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Program Structures & Algorithms
Fall 2021
Assignment No. 3

Task

- (Part 1) Implement height-weighted Quick Union with Path Compression

- (Part 2) Develop a UF client that takes an integer value n from the command line to determine the number of "sites." Then generates random pairs of integers between 0 and $n-1$, calling $connected()$ to determine if they are connected and $union()$ if not. Loop until all sites are connected then print the number of connections generated. Package your program as a static method $count()$ that takes n as the argument and returns the number of connections; and a $main()$ that takes n from the command line, calls $count()$ and prints the returned value.

- (Part 3) Determine the relationship between the number of objects (n) and the number of pairs (m)

Relationship Conclusion:

The relationship between the number of objects (n) and the number of pairs (m) generated to reduce the number of components from n to 1 is:

$$m = f(n) = \frac{1}{2} \times n \times \ln(n)$$

Where,

$$m = number\ of\ pairs\ generated\ to\ reduce\ the\ number\ of\ components$$
$$n = number\ of\ objects$$

Evidence to support the conclusion:

Let $f(N)$ be the number of pairs (m) generated to reduce the number of components from n to 1.

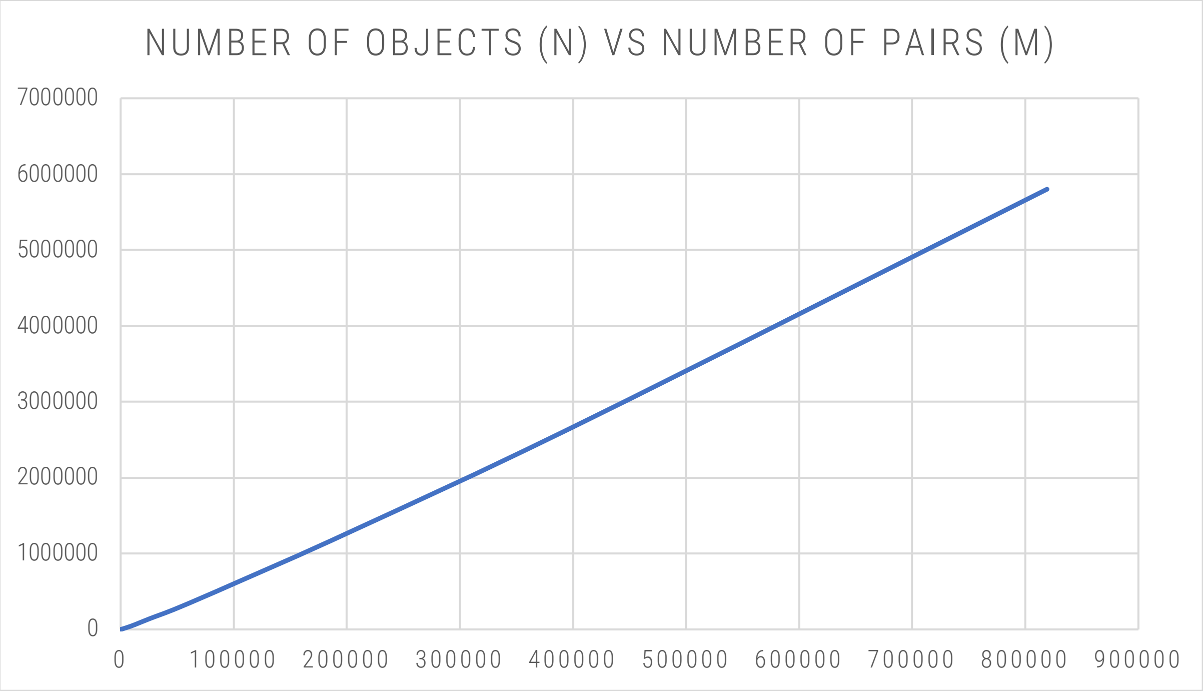
Taking initial value of n as 100 and using the doubling method, we can run calculate the number of pairs (m) generated to reduce the number of components from n to 1, and compute the average number of pairs generated to accomplish this for each value of n.

