

CDC 2015

Outline

Motivation

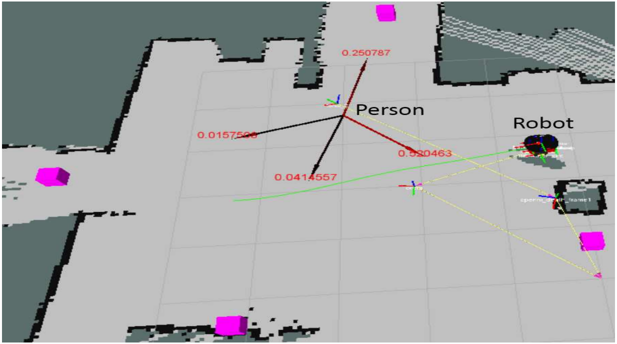
- Robots and humans are beginning to occupy the same work spaces.
- Account for human intent in robot's navigation and avoidance Maneuver.
- Uncertain and Haphazard local movements of human.

Outline

Human Intention prediction

- Characterize intents as the final destinations a person might reach.
- Let $D = \{\mathbf{d^1}, \mathbf{d^2}, ..., \mathbf{d^m}\}$ be the set of final destinations a person can go to in a given environment.
- Compute the probability of each of these intents Using Hidden Markov Model.
- Characterize local Haphazard movements as a gaussian $\mathcal{N}(\mu_i(\mathbf{x}^t), \sigma_t)$.

Human Intention prediction

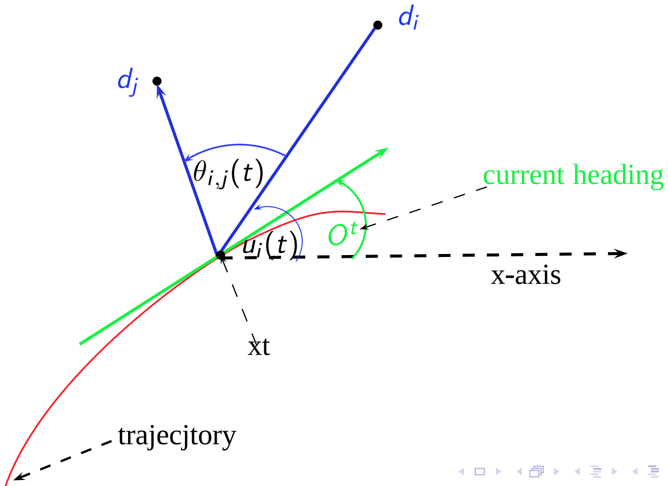


HMM for Intention prediction

- Let $S^t \in D$ represent the intent of a person to reach destination S^t at time t.
- D represents set of states in HMM.
- Human trajectories are represented as $X(T) = \{\mathbf{x^1}, \mathbf{x^2}, ..., \mathbf{x^T}\}$.

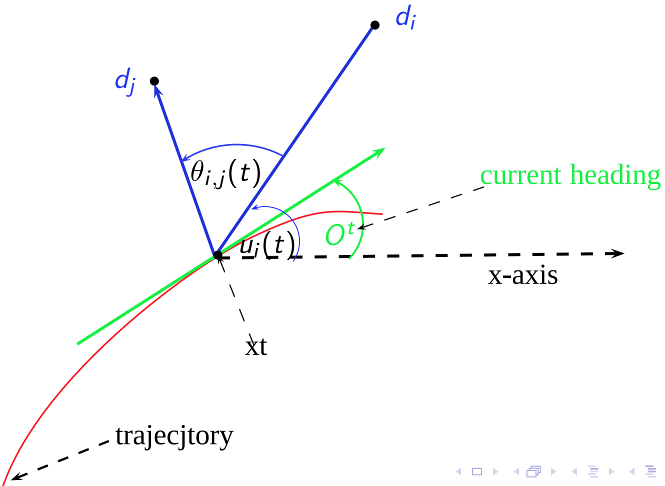
HMM for Intention prediction

- O^t is the angle defined by the first derivative of the trajectory at point $\mathbf{x^t}$.
- Given the current position and orientation we compute the probability of reaching each of the destination $d^i \in D$.



