EE5603:Concentration Inequalities

Sumohana Chennappayya†

1 Convergence

1.1 Definitions

If

$$X_1, \ldots, X_n \tag{1.1}$$

is a sequence of i.i.d RVs then,

- Convergence in probability

$$X_n \xrightarrow{p} X \implies \Pr(\lim_{n\to\infty} |X_n - X| < \varepsilon) = 0, \varepsilon > 0 \tag{1.2}$$

- Almost sure convergence

$$X_n \xrightarrow{a.s} X \implies \Pr\left(\lim_{n\to\infty} |X_n = X|\right) = 1 \tag{1.3}$$

- Convergence in distribution

$$X_n \xrightarrow{d} X \implies F_{X_n}(\lambda) = F_X(\lambda), \quad n \to \infty \tag{1.4}$$

- Convergence in mean square

$$X_n \xrightarrow{m.s} X \implies [(X_n - X)^2] = 0 \tag{1.5}$$

1.2 Law of large numbers (LLN)

If

$$X_1, \ldots, X_n \tag{1.6}$$

are i.i.d observations of a random varaiable with mean μ and $\bar{X}_n = \frac{1}{n}\sum_i^N x_i$, the weak and strong LLNs are respectively defined as

$$(WLLN) : \bar{X}_n \xrightarrow{p} \mu \tag{1.7}$$

$$(SLLN) : \bar{X}_n \xrightarrow{a.s} \mu \tag{1.8}$$

1.3 Central Limit Theroem

If

$$X_1, \ldots, X_n \tag{1.9}$$

is a sequence of i.i.d RVS with mean μ and finite variance σ^2 with

$$\bar{X}_n = \frac{1}{n}, \quad \text{then} \sum_i^N \sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} \mathcal{N}(0, \sigma^2) \tag{1.10}$$

2 Some Inequalities

- **Markov inequality**: For a non-negative RV X, and for any $\varepsilon > 0$

$$\Pr(X \geq \varepsilon) \leq \frac{E[X]}{\varepsilon} \tag{2.1}$$

- **Chebyschev inequality:** For a random variable X, for any $\varepsilon > 0$

$$\Pr(|X - E[X]| \geq \varepsilon) \leq \frac{\text{var}[X]}{\varepsilon^2} \tag{2.2}$$

- **Chernoff bound:** For a random variable X, for any $(t > 0)$

$$\Pr\left(e^{Xt} \geq e^{\varepsilon t}\right) \leq \frac{E[e^{Xt}]}{e^{at}} \tag{2.3}$$

† The author is with the Department of Electrical Engineering, IIT Hyderabad. IIT, Hyderabad 502285 India e-mail: {sumohana}@iith.ac.in.