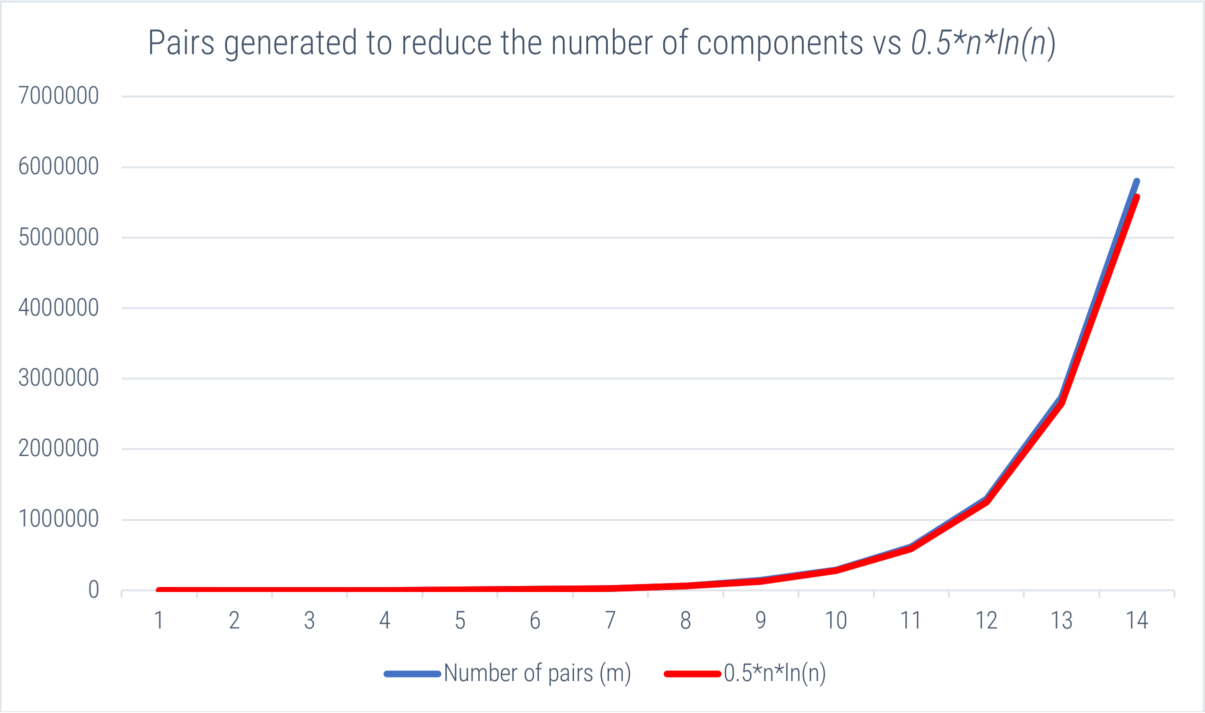
For larger values of n, although not equal, the average number of pairs needed to reduce the component 1 is pretty close to $\frac{1}{2} \times n \times \ln(n)$.

We can consider a union-find operation similar to the sorting of a list. Instead, we check if the pairs are connected or disconnected $(n\ ln\ n)$. There are only two possibilities for each pair. Hence, the relationship between m and n is almost identical to $\frac{1}{2} \times n \times \ln(n)$.