HMM for Intention prediction

- $\mu_i(t)$ is the measure relative to the destination $\mathbf{d^i}$.
- O^t is the global measure of the target orientation.
- $\theta_{ij}(t)$ is the measure between final destinations $\mathbf{d^i}$ and $\mathbf{d^j}$ relative to the current position $\mathbf{x^t}$.



HMM for Intention prediction

- $b_i(O^t)$ is the probability of observing heading O^t given that the person is following the intent $\mathbf{d^i}$ at time t.

$$b_i(O^t) = p(O^t|S^t = \mathbf{d^i}) = \mathcal{N}(O^t|\mu_i(t), \sigma_o)$$

- $a_{ij}(t)$ is the probability that the human changes his intent from $\mathbf{d^i}$ to $\mathbf{d^j}$ at any discrete instant t

$$a_{ij}(t) = p(S^{t+1} = \mathbf{d^j}|S^t = \mathbf{d^i}) = \eta\mathcal{N}(\theta_{ij}(t)|0, \sigma_a)$$

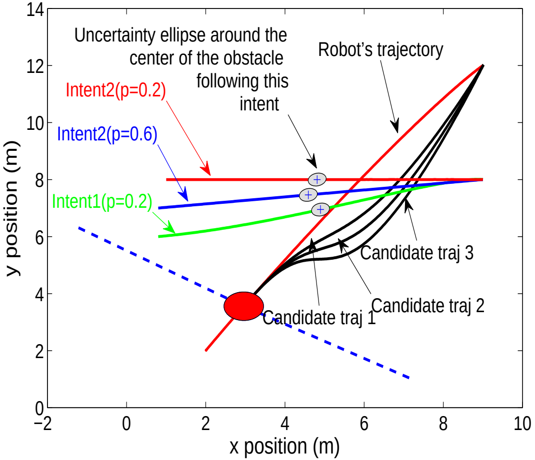
HMM for Intention prediction

- Let $O^{1:T} = \{O^1, O^1, ..., O^T\}$ is the set of measurements obtained till time T.
- Our task is to calculate $p(S^t = \mathbf{d^i} | O^{1:T}, \lambda)$.
- In HMM this term is usually referred to as $\gamma_t(i)$ To find this we use standard forward and backward algorithms.

Proactive collision avoidance in intent space

- To propose an optimization framework, That achieves an elegant balance between minimizing risk and ease of collision avoidance maneuver.
- Ease of Collision avoidance maneuver directly relates to factors like deviation from current path and acceleration and deceleration capabilities of robot.
- Minimizing risk boils down to biasing the maneuver towards avoiding the most likely intent with higher confidence.

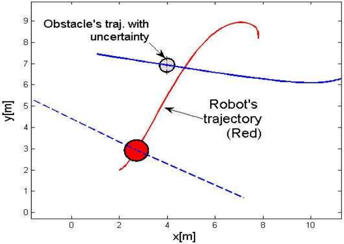
Proactive collision avoidance in intent space



Proactive collision avoidance in intent space

Formulation steps

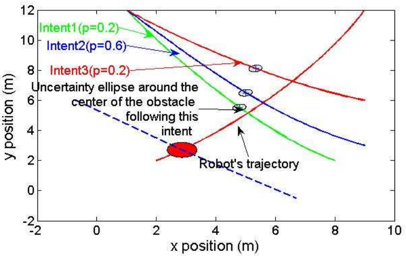
- Formulation for finding a relation between a particular collision avoidance maneuver and its confidence of safety, for a particular obstacle/intent.



