PHYS 225 Winter 2026

Homework #2

1. McIntyre, Chapter 1, Problem 1.16

2. A quantum object whose state is given by $|\psi\rangle = \frac{1}{2}|+\rangle_z - \frac{\sqrt{3}}{2}|-\rangle_z$ is sent through a Stern-Gerlach device with the magnetic field oriented in the *y*-direction. What is the probability that this object will emerge from the + side of this device?

3. McIntyre, Chapter 2, Problem 2.3

4. Suppose that operators $\hat{A}$ and $\hat{B}$ are both Hermitian, i.e., $\hat{A}^{\dagger} = \hat{A}$ and $\hat{B}^{\dagger} = \hat{B}$. Answer the following and show your work:

(a) Is $\hat{A}^2$ Hermitian?

(b) Is $\hat{A}\hat{B}$ Hermitian?

(c) Is $\hat{A}\hat{B} + \hat{B}\hat{A}$ Hermitian?

(d) Is it possible for $\hat{A}$ to have complex eigenvalues, or must they be real?

5. Complete the next page of questions about Dirac Notation.

Change of basis

Suppose an electron is described by the state vector $|\psi\rangle = \frac{1}{2}|+\rangle + \frac{\sqrt{3}}{2}|-\rangle$, where $|+\rangle$ and $|-\rangle$ are the basis states corresponding to spin-up and spin-down, respectively, in the *z*-direction.

A. Calculate the inner product $\langle +|\psi\rangle$.

B. Is this state vector normalized? Explain by discussing the probabilities of measuring spin up or down in the *z*-direction, as done in the Worksheet 2.

C. *Predict* (without performing explicit calculations) whether the magnitude of the inner product ${}_x\langle +|\psi\rangle$ is *greater than, less than,* or *equal to* the magnitude of the inner product $\langle +|\psi\rangle$, where $|+\rangle_x$ is the basis state corresponding to spin-up in the *x*-direction. Explain.

D. Consider the dialogue below between three students who are attempting to determine the inner product ${}_x\langle +|\psi\rangle$.

Student 1: "This is just like the first inner product: ${}_x\langle +|-\rangle$ is zero because the plus and minus states are always orthogonal to each other. Therefore, ${}_x\langle +|\psi\rangle$ is 1/2."

Student 2: "The *x*- and *z*-directions are orthogonal to each other, so $|+\rangle_x$ is orthogonal to both $|+\rangle$ and $|-\rangle$. That means the answer is zero."

Student 3: "I don't think the inner product of ${}_x\langle +|+\rangle$ is zero. If we send an electron prepared in the state $|+\rangle$ through a S-G apparatus with magnetic field in the *x*-direction, we would have a 50/50 chance of measuring spin up and spin down. Thus, the inner product can't be zero."

Which student(s), if any, do you agree with? Explain your reasoning.

E. Determine the inner product, ${}_x\langle+|\psi\rangle$, using Dirac notation. Clearly show each step in your calculation.

Resolve any inconsistencies with your prediction in question C.

F. Is it possible to rewrite $|\psi\rangle$ in terms of the basis state vectors $|+\rangle_x$ and $|-\rangle_x$? Explain why or why not.

If it is possible, do you expect the coefficients in terms of $|+\rangle_x$ and $|-\rangle_x$ to be the same as the coefficients in terms of $|+\rangle$ and $|-\rangle$? Explain.

If it is possible to write $|\psi\rangle$ in terms of $|+\rangle_x$ and $|-\rangle_x$, do so. Clearly show your work.