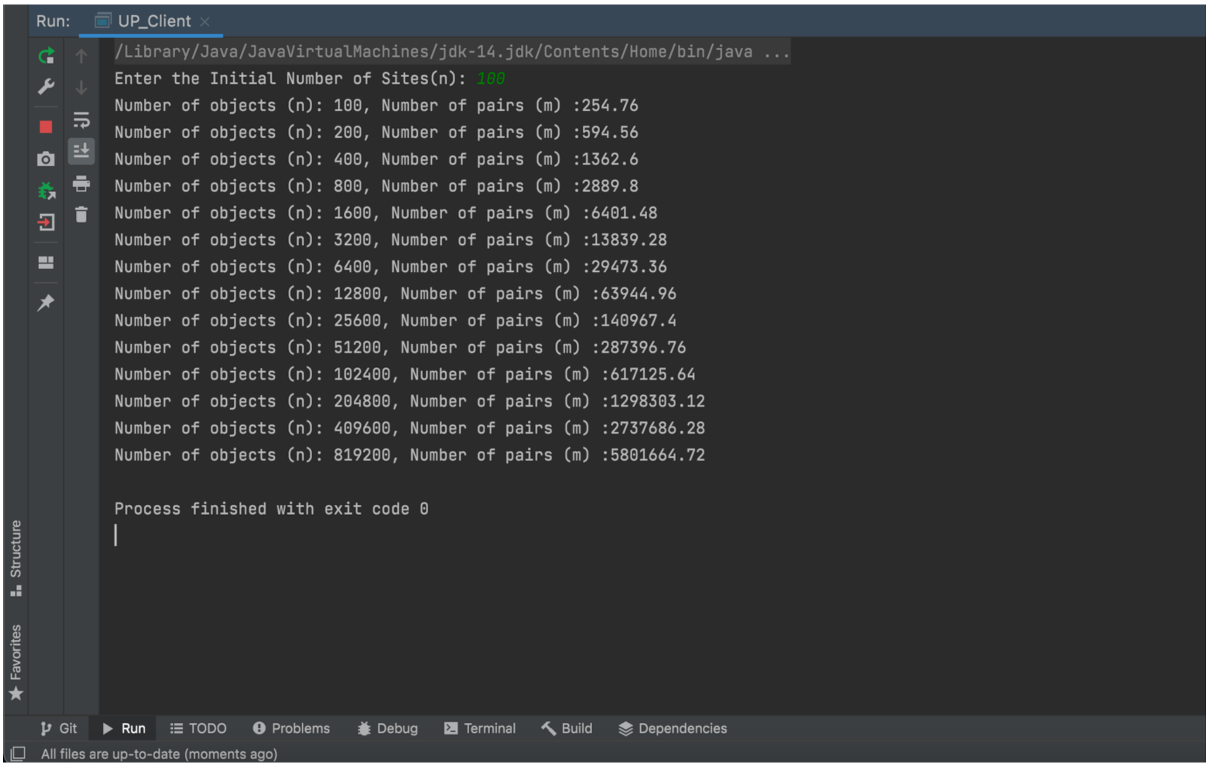
Number of Objects (n)	Number of pairs (m)	0.5*n*ln(n)
100	254.76	230.2585093
200	594.56	529.8317367
400	1362.6	1198.292909
800	2889.8	2673.844691
1600	6401.48	5902.207127
3200	13839.28	12913.44974
6400	29473.36	28044.97046
12800	63944.96	60526.08288
25600	140967.4	129924.4497
51200	287396.76	277593.4672
102400	617125.64	590676.07
204800	1298303.12	1252330.411
409600	2737686.28	2646617.365
819200	5801664.72	5577147.815

The below diagrams show the result of plotting the above table data, on a standard scale, with the number of objects (n) on the $x - axis$ and number of pairs (m) generated to reduce the number of components from n to 1 on the $y - axis$.




Pairs generated to reduce the number of components vs 0.5*n*ln(n)
7000000
6000000
5000000
4000000
3000000
2000000
1000000
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
Number of pairs (m)
0.5*n*ln(n)

Console Output:

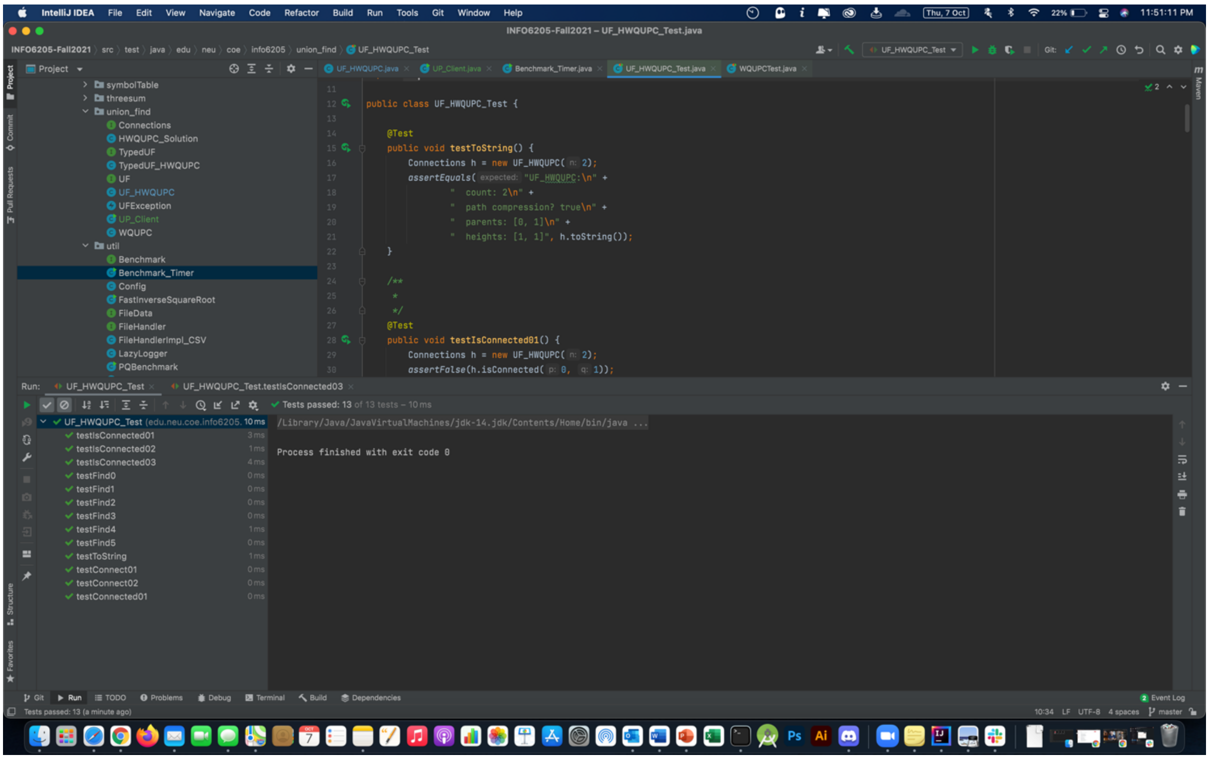


```
/Library/Java/JavaVirtualMachines/jdk-14.jdk/Contents/Home/bin/java ...
Enter the Initial Number of Sites(n): 100
Number of objects (n): 100, Number of pairs (m) :254.76
Number of objects (n): 200, Number of pairs (m) :594.56
Number of objects (n): 400, Number of pairs (m) :1362.6
Number of objects (n): 800, Number of pairs (m) :2889.8
Number of objects (n): 1600, Number of pairs (m) :6401.48
Number of objects (n): 3200, Number of pairs (m) :13839.28
Number of objects (n): 6400, Number of pairs (m) :29473.36
Number of objects (n): 12800, Number of pairs (m) :63944.96
Number of objects (n): 25600, Number of pairs (m) :140967.4
Number of objects (n): 51200, Number of pairs (m) :287396.76
Number of objects (n): 102400, Number of pairs (m) :617125.64
Number of objects (n): 204800, Number of pairs (m) :1298303.12
Number of objects (n): 409600, Number of pairs (m) :2737686.28
Number of objects (n): 819200, Number of pairs (m) :5801664.72

Process finished with exit code 0
```

```
Enter the Initial Number of Sites(n): 100
Number of objects (n): 100, Number of pairs (m) :254.76
Number of objects (n): 200, Number of pairs (m) :594.56
Number of objects (n): 400, Number of pairs (m) :1362.6
Number of objects (n): 800, Number of pairs (m) :2889.8
Number of objects (n): 1600, Number of pairs (m) :6401.48
Number of objects (n): 3200, Number of pairs (m) :13839.28
Number of objects (n): 6400, Number of pairs (m) :29473.36
Number of objects (n): 12800, Number of pairs (m) :63944.96
Number of objects (n): 25600, Number of pairs (m) :140967.4
Number of objects (n): 51200, Number of pairs (m) :287396.76
Number of objects (n): 102400, Number of pairs (m) :617125.64
Number of objects (n): 204800, Number of pairs (m) :1298303.12
Number of objects (n): 409600, Number of pairs (m) :2737686.28
Number of objects (n): 819200, Number of pairs (m) :5801664.72

Process finished with exit code 0
```