Proactive collision avoidance in intent space

Formulation steps

- Formulation extending it to multiple intent space



Proactive collision avoidance in intent space

Explanation of Formulation one

- Finding a relation between a particular collision avoidance maneuver and its confidence of safety, for a particular obstacle/intent [1].

[1]: Bharath Gopalakrishnan*, Arun Kumar Singh*, K.Madhava Krishna, Closed form characterization of Collison free velocities and confidence boinds for Non- holonomic robots in uncertain dynamic environments- To appear in IEEE Proc of IROS 2015.

Proactive collision avoidance in intent space

Recap of time scaled collision cone:

- Time scaled collision cone constraint takes the following from.

$$f_i^s \geq 0$$

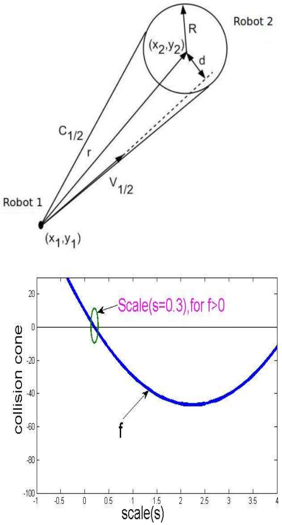
- where f_i^s is given by

$$f_i = (x^{t_c} - x_i^{t_c})^2 + (y^{t_c} - y_i^{t_c})^2 - R^2 \tag{1}$$
$$- \frac{(s\dot{x}^{t_c} - \dot{x}_i^{t_c})(x^{t_c} - x_i^{t_c}) + (s\dot{y}^{t_c} - \dot{y}_i^{t_c})(y^{t_c} - y_i^{t_c})^2}{(s\dot{x}^{t_c} - \dot{x}_i^{t_c})^2 + (s\dot{y}^{t_c} - \dot{y}_i^{t_c})^2}$$
$$, \forall i = 1, 2 \ldots n$$

- f_i^s denotes the collision cone constraint for the i^{th} obstacle as a function of scale s. which depends on the state of the robot and obstacle at time $t = t^c$ which gets reduced to

$$a_i s^2 + b_i s + c_i \geq 0$$

Proactive collision avoidance in intent space



Proactive collision avoidance in intent space

Probabilistic version of time scaled collision cone

- If at time $t = t_c$ the obstacles state are given by

$$x_i^{t_c} = \mathcal{N}(\mu_i^x, \sigma_i^x), \dot{x}_i^{t_c} = \mathcal{N}(\mu_i^{\dot{x}}, \sigma_i^{\dot{x}})$$

$$y_i^{t_c} = \mathcal{N}(\mu_i^y, \sigma_i^y), \dot{y}_i^{t_c} = \mathcal{N}(\mu_i^{\dot{y}}, \sigma_i^{\dot{y}})$$

- Then the objective would be to find the scale that maximizes

$$P(f_i^s \geq 0)$$

Proactive collision avoidance in intent space

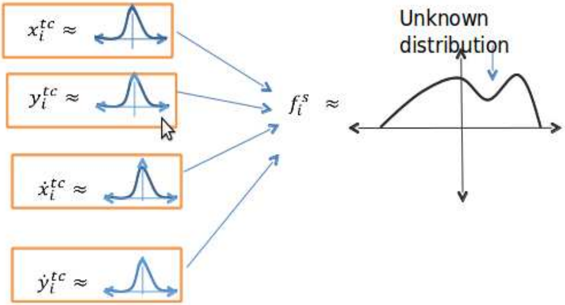
Objective

$$\underset{s}{\text{argmax}}\{P(f_i^s \geq 0)\}$$

Challenge

- f_i^s *is* a random variable with unknown analytical expression for its probability distribution.

