

# EE5603:Concentration Inequalities

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## 1 CONVERGENCE

### 1.1 Definitions

If

$$X_1, \dots, X_n \quad (1.1)$$

is a sequence of i.i.d RVs then,

- Convergence in probability

$$X_n \xrightarrow{p} X \implies \Pr(\lim_{n \rightarrow \infty} |X_n - X| < \varepsilon) = 0, \varepsilon > 0 \quad (1.2)$$

- Almost sure convergence

$$X_n \xrightarrow{a.s} X \implies \Pr\left(\lim_{n \rightarrow \infty} |X_n - X| = 0\right) = 1 \quad (1.3)$$

- Convergence in distribution

$$X_n \xrightarrow{d} X \implies F_{X_n}(\lambda) = F_X(\lambda), \quad n \rightarrow \infty \quad (1.4)$$

- Convergence in mean square

$$X_n \xrightarrow{m.s} X \implies [(X_n - X)^2] = 0 \quad (1.5)$$

### 1.2 Law of large numbers (LLN)

If

$$X_1, \dots, X_n \quad (1.6)$$

are i.i.d observations of a random variable with mean  $\mu$  and  $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n x_i$ , the weak and strong LLNs are respectively defined as

$$(WLLN) : \bar{X}_n \xrightarrow{p} \mu \quad (1.7)$$

$$(SLLN) : \bar{X}_n \xrightarrow{a.s} \mu \quad (1.8)$$

### 1.3 Central Limit Theorem

If

$$X_1, \dots, X_n \quad (1.9)$$

is a sequence of i.i.d RVS with mean  $\mu$  and finite variance  $\sigma^2$  with

$$\bar{X}_n = \frac{1}{n}, \quad \text{then } \sum_{i=1}^n \sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} \mathcal{N}(0, \sigma^2) \quad (1.10)$$

## 2 SOME INEQUALITIES

- **Markov inequality:** For a non-negative RV X, and for any  $\varepsilon > 0$

$$\Pr(X \geq \varepsilon) \leq \frac{E[X]}{\varepsilon} \quad (2.1)$$

- **Chebyshev inequality:** For a random variable X, for any  $\varepsilon > 0$

$$\Pr(|X - E[X]| \geq \varepsilon) \leq \frac{\text{var}[X]}{\varepsilon^2} \quad (2.2)$$

- **Chernoff bound:** For a random variable X, for any  $(t > 0)$

$$\Pr(e^{Xt} \geq e^{\varepsilon t}) \leq \frac{E[e^{Xt}]}{e^{\varepsilon t}} \quad (2.3)$$

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