Unit Test Results:

UF_HWQUPC_Test.java



INFO6205-Fall2021 – UF_HWQUPC_Test.java

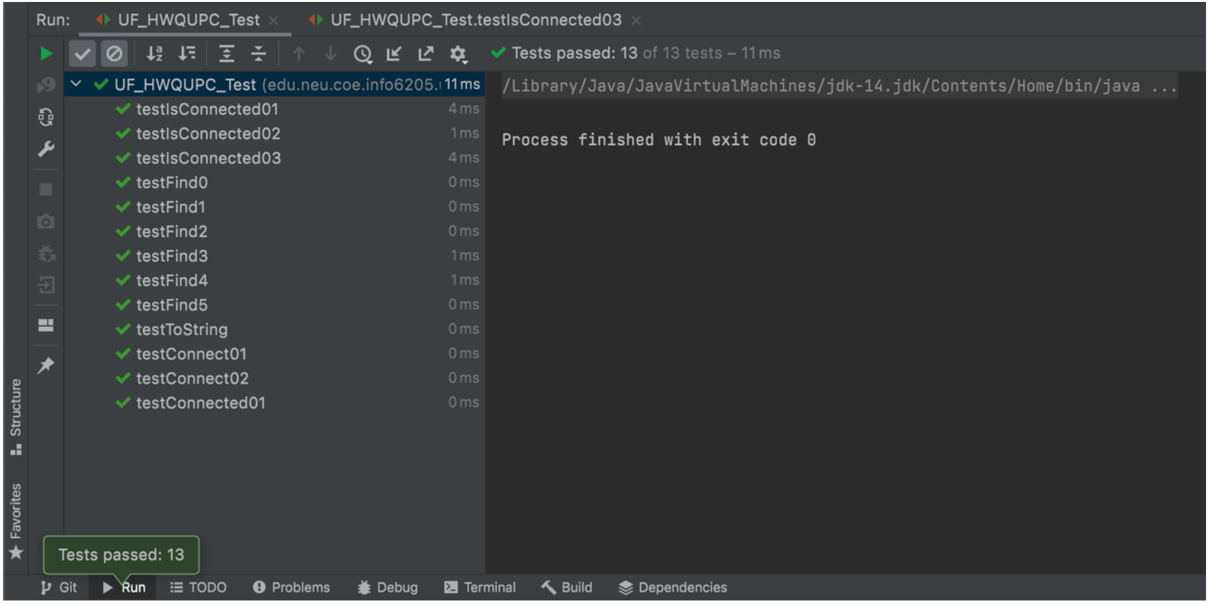
```java
public class UF_HWQUPC_Test {

    @Test
    public void testToString() {
        Connections h = new UF_HWQUPC( n: 2);
        assertEquals( expected: "UF_HWQUPC:\n" +
                "  count: 2\n" +
                "  path compression? true\n" +
                "  parents: [0, 1]\n" +
                "  heights: [1, 1]", h.toString());
    }

    /**
     *
     */
    @Test
    public void testIsConnected01() {
        Connections h = new UF_HWQUPC( n: 2);
        assertFalse(h.isConnected( p: 0,  q: 1));
```

Tests passed: 13 of 13 tests – 10 ms

```
UF_HWQUPC_Test (edu.neu.coe.info6205. 10 ms
  testIsConnected01   3 ms
  testIsConnected02   1 ms
  testIsConnected03   4 ms
  testFind0           0 ms
  testFind1           0 ms
  testFind2           0 ms
  testFind3           0 ms
  testFind4           1 ms
  testFind5           0 ms
  testToString        1 ms
  testConnect01       0 ms
  testConnect02       0 ms
  testConnected01     0 ms
```

```
/Library/Java/JavaVirtualMachines/jdk-14.jdk/Contents/Home/bin/java ...

Process finished with exit code 0
```

Tests passed: 13 (a minute ago)