Proactive collision avoidance in intent space



Proactive collision avoidance in intent space

Solution

- Though the pdf of f_i^s *is* does not have an analytical expression we can get its mean and standard deviation in closed form as a function of s.
- By the law of unconscious statistician.

$$E[f_i^s] = \mu_{f_i^s} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_i^s(.)P_i(.)dx_i^{t_c}dy_it_c d\dot{x}_i{}^{t_c}d\dot{y}_i{}^{t_c}$$

- Which evaluates as.

$$\mu_{f_i^2} = A_i s^2 + B_i s + C_i$$

Where A_i, B_i and C_i are the function of robot states and obstacle distribution parameters , $\mu_i^1, \mu_i^2, \sigma_i^1, \sigma_i^2$

Proactive collision avoidance in intent space

Solution

- Similarly

$$\sigma_{f_i^s} = \sqrt{E[(f_i^s - E[f_i^s])^2} = \sqrt{D_i s^4 + E_i^3 + F_i^2 + G_i s + H}$$

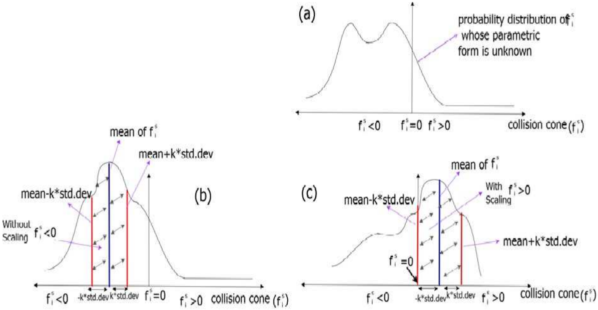
Where D_i, E_i, F_i, G_i, and H_i are the function of robot states and obstacle distribution parameters , $\mu_i^1, \mu_i^2, \sigma_i^1, \sigma_i^2$

Proactive collision avoidance in intent space

Solution

$$\underset{s}{\text{argmax}}\{P(f_i^s \geq 0)\} \implies \mu_{f_i^s} \pm k * \sigma_{f_i^s}$$

This can be suitably achieved by changing the value of k



Proactive collision avoidance in intent space

Lower bound on $P(f_i^s) \geq 0)$

- In the previous section we found out on how to obtain scale s for various values of k that would end up maximizing $P(f_i^s) \geq 0$.
- Since the pdf of $P(f_i^s) \geq 0$ does not have an analytical form ,it is not possible to get the probability of f_i^s for a particular value of s.
- Hence we can only bound $P(f_i^s) \geq 0$ by a lower bound and this can be done by Cantelli's inequality.

Proactive collision avoidance in intent space

Lower bound on $P(f_i^s) \geq 0)$

- The lower bound are thus obtained through

$$P(f_i^s \geq 0 | \mu_{f_i^s} - k * \sigma_{f_i^s} > 0) \geq \frac{k^2}{k^2 + 1}$$

- Thus solving for larger k increases the lower bounds and thus improves the confidence measures.

Proactive collision avoidance in intent space

Collision Avoidance in Multiple Intent space

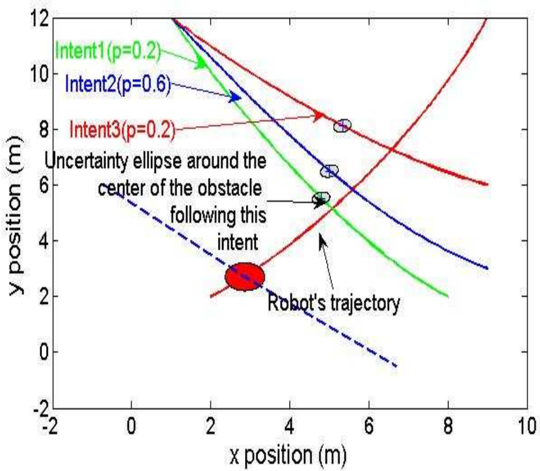
- As stated earlier, the objective is to Find maneuvers that maximize the confidence of safety for more probable intents.
- As seen in formulation one, the expression on the Lower bound of confidence is.