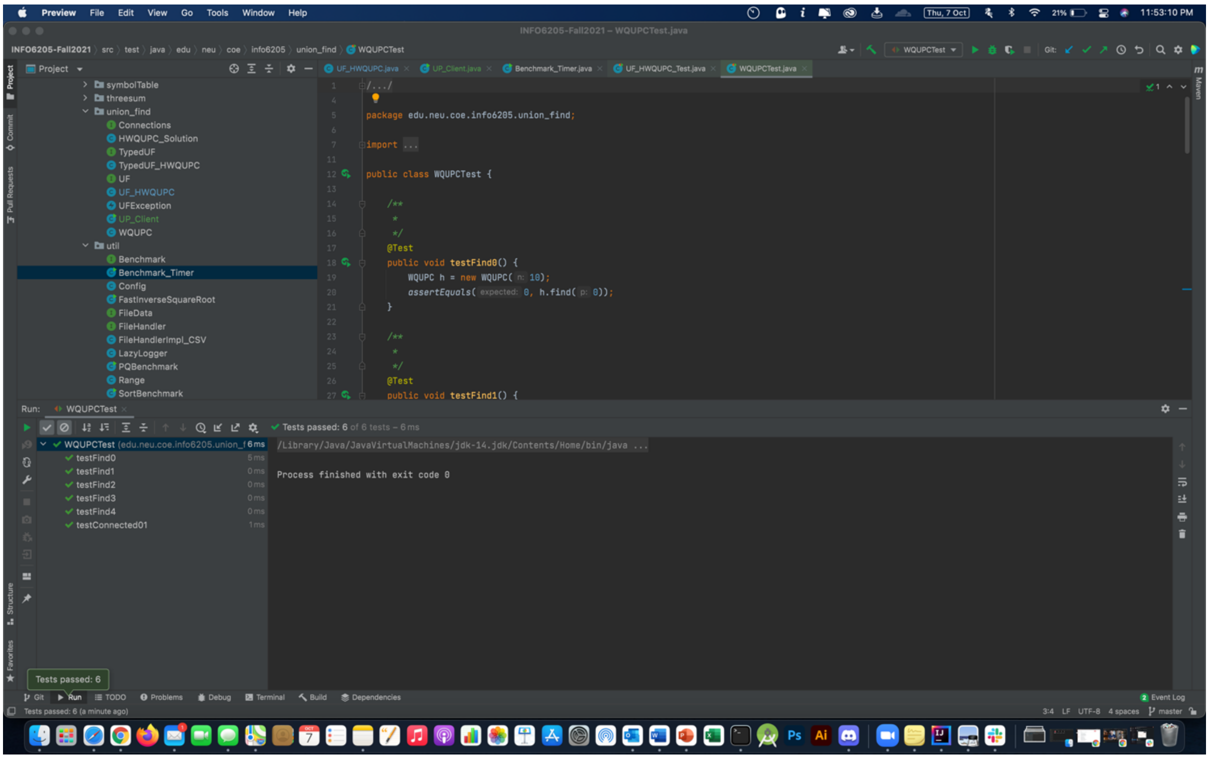
Tests passed: 13 of 13 tests – 11 ms

```
UF_HWQUPC_Test (edu.neu.coe.info6205.i 11 ms
  testIsConnected01   4 ms
  testIsConnected02   1 ms
  testIsConnected03   4 ms
  testFind0           0 ms
  testFind1           0 ms
  testFind2           0 ms
  testFind3           1 ms
  testFind4           1 ms
  testFind5           0 ms
  testToString        0 ms
  testConnect01       0 ms
  testConnect02       0 ms
  testConnected01     0 ms
```

```
/Library/Java/JavaVirtualMachines/jdk-14.jdk/Contents/Home/bin/java ...

Process finished with exit code 0
```

Tests passed: 13

WQUPC_Test.java



INFO6205-Fall2021 – WQUPCTest.java

INFO6205-Fall2021 〉 src 〉 test 〉 java 〉 edu 〉 neu 〉 coe 〉 info6205 〉 union_find 〉 WQUPCTest

- symbolTable
- threesum
- union_find
  - Connections
  - HWQUPC_Solution
  - TypedUF
  - TypedUF_HWQUPC
  - UF
  - UF_HWQUPC
  - UFException
  - UF_Client
  - WQUPC
- util
  - Benchmark
  - Benchmark_Timer
  - Config
  - FastInverseSquareRoot
  - FileData
  - FileHandler
  - FileHandlerImpl_CSV
  - LazyLogger
  - PQBenchmark
  - Range
  - SortBenchmark

```java
/.../

package edu.neu.coe.info6205.union_find;

import ...

public class WQUPCTest {

    /**
     *
     */
    @Test
    public void testFind0() {
        WQUPC h = new WQUPC( n: 10);
        assertEquals( expected: 0, h.find( p: 0));
    }

    /**
     *
     */
    @Test
    public void testFind1() {
```

Run: WQUPCTest

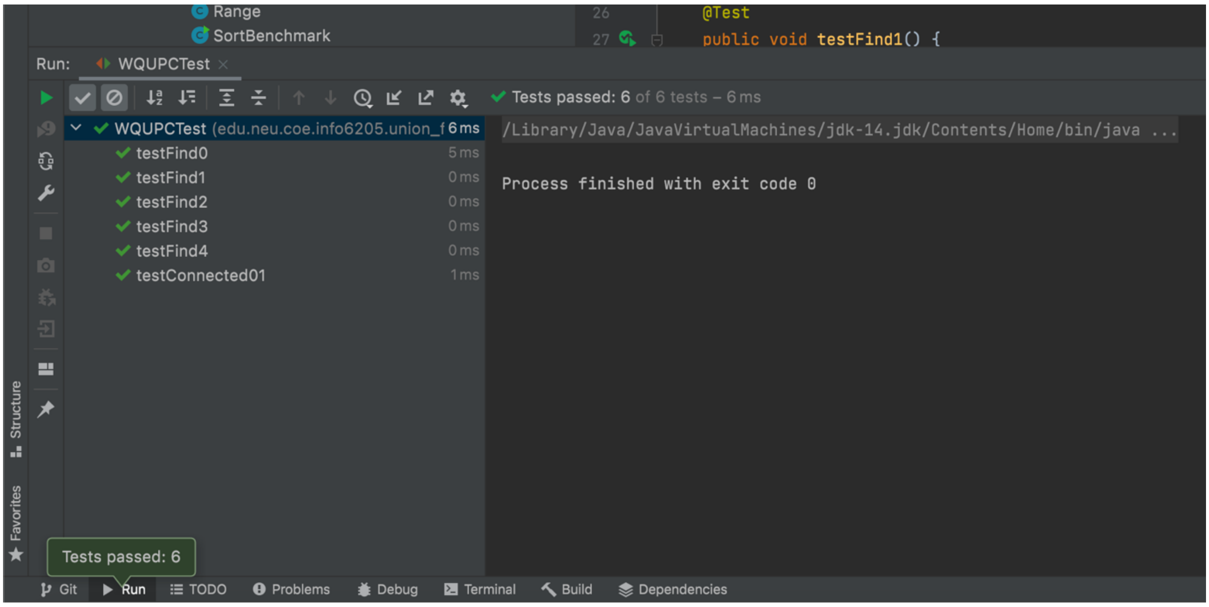
✓ Tests passed: 6 of 6 tests – 6 ms

```
✓ WQUPCTest (edu.neu.coe.info6205.union_f 6 ms
    ✓ testFind0                          5 ms
    ✓ testFind1                          0 ms
    ✓ testFind2                          0 ms
    ✓ testFind3                          0 ms
    ✓ testFind4                          0 ms
    ✓ testConnected01                    1 ms
```

```
/Library/Java/JavaVirtualMachines/jdk-14.jdk/Contents/Home/bin/java ...

Process finished with exit code 0
```

Tests passed: 6

Tests passed: 6 (a minute ago)



Run: WQUPCTest

✓ Tests passed: 6 of 6 tests – 6 ms

```
✓ WQUPCTest (edu.neu.coe.info6205.union_f 6 ms
    ✓ testFind0                          5 ms
    ✓ testFind1                          0 ms
    ✓ testFind2                          0 ms
    ✓ testFind3                          0 ms
    ✓ testFind4                          0 ms
    ✓ testConnected01                    1 ms
```

```
/Library/Java/JavaVirtualMachines/jdk-14.jdk/Contents/Home/bin/java ...

Process finished with exit code 0
```

Tests passed: 6