$$P(f_i^s \geq 0 | \mu_{f_i^s} - k * \sigma_{f_i^s} > 0) \geq \frac{k^2}{k^2 + 1}$$

- Hence confidence of safety is directly related to variable k

Proactive collision avoidance in intent space

Collision Avoidance in Multiple Intent space



Proactive collision avoidance in intent space

Cost function for Collision Avoidance in Multiple Intent space

- Optimization Formulation

$$\begin{aligned} \text{minimize} \quad & w_t \Delta t + w_r \Delta r \\ \text{subject to} \quad & \mu_{f_i^s} \pm k_i * \sigma_{f_i^s} \geq 0, \; i = 1, \ldots, n, \; . \end{aligned}$$

- Where $\Delta t = (s-1)^2$ and $\Delta r = -\sum_i^n \gamma_i * k_i$
- Here $\mu_{f_i^s}$ and $\sigma_{f_i^s}$ are the mean and the standard deviation of the collision cone respectively.
- k is the variable directly relating to the confidence of safety as described in Cantelli's inequality.
- γ_i is the probability of intent i

Proactive collision avoidance in intent space

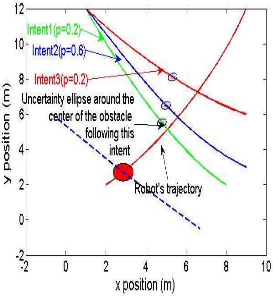
Cost function for Collision Avoidance in Multiple Intent space

- Δt ensures a collision free velocity very close to the current velocity.
- Δr relates the risk associated with the avoidance maneuver.
- This biases the solution space towards avoiding the most likely intent with higher confidence.

Proactive collision avoidance in intent space

Illustration,Scale along the current robot path

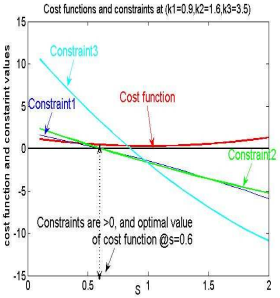
- Consider the Scenario shown in the following figure



Proactive collision avoidance in intent space

Illustration, Scale along the current robot path

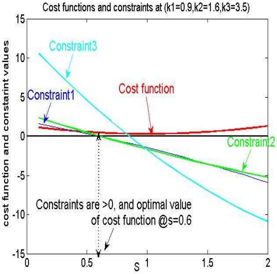
- The cost function and the constraint for the above scenario is shown in the following figure.



Proactive collision avoidance in intent space

Illustration, Scale along the current robot path

- It can be noticed from the above figure, that the values of scale s, k_1, k_2,k_3, at which the cost function has an optimal value are $s = 0.6$, $k_1 = 0.9, k_2 = 1.6, k_3 = 2$.

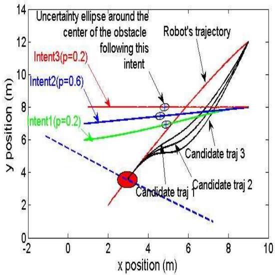


- From the values of k_1, k_2, k_3, we can say that the scale $s = 0.6$ corresponds to 75% of safety/confidence for intent two (most probable) and 30% confidence / safety for intent one.

Proactive collision avoidance in intent space

Illustration, for multiple candidate trajectories

- In some cases, it may become imperative to deviate from the current path to avoid collisions.



- For example in the figure, and in the table below, it is concluded that there is no solution (scale:s) possible along the robots current path(red).

Proactive collision avoidance in intent space

Illustration, for multiple candidate trajectories

Candidate Trajectory	k_1	k_2	k_3	scale
Robot Original	NULL	NULL	NULL	NULL
Candidate 1	0.2	2	2	0.6
Candidate 2	0.67	2	2	0.916
Candidate 3	1.73	2	2	1

- From the above table it is clear that Candidate Trajectory 2 can be preferred .